CHAPTER-3
Linear Equations in Two Variables

DAY 1

INTRODUCTION

We know that a linear equation in one variable is of the form is $ax + b = 0$, a, b are real numbers, $a \neq 0$

Also $ax + b = 0 \quad \Rightarrow \quad x = \frac{-b}{a}$

Thus $\frac{-b}{a}$ is a solution of equation $ax + b = 0$

In 9th class, we have studied about an equation of the type $ax + by + c = 0$ is a linear equation in two variables x & y where a, b & c are real numbers such that $\boldsymbol{a^2 + b^2 \neq 0}$

$e.g.\ 2x - 3y = 5,\ x + 2y + 3 = 0$

Pair of Linear Equations in two variables :-

In this chapter, we shall discuss a pair of linear equations in two variables the general form of a pair of linear equations in two variables x & y is $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$

Where a_1, b_1, c_1, a_2, b_2 & c_2 are real numbers & ${a_1}^2 + {b_1}^2 \neq 0$, ${a_2}^2 + {b_2}^2 \neq 0$

Solution of a pair of linear Equations in two variables:-

A pair of linear equations have two types of solutions.

i) ***Consistent system :-*** A system consisting of a pair of linear equations in two variables is said to be consistent if it has atleast one solution.

- Pair of linear equation have infinitely many solutions it also called dependent system.

ii) ***Inconsistent System :-*** A system consisting of a pair of linear equations in two variables is said to be inconsistent if it has no solution.

A pair of linear equations in two variables can be represented a solved by two methods.

i) Graphical method ii) Algebraic method

Graphical method for solving a pair of linear equations in two variables:-

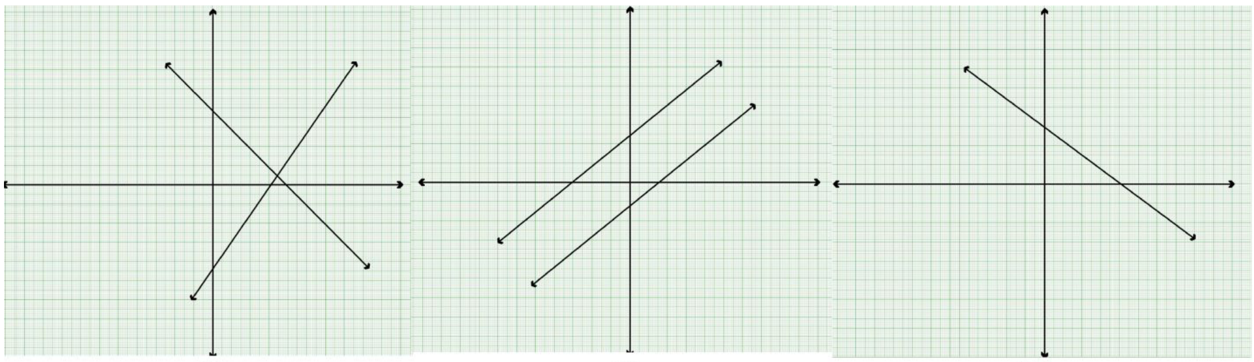
In 9th class, we learnt to draw the graph of linear equation in two variables. We know that a linear equation has infinitely many solutions.

Now we shall discuss about solutions of pair of linear equations in two variables. Here we draw the graphs of two equations in x & y on the same graph it will be in the form of straight line. We know that when two lines are drawn on a surface then the following possibilities are there:

- Both lines intersect at one point.
- Both lines will be parallel.
- Both lines are coincident lines.

Two linear equations have common solution means they have common intersection point on the graph.

- If the two lines intersect at one point then the equations are **consistent and unique** solution given by co-ordinates of point of intersection.
- If lines are parallel then the equations are **inconsistent** & have **no common solution**.
- If the lines are coincident, the equations are **consistent** and have **infinitely many solutions.**



(i) (ii) (iii)

Steps to Draw the Graph of a linear equation $ax + by + c = 0$ where a, b, c are real numbers.

1. Let the given equation be $ax + by + c = 0$
2. Either Find value of x or y : $y = \frac{-(ax+c)}{b}$ or $x = \frac{-(by+c)}{a}$
3. Take any convenient value of x & Find the corresponding value of y
   Suppose that value be $x = x_1, y = y_1$
4. Give another convenient value of x & find corresponding value of y.Suppose that values be $x = x_2, y = y_2$
5. Put these values in a table

X	x_1	x_2
Y	y_1	y_2

Plot the points (x_1, y_1) and (x_2, y_2) on the graph & join both points & extend in both directions.

