Task 1 (T1) - Given arbitrary trajectory of end effector (given x, y as functions of time) make the robot follow this trajectory

Task 2 (T2) - Given a location of a wall, make the robot touch the wall and apply a constant force against the wall.

Task 3 (T3) - Make the robot behave like a virtual spring connected to a given point (x_0, y_0).

Task 4 (T4) - Given mechanical constraints on the angles determine the range of possible positions of E (workspace)

Now,

$$x = l_1 \cos q_1 + l_2 \cos q_2$$

& $$y = l_1 \sin q_1 + l_2 \sin q_2$$

or using simplified notation.

$$\left.\begin{aligned} x &= l_1 c q_1 + l_2 c q_2 \\ \& \; y &= l_1 s q_1 + l_2 s q_2 \end{aligned}\right] \quad (1)$$

Differentiating (1) we get,

$$\dot{x} = -l_1 sq_1 \cdot \dot{q}_1 - l_2 sq_2 \cdot \dot{q}_2$$

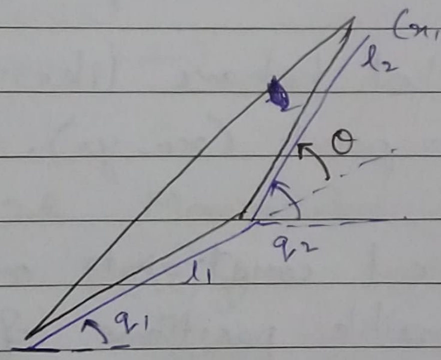
& $$\dot{y} = l_1 cq_1 \cdot \dot{q}_1 + l_2 cq_2 \cdot \dot{q}_2$$

End effector velocity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & -l_2 sq_2 \\ l_1 cq_1 & l_2 cq_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{— (2)}$$

We will also need the reverse relationships
Given x & y, we need to be able to solve for q_1 & q_2 using (1).

option 1 - solve numerically
option 2 - Derive closed form expression
- Hard in general
- Multiple solutions

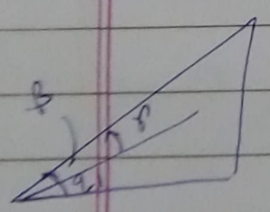


Cosine rule using black triangle + switching to the acute angle

$$\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)$$

$$(\beta - \gamma)$$



$$q_2 = \theta + q_1$$

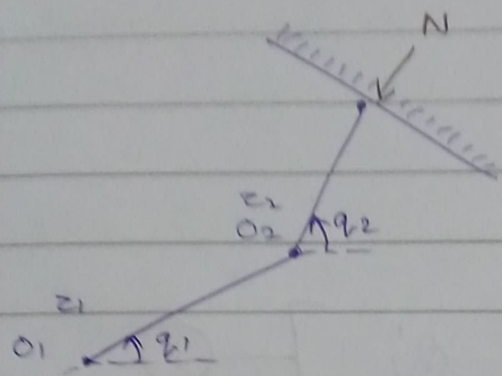
— (3)

Control both motors in position control mode to achieve above q_1 & q_2 at each time step

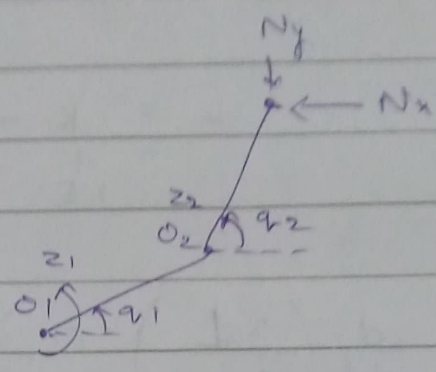
First level answer to T1.

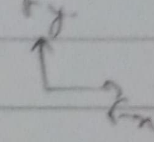
We may later start using x_d, y_d and q_{1d}, q_{2d} in the above equations to denote desired values.

