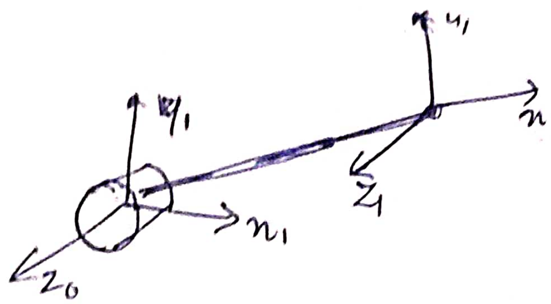
~~7~~

Qu

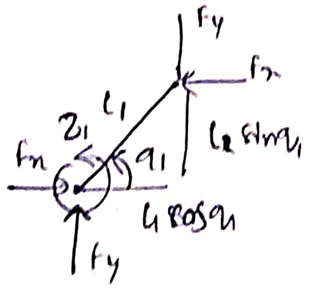

The DH parameters are basic.

| | $a_1$ | $d_1$ | $\alpha$ | $\theta_1$ |
|---|---|---|---|---|
| | $L_1$ | 0 | 0 | $\dot{\theta}$ |

So corresponding $T_0^1 = \begin{bmatrix} C_1 & -S_1 & 0 & a_1C_1 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) For the joint to behave like a virtual torsional stiffness with linear characteristics.

Now neglecting gravity:




$f_y = K(y - y_0)$ $\quad f_x = K(x - x_0)$

$\Sigma M_{01} = 0$ $\quad y = l_1 \sin q_1$ $\quad x = l_2 \cos q$

$F_y l_1 \cos q_1 - F_x l_1 \sin q_1 = Z_1$

$K l_1 \sin q_1 l_1 \cos q_1 - K l_2 \cos q_1 l_1 \sin q_1 = Z_1$

$K l_1^2 (\sin q_1 \cos q_1) - K y_0 l_1 \cos q_1$

$- K l_1^2 \cos q_1 \sin q_1 + K x_0 \sin q_1 =$

$K l_1 x_0 \sin q_1 - K y_0 l_1 \cos q_1 = Z_1$

4b The dynamics of the single link with gravity can be calculated applying Lagrangian Equations.

$$\mathcal{L} = K - V$$

K.E P.E

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) = Q \text{ generalized forces.}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \left(\frac{\partial L}{\partial q_1}\right) = \tau_1$$

$$\mathcal{L} = KE - PE$$

$$KE = \tfrac{1}{2}(I)\,\dot{q}_1^2$$

$$= \tfrac{1}{2}\left(\tfrac{1}{3} m_1 l_1^2\right)\dot{q}_1^2$$

$$= \tfrac{1}{6} m_1 l_1^2 \dot{q}_1^2$$

$$PE = mg\tfrac{l}{2}\sin(q_1)$$

$$L = \left(\tfrac{1}{6} m_1 l_1^2 \dot{q}_1^2 - mg\tfrac{l}{2}\sin q_1\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) = \frac{d}{dt}\left(\frac{1}{3} m_1 l_1^2 \dot{q}_1\right) = \tfrac{1}{3} m_1 l_1^2 \ddot{q}_1$$

$$\frac{\partial L}{\partial q_1} = -mg\tfrac{l}{2}\cos q_1$$

$$= \boxed{\tfrac{1}{3} m_1 l_1^2 \ddot{q}_1 + mg\tfrac{l}{2}\cos q_1 = \tau_1}$$

$$\tau = \tfrac{1}{3} m l_1^2 \ddot{q}_1 + mg\,l\cos q_1 + K y_0 x_0 \sin q_1 - K x_0 l_1 \cos q_1 =$$