Same procedure use to other equation & draw the graph of this equation on same one then check which type of result (as discussed above) you will get.

SOLVED EXAMPLES

1. Find the solution by graphs of the equations $x - 2y = 0$ and $3x + 4y = 20$.

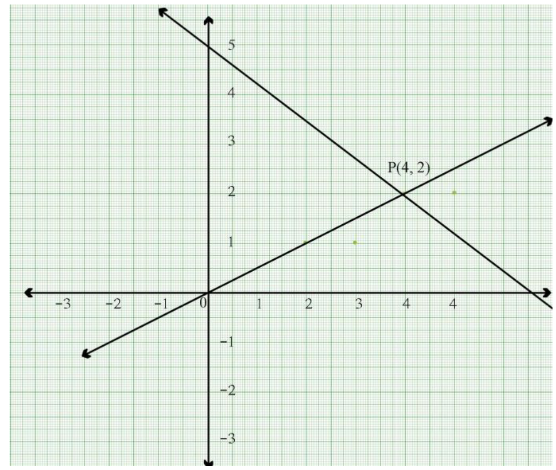
Sol :- To draw the graphs of given equations, we need minimum two points of each equation.

First equation is $x - 2y = 0$

If $x = 0$ then $0 - 2y = 0 \quad \Rightarrow -2y = 0 \Rightarrow y = \frac{0}{-2} = 0$

If $y = 1$ then $x - 2(1) = 0 \quad \Rightarrow x - 2 = 0 \Rightarrow x = 2$

x	0	2
y	0	1



Second equation is $3x + 4y = 20$

If $x = 0$ then $3(0) + 4y = 20$

$\Rightarrow 4y = 20 \Rightarrow y = \frac{20}{4} = 5$

If $y = 2$ then $3x + 4(2) = 20$

$\Rightarrow 3x + 8 = 20 \Rightarrow 3x = 20 - 8 = 12$

$\Rightarrow x = \frac{12}{3} = 4$

x	0	4
y	5	2

Plot-these points and Join them.

Now it is observed that Both lines Intersect at (4,2)

So $\boldsymbol{x = 4, y = 2}$ is the required solution

2. Find the solution by graphs of the equations $\boldsymbol{5x - y = 2}$ and $\boldsymbol{x - y = -2}$.

Sol :- To draw the graphs of given equations, we need minimum two points of each equation.

First equation is $5x - y = 2$

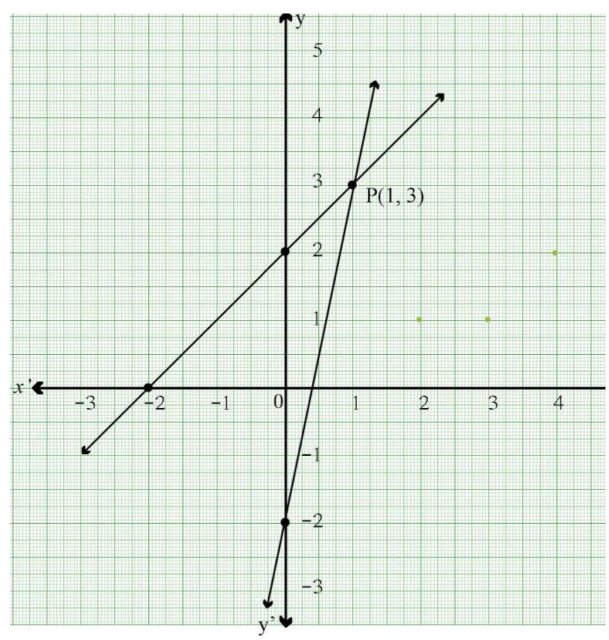
If $x = 0$ then $5(0) - y = 2$

$\Rightarrow 0 - y = 2 \Rightarrow y = -2$

If $x = 1$ then $5(1) - y = 2$

$\Rightarrow 5 - y = 2 \Rightarrow y = 5 - 2 = 3$

x	0	1
y	-2	3



Second equation is $x - y = -2$

If $x = 0$ then $0 - y = -2$

$\Rightarrow -y = -2 \Rightarrow y = 2$

If $y = 0$ then $x - 0 = -2 \quad \Rightarrow x = -2$

x	0	-2
y	2	0

Plot-these points and Join them.

