Sep. 11th

1. a) mean = 4.4444 | median = 5 | Variance = 6.2469

~~5+3+2+6+9+1+4+2+4+5~~ = 40

b) mean = ~~3.92307~~ ≈ 3.9231 Var = 5.4556 median = 4.

c) mean = ~~1.54545~~ = 1.5455 Var = ~~18.49338~~ = 18.7934 median = 3

2. a) $\frac{85}{100} \times \frac{85}{100} \times \frac{85}{100} \times \frac{85}{100} \times \frac{15}{100} = 0.0783$

b) $\frac{15}{100} + \left(\frac{85}{100}\right)^1 \times \frac{15}{100} + \left(\frac{85}{100}\right)^2 \times \frac{15}{100} = \cancel{0.38587}$ 0.3859

3. 15/20 → buy new iphone the # of ~~persons~~

3/4 X : ~~A person~~ buy n)

$P(X \geq 48)$

$= (P(X=49) + P(X=50))$

$\binom{50}{49}$ ~ $_{50}C_{49} \left(\frac{3}{4}\right)^{49} \left(\frac{1}{4}\right)^1 + \binom{50}{50} \left(\frac{3}{4}\right)^{50} \left(\frac{1}{4}\right)^0$

$= 50 \cdot \left(\frac{3}{4}\right)^{49} \left(\frac{1}{4}\right)^1 + 1 \cdot \left(\frac{3}{4}\right)^{50}$

0.000001, 000501 ...

b. $P(X=36) = \binom{50}{36} \cdot \left(\frac{3}{4}\right)^{36} \cdot \left(\frac{1}{4}\right)^{14}$

$= 0.11104$

c. reuse x.

$Var = npq = 50 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{75}{8} = 9.375$

d. Binomial Dist.

4. 5/month ~ Fall

$X \sim Pois(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$ $\lambda = 5$ for 1 month

$\lambda = 10$ for 2 month

$\Rightarrow Pois(x;10) = \frac{10^x e^{-10}}{x!}$

$x : 0 \sim 6$

Ans $= \frac{10^0 e^{-10}}{0!} + \frac{10^1 e^{-10}}{1!} + \frac{10^2 e^{-10}}{2!} + \frac{10^3 e^{-10}}{3!} + \frac{10^4 e^{-10}}{4!} + \frac{10^5 e^{-10}}{5!} + \frac{10^6 e^{-10}}{6!}$

$= 0.13014$

$e^{-10}\left(1 + 10 + \frac{100}{2} + \frac{1000}{6} + \frac{10^4}{24} + \frac{10^5}{5!} + \frac{10^6}{6!}\right)$

Oct. 2nd

1. B G
B B
G B
G G

⇒ 1/3

2.

Better strategy: If you see two blue or two green hats, then write down the opposite color, otherwise write down 'pass'.

It works like this ('-' means 'pass'):

Hats: GGG, Guess: BBB, Result: Lose
Hats: GGB, Guess: --B, Result: Win
Hats: GBG, Guess: -B-, Result: Win
Hats: GBB, Guess: G--, Result: Win
Hats: BGG, Guess: B--, Result: Win
Hats: BGB, Guess: -G-, Result: Win
Hats: BBG, Guess: --G, Result: Win
Hats: BBB, Guess: GGG, Result: Lose

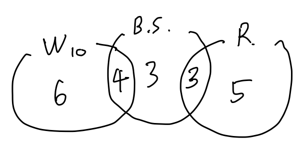
Result: 75% chance of winning!

3. G. B. Y. Y.

$\frac{1}{_2C_1 \times _2C_1 + _2C_2} = \frac{1}{4+1} = \frac{1}{5}$

4. 10 - W. 8 - R.

