AN INTRODUCTION TO

Reliability and Maintainability Engineering

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CONTENTS

COURSE SOFTWARE

INSTRUCTIONS ON THE USE OF THE SOFTWARE TO ACCOMPANY THE TEXT

Available for use with this text is an MS-DOS executable file (REL.EXE) that may be run under DOS or under Windows as a non-Windows application. This software is available from the instructor and is included as part of the *Solutions Manual.* To execute the file, simply (1) type REL at the DOS prompt while in the directory the file is resident in; or (2) if not in the same directory as the file, include the path to the file (for example, >A:REL); or (3) when operating from Windows, double click on REL.EXE while in the file manager or Windows Explorer (Windows 95).

This software is intended for use in Part II of the text. It performs analysis on failure and repair data. Analysis options include:

Empirical models for ungrouped and grouped complete and singly censored data and for multiply censored data, including life tables (multiply censored grouped data). For multiply censored data the models include the incremental rank method, the Kaplan-Miers product-limit estimator, and an alternative product-limit estimator.

Least-squares analysis for fitting exponential, Weibull, normal, and lognormal distributions to either complete or censored data.

The Duane reliability growth model.

Nonhomogeneous Poisson processes (NHPP) (such as the AMSAA growth model).

Maximum likelihood estimation for exponential, Weibull, normal, and lognormal distributions with complete or censored data.

Goodness-of-fit tests including the chi-square, Bartlett (exponential), Mann (Weibull), Komogorov-Smirnov (normal and lognormal), and Cramer–von Mises (NHPP) with a test for trend.

Upon execution, the following main menu will appear:

```
MAIN MENU
INPUT/SAVE/OUTPUT OPTIONS
EMPIRICAL ANALYSIS
REL GROWTH NHPP POWER/LAW MODELS
LEAST-SQUARES CURVE FIT (PROB-PLOT)
MAXIMUM LIKELIHOOD ESTIMATE (MLE)
GOODNESS-OF-FIT TEST
QUIT
```

Entering data to the program is accomplished by selecting **INPUT/SAVE/OUTPUT OPTIONS** from the main menu. The **DATA/INPUT/DISPLAY MENU** shown below will appear. Initially data must be entered from the keyboard or from a compatible file provided with the text or created by the instructor. However, once you have entered

effect of both the failure and the repair process and is an important characteristic of the system.

1.1 THE STUDY OF RELIABILITY AND MAINTAINABILITY

As engineering disciplines, reliability and maintainability are relatively new. Their growth has been motivated by several factors, which include the increased complexity and sophistication of systems, public awareness of and insistence on product quality, new laws and regulations concerning product liability, government contractual requirements to meet reliability and maintainability performance specifications, and profit considerations resulting from the high cost of failures, their repairs, and warranty programs.

A Gallup poll conducted in 1985 for the American Society for Quality Control interviewed over 1000 individuals to determine what attributes were most important to them in selecting a product. The 10 attributes listed in Table 1.1 were ranked by each individual on a scale from 1 (least important) to 10 (most important); the average scores are as shown in the table. Obviously, both reliability and maintainability are important considerations in consumer purchasing.

Reliability and maintainability are not only an important part of the engineering design process but also necessary functions in life-cycle costing, cost benefit analysis, operational capability studies, repair and facility resourcing, inventory and spare parts requirement determinations, replacement decisions, and the establishment of preventive maintenance programs.

1.1.1 Reliability Improvement

A product has value as a result of its utility or performance in satisfying a need or requirement. Factors that contribute to a high value for a product are its versatility,

TABLE 1.1
Ten most important product attributes

Attribute	Average score
Performance	9.5
Lasts a long time (reliability)	**9.0**
Service	8.9
Easily repaired (maintainability)	**8.8**
Warranty	8.4
Easy to use	8.3
Appearance	7.7
Brand name	6.3
Packaging/display	5.8
Latest model	5.4

Source: *Quality Progress*, vol. 18 (Nov.), pp. 12–17, 1985.

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TABLE 1.2
Reliability test data results

	Motor					
	1–100	**101–200**	**201–300**	**301–400**	**401–500**	**Total**
Number tested	12	11	12	12	15	62
Hours on test	2540	2714	2291	1890	2438	11873
Number failed	1	0	1	5	7	14
Failure rate	0.000394	0	0.000436	0.002646	0.002871	0.001179

1.3 APPLICATIONS

The types of problems solved through the use of reliability and maintainability concepts are illustrated by the following examples.

EXAMPLE 1.1. The B. A. Miller Company manufactures small motors for use in household appliances such as washing machines, dryers, refrigerators, and vacuum cleaners. It has recently designed a new motor that has experienced the abnormally high failure rate of 43 failures among the first 1000 motors produced. Several of these failures were observed during final product testing by the appliance manufacturer. These motors were inspected, and it appeared that the bearing case was turning in its seat. The sealed ball bearings, however, appeared to be okay. Possible causes of these failures included faulty design, defective material, and a manufacturing (tolerance) problem. The company initiated an aggressive program of accelerated life testing of motors randomly selected from the production line. As a result of the testing program, it observed that those motors produced near the end of a production run were failing at a higher rate than those at the start of the run. Table 1.2 summarizes the results of the testing program, which are displayed graphically in Fig. 1.1.

The failure rate is computed by dividing the number of failures by the total number of hours on test. Dividing total hours on test by the number of failures provides an

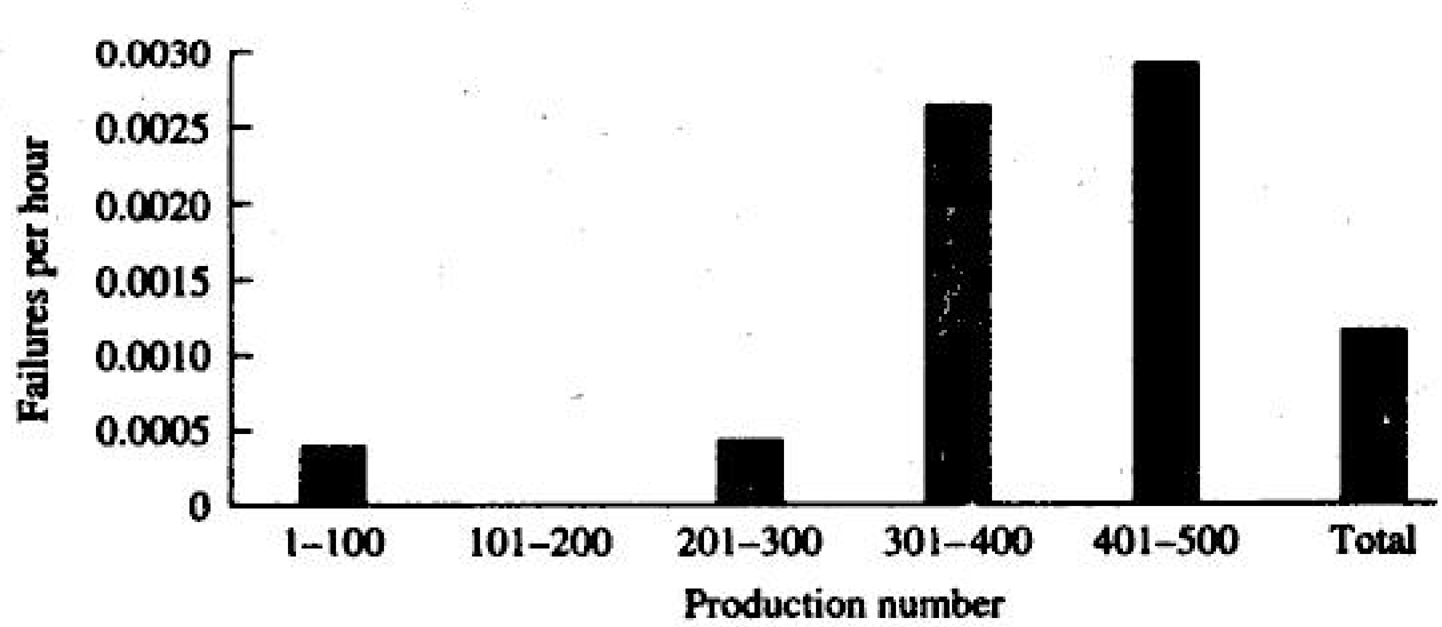

FIGURE 1.1
Failure rates of motors.

Reliability Review
Naval Research Logistics
International Journal of Reliability, Quality and Safety Engineering
Microelectronics and Reliability
Reliability Engineering
Journal of Applied Reliability

1.5 SCOPE OF THE TEXT

Figure 1.4 shows the organization of this book. The book is divided into three parts. Part I, "Reliability Models," develops mathematical models useful in analyzing component and system reliability, maintainability, and availability. Chapters 2–4

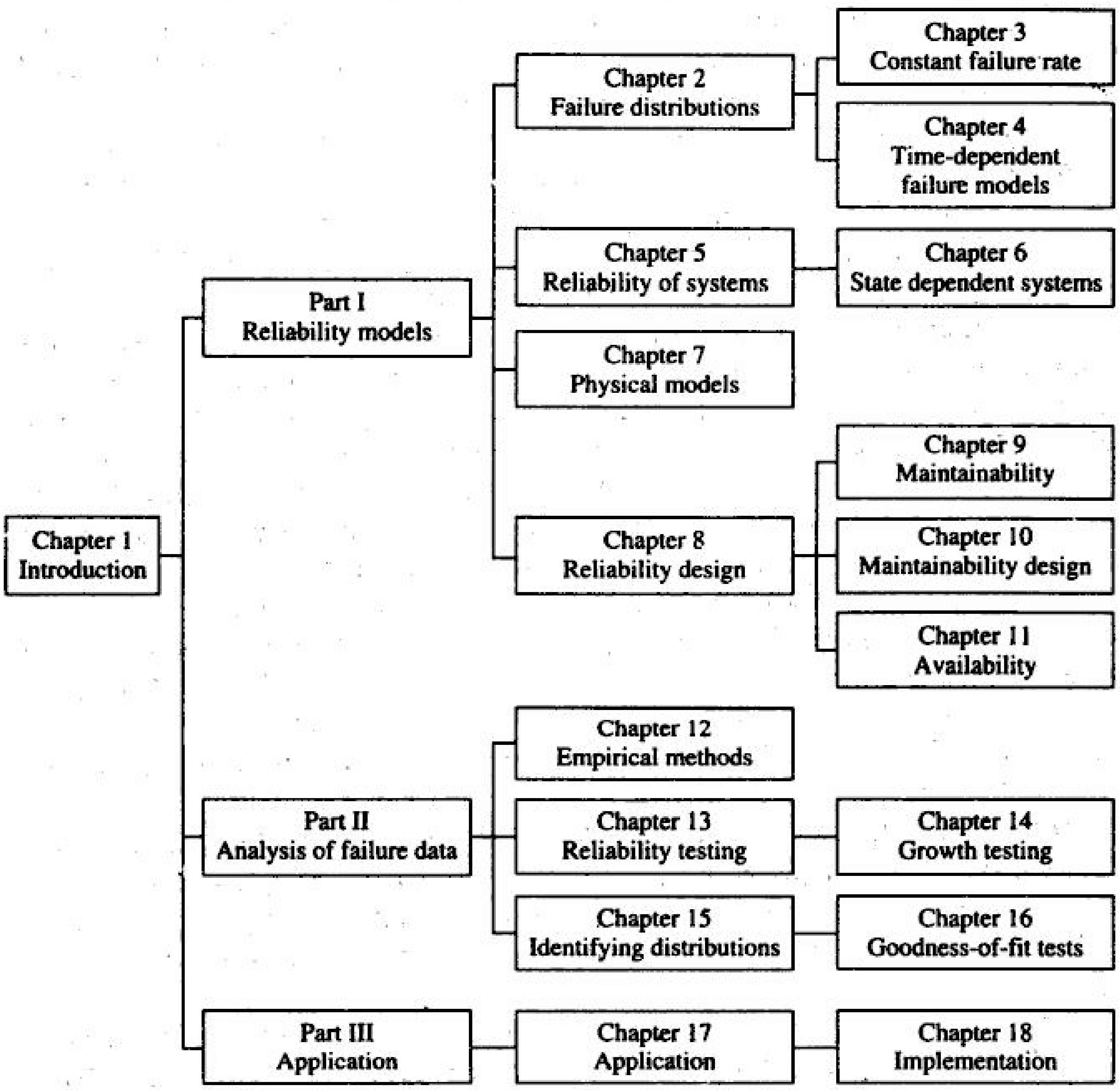

FIGURE 1.4
Organization of the text.

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fails and the event that component 2 fails, respectively. If $P(A) = P(B) = 0.05$ and $P(A \mid B) = P(B \mid A) = 0.10$, we are interested in the event that both components fail, or using Eq. (1.6),

$$P(A \cap B) = P(B)P(A \mid B) = P(A)P(B \mid A) = 0.05(0.10) = 0.005$$

EXAMPLE 1A.4. A two-component parallel system (repairable) is in a failure state (both components have failed) 3 percent of the time. Component 1 is in a failed state 8 percent of the time, and component 2 is in a failed state 6 percent of the time. If we let A = the event that component 1 is in a failed state and B = the event that component 2 is in a failed state, then $P(A \mid B) = 3/6 = 0.5$ and $P(B \mid A) = 3/8 = 0.375$.

The above discussion provides a general approach to analyzing the intersection of events. The following rule, referred to as the addition rule, does much the same for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{1.7}$$

Equation (1.7) can be explained by observing that $P(A)$ and $P(B)$ both include $P(A \cap B)$. Therefore $P(A \cap B)$ must be subtracted out once. If A and B are independent, Eq. (1.7) becomes

$$P(A \cup B) = P(A) + P(B) - P(A)P(B) \tag{1.8}$$

otherwise

$$P(A \cup B) = P(A) + P(B) - P(A \mid B)P(B) \tag{1.9}$$

EXAMPLE 1A.5. Given the two events A and B in Example 1A.3 and using Eq. (1.9), $P(A \cup B) = 0.05 + 0.05 - 0.005 = 0.095$ is the probability that at least one of the two components fails. Also, $P(A \cup B)^c = 1 - 0.095 = 0.905$ is the probability that neither component fails. In this example the reliability of the system is found from $P(A \cap B)^c = 1 - 0.005 = 0.995$, which is the probability that at least one of the two components does not fail.

1A.2 BAYES' FORMULA

Bayes' formula involves conditional probabilities of two events. It can be derived by observing that

$$\begin{aligned} P(A \mid B) &= \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P[(B \cap A) \cup (B \cap A^c)]} \\ &= \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A^c)P(A^c)} \end{aligned} \tag{1.10}$$

where the events $B \cap A$ and $B \cap A^c$ are mutually exclusive.

EXAMPLE 1A.6. A smoke detector is routinely inspected. Eighty percent of the detectors found inoperative had experienced a power surge, and 10 percent of those found in operating condition had experienced a power surge. Twenty percent of the detectors

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5. $\Pr\{a \leq X \leq b\} = \int_a^b f(x)\,dx = F(b) - F(a)$ (1.20)

6. $E(X) = \mu = \int_{-\infty}^{\infty} x f(x)\,dx$ (1.21)

where μ is the mean or expected value of the random variable X.

7. $\mathrm{Var}(X) = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\,dx$ (1.22)

where σ^2 is the variance of the distribution.

8. If $Y = \sum_{i=1}^{n} a_i X_i$ where the X_i are independent random variables having means μ_i and variances σ_i^2 and the a_i are constants, then

$$E(Y) = \sum_{i=1}^{n} a_i \mu_i \qquad \text{and} \qquad \mathrm{Var}(Y) = \sum_{i=1}^{n} a_i^2 \sigma_i^2 \tag{1.23}$$

In general we may describe the distribution of a random variable in terms of its PDF, $f(x)$, or its CDF, $F(x)$. Although a random variable may be defined over the open interval $(-\infty, \infty)$, when the random variable represents a failure time or repair time, only nonnegative values are permitted. Therefore the domain of the random variable would normally be $[0, \infty)$, and the lower limits of the integrals in relations 2, 4, 6, and 7 would reflect this change. The concepts of random variables and probability distributions are explored more fully in the next several chapters as the failure probability distribution is developed as the primary focus of Part I. Examples of these distributions will therefore be presented in the context of their use in developing reliability models. More detailed coverage of probability theory may be found in Ross [1987].

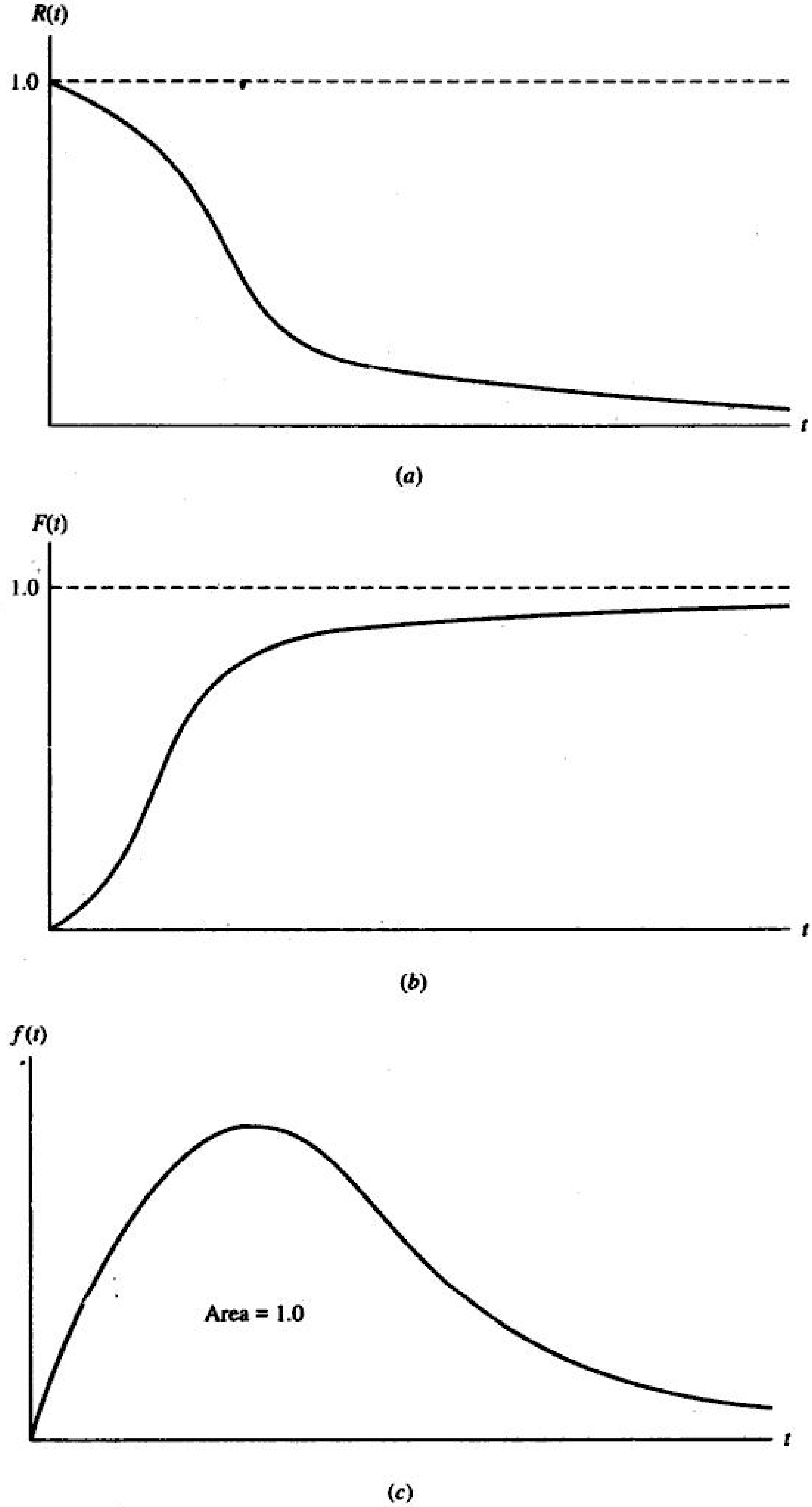

FIGURE 2.1
(*a*) The reliability function. (*b*) The cumulative distribution function. (*c*) The probability density function.

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Therefore $$\frac{dR(t \mid T_0)}{dT_0} = \exp\left[-\int_{T_0}^{t+T_0} \lambda(t')\,dt'\right] \frac{d}{dT_0}\left[\int_{T_0}^{t+T_0} \lambda(t')\,dt'\right]$$
$$= \quad R(t \mid T_0)[-\lambda(t+T_0) + \lambda(T_0)]$$

In order for the reliability to improve as a function of T_0, the derivative, or slope, of $R(t \mid T_0)$ with respect to T_0 must be positive. From the above result, this will occur only if $\lambda(T_0) > \lambda(t + T_0)$. In other words, the failure rate must be decreasing.

APPENDIX 2D INTERMEDIATE CALCULATIONS FOR THE LINEAR BATHTUB CURVE

For $t \leq c_0/c_1$:

$$R(t) = \exp\left[-\int_0^t (c_0 - c_1 t' + \lambda)\,dt'\right]$$
$$= \exp\left[-(c_0 t - \frac{c_1 t^2}{2} + \lambda t)\right]$$

For $c_0/c_1 < t \leq t_0$:

$$R(t) = R\left(\frac{c_0}{c_1}\right) \exp\left[-\int_{c_0/c_1}^{t} \lambda\,dt'\right]$$
$$= \exp\left(-\frac{c_0^2}{c_1} + c_1\frac{c_0^2}{2c_1^2} - \lambda\frac{c_0}{c_1}\right) \exp -\left(\lambda t - \lambda\frac{c_0}{c_1}\right)$$
$$= \exp -\left(\lambda t + \frac{c_0^2}{2c_1}\right)$$

For $t_0 < t$:

$$R(t) = R(t_0) \exp\left[-\int_{t_0}^{t} [c_2(t' - t_0) + \lambda]\,dt'\right]$$
$$= \exp -\left(\lambda t_0 + \frac{c_0^2}{2c_1}\right) \exp -\left(\frac{c_2}{2}(t - t_0)^2 + \lambda t - \lambda t_0\right)$$
$$= \exp -\left(\frac{c_2}{2}(t - t_0)^2 + \lambda t + \frac{c_0^2}{2c_1}\right)$$

CHAPTER 3

Constant Failure Rate Model

These next two chapters will develop several probability models useful in describing a failure process. These models are based upon the exponential, Weibull, normal, and lognormal probability distributions. They are often referred to as theoretical distributions since they are derived mathematically and not empirically. An important question to be answered is that of the ability of a particular theoretical distribution to describe the failures and reliability of a component or a system. The answer to that question is a central theme in Part II of the text. The immediate concern, however, is the development and use of these theoretical models in analyzing a failure process.

A failure distribution that has a constant failure rate is called an *exponential probability distribution.* The exponential distribution is one of the most important reliability distributions. Many systems exhibit constant failure rates, and the exponential distribution is in many respects the simplest reliability distribution to analyze. Another useful concept, failure modes, is also discussed. If the failure rates of all failure modes of a component are constant and independent, then the overall failure rate of the component is also constant.

3.1 THE EXPONENTIAL RELIABILITY FUNCTION

One of the most common failure distributions in reliability engineering is the exponential, or CFR, model. Failures due to completely random or chance events will follow this distribution. It should dominate during the useful life of a system or component. It is also one of the easiest distributions to analyze statistically. For example,

3.2 FAILURE MODES

Complex systems will fail through various means resulting from different physical phenomena or different failure characteristics of individual components. A useful analysis approach in reliability engineering is to separate these failures according to the mechanisms or components causing the failures. These categories of failures are then referred to as failure modes.[2]

If $R_i(t)$ is the reliability function for the ith failure mode, then, assuming *independence* among the failure modes, the system reliability $R(t)$ is obtained as

$$R(t) = \prod_{i=1}^{n} R_i(t) \tag{3.6}$$

$R_i(t)$ is the probability that the ith failure mode does not occur before time t, so $R(t)$, defined as the product of these probabilities, is the probability that none of the n failure modes occurs before time t. The system hazard rate function may be derived from Eq. (3.6), or it may be obtained directly by summing the hazard rate functions of all failure modes, as shown below.

Let $\lambda_i(t)$ be the failure rate function for the ith failure mode. Then

$$R_i(t) = \exp\left[-\int_0^t \lambda_i(t')\,dt'\right]$$

and

$$\begin{aligned} R(t) &= \prod_{i=1}^{n} \exp\left[-\int_0^t \lambda_i(t')\,dt'\right] \\ &= \exp\left[-\int_0^t \sum_{i=1}^{n} \lambda_i(t')\,dt'\right] \\ &= \exp\left[-\int_0^t \lambda(t')\,dt'\right] \end{aligned}$$

where

$$\lambda(t) = \sum_{i=1}^{n} \lambda_i(t) \tag{3.7}$$

This is an important result stating that the hazard rate function for the system is determined by summing the hazard rate functions of the n independent failure modes. Reliability testing and analysis can be conducted for different failure modes and a composite reliability function determined. This result also provides an alternative model of the bathtub curve, as shown in the following example.

---

[2]Failure modes are an example of a series relationship. Serial relationships are discussed further in Chapter 5.

TABLE 3.1
System renewals

Cycle	Number of failures
1	$\frac{1}{15}(1000) = 67$
2	$\frac{2}{15}(1000) + \frac{1}{15}(67) = 138$
3	$\frac{3}{15}(1000) + \frac{2}{15}(67) + \frac{1}{15}(138) = 218$
4	$\frac{4}{15}(1000) + \frac{3}{15}(67) + \frac{2}{15}(138) + \frac{1}{15}(218) = 313$
5	$\frac{5}{15}(1000) + \frac{4}{15}(67) + \frac{3}{15}(138) + \frac{2}{15}(218) + \frac{1}{15}(313) = 429$
6	$\frac{5}{15}(67) + \frac{4}{15}(138) + \frac{3}{15}(218) + \frac{2}{15}(313) + \frac{1}{15}(429) = 173$
7	$\frac{5}{15}(138) + \frac{4}{15}(218) + \frac{3}{15}(313) + \frac{2}{15}(429) + \frac{1}{15}(173) = 235$

Continuing in this manner:

Cycle	Number of failures	Cycle	Number of failures
8	281	18	277
9	303	19	274
10	294	20	271
11	236	21	272
12	269	22	275
13	283	23	274
14	281	24	273
15	271	25	273
16	263	26	273
17	275		

A steady-state constant rate of 273 failures per cycle is reached by the 24th cycle.

of time, an approximate exponential reliability function results.[5] Let $R = (1 - p)$, the reliability of a single load. Then

$$R_n = (1 - p)^n = e^{n \ln(1-p)}$$

is the reliability given n loads. Since[6] $\ln(1 - p) \approx -p$ for very small p,

$$R_n = e^{-np}$$

Let $n = t/\Delta t$, with Δt being the fixed time between loads. Then

$$R(t) = e^{-(p/\Delta t)t} = e^{-\lambda t}$$

with $\lambda = p/\Delta t$, a constant failure rate.

[5]A similar result is obtained if the loads are applied at random if the number of loads per time period has a Poisson distribution.

[6]This follows from a first-order Taylor series approximation around the origin; i.e., $f(x) = \ln(1 - x) \approx f(0) + xf'(0) = 0 + x(-1) = -x$.

$$\Pr\{Y_k \leq t\} = F_{Y_k}(t) = 1 - e^{-\lambda t}\sum_{i=0}^{k-1}\frac{(\lambda t)^i}{i!} \tag{3.22}$$

The cumulative probability given by Eq. (3.22) is the probability that the kth failure will occur by time t. The mean value for Y_k is k/λ, and the variance is k/λ^2. The mode is $(k-1)/\lambda$. We have

$$P_n(t) = \Pr\{Y_n \leq t\} - \Pr\{Y_{n+1} \leq t\} = F_{Y_n}(t) - F_{Y_{n+1}}(t)$$

Equation (3.21) follows from the above and from Eq. (3.22).

The relationship between the two probability distributions can also be seen by determining the probability of no failures occurring in time t, which is equivalent to $\Pr\{T \geq t\}$. That is,

$$p_0(t) = \frac{e^{-\lambda t}(\lambda t)^0}{0!} = e^{-\lambda t} = R(t)$$

The Poisson process is often used in inventory analysis to determine the number of spare components when the time between failures is exponential. For example, if S spare components are available to support a continuous operation over a time period t, then

$$R_S(t) = \sum_{n=0}^{S} p_n(t) \tag{3.23}$$

is the cumulative probability of S or fewer failures occurring during time t. Equation (3.23) therefore represents the probability of satisfying all demands for spare components during time t. Therefore, $R_S(t)$ is the component reliability if there are S spares available for immediate replacement when a failure occurs.

EXAMPLE 3.9. A specially designed welding machine has a nonrepairable motor with a constant failure rate of 0.05 failure per year. The company has purchased two spare motors. If the design life of the welding machine is 10 yr, what is the probability that the two spares will be adequate?

Solution. The expected number of failures over the life of the machine is $\lambda t = 0.05(10) = 0.5$. From Eq. (3.23),

$$R_2(10) = \sum_{n=0}^{2}\frac{e^{-0.5}0.5^n}{n!} = e^{-0.5}\left(1 + 0.5 + \frac{0.25}{2}\right) = 0.9856$$

is the probability of 2 or fewer failures occurring over the 10 yr.

Let Y_3 be the time of the third failure. Y_3 has a gamma distribution with $k = 3$ and $\lambda = 0.05$. Therefore, the expected, or mean, time to obtain 3 failures is $3/0.05 = 60$ yr. The probability that the third failure will occur within 10 yr is obtained from Eq. (3.22):

$$F_{Y_3}(10) = 1 - e^{-0.05\times 10}\left(1 + 0.05 \times 10 + \frac{(0.05 \times 10)^2}{2!}\right) = 0.0144$$

Observe that $0.0144 = 1 - 0.9856$ since the probability of two or fewer failures in 10 yr is complementary to the event that the third failure occurs within 10 yr.

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3.15 Consider two identical and redundant CFR components having a guaranteed life of 2 months and a failure rate of 0.15 failure per year. What is the system reliability for 10,000 hr of continuous operation?

3.16 Derive a general expression for $R(t)$ and the MTTF for the two-component system described in Exercise 3.15.

3.17 A microwave link in a communications network has a high failure rate. Although several pieces of equipment have been used in the link, the link always seems to fail at about the same rate regardless of the age of the equipment and its prior maintenance history. In general, microwave transmissions are subject to fading. Selective fading occurs when atmospheric conditions bend a transmission to the extent that signals reach the receiver in slightly different paths. The merging paths can cause interference and create data errors. Other channels in the microwave transmission are not affected by selective fading. Selective fading occurs when there is an electrical storm. Flat fading occurs during fog and when the surrounding ground is very moist. It is more serious since it may last several hours and affect surrounding channels. If during the current season, electrical storms occur about once every week and fog alerts are issued at the rate of one every two months, what is the reliability of the link over a 24-hour period? What assumptions, if any, are necessary?

3.18 A 60-watt outdoor lightbulb is advertised as having an average life (i.e., MTTF) of 1000 (operating) hours. However, experience has shown that it will also fail on demand an average of once every 120 cycles. A particular bulb is turned on once each evening for an average of 10 hr. If it is desired to have a reliability of 90 percent, what is its design life in days?

3.19 A more general exponential reliability model may be defined by

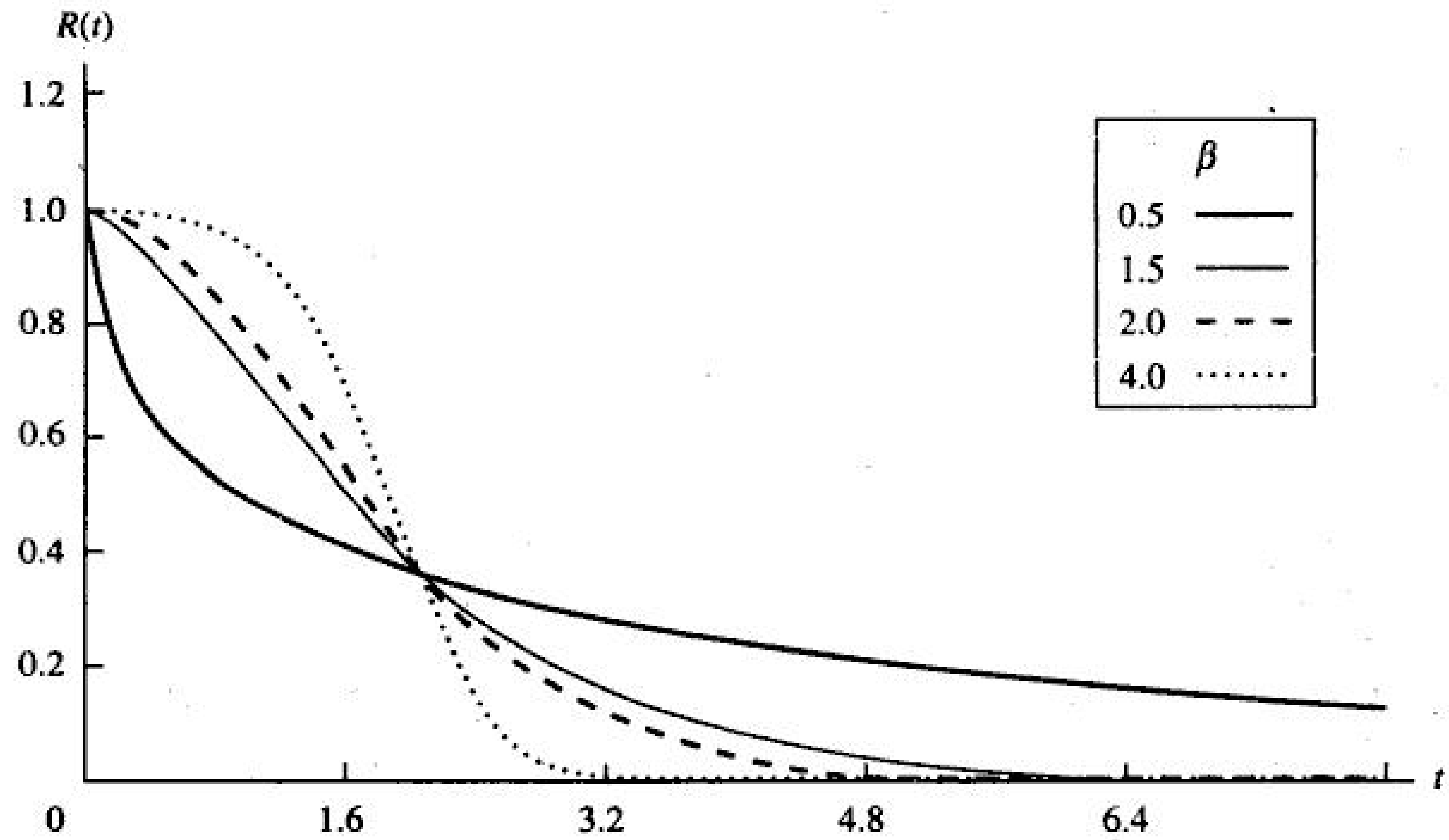
$$R(t) = a^{-bt} \qquad \text{where } a > 1, \qquad b > 0$$

and a and b are parameters to be determined. Find the hazard rate function, and show how this model is equivalent to $R(t) = e^{-\lambda t}$.

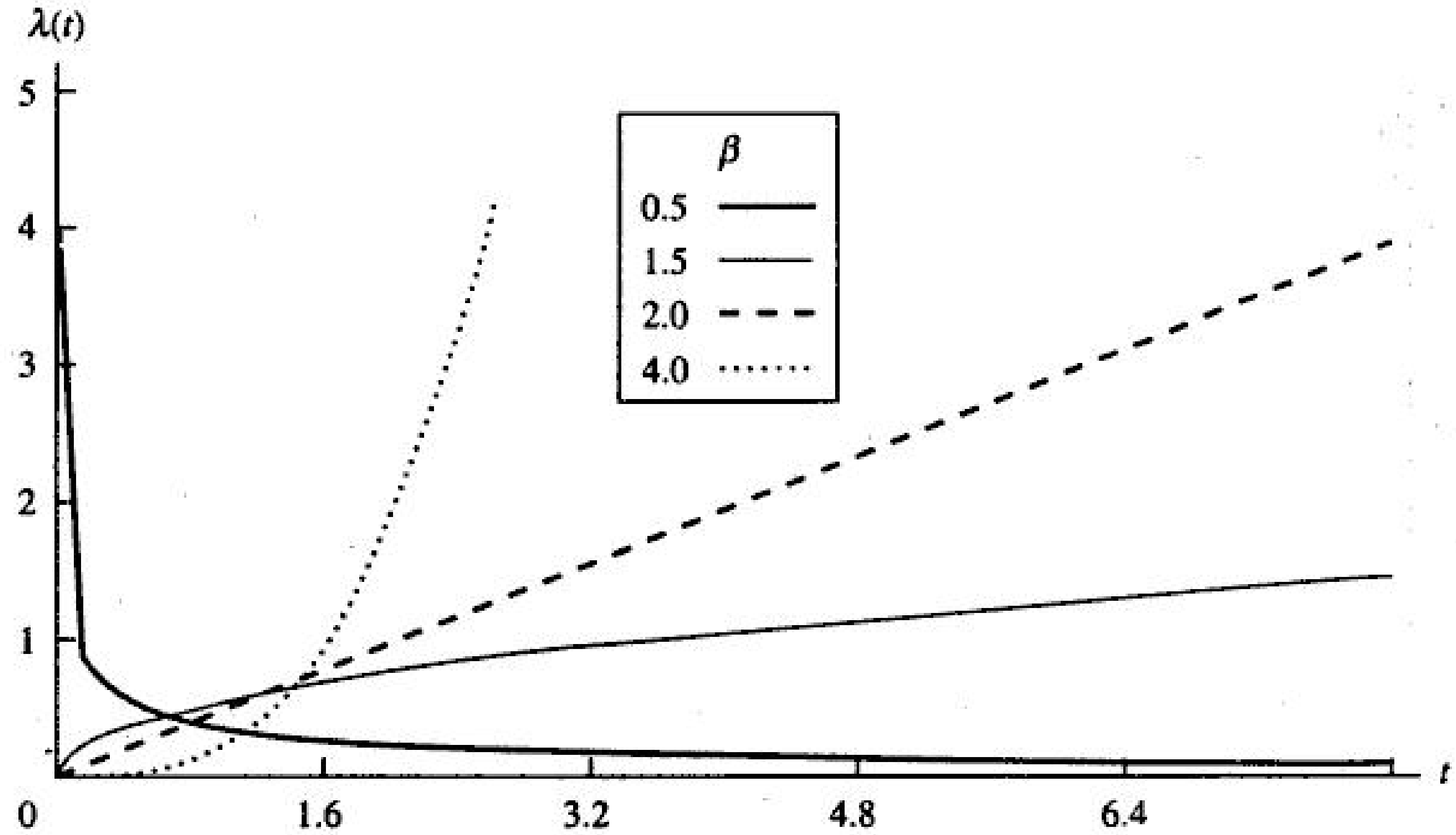
3.20 **Repetitive loading.** A packaging machine (cartoner) in a food processing facility will jam with a constant probability of 0.005 per application (per carton). Twelve cans of coffee are combined into a single case for shipment to buyers. The production rate is 30 cans of coffee every minute. What is the probability (reliability) of no jams during a 1-hr production run?

3.21 For the reliability function $R(t) = e^{-(t/1000)^2}$, use Eqs. (3.13) and (3.14) and compare the upper and lower bounds with the actual reliabilities at 100, 200, 500, 800, 1000, 2000, 5000, and 10,000 hr. This failure distribution has an IFR with an MTTF of 1772.46.

3.22 Show for the exponential distribution that the residual mean life is $1/\lambda$ regardless of the length of time the system has been operating.



(c)



(d)

FIGURE 4.1 *(continued)*
The effect of β *(c)* on the Weibull reliability function; *(d)* on the Weibull hazard rate curve.

In this case, $\beta = 2$ and $\theta = 1000$ hr. For a desired 0.99 reliability,

$$R(t) = e^{-(t/1000)^2} = 0.99$$

The design life is given by

$$t_R = 1000\sqrt{-\ln 0.99} = 100.25 \text{ hr}$$

From Eqs. (4.4) and (4.5),

$$\text{MTTF} = 1000\,\Gamma\left(1 + \tfrac{1}{2}\right) = 886.23 \text{ hr}$$

5. The characteristic life is 16,000 hr. Therefore 63 percent of the failures will occur by this time.
6. If a 90 percent reliability is desired, the design life is

$$t_R = 16{,}000(-\ln 0.90)^3 = 18.71 \text{ hr}$$

7. Its B1 life is $(16{,}000)(-\ln 0.99)^3 = 0.0162$ hr, indicating a high percentage of early failures.

4.1.2 Burn-In Screening for Weibull

The component depicted in Example 4.2 would be a good candidate for burn-in screening. Using the conditional reliability as defined by Eq. (2.17),

$$R(t \mid T_0) = \frac{\exp\{-[(t+T_0)/\theta]^\beta\}}{\exp[-(T_0/\theta)^\beta]} = \exp\left[-\left(\frac{t+T_0}{\theta}\right)^\beta + \left(\frac{T_0}{\theta}\right)^\beta\right]$$

for the Weibull model. If, in the previous example, a 10-hr burn-in period is accomplished, then

$$R(t \mid 10) = \exp\left[-\left(\frac{t+10}{16{,}000}\right)^{1/3} + \left(\frac{10}{16{,}000}\right)^{1/3}\right]$$

For a 90 percent reliability, the design life is found by solving for t_R:

$$R(t_R \mid 10) = 0.90$$

$$\begin{aligned} t_R &= 16{,}000\left[-\ln 0.90 + \left(\frac{10}{16{,}000}\right)^{1/3}\right]^3 - 10 \\ &= 101.24 \text{ hr} \end{aligned}$$

This is a significant increase in the component's design life over the original 18.71 hr.

