I. Mathematical Prerequisites
3. Transformations

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Transformation

- Examples of geometric transformations are translation, rotation, and scaling.
- Transformation function
  - Input and output: vector or position

$$\tau(\mathbf{v}) = \tau(x, y, z) = (x', y', z')$$

  - Linear transformation
    - Transformation between two vector spaces preserves the operations of vector addition (additivity) and scalar multiplication (homogeneity).

$$\tau(\mathbf{u}+\mathbf{v}) = \tau(\mathbf{u}) + \tau(\mathbf{v})$$
$$\tau(k\mathbf{u}) = k\tau(\mathbf{u})$$

$$\tau(\mathbf{u}) = \mathbf{uA} = \begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

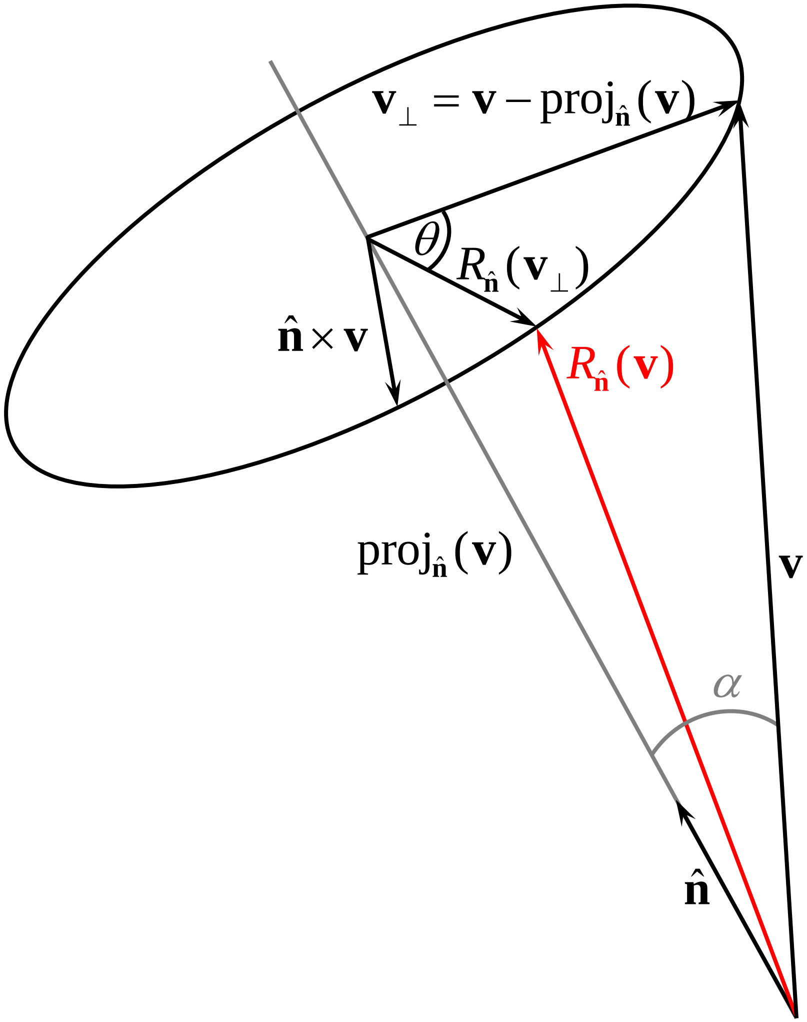
pre-multiplication

Scaling

$$S(\mathbf{v}) = S(x, y, z) = (s_x x, s_y y, s_z z)$$

$$S(\mathbf{u}) = \mathbf{uS} = \begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{pmatrix}$$

Rotation (1)


