

DAY 3

- 1. Find the value of y for which the distance between the points $P(2, -3)$ and $Q(10, y)$ is 10 units.** **[Ex 7.1, Q8]**

Sol :- Given $P(2, -3)$ and $Q(10, y)$ and $PQ = 10$ units

$$\begin{aligned} \Rightarrow \sqrt{(2-10)^2 + (-3-y)^2} &= 10 \\ \Rightarrow (-8)^2 + (9 + y^2 + 6y) &= 10^2 & \Rightarrow 64 + 9 + y^2 + 6y &= 100 \\ \Rightarrow y^2 + 6y + 73 - 100 &= 0 & \Rightarrow y^2 + 6y - 27 &= 0 \\ \Rightarrow y^2 + 9y - 3y - 27 &= 0 & \Rightarrow y(y+9) - 3(y+9) &= 0 \\ \Rightarrow (y+9)(y-3) &= 0 & \Rightarrow y &= 3, -9 \end{aligned}$$

- 2. Find a point on x – axis which is equidistant from points $(3, 4)$ and $(2, -3)$.**

Sol:- We know any point on x – axis is $P(x, 0)$

Given condition Let $P(x, 0)$ is equidistant from $Q(3, 4)$ and $R(2, -3)$

$$\begin{aligned} \Rightarrow PQ &= PR & \Rightarrow PQ^2 &= PR^2 \\ \Rightarrow (x-3)^2 + (0-4)^2 &= (x-2)^2 + (0-(-3))^2 \\ \Rightarrow (x^2 + 3^2 - 2 \times x \times 3) + (-4)^2 &= (x^2 + 2^2 - 2 \times x \times 2) + (3)^2 \\ \Rightarrow x^2 + 9 - 6x + 16 &= x^2 + 4 - 4x + 9 \\ \Rightarrow x^2 - 6x + 25 &= x^2 - 4x + 13 \\ \Rightarrow -6x + 4x &= 13 - 25 & \Rightarrow -2x &= -12 \\ \Rightarrow x &= \frac{-12}{-2} = 6 \\ (6, 0) &\text{ is point which is equidistant from } (3, 4) \text{ and } (2, -3) \end{aligned}$$

- 3. Find a point on y – axis which is equidistant from points $A(6, 5)$ and $B(-4, 3)$.**

Sol:- We know any point on y – axis is $P(0, y)$

Given condition Let $P(0, y)$ is equidistant from $A(6, 5)$ and $B(-4, 3)$

$$\begin{aligned} \Rightarrow PA &= PB & \Rightarrow PA^2 &= PB^2 \\ \Rightarrow (0-6)^2 + (y-5)^2 &= (0-(-4))^2 + (y-3)^2 \\ \Rightarrow (-4)^2 + (y^2 + 5^2 - 2 \times y \times 5) &= (-4)^2 + (y^2 + 3^2 - 2 \times y \times 3) \\ \Rightarrow 16 + y^2 + 25 - 10y &= 16 + y^2 + 9 - 6y \\ \Rightarrow y^2 - 10y + 41 &= y^2 - 6y + 25 \\ \Rightarrow -10y + 6y &= 25 - 41 & \Rightarrow -4y &= -16 \\ \Rightarrow y &= \frac{-16}{-4} = 4 \\ (0, 4) &\text{ is the point which is equidistant from } A(6, 5) \text{ and } B(-4, 3) \end{aligned}$$

- 4. Find the relation between x and y if the point (x, y) is equidistant from points $(7, 1)$ and $(3, 5)$.** **[Example 4]**

Sol:- Let $P(x, y)$ is equidistant from points $A(7, 1)$ and $B(3, 5)$

$$\begin{aligned} \Rightarrow PA &= PB & \Rightarrow PA^2 &= PB^2 \\ \Rightarrow (x-7)^2 + (y-1)^2 &= (x-3)^2 + (y-5)^2 \\ \Rightarrow (x^2 + 7^2 - 2 \times x \times 7) + (y^2 + 1^2 - 2 \times y \times 1) &= (x^2 + 3^2 - 2 \times x \times 3) + (y^2 + 5^2 - 2 \times y \times 5) \end{aligned}$$

$$\begin{aligned}
 &= (x^2 + 3^2 - 2 \times x \times 3) + (y^2 + 5^2 - 2 \times y \times 5) \\
 \Rightarrow &x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y \\
 \Rightarrow &-14x + 6x - 2y + 10y + 50 - 34 = 0 \\
 \Rightarrow &-8x + 8y + 16 = 0 \\
 &\text{Divide by } -8, \text{ we get} \\
 \Rightarrow &x - y - 2 = 0 \text{ which is the required relation.}
 \end{aligned}$$

When vertices of Quadrilateral are given:

In last section, we have discussed about distance formula of collinear points and non collinear points. Now we shall discuss when vertices of quadrilateral are given.

- **Parallelogram:** When opposite sides/distances are equal.
- **Rectangle:** When opposite sides/distances and diagonals are equal.
- **Rhombus:** When all sides/distances are equal.
- **Square:** When all sides/distances and diagonals are equal.

5. Show that (1, 7), (4, 2), (-1, -1) and (-4, 4) are the vertices of the square.

Sol: -Let A(1,7), B(4,2), C(-1, -1) and D(-4,4) are the vertices of the quadrilateral ABCD.

$$\text{Now } AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{(-3)^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$$

$$BC = \sqrt{(4-(-1))^2 + (2-(-1))^2} = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-1-(-4))^2 + (-1-4)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$$

$$AD = \sqrt{(1-(-4))^2 + (7-4)^2} = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

$$\text{Diagonal } AC = \sqrt{(1-(-1))^2 + (7-(-1))^2} = \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(4-(-4))^2 + (2-4)^2} = \sqrt{8^2 + (-2)^2} = \sqrt{64+4} = \sqrt{68}$$

\therefore All sides and diagonals are equal, So ABCD is a square.

EXERCISE

1. Ex 7.1, Q 6,7,9,10
2. Show that A(2,4), B(-2,3), C(-1, -1) and D(3,0) are the vertices of the square.
3. Find a point on y axis which is equidistant from (4,0) and (4,12).