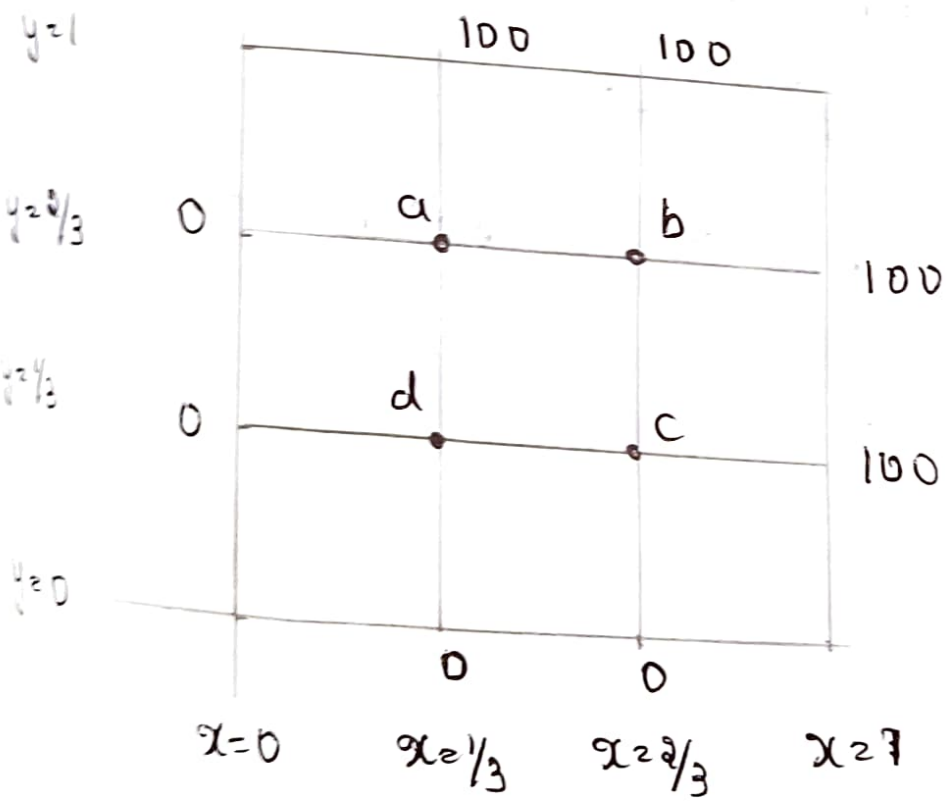
$$d = \frac{a + \cancel{d}^{\,c} + 2}{4} = \underline{1}$$

Solve this with step size $h = 1/3$, $U_{xx} + U_{yy} = -81xy$ where $0 < x < 1$ and $0 < y < 1$ and boundary conditions are $u(0, y) = u(x, 0) = 0$ and $u(1, y) = u(x, 1) = 100$.

Soln:-



It is Poisson Eqn so use 2nd formula.

$$u_{ij} = \frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - h^2 f(x_i, y_j)}{4}$$

$$a = \frac{1}{4}\left[100 + b + d + 0 - \frac{1}{9} f(1/3, 2/3)\right] = 1/4\,[b + d + 100 + 2]$$

$$= \frac{1}{4}[b + d + 102]$$

$$b = 1/4\,\left[100 + 100 + c + a - \frac{1}{9} f(2/3, 2/3)\right]$$

$$= 1/4\,[c + a + 204]$$

$$c = 1/4\,[b + 100 + 0 + d - 1/9\, f(2/3, 1/3)]$$

$$= 1/4\,[b + d + 102]$$

$$d = 1/4\,[a + c + 0 + 0 - 1/9\, f(1/3, 1/3)]$$

$$= 1/4\,[a + c + 1]$$

Since, $a = c$

$b = 1/4\,[c + c + 204] \Rightarrow 4b = 2c + 204 \Rightarrow 4b - 2c = 204$ — 1

$d = 1/4\,[c + c + 1] \Rightarrow 4d = 2c + 1 \Rightarrow 4d - 2c = 1$ — 2

$c = \frac{1}{4}(b + d + 102) \Rightarrow 4c = b + d + 102 \Rightarrow 4c - b - d = 102$ — 3

1 ust 2 to 3 class concept

Wave Eqn:-

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$\left[\frac{u_{i,j+1} - 2u_{ij} + u_{i,j-1}}{k^2}\right] = c^2 \left[\frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{h^2}\right]$$

