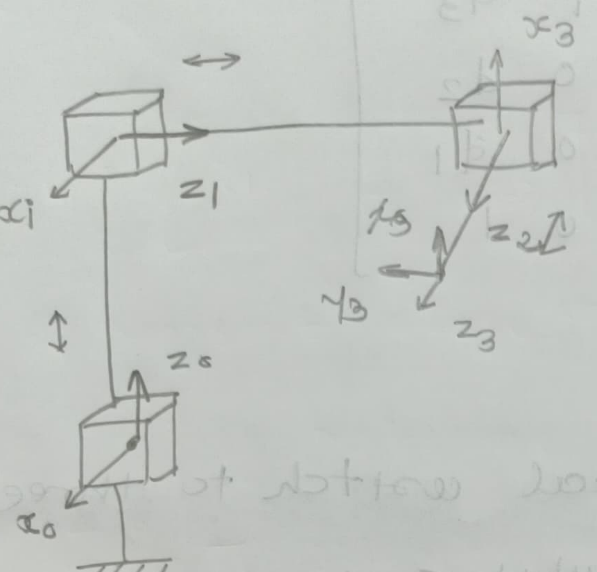
Kakadiya Jaydeep (18110078)

Assignment ③

Task ⑤



→ Link parameter table

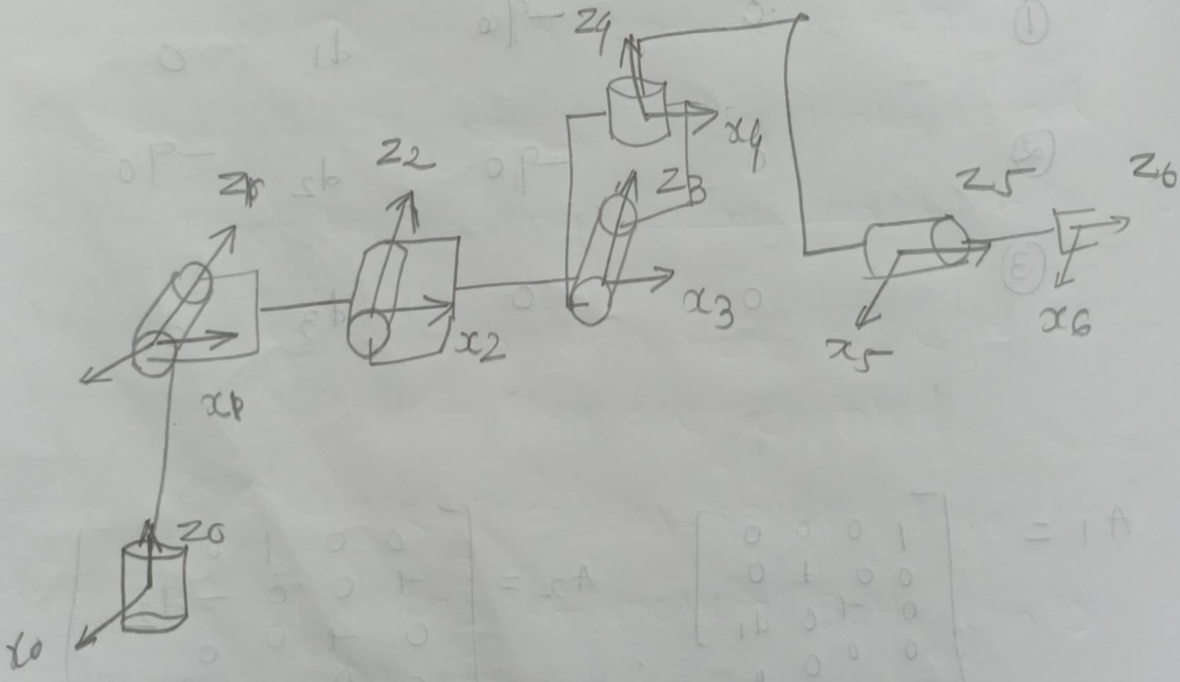
Link	a_i	α_i	d_i	θ_i
①	0	-90	d_1	0
②	0	-90	d_2	-90
③	0	0	d_3	0

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -d_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_3 A_2 A_1 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 6 Attach a spherical wrist to three link articulated Manipulator (3-18.) Derive forward kinematic equation?