4.1.3 Failure Modes

For a system comprised of n serially related components or having n independent failure modes, each having an independent Weibull failure distribution with shape parameter β and scale parameter θ_i, the system failure rate function can be determined from Eq. (3.7):

$$\begin{aligned} \lambda(t) &= \sum_{i=1}^{n} \frac{\beta}{\theta_i}\left(\frac{t}{\theta_i}\right)^{\beta-1} \\ &= \beta t^{\beta-1}\left[\sum_{i=1}^{n}\left(\frac{1}{\theta_i}\right)^\beta\right] \end{aligned} \tag{4.9}$$

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Solution. Using Eq. (4.17),

$$R_s(100) = 2\exp\left[-\left(\frac{100}{1000}\right)^{1/2}\right] - \exp\left[-2\left(\frac{100}{1000}\right)^{1/2}\right] = 0.9265$$

Then from Eq. (4.18),

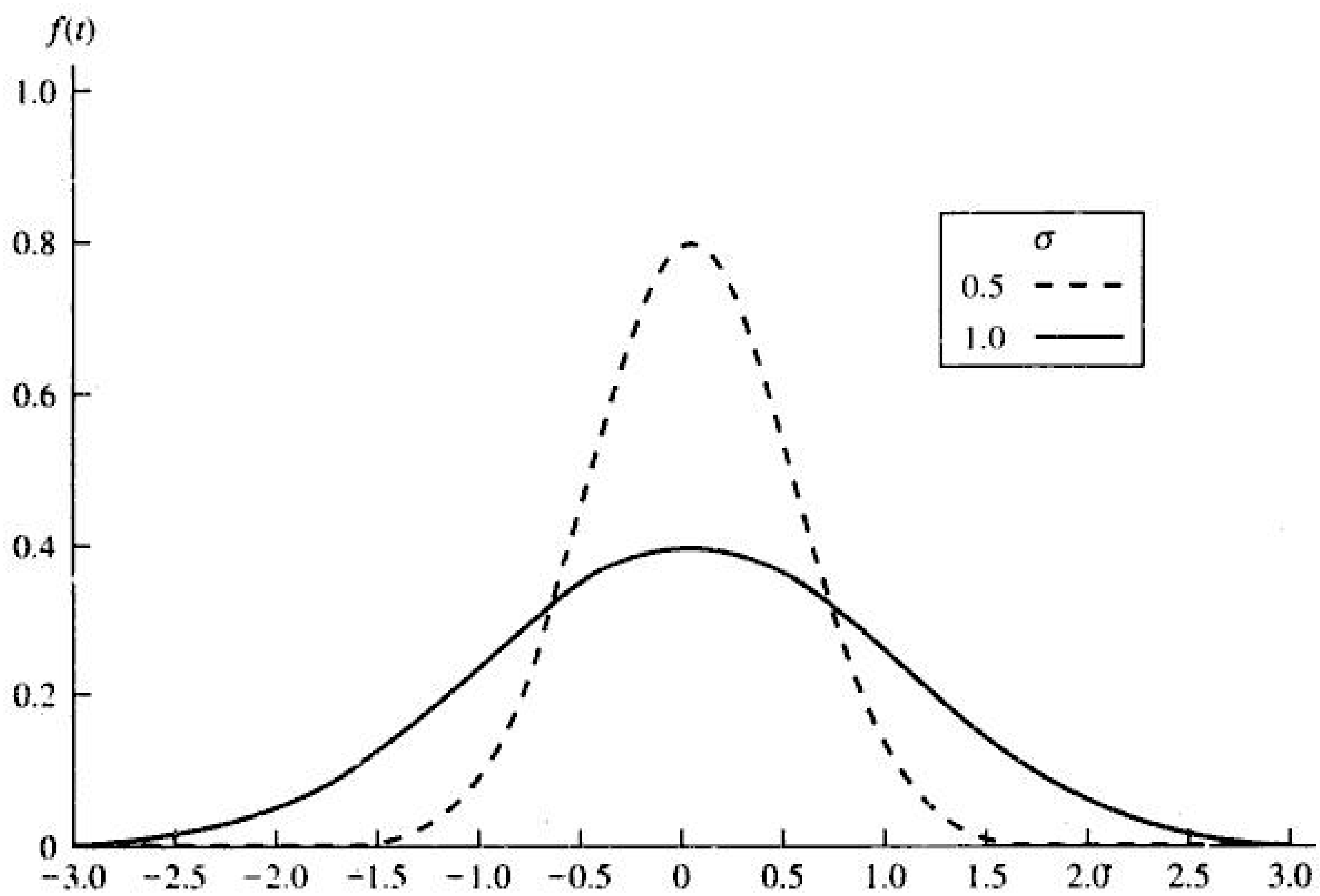
$$\text{MTTF} = 1000\,\Gamma\,(3)(2 - 2^{-2}) = 3500 \text{ hr}$$

As a comparison, a single fuel pump will have a mission reliability of 0.7288 and an MTTF equal to 2000.

4.2 THE NORMAL DISTRIBUTION

The normal distribution has been used successfully to model fatigue and wearout phenomena. Because of its relationship with the lognormal distribution, it is also useful in analyzing lognormal probabilities. The density function of the normal provides the familiar bell-shaped curve shown in Fig. 4.3(a). The formula for the PDF is

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\frac{(t-\mu)^2}{\sigma^2}\right] \qquad -\infty < t < \infty \tag{4.20}$$



(a)

FIGURE 4.3
The effect of the standard deviation σ *(a)* on the normal probability density function.

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The CLT implies that each random variable contributes a small amount to the total, with no single variable dominating. This result holds regardless of the distribution of the random variables. From a reliability perspective, if a failure is a result of many small cumulative effects, the normal failure process may be appropriate. Later, the CLT will enable us to compute approximate confidence intervals about an estimate of the MTTF obtained from sample failure data. However, our interest now in the normal distribution is also motivated by its use in the development of the lognormal failure process.

4.3 THE LOGNORMAL DISTRIBUTION

If the random variable T, the time to failure, has a lognormal distribution, the logarithm of T has a normal distribution. This is a very useful relationship in working with the lognormal distribution. The density function for the lognormal is

$$f(t) = \frac{1}{\sqrt{2\pi}st} \exp\left[-\frac{1}{2s^2}\left(\ln\frac{t}{t_{\text{med}}}\right)^2\right] \qquad t \geq 0 \tag{4.27}$$

where the parameter s is a shape parameter and t_{med}, the location parameter, is the median time to failure.[2] The distribution is defined for only positive values of t and is therefore more appropriate than the normal as a failure distribution. Examples of the lognormal probability density function for different values of the shape parameter are provided in Fig. 4.4(a). Like the Weibull distribution, the lognormal can take on a variety of shapes. It is frequently the case that data that fit a Weibull distribution will also fit a lognormal distribution.

The mean, variance, and mode of the lognormal are

$$\text{MTTF} = t_{\text{med}} \exp(s^2/2) \tag{4.28}$$

$$\sigma^2 = t_{\text{med}}^2 \exp(s^2)[\exp(s^2) - 1] \tag{4.29}$$

$$t_{\text{mode}} = \frac{t_{\text{med}}}{\exp(s^2)} \tag{4.30}$$

To compute failure probabilities, the lognormal's relationship to the normal is utilized. This relationship between the lognormal and normal distributions is summarized in Table 4.2.

---

[2] An alternative form of the lognormal density function uses the mean and standard deviation of the logarithm of T as the distribution parameters. In this case,

$$f(t) = \frac{1}{\sqrt{2\pi}t\sigma_n} \exp\left[-\frac{(\ln t - \mu_n)^2}{2\sigma_n^2}\right]$$

where μ_n and σ_n are the mean and standard deviation of $\ln t$.

or
$$\Phi\left(\frac{1}{s}\ln\frac{t_R}{t_{\text{med}}}\right) = 1 - R$$

and
$$\frac{1}{s}\ln\frac{t_R}{t_{\text{med}}} = z_{1-R}$$

where z_{1-R} is found in Table A.1 such that

$$\Phi(z_{1-R}) = 1 - R$$

Solving for t_R:

$$t_R = t_{\text{med}}\, e^{s z_{1-R}} \tag{4.32}$$

EXAMPLE 4.10. From the previous example, for a reliability of 0.95,

$$t_{0.95} = 5000\, e^{0.20(-1.64)} = 3602 \text{ hr}$$

APPENDIX 4A
DERIVATION OF THE MTTF FOR THE WEIBULL DISTRIBUTION

By definition,

$$\text{MTTF} = \int_0^\infty \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-(t/\theta)^\beta}\, t\, dt$$

Let
$$y = \left(\frac{t}{\theta}\right)^\beta$$

then
$$dy = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} dt$$

or
$$\text{MTTF} = \int_0^\infty t e^{-y} dy$$

Since
$$t = \theta y^{1/\beta}$$

we have
$$\text{MTTF} = \theta \int_0^\infty y^{1/\beta} e^{-y}\, dy$$
$$= \theta\,\Gamma\left(1 + \frac{1}{\beta}\right)$$

since
$$\Gamma(x) = \int_0^\infty y^{x-1} e^{-y}\, dy$$

(a) Determine its design life if specifications call for a reliability of 0.98.
(b) The component is to be used in a pumping device that will require 5 weeks of continuous use. What is the probability of a failure occurring because of the valve?
(c) Compute the MTTF.
(d) Find the standard deviation of the time to failure.
(e) Determine the mode.

4.11. A pressure gauge has been observed to have a Weibull failure distribution with a shape parameter of 2.1 and a characteristic life of 12,000 hr. Find the following:
(a) $R(5000 \text{ hr})$
(b) The B1 and B.1 life
(c) The MTTF and the standard deviation
(d) The median and the mode
(e) The probability of failure in the first year of continuous operation

4.12. A component has a Weibull failure distribution with β equal to 0.86, and its characteristic life is 2450 days. By how many days will the design life for a 0.90 reliability specification be extended as a result of a 30-day burn-in period?

4.13. An automobile engine has four belts each showing the identical wearout effect with a Weibull shape parameter of 1.34. However, their scale parameters are 2500, 3400, 8000, and 6100 operating hours.
(a) If the automobile is new, determine the probability of a belt failure on a 72-hr trip.
(b) If the car (with the belts) has had 4000 hr of use, what is the probability of a belt failure during the next 72 hr of use?

4.14. For the pressure gauge in Exercise 4.11, assume that two identical gauges are used in parallel (redundant) configuration.
(a) Find the system reliability for 5000 hr.
(b) Compute the system MTTF.
(c) Find the probability of failure in the first year.
(d) Determine, by trial and error, the B1 life.

4.15. A cutting tool wears out with a time to failure that is normally distributed with a mean of 10 working days and a standard deviation of 2.5 days.
(a) Determine its design life for a reliability of 0.99.
(b) Find the reliability if the tool is replaced every (i) day; (ii) two days; (iii) five days.
(c) Determine the probability that the cutting tool will last one more day given it has been in use for 5 days.

4.16. A complex machine has a high number of failures. The time to failure was found to be lognormal with $s = 1.25$. Specifications call for a reliability of 0.95 at 1000 cycles.
(a) Determine the median time to failure that must be achieved by engineering modifications to meet the specifications. Assume that design changes do not affect the shape parameter s.
(b) What is the corresponding MTTF and standard deviation?
(c) If the desired median time to failure is obtained, what is the reliability during the next 1000 cycles given it has operated for 1000 cycles?

TABLE 5.1
Serially related system reliability

Component reliability	Number of components 10	100	1000
0.900	0.3487	0.266×10^{-4}	0.17479×10^{-45}
0.950	0.5987	0.00592	0.52918×10^{-22}
0.990	0.9044	0.3660	0.432×10^{-4}
0.999	0.9900	0.9048	0.3677

EXAMPLE 5.1. Consider a four-component system of which the components are independent and identically distributed with CFR. If $R_s(100) = 0.95$ is the specified reliability, find the individual component MTTF.

Solution

$$R_s(100) = e^{-100\lambda_s} = e^{-100(4)\lambda} = 0.95$$

or

$$\lambda = \frac{-\ln 0.95}{400} = 0.000128$$

and

$$\text{MTTF} = \frac{1}{0.000128} = 7812.5$$

In general, for CFR components

$$\text{MTTF}_s = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\sum_{i=1}^{n} 1/\text{MTTF}_i} \tag{5.5}$$

where MTTF_i = mean time to failure of the ith component.

EXAMPLE 5.2. A system is comprised of four serially related components each having a Weibull time to failure distribution with parameters as shown in the accompanying table.

Component	Scale parameter	Shape parameter
1	100	1.20
2	150	0.87
3	510	1.80
4	720	1.00

The system reliability is therefore given by

$$R_s(t) = \exp\left\{-\left[\left(\frac{t}{100}\right)^{1.2} + \left(\frac{t}{150}\right)^{0.87} + \left(\frac{t}{510}\right)^{1.8} + \left(\frac{t}{720}\right)^{1.0}\right]\right\}$$

and, for example, $R(10) = e^{-0.172627} = 0.8415$.

5.2 PARALLEL CONFIGURATION

Two or more components are in parallel, or redundant, configuration if all units must fail for the system to fail. If one or more units operate, the system continues to operate. Parallel units are represented by the block diagram of Fig. 5.2.

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5.3.2 *k*-out-of-*n* Redundancy

A generalization of n parallel components occurs when a requirement exists for k out of n identical and independent components to function for the system to function. Obviously $k \leq n$. If $k = 1$, complete redundancy occurs, and if $k = n$, the n components are, in effect, in series. The reliability can be obtained from the binomial probability distribution.

If each component is viewed as an independent trial with R (its reliability) as a constant probability of success, then

$$P(x) = \binom{n}{x} R^x (1 - R)^{n-x} \tag{5.8}$$

is the probability of exactly x components operating. This is true since

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

is the number of ways (arrangements) in which x successes (nonfailures) can be obtained from n components. $R^x(1 - R)^{n-x}$ is the probability of x successes and $n - x$ failures for a single arrangement of successes and failures. Therefore

$$R_s = \sum_{x=k}^{n} P(x) \tag{5.9}$$

is the probability of k or more successes from among the n components.

EXAMPLE 5.6. A space vehicle requires three out of its four main engines to operate in order to achieve orbit. If each engine has a reliability of 0.97, determine the reliability of achieving orbit.

Solution

$$\begin{aligned} R_s &= \sum_{x=3}^{4} \binom{4}{x} 0.97^x 0.03^{4-x} \\ &= 4(0.97)^3(0.03) + 0.97^4 = 0.9948 \end{aligned}$$

Exponential failures

If the failure distribution is exponential,

$$R_s(t) = \sum_{x=k}^{n} \binom{n}{x} e^{-\lambda x t}[1 - e^{-\lambda t}]^{n-x}$$

Jumonville and Lesso [1969] have shown that in this case the MTTF can be expressed as

$$\text{MTTF} = \int_0^\infty R_s(t)\, dt = \frac{1}{\lambda} \sum_{x=k}^{n} \frac{1}{x} \tag{5.10}$$

EXAMPLE 5.6 (CONTINUED). The main engines in Example 5.6 require an 8-minute burn time. Assume a constant failure rate of 0.0038074 for each engine. Then $R(8) = e^{-0.0038074(8)} = 0.97$, and a single-engine MTTF is 262.65 minutes. Therefore

$$\text{MTTF}_s = 262.65\left(\tfrac{1}{3} + \tfrac{1}{4}\right) = 153.21$$

5.4 SYSTEM STRUCTURE FUNCTION, MINIMAL CUTS, AND MINIMAL PATHS (OPTIONAL)

A very general alternative approach for analyzing the reliability of complex systems is through the use of the system structure function. To define the system structure function, let

$$X_i = \begin{cases} 1 & \text{if component } i \text{ operates} \\ 0 & \text{if component } i \text{ has failed} \end{cases}$$

Then the system structure function is defined by

$$\Psi(X_1, X_2, \ldots, X_n) = \begin{cases} 1 & \text{if system operates} \\ 0 & \text{if system has failed} \end{cases} \tag{5.11}$$

Therefore, for a series system,

$$\Psi(X_1, X_2, \ldots, X_n) = X_1 X_2 \cdots X_n = \min[X_1, \ldots, X_n] \tag{5.12}$$

and for a parallel system,

$$\Psi(X_1, X_2, \ldots, X_n) = 1 - (1 - X_1)(1 - X_2)\cdots(1 - X_n) = \max[X_1, X_2, \ldots, X_n] \tag{5.13}$$

For a k-out-of-n system, the system structure function is given by

$$\Psi(X_1, X_2, \ldots, X_n) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} X_i \geq k \\ 0 & \text{if } \sum_{i=1}^{n} X_i < k \end{cases} \tag{5.14}$$

Our interest is in finding $R_s = \Pr\{\Psi(X_1, X_2, \ldots, X_n) = 1\} = E[\Psi(X_1, X_2, \ldots, X_n)]$. The second equality is a result of the binary form of the structure function since

$$\begin{aligned} E[\Psi(X_1, X_2, \ldots, X_n)] &= 0 \cdot \Pr\{\Psi(X_1, X_2, \ldots, X_n) = 0\} \\ &\quad + 1 \Pr\{\Psi(X_1, X_2, \ldots, X_n) = 1\} \end{aligned}$$

Assuming independence, for a series system,

$$\begin{aligned} \Pr\{\Psi(X_1, X_2, \ldots, X_n) = 1\} &= \Pr\{X_1 = 1, X_2 = 1, \ldots, X_n = 1\} \\ &= \Pr\{X_1 = 1\}\Pr\{X_2 = 1\}\cdots\Pr\{X_n = 1\} \\ &= R_1 R_2 \cdots R_n \end{aligned} \tag{5.15}$$

and for a parallel system,

$$\begin{aligned} \Pr\{\Psi(X_1, X_2, \ldots, X_n) = 1\} &= \Pr\{\max(X_1, X_2, \ldots, X_n) = 1\} \\ &= 1 - \Pr\{\text{all } X_i = 0\} \\ &= 1 - \Pr\{X_1 = 0, X_2 = 0, \ldots, X_n = 0\} \\ &= 1 - (1 - R_1)(1 - R_2)\cdots(1 - R_n) \end{aligned} \tag{5.16}$$

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Minimal cuts	Probability = $\prod(1 - R_k)$
A, B	$0.1(0.1) = 0.01$
C, D	$0.05(0.05) = 0.0025$
A, E, D	$0.1(0.2)(0.05) = 0.001$
B, E, C	$0.1(0.2)(0.05) = 0.001$

By configuring the set of minimal cuts in series, a lower bound on the system reliability is obtained since the system reliability cannot be any worse than the reliability that at least one component in each cut set operates. Therefore, from Eq. (5.18),

$$R_l = (1 - 0.01)(1 - 0.0025)(1 - 0.001)^2 = 0.98555$$

The minimal path sets and their reliabilities are as given here:

Minimal paths	Reliability = $\prod(R_k)$
A, C	$0.9(0.95) = 0.855$
B, D	$0.9(0.95) = 0.855$
A, E, D	$0.9(0.8)(0.95) = 0.684$
B, E, C	$0.9(0.8)(0.95) = 0.684$

By configuring the set of minimal paths in parallel, an upper bound on the system is obtained since the system cannot be more reliable than the union of all of its paths. Therefore, from Eq. (5.19),

$$R_u = 1 - (1 - 0.855)^2(1 - 0.684)^2 = 0.9979$$

5.5 COMMON-MODE FAILURES

The assumption of independence of failures among n components within a system may be easily violated. For example, several components may share the same power source, or external environmental conditions such as excessive heat or vibration may affect several components in the same manner. Operations or maintenance errors, design flaws, and substandard material or parts may also contribute to a common-mode failure. A common-mode failure can be depicted in series with those components sharing the failure mode. Figure 5.10 illustrates a common-mode failure associated with a three-component redundant system. The system reliability is given by $R_s = [1 - (1 - R_1)(1 - R_2)(1 - R_3)]R'$. In order to represent the system in this way, it must be possible to separate independent failures from the common-mode failures. In order for the redundancy network to have an effect, the common-mode failure must have a high reliability

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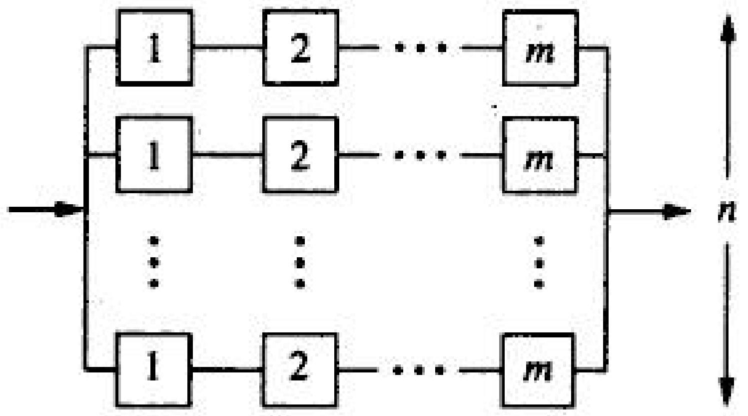


FIGURE 5.12
Block diagram of m three-state devices in a high-level redundant configuration.

5.6.4 High-Level Redundancy

Figure 5.12 represents a high-level redundant configuration having n redundant systems each with m serial components. The system reliability is a generalization of the parallel configuration (Eq. 5.21) and is found by computing the probability that no path is completely shorted minus the probability that there is at least one open on each path. Mathematically,

$$R_{\mathrm{H}} = \left(1 - \prod_{i=1}^{m} q_{si}\right)^{n} - \left[1 - \prod_{i=1}^{m}(1 - q_{oi})\right]^{n} \tag{5.23}$$

EXAMPLE 5.13. With three-state devices, it is not necessarily true that low-level redundancy provides a greater reliability than high-level redundancy. Consider a system composed of the following three components with two redundant units available: $m = 3$ and $n = 2$.

Component	q_s	q_o
1	0.15	0.05
2	0.10	0.06
3	0.20	0.01

Then

$$\begin{aligned} R_{\mathrm{L}} &= (1 - 0.05^2)(1 - 0.06^2)(1 - 0.01^2) \\ &\quad - [1 - (1 - 0.15)^2][1 - (1 - 0.10)^2[1 - (1 - 0.20)^2] \\ &= 0.9748 \end{aligned}$$

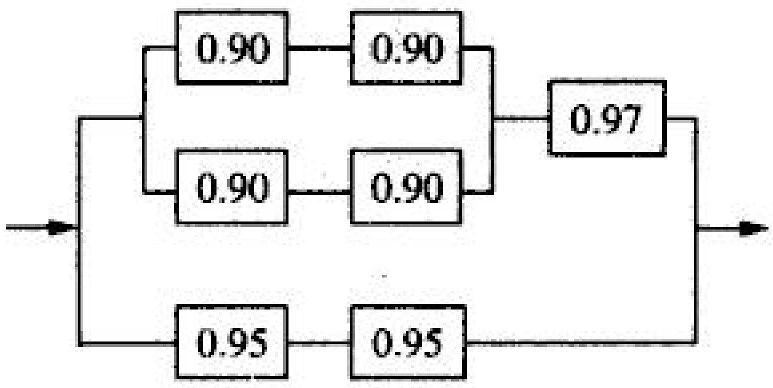
$$R_{\mathrm{H}} = [1 - (0.15)(0.10)(0.20)]^2 - [1 - (1 - 0.05)(1 - 0.06)(1 - 0.01)]^2 = 0.9806$$

The reliability of the above networks could have been determined from an opposite viewpoint. For example, the low-level redundant network could have been analyzed by finding the system reliability with respect to a short minus the system probability of failing open. This approach is equivalent to Eq. (5.22) since

$$\begin{aligned} R\{\text{short}\} - \Pr\{\text{failing open}\} &= 1 - \Pr\{\text{failing short}\} - [1 - R\{\text{failing open}\}] \\ &= R\{\text{failing open}\} - \Pr\{\text{failing short}\} \end{aligned}$$

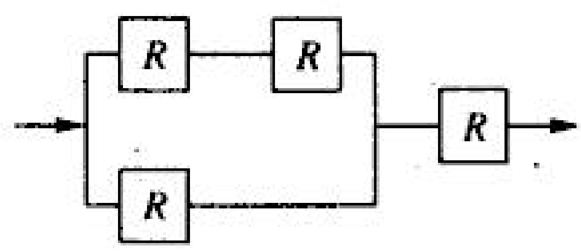
Because the events "failing open" and "failing short" are mutually exclusive, this approach works.

(b)



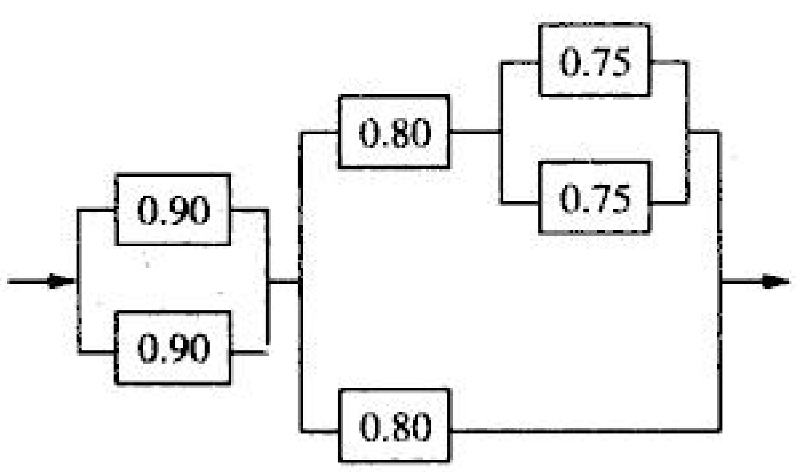
(*b*)

For $R_s = 0.99$, find R in (c). (*Hint:* Find R_s in the simplest terms of R and use trial and error).

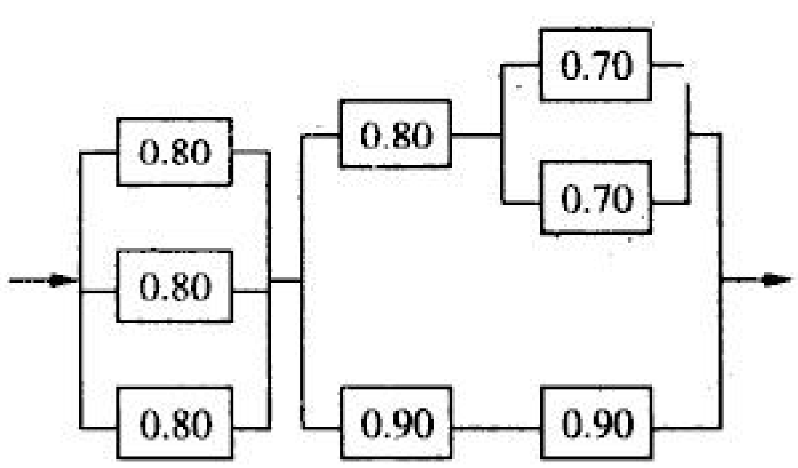


(*c*)

5.8 Find the system reliability of the following series-parallel configurations. Component reliabilities are given.

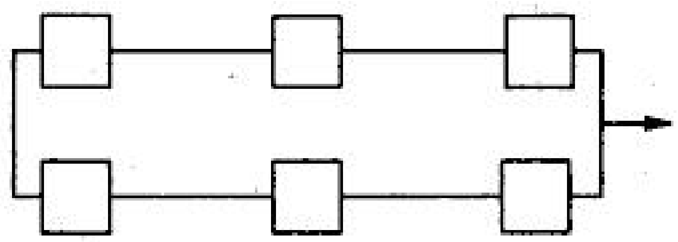


(*a*)

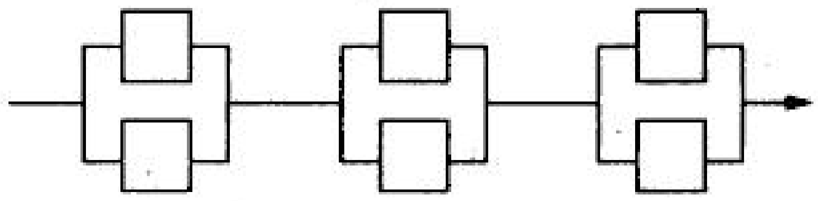


(*b*)

5.9 For each of the following redundant systems, determine the component MTTF necessary to provide a system reliability of 0.90 after 100 hr of operation. The components have the same constant failure rate.



(*a*) High-level redundancy



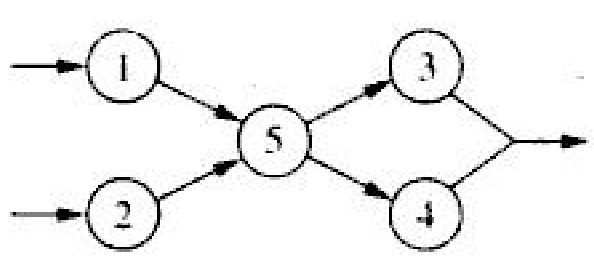
(*b*) Low-level redundancy

5.10 Itsa Failing, a reliability engineer, has determined that the reliability function for a critical solid-state power unit for use in a communications satellite is $R(t) = 10/(10 + t)$, $t \geq 0$ and t measured in years.
(a) How many units must be placed in parallel in order to achieve a reliability of 0.98 for 5 yr of operation?
(b) If there is an additional common mode, constant failure rate of 0.002 as a result of environmental factors, how many units should be placed in parallel? Compute the achieved system reliability.

5.11 Derive the reliability function and MTTF for three CFR components in parallel. If three components, each with CFR, are placed in parallel, determine the system reliability for 0.1 yr and the MTTF. Their failure rates are 5 per year, 10 per year, and 15 per year.

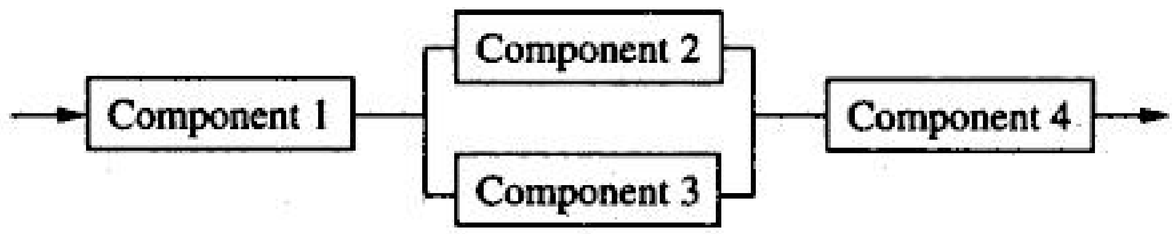
5.12 A signal processor has a reliability of 0.90. Because of the low reliability a redundant signal processor is to be added. However, a signal splitter must be added before the signal processors, and a comparator must be added after the signal processors. Each of these new components has a reliability of 0.95. Does adding a redundant signal processor increase the system reliability?

5.13 The following natural gas distribution network contains five shut-off valves configured as shown. Valves 1–4 have a probability of 0.02 of failing open and a probability of 0.15 of failing short. Valve 5 has a probability of 0.05 of failing open and a probability of 0.20 of failing short. Find the system reliability.



5.14 Compare the reliability between a high-level redundant network and a low-level redundant network comprised of two serially related components with three redundant units of each available. The probability of any component failing open is 0.05, and the probability of any component failing short is 0.1.

5.15 Determine the reliability of the following series-parallel configuration, in which the ith component has a probability q_{oi} of failing open and a probability q_{si} of failing short.



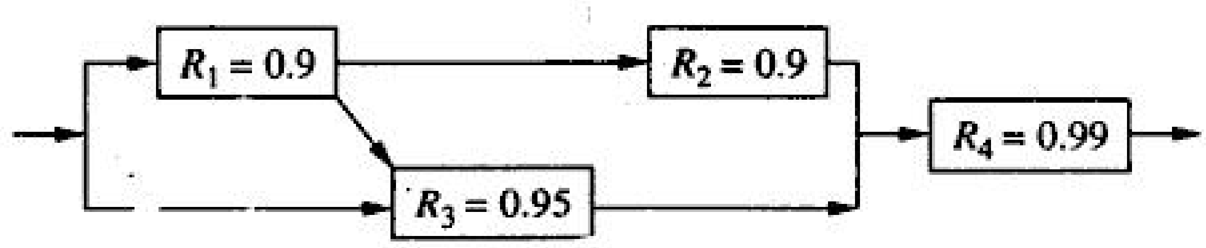
5.16 A system is designed to operate for 100 days. The system consists of three components in series. Their failure distributions are (1) Weibull with shape parameter 1.2 and scale parameter 840 days; (2) lognormal with shape parameter (s) 0.7 and median 435 days; (3) constant failure rate of 0.0001.

(a) Compute the system reliability.

(b) If two units of components 1 and 2 are available, determine the high-level redundancy reliability. Assume that components 1 and 2 can be configured as a subassembly.

(c) If two units of components 1 and 2 are available, determine the low-level redundancy reliability.

5.17 Determine the reliability of the following linked system using the decomposition method.



5.18 For k-out-of-n systems there may be a crossover point, where the single-component reliability is greater than the k-out-of-n system reliability. If R is the single-component reliability, then the crossover point is found from solving

$$R = \sum_{x=k}^{n} \binom{n}{x} R^x (1 - R)^{n-x}$$

(a) Find the crossover point for a 2-out-of-3 redundant system.

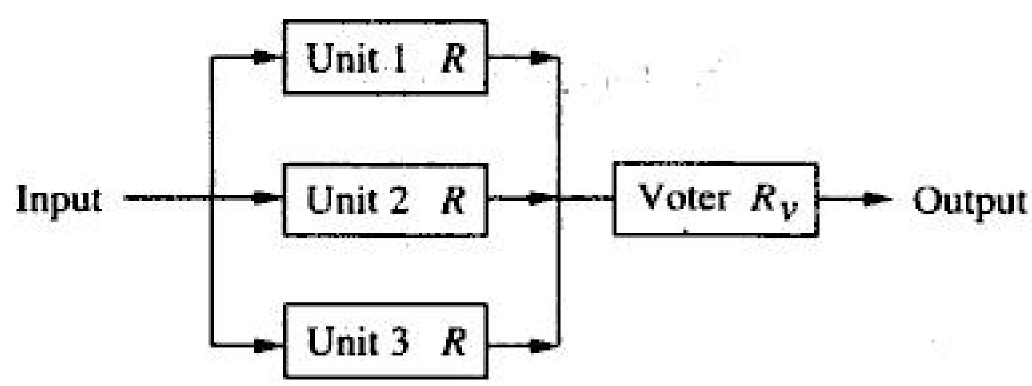
(b) Repeat for a 3-out-of-4 system.

5.19 An alternative approach for analyzing reliability networks is to identify all possible combinations of failure events of the components, compute the probabilities of those outcomes resulting either in system failure or in a system success (nonfailure), and sum the individual probabilities to obtain the system failure probability or reliability. Since each component has two states, success (S) or failure (F), there are 2^n mutually exclusive combinations to be considered, where n is the number of components composing the network. For the system of Exercise 5.17, the analysis will take the following form:

R_1	R_2	R_3	R_4	System	Probability
S	S	S	S	S	$(0.9)^2(0.95)(0.99) = 0.761805$
S	S	S	F	F	
S	S	F	S	S	$(0.9)^2(0.05)(0.99) = 0.040095$

Complete this problem by identifying all possible combinations and summing the probabilities of those resulting in a system success. Compare your result with that obtained from the decomposition approach used in Exercise 5.17.

5.20 In the design of computer systems, increased reliability may be achieved through the use of triple modular redundancy. This consists of three identical units (logic or binary variables) feeding into a common voting system, as shown in the figure. The value of the output variable is determined by majority voting. If the single-unit reliability is R and the reliability of the voting system is R_v, show that the system reliability is given by $R^2(3 - 2R)R_v$. If the reliability of the voting system is 0.95, what is the crossover reliability (see Exercise 5.18) of a single unit?



5.21 Show that

$$\text{MTTF} = \frac{1}{\lambda}\sum_{i=1}^{n}\binom{n}{i}\frac{(-1)^{i-1}}{i}$$

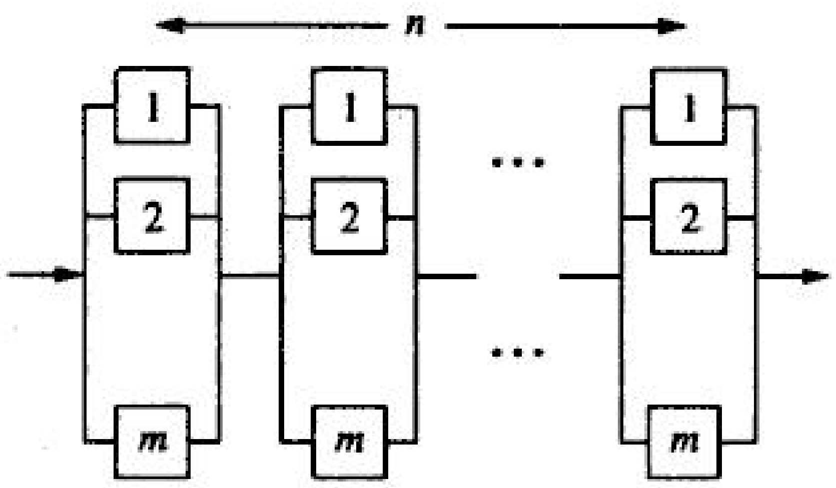
for n redundant and independent components each with $\lambda(t) = \lambda$. *Hint:* From the binomial theorem,

$$(p + q)^n = \sum_{i=0}^{n}\binom{n}{i}p^{n-i}q^i$$

5.22 (a) For three-state devices, write an expression for the reliability of a low-level redundant system as the system reliability with respect to a short minus the system probability of an open. Show that this is equivalent to Eq. (5.22).

(b) Repeat for a high-level redundant system by writing an expression for the system reliability with respect to an open minus the system probability of a short. Show that this expression is equivalent to Eq. (5.23).

5.23 Write an expression for the system reliability of the following series-parallel network, where q_{oi} is the probability that the ith component fails open and q_{si} is the probability that the ith component fails short.



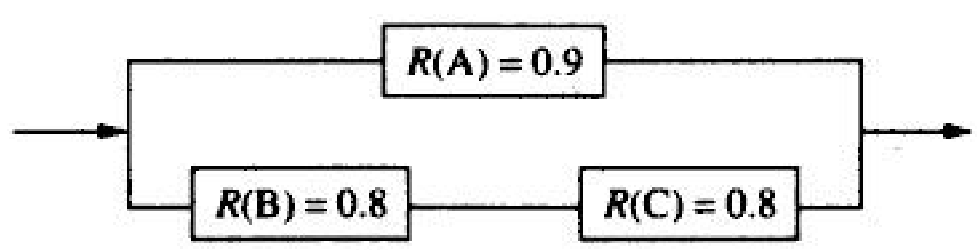
5.24 Show that the MTTF for n serially related and independent components each having a linear hazard rate function is

$$\text{MTTF} = \left(\frac{\pi}{2\sum_{i=1}^{n} a_i}\right)^{1/2}$$

where $\lambda_i(t) = a_i t$ and $a_i > 0$.

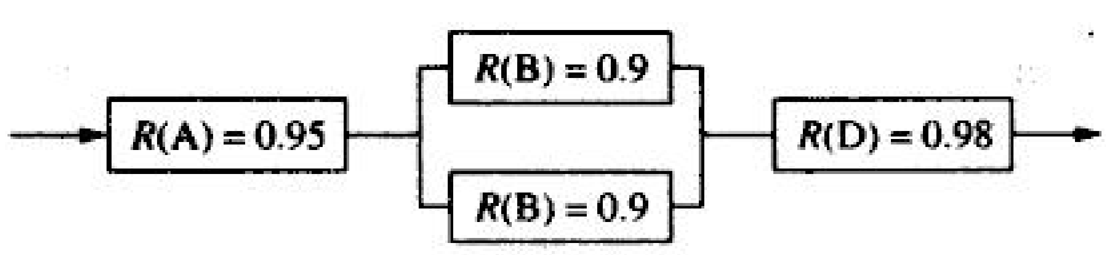
Hint:
$$\int_0^\infty e^{-a^2x^2}\,dx = \frac{\sqrt{\pi}}{2a}$$

5.25 Structure function (optional). Consider the network shown below.



(a) Write the structure function.
(b) Find the system reliability using the structure function.
(c) Identify the minimal path sets and minimal cut sets.
(d) Find the lower- and upper-bound reliabilities using the minimal sets in (c).

5.26 Structure function (optional). Consider the network shown below.



(a) Write the structure function.
(b) Find the system reliability using the structure function.
(c) Identify the minimal path sets and minimal cut sets.
(d) Find the lower- and upper-bound reliabilities using the minimal sets in (c).

CHAPTER 6

State-Dependent Systems

As discussed in the previous chapter, a fundamental computation in reliability engineering is the determination of system reliability from a knowledge of component reliabilities and their system configuration. On the basis of the critical assumption of independent failures among components, Eqs. (5.1) and (5.6) were easily derived using the basic rules of probability. However, when component failures are in some way dependent, more powerful methods, such as Markov analysis, may be needed.

6.1 MARKOV ANALYSIS

Markov analysis looks at a system as being in one of several states. One possible state, for example, is that in which all the components composing the system are operating. Another possible state is that in which one component has failed but the other components continue to operate. The fundamental assumption in a Markov process is that the probability that a system will undergo a transition from one state to another state depends only on the current state of the system and not on any previous states the system may have experienced. In other words, the transition probability is not dependent on the past (state) history of the system. This is equivalent to the memorylessness of the exponential distribution, and it is therefore not surprising that exponential times to failure satisfy this Markovian property. We will express the transition from one state to another as an instantaneous (failure) rate. Assuming the process is also stationary (i.e., the transition probabilities do not change over time), the transition rates will be constant. Again, this is equivalent to assuming exponential failure times.

The methodology is presented through the use of an example. We begin by using Markov analysis to derive Eqs. (5.2) and (5.7) for the two-component system. Although independence among components is assumed here, this derivation provides a good example of the technique. In applying Markov analysis to this problem, we assume that each of n components will be in one of two states—operating or failed. The system state is then defined to be one of the 2^n possible combinations of operating and failed components. For our two-component system we define the following four system states:

State	Component 1	Component 2
1	operating	operating
2	failed	operating
3	operating	failed
4	failed	failed

If the two components are in parallel (redundant), only state 4 results in a system failure. On the other hand, if the two components are in series, then states 2, 3, and 4 would each constitute a failure state. The objective is to find the probability of the system being in each state as a function of time. We denote the probability of being in state i at time t as $P_i(t)$. Then for a two-component series system,

$$R_s(t) = P_1(t)$$

and for a two-component parallel system,

$$R_p(t) = P_1(t) + P_2(t) + P_3(t)$$

Observe that the system must be one of the four states at any given time. Therefore,

$$P_1(t) + P_2(t) + P_3(t) + P_4(t) = 1 \tag{6.1}$$

What remains is to find $P_i(t)$, $i = 1, 2, 3, 4$.

