

Day – 9

ਅਸੀਂ ਜਾਣਦੇ ਹਾਂ ਕਿ $\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta - \tan^2\theta = 1$, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

- ਸਵਾਲਾਂ ਨੂੰ ਸੌਖਾ ਬਣਾਉਣ ਲਈ ਕੋਸ਼ਿਸ ਕੀਤੀ ਜਾਵੇ ਕਿ ਹਰੇਕ ਤਿਕੋਣਮਿਤਈ ਅਨੁਪਾਤ ਨੂੰ $\sin\theta$ ਜਾਂ $\cos\theta$ ਵਿੱਚ ਬਦਲਿਆ ਜਾਵੇ।

$$\text{ਭਾਵ } \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

1. ਸਿੱਧ ਕਰੋ ਕਿ $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

ਹੱਲ: : $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

($\sec A$ ਅਤੇ $\tan A$ ਨੂੰ $\sin A$ ਅਤੇ $\cos A$ ਵਿੱਚ ਬਦਲਿਆ ਜਾਵੇ)

$$\begin{aligned} &= \frac{1}{\cos A} (1 - \sin A) \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) \\ &= \left(\frac{1 - \sin A}{\cos A} \right) \left(\frac{1 + \sin A}{\cos A} \right) = \frac{1^2 - \sin^2 A}{\cos^2 A} = \frac{1 - \sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1 \\ &\quad \{1 - \sin^2 A = \cos^2 A\} \end{aligned}$$

2. ਸਿੱਧ ਕਰੋ ਕਿ $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

ਹੱਲ: ਖੱਬਾ ਪਾਸਾ : $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$

($\cot A$ ਨੂੰ $\sin A$ ਅਤੇ $\cos A$ ਵਿੱਚ ਬਦਲਿਆ ਜਾਵੇ)

$$= \frac{\cos A \left(\frac{1}{\sin A} - 1 \right)}{\cos A \left(\frac{1}{\sin A} + 1 \right)} = \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \text{ ਸੱਜਾ ਪਾਸਾ}$$

3. ਸਿੱਧ ਕਰੋ ਕਿ $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$

ਹੱਲ : ਖੱਬਾ ਪਾਸਾ : $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A} = \frac{\cos^2 A + (1 + \sin^2 A + 2 \sin A)}{(1 + \sin A) \cos A}$

$$\begin{aligned} &= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A) \cos A} = \frac{(\cos^2 A + \sin^2 A) + 1 + 2 \sin A}{(1 + \sin A) \cos A} \\ &= \frac{1 + 1 + 2 \sin A}{(1 + \sin A) \cos A} = \frac{2(1 + \sin A)}{(1 + \sin A) \cos A} \\ &= \frac{2}{\cos A} = 2 \sec A \end{aligned}$$

4. ਸਿੱਧ ਕਰੋ ਕਿ $\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = (\operatorname{cosec}\theta - \cot\theta)$ ਜਾਂ $\frac{1 - \cos\theta}{1 + \cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$

ਹੱਲ: ਸੱਜਾ ਪਾਸਾ : $(\operatorname{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \right)^2$

$$= \left(\frac{1-\cos\theta}{\sin\theta} \right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{ਖੱਬਾ ਪਾਸਾ}$$

ਜਾਂ ਖੱਬਾ ਪਾਸਾ : $\frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{\sin^2\theta} = \left(\frac{1-\cos\theta}{\sin\theta} \right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \right)^2 (\operatorname{cosec}\theta - \cot\theta)^2$$

ਅਭਿਆਸ

ਸਿੱਧ ਕਰੋ :

1. $(\sec A + \tan A)(1 - \sin A) = \cos A$

2. $\frac{1+\tan^2 A}{1+\cot^2 A} = \tan^2 A$

3. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

come-become-educated

37bhyas