Task - 2



F.B.D.



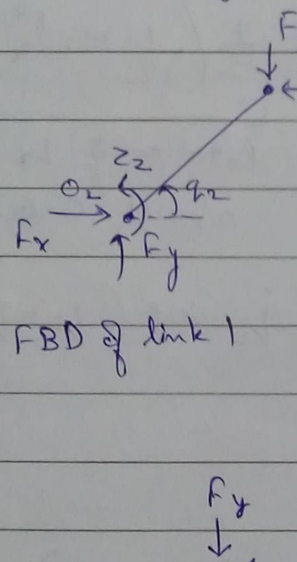


Static Equilibrium

$\Rightarrow \Sigma M_{O_1} = 0$ & $\Sigma M_{O_2} = 0$

FBD of each link separately

FBD of link 2



Ignore gravity

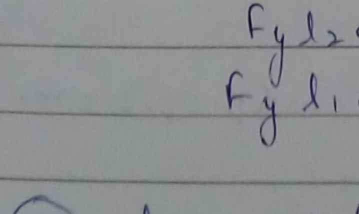
$\Sigma M_{O_2} = 0$

$\Rightarrow$

$F_y l_2 cq_2 - F_x l_2 sq_2 = z_2$

FBD of link 1

$\Rightarrow \Sigma M_{O_1} = 0$



$F_y l_1 cq_1 - F_x l_1 sq_1 = z_1$

$$\left.\begin{aligned} F_y l_2 cq_2 - F_x l_2 sq_2 &= z_2 \\ F_y l_1 cq_1 - F_x l sq_1 &= z_1 \end{aligned}\right] \quad (4)$$

(3) along with (4) answers T_2.

Apply torques z_1 & z_2 at the motors after reaching the wall.

For T3 and next level answer to T1 need to understand the dynamics of the robot.

Langrange's Equations:

$$L = \underset{KE}{K} - \underset{PE}{V}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i' \quad (5)$$

Q_i' are generalized forces derived using principle of virtual work.

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation about } O_1} + \underbrace{\frac{1}{2} m_2 V_{c_2}^2}_{\text{translation of } L_2} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{Rotation of } L_2 \text{ about C.G.}}$$

$$V_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_1)$$

Let us bring back gravity

$$V = m_1 g \frac{l_1}{2} s q_2 + m_2 g\left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

go through the steps

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2 \times$$

$$(\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1) + m_1 g \frac{l_1}{2} c q_2 + m_2 g l_1 c q_1 = \tau_1$$

$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_1 \times$

$(\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} s q_2 = \tau_2$ ——— (6)

Note :- (4) is valid for any end-effector f_x & f_y (not just wall reactions)

$f_x = kx$ — more generally $f_x = k_x (x - x_0)$

$f_y = ky$ $\quad$ $f_y = k_y (y - y_0)$

for now keep it simple.

$f_x = kx$ | This is what we want

$f_y = ky$

from (1) →

$f_x = k(l_1 c q_1 + l_2 c q_2)$

$f_y = k(l_1 s q_1 + l_2 s q_2)$

from (4)

