DAY 7

APPLICATIONS OF SUM OF n TERMS of AP

1. **If the sum of first n terms of an A.P. is $4n - n^2$, what is the first term? What is the sum of first two terms? What is the second term? Find the 3rd , 10th and n^{th} terms.** **[Ex 5.2, Q 11]**

Sol :- Given $S_n = 4\boldsymbol{n} - \boldsymbol{n}^2$

Put $n = 1,2,3,4$ … …… we get

For $n = 1, S_1 = 4(\mathbf{1}) - (\mathbf{1})^2 = 4 - 1 = 3$

$n = 2, S_2 = 4(\mathbf{2}) - (\mathbf{2})^2 = 8 - 4 = 4$

Now First term $\boldsymbol{a = S_1 = 3}$

$\boldsymbol{S_2 = 4}$

$\boldsymbol{a_2} = S_2 - S_1 = 4 - 3 = \mathbf{1}$ (By $S_n = t_n - t_{n-1}$)

$\Rightarrow a + d = 1 \quad \Rightarrow 3 + d = 1$

$\Rightarrow d = 1 - 3 = \mathbf{-2}$

$\boldsymbol{a_3} = a + 2d = 3 + 2(-2) = 3 - 4 = -1$

$\boldsymbol{a_{10}} = a + 9d = 3 + 9(-2) = 3 - 18 = -15$

$\boldsymbol{a_n} = a + (n-1)d = 3 + (n-1)(-2) = 3 - 2n + 2 = 5 - 2n$

2. **How many terms the A.P. 24, 21, 18, ……must be taken so that sum is 78?** **[Example 13]**

Sol :- A.P. 24,21,18, …… with $a = 24, d = 21 - 24 = -3$ and $S_n = 78$

Here we have to find number of terms so Let the number of terms in A.P. be n

Now $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 78 = \frac{n}{2}\{2 \times 24 + (n-1)(-3)\}$

$\Rightarrow 78 \times 2 = n\{48 - 3n + 3\} \quad \Rightarrow 156 = n(51 - 3n)$

$\Rightarrow 156 = 51n - 3n^2$ or $n^2 - 17n + 52 = 0$ (Divide by -3)

$\Rightarrow n^2 - 4n - 13n + 52 = 0 \quad \Rightarrow n(n-4) - 13(n-4) = 0$

$\Rightarrow (n-4)(n-13) = 0 \quad \Rightarrow n - 4 = 0$ or $n - 13 = 0$

$\Rightarrow n = 4,13$

Both values are acceptable. So the numbers of terms is either 4 or 13.

3. **Find the sum of first 1000 positive numbers?** **[Example 14]**

Sol:- First 1000 positive integers are 1,2,3,4, … … …. . ,1000

Here $a = 1, d = 2 - 1 = 1$ and $n = 1000$

Now $S_n = \frac{n}{2}\{a + l\} = \frac{1000}{2}\{1 + 1000\} = 500 \times 1001 = 500500$

4. **Find the sum of first 15 multiples of 8?** **[Ex 5.3 Q 13]**

Sol :- **first 15 multiples of 8** $= 8, 16, 24, \ldots\ldots, 120$

Here $a = 8, d = 16 - 8 = 8, l = 120$ and $n = 15$

Now $S_n = \frac{n}{2}\{a + l\} = \frac{15}{2}(8 + 120) = \frac{15}{2} \times 128 = 15 \times 64 = 960$

5. **A manufacturer of radio sets, produced 600 units in the third year and 700 units in the 7th year. Assuming the production uniformly increases by a fixed number every year, find a) The production of the first year b)The total production in 7 years and c) The production in 10th year.** **[Example 16]**

Sol:- a) Given the production uniformly increases by a fixed number every year. So the production in different years forms an AP.

Given conditions:

Production in 3rd year $(a_3) = 600$ $\Rightarrow a + 2d = 600 \ldots\ldots\ldots$ i)

Production in 7th year $(a_7) = 700$ $\Rightarrow a + 6d = 700 \ldots\ldots\ldots$ ii)

Subtracting i) from ii), we get

$(a + 6d) - (a + 2d) = 700 - 600$ $\Rightarrow 4d = 100$

$\Rightarrow d = \frac{100}{4} = 25$ Put this value in i), we get

i) $\Rightarrow$ $a + 2(\mathbf{25}) = 600$ $\Rightarrow$ $a = 600 - 50 = 550$

b) The total production in 7 years $= S_7 = S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{7}{2}\{2 \times 550 + (7-1)50\} = \frac{7}{2}\{1100 + 6 \times 25\}$

$= \frac{7}{2} \times 1250 = 7 \times 625 = 4375$

c) The production in 10th year $= a_{10} = a + 9d$

$= 550 + 9 \times 25 = 550 + 225 = 775$

6. **A sum of ₹700 is to be used to give cash prizes to students of a school for their overall academic performance. If each prize is 20 less than its preceding prize, find the value of the prizes.**

Sol:- Given: Total prize money for 7 prizes = 700 $\Rightarrow S_7 = 700$

and **each prize is 20 less than its preceding prize so** $d = -20$

Now $\Rightarrow S_7 = 700$ $\Rightarrow \frac{7}{2}\{2a + 6d\} = 700$

$\Rightarrow \frac{7}{2} \times 2a + \frac{7}{2} \times 6d = 700$ $\Rightarrow 7a + 21d = 700$

$\Rightarrow 7a + 21(-20) = 700$ $\Rightarrow 7a = 700 + 42 = 1120$

$$\Rightarrow \ a = \frac{1120}{7} = 160$$

Hence value of each prize is 160, 140, 120, 100, 80, 60, 40

EXERCISE

1. **Ex 5.3, Q** 14,15,17,18,19,20