Development of a Monte Carlo algorithm for optimal control problems

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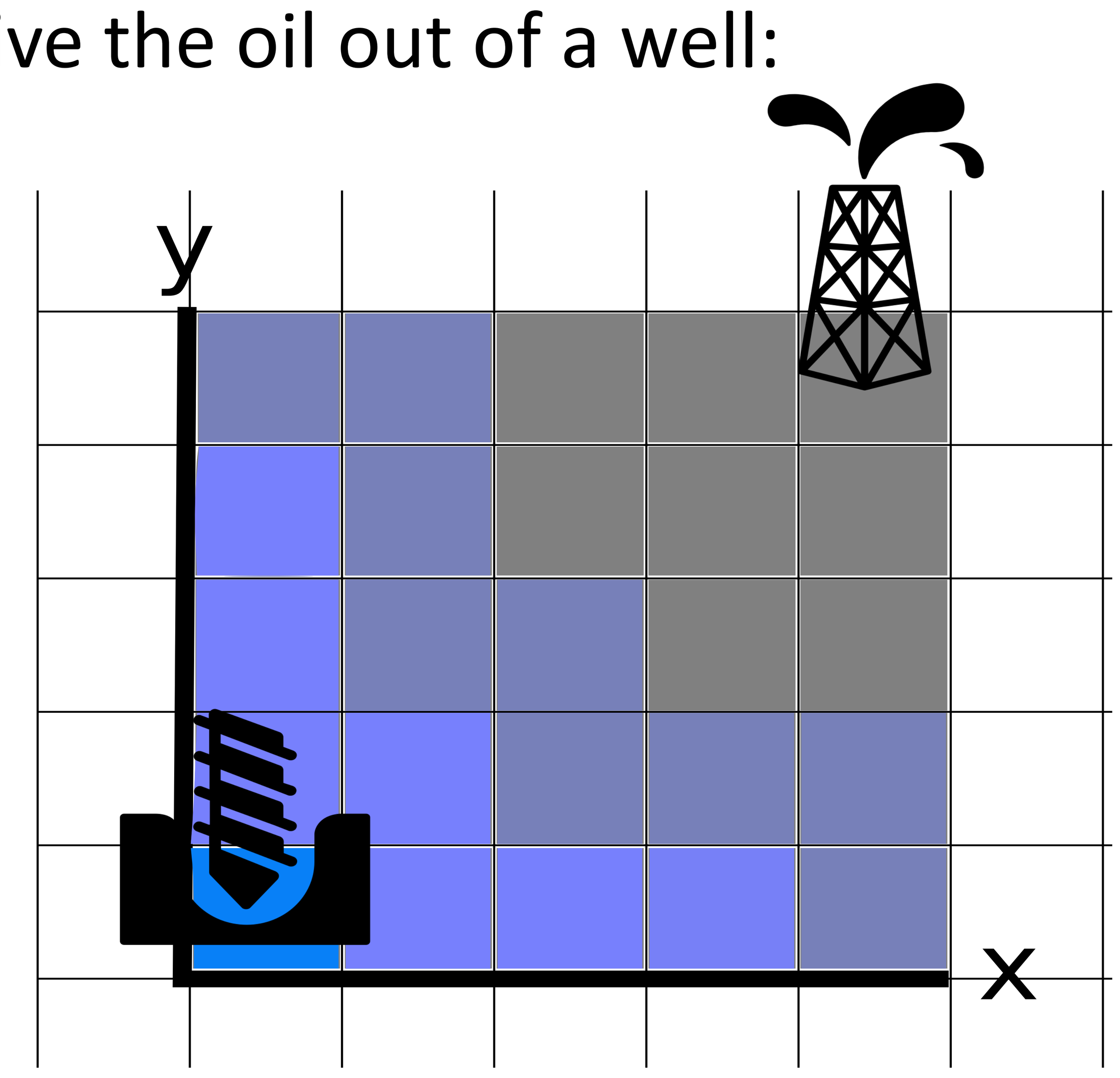
Description

We applied a Monte Carlo algorithm for solving the discretized adjoint equation of a quarter five spot configuration in petroleum engineering, where a 2D oilfield is filled with water from a drill to drive the oil out of a well:



Minimize

$$\sum_{i=0}^{T} \left(p_{\text{drill, measured}}(i\Delta t) - p^{(i)}_{\text{drill, computed}}\right)^2$$

subject to the differential equations

$$\operatorname{div}(\lambda_{\text{tot}} \operatorname{grad}(p)) = -q_{\text{tot}}$$
$$\phi \partial_t (S_{\text{w}}) + \operatorname{div}(\vec{v}_{\text{w}}) = q_{\text{w}}$$

by controlling the log permeabilities $\ln(k)$.
Boundary conditions are

- Initial values for S_{w} are given
- No flow boundary conditions on the boundary except at the drill and well

p	pressure
λ_{tot}	proportionality factor from Darcy's law $\vec{v} \propto -\operatorname{grad}(p)$ for both oil and water together
q_{tot}	sink term for oil and water together
ϕ	porosity, fraction of a cell which can be filled by a liquid
S_{w}	water saturation, how much of the liquid is water in a cell
$\vec{v}_{\text{w}}, \vec{v}_{\text{tot}}$	flow rate per unit area (Darcy velocity)
k	permeability, higher values mean more flow with the same pressure gradient

