DAY 7

1. **If $\tan A = \cot B$ then prove that $A + B = 90^0$**

Sol: - $\tan A = \cot B$

$\Rightarrow \tan A = \tan(90^0 - B)$

{If T Ratios are equal then angles are also equal}

$\Rightarrow A = 90^0 - B$ $\Rightarrow A + B = 90^0$

Note:- This is very important standard result which can be used directly

- If $\left\{\begin{matrix} \tan A = \cot B \\ \sin A = \cos B \\ \sec A = \operatorname{cosec} B \end{matrix}\right\}$ then $A + B = 90^0$
- *i.e.* **If T Ratios are complementary then angles are also complementary.**

Lets discuss some examples on it.

2. **If $\sin 3A = \cos(A - 26^0)$ then find A.**

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Sol: - Given $\sin 3A = \cos(A - 26^0)$

{If T Ratios are complementary then angles are also complementary}

$\Rightarrow 3A + (A - 26^0) = 90^0$ $\Rightarrow 4A - 26^0 = 90^0$

$\Rightarrow 4A = 90^0 + 26^0 = 116^0$ $\Rightarrow A = \frac{116^0}{4} = 29^0$

3. **If $\tan 4\theta = \cot(\theta + 20^0)$ then find θ.**

Sol: - Given $\tan 4\theta = \cot(\theta + 20^0)$

{If T Ratios are complementary then angles are also complementary}

$\Rightarrow 4\theta + (\theta + 20^0) = 90^0$ $\Rightarrow 5\theta + 20^0 = 90^0$

$\Rightarrow 5\theta = 90^0 - 20^0 = 70^0$ $\Rightarrow \theta = \frac{70^0}{5} = 14^0$

4. **If $\sec 2A = \operatorname{cosec}(A - 18^0)$ then find A.**

Sol: - Given $\sec 2A = \operatorname{cosec}(A - 18^0)$

{If T Ratios are complementary then angles are also complementary}

$\Rightarrow 2A + (A - 18^0) = 90^0$ $\Rightarrow 3A - 18^0 = 90^0$

$\Rightarrow 3A = 90^0 + 18^0 = 108^0$ $\Rightarrow A = \frac{108^0}{3} = 36^0$

5. **In ΔABC, prove that $\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$**

Sol:- In ΔABC, $A + B + C = 180^0$ … … … … i)

Now **LHS** $\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{180^0 - C}{2}\right)$ {from i)}

$$= \sin\left(\frac{180^0}{2} - \frac{C}{2}\right) = \sin\left(90^0 - \frac{C}{2}\right) = \cos\frac{C}{2}$$

6. Prove that $\tan 20^0 . \tan 40^0 . \tan 50^0 . \tan 70^0 = 1$

Sol:- LHS $\tan 20^0 . \tan 40^0 . \tan 50^0 . \tan 70^0$

{Here 20^0 and 70^0, 40^0 and 50^0 are complementary pairs}

$= (\tan 20^0 . \tan 70^0).(\tan 40^0 . \tan 50^0)$

$= \{\cot(90^0 - 20^0) . \tan 70^0\}.\{\cot(90^0 - 40^0) . \tan 50^0\}$

$= (\cot 70^0 . \tan 70^0).(\cot 50^0 . \tan 50^0)$

$= \left(\frac{1}{\tan 70^0} . \tan 70^0\right).\left(\frac{1}{\tan 50^0} . \tan 50^0\right) = 1 \times 1 = 1 = RHS$

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EXERCISE

1. If $\sin A = \cos B$ then prove that $A + B = 90^0$
2. If $\sec P = \operatorname{cosec} Q$ then prove that $P + Q = 90^0$
3. If $\sin 3A = \cos (A - 10^0)$ then find A.
4. If $\sec 4\theta = \operatorname{cosec} (\theta - 40^0)$ then find θ.
5. If $\tan 2A = \cot (A + 15^0)$ then find A.
6. Prove that $\tan 23^0 . \tan 42^0 . \tan 48^0 . \tan 67^0 = 1$
7. In ΔABC, prove that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$
8. In ΔABC, prove that $\tan\left(\frac{A+C}{2}\right) = \cot\frac{B}{2}$