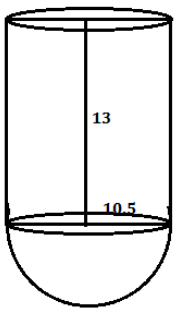
DAY 4

Volume of Combination of Solids

In last section, we have discussed about surface areas of combination of solids. In this section, we shall discuss about volumes of solids.

1. A container is in the form of a hemisphere surmounted by a right circular cylinder with same radius. The radius of the base of cylindrical portion is 10.5 cm and height 13cm. Find the capacity of container.



Sol:-According to figure, Radius of cylinder $=$ Radius of the hemisphere$(r) = 10.5\ cm$

and height of the cylinder $(h) = 13\ cm$

Capacity of the container = (Volume of the cylinder) +(Volume of Hemisphere)

$$= \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{2}{3}r\right)$$

$$= \frac{22}{7} \times 10.5 \times 10.5\left(13 + \frac{2}{3} \times 10.5\right)$$

$$= \frac{22}{7} \times 10.5 \times 10.5(13 + 7)$$

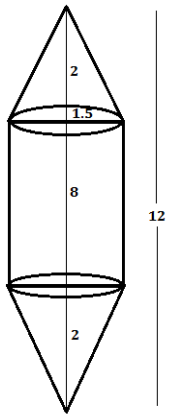
$$= \frac{22}{7} \times 10.5 \times 10.5 \times 20 = \mathbf{6930\ cm^3}$$

2. A model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3cm and its height is 12cm. If each cone has a height of 2 cm, find the volume of the model. [Ex 13.2, Q2]

Sol:- Diameter of the model = 3cm

$\therefore$ **Radius of cylinder** $=$ Radius of the Cone$(r) = \frac{3}{2}\ cm$

and height of the cone $(h_1) = 2$



According to figure,

height of the cylinder $(h_2) = \left(\begin{matrix}\textbf{Total Height}\\ \textbf{of the model}\end{matrix}\right) - \mathbf{2} \times \left(\begin{matrix}\textbf{Height of}\\ \textbf{the Cone}\end{matrix}\right)$

$$= 12 - 2 \times 2 = 12 - 4 = 8\ cm$$

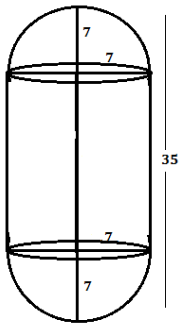
Volume of the model = (Volume of the cylinder) + 2 × (Volume of Cone)

$$= \pi r^2 h_2 + 2 \times \frac{1}{3}\pi r^2 = \pi r^2\left(h_2 + \frac{2}{3}h_1\right)$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}\left(8 + \frac{4}{3}\right) = \frac{99}{14} \times \left(\frac{24+4}{3}\right)$$

$$= \frac{99}{14} \times \frac{28}{3} = \mathbf{66\ cm^3}$$

3. **Find the volume of a solid in the form of a right circular cylinder with hemispherical ends whose total length is 35 cm and the diameter of each hemispherical end is 14 cm.**



Sol:- Given Diameter of the Hemisphere = Diameter of cylinder = 14cm

$\therefore$ Radius of Hemisphere = Radius of the Cylinder$(r) = 7\ cm.$

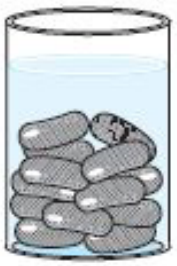
According to figure,

$$\text{height of cylinder}(h) = \begin{pmatrix}\textbf{Total Height}\\ \textbf{of the solid}\end{pmatrix} - \mathbf{2} \times \begin{pmatrix}\textbf{Radius of}\\ \textbf{the Hemisphere}\end{pmatrix}$$
$$= 35 - 2 \times 7 = 35 - 14 = 21\ cm$$

Volume of the solid = (Volume of the cylinder) + 2 × (Volume of Hemisphere)

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 \left(h + \frac{4}{3}r\right)$$
$$= \frac{22}{7} \times 7 \times 7 \left(21 + \frac{4}{3} \times 7\right)$$
$$= 154 \times \left(\frac{21}{1} + \frac{28}{3}\right) = 154 \times \left(\frac{63+28}{3}\right)$$
$$= 154 \times \frac{91}{3} = \mathbf{\frac{14014}{3}\ cm^3}$$

