Introduction to Robotics
(Mini Project notes)

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2R Manipulator – (Elbow)

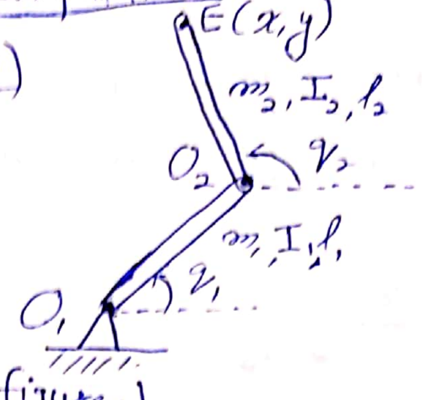


figure.1

E – end effector

(x,y) – end effector position

(q_1, q_2) = joint angles

Note: absolute angles

assume origin at O_1.

Let us assume that motors are connected to both joints O_1 & O_2. and we have the ability to control either torques τ_1 and τ_2 applied at these joints or control the angles q_1 and q_2.

Task-1 (T1) – Given arbitrary trajectory of end effector (Given x, y as functions of time) make the robot follow this trajectory.

Using figure.1

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2$$

or using simplified notations

$$\left.\begin{aligned} x &= l_1 c q_1 + l_2 c q_2 \\ y &= l_1 s q_1 + l_2 s q_2 \end{aligned}\right\} \quad (1)$$

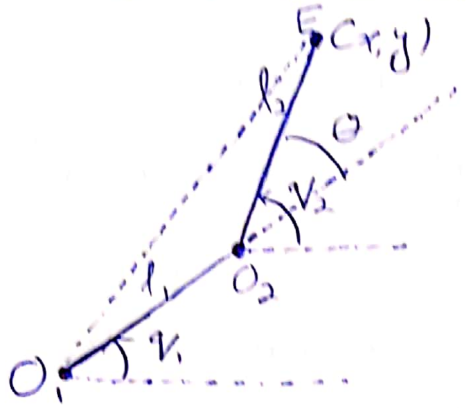
Differentiating (1), we get

$$\dot{x} = -l_1 s q_1 \cdot \dot{q}_1 - l_2 s q_2 \cdot \dot{q}_2$$
$$\dot{y} = l_1 c q_1 \cdot \dot{q}_1 + l_2 c q_2 \cdot \dot{q}_2$$

$\Rightarrow$ End-effector velocity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad (2)$$

We will also need the reverse relationships. Given x, y, we need to be able to solve for q_1 and q_2 using (1).



Cosine rule using $\Delta O_1 O_2 E$
+ switching to the acute angle

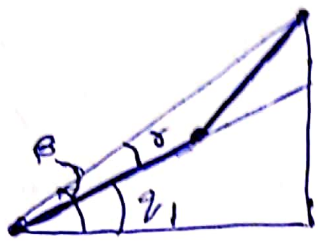
$$\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2\cos\theta}\right)$$

$$(\beta - \gamma)$$

$$q_2 = \theta + q_1$$

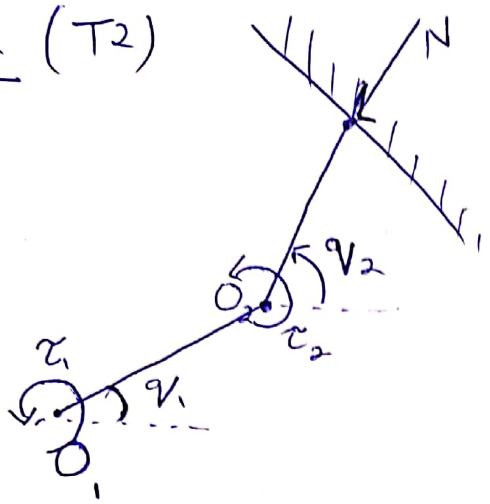
(3)



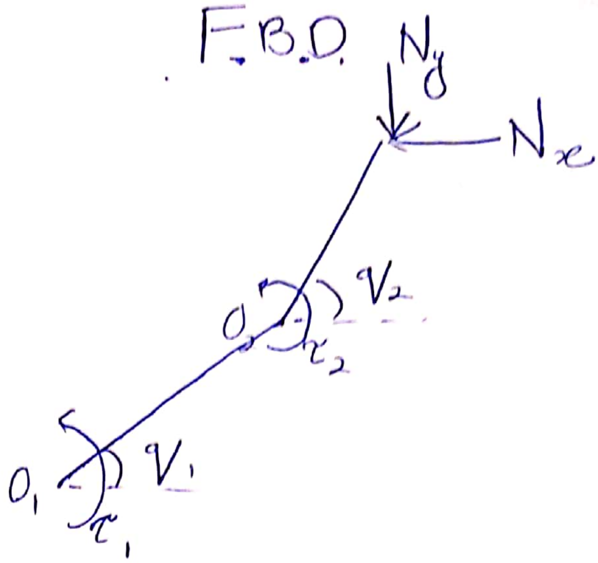
By rotating motors by q_1 and q_2 respectively, the manipulator can reach to any point (x, y) of a given trajectory.

