DAY 6

AREA Of A TRIANGLE

In earlier classes you've already learnt the formula for finding the area of a triangle i.e. Area of a triangle = $\frac{1}{2}$ ×base ×height or HERON's Formula

These formulas are applicable when we have base, height or sides of triangle. In this section, we shall learn the method of finding the area of the triangle when the co-ordinates of its vertices are given:

Suppose vertices $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ of ΔABC are given.
Draw AM, BL and CN perpendiculars to $x - axis$. From the figure it is observed that

$$\boldsymbol{ar}(\Delta \mathrm{ABC}) = (\boldsymbol{area\ of\ trapezium}\ \mathbf{BLMA} + \boldsymbol{area\ of\ trapezium}\ \mathbf{AMNC}) - \boldsymbol{area\ of\ trapezium}\ \mathbf{BLNC}$$

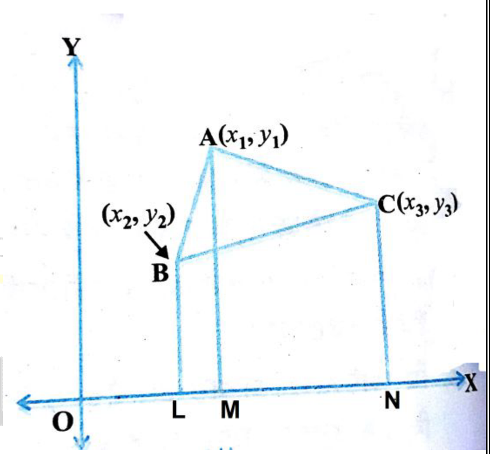
$$ar(\Delta ABC) = \frac{1}{2} \times (\mathrm{BL} + \mathrm{AM}) \times \mathrm{LM} + \frac{1}{2} \times (\mathrm{AM} + \mathrm{CN}) \times \mathrm{MN} - \frac{1}{2} \times (\mathrm{BL} + \mathrm{CN}) \times \mathrm{LN}$$

$$= \frac{1}{2}(y_2 + y_1)(x_1 - x_1) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$$

$$= \frac{1}{2}[y_1x_3 - y_3x_1 + y_3x_2 - y_2x_3 - y_1x_2 + y_2x_1]$$

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Which is the required area and it has always positive value.



- To write down this we have one easy method ***Arrow method***

$$\frac{1}{2}\left| \begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{array} \right|$$

$$= \frac{1}{2}\begin{bmatrix} (Sum\ of\ multiply\ of\ downwards\ arrow) - \\ (Sum\ of\ multiply\ of\ upwards\ arrow) \end{bmatrix}$$

$$= \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)]$$

1. **Find the area of triangle whose vertices are $(\mathbf{1}, -\mathbf{1}), (-\mathbf{4}, \mathbf{6})$ and $(-\mathbf{3}, -\mathbf{5})$.**

[Example 11]

Sol :- Area of triangle =

$$\frac{1}{2}\left| \begin{array}{cccc} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{array} \right|$$

$$= \frac{1}{2}[\{1 \times 6 \ + (-4) \times (-5) + (-3) \times (-1)\} - \ \{(-1) \times (-4) + 6 \times (-3) + (-5) \times 1\}]$$
$$= \frac{1}{2}[(6 + 20 + 3) - (4 - 18 - 5)] = \frac{1}{2}[29 - (-19)]$$
$$= \frac{1}{2}[29 + 19] = \frac{1}{2} \times 48 = 24 \text{ sq.units}$$

2. **Find the area of triangle whose vertices are $\mathbf{A(5, 1), B(4, 7)}$ and $\mathbf{C(7, -4)}$.**

Sol :- Area of triangle =

$$\frac{1}{2}\left|\begin{array}{cccc} 5 & 4 & 7 & 5 \\ 1 & 7 & -4 & 1 \end{array}\right|$$

$$= \frac{1}{2}[\{5 \times 7 \ + 4 \times (-4) + 7 \times 2\} - \ \{2 \times 4 + 7 \times 7 + (-4) \times 5\}]$$
$$= \frac{1}{2}[(35 - 16 + 14) - (8 + 49 - 20)] = \frac{1}{2}[33 - 37]$$
$$= \frac{1}{2}[-4] = -2$$

But area is always positive so required area is 2 sq.units

Collinear Points: In Ex 7.1, we have discussed about collinear points by Distance Formula. $\boldsymbol{i.e}$. Sum of any two distances is same as third distance.

Here we shall discuss this topic with the help of Are of Triangle formula. We know Triangle is a closed figure made by non-collinear points.

$\boldsymbol{If\ Points\ are\ collinear\ then\ Area\ of\ triangle = 0}$

3. **Check whether the points $\mathbf{P(-1.5, 3), Q(6, -2)}$ and $\mathbf{R(-3, 4)}$ are collinear.**

[Example13]

Sol:- Area of triangle =

$$\frac{1}{2}\left|\begin{array}{cccc} -1.5 & 6 & -3 & -1.5 \\ 3 & -2 & 4 & 3 \end{array}\right|$$

$$= \frac{1}{2}[\{(-1.5) \times (-2) \ + 6 \times 4 + (-3) \times 3\} - \{3 \times 6 + (-2) \times (-3) + 4 \times (-1.5)\}]$$
$$= \frac{1}{2}[(3 + 24 - 9) - (18 + 6 - 6)] = \frac{1}{2}[18 - 18] = 0$$

Hence given points are collinear.

4. **Show that $\mathbf{(1, -1), (2, 1)}$ and $\mathbf{(4, 5)}$ are collinear.**

Sol:- Area of triangle =

$$\frac{1}{2}\left|\begin{array}{cccc} 1 & 2 & 4 & 1 \\ -1 & 1 & 5 & -1 \end{array}\right|$$

$= \frac{1}{2}[\{1 \times 1 + 2 \times 5 + 4 \times (-1)\} - \{(-1) \times 2 + 1 \times 4 + 5 \times 1\}]$

$= \frac{1}{2}[(1 + 10 - 4) - (-2 + 4 + 5)] = \frac{1}{2}[7 - 7] = 0$

Hence given points are collinear.

5. For what value of k will the points $(2, 3)$, $(4, k)$ and $(6, -3)$ be collinear.

Sol:- Given points are collinear

$\Rightarrow$ **Area of triangle $= 0$**

$\Rightarrow \frac{1}{2}[\{2 \times k + 4 \times (-3) + 6 \times 3\} - \{3 \times 4 + k \times 6 + (-3) \times 2\}] = 0$

$\Rightarrow \frac{1}{2}[(2k - 12 + 18) - (12 + 6k - 6)] = 0$

$\Rightarrow \frac{1}{2}[(2k + 6) - (6k + 6)] = 0 \qquad \Rightarrow 2k + 6 - 6k - 6 = 0$

$\Rightarrow -4k = 0 \qquad \Rightarrow k = \frac{0}{-4} = 0$

Hence required value of $k = 0$

EXERCISE

1. **Ex 7.3. Q1,2**