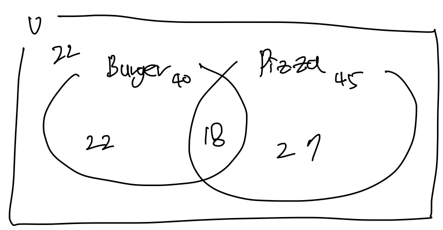
4 - B.S. ⊕ W. 3 - B.S. ⊕ R.



⇒ 21

21 - 18 = 3

5.



40 + ~~27 + 22~~ 49 = 89

6. $P_E = \frac{2}{6} = \frac{1}{3}$

7.

-1) $P(1 \leq X < 4)$

① k-2 = 1 → k=3 → 1/8

② k-2 = 2 → k=4 → 1/16

... k=5 → 1/32

~~k=5 → 1/32~~

$\frac{4+2+1}{32} = \frac{7}{32}$

-2) $P(X > 2)$

$= 1 - (P(X=-1) + P(X=0) + P(X=1) + P(X=2))$

X=-1 → k=1 → 1/2

0 → k=2 → 1/4

1 → k=3 → 1/8

2 → k=4 → 1/16

~~$\frac{4+2+1}{8} = \frac{7}{8}$~~

$\frac{8+4+2+1}{16} = \frac{15}{16}$

$\Rightarrow \frac{1}{16}$

8.

$P(X_1 + X_2 = 8)$

$(2,6), (3,5), (4,4), (5,3), (6,2)$

$\frac{5}{36}$

9.

$X = \{0, 1, 2\}$ $X : 0 \sim 2$

$R_X = \{0, 1, 2\}$

$$P_X = \begin{cases} 0 & \frac{1}{4} \\ 1 & \frac{1}{2} \\ 2 & \frac{1}{4} \end{cases}$$

10.

* If a problem asked the probability of exact value in Continuous Random Variables, the answer should be 0

11.

$$\frac{4}{3 \times 4} = \frac{1}{3}$$

12.

$1, 3, \cdots, 19$

$2n-1 = 19 \rightarrow n = 10$

$$\Rightarrow \frac{\cancel{10}}{\cancel{20}} \times \frac{\cancel{9}}{19} \times \frac{\cancel{8}\cancel{4}\,2}{\cancel{18}} = \frac{2}{19}$$

13.

Example: Picnic Day

You are planning a picnic today, but the morning is cloudy

- Oh no! 50% of all rainy days start off cloudy!
- But cloudy mornings are common (about 40% of days start cloudy)
- And this is usually a dry month (only 3 of 30 days tend to be rainy, or 10%)

What is the chance of rain during the day?

We will use Rain to mean rain during the day, and Cloud to mean cloudy morning.

The chance of Rain given Cloud is written P(Rain|Cloud)

So let's put that in the formula:

$$P(\text{Rain}|\text{Cloud}) = \frac{P(\text{Rain})\,P(\text{Cloud}|\text{Rain})}{P(\text{Cloud})}$$

- P(Rain) is Probability of Rain = 10%
- P(Cloud|Rain) is Probability of Cloud, given that Rain happens = 50%
- P(Cloud) is Probability of Cloud = 40%

$$P(\text{Rain}|\text{Cloud}) = \frac{0.1 \times 0.5}{0.4} = .125$$

Or a 12.5% chance of rain. Not too bad, let's have a picnic!

P(A|

14.

Example: The Art Competition has entries from three painters: Pam, Pia and Pablo

- Pam put in 15 paintings, 4% of her works have won First Prize.
- Pia put in 5 paintings, 6% of her works have won First Prize.
- Pablo put in 10 paintings, 3% of his works have won First Prize.

What is the chance that Pam will win First Prize?

$$P(\text{Pam}|\text{First}) = \frac{P(\text{Pam})P(\text{First}|\text{Pam})}{P(\text{Pam})P(\text{First}|\text{Pam}) + P(\text{Pia})P(\text{First}|\text{Pia}) + P(\text{Pablo})P(\text{First}|\text{Pablo})}$$

Put in the values:

$$P(\text{Pam}|\text{First}) = \frac{(15/30) \times 4\%}{(15/30) \times 4\% + (5/30) \times 6\% + (10/30) \times 3\%}$$

Multiply all by 30 (makes calculation easier):

$$P(\text{Pam}|\text{First}) = \frac{15 \times 4\%}{15 \times 4\% + 5 \times 6\% + 10 \times 3\%} = \frac{0.6}{0.6 + 0.3 + 0.3} = 50\%$$

A good chance!

