ME 639

Assignment - 2

Videh Patel
19110192

1) Let R_0^1 be a rotational matrix from $(\hat{i}_1, \hat{j}_1, \hat{k}_1)$ to $(\hat{i}_0, \hat{j}_0, \hat{k}_0)$.

~~Thus~~,

$$\cancel{\hat{i}_0 = R_0^1 \hat{i}_1}$$
$$\cancel{\hat{j}_0 = R_0^1 \hat{j}_1}$$
$$\cancel{\hat{k}_0 = R_0^1 \hat{k}_1}$$

~~Also~~

Thus, in $(\hat{i}_1, \hat{j}_1, \hat{k}_1)$ basis,

$$\hat{i}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \hat{j}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \hat{k}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Thus,

$$R_0^1 \hat{i}_1 = R_0^1 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1^{st} \text{ column of } R_0^1.$$

S'lly,

$$R_0^1 \hat{j}_1 = 2^{nd} \text{ column of } R_0^1$$

& $R_0^1 \hat{k}_1 = 3^{rd}$ column of R_0^1.

Also,

$$\hat{i}_0 = R_0^1 \hat{i}_1 = 1^{st} \text{ column}$$
$$\hat{j}_0 = R_0^1 \hat{j}_1 = 2^{nd} \text{ column}$$
$$\hat{k}_0 = R_0^1 \hat{k}_1 = 3^{rd} \text{ column.}$$

Since, $\hat{i}_0, \hat{j}_0, \hat{k}_0$ are othogonal basis, the column vectors of R_0^1 are also ~~or~~ mutually orthogonal.

~~8) R_0^1 since R_0^1.~~

9) We know that

$$R_0^1 \cdot (R_0^1)^T = I$$

Also, $\det(R_0^1) = \det((R_0^1)^T)$

Thus,

$$\det(I) = \det(R_0^1 (R_0^1)^T)$$
$$1 = \det(R_0^1) \cdot \det((R_0^1)^T)$$
$$1 = (\det(R_0^1))^2$$

Thus,

$\det(R'_o) = \pm 1$

~~when $\det(R'_o) = -1$, a change in the~~

For right-handed axes to right handed rotation,

$\underline{\det(R'_o) = 1}$.

5) For any orthonormal matrix, R belonging to SO_3, if a & b are two 3x1 vectors,

