DAY 11

1. Ritu can row downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of current. [Ex 3.6, Q2(i)]

Sol :- Let the speed of boat in still water $= x$ km/h and the speed of stream $= y$ km/h

$\therefore$ The **speed of boat in upstream** $= \left(\begin{matrix}\textbf{Speed of boat in}\\ \textbf{still water}\end{matrix}\right) - (\textbf{Speed of current})$

$= (x - y)$

and the speed of boat in downstream $= \left(\begin{matrix}\textbf{Speed of boat in}\\ \textbf{still water}\end{matrix}\right) - (\textbf{Speed of current})$

$= (x + y)$

First Condition: In 2 hours, she **can row downstream = 20 km**

$\Rightarrow 2 \times (x + y) = 20$ (Distance = speed × time)

$\Rightarrow x + y = \frac{20}{2} = 10$ ……. i)

Second Condition: In 2 hours, she **can row upstream = 4 km**

$\Rightarrow 2 \times (x - y) = 4$ (Distance = speed × time)

$\Rightarrow x - y = \frac{4}{2} = 2$ ……. ii)

Adding i) and ii), we get

$(x + y) + (x - y) = 10 + 2 \quad \Rightarrow 2x = 12 \quad \Rightarrow x = \frac{12}{2} = 6$

Put value of x in i), we get

i) $\Rightarrow \mathbf{6} + y = 10 \quad \Rightarrow y = 10 - 6 = 4$

Hence Speed of boat in still water be **6** km/h & speed of stream be **4** km/h

2. A boat goes 30 km upstream and 44 km downstream in 10 hours. It can go 40 km upstream and 55 km downstream in 13 hours. Find the speed of the boat in still water & speed of the stream. [Example 19]

Sol :-Let the speed of boat in still water $= x$ km/h & The speed of stream $= y$ km/h

$\therefore$ The speed of boat in upstream $= \left(\begin{matrix}\text{Speed of boat in}\\ \text{still water}\end{matrix}\right) - (\text{Speed of current}) = (x - y)$

and the speed of boat in downstream $= \left(\begin{matrix}\text{Speed of boat in}\\ \text{still water}\end{matrix}\right) - (\text{Speed of current}) = (x + y)$

First Condition: Time taken to go 30 km upstream $= \frac{30}{x-y}$ $\left(\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right)$

and time taken to go 44 km downstream $= \frac{44}{x+y}$

$\left(\begin{matrix}\textbf{Time taken to cover}\\ \textbf{30 km upstream}\end{matrix}\right) + \left(\begin{matrix}\textbf{Time taken to cover}\\ \textbf{44 km downstream}\end{matrix}\right) = \mathbf{10}$

$\Rightarrow \frac{30}{x-y} + \frac{44}{x+y} = 10$ ……. i)

Second Condition: Time taken to go 40 km upstream $= \frac{40}{x-y}$

and time taken to go 55 km downstream = $\frac{55}{x+y}$

$\left(\begin{matrix}\textbf{Time taken to cover}\\ \textbf{40 km upstream}\end{matrix}\right) + \left(\begin{matrix}\textbf{Time taken to cover}\\ \textbf{55 km downstream}\end{matrix}\right) = \mathbf{13}$

$\Rightarrow \quad \frac{40}{x-y} + \frac{55}{x+y} = 13$ ..... ii)

Substitute $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations i) & ii) , we get

i) $\Rightarrow 30u + 44v = 10$ ..... iii)

i) $\Rightarrow 40u + 55v = 13$ ..... iv)

To equate the coefficients of u, Multiply equation iii) by 4 & iv) by 3 and subtract, we get

$(120u + 176v) - (120u + 165v) = 40 - 39$

$\Rightarrow 11v = 1 \qquad \Rightarrow \boldsymbol{v = \frac{1}{11}}$ Replace in equation iii), we get

iii) $\Rightarrow \quad 30u + 44 \times \frac{\mathbf{1}}{\mathbf{11}} = 10 \qquad \Rightarrow \quad 30u = 10 - 4 = 6 \qquad \Rightarrow \quad u = \frac{6}{30} = \frac{1}{5}$

Since $\frac{1}{x-y} = u = \frac{1}{5} \qquad \Rightarrow \quad \boldsymbol{x - y = 5}$ ..... v)

and $\frac{1}{x+y} = v = \frac{1}{11} \qquad \Rightarrow \quad \boldsymbol{x + y = 11}$ ..... vi)

Adding both equations, we get $\qquad (x - y) + (x + y) = 5 + 11 = 16$

$\Rightarrow \quad 2x = 16 \qquad \Rightarrow \boldsymbol{x = 8}$ Replace this value in v)

v) $\Rightarrow \quad 8 - y = 5 \qquad \Rightarrow \boldsymbol{y = 3}$

Speed of boat in still water be 8 km/h & speed of stream be 3 km/h.