Now it is observed that both lines Intersect at (1,3)

So $\boldsymbol{x = 1, y = 3}$ is the required solution

3. Find the solution by graphs of the equations $2x + 3y = 6$ and $4x + 6y = 24$.

Sol :- To draw the graphs of given equations, we need minimum two points of each equation.

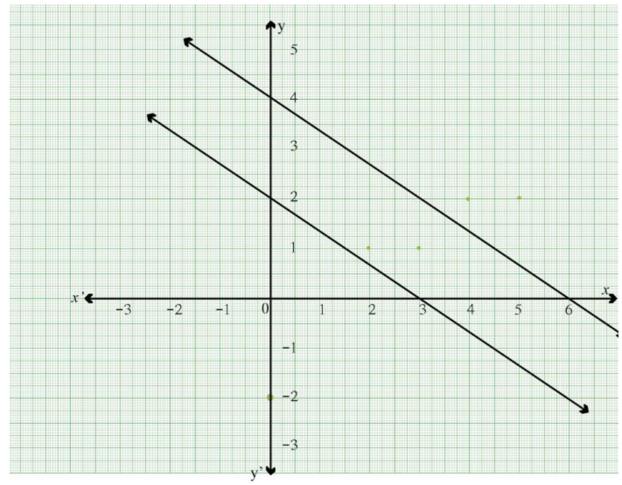
First equation is $2x + 3y = 6$

If $x = 0$ then $2(0) + 3y = 6$

$\Rightarrow 0 + 3y = 6 \Rightarrow 3y = 6 \Rightarrow y = \frac{6}{3} = 2$

If $y = 0$ then $2x + 3(0) = 6$

$\Rightarrow 2x + 0 = 6 \Rightarrow 2x = 6 \Rightarrow x = \frac{6}{2} = 3$



x	0	3
y	2	0

Second equation is $4x + 6y = 24$

If $x = 0$ then $4(0) + 6y = 24$

$\Rightarrow 0 + 6y = 24 \Rightarrow 6y = 24$

$\Rightarrow y = \frac{24}{6} = 4$

If $y = 0$ then $4x + 6(0) = 24 \quad \Rightarrow 4x + 0 = 24 \Rightarrow 4x = 24 \Rightarrow x = \frac{24}{4} = 6$

x	0	6
y	4	0

Plot-these points and Join them.

Now it is observed that both lines are **parallel**, So this system has **no solution**.

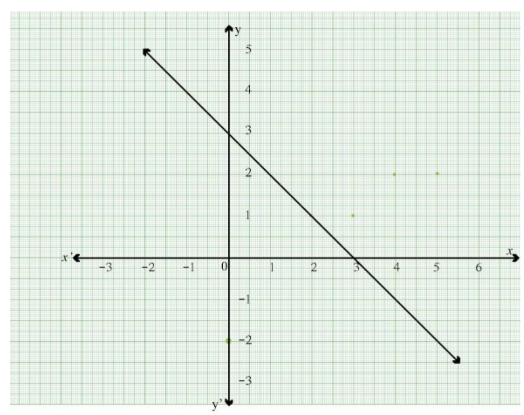
4. Find the solution by graphs of the equations $x + y = 3$ and $4x + 4y = 12$.

Sol :- To draw the graphs of given equations, we need minimum two points of each equation.

First equation is $x + y = 3$

If $x = 0$ then $0 + y = 3 \quad \Rightarrow y = 3$

If $y = 0$ then $x + 0 = 3 \quad \Rightarrow x = 3$



x	0	3
y	3	0

Second equation is $4x + 4y = 12$

If $x = 0$ then $4(0) + 4y = 12$

$\Rightarrow 0 + 4y = 12 \Rightarrow 4y = 12 \Rightarrow y = \frac{12}{4} = 3$

If $y = 0$ then $4x + 4(0) = 12$

$\Rightarrow 4x + 0 = 12 \Rightarrow 4x = 12 \Rightarrow x = \frac{12}{4} = 3$

x	0	3
y	3	0

Plot-these points and Join them. Now it is observed that both lines are on one another $i.e.$ both are **coincident**. So this system has **infinitely many solutions**.

EXERCISE

Solve the following equations by graphical method:

1. $x + y = 3$ and $2x + 5y = 12$
2. $2x + y = 4$ and $x + y = 2$
3. $x + 3y = 6$ and $2x - 3y = 12$
4. $2x - y = 2$ and $4x - y = 4$
5. $x + y = 5$ and $2x + 2y = 10$
6. $x - y = 4$ and $3x - 3y = 8$
7. Exercise 3.2, Q no. 4,7