$\mathbf{v}_\perp = \mathbf{v} - \mathrm{proj}_{\hat{\mathbf{n}}}(\mathbf{v})$
θ
$R_{\hat{\mathbf{n}}}(\mathbf{v}_\perp)$
$\hat{\mathbf{n}} \times \mathbf{v}$
$R_{\hat{\mathbf{n}}}(\mathbf{v})$
$\mathrm{proj}_{\hat{\mathbf{n}}}(\mathbf{v})$
$\mathbf{v}$
α
$\hat{\mathbf{n}}$

$$R_{\hat{\mathbf{n}}}(\mathbf{v}) = \mathrm{proj}_{\hat{\mathbf{n}}}(\mathbf{v}) + R_{\hat{\mathbf{n}}}(\mathbf{v}_\perp)$$

Both reference vectors have the same length and lie on the circle of rotation.

$$R_{\hat{\mathbf{n}}}(\mathbf{v}_\perp) = \cos\theta \mathbf{v}_\perp + \sin\theta(\hat{\mathbf{n}} \times \mathbf{v})$$

$$\begin{aligned} R_{\hat{\mathbf{n}}}(\mathbf{v}) &= (\hat{\mathbf{n}} \cdot \mathbf{v})\hat{\mathbf{n}} + \cos\theta \mathbf{v}_\perp + \sin\theta(\hat{\mathbf{n}} \times \mathbf{v}) \\ &= \cos\theta \mathbf{v} + \left(1 - \cos\theta\right)(\hat{\mathbf{n}} \cdot \mathbf{v})\hat{\mathbf{n}} + \sin\theta(\hat{\mathbf{n}} \times \mathbf{v}) \end{aligned}$$

$$R_{\hat{\mathbf{n}}} = \begin{pmatrix} c+(1-c)x^2 & (1-c)xy+sz & (1-c)xz-sy \\ (1-c)xy-sz & c+(1-c)y^2 & (1-c)yz+sx \\ (1-c)xz+sy & (1-c)yz-sx & c+(1-c)z^2 \end{pmatrix}$$

$$c = \cos\theta, s = \sin\theta$$

Rotation (2)

- Rotation matrices
  - Each row vector is a unit length, and the row vectors are mutually orthogonal.
  - The row vectors are orthonormal (i.e., mutually orthogonal and unit length).
  - A matrix whose rows are orthonormal is said to be an orthogonal matrix.
  - The inverse of an orthogonal matrix is equal to its transpose.

$$R_{\hat{\mathbf{n}}}^{-1} = R_{\hat{\mathbf{n}}}^{T}$$

$$\hat{\mathbf{n}} = (1,0,0) \qquad R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

Homogeneous Coordinates

- Homogeneous coordinates provide a convenient notational mechanism to handle points and vectors uniformly.
- With homogeneous coordinates, we augment to 4-tuples and what we place in the fourth w-coordinate depends on whether we are describing a point or vector.
- Using the homogeneous notation, transformations of vectors and positions are handled in a unified way.
    - $(x, y, z, 0)$ for vectors.
    - $(x, y, z, 1)$ for points.

Affine Transformation

- A linear transformation cannot describe the translation.

$$\alpha(\mathbf{u}) = \tau(\mathbf{u}) + \mathbf{b}$$

- Linear transformation + translation ➔ affine transformation

$$\alpha(\mathbf{u}) = \tau(\mathbf{u}) + \mathbf{b} = \begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} + \begin{pmatrix} b_x & b_y & b_z \end{pmatrix}$$

$$= \begin{pmatrix} x & y & z & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ b_x & b_y & b_z & 1 \end{pmatrix}$$

Composition of Transformations

- Matrix multiplication is associative.

