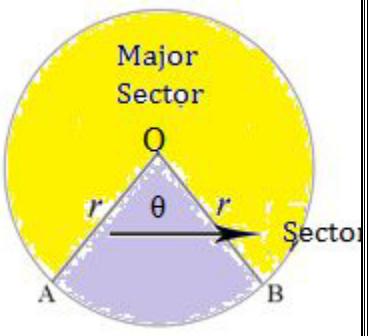
DAY 2

AREA OF A SECTOR OF A CIRCLE

Sector is the part which is covered with radii and their corresponding arc. Let AB be an arc of a circle whose centre is O then the region bounded by radii OA, OB and arc AB is called ***sector*** of the circle .The angle subtended by the two radii i.e $\angle AOB$ is the angle of the sector.

- If $\angle AOB$ is less than 180^0, the sector OACB is called ***minor sector*** and sector OADB is called ***major sector***. Angle of major sector is $(360^0 - \angle AOB)$
- Let us find the area of sector OAB of the circle having radius $\boldsymbol{r}$ and $\boldsymbol{\angle AOB} = \theta^0$



Area of circular region = πr^2

$i.e.$ Area of sector of degree measures $360^0 = \pi r^2$

$\therefore$ Area of sector of degree measures $\theta = \pi r^2 \times \frac{\theta}{360^0}$

Hence

$$\boldsymbol{Area\ of\ sector}(OAPB) = \frac{\boldsymbol{\pi r^2 \theta}}{\boldsymbol{360^0}} = (\boldsymbol{Area\ of\ Circle}) \times \frac{\boldsymbol{\theta}}{\boldsymbol{360^0}}$$

- Area of **Major** sector (OAQB) of degree measures $(360^0 - \theta) = \frac{\boldsymbol{\pi r^2 (360^0 - \theta)}}{\boldsymbol{360^0}}$

or (Area of major sector OAQB) = (Area of circle) − (Area of minor sector OAPB)

LENGTH OF AN ARC :-

Let APB is the corresponding arc of sector OAPB of circle with centre O subtends $\angle AOB$ at the centre.

Circumference of circular region = $2\pi r$

$i.e.$ Length of arc of degree measures $360^0 = 2\pi r$

$\therefore$ Length of arc of degree $\theta = 2\pi r \times \frac{\theta}{360^0} = \frac{\pi r \theta}{180^0}$

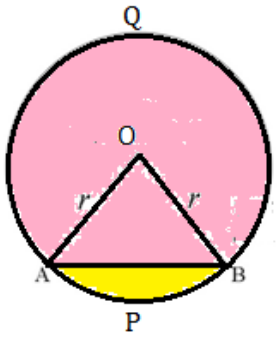
Hence $$\boldsymbol{Length\ of\ arc}(l) = \boldsymbol{Circumference\ of\ circle} \times \frac{\boldsymbol{\theta}}{\boldsymbol{360^0}} = \frac{\boldsymbol{\pi r \theta}}{\boldsymbol{180^0}}$$

- If length of arc l is given then area of sector $= \frac{1}{2} \times \boldsymbol{l} \times \boldsymbol{r}$

SEGMENT OF A CIRCLE:-

We know a chord divides the circular region into two regions. The segment which is less than semi circular region is called ***minor segment*** otherwise ***major segment***.

In the given figure, APBA is minor segment and AQBA is major segment of the circle.



- $\textbf{Area of Minor Segment APBA} = (\textbf{Area of sector OAPB}) - (\textbf{Area of } \Delta OAB)$

$$= \frac{\pi r^2 \theta}{360^0} - \frac{1}{2} r^2 sin\theta$$

[where $\frac{1}{2} r^2 sin\theta$ is the area of ΔOAB]

- $\textbf{Area of Major Segment AQBA} = (\textbf{Area of Circle}) - \left(\begin{matrix}\textbf{Area of minor} \\ \textbf{segment APBA}\end{matrix}\right)$

$$= \pi r^2 - \left(\frac{\pi r^2 \theta}{360^0} - \frac{1}{2} r^2 sin\theta\right)$$

Or

$$\textbf{Area of Major Segment AQBA} = (\textbf{Area of } \Delta OAB) + \left(\begin{matrix}\textbf{Area of major} \\ \textbf{sector AQBA}\end{matrix}\right)$$

$$= \frac{1}{2} r^2 sin\theta + \frac{\pi r^2 (360^0 - \theta)}{360^0}$$

Now lets discuss some examples.