3. 2 women and 5 men can do a piece of work in 4 days. The same work is done in 3 days by 3 women and 6 men. How long would it take one woman & one man to do it?
[Ex 3.6, Q2(ii)]

Sol. Let 1 woman can finish the work in x days

So 1 woman's one day's work = $\left(\frac{1}{x}\right)^{th}$ part of work

and 1 boy can finish the work in y days

1 boy's one day's work = $\left(\frac{1}{y}\right)^{th}$ part of work

{This problem is based on inverse proportion in which if we increase number of persons to do any work that day will be completed early means it take less days, so first change this problem in direct proportion as mentioned above and then solve}

First Condition: 2 women and 5 men can do a piece of work = 4 days

In 1 day, 2 women and 5 men can do a piece of work = $\left(\frac{1}{4}\right)^{th}$ part of work

$\Rightarrow \quad \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$ ..... i)

Second Condition: **3 women and 6 men can do a piece of work = 3 days**

In 1 day, 3 women and 6 men can do a piece of work = $\left(\frac{1}{3}\right)^{th}$ part of work

$\Rightarrow \frac{3}{x}+\frac{6}{y}=\frac{1}{3}$..... ii)

To equate the coefficient of x, Multiply i) by 3 and ii) by 2 then subtract, we get.

$\Rightarrow \left(\frac{6}{x}+\frac{15}{y}\right)-\left(\frac{6}{x}+\frac{12}{y}\right)=\frac{3}{4}-\frac{2}{3} \Rightarrow \frac{3}{y}=\frac{9-8}{12}=\frac{1}{12} \Rightarrow \boldsymbol{y=36}$

Replace this value in equation ii), we get

ii) $\Rightarrow \frac{3}{x}+\frac{6}{\mathbf{36}}=\frac{1}{3} \Rightarrow \frac{3}{x}+\frac{1}{6}=\frac{1}{3}$

$\Rightarrow \frac{3}{x}=\frac{1}{3}-\frac{1}{6}=\frac{2-1}{6}=\frac{1}{6} \Rightarrow \boldsymbol{x=18}$

Hence one woman can do this work in 18 days and a man can do in 36 days.

4. Deepika travel 300 km to her home partly by train and partly by bus. She takes 4 hours if he travel 60 km by train and the rest by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately. **[Ex 3.6, Q2(iii)]**

Sol : Let the speed of the train be x km/h and Speed of bus be y km/h

Total distance travelled = 300 km

First condition: $\left(\begin{matrix}\textbf{Time taken to travel}\\ \textbf{60 km by train}\end{matrix}\right)+\left(\begin{matrix}\textbf{Time taken to travel}\\ \textbf{240 km by bus}\end{matrix}\right)=\mathbf{4}$

$\Rightarrow \frac{60}{x}+\frac{240}{y}=4$ or $\frac{15}{x}+\frac{60}{y}=1$..... i) {Divide both sides by4}

Second condition: $\left(\begin{matrix}\textbf{Time taken to travel}\\ \textbf{100 km by train}\end{matrix}\right)+\left(\begin{matrix}\textbf{Time taken to travel}\\ \textbf{200 km by bus}\end{matrix}\right)=\boldsymbol{4h\ 10\ min}$

$\Rightarrow \frac{100}{x}+\frac{200}{y}=4\frac{10}{60}=4\frac{1}{6}=\frac{25}{6}$

or $\frac{4}{x}+\frac{8}{y}=\frac{1}{6}$..... ii) {Divide both sides by 25}

To equate the co-efficients of x, multiply i) by 4 and ii) by 15 then subtract, we get

$\Rightarrow \left(\frac{60}{x}+\frac{240}{y}\right)-\left(\frac{60}{x}+\frac{120}{y}\right)=4-\frac{15}{6}=\frac{24-15}{6}=\frac{9}{6}=\frac{3}{2}$

$\Rightarrow \frac{120}{y}=\frac{3}{2} \Rightarrow \boldsymbol{y=80}$

Replace this value of y in i), we get $\frac{15}{x}+\frac{60}{80}=11$

$\Rightarrow \frac{15}{x}+\frac{3}{4}=1 \Rightarrow \frac{15}{x}=1-\frac{3}{4}=\frac{4-3}{4}=\frac{1}{4} \Rightarrow \boldsymbol{x=60}$

Hence speed of train is 60 km/h and speed of bus is 80 km/h