DAY 6

In previous section we have discussed about irrational numbers, now we will discuss decimal representation of rational numbers without actual division.

Decimal Representation of Rational Numbers:-

Every rational number can be expressed in the form of terminating and recurring decimal.

Terminating Decimals:- The rational numbers for which the long division terminate after a finite number of steps are called Terminating decimals. e.g. $\frac{5}{16} = 0.3125$

Recurring Decimals:- The rational numbers for which the long division does not terminate after a number of steps are called recurring decimals. e.g. $\frac{10}{3} = 3.333\ldots\ldots$

Remarks

If $\frac{p}{q}$ is a rational number in its **standard (simplest) form**, then it will have terminating decimal when the denominator must be in form of 10,100,1000…..so on. So prime factors of denominator(q) are only 2's & 5's e.g. $\frac{1}{8}, \frac{2}{25}, \frac{13}{20}$ all are terminating decimals because their denominator have only prime factors 2 or 5 or both , thus we can say that where denominator can be expressed in the **form of $2^m \times 5^n$** then it is terminating decimal.

1. **State whether following rational numbers will have a terminating decimals expansion or non-terminating repeating decimal expansion without using long division:**

   i) $\frac{17}{8}$ **ii)** $\frac{33}{60}$ **iii)** $\frac{11}{28}$ **iv)** $\frac{151}{27}$ **v)** $\frac{111}{120}$

Sol:- **i)** $\frac{17}{8}$ here Denominator $= 8$ & 17, 8 are co-prime

Now $8 = 2^3$ So Prime factors of denominator is only 2's

Rational Number $\frac{17}{8}$ has terminating decimal representation

ii) $\frac{33}{60} = \frac{11}{20}$ now Denominator=30 & 11, 30 are co-prime

Now $20 = 2^2 \times 5$ Here prime factors of denominator is only in 2's and 5's

So it is terminating decimal.

iii) $\frac{11}{28}$ here Denominator $= 28$; 11 & 28 are co-prime

Now $28 = 2^2 \times 7$ Prime factors of denominator contains a factor other than 2's & 5's

It has non-terminating decimal representation.

iv) $\frac{151}{27}$ now Denominator=27 & 151, 27 are co-prime

Now $27 = 3^3$ Here prime factors of denominator is only in 2's and 5's

So it is non-terminating decimal.

v) $\frac{111}{120} = \frac{37}{40}$ now Denominator=40 & 37, 40 are co-prime

Now $40 = 2^3 \times 5$ Here prime factors of denominator is only in 2's and 5's

So it is terminating decimal.

2. **Write the terminating decimal expansion of the following:**

i) $\frac{17}{8}$ ii) $\frac{33}{60}$ iii) $\frac{11}{50}$

Sol:- i) $\frac{17}{8} = \frac{17}{2^3} = \frac{17}{2^3} \times \frac{5^3}{5^3} = \frac{2125}{10^3} = 2.125$

{For decimal expansion, there is need of power of 10 in denominator. Here we've 2^3 so for power of 10, we need to multiply it with 5^3}

ii) $\frac{33}{60} = \frac{11}{20} = \frac{11}{2^2 \times 5} = \frac{11}{2^2 \times 5} \times \frac{5}{5} = \frac{55}{10^2} = 0.55$

{For decimal expansion, there is need of power of 10 in denominator. Here we've $2^2 \times 5$ so for power of 10, we need to multiply it with 5}

iii) $\frac{11}{50} = \frac{11}{2 \times 5^2} = \frac{11}{2 \times 5^2} \times \frac{2}{2} = \frac{222}{10^2} = 2.22$

{For decimal expansion, there is need of power of 10 in denominator. Here we've 2×5^2 so for power of 10, we need to multiply it with 5}

Exercise

1. **EXERCISE 1.4**