ME639

ASSIGNMENT - 3

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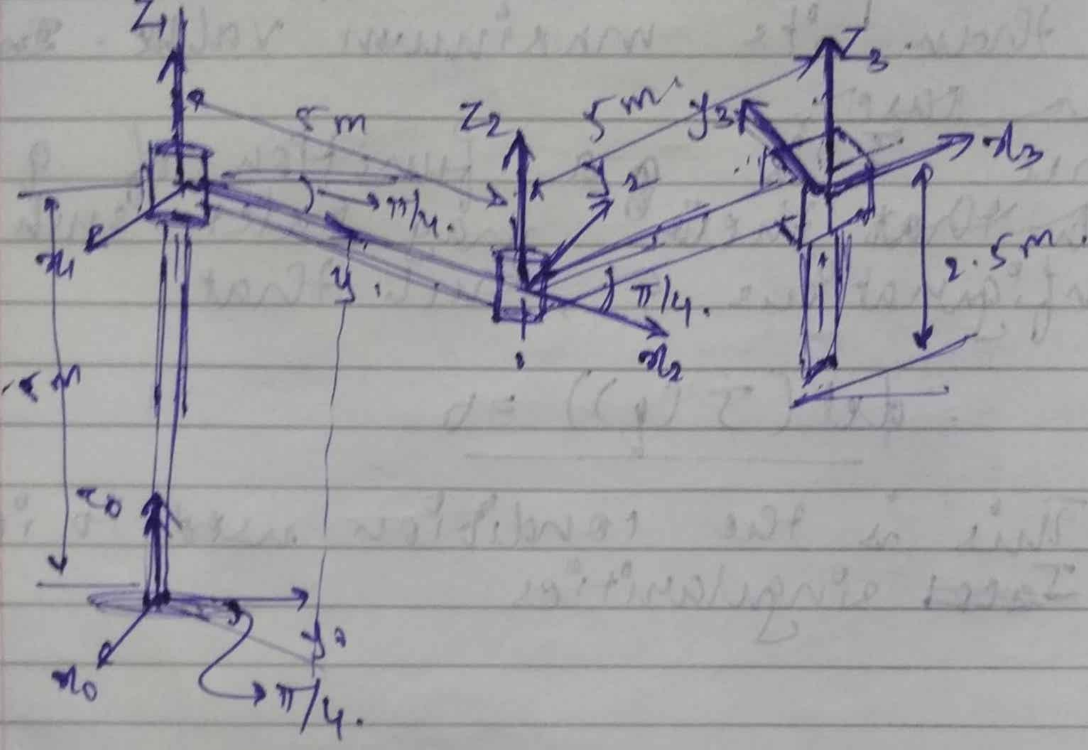
1) A singularity is a configuration of the manipulator such that the rank is less than its maximum value. ~~In any such case~~

→ Since J is a function of q, J(q), ~~such that~~ there may exist such configurations q, such that

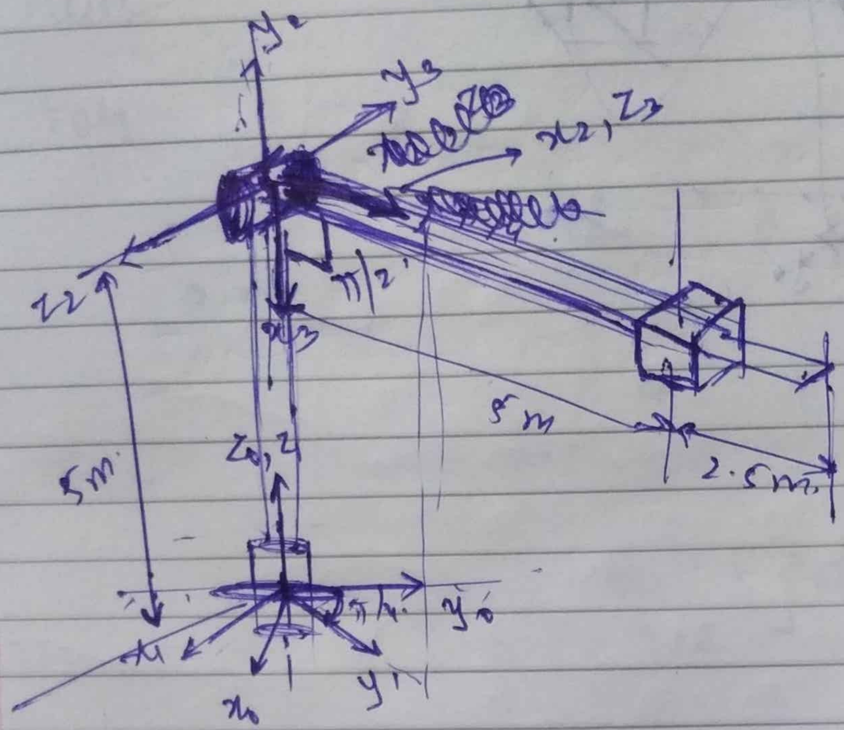
$$\det(J(q)) = 0$$

→ This is the condition used to identify ~~Jacob~~ singularities.

