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ME639 - Introduction to Robotics

Assignment (2)

Task (2) Show that columns of rotation matrix R_0^1 are orthogonal.

⇒ Let's take a base reference frame XYZ with origin at O.

→ $\vec{P}$ is a vector denoting point P in XYZ frame.

$$\vec{P} = (P_1, P_2, P_3) = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

→ Length of $\vec{P}$ can be calculated as,

$$\|\vec{P}\|^2 = P^T P = \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \quad \text{— (A)}$$

⇒ Now, for rotation transformation, it preserves the length ~~of original~~ between two points.

⇒ So, we rotate the frame with rotation matrix, R_0^1. New ~~distance~~ vector denoting P point w.r.t. x, y, z,

$$\vec{P_0} = R_0^1 \vec{P_1}$$

→ As distance between two points will be preserved

$$\vec{P_1}^T \vec{P_1} = (\vec{P_0})^T (\vec{P_0})$$

$$\vec{P_1}^T \vec{P_1} = (R_0^1 \vec{P_1})^T (R_0^1 \vec{P_1})$$

$$\vec{P_1}^T (I) \vec{P_1} = \vec{P_1}^T (R_0^{1T} R_0^1) \vec{P_1} \quad -(B)$$

→ from (B),

$R_0^{1T} R_0^1 = I$, so every rotation matrix satisfies this relation and they are orthogonal matrices.

→ Now $$(R_0^1)^T (R_0^1) = I$$

$$\Rightarrow \det(R_0^1)^2 = 1$$

$$\therefore \det(R_0^1) = \pm 1$$

∴ Rotation matrix do not change the "handness" of coordinate system so, it has determinant value = 1.

$$\therefore \det(R_0^1) = 1$$

5) Show that $R(S(\vec{a}))R^T = S(R(\vec{a}))$

→ from the properties of Rotation matrix,

$$S(\vec{a}) \cdot b = \vec{a} \times \vec{b}$$

and $R(\vec{a} \times \vec{b}) = R\vec{a} \times R\vec{b}$.

Now, to prove $R(S(\vec{a}))R^T = S(R\vec{a})$, lets take a vector $\vec{p}$,

so,

$$(R(S(\vec{a}))R^T \cdot \vec{p} = R(\vec{a} \times R^T\vec{p})$$

$$= R\vec{a} \times RR^T\vec{p}$$

$$= R\vec{a} \times \vec{P}$$

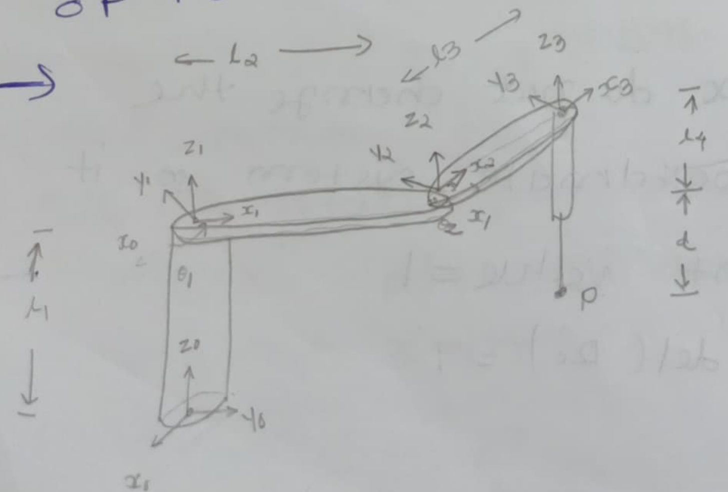
$$R(S(\vec{a}))R^T \cdot \vec{p} = S(R\vec{a}) \cdot \vec{P}$$

from above equation,

$$\therefore R(S(\vec{a}))R^T = S(R\vec{a})$$

TASK 6) Work out various coordinate frames and workout P_0 using a composition of homogeneous transformation for RRP SCARA

→



$$P_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \qquad R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ +\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ +\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

