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Assignment 2 Introduction to robotics

Q1)

$$R_0^1 = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix}$$

Orthogonal column's means columns $a_i^T a_j = 0$ where $i \neq j$

$i = 1 \quad j = 2$

$$\begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 \end{bmatrix}^T \begin{bmatrix} \hat{j}_1 \cdot \hat{i}_0 \\ \hat{j}_1 \cdot \hat{j}_0 \\ \hat{j}_1 \cdot \hat{k}_0 \end{bmatrix} = \hat{i}_1 \cdot \hat{i}_0 \cdot \hat{j}_1 \hat{i}_0 + \hat{i}_1 \hat{j}_0 \cdot \hat{j}_1 \hat{j}_0 + \hat{i}_1 \hat{k}_0 \cdot \hat{j}_1 \hat{k}_0$$

$$= \cancel{\quad} = \underline{\underline{0}}$$

for $i = 2 \quad j = 3$

$$\begin{bmatrix} \hat{j}_1 \cdot \hat{i}_0 \\ \hat{j}_1 \cdot \hat{j}_0 \\ \hat{j}_1 \cdot \hat{k}_0 \end{bmatrix}^T \begin{bmatrix} \hat{k}_1 \cdot \hat{i}_0 \\ \hat{k}_1 \cdot \hat{j}_0 \\ \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix} = \hat{j}_1 \hat{i}_0 \cdot \hat{k}_1 \hat{i}_0 + \hat{j}_1 \hat{j}_0 \cdot \hat{k}_1 \hat{k}_0 + \hat{j}_1 \hat{k}_0 \cdot \hat{k}_1 \hat{k}_0$$

$$= \underline{\underline{0}}$$

for $i = 3 \quad j = 1$

$$\begin{bmatrix} \hat{k}_1 \cdot \hat{i}_0 \\ \hat{k}_1 \cdot \hat{j}_0 \\ \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix}^T \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 \end{bmatrix} = \hat{k}_1 \hat{i}_0 \hat{i}_1 \hat{i}_0 + \hat{k}_1 \hat{j}_0 \hat{i}_1 \hat{j}_0 + \hat{k}_1 \hat{k}_0 \hat{i}_1 \hat{k}_0$$

$$= \underline{0}$$

$\therefore$ The columns of R_0^1 are mutually orthogonal.

Q4) $\det[R_0^1]$

We know that, $\det(R_0^1) = \det(R_0^{1\,T})$

from the properties of determinants and Rotation matrices

$$\det(R_0^1)\det(R_0^{1\,T}) = \det(R_0^1 R_0^{1\,T}) = (\det(R_0^1))^2$$

$$(\det(R_0^1))^2 = \det(I) = 1$$

$\therefore$ $\underline{\underline{\det(R_0^1) = 1}}$ (-1 if left hand system)

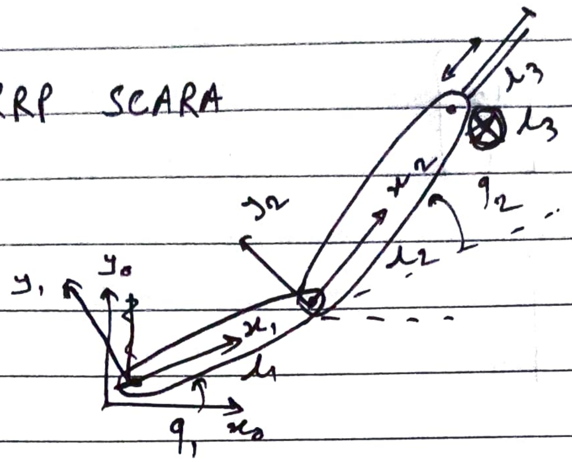
Q5) Show that $R\,S(a)R^T = S(Ra)$

from properties. $S(a)b = a \times b$

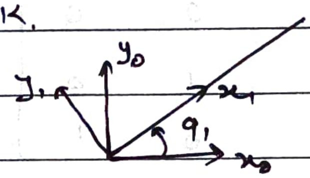
$$
\begin{aligned}
R\,S(a)R^T &= R(a \times R^T) \\
&= Ra \times RR^T \\
&= Ra \times I \\
&= S(Ra)\,I \\
&= S(Ra)
\end{aligned}
$$

$\therefore$ $\boxed{R\,S(a)R^T = S(Ra)}$

Q6) RRP SCARA



first link.



$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

from frame 1 - 2.

$$R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

for frame 2 - 3

$$R_2^3 = \begin{bmatrix} \cos \pi/2 & 0 & \sin \pi/2 \\ 0 & 1 & 0 \\ -\sin \pi/2 & 0 & \cos \pi/2 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} l_3+d \\ 0 \\ 0 \end{bmatrix}$$

