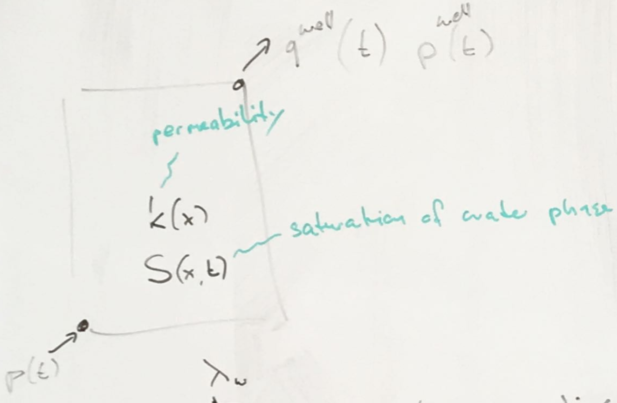


volumetric flow rates of water, oil and in total

$$q_w = -\underbrace{\frac{k k_{rw}}{\mu_w}}_{\lambda_w} \nabla p$$

$$q_o = -\frac{k k_{ro}}{\mu_o} \nabla p$$

$$q_{tot} = -\underbrace{k\left(\frac{S^2 + (1-S)^2}{1}\right)}_{\lambda_{tot}} \nabla p$$

$$\nabla \cdot q = q_{tot}^{well}$$

assumptions

$\Phi = \phi$ porosity

$\mu_w = \mu_o = \mu$ dynamic viscosity

$S_w = S = 1 - S_o$

$k_{rw} = S^2$ relative permeability of water

$k_{ro} = (1-S)^2$

$$(*) \quad \boxed{-\nabla \cdot \left(\lambda_{tot} \nabla p\right) = q_{tot}^{well}}$$

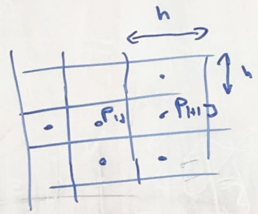
mass balance (flow or pressure problem)

$$\frac{\partial \phi S}{\partial t} + \nabla \cdot q_w = q_w^{well}$$

$$(**) \quad \boxed{\frac{\partial \phi S}{\partial t} + \nabla \cdot \left(\frac{S^2}{S^2 + (1-S)^2} q_{tot}\right) = q_w^{well}}$$

saturation transport

discretization at $(*)$



$\Rightarrow$

transmissibility $$T_{I+\frac{1}{2},J} = \frac{2}{\frac{1}{\lambda_{IJ}} + \frac{1}{\lambda_{I+1,J}}}$$

$$= \frac{2\lambda_{IJ}\lambda_{I+1,J}}{\lambda_{IJ} + \lambda_{I+1,J}}$$

$$\Rightarrow \quad T_{I,J-I+1,J}\left(p_{I,J} - p_{I+1,J}\right) + T_{I,J-I-1,J}\left(p_{IJ} - p_{I-1,J}\right) + T_{I,J-I,J+1}\left(p_{IJ} - p_{I,J+1}\right) + T_{I,J-I,J-1}\left(p_{IJ} - p_{I,J-1}\right) = q_{tot}^{well}$$

$$q_{I+\frac{1}{2},J} = T_{I+\frac{1}{2},J}\left(p_{I,J} - p_{I+1,J}\right)$$