Pam isn't the most successful artist, but she did put in lots of entries.

15.

$P(M) = \frac{51}{100}$ $P(F) = \frac{49}{100}$

a.

$$P(M \cap C) = \frac{51 \times \frac{9.5}{100}}{100} \qquad P(F \cap C) = \frac{49 \times \frac{1.7}{100}}{100}$$

$P(M | C) = ?$

$$= \frac{P(M \cap C)}{P(C)}$$

$$\frac{\cancel{95}\,19}{\cancel{1000}\,200} \times \frac{51}{100} = \frac{19 \times 51}{20000} \qquad \frac{17}{1000} \times \frac{49}{100} = \frac{17 \times 49}{100000}$$

$$\Rightarrow \frac{\frac{95}{1000} \times \frac{51}{100}}{\frac{19 \times 51 \times 5}{100{,}000} + \frac{17 \times 49}{100{,}000}}$$

$$= \frac{95 \times 51}{19 \times 51 \times 5 + 17 \times 49} = \frac{\cancel{17}(95 \times 3)}{\cancel{17}(19 \times 3 \times 5 + 49)}$$

$$= \frac{285}{334}$$

4 9 285
15 + 49
95 334
19
285

16.

X	\$1	\$2
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{2}$

$E(X) = 1\times\frac{1}{2} + 2\times\frac{1}{2} = 1.5\$$.

17.

$E(X) = 0.15 + 0.8 + 0.75 + 0.4$

$= 1.7 + 0.4 = 2.1$

18.

$E(X) = 1\cdot\frac{1}{6} + 2\cdot\frac{1}{6} + \cdots + 6\cdot\frac{1}{6}$

$= \frac{\frac{6\cdot 7}{2}}{6} = \frac{7}{2}$

$Var(X) = E(X^2) - E(X)^2$

i) $E(X^2) = 1^2\cdot\frac{1}{6} + 2^2\cdot\frac{1}{6} + \cdots + 6^2\cdot\frac{1}{6}$

$= \frac{1+4+9+16+25+36}{6}$

$= \frac{91}{6}$

ii) $E(X)^2 = \frac{49}{4}$

$\Rightarrow \frac{91}{6} - \frac{49}{4} = \frac{182-147}{12} = \frac{35}{12}$

19.

X : Error

$Var(X) = E(X^2) - E(X)^2$

i) $2^2\cdot\frac{1}{100} + 3^2\cdot\frac{25}{100} + 4^2\cdot\frac{40}{100} + 5^2\cdot\frac{30}{100} + 6^2\cdot\frac{4}{100}$

$= \frac{4+225+640+750+144}{100}$

$= \frac{1763}{100}$

ii) $E(X) = 0.02 + 0.75 + 1.6 + 1.5 + 0.24$

$= 4.11$

iii) $17.63 - (4.11)^2$

20.

a. $\int_0^1 2(1-x)\,dx$ ($2-2x$)

$2x - x^2 \Big|_0^1 = 2 - 1 = 1$

b.

$E(X) = \int_0^1 x\cdot(2-2x)\,dx$

$= \int_0^1 2x - 2x^2\,dx$

$= x^2 - \frac{2}{3}x^3 \Big|_0^1$

$= 1 - \frac{2}{3} = \frac{1}{3}$

$E(X^2) = \int_0^1 2x^2 - 2x^3\,dx$

$= \frac{2}{3}x^3 - \frac{2}{4}x^4 \Big|_0^1$

$= \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$

$V(X) = E(X^2) - E(X)^2$

$= \frac{1}{6} - \frac{1}{9} = \frac{3-2}{18} = \frac{1}{18}$

21.