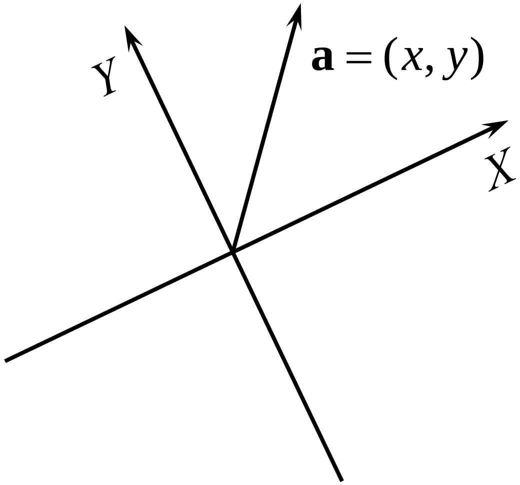
$$\left(\left(\mathbf{vS}\right)\mathbf{R}\right)\mathbf{T} = \mathbf{v}\left(\mathbf{SRT}\right)$$

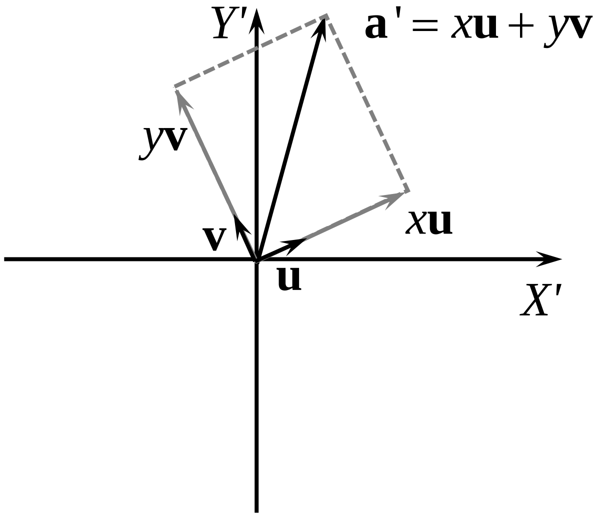
- Matrix multiplication is not commutative.

$$\left(\mathbf{vR}\right)\mathbf{T} \neq \left(\mathbf{vT}\right)\mathbf{R}$$

Change of Coordinate Transformations

- A vector $\mathbf{a}$ in XY-plane.
- The vector $\mathbf{a}$ in $X'Y'$-plane ($\mathbf{a}'$).





- Points

$$\mathbf{p} = x\mathbf{u} + y\mathbf{v} + z\mathbf{w} + \mathbf{q}$$

DirectX Math Transformation Functions (1)

```
// Constructs a scaling matrix:
XMMATRIX XM_CALLCONV XMMatrixScaling(
   float ScaleX, float ScaleY, float ScaleZ); // Scaling factors

// Constructs a scaling matrix from components in vector:
XMMATRIX XM_CALLCONV XMMatrixScalingFromVector(FXMVECTOR Scale);
   // Scaling factors (sx, sy, sz)

// Constructs a x-axis rotation matrix Rx: clockwise angle θ
XMMATRIX XM_CALLCONV XMMatrixRotationX(float Angle);

// Constructs a y-axis rotation matrix Ry:
XMMATRIX XM_CALLCONV XMMatrixRotationY(float Angle);

// Constructs a z-axis rotation matrix Rz:
XMMATRIX XM_CALLCONV XMMatrixRotationZ(float Angle);

// Constructs an arbitrary axis rotation matrix Rn:
XMMATRIX XM_CALLCONV XMMatrixRotationAxis(
   FXMVECTOR Axis,      // Axis n to rotate about
   float Angle);        // Clockwise angle θ to rotate
```

DirectX Math Transformation Functions (2)

```
// Constructs a translation matrix:
XMMATRIX XM_CALLCONV XMMatrixTranslation(
   float OffsetX, float OffsetY, float OffsetZ);
   // Translation factors

// Constructs a translation matrix from components in a vector:
XMMATRIX XM_CALLCONV XMMatrixTranslationFromVector(FXMVECTOR Offset);
   // Translation factors

// Computes the vector-matrix product vM where vw  = 1
// for transforming points:
XMVECTOR XM_CALLCONV XMVector3TransformCoord(
   FXMVECTOR V, CXMMATRIX M);         // Input V and M

// Computes the vector-matrix product vM where vw  = 0
// for transforming vectors:
XMVECTOR XM_CALLCONV XMVector3TransformNormal(
   FXMVECTOR V, CXMMATRIX M);         // Input V and M
```

