

Day – 10

1. ਸਿੱਧ ਕਰੋ : $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

ਹੱਲ: ਖੱਬਾ ਪਾਸਾ : $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$
 $= \tan\theta \frac{[1 - 2(1 - \cos^2\theta)]}{(2\cos^2\theta - 1)} = \tan\theta \left[\frac{1 - 2 + 2\cos^2\theta}{2\cos^2\theta - 1} \right]$
 $= \tan\theta \left[\frac{2\cos^2\theta - 1}{2\cos^2\theta - 1} \right] = \tan\theta$

2. ਸਿੱਧ ਕਰੋ : $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$

ਹੱਲ: ਖੱਬਾ ਪਾਸਾ : $\frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$
 $= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}} = \frac{\cos A + 1}{\cos A} \times \frac{\cos A}{1} = 1 + \cos A$
 ਸੱਜਾ ਪਾਸਾ : $\frac{\sin^2 A}{1 - \cos A} = \frac{1^2 - \cos^2 A}{1 - \cos A} = \frac{(1 - \cos A)(1 + \cos A)}{1 - \cos A} = 1 + \cos A$
 ਖੱਬਾ ਪਾਸਾ = ਸੱਜਾ ਪਾਸਾ

3. ਸਿੱਧ ਕਰੋ : $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

ਹੱਲ: ਖੱਬਾ ਪਾਸਾ : $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$
 $= \left(\frac{1}{\sin A} - \frac{\sin A}{1} \right) \left(\frac{1}{\cos A} - \frac{\cos A}{1} \right)$
 $= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) = \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} = \sin A \cdot \cos A$
 ਸੱਜਾ ਪਾਸਾ : $\frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}} = \sin A \cdot \cos A$
 ਖੱਬਾ ਪਾਸਾ = ਸੱਜਾ ਪਾਸਾ

4. ਸਿੱਧ ਕਰੋ : $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

ਹੱਲ: ਖੱਬਾ ਪਾਸਾ :

$$\begin{aligned} & (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\ &= (\sin^2 A + \operatorname{cosec}^2 A + 2\sin A \cdot \operatorname{cosec} A) + (\cos^2 A + \sec^2 A + 2\cos A \cdot \sec A) \\ &= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \cdot \frac{1}{\sin A} + \cos^2 A + \sec^2 A + 2 \cdot \cos A \cdot \frac{1}{\cos A} \\ &= (\sin^2 A + \cos^2 A) + 2 + \operatorname{cosec}^2 A + \sec^2 A + 2 \end{aligned}$$

$$= 1 + 2 + 2 + (1 + \cot^2 A) + (1 + \tan^2 A)$$

$$= 7 + \cot^2 A + \tan^2 A = \text{ਸੱਜਾ ਪਾਸਾ}$$

5. ਸਿੱਧ ਕਰੋ : $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta$

ਹੱਲ: ਖੱਬਾ ਪਾਸਾ :

$$\begin{aligned} \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \frac{\tan \theta}{1 - \frac{1}{\tan \theta}} + \frac{\frac{1}{\tan \theta}}{1 - \tan \theta} \\ &= \frac{\tan^2 \theta}{\tan \theta - 1} + \frac{\tan \theta}{\tan \theta(1 - \tan \theta)} = \frac{\tan^2 \theta}{\tan \theta - 1} - \frac{1}{\tan \theta(\tan \theta - 1)} \\ &= \frac{1}{\tan \theta - 1} \left[\frac{\tan^3 \theta - 1}{\tan \theta} \right] = \frac{1}{\tan \theta - 1} \left[\frac{(\tan \theta - 1)(\tan^2 \theta + 1 + \tan \theta)}{\tan \theta} \right] \\ &= \frac{\tan^2 \theta + 1 + \tan \theta}{\tan \theta} = \frac{\tan^2 \theta}{\tan \theta} + \frac{1}{\tan \theta} + \frac{\tan \theta}{\tan \theta} \\ &= \tan \theta + \cot \theta + 1 = \text{ਸੱਜਾ ਪਾਸਾ} \end{aligned}$$

6. ਸਿੱਧ ਕਰੋ : $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$

ਹੱਲ: ਖੱਬਾ ਪਾਸਾ : $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$

come-become-educated

(ਸੱਜੇ ਪਾਸੇ $\sec A$ ਅਤੇ $\tan A$ ਦੀ ਜ਼ਰੂਰਤ ਹੋਣ ਕਾਰਨ ਇਸਦੇ ਅੰਸ਼ ਅਤੇ ਹਰ ਨੂੰ $\cos A$ ਨਾਲ ਵੰਡਿਆ ਜਾਵੇ)

$$\begin{aligned} &= \frac{\frac{\sin A}{\cos A} - \frac{\cos A}{\cos A} + \frac{1}{\cos A}}{\frac{\sin A}{\cos A} + \frac{\cos A}{\cos A} - \frac{1}{\cos A}} = \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A} \\ &= \frac{\tan A + \sec A - 1}{\tan A - \sec A + (\sec^2 A - \tan^2 A)} \\ &= \frac{\tan A + \sec A - 1}{\tan A - \sec A + [(\sec A - \tan A)(\sec A + \tan A)]} \\ &= \frac{\tan A + \sec A - 1}{(\sec A - \tan A)[-1 + (\sec A + \tan A)]} \\ &= \frac{\tan A + \sec A - 1}{(\sec A - \tan A)(\tan A + \sec A - 1)} = \frac{1}{\sec A - \tan A} \end{aligned}$$