DAY 4

In last section, we have discussed completing the square method, whose process is lengthy and tedious. In this section, we shall discuss direct application of completing the square method which is known as QUADRATIC FORMULA

QUADRATIC FORMULA (SHREEDHAR ACHARAYA FORMULA):-

By method of completing square, we get the solutions of quadratic equations are

$$ax^2 + bx + c = 0 \ ; a \neq 0 \text{ are } \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When we apply this formula directly to the quadratic equations then it becomes ***QUADRATIC FORMULA.***

- First we find $\mathbf{D(Discriminant)} = \boldsymbol{b^2 - 4ac}$
- then $\boldsymbol{x = \frac{-b \pm \sqrt{D}}{2a}}$ $\quad$ **(if $\boldsymbol{D \geq 0}$)**
- If $\boldsymbol{D > 0}$ then the given equation has **two distinct** real roots.
- If $\boldsymbol{D = 0}$ then the given equation has **two equal** real roots.

1. **Find the discriminant of the following quadratic equations :-**

i) $\boldsymbol{x^2 - 7x + 12 = 0}$ **ii)** $\boldsymbol{3x^2 - 5x + 2 = 0}$ **iii)** $\boldsymbol{9x^2 + 6x + 1 = 0}$

iv) $\boldsymbol{5x^2 + 3x + 4 = 0}$ **v)** $\boldsymbol{16x^2 - 24x + 9 = 0}$

Sol :-

i) Given equation is $x^2 - 7x + 12 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = 1,\ b = -7, c = 12$

$D = b^2 - 4ac = (-7)^2 - 4 \times 1 \times 12 = 49 - 48 = 1$

ii) Given equation is $3x^2 - 5x + 2 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = 3, b = -5, c = 2$

$D = b^2 - 4ac = (-5)^2 - 4 \times 3 \times 2 = 25 - 24 = 1$

iii) Given equation is $9x^2 + 6x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = 9, b = 6\ , c = 1$

$D = b^2 - 4ac = (6)^2 - 4 \times 9 \times 1 = 36 - 36 = 0$

iv) Given equation is $5x^2 + 3x + 4 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = 5, b = 3, c = 4$

$D = b^2 - 4ac = (3)^2 - 4 \times 5 \times 4 = 9 - 80 = -71$

v) Given equation is $16x^2 - 24x + 9 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = 16, b = -24, c = 9$

$D = b^2 - 4ac = (-24)^2 - 4 \times 16 \times 9 = 576 - 576 = 0$

2. Whether the following equations has real roots, if yes then find the roots:

i) $\mathbf{5x^2 - 2x - 3 = 0}$ **ii)** $\mathbf{6x^2 - x - 2 = 0}$ **iii)** $\mathbf{2x^2 - 7x + 3 = 0}$

iv) $\mathbf{2x^2 - 2\sqrt{2}x + 1 = 0}$ **v)** $\mathbf{x^2 + 4x + 5 = 0}$

Sol :-

i) Given equation is $5x^2 - 2x - 3 = 0$

Compare it with $ax^2 + bx + c = 0$ we get $a = 5,\ b = -2, c = -3$

$D = b^2 - 4ac = (-2)^2 - 4 \times 5 \times (-3) = 4 + 60 = 64 > 0$

$\therefore$ The given equation has real and distinct roots.

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-2) \pm \sqrt{64}}{2 \times 5}$$

$$= \frac{2 \pm 8}{10} = \frac{2+8}{10}, \frac{2-8}{10} = \frac{10}{10}, \frac{-6}{10} = 1, \frac{-3}{5}$$

$\mathbf{x = 1, \frac{-3}{5}}$ are required roots of the given equation

ii) Given equation is $6x^2 - x - 2 = 0$

Compare it with $ax^2 + bx + c = 0$ we get $a = 6, b = -1, c = -2$

$D = b^2 - 4ac = (-1)^2 - 4 \times 6 \times (-2) = 1 + 48 = 49$

$\therefore$ The given equation has real and distinct roots

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-1) \pm \sqrt{49}}{2 \times 6}$$

$$= \frac{1 \pm 7}{12} = \frac{1+7}{12}, \frac{1-7}{12} = \frac{8}{12}, \frac{-6}{12} = \frac{2}{3}, \frac{-1}{2}$$

$\mathbf{x = \frac{2}{3}, \frac{-1}{2}}$ are required roots of given equation.

iii) Given equation is $2x^2 - 7x + 3 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = 2, b = -7, c = 3$

$D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 3 = 49 - 48 = 1$

$\therefore$ The given equation has real and distinct roots

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-7) \pm \sqrt{1}}{2 \times 2}$$

$$= \frac{7 \pm 1}{4} = \frac{7+1}{4}, \frac{7-1}{4} = \frac{8}{4}, \frac{6}{4} = 2, \frac{3}{2}$$

$\mathbf{x = 2, \frac{3}{2}}$ are required roots of given equation.

iv) Given equation is $2x^2 - 2\sqrt{2}x + 1 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = 2, b = -2\sqrt{2}, c = 1$

$D = b^2 - 4ac = \left(-2\sqrt{2}\right)^2 - 4 \times 2 \times 1 = 8 - 8 = 0$

$\therefore$ The given equation has real and equal roots

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-2\sqrt{2}) \pm \sqrt{0}}{2 \times 2}$

$= \frac{2\sqrt{2} \pm 0}{4} = \frac{2\sqrt{2}+0}{4}, \frac{2\sqrt{2}-0}{4} = \frac{2\sqrt{2}}{4}, \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$

$\boldsymbol{x = \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}}$ are required roots of given equation.

v) Given equation is $x^2 + 4x + 5 = 0$

Compare it with $ax^2 + bx + c = 0$ we get $a = 1, b = 4, c = 5$

$D = b^2 - 4ac = (4)^2 - 4 \times 1 \times 5 = 16 - 20 = -4 < 0$

$\therefore$ The given equation has no real roots

EXERCISE

1. **Find the discriminant of the following quadratic equations :-**

   i) $x^2 - 5x + 6 = 0$ ii) $3x^2 + 4x + 7 = 0$ iii) $5x^2 - x - 2 = 0$

   iv) $25x^2 - 30x + 9 = 0$ v) $x^2 - x + 1 = 0$ vi) $2x^2 + x - 1 = 0$

2. **Find the roots of the following quadratic equations, if they exist:**

   i) $3x^2 - 5x - 2 = 0$ ii) $x^2 + 2x + 3 = 0$ iii) $2x^2 + x - 4 = 0$

   iv) $4x^2 + 4\sqrt{3}x + 3 = 0$ v) $6x^2 - x - 7 = 0$