Re-entry trajectory optimization of novel Spear-In-Sand (SIS) vertical landing stage recovery approach with reliable terminal state constraints

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Abstract. The present study focuses on the re-entry trajectory optimization of the novel Spear-In-Sand (SIS) recovery approach, which eliminates the deep throttling of the engine for stage recovery using vertical landing. Due to no deep throttling, uncertainties are inevitable in the terminal state of the trajectory. In this paper, we attempt to investigate the characteristics of re-entry trajectory of SIS approach under uncertainties. A two-degree-of-freedom trajectory segmented into boostback burn, freefall, and landing burn phases is formulated, with phase durations and throttling parameters as design variables. The reliable terminal state constraints are reformulated as deterministic equivalents, and the re-entry trajectory is optimized across varying reliability levels. The results show that higher reliability probabilities influence the trajectory states, increasing total time and free-fall duration due to throttling limitations. Furthermore, the optimized trajectories satisfy the reliability constraints within acceptable bounds, demonstrating the feasibility of the SIS recovery approach.

Keywords: Spear-In-Sand recovery approach, re-entry trajectory, uncertainties, terminal states.

1 Introduction

The Vertical Takeoff Vertical Landing (VTVL) method has emerged as one of the most prominent recovery techniques in recent years. The SpaceX Falcon 9 demonstrated the first successful implementation of VTVL stage recovery for an operational launch vehicle. The thrust-to-weight ratio during descent is vital for the VTVL recovery approach. During the final descent phase, Falcon 9 operates one engine out of its nine-engine array at 40–50 % throttle, reducing the descent thrust to approximately 6.3 % of the total ascent thrust [1]. This process is also known as Deep Throttling. However, this can cause serious challenges, particularly the combustion instabilities in larger engine configurations.

To address these limitations, a novel approach called the Spear In Sand (SIS) method has been introduced at VSSC [2]. The proposed method allows vertical landing while eliminating the requirement for deep throttling. The spent stage decelerates to the lowest feasible velocity and body rates, with no need for an accurate vertical attitude. The stage then lands on a specifically prepared sand bed (see Fig. 1), where the spear-shaped landing legs penetrate the surface and dissipate the remaining kinetic energy. This penetration simultaneously provides anchoring that stabilizes the stage against toppling under environmental disturbances.

The initial and most critical step in developing the recovery approach is optimizing its re-entry trajectory. The typical vertical landing re-entry trajectory of a reusable stage

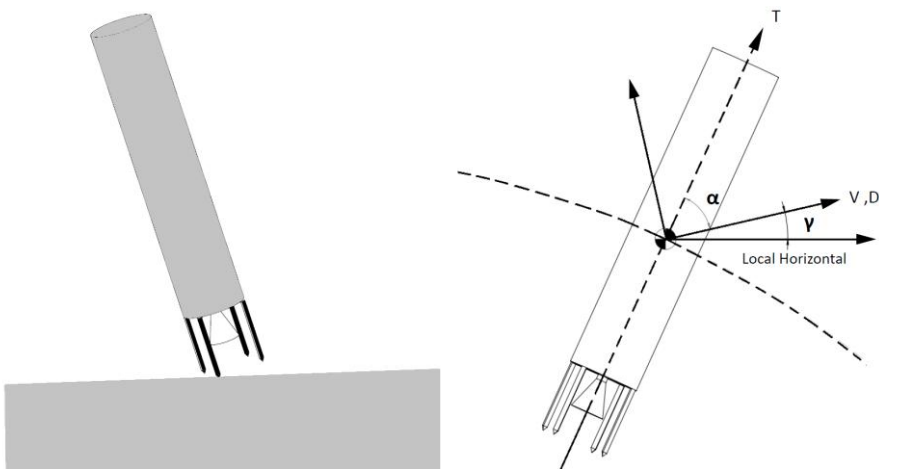

Fig. 1. Schematic (left) and free body diagram of the Spear-In-Sand Spent (SIS) reusable spent stage recovery approach.

begins after separation from the upper stage(s). After separation, the reusable stage performs a flip maneuver, changing the angle of attack from near zero to approximately 180° using aerodynamic grid fins and roll control thrusters. A boostback burn followed by an entry burn are then performed to steer the stage toward the landing site and reduce its velocity. The stage subsequently enters free-fall or aerobraking phase, where aerodynamic drag decelerates and guides it to the precise landing location. Finally, a landing burn brings the velocity to zero while aligning the vehicle vertically [3].