DirectX Math Transformation Functions (3)

```
#include <windows.h>
#include <DirectXMath.h>
#include <iostream>
#include <iomanip>
std::ostream& XM_CALLCONV operator <<(std::ostream& os, DirectX::FXMVECTOR v) {
    DirectX::XMFLOAT4 dest;
    DirectX::XMStoreFloat4(&dest, v);
    os << "(" << dest.x << ", " << dest.y << ", " << dest.z << ", "
       << dest.w << ")";
    return os;
}
std::ostream& XM_CALLCONV operator <<(std::ostream& os, DirectX::FXMMATRIX m) {
    for (int i = 0; i < 4; ++i) {
        os << DirectX::XMVectorGetX(m.r[i]) << "\t";
        os << DirectX::XMVectorGetY(m.r[i]) << "\t";
        os << DirectX::XMVectorGetZ(m.r[i]) << "\t";
        os << DirectX::XMVectorGetW(m.r[i]) << std::endl;
    }
    return os;
}
```

DirectX Math Transformation Functions (4)

```
int main() {
    if (!DirectX::XMVerifyCPUSupport()) {
        std::cout << "directx math not supported" << std::endl;
        return 0;
    }

    std::cout << std::fixed << std::setprecision(10);

    DirectX::XMMATRIX A = DirectX::XMMatrixAffineTransformation(
        DirectX::XMVECTOR({ 1.0f, 1.0f, 1.0f, 0.0f }),
        DirectX::XMVECTOR({ 0.0f, 0.0f, 0.0f, 0.0f} ),
        DirectX::XMVECTOR({ DirectX::XM_PIDIV4, DirectX::XM_PIDIV4,
          0.0f, 0.0f}),
        DirectX::XMVECTOR({ 1.0f, 0.0f, 0.0f, 0.0f} ));

    std::cout << "A = " << std::endl << A << std::endl;
```

DirectX Math Transformation Functions (5)

```
    DirectX::XMVECTOR X, Y;
    X = DirectX::XMVectorSet(-2.0f, 1.0f, -3.0f, 0.0f);
    Y = DirectX::XMVector3TransformNormal(X,  A);

    std::cout << "X = " << std::endl << X << std::endl;
    std::cout << "Y = " << std::endl << Y << std::endl;

    DirectX::XMMATRIX A2 = DirectX::XMMatrixInverse(nullptr, A);
    X = DirectX::XMVector3TransformNormal(Y, A2);

    std::cout << "X = " << std::endl << X << std::endl;

    return 0;
}
```

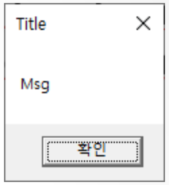
DirectX Math Transformation Functions (6)

```
A =
-0.2337006330   1.2337006330    0.0000000000    0.0000000000
1.2337006330    -0.2337006330   0.0000000000    0.0000000000
0.0000000000    0.0000000000    -1.4674012661   0.0000000000
1.0000000000    0.0000000000    0.0000000000    1.0000000000

X =
(-2.0000000000, 1.0000000000, -3.0000000000, 0.0000000000)
Y =
(1.7011018991, -2.7011017799, 4.4022035599, 0.0000000000)
X =
(-2.0000000000, 1.0000001192, -3.0000000000, 0.0000000000)
```

