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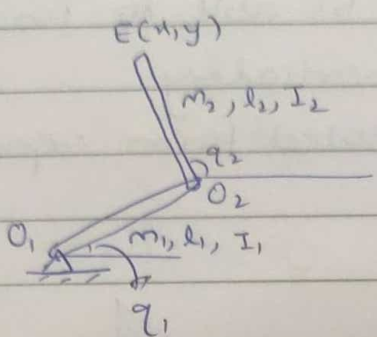
Introduction to Robotics

2R Manipulator [Elbow Manipulator]



O_1, O_2 ⇒ Revolute joints
E ⇒ End effector
(x, y) ⇒ End effector position
q_1, q_2 ⇒ Joint angles (Absolute)

Assume origin at O_1
Assume motors are connected to joints O_1 & O_2.
we can control torques, τ_1 & τ_2, or angles, q_1 & q_2
⇓ Torque control ⇓ Angle control

Task 1 ⇓ Make robot follow arbitrary trajectory ; Task 2 ⇓ Make robot touch given wall & apply cons. force ; Task 3 ⇓ Make robot behave like a virt. spring ; Task 4 ⇓ Det. range of possible pos. of E [workspace]

Now,

$$\left.\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 = l_1 cq_1 + l_2 cq_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 = l_1 sq_1 + l_2 sq_2 \end{aligned}\right\} \text{①}$$

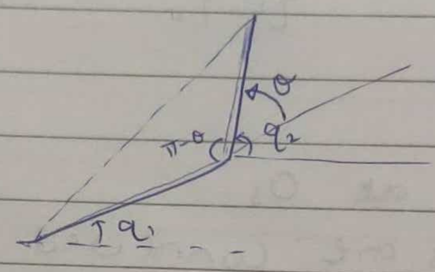
Differenciating ①

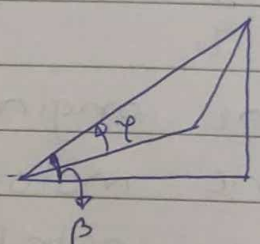
$$\dot{x} = -l_1 sq_1 \cdot \dot{q}_1 - l_2 sq_2 \cdot \dot{q}_2$$
$$\dot{y} = l_1 cq_1 \cdot \dot{q}_1 + l_2 cq_2 \cdot \dot{q}_2$$

End effector velocity ⇒

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{— (2)}$$

The reverse relationships ⇒
Given x & y, we need to be able to find q_1 & q_2.
→ Solve numerically
→ Derive closed form expression





Apply cosine rule

(3)

$$\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2\cos\theta}\right)$$

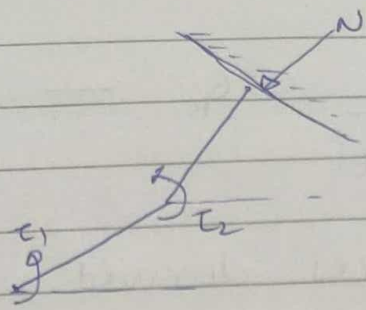
$$q_1 = \cancel{\gamma}\ \beta - \gamma$$

$$q_2 = \theta + q_1$$

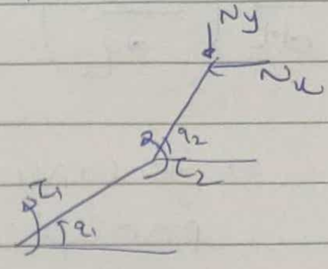
For T_1: Control motors in position control mode to achieve above q_1 & q_2 at each time step

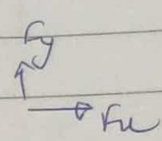
x_d, y_d ⇒ Desired position
q_{1d}, q_{2d} ⇒ Desired angles

for τ_2



F.B.D.

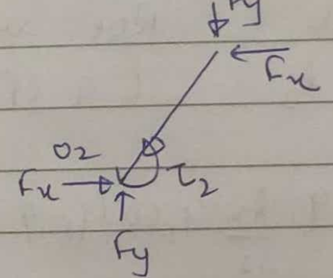




Assuming static equilibrium

$\Sigma M_{O_1} = 0$ & $\Sigma M_{O_2} = 0$

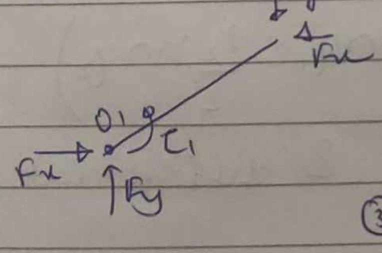
FBD of link 2 [Ignore gravity]



$\Sigma M_{O_2} = 0$

(4) — $F_y l_2 \cos q_2 - F_x l_2 \sin q_2 = \tau_2$

for link 1



(4) — $F_y l_1 \cos q_1 - F_x l_2 \sin q_1 = \tau_1$

(3) + (4) $\Rightarrow$ Solves T_2

for T3. Understanding the dynamics of robot

Lagrange's Equations :

$$\mathcal{L} = \underset{\text{K.E.}}{K} - \underset{\text{P.E.}}{V}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \quad \text{—} \quad (5)$$

$Q_i \Rightarrow$ Generalized forces derived using principle of virtual work.

$$K = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} m_2 v_{c_2}^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2$$

$$= \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2} m_2 v_{c_2}^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2$$

$\Downarrow$ Pure rotation about O_1 — $\Downarrow$ Trans. of l_2 — $\Downarrow$ Rot. about C.G. of l_2

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

Considering gravity,

$$V = mg\frac{l_1}{2}\sin q_2 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)$$

Substitute in $L = K - V$ & differenciate

A) $$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1)$$
$$- m_2 \frac{l_1 l_2}{2} \ddot{q}_2 (\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1) + m_1 g \frac{l_1}{2} c q_2 + m_2 g l_1 c q_1$$
$$= \tau_1$$

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B) $\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1)$

$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} s q_2 = \tau_2$

We note that (4) is valid for any end-effector forces F_x & F_y (not just wall reactions)

For spring effect $\Rightarrow$ $F_x = Kx = K_x(x - x_0)$

$F_y = ky = K_y(y - y_0)$

$(x_0, y_0) \Rightarrow$ Need not be in end-effector space

Using (1)

$F_x = K(l_1 c q_1 + l_2 c q_2)$

$F_y = K(l_1 s q_1 + l_2 s q_2)$

Substituting in (4)

$K(l_1 s q_1 + l_2 s q_2) l_2 c q_2 - K(l_1 c q_1 + l_2 c q_2) l_2 s q_2 = \tau_{2s}$

&

$K(l_1 s q_1 + l_2 s q_2) l_1 c q_1 - K(l_1 c q_1 + l_2 c q_2) l_1 s q_1 = \tau_{1s}$

For Spring Effect — (7)

Set motor torques to be $\underset{(A)}{\tau_1} + \underset{(7)}{\tau_{1s}}$ & $\underset{(B)}{\tau_2}$ & $\underset{(7)}{\tau_{2s}}$ respectively.

For T1 $\Rightarrow$ Use q_{1d} & q_{2d} calculated using (3), then sub in A) & B) & motor torques τ_1 & τ_2 set using A) & B)