4. **A gulab jamun contains sugar syrup upto about 30% of its volume. Find how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5cm and diameter 2.8 cm.** **[Ex 13.2, Q4]**



Sol:- Given Diameter of the Cylindrical part = Diameter of hemisphere = 2.8cm

$\therefore$ Radius of Cylindrical part = Radius of the Hemisphere$(r) = 1.4\ cm.$

Total length of gulabjamun=5 cm

According to figure,

$$\text{Height of cylinder}(h) = \begin{pmatrix}\textbf{Total Height}\\ \textbf{of the gulab jamun}\end{pmatrix} - \mathbf{2} \times \begin{pmatrix}\textbf{Radius of}\\ \textbf{the Hemisphere}\end{pmatrix}$$
$$= 5 - 2 \times 1.4 = 5 - 2.8 = 2.2\ cm$$

Volume of the a gulab jamun $= \begin{pmatrix}\textbf{Volume of the}\\ \textbf{cylinder}\end{pmatrix} + 2 \times \begin{pmatrix}\textbf{Volume of}\\ \textbf{Hemisphere}\end{pmatrix}$

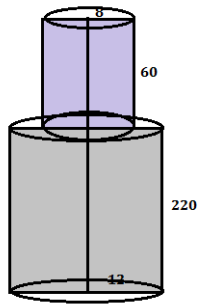
$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 \left(h + \frac{4}{3}r\right)$$
$$= \frac{22}{7} \times 1.4 \times 1.4 \left(2.2 + \frac{4}{3} \times 1.4\right)$$
$$= \frac{22}{7} \times 1.4 \times 1.4 \times (2.2 + 1.87)$$
$$= \frac{22}{7} \times 1.4 \times 1.4 \times 4.07 = 25.07 cm^3$$

Acc to question: Syrup of a gulab jamun = 30% of volume of a gulab jamun

$$= 30\% \text{ of } 25.07 = 7.52 cm^3$$

Now Syrup of 45 gulab jamuns $= 7.52 \times 45 =$ **338.445** $\boldsymbol{cm^3}$

5. **A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1** $\boldsymbol{cm^3}$ **of iron has approximately 8g mass. (Use** $\boldsymbol{\pi = 3.14}$**)**
[Ex 13.2, Q6]



Sol:- Given Height of larger cylinder (H) = 220 cm,
Diameter = 24 cm and radius of larger cylinder (R) $= 12cm.$
and Height of smaller cylinder (h)= 60 cm,
radius of smaller cylinder $(r) = 8\ cm$

Volume of the pole = (Volume of larger cylinder)
+(Volume of smaller cylinder)

$$= \pi R^2 H + \pi r^2 h = \pi(R^2 H + r^2 h)$$
$$= 3.14(12 \times 12 \times 220 + 8 \times 8 \times 60)$$
$$= 3.14 \times (31680 + 3840)$$
$$= 3.14 \times 35520 = \boldsymbol{111532.8\ cm^3}$$

Now Mass of 1 cm^3 iron = 8 g
$\therefore$ Mass of $111532.8\ cm^3$ iron $= 111532.8 \times 8 = 892262.4\ g$
$$= \frac{892262.4}{1000} kg = \boldsymbol{892.2624\ kg}$$

EXERCISE

1. A vessel is in the form of a cylindrical roller, whose base is a hemisphere with same radius of its base. If radius of base of cylindrical part is 6 cm and height is 10 cm, find the volume of the vessel.
2. Find the volume of the solid in the form of a right circular cylinder with hemispherical ends whose total length is 35 cm and the diameter of each hemispherical end is 14 cm.
3. A circus tent has cylindrical shape surmounted by a conical proof. The radius of the cylindrical base is 14 m and height also is 14 m. If the total height of tent is 9m, find the volume of tent.
4. An iron pole consisting of a cylindrical portion 110 cm high and of base radius 6 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that 1 cm^3 of iron has 10 g mass. $\left(\text{Use } \pi = \frac{355}{113}\right)$