CSE_223_1.pdf: Signals, Systems, and Signal Processing

Introduction to Signals

- A **signal** is defined as any physical quantity that varies with time, space, or any other independent variable.
- Mathematically, a signal is a function of one or more independent variables:
  - Examples include sound waves, vibrations of a spring, tsunami waves, surface ripples on water, seismic waves, and electromagnetic waves.
  - An image is an example of a signal with two independent variables.
- **Signal Generation** is associated with a system responding to a stimulus or force. For example, in speech, the system includes the vocal cords and the vocal tract.

Systems and Signal Processing

- A **system** is a device or software that performs an operation on a signal.
  - For instance, a filter reducing noise is an example of a system.
- **Signal processing** is the operation performed by a system on a signal, which is characterized by the type of operation the system performs.
  - For example, a filter eliminates noise.
- Signal processing can be done using:
  - **Analog signal processing systems**.
  - **Digital signal processing systems**.
- Most signals encountered in science and engineering are **analog** in nature. Analog signals are functions of a continuous variable (time or space) and take on values in a continuous range. These can be processed directly by analog systems.
- **Digital signal processing systems** perform processing digitally and may use a large programmable computer, a small microprocessor, or a hardwired digital processor.

Advantages of Digital over Analog Signal Processing

- **Flexibility**: Digital processing allows easy reconfiguration by changing the program, while analog requires hardware redesign.
- **Accuracy**: Digital systems provide better control of accuracy requirements, while analog components have tolerances.
- **Storage and Transportability**: Digital signals can be stored and processed offline, which is not possible with analog signals.
- **Sophisticated Algorithms**: Digital systems allow the implementation of more sophisticated signal processing algorithms.

Classification of Signals

- **Multichannel Signals**: Signals generated by multiple sources or sensors (e.g., ground acceleration due to an earthquake).
- **Multidimensional Signals**:
  - A signal that is a function of a single independent variable is called a **one-dimensional signal**.
  - A signal is called an **M-dimensional signal** if its value is a function of M independent variables.
  - A black-and-white TV picture can be represented as $I(x, y, t)$, a three-dimensional signal.
  - A color TV picture is a three-channel, three-dimensional signal with intensity functions $I_r(x, y, t)$, $I_g(x, y, t)$, and $I_b(x, y, t)$.
- **Continuous-Time Signals**: Defined for every value of time and take on values in a continuous interval.
- **Discrete-Time Signals**: Defined only at specific values of time, which are usually equidistant. They can arise by sampling an analog signal or by accumulating a variable over time.
- **Continuous-Valued Signals**: Can take on all possible values in a finite or infinite range.
- **Discrete-Valued Signals**: Can take on values from a finite set of possible values.
- An **analog signal** is a continuous-time, continuous-valued signal.
- A **digital signal** is a discrete-time signal having a set of discrete values.
- **Deterministic Signals**: Can be uniquely described by an explicit mathematical expression, a table of data, or a well-defined rule. All past, present, and future values are known precisely without any uncertainty.
- **Random Signals**: Cannot be accurately described by explicit mathematical formulas and evolve unpredictably.

Concept of Frequency in Continuous-Time and Discrete-Time Signals

- The nature of time (continuous or discrete) affects the nature of frequency.
- **Continuous-Time Sinusoidal Signals**:
  - A simple harmonic oscillation can be described as:

$$S(t) = A \sin(\omega t + \theta)$$

  - For every fixed frequency F, $x_a(t)$ is periodic.
  - Distinct frequencies result in distinct signals.

- Increasing the frequency $ F $ increases the rate of oscillation.
- Mathematically, negative frequencies are introduced for convenience.

- **Discrete-Time Sinusoidal Signals**:
  - Expressed as:

    $$x(n) = A\cos(\omega n + \theta) \quad \text{or} \quad x(n) = A\cos(2\pi f n + \theta)$$

    where $ n $ is the sample number, $ A $ is the amplitude, $ $ is the frequency in radians per sample, and $ $ is the phase in radians.
  - A discrete-time sinusoid is periodic only if its frequency $ f $ is a rational number.
  - Discrete-time sinusoids whose frequencies are separated by an integer multiple of $ 2 $ are identical.
  - The highest rate of oscillation is attained when $ = $ or equivalently, $f = \frac{1}{2}$.

