DAY 8

TRIGONOMETRIC IDENTITIES :-

By Pythagoras theorem, we've

$$H^2 = P^2 + B^2 \dots\dots\dots\dots\dots\dots \text{i)}$$

i) . Dividing i) both sides by H^2,we get

$$\left(\frac{H}{H}\right)^2 = \left(\frac{P}{H}\right)^2 + \left(\frac{B}{H}\right)^2$$

$i.e.\quad 1 = \sin^2\theta + \cos^2\theta \qquad\qquad i.e.\quad \mathbf{\sin^2\theta + \cos^2\theta = 1}$

ii) Dividing i) both sides by P^2, we get

$$\left(\frac{H}{P}\right)^2 = \left(\frac{P}{P}\right)^2 + \left(\frac{B}{P}\right)^2 \qquad i.e.\quad \mathbf{cosec^2\theta = 1 + \cot^2\theta}$$

iii). Dividing i) both sides by B^2, we get

$$\left(\frac{H}{B}\right)^2 = \left(\frac{P}{B}\right)^2 + \left(\frac{B}{B}\right)^2 \qquad i.e.\quad \mathbf{\sec^2\theta = \tan^2\theta + 1}$$

As a consequence of above identities, we've

i) $\mathbf{\sin^2\theta + \cos^2\theta = 1}$; $\mathbf{\cos^2\theta = 1 - \sin^2\theta}$; $\mathbf{\sin^2\theta = 1 - \cos^2\theta}$

ii) $\mathbf{\sec^2\theta = 1 + \tan^2\theta}$; $\mathbf{\sec^2\theta - \tan^2\theta = 1}$; $\mathbf{\tan^2\theta = \sec^2\theta - 1}$

iii) $\mathbf{cosec^2\theta - \cot^2\theta = 1}$; $\mathbf{cosec^2\theta = 1 + \cot^2\theta}$; $\mathbf{cosec^2\theta - 1 = \cot^2\theta}$

1. **Evaluate**

   i) $\mathbf{5\sin^2\theta + 5\cos^2\theta}$ ii) $\mathbf{7\sec^2 A - 7\tan^2 A}$ iii) $\mathbf{9\cot^2\theta - 9 cosec^2\theta}$

Sol:- i) $5\sin^2\theta + 5\cos^2\theta = 5(\sin^2\theta + \cos^2\theta) = 5(1) = 5$

ii) $7\sec^2 A - 7\tan^2 A = 7(\sec^2 A - \tan^2 A) = 7(1) = 7$

iii) $9\cot^2\theta - 9cosec^2\theta = 9(\cot^2\theta - cosec^2\theta) = 9(-1) = -9$

2. **Evaluate**

   i) $\mathbf{\sin^2 17^0 + \sin^2 73^0}$ ii) $\mathbf{\sec^2 40^0 - \cot^2 50^0}$

   iii) $\mathbf{\dfrac{\sin^2 20^0 + \sin^2 70^0}{\cos^2 44^0 + \cos^2 46^0}}$

Sol:-i) $\sin^2 17^0 + \sin^2 73^0$

{Here $17^0 + 73^0 = 90^0$ are complementary each other, So Change only one **T Ratio**}

$$= \cos^2(90^0 - 17^0) + \sin^2 73^0 = \cos^2 73^0 + \sin^2 73^0 = 1$$

ii) $\sec^2 40^0 - \cot^2 50^0$

{Here $40^0 + 50^0 = 90^0$ are complementary each other, So Change only one **T Ratio**}

$$= cosec^2(90^0 - 40^0) - \cot^2 50^0 = cosec^2 50^0 - \cot^2 40^0 = 1$$

iii) $\frac{\sin^2 20^0+\sin^2 70^0}{\cos^2 44^0+\cos^2 46^0} = \frac{\cos^2(90^0-20^0)+\sin^2 70^0}{\sin^2(90^0-44^0)+\cos^2 46^0} = \frac{\cos^2 70^0+\sin^2 70^0}{\sin^2 46^0+\cos^2 46^0} = \frac{1}{1} = 1$

EXERCISE

Evaluate

1. $3\sin^2\theta + 3\cos^2\theta$
2. $5\sec^2 A - 5\tan^2 A$
3. $4\tan^2\theta - 4\sec^2\theta$
4. $9\text{cosec}^2\theta - 9\cot^2\theta$
5. $\sin^2 40^0 + \sin^2 50^0$
6. $5\sec^2 23^0 - 5\cot^2 67^0$
7. $\frac{\sin^2 20^0+\sin^2 70^0}{\cos^2 44^0+\cos^2 46^0}$
8. $\frac{\sin^2 28^0+\sin^2 62^0}{\cos^2 40^0+\cos^2 50^0}$
9. $\frac{\sec^2 38^0-\cot^2 52^0}{\cos^2 31^0+\cos^2 59^0}$

Express One T–Ratio in other T-Ratios:

1. Express tanA, cosA in terms of sinA

Sol :- We know that $\sin^2 A + \cos^2 A = 1$

$\Rightarrow \cos^2 A = 1 - \sin^2 A \qquad \Rightarrow \cos A = \sqrt{1 - \sin^2 A}$

and $\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$

Alter Method;

Here, we have to change all T Ratios in **sin A**

$\therefore \ \sin A = \frac{\sin A}{1} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Let Perpendicular(P) = sin A and Hypotenuse(H) = 1

$\therefore$ By Pythagoras Theorem, we 've

$\text{Base(B)} = \sqrt{H^2 - P^2} = \sqrt{1 - \sin^2 A}$

$$\Rightarrow \tan A = \frac{P}{B} = \frac{\sin A}{\sqrt{1-\sin^2 A}} \text{ and } \cos A = \frac{B}{H} = \frac{\sqrt{1-\sin^2 A}}{1} = \sqrt{1-\sin^2 A}$$

2. Express secA, sinA in terms of cotA

Sol :-

Here, we have to change all T Ratios in **cot A**

$$\therefore \cot A = \frac{\cot A}{1} = \frac{\text{Base}}{\text{Perpendicular}}$$

Let Base(B) $= \cot A$ and Perpendicular(P) $= 1$

$\therefore$ By Pythagoras Theorem, we 've

Hypotenuse(H) $= \sqrt{P^2 + B^2} = \sqrt{1 + \cot^2 A}$

$$\Rightarrow \sec A = \frac{H}{B} = \frac{\sqrt{1+\cot^2 A}}{\cot A} \text{ and } \sin A = \frac{P}{H} = \frac{1}{\sqrt{1+\cot^2 A}}$$

EXERCISE

1. Express sin A in terms of tanA.
2. Express cosecA, cosA in terms of cotA.
3. Express secθ in terms of sinθ.