Scanned with CamScanner

Task 2 (T2)



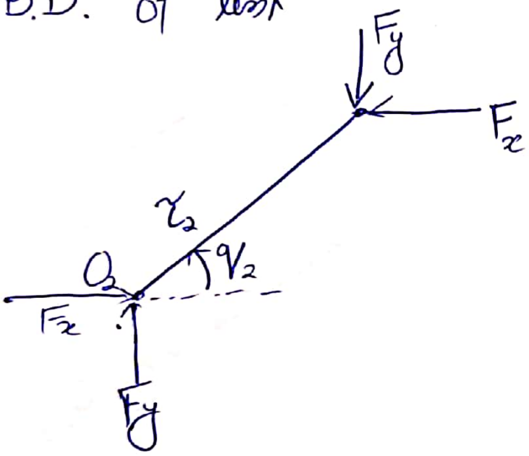
F.B.D



Static equilibrium

$$\Rightarrow \sum M_{O_1} = 0 \quad \& \quad \sum M_{O_2} = 0$$

F.B.D. of link 2

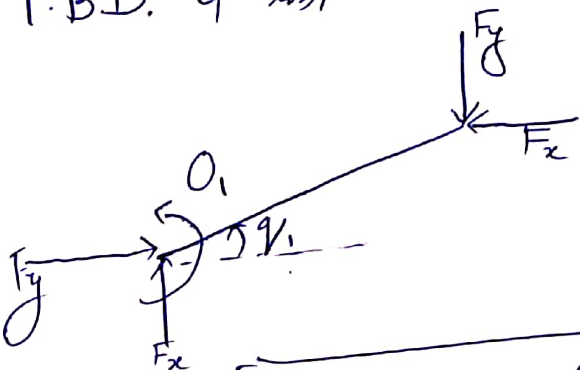


Ignore gravity

$$\sum M_{O_2} = 0$$

$$\Rightarrow F_y l_2 c q_2 - F_x l_2 s q_2 = \tau_2$$

F.B.D. of link 1



$$\sum M_{O_1} = 0$$

$$\Rightarrow F_y l_1 c q_1 - F_x l_1 s q_1 = \tau_1$$

$$\therefore \boxed{\begin{aligned} F_y l_1 c q_1 - F_x l_1 s q_1 &= \tau_1 \\ F_y l_2 c q_2 - F_x l_2 s q_2 &= \tau_2 \end{aligned}} \quad (4)$$

τ_1 and τ_2 must be applied by the motors respectively to balance the wall reaction forces.

Task-3 Lagrangian's equations -

Lagrangian : $\mathcal{L} = K - V$

K.E P.E

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i'} \quad — (5)$$

Q_i' are generalized forces derived using principle of virtual work.

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) \dot{q}_1^2}_{\text{pure rotation about } O_1} + \underbrace{\frac{1}{2} m_2 V_{c_2}^2}_{\text{translation of } l_2} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) \dot{q}_2^2}_{\text{rotation of } l_2 \text{ about C.G.}}$$

$$V_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2} \dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

Let us bring back gravity

$$V = mg \frac{l_1}{2} s q_2 + \frac{mg}{2}\left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

Using the above steps:

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2(\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_1}{2} c q_2 + m_2 g l_1 c q_1 = \tau_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + \frac{m_2 l_2^2}{2} \ddot{q}_2 + \frac{m_2 l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) - m_2 l_1 l_2 \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} s q_2 = \tau_2$$

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Equation (4) is valid for any end effector F_x and F_y (Not just wall reactions)

$F_x = kx$ (Spring forces)

$F_y = ky$

from (1) $\Rightarrow$

$F_x = k(l_1 cq_1 + l_2 cq_2)$

$F_y = k(l_1 sq_1 + l_2 sq_2)$

from (4)

$$k(l_1 sq_1 + l_2 sq_2) l_2 cq_2 - k(l_1 cq_1 + l_2 cq_2) l_2 sq_2) = \tau_{2s}$$

$$k(l_1 sq_1 + l_2 sq_2) l_1 cq_1 - k(l_1 cq_1 + l_2 cq_2) l_2 sq_2) = \tau_{1s}$$

If the motor torques are set to be be $\tau_1 + \tau_{1s}$ & $\tau_2 + \tau_{2s}$ respectively, the manipulator ~~would~~ & will behave like a spring.