

Introduction to the Discrete Fourier Transform (DFT)

What is the DFT?

- The **Discrete Fourier Transform (DFT)** is a mathematical technique used to **decompose** a signal into its constituent **sine and cosine waves**.
- It converts a signal from its **time domain** representation (how the signal changes over time) into its **frequency domain** representation (how much of each frequency is present in the signal). This is also known as **analysis** or **forward DFT**.
- The reverse process, converting from the frequency domain back to the time domain, is called **synthesis** or **inverse DFT**.

Fourier Transform Categories:

- The Fourier transform can be divided into four categories, which can further be split into **real** and **complex** versions. All these transformations apply to signals that extend from negative infinity to positive infinity.

Time Domain vs. Frequency Domain:

- The **time domain** shows how the signal varies over time, while the **frequency domain** shows how much of each frequency is present in the signal.

Input and Output:

- For an **N-point input** signal, the DFT produces an **N/2+1 point output** in the frequency domain. The value **N** refers to the number of data points, and it is usually a power of two, though it can be any positive integer.

Understanding the Frequency Domain

Horizontal Axis:

The horizontal axis of the frequency domain can be represented in four different ways: 1. **Index k**: The index k varies from 0 to N/2 samples. 2. **Frequency f**: $f = k/N$. 3. **Frequency in radians**, $\omega = \frac{k}{N} * \pi$. 4. **Analog frequency, F**: $F = f * Fs = \frac{k}{N} * Fs$, where **Fs** is the sampling frequency.

Real and Imaginary Parts:

- The output of the DFT is represented by **two parts**: a **real part** and an **imaginary part**.
 - The **real part** corresponds to the amplitudes of the **cosine** waves.
 - The **imaginary part** corresponds to the amplitudes of the **sine** waves.
 - For example, in a 64-point signal, both the real and imaginary parts of the frequency domain contain 33 points each.
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DFT Basis Functions

Sine and Cosine Waves:

- The DFT uses **sine** and **cosine waves**, called **basis functions**, to decompose a signal. These waves serve as building blocks for the signal.

Frequency Parameter k :

- The parameter k determines the **frequency** of the sine and cosine waves.

Amplitude:

- The amplitudes of these sine and cosine waves are stored in the **real part** ($\text{Re } X[k]$) for the cosine waves and the **imaginary part** ($I_m X[k]$) for the sine waves.
 - For example, $I_m X$ refers to the amplitude of the sine wave that completes three cycles between points 0 and 63 in the time domain.
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Methods to Calculate the DFT

There are three primary methods to calculate the DFT:

1. DFT by Simultaneous Equations:

- This method helps in understanding the DFT but is inefficient for practical use, especially for large datasets.

2. DFT by Correlation:

- This method is preferred when the DFT has fewer than 32 points. It calculates the DFT by correlating the input signal with sine and cosine waves.

3. Fast Fourier Transform (FFT):

- The **FFT** is the most efficient method for computing the DFT, especially for larger datasets. It is typically hundreds of times faster than the other methods.

DFT by Correlation Example:

- To calculate the DFT of a 64-point signal using correlation, you'll compute 33 points in both the **real** and **imaginary** parts of the frequency domain.
 - The process involves multiplying the input signal by sine and cosine waves (basis functions), summing the products, and storing the result as the amplitude of that sine or cosine wave. All other frequency domain values are calculated similarly.
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Key Concepts

- **Decomposition/Analysis/Forward DFT:** Breaking down a signal into its frequency components.
 - **Synthesis/Inverse DFT:** Reconstructing the signal from its frequency components.
 - **Basis Functions:** The sine and cosine waves used to decompose the signal.
 - **N-point input:** The number of data points in the time-domain signal input to the DFT.
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In Summary

- The **Discrete Fourier Transform (DFT)** is a powerful tool for analyzing signals by decomposing them into their frequency components. It transforms a signal from the **time domain** to the **frequency domain** using sine and cosine waves.
 - The process can be performed using different methods, such as **DFT by simultaneous equations**, **DFT by correlation**, and the most efficient method, the **Fast Fourier Transform (FFT)**.
 - The frequency domain is represented by a **real part** (cosine wave amplitudes) and an **imaginary part** (sine wave amplitudes).
 - The **horizontal axis** in the frequency domain can be represented in various ways, including by frequency, radians, or analog frequency.
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