(b)

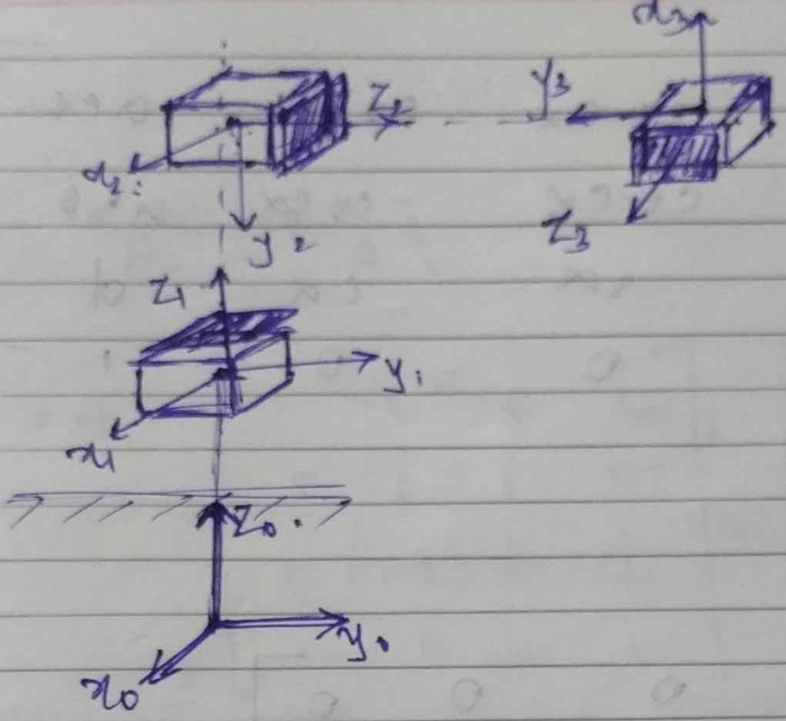


SCARA configuration used in Q.4.



Stanford Configuration used in 8th

5)



Thus,

For $0 \longrightarrow 1$,

$\theta = 0$; $d = d_0$ (a parameter)
$a = 0$; $\alpha = 0$ ..

For $1 \longrightarrow 2$,

$\theta = 0$; $d = d_1$ (joint variable)
$a = 0$; $\alpha = -\pi/2$

For $2 \longrightarrow 3$,

$\theta = -\pi/2$; $d = d_2$ (joint variable)
$a = 0$; $\alpha = -\pi/2$

$$A = \begin{bmatrix} c\theta & -s\theta c\alpha & s\theta s\alpha & a c\theta \\ s\theta & c\theta c\alpha & -c\theta s\alpha & a s\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$A_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$A_0^3 = A_0^1 \, A_1^2 \, A_2^3$$

$$= A_0^1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= A_0^1 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & d_0+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let d_3 be extension of 3rd prismatic.

Thus,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = A_0^3 \cdot \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & d_0+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_3 \\ d_2 \\ d_0+d_1 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} d_3 \\ d_2 \\ d_0+d_1 \end{bmatrix}$$

Let

$d_0 = 1\ m$
$d_1 = 2\ m$
$d_2 = 4\ m$
$d_3 = 9\ m$

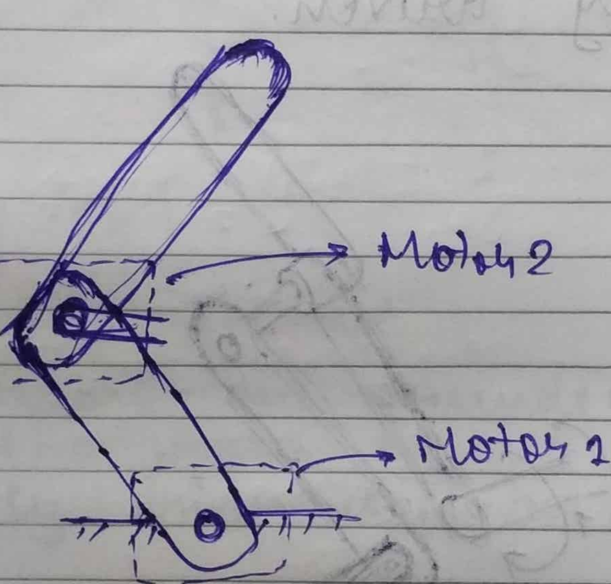
[for verificatn.

