

Assignment - 6, 7

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~~SECRET~~

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1 Applying the Quintic Polynomial Trajectory to move from point A to point B(0.4, 0.01, 0.1)

$$p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

where $e(t)$ end effector position.

The boundary conditions are given as

$$p_0 = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 + a_4 t_0^4 + a_5 t_0^5$$

$$v_0 = a_1 + 2a_2 t_0 + 3a_3 t_0^2 + 4a_4 t_0^3 + 5a_5 t_0^4$$

$$L_0 = 2a_2 + 6a_3 t_0 + 12a_4 t_0^2 + 20a_5 t_0^3$$

$$p_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5$$

$$v_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4$$

$$L_f = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3$$

For our case at $t = t_0$, $p_0 = (0.4, 0.06, 0.1)$, $v_0 = 0$, $L_0 = 0$
 at $t = t_f$, $p_f = (0.4, 0.01, 0.1)$, $v_f = 0$, $L_f = 0$

Also, $t_0 = 0$

$$p_0 = a_0 = [0.4, 0.06, 0.1]$$

$$v_0 = a_1 = 0$$

$$L_0 = 2a_2 = 0$$

$t_f = 1$, i.e. reaching point B in 1 sec

$$p_f = a_0 + a_3 + a_4 + a_5 = [0.4, 0.01, 0.1]$$

$$v_f = 3a_3 + 4a_4 + 5a_5 = 0$$

$$L_f = 6a_3 + 12a_4 + 20a_5 = 0$$

$$4a_4 + 10a_5 = 0$$

$$a_4 = -5/2 a_5$$

$$a_3 = 5/3 a_5$$

$$\therefore a_0 + \frac{5a_5}{3} + \frac{-5a_5}{2} + a_5 = [0.4, 0.01, 0.1]$$

$$[0.4, 0.06, 0.1] + \frac{a_5}{6} = [0.4, 0.01, 0.1]$$

~~$$a_5 = [0, -0.3, 0]$$~~

~~$$a_4 = [0, 0.625, 0]$$~~

~~$$a_3 = [0, -1.25, 0]$$~~

$$a_5 = [0, -0.3, 0]$$

$$a_4 = [0, 0.75, 0]$$

$$a_3 = [0, -0.5, 0]$$

3. Using the SCARA Manipulator to perform the given task.

We have the dynamic equations for the SCARA manipulator as

$$d_{11} \ddot{q}_1 + d_{12} \ddot{q}_2 - (m_2 l_1 l_2 + 2m_3 l_1 l_2) \sin q_2 \dot{q}_1 \dot{q}_2 - \dot{q}_1^2 (m_2 l_1 l_2 + 2m_3 l_1 l_2) \sin q_2 = \tau_1$$

$$d_{21} \ddot{q}_1 + d_{22} \ddot{q}_2 + d_{23} \ddot{q}_3 + \frac{1}{2} \dot{q}_1^2 (m_2 l_1 l_2 + 2m_3 l_1 l_2) \sin q_1 = \tau_2$$

$$d_{32} \ddot{q}_2 + d_{33} \ddot{q}_3 + 1 = \tau_3$$

where

$$d_{11} = \frac{m_1 l_1^2}{4} + \frac{m_2 l_1^2}{4} + \frac{m_2 l_2^2}{4} + m_2 l_1 l_2 \cos q_2 + m_3 l_1^2 + m_3 l_2^2 + 2m_3 l_1 l_2 \cos q_2 + I_1 + I_2 + I_3$$

$$d_{12} = d_{21} = m_2 l_1 l_2 \cos q_2 + \frac{m_2 l_2^2}{4} + 2m_3 l_1 l_2 \cos q_3 + m_3 l_2^2 + I_2 + I_3$$

