In last section, we have discussed about general term of AP, In this section we shall discuss about sum of n terms of AP.

Sum of first n terms of an A.P.

Let a be the first term and d be the common difference of an A.P. and S_n be the sum of n terms of an AP.

$$S_n = a_1 + a_2 + a_3 + \cdots \ldots \ldots \ldots \ldots + a_{n-1} + a_n$$

Rewrite it as

$$S_n = a_n + a_{n-1} + \cdots \ldots \ldots \ldots + a_3 + a_2 + a_1$$

Adding both term wise, we get

$$2S_n = (a_1 + a_n) + (a_2 + a_{n-1}) + \cdots \ldots \ldots \ldots \ldots + (a_{n-1} + a_2) + (a_n + a_1)$$

$\Rightarrow \quad 2S_n = [a + \{a + (n-1)d\}] + [\{a + d\} + \{a + (n-2)d\}] + \cdots \ldots \ldots \ldots \ldots \ldots ..$
$\qquad \qquad \ldots \ldots . + [(a + (n-2)d\} + \{a + d\}] + [\{a + (n-1)d\} + a]$

$\Rightarrow \quad 2S_n = [2a + (n-1)d] + [2a + (n-1)d] + \cdots \ldots \ldots \ldots \ldots \ldots ..$
$\qquad \qquad \ldots \ldots \ldots . . + [2a + (n-1)d] + [2a + (n-1)d]$

$\Rightarrow \quad 2S_n = \{2a + (n-1)d\}\{1 + 1 + 1 + \cdots \ldots \ldots . upto\ n\ terms\}$

$\Rightarrow \quad 2S_n = n\{2a + (n-1)d\}$

$\Rightarrow \quad \boldsymbol{S_n = \dfrac{n}{2}\{2a + (n-1)d\}}$

Or $\quad \boldsymbol{S_n} = \dfrac{n}{2}\{a + a + (n-1)d\} \quad$ or $\dfrac{n}{2}\{a + a_n\}$

$$\boldsymbol{S_n = \dfrac{n}{2}\{a + l\}} \quad \text{given by } l = a + (n-1)d$$

Remark :- In an A.P. $t_n = S_n - S_{n-1} \ldots \ldots \ldots \ldots \ldots$ i)

Consider $\quad S_n = 2 + 4 + 6 + \cdots \ldots + t_{n-1} + t_n$

Here $t_1 = 2, t_2 = 4, t_3 = 6, t_4 = 8$ and So-on .... ii)

From i) & ii) ; $\quad S_1 = 2 = t_1$; $\quad S_2 = 2 + 4 = 6$

$\quad S_3 = 2 + 4 + 6 = 12$; $\quad S_4 = 2 + 4 + 6 + 8 = 20$ and So – on

Thus $\qquad S_2 - S_1 = 6 - 2 = 4 = t_2$

$\qquad \qquad S_3 - S_2 = 12 - 6 = 6 = t_3$

$\qquad \qquad S_4 - S_3 = 20 - 12 = 8 = t_4$

And so on $\quad \boldsymbol{S_n - S_{n-1} = t_n}$

This formula is used when sum is given in terms of n & we have to find A.P.

1. **Find the sum of first 15 terms of the AP: $5, 9, 13, \ldots \ldots \ldots$**

   Sol :- Given $a = 5, d = 9 - 5 = 4. n = 15$

   We know $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

$$S_{15} = \frac{15}{2}\{2 \times 5 + (15-1)4\} = \frac{15}{2}\{10 + 14 \times 4\}$$
$$= \frac{15}{2}\{10 + 56\} = \frac{15}{2} \times 66 = 15 \times 33 = 495$$

2. **Find the sum of first 24 terms of the AP: 2, 6, 10, … … …**

   Sol :- Given $a = 2, d = 6 - 2 = 4. n = 24$

   We know $S_n = \frac{n}{2}\{2a + (n-1)d\}$

   $$S_{24} = \frac{24}{2}\{2 \times 2 + (24-1)4\} = 12 \times \{4 + 23 \times 4\}$$
   $$= 12 \times \{4 + 92\} = 12 \times 96 = 1152$$

3. **Find the sum of first 22 terms of the AP: 8, 3, −2, … … …**   **[Example 11]**

   Sol :- Given $a = 8, d = 3 - 8 = -5, n = 22$

   We know $S_n = \frac{n}{2}\{2a + (n-1)d\}$

   $$S_{22} = \frac{22}{2}\{2 \times 8 + (22-1)(-5)\} = 11 \times \{16 - 21 \times 5\}$$
   $$= 11 \times \{16 - 105\} = 11 \times (-89) = -979$$

4. **Find the sum of AP $2 + 6 + 10 + \cdots …… + 62$?**

   Sol :- Here $a = 2, d = 6 - 2 = 4$ and last term $(l) = 62$

   Since last term$(l)$is considered as n^{th}term

   So $l = a_n = 62$

   $\Rightarrow a + (n-1)d = 62$ $\Rightarrow 2 + (n-1)4 = 62$

   $\Rightarrow 2 + 4n - 4 = 62$ $\Rightarrow 4n - 2 = 62$

   $\Rightarrow 4n = 62 + 2 = 64$ $\Rightarrow n = \frac{64}{4} = 16$

   So in given AP, there are 16 terms.

   Now $S_n = \frac{n}{2}\{2a + (n-1)d\}$

   $$S_{16} = \frac{16}{2}\{2 \times 2 + (16-1)4\} = 8 \times \{4 + 15 \times 4)\}$$
   $$= 8 \times \{4 + 60)\} = 8 \times 64 = 512$$

   ALTERNATE METHOD $S_n = \frac{n}{2}\{a + l\}$

   $$S_{16} = \frac{16}{2}\{2 + 62\} = 8 \times 64 = 512$$

5. **Find the sum of AP $34 + 32 + 30 + \cdots …… + 10$?**   **[Ex 5.2, Q2 ii)]**

   Sol :- Here $a = 34, d = 32 - 34 = -2$ and last term $(l) = 10$

Since last term (l) is considered as n^{th} term

So $l = a_n = 10$

$\Rightarrow a + (n - 1)d = 10$ $\qquad \Rightarrow 34 + (n - 1)(-2) = 10$

$\Rightarrow 34 - 2n + 2 = 10$ $\qquad \Rightarrow -2n + 36 = 10$

$\Rightarrow -2n = 10 - 36 = -26$ $\qquad \Rightarrow n = \dfrac{-26}{-2} = 13$

So in given AP, there are 13 terms.

Now $S_n = \dfrac{n}{2}\{2a + (n - 1)d\}$

$$S_{13} = \frac{13}{2}\{2 \times 34 + (13 - 1)(-2)\} = \frac{13}{2}\{68 + (12)(-2)\}$$

$$= \frac{13}{2}\{68 - 24)\} = \frac{13}{2} \times 44 = 13 \times 22 = 286$$

ALTERNATE METHOD $\quad S_n = \dfrac{n}{2}\{a + l\}$

$$S_{13} = \frac{13}{2}\{34 + 10\} = \frac{13}{2} \times 44 = 13 \times 22 = 286$$

EXERCISE

1. Find the sum of first 20 terms of the AP: 7,11,15, … … …
2. Find the sum of first 15 terms of the AP: 1,5,9,13, … … …
3. Find the sum of AP $10 + 20 + 30 + \cdots \ldots +200$?
4. Find the sum of AP $6 + 12 + 18 + \cdots \ldots +96$?
5. Ex 5.3, Q 1,2