ME-639- ASSIGNMENT 2

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T2)

ANS) Let $\vec{P}$ be any vector in space.

$\vec{P}_0 \rightarrow$ is $\vec{P}$ expressed in one Co-ordinate frame $O_{x_0 y_0 z_0}$

$\vec{P}_1 \rightarrow$ is $\vec{P}$ expressed in another Co-ordinate frame $O_{x_1 y_1 z_1}$

Now, for some Rotation matrices R_0^1 we can write

$$\vec{P}_0 = R_0^1 \vec{P}_1 \quad \text{———} \quad (1)$$

Since rotation of a vector does not change its length (magnitude), we can write

$$\|\vec{P}_1\| = \|\vec{P}_0\|$$

$$\Rightarrow \vec{P}_1^T \vec{P}_1 = \vec{P}_0^T \vec{P}_0$$

using (1):-

$$\vec{P}_1^T \vec{P}_1 = (R_0^1 \vec{P}_1)^T (R_0^1 \vec{P}_1)$$

So we can write :-

$$\Rightarrow (\vec{P}_1)^T (I) \vec{P}_1 = (\vec{P}_1)^T (\vec{R}_0^{1\,T} \vec{R}_0^1) (\vec{P}_1)$$

On comparing both sides :-

$$\vec{R}_0^{1\,T} R_0^1 = \vec{I}$$

This is the condition for orthogonality.
Thus any Rotation matrix will satisfy this relation, & hence will be orthogonal.

~~ANS~~

Further :-

$$\|R_0^1\|^2 = \|\vec{I}\|$$

$$\Rightarrow (\det(R_0^1))^2 = 1$$

$$\Rightarrow \det(R_0^1) = \pm 1$$

Rotation matrices ~~do not~~ simply rotate vectors & do not affect the left-handedness or right-handedness of reference frame.

thus, $\det(R_0^1) = +1$

TASK-1

ANS:- Let r_1, r_2 & r_3 be 3 column vectors. a Rotⁿ matrix can be written as:-

$$R_0^1 = [r_1 \quad r_2 \quad r_3] \text{ --- } (A)$$

Now, we know

$$R_0^{1^T} R_0^1 = I$$

Substituting (A)

$$\begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix} \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r_1^T r_1 & r_1^T r_2 & r_1^T r_3 \\ r_2^T r_1 & r_2^T r_2 & r_2^T r_3 \\ r_3^T r_1 & r_3^T r_2 & r_3^T r_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing both sides; we see

This implies the columns of Rot^n matrix are orthogonal

$$\boxed{r_1^T r_1 = 1 \; ; \; r_2^T r_2 = 1 \; ; \; r_3^T r_3 = 1}$$

T5)

Ans) Let $\vec{P}$ be any vector $\vec{P} \in \mathbb{R}^3$

Since we know that: ~~$S(\vec{a})\vec{p} = \vec{a} \times p$~~

$$S(\vec{a})\vec{b} = \vec{a} \times \vec{b} \quad \text{--- (1)}$$

and $$R(\vec{a} \times \vec{b}) = R\vec{a} \times R\vec{b} \quad \text{--- (2)}$$

We can write:

~~$RS(\vec{a})R^T\vec{b}$~~ $$RS(\vec{a})R^T\vec{P} = R(\vec{a} \times R^T\vec{P})$$

{using (1)}

$$= (R\vec{a}) \times (RR^T\vec{P}) \quad \{\text{using (2)}\}$$

$$= (R\vec{a}) \times \vec{P} \qquad \{RR^T = \vec{I}\}$$

using (2) again, we can write RHS as

$$= S(R\vec{a})\vec{P}$$

thus

$$RS(\vec{a})R^T\vec{P} = S(R\vec{a})\vec{P}$$

Now since $\vec{P}$ is any general vector, the equality holds true for all $\vec{P} \in \mathbb{R}^3$

