DAY 4

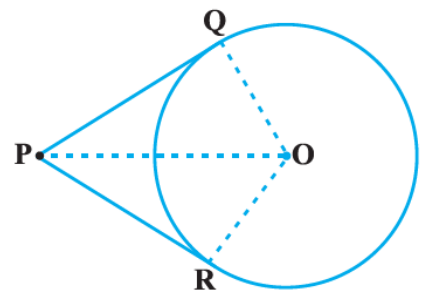
THEOREM: Prove that the lengths of tangents drawn from an external point to a circle are equal.

Given : A circle with centre O and PA and PB are two tangents drawn from point P.

To Prove: - $PA = PB$

Construction Join OP, OA and OB

Proof:- Now In right angled ΔOAP and ΔOBP, we've



$OA = OB$ (Equal Radii)

$\angle OAP = \angle OBP = 90^0$

$OP = OP$ (Common)

$\therefore\ \Delta OAP \cong \Delta OBP$

$\therefore\ PA = PB$ [C.P.C.T.]

ALITER METHOD :-

This theorem can be proved by Pythagoras theorem.

In ΔOAP, we've $OP^2 = OA^2 + AP^2$

$\Rightarrow OA^2 = OP^2 - AP^2 \ldots\ldots\ldots\ldots\ldots i)$

and In ΔOBP, we've $OP^2 = OB^2 + BP^2$

$\Rightarrow OB^2 = OP^2 - BP^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots ii)$

Since $OA = OB$ (Equal Radii) $\Rightarrow OA^2 = OB^2$

$\Rightarrow OP^2 - AP^2 = OP^2 - BP^2$ [From i) & ii)]

$\Rightarrow AP^2 = BP^2$ or $AP = BP$

RESULT: Centre of the circle lies on the bisector of the angle between the two tangents.

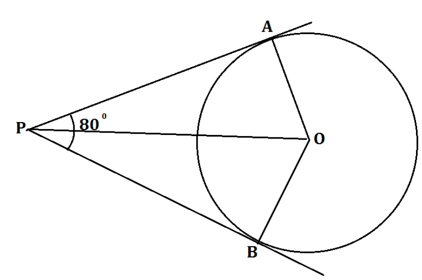
Since In above theorem $\Delta OAP \cong \Delta OBP$

$\therefore\ \angle OPA = \angle OPB$ [CPCT]

Hence the proof

1. **If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80^0, then determine $\angle POA$.** **[Ex 10.2, Q 3]**

Sol:- $\angle APO = \angle BPO = \frac{1}{2}\angle APB = \frac{1}{2} \times 80^0 = 40^0$



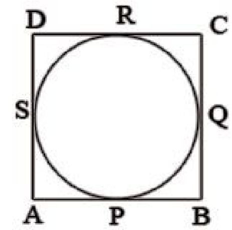
In rt. $\angle$d ΔPAO, we've $\angle PAO = 90^0$

$\Rightarrow \angle POA + \angle APO + 90^0 = 180^0$

$\Rightarrow \angle POA + 40^0 + 90^0 = 180^0$

$\Rightarrow \angle POA = 180^0 - 130^0 = 50^0$

2. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$ **[Ex 10.2, Q 8]**



Sol: - LHS: $AB + CD$

$= (AP + PB) + (CR + RD)$

$\left\{\begin{matrix} AP = AS, BP = BQ, CR = CQ, RD = DS \\ \text{Tangents from external point are equal} \end{matrix}\right\}$

$= AS + BQ + CQ + DS$

$= (AS + DS) + (BQ + CQ) = AD + BC$

3. Prove that the parallelogram circumscribing a circle is rhombus. **[Ex 10.2, Q 11]**

Sol:- Result of above example can be used directly.

$AB + CD = AD + BC$ … … … … i)

Since ABCD is ||gm (given)

$\therefore$ $AB = CD$ and $AD = BC$ … … … … … … . ii)

From i) & ii), we've

$AB + AB = AD + AD \Rightarrow 2AB = 2AD \Rightarrow AB = AD$

i.e. Two adjacent sides of a || gm are equal.

Hence ABCD is a rhombus