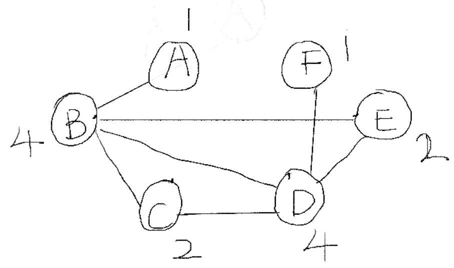
Problem 1

1.1 The vertices = A B C D E F

The edges = AB, BC, BD, BE, CD, DE, DF



1.2. 6 vertices = A B C D E F

1.3. 7 edges

1.4 It is connected

1.5 It is connected. So there is only one component in the graph.

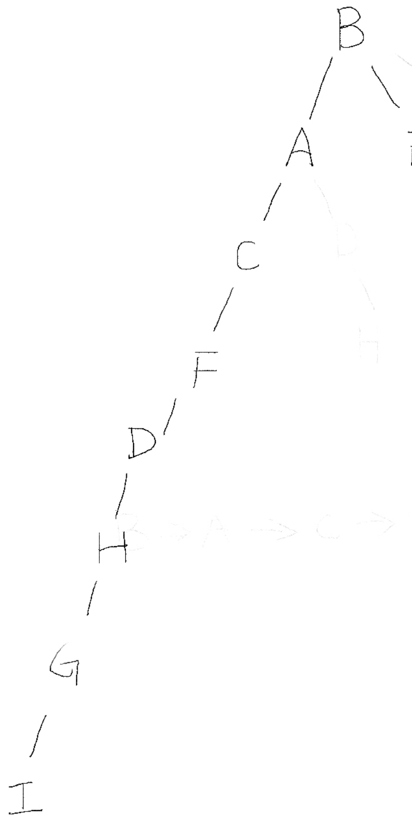
1.6 2, A and F

1.7 Yes. A → B → E → D → B → C → D → F

1.8 No. Not all vertices have even degrees.

Problem 2

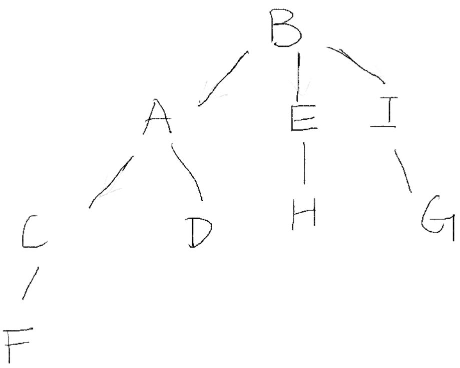
2.1



order
B→A→C→F→D→H G→I→E

2.2 B→A→C→F

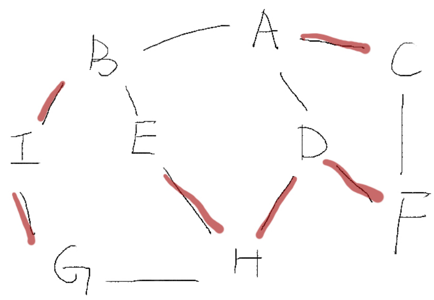
2.3



order
B→A→E→I C D H G F

2.4 B→A→C→F

2.5 All edges: AC = 10
IB = 11
DF = 13
EH = 17
IG = 19
DH = 20



$10+11+13+17+19+20 = 90$

The lowest cost = 90.

It can be called Minimum spanning tree.

2.6

		L(A)	L(C)	L(D)	L(E)	L(F)	L(G)	L(H)	L(I)
Start B		$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$	∞
BI	I	21_B	$\cancel{\infty}$	$\cancel{\infty}$	23_B	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$	11_B
BA	A	21_B	$\cancel{\infty}$	$\cancel{\infty}$	23_B	$\cancel{\infty}$	30_I	$\cancel{\infty}$	11_B
BAC	C	21_B	31_A	37_A	23_B	$\cancel{\infty}$	30_I	$\cancel{\infty}$	11_B
BAD	D	21_B	31_A	37_A	23_B	$\cancel{53_C}$	30_I	∞	11_B
BADH	H	21_B	31_A	37_A	23_B	50_C	30_I	51_D	11_B

Problem 3

3.1 There are V vertices in a complete graph.

You can start from every vertice.

It's a full permutation.

And the same circuit will be added twice.

Thus, $\frac{(V-1)!}{2}$

3.2 $\Sigma \text{degree}(V) = 2|E| = 4V$

$E = 2V$

3.3 The complete graph $= \frac{V(V-1)}{2}$

Thus, the answer is $\frac{V(V-1)}{2} - E$

Problem 4

4.1 There's only one edge.

Thus $|E| = p \times 1 = p$

4.2 First find all possible edges.

For complete graph (any pair of vertices in the graph), there are $\frac{n(n-1)}{2}$ edges.

Thus, the expectation is $\frac{n(n-1)}{2}p$.

Problem 5

5.1 $5 \times 9 = 45$

45 is an odd number.

$\Sigma(\text{degree}) = 2|E|$ is an even number.

So it is impossible.

5.2.

If all degrees are different. (0–71)

But if one vertex's degree is 0,

it is impossiple there is one vertex's degree is 71.

But there are 72 people (vertices)

So there exist two vertices have same degree.

5.3. According to handshake lemma, the number of vertices of odd degree must be even.

So if there is one odd-degree vertex, in the component, there must be another odd-degree vertex.

And a component is connected.

Thus any vertex with odd degree there exists a path from it to another vertex with odd degree.