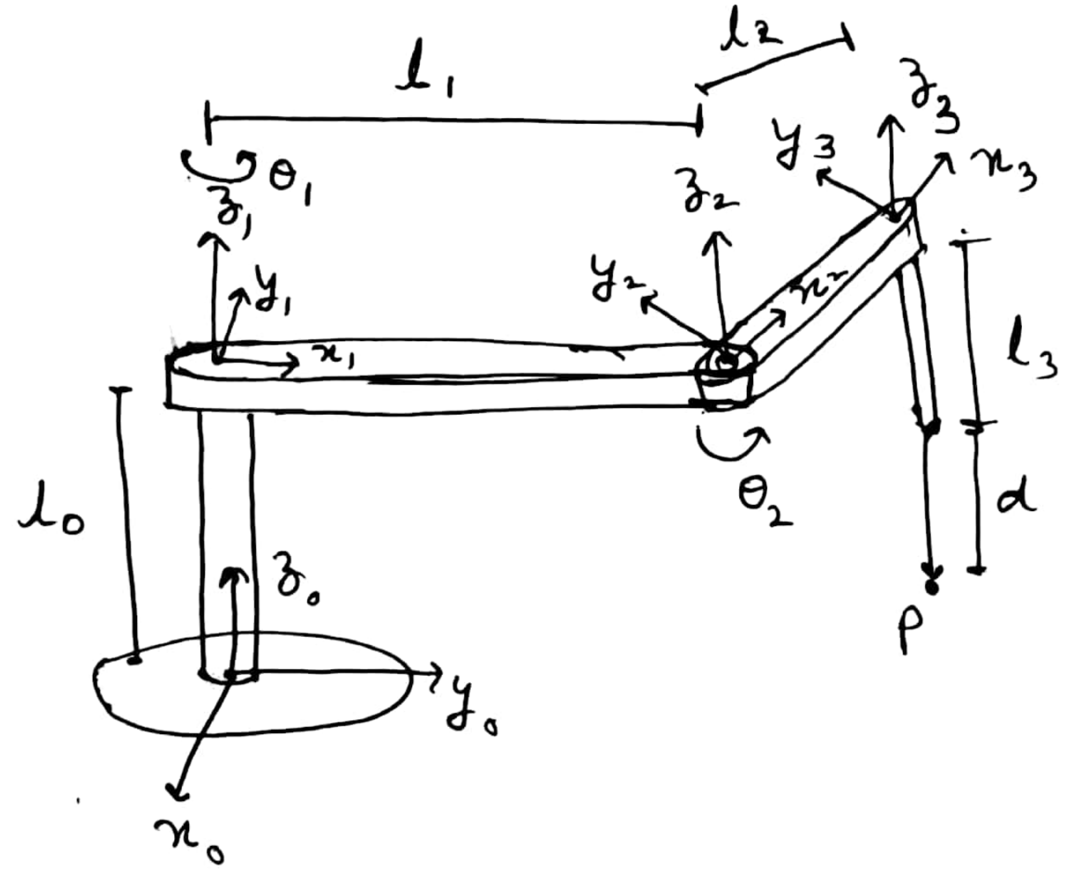
~~this proves the~~

thus we have proven the relation

$$\boxed{R\,S(\vec{a})\,R^T = S(R\vec{a})}$$

TG

Ans



The lengths, base and the reference frames are as indicated.

From base :

$$P_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix} \qquad R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For link 1 :

$$P_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For link 2:

$$P_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \qquad R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For 3rd link:

$$P_3^4 = \begin{bmatrix} 0 \\ 0 \\ l_3 + d \end{bmatrix}$$

Thus ~~and~~ P_0 will be:

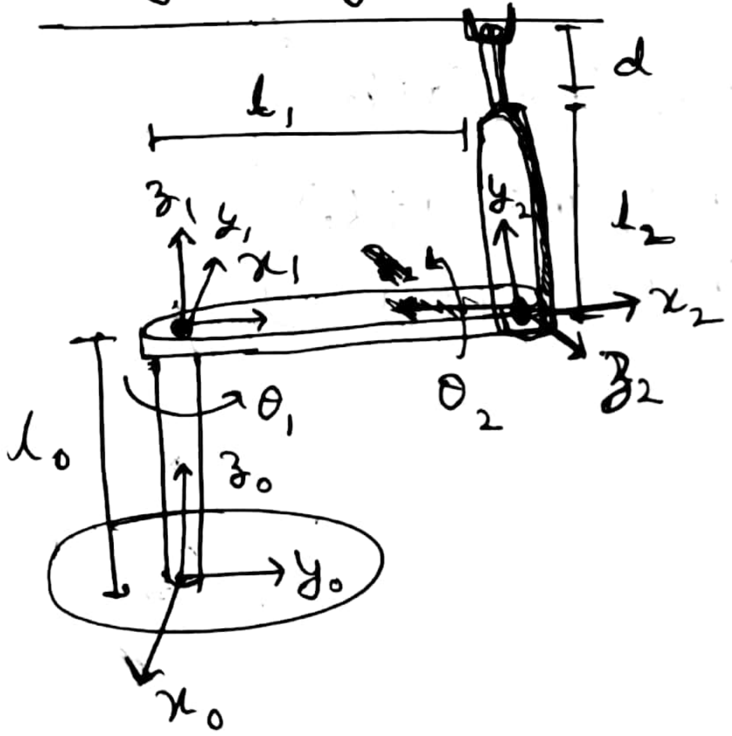
$$P_0 = P_0^1 + R_0^1 P_1^2 + R_0^1 R_1^2 P_2^3 + R_0^1 R_1^2 R_2^3 P_3^4$$

