CHAPTER-1

REAL NUMBERS
DAY 1

1.1 INTRODUCTION

In class IX we have discussed about real numbers. To recall, we have been dealing counting numbers are often called Natural Numbers.

$$\mathbf{N} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \dots \dots \dots\}$$

After natural numbers 0 is formed and a new set of numbers are formed called **whole numbers as** $\boldsymbol{W} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \dots \dots.\}$

While discussing whole numbers it is found that in whole numbers we could not subtract larger from smaller.

e.g. $3 - 5 =?$. We don't have any whole number in its answer. To overcome this difficulty, new numbers were introduced called Integers as Z or I.

i.e. $\boldsymbol{Z} \ \boldsymbol{or} \ \boldsymbol{I} = \{\dots \dots .., \mathbf{-3}, \mathbf{-2}, \mathbf{-1}, \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \dots \dots.\}$

Z^+ or I^+ are called positive integers. Z^- or I^- are called negative integers.

In the integers, we can define subtraction very easily but could define division. To overcome this difficulty, new numbers were introduced called rational numbers.

The numbers of this system can be represented either by terminating decimal recurring decimal.

All the four operations-addition, subtraction, multiplication and division can be performed in rational numbers. Which numbers are cannot be expressed in the form of $\frac{p}{q}$, p & q are integers & $q \neq 0$ are called **irrational numbers.**

Or "***A number which can neither be expressed as a termination decimal nor a repeating decimal is called an irrational number***"

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{11} \dots ....$ are examples of irrational numbers.

1.2 REAL NUMBERS

The totality of rational numbers and irrational numbers is called the set of real numbers. Every real number is either a rational or irrational number.

Euclid's Division Algorithm

With this background, in this chapter, we shall first discuss Euclid's division algorithm and Euclid's division lemma. As its name tells that this lemma is related with division, we know that

- If any number is divided by 2 then possible remainders are 0 or 1.
- If any number is divided by 3 then possible remainders are 0, 1 or 2.
- If any number is divided by 4 then possible remainders are 0, 1, 2 or 3.
- If any number is divided by 5 then possible remainders are 0,1,2,3 or 4.

And so on……

If any number(dividend) is divided by another number (divisor) then remainder would be 0 or less than divisor.

Euclid's Division Algorithm ***is stated below*** :

Given two positive integers a and b, there exists a unique pair of integers q and r such that $\boldsymbol{a = bq + r}$; $\boldsymbol{0 \leq r < b}$

Here a and b are called the dividend and the divisor respectively, q is called the quotient and r the remainder

We observe that if $r = 0$ then a is divisible by b or a is multiple of b. we can say that b is divisor or factor of a

Thus a is divisible by b if the remainder in the division of a and b is zero.

SOLVED EXAMPLES:-

1. **Show that every positive even integer is of form $\boldsymbol{2q}$ and every positive odd integer is of form $\boldsymbol{2q + 1}$, where $\boldsymbol{q}$ is some integer.** **(NCERT Ex 2)**

Sol:- By Euclid Division Algorithm $a = bq + r$; $0 \leq r < b$

[*Since number is required in the form of* $\mathbf{2}q$ *or* $\mathbf{2}q + 1$ ***i.e. divisor is 2***]

Take $b = 2$, so that $a = 2q + r$; $0 \leq r < 2$ (i.e. $r = 0$ or $r = 1$)

If $r = 0$ then $a = 2q$ and if $r = 1$ then $a = 2q + 1$

$\Rightarrow$ If a is of form $2q$ then it is even number and if a is of form $2q + 1$ then it is an odd number.