4가 2번 나올 확률

${}_6C_2\,(\frac{1}{6})^2\cdot(\frac{5}{6})^3 = \frac{6\cdot 5}{2!}\cdot\frac{5^3}{6^5}$

22.

$P(X \le 45)$ X: Head $n=100$

$\sim B(100, \frac{1}{2})$

=

23.

$X \sim B(5, \frac{3}{10})$

$P(X \le 2) = ?$

$p(X=0) + p(X=1) + p(X=2)$

$= {}_5C_0 (\frac{3}{10})^0 \cdot (\frac{7}{10})^5 + {}_5C_1 (\frac{3}{10})^1 \cdot (\frac{7}{10})^4 + {}_5C_2 (\frac{3}{10})^2 \cdot (\frac{7}{10})^3$

24.

Unit : 100 years $\rightarrow \mu = 1 \rightarrow \lambda = 1$

* Poisson Distribution

$P(X=k) = \dfrac{\lambda^k \cdot e^{-\lambda}}{k!}$

25. Unit : 1 match $\rightarrow \mu = 2.5$ goals

$\rightarrow \lambda = 2.5$

26.

$\dfrac{{}_5C_4 \; {}_{45}C_6}{{}_{50}C_{10}}$

* Hypergeometric Distribution

$\dfrac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$

27.

$1010 - 2.5x = 1000$

$10 = 2.5x \qquad x = 4$

28.

$\mu = 4$ minutes $\rightarrow \lambda = 1/4$

* Exponential Distribution : $f(x) = \lambda e^{-\lambda x}$

$P(4 < X < 5)$

$= \int_4^5 \frac{1}{4} e^{-\frac{1}{4}x} dx$

$= \frac{1}{4}(-4) e^{-\frac{1}{4}x} \Big|_4^5$

$= -e^{-\frac{5}{4}} - (-e^{-1})$

$= -e^{-\frac{5}{4}} + e^{-1}$

$= 0.08137$

29.

b. $f(x) = \frac{1}{4} e^{-\frac{1}{4}x}$

$\int_0^k \frac{1}{4} e^{-\frac{1}{4}x} dx = \frac{1}{2}$

$(-1) e^{-\frac{1}{4}x} \Big|_0^k = \frac{1}{2}$

$-e^{-\frac{1}{4}k} + (+1) = \frac{1}{2}$

$\frac{1}{2} = e^{-\frac{1}{4}k}$

$\ln \frac{1}{2} = -\frac{1}{4}k \qquad k = -4 \ln \frac{1}{2}$

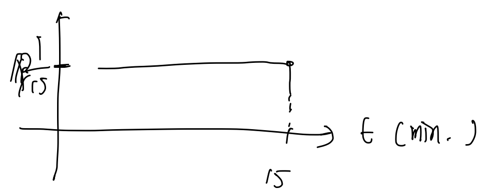
c) Median $= k = 2.773$

Mean $= \cancel{\int_0^\infty x \frac{1}{4} e^{-\frac{1}{4}x} dx}$

$= \frac{1}{\lambda} = 4$

30.

a.



$P(X < 12.5) \qquad p \times 15 = 1$

$p = \frac{1}{15}$

$= \int_0^{12.5} \frac{1}{15} dx$

$\frac{1}{15}(12.5 - 0) = \frac{1}{15} \times \frac{125}{10} = \frac{5}{6}$

b. * Uniform distribution.

PDF/CDF ⊕ μ, Var, σ

31.

4) Gamma Distributions

The gamma distribution is a two-parameter family of continuous probability distributions, particularly useful when dealing with waiting times. The gamma distribution is a generalization of the exponential distribution. In other words, they have same purpose, to predict the wait time until future events, but while the exponential distribution predicts the wait time until the first event, the gamma distribution predicts the wait time until the k-th event.

