Homework # 7

Assigned: Thursday October 24, 2024
Due: Thursday October 29, 2024 @ 11:59:00 pm

Instructions:

- Homework is due on Tuesday at 11:59 pm Boston Time. Homeworks received up to 24 hours late will be accepted with no penalty. *NO* assignment will be accepted after that.

- We expect that you will study with friends and often work out problem solutions together, but *you must write up your own solutions, in your own words*. **Cheating will not be tolerated**. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

- Assignments should be typed using Word or LaTeX, or hand-written *neatly*. Do write and submit your answers as if they were a professional report. There will be point deductions if the submission isn't neat (is disordered, difficult to read, scanned upside down, etc. . . .). When submitting to gradescope be sure to indicate the page containing your answer to each problem.

- *To get full credit, SHOW YOUR WORK!*

Problem 1 [20 pts (4,4,6,6)]: **Boston Cold Weather**

   Steve has lived in Boston for a long time. From experience, he knows that each day has a 5% chance to be snowy. Steve looks ahead to the next 10 days.

   i. What is the probability none of those days will be snowy?

   ii. What is the probability at least one of those days will be snowy?

   iii. What is the probability exactly 3 of those days will be snowy?

   iv. Steve decides that, without checking the weather, he will wear his heavy coat to work 3 of those days. What is the probability Steve will wear his coat on at least one snowy day?

Problem 2 [20 pts (6,7,7)] **Reading is probably good for you.**

   A book show consists of exhibits by three publishers: Penguin, Random House, and Springer. Penguin is providing 35% of the titles, Random House providing 25%, and Springer is providing the remainder. Each publisher's exhibit consists of fiction and non-fiction titles. Penguin's exhibit has 80% fiction titles, whereas Random House's only has 50% and Springer's just 30%. (Drawing a tree of possibilities might be helpful.)

   1. Calculate the probability that a randomly selected title is a work of fiction published by Springer.

   2. Calculate the probability that a randomly selected title is a work of fiction.

   3. Suppose that a randomly selected title is a fiction book. What is the probability that it was published by Penguin?

Problem 3 [20 pts (5,7,8)]: **Time Travel**

   Scientists recently discovered that time travel might be possible. In fact, there is a 2% chance it is already being used. They suggest that the appearance of an aurora borealis could indicate that time travel has happened. Last year, there were 13 days where an aurora borealis appeared. The scientists estimate that, if time travel occured, there is a 20% chance that it caused an aurora borealis.
The events are defined as follows:
A = Aurora borealis appeared
T = Time travel occured

   i. what is the probability an aurora borealis did not appear knowing that time travel occurred?

   ii. What is the probability time travel occurred given that an aurora borealis appeared in the sky?

   iii. What is the probability an aurora borealis appeared knowing that no time travel occurred?

Problem 4 [20 pts (10,10)]: **Thinking Independently.**

1. A fair coin is flipped 4 times. Let E be the event that the first flip is heads. Let F be the event that the coin landed on heads an even number of times. Are E and F independent? Show your work.

2. A family has three children. Assume that having either a boy or a girl is equally likely. Let the event E be the event that the family has children of both sexes, and F be the event that the family has at most 1 boy. Are the events E and F independent?

Problem 5 [20pts (10,10)]**: Johnny's Coin**
   Johnny has a coin that is three times more likely to land on heads than tails.

   i. How many heads should Johnny expect to see after 4 coin flips?

   ii. Johnny likes to play a game with strangers. He chooses between his biased coin and a fair coin at random. He then flips the chosen coin four times, recording the result of each flip. Finally, he has strangers guess which coin he used.
   You choose to play. Johnny chooses his coin and gets HHTH. Which coin was he most likely to use given the outcome of the flips? Support your answer using probability. (Hint: Compare the probabilities of getting the outcome of the flips given you chose an unbiased coin versus a fair coin.)