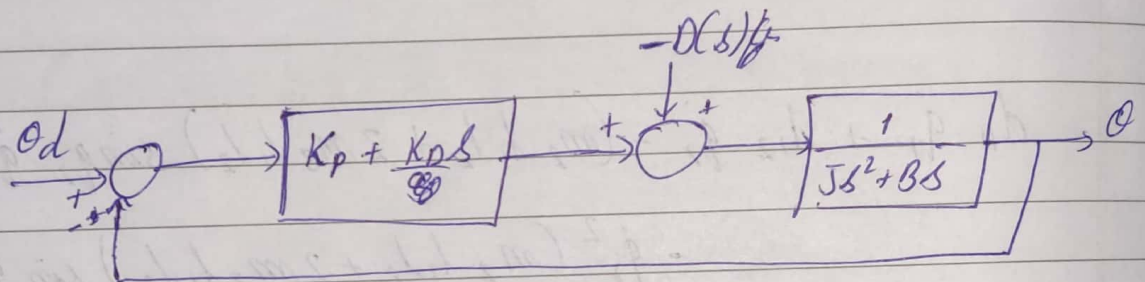
$$d_{23} = d_{32} = I_3$$

$$d_{22} = \frac{m_2 l_2^2}{4} + m_3 l_2^2 + I_2 + I_3$$

$$d_{33} = \frac{m_3}{4}$$

Here m_1 , m_2 & m_3 are mass of the links L_1 , L_2 & L_3 respectively.

a) P D Controller



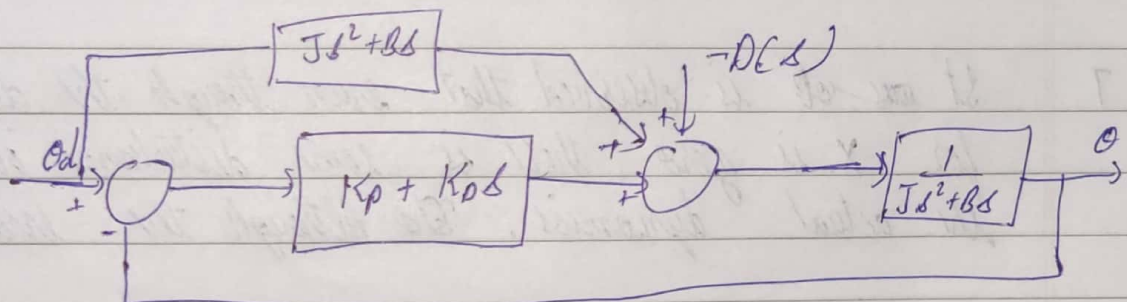
Control Equations

$$\frac{(\theta_d - \theta)(K_p + K_D s) - D(s)}{Js^2 + Bs} = 0$$

$$\therefore K_p \theta_d - K_p \theta + \theta_d K_D s - \theta K_D s - D(s) = 0(Js^2 + Bs)$$

$$\Rightarrow J\ddot{\theta} + B\dot{\theta} + K_D(\dot{\theta}_d - \dot{\theta}) - K_p(\theta_d - \theta) = d(t)$$

b) PD controller with Feed Forward

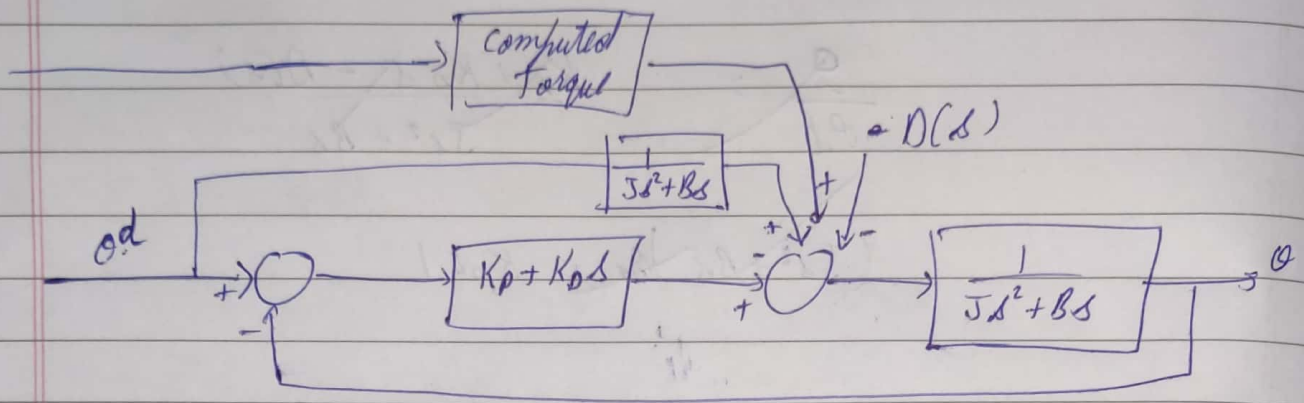


Control Equation

$$(\theta_d - \theta)(K_p + K_D s) + \theta_d(Js^2 + Bs) - D(s) = 0(Js^2 + Bs)$$

$$J\ddot{\theta}_d - J\ddot{\theta} + B(\dot{\theta}_d - \dot{\theta}) + K_D(\dot{\theta}_d - \dot{\theta}) + K_p(\theta_d - \theta) = d(t)$$

c) Computed Torque



$$(\theta_d - \theta) (K_p + K_D s) + \theta_d (Js^2 + Bs) - D(s) \overset{+ T_{computed}}{\wedge} = \theta (Js^2 + Bs)$$