In order to define the gamma distribution, the gamma function is priorly needed. The gamma function $\Gamma(x)$ is defined with the following five characteristics for any positive real number α:

$e^{-\lambda x}$

1. $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx, \quad \alpha > 0$
2. $\int_0^\infty x^{\alpha-1}e^{-\lambda x}dx = \frac{\Gamma(\alpha)}{\lambda^\alpha}, \quad \alpha > 0, \lambda > 0$
3. $\Gamma(\alpha+1) = \alpha\Gamma(\alpha), \quad \alpha > 0$
4. $\Gamma(n) = (n-1)!, \quad n \in \mathbb{N}$
5. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

With these characteristics, the gamma distribution, X~Gamma(α, λ) followed by a continuous random variable X with parameters $\alpha > 0$ and $\lambda > 0$, is defined as follows:

$$f(x) = \begin{cases} \frac{\lambda^\alpha x^{\alpha-1}e^{-\lambda x}}{\Gamma(\alpha)}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

The specific case, Exponential Distribution, can be checked by setting $\alpha = 1 \rightarrow f(x) = \lambda e^{-\lambda x}$. In other words, Gamma(1, λ) = Exponential(λ).

The moments of the gamma distribution are as follows:

1. $E(X) = \frac{\alpha}{\lambda}$
2. $Var(X) = \frac{\alpha}{\lambda^2}$
3. $SD(X) = \sqrt{\frac{\alpha}{\lambda^2}}$

☆Gamma Distribution
Mean, Var. 유도과정

Problem 8

Let $X \sim Gamma(\alpha, \lambda)$, where $\alpha, \lambda > 0$. Find EX, and $Var(X)$.

Solution

To find EX we can write

$$\begin{aligned} EX &= \int_0^\infty x f_X(x)dx \\ &= \int_0^\infty x \cdot \frac{\lambda^\alpha}{\Gamma\alpha} x^{\alpha-1}e^{-\lambda x}dx \\ &= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty x \cdot x^{\alpha-1}e^{-\lambda x}dx \\ &= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty x^\alpha e^{-\lambda x}dx \\ &= \frac{\lambda^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha+1)}{\lambda^{\alpha+1}} && \text{(using Property 2 of the gamma function)} \\ &= \frac{\alpha\Gamma(\alpha)}{\lambda\Gamma(\alpha)} && \text{(using Property 3 of the gamma function)} \\ &= \frac{\alpha}{\lambda}. \end{aligned}$$

Similarly, we can find EX^2:

$$\begin{aligned} EX^2 &= \int_0^\infty x^2 dx \\ &= \int_0^\infty x^2 \cdot \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1}e^{-\lambda x}dx \\ &= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty x^2 \cdot x^{\alpha-1}e^{-\lambda x}dx \\ &= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty x^{\alpha+1}e^{-\lambda x}dx \\ &= \frac{\lambda^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha+2)}{\lambda^{\alpha+2}} && \text{(using Property 2 of the gamma function)} \\ &= \frac{(\alpha+1)\Gamma(\alpha+1)}{\lambda^2\Gamma(\alpha)} && \text{(using Property 3 of the gamma function)} \\ &= \frac{(\alpha+1)\alpha\Gamma(\alpha)}{\lambda^2\Gamma(\alpha)} && \text{(using Property 3 of the gamma function)} \\ &= \frac{\alpha(\alpha+1)}{\lambda^2}. \end{aligned}$$

So, we conclude

$$\begin{aligned} Var(X) &= EX^2 - (EX)^2 \\ &= \frac{\alpha(\alpha+1)}{\lambda^2} - \frac{\alpha^2}{\lambda^2} \\ &= \frac{\alpha}{\lambda^2}. \end{aligned}$$

32.

