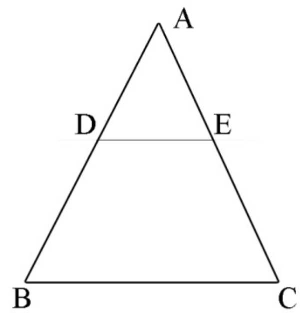
DAY 3

In last section we have discussed about Thales Theorem and its applications, In this section we shall discuss about converse of Thales theorem, as its proof is not in our syllabus so we shall focus on its applications.

CONVERSE OF BASIC PROPORTIONALITY THEOREM

Statement :- If a line divides any two sides of a triangles in the same ratio, then the line must be parallel to the third side

In ΔABC and a line DE intersecting AB in D and AC in E, such that $\frac{AD}{DB} = \frac{AE}{EC}$ then DE||BC



1. In ΔABC, X and Y are any points on sides AB and AC respectively then prove that DE||BC

i) $\mathbf{AX = 6, XB = 5, AY = 12, YC = 10}$

ii) $\mathbf{AX = 1.2, AB = 3, AY = 6, AC = 14}$

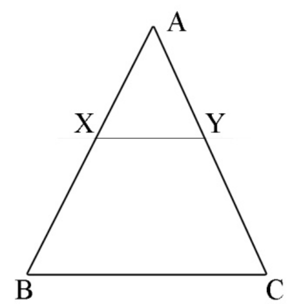
iii) $\mathbf{AX = 2, XB = 1.3, AY = 6, YC = 3.9}$

Sol:- i) In ΔABC

$$\frac{AX}{XB} = \frac{6}{5} \text{ and } \frac{AY}{YC} = \frac{12}{10} = \frac{6}{5}$$

$$\Rightarrow \frac{AX}{XB} = \frac{AY}{YC}$$

$\therefore$ Converse of Thales Theorem, XY||BC



ii) In ΔABC

$$\frac{AX}{AB} = \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5} \text{ and } \frac{AY}{YC} = \frac{6}{14} = \frac{3}{7}$$

$$\Rightarrow \frac{AX}{AB} \neq \frac{AY}{AC}$$

$\therefore$ XY is not parallel to BC.

iii) In ΔABC

$$\frac{AX}{XB} = \frac{2}{1.3} = \frac{20}{13} \text{ and } \frac{AY}{YC} = \frac{6}{3.9} = \frac{60}{39} = \frac{20}{13}$$

$$\Rightarrow \frac{AX}{XB} = \frac{AY}{YC}$$

$\therefore$ Converse of Thales Theorem, XY||BC

2. In given figure, DE||AQ and DF||AR. Prove that EF||QR. **[Ex 6.2. Q5]**

Sol :- Given DE||AQ and DF||AR

In ΔPOQ, DE||OQ

By Thales Theorem $\frac{PE}{EQ} = \frac{\mathbf{PD}}{\mathbf{DO}}$ ………. i)

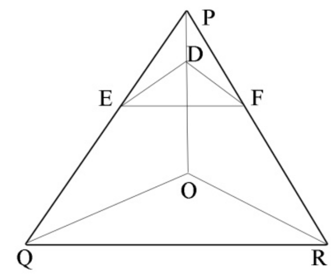
In ΔPOR, $DF || OR$

By Thales Theorem $\frac{PD}{DO} = \frac{PF}{FR}$ ……… ii)

From i) & ii), we get $\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore$ Converse of Thales Theorem in ΔPQR, $EF || QR$

Hence the result



3. The diagonals AC & BD of a quadrilateral ABCD intersect each other at O such that $\frac{AO}{OB} = \frac{CO}{OD}$, Prove that ABCD is a trapezium [Ex 6.3,Q10]

Sol :- Given $\frac{AO}{OB} = \frac{CO}{OD}$ or $\frac{AO}{OC} = \frac{BO}{OD}$ ……….. i)

To Prove: ABCD is a trapezium i. e. $AB || CD$

Construction Draw $OE || AB$

Proof :- In ΔABC, $OE || AB$

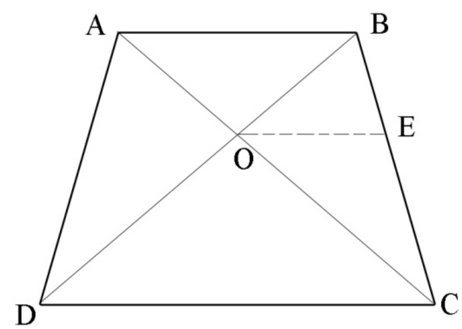
By Thales Theorem $\frac{AO}{OC} = \frac{BE}{EC}$ ……..ii)

From i) & ii), we get $\frac{BO}{OD} = \frac{BE}{EC}$

By converse of Thales Theorem In ΔBCD, $OE || CD$

But $OE || AB$ (Construction)

$\Rightarrow$ **$AB || CD$ Hence ABCD is a trapezium.**



4. Prove that the line joining the mid points of any two sides of a triangle is parallel to the third side. [Ex 6.3,Q 8]

Sol : -Given: In ΔABC, D and E are the mid-points of AB and AC respectively.

To prove $DE || BC$

Proof :- Since D is the mid-point of AB

$\therefore$ $AD = DB$ $\Rightarrow$ $\frac{AD}{DB} = 1$…………… i)

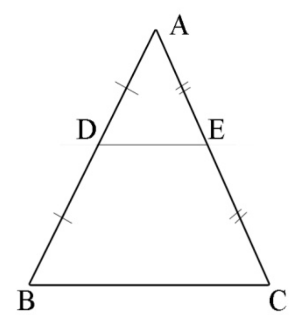
and E is the mid-point of AC

$\therefore$ AE = EC $\Rightarrow$ $\frac{AE}{EC} = 1$………ii)

From i) & ii), we' ve

$\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore$ By converse of Thales Theorem, **$DE || BC$**



5. In fig, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that ΔPQR is an isosceles triangle.

[Example 3]

Sol: - Given $\frac{PS}{SQ} = \frac{PT}{TR}$

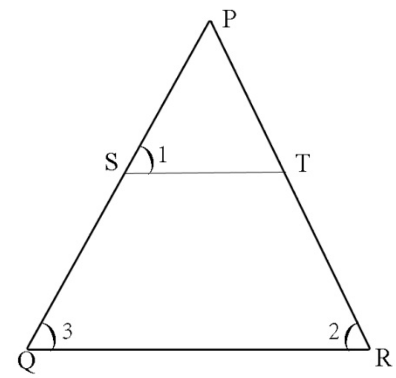
$\therefore$ By converse of Thales Theorem, ST || QR

$\Rightarrow \angle \mathbf{1} = \angle 3$ (Corresponding angles)

Given $\angle PST = \angle PRQ$ i.e. $\angle \mathbf{1} = \angle 2$

$\Rightarrow \angle 2 = \angle 3$

Hence ΔPQR is an isosceles triangle.



EXERCISE

1. Ex 6.3, Q2,6