↓ t-axis (step size k) ↓ x-axis (step size h)

choose $k = \frac{h}{c}$ and $\frac{k^2 c^2}{h^2} = 1$

$$\therefore u_{i,j+1} - \cancel{2u_{ij}} + u_{i,j-1} = u_{i+1,j} - \cancel{2u_{ij}} + u_{i-1,j}$$

$$\therefore \underline{u_{i,j+1}} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1}$$

$$\therefore u_{i0} = f_i = f(x_i, y_j)$$

$$u_{i1} = \frac{1}{2}[f_{i+1} + f_{i-1}] + k g_i$$

where $g_i = \frac{\partial u}{\partial t}(x_i, 0)$

Solve, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $0 < x < 1$; $t > 0$ and initial conditions

are $u(x, 0) = 100(x - x^2)$

$u(0, t) = u(1, t) = 0$

$\frac{\partial u}{\partial t}(x, 0) = 0$

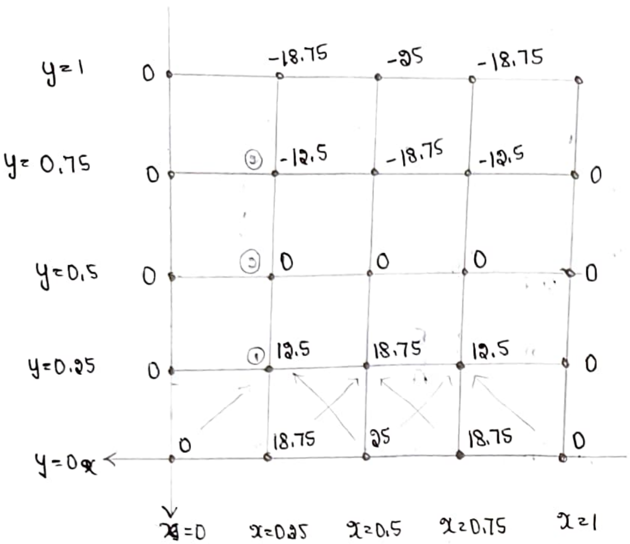
by taking $h = 0.25$ and compute 'u' for 4 times steps.

Soln:-

$h = 0.25$

$c = 1$

hence step size $k = \frac{h}{c} = \frac{0.25}{1} = 0.25$



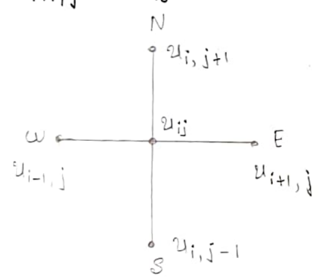
① is the average of 0 & 25 and add it with kg_i where $g_i = 0$ (given $\frac{\partial u}{\partial t}(x,0) = 0$).

: for same problem change $\frac{\partial u}{\partial t}(x,0) = x_i = g_i$

Repeat for $y=0.25$ all points.

For $y=0.5$ & 0.75 use NESW formula by ~~using~~ substituting in

② $\Rightarrow$ $u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1}$



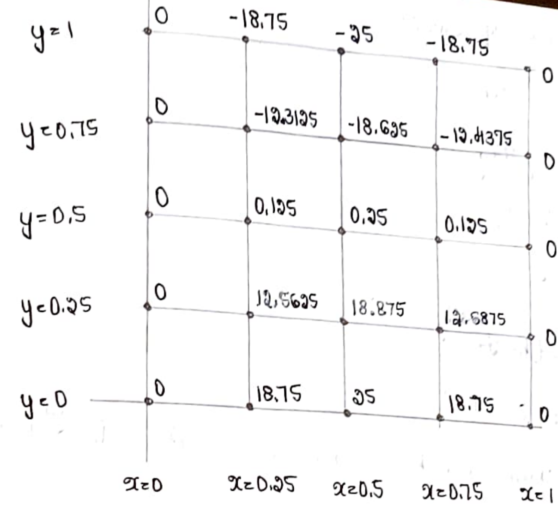
For the same question the initial conditions are,

$u(x,0) = 100(x - x^2)$

$u(0,t) = u(1,t) = 0$

$\frac{\partial u}{\partial t}(x,0) = x_i = g_i$

Soln:-



$h = 0.25$,

$c = 1$

$\therefore k = \frac{h}{c} = 0.25$

$\therefore u_{i1} = \frac{1}{2}(f_{i+1} + f_{i-1}) + kg_i$ & $g_i = x_i$

Solve System of Equations:-

Liebman's Method:-

Ex: If eqn getting as

$a = \frac{1}{4}[b+d]$

$b = \frac{1}{4}[2+c+a]$

$c = \frac{1}{4}[b+\cancel{2}4+d]$

$d = \frac{1}{4}[c+2+a]$

Soln: If values not known then consider it as 0 ~~and for next~~ and do 3 iterations

