DAY 2

Roots Or (Solutions) of Quadratic Equation :-

As in polynomials, we've discussed about zeros of polynomial. i.e. There are atmost two zeros of a quadratic polynomial $p(x)$.

If $p(x) = 0$ then it is a quadratic equation and the zeros of quadratic polynomial $p(x)$ are called the roots (solutions) of quadratic equation $p(x) = 0$

If α is the root of quadratic equation (solution) $p(x) = ax^2 + bx + c = 0$

$i.e.$ **That values of x which make polynomial zero are called roots or solutions of polynomial.**

SOLVING A QUADRATIC EQUATION :-

There are three methods for solving a quadratic equation $ax^2 + bx + c = 0; a \neq 0$

- Factorisation with splitting the middle term
- Completing the perfect square
- Quadratic formula

Factorisation method:-

In earlier classes, we have already done factorisation of quadratic polynomial $ax^2 + bx + c$ with *splitting the middle term* in two parts. Now in this section using that factorisation, we shall find the possible solutions of quadratic equations.

Before that, we shall know about use of property of real numbers called ***zero – product rule***,

Zero Product rule

If multiply of two numbers is 0 then out of that numbers atleast one number must be 0 $\boldsymbol{i.e.}$ **If $\boldsymbol{ab = 0}$ then either $\boldsymbol{a = 0}$ or $\boldsymbol{b = 0}$**

Similarly If $\boldsymbol{(x-a)(x-b) = 0}$ then either $\boldsymbol{x - a = 0}$ or $\boldsymbol{x - b = 0}$

$$\Rightarrow \boldsymbol{x = a} \text{ or } \boldsymbol{x = b}$$

- Sometimes we get the equations of type $ax^2 + bx = 0$ then How to solve such equations $\boldsymbol{ax^2 + bx = 0} \Rightarrow x(ax + b) = 0$

  Either $x = 0$ or $ax + b = 0 \Rightarrow x = \frac{-b}{a}$
- Sometimes we get the equations of type $ax^2 - c = 0$ then How to solve such equations $\boldsymbol{ax^2 - c = 0} \Rightarrow x^2 = \frac{c}{a} \Rightarrow x = \pm\sqrt{\frac{c}{a}}$

Now we shall discuss some examples:

1. **Solve the following quadratic equations by factorisation method:**

   i) $\boldsymbol{2x^2 - 4x = 0}$ **ii)** $\boldsymbol{3x^2 - 9x = 0}$ **iii)** $\boldsymbol{x^2 - 9 = 0}$

   iv) $\boldsymbol{x^2 - 3 = 0}$ **v)** $\boldsymbol{2x^2 - 5x + 3 = 0}$ **vi)** $\boldsymbol{6x^2 - x - 2 = 0}$

   vii) $\boldsymbol{4x^2 - 25 = 0}$ **viii)** $\boldsymbol{3x^2 - 2\sqrt{6}x + 2 = 0}$

Sol : -

i) Given equation is $2x^2 - 4x = 0$

$\Rightarrow \quad 2x(x-2)=0$

$\Rightarrow$ Either $2x=0$ or $x-2=0$

$\Rightarrow x=\frac{0}{2}=\mathbf{0}$ or $x=\mathbf{2}$ are roots of equation $2x^2+4x=0$

ii) Given equation is $3x^2-9x=0 \quad \Rightarrow 3x(x-3)=0$

Either $3x=0$ or $x-3=0$

$\Rightarrow x=\frac{0}{3}=\mathbf{0}$ or $x=\mathbf{3}$ are two roots of equation $3x^2-9x=0$

iii) Given equation is $x^2-9=0$

$\Rightarrow x^2-3^2=0 \quad \Rightarrow (x-3)(x+3)=0 \quad \{a^2-b^2=(a-b)(a+b)\}$

$\Rightarrow$ Either $x-3=0$ or $x+3=0$

$\Rightarrow x=\mathbf{3}$ or $x=\mathbf{-3}$ are two roots of equation $x^2-9=0$

iv) Given equation is $x^2-3=0$

$\Rightarrow x^2-\left(\sqrt{3}\right)^2=0 \Rightarrow (x-\sqrt{3})(x+\sqrt{3})=0 \quad \{a^2-b^2=(a-b)(a+b)\}$

$\Rightarrow$ Either $x-\sqrt{3}=0$ or $x+\sqrt{3}=0$

$\Rightarrow x=\mathbf{\sqrt{3}}$ or $x=\mathbf{-\sqrt{3}}$ are two roots of equation $x^2-3=0$

v) Given equation is $2x^2-5x+3=0$

$\Rightarrow 2x^2-2x-3x+3=0 \quad \Rightarrow 2x(x-1)-3(x-1)=0$

$\Rightarrow (x-1)(2x-3)=0$

$\Rightarrow$ Either $x-1=0$ or $2x-3=0$

$\Rightarrow x=\mathbf{1}$ or $x=\mathbf{\frac{3}{2}}$ are two roots of equation $2x^2-5x+3=0$

vi) Given equation is $6x^2-x-2=0$

$\Rightarrow 6x^2-4x+3x-2=0 \quad \Rightarrow 2x(3x-2)+1(3x-2)=0$

$\Rightarrow (3x-2)(2x+1)=0$

$\Rightarrow$ Either $3x-2=0$ or $2x+1=0$

$\Rightarrow x=\mathbf{\frac{2}{3}}$ or $x=\mathbf{\frac{-1}{2}}$ are two roots of equation $6x^2-x-2=0$

vii) Given equation is $4x^2-25=0$

$\Rightarrow (2x)^2-5^2=0 \Rightarrow (2x-5)(2x+5)=0 \quad \{a^2-b^2=(a-b)(a+b)\}$

$\Rightarrow$ Either $2x-5=0$ or $2x+5=0$

$\Rightarrow x=\mathbf{\frac{5}{2}}$ or $x=\mathbf{-\frac{5}{2}}$ are two roots of equation $4x^2-25=0$

viii) Given equation is $3x^2-2\sqrt{6}x+2=0$

$\Rightarrow 3x^2-\sqrt{6}x-\sqrt{6}x+2=0$

$\Rightarrow (\sqrt{3}x)^2-\sqrt{3}\times\sqrt{2}x-\sqrt{3}\times\sqrt{2}x+(\sqrt{2})^2=0$

$\Rightarrow \sqrt{3}x(\sqrt{3}x-\sqrt{2})-\sqrt{2}(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow (\sqrt{3}x-\sqrt{2})(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow$ Either $\sqrt{3}x-\sqrt{2}=0$ or $\sqrt{3}x-\sqrt{2}=0$

$\Rightarrow x=\mathbf{\frac{\sqrt{2}}{\sqrt{3}}}$ or $x=\mathbf{\frac{\sqrt{2}}{\sqrt{3}}}$ are two roots of equation $3x^2-2\sqrt{6}x+2=0$

EXERCISE

Solve the following quadratic equations by factorisation:

1. $x^2 - 3x - 10 = 0$

2. $2x^2 + x - 6 = 0$

3. $2x^2 - x + \frac{1}{8} = 0$

4. $100x^2 - 20x + 1 = 0$

5. $2x^2 - 7x + 3 = 0$

6. $4x^2 + 8x = 0$

7. $7x^2 - 21x = 0$

8. $16x^2 - 9 = 0$

9. $x^2 - 4 = 0$

10. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$