$$R\cdot(a\times b) = (Ra)\times(Rb) \quad \text{--- (1)}$$

Thus,

$$R\,S(a)R^T b = R\cdot\left(S(a)\cdot(R^T\cdot b)\right)$$

$$= R\cdot(a\times R^T b) \quad [\because S(a)b = a\times b]$$

$$= Ra\times RR^T b \quad [\text{From (1)}]$$

$$= Ra\times b = S(Ra)\,b$$

Thus,

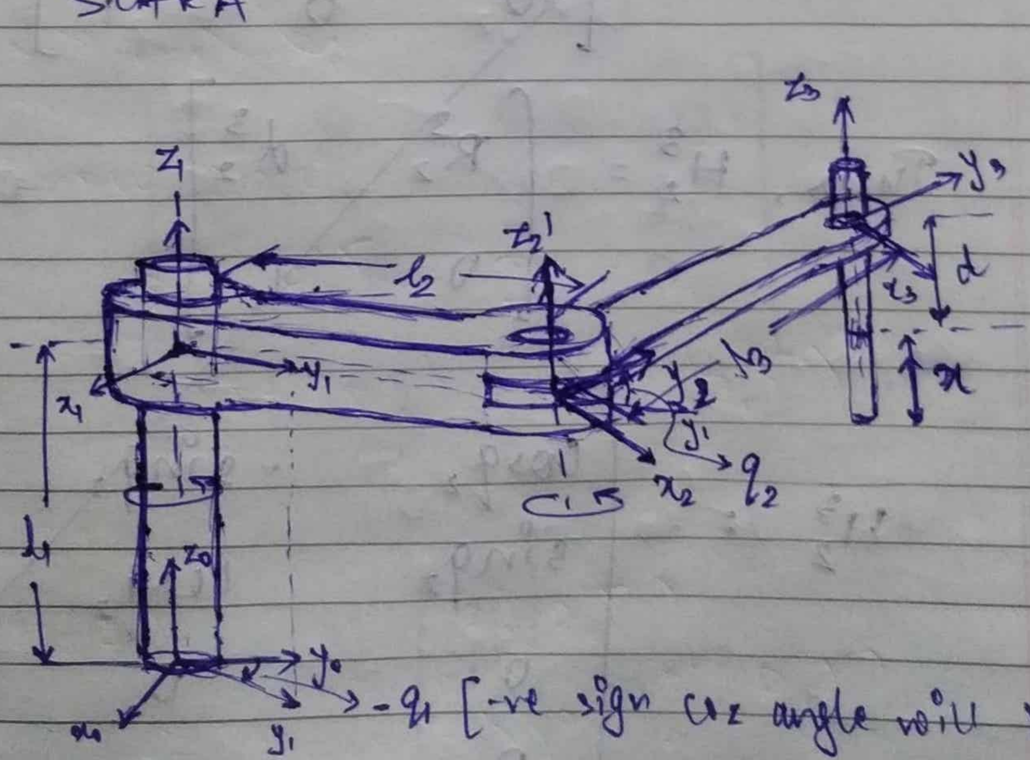
$$\left[R\,S(a)\,R^T - S(Ra)\right] b = 0 \quad \forall\ b \in \mathbb{R}^3$$

Thus,

$$R\,S(a)\,R^T - S(Ra) = 0$$

$$\therefore \quad \boxed{R\,S(a)\,R^T = S(Ra)}$$

i) RRP SCARA



$-q_1$ [-ve sign coz angle will be -ve].

Thus,

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ -(x+d) \end{bmatrix} = -(x+d)\hat{k}_3$$

$$d_2^3 = l_3 \hat{j}_2 = \begin{bmatrix} 0 \\ l_3 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$d_1^2 = l_2 \hat{j}_1 = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ \sin q_2 & \cos q_2 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$d_0^1 = l_1 \hat{k}_0 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

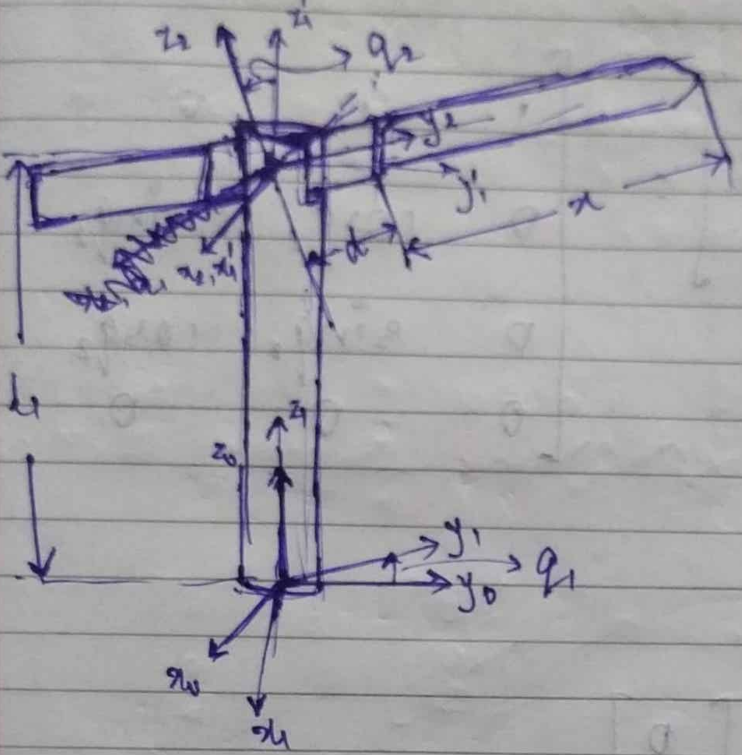
Thus,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

PTO.