If we assume that individual components have constant failure rates λ_i, we can represent the two-component system using the rate diagram in Fig. 6.1. The nodes in Fig. 6.1 represent the four system states, and the branches show the transition rate (λ_i) from one node to another. From the rate diagram we can derive the following equation:

$$P_1(t + \Delta t) = P_1(t) - \lambda_1 \Delta t P_1(t) - \lambda_2 \Delta t P_1(t) \tag{6.2}$$

Equation 6.2 states that the probability of the system being in state 1 at time $t + \Delta t$ is equal to the probability of it being in state 1 at time t minus the probability of it

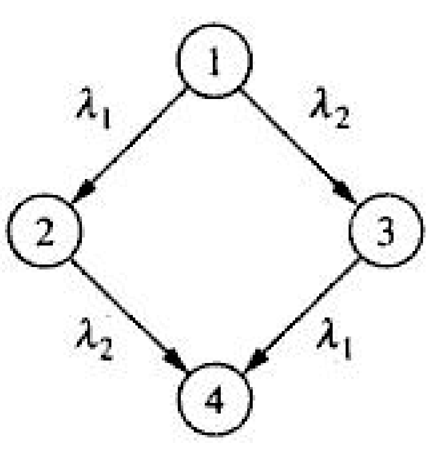


FIGURE 6.1
Rate diagram for a two-component system with independence.

being in state 1 at time t times the probability of transitioning ($\lambda_i \Delta t$) to either state 2 or 3. Observe that $\lambda_1 \Delta t$ is the conditional probability of a transition to state 2 occurring during time Δt given that the system is currently in state 1. Therefore, $\lambda_1 \Delta t P_1(t)$ is the joint probability of the system being in state 1 at time t and making a transition to state 2 during time Δt. A similar argument holds for $\lambda_2 \Delta t P_1(t)$ for transitioning to state 3.

A second equation is obtained from state 2.

$$P_2(t + \Delta t) = P_2(t) + \lambda_1 \Delta t P_1(t) - \lambda_2 \Delta t P_2(t) \tag{6.3}$$

is the probability of the system being in state 2 at time $t + \Delta t$ and is equal to the probability of being in state 2 at time t plus the probability of being in state 1 at time t and making a transition ($\lambda_1 \Delta t$) to state 2 in time Δt minus the probability of being in state 2 at time t and making a transition to state 4 ($\lambda_2 \Delta t$) in time Δt. Similarly for state 3,

$$P_3(t + \Delta t) = P_3(t) + \lambda_2 \Delta t P_1(t) - \lambda_1 \Delta t P_3(t) \tag{6.4}$$

and for state 4,

$$P_4(t + \Delta t) = P_4(t) + \lambda_2 \Delta t P_2(t) + \lambda_1 \Delta t P_3(t) \tag{6.5}$$

Rewriting Eq. 6.2,

$$\frac{P_1(t + \Delta t) - P_1(t)}{\Delta t} = -(\lambda_1 + \lambda_2) P_1(t)$$

Then

$$\lim_{\Delta t \to 0} \frac{P_1(t + \Delta t) - P_1(t)}{\Delta t} = \frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2) P_1(t) \tag{6.6}$$

In a similar fashion, Eqs. (6.3) and (6.4) lead to the following differential equations:

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2 P_2(t) \tag{6.7}$$

$$\frac{dP_3(t)}{dt} = \lambda_2 P_1(t) - \lambda_1 P_3(t) \tag{6.8}$$

Equations (6.6), (6.7), and (6.8) along with equation (6.1) can be solved simultaneously (see Appendix 6A for details). Their solution is

$$P_1(t) = e^{-(\lambda_1 + \lambda_2)t} \tag{6.9}$$

$$P_2(t) = e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t} \tag{6.10}$$

$$P_3(t) = e^{-\lambda_1 t} - e^{-(\lambda_1 + \lambda_2)t} \tag{6.11}$$

$$P_4(t) = 1 - P_1(t) - P_2(t) - P_3(t) \tag{6.12}$$

Then for a series system we have

$$R_s(t) = P_1(t) = e^{-(\lambda_1 + \lambda_2)t}$$

and for a parallel system,

$$R_p(t) = P_1(t) + P_2(t) + P_3(t)$$
$$= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

which are equivalent to Eqs. (5.2) and (5.7) for a two-component ($n = 2$) system.

6.2 LOAD-SHARING SYSTEM

A rather straightforward application of Markov analysis is to a load-sharing system. We are given two components in parallel as before except there is now a dependency between the two components. If one component fails, the failure rate of the other component increases as a result of the additional load placed on it. Because of this dependency, the reliability block diagram techniques of Chapter 5 cannot be applied. Instead, we must use Markov analysis to determine the system reliability.

Define the four states of the system as before. The rate diagram is shown in Fig. 6.2, where λ_1^+ and λ_2^+ represent the increased failure rates of components 1 and 2, respectively, as a result of the increased load. The resulting differential equations are

$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2)P_1(t) \tag{6.13}$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2^+ P_2(t) \tag{6.14}$$

$$\frac{dP_3(t)}{dt} = \lambda_2 P_1(t) - \lambda_1^+ P_3(t) \tag{6.15}$$

The solution to these equations are found using the same approach as before (see Appendix 6B) and is as follows:

$$P_1(t) = e^{-(\lambda_1+\lambda_2)t} \tag{6.16}$$

$$P_2(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2 - \lambda_2^+}\left[e^{-\lambda_2^+ t} - e^{-(\lambda_1+\lambda_2)t}\right] \tag{6.17}$$

$$P_3(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_1^+}\left[e^{-\lambda_1^+ t} - e^{-(\lambda_1+\lambda_2)t}\right] \tag{6.18}$$

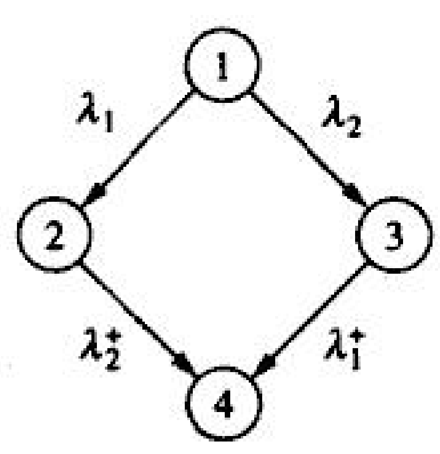


FIGURE 6.2
Rate diagram for a two-component load-sharing system.

and $$R(t) = P_1(t) + P_2(t) + P_3(t)$$

If we let $\lambda_1 = \lambda_2 = \lambda$ and $\lambda_1^+ = \lambda_2^+ = \lambda^+$, then

$$R(t) = e^{-2\lambda t} + \frac{2\lambda}{2\lambda - \lambda^+}\left[e^{-\lambda^+ t} - e^{-2\lambda t}\right] \tag{6.19}$$

and $$\text{MTTF} = \int_0^\infty R(t)\,dt = \frac{1}{2\lambda} + \frac{2\lambda}{2\lambda - \lambda^+}\left[\frac{1}{\lambda^+} - \frac{1}{2\lambda}\right] \tag{6.20}$$

EXAMPLE 6.1. Two generators provide needed electrical power. If either fails, the other can continue to provide electrical power. However, the increased load results in a higher failure rate for the remaining generator. If $\lambda = 0.01$ failure per day and $\lambda^+ = 0.10$ failure per day, determine the system reliability for a 10-day contingency operation and determine the system MTTF.

Solution. From Eq. 6.19,

$$R(t) = e^{-2(0.01)t} + \frac{2(0.01)}{2(0.01) - 0.10}\left[e^{-0.10t} - e^{-2(0.01)t}\right]$$

and $$R(10) = e^{-0.2} + \frac{0.02}{-(0.08)}\left[e^{-1} - e^{-0.2}\right] = 0.9314$$

From Eq. (6.20),

$$\text{MTTF} = \frac{1}{0.02} + \frac{0.02}{-0.08}\left[\frac{1}{0.10} - \frac{1}{0.02}\right] = 60 \text{ days}$$

6.3 STANDBY SYSTEMS

Standby systems are an important area of study within reliability. Depending on the probability of a failure occurring when switching to a standby unit, these systems are generally much more reliable than an active redundant system. The two-component standby system differs from the active redundant system discussed earlier in that the standby unit will have no failures or a reduced failure rate while in its standby mode. Once active, the backup unit may experience the same failure rate as the on-line (primary) system (if they are identical units) or may have a different failure rate. The dependency arises because the failure rate of the standby unit depends on the state of the primary unit.

The rate diagram is shown in Fig. 6.3, where state 3 represents a failure (perhaps undetected) of the standby unit while in standby with λ_2^- being the corresponding failure rate. The resulting system of equations is

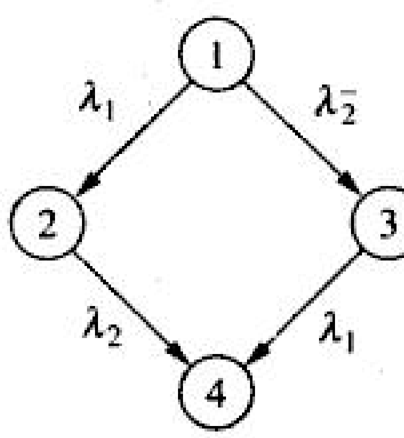


FIGURE 6.3
Rate diagram for a two-component standby system with failures in standby.

$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2^-)P_1(t) \tag{6.21}$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2 P_2(t) \tag{6.22}$$

$$\frac{dP_3(t)}{dt} = \lambda_2^- P_1(t) - \lambda_1 P_3(t) \tag{6.23}$$

having solution (see Appendix 6C)

$$P_1(t) = e^{-(\lambda_1+\lambda_2^-)t} \tag{6.24}$$

$$P_2(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2^-)t}\right] \tag{6.25}$$

$$P_3(t) = e^{-\lambda_1 t} - e^{-(\lambda_1+\lambda_2^-)t} \tag{6.26}$$

with
$$R(t) = P_1(t) + P_2(t) + P_3(t)$$

$$= e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2^-)t}\right] \tag{6.27}$$

and

$$\text{MTTF} = \frac{1}{\lambda_1} + \frac{\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2^-}\right] = \frac{1}{\lambda_1} + \frac{\lambda_1}{\lambda_2(\lambda_1 + \lambda_2^-)} \tag{6.28}$$

If there are no failures of the standby unit, set $\lambda_2^- = 0$ in Eqs. (6.27) and (6.28). If $\lambda_1 = \lambda_2 = \lambda$ and $\lambda_2^- = \lambda^-$, then Eqs. (6.27) and (6.28) simplify to

$$R(t) = e^{-\lambda t} + \frac{\lambda}{\lambda^-}\left[e^{-\lambda t} - e^{-(\lambda+\lambda^-)t}\right] \tag{6.29}$$

and
$$\text{MTTF} = \frac{1}{\lambda} + \frac{\lambda}{\lambda^-}\left[\frac{1}{\lambda} - \frac{1}{\lambda + \lambda^-}\right]$$

$$= \frac{1}{\lambda} + \frac{1}{\lambda^-} - \frac{\lambda}{(\lambda + \lambda^-)\lambda^-} = \frac{1}{\lambda} + \frac{1}{\lambda + \lambda^-} \tag{6.30}$$

EXAMPLE 6.2. An active generator has a failure rate (failures per day) of 0.01. An older standby generator has a failure rate of 0.001 while in standby and a failure rate of 0.10 when on-line. Determine the system reliability for a planned 30-day use and compute the system MTTF.

Solution

$$R(t) = e^{-0.01t} + \frac{0.01}{0.01 + 0.001 - 0.1}\left[e^{-0.1t} - e^{-0.011t}\right]$$

Therefore
$$R(30) = 0.741 - 0.11236[0.04978 - 0.7189] = 0.8162$$

$$\text{MTTF} = \frac{1}{0.01} + \frac{0.01}{0.1(0.01 + 0.001)} = 109.09 \text{ days}$$

EXAMPLE 6.3. Both units of a two-component standby system are identical with $\lambda =$ 0.002 failure per hour and $\lambda^- = 0.0001$ failure per hour. Determine the design life on the basis of a 95 percent reliability.

Solution. Us'ng Eq. (6.29),

$$0.95 = R(t) = e^{-0.002t} + \frac{0.002}{0.0001}\left[e^{-0.002t} - e^{-0.0021t}\right]$$

Solving for t by trial and error:

$$R(100) = 0.982$$

$$R(200) = 0.935$$

$$R(150) = 0.961$$

$$R(175) = 0.949$$

$$R(173) = 0.950$$

The answer is 173 hours.

6.3.1 Identical Standby Units

If the primary and backup units have identical constant failure rates with no failures in the standby mode, then Eq. 6.27 is undefined. This is a special case in which the system of differential equations must be solved for separately. However, a simpler and more general approach is possible under this condition. Assume that there are k identical units of which one is on-line and the remaining are backup. When the on-line component fails, the first backup component is placed on-line. When it fails, the next is placed on-line, and so on. Therefore, the time in which the kth failure is observed is the sum of k identical and independent exponential distributions. As discussed in Section 3.5, the time of the kth failure has a gamma distribution with parameters λ and k. Therefore

$$R_k(t) = e^{-\lambda t}\sum_{i=0}^{k-1}\frac{(\lambda t)^i}{i!} \tag{6.31}$$

and MTTF $= k/\lambda$.

EXAMPLE 6.4. The Rey Lie Able Printing Company has four presses: one operating and three in standby. Each press has an identical constant failure rate where the MTTF is 50 operating hours. The company has received a rush order requiring 75 hr of continuous time on a press. If a standby is utilized whenever the on-line press fails, determine the probability of there being continuous printing support while the order is being processed.

Solution. The time to failure of the four-unit standby system has a gamma distribution with $\lambda = \frac{1}{50}$ and $k = 4$. Therefore

$$R_4(75) = e^{-75/50}\sum_{i=0}^{3}\frac{(75/50)^i}{i!} = e^{-1.5}\left[1 + \frac{3}{2} + \frac{9}{8} + \frac{27}{48}\right] = 0.9344$$

and MTTF $= 4/(1/50) = 200$ hr.

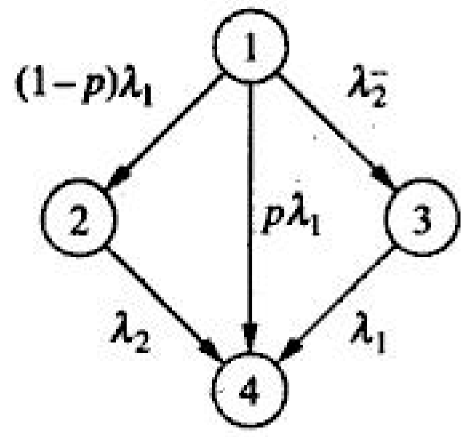


FIGURE 6.4
Rate diagram for a two-component standby system with failures in standby and with switching failures.

6.3.2 Standby System with Switching Failures

It is not uncommon in a standby system to have some probability p of there being an on-demand failure of a switching device designed to place the standby system on-line. The rate diagram in Fig. 6.3 is modified as shown in Fig. 6.4.

The resulting differential equation for state 1 does not change since

$$\frac{dP_1(t)}{dt} = -[(1-p)\lambda_1 + p\lambda_1 + \lambda_2^-]P_1(t) = -(\lambda_1 + \lambda_2^-)P_1(t)$$

It is also apparent by comparing Fig. 6.3 and Fig. 6.4 that Eq. (6.23) will not change either. Equation (6.22) is modified as follows:

$$\frac{dP_2(t)}{dt} = (1-p)\lambda_1 P_1(t) - \lambda_2 P_2(t)$$

The solution to these three equations is given by Eqs. (6.24) and (6.26) and by

$$P_2(t) = \frac{(1-p)\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2^-)t}\right] \tag{6.32}$$

Therefore the reliability function is

$$R(t) = e^{-\lambda_1 t} + \frac{(1-p)\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2^-)t}\right] \tag{6.33}$$

Observe that if $p = 1$, the standby system has no effect and the overall system reliability is that of the primary unit only.

EXAMPLE 6.5. Consider the standby system described in Example 6.2. If there is a 10 percent probability of a switching failure, the system reliability becomes

$$R(30) = 0.741 + \frac{(0.90)(0.01)}{0.01 + 0.001 - 0.1}[0.04978 - 0.7189] = 0.8087$$

This is a slight decrease from the perfect switching case.

6.3.3 Three-Component Standby System

Consider a system with one active unit and two standby units. For simplicity assume that no units fail while in standby and that all three systems have the same constant failure rate when on-line. Define the following states:

State	Unit 1	Unit 2	Unit 3
1	on-line	standby	standby
2	failed	on-line	standby
3	failed	failed	on-line
4	failed	failed	failed

This leads to the following differential equations:

$$\frac{dP_1(t)}{dt} = -\lambda P_1(t) \tag{6.34}$$

$$\frac{dP_2(t)}{dt} = \lambda P_1(t) - \lambda P_2(t) \tag{6.35}$$

$$\frac{dP_3(t)}{dt} = \lambda P_2(t) - \lambda P_3(t) \tag{6.36}$$

with initial conditions $P_1(0) = 1$, $P_2(0) = 0$, and $P_3(0) = 0$. The solution follows the methodology given in Appendix 6A, resulting in

$$P_1(t) = e^{-\lambda t} \tag{6.37}$$

$$P_2(t) = \lambda t e^{-\lambda t} \tag{6.38}$$

$$P_3(t) = \frac{\lambda^2 t^2}{2} e^{-\lambda t} \tag{6.39}$$

Since the system is functioning while in any of the first three states,

$$R(t) = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2 t^2}{2}\right] \tag{6.40}$$

A comparison of Eq. 6.40 with the reliability function computed for the case of identical standby units based on the gamma distribution (Eq. 6.31) will reveal that the two are equivalent. The MTTF may be found from[1]

$$\text{MTTF} = \int_0^\infty e^{-\lambda t}\,dt + \int_0^\infty \lambda t e^{-\lambda t}\,dt + \int_0^\infty \frac{\lambda^2 t^2}{2} e^{-\lambda t}\,dt = \frac{3}{\lambda} \tag{6.41}$$

It is not surprising that the MTTF is three times a single-unit MTTF since each unit will have an MTTF of $1/\lambda$ while active. In general, if there are k identical and independent units with $k-1$ on standby, then the system MTTF is k/λ (see Exercise 6.15).

[1]The definite integrals may be solved from the following:

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

for $a > 0$ and n a positive integer.

EXAMPLE 6.6. Three identical transmitters are available each having a constant failure rate of 0.0035 per operating hour. A mission requires 500 hr of continuous transmission. Determine the reliability of the system.

Solution

$$R(500) = e^{-0.0035\times 500}\left[1 + 0.0035 \times 500 + \frac{(0.0035 \times 500)^2}{2}\right] = 0.744$$

In comparison, a single unit has a mission reliability of 0.174.

6.4 DEGRADED SYSTEMS

Some systems may continue to operate in a degraded mode following certain types of failures. The system may continue to perform its function but not at a specified operating level. For example, a computer system may not be able to access all of its direct access storage devices, a copying machine may not be able to automatically feed originals and may thereby require manual operation, or a multi-engine aircraft may experience a problem in one of its engines. Whether the degraded mode is considered a failure or not must be determined as part of the reliability specification. However, if it is important to distinguish the degraded state from that of a complete failure, then Markov analysis can be utilized if constant failure rates are assumed.

Defining the states of a system as fully operational (state 1), degraded (state 2), and failed (state 3), the rate diagram of Fig. 6.5 is constructed. The differential equations are

$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2)P_1(t) \tag{6.42}$$

$$\frac{dP_2(t)}{dt} = \lambda_2 P_1(t) - \lambda_3 P_2(t) \tag{6.43}$$

The solution to Eq. (6.42) is straightforward and is given by

$$P_1(t) = e^{-(\lambda_1+\lambda_2)t} \tag{6.44}$$

An integrating factor must be used in solving Eq. (6.43); however, the solution procedure is similar to that of the standby system (Eq. 6.22):

$$P_2(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_3}\left[e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t}\right] \tag{6.45}$$

Finally, $P_3(t) = 1 - P_1(t) - P_2(t)$. The mean time to a complete failure is found from

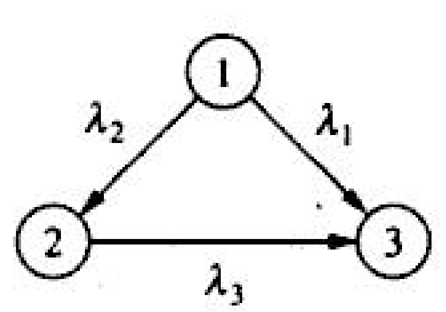


FIGURE 6.5
Rate diagram for modeling a degraded system.

$$\text{MTTF} = \int_0^\infty [P_1(t) + P_2(t)]\,dt = \frac{1}{\lambda_1 + \lambda_2} + \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_3}\left[\frac{1}{\lambda_3} - \frac{1}{\lambda_1 + \lambda_2}\right]$$

EXAMPLE 6.7. A machine used in a manufacturing process experiences complete failures at a constant rate of 0.01 per day. However, the machine may degrade randomly, producing substandard parts (out of tolerances) at a constant rate of 0.05 per day. Once it has degraded, it will fail completely at a constant rate of 0.07 per day. Therefore,

$$P_1(t) = e^{-0.06t} \qquad P_2(t) = \frac{0.05}{0.06 - 0.07}\left[e^{-0.07t} - e^{-0.06t}\right]$$

and over a one-day operation, $P_1(1) = 0.942$, $P_2(1) = 0.047$, and $P_3(1) = 0.011$. The MTTF is found to be 28.6 days. Of interest perhaps is the mean number of days the machine may operate in a degraded mode until it fails, which is given by $1/0.07 = 14.3$ days. Predictive maintenance attempts to determine when a machine will fail and, as a result, perform corrective maintenance just prior to a failure. From this analysis, once the machine begins to produce substandard parts, maintenance of the machine should be accomplished on the average within 14 days. Notice that

$$\int_0^\infty P_1(t)\,dt = \frac{1}{0.01 + 0.05} = 16.67 \text{ days}$$

is the mean number of days the machine will spend in state 1 prior to degrading or experiencing a complete failure.

6.5 THREE-STATE DEVICES

Components having three states, or two dependent failure modes, were introduced in Chapter 5. These components are in either an operating state, a failed open state, or a failed short state. Since there is a dependency between the two failure states (i.e., they are mutually exclusive), Markov analysis can be used to determine the component reliability assuming each failure state has a constant failure rate.

In the rate diagram of Fig. 6.6, state 1 is the operating state, state 2 is the failed open state, and state 3 is the failed short state. The differential equations are

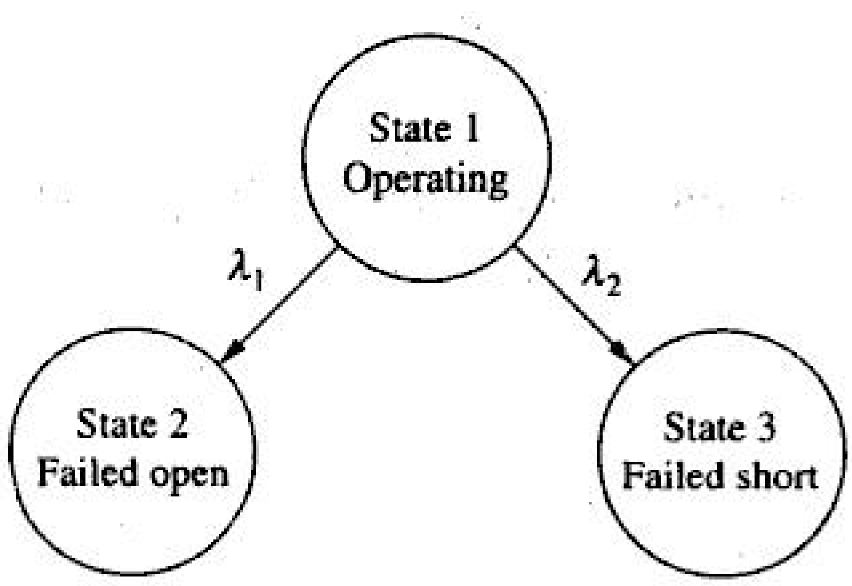


FIGURE 6.6
Rate diagram for a three-state device.

$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2)P_1(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t)$$

$$\frac{dP_3(t)}{dt} = \lambda_2 P_1(t)$$

The solution is straightforward and given by

$$R(t) = P_1(t) = e^{-(\lambda_1+\lambda_2)t} \tag{6.46}$$

with

$$P_2(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2}[1 - e^{-(\lambda_1+\lambda_2)t}]$$

and

$$P_3(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2}[1 - e^{-(\lambda_1+\lambda_2)t}]$$

Equation (6.46) may also be interpreted as the reliability of a component having two failure modes both with constant failure rates.

APPENDIX 6A
SOLUTION TO TWO-COMPONENT REDUNDANT SYSTEM

From Eq. (6.6),

$$\frac{dP_1(t)}{P_1(t)} = -(\lambda_1 + \lambda_2)\,dt$$

Integrating both sides,

$$\ln P_1(t) = -(\lambda_1 + \lambda_2)t$$

or

$$P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$

From Eq. (6.7),

$$\frac{dP_2(t)}{dt} = \lambda_1 e^{-(\lambda_1+\lambda_2)t} - \lambda_2 P_2(t)$$

with $e^{+\lambda_2 t}$ as an integrating factor,

$$P_2(t)e^{+\lambda_2 t} = +\lambda_1 \int e^{-(\lambda_1+\lambda_2)t} e^{+\lambda_2 t}\,dt + C$$

or

$$P_2(t) = -e^{-(\lambda_1+\lambda_2)t} + ce^{-\lambda_2 t}$$

The initial conditions are $P_1(0) = 1$, $P_2(0) = 0$, and $P_3(0) = 0$. Therefore, $c = 1$. $P_3(t)$ is derived in a similar manner.

APPENDIX 6B
SOLUTION TO LOAD-SHARING SYSTEM

$P_1(t)$ is solved for as shown in Appendix 6A. Then

$$\frac{dP_2(t)}{dt} = \lambda_1 e^{-(\lambda_1+\lambda_2)t} - \lambda_2^+ P_2(t)$$

With $e^{\lambda_2^+ t}$ as an integrating factor, the solution is

$$e^{\lambda_2^+ t} P_2(t) = \int \lambda_1 e^{-(\lambda_1+\lambda_2)t} e^{\lambda_2^+ t}\, dt + C$$

or

$$P_2(t) = e^{-\lambda_2^+ t}\left\{\int \lambda_1 e^{[\lambda_2^+ - (\lambda_1+\lambda_2)]t}\, dt + C\right\}$$

$$= e^{-\lambda_2^+ t}\left[\frac{\lambda_1 e^{[\lambda_2^+ - (\lambda_1+\lambda_2)]t}}{\lambda_2^+ - (\lambda_1+\lambda_2)} + c\right]$$

$$= \frac{\lambda_1 e^{-(\lambda_1+\lambda_2)]t}}{\lambda_2^+ - (\lambda_1+\lambda_2)} + ce^{-\lambda_2^+ t}$$

Since $P_2(0) = 0$, then

$$c = \frac{-\lambda_1}{\lambda_2^+ - (\lambda_1+\lambda_2)}$$

and

$$P_2(t) = \frac{\lambda_1}{\lambda_2^+ - (\lambda_1+\lambda_2)}\left[e^{-(\lambda_1+\lambda_2)t} - e^{-\lambda_2^+ t}\right]$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2 - \lambda_2^+}\left[e^{-\lambda_2^+ t} - e^{-(\lambda_1+\lambda_2)t}\right]$$

$P_3(t)$ is solved for in a similar manner.

APPENDIX 6C
SOLUTION TO STANDBY SYSTEM MODEL

Equation (6.21) has as its solution

$$P_1(t) = e^{-(\lambda_1+\lambda_2^-)}$$

by the same procedure as that of Appendix 6A. Substituting this solution into Eq. (6.22),

$$\frac{dP_2(t)}{dt} = \lambda_1 e^{-(\lambda_1+\lambda_2^-)t} - \lambda_2 P_2(t)$$

With $e^{\lambda_2 t}$ as an integrating factor, the solution is

$$e^{\lambda_2 t}P_2(t) = \int \lambda_1 e^{-(\lambda_1+\lambda_2^-)t}e^{\lambda_2 t}\,dt + C$$

$$= \frac{\lambda_1}{\lambda_1+\lambda_2^- - \lambda_2}e^{-(\lambda_1+\lambda_2^- - \lambda_2)t} + c$$

or

$$P_2(t) = \frac{-\lambda_1}{\lambda_1+\lambda_2^- - \lambda_2}e^{-(\lambda_1+\lambda_2^-)t} + ce^{-\lambda_2 t}$$

With the initial condition $P_2(0) = 0$,

$$c = \frac{\lambda_1}{\lambda_1+\lambda_2^- - \lambda_2}$$

giving Eq. (6.25).

To obtain $P_3(t)$, use Eq. (6.23) and the solution to $P_1(t)$:

$$\frac{dP_3(t)}{dt} = \lambda_2^- e^{-(\lambda_1+\lambda_2^-)t} - \lambda_1 P_3(t)$$

With $e^{\lambda_1 t}$ as the integrating factor,

$$e^{\lambda_1 t}P_3(t) = \int \lambda_2^- e^{-(\lambda_1+\lambda_2^-)t}e^{\lambda_1 t}\,dt + C$$

$$= \frac{-\lambda_2^-}{\lambda_2^-}e^{-(\lambda_2)t} + c$$

or

$$P_3(t) = -e^{-(\lambda_1+\lambda_2^-)t} + ce^{-\lambda_1 t}$$

With $P_3(0) = 0$, we know $c = 1$, giving Eq. (6.26).

EXERCISES

6.1 Two nickel-cadmium batteries provide electrical power to operate a satellite transceiver. If both batteries are operating in parallel, they have an individual failure rate of 0.1 per year. If one fails, the other can operate the transceiver (at a reduced power output). However, the increased electrical demand will triple the failure rate of the remaining battery. Determine the system reliability at 1, 2, 3, 4, and 5 yr. What is the system MTTF?

6.2 An engine health monitoring system consists of a primary unit and a standby unit. The MTTF of the primary unit is 1000 operating hours, and the MTTF of the standby unit is 333 hr when in operation and 2000 hr while in standby status. Estimate the design life of the system if specifications require a reliability of 0.90. What is the system MTTF?

6.3 A computerized airline reservation system has a main computer on-line and a secondary standby computer. The on-line computer fails at a constant rate of 0.001 failure per hour, and the standby unit fails when on-line at the constant rate of 0.005 failure per hour. There are no failures while the unit is in the standby mode.

(a) Determine the system reliability over a 72-hr period.
(b) The airline desires to have a system MTTF of 2000 hr. Determine the (minimum) MTTF of the main computer to achieve this goal assuming that the standby computer MTTF does not change.

6.4 Repeat Exercise 6.3 assuming a 0.005 probability of a switching failure.

6.5 An alarm system may fail safe (false alarm) or may fail to danger (failure on demand). If the fail-safe failure rate is a constant 0.00034 failure per operating hour and the fail-to-danger failure rate is a constant 0.000021 failure per operating hour, what is the design life of the alarm if an operating reliability of 0.99 is desired? What is the probability of a fail-to-danger failure occurring over 1000 operating hours?

6.6 Consider a three-component standby system in which two units are normally on-line. Both on-line units must fail before the standby unit is placed on-line. Compute the system reliability function and the MTTF. Assume no failures in standby and a constant failure rate of λ when the unit is on-line.

6.7 The Brake A. Bac Trucking Company requires two workers to unload a truck in 5 hr. Working together, the two workers have a mortality rate (due to heart failure) of 0.05 per hour. However, if one experiences a heart attack and dies while unloading a truck, the other's mortality rate increases to 0.2 per hour as a result of the increase in the unloading rate necessary in order to complete the task on time. Compute the probability (reliability) that a truck will be successfully unloaded by a two-person crew. What is the expected time to failure (in working hours) of the two-person crew?

6.8 A contractor must decide between two different sump pump systems to be installed in a new housing development. The option is to install a single 1000 gallon per minute (gpm) system or two 500-gpm pumps. If the two-pump system is used, one pump can carry most of the load in the event the other pump fails. Both of the 500-gpm pumps have an MTTF of 800 hr when working together. Their individual MTTF is 200 hr. The 1000-gpm system has a rated MTTF of 700 hr. Which system is preferred on the basis of system MTTF? Which system has the best design life for a reliability of 0.80?

6.9 The contractor in Exercise 6.8 can obtain used 1000-gpm sump pumps to back up the primary unit. Because of their age, these pumps could fail while in standby with a failure rate of 0.001 failure per hour. On-line, they have experienced an MTTF of 200 hr. Would this (standby) system be preferred to that in Exercise 6.8?

6.10 Determine which of the following systems is the most reliable at 100 hr.
(a) Two parallel and CFR units with $\lambda_1 = 0.0034$ and $\lambda_2 = 0.0105$
(b) A standby system with $\lambda_1 = 0.0034$, $\lambda_2 = 0.0105$, $\lambda_2^- = 0.0005$, and a switching failure probability of 15 percent
(c) A load-sharing system with $\lambda_1 = 0.0034$ and $\lambda_2 = 0.0105$ in which the single-component failure rate increases by a factor of 1.5
Compare the MTTF of all three systems.

6.11 A hospital has three identical generators, one on-line and the other two in standby. In the primary failure mode, which is due to external weather conditions, the failure rat is constant and equal to 0.02 failure per hour during bad weather. Assuming that no failures occur in standby and that the average duration of a storm is 10 hr, what is the reliability of the generator system?

6.12 Derive the reliability function for the case of a three-component standby system (i.e., one active and two standby units) in which the standby units have a different failure rate (λ_s) than the active unit (λ_a). What is the system MTTF? Assume that there are no switching failures and no failures while in standby.

6.13 Derive the expression for the MTTF for the load-sharing system defined by Eqs. (6.13) through (6.15).

6.14 A manufacturing company operates two production lines. When both lines are operating, the production rate on each line is 500 units per hour. At this production rate the failure rate of line 1 is 3 failures per 8-hr day (CFR) and the failure rate of line 2 is 2 failures per 8-hr day. When one line fails, the production rate of the second line must be increased in order to make production quotas. At the increased rate of 800 units per hour, the failure rate of line 1 is 6 per 8-hr day and the failure rate of line 2 is 3 per 8-hr day. Find the MTTF and the reliability of the production system over a 1-hr and over an 8-hr production run.

6.15 By integrating the reliability function given by Eq. 6.31, show that the MTTF of the standby system having k identical and independent units of which $k - 1$ are on standby is k/λ. *Hint:* Use integration formula 9, Appendix 2E.

6.16 Find the general solution for a degraded system that fails completely from the degraded state only. In other words, the system must degrade before it can fail. Also show that the MTTF for this system is given by $1/\lambda_1 + 1/\lambda_2$ where λ_1 is the rate at which the system degrades and λ_2 is the rate at which the system fails from the degraded state.

6.17 Define the states, construct the rate diagram, and write the corresponding system of equations for a three-component standby system in which two units must always be operating on-line. If either unit fails, the standby unit will replace the failed unit. When a second component failure occurs, a system failure is observed. Assume no switching failures and no failures in standby. Assume that each component has the same failure rate.

CHAPTER 7

Physical Reliability Models

The previous chapters have focused on the development of reliability models in which system or component reliability was considered as a function of time only. In many applications other factors may be equally important. For example, electronic component failures may depend on the applied voltage or on the operating temperature of the equipment. The strength (and therefore the reliability) of a precast concrete support beam may depend on the impurities found in the water and in other materials used in the mixture. In general, a more accurate reliability model may be one in which the inherent characteristics or the external operating conditions of a component are included. Covariate models incorporate these additional factors into the failure distribution generally by expressing one or more of the distribution parameters as a function of the covariates. This chapter begins with a discussion of several covariate models. Static stress-strength models, in which time is not a factor in determining reliability, are then introduced. In these models only the physical (internal) strength and applied loads are involved. This is followed with the development of dynamic stress-strength models, in which the application of loads over time is also considered. The chapter concludes with the physics-of-failure modeling approach, in which the (mean) time to failure is estimated by determining the root causes of failures through analysis of the physical properties and operating conditions of the components.

7.1 COVARIATE MODELS

Our interest is in developing failure distributions involving one or more covariates or explanatory variables. To do this, we will build on the previous failure distributions in an obvious manner. The approach is to define one or more of the distribution parameters as a function of the explanatory variables. In general, if α is a distribution parameter, such as a percentile, representing a nominal lifetime, then

$$\alpha(\mathbf{x}) = f(x_1, x_2, \ldots, x_k)$$

where $\mathbf{x} = (x_1, \ldots, x_k)$ and x_i = the ith covariate. A covariate may be a voltage, current, temperature, humidity, or other measure of stress or environment. There should be an obvious correlation between the covariates and the parameter value, although there may not necessarily be a cause-and-effect relationship. The functional form of $f(\mathbf{x})$ may be determined by the physical process relating the covariates to the parameters. However, if these relationships are unknown, a simple functional form (e.g., linear) is assumed.

7.1.1 Proportional Hazards Models

Models having the property that individual component hazard rate functions are proportional to each other are referred to as *proportional hazards models.*

Exponential case

For the constant failure rate model, the simplest covariate model is given by

$$\lambda(\mathbf{x}) = \sum_{i=0}^{k} a_i x_i \tag{7.1}$$

where the a_i are unknown parameters to be determined and by convention $x_0 = 1$. The x_i may be transformed variables (e.g., squares or reciprocals), thus allowing, for example, polynomials to be used. The failure rate remains constant over time but does depend on the particular covariate values. For example, the failure rate of a circuit board may be linearly related to the operating temperature of the equipment and the ambient relative humidity. Clearly, other functional forms could be assumed. One popular model is that in which the covariates affect the parameter (failure rate) multiplicatively. It has provided good correlation with observed data.

EXAMPLE 7.1. Ploe and Skewis [1990] provide the following multiplicative model for predicting the failure rate of ball bearings:

$$\lambda = \lambda_b \left(\frac{L_a}{L_s}\right)^y \left(\frac{A_e}{0.006}\right)^{2.36} \left(\frac{\nu_0}{\nu_1}\right)^{0.54} \left(\frac{C_1}{60}\right)^{0.67} \left(\frac{M_b}{M_f}\right) C_w$$

where λ_b = base failure rate of a bearing per 10^6 hr of operation (obtained from the manufacturer)
L_a = actual radial load in pounds
L_s = specification radial load in pounds
y = 3.33 for roller bearings; 3.0 for ball bearings
A_e = alignment error in radians
ν_0 = specification lubricant viscosity
ν_1 = operating lubricant viscosity
C_1 = actual contamination level ($\mu g/m^3$)

M_b = material factor of base material in PSI (yield strength)
M_f = material factor of operating material in PSI (yield strength)
C_w = water contamination factor (leakage of water into the oil lubricant)

$$= \begin{cases} 1 + 460x & \text{for } x < 0.002 \\ 2.036 + 1.029x - 0.0647x^2 & \text{for } x \geq 0.002 \end{cases}$$

where x is the percentage of water present in the oil

A popular form of the multiplicative model is obtained by letting

$$\lambda(\mathbf{x}) = \prod_{i=0}^{k} \exp(a_i x_i) = \exp\left(\sum_{i=0}^{k} a_i x_i\right) \tag{7.2}$$

This model has the desirable property that $\lambda(\mathbf{x}) > 0$ and is linear in the logarithm of $\lambda(\mathbf{x})$. Regardless of the model used,

$$R(t) = e^{-\lambda(\mathbf{x})t}$$

EXAMPLE 7.2. To account for voltage stress, *Military Handbook: Reliability Prediction of Electronic Equipment* [1986] provides the following multiplicative factor for certain types of monolithic microelectronic devices:

$$g(x) = 0.11e^{0.916+0.0005638x_1x_2}$$

where x_1 is the operating supply voltage and x_2 is the worst-case junction temperature (in degrees centigrade). This factor is multiplied by other similar factors in arriving at a (constant) failure rate.

EXAMPLE 7.3. A popular approach in the aerospace industry for estimating life-cycle costs and failure rates or mean failure times is to use parametric estimating equations. Parametric equations relate the mean failure time (the dependent variable) to one or more independent variables (parameters), which are often surrogate variables for factors that may cause failures. There is not necessarily a cause-and-effect relationship present. Instead, the independent variables are useful in explaining the dependent variable. Examples include the use of component weight as a substitute for complexity, or surface area as a substitute for the number of parts. The following parametric regression equation was derived from mean time between maintenance (MTBM) data from 33 aircraft over a two-year period; the dependent variable is the mean number of flying hours between maintenance on the propulsion subsystem of the aircraft, and the engine weight is in pounds.

$$\text{MTBM} = 34.104 + 0.0009853 \times (\text{engine weight}) - 0.31223\sqrt{\text{engine weight}}$$

Other aircraft parametric equations include the electrical subsystem MTBM as a function of the subsystem weight and the maximum power output of the generators, and landing gear subsystem MTBM as a function of the number of wheels and the vehicle weight. Generally, exponential failure distributions are then assumed.

Weibull case

For the Weibull distribution it is common to assume that only the characteristic lifetime and not the shape parameter depends on the covariates. For the multiplicative model, let

$$\theta(\mathbf{x}) = \exp\left(\sum_{i=0}^{k} a_i x_i\right) \tag{7.3}$$

Then
$$R(t) = \exp\left[-\left(\frac{t}{\theta(\mathbf{x})}\right)^{\beta}\right]$$

and
$$\lambda(t \mid \mathbf{x}) = \frac{\beta t^{\beta-1}}{\theta(\mathbf{x})^{\beta}} = \beta t^{\beta-1}\left[\exp\left(\sum_{i=0}^{k} a_i x_i\right)\right]^{-\beta} \tag{7.4}$$

The ratio of two Weibull hazard rates having different covariate vectors is

$$\frac{\lambda(t \mid \mathbf{x}_1)}{\lambda(t \mid \mathbf{x}_2)} = \left[\frac{\theta(\mathbf{x}_2)}{\theta(\mathbf{x}_1)}\right]^{\beta} \tag{7.5}$$

which does not depend on time. Therefore, this model is a proportional hazards model since the component hazard rate functions are proportional to one another. This also suggests that a general form of the hazard rate function may be

$$\lambda(t \mid \mathbf{x}) = \lambda_0(t) g(\mathbf{x}) \qquad \text{with } g(\mathbf{x}) = \exp\left(\sum_{i=1}^{k} a_i x_i\right) \tag{7.6}$$

where $\lambda_0(t)$ is a baseline hazard rate function when $g(\mathbf{x}) = 1$. For example, the exponential baseline failure rate in Eq. (7.2) is $\lambda_0(t) = e^{a_0}$.

