*Algebraic Method for solving a pair of linear equations in two variables:-*

In last section, we have discussed solving a pair of linear equations by graphical method. But there are some drawbacks of this method if our solution is non-integers $e.g.\ x = \frac{1}{4}, y = \frac{2}{3}$ or $x = \frac{-2}{7}, y = \frac{11}{7}$ because on graph we are not able to read these solutions exactly. So to solve these types of problems, algebraic methods are very helpful. In our syllabus, there are three methods:

- Elimination method by Substitution
- Elimination method by Equating the coefficients
- Cross Multiplication method

In this section we shall discuss Substitution method:

## SUBSTITUTION METHOD
*Procedure:-*
- Firstly find the value of one variable (suppose $x$) in the form of other (suppose $y$) from any one of the equation.
- Substitute the value of that variable ($x$) in other equation.
- After this solve the equation & Find the value of the variable ($y$)
- Substitute the value of the variable ($y$) in any one of the equation to find the $x$.
  Lets discuss some examples:

## *SOLVED EXAMPLES*

**1. Solve $x + 3y = 5$ and $2x + 3y = 7$.**
**Sol:** Given equations are
$\quad x + 3y = 5 \dots\dots\dots\dots$ (i) & $\quad 2x + 3y = 7 \ \dots\dots\dots\dots$ (ii)
$\quad$ Find the value of $x$ from (i):
i) $\Rightarrow \ x = 5 - 3y \dots\dots\dots\dots\dots$ (iii)
$\quad$ Substitute the value of $x$ in ii), we get
ii) $\Rightarrow \ 2x + 3y = 7 \ \Rightarrow \ 2(5 - 3y) + 3y = 7 \ \Rightarrow 10 - 6y + 3y = 7$
$\quad\quad \Rightarrow -3y = 7 - 10 = -3 \quad \Rightarrow y = \frac{-3}{-3} = 1$
$\quad$ Now substitute this value of $y$ in iii), we get
iii) $\Rightarrow \ x = 5 - 3(1) = 5 - 3 = 2$
$\quad\quad x = 2, y = 1$ is required solution.

**2. Solve $x + 2y = 3$ and $7x - 15y = 2$.**
**Sol:** Given equations are
$\quad x + 2y = 3 \dots\dots\dots\dots$ (i) & $\quad 7x - 15y = 2 \ \dots\dots\dots\dots$ (ii)
Find the value of $x$ from (i): $\quad$ i) $\Rightarrow \ x = 3 - 2y \dots\dots\dots\dots\dots$ (iii)
$\quad$ Substitute the value of $x$ in ii), we get

ii) $\Rightarrow\ 7x - 15y = 2\quad \Rightarrow 7(3 - 2y) - 15y = 2\ \Rightarrow 21 - 14y - 15y = 2$

$\Rightarrow -29y = 2 - 21 = -19\quad \Rightarrow y = \dfrac{-19}{-29} = \dfrac{19}{29}$

Now substitute this value of $y$ in iii), we get

iii) $\Rightarrow\ x = 3 - 2y = 3 - 2\left(\dfrac{19}{29}\right) = 3 - \dfrac{38}{29} = \dfrac{3 \times 29 - 38}{29} = \dfrac{87 - 38}{29} = \dfrac{49}{29}$

$\boldsymbol{x = \dfrac{49}{29}, y = \dfrac{19}{29}}$ is required solution.

**3. Solve $3x - y = 3$ and $7x + 2y = 20$.**

**Sol:** Given equations are

$\qquad 3x - y = 3 \ldots\ldots\ldots\ldots$ (i) $\quad \& \quad 7x + 2y = 20 \ldots\ldots\ldots\ldots$ (ii)

Find the value of $x$ from (i): $\quad$ i) $\Rightarrow 3x = 3 + y \quad \Rightarrow \quad x = \dfrac{3+y}{3} \ldots\ldots\ldots\ldots\ldots$ (iii)

Substitute the value of $x$ in ii), we get

ii) $\Rightarrow\ 7x + 2y = 20 \quad \Rightarrow 7\left(\dfrac{3+y}{3}\right) + 2y = 20$

{Multiply complete equation by 3 instead of taking LCM}

$\Rightarrow 7\left(\dfrac{3+y}{3}\right) \times \mathbf{3} + 2y \times \mathbf{3} = 20 \times \mathbf{3}$

$\Rightarrow 7(3 + y) + 6y = 60 \quad \Rightarrow 21 + 7y + 6y = 60 \quad \Rightarrow 13y = 60 - 21 = 39$

$\Rightarrow y = \dfrac{39}{13} = 3$

Now substitute this value of $y$ in iii), we get

iii) $\Rightarrow\ x = \dfrac{3+y}{3} = \dfrac{3+3}{3} = \dfrac{6}{3} = 2$

$\qquad \boldsymbol{x = 2, y = 3}$ is required solution.

**4. Solve $2x - 7y = 1$ and $4x + 3y = 15$.**

**Sol:** Given equations are

$\qquad 2x - 7y = 1 \ldots\ldots\ldots\ldots$ (i) $\quad \& \quad 4x + 3y = 15 \ldots\ldots\ldots\ldots$ (ii)

Find the value of $x$ from (i): $\quad$ i) $\Rightarrow 2x = 1 + 7y \quad \Rightarrow \quad x = \dfrac{1+7y}{2} \ldots\ldots\ldots\ldots\ldots$ (iii)

Substitute the value of $x$ in ii), we get

ii) $\Rightarrow\ 4x + 3y = 15 \quad \Rightarrow 4\left(\dfrac{1+7y}{2}\right) + 3y = 15$

$\Rightarrow 2(1 + 7y) + 3y = 15 \quad \Rightarrow 2 + 14y + 3y = 15 \quad \Rightarrow 17y = 15 - 2 = 13$

$\Rightarrow y = \dfrac{13}{17}$

Now substitute this value of $y$ in i), we get

i) $\Rightarrow\ 2x - 7y = 1 \Rightarrow 2x - 7\left(\dfrac{13}{17}\right) = 1$

{Multiply complete equation by 17 instead of taking LCM}

$\Rightarrow 2x \times \mathbf{17} - 7\left(\dfrac{13}{17}\right) \times \mathbf{17} = 1 \times \mathbf{17} \quad \Rightarrow 34x - 91 = 17$

$\Rightarrow 34x = 17 + 91 = 108 \quad \Rightarrow \quad x = \dfrac{108}{34} = \dfrac{54}{17}$

$$x = \frac{54}{17}, y = \frac{13}{17} \text{ is required solution.}$$

## EXERCISE
**Find the possible solutions of the followings:**
1. $7x + 11y = 3$ and $8x + y = 15$
2. $3x + 4y = 7$ and $2x + y = -2$
3. $x + y = 7$ and $2x - 3y = 11$
4. $3x - 5y = 1$ and $5x + 2y = 19$
5. $5x + 8y = 9$ and $2x + 3y = 4$
6. Exercise 3.3, Q 1 and 2