



MANIPAL
ACADEMY of HIGHER EDUCATION
(Institution of Eminence Deemed to be University)

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I SEMESTER M. Tech. (CSE/CSIS)
MID TERM EXAMINATION, October 2023
Computational Methods and Stochastic Processes [MAT 5128]

Time: 10:30 AM to 12:30 PM (2 Hours)

Date: 09 October, 2023

MAX. MARKS: 30

Note (i) Answer ALL questions

(ii) Draw diagrams, and write equations wherever necessary

Q.1 If $P(A) = \frac{1}{5}$, $P(A \cap B) = \frac{3}{25}$, A and B are independent then test whether A^c and B^c are independent.

(2 Marks; CO: 2; BL: 2)

Q.2 Explain the concept of a random variable using an illustration.

(2 Marks; CO: 2; BL: 2)

- Q.3** Express the following matrix A as product of elementary matrices and then describe the geometric effect of multiplication by A .

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(3 Marks; CO: 1; BL: 4)

- Q.4** Six real numbers are selected from the interval $[0, 4]$. What is the probability that at least two of them are higher than 1?

(3 Marks; CO: 2; BL: 3)

- Q.5** Illustrate the concept of a linear transformation T with an appropriate example and give the corresponding geometric interpretation for T . Also find the matrix of the linear transformation T .

(3 Marks; CO: 1; BL: 4)

- Q.6** A two dimensional random variable (X, Y) is uniformly distributed over the region R given as follows: $R = \{(x, y) | 0 < y < x < 1\}$.

Find the coefficient of correlation between X and Y . Are X and Y independent?

(3 Marks; CO: 2; BL: 4)

- Q.7** A bag contains 90 fair coins ($P(H) = 0.5 = P(T)$) and 10 unfair coins which flip with $P(H) = 0.75$, $P(T) = 0.25$. A coin is picked at random and tossed n times and each one of the n tosses were heads.

- (i) What is the probability that the picked coin is an unfair coin?
- (ii) Find the least value of n that gives probability that the picked coin is unfair to be at least 70 percent.

(3 Marks; CO: 2; BL: 3)

Q.8 Draw the Markov chain and find the stationary distribution for the Markov chain using the graph theoretic method, given the following transition probability matrix:

$$\begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

(3 Marks; CO: 3; BL: 4)

Q.9 Find the n th power of the following matrix:

$$\begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}$$

(4 Marks; CO: 1; BL: 4)

Q.10 A binary communication channel is a model that consists of a transmitter sending a binary signal 0 or 1 together with a receiver. Suppose a 100 bit message was sent with a probability that 0.001 that error will occur in each co-ordinate, then find the probability that the message was sent

- (i) error free
- (ii) with exactly one error
- (iii) with exactly 2 errors
- (iv) with 3 or more errors

(4 Marks; CO: 2; BL: 3)