Question 8

Mathematics and Statistics Research Competition

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The Question

Problem 1

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A particle generator is emitting two types of particles (called X and Y) into a long tube. The particles will line up in order after entering the tube. Initially, the tube is empty. At each shot, either an X- or Y-particle is randomly emitted into the tube with equal probability. Different shots are assumed to be independent from each other. Suppose that n shots have been emitted.

Problem 1

- What is the probability that no two X-particles are next to each other?

$$\Pr(\text{No consecutive X-particles}) = \frac{\#\text{No consecutive X-particles}}{\#\text{Total arrangements}}$$

Claim

The number of arrangements with no consecutive X-particles is

$$\sum_{k=0}^{n} \binom{n-k+1}{k}. \tag{1}$$

Proof.

Consider a tube with n particles, k of them are X, $n-k$ are Y.

$$\underbrace{\mathrm{YY}\ldots\mathrm{YY}}_{n-k}$$

Placing all k of the X particles in the $n-k+1$ gaps will ensure no consecutive X's. $\square$