Day – 10

1. Prove That: $\mathbf{\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta}$

Sol: LHS : $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \dfrac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta-1\right)}$

$= \tan\theta \dfrac{\left[1-2\left(1-\cos^2\theta\right)\right]}{\left(2\cos^2\theta-1\right)} = \tan\theta\left[\dfrac{1-2+2\cos^2\theta}{2\cos^2\theta-1}\right]$

$= \tan\theta\left[\dfrac{2\cos^2\theta-1}{2\cos^2\theta-1}\right] = \tan\theta$

2. Prove that: $\mathbf{\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}}$

Sol: LHS: $\dfrac{1+\sec A}{\sec A} = \dfrac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$= \dfrac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}} = \dfrac{\cos A+1}{\cos A} \times \dfrac{\cos A}{1} = 1 + \cos A$

RHS: $\dfrac{\sin^2 A}{1-\cos A} = \dfrac{1^2-\cos^2 A}{1-\cos A} = \dfrac{(1-\cos A)(1+\cos A)}{1-\cos A} = 1 + \cos A$

LHS = RHS

3. Prove that: $\mathbf{(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}}$

Sol: LHS: $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \dfrac{\sin A}{1}\right)\left(\dfrac{1}{\cos A} - \dfrac{\cos A}{1}\right)$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right) = \dfrac{\cos^2 A}{\sin A} \cdot \dfrac{\sin^2 A}{\cos A} = \sin A . \cos A$

RHS: $\dfrac{1}{\tan A+\cot A} = \dfrac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}} = \dfrac{1}{\frac{\sin^2 A+\cos^2 A}{\sin A.\cos A}} = \sin A . \cos A$

LHS = RHS

4. Prove That : $\mathbf{(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A}$

Sol: LHS: $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= (\sin^2 A + \operatorname{cosec}^2 A + 2\sin A . \operatorname{cosec} A) + (\cos^2 A + \sec^2 A + 2\cos A . \sec A)$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A . \dfrac{1}{\sin A} + \cos^2 A + \sec^2 A + 2 . \cos A . \dfrac{1}{\cos A}$

$= (\sin^2 A + \cos^2 A) + 2 + \operatorname{cosec}^2 A + \sec^2 A + 2$

$$= 1 + 2 + 2 + (1 + \cot^2 A) + (1 + \tan^2 A)$$
$$= 7 + \cot^2 A + \tan^2 A = \text{RHS}$$

5. Prove that : $\mathbf{\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta}$

Sol: LHS: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$$= \frac{\tan^2\theta}{\tan\theta-1} + \frac{\tan\theta}{\tan\theta(1-\tan\theta)} = \frac{\tan^2\theta}{\tan\theta-1} - \frac{1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{1}{\tan\theta-1}\left[\frac{\tan^3\theta-1}{\tan\theta}\right] = \frac{1}{\tan\theta-1}\left[\frac{(\tan\theta-1)\left(\tan^2\theta+1+\tan\theta\right)}{\tan\theta}\right]$$

$$= \frac{\tan^2\theta+1+\tan\theta}{\tan\theta} = \frac{\tan^2\theta}{\tan\theta} + \frac{1}{\tan\theta} + \frac{\tan\theta}{\tan\theta}$$

$$= \tan\theta + \cot\theta + 1 = \text{RHS}$$

6. Prove that : $\mathbf{\frac{\sin A-\cos A+1}{\sin A+\cos A-1} = \frac{1}{\sec A-\tan A}}$

Sol: LHS: $\frac{\sin A-\cos A+1}{\sin A+\cos A-1}$

$\left\{ \begin{array}{c} \text{In RHS, we need of } \mathbf{\sec A = \frac{1}{\cos A}} \text{ and } \mathbf{\tan A = \frac{\sin A}{\cos A}}, \\ \textbf{So divide Numerator and Denominator of LHS by cosA} \end{array} \right\}$

$$= \frac{\frac{\sin A}{\cos A}-\frac{\cos A}{\cos A}+\frac{1}{\cos A}}{\frac{\sin A}{\cos A}+\frac{\cos A}{\cos A}-\frac{1}{\cos A}} = \frac{\tan A-1+\sec A}{\tan A+\mathbf{1}-\sec A}$$

$$= \frac{\tan A+\sec A-1}{\tan A-\sec A+\left(\mathbf{\sec^2 A-\tan^2 A}\right)}$$

$$= \frac{\tan A+\sec A-1}{\tan A-\sec A+[(\sec A-\tan A)(\sec A+\tan A)]}$$

$$= \frac{\tan A+\sec A-1}{(\sec A-\tan A)[-1+(\sec A+\tan A)]}$$

$$= \frac{\tan A+\sec A-1}{(\sec A-\tan A)(\tan A+\sec A-1)} = \mathbf{\frac{1}{\sec A-\tan A}}$$