Sample Program

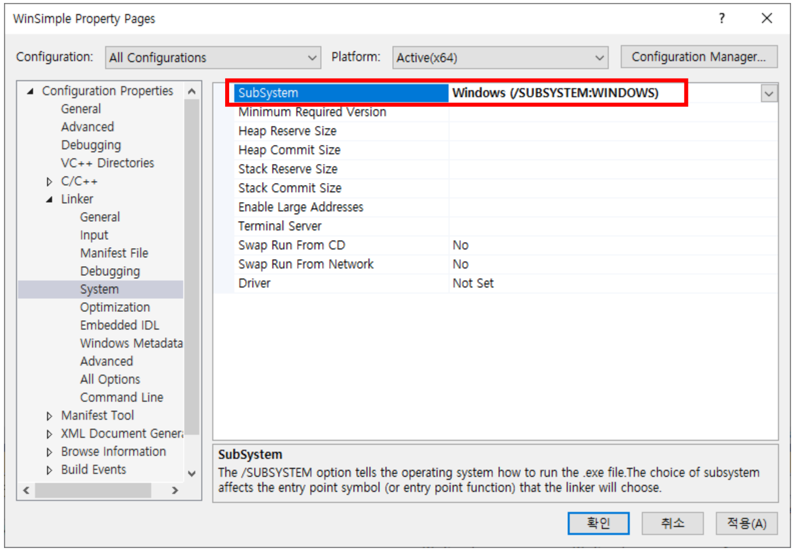
- New project (empty)
- [Linker] – [System] – [SubSystem]: Windows



Title

Msg

```
#include <windows.h>
int WINAPI WinMain(HINSTANCE hInstance,
  HINSTANCE prevInstance,PSTR cmdLine, int showCmd) {
   return MessageBox(nullptr, L"Msg", L"Title", MB_OK);
}
```



WinSimple Property Pages

Configuration: All Configurations Platform: Active(x64)

SubSystem	Windows (/SUBSYSTEM:WINDOWS)
Minimum Required Version	
Heap Reserve Size	
Heap Commit Size	
Stack Reserve Size	
Stack Commit Size	
Enable Large Addresses	
Terminal Server	
Swap Run From CD	No
Swap Run From Network	No
Driver	Not Set

SubSystem

The /SUBSYSTEM option tells the operating system how to run the .exe file.The choice of subsystem affects the entry point symbol (or entry point function) that the linker will choose.

```
#ifdef UNICODE
#define MessageBox  MessageBoxW
#else
#define MessageBox  MessageBoxA
#endif // !UNICODE
```

Windows Data Types

- Windows data types
  - https://learn.microsoft.com/en-us/windows/win32/winprog/windows-data-types
  - `BOOL`: A Boolean variable (should be `TRUE` or `FALSE`). `typedef int BOOL;`
  - `BOOLEAN`: A Boolean variable. `typedef BYTE BOOLEAN;`
  - `BYTE`: A byte (8 bits). `typedef unsigned char BYTE;`
  - `DWORD32`: A 32-bit unsigned integer. `typedef unsigned int DWORD32;`
  - `DWORD64`: A 64-bit unsigned integer. `typedef unsigned __int64 DWORD64;`
  - `FLOAT`: A floating-point variable. `typedef float FLOAT;`
  - `UINT`: An unsigned INT. The range is 0 through 4294967295 decimal. `typedef unsigned int UINT;`
  - `INT_PTR`: A signed integer type for pointer precision.

    -
      ```
      #if defined(_WIN64)
       typedef __int64 INT_PTR;
      #else
       typedef int INT_PTR;
      #endif
      ```
  - …

API References

- API references
  - https://learn.microsoft.com/en-us/windows/win32/api/d3d12/
  - Interfaces
  - Functions
    - **`[in]`**, **`[in, out]`**, **`[in, optional]`** (default **`NULL`**)
  - Callback functions
  - Structures
  - Enumerations

WinMain

- **`WinMain`** is the conventional name used for the application entry point.

-
  ```
  int __stdcall WinMain(
    [in]           HINSTANCE hInstance,
    [in, optional] HINSTANCE hPrevInstance,
    [in]           LPSTR     lpCmdLine,
    [in]           int       nShowCmd
  );
  ```
  - **`__stdcall`** calling convention is used to call Win32 API functions. The callee cleans the stack.
  - **`__cdecl`** is the default calling convention for C and C++ programs. The stack is cleaned up by the caller.
  - Handle is a variable that identifies an object; an indirect reference to an operating system resource.