Results

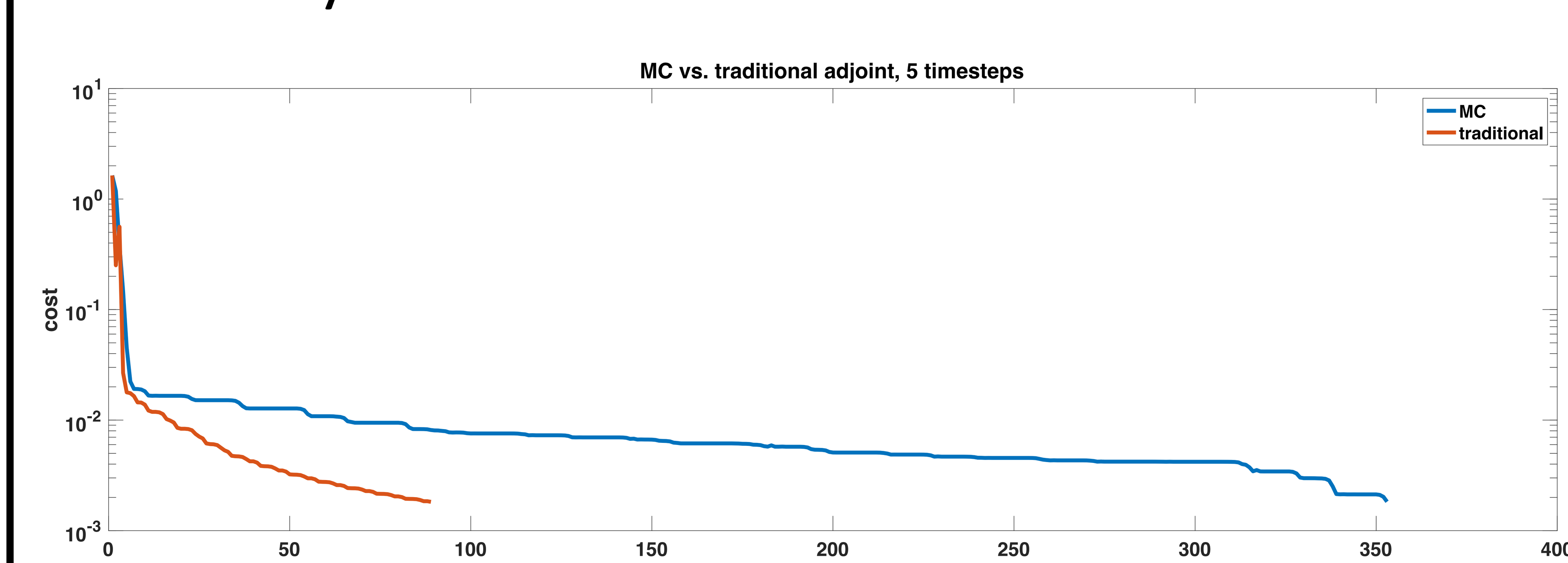
The adjoints corresponding to pressure states can be well approximated by Monte Carlo (the discrepancy in the gray value is a artifact of the discretization and unimportant).

$$\begin{array}{ccc} & \text{Monte Carlo} & \text{Traditional} \\ \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \\ \psi^{(4)} \\ \psi^{(5)} \\ \psi^{(6)} \\ \psi^{(7)} \\ \psi^{(8)} \\ \psi^{(9)} \end{pmatrix} & \begin{pmatrix} -0.201 \\ -0.267 \\ -0.333 \\ -0.134 \\ -0.201 \\ -0.267 \\ -1.4 \cdot 10^{-16} \\ -0.134 \\ -0.201 \end{pmatrix} & \begin{pmatrix} -0.201 \\ -0.267 \\ -0.325 \\ -0.134 \\ -0.201 \\ -0.267 \\ -0.424 \\ -0.134 \\ -0.201 \end{pmatrix} \end{array}$$

On the other hand, the adjoints corresponding to saturation states are completely off without a good preconditioner.

$$\begin{array}{ccc} & \text{Monte Carlo} & \text{Traditional} \\ \begin{pmatrix} \psi^{(10)} \\ \psi^{(11)} \\ \psi^{(12)} \\ \psi^{(13)} \\ \psi^{(14)} \\ \psi^{(15)} \\ \psi^{(16)} \\ \psi^{(17)} \\ \psi^{(18)} \end{pmatrix} & \begin{pmatrix} -3.33 \cdot 10^{5} \\ 3.84 \cdot 10^{8} \\ -1.26 \cdot 10^{11} \\ -3933.0 \\ 2.14 \cdot 10^{5} \\ -1.87 \cdot 10^{10} \\ 6.05 \cdot 10^{6} \\ 5.77 \cdot 10^{4} \\ -1.95 \cdot 10^{5} \end{pmatrix} & \begin{pmatrix} 33.7 \\ 47.2 \\ 0.0223 \\ 47.1 \\ 55.6 \\ 47.2 \\ 37.7 \\ 47.1 \\ 33.7 \end{pmatrix} \end{array}$$

We haven't found a good preconditioner. Our best try leads to poor results even for small timelevels and small grid sizes which is easy



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