Y \ X	1	2	3	4
1	$\frac{1}{10}$			
2				
3				
4				$\frac{1}{10}$

33. a. $\frac{5}{12}$

b. $P(X=x) = \begin{cases} \frac{13}{24} & x=0 \\ \frac{11}{24} & x=1 \\ 0 & \text{otherwise} \end{cases}$

$\frac{20+6}{48} = \frac{26}{48}$

$\frac{\cancel{3+4+4+1}\ 8}{24}$

$P(Y=y) = \begin{cases} \frac{2}{24} & Y=0 \\ \frac{5}{12} & Y=1 \\ \frac{7}{24} & Y=2 \\ 0 & \text{otherwise} \end{cases}$

c. $\frac{\frac{1}{4}}{\frac{13}{24}} = \frac{1}{\cancel{4}} \cdot \frac{\cancel{24}\,6}{13} = \frac{6}{13}$

d. No.

34. a. $\frac{2+1+4+2}{24} = \frac{9}{24}$

b. $P(X=x) = \begin{cases} \cancel{\frac{4}{24}}\ \frac{1}{6} & x=1 \\ \cancel{\frac{4+2+3}{24}}\ \frac{3}{8} & x=2 \\ \cancel{\frac{12+6+4}{48}}\ \frac{11}{24} & x=3 \\ 0 & \text{otherwise} \end{cases}$

$P(Y=y) = \begin{cases} \cancel{\frac{1+2+3}{12}}\ \frac{1}{2} & Y=2 \\ \cancel{\frac{1+2+3}{24}}\ \frac{1}{4} & Y=4 \\ \cancel{\frac{1+3+2}{24}}\ \frac{1}{4} & Y=5 \\ 0 & \text{otherwise} \end{cases}$

c. $\frac{\frac{1}{12}}{\frac{1}{6}} = \frac{1}{2}$

d. No.

35. X : Blue marble Y : Red marble

$P_{XY} = \begin{cases} \frac{{}_{40}C_i \ {}_{60}C_{10-i}}{{}_{100}C_{10}} & i = 0 \text{ to } 10 \\ 0 & \text{otherwise} \end{cases}$

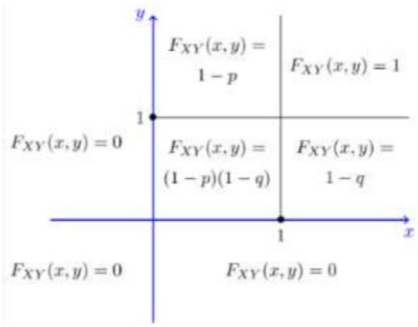
36

a. Let X~Bernoulli(p) and Y~Bernoulli(q) are independent. Find joint PMF and joint CDF for X and Y.

Note that $P(X=0)=1-p, P(X=1)=p, P(Y=0)=1-q, P(Y=1)=q$.

$$R_{XY}=\{(0,0),(0,1),(1,0),(1,1)\}, R_X=\{0,1\}, R_Y=\{0,1\}$$
$$F_{XY}(x,y)=F_X(x)\cdot F_Y(y)$$

Joint PMF	Joint CDF
$P_{XY}(x,y)=\begin{cases}(1-p)(1-q) & \text{if } x=0, y=0\\ (1-p)q & \text{if } x=0, y=1\\ p(1-q) & \text{if } x=1, y=0\\ pq & \text{if } x=1, y=1\\ 0 & \text{otherwise}\end{cases}$	$P_{XY}(x,y)=\begin{cases}0 & \text{if } x<0, y<0\\ (1-p)(1-q) & \text{if } 0\le x,y<1\\ (1-p) & \text{if } 0\le x<1, y\ge 1\\ (1-q) & \text{if } x\ge 1, 0\le y<1\\ 1 & \text{if } x\ge 1, y\ge 1\end{cases}$



$\cancel{E}(X) = p(H)\cdot P(X|H) + E(T)\cdot p(X|T)$

$= p \cdot 1 + (1-p)\cdot(1+E(X))$

$= \cancel{p}+1+E(X)-\cancel{p}-pE(X)$

$= E(X)(1-p)+1$

$E(X)\cdot p = 1 \qquad E(X)=\frac{1}{p}$

← Geometric Mean 유도과정

Example 5.6

Let $X \sim Geometric(p)$. Find EX by conditioning on the result of the first "coin toss."

