ME 639

Mid-Semester Exam

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5) Yes, in D-H convention, all joint axes are always aligned with respective z-axis.

6) No, there are several instances that origins of coordinate frames do not align with the joint centres. For eg., spherical wrist.

→ However, even in this case, the z-axes of ~~joints~~ frames do align with their respective joint axes.

7) Yes, a homogeneous transformation accounts for both, rotation & translation.

rotation ~~from~~ of b w.r.t. a

$$H_a^b = \begin{bmatrix} R_a^b & d_a^b \\ 0 & 1 \end{bmatrix}$$

translation from a to b w.r.t. frame a.

8) Yes, for a sequence of rotations performed one after the other. the rotation matrices for each individual rotation can be multiplied together to form the overall rotation matrix.

9) Yes, a composite multiplication matrix of several rotation matrices is still an orthogonal matrix with determinant equal to 1.

Let R_1, R_2 be 2 rotational, orthogonal matrices.

Thus,

$$R_1 R_1^T = R_1^T R_1 = I \Rightarrow \det(R_1) = 1$$
$$R_2 R_2^T = R_2^T R_2 = I \Rightarrow \det(R_2) = 1$$

If $R = R_1 R_2$

$$\Rightarrow R^T = R_2^T R_1^T$$

$$\Rightarrow RR^T = R_1 R_2 R_2^T R_1^T = R_1(I)R_1^T = I$$

Slly,

$$R^T R = R_2^T R_1^T R_1 R_2 = R_2^T (I) R_2 = I$$

$$\cancel{\det(R^T) \cdot \det(R) = 1}$$
$$\cancel{\Rightarrow \det^2(R) = 1}$$
$$\cancel{\rightarrow \det(R) = 1}$$

$$\det(R) = \det(R_1)\det(R_2) = 1.$$

& $R^T R = RR^T = I$

Thus, by induction, we can prove that any matrix obtained by multiplying 2 or more orthogonal matrices is orthogonal.

Answer 1.d

→ For any of the given 3 manipulators, we can calculate a corresponding Jacobian (let's say J).

~~Thus,~~ → Let J_v be the velocity Jacobian.

Thus,

$$v = J_v \cdot \dot{q}$$

Thus, we can define pseudoinverse of J_v such that

$$\dot{q} = J_v^+ \cdot v \quad ; \quad \underline{J_v^+ = J_v^T (J_v J_v^T)^{-1}}$$

~~Thus, given any velocity v of end effector, we can calculate J_v,~~

→ ~~Thus~~, at specific ~~intervals, we~~

→ Thus, at specific time intervals (Δt), we can ~~calculate~~ measure corresponding joint variables, calculate J_v, use J_v to calculate J_v^+ & use $\dot{q} = J_v^+ v$ to calculate joint velocities corresponding to that moment.

Δt → can be the sampling rate of encoders at joints

4)

Answer 2.a

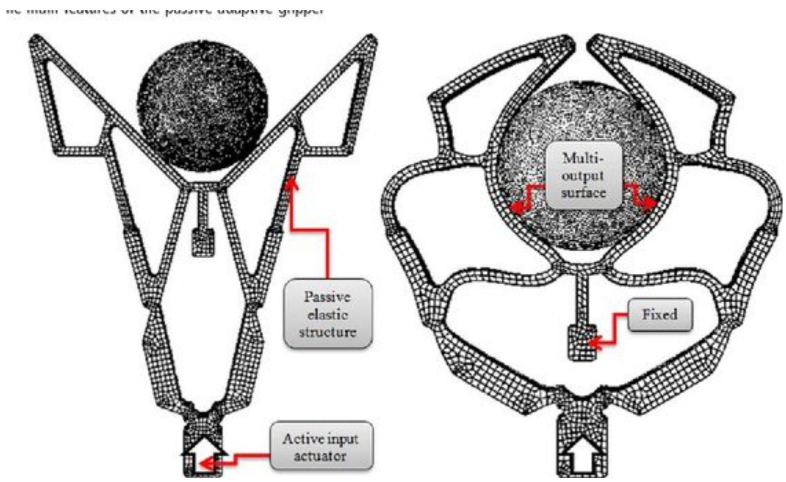
→ According to me, a compliant gripper will be more suitable for the task. ~~thy~~ ~~that~~ The benefits of using a compliant gripper (over hard gripper) are listed as follows:

i) Small size of pills: Since we ~~need~~ need to pick up pills from ~~of~~ a cup, gripper should be small enough to go inside cup and even adjust its orientation. A hard gripper will require a lot of minute parts to be ~~manufactured~~ manufactured. whereas, in compliant ~~aa~~ gripper, whole gripper can be ~~man~~ manufactured as a single part.

ii) Grabbing force: Crushing the pills due to extra force of hard gripper will be un-desirable. ~~Ea~~ A compliant gripper can be designed according to never exceed the grabbing force than the specified value.

