

Report for the semester thesis “Development of a Monte Carlo algorithm for optimal control problems”

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Abstract—This document is a model and instructions for L^AT_EX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.

Index Terms—a, b, c

I. PROBLEM DESCRIPTION

We model a cross section of an oil field as a two dimensional square $\Omega := [0, 1]^2$. In the oilfield, there are two phases: water and oil. At the lower left corner $(0, 0)$, we know the pressure $p(t)$. Opposite of that, at $(1, 1)$ a well is located. There we can measure the pressure $p_{\text{well}}(t)$ as well as the volumetric outflow $Q_o(t)$, $Q_w(t)$ per unit depth.

The flow rates for both phases are described by Darcy’s law

$$\mathbf{v}_w = -\frac{k k_{\text{rel}, w}}{\mu_w} \text{grad}(p), \quad (1)$$

for water and

$$\mathbf{v}_o = -\frac{k k_{\text{rel}, o}}{\mu_o} \text{grad}(p) \quad (2)$$

for oil. Here, $p(\mathbf{x}, t)$ is the pressure, μ_o , μ_w are dynamic viscosities for oil and water, $k(\mathbf{x}, t)$ is the permeability. $k_{\text{rel}, o}(S)$, $k_{\text{rel}, w}(S)$ are relative permeabilities and are assumed to depend quadratically on the saturation of water $S_w \in [0, 1]$ and the saturation of oil $S_o \in [0, 1]$:

$$k_{\text{rel}, o} = S_o^2 \quad (3)$$

$$k_{\text{rel}, w} = S_w^2. \quad (4)$$

The saturations are linked by the constitutive relation

$$S_o + S_w = 1. \quad (5)$$

$\mathbf{v}_o(\mathbf{x}, t)$, $\mathbf{v}_w(\mathbf{x}, t)$ finally are the volumetric flow rates per unit area (Darcy velocities).

The saturation S is not assumed constant but instead is transported according to the equation

$$\frac{\partial}{\partial t} \phi S_w + \text{div}(\mathbf{v}_w) = q_w. \quad (6)$$

The term

$$q_w := Q_w \delta(\mathbf{x} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}) \quad (7)$$

describes a line sink of water located at the well. Similarly, we use

$$q_o := Q_o \delta(\mathbf{x} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}) \quad (8)$$

$$q_{\text{tot}} := q_o + q_w. \quad (9)$$

Conservation of the total mass then reads

$$\text{div}(\mathbf{v}_{\text{tot}}) = q_{\text{tot}}, \quad (10)$$

where

$$\mathbf{v}_{\text{tot}} := \mathbf{v}_o + \mathbf{v}_w \quad (11)$$

is the total Darcy velocity.

We then introduce some additional simplifications: We set

$$\mu_w = \mu_o =: \mu, \quad (12)$$

and introduce the abbreviations

$$\lambda_o := \frac{k k_{\text{rel}, o}}{\mu_o} \quad (13)$$

$$\lambda_w := \frac{k k_{\text{rel}, w}}{\mu_w} \quad (14)$$

$$\lambda_{\text{tot}} := \lambda_o + \lambda_w. \quad (15)$$

Substituting in the λ from (13) into Darcy’s law (2), (1) and adding both sides of the results we get the total Darcy’s law

$$\mathbf{v}_{\text{tot}} = -\lambda_{\text{tot}} \text{grad}(p). \quad (16)$$

Plugging this (16) into the conservation of mass (10) leads to the pressure equation

$$\text{div}(\lambda_{\text{tot}} \text{grad}(p)) = q_{\text{tot}}. \quad (17)$$