Problem 5

5.1

$|C| = 52$

$|R| = 26$

$|P| = 16$

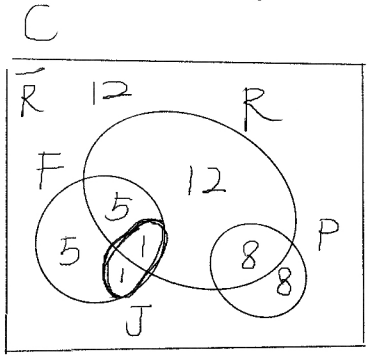
$|F| = 12$

$|J| = 2$

① $J \subset F$, $|F \cap R| = 6$, $|J \cap R| = 1$, $|F \cap J| = 2$

② $P \cap F = \emptyset$, $|P \cap R| = 8$

The Venn Diagram:



5.2 $|R \cup F \cup P \cup J| = |R| + |F| + |P| + |J| - |R \cap F| - |R \cap P| - |R \cap J| - |F \cap P| - |F \cap J| - |P \cap J| + |R \cap F \cap P| + |R \cap F \cap J| + |R \cap P \cap J| + |F \cap P \cap J| - |R \cap F \cap P \cap J|$

$= 26 + 12 + 16 + 2 - 6 - 8 - 1 - 0 - 2 - 0 + 0 + 1 + 0 + 0 - 0$

$= 40$

5.3 $|C - (F \cup P)| = |C| - |F \cup P|$

$\because F \cap P = \emptyset$

$\therefore |C| - |F \cup P| = 52 - 28 = 24$

5.4 $|C - R \cup P \cap \bar{J}| = |C| - |R \cup P \cap \bar{J}| = 52 - |R \cup P \cap \bar{J}|$

$= 52 - (|R| + |P| - |R \cap P| - |R \cap J|)$

$= 52 - (26 + 16 - 8 - 1) = 19$

5.5 $|(R - P) \Delta J| = |R| - |R \cap P| + |J| - |J \cap R| = 26 - 8 + 2 - 1 = 17$

Problem 4

4.1 $|P|$ = Number of students who liked Python

$|J|$ = Number of students who liked Java

$|C|$ = Number of Students who liked C++.

$\therefore |J \cap C| = 13$

$|P \cap C| = 25$

$|J \cap P| = 15$

$|P \cap J \cap C| = 10$

$\therefore |U| - |P \cup J \cup C| = 15$

$|P \cup J \cup C| = 85$

$|P|+|J|+|C|-|P \cap J|-|J \cap C|-|J \cap C|+|P \cap J \cap C| = |P|+|J|+|C|-15-13-25+10 = 85$

$|P|+|J|+|C| = 128$

$\therefore |P| = 100/2 = 50$

$2|J| = |C|$

$\therefore |P|+|J|+|C| = 128$

$50 + 2|J| = 128$

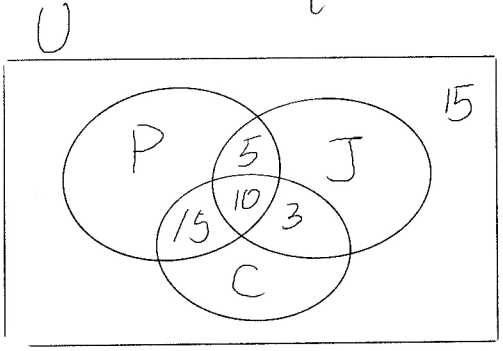
$2|J| = 78$

$|J| = 26$

$|C| = 2|J| = 52$

So, 26 Students like Java, 52 Students like C++.

4.2 Based on the question, we can draw Venn Diagram.



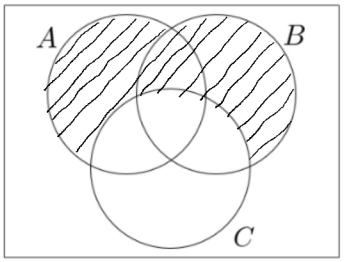
The students only like Python = $50-5-10-15 = 20$

The students only like Java = $26-5-10-3 = 8$

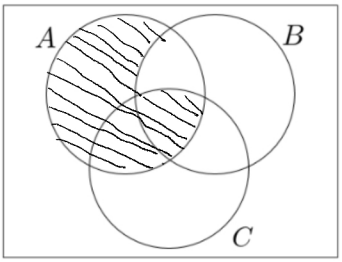
The students only like C++ = $52-10-15-3 = 24$

So, students like only one language = $20+8+24 = 52$

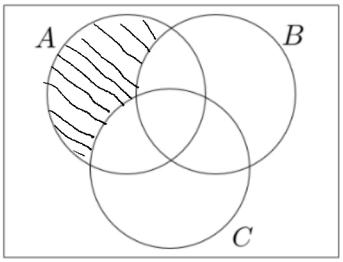
Problem 3



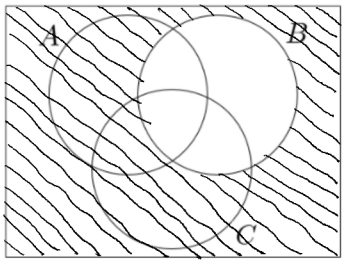
$(A \cup B) \cap \overline{C}$



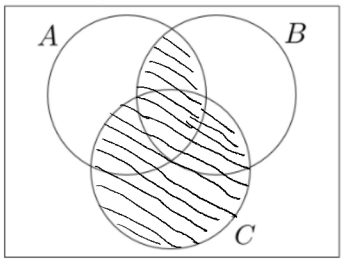
$A \cap \overline{(B - C)}$



$(A - B) - C$



$\overline{B}$



$(B \cap \overline{(U - A)}) \cup C$

Problem 2

2.1 $A \cup A = A$ (Idempotent)

2.2 $\overline{\overline{A} \cap \overline{B}} \cap U = \overline{\overline{A} \cap \overline{B}}$ (Identity)

$= A \cup B$ (De Morgan)

2.3 $(B - A) \cup (C - A) = (B \cap \overline{A}) \cup (C \cap \overline{A})$

$= \overline{A} \cap (B \cup C)$ (Distributive)

Homework 4

Problem 1

1.1

i. $\{x \mid x \in \mathbb{N} \wedge x < 5\} = \{0, 1, 2, 3, 4\}$

ii. $\{x \in \mathbb{N} \mid x \leq 55 \wedge x \text{ is even}\} = \{0, 2, 4, 6, 8, \dots, 54\}$

iii. $\{x \in \mathbb{N} \mid x \text{ is odd}\} = \{1, 3, 5, 7, 9, \dots\}$

iv. $\{x \in S \mid 3x \in S\}$ where $S = \{2, 3, 6, 9\} = \{2, 3\}$

v. $\{3x \mid x \in S\}$ where $S = \{2, 3, 6, 9\} = \{6, 9, 18, 27\}$

1.2

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{\{1\}\}, \{1, \{1\}\}, \{1, \{\{1\}\}\}, \{\{1\}, \{1, \{1\}\}\}, \{1, \{\{1\}, \{1, \{1\}\}\}\}\}$$

$|A| = 3$

$|\mathcal{P}(A)| = 2^{|A|} = 2^3 = 8$

1.3

i. $-35 \in \{4x + 5 : x \in \mathbb{Z}\}$ True

explanation:

$-35 = 4x + 5$

$-40 = 4x$

$x = -10$

$x \in \mathbb{Z}$

ii. $-35 \in \{4x + 5 = x \in \mathbb{N}\}$ False

explanation:

$-35 = 4x + 5$

$-40 = 4x$

$x = -10$

$x \notin \mathbb{N}$