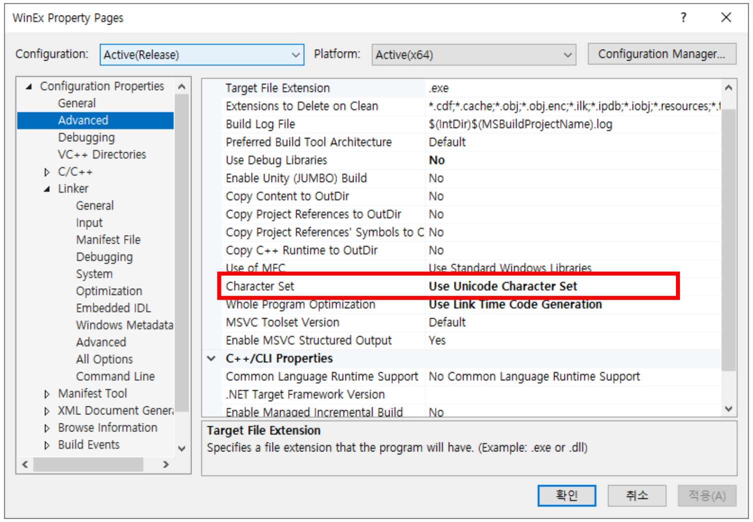
MessageBox

- **MessageBox** displays a modal dialog box that contains a system icon, a set of buttons, and a brief application-specific message, such as status or error information. The message box returns an integer value that indicates which button the user clicked.

-
  ```
  int MessageBox(
     [in, optional] HWND    hWnd,
     [in, optional] LPCTSTR lpText,
     [in, optional] LPCTSTR lpCaption,
     [in] UINT    uType
  );
  ```

- **L**: wide literal encoding (for Unicode)
  - `L'A', L"Ex"`

```
#ifdef UNICODE
#define MessageBox  MessageBoxW
#else
#define MessageBox  MessageBoxA
#endif // !UNICODE
```



Configuration: Active(Release)   Platform: Active(x64)   Configuration Manager...

Target File Extension	.exe
Extensions to Delete on Clean	*.cdf;*.cache;*.obj;*.obj.enc;*.ilk;*.ipdb;*.iobj;*.resources;*.t
Build Log File	$(IntDir)$(MSBuildProjectName).log
Preferred Build Tool Architecture	Default
Use Debug Libraries	**No**
Enable Unity (JUMBO) Build	No
Copy Content to OutDir	No
Copy Project References to OutDir	No
Copy Project References' Symbols to C	No
Copy C++ Runtime to OutDir	No
Use of MFC	Use Standard Windows Libraries
Character Set	**Use Unicode Character Set**
Whole Program Optimization	**Use Link Time Code Generation**
MSVC Toolset Version	Default
Enable MSVC Structured Output	Yes
C++/CLI Properties	
Common Language Runtime Support	No Common Language Runtime Support
.NET Target Framework Version	
Enable Managed Incremental Build	No

Target File Extension
Specifies a file extension that the program will have. (Example: .exe or .dll)

Windows Application (1)

- Every Windows desktop application must have a window-procedure function.
    - A function that processes all messages sent or posted to all windows of the class.

    -
      ```
      LRESULT CALLBACK WndProc(
          [in] HWND   hWnd,
          [in] UINT   message,
          [in] WPARAM wParam,
          [in] LPARAM lParam
      );
      ```
    - This function is called a **`WndProc`**, but you can give it whatever name you like in your code.
    - It is called handling an event.

A callback function is a function that is passed (as an argument) to another (function) and is executed after the completion of some operations.

