

DAY 4

SECTION FORMULA

When three points are collinear then one point divides the other two points in some ratio.
Let AB be a line segment such that $A(x_1, y_1)$ and $B(x_2, y_2)$ and $P(x, y)$ is any point on the line AB and divides it in the ratio $m : n$

Draw AL, PM, BN perpendiculars on the x-axis

Thus $AL \parallel PM \parallel BN$

$$\therefore OL = x_1, OM = x, ON = x_2$$

$$\Rightarrow LM = OM - OL = x - x_1 \text{ and}$$

$$NM = ON - OM = x_2 - x$$

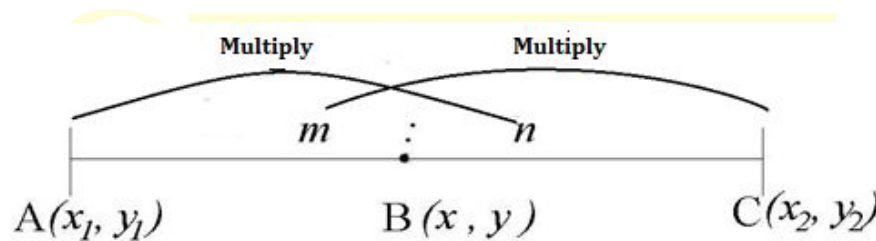
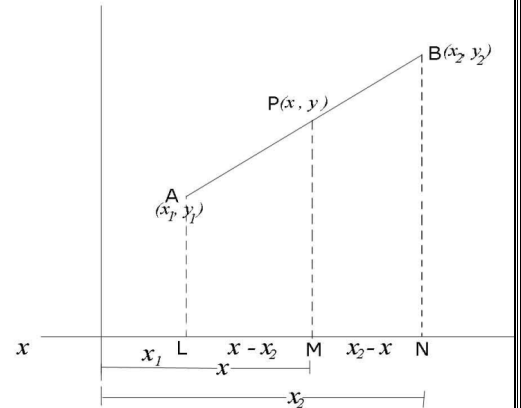
Hence by Intercept theorem ; we've $\frac{AP}{BP} = \frac{LM}{MN}$

$$\Rightarrow \frac{m}{n} = \frac{x - x_1}{x - x_2} \quad \Rightarrow m(x_2 - x) = n(x - x_1)$$

$$\Rightarrow mx_2 - mx = nx - nx_1 \Rightarrow mx_2 + nx_1 = nx + mx$$

$$\text{Or } mx_2 + nx_1 = (m + n)x \quad \text{or } x = \frac{mx_2 + nx_1}{m + n}$$

Similarly $y = \frac{my_2 + ny_1}{m + n}$ by drawing perpendiculars on y-axis



Thus the point of division P is $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$

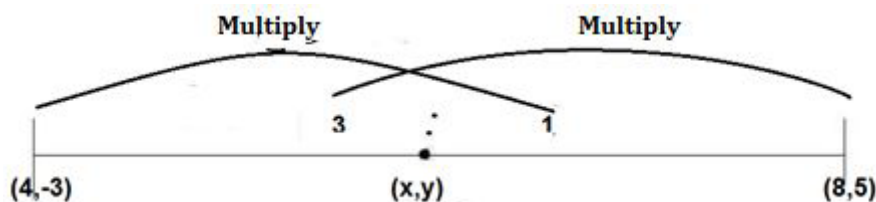
➤ **MID-POINT FORMULA**

In mid-point of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ divides the line segment in the ratio 1:1

Hence putting $m = 1$ and $n = 1$ in section formula, we get the co-ordinates of the mid-point as $\left(\frac{1 \times x_2 + 1 \times x_1}{1 + 1}, \frac{1 \times y_2 + 1 \times y_1}{1 + 1} \right)$ or $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

1. Find the co-ordinates of a point which divides the line segment joining the points (4, -3) and (8, 5) in the ratio 3:1 [Example 6]

Sol :- Let the point P(x, y) divides the line segment joining (4, -3) and (8, 5) in 3:1

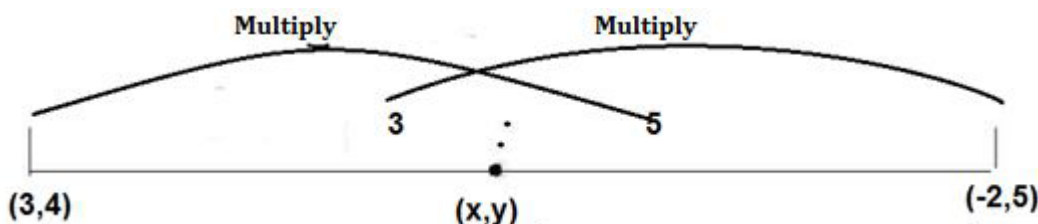


$$x = \frac{4+24}{3+1} = \frac{28}{4} = 7 \quad \text{and} \quad y = \frac{-3+15}{3+1} = \frac{12}{4} = 3$$

\therefore Required point is (7, 3).

