DAY 10

1. **A container, open from the top made up of a metal sheet is in the form of a frustum of a cone of height 16cm with radii of its lower and upper ends as 8cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹20 per litre and the cost of the metal sheet used, if it costs ₹ 8 per 100 cm² (Use $\pi = 3.14$)** **[Ex 13.4. Q4]**

Sol:- Here $R = 20\ cm, r = 8\ cm, h = 16\ cm$

i) Volume of milk in the container $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$$= \frac{1}{3} \times 3.14 \times 16(20^2 + 8^2 + 20 \times 8)$$
$$= \frac{1}{3} \times 3.14 \times 16(400 + 64 + 160)$$
$$= \frac{1}{3} \times 3.14 \times 16 \times 624 = 10449.92\ cm^3 = \frac{10449.92}{1000} l = 10.45\ l \text{ (app)}$$

$\therefore$ Quantity of milk $= 10.45\ l$

Cost of the milk $= 10.45 \times 20 = 209$ ₹

ii) Now $l = \sqrt{h^2 + (R-r)^2} = \sqrt{16^2 + (20-8)^2} = \sqrt{256 + 144} = \sqrt{400} = 20\ cm$

Surface Area of the container (Excluding upper end) $= \pi l(R + r) + \pi r^2$

$$= 3.14 \times 20 \times (20 + 8) + 3.14 \times 8^2$$
$$= 3.14 \times 20 \times 28 + 3.14 \times 64 = 1959.36\ cm^2$$

$\therefore$ Cost of the metal sheet used $= 1959.36 \times \frac{8}{100} = 156.75$ ₹

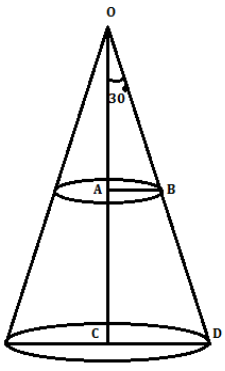
2. **A metallic right circular cone 20 *cm* high and whose vertical angle is 60⁰ is cut into two parts at the middle point of its height by a plane parallel to the base. If the frustum, so obtained, be drawn into a wire of diameter $\frac{1}{16}$ *cm*, Find the length of the wire.** **[Ex 13.4, Q5]**

Sol:- Given cone is divided into two equal parts and the frustum of cone is drawn into a wire.

First to find both radii of bases of frustum of cone.

Given vertical angle is 60^0 $\Rightarrow \angle AOB = 30^0$

and height of the cone $= 20\ cm$ then height of each part $= 10cm$.



In ΔAOB

$$\frac{AB}{OA} = \tan 30^0 \quad \Rightarrow \frac{r}{10} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad r = \frac{10}{\sqrt{3}}$$

In ΔOCD

$$\frac{CD}{OC} = \tan 30^0 \quad \Rightarrow \frac{R}{20} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad R = \frac{20}{\sqrt{3}}$$

So Frustum of cone has dimensions $r = \frac{10}{\sqrt{3}} cm$, $R = \frac{20}{\sqrt{3}} cm$

and $h = 10\ cm$

Given frustum of cone is drawn into a wire of diameter $\frac{1}{16}\,cm$

Let r_1 be radius of the wire (cylinder) $= \frac{1}{16\times 2} = \frac{1}{32}$ and height be H

Given condition: Volume of frustum of cone = Volume of wire

$$\Rightarrow \frac{1}{3}\pi h(R^2 + r^2 + Rr) = \pi {r_1}^2 \mathrm{H}$$

$$\Rightarrow \frac{1}{3}\times \pi \times 10\left\{\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \frac{20}{\sqrt{3}}\times\frac{10}{\sqrt{3}}\right\} = \pi \times \left(\frac{1}{32}\right)^2 \times \mathrm{H}$$

$$\Rightarrow \frac{1}{3}\times \pi \times 10\left\{\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right\} = \pi \times \frac{1}{32}\times\frac{1}{32}\times \mathrm{H}$$

$$\Rightarrow \frac{1}{3}\times 10 \times \frac{700}{3} = \frac{1}{32}\times\frac{1}{32}\times \mathrm{H}$$

$$\Rightarrow \mathrm{H} = \frac{10\times 700\times 32\times 32}{3\times 3} = \frac{7168000}{9} = 796444.4\ cm \text{ or } \boldsymbol{7964.4\ m}$$