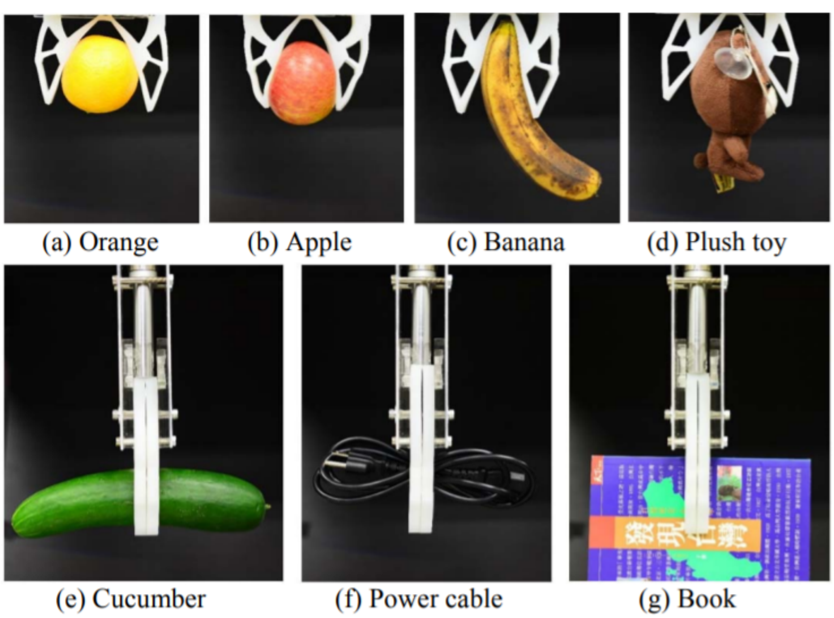
iii) Freedom of grabbing shape: Due to their flexible nature, we can design the compliant gripper to be responsive to shape and orientation of pill. This will be crucial if the ~~grasp~~ gripper has to be 1-DOF, ~~sin~~ otherwise orientation of gripper will require another DOF.

Answer 2.d



Source: https://in.pinterest.com/pin/654147914608801418/

As explained in the previous question, a gripper with an adjustable grabbing shape will be best suitable for the job. As we can see in the figure, the gripper above has one degree of freedom. The active input actuator only moves back and forth, making it ideal for use.
Further, its surface will change accordingly to grab the pill in any orientation. Thus, no extra DOF is required for adjusting the orientation of the gripper.

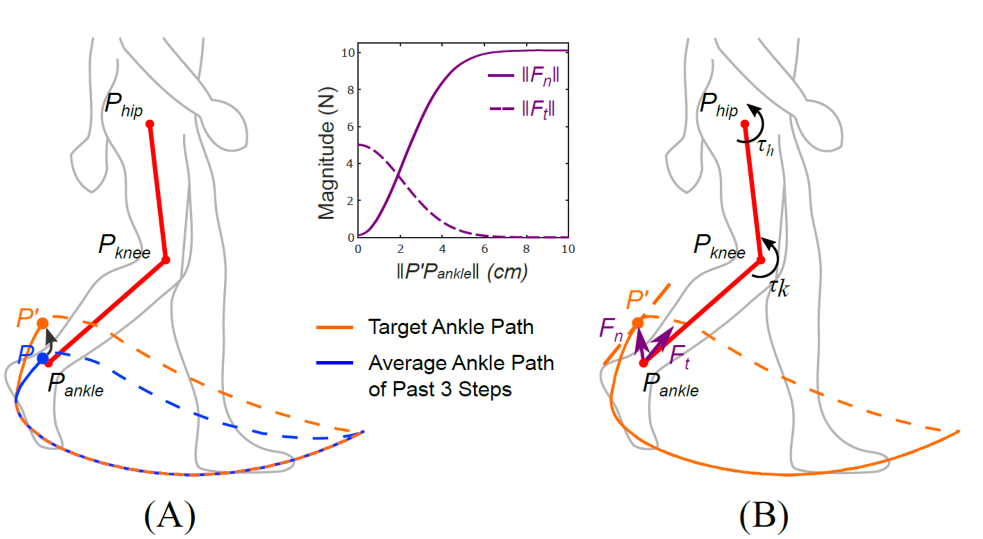


Source: https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=7989332

The holding capabilities of this type of grippers have been tested in this paper. As we can in the figure from the paper, the gripper can be used to hold objects of various shapes and hold them against their weight. A suitable gripper can be formed for the pill picking job by sufficiently reducing the size and power of the gripper above.