Solution

Remember that the random experiment behind $Geometric(p)$ is that we have a coin with $P(H)=p$. We toss the coin repeatedly until we observe the first heads. X is the total number of coin tosses. Now, there are two possible outcomes for the first coin toss: H or T. Thus, we can use the law of total expectation (Equation 5.3):

$$\begin{aligned}EX &= E[X|H]P(H)+E[X|T]P(T)\\ &= pE[X|H]+(1-p)E[X|T]\\ &= p\cdot 1+(1-p)(EX+1).\end{aligned}$$

In this equation, $E[X|T]=1+EX$, because the tosses are independent, so if the first toss is tails, it is like starting over on the second toss. Solving for EX, we obtain

$$EX=\frac{1}{p}.$$

Example 5.11 Consider two random variables X and Y with joint PMF given in Table 5.2. Let $Z=E[X|Y]$.

a. Find the Marginal PMFs of X and Y.
b. Find the conditional PMF of X given $Y=0$ and $Y=1$, i.e., find $P_{X|Y}(x|0)$ and $P_{X|Y}(x|1)$.
c. Find the PMF of Z.
d. Find EZ, and check that $EZ=EX$.
e. Find Var(Z).

Table 5.2: Joint PMF of X and Y in example 5.11

	$Y=0$	$Y=1$
$X=0$	$\frac{1}{5}$	$\frac{2}{5}$
$X=1$	$\frac{2}{5}$	0

37.

$\left(\frac{17}{18}\right)^5\left(\frac{1}{18}\right)^1$

38.

$p(\text{hit}) = \frac{3}{4}$

${}_6C_2\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^2$

39.

a. $\mu=3$ / week $\to \lambda=3$

$1-p(X=0)$

$=1-\left(\frac{e^{-3}\cdot 3^0}{0!}\right)$

b.

$p(2\le X<5) = p(2\le X\le 4)$

c.

$\mu=3/5$ / day $\to \lambda=3/5$

$p(X=1) = \frac{e^{-\frac{3}{5}}\left(\frac{3}{5}\right)^1}{1!} =$

$\therefore k=1$

40.

c.

~~Z | $Y=0, X=0$~~

~~$E(0|0)=\frac{E(X=0,Y=0)}{E(Y=0)}$~~

$=$

a. $f_Y(y)=\begin{cases}\frac{3}{5} & Y=0\\ \frac{2}{5} & Y=1\end{cases}$ $\qquad f_X(x)=\begin{cases}\frac{3}{5} & X=0\\ \frac{2}{5} & X=1\end{cases}$

Y	0	1
$E(Y)$	0	$\frac{2}{5}$

X	0	1
$E(X)$	0	$\frac{2}{5}$

b. $P(X|Y=0)$ $\qquad p(X|Y=1)$

$=\begin{cases}\frac{1}{3} & x=0\\ \frac{2}{3} & x=1\end{cases}$ $\qquad =\begin{cases}1 & x=0\\ 0 & x=1\end{cases}$

$\frac{\frac{1}{5}}{\frac{3}{5}}$

c. $E(X|Y=0)=\frac{2}{3}$ $\qquad E(X|Y=1)=0$

$P_Z=\begin{cases}\frac{3}{5} & Z=\frac{2}{3}\\ \frac{2}{5} & Z=0\\ 0 & \text{otherwise}\end{cases}$ ⇐

