DAY 10

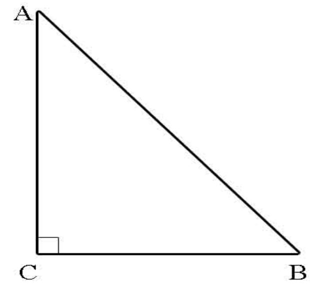
1. ABC is an isosceles right angled at C. Prove that $AB^2 = 2AC^2$. **[Ex 6.5, Q4]**

Sol:- In ΔABC, $AC = BC$ … … … … … … … … i)

By Pythagoras Theorem, we get

$$AB^2 = AC^2 + BC^2 = AC^2 + AC^2 \quad \text{[By i)]}$$

$$\Rightarrow AB^2 = 2AC^2.$$



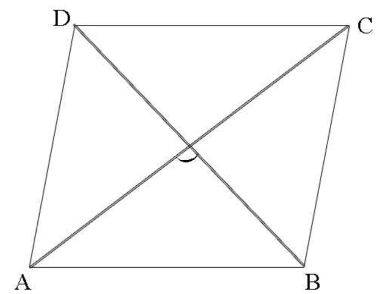
2. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals. **[Ex 6.5, Q7]**

Sol:- Given ABCD is a rhombus where $AB = BC = CD = DA$ … … … . i)

and diagonals AC and BD bisect at right angles at O

To prove: $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Proof : - Now In ΔOAB, we've



$$AB^2 = OA^2 + OB^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$\left\{\begin{array}{l} \text{As } OA = OC \text{ So } AC = 2OA \\ \text{and } OB = OD \text{ So } BD = 2OB \end{array}\right\} \qquad = \frac{AC^2}{4} + \frac{BD^2}{4}$$

$$\Rightarrow \quad 4AB^2 = AC^2 + BD^2$$

$$\text{or } AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2 \quad \text{[by i)]}$$

3. D and E are points on the sides CA and CB respectively of ΔABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$. **[Ex 6.5, Q13]**

Sol:- {In this sum, take right triangles according to what to prove *e. g.* **for AE^2 take ΔACE, for BD^2 take ΔBDC, for AB^2 take ΔABC, for DE^2 take ΔCDE}**

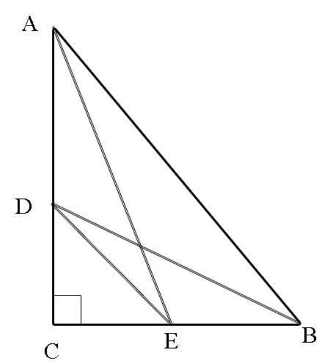
Now In right ΔACE, $AE^2 = AC^2 + CE^2$ … … … … … … … … . . i)

In right ΔBDC, $BD^2 = BC^2 + CD^2$ … … … … ii)

Adding i) and ii), we get

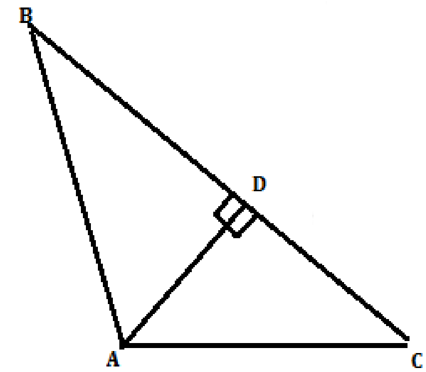
$$AE^2 + BD^2 = (AC^2 + CE^2) + (BC^2 + CD^2)$$
$$= (AC^2 + BC^2) + (CE^2 + CD^2)$$
$$= AB^2 + CD^2$$

{As In right ΔABC, $AB^2 = AC^2 + BC^2$ and In right ΔDCE, $DE^2 = CE^2 + CD^2$}



Hence the result

4. In the figure, if AD ⊥ BC, prove that $\mathbf{AB^2 + CD^2 = BD^2 + AC^2}$ **[Example 12]**



Sol:- In right ΔADC, we have

$$AC^2 = AD^2 + CD^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. i)$$

In right ΔADB, we have

$$AB^2 = AD^2 + BD^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. ii)$$

Subtracting i) from ii),we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

or $\mathbf{AB^2 + CD^2 = BD^2 + AC^2}$

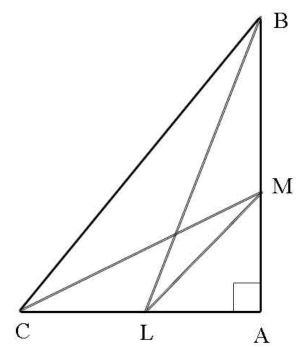
5. BL and CM are medians of a ΔABC right angled at A. Prove that $\mathbf{4(BL^2 + CM^2) = 5BC^2}$

[Example 13]

Sol:- In ΔABC, $\angle A = 90^0$ and BL, CM are medians, so $AM = BM = \frac{1}{2}AB, AL = LC = \frac{1}{2}AC \ldots\ldots i)$

Now In ΔABL, we have $BL^2 = AB^2 + AL^2 \ldots\ldots\ldots\ldots. ii)$

In ΔAMC, we have $CM^2 = AM^2 + AC^2 \ldots\ldots\ldots\ldots. iii)$



Adding ii) and iii), we get

$$BL^2 + CM^2 = AB^2 + AL^2 + AM^2 + AC^2$$

$$= AB^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{AB}{2}\right)^2 + AC^2 \quad \{\text{by i)}\}$$

$$= AB^2 + \frac{AC^2}{4} + \frac{AB^2}{4} + AC^2 = \frac{4AB^2 + AC^2 + AB^2 + 4AC^2}{4}$$

$$= \frac{5AB^2 + 5AC^2}{4} = \frac{5(AB^2 + AC^2)}{4} = \frac{5}{4}BC^2$$

$$\Rightarrow \quad \mathbf{4(BL^2 + CM^2) = 5BC^2}$$