Mathematics and Statistics Research Competition Question 1

Jiamu Li & Frank Tang & Edward Wang

Scotch College

July 23, 2024

Contents

Let $\mathcal{N}_{10,2}$ be the set of positive integers whose digits in base-10 comprise only 0s and 1s. Examples of elements in $\mathcal{N}_{10,2}$ are: 1001, 110, and 11. Examples of elements not in $\mathcal{N}_{10,2}$ are: 4201, 690, and 12.

Consider a positive integer N. It can be constructed as the sum of elements in $\mathcal{N}_{10,2}$. For example, one construction of 1337 with 8 summands which are elements in $\mathcal{N}_{10,2}$ is as follows

$$1337 = 1000 + 111 + 111 + 111 + 1 + 1 + 1 + 1.$$

1   Problem 1

Problem. What are all of the constructions of 1337 using elements of $\mathcal{N}_{10,2}$?

We interpret the question as asking for how many unique ways there are to obtain 1337 as a sum of elements from $\mathcal{N}_{10,2}$. To do this, we look at the general case which seeks to find the number of unique ways to obtain a positive integer n as a sum of elements from $\mathcal{N}_{10,2}$.

Definition 1.1. For sake of convenience, we define a function $C(n)$ that counts the number of unique ways of constructing n as a sum of elements in $\mathcal{N}_{10,2}$. That is, $C(n)$ is the number of unique constructions such that

$$n = \sum_{a_i \in \mathcal{N}_{10,2}} a_i.$$

We start with an elementary example. Consider $n = 15$, and suppose we wish to find $C(15)$.