EXAMPLE 7.4. An AC motor is known to have a Weibull failure distribution with a shape parameter of 1.5. Reliability test results have shown that the characteristic life in operating hours depends on the load placed on the motor in the following manner:

$$\theta(x) = e^{23.2 - 0.134x}$$

Find the design life for a reliability of 0.95 of a particular motor that is to be placed under a load of 115. If the load is reduced to 100, how much improvement in design life should be expected?

Solution

$$\theta(115) = 2416.3 \qquad t_{0.95} = 2416.3(-\ln 0.95)^{0.6667} = 333.5 \text{ hr}$$
$$\theta(100) = 18{,}033.7 \qquad 18{,}033.7(-\ln 0.95)^{0.6667} = 2489.3 \text{ hr}$$

7.1.2 Location-Scale Models

A separate family of covariate models referred to as *location-scale models* is obtained by setting

$$\mu(\mathbf{x}) = \sum_{i=0}^{k} a_i x_i$$

and letting

$$Y = \mu(\mathbf{x}) + \sigma z \tag{7.7}$$

where $\sigma > 0$ and z has a specified probability distribution not dependent on the covariate vector $\mathbf{x}$.

Normal case

The most common example of this model assumes that z is normally distributed with a mean of 0 and a variance of 1. Therefore Y is normally distributed with mean $\mu(\mathbf{x})$ and standard deviation σ.

Lognormal case

By setting $T = e^Y$, we obtain

$$R(t) = 1 - \Phi\left(\frac{\ln t - \sum_{i=0}^{k} a_i x_i}{s}\right) \tag{7.8}$$

T is lognormal with shape parameter $s = \sigma$ and $t_{med} = e^{u(\mathbf{x})}$.

In the location-scale model, the covariates act in a linear manner on the (mean) failure time when failures are normal, and they act in a multiplicative manner on the (median) failure times when failures are lognormal.

EXAMPLE 7.5. The time to failure of an electrical connector is lognormal with a shape parameter of 0.73. Since failures, measured in operating hours, were observed to be related to the operating temperature of the connector and the number of electrical contacts, the following covariate model was derived:

$$u(\mathbf{x}) = -3.86 + 0.1213x_1 + 0.2886x_2$$

where x_1 is the operating temperature in degrees centigrade and x_2 is the number of contacts. A particular connector used in a personal computer will operate at 80°C and has 16 contact pins. Therefore its reliability over a 5000-hr life is found from the following:

$$u(80, 16) = -0.386 + 0.1213(80) + 0.2886(16) = 10.46$$

$$R(5000) = 1 - \Phi\left(\frac{\ln(5000) - 10.46}{0.73}\right) = 1 - \Phi(-2.66) = 0.996$$

In this case $t_{med} = e^{10.46} = 34{,}891.55$ hr.

The more difficult task of estimating the parameters of these models will be covered in Chapter 15. Lawless [1982] provides additional detail on the use of covariate models. These covariate models are also useful in accelerated life testing, discussed in Chapter 13.

7.2 STATIC MODELS

In many situations it is not appropriate to assume that reliability is a function of time. This section considers a single stress placed on a system during a relatively short time period. A stress is any load that may produce a failure. A failure occurs if the stress exceeds the strength of the system. The strength is the highest stress value that the system can endure without failing. Reliability is therefore viewed as static and not as a function of time, as it was previously viewed. Either the system will bear the load or it will not, resulting in a failure. Loads may be electrical, thermal, chemical, or mechanical. A load may be measured in volts, degrees, pounds, operations per second, miles per hour, or any other units. Examples of a static stress being applied to a system include the landing gear of an aircraft on landing, a rocket being fired, and a building withstanding a hurricane. After the development of a static reliability, we will return to the dynamic case, in which loads are applied either periodically or randomly over time.

Static models are a result of failures due to (nearly) instantaneous stress placed on a system and not a result of any prior effects or history. To quantify stress and strength, let X be the random variable representing the stress placed on a system such that $f_X(x)$ is the probability density function, and let Y be the random variable representing the capacity of the system such that $f_Y(y)$ is the probability density function. Then the probability that the stress does not exceed the value x is given by

$$\Pr\{X \leq x\} = F_X(x) = \int_0^x f_{X'}(x')\,dx' \tag{7.9}$$

and the probability that the capacity does not exceed the value y is given by

$$\Pr\{Y \leq y\} = F_Y(y) = \int_0^y f_{Y'}(y')\,dy' \tag{7.10}$$

7.2.1 Random Stress and Constant Strength

If the system strength is a known constant k and the stress is a random variable with PDF as defined above, then the system (static) reliability can be defined as the probability that stress does not exceed strength. That is,

$$R = \int_0^k f_X(x)\,dx = F_X(k) \tag{7.11}$$

Figure 7.1 illustrates the reliability graphically.

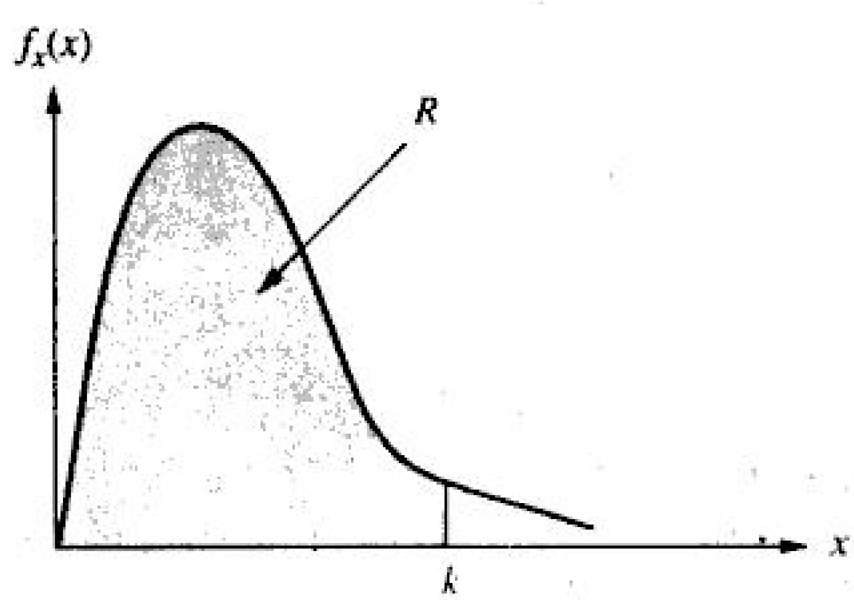


FIGURE 7.1
Reliability for a component under a random load with fixed strength.

EXAMPLE 7.6. The stress placed on a motor mount has the following PDF:

$$f_x(x) = \begin{cases} \dfrac{x^2}{1125} & \text{for } 0 \le x \le 15 \text{ lb} \\ 0 & \text{otherwise} \end{cases}$$

The motor mount has been found through laboratory testing to have a fixed tolerance of 14 lb. Therefore, its static reliability is given by

$$R = \Pr\{X \le 14\} = \int_0^{14} \frac{x^2}{1125}\, dx = \frac{14^3}{3375} = 0.813$$

7.2.2 Constant Stress and Random Strength

If the stress, or load, is fixed at a known constant s and the strength is a random variable having a PDF as given above, then the system (static) reliability is the probability that the strength exceeds the fixed load, as shown in Fig. 7.2. That is,

$$R = \Pr\{Y \ge s\} = \int_s^{\infty} f_y(y)\, dy = 1 - F_y(s) \tag{7.12}$$

EXAMPLE 7.7. The strength of a new superglue has been found to be random, its value depending on the exact mixture of the compounds used in the manufacturing process. It has the following PDF:

$$f_y(y) = \begin{cases} 10/y^2 & \text{for } y \ge 10 \text{ lb} \\ 0 & \text{otherwise} \end{cases}$$

A fixed load of 12 lb is to be applied. What is the reliability?

$$R = \Pr\{Y \ge 12\} = \int_{12}^{\infty} \frac{10}{y^2}\, dy = \frac{10}{12} = 0.833$$

7.2.3 Random Stress and Random Strength

If both stress and strength are random variables, the reliability remains the probability that stress is less than strength (or equivalently, that strength is greater than stress). However, to compute the reliability, the following double integral must be solved:

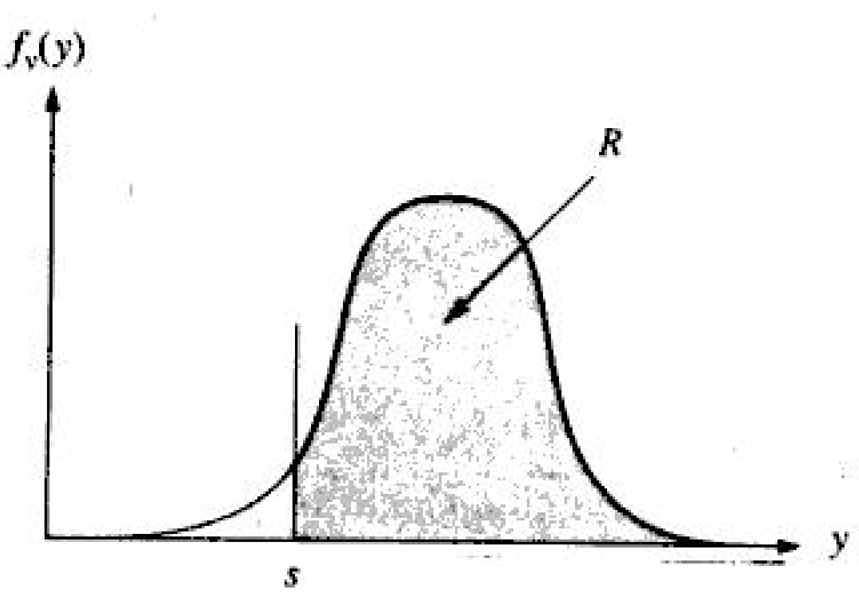


FIGURE 7.2
Reliability for a component under a fixed load with random strength.

$$R = \Pr\{X \le Y\} = \int_0^\infty \left[\int_0^y f_x(x)\,dx\right] f_y(y)\,dy$$
$$= \int_0^\infty F_x(y) f_y(y)\,dy \tag{7.13}$$

since for a given y,

$$R(y) = \int_0^y f_x(x)\,dx = F_x(y)$$

The static reliability can then be obtained from

$$R = \int_0^\infty R(y) f_y(y)\,dy$$

The reliability depends on the region of the two curves in which the tails overlap, or interfere with one another, as shown in Fig. 7.3. For this reason the analysis of stress versus strength is sometimes referred to as *interference theory*.

EXAMPLE 7.8. Let $f_x(x) = \frac{1}{50}$, $0 \le x \le 50$, and let $f_y(y) = 0.0008y$, $0 \le y \le 50$. Then

$$F_x(y) = \int_0^y \frac{1}{50}\,dx = \frac{y}{50}$$

$$R = \int_0^{50} \frac{y}{50} 0.0008y\,dy = \int_0^{50} 0.000016y^2\,dy = 0.0000053y^3\Big|_0^{50} = 0.667$$

An equivalent method for finding R is given by

$$R = \Pr\{Y > X\} = \int_0^\infty \left[\int_x^\infty f_y(y)\,dy\right] f_x(x)\,dx \tag{7.14}$$

For some problems, performing the integration in the order given by the above expression may be easier than solving Eq. (7.13). If the stress and strength are not defined over similar intervals, the integration may have to be performed over disjoint intervals, as shown in the following example.

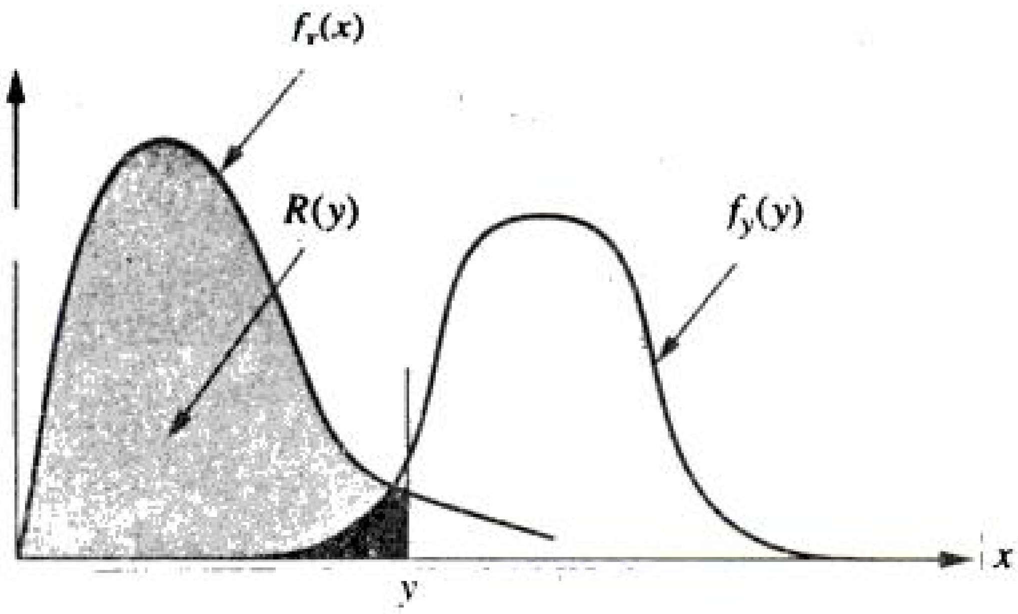


FIGURE 7.3
Reliability for a component under a random load with random strength.

EXAMPLE 7.9. Let $f_x(x) = 3x^2/10^9$, $0 \le x \le 1000$ lb, and let $f_y(y) = (5 \times 10^{-7})y$, $0 \le y \le 2000$ lb. Then

$$R = \int_0^{1000}\left[\int_0^y \frac{3x^2}{10^9}\,dx\right](5 \times 10^{-7})y\,dy + (1)\int_{1000}^{2000}(5 \times 10^{-7})y\,dy = 0.85$$

where the second integral is simply the probability that Y is greater than 1000 ($\Pr\{Y > 1000\}$) in which case $\Pr\{X < Y\} = 1$. However, if the order of the integration is reversed, as suggested by the above form for R, then

$$R = \int_0^{1000}\left[\int_x^{2000}(5 \times 10^{-7})y\,dy\right]\frac{3x^2}{10^9}\,dx = 0.85$$

Exponential case

One of the simplest cases involving both random stress and random strength occurs when both distributions are exponential. In this case their PDFs are given by

$$f_x(x) = \frac{1}{\mu_x}e^{-x/\mu_x} \qquad \text{and} \qquad f_y(y) = \frac{1}{\mu_y}e^{-y/\mu_y}$$

where μ_x is the mean stress and μ_y is the mean strength. Therefore,

$$R = \int_0^\infty\left[\int_0^y \frac{1}{\mu_x}e^{-x/\mu_x}\,dx\right]\frac{1}{\mu_y}e^{-y/\mu_y}\,dy = \int_0^\infty\left[1 - e^{-y/\mu_x}\right]\frac{1}{\mu_y}e^{-y/\mu_y}\,dy$$

$$= \int_0^\infty \frac{1}{\mu_y}e^{-y/\mu_y}\,dy - \int_0^\infty \frac{1}{\mu_y}\exp\left[-y\left(\frac{1}{\mu_x} + \frac{1}{\mu_y}\right)\right]dy$$

$$= 1 - \frac{1}{\mu_y}\,\frac{\exp[-y(1/\mu_x + 1/\mu_y)]}{-(1/\mu_x + 1/\mu_y)}\Bigg|_0^\infty$$

where the first integral is the total area under the density function,

$$R = 1 - \frac{1}{\mu_y}\left(\frac{\mu_x\mu_y}{\mu_x + \mu_y}\right) = 1 - \frac{\mu_x}{\mu_x + \mu_y} = \frac{\mu_y}{\mu_x + \mu_y} = \frac{1}{1 + \mu_x/\mu_y} \tag{7.15}$$

Selected values of the ratio of the mean stress to the mean strength and the corresponding reliabilities calculated from Eq. 7.15 are given in Table 7.1. It is obvious from Table 7.1 that when both distributions are exponential, the mean strength must be *more* than 10 times the mean stress in order to achieve an acceptable reliability.

Normal case

When both stress and strength are normally distributed, system reliability may be obtained as follows. Let X be a normally distributed stress with mean μ_x and standard deviation σ_x, and let Y be a normally distributed strength with mean μ_y and standard deviation σ_y. Then

$$R = \Pr\{Y \ge X\} = \Pr\{Y - X \ge 0\} = \Pr\{W \ge 0\}$$

TABLE 7.1
Ratio of stress to strength for two exponentials

μ_x/μ_y	Reliability
1.0	0.50
0.9	0.53
0.8	0.56
0.7	0.59
0.6	0.63
0.5	0.67
0.4	0.71
0.3	0.77
0.2	0.83
0.1	0.91

where $W = Y - X$, $E(W) = E(Y - X) = \mu_y - \mu_x$, and $\mathrm{Var}(W) = \mathrm{Var}(Y - X) = \sigma_y^2 + \sigma_x^2$ (assuming Y and X are independent). Then

$$R = \Pr\{W \geq 0\} = \Pr\left\{\frac{W - \mu_W}{\sigma_W} \geq \frac{-(\mu_y - \mu_x)}{\sqrt{\sigma_y^2 + \sigma_x^2}}\right\}$$

$$= \Pr\left\{\frac{W - \mu_W}{\sigma_W} \leq \frac{(\mu_y - \mu_x)}{\sqrt{\sigma_y^2 + \sigma_x^2}}\right\} = \Phi\left(\frac{\mu_y - \mu_x}{\sqrt{\sigma_x^2 + \sigma_y^2}}\right) \tag{7.16}$$

EXAMPLE 7.10. If stress is normally distributed with a mean of 10.3 and a standard deviation of 2.1, and strength is normally distributed with a mean of 25.8 and a standard deviation of 8.2, determine the system reliability.

Solution

$$R = \Phi\left(\frac{25.8 - 10.3}{\sqrt{67.24 + 4.41}}\right) = \Phi(1.83) = 0.96638$$

using the normal table in the Appendix, Table A.1.

Lognormal case

Because of the relationship between the normal and lognormal distributions, similar results may be obtained when both stress and strength follow lognormal distributions. Let X be a lognormally distributed stress with median m_x and shape parameter s_x, and let Y be a lognormally distributed strength with median m_y and shape parameter s_y. Then

$$R = \Pr\{Y \geq X\} = \Pr\left\{\frac{Y}{X} \geq 1\right\}$$

Let $W = \ln(Y/X) = \ln Y - \ln X$; W is normal with mean $\mu_W = \ln(m_y/m_x)$ and variance $\mathrm{Var}(W) = s_y^2 + s_x^2$, assuming Y and X are independent. Then

$$
\begin{aligned}
R &= \Pr\{W \geq \ln 1\} = \Pr\{W \geq 0\} = \Pr\left\{\frac{0-\mu_w}{\sigma_W} \geq \frac{0-\mu_W}{\sigma_W}\right\} \\
&= \Pr\left\{\frac{W-\mu_W}{\sigma_W} \leq \frac{\mu_W}{\sigma_W}\right\} = \Phi\left(\frac{\ln(m_y/m_x)}{\sqrt{s_y^2+s_x^2}}\right)
\end{aligned}
\tag{7.17}
$$

EXAMPLE 7.11. A structure has a capacity to withstand earthquakes, which are lognormally distributed with a median of 8.1 on the Richter scale and with $s_y = 0.07$. Historically, the magnitude of earthquakes in this region has been lognormal with a median of 5.5 and $s_x = 0.15$. Therefore the static reliability to withstand a single random occurrence of an earthquake is found from Eq. (7.17):

$$R = \Phi\left(\frac{\ln(8.1/5.5)}{\sqrt{0.15^2 + 0.07^2}}\right) = \Phi(2.33) = 0.99$$

Table 7.2 provides formulas for calculating the static reliability when stress or strength is modeled using the theoretical distributions. Other stress-strength models may be developed by assuming other combinations of probability distributions for stress and strength. However, the integrals resulting from applying Eq. (7.13) or Eq. (7.14) generally cannot be solved in closed form. Therefore, some form of numerical integration must be performed or use made of tables in which the integration has already been accomplished. Kapur and Lamberson [1977] provide reliability expressions for normal and exponential combinations, gamma stress and strength, and normal stress and Weibull strength distributions. Evaluating the gamma reliability requires the use of tables of the incomplete beta function. For the normal-Weibull combination, Kapur and Lamberson provide table values for selected parameter values. Other discussions on stress-strength analysis may be found in Carter [1972] and Dhillon and Singh [1981].

TABLE 7.2
Static reliability for specified distributions

Distribution	Constant strength k	Constant stress s	Random stress and strength
Exponential	$R = 1 - \exp\left(-\frac{k}{\mu_x}\right)$	$R = \exp\left(-\frac{s}{\mu_y}\right)$	$R = \frac{\mu_y}{\mu_x + \mu_y}$
Weibull	$R = 1 - \exp\left[-\left(\frac{k}{\theta_x}\right)^{\beta_x}\right]$	$R = \exp\left[-\left(\frac{s}{\theta_y}\right)^{\beta_y}\right]$	Solve numerically
Normal	$R = \Phi\left(\frac{k-\mu_x}{\sigma_x}\right)$	$R = 1 - \Phi\left(\frac{s-\mu_y}{\sigma_y}\right)$	$R = \Phi\left(\frac{\mu_y - \mu_x}{\sqrt{\sigma_x^2+\sigma_y^2}}\right)$
Lognormal	$R = \Phi\left(\frac{1}{s_x}\ln\frac{k}{m_x}\right)$	$R = 1 - \Phi\left(\frac{1}{s_y}\ln\frac{s}{m_y}\right)$	$R = \Phi\left(\frac{\ln(m_y/m_x)}{\sqrt{s_x^2+s_y^2}}\right)$

7.3 DYNAMIC MODELS

If a load is placed repetitively over time on a system, then, under certain conditions, a dynamic reliability may be derived. Two cases will be discussed. The first case occurs when loads occur at regular (or known) intervals of time. The second case corresponds to loads occurring at completely random times as characterized by the Poisson probability distribution. In both cases the assumption is made that the distribution of the strength or capacity of the system does not change over time (a stationary process). This precludes those situations in which aging or wearout occurs.

7.3.1 Periodic Loads

For the first case assume that n loads (cycles) occur at times $t_1, t_2, \ldots, t_n$, that each load has an identical and independent distribution represented by Eq. (7.9), and that the strength of the system at each cycle has an identical and independent probability distribution represented by Eq. (7.10). Either the load or the strength may also be a known constant rather than a random variable. Let X_i be the load and Y_i be the strength observed on the ith cycle. After n cycles the reliability R_n is found from

$$\begin{aligned} R_n &= \Pr\{X_1 < Y_1, X_2 < Y_2, \ldots, X_n < Y_n\} \\ &= \Pr\{X_1 < Y_1\}\Pr\{X_2 < Y_2\}\cdots\Pr\{X_n < Y_n\} \end{aligned} \tag{7.18}$$

assuming independent load and strength applications each cycle. If the distributions of X and Y are identical for each cycle (a stationary process), then $\Pr\{X_i < Y_i\} = R$ where R is the static reliability for a single load-versus-strength application and may be computed using Eq. (7.11), (7.12), (7.13), or (7.14) as appropriate. Therefore, for n independent loads applied to a system, $R_n = R^n$.

EXAMPLE 7.12. If the strength of a system is a constant k and the load can be represented by an exponential distribution with parameter α, then

$$R_n = \left(1 - e^{-\alpha k}\right)^n$$

where $1/\alpha$ is the mean load per application.

EXAMPLE 7.13. If the load is a constant s and the strength of the system can be described by a Weibull distribution with parameters θ and β, then

$$R_n = e^{-n(s/\theta)^\beta}$$

EXAMPLE 7.14. The breaking strength of a support beam has a Weibull distribution with $\beta = 2.1$ and $\theta = 1200$ lb. Four beams are used to support a structure that places 100 pounds on each beam. What is the reliability of the structure?

$$R_4 = e^{-4(100/1200)^{2.1}} = 0.9785$$

If load application times are known constants, a dynamic reliability may be found from

$$R(t) = R^n \qquad \text{for } t_n \leq t < t_{n+1} \tag{7.19}$$

with $t_0 = 0$. If the cycle (load) time is uniformly spaced with $\Delta t = t_{i+1} - t_i$, then

$$R(t) = R^{\lfloor t/\Delta t \rfloor} \tag{7.20}$$

where $\lfloor z \rfloor$ is the integer portion of z.

EXAMPLE 7.15. A die is designed to withstand a force of 10,000 lb. A hydraulic forge has been found to exert a force that has an exponential distribution with a mean of 1000 lb. If castings are made at the fixed rate of one every two minutes ($\Delta t = 1/30$ hr), what is the reliability of completing an 8-hr shift without the die failing under the load?

Solution

$$R = \Pr\{X < Y\} = F_x(10{,}000) = 1 - e^{-10{,}000/1{,}000} = 0.9999546$$

$R(t) = 0.9999546^{\lfloor 30t \rfloor}$ where t is measured in hours. Then $R(8) = 0.9999546^{\lfloor 240 \rfloor} =$ 0.98916.

7.3.2 Random Loads

If loads are applied at random so that the number of loads per unit of time has a Poisson distribution, then

$$P_n(t) = (\alpha t)^n e^{(-\alpha t)}/n! \qquad n = 0, 1, 2, \ldots$$

is the probability of n loads occurring during time t. Alpha (α) is the mean number of loads per unit of time, and αt is therefore the mean number of loads during time t. The reliability can be found from

$$\begin{aligned} R(t) &= \sum_{n=0}^{\infty} R^n P_n(t) = \sum_{n=0}^{\infty} R^n \left[\frac{(\alpha t)^n e^{-\alpha t}}{n!}\right] \\ &= e^{-\alpha t} \sum_{n=0}^{\infty} \frac{(\alpha t R)^n}{n!} \\ &= e^{-(1-R)\alpha t} \end{aligned} \tag{7.21}$$

This last expression results from the infinite series

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

R is again computed using Eq. (7.11), (7.12), (7.13), or (7.14).

EXAMPLE 7.16. A building is designed to withstand winds of speeds up to 120 mph. Hurricane winds are normally distributed with a mean of 86 mph and a standard deviation of 9 mph. In this region hurricanes occur at random (Poisson process) at the mean rate of 2 per year. Derive the reliability function.

Solution. Using Eq. (7.11) with $F_x(x)$ normally distributed,

$$R = \Phi\left(\frac{120 - 86}{9}\right) = \Phi(3.78) = 0.99992 \qquad \text{and} \qquad R(t) = e^{-(0.00008)2t}$$

If a reliability of 0.99 is desired, one would expect buildings of this type to last

$$t = \frac{\ln 0.99}{-0.00016} = 62.8 \text{ yr}$$

7.3.3 Random Fixed Stress and Strength

A different result is obtained if stress and strength are randomly determined once and then fixed for each cycle. In the case of random cycle times (e.g., Poisson),

$$R(t) = \sum_{n=0}^{\infty} R_n P_n(t) = P_0(t) + R \sum_{n=1}^{\infty} P_n(t)$$

since $R_0 = 1$ and $R_n = R = \Pr\{x < y\}$ for $n = 1, 2, \ldots$. Thus

$$R(t) = P_0(t) + R(1 - P_0(t))$$

using the fact that $\sum_{i=0}^{\infty} P_i(t) = 1$. Since $P_0(t) = e^{-\alpha t}$ for the Poisson process,

$$\begin{aligned} R(t) &= e^{-\alpha t} + R(1 - e^{-\alpha t}) \\ &= R + (1 - R)e^{-\alpha t} \end{aligned} \tag{7.22}$$

Equation (7.22) is simply the weighted average of a reliability of 1 times the probability that no load is applied and the static reliability times the probability that one or more loads are applied.

EXAMPLE 7.17. A single emergency shut-off valve has an exponentially distributed strength having a mean of 3700 lb. The force, or load, is also exponentially distributed with a mean of 740 lb. Once applied, the load will then remain constant. Emergency shut-off procedures occur randomly at the rate of 1 per year. We have $\mu_x/\mu_y = 740/3700 = 0.20$, so from Table 7.1, $R = 0.83$ and $R(t) = 0.83 + 0.17e^{-t}$ with t measured in years. Therefore, the reliability for 1 yr is obtained from $R(1) = 0.83 + 0.17e^{-1} = 0.8925$.

7.4 PHYSICS-OF-FAILURE MODELS

The primary approach taken so far is to treat the occurrence of failures as a random process. As discussed in Chapter 1, this view results from our lack of knowledge of the physical processes resulting in a failure. As a consequence, we must develop reliability models statistically. Through the collection and analysis of failure data, we can estimate parameters and perform goodness-of-fit tests in deriving an acceptable reliability model. Indeed, this is the focus of the second part of this book. However, in taking this approach, we can only infer from our sample of failure data to the general population. Our reliability estimates are valid predictions for the general population but say very little concerning an individual component or failure occurrence. In fact, if failures are distributed exponentially (or have almost any distribution, for that matter), then the time to failure of a single occurrence can be any $t \geq 0$. It is only over a large number of failures that we can begin to see the exponential pattern. Therefore, it is only over a large number of failures that we can make reliable reliability predictions. A second limitation of this approach is that it does not consider the effect of individual stresses and operating conditions on components. Not all components will experience the same voltage, ambient temperature, vibration, shock, or humidity. Therefore we would not expect all of the components necessarily to exhibit the same failure pattern. The covariate models presented earlier address this problem to some degree, but they are still statistical models developed from samples and applied over an entire population.

An alternative approach to reliability estimation is called physics of failure. We define the physics-of-failure models as mathematically derived, usually deterministic models based on knowledge of the failure mechanisms and the root causes of failures. A failure is not viewed as a stochastic event. Instead, a time to failure is found for each component failure mode and failure site based on the stresses, material properties, geometry, environmental conditions, and conditions of use. The failure times can then be ranked, and the most dominant one (i.e., the shortest time) provides the component time to failure estimate. Once the model has been developed, individual use and environmental conditions can be considered to develop a reliability estimate tailored to a particular application. The disadvantages of this approach are that the models are very specific to the failure mechanism and failure site, a detailed understanding of the failure processes is required, and experimental data as well as engineering analysis may be required in order to derive the equations. As a result, only a limited number of useful models are available.

Although there is no well-defined approach for developing physical failure models, several general steps can be identified:

1. Identify failure sites and mechanisms.
2. Construct mathematical models.
3. Estimate reliability for a given operating and environmental profile and for given component characteristics.
4. Determine dominant service life.
5. Redesign to increase service (design) life.

The significant activity in this methodology is the development of the physical model. Figure 7.4 identifies the generic inputs to the model.

Ideally the effect of the variables that collectively describe the failure mechanism can be mathematically modeled on the basis of known physical principles and

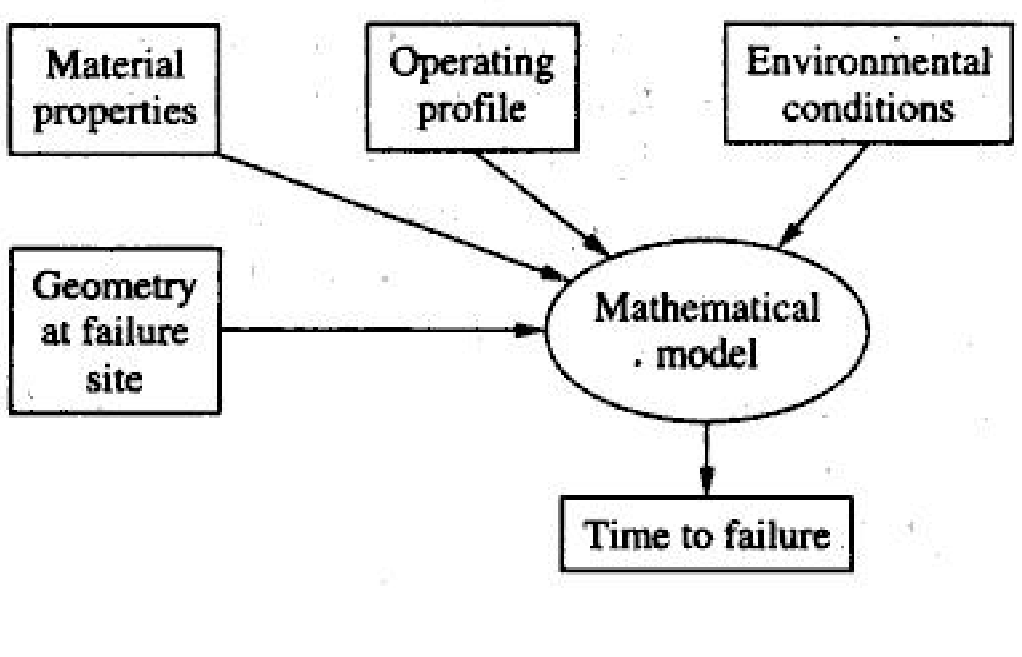


FIGURE 7.4
A physics-of-failure conceptual model.

constants or empirically derived through experimentation and observation. Typical failure mechanisms that have been modeled include fatigue, friction, corrosion, dielectric breakdown, electromigration, contamination, molecular migration, temperature cycling, and mechanical stress. Several simple examples in the spirit of this approach follow.

EXAMPLE 7.18. The useful life of cutting tools, such as drill bits or saw blades, may be modeled on the basis of the geometry and operating characteristics of the cut and the hardness of the material. There are several failure modes that may be identified, including fracture, plastic deformation, and gradual wear. The life of a cutting tool with respect to gradual wear was first expressed mathematically by Frederick Taylor in 1907 and has been enhanced since then. A typical form of this model is given by

$$t = \frac{c(B_{\text{hn}})^m}{v^\alpha f^\beta d^\gamma}$$

where t = the life of a tool in minutes
B_{hn} = the Brinell hardness number of the work material
v = cutting speed in feet per minute
f = the feed in inches per revolution or inches per sawtooth
d = depth of the cut in inches
c, m, α, β, and γ are constants to be determined empirically

Usually $\alpha > \beta > \gamma > m$, indicating that tool life is most sensitive to cutting speed, then feed, depth of cut, and finally material hardness.

Consider a milling operation performed on cast iron in which the cutting tool advances at a feed rate of 0.02 inch per revolution with a cutting speed of 40 feet per minute. The depth of the cut is 0.011 inch. For the particular operation and cutting tool used, the following model parameters were determined from a least-squares fit to data generated in a laboratory:

$$t = \frac{0.023(B_{\text{hn}})^{1.54}}{v^{7.1} f^{4.53} d^{2.1}}$$

The cast iron has a Brinell hardness number of 180. Therefore

$$t = \frac{0.023(180)^{1.54}}{(40)^{7.1}(0.02)^{4.53}(0.011)^{2.1}} = 186 \text{ minutes}$$

EXAMPLE 7.19. Ploe and Skewis [1990] report the following model, developed by T. P. Newcomb for estimating brake-pad wear and used in determining the pad life:

$$W = \left(\frac{10^4 W_b}{2A}\right)\left(\frac{\text{Wt}(\Delta v^2 N)y}{4g}\right)$$

where W = pad wear per mile in inches
W_b = specific wear rate of friction material (in^3/ft-lb)
A = lining area (in^2)
Δv = average change in velocity per brake action (ft/sec)
Wt = weight of vehicle (lb)
g = acceleration due to gravity (32.2 ft/sec^2)
N = frequency of brake applications per mile
y = proportion of total braking effort transmitted through the lining

7.5 A load is exponential with a mean of 25. The strength is exponential. Determine the minimum value of the mean strength in order to achieve a reliability of 0.95.

7.6 Consider the following load distribution:

$$f_x(x) = \begin{cases} 3x^2/10^9 & 0 \le x \le 1000 \quad \text{kg} \\ 0 & \text{otherwise} \end{cases}$$

If the strength is a constant 950 kg, determine the static reliability.

7.7 A brake assembly has been tested and found to have a capacity that is normally distributed with a mean of 275 lb and a standard deviation of 25 lb. If a normally distributed force having a mean of 180 lb and a standard deviation of 30 lb is applied, what is the static reliability? As the variability (variance) of either the strength or load increases, what happens to the reliability?

7.8 A dam has a fixed water capacity of 20 feet. The distribution of flood levels is given by the following probability density function with x measured in feet:

$$f_x(x) = 0.25e^{-0.25x} \qquad x \ge 0$$

Floods occur at random following a Poisson process with a mean rate of 1 every 2 yr. Compute the reliability of the dam over:
(a) A 10-yr period
(b) A 20-yr period

7.9 A capacitor for use in an emergency signal transmitter is selected at random from a shipment in which the rated strength of the capacitors as measured in applied voltage is normal with a mean of 200 V and a standard deviation of 45 V. If the transmitter is needed, a fixed load of 120 V will be applied. Frequency of use of the transmitter is random with a Poisson distribution having a mean rate of once every 48 months. What is the reliability over a 5-yr design life? *Hint:* Does this problem fit the conditions of the model discussed in Section 7.3.3?

7.10 Determine the expected reliability (static) of a system having the following load and capacity distributions: $f_x(x) = \frac{1}{2}, 15 \le x \le 17; f_y(y) = 0.04(y-15), 15 \le y \le 20.$

7.11 Each day during the peak demand period, an extra electrical generator powers up. It has a maximum output of 1200 watts. The demand for this additional power is random, having an exponential distribution with a mean of 300 watts. What is the reliability of the generator over a week's time (7 days) in meeting the peak demands?

7.12 The strength of a concrete structure designed to support a fixed load of 464 lb has the following probability density function:

$$f_y(y) = \frac{3y^2}{10^9} \qquad 0 \le y \le 1000 \quad \text{lb}$$

(a) Compute the static reliability.
(b) If the load is also random having the probability density function below, find the static reliability.

$$f_x(x) = 2(0.001 - 0.000001x) \qquad 0 \le x \le 1000 \quad \text{lb}$$

(c) If the load in part (a) is applied at random according to a Poisson process with a mean occurrence rate of 0.01 per year, compute the design life for a reliability of 0.99.

7.13 The Weibull Building, a marvel in engineering design, is subject to random (Poisson-distributed) wind gusts at an average rate of two per day. The strength of the building is such that it can withstand winds of up to 100 mph (deterministic). Wind speed, however, during gusts, is random with a Weibull distribution having a shape parameter of 2 and a scale parameter of 50 mph. Determine the building's reliability function and the mean number of days to failure.

7.14 The peak daily load in megawatt-hours on an electric utility substation is normally distributed with a mean of 10,000 and a standard deviation of 1000. The capacity of the system is 13,500 megawatt hours. What is the reliability of the station over a 100-day period?

7.15 The breaking strength of a cutting tool is a constant 25 lb. If the load being placed on the tool has the following probability density function, compute the tool's static reliability.

$$f_x(x) = \frac{200}{(x+10)^3} \qquad x \geq 0$$

7.16 The Fawlty Construction Company has built a condo for Mr. Herr A. Cane on the Outer Banks of North Carolina having a static reliability to withstand a major storm in this area of 0.992. If major storms hit the Outer Banks with an average frequency of 1 per year, what is the probability that the condo will incur major damage during Mr. Cane's 25-yr mortgage on the condo? If a reliability of 0.95 is desired, what is the condo's design life?

7.17 Determine the reliability of a system having a capacity that is lognormal with a median of 100 and $s = 0.6$ subject to a load that is lognormal with a median of 20 and $s = 0.8$.

7.18 A remote sensor communicates with a central processing facility using a 2400-bit-per-second modem. Each event detected by the sensor requires one bit to relay the information. The sensor must report 100 percent of all events. Events occur according to the following probability density function:

$$f_x(x) = \frac{x}{3.125 \times 10^6} \qquad 0 \leq x \leq 2500 \quad \text{events/sec}$$

What is the static reliability of the system?

7.19 Refer back to Exercise 3.17. The moisture content in the air necessary to cause flat fading is 100 PPM. When fog occurs, the moisture content of the air is normally distributed with a mean of 75 PPM and a standard deviation of 25 PPM. Fog occurs at the rate of once every two months. The charge density of the air necessary to cause selective fading is 5×10^{12} electrons per cubic centimeter. The charge density of an electrical storm is lognormal with a median of 3×10^{12} electrons per cubic centimeter and a mean of 4×10^{12} electrons per cubic centimeter. Electrical storms occur at the rate of 4 per month. Recompute the reliability for a 24-hr period.

7.20 The buckling force under an axial compressive load for a column made from timber and hinged at both ends is given by Euler's formula:

$$F_c = \frac{\pi^2 EI}{L^2}$$

where F_c = critical load in pounds
E = modulus of elasticity = 1.6×10^6 pounds per square inch for timber
I = moment of inertia of the cross section of the column = 5.359 inches^4 for a 2-inch by 4-inch column
L = the length of the column in inches

A 10-foot column is used to support a load that has a Weibull distribution with a shape parameter of 1.76 and a scale parameter of 2,500 lb.

(a) What is the static reliability?
(b) If a material stronger than timber is used that has a safety factor of 3 (i.e., F_c is three times the mean load), what is the static reliability?

7.21 The thermal protection system on the space shuttle consists in part of approximately 30,000 ceramic and reinforced carbon–carbon tiles. These tiles are designed to withstand reentry temperatures of up to 1260°C. Assume that the temperature of the tile system on reentry is lognormally distributed with a shape parameter of 0.15 and a median value of 835°C. If a failure occurs when the reentry temperature exceeds the design capacity of the tile system, determine the reliability of the tile system over a 5-yr period in which the shuttle averages 12 flights a year. A failure in this case requires the removal and replacement of one or more tiles during the ground recovery and restoration process.

