CHAPTER-5

Arithmetic Progression

DAY 1(Introduction)

In last classes, we have studied some pattern of numbers, which are inter related with some fixed pattern. In this chapter, we shall study an interesting pattern of numbers where in a quantity increases or decreases by a fixed amount. In our daily life, we see many sequences of numbers which are arranged in a certain fixed number. Before discussing about Arithmetic Progression you should know about sequence.

"A sequence is an arrangement of numbers according to a certain fixed pattern."

- 2,4,6,8,10……..
- 2,4,8,16,32……..
- $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots\ldots\ldots\ldots\ldots$are Examples of sequence

In example i) terms are arranged in this successive way such that difference between two terms i.e. 2 is same.

In Example ii), Ratio between two successive term i.e.2 is same.

In Example iii) Ratio between two successive terms i.e. $\frac{1}{2}$ is same.

Example i) is representing Arithmetic Progression in which difference sequence is increasing or decreasing progressively by a fixed amount.

e.g. In our daily life.

- Monday comes after every 7 days.
- Your Birthday comes after every 12 months.
- In a cricket match, every over is bowled after every 6 balls etc.

In order to study such situations of daily life, we introduce & study the concept of Arithmetic Progression (A.P.).

Take a look at the following examples:-

- 4, 7,10,13,16 ……..
- 8, 13, 18, 23 ……
- $-10, -2, 6, 14, \ldots \ldots$
- $12, 3, -6, -15$……
- 28, 13, 18, 13, 8 ……

All these sequences contain different numbers. It is observed that all the sequences are increasing or decreasing by a fixed term.

Now we can say

- **Each Series starts will an arbitrary number.**
- **Every successive term in the series is formed by adding/subtracting a fixed number to the previous number. All series are in Arithmetic Progression (A.P.)**

Arithmetic Progression :- *"A sequence or series is called an Arithmetic Progression (A.P.) if the difference of any term from its preceding term is same."*

The first term of an A.P. is denoted by 'a' and the difference (usually called as common Difference) is denoted by 'd'

Lets take one example: 4,7,10,13,.....

This can be written as $a, a+d, (a+d)+d, (a+2d)+d$ ……

or $a, a+d, a+2d, a+3d$ ……

Here 1st term $(a_1) = a = 4$

2nd term$(a_2) = a + \mathbf{1}d = 7$

3rd term$(a_3) = a + \mathbf{2}d = 10$

4th term$(a_4) = a + \mathbf{3}d = 13$

……

n^{th}term$(a_n) = a + (\boldsymbol{n-1})d$

General term of an A.P. From above discussion, we have

1st term $(a_1) = T_1 = a$

Second term $(a_2) = T_2 = a + d = a + (2-1)d$

Third term $(a_3) = T_3 = a + 2d = a + (3-1)d$

Fourth term $(a_4) = \mathrm{T}_4 = \mathrm{a} + 3\mathrm{d} = \mathrm{a} + (4-1)\mathrm{d}$

……

……

n^{th} term $(a_n) = T_n = a + (n-1)d$ is called general term of an A.P. and

$d = T_2 - T_1$;

$d = T_3 - T_2$ … … … … … … … … … … … … … … …

$d = T_n - T_{n-1}$

- **In general term power of n is always one. *i. e.* general term is always linear in n.**
- **Coefficient of n is difference(d) of AP**
- **n^{th} term is also considered as *last term*(l).**

If the last term of an A.P. consisting of n terms is denoted by l then

$$l = a_n = a + (n-1)d.$$

Lets discuss some examples

1. Which of the following are A.P.'s ? If they form an A.P. find the common difference:

i) 6,10,14,17…… **ii) 5,11,17,23,……**

iii) 0.2, 0.22, 0.222, 0.2222 …… **iv)** $1^2, 2^2, 3^2, 4^2$, … … … … … … … … … ….

v) 3, 3, 3, 3 …… **vi)** $-21, -18, -15$, … …

Sol :-

i) The given numbers are 6,10,14,17,……..

Here $a_1 = 6, a_2 = 10, a_3 = 14, a_4 = 17$

Now $a_2 - a_1 = 10 - 6 = 4$;

$a_3 - a_2 = 14 - 10 = 4$;

$a_4 - a_3 = 17 - 14 = 3$

Difference of all given terms is not same, So given sequence does not form an A.P.

ii) Given numbers are 5, 11, 17, 23, ……..

Here $a_1 = 5, a_2 = 11, a_3 = 17, a_4 = 23$

Now $a_2 - a_1 = 11 - 5 = 6$

$a_3 - a_2 = 17 - 11 = 6$

$a_4 - a_3 = 23 - 17 = 6$

Hence difference of given terms are same, So given sequence forms an A.P. with common difference 6.

iii) Given numbers are 0.2, 0.22, 0.222, 0.2222 ……..

Here $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$

Now $a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

Hence difference of given terms are not same, So given sequence does not form an A.P.

iv) Given numbers are $1^2, 2^2, 3^2, 4^2$, … … … … … … … … … … . = 1,4,9,16, … … … … … … ..

Here $a_1 = 1, a_2 = 4, a_3 = 9, a_4 = 16$

Now $a_2 - a_1 = 4 - 1 = 3$

$a_3 - a_2 = 9 - 4 = 5$

$a_4 - a_3 = 16 - 9 = 7$

Hence difference of given terms are not same, So given sequence does not form an A.P

v) Given numbers are 3,3,3,3, … … … … … … … … … … . … ..

Here $a_1 = 3, a_2 = 3, a_3 = 3, a_4 = 3$

Now $a_2 - a_1 = 3 - 3 = 0$

$a_3 - a_2 = 3 - 3 = 0$

$a_4 - a_3 = 3 - 3 = 0$

Hence difference of given terms are same, So given sequence forms an A.P with common difference 0.

vi) Given numbers are $-21, -18, -15, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Here $a_1 = -21, a_2 = -18, a_3 = -15$

Now $a_2 - a_1 = (-18) - (-21) = -18 + 21 = 3$

$a_3 - a_2 = (-15) - (-18) = -15 + 18 = 3$

Hence difference of given terms are same, So given sequence forms an A.P with common difference 3.

2. Find next two terms of the following APs:

i) **4, 10, 16, 22……** **ii**) **3, 7, 11, 15,……**

iii) **1, −1, −3, −5 ……** **iv**)**110, 100, 90,** $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Sol :-

i) The given AP is 4, 10, 16, 22,……

Here $d = 10 - 4 = 6$

$\therefore$ next two terms are $22 + 6 = 28$ and $28 + 6 = 34$

ii) The given AP is 3, 7, 11, 15,……

Here $d = 7 - 3 = 4$

$\therefore$ next two terms are $15 + 4 = 19$ and $19 + 4 = 23$

iii) The given AP is $1, -1, -3, -5$ …… .

Here $d = (-1) - 1 = -1 - 1 = -2$

$\therefore$ next two terms are $(-5) + (-2) = -7$ and $(-7) + (-2) = -9$

iv) The given AP is 110, 100, 90, ……

Here $d = 100 - 110 = -10$

$\therefore$ next two terms are $90 + (-10) = 80$ and $80 + (-10) = 70$

EXERCISE

1. Exercise 5.1