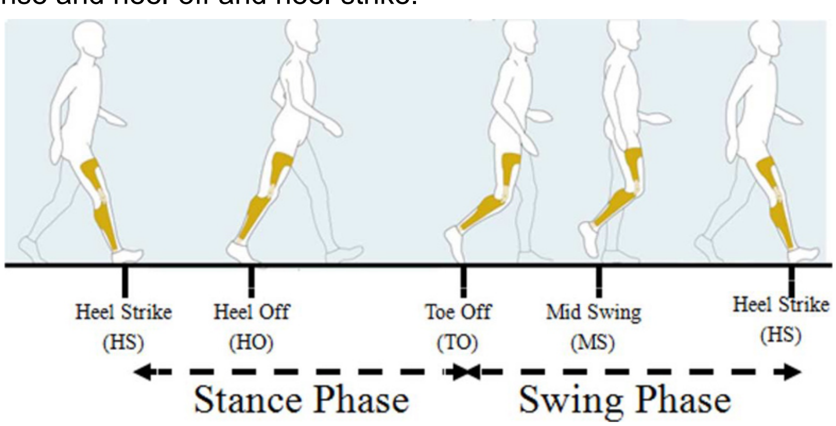
Answer 3.a

Taking the measurements for legs from [this anthropometry journal](),
Knee-length: Distance from ankle to knee in sitting position = 507 mm = 50.7 cm
Thigh-length: Distance from knee to hip in sitting position = 549 mm = 54.9 cm



Source: https://roar.me.columbia.edu/news/retraining-human-gait-are-lightweight-cable-driven-leg-exoskeleton-designs-effective

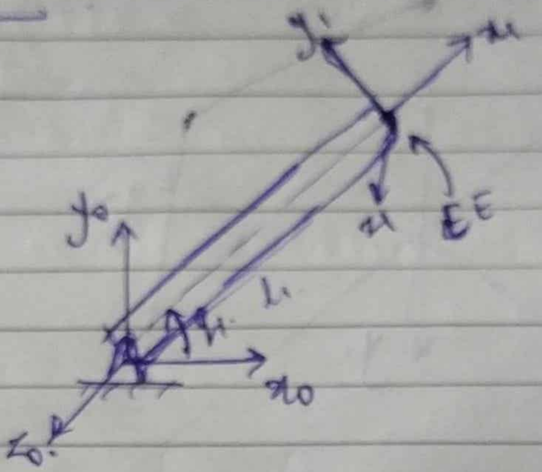
Gait Trajectory is the trajectory followed by the ankle of a human during one complete swing and support phase while walking. The trajectory of one ankle is measured with respect to the body (torso).
Step Height is characterised as the height of the highest point the ankle achieves during one cycle.
Step Length is characterized by the distance covered by the ankle during the swing phase. Given the gait trajectory, it can be given by the horizontal distance between the points of heel rise and heel-off and heel-strike.



Source:
https://www.embs.org/tnsre/articles/assessment-of-foot-trajectory-for-human-gait-phase-detection-using-wireless-ultrasonic-sensor-network/

4)

Answer 4.A



→ Considering the frames as shown in the figure,

DH Params

q rotat^n abt z	d translat^n abt z	a trans^n abt x	α rotat^n abt x
q_1	0	l_1	0

Answer 4.B

Given:

$$ml^2 \frac{d^2q_1}{dt^2} + mgl\sin(q_1) = \tau_1$$ [mass of link assumed to be at the end.

Thus, neglecting gravity,

$$ml^2 \frac{d^2q_1}{dt^2} = \tau_1$$ → torque required to induce accelerat^n of $\frac{d^2q_1}{dt^2}$

→ Torque required to make it act like a ~~spring~~ torsion spring.

$$\tau_{ex} = -K(q_1 - q_0)$$

τ_{ex}: extra torque to be applied.

K: torsion constant

q_0: mean position to act as torsion.

$$\tau_{net} = \tau_1 + \tau_{ex} = ml^2\ddot{q}_1 - K(q_1 - q_0)$$

$\ddot{q}_1$ → induced acceleration.

→ torque to induce $\ddot{q}_1$ at an angle of q_1

However, if no acceleration is to be induced, $\ddot{q}_1 = 0$,

$$\therefore \tau_{net} = -k(q_1 - q_0)$$