2. Find the co-ordinates of a point which divides the line segment joining the points (3, 4) and (-2, 5) in the ratio 3:5

Sol :- Let the required point is P(x, y)



$$x = \frac{15+(-6)}{3+5} = \frac{15-6}{8} = \frac{9}{8} \quad \text{and} \quad y = \frac{20+15}{3+5} = \frac{35}{8}$$

\therefore Required point is $\left(\frac{9}{8}, \frac{35}{8}\right)$

3. Find the coordinates of mid point of line joining A(3, 4) and B(-1, 6)

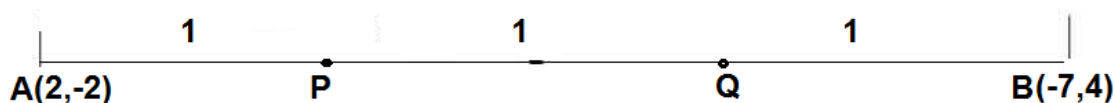
Sol :- Required mid point is $\left(\frac{3+(-1)}{2}, \frac{4+6}{2}\right) = \left(\frac{3-1}{2}, \frac{10}{2}\right) = \left(\frac{2}{2}, 5\right) = (1, 5)$

4. Find the coordinates of mid point of line joining A(-4, 5) and B(3, -2)

Sol :- Required mid point is $\left(\frac{-4+3}{2}, \frac{5+(-2)}{2}\right) = \left(\frac{-1}{2}, \frac{5-2}{2}\right) = \left(\frac{-1}{2}, \frac{3}{2}\right)$

5. Find the co-ordinates of the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4). [Example 8]

Sol :- Let P and Q be the points of trisection such that AP = PQ = QB



i) Clearly P divides AB in the ratio 1:2

$$\text{Co-ordinates of P are } \left(\frac{4+(-7)}{1+2}, \frac{-4+4}{1+2} \right) = \left(\frac{4-7}{3}, \frac{0}{3} \right) = \left(\frac{-3}{3}, 0 \right) = (-1, 0)$$

ii) Now Clearly Q divides the line segment AB in ratio 2:1

$$\text{Co-ordinates of Q are } \left(\frac{2+(-14)}{1+2}, \frac{-2+8}{1+2} \right) = \left(\frac{2-14}{3}, \frac{6}{3} \right) = \left(\frac{-12}{3}, 2 \right) = (-4, 2)$$

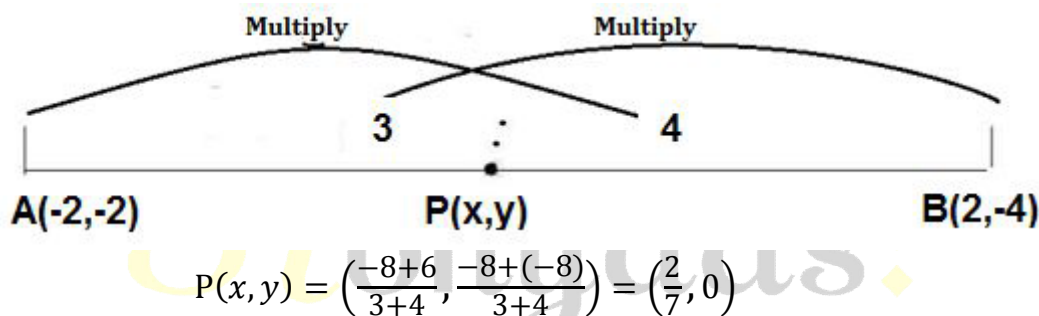
Hence the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4) are **(-1, 0) and (-4, 2)**

6. If A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that

$AP = \frac{3}{7}AB$ and P lies on the line segment AB.

[Ex 7.2, Q8]

Sol:- Given $\frac{AP}{AB} = \frac{3}{7} \Rightarrow$ AP has 3 parts and AB has 7 parts such that PB has left with 4 parts.
 \Rightarrow P divides AB in 3:4.



EXERCISE

- Find the co-ordinates of a point which divides the line segment joining the points (5, 4) and (10, 9) in the ratio 2:3
- Find the co-ordinates of a point which divides the line segment joining the points (-2, 3) and (4, -6) in the ratio 5:2
- Find the mid point of line joining the points (-3, -2) and (6, 9).
- Find the mid point of line joining the points (2, -3) and (-4, 5).
- Find the co-ordinates of the points of trisection of the line segment joining the points A(3, 4) and B(5, -2).
- Ex 7.2, Q1, 2, 9