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19/09/2021

ASSIGNMENT - 3

1. Singularities: configurations of manipulators for which the rank of J decreases / is less than its max value.

Near singularities, a unique solution to the inverse kinematics problem won't exist.

Singularity can be identified by:

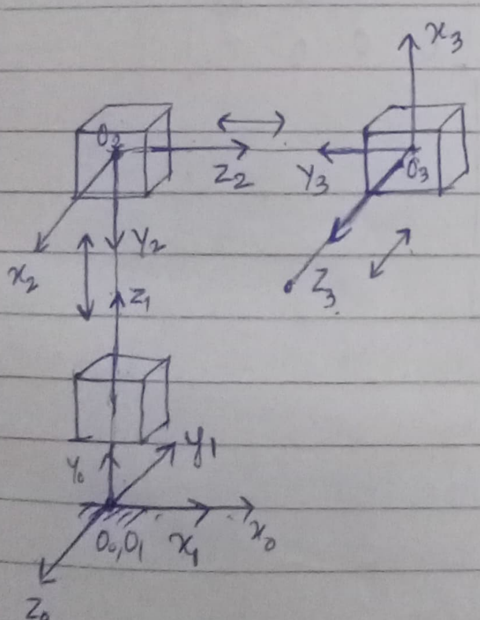
$$\det(J(q)) = 0$$

2. Reading Assignment

3. Code seperately submitted.

~~4.~~

5.



	a_i	α_i	d_i	θ_i
1	0	-90°	d_1^*	0
2	0	-90°	d_2^*	-90°
3	0	90°	d_3^*	90°

$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & d_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$T_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & d_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

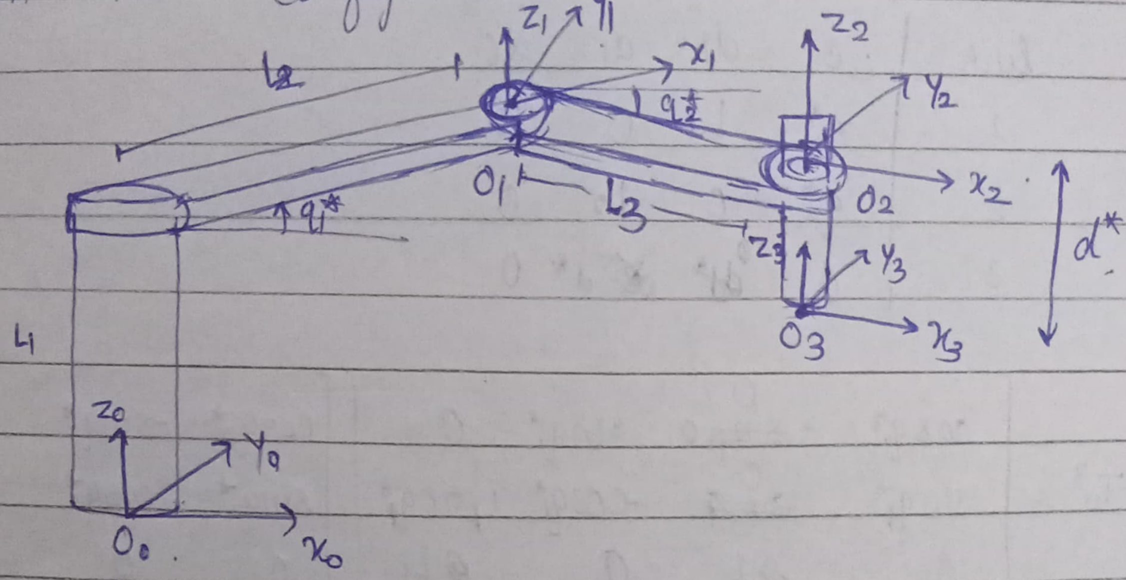
$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2^* \\ 0 & -1 & 0 & d_1^* + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & d_3^* \\ -1 & 0 & 0 & d_2^* \\ -1 & 0 & 0 & d_1^* + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End effector position

$$P = \begin{bmatrix} d_3^* \\ d_2^* \\ d_1^* + d_2^* \end{bmatrix}$$

4. SCARA RRP Configuration



Link	θ_i	d_i	a_i	α_i
1	q_1^*	L_1	L_2	0
2	q_2^*	0	L_3	0
3	0	d^*	0	0

DH Parameters

Let's take following values:

$q_1^* = \pi/4$

$q_2^* = -\pi/6$

$L_1 = 3$

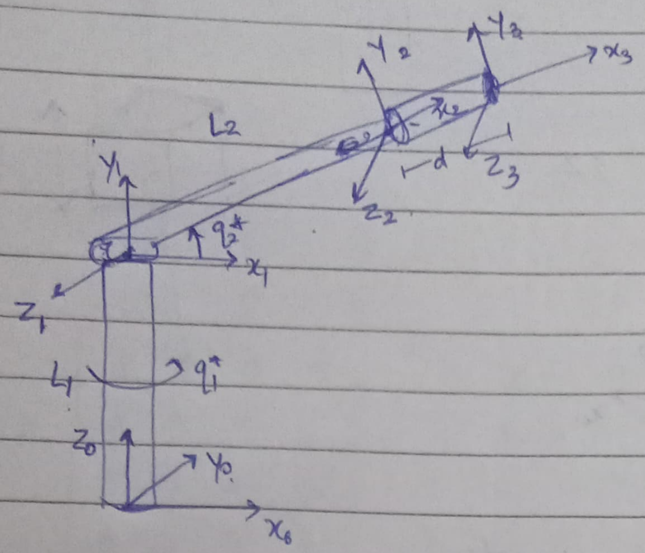
$L_2 = 5$

$L_3 = 8$

$d^* = 2$

The numerical & calculated values are quite similar but not same due to the approximations made in hand-calculations

Standford configuration



link	θ_i	d_i	a_i	α_i
1	q_1^*	L_1	0	90°
2	q_2^*	0	L_2	0
3	0	$\cancel{d^*}$ 0	$\cancel{d^*}$ d^*	0

$$T_0^3 = \begin{bmatrix} \cos q_1^* & \cancel{-\sin q_1} \; 0 & \sin q_1^* & 0 \\ \sin q_1^* & \cancel{\cos q_1} \; 0 & -\cos q_1^* & L_1 \sin q_1^* \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos q_2^* & -\sin q_2^* & 0 & L_2 \cos q_2^* \\ \sin q_2^* & \cos q_2^* & 0 & L_2 \sin q_2^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d^* \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 cq_2 & -cq_1 sq_2 & sq_1 & L_2 cq_1 cq_2 \\ cq_2 sq_1 & -sq_1 sq_2 & -cq_1 & L_2 cq_2 sq_1 + L_1 sq_1 \\ sq_2 & cq_2 & 0 & L_2 sq_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d^* \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 cq_2 & -cq_1 sq_2 & sq_1 & cq_1 cq_2 d^* + L_2 cq_1 cq_2 \\ cq_2 sq_1 & -sq_1 sq_2 & -cq_1 & cq_2 cq_1 d^* + L_2 cq_2 sq_1 + L_1 sq_1 \\ sq_2 & cq_2 & 0 & sq_2 d^* + L_2 sq_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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End effector position = $\begin{bmatrix} cq_1 cq_2 d^* + L_2 cq_1 cq_2 \\ cq_2 cq_1 d^* + L_2 cq_2 sq_1 + L_1 sq_1 \\ sq_2 d^* + L_2 sq_2 + L_1 \end{bmatrix}$

Results same as for SCARA
(comparision with values obtained from code)

7 Direct Drive Manipulator:

→ No gears at joints → low costs, no backlash, faster response time

→ Motor with high torque is directly attached to the ~~main j~~ joint.

Remotely- Driven

→ Gear mechanism is used to drive joints

→ Low torque motors suffice

→ Implementation & control is easy

5- Bar Parallelogram Arrangement.

→ This is an extension to remotely driven joint. Motors stay on ground and ~~ta~~ configuration of links is such ~~to~~ that 2R- Manipulator type workspace is achieved

→ Size smaller than serial manipulator.

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10 Given: $D(\vec{q})$ and $V(\vec{q})$

Derive equations of motions.

d_{ij} are elements of matrix $D(\vec{q})$.

Then kinetic energy will be

$$K = \frac{1}{2} \sum_{i,j}^{n} d_{ij}(\phi) \dot{q}_i \dot{q}_j$$

$$= \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

Lagrangian, $L = K - V$

$$= \frac{1}{2} \sum_{i,j}^{n} d_{ij}(\phi) \dot{q}_i \dot{q}_j - V(q)$$

$$\frac{\partial L}{\partial \dot{q}_k} = \sum d_{kj}(q) \dot{q}_j$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_j d_{kj}(q) \dot{q}_j + \sum_j \frac{d}{dt}(d_{kj}(q)) \dot{q}_j$$

$$= \sum_j d_{kj}(q) \ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i} \dot{q}_i \dot{q}_j$$

$$\frac{\partial L}{\partial q_k} = \frac{1}{2} \sum_{i,j} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

