Assignment 3

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Ques 1)
Ans)

Singularities are points in the workspace of a robot where the joints are unable to cause any change at end-effector. As a result, the manipulator end-effector cannot move in certain directions. Such configurations of the robot when it reaches these singularities, are called Singular configurations. At these configurations, one or more degrees of freedom of the system become uncontrollable. These configurations are highly unstable and may cause the system to fail. Thus while planning the path/trajectory, it is important to avoid the singularities as much as possible. Mathematically, the inverse of Jacobian matrix ceases to exist and the matrix loses it's rank.

Yes by checking the rank of the Manipulator Jacobian in some configuration, it is possible to detect whether that configuration is close to singularity or not.

Ques 7)
Ans)

For a **Serial chain 2R (direct drive configuration)**, the equations of motion are:

$$
\begin{aligned}
d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + \phi_1 &= \tau_1 \\
d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 &= \tau_2.
\end{aligned}
$$

The equations of motion contain coriolis force terms & centrifugal terms. They seem fairly long & computationally expensive to be executed on small microprocessors. However, this setup is very easy & simple to execute practically. The motors are directly attached the the ends of the links & drive them. One requires only links, Motors & microprocessor to execute this configuration.

Now, In a **remotely driven configuration** of 2R, the first joint is directly actuated by one motor but the other is actuated by a motor that is situated on the base, by using certain gear or timing belt mechanism. The equations of motion in this case are:

$$
\begin{aligned}
d_{11}\ddot{p}_1 + d_{12}\ddot{p}_2 + c_{221}\dot{p}_2^2 + \phi_1 &= \tau_1 \\
d_{21}\ddot{p}_1 + d_{22}\ddot{p}_2 + c_{112}\dot{p}_1^2 + \phi_2 &= \tau_2 .
\end{aligned}
$$

These are relatively simpler than those in direct drive configuration. By remotely driving one of the links, the coriolis terms from the 1st equation have been eliminated, but the centrifugal coupling still remains between the links. This coupling restricts independent control of individual quantity q1 & q2. Changing one affects the other and that has to be changed accordingly.
The advantage of this over the direct drive configuration is that this eliminates the coriolis forces coulpling the two links which makes controlling the system easier. However, execution of this configuration requires a high performance timing belt or accurate gearing mechanism, which makes it a bit complex to execute in real life.

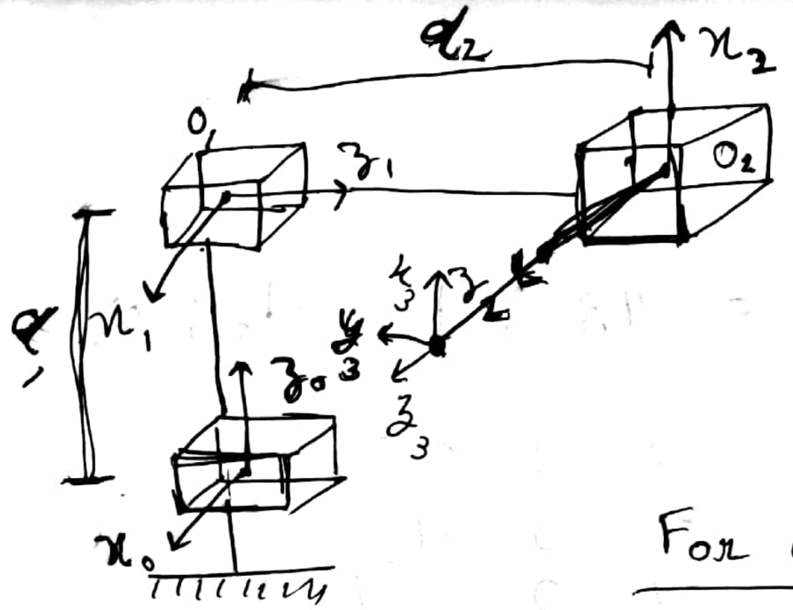
The **Five-bar linkage configuration** takes care of this problem. The final equations in Five-baar linkage configuration are:

$$
d_{11}\ddot{q}_1 + \phi_1(q_1) = \tau_1, \qquad d_{22}\ddot{q}_2 + \phi_2(q_2) = \tau_2 .
$$

This set of completely decoupled equations is very easy to understand and code. These decoupled equations allow one to **independently** control the parameters q1 & q2 as they are decoupled in this system. This is a huge advantage over the previous two configurations (direct-driven & remote-driven) in terms of controls & dynamics. However, this setup is very complex & difficult to execute. While it is fairly easy on the controls side, the practical execution part of it is difficult. The linkages have to be designed and manufactured properly to make things work smoothly. This configuration has more number of joints as compared to the other ones. This can become a source of unnecessary damping (frictional) in the system, which can create problems.

