DAY 11

1. In an equilateral ΔABC, D is a point on BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$. **[Ex 6.5, Q15]**

Sol:- Let side of equilateral ΔABC is a. $\therefore BD = DC = \frac{1}{3}BC = \frac{1}{3}a$

Draw $AL \perp BC$ $\Rightarrow BL = LC = \frac{a}{2}$

Now $DL = BL - BD = \frac{a}{2} - \frac{a}{3} = \frac{3a-2a}{6} = \frac{a}{6}$

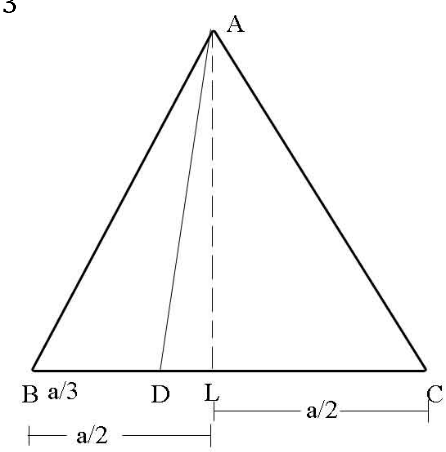
and In ΔACL, we have

$$AC^2 = AL^2 + CL^2$$

$$\Rightarrow a^2 = AL^2 + \left(\frac{a}{2}\right)^2$$

$$\Rightarrow a^2 = AL^2 + \frac{a^2}{4}$$

$$\Rightarrow AL^2 = a^2 - \frac{a^2}{4} = \frac{4a^2-a^2}{4} = \frac{3a^2}{4}$$



In ΔADL, we have

$$AD^2 = AL^2 + DL^2 = \frac{3a^2}{4} + \left(\frac{a}{6}\right)^2$$

$$= \frac{3a^2}{4} + \frac{a^2}{36} = \frac{27a^2+a^2}{36} = \frac{28a^2}{36}$$

$$\Rightarrow AD^2 = \frac{7a^2}{9} \quad \Rightarrow \quad 9AD^2 = 7a^2 = 7AB^2$$

2. O is any point in the interior of rectangle ABCD. Prove that $OB^2 + OD^2 = OC^2 + OA^2$ **[Example 14]**

Sol : Given :- O is any point in the interior of rectangle ABCD.

To Prove : $OB^2 + OD^2 = OC^2 + OA^2$

Construction Through O, Draw a segment $EF \| BC \| AD$.

Proof :- Since $EF \| BC \| AD$ and ADFE and BCFE are rectangles

{In this sum, take right triangles according to what to prove *e.g.* for OB^2 take ΔOBE, for OD^2 take ΔODF, for OA^2 take ΔOAE, for OC^2 take ΔOCF}

In right ΔOBE, we've

$OB^2 = OE^2 + EB^2$ (Pythagoras Theorem) i

and In ΔODF, we've

$OD^2 = OF^2 + FD^2$ ....ii)

Adding i) & ii), we get

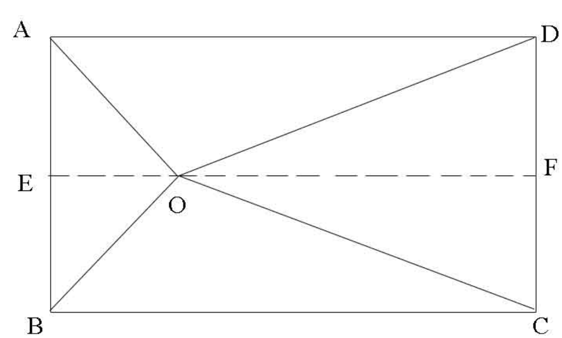
$OB^2 + OD^2 = OE^2 + EB^2 + OF^2 + FD^2$

$\left\{\begin{array}{l}\text{ADFE and BCFE are rectangles} \\ \text{So } EB = FC \text{ and } AE = FD\end{array}\right\}$

$= OE^2 + FC^2 + OF^2 + AE^2$

$= (OE^2 + AE^2) + (OF^2 + FC^2) = OA^2 + OC^2$

Hence the result



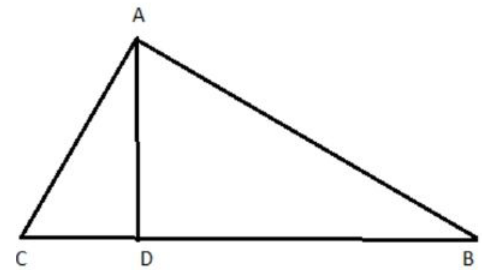
3. **The perpendicular from A on the side BC of ΔABC intersect BC at D such that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$ [Ex 6.5, Q14]**

Sol :

Prove :$2AB^2 = 2AC^2 + BC^2$ or $2AB^2 - 2AC^2 = BC^2$ or $2(AB^2 - AC^2) = BC^2$

Proof :- In right ΔABD, we've

$AB^2 = AD^2 + BD^2$ (Pythagoras Theorem) ……i)



and In ΔACD, we've

$AC^2 = AD^2 + CD^2$ ………..ii)

Subtracting ii) from ii), we get

$AB^2 - AC^2 = (AD^2 + BD^2) - (AD^2 + CD^2)$

$= AD^2 + BD^2 - AD^2 - CD^2$

$= BD^2 - CD^2 = (3CD)^2 - CD^2$

$= 9CD^2 - CD^2 = 8CD^2$ {Since $BC = BD + CD = 3CD + CD = 4CD$}

$= 8\left(\frac{BC}{4}\right)^2 = 8 \times \frac{BC^2}{16} = \frac{BC^2}{2}$

$\Rightarrow$ $\mathbf{2(AB^2 - AC^2) = BC^2}$

Hence the result