

DAY 6

In last section, we have discussed about combining of two or more than two solids. Now in this section, we shall discuss about conversion of one solid into another solid or melting one into other.

CONVERSION OF SOLIDS FROM ONE SHAPE TO ANOTHER:-

In our daily life, many times we need to convert a solid into another solid of different shape. We melt gold and silver to form various types of ornaments. In such process, we assume that there is no wastage when such conversions are done. In conversion, **the volume of the first solid is equal to the volume of the new formations.** In this section, we shall discuss such problems:-

1. A cone of height 24cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere. [Example 8]

Sol:- Given height of the cone (h) = 24cm and radius (r) of base of cone = 6 cm

Let R be the radius of the sphere.

According to Given condition: The child reshapes the cone into the sphere

\therefore Volume of the Cone = Volume of the sphere

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow \frac{1}{3} \times \pi \times 6 \times 6 \times 24 = \frac{4}{3}\pi R^3$$

$$\Rightarrow R^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24 \times \frac{3}{4\pi}$$

$$\Rightarrow R^3 = 6^3 \quad \Rightarrow R = 6 \text{ cm}$$

\therefore Radius of sphere is **6 cm**

2. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire. [Example 10]

Sol:- Given Diameter of the rod = 1cm \Rightarrow radius of rod = $\frac{1}{2}$ cm

and length(height) of wire = 8cm

Now length (height) of wire = 18 m = 1800 cm and let R be the radius of the wire

According to given condition: Copper rod is drawn into a wire

\therefore Volume of the rod = Volume of the wire

$$\Rightarrow \pi r^2 h = \pi R^2 H$$

$$\Rightarrow \pi \times \frac{1}{2} \times \frac{1}{2} \times 8 = \pi \times R^2 \times 1800$$

$$\Rightarrow R^2 = \frac{2}{1800} = \frac{1}{900} = \left(\frac{1}{30}\right)^2$$

$$\Rightarrow R = \frac{1}{30} \text{ cm}$$

\therefore Thickness of wire = $2r = 2 \times \frac{1}{30} = \frac{1}{15} \text{ cm}$

3. A metallic sphere whose radius is 3 cm, is melted and recast into a right circular cone. If the radius of base of cone is $\frac{3}{2}$ cm, find the height of the cone.

Sol:- Given radius of the sphere(R) = 3cm.

and radius (r) of cone = $\frac{3}{2}$ cm and Let h be the height of the cone.

According to given condition: Metallic sphere is melted into a cone

\therefore Volume of the sphere = Volume of the Cone

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \frac{4}{3} \times \pi \times 3 \times 3 \times 3 = \frac{1}{3} \pi \times \frac{3}{2} \times \frac{3}{2} \times h$$

$$\Rightarrow h = 36\pi \times \frac{4}{3\pi} = 48 \text{ cm}$$

\therefore Height of cone is **48 cm**

4. A metallic right circular cylinder whose radius of base is 7 cm and height 20 cm is melted and recast into a cuboid. If the length of the cuboid is 22cm, breadth is 14 cm then find the height of the cuboid.

Sol:- Given radius of the cylinder (R) = 7cm, height of cylinder (H) = 20cm

and cuboid has length (l) = 22 cm and breadth(b) = 14cm and h be the height.

According to given condition: Metallic Cylinder is melted into a Cuboid

\therefore Volume of the Cylinder = Volume of the Cuboid

$$\Rightarrow \pi R^2 H = lbh$$

$$\Rightarrow \frac{22}{7} \times 7 \times 7 \times 20 = 22 \times 14 \times h$$

$$\Rightarrow h = \frac{22 \times 7 \times 20}{22 \times 14} = 10 \text{ cm}$$

\therefore Height of cuboid is **10 cm**

5. Metallic spheres of radii 6cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol:- Given radii of small spheres $r_1 = 6 \text{ cm}$, $r_2 = 8 \text{ cm}$ and $r_3 = 10 \text{ cm}$, and R be radius of larger sphere.

According to given condition: 3 small spheres melted to a larger sphere.

\therefore Volume of 3 smaller spheres = Volume of the larger sphere

$$\Rightarrow \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi R^3$$

Divide by $\frac{4}{3}\pi$ both sides, we get

$$r_1^3 + r_2^3 + r_3^3 = R^3$$

$$\Rightarrow 6^3 + 8^3 + 10^3 = R^3$$

$$\Rightarrow R^3 = 216 + 512 + 1000 = 1728 = 12^3$$

$$\Rightarrow R = 12 \text{ cm}$$

\therefore Radius of larger sphere is **12 cm**

EXERCISE

1. A metallic sphere of radius 5 cm is melted and recast into a right circular cone of height 20 cm. Find the radius of base of the cone.
2. A cone is $\frac{32}{9}$ cm high and the radius of base is 3 cm. It is melted and recast into a sphere. Find the radius of sphere.
3. The diameter of a metallic sphere is 3 cm. it is melted and drawn into a wire having diameter of the circular cross section as 0.2 cm. Find the length of the wire.
4. Ex 13.3, Q1,2

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