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} 0 \\ 0 \\ -(r+d) \\ 1 \end{bmatrix}$$

8) RRP Stanford.



Here,

$$p_2 = (x+d)\hat{j}_2 = \begin{bmatrix} 0 \\ x+d \\ 0 \\ 1 \end{bmatrix}$$

$$d_1^2 = l_1 \hat{k}_1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_2 & -\sin q_2 \\ 0 & \sin q_2 & \cos q_2 \end{bmatrix}$$

Thus,

$$H^2_1 = \begin{bmatrix} R^2_1 & d^2_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos q_2 & -\sin q_2 & 0 \\ 0 & \sin q_2 & \cos q_2 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$d^1_0 = 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R^1_0 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus,

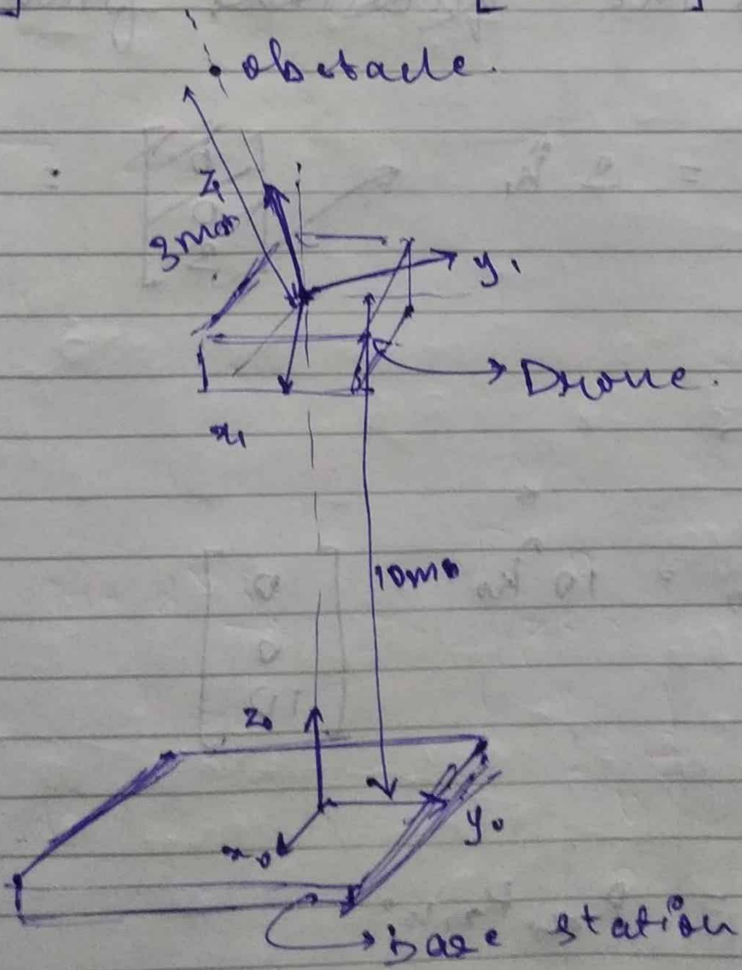
$$H^1_0 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} 0 \\ a+d \\ 0 \\ 1 \end{bmatrix}$$

9)



Thus, here first frame is attached to base station as explained in the question.

→ The other frame $D(x_1, y_1, z_1)$ is attached to the drone that is tilted as mentioned.

