MANIPAL
ACADEMY of HIGHER EDUCATION
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I SEMESTER M. Tech. (CSE/CSIS)
END SEMESTER EXAMINATION, December 2023
Computational Methods and Stochastic Processes [MAT 5128]

Time: 09:30 to 12:30 PM (3 Hours) Date: 09 December, 2023 MAX. MARKS: 50

Note (i) Answer ALL questions
(ii) Draw diagrams, and write equations wherever necessary

Q.1A When you will say that two events A and B are independent? Assuming that a year has 365 days, what is the probability that in a room with four people there are two of them with the same birthday?

(3 Marks; CO: 2; BL: 3)

Q.1B Express the following matrix A as product of elementary matrices and then describe the geometric effect of multiplication by A.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

(3 Marks; CO: 1; BL: 3)

Q.1C Find the Singular Value Decomposition (SVD) of the following matrix:

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

Mention some applications of SVD.

(4 Marks; CO: 1; BL: 4)

Q.2A Find the nth power of the following matrix:

$$A = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$$

(3 Marks; CO: 1; BL: 3)

Q.2B A random variable (X, Y) is uniformly distributed over a square with vertices $(1,0), (0,1), (-1,0), (0,-1)$. Find the correlation coefficient between X and Y.

(3 Marks; CO: 2; BL: 4)

Q.2C Let a pair of unbiased dice be thrown and X denote the sum of faces of dice. Suppose the income function is defined as follows: You gain an amount X if X is even and you loose an amount X if X is odd. Then find the expectation and variance of the income function.

(4 Marks; CO: 2; BL: 4)

Q.3A A bag contains 40 fair coins (P(H) = 0.5 = P(T)) and 10 unfair coins which flip with P(H) = 0.75, P(T)= 0.25. A coin is picked at random and tossed n times and each one of the n tosses were heads.

(i) What is the probability that the picked coin is a fair coin?

(ii) Find the least value of n that gives probability that the picked coin is fair to be at least 60 percent.

(3 Marks; CO: 2; BL: 3)

Q.3B Solve the game with following payoff matrix:

$$A = \begin{bmatrix} 0 & 4 & 6 \\ 5 & 7 & 4 \\ 9 & 6 & 3 \end{bmatrix}$$

(3 Marks; CO: 5; BL: 4)

Q.3C Draw the Markov chain and find the stationary distribution for the Markov chain using the graph theoretic method, given the following transition probability matrix:

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

Validate your answer with another technique.

(4 Marks; CO: 3; BL: 5)

Q.4A Illustrate that there is one-one correspondence between a labelled tree and its Prufer sequence. Hence find the number of labelled trees that can be formed from 10 vertices.

(3 Marks; CO: 5; BL: 3)

Q.4B Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes,

(i) exactly 4 customers arrive.

(ii) more than 4 customers arrive.

(3 Marks; CO: 3; BL: 4)

Q.4C Consider a stochastic process with $X(t) = A\cos wt + B\sin wt$ where A, B are uncorrelated random variables with mean 0 and variance 1; w is a positive constant. Find

(i) $E(X(t))$.

(ii) $V(X(t))$.

(iii) Auto covariance $c(s,t)$.

(iv) Auto correlation coefficient $r(s,t)$.

(4 Marks; CO: 3; BL: 4)

Q.5A Solve the game with following payoff matrix:

$$\begin{bmatrix} -1,-1 & -3,0 \\ 0,-3 & -2,-2 \end{bmatrix}$$

Explain the above game with its salient points.

(3 Marks; CO: 5; BL: 4)

Q.5B With step size $h = \frac{1}{3}$, solve $u_{xx}+u_{yy} = -81xy$; $0 \le x \le 1$, $0 \le y \le 1$; $u(0,y) = u(x,0) = 0$; $u(1,y) = u(x,1) = 100$.

(3 Marks; CO: 5; BL: 3)

Q.5C Solve the following linear programming problem using the Simplex Method:

$$\begin{gathered} \text{Maximize } Z = 3x + 2y \text{ subject to} \\ x + y \le 4; \\ x - y \le 2; \\ x \ge 0; y \ge 0. \end{gathered}$$

(4 Marks; CO: 5; BL: 4)