$Z=E(X|Y)$	$\frac{2}{3}$	0
	$\frac{3}{5}$ $P(Y=0)$	$\frac{2}{5}$ $P(Y=1)$

$P(X \mid X>1) \qquad X \sim Exp(1),\ \lambda = 1$

$$P(X>1) = \int_1^\infty e^{-x}\,dx$$

$$= -e^{-x}\Big|_1^\infty = 0 - (-e^{-1})$$

$$= \frac{1}{e}$$

$$P(X \mid X>1) = \begin{cases} e^{-x+1} & x>1 \\ 0 & \text{otherwise} \end{cases}$$

$$E(X \mid X>1) = \int_1^\infty x \cdot e^{-x+1}\,dx$$

$$= x(-e^{-x+1})\Big|_1^\infty - \int_1^\infty -e^{-x+1}\,dx$$

$$= 0 - (-1) + \int_1^\infty e^{-x+1}\,dx$$

$$= \cancel{1} - e^{-x+1}\Big|_1^\infty = \cancel{1} + 0 - (-e^0)$$

$$= 1$$

$$= 1 + 1 = 2.$$

$$E(X^2 \mid X>1) = \int_1^\infty x^2 e^{-x+1}\,dx$$

$$= x^2(-e^{-x+1})\Big|_1^\infty + \int_1^\infty 2x e^{-x+1}\,dx$$

$$= (0-(-1)) + \left(2x(-e^{-x+1})\Big|_1^\infty + \int_1^\infty 2(+e^{-x+1})\,dx \right)$$

$$= 1 + \left(0 + \cancel{(2(+1))}\ 2\right) + 2(-e^{-x+1})\Big|_1^\infty$$

$$= 3 + (0 - (-2))$$

$$= 5$$

$$Var(X \mid X>1) = E(X^2 \mid X>1) - E(X \mid X>1)^2$$

$$= 5 - 2^2 = 1$$

41.

$$p(X \mid Y=y) = \frac{p(X,Y)}{p(Y=y)}$$

$$\int_0^1 \frac{1}{4}x^2 + \frac{y^2}{4} + \frac{y}{6}x\,dx$$

$$\frac{1}{12}x^3 + \frac{y^2}{4}x + \frac{y}{12}x^2\Big|_0^1$$

$$= \frac{1}{12} + \frac{y^2}{4} + \frac{y}{12} = \frac{3y^2+y+1}{12}$$

$$\Rightarrow \frac{\frac{3x^2+3y^2+2xy}{12}}{\frac{3y^2+y+1}{12}} = \begin{cases} \frac{3x^2+3y^2+2xy}{3y^2+y+1} & 0 \le x \le 1,\ 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

b.

$$\cancel{\int_0^{\frac{1}{2}} \frac{1}{4}x^2 + \frac{y^2}{4} + \frac{y}{6}x\,dx} \qquad \int_0^{\frac{1}{2}} \frac{3x^2+3y^2+2xy}{3y^2+y+1}\,dx$$

$$\cancel{\frac{1}{12}x^3 + \frac{y^2}{4}x + \frac{y}{12}x^2\Big|_0^{\frac{1}{2}}}$$

$$\cancel{= \frac{1}{12}\cdot\frac{1}{8} + \frac{y^2}{4}\cdot\frac{1}{2} + \frac{y}{12}\cdot\frac{1}{4}}$$

$$\cancel{= \frac{1+12y^2+2y}{96}}$$

$$\cancel{\frac{\frac{1+12y^2+2y}{96\ 8}}{\frac{3y^2+y+1}{12}} = \frac{12y^2+2y+1}{8(3y^2+y+1)}}$$

42. $0<x<y<1$

$$\cancel{\int_0^1 8xy\,dx = 4yx^2\Big|_0^1 = 4y}$$

$$\cancel{\int_0^1 8xy\,dy = 4xy^2\Big|_0^1 = 4x}$$

$$\int_x^1 8xy\,dy = 4xy^2\Big|_x^1 = \begin{cases} 4x(1-x^2) & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^y 8xy\,dx = 4yx^2\Big|_0^y = \begin{cases} 4y(y^2) = 4y^3 & 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$