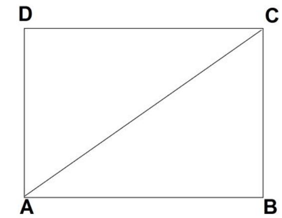
## DAY 7

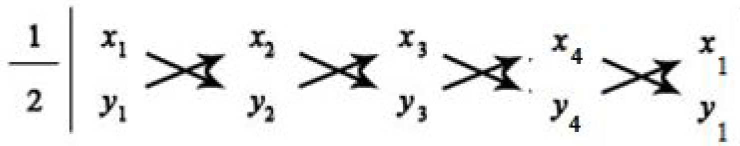
In this section, we shall discuss Area of quadrilateral when its vertices are given.

To find the area of quadrilateral, we have two options:




- Join any diagonal. Now quadrilateral divided into two triangles and find areas of both triangles and by adding, we get the area of the quadrilateral.

- With **Arrow Method** we can find directly area of quadrilateral.

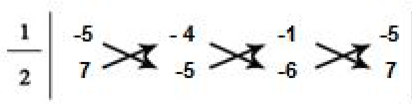


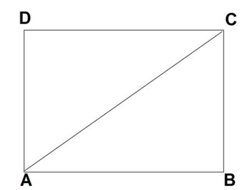
1. **If $\mathbf{A(-5,7), B(-4,-5), C(-1,-6)}$ and $\mathbf{D(4,5)}$ are the vertices of a quadrilateral, find the area of the quadrilateral.** **[Example 15]**

**Sol:-** Join diagonal AC**,** now quadrilateral ABCD divides into $\Delta$ABC and $\Delta$ADC

$ar(\Delta\text{ABC}) =$



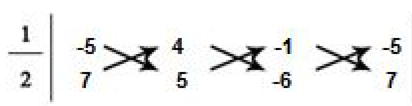





$$= \frac{1}{2}\begin{bmatrix} \{(-5)\times(-5)+(-4)\times(-6)+(-1)\times 7\} \\ -\{7\times(-4)+(-5)\times(-1)+(-6)\times(-5)\} \end{bmatrix}$$

$$= \frac{1}{2}[(25+24-7)-(-28+5+30)] = \frac{1}{2}[42-7] = \frac{35}{2} = 17.5$$

$ar(\Delta\text{ADC}) =$



$$= \frac{1}{2}\begin{bmatrix} \{(-5)\times(-6)+(-1)\times 5+4\times 7\} \\ -\{7\times(-1)+(-6)\times 4+5\times(-5)\} \end{bmatrix}$$

$$= \frac{1}{2}[(30-5+28)-(-7-24-25)] = \frac{1}{2}[53-(-56)]$$

$$= \frac{1}{2}[53+56] = \frac{109}{2} = 54.5$$

$$\therefore \boldsymbol{ar(\text{ABCD}) = ar(\Delta\text{ABC}) + ar(\Delta\text{ADC}) = 17.5 + 54.5 = 72 sq.units}$$

**ALTER METHOD**

$ar(\text{ABCD}) =$

$$\frac{1}{2}\left|\begin{array}{ccccc} -5 & -4 & -1 & 4 & -5 \\ 7 & -5 & -6 & 5 & 7 \end{array}\right|$$

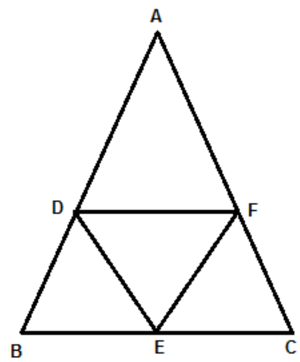
$$= \frac{1}{2}\begin{bmatrix} \{(-5)\times(-5)+(-4)\times(-6)+(-1)\times 5+4\times 7\} \\ -\{7\times(-4)+(-5)\times(-1)+(-6)\times 4+5\times(-5)\} \end{bmatrix}$$

$$= \frac{1}{2}[(25+24-5+28)-(-28+5-24-25)] = \frac{1}{2}[72-(-72)]$$

$$= \frac{1}{2}[72+72] = \frac{144}{2} = 72\ sq.units$$

2. **Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are $(0,-1)$, $(2,1)$ and $(0,3)$. Find the ratio of this area to the area of the given triangle.** **[Ex 7.3, Q 3]**

**Sol:-** Let A$(0,-1)$, B$(2,1)$ and C$(0,3)$ are the vertices of $\Delta$ABC and D, E, F are mid points of sides AB,BC and AC respectively.




