CS5002 Homework # 1

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1 Problem 1

1.1 i. Convert 200_{10} to binary and hexadecimal

Step 1: Convert the decimal number to binary number
To convert the decimal number to binary, we subtract powers of 2:

$$200 - 2^7 = 200 - 128 = 72$$

$$72 - 2^6 = 72 - 64 = 8$$

$$8 - 2^3 = 8 - 8 = 0$$

So, the 7th, 6th, and 3rd digits are 1, and others are 0. Thus:

$$200_{10} = 11001000_2$$

Step 2: Convert the binary number to hexadecimal
We can group the binary digits into sets of four from right to left:

$$1100\,1000_2$$

Then convert each group to hexadecimal:

$$1100_2 = C_{16}$$

$$1000_2 = 8_{16}$$

Thus:

$$200_{10} = 11001000_2 = C8_{16}$$

1.2 ii.Convert 1010110110_2 to decimal and hexadecimal

Step 1: Convert the binary number to decimal number
We can add powers of 2, and the power of 2 is based on the digits of 1 in the binary number:

$$1 \times 2^1 + 1 \times 2^2 + 1 \times 2^4 + 1 \times 2^5 + 1 \times 2^7 + 1 \times 2^9 = 694_{10}$$

Thus:

$$1010110110_2 = 694_{10}$$

Step 2: Convert the binary number to hexadecimal number
We can group the binary digits into sets of four from right to left, then convert each group to hexadecimal:

$$0110_2 = 6_{16}$$
$$1011_2 = B_{16}$$
$$0010_2 = 2_{16}$$

Thus:

$$1010110110_2 = 2B6_{16}$$

1.3 iii.Convert ABC_{16} to 12-bit unsigned binary and get the signed corresponding(negative) number in decimal

Step 1: Convert the hex number to binary number
We can switch every digit of the hex number to binary number and combine them:

$$A_{16} = 1010_2$$
$$B_{16} = 1011_2$$
$$C_{16} = 1100_2$$

Thus:

$$ABC_{16} = 101010111100_2$$

Step 2: Convert the unsigned to signed
Firstly, we find the complement. The complement of the original number is:

$$010101000011_2$$

Then, we add 1 to the complement:

$$010101000011_2 + 1 = 010101000100_2$$

Then, we convert this binary number to decimal number:

$$0101010000100_2 = 4 + 64 + 256 + 1024 = 1348$$

At last, we switch it to the negative number. Thus, the answer is:

$$-1348$$

1.4 iv. Use a substitution trick to convert 110011110_3 to base-9

We can group the digits into sets of 2 from right to left, since the 9 is 3^2:

$$\begin{aligned} 10_3 &= 3_9 \\ 11_3 &= 3 + 1 = 4_9 \\ 01_3 &= 1_9 \\ 10_3 &= 3_9 \\ 01_3 &= 1_9 \end{aligned}$$

Thus:

$$110011110_3 = 13143_9$$

1.5 v. Convert 1801_{18} to base-20

Proof.
Step 1: Convert base-18 to decimal

$$1801_{18} = 1 \times 18^0 + 0 \times 18^2 + 8 \times 18^2 + 1 \times 18^3 = 1 + 0 + 2592 + 5832 = 8425$$

Step 2: Convert decimal to base-20

$$\begin{aligned} 8425 &= 20 \times 421 + 5 \\ 421 &= 20 \times 21 + 1 \\ 21 &= 20 \times 1 + 1 \\ 1 &= 20 \times 0 + 1 \end{aligned}$$

So we combine the remainder from bottom to top. Thus:

$$1801_{18} = 1115_{20}$$

2 Problem 2

2.1 i. Convert the IP address 179.55.223.12 from dotted decimal format to the hexadecimal and the decimal formats

Step 1: From dotted decimal to hexadecimal
Firstly, we convert each part of the dotted decimal to 8-digit binary format:

$$12_{10} = 8 + 4 = 00001100_2$$

$$223_{10} = 128 + 64 + 16 + 8 + 4 + 2 + 1 = 11011111_2$$

$$55_{10} = 32 + 16 + 4 + 2 + 1 = 00110111_2$$

$$179_{10} = 128 + 32 + 16 + 2 + 1 = 10110011_2$$

And then, we can convert each 8-digit binary to 2-digit hexadecimal:

$$\begin{aligned} 00001100_2 &= 0C_{16} \\ 11011111_2 &= DF_{16} \\ 00110111_2 &= 37_{16} \\ 10110011_2 &= B3_{16} \end{aligned}$$

