**DAY 5**

**1. Find the roots of the equation $x + \frac{1}{x} = 3$**

**Sol:-** Given equation is $x + \frac{1}{x} = 3$

Instead of taking LCM, multiply complete equation by $x$, we get

$$\left\{x + \frac{1}{x} = 3\right\} \times x$$

$$\Rightarrow x \times x + \frac{1}{x} \times x = 3 \times x \qquad \Rightarrow x^2 + 1 = 3x$$

$$\Rightarrow x^2 - 3x + 1 = 0$$

Compare it with $ax^2 + bx + c = 0$, we get $a = 1, b = -3, c = 1$

$$D = b^2 - 4ac = (-3)^2 - 4 \times 1 \times 1 = 9 - 4 = 5$$

$\therefore$ The given equation has real and distinct roots

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-3) \pm \sqrt{5}}{2 \times 1} = \frac{3 \pm \sqrt{5}}{2}$$

$\therefore$ $\boldsymbol{x = \frac{3 \pm \sqrt{5}}{2}}$ are required roots of given equation.



**2. Find the roots of the equation $\frac{1}{x} - \frac{1}{x-2} = 3$**

**Sol:-** Given equation is $\frac{1}{x} - \frac{1}{x-2} = 3$

Instead of taking LCM, multiply complete equation by $x(x-2)$, we get

$$\left\{\frac{1}{x} - \frac{1}{x-2} = 3\right\} \times x(x-2)$$

$$\Rightarrow \frac{1}{x} \times x(x-2) - \frac{1}{x-2} \times x(x-2) = 3 \times x(x-2)$$

$$\Rightarrow (x-2) - (x) = 3x^2 - 6x \qquad \Rightarrow 3x^2 - 6x + 2 = 0$$

Compare it with $ax^2 + bx + c = 0$, we get $a = 3, b = -6, c = 2$

$$D = b^2 - 4ac = (-6)^2 - 4 \times 3 \times 2 = 36 - 24 = 12$$

$\therefore$ The given equation has real and distinct roots

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 3}$$

$$= \frac{6 \pm \sqrt{2 \times 2 \times 3}}{6} = \frac{6 \pm 2\sqrt{3}}{6} = \frac{3 \pm \sqrt{3}}{3}$$

$\therefore$ $\boldsymbol{x = \frac{3 \pm \sqrt{3}}{3}}$ are required roots of given equation.

***RELATIONSHIP BETWEEN DISCRIMINANT & NATURE OF ROOTS:***

By quadratic formula $ax^2 + bx + c = 0$ ; $a \neq 0$ will have real roots if $b^2 - 4ac \geq 0$ or $D \geq 0$

Thus If $b^2 - 4ac < 0$ then the equation will have no real roots (It has imaginary roots which you will study in XI class)

Here three cases arise

***Case I.*** If $D > 0$ then equation has two real & distinct roots, say

$$\alpha = \frac{-b + \sqrt{D}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{D}}{2a}$$

***Case II.*** If $D = 0$ then the equation has two real & equal roots, say ***the quadratic equation is perfect square***.

$$\alpha = \frac{-b \pm 0}{2a} = \frac{-b}{2a} = \beta$$

***Case III.*** If $D < 0$ then the equation has no real roots.

(Note: If only roots are mentioned then take $D \geq 0$)

Lets discuss some examples

**3. Find the value of $k$ for equation $3x^2 + kx + 4 = 0$ so that it has equal roots.**

**Sol:-** Given equation is $3x^2 + kx + 4 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = 3, b = k, c = 4$

$D = b^2 - 4ac = (k)^2 - 4 \times 3 \times 4 = k^2 - 48$

Since the equation has equal roots.

$\therefore D = 0 \Rightarrow k^2 - 48 = 0$

$\Rightarrow k^2 = 48 \quad \Rightarrow k = \pm\sqrt{48} = \pm 4\sqrt{3}$

So $\boldsymbol{k = \pm 4\sqrt{3}}$ is the required solution.



**4. Find the value of $k$ for equation $kx(x-2) + 6 = 0$ so that it has equal roots.**

**Sol:-** Given equation is $kx(x-2) + 6 = 0 \quad \Rightarrow kx^2 - 2kx + 6 = 0$

Compare it with $ax^2 + bx + c = 0$, we get $a = k, b = -2k, c = 6$

$D = b^2 - 4ac = (-2k)^2 - 4 \times k \times 6 = 4k^2 - 24k$

Since the equation has equal roots.

$\therefore D = 0 \Rightarrow 4k^2 - 24k = 0 \quad \Rightarrow 4k(k-6) = 0$

$\Rightarrow$ Either $4k = 0$ or $k - 6 = 0$

$\Rightarrow k = 0, 6 \quad \{\text{but } k \neq 0\}$

$\therefore \boldsymbol{k = 6}$ is the required solution.

**EXERCISE**

1. Find the roots of the equation $x - \frac{1}{x} = 3$
2. Find the roots of the equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$
3. Find the value of $k$ for equation $2x^2 + kx + 3 = 0$ so that it has equal roots.