DAY 10

Equations reducible to a pair of linear Equations :-

In last section we have discussed about solving a pair of linear equations with different methods. Now we have some different type of equations which are not linear but that can be reduced to linear and solved according to discussed last.

Equations which has variables in the denominator are called reciprocal equations.

e.g. $\frac{3}{x}+\frac{2}{y}=5$ or $\frac{1}{2x+3y}+\frac{2}{3x-2y}=5$ are reciprocal equations

1. Solve $\frac{2}{x}+\frac{3}{y}=13$ and $\frac{5}{x}-\frac{4}{y}=-2$ where $x\neq 0, y\neq 0$ [Example 17]

Sol:- Given equations are $\frac{2}{x}+\frac{3}{y}=13$ ..... i) and $\frac{5}{x}-\frac{4}{y}=-2$ ..... ii)

To equate the coefficient of $\frac{1}{x}$, multiply i) by 5 & ii) by 2 and then subtract, we get

$$\left(\frac{10}{x}+\frac{15}{y}\right)-\left(\frac{10}{x}-\frac{8}{y}\right)=65-(-4)$$

$$\Rightarrow \quad \frac{23}{y}=69 \quad \Rightarrow \quad y=\frac{23}{69}=\frac{1}{3}$$

Replace this value in i), we get

$$\text{i)} \Rightarrow \frac{2}{x}+\frac{3}{\frac{1}{3}}=13 \quad \Rightarrow \quad \frac{2}{x}+9=13 \quad \Rightarrow \frac{2}{x}=13-9=4 \quad \Rightarrow \quad x=\frac{2}{4}=\frac{1}{2}$$

$\therefore \quad x=\frac{1}{2}, y=\frac{1}{3}$ is required solution

2. Solve $\frac{5}{x-1}+\frac{1}{y-2}=2$ and $\frac{6}{x-1}-\frac{3}{y-2}=1$ where $x\neq 1, y\neq 2$ [Ex 3.1, Q1(iv)]

Sol:- Given equations are $\frac{5}{x-1}+\frac{1}{y-2}=2$ ..... i) and $\frac{6}{x-1}-\frac{3}{y-2}=1$ ..... ii)

To equate the coefficient of $\frac{1}{x-1}$, multiply i) by 6 & ii) by 5 and then subtract, we get

$$\left(\frac{30}{x-1}+\frac{6}{y-2}\right)-\left(\frac{30}{x-1}-\frac{15}{y-2}\right)=12-5$$

$$\Rightarrow \frac{30}{x-1}+\frac{6}{y-2}-\frac{30}{x-1}+\frac{15}{y-2}=7$$

$$\Rightarrow \frac{21}{y-2}=7 \quad \Rightarrow \quad y-2=\frac{21}{7}=3 \quad \Rightarrow \quad y=3+2=5$$

Replace this value in i), we get

$$\text{i)} \Rightarrow \frac{5}{x-1}+\frac{1}{3}=2 \quad \Rightarrow \quad \frac{5}{x-1}=2-\frac{1}{3} \quad \Rightarrow \frac{5}{x-1}=\frac{6-1}{3}=\frac{5}{3}$$

$$\Rightarrow \quad x-1=3 \quad \Rightarrow \quad x=3+1=4$$

$\therefore \quad x=4, y=5$ is required solution

3. Solve $\frac{10}{x+y}+\frac{2}{x-y}=4$ and $\frac{15}{x+y}-\frac{5}{x-y}=-2$ **[Ex 3.1, Q1(vii)]**

Sol :- Given Equations are

$\frac{10}{x+y}+\frac{2}{x-y}=4$ and $\frac{15}{x+y}-\frac{5}{x-y}=-2$

Change both equations in linear form so

Replace $\frac{1}{x+y}=u$ and $\frac{1}{x-y}=v$

$10u+2v=4$ ... ... i)

and $15u-5v=-2$ .... ii)

Multiply i) by 5 and ii) by 2 then add both, we get

$(50u+10v)+(30u-10v)=20+(-4)$

$\Rightarrow 80u=16 \quad \Rightarrow u=\frac{16}{80}=\frac{1}{5}$ Replace in i), we get

i) $\Rightarrow 10\left(\frac{1}{5}\right)+2v=4 \quad \Rightarrow 2+2v=4$

$\Rightarrow 2v=4-2=2 \quad \Rightarrow v=\frac{2}{2}=1$

Since $\frac{1}{x+y}=u=\frac{1}{5} \quad \Rightarrow \boldsymbol{x+y=5}$

and $\frac{1}{x-y}=v=1 \quad \Rightarrow \boldsymbol{x-y=1}$

Solve both equations, we get $\boldsymbol{x=3, y=2}$

EXERCISE

1. Exercise 3.6, Q 1