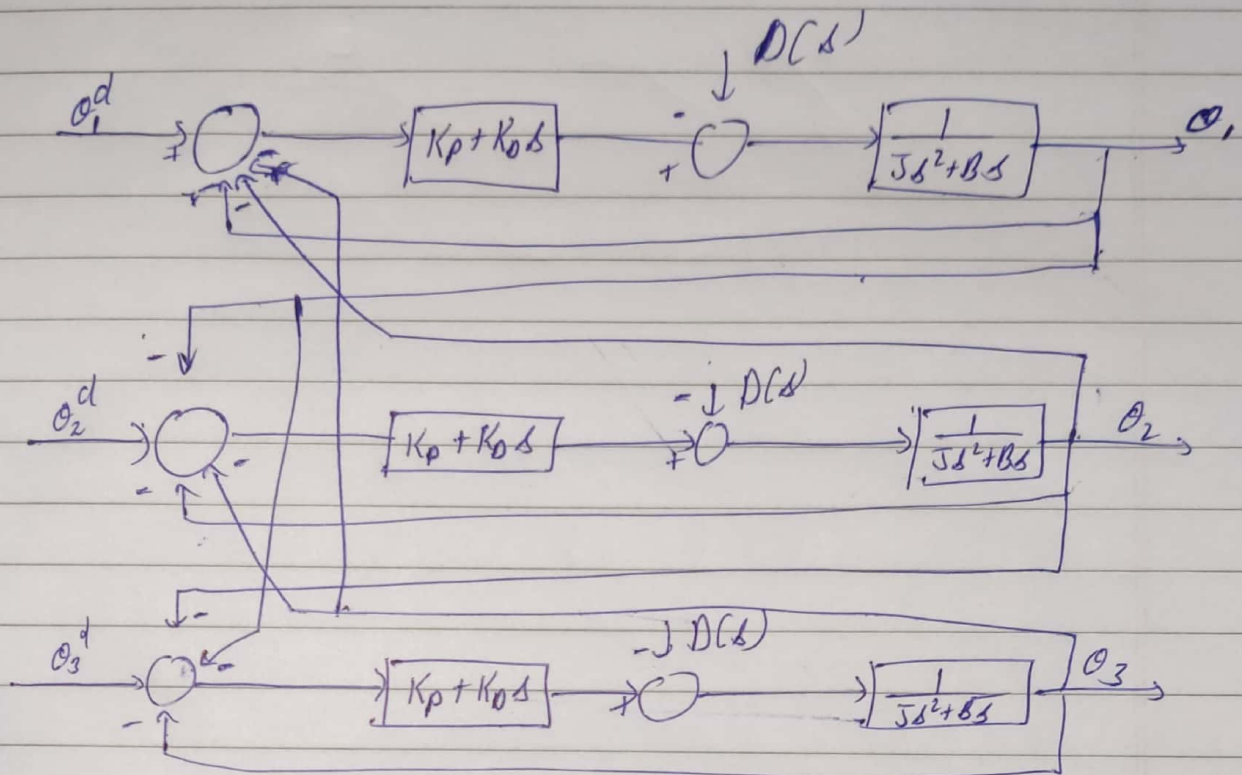
$$J(\ddot{\theta}_d - \ddot{\theta}) + (B + K_D)(\dot{\theta}_d - \dot{\theta}) + K_P(\theta_d - \theta) + T_{computed} = \theta \frac{d}{dt}$$

The gains K_P & K_D chosen for the given task is

$$K_P = 1000$$

$$K_D = 9$$

1) Multivariable Control



Control Equations

Eq

$$J_{eff} \ddot{\theta}_1 + (B + K_D) \dot{\theta}_1 + K_P \theta_2 + K_D \dot{\theta}_3 + K_P (\theta_1^d - \theta_1) + K_P \theta_2 + K_P \theta_3 = -d(t)$$

Similarly for θ_2 & θ_3

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It ~~was~~ is observed that even though the desired trajectory for x is zero, there is some disturbance observed during the actual dynamics. ~~But~~ Although the error is very less.