DAY 4

In last section, we have discussed about elimination by substitution and equating the coefficients. In this section, we shall discuss another algebraic method which is as follows:

CROSS MULTIPLICATION METHOD

Consider the system of linear of equations:-

$a_1x + b_1y + c_1 = 0 \ldots\ldots\ldots\ldots$ i) $\quad a_2x + b_2y + c_2 = 0 \ldots\ldots\ldots\ldots$ ii)

{Before applying this method, make it sure that RHS $= 0$}

- $$\frac{x}{\begin{matrix}\textbf{Coefficient of } y \textbf{ and}\\ \textbf{constant term}\\ (\textbf{except } x)\end{matrix}} = \frac{-y}{\begin{matrix}\textbf{Coefficient of } x \textbf{ and}\\ \textbf{constant term}\\ (\textbf{except } y)\end{matrix}} = \frac{1}{\begin{matrix}\textbf{Coefficient of } x\\ \textbf{and coefficient of } y\\ (\textbf{except constant term})\end{matrix}}$$

$$\frac{x}{\begin{matrix}b_1 & c_1\\ b_2 & c_2\end{matrix}} = \frac{-y}{\begin{matrix}a_1 & c_1\\ a_2 & c_2\end{matrix}} = \frac{1}{\begin{matrix}a_1 & b_1\\ a_2 & b_2\end{matrix}}$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

- Take 1st & last $\quad \frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1} \quad \Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$
- Take 2nd & last $\quad \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_2b_1 - a_1b_2} \quad \Rightarrow \quad y = \frac{a_1c_2 - a_2c_1}{a_2b_1 - a_1b_2}$

Lets discuss some examples

1. Solve the equations $2x - 3y = -1$ and $3x + 4y = 5$ by cross multiplication method.

Sol : - Given Equations are $2x - 3y = -1 \quad \Rightarrow \quad 2x - 3y + 1 = 0$

and $\quad 3x + 4y = 5 \quad \Rightarrow \quad 3x + 4y - 5 = 0$

By Cross Multiplication, we get

$$\frac{x}{\begin{matrix}-3 & 1\\ 4 & -5\end{matrix}} = \frac{-y}{\begin{matrix}2 & 1\\ 3 & -5\end{matrix}} = \frac{1}{\begin{matrix}2 & -3\\ 3 & 4\end{matrix}}$$

$$\Rightarrow \quad \frac{x}{(-3\times-5)-(1\times4)} = \frac{-y}{(2\times-5)-(1\times3)} = \frac{1}{(2\times4)-(-3\times3)}$$

$$\Rightarrow \quad \frac{x}{(15)-(4)} = \frac{-y}{(-10)-(3)} = \frac{1}{(8)-(-9)}$$

$$\Rightarrow \quad \frac{x}{15-4} = \frac{-y}{-10-3} = \frac{1}{8+9} \quad \Rightarrow \quad \frac{x}{11} = \frac{-y}{-13} = \frac{1}{17}$$

From 1st and last $\quad \Rightarrow \quad \frac{x}{11} = \frac{1}{17} \quad \Rightarrow \quad \boldsymbol{x = \frac{11}{17}}$

From 2nd and last $\Rightarrow \quad \frac{-y}{-13} = \frac{1}{17} \quad \Rightarrow \quad \boldsymbol{y = \frac{13}{17}}$.

2. Solve the equations $6x - y - 3 = 0$ and $7x + 4y - 9 = 0$.

Sol : - Given Equations are $6x - y - 3 = 0$

and $7x + 4y - 9 = 0$

By Cross Multiplication, we get

$$\frac{x}{\begin{matrix}-1 & -3\\ 4 & -9\end{matrix}} = \frac{-y}{\begin{matrix}6 & -3\\ 7 & -9\end{matrix}} = \frac{1}{\begin{matrix}6 & -1\\ 7 & 4\end{matrix}}$$

$$\Rightarrow \quad \frac{x}{(-1\times-9)-(-3\times4)} = \frac{-y}{(6\times-9)-(-3\times7)} = \frac{1}{(6\times4)-(-1\times7)}$$

$$\Rightarrow \quad \frac{x}{(9)-(-12)} = \frac{-y}{(-54)-(-21)} = \frac{1}{(24)-(-7)}$$

$$\Rightarrow \quad \frac{x}{9+12} = \frac{-y}{-54+21} = \frac{1}{24+7} \qquad \Rightarrow \quad \frac{x}{21} = \frac{-y}{-33} = \frac{1}{31}$$

From 1st and last $\Rightarrow \quad \frac{x}{21} = \frac{1}{31} \quad \Rightarrow \quad \boldsymbol{x = \frac{21}{31}}$

From 2nd and last $\Rightarrow \quad \frac{-y}{-33} = \frac{1}{31} \quad \Rightarrow \quad \boldsymbol{y = \frac{33}{31}}$.

3. Solve the equations $4x - 5y = 13$ and $3x + 2y = 4$.

Sol : - Given Equations are $4x - 5y = 13 \quad \Rightarrow \quad 4x - 5y - 13 = 0$

and $3x + 2y = 4 \quad \Rightarrow \quad 3x + 2y - 4 = 0$

By Cross Multiplication, we get

$$\frac{x}{\begin{matrix}-5 & -13\\ 2 & -4\end{matrix}} = \frac{-y}{\begin{matrix}4 & -13\\ 3 & -4\end{matrix}} = \frac{1}{\begin{matrix}4 & -5\\ 3 & 2\end{matrix}}$$

$$\Rightarrow \quad \frac{x}{(-5\times-4)-(-13\times2)} = \frac{-y}{(4\times-4)-(-13\times3)} = \frac{1}{(4\times2)-(-5\times3)}$$

$$\Rightarrow \quad \frac{x}{(20)-(-26)} = \frac{-y}{(-16)-(-39)} = \frac{1}{(8)-(-15)}$$

$$\Rightarrow \quad \frac{x}{20+26} = \frac{-y}{-16+39} = \frac{1}{8+15} \qquad \Rightarrow \quad \frac{x}{46} = \frac{-y}{23} = \frac{1}{23}$$

From 1st and last $\Rightarrow \quad \frac{x}{46} = \frac{1}{23} \quad \Rightarrow \quad x = \frac{46}{23} \quad \Rightarrow \quad \boldsymbol{x = 2}$

From 2nd and last $\Rightarrow \quad \frac{-y}{23} = \frac{1}{23} \quad \Rightarrow \quad y = \frac{-23}{23}. \Rightarrow \quad \boldsymbol{y = -1}$

EXERCISE

Solve the following by cross multiplication method:

1. $2x + 3y = 7$ and $6x - 5y = 11$

2. $10x + 7y = 25$ and $20x - 35y = 50$

3. $3x - 5y - 20 = 0$ and $7x + 2y - 17 = 0$

4. $5x - 4y = 9$ and $3x - 2y = 5$

5. $5x + 2y = -8$ and $4x - 3y = -11$