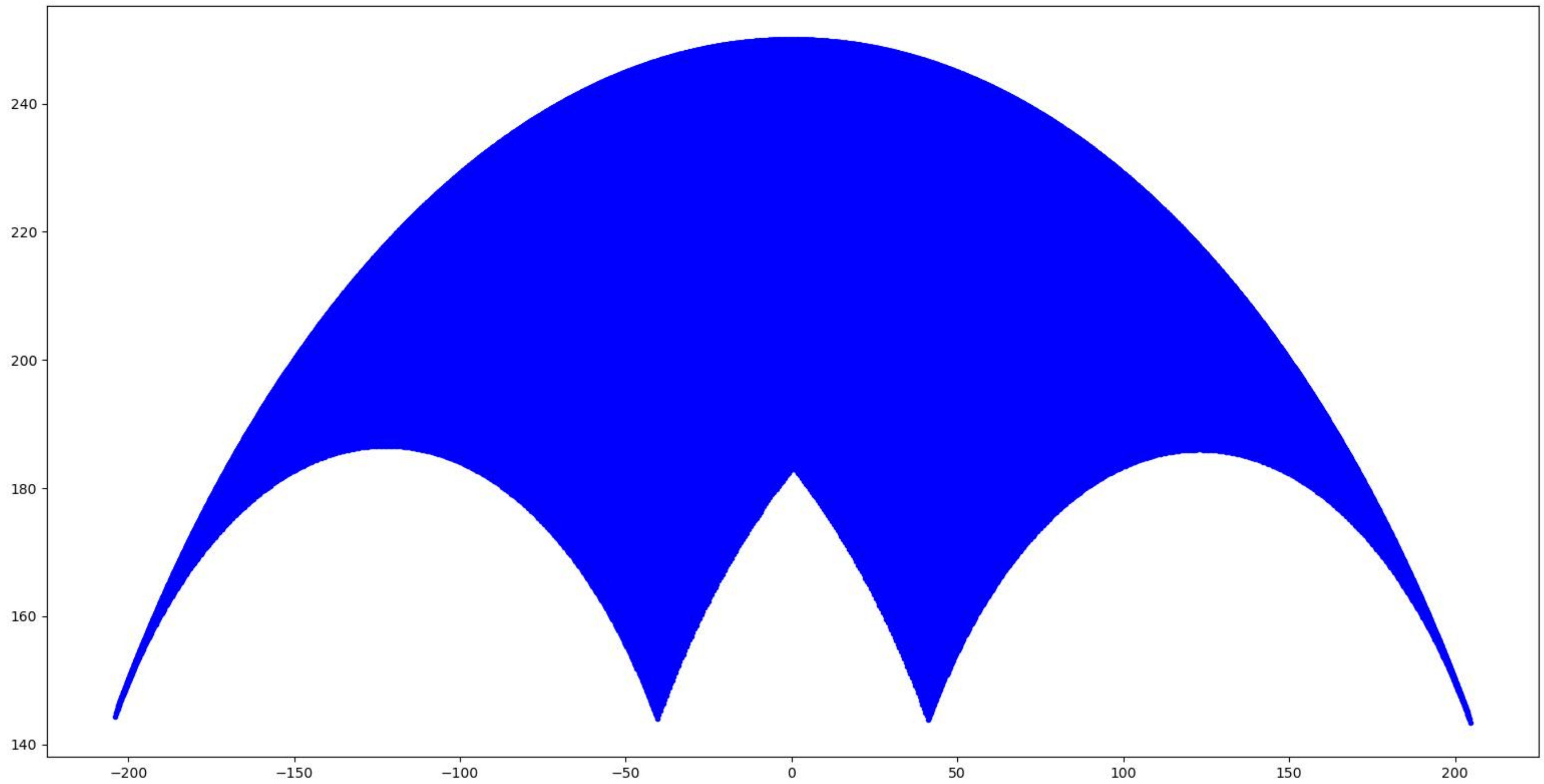
$k(l_1 s q_1 + l_2 s q_2) l_2 c q_2 - k(l_1 c q_1 + l_2 c q_2) l_2 s q_2 = \tau_{2s}$

$k(l_1 s q_1 + l_2 s q_2) l_1 c q_1 - k(l_1 c q_1 + l_2 c q_2) l_1 s q_1 = \tau_{1s}$

——— (7)

Set the motor torques to be $\tau_1 + \tau_{1s}$ & $\tau_2 + \tau_{2s}$. respectively — from (6) & (7)

Another approach to T1 is to use q_{1d} & q_{2d} using (3), then subsititute them in (6) $q_{set\,motor}$ & torques to be τ_1 & τ_2 calculated using (6).



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