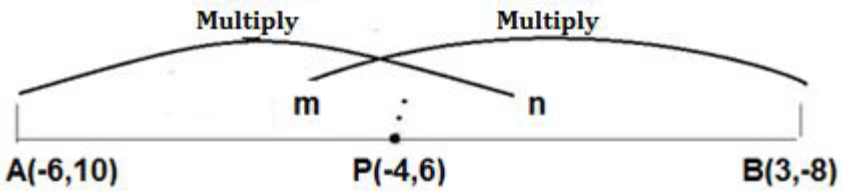
DAY 5

In last section, we have discussed to find the point of division when ratio and extreme points are given. In this section, we shall find the ratio which divide the given line segment by a point of division.

When a point P(x, y) divides the line segment joining A(x_1, y_1) and B (x_2, y_2) in certain ratio then to find ratio, we take $\boldsymbol{m:n}$ **or** $\frac{\boldsymbol{m}}{\boldsymbol{n}}:\boldsymbol{1}$ **or** $\boldsymbol{k:1}$ **where** $\boldsymbol{k=\frac{m}{n}}$

1. **In what ratio does the point $(-4, 6)$ divide the line-segment joining the points $(-6, 10)$ and $(3, -8)$?** **[Example 7]**

Sol :- Let P$(-4,6)$ divide the line-segment joining the points A$(-6,10)$ and B$(3,-8)$ in ratio $m:n$



$\Rightarrow \ -4 = \frac{-6n+3m}{m+n}$

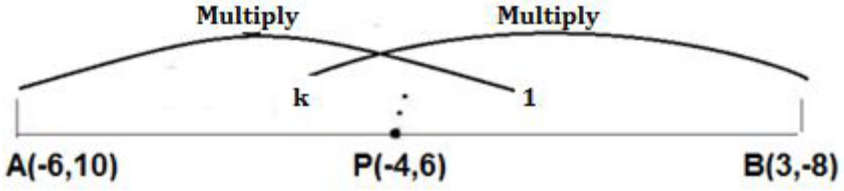
$\Rightarrow -4(m+n) = -6n+3m \qquad \Rightarrow -4m-4n = -6n+3m$

$\Rightarrow \ -4m-3m = -6n+4n \qquad \Rightarrow -7m = -2n$

$\Rightarrow \ \frac{m}{n} = \frac{-2}{-7} = \frac{2}{7}$ i.e. $m:n = 2:7$

ALITER METHOD :-

Suppose required ratio is $k:1$



$\Rightarrow \ -4 = \frac{-6+3k}{k+1}$

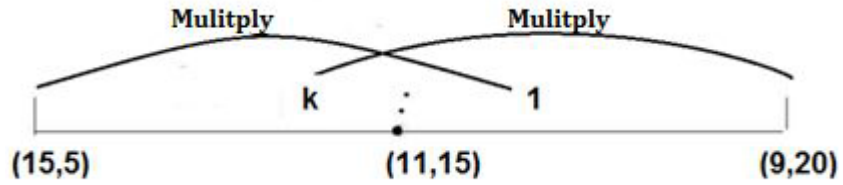
$\Rightarrow -4(k+1) = -6+3k \qquad \Rightarrow -4k-4 = -6+3k$

$\Rightarrow \ -4k-3k = -6+4 \qquad \Rightarrow -7k = -2$

$\Rightarrow \ k = \frac{-2}{-7} = \frac{2}{7}$ i.e. $k:1 = 2:7$ Required ratio is 2:7

2. **In what ratio does the point $(11, 15)$ divide the line-segment joining the points $(15, 5)$ and $(9, 20)$?**

Sol :- Let P$(11,15)$ divide the line-segment joining the points A$(15,5)$ and B$(9,20)$ in ratio $k:1$



$\Rightarrow\ 11 = \frac{15+9k}{k+1}$

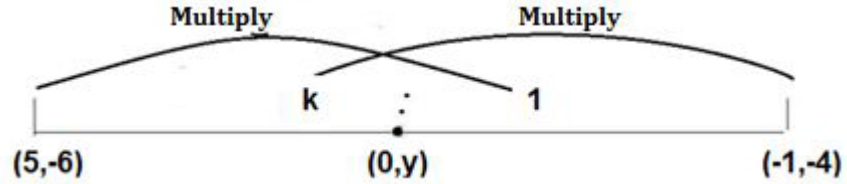
$\Rightarrow 11(k+1) = 15+9k \qquad \Rightarrow 11k+11 = 15+9k$