Therefore, Euler-Lagrange's equations become

$$\sum_j d_{kj}\ddot{q}_j + \sum_{ij}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k$$

By symmetry

$$\sum_{ij}\left(\frac{\partial d_{kj}}{\partial q_i}\right)\dot{q}_i\dot{q}_j = \frac{1}{2}\sum_{ij}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}\right]\dot{q}_i\dot{q}_j$$

Hence

$$\sum_{ij}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j = \sum_{i,j}\frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j$$

The terms $C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$ Are known as Christoffel Symbols (of the first kind)

Then Euler-Lagrange's Equations Become

$$\boxed{\sum_j d_{kj}(q)\,\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\,\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k \qquad \text{Where} \quad \phi_k(q) = \frac{\partial V}{\partial q_k}}$$

More common to write

$$\boxed{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau}$$

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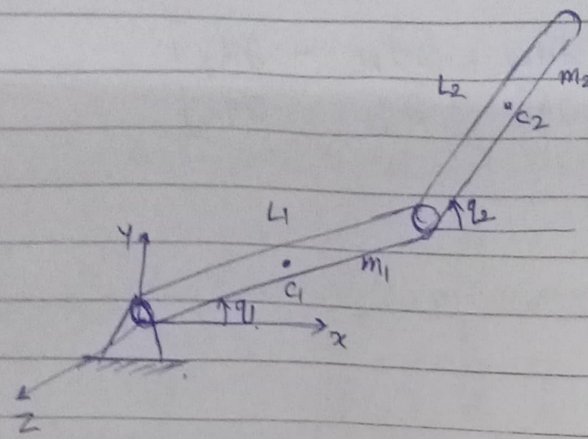
8 Derive dynamic equations of 2R Manipulator



Angular velocities:

$\omega_1 = \dot{q}_1 \hat{k}$ $\qquad \omega_2 = \dot{q}_2 \hat{k}$

Linear velocities about center of mass.

$$V_{c1} = \begin{bmatrix} -l_1/2 \sin q_1 \\ l_1/2 \cos q_1 \\ 0 \end{bmatrix} \dot{q}_1 \qquad V_{c2} = \begin{bmatrix} -l_1 s q_1 & -l_2/2 \, s q_2 \\ l_1 c q_1 & l_2/2 \, c q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Kinetic Energy $K = \frac{1}{2} \sum_{i=1}^{n} m_i v_{ci}^T v_{ci} + \frac{1}{2} \sum_{i=1}^{n} w_i^T I_i w_i$

Also

$v_{ci} = J_{vci}(q) \cdot \dot{q}$ $\qquad w_i = R_i^T J_{wi}(q) \dot{q}$

$$K = \frac{1}{2} \dot{q}^T \sum_{i=1}^{n} \left[m_i J_{vci}(q)^T J_{vci}(q) + J_{wi}(q)^T R_i(q) I_i R_i(q)^T J_{wi}(q) \right] \dot{q}$$

$$K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

So, $D(q)$ for 2R Manipulator is

$$D(q) = \begin{bmatrix} m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 \frac{l_1 l_2}{2} c(q_2 - q_1) \\ m_2 \frac{l_1 l_2}{2} c(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

Computing the Christo~~pher~~ffel Symbols

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0 \qquad c_{ijk} = c_{jik}$$

$$c_{121} = c_{211} = \frac{1}{2}\left[\frac{\partial d_{21}}{\partial q_1} + \frac{\partial d_{11}}{\partial q_2} - \frac{\partial d_{12}}{\partial q_1}\right]$$

$$= 0$$

$$c_{221} = \frac{1}{2}\left[\frac{\partial d_{12}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_2} - \frac{\partial d_{22}}{\partial q_1}\right]$$

$$= \frac{m_2 l_1 l_2}{2}\left[-\sin(q_2 - q_1)\right]$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = \frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1)$$

$$c_{212} = c_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

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Potential Energy $V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$