The trajectory can be segmented into several phases depending on the stage configuration. Anglim et al. [3] categorized the flight into three phases: boostback burn, free-fall, and landing burn. The authors employed a bang-off-bang thrust profile with fixed attitude, reducing the design variables to phase durations. The optimization is formulated as a minimum-fuel, minimum-terminal-error problem and solved using Particle Swarm Optimization (PSO). Zhao et al. [4] proposed a multi-phase, three-dimensional trajectory optimization framework comprising attitude-adjusting, powered descent, aerobraking, and vertical landing phases. The trajectory is optimized with respect to the phase durations, aerodynamic angles, and throttle parameters as design variables. A clustering-based adaptive optimization technique is employed to solve the problem.

The re-entry trajectory of the SIS recovery method differs primarily in the terminal states of the trajectory. With no deep throttling, the terminal states, impact velocity, attitude, and attitude rate, are prone to uncertainties. To this end, the authors carried out a prior inverse study deriving the reliable terminal states with required probabilities, such that a safe landing is guaranteed in the presence of uncertainties. This prior study focused solely on the landing dynamics of the spent stage in sand. The current study evaluates the feasibility of these reliable terminal states for the re-entry trajectory under throttling constraints. The reliable terminal state constraints are converted to equivalent deterministic constraints, and the re-entry trajectory is optimized for different levels of reliable probabilities.

2 Vertical Landing Re-entry Trajectory

2.1 Methodology

As discussed earlier, the vertically landing re-entry trajectory can divided into several phases, one can use any number of phases and as the number increases the complexity of the problem increases. In the current work, we assume that the re-entry trajectory is starting after the flipover maneuver by roll control thrusters till the angle of attack becomes 180°, and a total of three phases are considered, *Boostback Burn Phase, Aerobraking Phase and Landing Burn Phase*.

a. Boostback Burn Phase: In this phase, the reusable stage uses propulsion to reduce the velocity of the stage. It is essential because, at the higher altitude the low atmospheric results in minimum drag, making it difficult to control the down range, leading to unfavorable recovery conditions.
b. Aerobraking or freefall Phase: In this phase, the engine shuts off and the stage is allowed to fall under gravity while maintaining its attitude (using aerodynamic surfaces). First, the acceleration due to gravity decreases the velocity component in gravity direction to zero. Then, the velocity increases again as the stage descends; however, the increasing atmosphere density and aerodynamic drag progressively reduce velocity as altitude decreases.
c. Landing Burn Phase: After the aerobraking phase, the final landing burn reduce the residual velocity to desired value and aerodynamics surfaces steer the reusable stage to required attitude.

2.2 Equations of motion

A globally accepted two-dimensional point mass launch vehicle dynamics with rotating Earth following oblete spheroid gravity model (Tewari [5]) is chosen for the current study. The state of the vehicle at any time can obtained from radial distance (r), latitude

(ϕ), velocity (v), Flight Path Angle (γ) and Attitude Angle or pitch angle (θ). The state equations are modeled as follows,

$$\dot{r} = v sin\gamma \tag{1}$$

$$\dot{\phi} = v cos\gamma / r \tag{2}$$

$$\dot{v} = (T cos\alpha - D)/m - g_r sin\gamma + g_{\phi} cos\gamma + r w_e^2 cos\phi(\sin(\gamma - \phi)) \tag{3}$$

$$\dot{\gamma} = T sin\alpha / mv + v cos\gamma / r - g_r cos\gamma / v - g_{\phi} sin\gamma / v + r w_e^2 cos\phi (cos(\gamma - \phi))/v \tag{4}$$

$$\dot{m} = -T / I_{sp} g_0 \tag{5}$$

where α is the angle of attack, T represents the thrust, w_e = 7.29211 × 10−5 rad/s is the angular velocity of the earth, m is the mass of the vehicle, g_r and g_ϕ are the oblate gravity components in radial and latitude directions and are given by,

$$g_r = (\mu_E / r^2)\, [1 - (3/2) J_2 (R_E / r)^2 (3 sin^2\phi - 1)] \tag{6}$$

$$g_{\phi} = (-3\mu_E / r^2) J_2 (R_E / r)^2\, sin\phi cos\phi \tag{7}$$

where, μ_E is the Earth's Gravitation Constant(μ_E = 3.98600448 × 10^{14} m^3/s^2), J_2 is the second zonal harmonic (J_2 = 1.0826368 × 10^{-3}) and R_E is the equatorial radius of earth (R_E = 6378137 m). The attitude of the vehicle at any time can be obtained from angle of attack and flight path angle,