$\Rightarrow\ 11k-9k = 15-11 \qquad \Rightarrow\ 2k = 4$

$\Rightarrow\ k = \frac{4}{2} = 2$ i.e. $k:1 = 2:1$ Required ratio is 2:7

3. Find the ratio in which the line segment joining A$(5,-6)$ and B$(-1,-4)$ is divided by the y −axis. Also find the co-ordinates of the point of division. [Example 9]

Sol :- Let the line segment joining A$(5,-6)$ and B$(-1,-4)$ be divided by point P on y −axis in the ratio k: 1.



Co-ordinates of P are $\left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$

Since point P lie on y −axis so coordinates of $x = 0$

$\Rightarrow\ \frac{-k+5}{k+1} = 0 \qquad \Rightarrow -k+5 = 0(k+1) = 0$

$\Rightarrow\ k = 5$

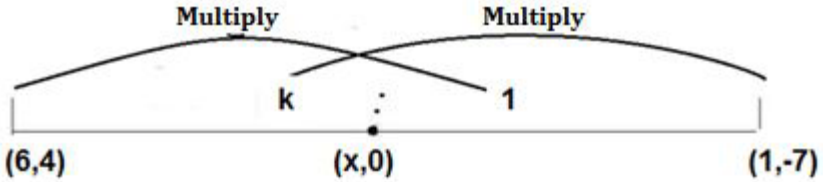
$\therefore$ Required ratio k: 1 or **5**: **1**

$\therefore$ Co-ordinates of y in P $= \frac{-4\times5-6}{5+1} = \frac{-20-6}{6} = \frac{-26}{6} = \frac{-13}{3}$

$\therefore$ Required point of division is $\left(0, \frac{-13}{3}\right)$

4. Find the ratio in which the line segment joining A$(6,4)$ and B$(1,-7)$ is divided by the x −axis. Also find the co-ordinates of the point of division.

Sol :- Let the line segment joining A(6,4) and B$(1,-7)$ be divided by point P on x −axis in the ratio k: 1.



Co-ordinates of P are $\left(\frac{k+6}{k+1}, \frac{-7k+4}{k+1}\right)$

Since point P lie on x −axis so coordinates of $y = 0$

$\Rightarrow \frac{-7k+4}{k+1} = 0 \quad \Rightarrow -7k + 4 = 0(k+1) = 0$

$\Rightarrow k = \frac{4}{7}$

$\therefore$ Required ratio k: 1 or **4**: **7**

$\therefore$ Co-ordinates of y in P $= \frac{\frac{4}{7}+6}{\frac{4}{7}+1} = \frac{\frac{4+42}{7}}{\frac{4+7}{7}} = \frac{\frac{46}{7}}{\frac{11}{7}} = \frac{46}{11}$

$\therefore$ Required point of division is $\left(\frac{46}{11}, 0\right)$

5. If A(6, 1), B(8, 2), C(9,4) and D(p, 3) are the vertices of a parallelogram taken in order, find the values of p. [Example 10]

Sol :- We know that diagonals AC and BD of parallelogram ABCD bisect each other

i.e. **Mid point of AC = Mid point of BD**

$\Rightarrow \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$

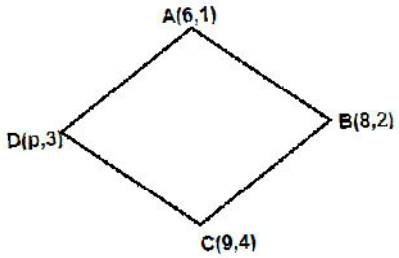
$\Rightarrow \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$

. Equating first coordinate from both sides, we get

$\frac{15}{2} = \frac{8+p}{2} \Rightarrow 15 = 8 + p$

$\Rightarrow p = 15 - 8 = 7$

Hence value of $p = 7$



EXERCISE

1. In what ratio does the point $(-2,3)$ divide the line-segment joining the points $(-3,5)$ and $(4,-9)$?
2. Find the ratio in which the line segment joining $(-1,3)$ and $(2,4)$ is divided by the y −axis. Also find the co-ordinates of the point of division.
3. Ex 7.2, Q4,5,6,7