Aliasing

- The sinusoids having the frequency $|\omega| > \pi$ are the alias of the corresponding sinusoids with frequestion $|\omega| < \pi$
- **Aliasing** occurs because periodic sampling of a continuous-time signal maps an infinite range of frequencies into a finite range for the discrete-time signal.
  - Frequencies $ F = F_0 + k F_s $, where $ k $ is an integer, are indistinguishable from $ F_0 $ after sampling and are aliases of $ F_0 $.
  - For example, with a sampling rate $ F_s = 40 $ Hz, a 50 Hz signal is an alias of a 10 Hz signal.

Sampling of Analog Signals

- **Sampling** is the process of selecting values of an analog signal at discrete-time instants.
- The relationship between continuous and discrete time variables is given by:

  $$t = nT$$

  where $ T $ is the sampling period.
- The reciprocal of $ T $ is the **sampling rate** $ F_s $ or **sampling frequency**:

  $$F_s = \frac{1}{T}$$

- The relationship between analog and digital frequencies is given by:

  $$f = \frac{F}{F_s}$$

The highest frequency in a discrete-time signal is $f = \frac{1}{2}$.

The Sampling Theorem

- **Shannon's sampling theorem** states that a continuous-time signal with no frequency components higher than $ F_{max} $ can be completely determined by samples taken at a rate $ F_s 2 F_{max} $.
  - The minimum sampling rate $ F_s = 2 F_{max} $ is called the **Nyquist rate**.
  - To avoid aliasing, the sampling rate must be selected such that $ F_s > 2 F_{max} $.

Quantization of Continuous-Amplitude Signals

- **Quantization** is the process of converting a discrete-time continuous-amplitude signal into a digital signal by expressing each sample as a finite number of digits.
  - The error introduced in this process is called **quantization error** or **quantization noise**, and is given by:

$$e_q(n) = x_q(n) - x(n)$$

  - Quantization is an irreversible process because samples within a certain distance of a quantization level are assigned the same value.

Digital-to-Analog Conversion

- This topic is mentioned but not detailed in the document.

CSE_223_2.pdf: ntroduction to Discrete-Time Signals

What is a Discrete-Time Signal?

- A **discrete-time signal** is a sequence of values, denoted as **x(n)**, where **n** represents the time index, typically an integer.
- Unlike continuous-time signals, discrete-time signals are only defined at integer values of **n** and are not defined for non-integer values of **n**.

Graphical Representation:

- Discrete-time signals are displayed as a series of **discrete points** on a graph. The time origin (**n=0**) is typically marked with a symbol (↑).

Sampling:

- Discrete-time signals are obtained by sampling an **analog signal** (xa(t)), where **x(n)** is defined as **xa(nT)**, with **T** being the sampling period.

Elementary Discrete-Time Signals

1. **Unit Sample Sequence (Unit Impulse):**
    - A signal that is **1** at $n = 0$ and **0** everywhere else.

2. **Unit Step Signal:**
    - A signal that is **0** for **n<0** and **1** for **n 0**.

3. **Unit Ramp Signal:**
    - A signal whose value increases **linearly** with time for $n \geq 0$.

4. **Exponential Signal:**
    - **Real Exponential:** When the parameter **a** is real, the signal is a simple exponential.
    - **Complex Exponential:** If **a** is complex (i.e., **a = re^(j))**, the signal is a complex exponential, represented as:
        - $r^n * e^(j\theta n)$ or equivalently $r^n(cos(\theta n) + jsin(\theta n))$.
    - The **magnitude** of the complex exponential is $A(n) = r^n$, and the **phase** is $\emptyset(n) = \theta n$.

Classification of Discrete-Time Signals

1. **Energy Signals and Power Signals:**
    - **Energy:** The capacity of a signal to create a change.

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

    - **Power:** The rate at which energy is used or transmitted.

$$P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} |x[n]|^2$$

Energy Signal:

- A signal **x(n)** is an **energy signal** if its **total energy E** (calculated by summing the squared magnitudes of the signal) is finite ($\mathbf{0 < E < \infty}$).

Power Signal:

- The **average power P** of a discrete-time signal **x(n)** is calculated as the average of the sum of the squared magnitudes of the signal. If **P** is finite and non-zero, the signal is a **power signal**.

Key Relationships:

- If **E** is finite, $\mathbf{P = 0}$.
- If **E** is infinite, **P** can be finite or infinite.
- A signal cannot be both an energy signal and a power signal; they are **mutually exclusive**.
- A signal is neither an energy nor a power signal if both energy and power are infinite.
- **Practical Signals:** Most practical signals are energy signals with finite duration and amplitude. Power signals require infinite duration, making them physically impossible to generate.
- Signals with **constant amplitude** over an infinite duration are **power signals**.