Q5)

Ans)



For Link 1 :-

$$a_1 = 0 \;;\; \alpha_1 = -90^\circ \text{ (clockwise dir}^n\text{)}$$

$$d_1 = d_1 \;;\; \theta_1 = 0^\circ$$

Thus;

$$\cancel{A_1 = \left[\cos 0 \quad -\sin 0 . \cos(-90) \quad \sin 0 \sin(-90)\right.}$$

$$A_1 = \begin{bmatrix} \cos 0 & -\sin 0 . \cos(-90) & \sin 0 \sin(-90) & 0 . \cos(0) \\ \sin(0) & \cos(0)\cos(-90) & -\cos(0)\sin(-90) & 0 . \sin(0) \\ 0 & \sin(-90) & \cos(-90) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For link 2 :

$a_2 = 0$; $\alpha_2 = -90^\circ$; $d_2 = d_2$; $\theta_2 = -90^\circ$

thus

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -d_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly for link 3 :

$a_3 = 0$; $\alpha_3 = 0$; $d_3 = d_3$; $\theta_3 = 0$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Table:

Link	a_i	α_i	d_i	θ_i
1	0	-90°	d_1	0
2	0	-90°	d_2	-90
3	0	0	d_3	0

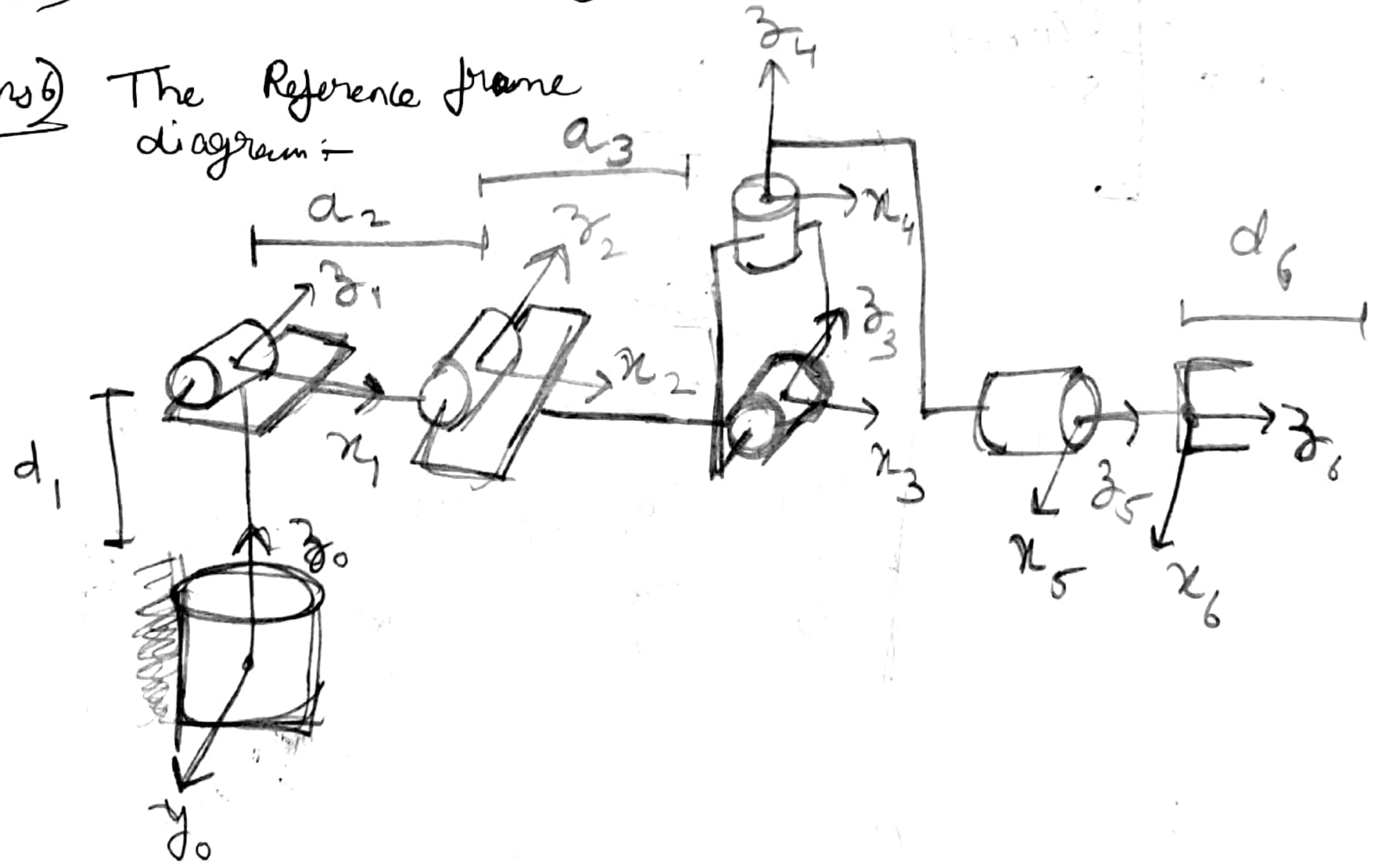
end - effector:-

$$T_0^3 = A_1 A_2 A_3$$

$$= \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q6) {3.8 from book}

Ans6) The Reference frame diagram:-



The table of joint Parameter for this will be :-

Link	a_i	α_i	d_i	θ_i
1	0	-90°	d_1	$\theta_1 + 90 = \theta_1'$
2	a_2	0	0	$-\theta_2 = \theta_2'$
3	a_3	0	0	$-\theta_3 = \theta_3'$
4	0	90	0	$-\theta_4 = \theta_4'$
5	0	-90	0	$-\theta_5 - 90^\circ = \theta_5'$
6	0	0	d_6	$\theta_6 = \theta_6'$