Thus,
position of obstacle as mentioned in the ~~question~~, frame $D(x_1, y_1, z_1)$.

$$P_1 = 3\hat{k}_1 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

Here,

$$d_0^1 = 10\hat{k}_0 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_0^1 = R_{z,60^\circ} R_{x,30^\circ}$$

$$= \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix}$$

$$R^1_0 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$R^1_0 = \begin{bmatrix} 1/2 & -3/4 & \sqrt{3}/4 \\ \sqrt{3}/2 & \sqrt{3}/4 & -1/4 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$H^1_0 = \begin{bmatrix} R^1_0 & d^1_0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -3/4 & \sqrt{3}/4 & 0 \\ \sqrt{3}/2 & \sqrt{3}/4 & -1/4 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H^1_0 \begin{bmatrix} P_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -3/4 & \sqrt{3}/4 & 0 \\ \sqrt{3}/2 & \sqrt{3}/4 & -1/4 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3\sqrt{3}/4 \\ -3/4 \\ 10 + 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

Thus, $P_0 = \frac{3\sqrt{3}}{4}\hat{i}_0 - \frac{3}{4}\hat{j}_0 + (10 + 3\sqrt{3}/2)\hat{k}_0$

iii) Refering to digram ~~for Q6~~, and matrices derived in Q6,

$\rho_1 = 1$ & $z_0 = \hat{k}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\rho_2 = 1$ & $z_1 = R^2_1 \cdot \hat{k}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\rho_3 = 0$

Thus,

$$J_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

SIlly,

$$\frac{\partial d_0^n}{\partial q_1} = z_0 \times (-R_0^0 \cdot d_0^n)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_2 \sin q_1 + l_3 \sin(q_2+q_1) \\ l_2 \cos q_1 + l_3 \cos(q_2-q_1) \\ l_1 - x - d \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} l_2 \sin q_1 + l_3 \sin(q_2+q_1) \\ l_2 \cos q_1 + l_3 \cos(q_2+q_1) \\ l_1 - x\, d \end{bmatrix}$$

$$\frac{\partial d_0^n}{\partial q_1} = \begin{bmatrix} -l_2 \cos q_1 - l_3 \cos(q_2+q_1) \\ l_2 \sin q_1 + l_3 \sin(q_2+q_1) \\ 0 \end{bmatrix}$$

$$\frac{\partial d_0^n}{\partial q_2} = \cancel{z_2 \times (R_{i-1}^{0}\, d_{i-1}^{n})}\,(R_0^2\, d)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} l_3 \cos(q_2+q_1) \\ l_3 \sin(q_2+q_1) \\ -x-d \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} l_3 \sin(q_2+q_1) \\ l_3 \cos(q_2+q_1) \\ -x-d \end{bmatrix}$$

$$\frac{\partial d_0^n}{\partial q_2} = \begin{bmatrix} -l_3 \cos(q_2+q_1) \\ l_3 \sin(q_2+q_1) \\ 0 \end{bmatrix}$$

$$\frac{\partial d_0^n}{\partial x} = z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

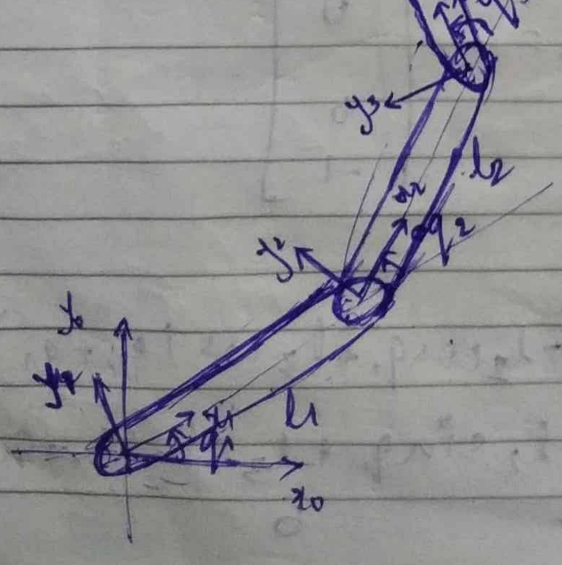
Thus,

$$J_v = \begin{bmatrix} -l_2 \cos q_1 - l_3 \cos(q_2+q_1) & -l_3 \cos(q_2+q_1) & 0 \\ l_2 \sin q_1 + l_3 \sin(q_2+q_1) & l_3 \sin(q_2+q_1) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Thus,

$$J = \begin{bmatrix} -l_2 \cos q_1 - l_3 \cos(q_2+q_1) & -l_3 \cos(q_2+q_1) & 0 \\ l_2 \sin q_1 + l_3 \sin(q_2+q_1) & l_3 \sin(q_2+q_1) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