$$\theta = \alpha + \gamma \tag{8}$$

For drag, we assume that C_D = 0.75 for circular cross-sectional area with radius 2.1 m. The thrust T can be written as,

$$T = s T_{avg} \tag{9}$$

where s is the throttling parameter which regulates the amount of thrust required for the re-entry. We assume the T_{avg} a constant with the magnitude of 680 kN. The exponential density model is considered for the current work,

$$\rho(h) = \rho_0 e^{\frac{-h(t)}{H_0}} \tag{10}$$

where ρ_0 = 1.225 kg/m^3, H_0 = 7500 m and $h(t)$ is the altitude as function of time. From all the above equations, the state dynamics of the vehicle can be described as follows,

$$\dot{x} = f(x, u) \tag{11}$$

$$x = [r, \phi, v, \gamma]^T; u = [s, \alpha]^T \tag{12}$$

Here x and u are the state and control variables of the spent stage.

2.3 Problem statement

For the current study, we consider the time lengths of the trajectory phases (t_{bb}, t_{ff}, t_{la}) and the throttling parameters in the boostback burn (s_{bb}) and landing burn (s_{la}) as the design variables. Furthermore, the angles of attack during the all the phases is assumed to be fixed at 180°. The re-entry trajectory is optimized using a direct method, formulating the problem as a nonlinear program (NLP) that minimizes fuel consumption subject to terminal state constraints. The mathematical formulation is as follows,

$$Minimize \qquad J = -m\{t_f\} \tag{23}$$

$$w.r.to \qquad \{s_{bb}, s_{la}, t_{bb}, t_{ff}, t_{la}\} \tag{14}$$

$$subject\ to$$

$$c1: r\{t_f\} = R_E \tag{15}$$

$$c2: m\{t_f\} \geq 8350\ Kg \tag{16}$$

$$c3:\ V_r - \sigma \leq v\{t_f\} \leq V_r + \sigma \tag{17}$$

$$c4:\ \theta_r - \sigma \leq \theta\{t_f\} \leq \theta_r + \sigma \tag{18}$$

$$c5:\ \dot{\theta}_r - \sigma \leq \dot{\theta}\{t_f\} \leq \dot{\theta}_r + \sigma \tag{19}$$

Here the objective is to minimize fuel consumption, expressed as maximizing the touchdown mass, which is the sum of empty mass and fuel mass. The equality constraint c1 ensures that the final radial distance is equal to the radius of Earth at touchdown, while c2 enforces that the touchdown mass is not less than the empty mass of the reusable stage. The constraints c3, c4, and c5 represent the reliable terminal state conditions derived from a prior probabilistic study based on landing dynamics. The terminal constraints guarantees safe landing with desired reliability when variations are within ±σ (listed in Table 3). In the current study, we imposed the same as inequality constraints c3, c4, and c5. The boundary values and limits of the state and design variables are listed in Table 1 and 2. The optimization problem is solved using the Genetic Algorithm (GA) solver in MATLAB, with total 200 generations.

Table 1. Boundary values of the state variables

Variables	Initial Value	Final Value
Radial distance (*r*)	R_E+60 km	R_E
Latitude (ϕ)	13.4191˚	Free
Velocity (*v*)	1500 m/s	Listed in Table 3
Flight path angle (γ)	-25˚	Constrained by pitch angle as listed in Table 3
Total mass	18350	≥8350 Kg

Table 2. Range of the design variables.

Variables	Range	Unit
s_{bb}	[0.5, 1]	-
s_{la}	[0.5, 1]	-
t_{bb}	[0, 60]	sec
t_{ff}	[100, 360]	sec
t_{la}	[0, 60]	sec

Table 3. Reliable terminal states.