7.22 A manufacturing lathe operation requires cutting a malleable iron rod having a Brinell hardness number 125. For this particular cutting operation, the following tool life equation was derived:

$$t = \frac{7.4B_{hn}^{1.1}}{v^{4.76} f^{2.11} d^{1.84}}$$

The depth of the cut is 0.02 inch, and the feed rate is 0.03 inch per revolution. Because of production requirements, the cutting tool can only be replaced once every 4 hr. What is the maximum cutting speed v permissible?

CHAPTER 8

Design for Reliability

To a large degree, reliability is an inherent attribute of a system, component, or product. As such, it is an important consideration in the engineering design process. When the life-cycle costs of a system are being analyzed, reliability plays an important role as a major driver of these costs and has considerable influence on system performance. As we will see, decisions made during conceptual and preliminary design will affect total life-cycle costs. The objective of this chapter is to describe a reliability design process that will establish and then achieve realistic reliability goals.

Reliability design is an iterative process that begins with the specification of reliability goals consistent with cost and performance objectives. This requires consideration of the life-cycle costs of the system and the effect that reliability has on overall costs and system effectiveness. Figure 8.1 outlines this process. Once the reliability goals have been established, these goals must be translated into individual component, subcomponent, and part specifications. This is not necessarily an easy task, and it generally requires a reliability block analysis. After individual component and part requirements have been determined, various design methods can be applied in order to meet the goals. These methods include the proper selection of parts and material, stress-strength analysis, derating, simplification, identification of technologies, and use of redundancy. Following completion of preliminary and detailed design and along with initial development and prototyping, a failure analysis may be performed to determine whether specifications are being met and to provide a systematic approach for identifying, ranking, and eliminating failure modes. This requires the use of reliability testing, including, perhaps, a formalized reliability growth testing program. Once reliability goals have been achieved, verification that safety margins are also being met must be made. Fault tree analysis can be a

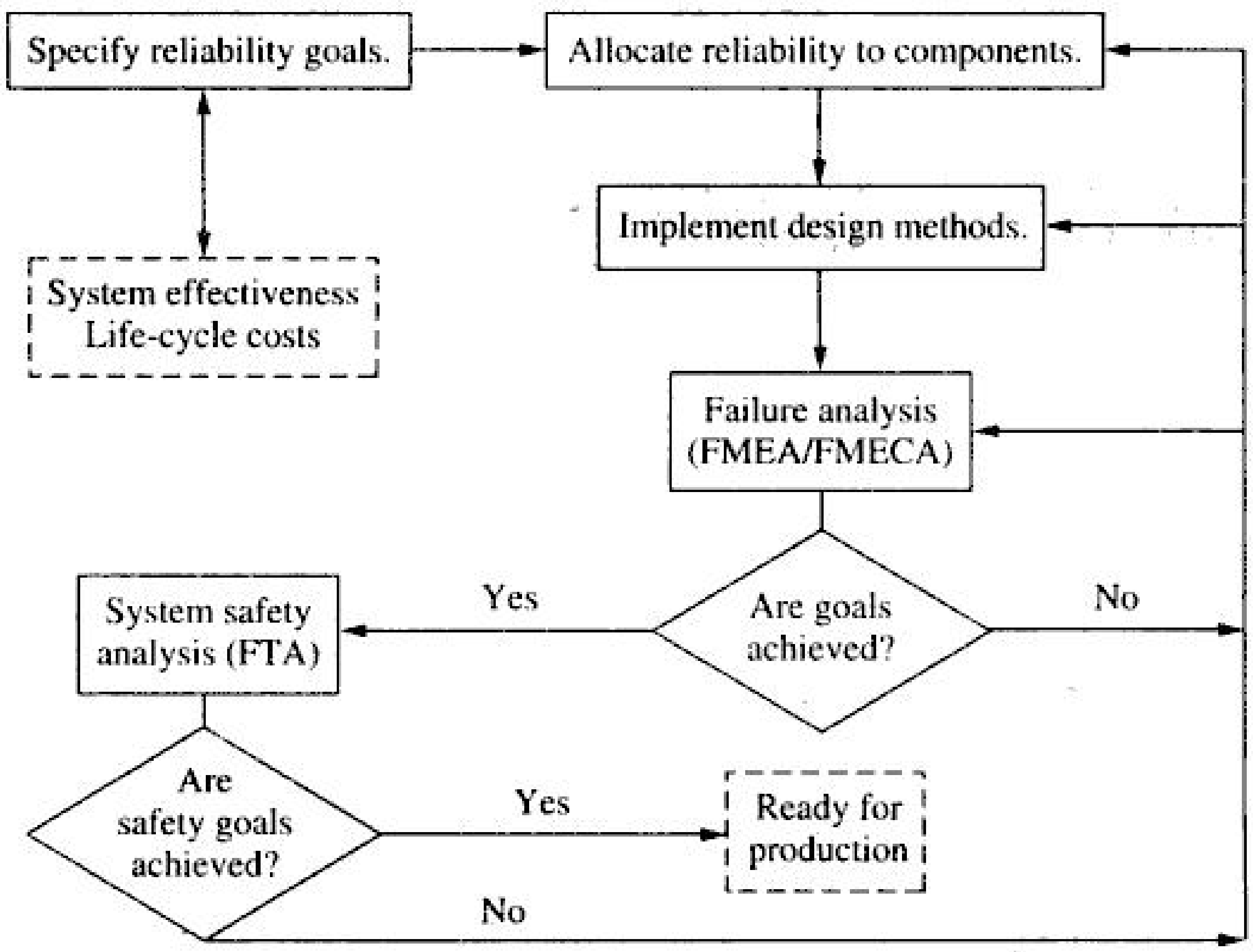


FIGURE 8.1
The reliability design process.

useful tool in identifying critical (catastrophic) failure modes. If either the reliability or safety goals are not met, the design process must continue. This may require reallocating reliability goals among the components if it is not possible to achieve a desired component reliability. More often it may require a redesign through the use of the design methods presented in Section 8.3. The effect of design changes should then be verified through continued use of failure analysis and reliability testing.

Although we are considering reliability as an inherent system or component attribute that can largely be determined during design, we cannot ignore the fact that reliability is influenced throughout a product life cycle by factors external to the product itself. Table 8.1 shows the typical product life cycle and the major activities

TABLE 8.1
Reliability activities and product life cycle

Development phase	Conceptual preliminary design	Detailed design, development, and prototyping	Production and manufacture	Product use and support
	Specification Allocation Design methods	Design methods Failure analysis Growth testing Safety analysis	Acceptance testing Quality control Burn-in and screen testing	Preventive and predictive maintenance Modifications Parts replacement

a wide range of failure distributions (and therefore reliabilities) are attainable by selecting various distributions and variances each having the same MTTF.

As discussed in Chapter 1, in specifying a reliability, it is necessary to also define what constitutes a failure. Many systems degrade before failing completely. At what point is performance no longer acceptable? It may also be necessary to exclude certain failure modes in the specification. For example, failures due to natural catastrophes may be difficult to eliminate as part of the inherent design process. On the other hand, certain systems may be designed specifically to operate in these environments, and their effect should then be an integral part of the design process. The measurement of time is equally important, especially if time to failure is not being measured in clock or calendar time. Jet engine failures, for example, have often been measured in terms of cycles to maximum power rather than operating hours. Finally, the normal operating and environmental conditions in which the product is to be evaluated should be clearly stated as part of the specification. This includes not only external stress factors such as temperature and humidity, but also policy and procedural conditions such as frequency and type of preventive maintenance and experience and training levels of the users or operators.

8.1.1 System Effectiveness

In establishing the system-level reliability requirements, both system performance (effectiveness) and life-cycle cost must be considered. *System effectiveness* is the probability that the system can successfully perform its intended purpose or mission when operated under specified conditions. System effectiveness includes reliability, as shown in Fig. 8.2. *Operational readiness* is the probability that the system is operational when first used or at the start of a mission. Recall that $R(0) = 1$. However, at time $t = 0$, when a system is first powered up, turned on, put on-line, etc., it may or may not work. For example, it may have been defective coming from the factory or experienced a dormant failure. *Mission availability* will be defined in Chapter 11. It is the percentage of time the system will be operating during the mission. Maintainability and availability will be discussed in the following three chapters. However, note here that if the system is not repairable, availability is the same as the system reliability. Design adequacy is the probability that the system will accomplish its

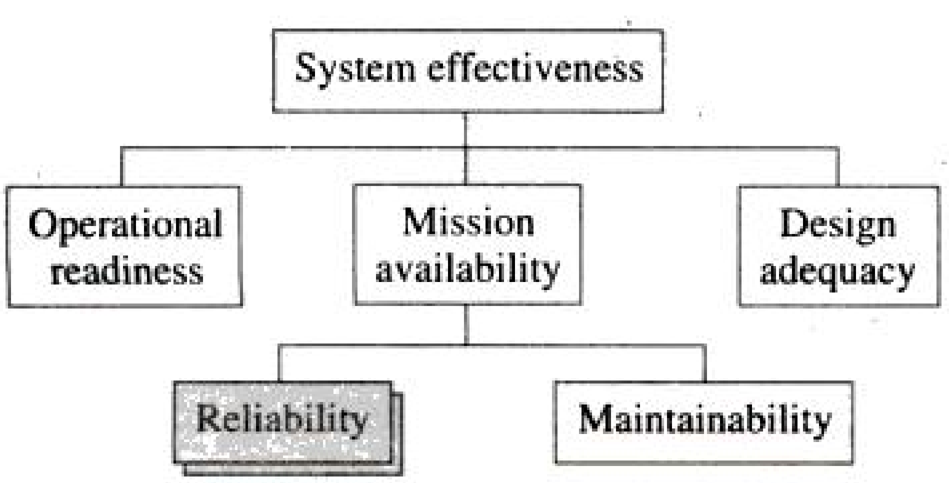


FIGURE 8.2
System effectiveness.

mission or fulfill its purpose given that the system is operating within its design parameters. For example, assume that a copying machine is working on demand and continues to work over the period of time in which copies are to be produced. However, in order to complete the job on time, it must operate at a speed of 45 copies a minute. If it is unable to maintain this rate, then the copier is inadequately designed for the job. Design adequacy in this example may be the probability or percentage of jobs in which the reproduction rate required is within the machine capability. Since all three components of system effectiveness are defined to be probabilities, then assuming independence among all three,

$$\text{System effectiveness} = \text{operational readiness} \times \text{availability} \times \text{design adequacy} \tag{8.1}$$

Obviously, all three probabilities must be high in order for system effectiveness to be at an acceptable level.

In selecting from among design alternatives, ideally we would want to either maximize system effectiveness subject to an upper bound on life-cycle cost or minimize life-cycle cost subject to a lower bound on system effectiveness.

8.1.2 Economic Analysis and Life-Cycle Costs

The other major consideration in establishing a system reliability is life-cycle cost. Life-cycle costing is the process of determining all relevant costs from conceptual development through production, utilization, and phase-out. It is the total cost of ownership. Our interest in discussing life-cycle cost is to ensure that those costs affected by our choice of design variables, especially reliability (and later maintainability), are properly accounted for. There are many different ways to establish life-cycle cost categories; a typical cost element structure is shown in Table 8.2.

TABLE 8.2
Cost categories

Acquisition costs	Operations and support costs	Phase-out
Research and development	Operations	Salvage value
Management	Facilities	Disposal costs
Engineering	Operators	
Design and prototyping	Consumables (energy and fuel)	
Engineering design	Unavailable time or downtime	
Fabrication	Support	
Testing and evaluation	Repair resources	
Production	Supply resources:	
Manufacturing	Repairables	
Plant facilities and overhead	Expendables	
Marketing and distribution	Tools, test, and support equipment	
	Failure costs	
	Training	
	Technical data	

In performing design trade-offs, total life cycle cost of each alternative design should be estimated and compared. At the highest level, a life-cycle cost model may take on the following form:

$$\begin{aligned}\text{Life-cycle cost} &= \text{acquisition costs} + \text{operations costs} + \text{failure cost}\\ &\quad + \text{support costs} - \text{net salvage value}\end{aligned} \tag{8.2}$$

where $\quad \text{Net salvage value} = \text{salvage value} - \text{disposal cost}$

Since the system will normally be operated over an extended period of time corresponding to its design life or economic life, the time value of money must be taken into account. The economic life is the number of years beyond which it is no longer economical to operate or maintain the system and replacement or discontinuance is justified on a cost basis. To discount monetary values over time, all revenues and costs can be expressed in present-day equivalent dollars.[2] Therefore, the following adjustments must be made. P is the present value, and i is the real, or effective, discount rate. If we assume a constant annual inflation rate of f and an annual return on investment rate of e, then $i \approx e - f$ for small values [3] of f and e.

Let $P_F(i, d) = 1/(1 + i)^d$ where F is a future amount at the end of year d, and $P_A(i, d) = [(1 + i)^d - 1]/[i(1 + i)^d]$, where A is an equal annual amount observed over d years. The term $P_A(i, d)$ is an annuity factor, which converts equal annual payments over d years to a single present-day equivalent amount. Writing Eq. (8.2) more explicitly,

$$\begin{aligned}\text{Life-cycle cost} &= C_u N + [F_o + P_A(i, t_d)C_o N] + \left[P_A(i, t_d)C_f \frac{t_0}{\text{MTTF}} N\right]\\ &\quad + [F_s + P_A(i, t_d)C_s N] - [P_F(i, t_d)SN]\end{aligned} \tag{8.3}$$

where C_u = unit acquisition cost
N = number of identical units to be procured
F_o = fixed cost of operating
C_o = annual operating cost per unit
F_s = fixed support cost
C_s = annual support cost per unit
C_f = cost per failure
t_o = operating hours per year per unit
t_d = design life (in years)
S = unit salvage value (a negative value is interpreted as a disposal cost)

The expression t_0/MTTF in Eq. (8.3) is the expected number of failures per year assuming replacement or repair to "as good as new" condition of the failed unit (a

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[2] Alternative methods of comparing two or more cash flows may be used, such as annual equivalent amounts and internal rate of return. Present value is used here since it is the simplest and most convenient for the problem at hand.

[3] A more precise relationship is given by $1/(1 + i) = (1 + f)/(1 + e)$.

renewal process which is discussed in the next chapter). The cost per failure, C_f, may be a repair cost, replacement cost, or a warranty cost. The unit acquisition cost includes the design, development, and production costs allocated over the total number produced. As the reliability goal increases, these costs will increase because of additional reliability growth testing, improved manufacturing quality control, more expensive parts and material, increased use of redundancy, and additional resources committed to reliability improvement.

Assuming that only the unit acquisition cost and failure cost are sensitive to the design reliability, we may wish to compare the following expected present equivalent unit cost for each alternative:

$$C_u + (P/A, i, t_d)C_f \frac{t_0}{\text{MTTF}}$$

EXAMPLE 8.1 Two designs are being considered for a new product operating throughout the year and having the following characteristics:

Design 1: $C_u = 1200$, $\lambda_1 = 0.02/\text{yr}$
Design 2: $C_u = 1300$, $\lambda_2 = 0.01/\text{yr}$

The design life of the product is 10 yr. A failure results in a replacement at the unit acquisition cost (i.e., $C_u = C_f$). Assuming an interest rate of 5 percent, $P/A(0.05, 10) = 7.7217$, and

$$\text{Design 1: } 1200[1 + (P/A, 0.05, 10)0.02] = 1385.32$$

$$\text{Design 2: } 1300[1 + (P/A, 0.05, 10)0.01] = 1400.38$$

Therefore design 1 is preferred.

The acquisition cost may be difficult to estimate during the early design of a product. The reader is encouraged to see Fabrycky and Blanchard [1991] for a detailed development of life-cycle costing. Discussions on life-cycle costs of repairable items will be deferred until Chapter 11. The reader is cautioned not to assume that the minimum-cost design that meets the system effectiveness goal is always the preferred one. Other constraints, for example, may be minimum acceptable reliability goals and product safety requirements.

8.2 RELIABILITY ALLOCATION

Once the system reliability goals have been defined, reliability must then be allocated to the components and possibly subcomponents in a manner that will support these goals. Reliability block diagrams and the relationships developed in Chapter 5 are useful tools in accomplishing this. In general, we want the following inequality to hold:

$$h(R_1(t), R_2(t), \ldots, R_n(t)) \geq R^*(t) \tag{8.4}$$

where $R_i(t)$ is the reliability at time t of the ith component, $R^*(t)$ is the system reliability goal at time t, and h is a function that relates component reliabilities to system reliability. If MTTF* is a system reliability goal and component failures are exponential, then we desire

$$g(\text{MTTF}_1, \text{MTTF}_2, \ldots, \text{MTTF}_n) \geq \text{MTTF}^* \tag{8.5}$$

where MTTF_i is the MTTF of component i and g is a function that provides the system MTTF. The form of the functions h and g depend on the component serial-parallel configuration. For example, if all the components are serially related and their failures are independent of one another, then

$$\prod_{i=1}^{n} R_i(t) \geq R^*(t) \tag{8.6}$$

8.2.1 Exponential Case

If all the components have constant failure rates, Eq. (8.6) can be written as

$$\prod_{i=1}^{n} e^{-\lambda_i t} \geq R^*(t) \tag{8.7}$$

or equivalently,

$$\sum_{i=1}^{n} \lambda_i \leq \lambda_S \tag{8.8}$$

based on Eq. (5.2) where λ_s is the system failure rate goal.

8.2.2 Optimal Allocations

Ideally, reliability allocation should be accomplished in a least-cost manner. For example, assume that Eq. (8.6) applies and that R* has been specified for some time t (we will drop the argument t as being understood). If each component has a current reliability R_i where $\prod R_i < R^*$, we may be interested in solving the following problem:

$$\min z = \sum_{i=1}^{n} C_i(x_i) \tag{8.9}$$

subject to

$$\prod_{i=1}^{n}(R_i + x_i) \geq R^* \tag{8.10}$$

$$0 < R_i + x_i \leq B_i < 1 \qquad i = 1, 2, \ldots, n \tag{8.11}$$

where x_i is the increase in reliability of the ith component, $C_i(x_i)$ is the corresponding cost of achieving this growth, and B_i is an upper bound on the attainable component reliability. In practice, specifying the cost functions may be quite difficult. However, let us assume that the cost of component i is determined by $c_i x_i^2$ over the range in which we are interested. A quadratic cost function is the simplest nonlinear function, and since it is convex, it shows reliability growth cost increasing at an increasing rate—a reasonable assumption. Equation (8.9) then becomes

$$\min z = \sum_{i=1}^{n} c_i x_i^2 \tag{8.12}$$

To solve our problem, we assume that Eq. (8.10) is a strict equality, relax (ignore) the inequalities (8.11) for the moment, and form the Lagrangian function:

$$x_i, \theta) \quad \sum_{i=1}^{n} c_i x_i^2 - \theta \left[\prod_{i=1}^{n} (R_i + x_i) - R^* \right] \tag{8.13}$$

where θ is the Lagrangian multiplier. Necessary conditions for a minimum are then found by taking the partial derivatives of Eq. (8.13) and setting them equal to zero:

$$\frac{\partial L(x_i, \theta)}{\partial x_i} = 2c_i x_i - \theta \prod_{\substack{j=1 \\ j \neq i}}^{n} (R_j + x_j) = 0 \qquad i = 1, 2, \ldots, n \tag{8.14}$$

$$\frac{\partial L(x_i, \theta)}{\partial \theta} = \prod_{i=1}^{n} (R_i + x_i) - R^* = 0 \tag{8.15}$$

Multiplying the ith Eq. (8.14) by $(R_i + x_i)$ and rearranging terms:

$$\theta \prod_{i=1}^{n} (R_i + x_i) = 2c_i x_i (R_i + x_i) = \theta R^*$$

where the last equality resulted from Eq. (8.15). Rearranging terms again:

$$2c_i x_i^2 + 2c_i x_i R_i - \theta R^* = 0$$

Solving for x_i by using the quadratic formula with the positive root results in

$$x_i = \frac{-2c_i R_i + \sqrt{4c_i^2 R_i^2 + 8c_i \theta R^*}}{4c_i} = -0.5R_i + \sqrt{0.25R_i^2 + \frac{0.5\theta R^*}{c_i}} \tag{8.16}$$

In order to solve for x_i in Eq. (8.16), θ must be known. The generalized Lagrangian multiplier technique can be used to search for the correct value of θ. For a given value of θ, Eq. (8.16) solves a related optimization problem in which the $R*$ value in Eq. (8.15) is equal to the product in the equation. In other words, if we guess at a value for θ and compute the x_i using Eq. (8.16), then we would have solved an optimization problem in which our reliability goal was equal to the product in Eq. (8.15). Therefore, if we systematically update θ and compute the resulting reliability, we should eventually converge to the desired solution. This is illustrated in the following example.

EXAMPLE 8.2 A system consists of three components in series having the following parameters:

Component	Current reliability	Cost coefficient
1	0.85	$25
2	0.80	20
3	0.90	40

The upper bound on component reliability is 0.99. Current system reliability is $(0.85)(0.80)(0.90) = 0.612$. The system reliability goal is $R^* = 0.90$. Varying θ in Eq. (8.16) and then computing the left-hand side of Eq. (8.10) results in the following system reliability values:

θ	System reliability	Total cost
4.00	0.791	$5.840
5.00	0.835	7.168
6.00	0.880	8.453
7.00	0.924	9.701
6.5	0.902	9.081
6.48	0.901	9.057
6.46	0.900	9.031

The final solution is as follows:

Component	Reliability increase x_i	Component reliability	Component cost
1	0.1199	0.9699	2.997
2	0.1526	0.9526	3.051
3	0.0746	0.9746	2.983

The solution obtained by this method may not satisfy the upper-bound constraints given by Eq. (8.11). If this is the case, the problem must be resolved with the upper-bound constraints included. The solution to the inequality-constrained problem must satisfy more general conditions (e.g., the Kuhn-Tucker conditions). To find a solution, however, will generally require a numerical procedure. Since θ must be positive, the x_i will always be positive, automatically satisfying the lower-bound constraint.

8.2.3 ARINC Method

There are several popular reliability allocation strategies discussed in the literature that do not require optimization. One of the earliest proposed and the simplest is the ARINC method. This method assumes that the components are in series, are independent, and have constant failure rates. If λ_i is the current estimated failure rate for the ith component and $\lambda*$ is the target system failure rate, then

$$\text{new } \lambda_i = w_i \lambda*$$

where
$$w_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i} \qquad i = 1, 2, \ldots, n$$

Therefore, allocated failure rates are proportional to the current failure rates. This method could be adopted if failure rates are not constant by using an average failure rate (see Exercise 8.4).

8.2.4 AGREE Method

The AGREE (Advisory Group on Reliability of Electronic Equipment) method assumes that a system is comprised of n components each having n_i modules, or subcomponents. This method is somewhat more sophisticated than the ARINC method, for it allows component operating times to be less than the system operating times and it allows the inclusion of an importance index. Let

t = system operating time
$R^*(t)$ = system reliability goal at time t
n = number of components
n_i = a complexity number, e.g., the number of modules within component i
$N = \sum n_i$ = total number of modules in system
t_i = operating time of the ith component ($t_i \leq t$)
λ_i = failure rate of the ith component
w_i = probability that the system will fail given component i has failed (importance index)

The approach is to allocate an equal share of the reliability to each module in the system. Therefore the ith component's contribution to system reliability is given by $[R^*(t)]^{n_i/N}$. This allows us, then, to establish the following relationship for the ith component:

$$w_i(1 - e^{-\lambda_i t_i}) = 1 - [R^*(t)]^{n_i/N} \tag{8.17}$$

The left side of Eq. (8.17) is the joint probability that the ith component fails and results in a system failure. The right side of the equation is the failure probability allocated to the ith component. Solving Eq. (8.17) for λ_i results in

$$\lambda_i = -\frac{1}{t_i} \ln\left(1 - \frac{1 - R^*(t)^{n_i/N}}{w_i}\right) \qquad i = 1, 2, \ldots, n \tag{8.18}$$

Observe that $\prod_{i=1}^{n} e^{-\lambda_i t_i} \leq R^*(t)$ since not all component failures result in system failures (if $w_i < 1$).

EXAMPLE 8.3 A transceiver has four components: a receiver, a power supply, a transmitter, and an antenna system. Reliability specifications require the transceiver to operate 1000 hr with a probability of 0.99. The following component data are available:

Component	Importance index w_i	Operating time t_i, hr	Number of modules, n_i
Receiver	0.8	1000	25
Antenna	1.0	1000	15
Transmitter	0.7	500	23
Power supply	1.0	1000	70

The total module count is 133. Therefore, $0.99^{n_i/133}$ is the reliability to be allocated to the ith component. From Eq. (8.18) the following results are obtained:

Component	Failure rate	MTTF	Reliability	System reliability
Receiver	2.362×10^{-6}	423,369	0.9976	0.9981
Antenna	1.1335×10^{-6}	882,227	0.9989	0.9989
Transmitter	4.9676×10^{-6}	201,303	0.9975	0.9983
Power supply	5.2896×10^{-6}	189,048	0.9947	0.9947
System	1.3753×10^{-5}	72,713	0.9888	0.9900

From the above table, the probability of a component failure is $1 - 0.9888$ and the probability of a system failure is $1 - 0.99$. The difference is that some component failures will not cause a system failure (i.e., $w_i < 1$).

8.2.5 Redundancies

A general strategy to follow in allocating reliability among the components is to consider only serially related components. Redundancy can then be used to achieve the allocated component reliability if necessary, as discussed in the next section. However, there may be instances in which redundancy is an integral part of the design. It may therefore be desirable to account for this redundancy as part of the reliability allocation process. Reliability block diagrams can be a useful tool in this case. For example, consider the block diagram of Fig. 8.3. If R^* is the system reliability goal, we can write $R^* = R_1 \times R' \times R_4$ and use one of the above allocation methods based on serially related components. Components 2 and 3 would initially be treated as a single component. Then, assuming R' is the allocated reliability, we have

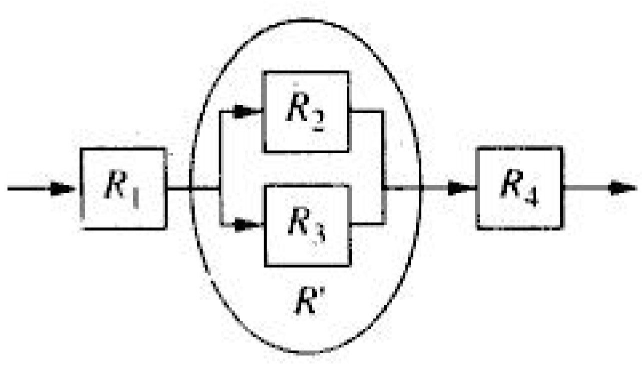

FIGURE 8.3
Reliability block diagram for reliability allocation.

$$R' = 1 - (1 - R_2)(1 - R_3) = R_2 + R_3 - R_2R_3$$

We can assign a reliability to one of the components, say component 3, such that $R_3 < R'$. (Why?) Then solving for the other,

$$R_2 = \frac{R' - R_3}{1 - R_3}$$

If both components receive the same reliability R, we have $R' = 2R - R^2$, which has the solution $R = 1 - (1 - R')^{0.5}$ from the quadratic formula. Obviously, if the system configuration is more complex, the allocation process is more difficult. However, generally it will be possible to reduce the system initially to serially related components and then further decompose as necessary.

8.3 DESIGN METHODS

In order to design reliability into a product, reasons for product failures must be considered. Generally, a product fails prematurely because of inadequate design features, manufacturing and parts defects, abnormal stresses induced during packaging or distribution, operator or maintenance error, or external conditions (environmental or operating) that exceed the design parameters. The reliability engineer should ensure that reliability is a primary consideration in the product design and during manufacturing. Several methods are available to the engineer to accomplish this, including parts and material selection, derating, stress-strength analysis, use of technology, simplification, and redundancy. Generally combinations of these methods will be utilized in the design process as trade-offs are made between performance and costs. Each of these methods is discussed below.

8.3.1 Parts and Material Selection

Often the designer can choose between selecting standard parts and manufacturing specialized parts having perhaps greater tolerances and reliability. The trade-off is usually in cost, but ease of repair, parts availability, energy requirements, weight, and size may also be considerations. Historical databases can assist in determining relative reliabilities among competing parts. For example, *Military Handbook: Reliability Prediction of Electronic Equipment* [1986] provides detailed information on various electronic parts and their failure rates.

Knowledge of material properties and the external stresses the system will experience is important. Stress is typically measured in pounds per square inch (psi) or megapascals (MPa) where 1 MPa = 10^6 Newtons per meter squared (N/m^2). The mechanical properties of materials such as metals, polymers, ceramics, and composites, include tensile strength, hardness, impact value, fatigue life, and creep. They will be discussed in the following.

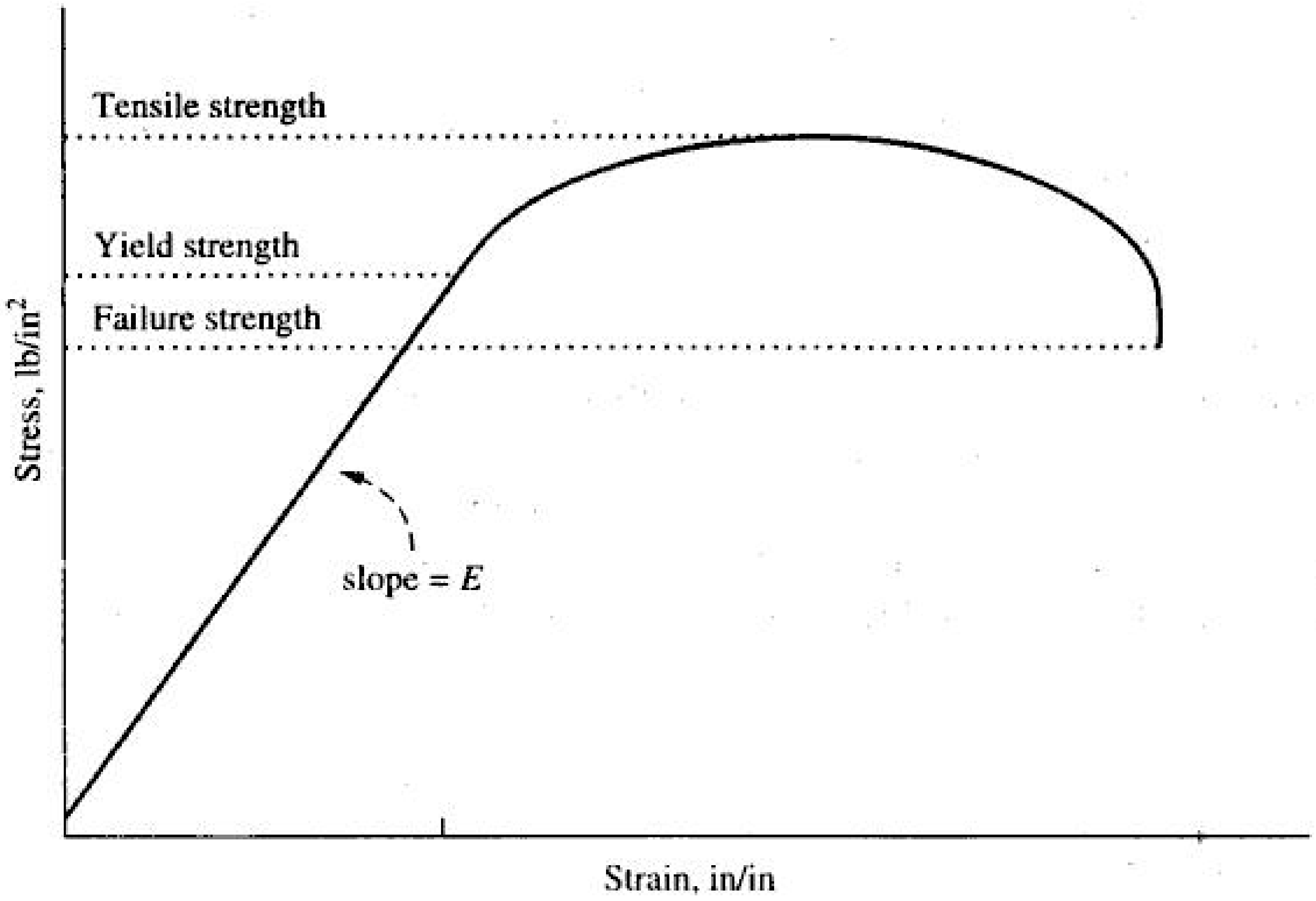

FIGURE 8.4
A stress-strain curve.

Tensile strength

Tensile strength is the ability to withstand a tensile or compressive load. Material will typically deform first elastically and then plastically. The deformation is elastic if after the load has been removed, the material returns to its original shape. When a material is stressed beyond its elastic limit, a permanent, or plastic, deformation results. See Fig. 8.4. The elastic range is represented by Hooke's law: stress $= E \times$ strain, where E, the slope of the stress-versus-strain line, is the modulus of elasticity (in psi or MPa) of the material. Beyond the elastic limit (yield strength) this relationship is nonlinear. At some point beyond the maximum stress level (tensile strength), the material will fracture. Empirically derived stress-strain diagrams may be utilized for specific materials. Strain is dimensionless since it represents the change in length per original length resulting from the load application. Yield strength is therefore the stress level at which plastic deformation occurs on a stress-strain curve.

Hardness

Hardness is the resistance of material to the penetration of an indenter. Hardness measurements are useful in analyzing the service wear of material. Hardness may be measured by several different relative scales, including Brinell (B_{hn})(kg/mm^2), Rockwell (R), and Vickers (V_{hn}). Specific tests are used in obtaining numerical values that are made readily available in published handbooks.

Impact value

Impact value is a measure of the toughness of the material under sudden impact. *Toughness* refers to the amount of energy absorbed before fracture. Impact testing will determine the ability of the material to withstand a sudden dynamic force. Since temperature may have a significant effect on the fracture behavior of the material, graphs of impact energy versus temperature have been established for certain types of material. For example, steel will become very brittle in very cold temperatures.

Fatigue life

Fatigue life is the number of cycles until failure for a part subjected to repeated stresses over an extended period of time. The fatigue strength is generally less than would be observed under a static load. Fatigue testing results in experimental data relating the number of cycles to failure (N) to the magnitude of the cyclical stress (S). Certain materials, such as steel, have an infinite life below a specific stress level (endurance limit). The fatigue strength is the maximum stress amplitude for a specified number of cycles until failure. Mathematically, an S-N curve (Fig. 8.5) may take the form $N = cS^{-m}$, where $c > 0$ and $m > 0$ are constants determined experimentally in laboratory tests that duplicate the amplitude, frequency, and pattern of specific stresses.

Creep

Creep is the progressive deformation of material under a constant stress. At temperatures within 40 percent of its absolute melting point, a metal or alloy begins to elongate continuously under a constant load until a fracture occurs. Creep is a design consideration when the component will be operating at moderate or high

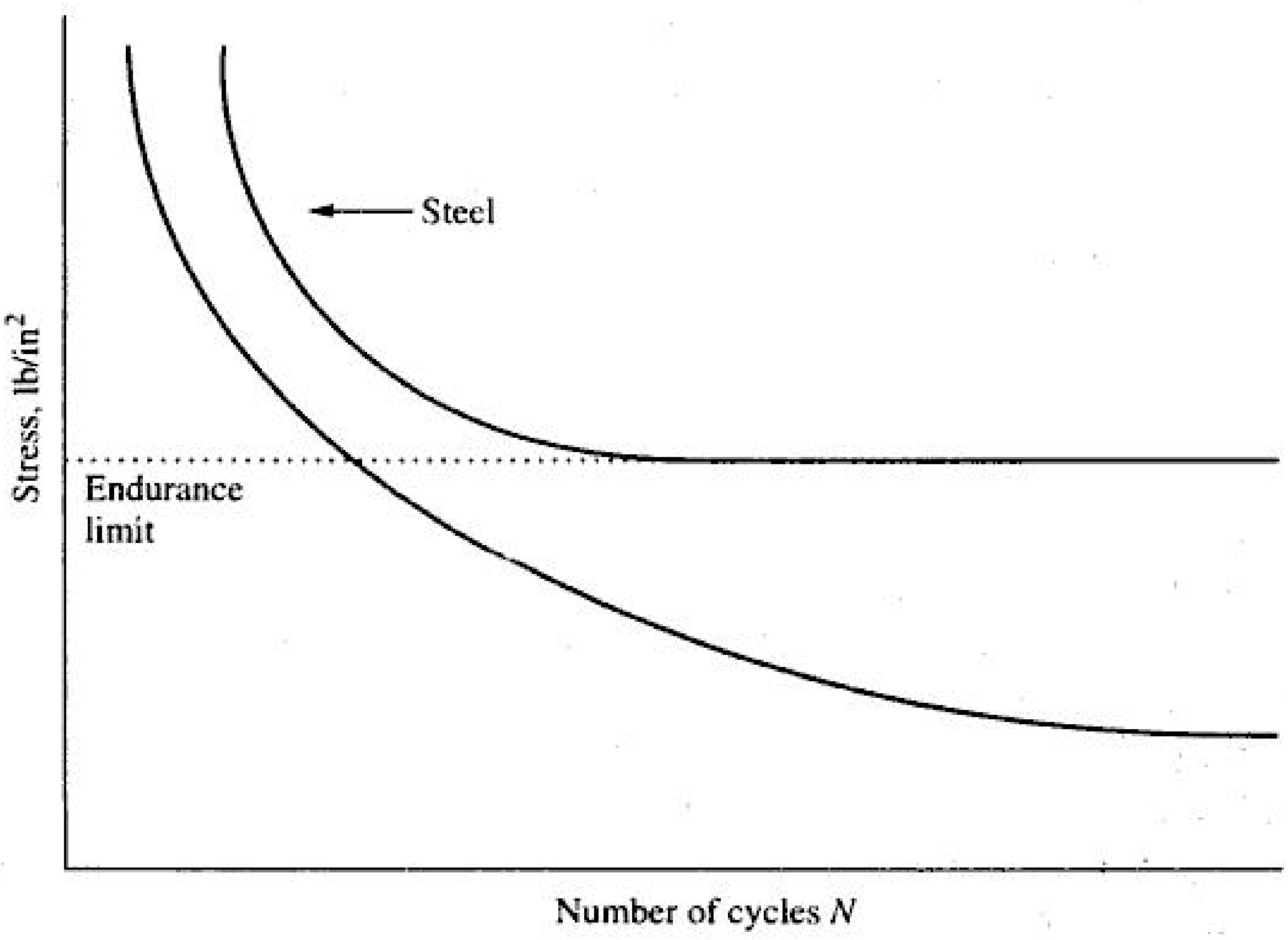

FIGURE 8.5
An S-N curve.

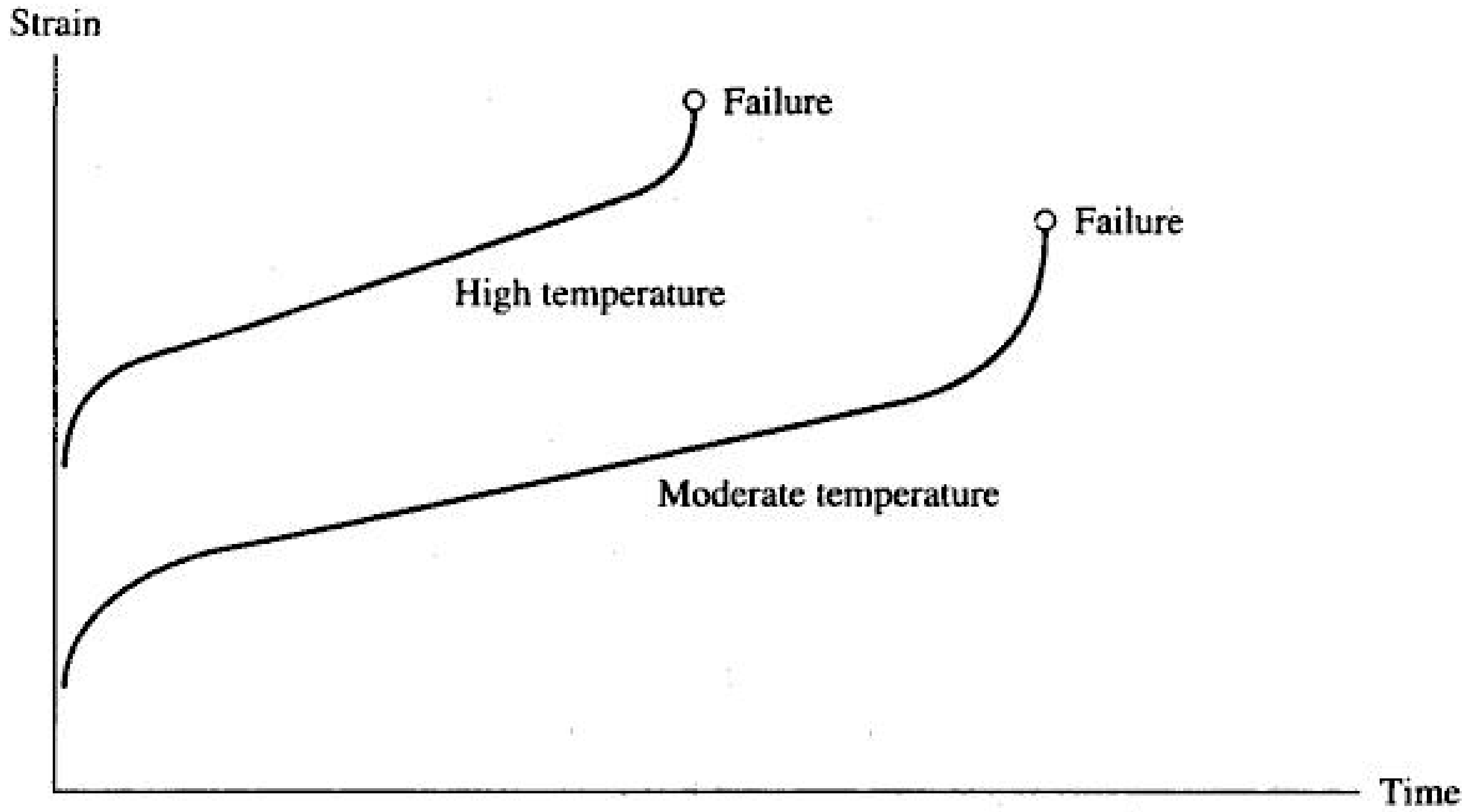

FIGURE 8.6
Typical creep curves at constant load.

temperatures. Andrade's formula, presented in Example 7.20, relates the strain to the time under the load for a particular material as a function of temperature. See Fig. 8.6.

