Report for the semester thesis “Development of a Monte Carlo algorithm for optimal control problems”

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

***Index Terms*—a, b, c**

I. Problem description

We model a cross section of an oil field as a two dimensional square $\Omega := [0,1]^2$. In the oilfield, there are two phases: water and oil. At the lower left corner $(0,0)$, we know the pressure $p(t)$. Opposite of that, at $(1,1)$ a well is located. There we can measure the pressure $p_{\text{well}}(t)$ as well as the volumetric outflow $\boldsymbol{q}_{\text{well}}(t)$ per unit area.

The flow rates for both phases are described by Darcy's law

$$\boldsymbol{q}_{\text{w}} = -\frac{k k_{\text{rel, w}}}{\mu_{\text{w}}} \operatorname{grad}(p), \tag{1}$$

for water and

$$\boldsymbol{q}_{\text{o}} = -\frac{k k_{\text{rel, o}}}{\mu_{\text{o}}} \operatorname{grad}(p) \tag{2}$$

for oil. Here, $p(\boldsymbol{x},t)$ is the pressure, μ_{o},μ_{w} are dynamic viscosities for oil and water, $k(\boldsymbol{x},t)$ is the permeability. $k_{\text{rel, o}}(S)$,$k_{\text{rel, w}}(S)$ are relative permeabilities and depend quadratically on the saturation of water $S \in [0,1]$:

$$k_{\text{rel, o}} = (1-S)^2 \tag{3}$$
$$k_{\text{rel, w}} = S^2. \tag{4}$$

$\boldsymbol{q}_{\text{o}}(\boldsymbol{x},t)$, $\boldsymbol{q}_{\text{o}}(\boldsymbol{x},t)$ finally are the volumetric flow rates per unit area.

The main difficulty in simulating this flow is that the permeability k is unknown and hard to determine. Here, we set the permeability k so that the measured outflow rate matches the calculated outflow rate at the well

$$\boldsymbol{q}_{\text{tot}}(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, t) \overset{!}{=} q_{\text{well}}(t). \tag{5}$$