2. **Show that every positive odd integer is of form $\boldsymbol{4q + 1}$ or $\boldsymbol{4q + 3}$, where $\boldsymbol{q}$ is some integer. (NCERT)**

Sol:- By Euclid Division Algorithm $a = bq + r$; $0 \leq r < b$

[*Since number is required in the form of* $\mathbf{4}q$ *or* $\mathbf{4}q + 3$ ***i.e. divisor is 4***]

Take $b = 4$ so that $a = 4q + r$; $0 \leq r < 2$ (i.e. $r = 0,1,2$ or3)

If $r = 0$ then $a = 4q$ (Even number as it is multiple of 2)

if $r = 1$ then $a = 4q + 1$ (Odd number as it is 1 more than even number)

if $r = 2$ then $a = 4q + 2$ (Even number as it is multiple of 2)

if $r = 3$ then $a = 4q + 3$ (Odd number as it is 1 more than even number)

$\Rightarrow$ Every positive odd integer is of form $4q + 1$ or $4q + 3$, where q is some integer.

3. **Show that square of any positive integer is of form $\boldsymbol{3m}$ or $\boldsymbol{3m + 1}$ for some positive integer.** **(NCERT Ex1.1, Q4)**

Sol:- By Euclid Division Algorithm $a = bq + r$; $0 \leq r < b$

[*Since number is required in the form of* $\mathbf{3}m$ *or* $\mathbf{3}m + 1$ ***i.e. divisor is 3***]

Take $b = 3$ then $a = 3q + r$; $0 \leq r < 3$ (i.e. $r = 0,1$, or 2)

If $r = 0$ then $a = 3q$ then

$a^2 = (3q)^2 = 9q^2 = 3.3q^2 = 3(m)$ (for $m = 3q^2$)

If $r = 1$ then $a = 3q + 1$ then

$$a^2 = (3q+1)^2 = 9q^2+6q+1 = 3(3q^2+2q)+1 = 3(m)+1$$
(for $m = 3q^2+2q$)

If $r = 2$ then $a = 3q+2$ then

$$a^2 = (3q+2)^2 = 9q^2+12q+4 = (9q^2+12q+3)+1 = 3(3q^2+4q)+1$$
$$= 3(m)+1 \qquad \text{(for } m = 3q^2+4q+1)$$

Hence square of any positive integer is of the form of $3m$ or $3m$ +1 for some positive integer m.

4. Use Euclid's division algorithm to show that cube of any positive integer m is of the form $9m$ *or* $9m+1$ *or* $9m+8$. **(NCERT Ex.1.1, Que. 5)**

Sol:- By Euclid Division Algorithm $a = bq + r$; $0 \le r < b$

(Here instead of taking divisor 9, we take 3 because cube of 3 is divisible by 9)

Take $b = 3$ then $a = 3q + r$; $0 \le r < 3$ (i.e. $r = 0,1,$ or 2)

If $r = 0$ then $a = 3q$ then

$$a^3 = (3q)^3 = 27q^3 = 9.3q^3 = 3(m) \qquad \text{(for } m = 3q^3)$$

If $r = 1$ then $a = 3q+1$ then

$$a^3 = (3q+1)^3 = 27q^3+27q^2+9q+1$$
$$= 9(3q^3+3q^2+1)+1 = 9(m)+1 \qquad \text{(for } m = 3q^3+3q^2+1)$$

If $r = 2$ then $a = 3q+2$ then

$$a^3 = (3q+2)^3 = 27q^3+54q^2+36q+8 = 9(3q^3+6q^2+4q)+8$$
$$= 9(m)+8 \qquad \text{(for } m = 3q^3+6q^2+4q)$$

Hence cube of any positive integer m is of the form $9m$ *or* $9m+1$ *or* $9m+8$.

EXERCISE

1. Show that any odd integer is of the form $6q+1$, *or* $6q+3$, *or* $6q+5$, where q is some integer.
   (NCERT)
2. Show that any even positive integer is of form $4q$ or $4q+2$, where q is positive integer.
3. Show that any even positive integer is of form $6q, 6q+2$ or $6q+4$ for some positive integer q.
4. Exercise 1.1, Que 2