The physical design of a part can affect its fatigue properties. For example, any irregularities or discontinuities such as seams, grooves, holes, and sharp corners may have weaker stress points and therefore become the site of a fatigue crack. Increasing the amount of material subject to wear or fatigue, improving the properties of the material used in a part, replacing one material with another, and redesigning the geometry of the part are options available for improving part reliability. Other nonmechanical properties of material that may be important design considerations include heat capacity, thermal conductivity, electrical resistivity, and magnetic permeability. Additional details on the properties of materials may be found in Callister [1994] and Dieter [1991].

8.3.2 Derating

Derating consists of using a component under stress significantly below its rated value. It has been most beneficial when applied to electronics, in which case the designed voltage or current strength of the part is well above the normal operating level. *Military Handbook: Reliability Prediction of Electronic Equipment* [1986] provides derating curves similar to the example in Fig. 8.7. The values given in the legend are the ratios of the applied voltage to the rated voltage. The covariate models discussed in the previous chapter may be useful for establishing curves such as these.

Voltage and temperature are common derating stresses for electrical components. Other types of derating stresses can be found in mechanical systems. For example, Ploe and Skewis [1990] present the following general formula for estimating the failure rate of a gear:

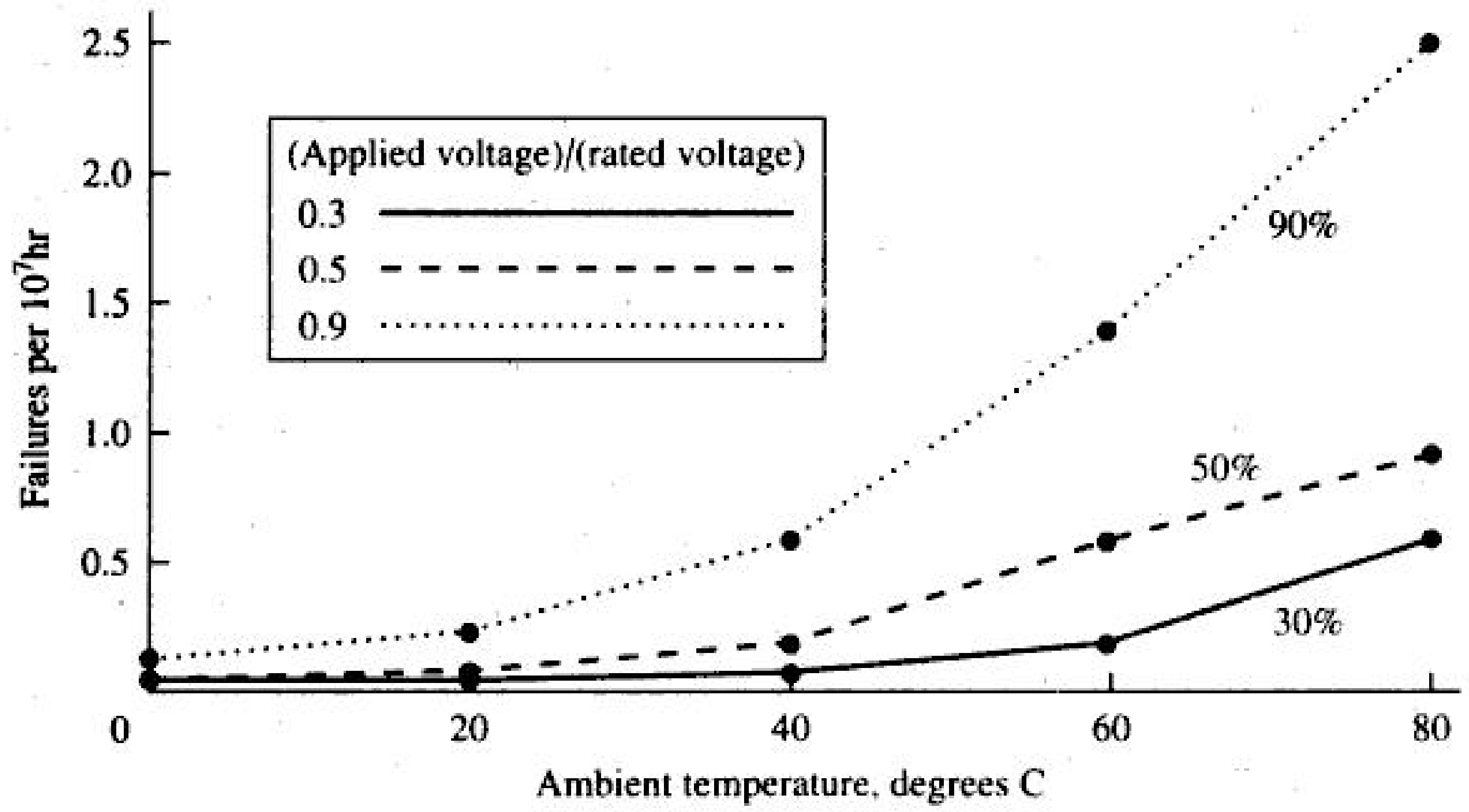

FIGURE 8.7
Examples of derating curves.

$$\lambda = \lambda_b \left(\frac{s}{s_d}\right)^{0.7} \left(\frac{L}{L_d}\right)^{4.69} \left(\frac{\nu_s}{\nu}\right)^{0.54} \left(\frac{c}{c_s}\right)^{0.67} \left(\frac{T}{T_s}\right)^{3}$$

where λ_b = base failure rate specified by the manufacturer
s = operating speed
s_d = design speed
L = operating load
L_d = design load
ν = viscosity of lubricant used
ν_s = viscosity of specification lubricant
c = concentration of contaminants
c_s = standard contamination level
T = operating temperature
T_s = specification temperature

In addition to the above, other factors that measure the effect of misalignment, vibration, and shock on the failure rate may be included.

8.3.3 Stress-Strength Analysis

When abnormal loads that may cause a failure are a possibility and a concern, then a probabilistic evaluation of the magnitude of the stress in comparison with the designed strength may be required. This method makes use of the techniques discussed in Chapter 7. Often this may be in response to a system safety analysis to ensure there are adequate safety margins designed into the system. There are four major categories of stress: electrical, thermal, mechanical, and chemical. Stresses can be environmental or operating; they include electrical loads, temperature, vibration,

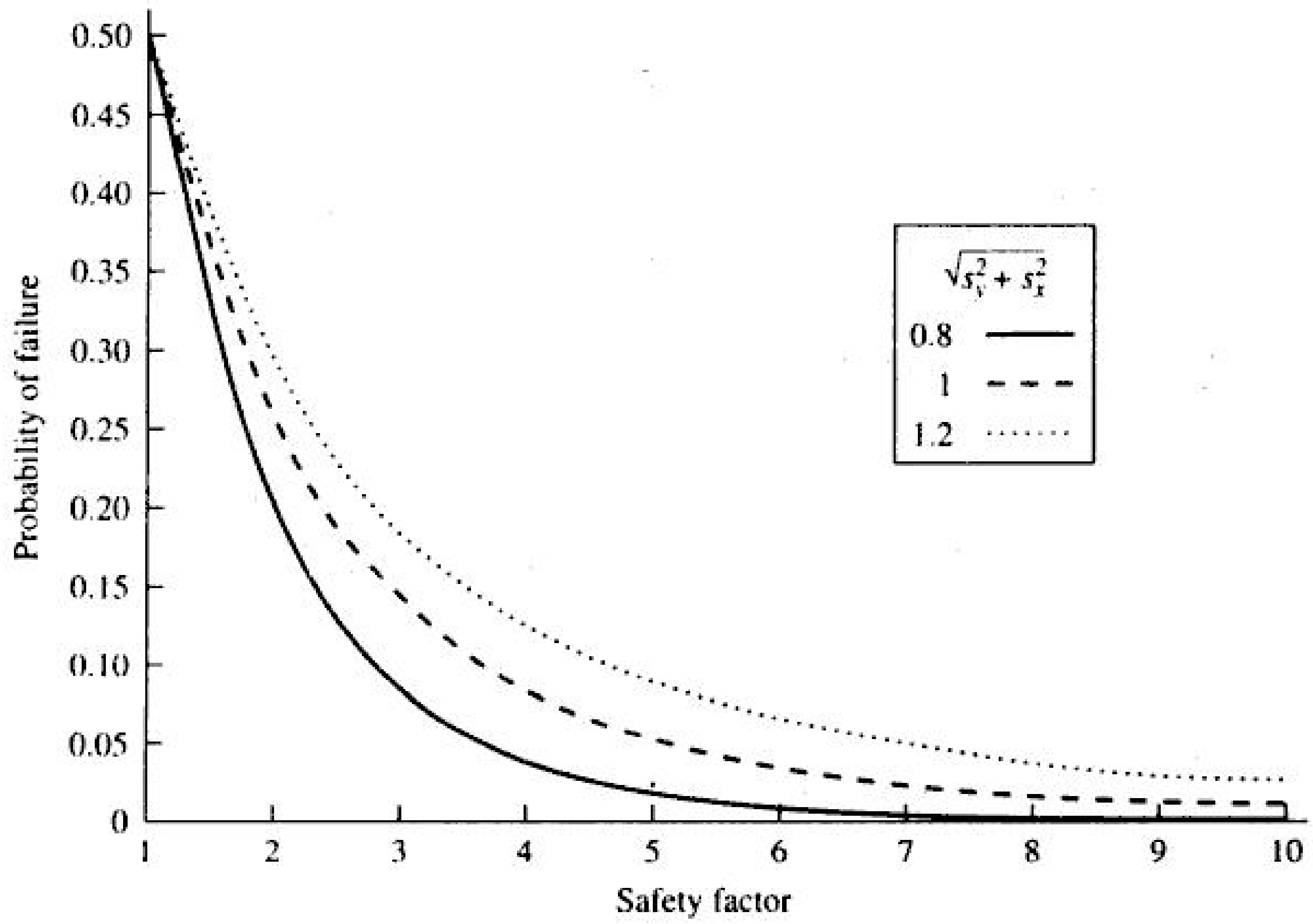

FIGURE 8.8
The probability of a failure versus the safety factor (lognormal distribution).

8.3.4 Complexity and Technology

The number of parts in a system is one measure of system complexity. Since most parts will be serially related with respect to the system reliability, each part is required to have a very high reliability. Any design alternative that can reduce the number of parts can lead to a significant improvement in reliability. As illustrated in Table 5.1, component reliabilities above 0.999 are necessary for systems having as few as 10 components in order to achieve a 0.99 system reliability. Component reliability increases to 0.9999 for a 100-component system. Part counts may be reduced by designing parts to serve more than one function or use. For example, a single switch may apply power to an entire system instead of a separate switch applying power to each powered component. It may also be possible to eliminate or reduce some function along with its components. Closely related to parts minimization is minimization of part variation. By using common parts and components, material quality and manufacturing tolerances can be better controlled.

When alternative technologies are available, the design engineer has additional flexibility in meeting the design objectives. Examples include electromechanical devices versus solid state devices, ink-jet versus laser versus impact printing, light-emitting diodes versus filament devices versus liquid crystals, discrete electronics components versus integrated circuits, and digital display versus analog display.

$R_i(t)$ = (known) reliability of component i at time t
n_i = number of parallel components i (decision variables)
c_i = unit cost of component i
B = budget available for additional units (redundancy)

The problem is to find values for n_i so that

$$\max \prod_{i=1}^{M} [1 - (1 - R_i(t))^{n_i}] \tag{8.19}$$

$$\text{subject to } \sum_{i=1}^{m} c_i n_i \leq B + \sum_{i=1}^{m} c_i$$

The summation on the right side of the inequality accounts for the sunk costs necessary to have at least one of each component in the design. Marginal analysis may be used to solve this problem if the (natural) logarithm of the reliability function is maximized rather than the function itself.[4] Therefore the objective function becomes

$$\max \sum_{i=1}^{m} \ln[1 - (1 - R_i(t))^{n_i}] \tag{8.20}$$

Eliminate the argument from $R_i(t)$, since the analysis is performed for a specified time t, and let

$$\Delta_i = \frac{\ln[1 - (1 - R_i)^{n_i+1}] - \ln[1 - (1 - R_i)^{n_i}]}{c_i}$$

Then the marginal analysis consists of the following steps:

1. Set $n_i = 1, i = 1, 2, \ldots, m$, and set cost = 0.
2. Compute Δ_i, $i = 1, 2, \ldots, m$.
3. Find $\max\{\Delta_1, \Delta_2, \ldots, \Delta_m\}$; call it Δ_k.
4. Set cost = cost + c_k.
5. If cost < B, then set $n_k = n_k + 1$, recompute Δ_k, and go to step 3; otherwise, stop.

The marginal values Δ_i represent the increase in the logarithm of the component reliability per dollar investment in the ith component. At each iteration the component with the largest (current) marginal value is selected for an additional redundant unit. The process is repeated until the budget target is met. Since the budget may not be precisely met by the last component to be added, the final iteration may require selecting an alternate component having a smaller unit cost. The component having the largest marginal value with a unit cost that will satisfy the budget is selected.

EXAMPLE 8.4 An engineer has an \$850 budget (per system) to be used to increase the reliability of a four-component series system. The reliability of each component at the

[4]Marginal analysis requires separability of the variables to ensure an optimal solution. If logarithms are used, the terms are additive rather than multiplicative. Since the logarithmic transformation is monotonically increasing, the optimal solution is not affected.

analysis (FMECA). This is an iterative process that influences design by identifying failure modes, assessing their probabilities of occurrence and their effects on the system, isolating their causes, and determining corrective action or preventive measures. It is often performed as a bottom-up analysis, though it may be applied at any level in which there is sufficient definition to provide the necessary data. Therefore, an FMECA program should be initiated early in the design phase. In the reliability design process, FMECA provides a design tool that measures progress toward the reliability goals and indicates areas for redesign. It is an inductive process in which individual failures are generalized into possible failure modes. Through early (in the design process) identification of significant failure modes, they can be eliminated or their probability reduced. Additional information on FMECA to that given below may be found in *Military Standards: Procedures for Performing a Failure Mode, Effects, and Criticality Analysis* (1980). Typical steps in conducting an FMECA include system definition, identification of failure modes, determination of cause, assessment of effect, classification of severity, estimation of probability of occurrence, computation of criticality index, and determination of corrective action. They will be discussed in what follows.

8.4.1 System Definition

The objective of this first step is to identify those system components that will be subject to failure. A functional and physical (hardware) description of the system provides the definitions and boundaries for performing the analysis. A functional description can be represented as a functional flow diagram consisting of blocks defining *what* is to be done along with the interfaces between the blocks. A functional analysis provides the initial description of the system without regard to *how* the system will operate and be maintained. It should be part of the preliminary system design. The reader interested in further detail on the general systems engineering process is referred to Blanchard and Fabrycky [1990]. The physical description of the system is represented by an indenture diagram showing subassemblies, components, and parts along with their hierarchical relationships. The level of detail available in defining the system will depend upon how early in the design phase the FMECA is initiated. As the system evolves from preliminary design to detailed design and development, more detailed functional analysis, schematics, and hardware specifications will be utilized. In order to define failures, acceptable performance specifications under expected operating and environmental conditions must be determined. Using the functional flow and indenture diagrams, a reliability block diagram will be constructed and used as the basis for performing the analysis. The reliability block diagram may be based upon the hardware configuration, the functional analysis, or a combination of the two. The hardware approach is usually a bottom-up analysis, whereas the functional approach requires a top-down analysis of the system.

8.4.2 Identification of Failure Modes

Failure modes will be identified either by component (hardware approach) or function. Through development and reliability testing and the analysis of the reliability

TABLE 8.3
Analysis of failure mode causes

Failure mode	Category	Cause	Failure mechanism	Possible corrective action
Capacitor short	Electrical	High voltage	Dielectric breakdown	Derating
Failure of metal contacts	Chemical	Humid and salty atmosphere	Corrosion	Use of a protective casing
Connector fractures	Mechanical	Excessive vibration	Fatigue	Redesign of mountings

The last three steps discussed—identification of failure modes, determination of cause, and assessment of effect—are related as illustrated in Table 8.4.

8.4.5 Classification of Severity

Various severity classifications may be used. A severity classification is assigned to each failure mode to be used as a basis for ranking corrective actions. One of the more common classifications places failures in one of the following four categories:

Category I: Catastrophic. Significant system failure occurs that can result in injury, loss of life, or major damage.
Category II: Critical. Complete loss of system occurs; performance is unacceptable.
Category III: Marginal. System is degraded, with partial loss in performance.
Category IV: Negligible. Minor failure occurs, with no effect on acceptable system performance.

TABLE 8.4
Failure mechanisms, modes, and effects

Failure mechanism*	Failure mode	Failure effect
Corrosion	Failure in tank wall seam	Tank rupture
Manufacturing defect in casing	Leaking battery	Failure of flashlight to light
Prolonged excessive vibration and fatigue	Break in a motor mount	Loss of engine power and excessive noise
Friction and excessive wear	Drive belt break	Shutdown of production line
Contamination (dust and dirt)	Loss of contact	Circuit board failure
Evaporation	Filament breaking	Light bulb burnout
Prolonged low temperatures	Brittle seals	Leakage in hydraulic system

*The failure mechanism causes the failure mode, which causes the failure effect.

8.4.6 Estimation of Probability of Occurrence

Initial probability estimates will be based on the reliability specification and allocation, experience (past history), existing databases, *Military Handbook: Reliability Prediction of Electronic Equipment* [1986] (for electronic parts), and comparability with components and parts having known reliabilities. As development progresses, functional and reliability testing will provide an alternative source for these probabilities. Reliability testing will be discussed at length in Chapters 13 and 14. The probability of occurrence will be based on the expected number of occurrences of each failure mode over a specific time interval. This interval may be a mission time, a scheduled maintenance interval, or the system design life. Using the reliability block diagram, one may group these probability estimates by component and roll them up to higher component levels, including the system level. This will then provide an assessment as to whether the design meets the reliability specifications.

When sufficient data does not exist for quantifying the probability of occurrence, *Military Standard: Procedures for Performing a Failure Mode, Effects, and Criticality Analysis* [1980] provides the following qualitative grouping of failure mode frequencies over the operating time interval:

Level A: Frequent. High probability of failure ($p \geq 0.20$)
Level B: Probable. Moderate probability of failure ($0.10 \leq p < 0.20$)
Level C: Occasional. Marginal probability of failure. ($0.01 \leq p < 0.10$)
Level D: Remote. Unlikely probability of failure ($0.001 \leq p < 0.01$)
Level E: Extremely unlikely. Rare event ($p < 0.001$)

8.4.7 Computation of Criticality Index

This is a quantitative measure of the criticality of the failure mode that combines the probability of the failure mode's occurrence with its severity ranking. For each severity classification, the criticality index is computed for each of the corresponding failure modes. The result is a rank ordering of failure modes within each severity classification. The index may be defined as follows:

$$C_k = \alpha_{kp}\beta_k\lambda_p t \tag{8.21}$$

where C_k = the critical index for failure mode k
α_{kp} = the fraction of the component p's failures having failure mode k (that is, the conditional probability of failure mode k given component p has failed)
β_k = the conditional probability that failure mode k will result in the identified failure effect
λ_p = the failure rate of component p
t = duration of time used in the analysis

The conditional probability, β_k, is a subjective estimate that may be quantified within the following guidelines [*Military Standards: Procedures for Performing a Failure Mode, Effects, and Criticality Analysis, 1980*]:

Failure effect	β
Certain	$\beta = 1.00$
Probable	$0.10 < \beta < 1.00$
Possible	$0 < \beta \leq 0.10$
No effect	$\beta = 0$

For a given p, the sum of α_{kp} over all its failure modes would normally equal 1. This probability may be derived from the estimated probability of occurrences. Component failure rates are obtained from existing data sources or aggregations from the failure mode probability of occurrences. In the later case, $\alpha_{kp}\lambda_p$ is the probability of occurrence of failure mode k and the distinction between the two probabilities is not necessary. The rationale for this distinction is to allow for the component failure rate, λ_p, to be estimated from other sources (such as *Military Handbook: Reliability Prediction of Electronic Equipment* [1986]).

Criticality numbers may be computed for each component by summing all of the component's failure mode criticality indices. A separate number is computed for each component and severity combination. A criticality matrix is then constructed, as shown in Fig. 8.10. The cells will contain the corresponding failure modes or

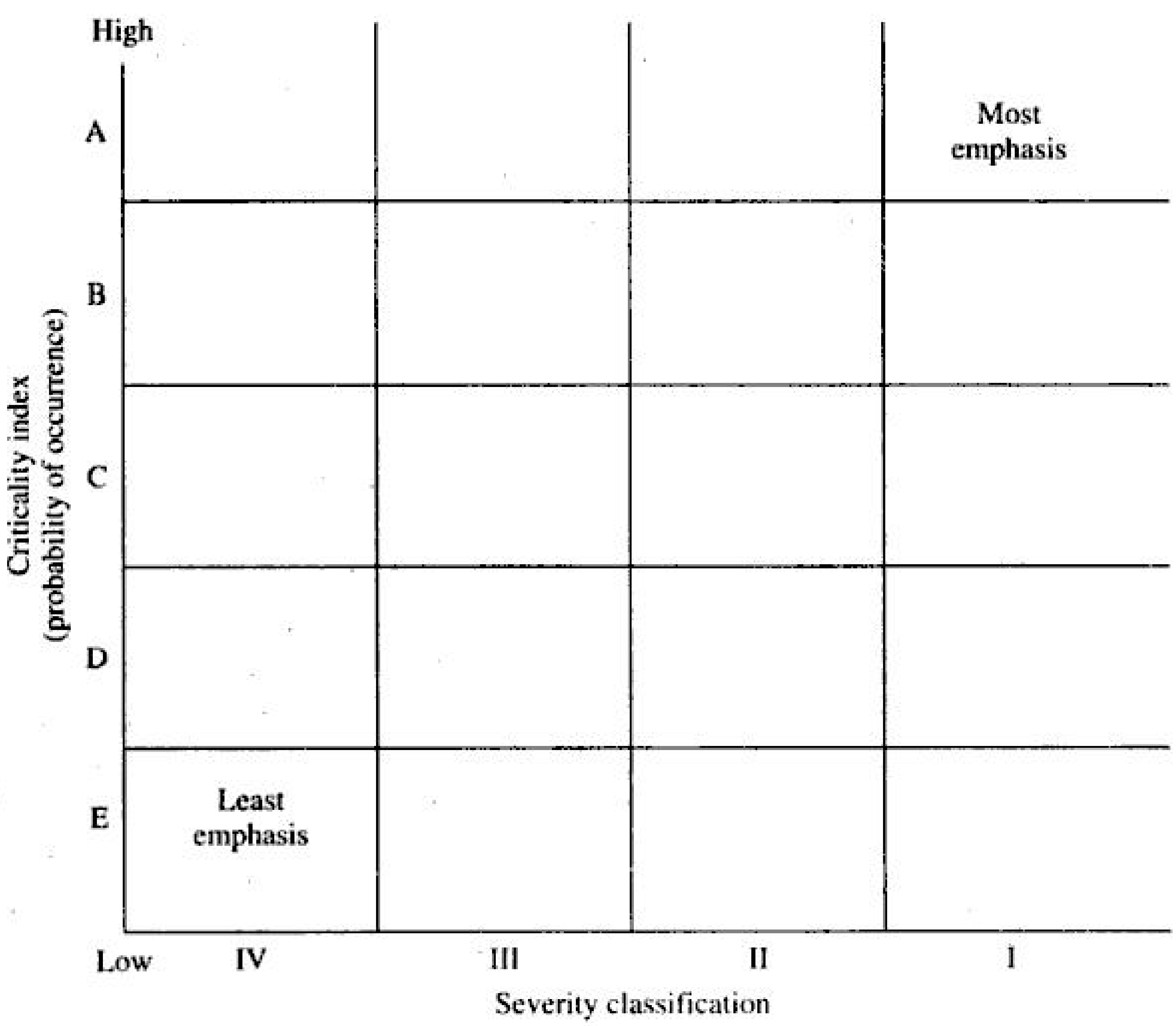

FIGURE 8.10
Failure mode classification matrix.

components. In a qualitative analysis the critical index on the vertical axis could be replaced with the qualitative assessment of the probability of occurrence. As we move from the lower left corner of the matrix to the upper right corner, the criticality and severity of the failure mode become greater. The matrix then provides a guide to be used during design for eliminating or mitigating failures.

8.4.8 Determination of Corrective Action

This is very dependent on the problem. Many times a solution will suggest itself as a result of a thorough analysis of the failure mode along with the identification of the cause and the failure mechanism. Obviously, those failure modes having a high criticality index and severity classification should receive the most attention. Design activity should be oriented toward removing the cause of the failure, decreasing the probability of occurrence, and reducing the severity of the failure.

The above analysis can be summarized in a worksheet, as shown in Table 8.5, in which each component has an operating time of 1000 hr in the computation of the criticality index. The worksheet is completed for all failure modes and all components, and a subtotal is computed for each combination of component and severity class.

An FMECA can be accomplished from several perspectives. The primary focus may be to improve reliability during design, but the technique may also be used in addressing system safety, availability, maintainability, or logistics support. Additional detail on FMECA can be found in the *Handbook of Reliability Engineering and Management* [Ireson and Coombs, 1988].

8.5 SYSTEM SAFETY AND FAULT TREE ANALYSIS

Reliability and product safety are obviously related. Safety can be broadly defined as the avoidance of conditions that can cause injury, loss of life, or severe damage to equipment and possibly the surrounding environment. Therefore the focus here is on failures that may create safety hazards. The objective is to determine during design how these failures are likely to occur, to estimate their probability of occurrence, and to take corrective action. Often safety-related failure modes have a low probability of occurrence and are therefore difficult to estimate. Reliability testing at the system level may fail to generate an unsafe condition. Additionally, because of designed safety features with backup or redundancy, a system safety failure is usually caused by a combination of events. For example, a combination of equipment failure, human error, and an alarm failure may be necessary before a boiler begins to overheat, causing pressure buildup.

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8.5.1 Fault Tree Analysis

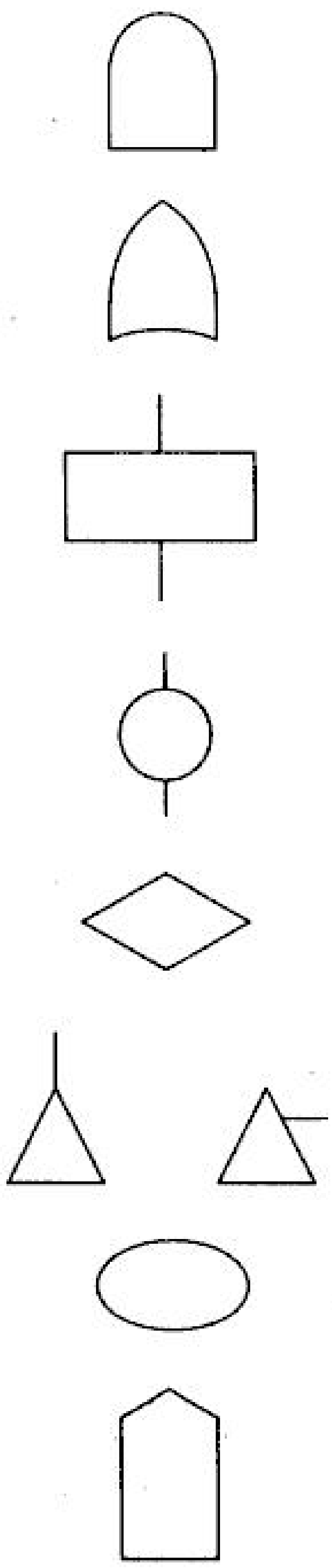
A useful tool in performing a system safety analysis is fault tree analysis. A fault tree analysis is a graphical design technique that provides an alternative to reliability block diagrams. It is broader in scope than a reliability block diagram and differs from reliability block diagrams in several respects. It is a top-down, deductive analysis structured in terms of events rather than components. The perspective is on faults rather than reliability. All failures are faults, but not all faults may be considered failures. For example, a human error resulting in an incorrect switch being set would be treated as a fault although it would not normally be an inherent equipment failure mode. An advantage of focusing on failures is that failures are usually easier to define than nonfailures and there may be far fewer ways in which a failure can occur, as opposed to the numerous ways in which nonfailures can occur. The focus is usually on a significant failure or a catastrophic event, which is referred to as the *top event* and appears at the top of the fault tree diagram. The qualitative analysis consists of identifying the various combinations of events that will cause the top event to occur. This may be followed by a quantitative analysis to estimate the probability of occurrence of the top event.

There are four major steps to a fault tree analysis:

1. Define the system, its boundaries, and the top event.
2. Construct the fault tree, which symbolically represents the system and its relevant events.
3. Perform a qualitative evaluation by identifying those combinations of events that will cause the top event.
4. Perform a quantitative evaluation by assigning failure probabilities or unavailabilities to the basic events and computing the probability of the top event.

Symbols frequently used in fault trees include those presented in Fig. 8.11. A typical fault tree has the structure shown in Fig. 8.12. In construction of a fault tree, the two logic gates, the OR gate and the AND gate, are used to relate the resultant, basic, and intermediate events, or faults, to the top event. Lower events are input to a gate, and a higher event is the gate's output. The type of gate determines whether all input events must occur for the output event to occur (AND gate) or whether only one of the input events must occur for the output event to occur (OR gate). The following example illustrates the use of the two gate types. For clarity, all fault trees will use the words AND and OR in place of the symbols shown in Fig. 8.11.

EXAMPLE 8.5 In Fig. 8.13(a) the OR gate indicates that a tank rupture is caused either by overpressure in a hot water tank or by an inherent fatigue failure in the wall of the tank. The fatigue failure is not developed further since it is represented as a basic event. On the other hand, overpressure is depicted as a resultant event. Overpressure is further developed in Fig. 8.13(b) through the use of the AND gate. If both excessive temperature and failure of a relief valve occur, then a tank rupture will result. In completion of the fault tree, both gates would be utilized as shown in Figure 8.13(c).



AND gate. A logic gate where an output event occurs only when all the input events have occurred. In Boolean algebra the output is the intersection (∩) of the input events.

OR gate. A logic gate where an output event occurs if at least one of the input events has occurred. In Boolean algebra the output is the union (∪) of the input events.

Resultant event. A fault event resulting from the logical combination of other fault events and usually an output to a logic gate.

Basic event. An independent elementary event representing a basic fault or component. The analysis ends with a basic event (there are no events below a basic event).

Incomplete event. An event that has not been fully developed because of lack of knowledge concerning its cause or because of its unimportance in the analysis.

Transfer-in and transfer-out. Used to link sections of the fault tree that are not contiguous or appear on separate pages.

Conditional event. A condition or restriction tied to a logic gate.

Normal event. A normally occurring event that is not a fault.

FIGURE 8.11
Fault tree symbols.

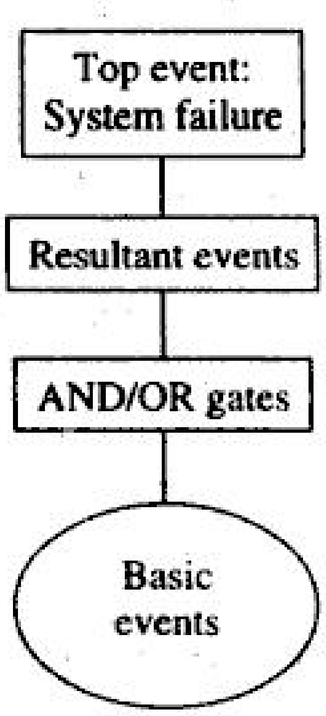


FIGURE 8.12
The general structure of a fault tree.

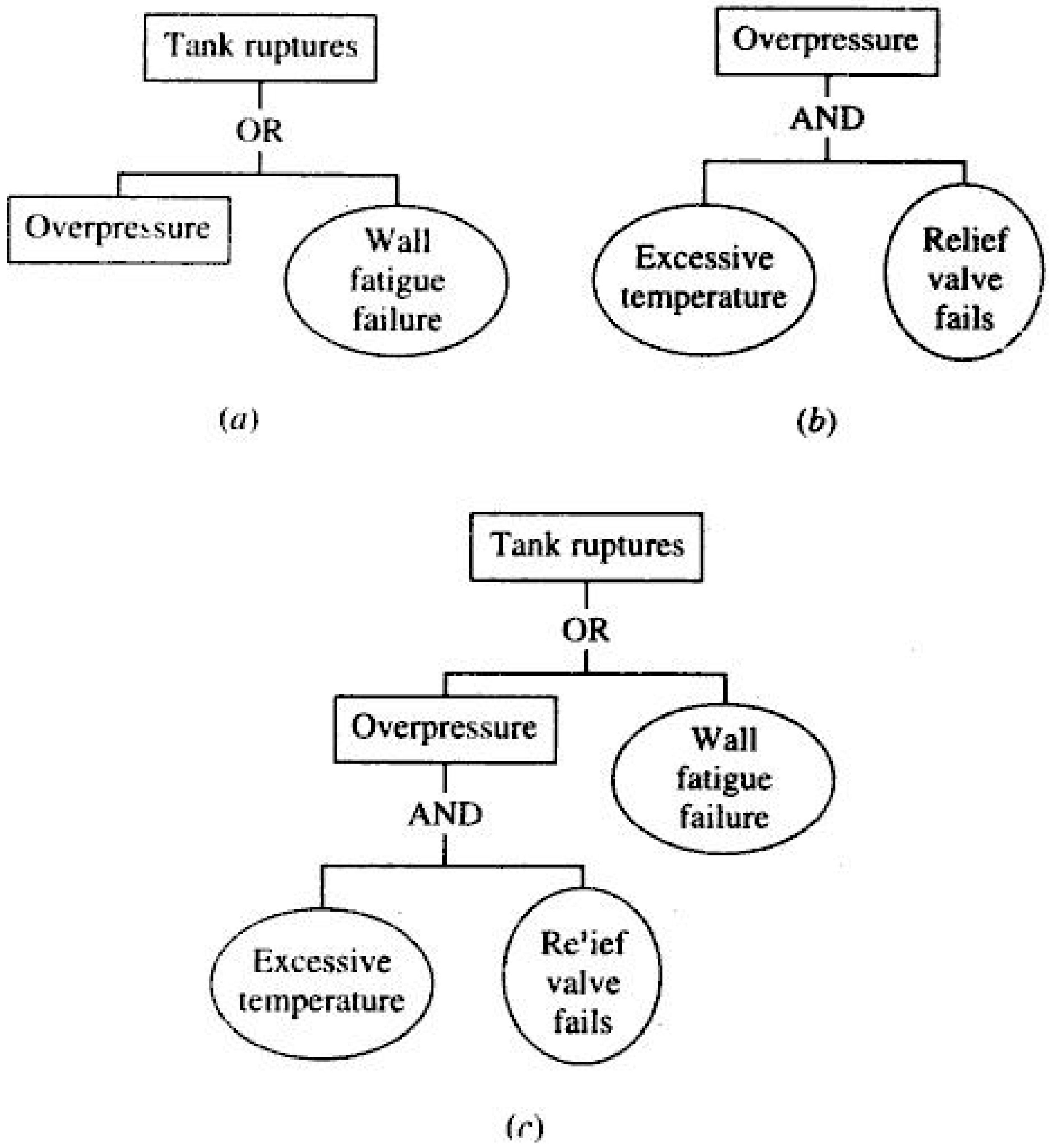


FIGURE 8.13
An example of the use of AND gates and OR gates.

Faults can be classified as primary, secondary, and command. A fault is primary if the component or part is functioning within its design parameters when an inherent failure occurs. A secondary failure occurs when an environmental stress or an excessive operational stress causes the failure (that is, it is not an inherent failure but an external event). A command fault is one that occurs as a result of a correct action being accomplished at a wrong time or place. For example, a command fault may occur when turning power on prematurely or turning off a cooling subsystem before the system has been shut down. Faults may also be classified as active or passive. Passive faults or events are typically related to a static transmitter of energy, material, loads, or signals such as pipes, bearings, structural beams, and wires. Active faults are related to dynamic events or components in operation, such as valves regulating flow, electrical switches, mechanical pumps, and relays or actuators. A component such as a valve may have a static failure mode in which a rupture occurs or an active failure mode in which the valve opens when it should remain closed. Active faults typically have probabilities of occurrence that are two to three times those of passive faults.

EXAMPLE 8.6 Construct a fault tree for the redundant manual and automated alarm system shown in Fig. 8.14.

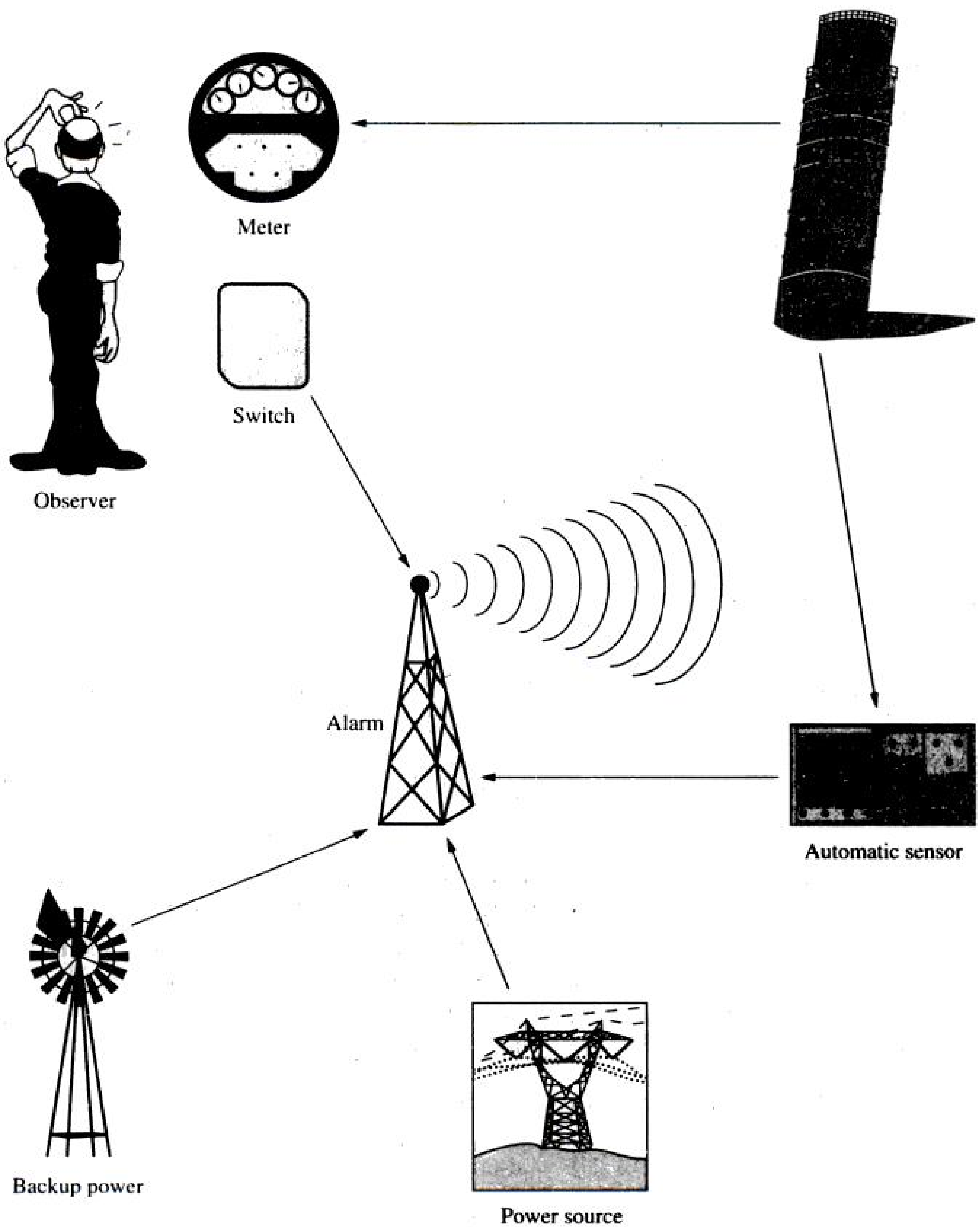


FIGURE 8.14
An example of an alarm system.