At last, we can combine all the hexadecimal numbers into one hexadecimal number. Thus:

$$179.55.223.12 = B337DF0C_{16}$$

Step 2: From dotted decimal to decimal
Because we have decimal format for each 8-digit binary number, separated by dots. So each dot means we should times 2^8 to the next decimal number. Since $2^8 = 256$:

$$\begin{aligned} 12 \times 256^0 &= 12 \\ 223 \times 256^1 &= 57088 \\ 55 \times 256^2 &= 3604480 \\ 179 \times 256^3 &= 3003121664 \end{aligned}$$

Then we add all of them:

$$12 + 57088 + 3604480 + 3003121664 = 3006783244$$

Thus:

$$179.55.223.12 = 3006783244_{10}$$

2.2 ii. Convert the IP address BAC2A78F from hexadecimal format to the dotted decimal and decimal formats.

Step 1: From hexadecimal format to the dotted decimal
We firstly group the hexadecimal number by 2 and turn them into decimal format.

$$8F_{16} = 8 \times 16^1 + 15 = 143_{10}$$
$$A7_{16} = 10 \times 16^1 + 7 = 167_{10}$$
$$C2_{16} = 12 \times 16^1 + 2 = 194_{10}$$
$$BA_{16} = 11 \times 16^1 + 10 = 186_{10}$$

We connect all the decimal numbers form bottom to top. Thus:

$$BAC2A78F_{16} = 186.194.167.143$$

Step 2: From hexadecimal format to the decimal
Because we have decimal format for each 8-digit binary number, separated by dots. So each dot means we should times 2^8 to the next decimal number. Since $2^8 = 256$:

$$143 \times 256^0 = 143$$
$$167 \times 256^1 = 42752$$
$$194 \times 256^2 = 12713984$$
$$186 \times 256^3 = 3120562176$$

At last we add all the numbers. Thus:

$$BAC2A78F_{16} = 186.194.167.143 = 3133319055$$

2.3 iii. Convert the IP address 768744140 from decimal format to the hexadecimal and dotted decimal formats.

Step 1: From decimal format to hexadecimal
To get each digit of hexadecimal form, we make divisions:

$$768744140 \div 16 = 48046508 \text{ with a remainder of } 12$$
$$48046508 \div 16 = 3002906 \text{ with a remainder of } 12$$
$$3002906 \div 16 = 187681 \text{ with a remainder of } 10$$
$$187681 \div 16 = 11730 \text{ with a remainder of } 1$$
$$11730 \div 16 = 733 \text{ with a remainder of } 2$$
$$733 \div 16 = 45 \text{ with a remainder of } 13$$
$$45 \div 16 = 2 \text{ with a remainder of } 13$$
$$2 \div 16 = 0 \text{ with a remainder of } 2$$

Then we switch the remainder to hexadecimal form and connect them from bottom to top. Thus:

$$768744140_{10} = 2DD21ACC_{16}$$

Step 2: From decimal format to dotted decimal
Use the hexadecimal just derived. Group each 2 of the hexadecimal number into decimal:

$$CC_{16} = 12 \times 16 + 12 = 204$$
$$1A_{16} = 1 \times 16 + 10 = 26$$
$$D2_{16} = 13 \times 16 + 2 = 210$$
$$2D_{16} = 2 \times 16 + 13 = 45$$

Thus:

$$768744140_{10} = 2DD21ACC_{16} = 45.210.26.204$$

3 Problem 3

3.1 i. Give the decimal value which is represented by each of the following 8-bit two's complement numbers

Step1: $(10000000)_2$ It is a negative number since the 8th digit is 1.
First, find the complement:

$$011111111_2$$

Then, add 1 to the 8-digit binary number and turn it into decimal:

$$011111111_2 + 1 = 10000000_2 = 2^7 = 128$$

Thus the negative number is :

$$-128$$

Step2: $(11110011)_2$
It is a negative number since the 8th digit is 1.
First, find the complement:

$$00001100_2$$

Then, add 1 to the 8-digit binary number and turn it into decimal:

$$00001100_2 + 1 = 00001101_2$$

$$00001101_2 = 8 + 4 + 1 = 13$$

Thus the negative number is:

$$-13$$

Step3: $(01111111)_2$
It is a positive number since the 8th digit is 0. So we can treat it as unsigned number. The decimal number is:

$$01111111_2 = 10000000_2 - 1 = 127$$

3.2 ii. Give the 8-bit two's complement representations of the following decimal values

