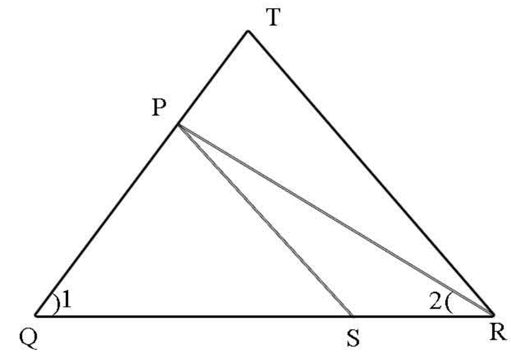
DAY 6

1. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$, show that $\Delta PQS \sim \Delta TQR$ **[Ex 6.3, Q4]**



Sol:- Given: $\frac{QR}{QS} = \frac{QT}{\mathbf{PR}} \ldots\ldots\ldots\ldots .\text{i)}$

and $\angle 1 = \angle 2 \quad \Rightarrow PR = PQ$ Put in i)

$\text{i)} \Rightarrow \frac{QR}{QS} = \frac{QT}{\mathbf{PQ}}$

In ΔTQR and ΔPQS, we've

$\left\{\begin{array}{l}\text{Here we have taken } \Delta TQR \text{ in first place} \\ \text{as its sides } \frac{\mathbf{QR}}{QS} = \frac{\mathbf{QT}}{PQ} \text{ are in numerator}\end{array}\right\}$

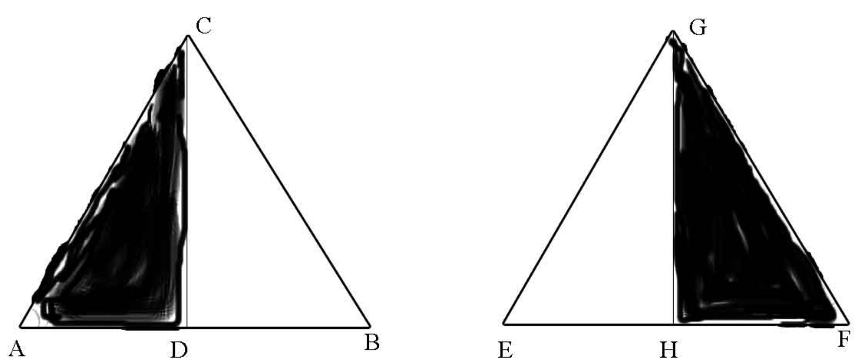
$\frac{QR}{QS} = \frac{QT}{PQ}$

$\angle Q = \angle Q$ (Common)

$\therefore \Delta TQR \sim \Delta PQS$ (AA Similarity)

2. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that

i) $\Delta DCB \sim \Delta HGF$ **ii) $\frac{CD}{GH} = \frac{AC}{FG}$** **[Ex 6.3, Q10]**



Sol:- Given $\Delta ABC \sim \Delta EFG$

$\Rightarrow \angle A = \angle E, \angle B = \angle F, \angle C = \angle G \ldots\ldots\ldots\ldots ..\text{i)}$

Now As CD and GH are angle bisectors of $\angle ACB$ and $\angle EGF$

$\therefore \quad \angle C = \angle G \quad \Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G \quad \Rightarrow \angle 1 = \angle 2$

In ΔDCA and ΔHGF, we've

$\angle 1 = \angle 2$

$\angle A = \angle F$ (by i))

$\therefore \Delta DCA \sim \Delta HGF$ (AA Similarity)

$\therefore$ Their corresponding sides are in proportional

$$\Rightarrow \frac{\text{AD}}{\text{HF}} = \frac{\textbf{CD}}{\textbf{GH}} = \frac{\textbf{AC}}{\textbf{GF}}$$

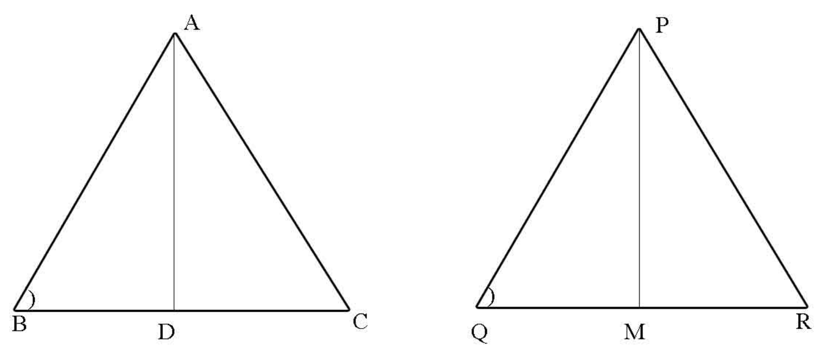
From 2nd and last $\frac{\textbf{CD}}{\textbf{GH}} = \frac{\textbf{AC}}{\textbf{GF}}$

