DAY 8

1. **In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in two subjects. [Ex 4.3, Q5]**

Sol:- Given $\textbf{(Math marks)} + \textbf{(English marks)} = \mathbf{30}$

and $(\textbf{Math marks} + \mathbf{2}) \times (\textbf{English marks} - \mathbf{3}) = \mathbf{210}$

Let marks in Math be x and in English be y

First condition: $x + y = 30 \ldots\ldots\ldots\ldots\ldots\ldots..\text{i)}$

Second condition: $(x+2)(y-3) = 210$

$\Rightarrow (x+2)(\mathbf{30}-\boldsymbol{x}-3) = 210$ {Replace value of y by i)}

$\Rightarrow (x+2)(27-x) = 210$ $\Rightarrow 27x + 54 - 2x - x^2 = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

$\Rightarrow x^2 - 13x - 12x + 156 = 0$ {Students can do this by quadratic formula}

$\Rightarrow x(x-13) - 12(x-13) = 0$ $\Rightarrow (x-13)(x-12) = 0$

$\Rightarrow x - 13 = 0$ or $x - 12 = 0$ $\Rightarrow x = 13$ or 12

If She has 13 marks in Math then in English $30 - 13 = 17$ marks.

If She has 12 marks in Math then in English $30 - 12 = 18$ marks.

Alternate Method:- This sum also can be solved in one variable.

Suppose marks in Math are x then in English she has $30 - x$

Solve as above

2. **A train travels 360 km at a uniform speed. If the speed had been 5km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train. [Ex 4.3, Q8]**

Sol :- Let the speed of the train be x km/h and Distance covered = 360 km

$\therefore$ Time taken by a train = $\frac{360}{x}$ hours

If speed of the train is increased by 5 km/h then time taken by train to cover distance 360 km = $\frac{360}{x+5}$ hours

Given: $\begin{pmatrix}\textbf{Time taken by train}\\ \textbf{at speed of } (\boldsymbol{x}+\mathbf{5})\boldsymbol{km/h}\end{pmatrix} = \begin{pmatrix}\textbf{Time taken by train}\\ \textbf{at speed of } \boldsymbol{x}\ \boldsymbol{km/h}\end{pmatrix} - \mathbf{1}$

$\Rightarrow \frac{360}{x+5} = \frac{360}{x} - 1$ $\Rightarrow \frac{360}{x+5} - \frac{360}{x} = -1$

$\Rightarrow \frac{360x - 360(x+5)}{(x+5)x} - 1$ $\Rightarrow \frac{360x - 360x - 1800}{(x+5)x} - 1$

$\Rightarrow -1800 = -(x^2 + 5x)$ Or $x^2 + 5x - 1800 = 0$

$\Rightarrow\ x^2+45x-40x-1800=0$ $\quad\Rightarrow\ x(x+45)-40(x+45)=0$

$\Rightarrow\ (x+45)(x-40)=0$ $\quad\Rightarrow\ x=40$ or -45

Hence speed of train = 40 km/h

3. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24km upstream than to return downstream to the same post. Find the speed of the stream. [Example 15]

Sol :- Let the speed of stream be x km/h

Given speed of boat in still water = 15 km/h and Distance covered in both cases=24km

We know

$$\begin{pmatrix}\textit{Speed of boat in}\\ \textit{downstream}\end{pmatrix}=\begin{pmatrix}\textit{Speed of boat in}\\ \textit{still water}\end{pmatrix}+(\textit{Speed of stream})=(18+x)km/h$$

and $$\begin{pmatrix}\textit{Speed of boat in}\\ \textit{upstream}\end{pmatrix}=\begin{pmatrix}\textit{Speed of boat in}\\ \textit{still water}\end{pmatrix}-(\textit{Speed of stream})=(18-x)km/h$$

Given Condition: $\begin{pmatrix}\text{Time taken in}\\ \text{upstream}\end{pmatrix}=\begin{pmatrix}\text{Time taken}\\ \text{in downstream}\end{pmatrix}+1$

$\Rightarrow\ \frac{24}{18-x}=\frac{24}{18+x}+1$ $\quad\Rightarrow\ \frac{24}{18-x}-\frac{24}{18+x}=1$

$\Rightarrow\ \frac{24(18+x)-24(18-x)}{(18-x)(18+x)}=1$

$\Rightarrow\ 432+24x-432+24x=(18-x)(18+x)$

$\Rightarrow\ 48x=324-x^2$ $\quad\Rightarrow x^2+48x-324=0$

$\Rightarrow x^2+54x-6x-324=0$ $\quad\Rightarrow (x+54)(x-6)=0$

$\Rightarrow\ x=-54,6$ (But speed can't be negative)

$\Rightarrow x=5$ km/h is required sol.

So speed of stream be 5 km/h

4. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. [Ex 4.3, Q9]

Sol :- Given $\begin{pmatrix}\text{Time taken by tap of}\\ \text{larger diameter}\end{pmatrix}=\begin{pmatrix}\text{Time taken by tap of}\\ \text{smaller diameter}\end{pmatrix}-10$

Let the time taken by smaller pipe to fill the tank x hours

and the time taken by larger pipe to fill the tank $(x-10)$ hours

Given, time taken by both pipes to fill the tank $=9\frac{3}{8}=\frac{75}{8}$ hours

In one hour, smaller pipe will fill $\left(\frac{1}{x}\right)^{th}$ part of tank.

In one hour, larger pipe will fill $\left(\frac{1}{x-10}\right)^{th}$ part of tank.

In one hour, both will fill $\frac{8}{75}$ hours.

$\Rightarrow \frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$

$\Rightarrow \frac{x-10+x}{x(x-10)}=\frac{8}{75}$

$\Rightarrow \frac{2x-10}{x^2-10x}=\frac{8}{75}$

$\Rightarrow 75(2x-10)=8(x^2-10x)$

$\Rightarrow 150x-750=8x^2-80x$

$\Rightarrow 8x^2-230x+750=0$

$\Rightarrow 4x^2-115x+375=0$ {Divide both sides by 2}

$\Rightarrow 4x^2-100x-15x+375=0$

$\Rightarrow 4x(x-25)-15(x-25)=0$

$\Rightarrow (x-25)(4x-15)=0$

$\Rightarrow x=25, \frac{15}{4}$ Rejected

If smaller pipe take 25 hours to fill then larger pipe takes $25-10=15$ hours

If smaller pipe take $\frac{15}{4}$ hours to fill then larger pipe takes $\frac{15}{4}-10=\frac{-25}{4}$ hours (which is not possible)

5. A pole has to be erected at a point on the boundary of a circular park of diameter 13 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 m. At what distances from the two gates should be erected? [Example 17]

Sol :- Suppose in the circular park, the pole is situated at P and AB $=13m$ is a diameter.

Given: **Difference between AP and BP is $7m$ *i.e.* AP − BP $=7m$**

Suppose BP $=x$ then AP $=7+x$

Since AB is a diameter, so $\angle APB=90^0$.

In Right Angled Triangle ΔABP, $AB^2=BP^2+AP^2$

$\Rightarrow 13^2=x^2+(7+x)^2$

$\Rightarrow 169=x^2+49+x^2+14x$

$\Rightarrow 2x^2+14x-120=0$ or $x^2+7x-60=0$ {Divide by 2}

$\Rightarrow x^2+12x-5x-60=0$

$\Rightarrow x(x+12)-5(x+12)=0$

$\Rightarrow (x+12)(x-5)=0$

$\Rightarrow x=5,-12$ (−12 Rejected, side can't be negative)

Hence BP $=x=5\ m$ and AP $=x+7=5+7=12\ m$