Solution. The fault tree may take the form shown in Fig. 8.15. From the fault tree, the top event, an alarm system failure, will occur if the alarm fails from either a basic (inherent) failure or a secondary failure or there is a power or sensor failure. The event D, secondary alarm failure, represents external failures to the alarm, perhaps resulting from a natural disaster. Both events D and I are diagrammed as incomplete events, presumably because

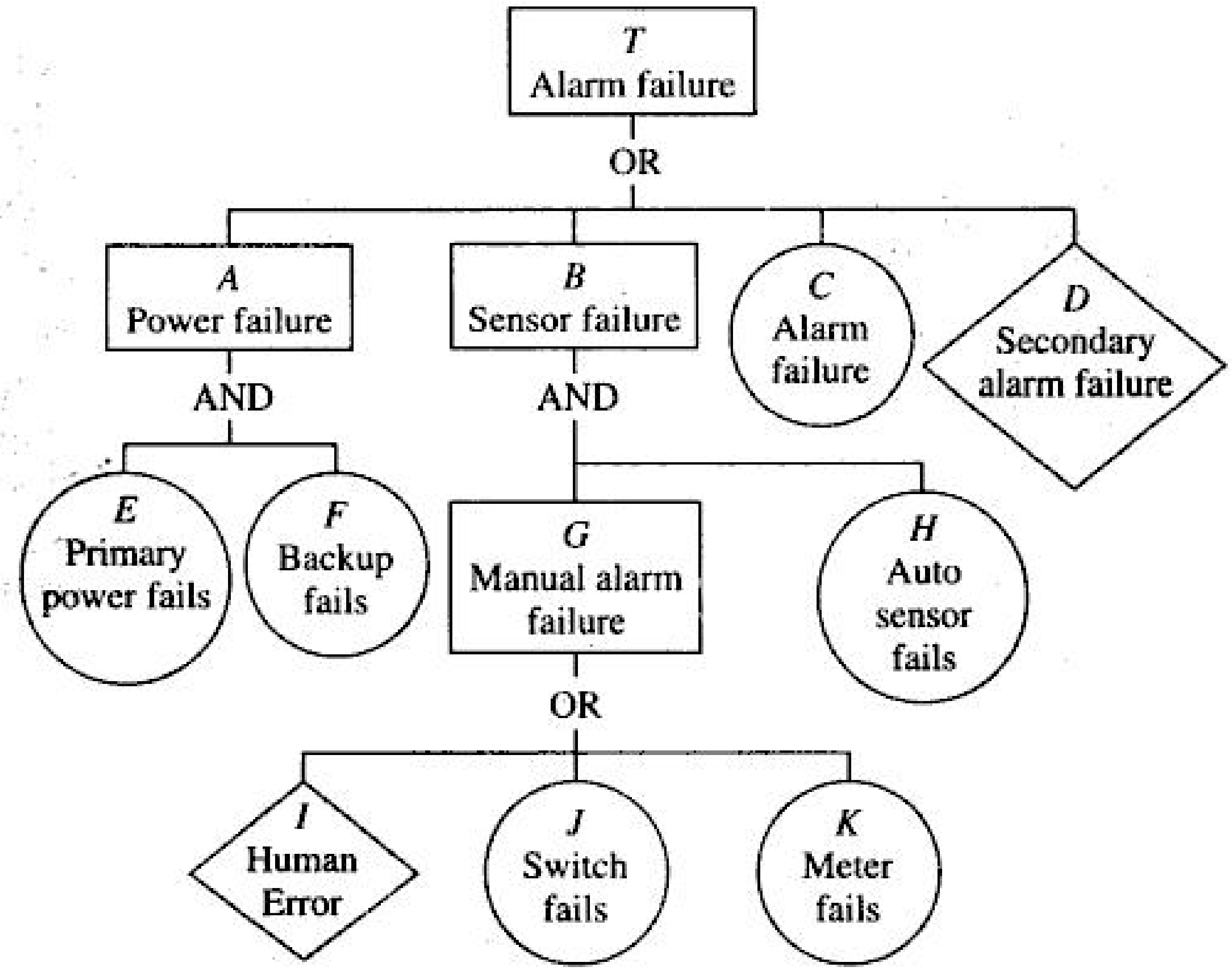


FIGURE 8.15
The alarm system fault tree.

there is incomplete data available to further define the events. The top event, T, can be written as

$$
\begin{aligned}
T &= A \cup B \cup C \cup D \\
&= (E \cap F) \cup (G \cap H) \cup C \cup D \\
&= (E \cap F) \cup [(I \cup J \cup K) \cap H] \cup C \cup D
\end{aligned}
$$

The objective of the Boolean algebra representation is to express the top event in terms of nonredundant basic events. Eliminating redundancies results in a simplified fault tree and should be accomplished prior to the quantitative analysis. In the above example there are no redundant (duplicate) events. Redundancy exists when the same event occurs more than once in the fault tree or when one event is a subset of another event. The following example illustrates the procedure.

EXAMPLE 8.7 For the fault tree of Fig. 8.16,

$$
\begin{aligned}
T = A \cup B &= A \cup (C \cap D) \\
&= A \cup [(E \cup F) \cap (E \cup A)] \\
&= A \cup [E \cup (F \cap A)] = A \cup E
\end{aligned}
$$

since $A \cup (F \cap A) = A$. Therefore the top event will occur if either the event A occurs or the event E occurs. At this point we can construct an equivalent fault tree, as shown in Fig. 8.17, that provides a much simpler analysis.

8.5.2 Minimal Cut Sets

Another form of qualitative analysis is that utilizing minimal cut sets. A cut set is a collection of basic events that will cause the top event. A minimal cut set is one

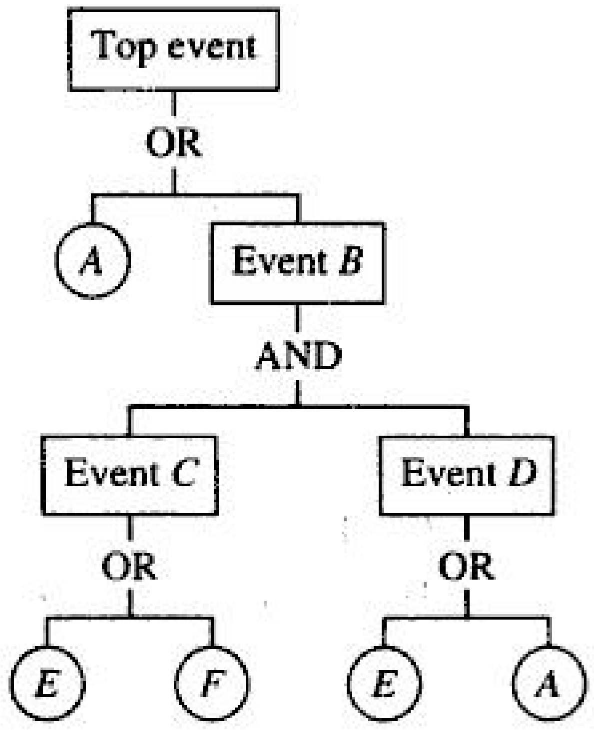


FIGURE 8.16
A fault tree with redundant events.

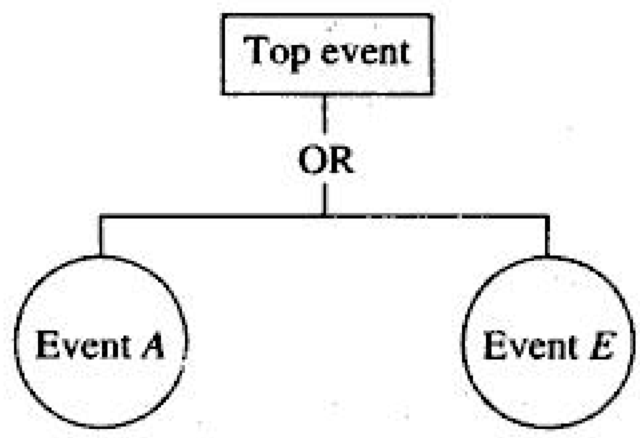


FIGURE 8.17
The equivalent fault tree.

with no unnecessary events. That is, all the events within the cut set must occur to cause the top event. Every fault tree has a finite number of minimal cut sets (since there are a finite number of events). A cut set can be characterized by the number of basic events comprising it. For example, a single event that will cause the top event is a singlet, and a two-event minimal cut set is a doublet. If M_i, $i = 1, 2, \ldots, k$, are all possible minimal cut sets, then

$$T = M_1 \cup M_2 \cup \cdots \cup M_k \tag{8.22}$$

where $M_i = E_1 \cap E_2 \cap \cdots \cap E_{ni}$ and E_i are basic events.

Minimal cut sets may be generated by a top-down expansion of events. Events input to OR gates are expanded by generating new rows (row-wise), and events input to AND gates are expanded by generating new columns (columnwise). This expansion continues until only basic events (or incomplete events) remain. At the completion, each row results in a cut set. Redundant cut sets, if present, may then be eliminated.

EXAMPLE 8.8 Consider the fault tree in Example 8.6 (Fig. 8.15):

Iteration		
1	**2**	**3**
A	E, F	E, F
B	G, H	I, H
C	C	J, H
D	D	K, H
		C
		D

Therefore $T = (E \cap F) \cup (I \cap H) \cup (J \cap H) \cup (K \cap H) \cup C \cup D$. An equivalent fault tree may be constructed using the minimal cut sets. The tree would have the general structure shown in Fig. 8.18.

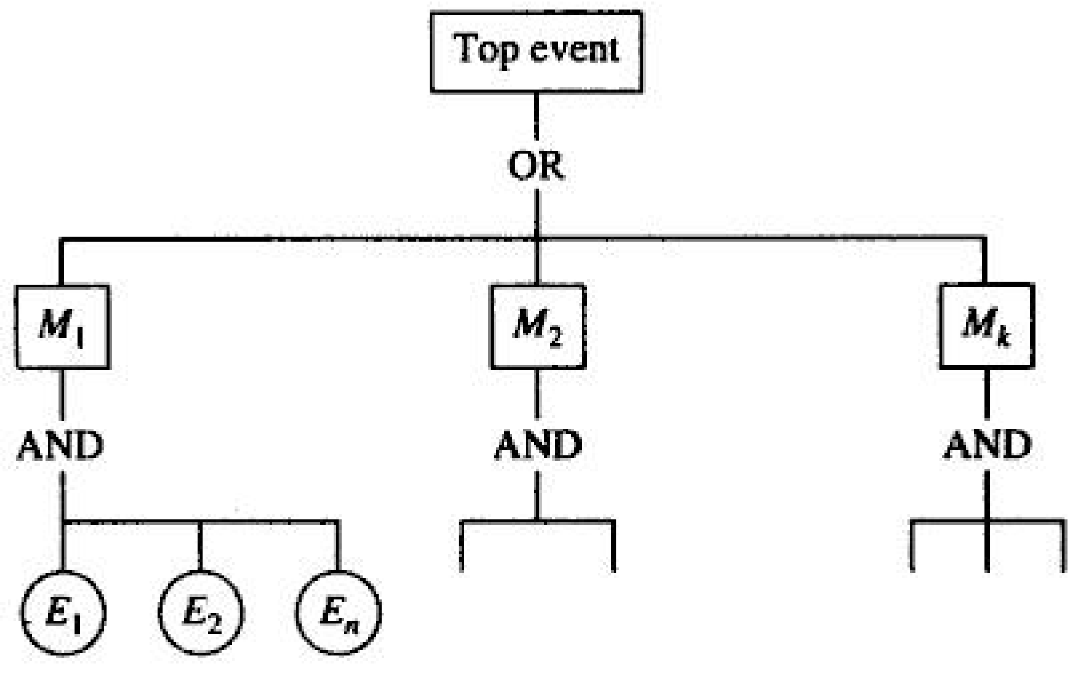


FIGURE 8.18
The minimal-cut-set fault tree.

EXAMPLE 8.9 For the fault tree shown in Fig. 8.16, the minimal cut set is found from the following:

Iteration				
1	2	3	4	5
A	A	A	A	A
B	C, D	E, D	E, E	E
		F, D	E, A	
			F, E	
			F, A	

Redundancies are eliminated in iteration 5 since $E \cap E = E, E \cap A \subseteq A, F \cap E \subseteq E$, and $F \cap A \subseteq A$. Therefore $T = A \cup E$, as found earlier when we applied Boolean algebra to eliminate repeated events.

In performing a qualitative evaluation of the minimal cut sets, one should place more importance on those sets having a single or double event since they are more likely to occur than cut sets having multiple events. For example, if a single event has a probability of occurrence on the order of 10^{-2}, then a double-event cut set could be expected to have a probability of occurrence on the order of 10^{-4}, and a triple-event cut set, a probability on the order of magnitude 10^{-6}. Therefore an objective would be to eliminate or minimize the failure probability of single-event cut sets. In general, ordering cut sets according to size will provide a qualitative measure of their importance. Events appearing in more than one cut set would also be candidates for elimination, especially if the cut sets were doublets or triplets. If a quantitative analysis is performed, those events in cut sets having the greatest probability of occurrence should receive the most attention. For simple systems such as those in the above examples, cut sets may be apparent from a system or fault tree diagram. However, for complex systems combinations of events leading to the top event are not as obvious. For such systems computer algorithms are available for performing this type of analysis.

8.5.3 Quantitative Analysis

Quantitative analysis consists of assigning failure probabilities, unavailabilities, failure-on-demand probabilities, or other measures to each basic event and then computing the corresponding measure for the top event. Obviously, this is easiest when the repeated events have been eliminated, since repeated events generate dependencies. If the top event has been defined in terms of the minimal cut sets (Eq. 8.22), then

$$P(T) = P(M_1 \cup M_2 \cup \cdots \cup M_k) = P(M_1) + P(M_2) + \cdots + P(M_k) \quad (8.23)$$

if the cut sets are mutually exclusive. Generally the events will not be mutually exclusive, and terms involving the probability of the intersection of two or more events must be included. Therefore, from Example 8.9,

$$P(T) = P(A \cup E) = P(A) + P(E) - P(A \cap E)$$

In general,

$$P(M_i) = P(E_1 \cap E_2 \cap \cdots \cap E_{ni}) = P(E_1)P(E_2) \cdots P(E_{n1}) \quad (8.24)$$

if the basic events are mutually independent. Therefore if the events A and E are independent,

$$P(T) = P(A) + P(E) - P(A)P(E)$$

and it is sufficient to estimate the failure probabilities of the basic events in order to find the probability of the top event. Equation 8.23 may be a useful approximation if the events are not mutually exclusive but their individual failure probabilities are quite small. This is because the remaining terms will consist (under independence) of products of the individual probabilities. If the probability of a basic event is on the order of 10^{-6}, the probability of the intersection of two or more events will be of order of magnitude 10^{-12} or smaller.

EXAMPLE 8.10 Using Eq. 8.23 as an approximation, we obtain the following for the fault tree in Example 8.6:

$$\begin{aligned} P(T) &= P\{(E \cap F) \cup [(I \cup J \cup K) \cap H] \cup C \cup D\} \\ &\approx P(E \cap F) + P[(I \cup J \cup K) \cap H] + P(C) + P(D) \\ &\approx P(E)P(F) + [P(I) + P(J) + P(K)]P(H) + P(C) + P(D) \end{aligned}$$

If each basic event has a probability of 0.01, then

$$P(T) \approx (0.01)^2 + (0.01 + 0.01 + 0.01)(0.01) + 0.01 + 0.01 = 0.0204$$

From both the qualitative and quantitative analysis, it is apparent that events C and D are the most important in causing an alarm system failure, and they should be considered for improved reliability. Although this conclusion may be obvious since the remainder of the basic events (sensor and power failures) have redundancy, for more complicated systems the insight provided by a detailed fault tree analysis can be considerable.

EXAMPLE 8.11 (EXAMPLE 5.7 REVISITED). An automobile braking system consists of a fluid braking subsystem (foot brake) and a mechanical braking subsystem (parking brake). Both subsystems must fail in order for the system to fail. The fluid braking sub system will fail if the master cylinder or a hydraulic line fails (event M) or all four wheel braking units fail. A wheel braking unit will fail if either the wheel cylinder fails (events WC_1, WC_2, WC_3, WC_4) or the brake pad assembly fails (events BP_1, BP_2, BP_3, BP_4). The mechanical braking system will fail if the cable system fails (event C) or both rear brake pad assemblies fail (events BP_3, BP_4).

The fault tree may be constructed as shown in Fig. 8.19. The top event may be expressed in Boolean algebra terms of the basic events as

$$T = \{M \cup [(WC_1 \cup BP_1) \cap (WC_2 \cup BP_2) \cap (WC_3 \cup BP_3) \cap (WC_4 \cup BP_4)]\} \cap \{C \cup (BP_3 \cap BP_4)\}$$

Minimal cut sets can be formed from $\{M, C\}$, $\{M, BP_3, BP_4\}$, {wheel subsystem failure, C}, {wheel subsystem failure, BP_3, BP_4}. Wheel subsystem failure can be decomposed into 16 combinations of wheel cylinder and brake-pad assembly failures. In the latter case, four of these decompositions include failure of both BP_3 and BP_4. Therefore, only 12 unique cut sets are formed. Assuming that each basic event has approximately the same probability of occurrence, the cut sets involving wheel subsystem failures are less likely since they contain four or five events. Therefore, the focus

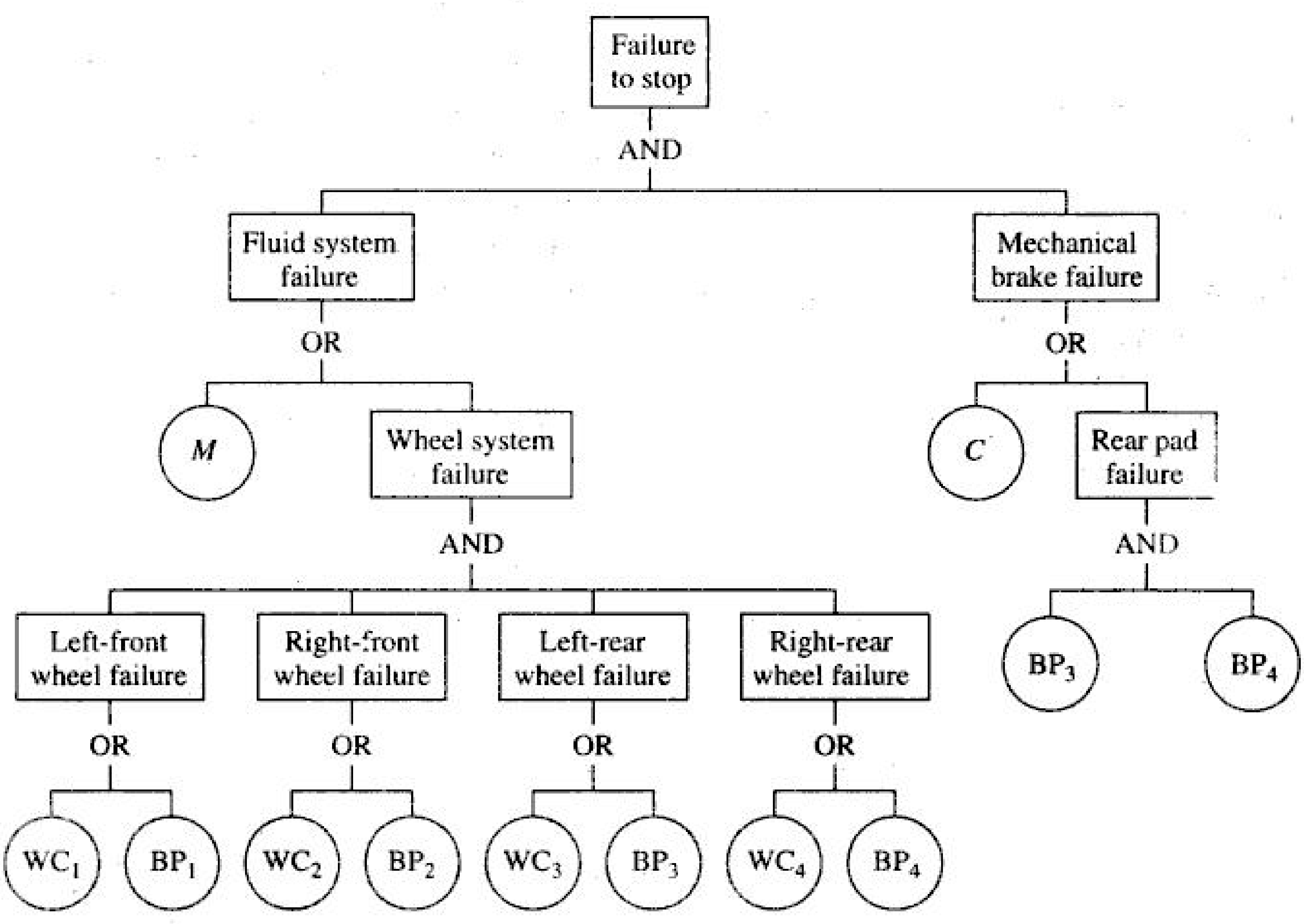


FIGURE 8.19
Fault tree for the automotive braking system.

of reliability improvement would be on the master cylinder subsystem and the cable subsystem.

The reader interested in more detail on fault tree analysis is referred to Dhillon and Singh [1981] or Roberts et al. [1981].

EXERCISES

8.1 Two alternative components are being considered for use in a fiber-optic cable communications system. One component has a unit cost of \$840 and a Weibull failure distribution with $\beta = 2$ and $\theta = 10{,}000$ hr. The other component has a unit cost of \$870 and an exponential failure distribution with an MTTF of 10,000 hr. It is estimated that the selected component will experience 2000 operating hours a year, and the system is being designed for a 20-yr life. Any component failures will require a replacement at the current unit cost. Assuming a 3 percent interest rate, which is the preferred component?

8.2 In the design of a space station, four major subsystems have been identified, each having a Weibull failure distribution with parameter values as given here:

Subsystem	Scale parameter, θ, yr	Shape parameter, β
Computer	3.5	0.91
Avionics	4.0	0.80
Structures	5.0	1.80
Life support	6.0	1.00

The reliability of the station must be 0.995 at the end of the first year. Determine the percentage increase in reliability for each of the major subsystems needed in order to reach the system goal. Assign equal reliability goals to all subsystems.

8.3 If redundancy is the only means of achieving further reliability growth for each of the subsystems in Exercise 8.2, what is the minimum number of redundant units of each necessary to achieve the component reliability goals?

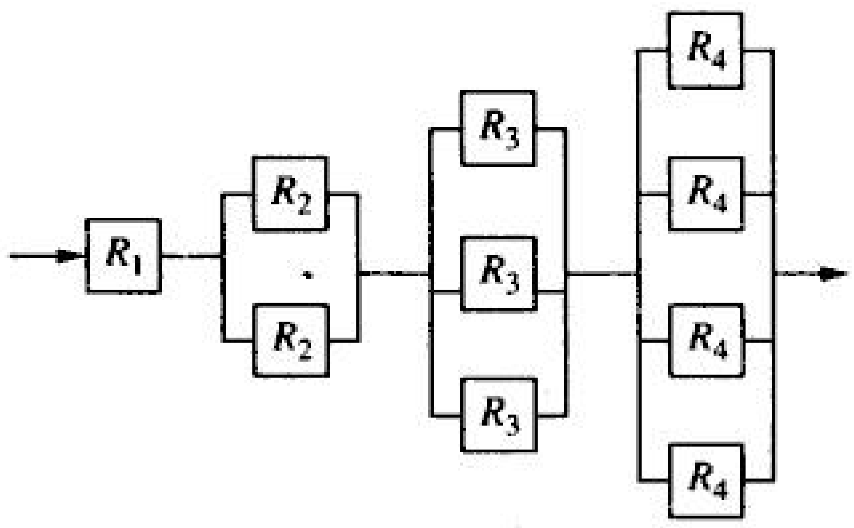
8.4 On the basis of the current average failure rate of each subsystem over the first year of the space station, apply the ARINC method to determine an average failure rate goal for each subsystem of Exercise 8.2. Assuming that reliability growth will improve only the characteristic life and not the shape parameter, what is the characteristic life goal for each subsystem?

8.5 Apply the AGREE allocation method to a personal computer system containing the following components:

Component	Parts count	Importance index	Operating time, hr/yr
System board	153	0.95	2000
Hard drive	28	0.90	1000
DC power pack	34	1.00	2000

The warranty program requires a reliability of 0.99 over the first year of use.

8.6 Allocate the system reliability goal of 0.95 to the components of the following reliability block diagram. Assume an equal allocation to each redundant subset. Each redundant subset contains identical components.



8.7 The following table (from *Military Handbook: Reliability Prediction of Electronic Equipment* [1986]) shows the effect of derating at various temperatures for a wire-wound resistor. The values are the number of failures per 10^6 hr.

Temperature, °C	Operating wattage/rated wattage					
	0.4	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
30	0.015	0.017	0.019	0.021	0.023	0.026
40	0.016	0.018	0.021	0.023	0.026	0.029
50	0.018	0.02	0.023	0.026	0.029	0.033
60	0.02	0.023	0.026	0.029	0.033	0.037
70	0.023	0.026	0.03	0.033	0.038	0.043

A system containing 73 resistors is being designed. With the selection of a system cooling fan, the operating temperature can be controlled:

Fan size	Unit cost	Operating temperature, °C
Small	\$ 50	60
Medium	90	50
Large	160	30

Operating power consumption is 180 watts. The following resistors may be used in the design of the system:

Resistor	Rating, W	Unit cost
1	200	$1.00
2	225	1.20
3	300	2.00

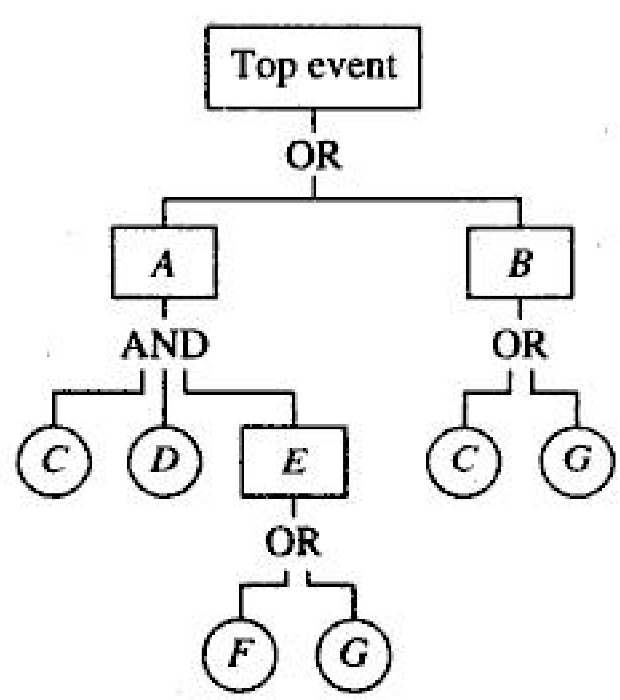
Select the fan size and resistor type that will provide the largest MTTF per dollar cost. The system of 73 resistors in series must have at least a 0.95 reliability after 10,000 operating hours. Assume constant failure rates.

8.8 Given a budget of $700 and the following data on three components that must operate in series, determine, using marginal analysis, the optimum number of redundant units. Compute the achieved reliability.

Component	Reliability	Unit cost
1	0.80	$200
2	0.90	100
3	0.95	75

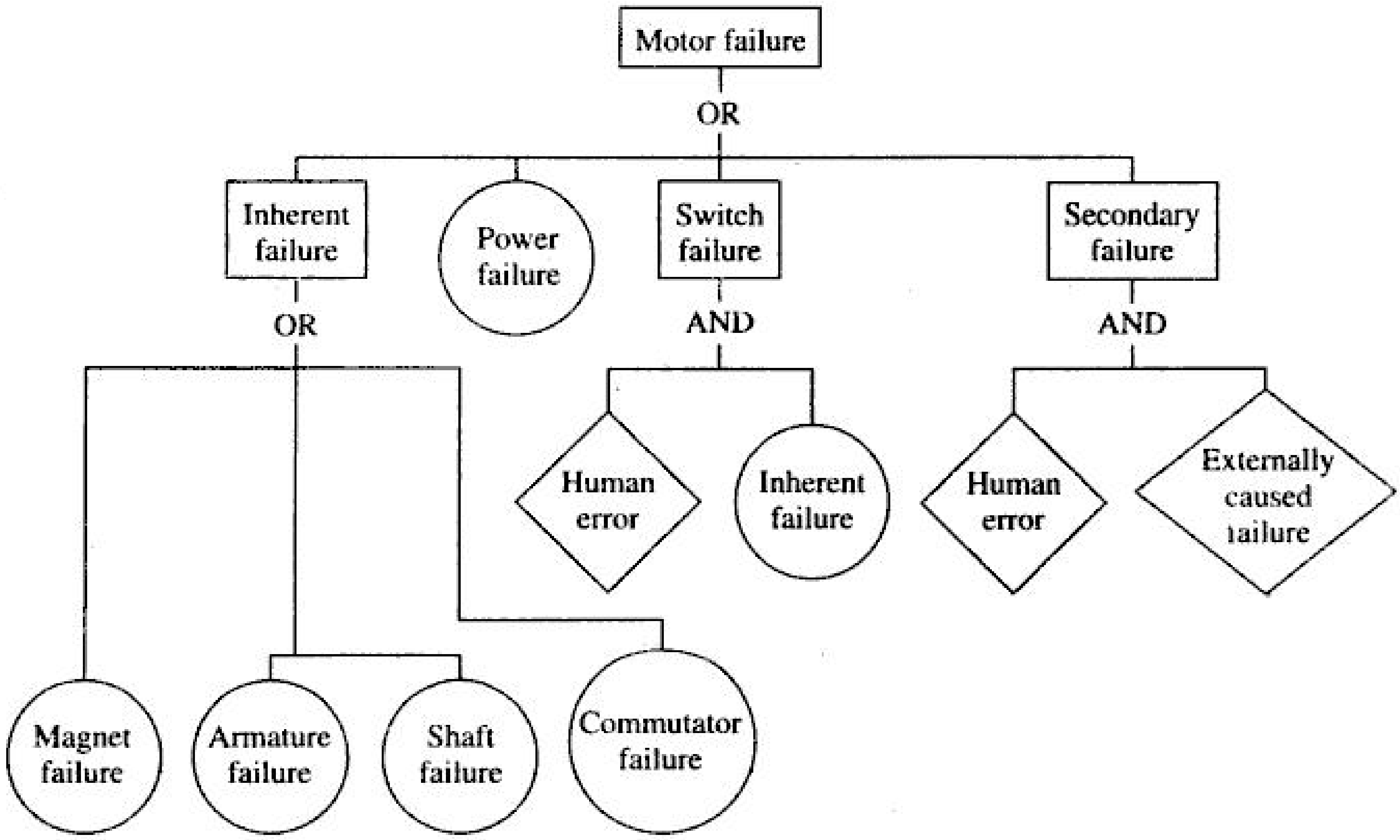
8.9 Establish a criticality index for the components in Exercise 8.5 assuming that the conditional probability β_k is equal to the importance index. Assume that failure modes equate to components.

8.10 Perform a qualitative analysis on the following fault tree by expressing the top event in terms of nonredundant basic events using Boolean algebra. If the probability of each basic event is 0.005 and events are independent, what is the probability of the top event?



8.11 A space probe to be launched by NASA will contain a module of three scientific experiments. Because of the high cost of the launch vehicle and the probe and the importance of the experiments, there will be redundant experimental packages within the module. Environmental stress testing has resulted in a (nonredundant) reliability estimate over the duration of the mission of 0.85, 0.90, and 0.95 for the respective experiments. There is a weight limitation of 350 pounds allocated to the module. How many redundant experiments of each type should be included in the module if their respective weights are 40, 30, and 50 pounds? Since a minimum of one unit of each experiment is required, 120 pounds of the 350 pounds allocated are already accounted for. The objective is to maximize the overall reliability of the module.

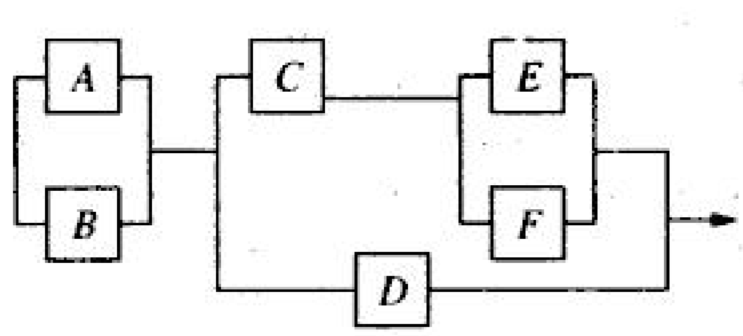
8.12 Consider the following fault tree.



(a) Express the failure of the top event in Boolean algebra terms consisting of basic events and incomplete events.
(b) Find the minimal cut set.
(c) Estimate the probability of the top event given that the probability of each basic event is 0.01 and that of each incomplete event is 0.02.

8.13 Construct a fault tree of a gas water heating system such that the top event is a safety-related failure.

8.14 Construct a fault tree from the following reliability block diagram such that the top event is a system failure and component failures are basic events. Express the top event in Boolean algebra terms of the basic events. If Pr{A} = Pr{B} = 0.9, Pr{C} = Pr{D} = 0.8, and Pr{E} = Pr{F} = 0.75, compute the probability of the top event. Compare your answer to that of Exercise 5.8(a).



8.15 Nether Fales, a reliability engineer, must decide which of two components to use in the design of a new product. According to the supplier, component A has a unit cost of \$225 and a Weibull failure distribution with a shape parameter of 1.7 and a characteristic life of 12 yr. Component B has a unit cost of \$245 and a constant failure rate of 0.11 failure per year according to the manufacturer's specifications. A failure of either component results primarily in the replacement of a comparable (same age) part. The average part cost of component A is \$40, and that of component B is \$35. In addition, if component A is selected, a special test unit must be purchased at a cost of \$4300 to be used in identifying the failed part. The product will have a 10-yr design life, at the end of which component A has a salvage value of \$60 and component B has a salvage value of \$40. Operating costs and other support costs are the same for either component. If 100 units of the product are to be manufactured, determine which component to use in the design of the product by comparing the life cycle costs. Assume a 5 percent effective discount rate.

8.16 Assuming that the load applied to a part is fixed at some value K and the design strength of the material used in the part has a Weibull distribution with a shape parameter $\beta = 0.8$, determine the probability of the part failing if a safety factor of 1.2 is used for establishing the part's strength. The safety factor is defined to be $SF = \mu/K$, where μ is the mean strength of the material used in manufacturing the part. Repeat your analysis for safety factors of 2 and 4. What conclusion would you make concerning the deterministic approach to using safety factors?

8.17 Fatigue testing of a steel alloy resulted in the following S-N curve:

$$N = (1.23 \times 10^{28})S^{-14.85}$$

where N is in 1000 cycles to failure and S is the stress amplitude in 1000 psi. The steel alloy is to be used in the design of an automobile engine that under normal use will experience 20 cycles per second under a stress amplitude of 35,000 psi. It is estimated that the typical driver will operate the vehicle 350 hr/yr. If the engine is being designed for a 10-yr life, is the selected material adequate?

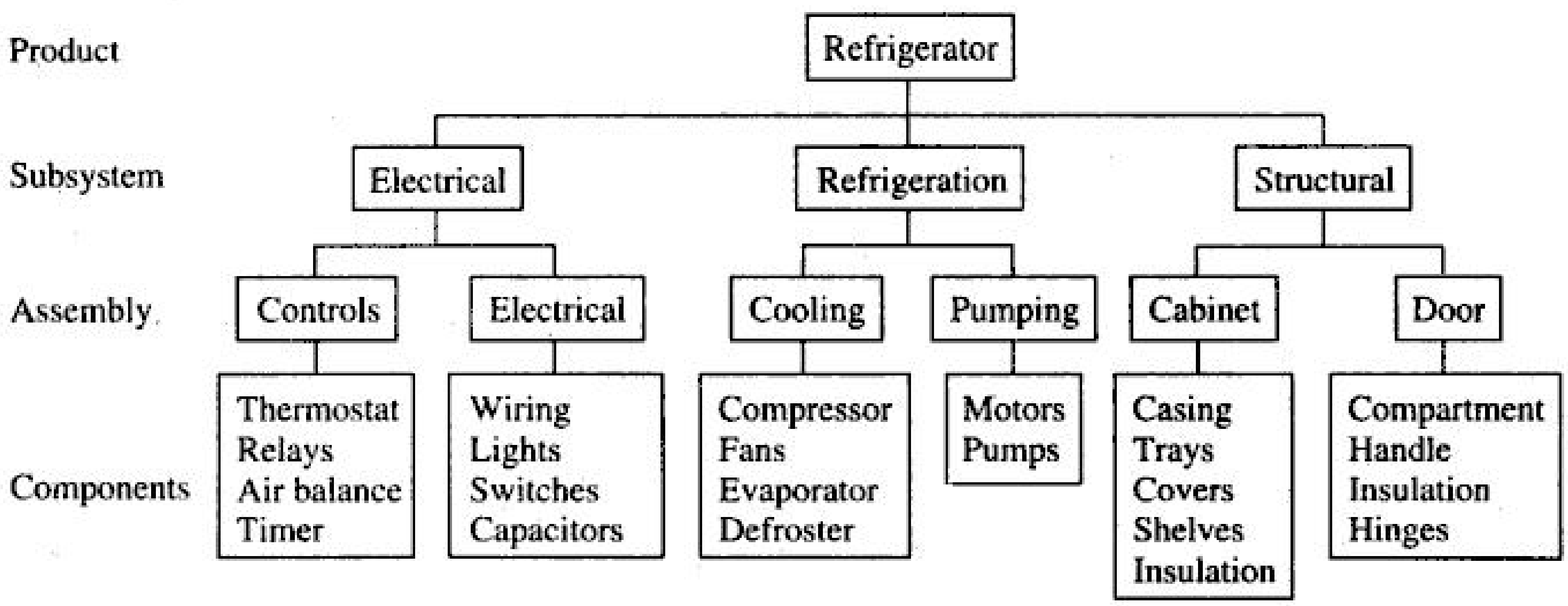
8.18 In an optimal reliability allocation based on Eqs. 8.9 and 8.10 where $C_i(x_i) = cx_i$, show that the solution is $x_i = \sqrt[n]{R^*} - R_i$.

8.19 A system is comprised of three components in series each having a Weibull failure distribution with the parameter as follows:

Component	Scale parameter, θ	Shape parameter, β
1	7,000	1.7
2	10,000	1.0
3	12,000	2.0

The cost of improving the current reliability, $C_i(x_i)$, is assumed to linear and equal for each of the components within the current (somewhat narrow) design range (see Exercise 8.18). The system reliability goal is 0.99 at 1000 hours of operation. Determine the percentage improvement required for each component in order to achieve the system goal.

8.20 The following is a hierarchical breakdown of a new refrigerator model. Discuss a scheme for establishing reliability goals at each indenture level assuming that the reliability is known for some components that are common to other products and models while other components are new with no previous failure history.



CHAPTER 9

Maintainability

The objective of this chapter is to characterize and quantify the repair or restoration of a failed item. We can distinguish between two general types of maintenance: reactive and proactive maintenance. Reactive maintenance is performed in response to unplanned or unscheduled downtime of the unit, usually as a result of a failure, whether it be internal (inherent) or external (for example, operator-induced). Proactive maintenance may be either preventive maintenance or predictive maintenance. *Preventive maintenance* is scheduled downtime, usually periodical, in which a well-defined set of tasks, such as inspection and repair, replacement, cleaning, lubrication, adjustment, and alignment, are performed. *Predictive maintenance* estimates, through diagnostic tools and measurements, when a part is near failure and should be repaired or replaced, thereby eliminating a presumably more costly unscheduled maintenance action. Proactive maintenance should be performed only when and to the extent it is cost-effective. It must reduce the number of unscheduled failures or extend the life of the item or both. It is generally assumed that a proactive maintenance action is less costly than a reactive maintenance action.

9.1 ANALYSIS OF DOWNTIME

When an item fails, it enters the repair process. The repair process itself can be decomposed into a number of different subtasks and delay times, as shown in Fig. 9.1.

Supply delay consists of the total delay time in obtaining necessary spare parts or components in order to complete the repair process. This time may consist of administrative lead times, production or procurement lead times, repair of the failed subcomponents themselves, and transportation times. To a large extent this time is influenced by the breadth and depth of the selection of spare parts and components

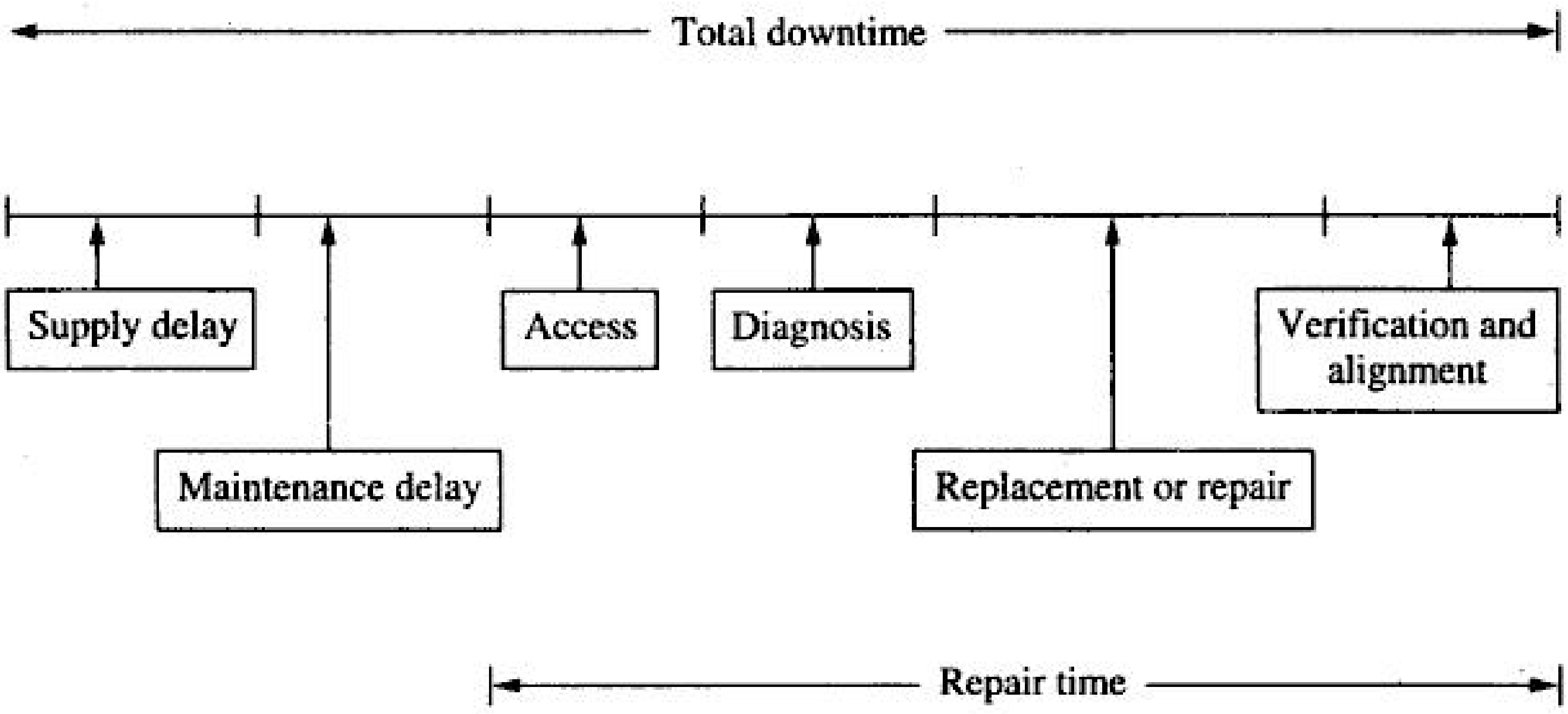


FIGURE 9.1
Maintenance downtime.

available at the repair facility. *Breadth* refers to the range of different components and parts that are stocked; *depth* refers to the number of spares of a given component or part. The supply delay time will not necessarily occur at the beginning of the repair cycle, but may occur after the diagnosis subtask has identified the failed component to be replaced. However, it will be advantageous to keep this time separate from the other categories. Obviously, the supply delay time will be zero if the needed replacement part is immediately available.