3. Two sides AB and BC and median AD of ΔABC are proportional to PQ and QR and median PM of another ΔPQR. Prove that ΔABC ~ ΔPQR. [Ex 6.3, Q12]

Sol :- Given : In two ΔABC and ΔPQR, $\frac{\text{AB}}{\text{PQ}} = \frac{\text{AC}}{\text{QR}} = \frac{\text{AD}}{\text{PM}} \dots \dots \dots \dots .\text{i)}$

and AD and PM are medians such that $\text{BD} = \text{DC}$ and $\text{QM} = \text{MR}$

To prove $\Delta\text{ABC} \sim \Delta\text{PQR}$



Proof – Since AD and PM are medians such that $\text{BD} = \text{DC}$ $i.e.$ $\text{BC} = 2\text{BD}$ and $\text{QM} = \text{MR}$ $i.e.$ $\text{QR} = 2\text{QM}$

Now $\frac{\text{BC}}{\text{QR}} = \frac{2\text{BD}}{2\text{QM}} = \frac{\text{BD}}{\text{QM}} \dots \dots \dots \dots \dots ..\text{ii)}$

From i) and ii), we get

$\Rightarrow \frac{\text{AB}}{\text{PQ}} = \frac{\text{BD}}{\text{QM}} = \frac{\text{AD}}{\text{PM}}$

$\Rightarrow \Delta\text{ABD} \sim \Delta\text{PQM}$ (SSS Similarity)

$\Rightarrow \angle\text{B} = \angle\text{Q}$

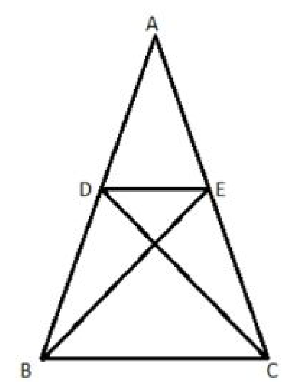
Now In ΔABC and ΔPQR

$\angle\text{B} = \angle\text{Q}$

and $\frac{\text{AB}}{\text{PQ}} = \frac{\text{BC}}{\text{QR}}$ (Given)

$\therefore \Delta\text{ABC} \sim \Delta\text{PQR}$ (SAS Similarity)

4. In the figure, If ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC [Ex 6.3, Q6]



Sol:- Given: ΔABE ≅ ΔACD

$\Rightarrow$ $\text{AB} = \text{AC} \dots \dots \dots \dots \dots \dots ..\text{i)}$

and $\text{AE} = \text{AD}$ or $\text{AD} = \text{AE} \dots \dots \dots \dots \dots ..\text{ii)}$

(**cpct of congruent triangles**)

Divide ii) by i), we get

$$\frac{AD}{AB} = \frac{AE}{AC}$$

By converse of Thales Theorem, $DE || BC$

$\Rightarrow \quad \angle ADE = \angle ABC \quad$ (corresponding angles)

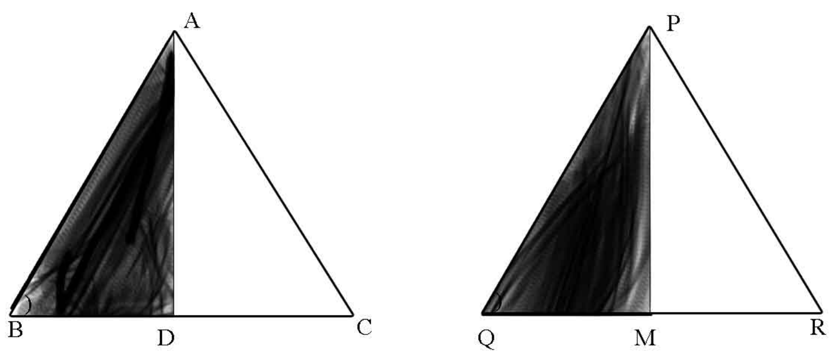
Now In ΔADE and ΔABC

$\angle A = \angle A \quad$ (common)

and $\angle ADE = \angle ABC$

$\therefore \Delta ADE \sim \Delta ABC$ (AA Similarity)

5. If AD and PM are medians of triangles ABC and PQR respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$



Sol :- In two $\Delta ABC \sim \Delta PQR \quad \Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \ldots\ldots\ldots\ldots.i)$

and AD and PM are medians such that $BD = DC$ and $QM = MR$

To prove $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof – Since AD and PM are medians such that $BD = DC$ $i.e.$ $BC = 2BD$ and $QM = MR$ $i.e.$ $QR = 2QM$

By i) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM}$

Now In ΔABD and ΔPQM

$\angle B = \angle Q \quad$ {By i)}

and $\frac{AB}{PQ} = \frac{BD}{QM}$

$\therefore \Delta ABD \sim \Delta PQM$ (SAS Similarity)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

From 1st and last, we get $\frac{AB}{PQ} = \frac{AD}{PM}$