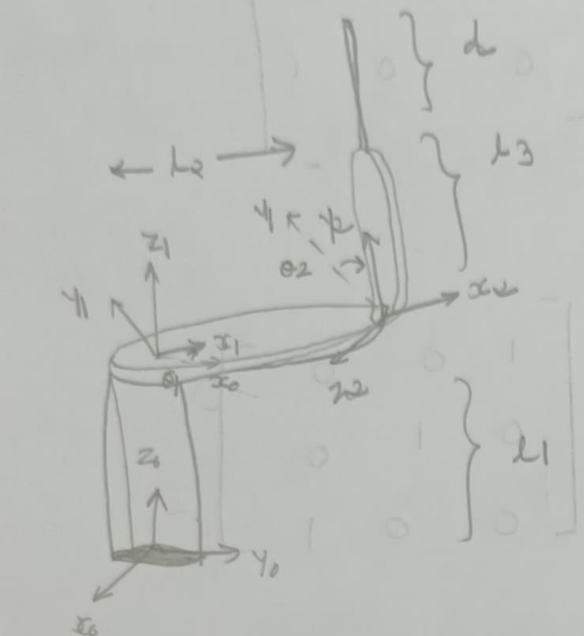
$$P_2^3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} \qquad R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{R}\ P_3^4 = \begin{bmatrix} 0 \\ 0 \\ l_4+d \end{bmatrix}$$

$$P_0 = P_0^1 + R_0^1 P_1^2 + R_0^2 P_2^3 + R_0^3 P_3^4$$

$$P_0 = \begin{bmatrix} l_3(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + l_2\cos\theta_1 \\ l_3(\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1) + l_2\sin\theta_1 \\ d + l_1 + l_4 \end{bmatrix}$$

Task ⑧ Stanford type RRP configuration:



$$p_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

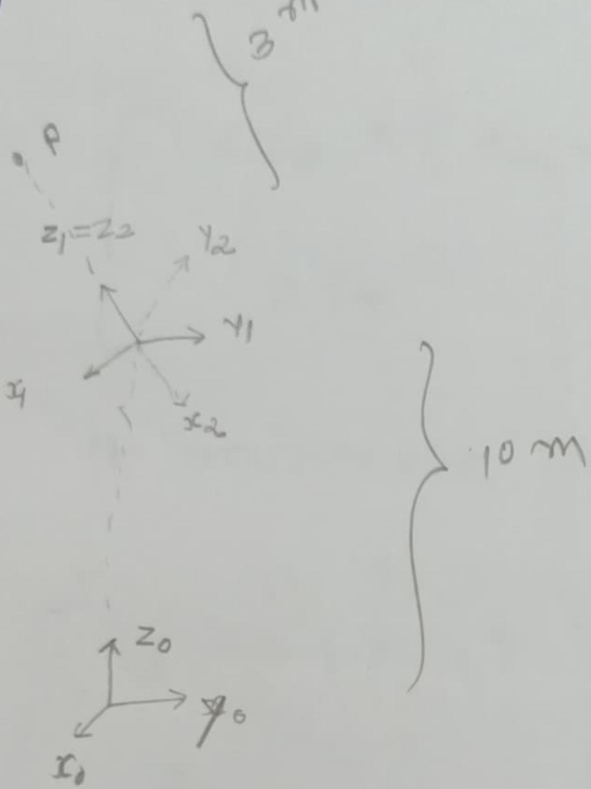
$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 \\ 0 & \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$p_2^3 = \begin{bmatrix} 0 \\ l_3 + d \\ 0 \end{bmatrix}$$

$$P_0 = P_0^1 + R_0^1 P_1^2 + R_0^2 P_2^3$$

$$P_0 = \begin{bmatrix} l_2\cos\theta_1 - \cos\theta_2\sin\theta_1(d+l_3) \\ l_1\sin\theta_1 + \cos\theta_1\sin\theta_2(d+l_3) \\ l_1 + \sin(\theta_2)(d+l_3) \end{bmatrix}$$

Task (9)



Base station

$$P_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_0^1 = (R_0^1)_1 \, (R_0^1)_2$$

$$(R_0^1)_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & +\sin 30 & \cos 30 \end{bmatrix}$$

$$(R_0^1)_2 = \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ +\sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_1^2 = \begin{bmatrix} 0 \\ a \\ 3 \end{bmatrix}$$

$$P_0 = P_0^1 + (R_0^1)(P_1^2)$$

$$P_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.5981 \end{bmatrix}$$

Task (11) Manipulator Jacobian for SCARA: RRP

⇒ Reference frames is taken as task (6).

