DAY 9

As we know $\sin^2\theta + \cos^2\theta = 1,\ \sec^2\theta - \tan^2\theta = 1, \operatorname{cosec}^2\theta - \cot^2\theta = 1$

- **To make the sums easy related to Trigonometry, Change every T Ratio in $\sin\theta$ and $\cos\theta$.**

$$i.e.\ \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

1. Prove that $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

Sol: $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

{Change **secA** and **tanA** in sinA and cosA}

$$= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$= \left(\frac{1-\sin A}{\cos A}\right)\left(\frac{1+\sin A}{\cos A}\right) = \frac{1^2-\sin^2 A}{\cos^2 A} = \frac{1-\sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$$

$\{$As $a^2 - b^2 = (a-b)(a+b)$: $1 - \sin^2 A = \cos^2 A\}$

2. Prove that $\frac{\cot A-\cos A}{\cot A+\cos A} = \frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$

Sol: LHS : $\frac{\cot A-\cos A}{\cot A+\cos A} = \frac{\frac{\cos A}{\sin A}-\cos A}{\frac{\cos A}{\sin A}+\cos A}$

{Change **cotA** in sinA and cosA}

$$= \frac{\cos A\left(\frac{1}{\sin A}-1\right)}{\cos A\left(\frac{1}{\sin A}+1\right)} = \frac{\frac{1}{\sin A}-1}{\frac{1}{\sin A}+1} = \frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1} = \text{RHS}$$

3. Prove that $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

Sol: LHS: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = \frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A} = \frac{\cos^2 A+\left(1+\sin^2 A+2\sin A\right)}{(1+\sin A)\cos A}$

$$= \frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\cos A} = \frac{\left(\cos^2 A+\sin^2 A\right)+1+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A} = 2\sec A$$

4. Prove that $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = (\operatorname{cosec}\theta - \cot\theta)$ or $\frac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$

Sol: RHS: $(\text{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{LHS}$$

ALTER METHOD

LHS: $\frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$

{Since LHS in power 1 and RHS in power 2, So Rationalise with $(1 - \cos\theta)$}

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{\sin^2\theta} = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = (\text{cosec}\theta - \cot\theta)^2$$

EXERCISE

Prove That

1. $(\sec A + \tan A)(1 - \sin A) = \cos A$
2. $\frac{1+\tan^2 A}{1+\cot^2 A} = \tan^2 A$
3. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$