using $$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

we can find

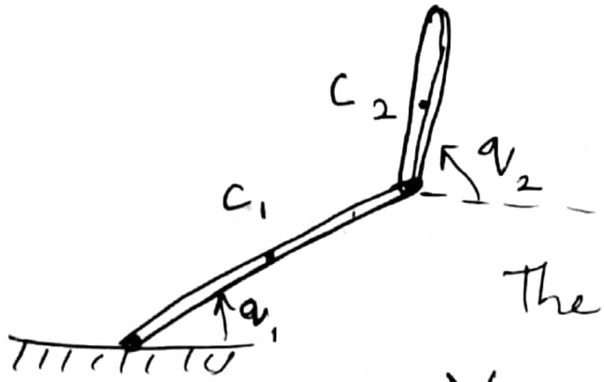
A_1, A_2, A_3, A_4, A_5 & A_6

the end-effector Pos^n will be

$$\boxed{T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6}$$

Q.8

Ans 8



The velocity of Com

$$V_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_2}{2}\cos q_1 \\ 0 \end{bmatrix}$$

$$\cancel{V_{c_2} = \begin{bmatrix} -l_1 \sin q_1 - (l_2/2)\sin q_2 \\ l_1 \cos q_1 \end{bmatrix}}$$

$$V_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -(l_2/2)\sin q_2 \\ l_1 \cos q_1 & (l_2/2)\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \quad , \quad \omega_2 = \dot{q}_2 \hat{k}$$

Kinetic energy

$$\Rightarrow K = \sum_{i=1}^{n} \frac{1}{2} m_i V_{c_i}^T V_{c_i} + \frac{1}{2} \sum_{i=1}^{n} \omega_i I_i \omega_i$$

$$V_{c_i} = J_{v_{c_1}}(q)\dot{q} \quad ; \quad \omega_i = R_i^T J_{\omega_1}(q)\dot{q}$$

$$K = \frac{1}{2} \dot{q}^T \sum_{i=1}^{n} \Big[m_1 J_{v_{c_1}}^T(q) J_{v_{c_1}}(q) + J_{\omega_i}(q)^T R_i(q) I_1 R_i(q)^T J_{\omega_1}(q) \Big] \dot{q}$$

$$\Rightarrow \underline{K = \tfrac{1}{2} \dot{q}\, D(q)\, \dot{q}}$$

thus

$$\Rightarrow D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & \frac{m_2 l_1 l_2}{2} \cos(q_2 - q_1) \\ \frac{m_2 l_1 l_2}{2} \cos(q_2 - q_1) & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix}$$

The Christoffel symbols become :-

$$c_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = 0$$

$$c_{221} = \frac{\partial d_{22}}{\partial q_1} \times \frac{1}{2} = \frac{\partial d_{12}}{\partial q_2} = -\frac{m_2 l_1 l_2}{2} \sin(q_2 - q_1)$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = \frac{m_2 l_1 l_2}{2} \sin(q_2 - q_1)$$

$C_{122} = 0$; $C_{222} = 0$

The final eq^n become :-

$$\boxed{\left(\frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1\right)\ddot{q}_1 + \frac{m_2 l_1 l_2}{2}\cos(q_1 - q_2)\,\ddot{q}_2 + \left(-\frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1)\right)\dot{q}_2^{\,2} + \left(\frac{m_1 l_1}{2} + m_2 l_1\right) g \cos q_1 = \tau_1}$$

$$\boxed{\left(\frac{m_2 l_1^2}{4} + I_2\right)\ddot{q}_2 + \left(\frac{m_1 l_1 l_2}{2}\cos(q_2 - q_1)\right)\ddot{q}_1 + \frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1) = \tau_2}$$

Q.10

Ans 10 The most important Eq^n is

$$\boxed{\sum_j d_{kj}(q)\,\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\,\dot{q}_i\dot{q}_j - \phi_k(q) = \tau_k}$$

special case of Euler-Lagrange Eq^n

where

$$C_{ijk}(q) = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

are the christoffel's symbols. and

$$\boxed{\phi_k(q) = \frac{\partial V(q)}{\partial q_k}}$$

Since it is given that $V(q)$ & $D(q)$ are known, the above quantities & thus the equations of motions (k- equations) can be worked out.