DAY 3

**SOLVING A QUADRATIC EQUATION BY METHOD OF COMPLETING THE SQUARE :-**

Sometimes some equations are given in this way whose factorisation is quite difficult or terms of polynomials are very large. At that time, factorisation method took so much time.

To make such equations easier we will try another method by which we can solve every quadratic equation.

***PROCEDURE: -***

Consider quadratic equation $ax^2 + bx + c = 0$ ; $a \neq 0$ ………..i)

- Divide both sides by $a$ (coefficient of $x^2$, we get)

  i) $\Rightarrow \quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

- Shift constant term to right side, we get

  $\Rightarrow \quad x^2 + \frac{b}{a}x = \frac{-c}{a}$

- Adding $\left(\frac{1}{2}\text{ coeff. of } x\right)^2$ on both sides, we get $\left(\frac{b}{2a}\right)^2$

  $\Rightarrow \quad x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$

- On left side, there will perfect square and solve right side

  $$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2} = \frac{b^2-4ac}{4a^2}$$

- Taking square root on both sides assuming $b^2 - 4ac \geq 0$

  $\Rightarrow \quad x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$ $\quad \Rightarrow \quad x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

  $\Rightarrow \quad x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ or $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$

  Thus If $b^2 - 4ac \geq 0$ then the quadratic equation has two real roots.

**1. Find the roots of the equation $2x^2 - 7x + 3 = 0$ by method of completing the square.**

**Sol :-** Given equation is $2x^2 - 7x + 3 = 0$

- Dividing both sides by 2, we get

  $\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

  $\Rightarrow x^2 - \frac{7}{2}x = \frac{-3}{2}$

- Adding $\left(\frac{1}{2}\text{ coeff. of } x\right)^2 = \left(\frac{1}{2} \times \frac{-7}{2}\right)^2 = \left(\frac{-7}{4}\right)^2$ on both sides, we get

$$\Rightarrow \quad x^2 - \frac{7}{2}x + \left(\frac{-7}{4}\right)^2 = \frac{-3}{2} + \left(\frac{-7}{4}\right)^2$$

$$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = \frac{-3}{2} + \frac{49}{16} = \frac{-24+49}{16} = \frac{25}{16}$$

$$\Rightarrow \quad x - \frac{7}{4} = \pm \frac{5}{4} \qquad \Rightarrow \quad x = \frac{7}{4} \pm \frac{5}{4}$$

$$\Rightarrow \quad x = \frac{7+5}{4} \text{ or } \frac{7-5}{4} = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\Rightarrow \quad \boldsymbol{x = 3 \text{ or } \frac{1}{2}} \text{ are required roots.}$$

**2. Find the roots of equation $5x^2 - 6x - 2 = 0$ by method of completing the square.**

**Sol :-** Given equation is $5x^2 - 6x - 2 = 0$

Divide by 5 both sides, we get

$$\Rightarrow x^2 - \frac{6x}{5} - \frac{2}{5} = 0 \qquad \Rightarrow x^2 - \frac{6x}{5} = \frac{2}{5}$$

Adding $\left(\frac{1}{2} \text{ coeff. of } x\right)^2 = \left(\frac{1}{2} \times \frac{-6}{5}\right)^2 = \left(\frac{-3}{5}\right)^2$ on both sides, we get

$$\Rightarrow \quad x^2 - \frac{6x}{5} + \left(\frac{-3}{5}\right)^2 = \frac{2}{5} + \left(\frac{-3}{5}\right)^2$$

$$\Rightarrow \left(x - \frac{3}{5}\right)^2 = \frac{2}{5} + \frac{9}{25} = \frac{10+9}{25} = \frac{19}{25}$$

$$\Rightarrow \quad x - \frac{3}{5} = \pm \frac{\sqrt{19}}{5}$$

$$\Rightarrow \quad x = \frac{3}{5} \pm \frac{\sqrt{19}}{5} = \boldsymbol{\frac{3 \pm \sqrt{19}}{5}} \text{ are required roots of given equation.}$$

**3. Find the roots of equation $4x^2 + 12x + 9 = 0$ by method of completing the square.**

**Sol :-** Given equation is $4x^2 + 12x + 9 = 0$

Divide by 4 both sides, we get

$$\Rightarrow x^2 + \frac{12x}{4} + \frac{9}{4} = 0 \qquad \Rightarrow x^2 + 3x = -\frac{9}{4}$$

Adding $\left(\frac{1}{2} \text{ coeff. of } x\right)^2 = \left(\frac{1}{2} \times 3\right)^2 = \left(\frac{3}{2}\right)^2$ on both sides, we get

$$\Rightarrow \quad x^2 + 3x + \left(\frac{3}{2}\right)^2 = -\frac{9}{4} + \left(\frac{-3}{5}\right)^2$$

$$\Rightarrow \left(x + \frac{3}{2}\right)^2 = -\frac{9}{4} + \frac{9}{4} = 0$$

$$\Rightarrow \quad x + \frac{3}{2} = \pm 0$$

$$\Rightarrow x = -\frac{3}{2} \pm 0 = \boldsymbol{-\frac{3}{2}, -\frac{3}{2}} \text{ are required roots of given equation.}$$

**4. Find the roots of equation $4x^2 + 3x + 5 = 0$ by method of completing the square.**

**Sol :-** Given equation is $4x^2 + 3x + 5 = 0$

Divide by 4 both sides, we get

$\Rightarrow x^2 + \frac{3x}{4} + \frac{5}{4} = 0 \qquad \Rightarrow x^2 + \frac{3}{4}x = -\frac{5}{4}$

Adding $\left(\frac{1}{2} \text{ coeff. of } x\right)^2 = \left(\frac{1}{2} \times \frac{3}{4}\right)^2 = \left(\frac{3}{8}\right)^2$ on both sides, we get

$\Rightarrow x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2 = -\frac{5}{4} + \left(\frac{3}{8}\right)^2$

$\Rightarrow \left(x + \frac{3}{4}\right)^2 = -\frac{5}{4} + \frac{9}{64} = \frac{-80+9}{4} = \frac{-71}{64}$

$\Rightarrow x + \frac{3}{4} = \pm\sqrt{\frac{-71}{64}}$ [Which is not possible]

Hence the given equation has no real roots.

**EXERCISE**

**Solve the following quadratic equations by completing the perfect square method:**

**1.** $2x^2 - 5x + 3 = 0$

2. $3x^2 - 5x + 2 = 0$

**3.** $2x^2 + 9x + 4 = 0$

4. $3x^2 + 4x - 3 = 0$

**5.** $25x^2 - 20x + 4 = 0$

**6.** $2x^2 - 9x - 5 = 0$

**7.** $4x^2 + 5x + 7 = 0$