DAY 5

In last section we have discussed about integers, prime numbers and composite numbers. In this section we will discuss about irrational numbers.

IRRATIONAL NUMBERS:-

As we already discussed that irrational numbers are those numbers which cannot be expressed in the form of $\frac{p}{q}$, p & q are integers, $q \neq 0$. $eg. \sqrt{2}, \sqrt{3}, \sqrt{5}, \ldots..$ OR Which can be expressed as non-terminating Or non-recurring form like 0.212112111..., 1.242442444... etc..

1. The square root of every non-prefect square is irrational. e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{8}$ etc.
2. The cube root of non-prefect cubes are irrational..
3. Constant number π is irrational

Now this questions came in mind that value of π is $\frac{22}{7}$ and $\frac{22}{7}$ is rational

then how we can say that π is irrational. Actually $\frac{22}{7}$ or 3.14 or $\frac{355}{113}$, all these are not exact values, these are approximate value of π. Value of π upto 10 decimals is 3.1415926535…… It has been calculated to trillion digits, but no sign of recurrence of digits was found. So π is irrational number. ***Infact π is ratio of circumference of a circle to the length of diameter.***

4. Constant number $\boldsymbol{e}$ is also an irrational number.

PROPERTIES OF IRRATIONAL NUMBERS:-

1. The sum of a rational number & an irrational number is always irrational.
2. The product of non-zero rational number & an irrational number is always irrational.
3. The sum of two irrational numbers is not always an irrational number.
4. If a & b are rational & ab is not a perfect square then $\sqrt{ab}$ always lie between a & b.

Before discussing this topic, we must know very important theorem

- **If p is a prime number and p divides a^2 then p also divides a.**

1. Prove that $\sqrt{2}$ is not a rational number.

Sol:- we shall prove this by contradiction..

Suppose if possible $\sqrt{2}$ is an rational number

$\therefore \quad \sqrt{2} = \frac{p}{q}$ where p & q are integers having no common factor other than1, $q \neq 0$

Squaring ; $2 = \frac{p^2}{q^2}$ i.e $p^2 = 2q^2$ ....(i)

Since 2 is a factor of p^2

$\therefore$ 2 divides p^2 i.e 2 divides p

So $p = 2m$ ………(A) Put in (i)

(Dividend(p) =Quotient $\times$ Divisor + remainder = $2m + 0$; m is any prime factor)

(i) $\Longrightarrow$ $(2m)^2 = 2q^2$ $\Longrightarrow$ $2q^2 = 4m^2$ $\Longrightarrow$ $q^2 = 2m^2$

2 is factor of q^2 i.e. 2 divides q^2 so 2 divides q

$q = 2n$, …………..(B)

From (A) & (B).It can be found that 2 is common factor of p and q.

Which contradicts that p and q having no common factor other than 1.

Our supposition is wrong. $\sqrt{2}$ is an irrational number.

2. **Prove that $\sqrt{3}$ is not a rational number**.

Sol:- we shall prove this by contradiction..

Suppose if possible $\sqrt{3}$ is an rational number

$\therefore$ $\sqrt{3} = \frac{p}{q}$ where p & q are integers having **no common factor other than 1**, $q \neq 0$

Squaring ; $3 = \frac{p^2}{q^2}$ i.e $p^2 = 3q^2$ ……………..(i)

Since 3 is a factor of p^2 $\therefore$ 3 divides p^2 i.e 3 divides p

So $p = 3m$ ………(A) Put in (i) (m is any prime factor)

(i) $\Longrightarrow$ $(3m)^2 = 3q^2$ $\Longrightarrow$ $3q^2 = 9m^2$ $\Longrightarrow$ $q^2 = 3m^2$

3 is factor of q^2 i.e. 3 divides q^2 so 3 divides q

$q = 3n$, …………..(B)

From (A) & (B).It can be found that 3 is common factor of p and q.

Which contradicts that p and q having no common factor other than 1.

Our supposition is wrong. $\sqrt{3}$ is an irrational number.

3. **Prove $5 + \sqrt{6}$ is an irrational number.**

Sol:- Let $5 + \sqrt{6}$ be a rational number say r then $r = 5 + \sqrt{6}$

$r - 5 = \sqrt{6}$

Since r is a rational number so $r - 5$ is also a rational number.

but $\sqrt{6}$ is an irrational number.

Thus Rational = irrational Which is not possible

$\therefore 5 + \sqrt{6}$ is irrational.

Alternate Method: Here 5 is rational number and $\sqrt{6}$ is an irrational number.

We know that sum of rational and irrational number is always irrational number.

$\therefore 5 + \sqrt{6}$ is an irrational number.

4. **Prove $3\sqrt{2}$ is an irrational number.**

Sol:- Let $3\sqrt{2}$ be a rational number say r then $r = 3\sqrt{2}$

$\frac{r}{3} = \sqrt{2}$

Since r is a rational number so $\frac{r}{3}$ is also a rational number.

but $\sqrt{2}$ is an irrational number.

Thus Rational = irrational Which is not possible

$\therefore$ $3\sqrt{2}$ is irrational.

Alternate Method: Here 3 is rational number and $\sqrt{2}$ is an irrational number.

We know that product of rational and irrational number is always irrational number.

$\therefore$ $3\sqrt{2}$ is an irrational number.

5. Prove $3 - 2\sqrt{5}$ is an irrational number.

Sol:- Let $3 - 2\sqrt{5}$ be a rational number say r then $r = 3 - 2\sqrt{5}$

$\frac{r-3}{-2} = \sqrt{5}$

Since r is a rational number so $\frac{r-3}{-2}$ is also a rational number.

but $\sqrt{5}$ is an irrational number.

Thus Rational = irrational Which is not possible

$\therefore$ $3-2\sqrt{5}$ is irrational.

Alternate Method: Here $\sqrt{5}$ is irrational number and 2 is a rational number.

$\therefore$ $2\sqrt{5}$ is an irrational number and 3 is rational number.

We know that difference of rational and irrational number is always irrational number.

$\therefore$ $3 - 2\sqrt{5}$ is an irrational number.

Exercise

1. Prove that $\sqrt{5}, \sqrt{7}$ are irrational numbers
2. Prove that the following numbers are irrational numbers:

   (i) $4 + \sqrt{2}$ (ii) $5 - \sqrt{3}$ (iii) $2 + 5\sqrt{3}$ (iv) $5\sqrt{3}$ (v) $\frac{1}{\sqrt{2}}$