Homework 5

Problem 1

1.1 There are 38^5 outcomes, since we count the sequence of outcomes.

$38^5 = 38 \times 38 \times 38 \times 38 \times 38 = 79235168$

1.2 The outcomes of the same color:

$18^5 + 18^5 + 2^5 = 3779168$

1.3 Five times. Because all 38 slots are numbered, and the marble would land on one of the slots after we spun the wheel. After 5 times, the marble would be landed on one of the slots for at least 5 times.

1.4 There are 52×51 ways to deal the cards. Since the cards are distinct from each other.

$52 \times 51 = 2652$

1.5 There are 4 Aces, $3 \times 4 = 12$ face cards, 4 ten-cards. Thus, there are $2 \times 4 \times 16 = 128$ ways.

1.6 There are 13 spades, and 12 face cards

If the first card is not spade-face:

$10 \times 12 = 120$

If the first card is spade-face:

$3 \times 11 = 33$

So all outcomes are $120 + 33 = 153$

1.7 There are 13 spades, 2 one-eyed jack, 11 valid face cards.

$13 \times 11 \times 2 = 286$

Problem 2

2.1 There are 11 songs in total.

$11^7 = 19487171$

2.2 $6^7 + 5^7 = 358061$

2.3 $6^7 + 6^8 + 6^9 + 6^{10} = 72503424$

2.4 We can calculate the total number and subtract the cases where only one artist is used.

$11^7 - (6^7 + 5^7) = 19129110$

2.5 There are 10 songs left. And the choices to play "US" in the sequence are 7.

$10 \times 7 = 70$

2.6 $n(11, 7) = 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 = 1663200$

2.7 ① Spektor first:

$6 \times 5 \times 5 \times 4 \times 4 \times 3 \times 3 = 21600$

② Michaelson first:

$5 \times 6 \times 4 \times 5 \times 3 \times 4 \times 2 = 14400$

③ So the total ways $= 21600 + 14400 = 36000$

Problem 3

3.1 $\binom{6}{4} = \frac{6!}{4!\,2!} = \frac{6\times 5}{2} = 15$

3.2 $\binom{6}{2} = \binom{6}{4} = 15$

3.3 $\binom{200}{199} = \frac{200!}{199!\cdot 1!} = 200$

3.4 $P(6,4) = \frac{6!}{2!} = 6\times 5\times 4\times 3 = 360$

3.6 $P(26,2) = \frac{26!}{24!} = 26\times 25 = 650$

Problem 4

4.1 $2^{10} = 1024$

4.2 There are no 1, all digits are 0 = 1 way.
The answer = $1024 - 1 = 1023$

4.3 There are only 1 or 0 = 2 ways.
The answer: $1024 - 2 = 1022$

4.4 It depends on where the ones are. There are total 10 places, and 5 ones.

$$\binom{10}{5} = \frac{10!}{5!\,5!} = 252$$

4.5 Begins with 0 = there are 9 places left.

$$2^9 = 512$$

And the ways start with 0 and exactly 5 ones =

$$\binom{9}{5} = 126$$

Thus = $252 + 512 - 126 = 638$