2. Periodic and Aperiodic Signals:

Periodic Signal:

- A signal $x(n)$ is periodic with **period N** if $x(n+N) = x(n)$ for all **n**. The smallest such **N** is called the **fundamental period**.

Aperiodic Signal:

- A signal is **aperiodic** (or non-periodic) if no such **N** exists.

3. Symmetric (Even) and Antisymmetric (Odd) Signals:

Symmetric (Even) Signal:

- A signal **x(n)** is **symmetric** (even) if **x(-n) = x(n)**.

Antisymmetric (Odd) Signal:

- A signal **x(n)** is **antisymmetric** (odd) if **x(-n) = -x(n)**.
- Any arbitrary signal can be expressed as the sum of an **even** and an **odd** signal:
  - **Even component:** $(x(n) + x(-n))/2$

– **Odd component:** $(x(n) - x(-n))/2$

Simple Manipulations of Discrete-Time Signals

1. Time Shifting:

- Replacing **n** with **n - k** shifts the signal in time.
  - If **k** is positive, it is a **delay**.
  - If **k** is negative, it is an **advance**.

2. Time Reversal (Folding):

- Replacing **n** with **-n** reflects the signal about the time origin ($n = 0$).

3. Time Scaling (Down-sampling):

- Replacing **n** with μn, where is an integer, scales the time base.
  - For example, $y(n) = x(2n)$ represents **down-sampling** the signal.

4. Amplitude Modifications:

Amplitude Scaling:

- Multiplying the signal by a constant **A** scales the amplitude of every sample.

Addition of Signals:

- Adding two signals, **x1(n)** and $x2(n)$, results in a new signal **y(n) = x1(n) + x2(n)**.

Multiplication of Signals:

- The product of two signals is calculated sample-by-sample: $y(n) = x1(n) * x2(n)$.

Discrete-Time Systems

Definition:

- A **discrete-time system** is a device or algorithm that transforms a **discrete-time input signal** into a corresponding **output signal**.

Input-Output Relationship:

- The system's behavior can be described by the relationship between its **input** and **output** signals.

Block Diagram Representation:

- Basic building blocks include:
  - **Adder:** Sums two or more signals.
  - **Constant Multiplier:** Multiplies a signal by a constant.
  - **Signal Multiplier:** Multiplies two signals.
  - **Unit Delay Element:** Delays the signal by one sample.
  - **Unit Advance Element:** Advances the signal by one sample.

Classification of Discrete-Time Systems

1. Static vs. Dynamic Systems:

Static (Memoryless) System:

- The output depends only on the current input, not on past or future inputs.

Dynamic System:

- The output depends on past or future inputs, implying the system has **memory**.

2. Time-Invariant vs. Time-Variant Systems:

Time-Invariant System:

- The system's behavior doesn't change with time. Shifting the input by **k** samples results in the same shift in the output.

Time-Variant System:

- The system's behavior changes with time.

3. Linear vs. Nonlinear Systems:

Linear System:

- A system that satisfies the **superposition principle** (the output is the sum of the individual responses to each input signal).

Nonlinear System:

- A system that does not satisfy the superposition principle.

4. Causal vs. Noncausal Systems:

Causal System:

- The output depends only on **present** and **past** inputs, not on future inputs.

Noncausal System:

- The output depends on **future** inputs.

5. Stable vs. Unstable Systems:

Stable (BIBO) System:

- A system is stable if **bounded input** results in a **bounded output**.

Unstable System:

- If a bounded input leads to an **unbounded** output, the system is unstable.

Interconnection of Discrete-Time Systems

- Systems can be interconnected to form larger systems using configurations such as **cascade** (series) and **parallel** arrangements.

Analysis of Linear Time-Invariant (LTI) Systems

Techniques for Analysis:

- Directly solve the input-output equations or decompose the input signal into elementary components and apply linearity to find the total output.

Impulse Response:

- The **impulse response** of an LTI system is the response to an input **(n-k)** (the unit sample).

Convolution Sum:

- The output **y(n)** of an LTI system is given by the **convolution sum** of the input signal **x(n)** and the system's impulse response **h(n)**:
  - $y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$

Correlation of Discrete-Time Signals

Objective:

- To measure the degree of similarity between two signals.

Applications:

- Used in areas such as radar, sonar, digital communications, and geology.

Crosscorrelation and Autocorrelation:

- Methods for measuring similarity between signals.
