ME 639 - Intro to Robotics

Assignment - 6, 7

1 Name :- Dev Patel
Roll No :- 18110113

1 Applying the Quintic Polynomial Trajectory to move from point A(0.4, 0.06, 0.1) to point B(0.4, 0.01, 0.1)

$$e(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

where $e(t)$ end effector position.

The boundary conditions are given as

$$e_0 = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 + a_4 t_0^4 + a_5 t_0^5$$
$$v_0 = a_1 + 2a_2 t_0 + 3a_3 t_0^2 + 4a_4 t_0^3 + 5a_5 t_0^4$$
$$\alpha_0 = 2a_2 + 6a_3 t_0 + 12a_4 t_0^2 + 20a_5 t_0^3$$

$$e_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5$$
$$v_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4$$
$$\alpha_f = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3$$

For our case at $t = t_0$, $e_0 = (0.4, 0.06, 0.1)$, $v_0 = 0$, $\alpha_0 = 0$
& at $t = t_f$, $e_f = (0.4, 0.01, 0.1)$, $v_f = 0$, $\alpha_f = 0$

Also, $t_0 = 0$

$\therefore$ $e_0 = a_0 = [0.4, 0.06, 0.1]$
$v_0 = a_1 = 0$
$\alpha_0 = 2a_2 = 0$

$t_f = 1$, i.e reaching point B in 1 sec

$e_f = a_0 + a_3 + a_4 + a_5 = [0.4, 0.01, 0.1]$
& $v_f = 3a_3 + 4a_4 + 5a_5 = 0$
$\alpha_f = 6a_3 + 12a_4 + 20a_5 = 0$

$\therefore \quad 4a_4 + 10a_5 = 0$

$a_4 = -5/2\, a_5$

$a_3 = \cancel{5a_5}\ 5/3\, a_5$

$\therefore\ a_0 + \frac{5}{3}a_5 + \frac{-5}{2}a_5 + a_5 = [0.4, 0.01, 0.1]$

$[0.4, 0.06, 0.1] + \frac{a_5}{6} = [0.4, 0.01, 0.1]$

$\cancel{a_5 = [0, -0.30, 0]}$

$\cancel{a_4 = [0, 0.625, 0]}$

$\cancel{a_3 = [0, -\frac{1.25}{3}, 0]}$

$a_5 = [0, -0.3, 0]$

$a_4 = [0, 0.75, 0]$

$a_3 = [0, -0.5, 0]$

3) Using the ~~scor~~ SCARA Manipulator to perform the given task.

We have the dynamic equations for the SCARA manipulator as

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 - (m_2 l_1 l_2 + 2 m_3 l_1 l_2)\sin q_2\, \dot{q}_1 \dot{q}_2$$
$$- \dot{q}_2^2 (m_2 l_1 l_2 + 2 m_3 l_1 l_2)\sin q_2 = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + d_{23}\ddot{q}_3 + \frac{1}{2}\dot{q}_1^2 (m_2 l_1 l_2 + 2 m_3 l_1 l_2)\sin q_1 = \tau_2$$

$$d_{32}\ddot{q}_2 + d_{33}\ddot{q}_3 + 1 = \tau_3$$

where $d_{11} = \frac{m_1}{4} l_1^2 + m_2 l_1^2 + \frac{m_2}{4} l_2^2 + m_2 l_1 l_2 \cos q_2$
$+ m_3 l_1^2 + m_3 l_2^2 + 2 m_3 l_1 l_2 \cos q_2$
$+ I_1 + I_2 + I_3$

$$d_{12} = d_{21} = m_2 l_1 l_2 \cos q_2 + \frac{m_2}{4} l_2^2 + 2 m_3 l_1 l_2 \cos q_3 + m_3 l_2^2 + I_2 + I_3$$

