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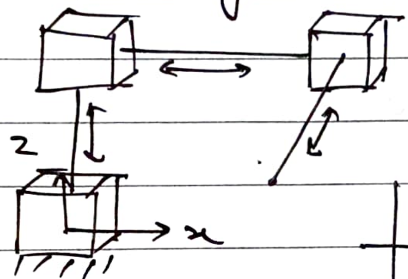
Assignment 3

Q5)

→ we know that for any joint i Transformation from i-1 to i

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where θ_i, α_i, a_i and d_i are D-H parameters for i th joint of the manipulator



D-H parameters

	a	α	d	θ
1	0	0	l_1	0
2	l_2	0	0	0
3	l_3	0	0	$-\pi/2$

for first joint.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for joint 2.

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for joint 3

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 A_2 A_3$$

$$A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} 0 & 1 & 0 & l_2 \\ -1 & 0 & 0 & -l_3 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∴

$$T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & 1 & 0 & l_2 \\ -1 & 0 & 0 & -l_3 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_2 \\ -l_3 \\ l_1 \end{bmatrix}$$

because all joints are prismatic, there are No θ values

Q6)

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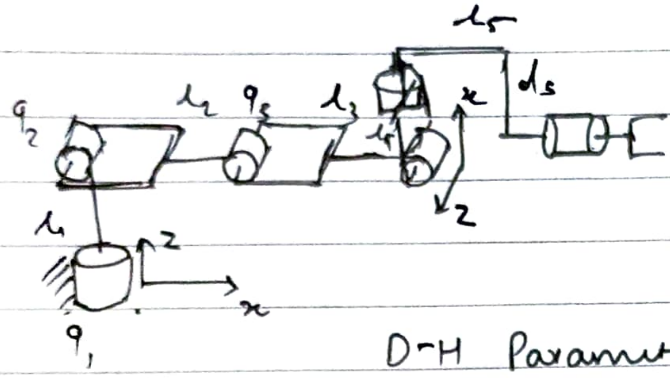

D-H Parameters.

	a	α	d	θ
1	0	$\pi/2$	l_1	q_1
2	l_2	0	0	q_2
3	l_3	0	0	$\pi/2 + q_3$
4	l_4	$-\frac{\pi}{2}$	0	$-\pi/2 + q_4$
5	l_5	0	$-d_5$	q_5

$$A_1 = \begin{bmatrix} \cos q_1 & \overset{0}{\cancel{-\sin q_1}} & \sin q_1 & 0 \\ \sin q_1 & 0 & -\cos q_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & L_2 \cos q_2 \\ \sin q_2 & \cos q_2 & 0 & L_2 \sin q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos(\frac{\pi}{2}+q_3) & -\sin(\frac{\pi}{2}+q_3) & 0 & L_3 \cos(\pi/2+q_3) \\ \sin(\pi/2+q_3) & \cos(\pi/2+q_3) & 0 & L_3 \sin(\pi/2+q_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} \cos(-\frac{\pi}{2}+q_4) & -\cancel{\sin}\ 0 & -\sin(-\pi/2+q_4) & L_4 \cos(-\pi/2+q_4) \\ \sin(-\pi/2+q_4) & 0 & \cos(-\pi/2+q_4) & L_4 \sin(-\pi/2+q_4) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} \cos(q_5) & -\sin q_5 & 0 & L_5 \cos q_5 \\ \sin(q_5) & \cos q_5 & 0 & L_5 \sin q_5 \\ 0 & 0 & 1 & -d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T_0^5 = A_1 A_2 A_3 A_4 A_5}$$

Q7)

→ 1) Direct drive 2-R manipulator.

In this configuration, ~~we~~ The first motor is on base and the second motor is on the joint between 1st and second link. In this config. we can use relative angles for oprating motors which is beneficial while using standard parameters like D-H.

But in this, as the motor is on the robot, which increases ~~of~~ the weight of arm and slows the movement speed.

2) Remotely driven 2-R manipulator.

In this configuration, ~~the~~ both motors are on base and the second link is driven by a belt or chain drive. For driving motors in this config, we have to use absolute angles for links. As the motors are on base, the arm becoms much lighter and easily movable.

3) 5-bar Parallelogram arrangement.