On Simplifying:

$$P_0 = \begin{bmatrix} l_3(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + l_2 \cos\theta_1 \\ l_3(\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1) + l_2 \sin\theta_1 \\ d + l_1 + d_4 \end{bmatrix}$$

T8

Ans) Stanford-type RRP



$$P_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix} \qquad R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 \\ 0 & \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$P_2^3 = \begin{bmatrix} 0 \\ l_2 + d \\ 0 \end{bmatrix}$$

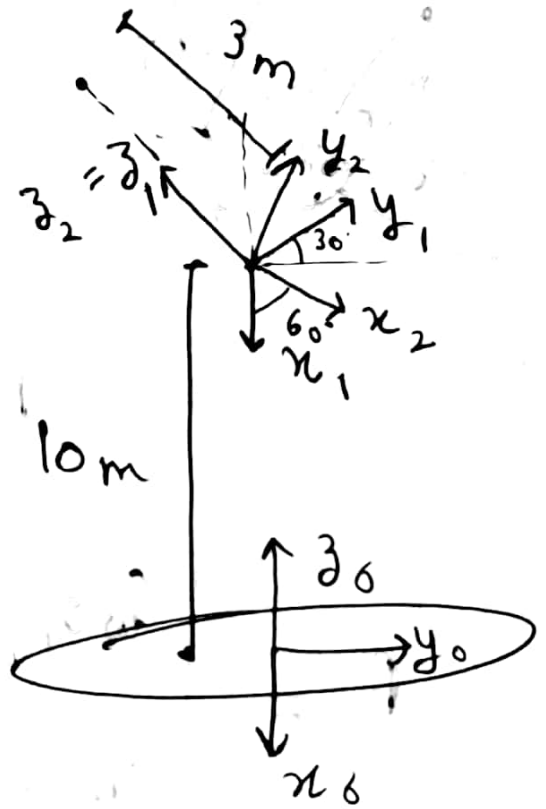
$$P_0 = P_0^1 + R_0^1 P_1^2 + R_0^2 P_2^3$$

On Simplifying

$$P_0 = \begin{bmatrix} l_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1 (d + l_3) \\ l_1 \sin\theta_1 + \cos\theta_1 \sin\theta_2 (d + l_3) \\ l_1 + \sin(\theta_2)(d + l_3) \end{bmatrix}$$

TASK-9

ANS)



The transformation matrix will be :-

$$P_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \qquad R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix}$$

