Physics 2: Advanced
PHYC10002

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Contents

1 Electricity

Definition 1. *Electrostatics* concerns forces between charges at rest.

Law 1 (Coulomb's law)**.** *The electrostatic force experienced by a charge q_1 in the vicinity of another charge q_2 is equal to*

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12},$$

where $\varepsilon_0 \approx 8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$.

Definition 2 (Coulomb's constant)**.** Define *Coulomb's constant* to be $k_e = \frac{1}{4\pi\varepsilon_0} \approx 8.99 \times 10^9\,\mathrm{N\,m^2/C^2}$. We can then write Coulomb's law as

$$\mathbf{F}_1 = k_e \frac{q_1 q_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12}.$$

Remark 1. The electromagnetic force at an atomic scale is far stronger than the gravitational force. Consider an electron and a proton about $10^{-10}\,\mathrm{m}$ apart. Given that $8.99 \times 10^9\,\mathrm{N\,m^2/C^2}$, $G \approx 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, $m_e \approx 9.1 \times 10^{-31}\,\mathrm{kg}$, $m_p \approx 1.6 \times 10^{-27}\,\mathrm{kg}$, and $e \approx 1.6 \times 10^{-19}\,\mathrm{C}$, we would have

$$|\mathbf{F}_E| = k_e \frac{q_1 q_2}{r^2} \approx 2.3 \times 10^{-8} \gg 9.7 \times 10^{-48} \approx G\frac{m_1 m_2}{r^2} = |\mathbf{F}_g|.$$

It should also be noted that the strong nuclear force, the force of the gluons binding the quarks together within nucleons, is far stronger than the electromagnetic force.

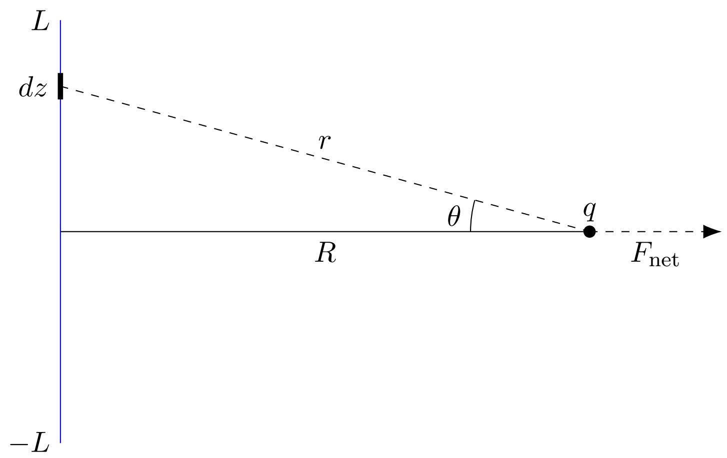

Figure 1: A wire of length $2L$ is at a distance of R away from a charge q.

Example 1. We can now calculate the force on a charge from a line of charges (for example, a wire). Consider the setup as depicted in Figure 1.

Suppose that the wire has a charge density of $\lambda\,\mathrm{C\,m^{-1}}$. Then, an infinitesimally small segment of the wire with length dz will have a charge λdz, and will be at a distance of $r = \frac{R}{\cos\theta}$ away from it (note that θ depends on the location of dz). The net force on q will then simply be the sum of all the forces from dz for all possible locations of dz along the length of the wire. Moreover, notice that the net force will have zero vertical component due to the symmetry of the setup. Hence, the net force on q will be the sum of all the horizontal components of the forces from each dz.

The horizontal force due to dz is calculated via Coulomb's Law:

$$\begin{aligned} F_{dz} &= \cos\theta \frac{1}{4\pi\varepsilon_0}\frac{q\lambda dz}{r^2} \\ &= \frac{\cos\theta}{4\pi\varepsilon_0}\frac{q\lambda dz\cos^2\theta}{R^2} \\ &= \frac{q\lambda}{4\pi\varepsilon_0 R^2}\cos^3\theta dz. \end{aligned}$$

Integrating from $z = -L$ to L will then give the desired net force on q:

$$F_{\text{net}} = \int_{-L}^{L} \frac{q\lambda}{4\pi\varepsilon_0 R^2}\cos^3\theta\, dz.$$

Note that z is simply the distance of dz to the origin, which is $z = R\tan\theta$. Using this substitution yields $dz = R\sec^2\theta\,d\theta$, so we get

$$
\begin{aligned}
F_{\text{net}} &= \frac{q\lambda}{4\pi\varepsilon_0 R^2}\int_{\theta=-\varphi}^{\theta=\varphi} R\cos^3\theta\sec^2\theta\,d\theta \\
&= \frac{q\lambda}{4\pi\varepsilon_0 R}\int_{-\varphi}^{\varphi}\cos\theta\,d\theta \\
&= \frac{q\lambda\sin\varphi}{2\pi\varepsilon_0 R}.
\end{aligned}
$$

As a special case, if the wire was infinitely long, we would have $\varphi = \frac{\pi}{2}$, so $\sin\varphi = 1$ which would yield a force of $\frac{q\lambda}{2\pi\varepsilon_0 R}$. In any case, notice that the force from the wire is proportional to $\frac{1}{R}$ and not $\frac{1}{R^2}$ like for a point charge — the force dropoff is less dramatic from a wire.