This configuration is combination of 2- 2-R manipulators with common end effector. The motors are at base controlling the movement of the base links which determine the end effector Position. The calculation and dynamics eqn's are complicated and sometimes workspace of the robot is less.

Q8)

→ 2R- elbow manipulator.
using absolute angles.

$$v_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q \\ \frac{l_1}{2}\cos q \\ 0 \end{bmatrix} \dot{q}_1 \qquad v_{c_2} = \begin{bmatrix} -l_1\sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1\cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \qquad \omega_2 = \dot{q}_2 \hat{k}$$

Kinetic energy.

$$\boxed{K = \frac{1}{2}\sum_{i=1}^{n} m_i v_{c_i}^T v_{c_i} + \frac{1}{2}\sum_{i=1}^{n} \omega_i^T I_i \omega_i}$$

$$v_{c_i} = J_{v_{c_i}}(q)\dot{q} \qquad \omega_i = \left(R_i^T J_{\omega_i}(q)\dot{q}\right)$$

$$K = \frac{1}{2}\dot{q}^T \sum_{i=1}^{n} \left[m_i J_{v_{c_i}}(q)^T J_{v_{c_i}}(q) + J_{\omega_i}(q)^T R_i(q) I_i R_i(q)^T J_{\omega_i}(q) \right] \times \dot{q}$$

$$\boxed{K = \frac{1}{2}\dot{q}^T D(q)\dot{q}}$$

$$D(q) = \begin{bmatrix} m_1\frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) & m_2\frac{l_2^2}{4} + I_2 \end{bmatrix}$$

Computing Christoffel Symbols

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ii}}{\partial q_k}\right]$$

$$C_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2} + \cancel{\frac{\partial d_{12}}{\partial q_1}} \cancel{- \frac{\partial d_{12}}{\partial q_1}}\right]$$

$$= \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \cancel{\frac{1}{2}\frac{\partial d_{22}}{\partial q_1}}^{0} = \frac{\partial d_{12}}{\partial q_2} = -m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\cancel{\frac{\partial d_{11}}{\partial q_2}}^{0} = + m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$$

$$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0 \qquad C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential energy $V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$

$$\phi_1 = \frac{\partial V}{\partial q_1} = m_1 g \frac{l_1}{2} \cos q_1 + m_2 g \left(l_1 \cos q_1 \cancel{+ \frac{l_2}{2}\cos q_2}\right)$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 g \frac{l_2}{2} \cos q_2$$

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we know that

$$\tau_k = \sum_j d_{kj}(q)\ddot{q}_j + \sum_{ij} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q)$$

$$\tau_1 = d_{11}(q)\ddot{q}_1 + d_{12}(q)\ddot{q}_2 + \cancel{c_{111}}(q)\dot{q}_1\dot{q}_1 + \cancel{c_{121}}(q)\dot{q}_1\dot{q}_2$$
$$+ \cancel{c_{211}}(q)\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2\dot{q}_2 + \phi_1$$

$$\therefore \boxed{\tau_1 = d_{11}(q)\ddot{q}_1 + d_{12}(q)\ddot{q}_2 + c_{221}\dot{q}_2^2 + \phi_1}$$

$$\tau_2 = d_{21}(q)\ddot{q}_1 + d_{22}(q)\ddot{q}_2 + c_{112}(q)\dot{q}_1\dot{q}_1 + \cancel{c_{122}}(q)\dot{q}_1\dot{q}_2$$
$$+ \cancel{c_{212}}(q)\dot{q}_2\dot{q}_1 + \cancel{c_{222}}\dot{q}_2\dot{q}_2 + \phi_2$$

$$\therefore \boxed{\tau_2 = d_{21}(q)\ddot{q}_1 + d_{22}(q)\ddot{q}_2 + c_{112}(q)\dot{q}_1\dot{q}_2 + \phi_2}$$

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Q10 For deriving the dynamics equation, given $D(q)$ $V(q)$

First we have to calculate the Christoffel Symbols. These can be calculated using elements of matrix $D(q)$ as d_{ij}

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

We have to calculate ϕ_k

$$\therefore \phi_k = \frac{\partial V_k}{\partial q_k}$$

from these values →

$$\boxed{\tau_k = \sum_j d_{kj}(q)\ddot{q}_j + \sum_{ij} C_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q)}$$

Then we can calculate all n equations using matrix form

$$\therefore \boxed{\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)}$$