Maintenance delay time is the time spent waiting for maintenance resources or facilities. It may also include administrative (notification) time and travel time. Resources may be personnel, test equipment, support equipment, tools, and manuals or other technical data. Facilities may be a repair dock such as an aircraft hangar, a service bay in an automotive repair shop, or a fixed test stand. Maintenance delay time is influenced by the number of assigned parallel repair channels. A repair channel is defined as all the necessary maintenance resources and facilities needed to initiate and complete the repair process (other than spare components and parts). If a repair channel is immediately available on failure of the item, the maintenance delay time is zero.

Because supply delay times and maintenance delay times are influenced by external parameters (such as resource levels) that are not part of the system itself, they are not considered part of the inherent repair time of the item. The inherent repair time of the item is defined to be the sum of the durations of the following subtasks: access, diagnosis, repair or replacement, validation, and alignment. Access time is the amount of time required to gain access to the failed component. It may, for example, require removal of panels or covers. Diagnosis, or troubleshoot, time is the amount of time required to determine the cause of the failure. It is also referred to as fault isolation time. The repair time or replacement time includes only the actual hands-on time to complete the restoration process once the problem has been identified and access to the failed component obtained. Any delay in waiting for spares, additional personnel, test equipment, and so on, is either supply or maintenance delay time and is not considered part of the task of replacement or repair. Following restoration, some failures may require validating the restoration or alignment check

$$\Pr\{T \le t\} = H(t) = \int_0^t h(t')\,dt' \tag{9.1}$$

Equation (9.1) is the probability that a repair will be accomplished within time t. The mean time to repair may be found from

$$\text{MTTR} = \int_0^\infty t\,h(t)\,dt = \int_0^\infty (1 - H(t))\,dt \tag{9.2}$$

and the variance of the repair distribution is found from

$$\sigma^2 = \int_0^\infty (t - \text{MTTR})^2 h(t)\,dt \tag{9.3}$$

EXAMPLE 9.1. The time to repair a failed widget has the following probability density function:

$$h(t) = 0.08333t \qquad 1 \le t \le 5\text{ hr}$$

where $$H(t) = \int_1^t 0.08333t'\,d't = 0.041665t^2 - 0.041665$$

The probability or completing a repair, in less than 3 hr, for example, is

$$H(3) = 0.041665 \times 9 - 0.041665 = 0.333$$

The mean time to repair is

$$\text{MTTR} = \int_1^5 0.08333t^2\,dt = \left.\frac{0.08333t^3}{3}\right|_1^5 = 3.44\text{ hr}$$

9.2.1 Exponential Repair Times

If the repair distribution is exponential, then

$$H(t) = \int_0^t \frac{e^{-t'/\text{MTTR}}}{\text{MTTR}}\,dt' = 1 - e^{-t/\text{MTTR}} \tag{9.4}$$

where the parameter of the distribution is the MTTR. For this distribution, $r = 1/\text{MTTR}$ is the rate of repair (number of repairs per unit of time). The repair rate is constant only for the exponential distribution.

EXAMPLE 9.2. A component can be repaired at the constant rate of 10 per 8-hr day. What is the probability of a single repair exceeding 1 hr?

Solution. MTTR = 0.1 day = 0.8 hr. Therefore

$$\Pr\{T > 1\} = 1 - H(1) = e^{-1/0.8} = e^{-1.25} = 0.2865$$

9.2.2 Lognormal Repair Times

The lognormal distribution is often used to represent the repair distribution. For the lognormal distribution,

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has been found to be Weibull with $\beta = 0.5$ and $\theta = 1.5$ hr. The decreasing failure rate is consistent with the observation that if the line is to fail, it is likely to fail soon after startup. The longer the line is operating, the more likely it will continue to operate without an operational failure occurring. Assuming that restoration of the line results in a renewal process, the probability of the kth failure occurring by time t is approximated by

$$\Pr\{T_k \le t\} \approx \Phi\left(\frac{t-3}{6.71\sqrt{k}}\right)$$

where the MTTF = 3 hr with $\sigma = 6.71$.

EXAMPLE 9.5. A cutting tool has a time-to-failure distribution that is normal with a mean of 5 operating hours and a standard deviation of 1 hr. Nine replacement tools are available with which to complete a production run requiring 40 hr of operation. Determine the probability (reliability) of completing the production run with the available tools.

Solution

$$\Pr\{T_9 \ge 40\} = 1 - \Phi\left(\frac{40-45}{\sqrt{9}}\right) = 1 - \Phi(-1.67) = 0.95254$$

A stochastic point process can equivalently be defined by the number of failures in the interval $(0, t)$. Let $N(t)$ be the discrete random variable representing the cumulative number of failures in the interval $(0, t)$. Then

$$\begin{aligned}\Pr\{N(t) = 0\} &= \Pr\{T_1 > t\}\\ \Pr\{N(t) = j\} &= \Pr\{T_j \le t < T_{j+1}\}\\ &= \Pr\{T_{j+1} \ge t\} - \Pr(T_j > t\} \qquad \text{for } j = 1, 2, \ldots\end{aligned} \tag{9.9}$$

The second line of Eq. (9.9) holds since the event that exactly j failures occur by time t is equivalent to failure j occurring by time t and failure $j + 1$ occurring after time t.

EXAMPLE 9.5. (CONTINUED). The probability distribution for the number of failures during the first 12 hr of operation may be found as follows:

$$\Pr\{N(12) = 0\} = \Pr\{T_1 > 12\} = 1 - \Phi\left(\frac{12-5}{1}\right) = 1 - \Phi(-7) = 0$$

$$\Pr\{N(12) = 1\} = \Pr\{T_2 \ge 12\} - \Pr\{T_1 > 12\} = 1 - \Phi\left(\frac{12-10}{\sqrt{2}}\right) - 0$$

$$= 1 - \Phi(1.414) = 0.07927$$

$$\Pr\{N(12) = 2\} = \Pr\{T_3 \ge 12\} - \Pr\{T_2 > 12\}$$

$$= 1 - \Phi\left(\frac{12-15}{\sqrt{3}}\right) - \left[1 - \Phi\left(\frac{12-10}{\sqrt{2}}\right)\right]$$

$$= -\Phi(-1.732) + \Phi(1.414) = 0.87891$$

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$$\text{MTBF}_{\text{aircraft}} = \left(\sum_{i=1}^{n} \frac{1}{\text{MTBF}_i}\right)^{-1} \quad \text{where MTBF}_i \text{ refers to component } i.$$

9.3.2 Minimal Repair Process

It is frequently the case that repair consists of replacing or restoring only a small percentage of the parts or components composing the system. This will leave the system in approximately the same state (age) it was in just prior to the failure. Therefore, as a result of minimal repair, the times between failures may no longer be independent and identically distributed. The system may continue to deteriorate over time, and therefore successive values of the X_i will be correlated and will display a trend. A useful and somewhat natural way to model this situation is to treat it also as a stochastic point process. To model this point process, we define an intensity function, $\rho(t)$, as the rate of change of the expected number of failures with respect to time, or

$$\rho(t) = \frac{dE[N(t)]}{dt} \tag{9.14}$$

A natural estimate for $\rho(t)$ is

$$\rho(t) \approx \frac{N(t + \Delta t) - N(t)}{\Delta t} \tag{9.15}$$

The intensity function may also be referred to as the renewal rate, failure intensity, peril rate, or the rate of occurrence of failure (ROCOF). It should not be confused with the hazard rate function, $\lambda(t)$, since $\lambda(t)\,\Delta t$ is the conditional probability of a failure in time Δt given that the unit has survived to time t, whereas $\rho(t)\,\Delta t$ is the unconditional probability of a failure in time Δt. The hazard rate function is a relative rate pertaining only to the first failure, whereas the intensity function is an absolute rate of failure for repairable systems. Ascher and Feingold [1984] provide additional discussion concerning the hazard rate and the intensity function and the confusion generated by the similar terminology. From the intensity function the expected number of failures is

$$E[N(t)] = \int_0^t \rho(t')\,dt' \tag{9.16}$$

An instantaneous MTBF is defined by

$$\text{MTBF} = \frac{1}{\rho(t)} \tag{9.17}$$

and an interval MTBF is given by

$$\text{MTBF}(t_1, t_2) = \frac{t_2 - t_1}{m(t_1, t_2)} \tag{9.18}$$

where $m(t_1, t_2) = E[N(t_2) - N(t_1)] = \int_{t_1}^{t_2} \rho(t)\,dt$ = the expected number of failures in the interval (t_1, t_2).

EXAMPLE 9.8. A manufacturing machine has an intensity function given by $\rho(t) = e^{-6.5+0.0002t}$ with t measured in operating hours. After 1 yr of operation (3000 hr), $\rho(3000) = 0.0027394$ and the instantaneous MTBF is $1/\rho(3000) = 365$ hr. The expected number of failures over the second year is

$$m(3000, 6000) = \int_{3000}^{6000} e^{-6.5+0.0002t}\, dt = 11.26$$

Nonhomogeneous Poisson process

A somewhat simple model of a stochastic point process is the *nonhomogeneous Poisson process,* in which the probability distribution of the number of failures in the interval (t_1, t_2) is given by

$$\Pr\{N(t_2) - N(t_1) = j\} = \frac{m(t_1, t_2)^j e^{-m(t_1,t_2)}}{j!} \tag{9.19}$$

where $N(0) = 0$. The reliability at time t with respect to the first failure is equivalent to the probability of no failures in time t, or

$$R(t) = \Pr\{N(t) = 0\} = e^{-m(0,t)} \tag{9.20}$$

The reliability with respect to the next failure can be found given that the system is restored at time T,

$$R(t \mid T) = \Pr\{N(T + t) - N(T) = 0\} = e^{-m(T+t,T)}$$

Observe that $m(t + T, T)$ is the expected number of failures occurring from time T to $T + t$. Note that the homogeneous Poisson process is the special case of the nonhomogeneous Poisson process in which $\rho(t) = \lambda$, a constant, since then $m(0, t) = \lambda t$. If several independent nonhomogeneous Poisson processes are superimposed (that is, if one forms the sequence of renewals from the union of all the nonhomogeneous Poisson process renewals), the intensity function of the superimposition is $\rho(t) = \sum \rho_i(t)$.

A common form for the intensity function is

$$\rho(t) = a\, b\, t^{b-1} \qquad a, b > 0 \tag{9.21}$$

which is called the power law process or the Weibull process.[3] The latter name is the result of the functional form of Eq. (9.21) being identical to the Weibull hazard rate function. However, except for the time to the first failure, the times between failures do not have Weibull distributions. For the intensity function given in Eq. (9.21), if $b < 1$, the system is improving over time, as experienced during reliability growth testing (see Section 14.4). If $b > 1$, the system is deteriorating over time, as might be observed under minimal system repair. Estimation techniques for determining the parameters a and b are presented in Chapter 14 in the context of the U.S. Army Material Systems Analysis reliability growth model. A statistical test for trend and a goodness-of-fit test for the power-law process are provided in Chapter 16. Bain and Engelhardt[1991] provide details concerning other inferential statistics, including

[3] Another common form is $\rho(t) = e^{(a+bt)}$ (see Crowder et al. [1991]).

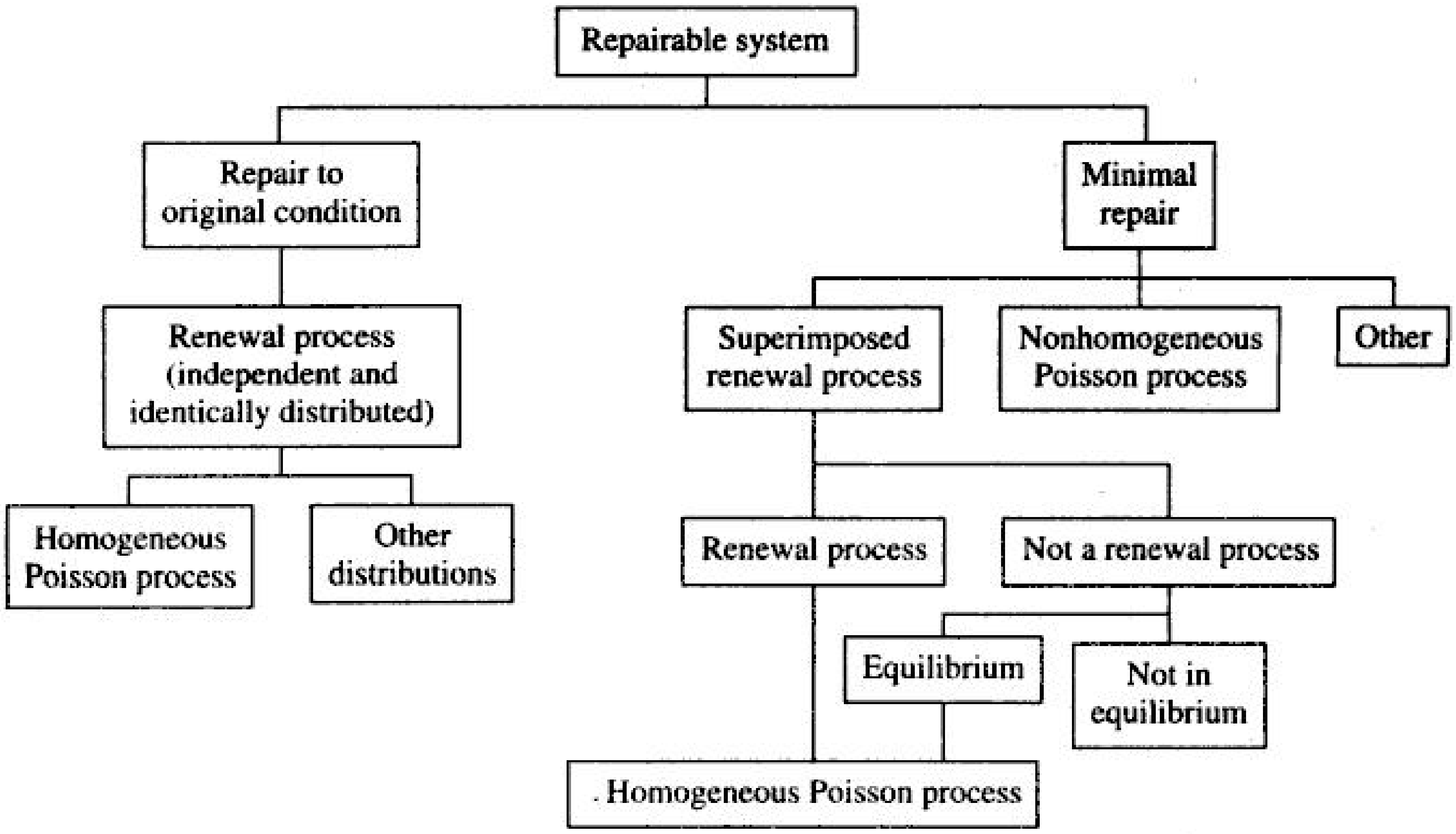

FIGURE 9.2
Stochastic point processes in modeling a repairable system.

estimation and hypothesis testing for the power-law process. Figure 9.2 summarizes the various conditions presented.

EXAMPLE 9.9. A six-year-old regional transit bus experiences minimal repair upon failure. It was found to have an intensity function given by $\rho(t) = 0.0464t^{2.1}$ with t measured in years. Then MTBF (instantaneous) $= 1/[(0.0464)(6)^{2.1}] = 0.5$ yr, and $m(6,7) = \int_6^7 0.0464t^{2.1}\,dt = 2.35$ is the expected number of failures over the coming year. Therefore, $R(1 \mid 6) = \Pr\{\text{No failure occurs in the seventh year}\} = \Pr\{N(7) - N(6) = 0\} = e^{-2.35} = 0.095$, and $\Pr\{\text{Exactly one failure occurs in the seventh year}\} = \Pr\{N(7) - N(6) = 1\} = 2.35e^{-2.35} = 0.224$. The reliability function can be written as $R(t) = \exp[-\int_0^t 0.0464y^{2.1}\,dy] = e^{-0.0149677t^{3.1}}$. This is a Weibull distribution with $\beta = 3.1$ and $\theta = 3.878$ hr. Of course, $R(t)$ represents the reliability relative to the first failure only.

The reader desiring additional information concerning the nonhomogeneous Poisson process should see Ascher and Feingold [1984], Crowder et al. [1991], or Lawless [1982]. As discussed in the first reference, sample values of the times between failures should be tested for trends prior to assuming a renewal process. If trends are present, the nonhomogeneous Poisson process will be a more appropriate model to consider.

9.3.3 Overhaul and Cycle Time

A particular type cf renewal process is observed if a system undergoes a complete overhaul either every T_0 time periods or on failure, whichever occurs first. Assume that the overhaul restores to the system to as good as new condition. Then letting T_{ov}

be the random variable representing the time between overhauls, and letting $R(t)$ and $f(t)$ be the system reliability and failure density function, respectively, the mean time between overhauls can be found from

$$E(T_{ov}) = T_0 R(T_0) + \int_0^{T_0} tf(t)\,dt = \int_0^{T_0} R(t)\,dt \tag{9.22}$$

Equation (9.22) shows that the mean time to the next overhaul will be T_0 if no failure occurs before T_0 (a probability of $R(T_0)$), or equal to the partial expectation of the failure distribution from 0 to T_0. The second equality in Eq. (9.22) is obtained from setting $\int_0^{T_0} tf(t)\,dt = -tR(t)\,|_0^{T_0} + \int_0^{T_0} R(t)\,dt$ on the basis of integration by parts. If the failure distribution is exponential, then

$$E(T_{ov}) = \int_0^{T_0} e^{-\lambda t} = \frac{1}{\lambda}(1 - e^{-\lambda T_0})$$

EXAMPLE 9.10. An aircraft engine is scheduled for a complete overhaul every 10,000 flying (operating) hours or on a failure requiring removal of the engine from the aircraft. Assuming a constant failure rate of 10^{-5} failure per flying hour for engine-removal failure modes, the mean time between overhauls will be

$$E(T_{ov}) = 10^5(1 - e^{-0.1}) = 9516 \text{ flying hours}$$

Cycle time

In the discussion on renewals and point processes, the repair time was considered to be negligible. In this discussion the repair time is explicitly included. If X_i is the random variable representing the time to the ith failure (following restoration) and S_i is the random variable representing the repair time of the ith failure, then $Y_i = X_i + S_i$ is the length of the ith renewal cycle. The actual time of the ith renewal, T_i, can then be found from

$$T_i = T_{i-1} + Y_i \qquad i = 1, 2, \ldots$$

with $T_0 = 0$. Y_i is a random variable with cumulative distribution function, $G(t) = \Pr\{Y_i \leq t\}$ and probability density function $g(t)$. If X_i has probability density function $f(x)$, S_i has probability density function $h(s)$, and S_i and X_i are independent, then

$$g(t) = \int_0^t f(x)h(t - x)\,dx$$

For most distributions this convolution must be solved numerically. However, if both the failure and the repair distribution are exponential, a closed-form solution can be obtained.

EXAMPLE 9.11. Let $f(t) = \lambda e^{-\lambda t}$ be the probability density function of the failure distribution and $h(t) = re^{-rt}$ be the probability density function of the repair distribution. Then with $\lambda \neq r$,

$$g(t) = \lambda r \int_0^t e^{-\lambda x - r(t-x)}\,dx = \lambda r \left[\frac{e^{-\lambda x - r(t-x)}}{-\lambda + r} \right]_0^t = \frac{\lambda r}{r - \lambda}\left[e^{-\lambda t} - e^{-rt} \right]$$

and $$G(t) = 1 - \frac{re^{-\lambda t} - \lambda e^{-rt}}{r - \lambda}$$

Although both the failure time and the repair time are exponential, the cycle time is not.

When repair time is a significant part of the cycle time, the MTBF in Eq. (9.11) and (9.12) must be replaced by MTBF + MTTR, the expected cycle length.

EXAMPLE 9.12. A system comprises four identical modules each having a time-to-failure distribution that is Weibull with $\beta = 1.4$ and $\theta = 200$ hr. Repair time is lognormal with $t_{\text{med}} = 2.5$ hr and $s = 0.87$. Assuming that a failure results in the failed unit's replacement with an identical unit (renewal process) and assuming that an equilibrium is achieved over a 5-yr period, find the expected number of repairs (failures).

Solution

$$\text{MTBF} = 200\Gamma\left(1 + \frac{1}{1.4}\right) = 182.1 \text{ hr} \qquad \text{and} \qquad \text{MTTR} = 2.5e^{0.37845} = 3.65 \text{ hr}$$

Therefore, the number of failures is $f = 4(365)(24)(5)/(182.1 + 3.65) = 943.2$ and the system MTBF is $(365)(24)(5)/943.2 = 46.4$ hr.

9.4 SYSTEM REPAIR TIME

We will frequently want to express system repair time as a function of the repair times of the components. To do this, we will compute an average (mean) system repair time from knowledge of the mean subsystem or component repair times. For example, the mean time to repair an aircraft depends on the repair distribution of each of the subsystems, such as the electrical, hydraulic, and environmental subsystems. The system MTTR may be computed as a weighted average of the subsystem MTTRs in which the weights are based on the relative number of failures. Let MTTR_i be the mean time to repair the ith unique subsystem, f_i be the expected number of failures of the ith unique subsystem over the system design life, and q_i be the number of identical subsystems of type i. Then the system mean time to repair is

$$\text{MTTR} = \frac{\sum_{i=1}^{n} q_i f_i \,\text{MTTR}_i}{\sum_{i=1}^{n} q_i f_i} \tag{9.23}$$

The expected number of failures of the ith subsystem can be computed from

$$f_i = \begin{cases} \dfrac{t_{oi}}{\text{MTTF}_i} & \text{for renewal process} \\ \int_0^{t_{oi}} \rho_i(t)\,dt & \text{for minimal repair} \end{cases}$$

where t_{0i} is the total number of operating hours of the ith component over the system design life. If all of the components have constant failure rates and the same number of operating hours, f_i can be replaced by λ_i. Identical subsystems may be serially related or active redundant. From Eq. (9.22) the role reliability plays in directly influencing repair times can be seen. Obviously, if a component with a high MTTR also generates a high number of failures, it will affect the system MTTR much more than a component with a high MTTR and a low number of failures. From a design trade-off perspective, high reliability should be a design objective for those components having long repair times, and short repair times should be the design objective of those components having high failure rates.

EXAMPLE 9.13. A radio consists of the following three subsystems:

Subsystem	λ_i	MTTR$_i$, hr
Power supply	0.00045	2.3
Amplifier	0.00130	3.7
Tuner	0.00007	4.6
Total	0.00182	

Then

$$\text{MTTR}_s = \frac{0.00045(2.3) + 0.00130(3.7) + 0.00007(4.6)}{0.00182} = 3.388 \text{ hr}$$

Systems having redundant components

When k out of n active redundant and identical components are present, repair may occur when any one of the n components fails or when $n - k + 1$ components fail (a system failure). If a failure is repaired when it occurs, the component mean repair time is MTTR. In the second case, however, the system may be restored when one is repaired or when all $n - k + 1$ are repaired. If restoration occurs when any one of the redundant components is repaired and only one component can be repaired at a time, MTTR is the system mean repair time. However, if all failures can be repaired at the same time, then under the assumption of a constant repair rate (exponential repair time), the system repair rate is $(n - k + 1)$ / MTTR and the system mean time to repair is MTTR / $(n - k + 1)$. On the other hand, if all units must be repaired before the system is restored and only one can be repaired at a time, the mean time to repair is $(n - k + 1)$MTTR. If all failed units can be repaired simultaneously (each having an exponential distribution) and all must be repaired, the repair time is given by

$$\text{MTTR} \sum_{i=1}^{n-k+1} \frac{1}{i} \tag{9.24}$$

This last expression holds because the repair rate is initially $(n - k + 1)$ / MTTR for the first repair, then it is $(n - k)$ / MTTR for the second repair, $(n - k - 1)$/ MTTR for the third, and so on, and finally 1/MTTR for the last. The sum of the reciprocals of these terms is the desired solution.

TABLE 9.1
System repair time

	Repair one at a time	Repair simultaneously
Restore when one is repaired	MTTR	MTTR/2
Restore when both are repaired	2 MTTR	1.5 MTTR

EXAMPLE 9.14. For a 2-out-of-3 system with each component having a constant repair rate equal to 1/MTTR, the system will fail when two units have failed. Assuming that repair begins when both units have failed, Table 9.1 provides the system mean repair time for each of the four possible cases.

9.5 RELIABILITY UNDER PREVENTIVE MAINTENANCE

For complex systems increased reliability can often be achieved through a preventive maintenance program. Such a program can reduce the effect of aging or wearout and have a significant impact on the life of the system. The following reliability model assumes that a system is restored to its original condition following preventive maintenance. Let $R(t)$ be the system reliability without maintenance, T be the interval of time between preventive maintenance, and $R_m(t)$ be the reliability of the system with preventive maintenance. Then

$$R_m(t) = R(t) \qquad \text{for } 0 \le t < T$$

and

$$R_m(t) = R(T)R(t-T) \qquad \text{for } T \le t < 2T$$

where $R(T)$ is the probability of survival until the first preventive maintenance and $R(t-T)$ is the probability of surviving the additional time $t-T$ given that the system was restored to its original condition at time T. Continuing, in general we have

$$R_m(t) = R(T)^n R(t-nT) \qquad \begin{array}{l} nT \le t < (n+1)T \\ n = 0, 1, 2, \ldots \end{array} \tag{9.25}$$

where $R(t)^n$ is the probability of surviving n maintenance intervals and $R(t-nT)$ is the probability of surviving $t - nt$ time units past the last preventive maintenance. The MTTF under preventive maintenance may be found using Eq. (2.8). The details of this derivation are found in Appendix 9A. The result is

$$\text{MTTF} = \int_0^\infty R_m(t)\,dt = \frac{\int_0^T R(t)\,dt}{1 - R(T)} \tag{9.26}$$

EXAMPLE 9.15. For the constant failure rate model,

$$R(t) = e^{-\lambda t}$$

and

$$R_m(t) = (e^{-\lambda T})^n e^{-\lambda(t-nT)}$$
$$= e^{-\lambda nT} e^{-\lambda t} e^{\lambda nT} = e^{-\lambda t} = R(t)$$

This again reflects the memorylessness of the exponential distribution. Under a constant failure rate, preventive maintenance has no effect!

EXAMPLE 9.16. For the Weibull failure distribution,

$$R_m(t) = \exp\left[-n\left(\frac{T}{\theta}\right)^\beta\right]\exp\left[-\left(\frac{t-nT}{\theta}\right)^\beta\right] \qquad nT \le t \le (n+1)T$$

A compressor has a Weibull failure process with $\beta = 2$ and $\theta = 100$ days. If we assume a 20-day preventive maintenance program ($T = 20$), then

$$R_m(t) = \exp\left[-n\left(\frac{20}{100}\right)^2\right]\exp\left[-\left(\frac{t-20n}{100}\right)^2\right] \qquad 20n \le t \le 20(n+1)$$

In Figure 9.3 are plots of $R(t)$ (no PM) and $R_m(t)$ (cumulative PM) showing the improvement in reliability over time as a result of preventive maintenance. The curve at the top (PM) shows preventive maintenance restoring the system to as good as new at the end of each preventive maintenance cycle, $R(t - nT)$, but does not take into account the cumulative effect over time, that is, $R(T)^n$. The reliability function, $R_m(t)$, must be a monotonically decreasing function. On the other hand, the conditional reliability $R_m(t \mid nT)$ equals $R(t - nT)$ for $nT \le t < (n+1)T$. The reliability for 90 days is found by first observing that $n = 4$. Then

$$R_m(90) = \exp\left[-4\left(\frac{20}{100}\right)^2\right]\exp\left[-\left(\frac{90-80}{100}\right)^2\right] = 0.8437$$

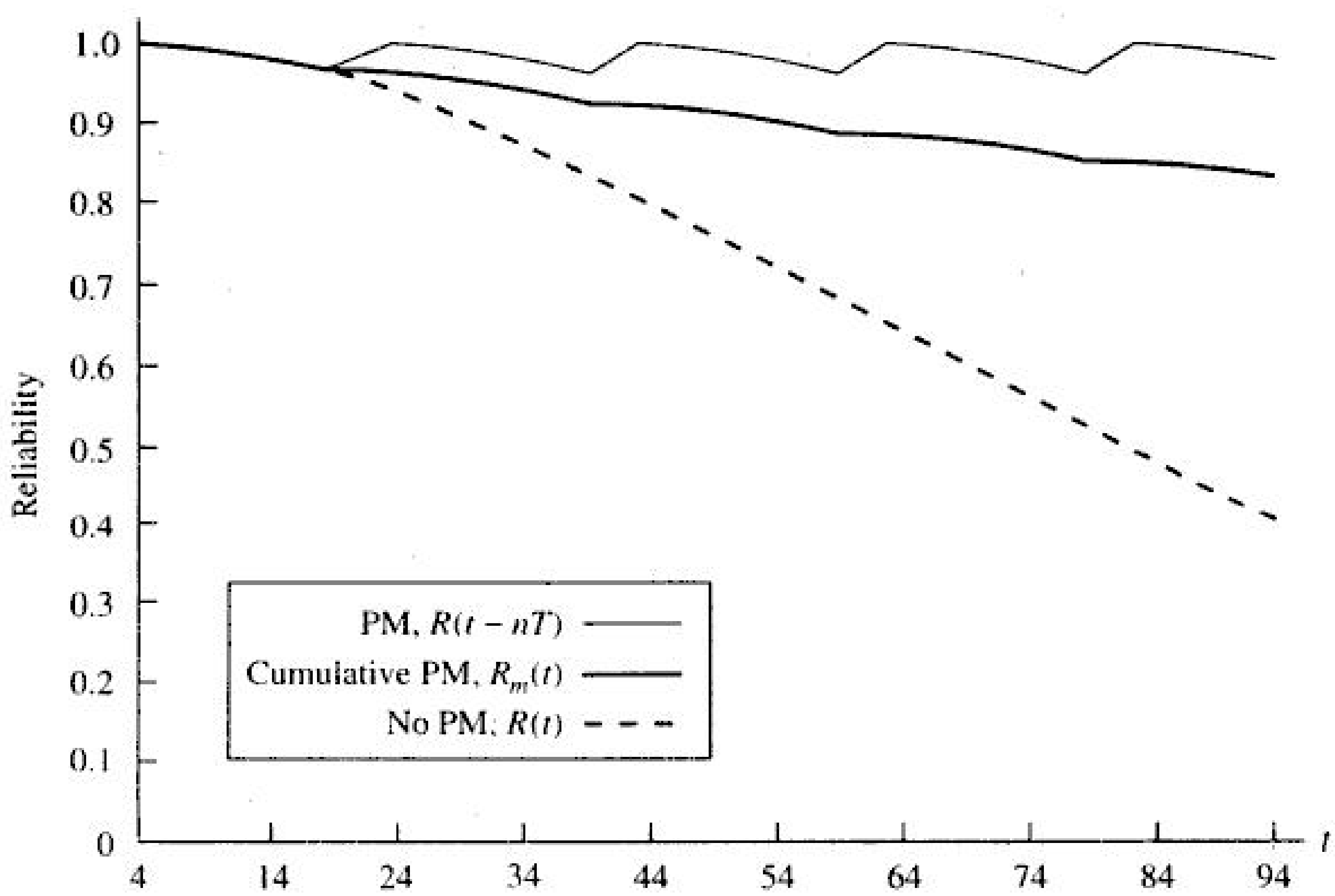

FIGURE 9.3
A periodic maintenance reliability curve for an increasing failure rate.

The design life at 0.90 reliability with no preventive maintenance is 32.5 days. Under preventive maintenance, if we consider the compressor reliability at the end of a maintenance interval, then we desire

$$\exp\left[-n\left(\frac{20}{100}\right)^2\right] \approx 0.90$$

Solving for n,

$$n = \frac{(-\ln 0.90)}{(20/100)^2} = 2.63$$

Letting $n = 2$,

$$R_m(t) = \exp\left[-2\left(\frac{20}{100}\right)^2\right]\exp\left[\left(-\frac{t-40}{100}\right)^2\right] \qquad 40 \le t < 60$$

$$= 0.9231 \exp\left[-\left(\frac{t-40}{100}\right)^2\right] = 0.90$$

Now solving for t:

$$t = 100\left[-\ln\left(\frac{0.90}{0.9231}\right)\right]^{1/2} + 40 = 55.9 \text{ days}$$

This is a 32 percent increase in the component's design life as a result of preventive maintenance.

In some preventive maintenance situations there is the possibility of a maintenance-induced failure. The model represented by Eq. (9.25) may be modified to account for these failures by letting p be the probability of a maintenance-induced failure during an individual preventive maintenance. Then

$$R_m(t) = R(T)^n(1-p)^n R(t-nT) \qquad \begin{matrix} nT \le t < (n+1)T \\ n = 0, 1, 2, \ldots \end{matrix} \tag{9.27}$$

EXAMPLE 9.17. For a component having a lognormal failure distribution,

$$R(T)^n = \left[1 - \Phi\left(\frac{1}{s}\ln\frac{T}{t_{med}}\right)\right]^n$$

$$R(t-nT) = 1 - \Phi\left(\frac{1}{s}\ln\frac{t-nT}{t_{med}}\right)$$

with $s = 1.00$ and $t_{med} = 5{,}000$ hr, the reliability at 5000 hr without preventive maintenance is

$$R(5000) = 1 - \Phi\left(\ln\frac{5000}{5000}\right) = 1 - 0.5 = 0.50$$

Assuming that $T = 500$ hr and $p = 0.005$,

$$R_m(5000) = \left[1 - \Phi\left(\ln\frac{500}{5000}\right)\right]^{10}(1-0.005)^{10} = 0.854$$

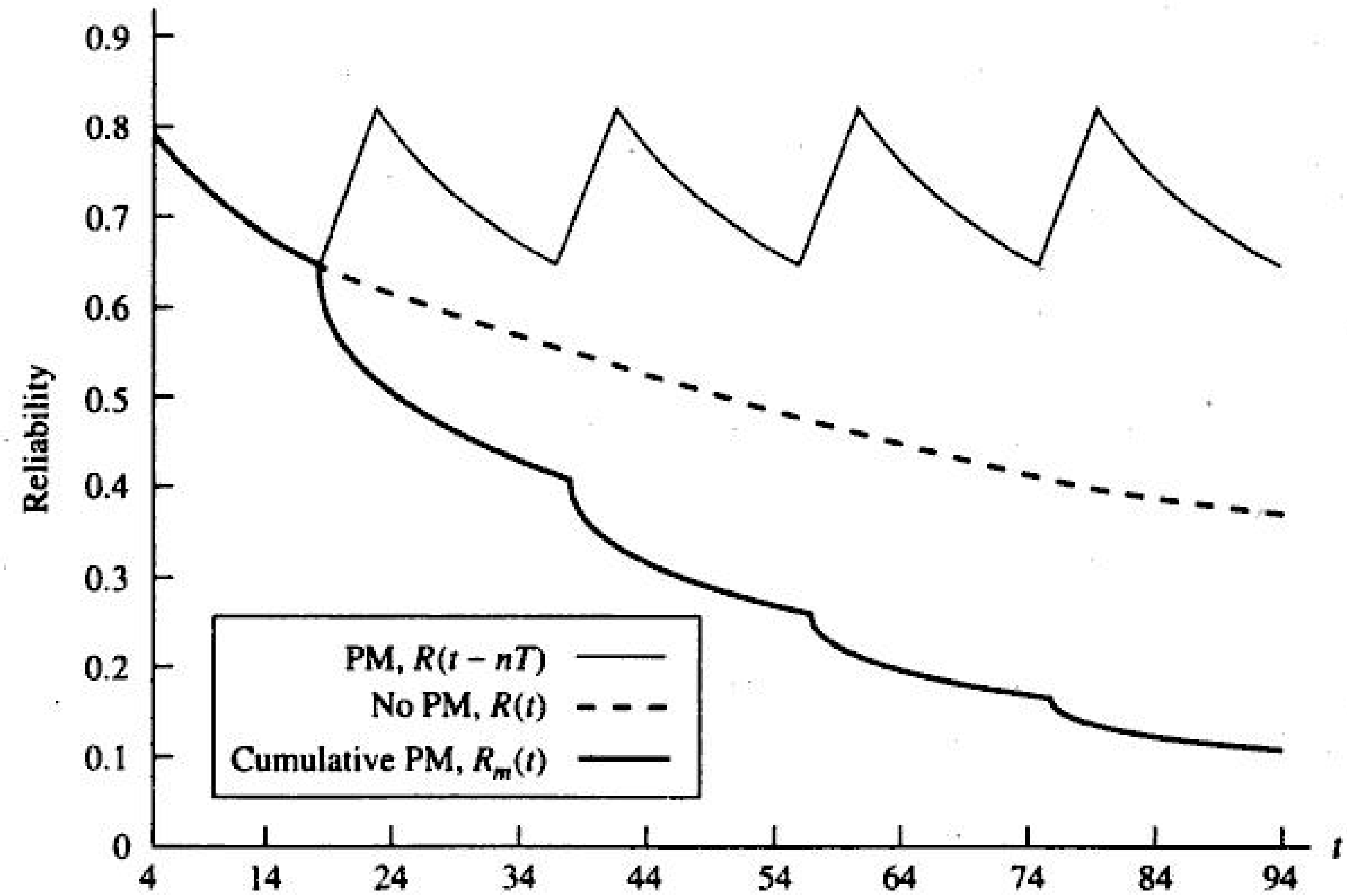

FIGURE 9.4
A periodic maintenance reliability curve for a decreasing failure rate.

EXAMPLE 9.18. If the failure rate is decreasing, preventive maintenance becomes counterproductive since restoring a system to as good as new condition introduces new wear-in failure modes (for example, manufacturing defects and marginal or substandard parts). This can be seen by changing the shape parameter of the Weibull failure distribution in Example 9.16 from 2 to 0.5. Figure 9.4 depicts the reliability curve with and without preventive maintenance.

9.6 STATE-DEPENDENT SYSTEMS WITH REPAIR

In Chapter 6 component dependencies were modeled using Markov analysis. This analysis procedure is here extended to include repairable components assuming that both the failure rate and the repair rate are constant (exponential distributions). Consider two active redundant components such that repair may be completed for a failed unit before the other unit has failed. As a result, no system failure is observed and the system reliability is improved. Assume that both units have the same failure rate, λ, and repair rate, r. The rate diagram appears in Fig. 9.5. State 1 is that of both units operating, state 2 is that of one operating and one being in repair, and state 3 is that of both units having failed. The resulting differential equations are as follows:

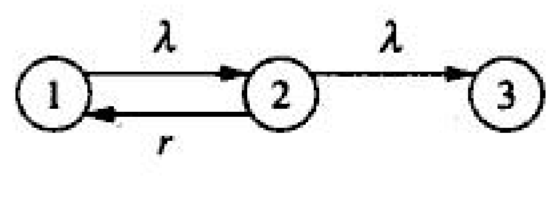

FIGURE 9.5
Rate diagram for a two-component system under repair.

$$
\begin{aligned}
\frac{dP_1(t)}{dt} &= -2\lambda P_1(t) + rP_2(t) \\
\frac{dP_2(t)}{dt} &= 2\lambda P_1(t) - (r+\lambda)P_2(t) \\
\frac{dP_3(t)}{dt} &= \lambda P_2(t)
\end{aligned}
\tag{9.28}
$$

They have the following solution (see Appendix 9B):[4]

$$P_1(t) = \frac{\lambda + r + x_1}{x_1 - x_2} e^{x_1 t} - \frac{\lambda + r + x_2}{x_1 - x_2} e^{x_2 t} \tag{9.29}$$

$$P_2(t) = \frac{2\lambda}{x_1 - x_2} e^{x_1 t} - \frac{2\lambda}{x_1 - x_2} e^{x_2 t} \tag{9.30}$$

$$P_3(t) = 1 + \frac{x_2}{x_1 - x_2} e^{x_1 t} - \frac{x_1}{x_1 - x_2} e^{x_2 t} \tag{9.31}$$

where

$$x_1, x_2 = \frac{1}{2}\left[-(3\lambda + r) \pm \sqrt{\lambda^2 + 6\lambda r + r^2}\right]$$

Therefore

$$R(t) = 1 - P_3(t) = \frac{x_1}{x_1 - x_2} e^{x_2 t} - \frac{x_2}{x_1 - x_2} e^{x_1 t} \tag{9.32}$$

and

$$
\begin{aligned}
\text{MTTF} &= \int_0^\infty \left(\frac{x_1}{x_1 - x_2} e^{x_2 t} - \frac{x_2}{x_1 - x_2} e^{x_1 t} \right) dt \\
&= \frac{-1}{x_1 - x_2}\left[\frac{x_1}{x_2} - \frac{x_2}{x_1}\right] = \frac{-(x_1 + x_2)}{x_1 x_2} = \frac{3\lambda + r}{2\lambda^2}
\end{aligned}
\tag{9.33}
$$

Observe that if $r = 0$ (no repair capability), the MTTF equals $1.5/\lambda$, as was shown in Chapter 3 for the two-component active redundant system. Equation (9.33) can also be written as

$$\text{MTTF} = \left(1.5 + 0.5\frac{\text{MTTF}_c}{\text{MTTR}_c}\right)\text{MTTF}_c$$

where MTTF_c and MTTR_c are the individual unit MTTF and MTTR, respectively. For this particular case, improving component maintainability will also increase the system reliability.

---

[4]This solution is more difficult than those of previous Markov processes since the two differential equations must be solved simultaneously. Laplace transforms provide the best solution technique.

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$$R(t) = \exp[-\int_0^t \lambda(t')dt']$$

An Introduction to Reliability and Maintainability Engineering is unique in its broad and practical coverage of reliability and maintainability. The fundamental concepts, models, and analysis techniques necessary to perform reliability and maintainability (R&M) engineering are presented in a well-organized and straightforward manner. Designed for use by students in engineering, engineering technology, operations research, and similar technical fields, the text provides a self-contained and comprehensive introduction of R&M, which includes the basic reliability models, covariate models, and hazard rate functions including the exponential, Weibull, normal, and lognormal; systems reliability including redundant, standby, and load sharing systems; failure mode, effect, and criticality analysis (FMECA) and fault tree analysis (FTA); R&M design methods based on availability and life cycle costs; preventive maintenance and maintenance and spares provisioning models; renewal and minimal repair models; the treatment of censored data; reliability testing including burn-in, accelerated life tests, acceptance sampling, and reliability growth testing; probability plots and curve fitting; and maximum likelihood estimation (MLE) and goodness-of-fit tests.

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