Link	a_i	α_i	d_i	θ_i
(1)	0	-90	d_1	θ_1+90
(2)	a_2	0	0	$-\theta_2$
(3)	a_3	0	0	$-\theta_3$
(4)	0	90	0	$-\theta_4$
(5)	0	-90	0	(θ_5+90)
(6)	0	0	d_6	θ_6

Problem: 7) 1) In direct drive configuration, acuators are mounted ~~at~~ directly at joint, this configuration has coriolis terms in equations of motion. They are also coupled equations

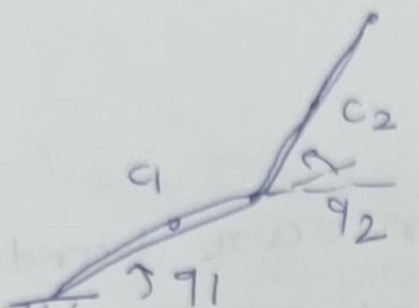
2) In remotely driven configuration, acuators are not directly mounted on links, and they will not be moving ~~along the~~ with links.

This configuration does not have coriolis term in EOM but they are coupled.

3) Five bar linkage provides coriolis free and decoupled equations of motion. Both joints can be controlled independently.

Problem(8) Complete derivation of dynamic equation of RR Manipulator discussed in class.

$\Rightarrow$



$$v_{c1} = \begin{bmatrix} -l_1/2 \sin q_1 \\ l_1/2 \cos q_1 \\ 0 \end{bmatrix}$$

$$v_{c2} = \begin{bmatrix} -l_1 \sin q_1 & -l_2/2 \sin q_2 \\ l_1 \cos q_1 & l_2/2 \cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix}$$

$\rightarrow \omega_1 = \dot q_1 \hat{k}, \quad \omega_2 = \dot q_2 \hat{k}.$

$$\rightarrow K = \sum_{i=1}^{n} \frac{1}{2} m_i v_{ci}^T v_{ci} + \frac{1}{2} \sum_{i=1}^{n} \omega_i^T I_i \omega_i$$

$$v_{ci} = J_{v_{ci}}(q)\, \dot q \quad , \quad \omega_i = R_i^T J_{\omega_i}(q)\, \dot q$$

$$\therefore K = \frac{1}{2} \dot q^T \sum_{i=1}^{n} \Big[m_i J_{v_{ci}}(q)^T J_{v_{ci}}(q) + J_{\omega_i}(q)^T R_i(q) I_i R_i(q)^T J_{\omega_i}(q) \Big] \dot q$$

$$\therefore K = \frac{1}{2} \dot q\, D(q)\, \dot q$$

$$\Rightarrow D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 l_2/2 \cos(q_2 - q_1) \\ m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix}$$

hence,

$$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = -\frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1)$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = \frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1)$$

$$c_{122} = 0$$

$$c_{222} = 0$$

equations,

$$\left(\frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1\right)\ddot{q}_1 + \frac{m_2 l_1 l_2 \cos(q_1 - q_2)}{2}\ddot{q}_2$$

$$+ \left(-\frac{m_2 l_1 l_2 \sin(q_2 - q_1)}{2}\right)\dot{q}_2^2$$

$$+ \left(\frac{m_1 l_1}{2} + m_2 l_1\right) g \cos q_1 = \tau_1$$

$$\left(\frac{m_2 l_1^2}{4} + I_2\right)\ddot{q}_2 + \left(\frac{m_1 l_1 l_2}{2}\cos(q_2 - q_1)\right)\ddot{q}_1 +$$

$$\frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1) = \tau_2$$

⑩ Summarize neatly in your own handwriting the key to derive equations of motion when provided with $D(q)$ and $V(q)$.

⇒ Given $D(q)_{K\times K}$ (K states), $V(q)$, we can write K equations of motion as,

$$\sum_j d_{kj}(q)\cdot \ddot{q}_j + \sum_{i,j} c_{ijk}(q)\,\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

for $k = 1, 2, \dots K$)

→ d_{kj} is kth row, j column element from $D(q)$

→ $\phi_k(q) = \dfrac{\partial V(q)}{\partial q_k}$

→ $c_{ijk} = \dfrac{1}{2}\left\{\dfrac{\partial d_{kj}}{\partial q_i} + \dfrac{\partial d_{ki}}{\partial q_j} - \dfrac{\partial d_{ij}}{\partial q_k}\right\}$

Task ① Describe in 3-4 sentences in your word

What is singular configuration?

→ Singularity is a point in Robot workspace, where jacobian matrix loses its rank ~~ext~~, which means that certain controlled variables are unable to change the end effector states.

→ At these points certain joints are unable to cause effect at end effecter.

⇒ Yes, we can check singular configuration by checking rank of manipulator jacobian at ~~given~~ particular configuration.