DAY 5

1. Find the area of the shaded region, if $\text{PQ} = 24cm$, $\text{PR} = 7cm$ and O is the centre of the circle. **[Ex 12.3, Q1]**

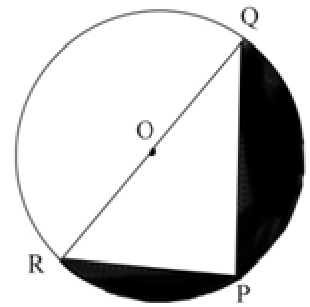
Sol:- We know that angle in semi circle is right angle. So $\angle \text{P} = 90^0$

In right ΔPQR,

$$\text{QR}^2 = \text{PR}^2 + \text{PQ}^2 = 7^2 + 24^2 = 49 + 576 = 625 = 25^2$$

$\Rightarrow \quad \text{QR} = 25$

$\Rightarrow$ Diameter of the circle $= 25cm \qquad \Rightarrow r = \frac{25}{2}cm$



Now Area of shaded part = (Area of semi circle QPRO) $- ar(\Delta \text{PQR})$

$$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times \text{PR} \times \text{PQ}$$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - \frac{1}{2} \times 7 \times 24$$

$$= \frac{6875}{28} - \frac{84}{1} = \frac{6875-2352}{28} = \mathbf{\frac{4523}{28}} \ \boldsymbol{cm^2}$$

2. Find the area of the shaded region, if the radii of the two concentric circles with centre O are 7 *cm* and 14*cm* respectively and $\angle \text{AOB} = 40^0$ **[Ex 12.3, Q2]**

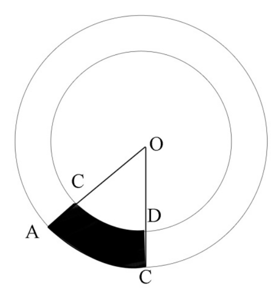
Sol:- Inner radius OB $(r) = 7\ cm$ and Outer radius OA $(R) = 14\ cm$. $\theta = 40^0$

Area of shaded portion = (Area of sector OAC) − (Area of sector OBD)

$$= \frac{\pi R^2\theta}{360^0} - \frac{\pi r^2\theta}{360^0} = \frac{\pi\theta}{360^0}(R^2 - r^2)$$

$$= \frac{22}{7} \times \frac{40^0}{360^0}(14^2 - 7^2) = \frac{22}{7} \times \frac{1}{9}(196 - 49)$$

$$= \frac{22}{7} \times \frac{1}{9} \times 147 = \mathbf{\frac{154}{3}} \ \boldsymbol{cm^2}$$



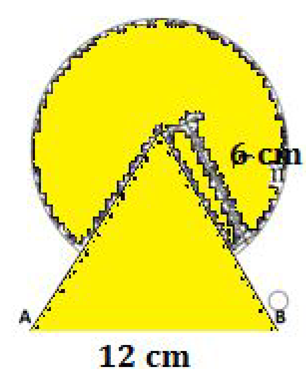
3. Find the area of the shaded region where a circular arc of radius 6 *cm* has been drawn with vertex O of an equilateral triangle OAB of side 12 *cm* as centre.

[Ex 12.3, Q4]

Sol:- Given $\angle \text{AOB} = 60^0$ and reflex $\angle \text{AOB} = 360^0 - 60^0 = 300^0$

Area of shaded portion = (Area of equilateral ΔOAB) + (Area of major sector having angle 300^0)

$$= \frac{\sqrt{3}}{4}a^2 + \pi r^2 \frac{300^0}{360^0}$$

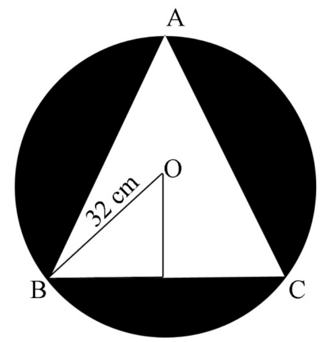


$$= \frac{\sqrt{3}}{4} \times 12 \times 12 + \frac{22}{7} \times 6 \times 6 \times \frac{5}{6}$$
$$= \left(\mathbf{36\sqrt{3} + \frac{660}{7}}\right) \boldsymbol{cm^2}$$

4. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle. Find the area of the design. [Ex 12.3, Q6]

Sol:- Given radius of circle $\text{OB}(r) = 32cm$, Draw $\text{OL} \perp \text{BC}$

In right ΔOBL, $\angle\text{OBL} = 30^0$



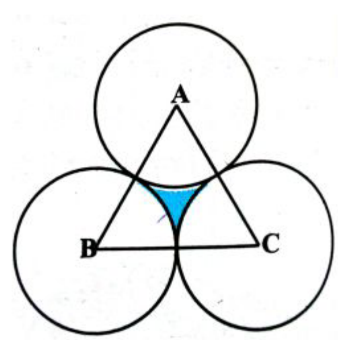
$$\Rightarrow \frac{\text{BL}}{\text{OB}} = \cos 30^0 = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \frac{\text{BL}}{32} = \frac{\sqrt{3}}{2} \qquad \Rightarrow \text{BL} = 16\sqrt{3}$$
$$\therefore \text{BC} = 2\text{BL} = 2 \times 16\sqrt{3} = 32\sqrt{3}\ cm$$

Area of Design = (Area of circle) − $ar(\Delta\text{ABC})$

$$= \pi r^2 - \frac{\sqrt{3}}{4} a^2$$
$$= \frac{22}{7} \times 32 \times 32 - \frac{\sqrt{3}}{4} \times 32\sqrt{3} \times 32\sqrt{3}$$
$$= 32 \times 32 \left(\frac{22}{7} - \frac{\sqrt{3}}{4} \times 3\right) = \mathbf{1024}\left(\mathbf{\frac{22}{7} - \frac{3\sqrt{3}}{4}}\right) \boldsymbol{cm^2}$$

5. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle. Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$) [Ex 12.3, Q10]

Sol:- Area of an equilateral triangle ABC = 17320.5 cm^2.



iii)

$$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 17320.5$$
$$\Rightarrow \frac{1.73205}{4} \times a^2 = 17320.5$$
$$\Rightarrow a^2 = \frac{173205}{10} \times \frac{4}{1.73205}$$
$$\Rightarrow a^2 = \frac{173205}{10} \times \frac{400000}{173205} = 40000 = 200^2$$
$$\Rightarrow a = 200\ cm$$

According to diagram, Side of triangle = 2 × radius of circle

$$\Rightarrow 200 = 2 \times \text{radius of circle} \qquad \Rightarrow \text{radius}(r) = 100\ cm$$

Area of shaded region = $ar(\Delta\text{ABC}) - 3 \times$ (Area of each sector)

$$= 17320.5 - 3 \times \frac{\pi r^2 \theta}{360^0}$$

$$= 17320.5 - 3 \times \frac{314}{100} \times 100 \times 100 \times \frac{60^0}{360^0}$$

$$= 17320.5 - 15700 = \mathbf{1620.5\ cm^2}$$