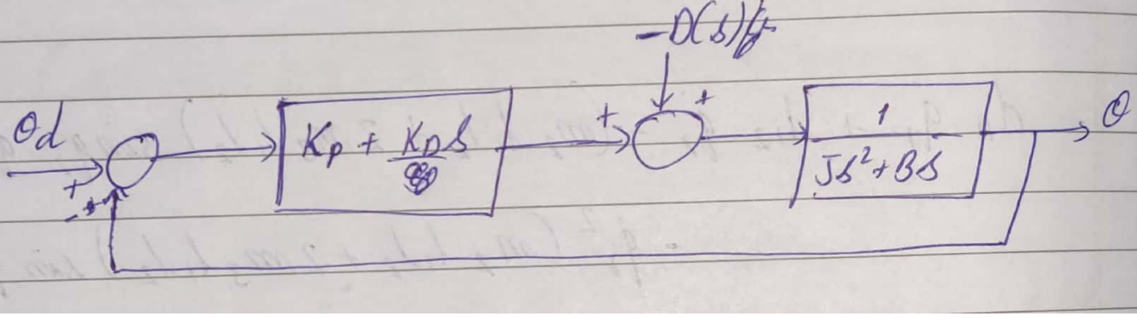
$$d_{23} = d_{32} = I_3$$

$$d_{22} = \frac{m_2}{4} l_2^2 + m_3 l_2^2 + I_2 + I_3$$

$$d_{33} = \frac{m_3}{4}$$

Here m_1, m_2 & m_3 are mass of the links l_1, l_2 & l_3 respectively.

a) P D controller



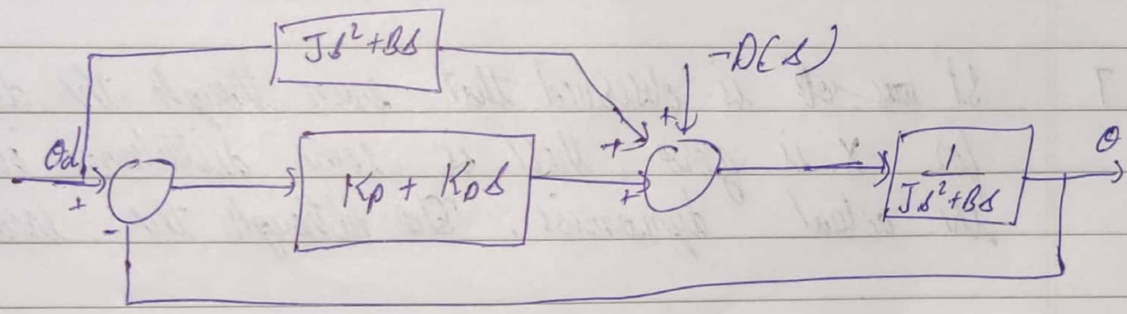
Control Equations

$$\frac{(\theta_d \mp \theta)(K_p + K_D s) - D(s)}{Js^2 + Bs} = \theta$$

$\therefore K_p \theta_d \mp K_p \theta + \theta_d K_D s \mp \theta K_D s - D(s) = \theta(Js^2 + Bs)$

$\Rightarrow J_{eff} \ddot{\theta} + (B_{eff} + K_D)\dot{\theta} - K_p(\theta_d - \theta_s) - K_d \dot{\theta}_d = -d(t)$

b) PD controller with Feed Forward

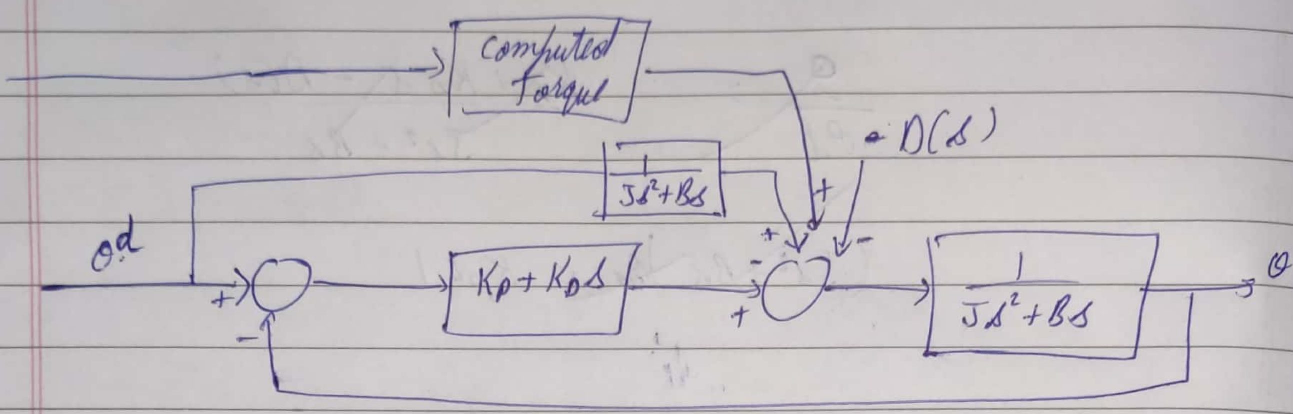


Control Equations

$(\theta_d - \theta)(K_p + K_D s) + \theta_d(Js^2 + Bs) - D(s) = \theta(Js^2 + Bs)$