Step1: 55
It is a positive number.

$$55 = 32 + 16 + 4 + 2 + 1 = (00110111)_2$$

Step2: 83
It is a positive number.

$$83 = 64 + 16 + 2 + 1 = (01010011)_2$$

Step3: -79
It is a negative number. So first, the absolute number is 79. Find the binary form of 79.

$$79 = 64 + 8 + 4 + 2 + 1 = (01001111)_2$$

Then minus 1.

$$(01001111)_2 - 1 = (01001110)_2$$

At last find the complement. Thus the answer is:

$$(10110001)_2$$

Step4: -88
It is a negative number. So first, the absolute number is 88. Find the binary form of 88.

$$88 = 64 + 16 + 8 = (01011000)_2$$

Then minus 1:

$$(01011000)_2 - 1 = (01010111)_2$$

At last find the complement. Thus the answer is:

$$(10101000)_2$$

3.3 iii. Computations

Step1: $-79 + 55$

$$-79 + 55 = (10110001)_2 + (00110111)_2 = (11101000)_2$$

Check the answer:
1. the complement : 00010111_2
2.add 1: 00011000_2
3.turn to decimal: $00011000_2 == 16 + 8 = 24_{10}$
4.get the negative : $-24 = -79 + 55$
Correct.
The answer is 8-digit. **So there is no overflow.**
Step2: $-79 - 88$

$$-79 + (-88) = (10110001)_2 + (10101000)_2 = (101011001)_2$$

Check the answer: the 8-digit answer is 01011001_2, is a positive number.$01011001_2 = 1 + 8 + 16 + 64 = 89$
Incorrect.
The answer is 9-digit. If it is 8-digit answer 01011001_2, then it is incorrect since two negative numbers' addition can't be positive. **So there is a overflow.**

Step3: $83 + 55$

$$83 + 55 = 01010011_2 + 00110111_2 = 10001010_2$$

Check the answer: 10001010_2 is a negative number.
1. The complement: 01110101_2
2. Add 1:$01110101_2 + 1 = 01110110_2$ 3.Turn it to decimal and get negative: $01110110_2 = 2 + 4 + 16 + 32 + 64 = 118$, the negative number is -118.
Incorrect.
The answer is 9-digit. And it is incorrect since two positive numbers' addition can't be negative. **So there is a overflow.**

3.4 iv. What range of numbers can be represented in 10-bit two's complement?

The range of numbers can be represented in n-bit two's complement is

$$[-2^{n-1}, 2^{n-1} - 1]$$

Thus, the range of numbers can be represented in 10-bit two's complement is

$$[-512, 511]$$

3.5 v. What are the minimum number of bits necessary to represent 2100 in two's complement

Because,

$$2^{11} = 2048 < 2100 < 4096 = 2^{12}$$

$$2100 < 4095 = 2^{12} - 1$$

$$n - 1 = 12$$

$$n = 13$$

Thus, the minimum number of bits necessary to represent 2100 in two's complement is 13.

4 Problem 4

4.1 i.What are the dimensions of the image

From the picture, find the line starting at 10_{16} . Four bytes for the width is $(00000500)_{16}$, and four bytes for the height is $(000000ec)_{16}$. To get the decimal form: Width:

$$(500)_{16} = 5 \times 16^2 = 1280$$

Height:

$$(ec)_{16} = 14 \times 16 + 12 = 236$$

Thus, the dimensions of the image is

$$1280 \times 236 \,\text{pixels}$$

4.2 ii.How many kilobytes of disk space would the image consume

First, the total number of the pixels are

$$1280 \times 236 = 302080$$

Second, each pixel consists a 24-bit color encoding. Then each pixel consists a 3-bytes color encoding, because 1 byte = 8 bits. So the total number of bytes are

$$302080 \times 3 = 906240$$

Then we need turn bytes to kilobyte

$$906240 \div 2^{10} = 906240 \div 1024 = 885$$

Thus, **the image consume would 885 kilobytes of disk space.**