13)



$$J_1 = \begin{bmatrix} Z_0 \times (O_n - O_0) \\ \\ Z_0 \end{bmatrix}$$

Here, $Z_0 = \hat{k}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$O_n - O_0 = R_0^0 d_0^n = d_0^n = R_0^1 d_1^n$$

$$= R_0^1 \left(d_1^2 + R_1^2 d_2^n \right)$$

$$= R_0^1 \left(d_1^2 + R_1^2 \left(d_2^3 + R_2^3 d_3^n \right) \right)$$

$$= R_0^1 d_1^2 + R_0^1 R_1^2 d_2^3 + R_0^1 R_1^2 R_2^3 d_3^n$$

$$d_3^n = l_3 \hat{i}_3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} \cos q_3 & -\sin q_3 & 0 \\ \sin q_3 & \cos q_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Page:
Date: / /

$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_2^3 = l_2 \hat{i}_2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$d_1^2 = l_1 \hat{i}_1 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^2 = R_0^1 \cdot R_1^2 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sly,

$$R_0^3 = \begin{bmatrix} c(q_1+q_2+q_3) & -s(q_1+q_2+q_3) & 0 \\ s(q_1+q_2+q_3) & c(q_1+q_2+q_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$O_n - O_0 = \begin{bmatrix} \cos q_1 & -s q_1 & 0 \\ s q_1 & c q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} c(q_1+q_2+q_3) & -s(q_1+q_2+q_3) & 0 \\ s(q_1+q_2+q_3) & c(q_1+q_2+q_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$O_n - O_1 = \begin{bmatrix} l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ l_1 s q_1 + l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

Thus,

$$z_0 \times (O_n - O_1) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ l_1 s q_1 + l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$z_1(O_n - O_1) = \begin{bmatrix} -(l_1 s q_1 + l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3)) \\ l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

Thus,

$$J_1 = \begin{bmatrix} -(l_1 s q_1 + l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3)) \\ +l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_1(O_n - O_2) \\ z_1 \end{bmatrix}$$

s||y,

$$J_2 = \begin{bmatrix} z_2 \times (O_n - O_2) \\ z_2 \end{bmatrix}$$

$$z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$O_n - O_2 = R_0^2 d_2^n = R_0^2 (d_2^3 + R_2^3 d_3^n)$$

$$= R_0^2 d_2^3 + R_0^3 d_3^n$$

$$= \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$ PTO.

$$+ \begin{bmatrix} c(q_1+q_2+q_3) & -s(q_1+q_2+q_3) & 0 \\ s(q_1+q_2+q_3) & c(q_1+q_2+q_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$O_n - O_2 = \begin{bmatrix} l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$z_2 \times (O_n - O_2) = \begin{bmatrix} -(l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3)) \\ l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} -(l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3)) \\ l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_3 \times (O_n - O_3) \\ z_3 \end{bmatrix}$$

$$z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_n - O_3 = R_0^3 \, d_3^n = \begin{bmatrix} c(q_1+q_2+q_3) & -s(q_1+q_2+q_3) & 0 \\ s(q_1+q_2+q_3) & c(q_1+q_2+q_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$O_n - O_3 = \begin{bmatrix} l_3 c(q_1 + q_2 + q_3) \\ l_3 s(q_1 + q_2 + q_3) \\ 0 \end{bmatrix}$$

$$z_2 \times (O_n - O_3) = \begin{bmatrix} -l_3 s(q_1 + q_2 + q_3) \\ l_3 c(q_1 + q_2 + q_3) \\ 0 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} -l_3 s(q_1 + q_2 + q_3) \\ l_3 c(q_1 + q_2 + q_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_1 s q_1 + l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3)) & -(l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3)) & -l_3 s(q_1+q_2+q_3) \\ l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) & l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) & l_3 c(q_1+q_2+q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$