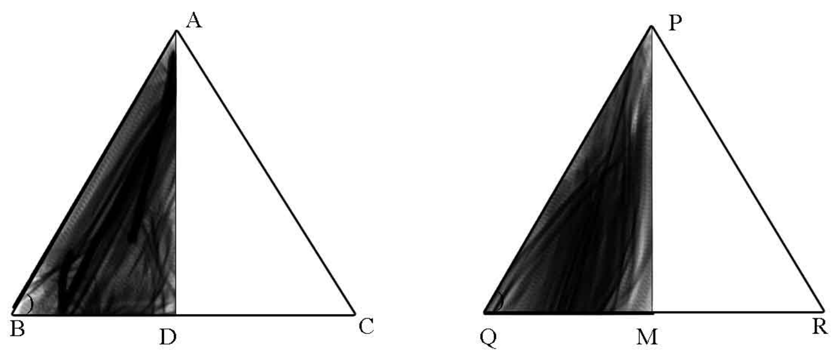
DAY 7

In last section, we have discussed about similarity of triangles and its applicability on sums. In this section we shall discuss about relationship between areas of similar triangles and their corresponding sides.

THEOREM: The ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



Proof:- **Given:** $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

and $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \ldots \ldots \ldots \ldots .$ i)

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Construction: Draw $AL \perp BC$ and $PM \perp QR$

Proof $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times QR \times PM} = \frac{BC}{QR} \times \frac{AL}{PM}$ ……… ii)

Now if we prove $\frac{BC}{QR} = \frac{AL}{PM}$ then we get our result; for this

In ΔABL & ΔPQM, we've

$\angle B = \angle Q$ (By i))

$\angle L = \angle M = 90^0$

$\therefore \Delta ABL \sim PQM$ (AA similarity)

$$\Rightarrow \frac{AL}{PM} = \frac{AB}{PQ} \ldots \ldots . . \text{iii)}$$

From i), ii) & ii), we get $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2}$…………. iii)

Using i), we get $\boldsymbol{\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}}$

Now lets discuss some examples.

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Abhyaas:

1. The areas of two similar triangles ΔABC and ΔPQR are 64 cm² and 121cm² respectively. If QR = 15.4 cm, find BC.

Sol :- Given $\Delta ABC \sim \Delta PQR$

By Area Theorem

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2} \qquad \Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \frac{8^2}{11^2} = \frac{BC^2}{(15.4)^2} \qquad \Rightarrow \frac{8}{11} = \frac{BC}{15.4} \qquad \Rightarrow BC = \frac{8 \times 15.4}{11} = 8 \times 1.4 = 11.2 \text{ cm.}$$

2. The areas of two similar triangles ΔABC and ΔDEF are 64 cm² and 196 cm² respectively. If DE = 8.4 cm, find AB.

Sol :- Given $\Delta ABC \sim \Delta DEF$

By Area Theorem

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} \qquad \Rightarrow \frac{64}{196} = \frac{AB^2}{(8.4)^2}$$

$$\Rightarrow \frac{8^2}{14^2} = \frac{AB^2}{(8.4)^2} \qquad \Rightarrow \frac{8}{14} = \frac{AB}{8.4} \qquad \Rightarrow AB = \frac{8 \times 8.4}{14} = 8 \times 0.6 = 4.8 \text{ cm.}$$

3. The areas of two similar triangles ΔABC and ΔPQR are 64 cm² and 100 cm² respectively. If EF = 10 cm, find BC.

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Sol :- Given $\Delta ABC \sim \Delta DEF$

By Area Theorem

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2} \qquad \Rightarrow \frac{64}{100} = \frac{BC^2}{(10)^2}$$

$$\Rightarrow \frac{8^2}{10^2} = \frac{BC^2}{(10)^2} \qquad \Rightarrow \frac{8}{10} = \frac{BC}{10} \qquad \Rightarrow BC = 8 \text{ cm.}$$

EXERCISE

1. Ex6.4, Q1,9