DAY 8

1. **In figure, ABCD is a trapezium in which AB||CD and $AB = 2CD$. Find the ratio of the areas of ΔAOB and ΔCOD.** **[Ex 6.4, Q 2]**

Sol:- Given AB||CD and $AB = 2CD$.

To find the ratio of area of ΔAOB and ΔCOD, we have to first prove them similar.

Now In ΔAOB and ΔCOD, we've

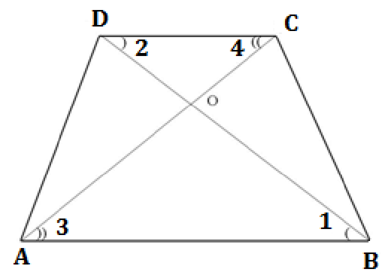
$\angle 1 = \angle 2$ (Alternate $\angle$s)

$\angle 3 = \angle 4$ (Alternate $\angle$s)

$\therefore \Delta AOB \sim \Delta COD$ (AA Similarity)

By Area Theorem, $\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2} = \frac{4CD^2}{CD^2} = \frac{4}{1}$

Hence $ar(\Delta AOB) : ar(\Delta COD) = 4 : 1$



2. **If the areas of two similar triangles are equal, prove that they are congruent.** **[Ex 6.4, Q4]**

Sol:- Suppose $\Delta ABC \sim \Delta DEF$

Given: $ar(\Delta ABC) = ar(\Delta DEF) \ldots\ldots\ldots\ldots\ldots..i)$

We know $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$ {By i)}

$\Rightarrow 1 = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

$\Rightarrow AB^2 = DE^2,\ BC^2 = EF^2,\ AC^2 = DF^2$

$\Rightarrow AB = DE,\ BC = EF,\ AC = DF$

$\Rightarrow$ **$\Delta ABC \cong \Delta DEF$ (SSS criterion)**

3. **In the figure, ΔABC and ΔDBC are two triangles on the same base BC, prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$** **[Ex 6.4, Q 3]**

Sol :-Given ΔABC and ΔDBC are on the same base

To prove :- $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw $AL \perp BC$ and $DM \perp BC$

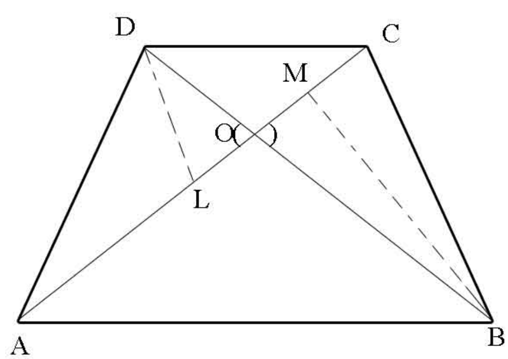
Proof:- $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}\times BC\times AL}{\frac{1}{2}\times BC\times DM} = \frac{AL}{DM} \ldots\ldots..i)$

Now In ΔALO and ΔDMO, we've

$\angle L = \angle M = 90^0$

$\angle 1 = \angle 2$ (Vertically opp. $\angle$s)

$\therefore \Delta ALO \sim \Delta DMO$ (AA Similarity)



$\Rightarrow \quad \frac{AL}{DM} = \frac{AO}{DO}$ ……………….. ii)

From i) & ii), we've $\frac{\mathbf{ar(\Delta ABC)}}{\mathbf{ar(\Delta DBC)}} = \frac{\mathbf{AO}}{\mathbf{DO}}$

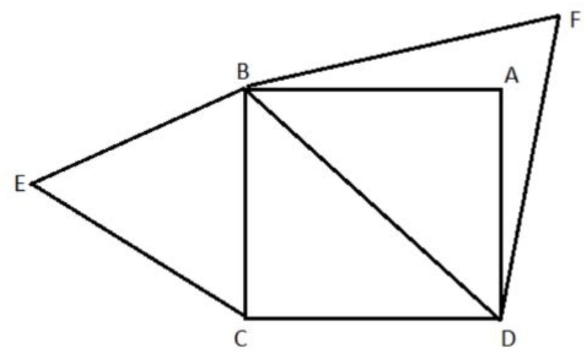
4. Prove that the area of the equilateral triangle described on the diagonal of a square is half the area of the equilateral triangle described on its side. [Ex 6.4, Q7]

Sol :- Let ΔBEC be equilateral triangle made on diagonal and ΔBFD is equilateral made on side of square.

To Prove: $\text{ar}(\Delta BEC) = \frac{1}{2}\text{ar}(\Delta BFD)$

Proof: Let side of square $= a$

$\therefore$ Diagonal of square$(BD) = \sqrt{a^2 + a^2} = \sqrt{2}a$……. i)



Now ΔBEC and ΔBFD both are equilateral triangles, so they are similar.

