

Achieving Precise Modeling of Geometric Transformations

Carson Wu September 2025

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- Lie Group Foundations

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- Lie Algebra (g): Tangent space at identity, e.g., $\mathfrak{se}(3)$ with generators for rotation and translation.
- Exponential Map: $\exp : \mathfrak{g} \to \mathcal{G}$, maps algebra to group.

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 - Optimize with FFT for efficiency.

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• **Self-Supervised Learning**: Contrastive loss:

$$L_{\mathsf{contrast}} = -\log \frac{\exp(\mathsf{sim}(f(x_i), f(T(g)x_i))/\tau)}{\sum_{j} \exp(\mathsf{sim}(f(x_i), f(x_j))/\tau)}$$



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- Tools: OpenCV, PyTorch Geometric, Sophus.

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- Implementation: PyTorch/JAX with Sophus.

• 3D Reconstruction: Chamfer distance reduced by 10-15%.

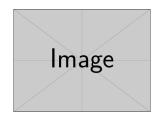
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• **Process**: Multi-view images \rightarrow Lie group features \rightarrow voxel/point cloud fusion.

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- Case Study: VR gamingindoor scenes with 2 pose accuracy.

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- Case Study: Hospital robots with 0.03m localization error.

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- Tools: ITK, MONAI, 3D Slicer.
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- Case Study: Brain tumor segmentation with 0.92 Dice score.

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- Steep learning curve for Lie group theory.

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Comparison with Existing Methods

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- Open-Source Tools: Standardize Lie group vision libraries.

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- Future: Optimize efficiency, expand multimodal integration.

Thank You! by
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