Prathamesh Vibhute 18110186

Assignment -3

Q.1

We know that,

$$\dot{x} = J(q)\dot{q}$$

$\dot{x}$ = end effector velocity vector

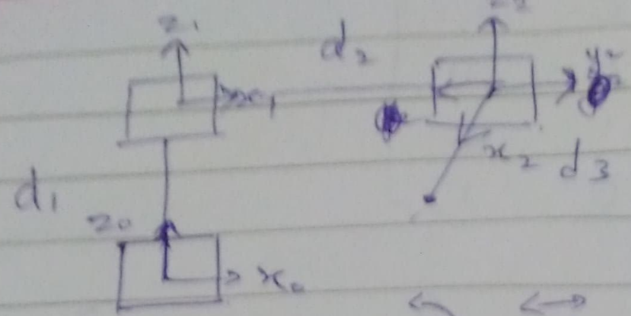
$\dot{q}$ = joint velocities vector

$J(q)$ => Jacobian matrix mapping $\dot{x}$ & $\dot{q}$

The configuration for which rank of Jacobian decreases, where robot loses one or more DOF and it comes impossible for it to move the end effector in a particular direction is called singularity configuration.

To find a singularity configuration put $\det J(q) = 0$ and obtain the values of q.

Q.5



	x	x	z	z
i	a	α	d	θ
1	0	0	d_1	0
2	d_2	0	0	0
3	d_3	0	0	-90°

We know that, transformation matrix

$$T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_1^2 = \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -d_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

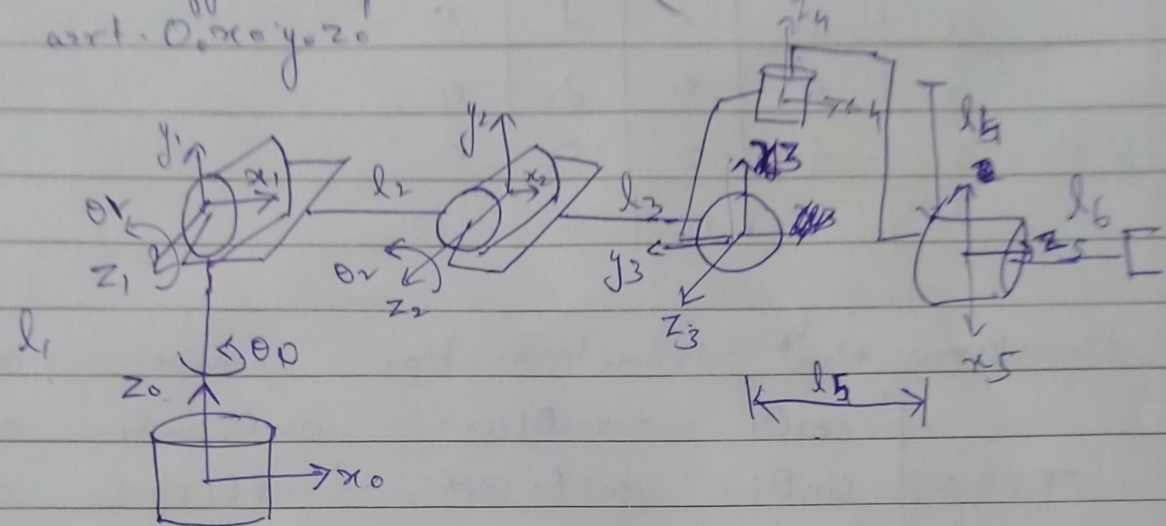
$$T_0^3 = T_0^1\, T_1^2\, T_2^3$$

$$= \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -d_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} 0 & 1 & 0 & d_2 \\ -1 & 0 & 0 & -d_3 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ End effector position $= (d_2, -d_3, d_1)$
w.r.t. $O_0 x_0 y_0 z_0$

Q.6



	$\overset{\leftrightarrow}{x}$	$\overset{\circlearrowleft}{x}$	$\overset{z}{\leftrightarrow}$	$\overset{\circlearrowleft}{z}$
i	a	α	d	θ
1	0	90	l_1	θ_0
2	l_2	0	0	θ_1
3	l_3	0	0	$\theta_2 + 90$
4	l_4	-90	0	$\theta_3 - 90$
5	l_5	-90	$-l_3$	$\theta_4 - 90$
6	0	0	l_6	0

We know

$$T_0^i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} \cos\theta_0 & 0 & \sin\theta_0 & 0 \\ \sin\theta_0 & 0 & \cos\theta_0 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad T_1^2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & l_2\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & l_2\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

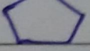
$$T_2^3 = \begin{bmatrix} -\sin\theta_2 & +\cos\theta_2 & 0 & -l_3\sin\theta_2 \\ \cos\theta_2 & -\sin\theta_2 & 0 & l_3\cos\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad T_3^4 = \begin{bmatrix} \sin\theta_3 & 0 & -\cos\theta_3 & l_4\sin\theta_3 \\ \cos\theta_3 & 0 & \sin\theta_3 & l_4\cos\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^5 = \begin{bmatrix} \sin\theta_4 & 0 & -\cos\theta_4 & l_5\sin\theta_4 \\ \cos\theta_4 & 0 & \sin\theta_4 & l_5\cos\theta_4 \\ 0 & -1 & 0 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad T_5^6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