1. Find the area of a sector of a circle with radius 6 *cm* if angle of the sector is 60^0. [Ex 12.2, Q1]

Sol:- Given $r = 6cm,\ \theta = 60^0$

Area of sector $= \frac{\pi r^2 \theta}{360^0} = \frac{22}{7} \times 7 \times 7 \times \frac{60^0}{360^0} = \frac{132}{7}$ cm²

2. Find the area of the sector of a circle with radius 4 *cm* if angle of the sector is 30^0. Also find the area of the corresponding major sector. [$\pi = 3.14$] [Example 2]

Sol:- Given $r = 4cm,\ \theta = 30^0$

Area of Minor sector $= \frac{\pi r^2 \theta}{360^0} = 3.14 \times 4 \times 4 \times \frac{30^0}{360^0} = \frac{1256}{300} = 4.187$ cm²

Area of Major sector $= \frac{\pi r^2 (360^0 - \theta)}{360^0} = 3.14 \times 4 \times 4 \times \frac{(360^0 - 30^0)}{360^0}$

$$= 3.14 \times 4 \times 4 \times \frac{330^0}{360^0} = 46.05 \text{ cm}^2$$

3. The length of a minute hand of a clock is 14 *cm.* Find the area swept by the minute hand in 5 minutes. [Ex 12.2, Q3]

Sol:- Length of a minute hand = radius of a circle $(r) = 14\ cm$

Minute hand complete one circle in 60 minutes.

Area covered by minute hand in 60 minutes $= \pi r^2$

Area covered by minute hand in 1 minute $= \frac{\pi r^2}{60}$

Area covered by minute hand in 5 minutes $= \frac{\pi r^2}{60} \times 5 = \frac{1}{12}\pi r^2$

$$= \frac{1}{12} \times \frac{22}{7} \times 14 \times 14 = \frac{154}{3}\ cm^2$$

4. A chord of a circle of radius 20*cm* subtends a right angle at the centre. Find i) Length of the arc ii) Area of the minor segment iii) Area of the major segment. $[\pi = 3.14]$

Sol:- Here $r = 20\ cm, \theta = 90^0$

i) Length of the arc $= \frac{\pi r\theta}{180^0} = 3.14 \times 20 \times \frac{90^0}{180^0} = \mathbf{31.4\ cm}$

ii) Area of Minor segment $= \frac{\pi r^2\theta}{360^0} - \frac{1}{2}r^2 sin\theta$

$$= 3.14 \times 20 \times 20 \times \frac{90^0}{360^0} - \frac{1}{2} \times 20 \times 20 \times sin90^0$$

$$= 314 - 200 = \mathbf{114\ cm^2}$$

iii) Area of Major segment $=$ (Area of circle) $-$ (Area of minor segment)

$$= \pi r^2 - 114 = 3.14 \times 20 \times 20 - 114$$

$$= 1256 - 114 = \mathbf{1142\ cm^2}$$

Alter Method:

Area of Major Segment $=$ (**Area of** Δ**OAB**) $+ \left(\begin{matrix}\textbf{Area of major}\\ \textbf{sector}\end{matrix}\right)$

$$= \frac{1}{2}r^2 sin\theta + \frac{\pi r^2(360^0 - \theta)}{360^0}$$

$$= \frac{1}{2} \times 20 \times 20 \times sin90^0 + 3.14 \times 20 \times 20 \times \frac{(360^0 - 90^0)}{360^0}$$

$$= 200 + 942 = \mathbf{1142\ cm^2}$$

EXERCISE

1. A chord of a circle of radius 10 *cm* subtends a right angle at the centre. Find
   i) Area of the sector ii) Length of the arc $[\pi = 3.14]$
2. The length of a minute-hand of a clock is 21 *cm* long. Find the area swept by the minute hand in 5 minutes.
3. Ex 12.2, Q 4,5,6,7