Windows Application (2)

- Register window-class

  -
    ```
    typedef struct tagWNDCLASSA {
      UINT      style;
      WNDPROC   lpfnWndProc;
      int       cbClsExtra;
      int       cbWndExtra;
      HINSTANCE hInstance;
      HICON     hIcon;
      HCURSOR   hCursor;
      HBRUSH    hbrBackground;
      LPCSTR    lpszMenuName;
      LPCSTR    lpszClassName;
    } WNDCLASSA;
    ```

  -
    ```
    ATOM RegisterClassA(
      [in] const WNDCLASSA *lpWndClass
    );
    ```

Windows Application (3)

- Create window

  -
    ```
    HWND CreateWindowA(
      [in, optional] LPCSTR    lpClassName,
      [in, optional] LPCSTR    lpWindowName,
      [in]           DWORD     dwStyle,
      [in]           int       x,
      [in]           int       y,
      [in]           int       nWidth,
      [in]           int       nHeight,
      [in, optional] HWND      hWndParent,
      [in, optional] HMENU     hMenu,
      [in, optional] HINSTANCE hInstance,
      [in, optional] LPVOID    lpParam
    );
    ```
- Show window

  -
    ```
    BOOL ShowWindow(
      [in] HWND hWnd,
      [in] int  nCmdShow
    );
    ```

Windows Application (4)

- Message loop

  -
    ```
    MSG msg;
    while (GetMessage(&msg, NULL, 0, 0)) {
    // Translates virtual-key messages into character messages.
      TranslateMessage(&msg);
    // Dispatches a message to a window procedure
      DispatchMessage(&msg);
    }
    ```

Simple Window Application (1)

```
#include <windows.h>
LRESULT CALLBACK WndProc(HWND hwnd, UINT message,
   WPARAM wParam, LPARAM lParam)
{
    switch (message) {
    case WM_CLOSE:
        PostQuitMessage(0);
        break;
    }

    return (DefWindowProc(hwnd, message, wParam, lParam));
}
```

Simple Window Application (2)

```
int WINAPI WinMain(HINSTANCE hInstance, HINSTANCE prevInstance,
    PSTR cmdLine, int showCmd) {
    HWND hWnd;
    WNDCLASS wc;
    MSG msg;

    wc.style = CS_HREDRAW | CS_VREDRAW;
    wc.lpfnWndProc = WndProc;
    wc.cbClsExtra = 0;
    wc.cbWndExtra = 0;
    wc.hInstance = hInstance;
    wc.hIcon = LoadIcon(0, IDI_APPLICATION);
    wc.hCursor = LoadCursor(0, IDC_ARROW);
    wc.hbrBackground = (HBRUSH)GetStockObject(NULL_BRUSH);
    wc.lpszMenuName = 0;
    wc.lpszClassName = L"ClassName";
```

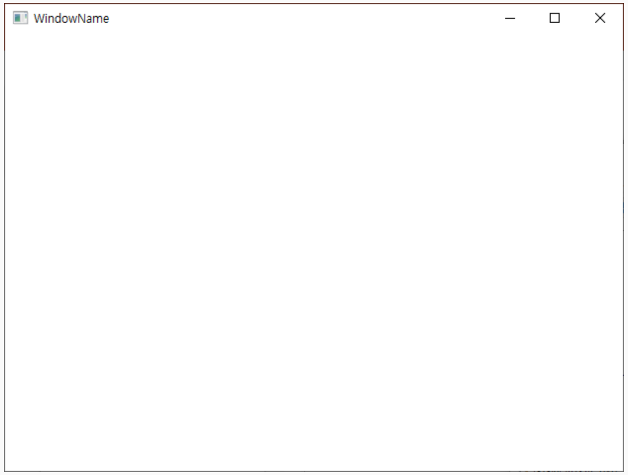
Simple Window Application (3)

```
    RegisterClass(&wc);

    hWnd = CreateWindow(L"ClassName", L"WindowName",
      WS_OVERLAPPEDWINDOW, 0, 0, 640, 480,
      0, 0, hInstance, nullptr);
    ShowWindow(hWnd, showCmd);

    while (1) {
        if (PeekMessage(&msg, 0, 0, 0, PM_REMOVE)) {
            TranslateMessage(&msg);
            DispatchMessage(&msg);
        }
    }

    return (int)msg.wParam;
}
```


WindowName