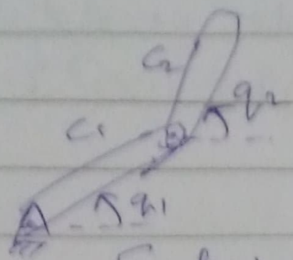
$$T_0^6 = T_0^1 \, T_1^2 \, T_2^3 \, T_3^4 \, T_4^5 \, T_5^6$$

Q.7 A) Direct - Driven - Does not involve any transmission element between the actuators and the joints.
Advantage - Very less power loss

B) Remotely driven - Drive shafts are used to transfer the motion to the actuated joints of the arm
Advantage - Only the rotation angle of motor needed.

C) 5 bar Parallel - Two inputs are given 
Advantage - More strength and firmness.

Q.8



$$v_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_1}{2}\cos q_1 \\ 0 \end{bmatrix} \qquad v_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1 \cos q_1 & \frac{l}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \qquad \omega_2 = \dot{q}_2 \hat{k}$$

$$K = \frac{1}{2}\sum_{i=1}^{n} m_i v_{c_i}^T v_{c_i} + \frac{1}{2}\sum_{i=1}^{n} \omega_i^T I_i \omega_i$$

$$v_{c_i} = J_{v_{c_i}}(q)\,\dot{q} \qquad \omega_i = R_i^T J_{\omega_i}(q)\,\dot{q}$$

$$K = \frac{1}{2}\dot{q}^T \sum_{i=1}^{n}\left[m_i J_{v_{c_i}}(q)^T J_{v_{c_i}}(q) + J_{\omega_i}(q)^T R_i(q) I_i R_i(q)^T J_{\omega_i}(q)\right]\dot{q}$$

$$K = \frac{1}{2}\dot{q}^T D(q)\,\dot{q}$$

For 2R Manipulator

$$D(q) = \begin{bmatrix} m_1\frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

Computing the christoffel symbols.

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ii}}{\partial q_k} \right]$$

$$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2} + \cancel{\frac{\partial d_{12}}{\partial q_1}} - \cancel{\frac{\partial d_{12}}{\partial q_1}}\right]$$

$$= \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\cancel{\frac{\partial d_{22}}{\partial q_1}}^{0} = \frac{\partial d_{12}}{\partial q_2} = -m_2\frac{l_1 l_2}{2}\sin(q_2-q_1)$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\cancel{\frac{\partial d_{11}}{\partial q_2}}^{0} = m_2\frac{l_1 l_2}{2}\sin(q_2-q_1)$$

$$c_{212} = c_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0, \quad c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential Energy

$$V = m_1 g\frac{l_1}{2}\sin q_1 + m_2 g\left(l_1\sin q_1 + \frac{l_2}{2}\sin q_2\right)$$

$$\phi_1 = \frac{\partial V}{\partial q_1} = m_1 g\frac{l_1}{2}\cos q_1 + m_2 g l_1\cos q_1$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 g\frac{l_2}{2}\cos q_2$$

$$z_k = \sum_{j=1}^{2} d_{kj}\,\ddot{q}_j + \sum_{i,j} c_{ijk}\,\dot{q}_i\,\dot{q}_j + \phi_k \qquad k = 1, 2.$$

$$z_1 = d_{11}\,\ddot{q}_1 + d_{12}\,\ddot{q}_2 + \cancelto{0}{c_{111}}\,\dot{q}_1\,\dot{q}_1 + \cancelto{0}{c_{121}}\,\dot{q}_1\,\dot{q}_2 + \cancelto{0}{c_{211}}\,\dot{q}_2\,\dot{q}_1 + c_{221}\,\dot{q}_2\,\dot{q}_2 + \phi_1$$

$$z_1 = d_{11}\,\ddot{q}_1 + d_{12}\,\ddot{q}_2 + c_{221}\,\dot{q}_2^{2} + \phi_1$$

$$\tau_2 = d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1\dot{q}_1 + \cancel{c_{122}}\dot{q}_1\dot{q}_2 + \cancel{c_{212}}\dot{q}_2\dot{q}_1 + \cancel{c_{222}}\dot{q}_2\dot{q}_2 + \phi_2$$

$$\tau_2 = d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2$$

Q.10

Given :- $D(q)$ and $V(q)$

1) Compute christofell symbols using the elements of $D(q)$

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

2) Using $V(q)$, calculate ϕ_k

$$\phi_k(q) = \frac{\partial V(q)}{\partial q_k}$$

3) Put the values calculated above in the Euler - Lagrange's Equation

$$\tau_k = \sum_{j=1}^{n} d_{kj}\ddot{q}_j + \sum_{i,j} c_{ijk}\dot{q}_i\dot{q}_j + \phi_k$$