Terminal state ([V_r (m/s), θ_r (deg), $\dot{\theta}_r$ (deg/s)])	Allowable deviation ([σ (m/s), σ (deg), σ (deg/s)])	Reliability probability (R_i)
[6.5838, 82.7204, -5.4624]	[±1, ±1, ±1]	R_i = 0.9
[7.8773, 83.7342, -6.0055]	[±1, ±1, ±1]	R_i = 0.99
[8.2469, 84.0261, -6.1469]	[±1, ±1, ±1]	R_i = 0.995

3 Result and Discussion

The Fig. 2 shows the altitude, velocity, pitch angle, and mass time histories of the reusable spent stage. The altitude initially increases during the boostback burn, which propels the stage in the direction opposite to the velocity vector. As the free-fall phase begins, the altitude decreases and eventually reaches zero during the landing phase. The velocity initially decreases during the boostback burn; however, it increases during free-fall due to gravitational acceleration. Finally, the landing burn decelerates the stage to the desired terminal velocity. The pitch angle evolves from its initial value to the desired terminal value over the course of the trajectory. Table 4 and 5 lists the optimal design variables and terminal states of the trajectory for different reliabilities.

A significant difference is observed in the trajectory states for reliability probabilities between 0.9 and 0.99, driven by the substantial increase in terminal velocity and pitch angle requirements. Additionally, the total trajectory time increases with reliability probability. Notably, the boostback burn duration decreases while the free-fall phase duration increases. This trend arises from engine throttling limitations, which require more time to decelerate the spent stage to meet the stricter terminal state constraints. This is further evidenced by the design variable s_{la}, which converges to its lower bound.

Furthermore, the pitch angle decreases with increasing reliability probability due to the greater negative pitch rate at the terminal state, which tends to pull the spent stage away from vertical alignment. However, the reliability constraints in the prior study are computed based on the assumption of Gaussian disturbance, the terminal states in the current study are in the allowable deviation bounds for achieving the desired reliability.

Table 4. Optimal design variables.

Reliability probability (R_i)	s_{bb}	s_{la}	t_{bb} (sec)	t_{ff} (sec)	t_{la} (sec)

R_i = 0.9	0.7986	0.5255	20.801	127.2195	28.7472
R_i = 0.99	0.5009	0.5	21.0289	134.6728	40.8130
R_i = 0.995	0.5598	0.5	16.3323	140.2235	40.3495

Table 5. Reliable terminal states.

Reliability probability (R_i)	Terminal state ([V_r (m/s), θ_r (deg), $\dot{\theta}_r$ (deg/s)])
R_i = 0.9	[6.1630, 83.3563, -4.9182]
R_i = 0.99	[7.3465, 83.0999, -5.2248]
R_i = 0.995	[7.9784, 82.8124, -5.2510]

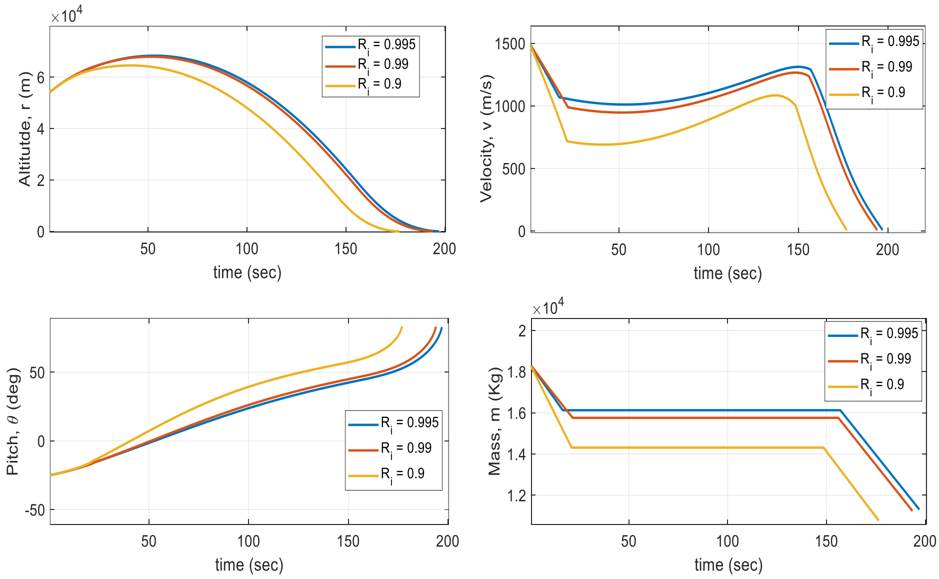

Fig. 2. Time histories of altitude (top left), velocity (top right), pitch angle (bottom left), and mass (bottom right) of the reusable spent stage for different cases of reliable terminal constraints.

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