$\therefore$ Coordinates of D $= \left(\frac{0+2}{2},\frac{-1+1}{2}\right) = \left(\frac{2}{2},\frac{0}{2}\right) = (1,0)$

Coordinates of E $= \left(\frac{2+0}{2},\frac{1+3}{2}\right) = \left(\frac{2}{2},\frac{4}{2}\right) = (1,2)$

Coordinates of F $= \left(\frac{0+0}{2},\frac{-1+3}{2}\right) = \left(\frac{0}{2},\frac{2}{2}\right) = (0,1)$

**Now** $ar(\Delta\text{DEF}) =$

$$\frac{1}{2}\left|\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{array}\right|$$

$$= \frac{1}{2}\begin{bmatrix} \{1\times 2+1\times 1+0\times 0\} \\ -\{0\times 1+2\times 0+1\times 1\} \end{bmatrix} = \frac{1}{2}[(2+1+0)-(0+0+1)]$$

$$= \frac{1}{2}[3-1] = \frac{2}{2} = 1\ sq.units$$

$ar(\Delta\text{ABC}) =$

$$\frac{1}{2}\left|\begin{array}{cccc} 0 & 2 & 0 & 0 \\ -1 & 1 & 3 & -1 \end{array}\right|$$

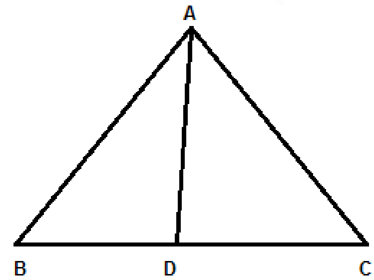
$$= \frac{1}{2}\begin{bmatrix} \{0\times 1+2\times 3+0\times(-1)\} \\ -\{(-1)\times 2+1\times 0+3\times 0\} \end{bmatrix} = \frac{1}{2}[(0+6+0)-(-2+0+0)]$$

$$= \frac{1}{2}[6-(-2)] = \frac{1}{2}[6+2] = \frac{8}{2} = 4\ sq.units$$

$\therefore \boldsymbol{ar(\Delta\text{DEF}) : ar(\Delta\text{ABC}) = 1:4}$

3. **Prove that the median of the triangle with vertices A$(4,-6)$, B$(3,-2)$ and C$(5,2)$ divides in into two triangles of equal areas.** **[Ex 7.3, Q 5]**

**Sol:-** Let AD is the median of ΔABC, so D is the mid point of BC.




Coordinates of D $= \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = \left(\frac{8}{2}, \frac{0}{2}\right) = (4,0)$

**Now** $ar(\Delta\text{ABD}) =$

$$\frac{1}{2}\left| \begin{array}{cccc} 4 & 3 & 4 & 4 \\ -6 & -2 & 0 & -6 \end{array} \right|$$

$$= \frac{1}{2}\begin{bmatrix} \{4 \times (-2) + 3 \times 0 + 4 \times (-6)\} \\ -\{(-6) \times 3 + (-2) \times 4 + 0 \times 4\} \end{bmatrix} = \frac{1}{2}[(-8 + 0 - 24) - (\text{—}18 - 8 + 0)]$$

$$= \frac{1}{2}[(-32) - (-26)] = \frac{1}{2}[-32 + 26] = \frac{-6}{2} = -3$$

But area is always positive, so $ar(\Delta\text{ABD}) = 3\ sq.units$

$ar(\Delta\text{ACD}) =$

$$\frac{1}{2}\left| \begin{array}{cccc} 4 & 5 & 4 & 4 \\ -6 & 2 & 0 & -6 \end{array} \right|$$



$$= \frac{1}{2}\begin{bmatrix} \{4 \times 2 + 5 \times 0 + 4 \times (-6)\} \\ -\{(-6) \times 5 + 2 \times 4 + 0 \times 4\} \end{bmatrix} = \frac{1}{2}[(8 + 0 - 24)\text{—}(-30 + 8 + 0)]$$

$$= \frac{1}{2}[(-16) - (-22)] = \frac{1}{2}[-16 + 22] = \frac{6}{2} = 3\ sq.units$$

$\therefore \boldsymbol{ar(\Delta\text{ABD}) = ar(\Delta\text{ACD}) = 3\ sq.units}$