~~Iteration-1 values~~ :-

$a^{(1)} = \frac{1}{4}[0+0] = 0$

$b^{(1)} = \frac{1}{4}(2+0+0) = 0.5$

$c^{(1)} = \frac{1}{4}(0.5+4+0) = 1.125$

$d^{(1)} = \frac{1}{4}(1.125+2+0) = 0.7813$

Iteration-2:-

$a^{(2)} = \frac{1}{4}(0.5+0.7813) = 0.3203$

$b^{(2)} = \frac{1}{4}(2+1.125+0.3203) = 0.8613$

$c^{(2)} = \frac{1}{4}(0.8613+4+0.7813) = 1.4107$

$d^{(2)} = \frac{1}{4}(1.4107+2+0.3203) = 0.9328$

Iteration-3:-

$a^{(3)} = \frac{1}{4}(0.8613+0.9328) = 0.4485$

$b^{(3)} = \frac{1}{4}(2+1.4107+0.4485) = 0.9648$

$c^{(3)} = \frac{1}{4}(0.9648+4+0.9328) = 1.4744$

$d^{(3)} = \frac{1}{4}(1.4744+2+0.4485) = \underline{0.9807}$

Optimization:-

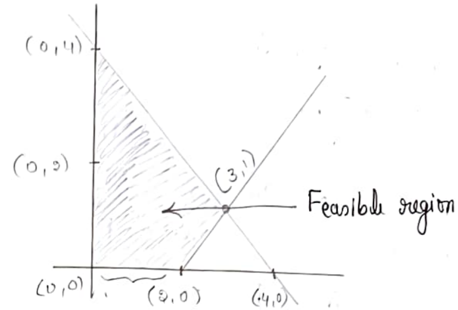
1) Maximize
2) Minimize

} Subjected to conditions.

Solve $\text{Max } Z = 3x_1 + 2x_2$, s.t $x_1 + x_2 \le 4$, $x_1 - x_2 \le 2$, $x_1 \ge 0$, $x_2 \ge 0$.

Soln:-

Graphical Method



Boundary points: (0,0), (2,0), (0,4), (3,1)

Z-at boundary points: 0, 6, 8, $\underline{11}$

$Z = 3x_1 + 2x_2$

$x_1 + x_2 = 4$

$x_1 - x_2 = 2$

(+) $2x_1 = 6$

$\underline{x_1 = 3}$

$\therefore x_2 = 1$

$\therefore$ Max - Z is $\underline{11}$ with $x_1 = 3$ and $x_2 = 1$

Simplex Method:-

LPP $\longrightarrow$ Linear Programming Problem

Solve; Max $Z = 3x_1 + 2x_2$ and S.t $x_1 + x_2 + S_1 = 4$,

$x_1 - x_2 + S_2 = 2$, x_1; x_2; S_1; $S_2 \geq 0$.

Soln:-

Using table method, fill the x_1, x_2, S_1 & S_2 with the coefficient

Table-1:- Initial Simplex Table

		C_j	3	2	0	0	
C_B	B	X_B	x_1	X_2	S_1	S_2	Ratio: $\frac{X_B}{X_1}$; $x_1 > 0$
0	S_1	4	1	1	1	0	$\frac{4}{1} = 4$
0	S_2	2	$\boxed{1}$ ② point	-1	0	1	$\frac{2}{1} = 2 \to$ Min
z_j		0	0	0	0	0	Outgoing Vector ②
$z_j - C_j$		—	-3 min	-2	0	0	

C_B

Dot product with C_B & all other values.

① $\to$ Incomming Vector

Table-2:-

		C_j	3	2	0	0	
C_B	B	X_B	X_1	X_2	S_1	S_2	Ration: $\frac{X_B}{X_2}$; $x > 0$
0	S_1	2	0	$\boxed{2}$	1	-1	$\frac{2}{2} = 1 \to$ Outgoing
3	X_1	2	1	-1	0	1	— (Already done in table-1)
z_j		6	3	-3	0	3	
$z_j - C_j$		—	0	-5	0	3	

Min

Incoming Vector

Table-3:- Use decomposition method to make initial values

		C_j	3	2	0	0
C_B	B	X_B	X_1	X_2	S_1	S_2
2	X_2	1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$
3	X_1	3	1	0	$\frac{1}{2}$	$\frac{1}{2}$
z_j		11	3	2	$\frac{5}{2}$	$\frac{1}{2}$
$z_j - C_j$		—	0	0	$\frac{5}{2}$	$\frac{1}{2}$

$\therefore z_j - C_j \geq 0$, we get optimum solution.

$\therefore z_j = 11$

$x_1 = 3$

$x_2 = 1$