Thus,

$$P_0 = \begin{bmatrix} 9 \\ 4 \\ 3 \end{bmatrix} m.$$

[Same as the one calcuted from code

7) ⇒ Direct Driven



→ There exists a housing at the end of link 1 where motor 2 is mounted.

→ ~~The weight of motors are considerably high in the~~

→ Thus, while rotating link 1, even motor 2 will have to be rotated. This increases the overall moment of inertia about motor 1, thus, requiring a high power motor at ~~the~~ joint 1.

→ ~~Also, the angles put in the jo motor 1~~

→ ~~Also~~, the angular orientation of ~~joint 2~~ motor 2 will be ~~not~~ the relative angle of 2 links.

⇒ Remotly driven.

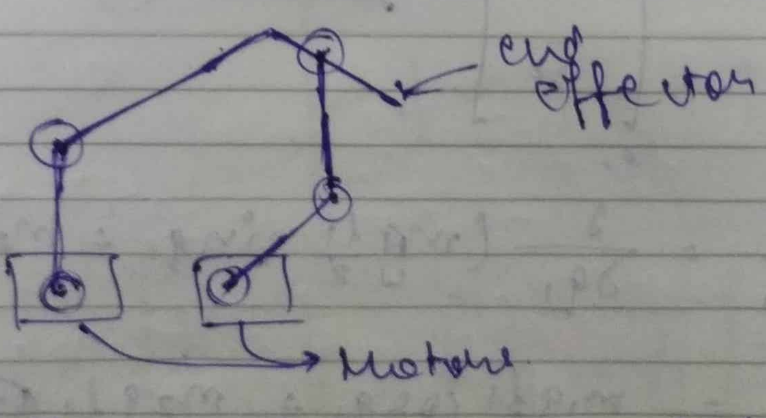
→ Here, both the motors are grounded
→ Thus, the moment of inertia

is reduced and motor 1 can move the manipulator even with lower power.

→ Also, the angular configuration of motor 2 will now have to be the absolute orientation of link 2 rather than a relative one.

→ Belt slippage might occur, to ~~give~~ result in inaccurate control.

⟹ Five bar linkage.

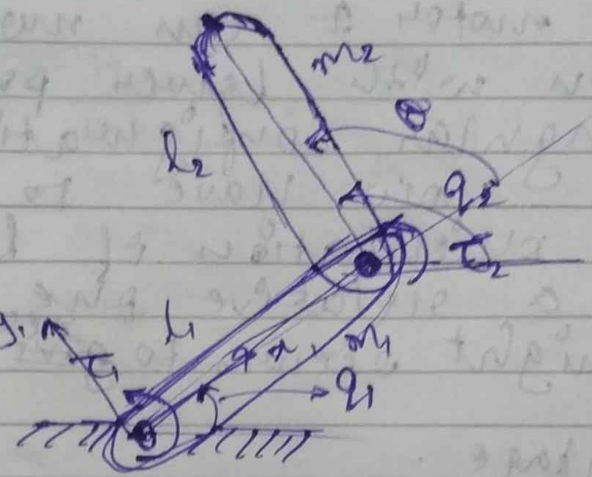


→ Both motor are grounded, thus, no extra ~~motor~~ torque required like directly driven configuratn!

→ No belt slippage can occur, thus, better control over end-effector.

→ However, workspace will be smaller than that of a 2-link manipulator.