$\therefore \Delta BEC \sim \Delta BFD$

$\therefore \frac{\text{ar}(\Delta BEC)}{\text{ar}(\Delta BFD)} = \frac{AB^2}{AC^2} = \frac{a^2}{(\sqrt{2}a)^2} = \frac{a^2}{2a^2} = \frac{1}{2}$

$\therefore \text{ar}(\Delta BEC) = \frac{1}{2}\text{ar}(\Delta BFD)$

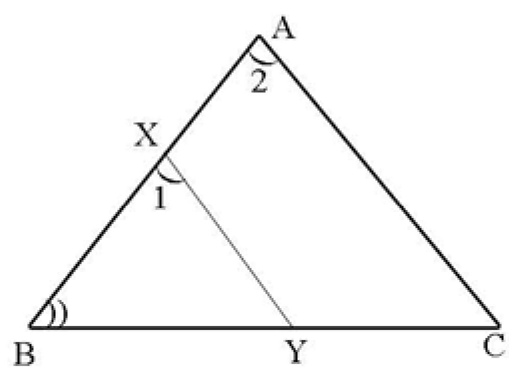
5. In figure, XY || AC and XY divides triangular region ΔABC into two equal parts in area. Determine $\frac{\mathbf{AX}}{\mathbf{AB}}$. [Example 9]

Sol:- Given XY || AC and XY divides triangular region ΔABC into two equal parts in area.

$\Rightarrow \quad \text{ar}(\Delta BXY) = \text{ar}(ACYX)$

$\Rightarrow \quad \text{ar}(\Delta BXY) = \frac{1}{2}\text{ar}(\Delta ABC)$

$\Rightarrow \frac{\text{ar}(\Delta BXY)}{\text{ar}(\Delta ABC)} = \frac{1}{2} \ldots\ldots\ldots\ldots.\ldots.$ i)



Now In ΔBXY and ΔABC, we've

$\angle 1 = \angle 2$ (Corrspomding angles)

$\angle B = \angle B$ (Common)

$\therefore \Delta BXY \sim \Delta ABC$ (AA Similarity)

$\therefore \frac{\text{ar}(\Delta BXY)}{\text{ar}(\Delta ABC)} = \frac{BX^2}{AB^2}$

$\Rightarrow \frac{1}{2} = \frac{BX^2}{AB^2}$

$\Rightarrow \frac{BX}{AB} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

Now $\frac{BX}{AB} = \frac{AB-AX}{AB} = \frac{AB}{AB} - \frac{BX}{AB} = 1 - \frac{BX}{AB} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$

6. D, E, F are the mid-points of the sides BC, CA and AB respectively of ΔABC.Determine the ratios of the areas of ΔDEF and ΔABC. [Ex 6.4, Q 5]

Sol :- Given D,E,F are the mid points of BC, CA and AB respectively.

To find $\frac{ar(\Delta DEF)}{ar(\Delta ABC)}$

Since D and F are mid points of sides AB and AC respectively of ΔABC.

$\therefore$ DF || BC and DF = $\frac{1}{2}$ BC (Mid point theorem)

$\Rightarrow$ DF || BE and DF=BE ……………… i)

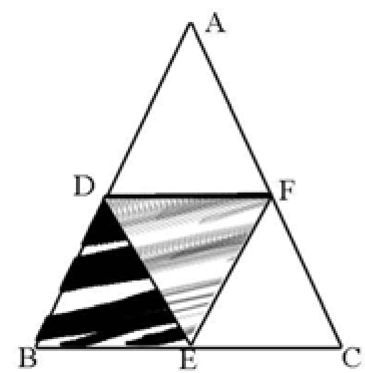
$\Rightarrow$ BDEF is a parallelogram

$\Rightarrow$ $\angle B = \angle F$ (opposite angles are equal)

Similarly $\angle C = \angle D$ and $\angle A = \angle E$

$\Delta DEF \sim \Delta ABC$ (AA similarity)

$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{DF^2}{BC^2} = \frac{DF^2}{(2DF)^2} = \frac{DF^2}{4DF^2} = \frac{1}{4}$



7. Prove that the ratio of the areas of similar triangles is equal to the ratio of the squares of their corresponding medians. [Ex 6.4, Q 6]

Sol :- **Given :** $\Delta ABC \sim \Delta DEF$ and AL & DM are medians.

To Prove :- $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AL^2}{DM^2}$

Proof:- We know, the ratio of the areas of two similar triangles is equal to ratio of square of their corresponding sides

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{DE^2} = \frac{BC^2}{DF^2} = \frac{AC^2}{EF^2}$ ………… i)

Also we know, the ratio of the corresponding sides of two similar triangles is equal to ratio of their corresponding medians.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM} \ldots\ldots\ldots\ldots\text{ ii)}$$

From i) & ii), we've $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AL^2}{DM^2}$