$J(\ddot{\theta}_d - \ddot{\theta}) + B(\dot{\theta}_d - \dot{\theta}) + K_D(\dot{\theta}_d - \dot{\theta}) + K_p(\theta_d - \theta) = d(t)$

c) Computed Torque



$$(\theta d-\theta)(K_p+K_D s) + \theta d(Js^2+Bs) - D(s) \overset{+\tau_{computed}}{\wedge} = \theta(Js^2+Bs)$$

$$J(\ddot{\theta}d-\ddot{\theta}) + (B+K_D)(\dot{\theta}d-\dot{\theta}) + K_P(\theta d-\theta) + \tau_{computed} = d(t)$$

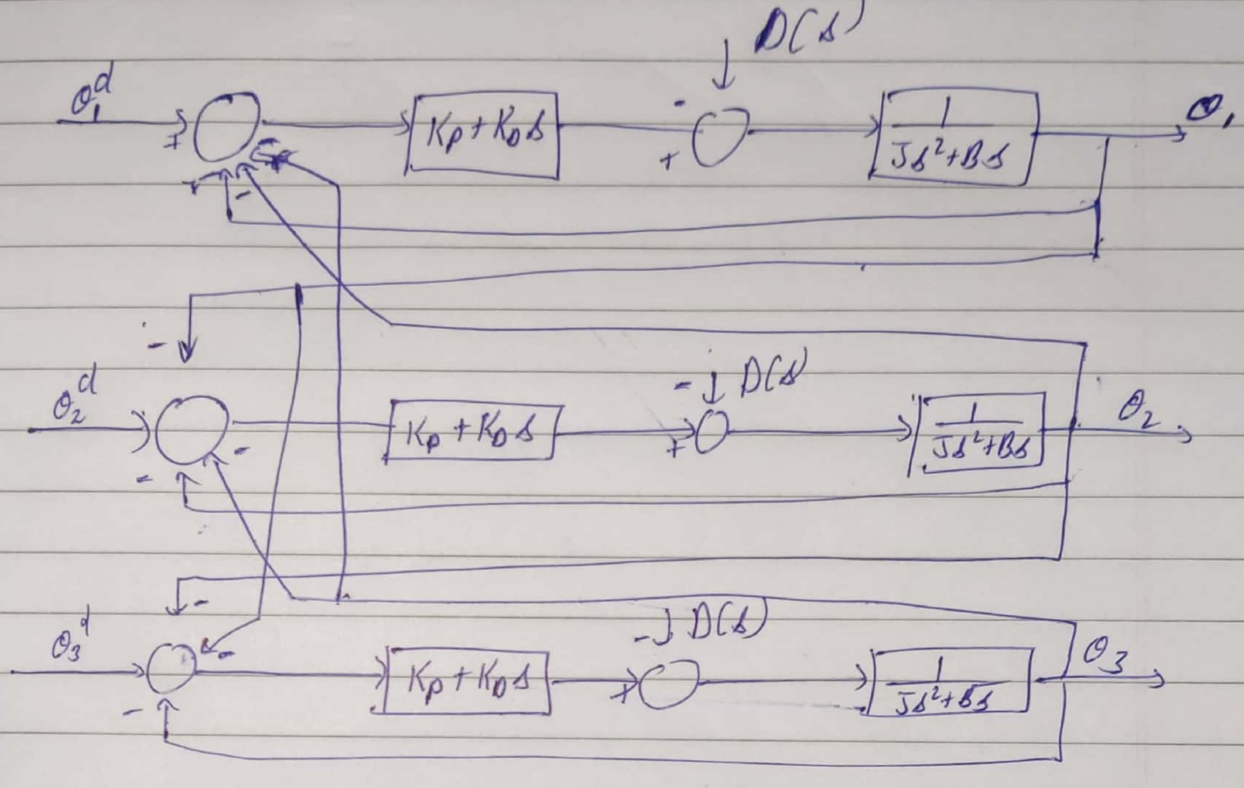
The Gains K_P & K_D chosen for the ~~gan~~ given task is

K_P = ~~10000~~ 1000

K_D = 9

classmate
Date
Page

D) Multivariable Control



Control Equations

τ_4

$$J_{eff}\ddot{\theta}_1 + (B+K_D)\dot{\theta}_1 + K_D\dot{\theta}_2 + K_D\dot{\theta}_3 + K_p(\theta_1^d - \theta_1) + K_p\theta_2 + K_p\theta_3 = -d(t)$$

Ally for θ_2 & θ_3

7 It ~~was of~~ is observed that even though the desired trajectory for x is zero; there is some disturbance observed during the actual dynamics. ~~The~~ Although the error is very less.