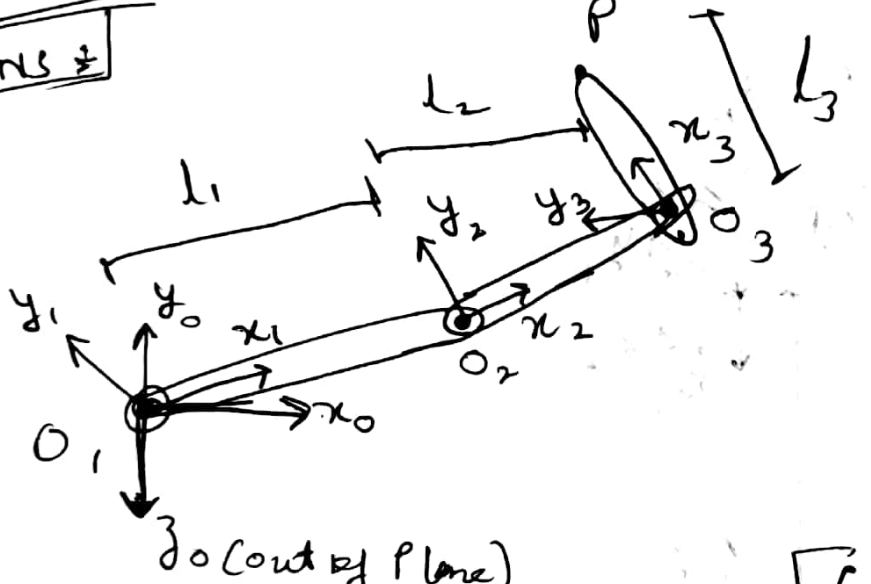
$$P_1^2 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_0 = p_0^1 + R_0^1 R_1^2 p_1^2$$

$$\Rightarrow p_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.6 \end{bmatrix}$$

TASK → 13

Ans :



$$p_3^4 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$p_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \qquad R_2^3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$O_2 = P_0^1 + R_0^1 P_1^2 = R_0^1 P_1^2$$

$$O_2 = \begin{bmatrix} \cos\theta_1 \, l_1 \\ \sin\theta_2 \, l_2 \\ 0 \end{bmatrix}$$

$$O_3 = P_0^1 + R_0^1 P_1^2 + R_0^1 R_1^2 P_2^3$$

$$= R_0^1 P_1^2 + R_0^1 R_1^2 P_2^3$$

$$O_3 = \begin{bmatrix} l_2 \cos\theta_1 + \theta_2 + l_1 \cos\theta_1 \\ l_2 \sin\theta_1 + \theta_2 + l_1 \sin\theta_1 \\ 0 \end{bmatrix}$$

~~The jacobian will be~~

$$\cancel{J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}}$$

$$\cancel{J_1 = \begin{bmatrix} Z_1 \times (P - O) \\ Z \end{bmatrix}}$$

$\cancel{P = R_0}$ $P = P_0^1 + R_0^1 P_1^2 + R_0^1 R_1^2 P_2^3 + R_0^1 R_1^2 R_2^3 P_3^4$

on simblifying : {using MATLAB}

(P.T.O)

$$P = \begin{bmatrix} l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 + l_3\cos(\theta_1+\theta_2+\theta_3) \\ l_2\sin(\theta_1+\theta_2) + l_1\sin\theta_1 + l_3\sin(\theta_1+\theta_2+\theta_3) \\ 0 \end{bmatrix}$$

The jacobian will be:

$$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_1 \times (P - O_1) \\ z_1 \end{bmatrix}$$

on calculating

$$J_1 = \begin{bmatrix} -l_2\sin(\theta_1+\theta_2) - l_1\sin\theta_1 - l_3\sin(\theta_1+\theta_2+\theta_3) \\ l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 + l_3\cos(\theta_1+\theta_2+\theta_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_2 \times (P - O_2) \\ z_2 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 \sin(\theta_1 + \theta_2) - l_3 \sin(\theta_1 + \theta_2 + \theta_3) \\ l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_3 \times (P - O_3) \\ z_3 \end{bmatrix}$$

$$= \begin{bmatrix} -l_3 \sin(\theta_1 + \theta_2 + \theta_3) \\ l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

TASK-11

ANS :- (Continued from Tas 6)

The Jacobian will be :-

$$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$$, where

$$J_1 = \begin{bmatrix} z_1 \times (P - O_1) \\ z_1 \end{bmatrix}$$ {revolute Joint}

$$= \begin{bmatrix} -l_2 \sin(\theta_1 + \theta_2) - l_1 \sin\theta_1 \\ l_2 \cos(\theta_1 + \theta_2) + l_1 \cos\theta_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Now :

$$z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

& $O_2 = P_0^1 + R_0^1 P_1^2$

$$= \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ l_0 \end{bmatrix}$$

So:

$$J_2 = \begin{bmatrix} z_2 \times (p - o_2) \\ z_2 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 \sin(\theta_1 + \theta_2) \\ l_2 \cos(\theta_1 + \theta_2) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \{\text{Prismatic joint}\}$$