8)



$$D(q)\cdot\ddot{q} + c(q,\dot{q})\dot{q} + g(q) = \tau$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad \text{——— (i)}$$

$$\frac{\partial V}{\partial q_1} = \frac{\partial}{\partial q_1}\left(mg\frac{l_1}{2}\sin q_1 + m_2 g\left(l_1\sin q_1 + \frac{l_2}{2}\sin\right.\right.$$

$$= m_1 g\frac{l_1}{2}\cos q_1 + m_2 g l_1 \cos q_1$$

$$= \cancel{\left(m_1 + \frac{m_2}{2}\right)}$$

$$\frac{\partial V}{\partial q_1} = \left(\frac{m_1}{2} + m_2\right) g l_1 \cos q_1 = \phi_1$$

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$$\phi_2 = \frac{\partial V}{\partial q_2} = \frac{\partial}{\partial q_2}\left(m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2 \right) \right)$$

$$\phi_2 = \frac{m_2 g l_2}{2} \cos q_2$$

$$g(q) = \begin{bmatrix} \left(\frac{m_1}{2} + m_2 \right) g l_1 \cos q_1 \\ \frac{1}{2} m_2 g l_2 \cos q_2 \end{bmatrix} \quad — (2)$$

In class, it was discussed that,

$$D(q_1) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) \\ m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_1 \end{bmatrix}$$

$$c_{221} = -m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$$

$$c_{112} = m_2 \frac{l_1 l_2}{2} s(q_2 - q_1)$$

&

$$d_{11} \ddot{q}_1 + d_{12} \ddot{q}_2 + c_{221} \dot{q}_1^2 + \phi_1 = \tau_1$$

$$d_{21} \ddot{q}_1 + d_{22} \ddot{q}_2 + c_{112} \dot{q}_1^2 + \phi_2 = \tau_2$$

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$$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{3} + m_2 l_1^2 & m_2 \frac{l_1 l_2}{2} c(q_2 - q_1) \\ m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) & \frac{m_2 l_2^2}{3} \end{bmatrix}$$

Thus,

$$\left(\frac{m_1 l_1^2}{3} + m_2 l_1^2\right)\ddot{q}_1 + \left(\frac{m_2 l_1 l_2}{2} c(q_2 - q_1)\right)\ddot{q}_2$$

$$- \frac{m_2 l_1 l_2}{2} \sin(q_2 - q_1)\dot{q}_1^2$$

$$+ \left(\frac{m_1}{2} + m_2\right) g l_1 \cos q_1 = \tau_1 \quad (1)$$

& $$\left(\frac{m_2 l_1 l_2}{2} c(q_2 - q_1)\right)\ddot{q}_1 + \left(\frac{m_2 l_2^2}{3}\right)\ddot{q}_2$$

$$+ \dot{q}_1^2\left(\frac{m_2 l_1 l_2}{2} \sin(q_2 - q_1)\right) + \frac{m_2 g l_2}{2} c_2 = \tau_2 \quad (2)$$

PTO.

→ Comparing with equations derived for miniproject,

(1')

$$\frac{1}{3}m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 c_{21}$$
$$- m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$
$$+ m_1 g \frac{l_1}{2} c_1 + m_2 g l_1 c_1 = \tau_1$$

(2')

$$\frac{1}{3}m_2 l_2^2 \ddot{q}_2 + \frac{m_2 l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 c_{21}$$
$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$
$$+ m_2 g \frac{l_2}{2} c_2 = \tau_2$$

→ One of the discripensies observed is that the older equations had an extra term of $\dot{q}_1 \cdot \dot{q}_2$, where as, the product of angular velocities does not show up in ~~the~~ the recently derived equations.

10) Once $P(q)$ & ~~$Q(q)$~~ $V(q)$ are given

→ Let D be $n \times n$ matrix.

→ Calculate $g(q)$ as:

$$g(q) = \begin{bmatrix} \frac{\partial V}{\partial q_1} \\ \frac{\partial V}{\partial q_2} \\ \vdots \\ \frac{\partial V}{\partial q_n} \end{bmatrix}$$

→ Calculate $C(q, \dot{q})$ as follows:

$$c_{jk} = \sum_{i=1}^{n} c_{ijk} \cdot \dot{q}_i$$

j,k^{th} coefficient of $C(q, \dot{q})$

Christoffel's co-efficient

$$c_{kj} = \sum_{i=1}^{n} c_{ijk} \cdot \dot{q}_i$$

$$c_{kj} = \sum_{i=1}^{n} \dot{q}_i \left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right)$$

Thus,

$$C(q, \dot{q}) = \begin{bmatrix} c_{11} & c_{21} & \cdots & c_{n1} \\ c_{12} & c_{22} & \cdots & c_{n2} \\ c_{13} & c_{23} & \cdots & c_{n3} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1n} & c_{2n} & \cdots & c_{nn} \end{bmatrix}$$

Apply,

$$D(q)\cdot\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}$$