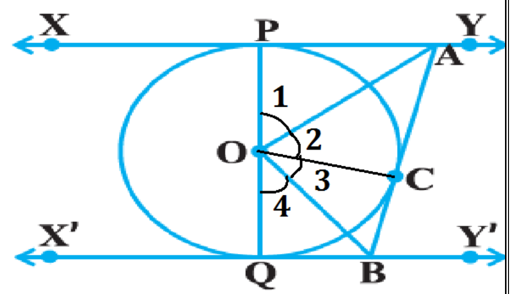
DAY 5

1. **In the given figure, XY and X'Y' are two parallel tangent to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^0$** **[Ex 10.2, Q9]**

Sol:- Join OC



In right angled ΔAPO and ΔACO, we've

$OP = OC$ [Equal Radii]

$OA = OA$ (common)

$AP = AC$ (Tangents from an external point)

$\therefore \Delta APO \cong \Delta ACO$ (SSS)

$\Rightarrow \angle 1 = \angle 2$ (C.P.C.T.) …………….i)

Similarly, $\Delta OCB \cong \Delta OQB$ and $\angle 3 = \angle 4$………………ii)

Since $XY || X'Y'$, So POQ is diameter

$\therefore \angle POQ = 180^0$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^0$

$\Rightarrow \angle 2 + \angle 2 + \angle 3 + \angle 3 = 180^0$

$\Rightarrow 2(\angle 2 + \angle 3) = 180^0$ $\Rightarrow \angle 2 + \angle 3 = \frac{180^0}{2} = 90^0$

$\Rightarrow \angle AOB = 90^0$

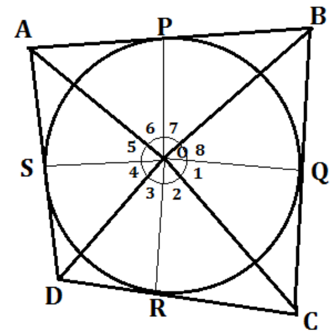
2. **Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.** **[Ex 10.2, Q13]**

Sol:- Given A quadrilateral ABCD circumscribing a circle having centre at O.

AB, BC, CD and DA touches the circle at S,R,Q and P respectively.

To Prove : $\angle AOD + \angle BOC = 180^0$ and $\angle AOB + \angle COD = 180^0$

Construction : Join OA, OB, OC, OD, OP, OQ, OR and OS



Proof : In ΔOCR and ΔOQC, we've

$CR = CQ$(tangents from an external point)

$OC = OC$ (common)

$OR = OQ$ (Equal Radii)

$\therefore \Delta APO \cong \Delta ASO$ (SSS)

$\Rightarrow \angle 1 = \angle 2$ … … … … … i)

Similarly $\angle 3 = \angle 4$ … … … … … . . ii)

$\angle 5 = \angle 6$ … … … … … . iii)

$\angle 7 = \angle 8$ … … … … … . iv)

Now We know $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^0$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^0$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^0$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = \frac{360^0}{2} = 180^0$

$\Rightarrow (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^0$

$\Rightarrow \angle COD + \angle AOB = 180^0$

Similarly $\angle AOD + \angle BOC = 180^0$

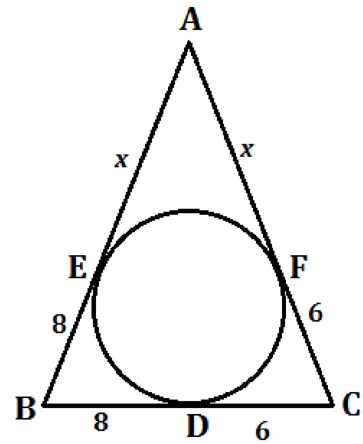
3. A ΔABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8cm and 6 cm respectively. Find the sides AB and AC. [Ex 10.2, Q12]

Sol:- Here $BD = BF = 8\ cm$ (tangents from an external point)

and $CF = CD = 6\ cm$ and Let $AE = AF = x\ cm$

We know $\boldsymbol{r} = \dfrac{\boldsymbol{ar}(\Delta \mathbf{ABC})}{\textbf{Semi perimeter of } \Delta\mathbf{ABC}}$ ……………………..i)

For $ar(\Delta ABC)$, we use Hero's formula with sides 14, $(8 + x)$, $(6 + x)$



$\Rightarrow$ s = semi-perimeter of $\Delta ABC = \dfrac{(x+8)+(14)+(6+x)}{2}$

$= \dfrac{2x+28}{2} = (x + 14)\ cm$

$\therefore\ ar(\Delta ABC) = \sqrt{s(s - AB)(s - BC)(s - CA)}$

$= \sqrt{(x + 14)(x + 14 - x - 8)(x + 14 - 14)(x + 14 - x - 6)}$

$= \sqrt{(x + 14)(6)(x)(8)} = \sqrt{48x(x + 14)}$

From i), we've

$\boldsymbol{r} = \dfrac{\boldsymbol{ar}(\Delta \mathbf{ABC})}{\textbf{Semi perimeter of } \Delta\mathbf{ABC}}$

$\Rightarrow \quad 4 = \dfrac{\sqrt{48x(x+14)}}{x+14}$

$\Rightarrow \sqrt{48x(x + 14)} = 4(x + 14)$

Squaring both sides, we've

$48x(x + 14) = 16(x + 4)^2$

$\Rightarrow \quad 3x = x + 14$ {Divide by $3(x + 14)$ both sides, we get}

$\Rightarrow \quad 3x - x = 14 \quad \Rightarrow \quad 2x = 14 \quad \Rightarrow \quad x = \dfrac{14}{2} = 7$

$\therefore$ **AB** $= \boldsymbol{x} + \mathbf{8} = \mathbf{7} + \mathbf{8} = \mathbf{15}\ \boldsymbol{cm}$ **and AC** $= \boldsymbol{x} + \mathbf{6} = \mathbf{7} + \mathbf{6} = \mathbf{13}\ \boldsymbol{cm}$