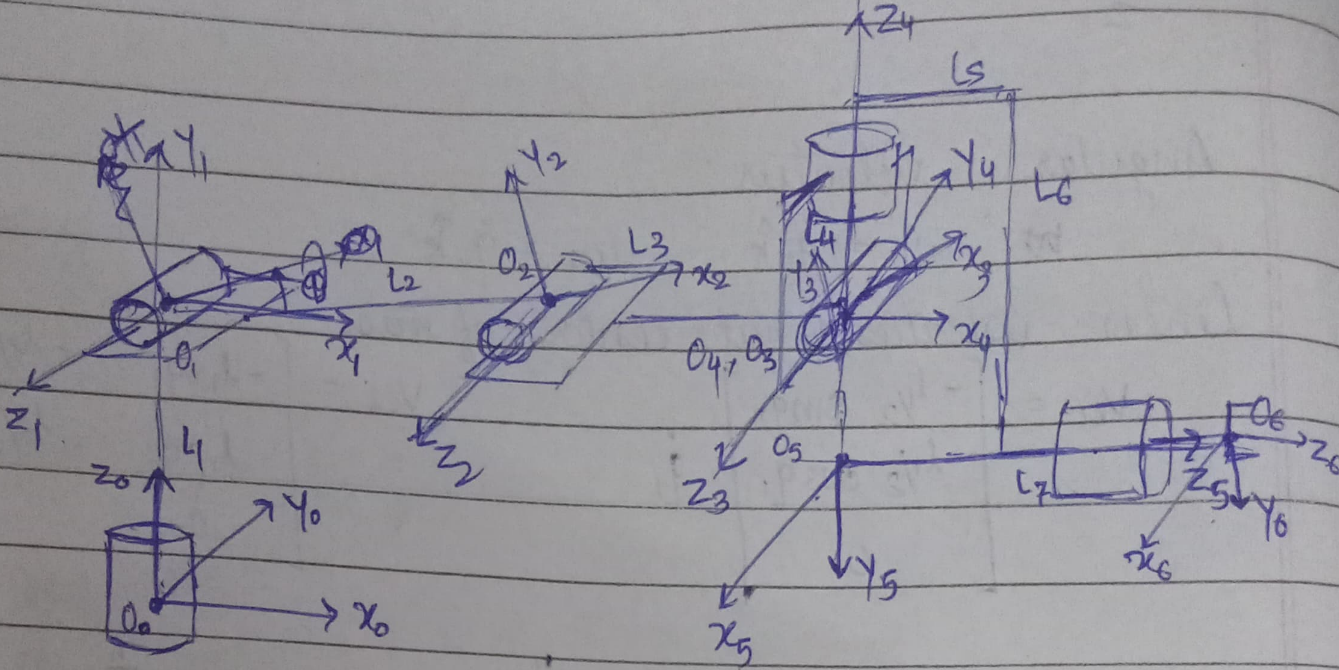
$\phi_1 = \frac{\partial V}{\partial q_1}$ $\qquad \phi_2 = \frac{\partial V}{\partial q_2} = \frac{m_2 g l_2}{2} \cos q_2$

$= \frac{m_1 g l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1$

$\Rightarrow$ Final equations are

$$\boxed{\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + C_{221}\dot{q}_1^2 + \phi_1 &= \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + C_{112}\dot{q}_1^2 + \phi_2 &= \tau_2 \end{aligned}}$$

6.



link	θ_i	d_i	α_i	a_i
1	q_1^*	L_1	90°	0
2	q_2^*	L_2	0	$-L_2$
3	q_3^*	L_3	0	$-L_3$
4	$q_4^* + 90^\circ$	0	-90°	0
5	$q_5^* + 90^\circ$	$-L_6 + L_4$	-90°	0
6	q_6^*	L_7	0	0

Transformation Matrix :

$$T_0^6 = \begin{bmatrix} cq_1^* & \cancel{sq_1^*}\,0 & sq_1^* & 0 \\ sq_1^* & \cancel{+}0 & -cq_1^* & 0 \\ 0 & sq_1^* & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2^* & -sq_2^* & 0 & -L_2 cq_2^* \\ sq_2^* & cq_2^* & 0 & -L_2 sq_2^* \\ 0 & sq_2^* & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} cq_3^* & -sq_3^* & 0 & -L_3 cq_3^* \\ sq_3^* & cq_3^* & 0 & -L_3 sq_3^* \\ 0 & sq_3^* & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -sq_4^* & 0 & -cq_4^* & 0 \\ cq_4^* & 0 & \mp sq_4^* & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -sq_5^* & \cancel{-cq_5^*}\,0 & -cq_5^* & 0 \\ cq_5^* & \cancel{+sq_5^*}\,0 & -sq_5^* & 0 \\ 0 & -1 & 0 & L_4 - L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_6^* & -sq_6^* & 0 & 0 \\ sq_6^* & cq_6^* & 0 & 0 \\ 0 & 0 & 1 & L_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The simplification was lengthy.
So the values were calculated numerically with code, without writting

The results showed the code gave very accurate outputs similar to handwritten calculations