→ Jacobian can be calculated as,

$$J = [\, J_1 \quad J_2 \quad J_3 \,]$$

$$J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$$

if revolute joint — if prismatic joint.

$$J_1 = \begin{bmatrix} z_0 \times (p - o_0) \\ z_0 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} \cancel{-l_3\cos(} -l_3\sin(\theta_1+\theta_2) - l_2\sin\theta_1 \\ l_3\cos(\theta_1+\theta_2) + l_2\cos\theta_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_2 \times (P - O_1) \\ \\ z_2 \end{bmatrix} \qquad z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

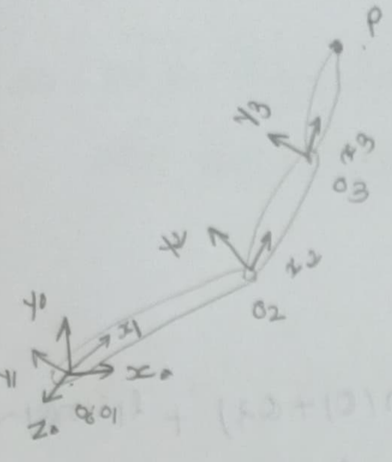
$$O_2 = \begin{bmatrix} l_2 \cos\theta_1 \\ l_2 \sin\theta_1 \\ l_1 \end{bmatrix}$$

$$= \begin{bmatrix} -l_3 \overset{\sin}{\cancel{\cos}}(\theta_1 + \theta_2) \\ l_3 \cos(\theta_1 + \theta_2) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} 0 \\ 0 \\ \dot{d} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ prismatic joint.

$$J = \begin{bmatrix} -l_3\sin(\theta_1+\theta_2) - l_2\sin\theta_1 & -l_3\sin(\theta_1+\theta_2) & 0 \\ l_3\cos(\theta_1+\theta_2) + l_2\cos\theta_1 & l_3\cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & \dot{d} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

TASK (13)



$$P_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_3^t = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

i) $O_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$,

$$O_2 = P_0^1 + R_0^1 P_1^2$$

$$= \begin{bmatrix} \cos\theta_1 l_1 \\ l_2 \sin\theta_1 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_2 \cos\theta_1 + \theta_2 + l_1 \cos\theta_1 \\ l_2 \sin\theta_1 + \theta_2 + l_1 \sin\theta_1 \\ 0 \end{bmatrix},$$

→ Jacobian

$$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$$

where,

$$J_1 = \begin{bmatrix} z_1 \times (P - O_1) \\ z_1 \end{bmatrix} = \begin{bmatrix} -(l_2 \sin(\theta_1 + \theta_2) + l_1 \sin\theta_1 + l_3 \sin(\theta_1 + \theta_2 + \theta_3)) \\ (l_2 \cos(\theta_1 + \theta_2) + l_1 \cos\theta_1 + l_3 \cos(\theta_1 + \theta_2 + \theta_3)) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_2 \times (P - O_2) \\ z_2 \end{bmatrix} = \begin{bmatrix} -l_1 \sin(\theta_1 + \theta_2) - l_3 \sin(\theta_1 + \theta_2 + \theta_3) \\ + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_3 \times (P - O_3) \\ z_3 \end{bmatrix} = \begin{bmatrix} -l_3 \sin(\theta_1 + \theta_2 + \theta_3) \\ + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Task ① show that columns of rotation matrix R_0^1 are orthogonal.

⇒ Let's assume that rotation matrix R_0^1 can be written as,

$$R_0^1 = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}$$

where, r_i represent whole column $r_{i(3\times1)}$.

→ Now, for any rotation matrix,

$$(R_0^1)^T (R_0^1) = I$$

$$\therefore \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} r_1^2 & r_1 \cdot r_2 & r_1 \cdot r_3 \\ r_2 \cdot r_1 & r_2^2 & r_2 \cdot r_3 \\ r_3 \cdot r_1 & r_2 \cdot r_3 & r_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ From above equation it is clear that,

$r_i \cdot r_j = 0$. $(i \neq j)$ so, columns of rotation matrix are orthogonal.

Task (10) Types of gearboxes.

⇒ Planetary gear box: Planetary gear box is widely used in various robotics applications. Planetary gear box allows to reduce the rotation speed and increase the torque out put in motors. Planetary gear box consists of sun gear, planet gear and ring gear. Sun gear is attached in the middle of planet gears and ring gear. This systems gives ~~poss~~ various possibilities to attach motor at any gear depending upon various goals.

⇒ Cycloidal gear box: Cycloidal gearbox is not widely used in motor applications. But sometimes it can be used to reduce speed significantly. It provides high ratio for speed and low backlash.

→ bevel gear box: Bevel gear box changes the output axis relative to motor core rotation axis. Bevel gear motor can be used to minimize the ~~design~~ robot volume where we have space constraints.