d is extainsion in the link 3
l_3 is narural length of link 3

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_1 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \pi/2 & 0 & \sin \pi/2 & l_2 \\ 0 & 1 & 0 & 0 \\ -\sin \pi/2 & 0 & \cos \pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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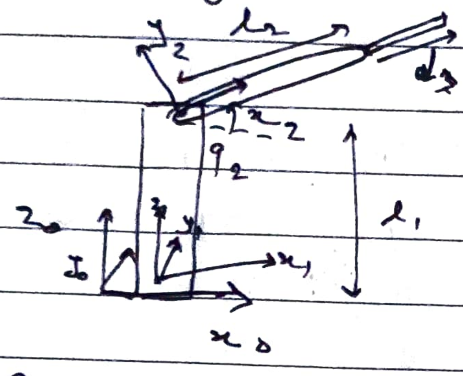
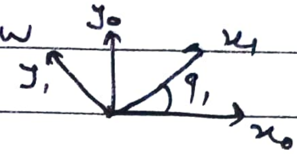
Q8) Stanford type RRP



top view



for frame 0-1

$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

for frame 1-2

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \pi/2 & -\sin \pi/2 \\ 0 & \sin \pi/2 & \cos \pi/2 \end{bmatrix} \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ 0 & 0 & -1 \\ \sin q_2 & \cos q_2 & 0 \end{bmatrix}$$

frame 2-3

$$R_2^3 = I \qquad d_2^3 = \begin{bmatrix} l_2 + d \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{matrix} \cancel{1} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{\cos} & \\ \cancel{0} & & \end{matrix}$$

$$H_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin q_2 & \cos q_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & l_2 + d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q9) Trajectory of drone

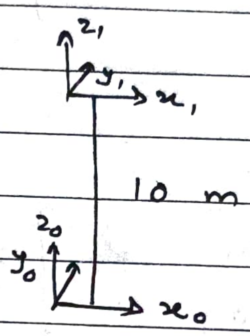
10 m z direction

30° about x - axis

60° about z axis

3 m in z direction (above)

for frame 0 - 1



$R_0^1 = I$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

for frame 1 - 2

80° x axis rotation

and 60° z axis rotation

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30° & -\sin 30° \\ 0 & \sin 30° & \cos 30° \end{bmatrix} \begin{bmatrix} \cos 60° & -\sin 60° & 0 \\ \sin 60° & \cos 60° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} \frac{1\cancel{\sqrt{3}}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 3/4 & \frac{\sqrt{3}}{4} & -1/2 \\ \sqrt{3}/4 & 1/4 & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 3/4 & \frac{\sqrt{3}}{4} & -1/2 & 0 \\ \frac{\sqrt{3}}{4} & 1/4 & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 H_1^2$$

$$= \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/4 & \frac{\sqrt{3}}{4} & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

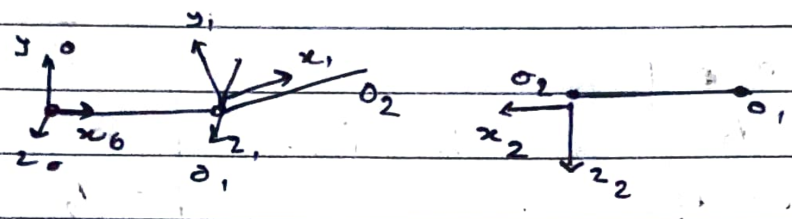
$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ 10 + \frac{3\sqrt{3}}{2} \\ 1 \end{bmatrix} \qquad \therefore \; P_0 = \begin{bmatrix} 0 \\ -3/2 \\ \frac{20+3\sqrt{3}}{2} \end{bmatrix}$$

Q11)

→ when joint is revolute $J_i = \begin{bmatrix} z_{i-1} \times (O_n - O_{i-1}) \\ z_{i-1} \end{bmatrix}$

when joint is prismatic $J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$

O are



$\therefore \quad O_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_0^1 = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix} \qquad O_0^2 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 \\ l_1 \sin q_1 + l_2 \sin q_2 \\ 0 \end{bmatrix}$

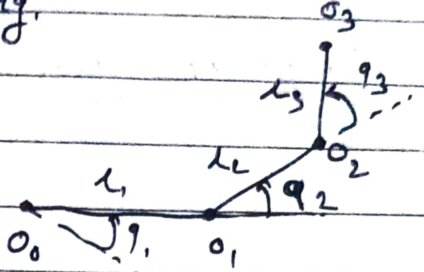
$O_0^3 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 \\ l_1 \sin q_1 + l_2 \sin q_2 \\ -d_3 \end{bmatrix} \qquad z_0^0 = z_0^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$z_0^2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

$J = \begin{bmatrix} J_v \\ J_w \end{bmatrix} = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$

$J = \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin q_2 & -l_2 \sin q_2 & 0 \\ l_1 \cos q_1 + l_2 \cos q_2 & l_2 \cos q_2 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Q12) Jacobian for RRR config.



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix} \qquad O_2 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 \\ l_1 \sin q_1 + l_2 \sin q_2 \\ 0 \end{bmatrix}$$

O are joint origins

$$O_3 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 + l_3 \cos q_3 \\ l_1 \sin q_1 + l_2 \sin q_2 + l_3 \sin q_3 \\ 0 \end{bmatrix}$$

z_i = z-axis direction

$$z_0 = z_1 = z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_1 \sin q_1 + l_2 \sin q_2 + l_3 \sin q_3) & -(l_2 \sin q_2 + l_3 \sin q_3) & -l_3 \sin q_3 \\ l_1 \cos q_1 + l_2 \cos q_2 + l_3 \cos q_3 & l_2 \cos q_2 + l_3 \cos q_3 & l_3 \cos q_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$