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Assignment - 2

Q.1

We know that Rotation matrix are orthogonal.

$$R^1_0 = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix}$$

Let $V_1 = \begin{pmatrix} \hat{i}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 \end{pmatrix}$; $V_2 = \begin{pmatrix} \hat{j}_1 \cdot \hat{j}_0 \\ \hat{j}_1 \cdot \hat{j}_0 \\ \hat{j}_1 \cdot \hat{k}_0 \end{pmatrix}$; $V_3 = \begin{pmatrix} \hat{k}_1 \cdot \hat{i}_0 \\ \hat{k}_1 \cdot \hat{j}_0 \\ \hat{k}_1 \cdot \hat{k}_0 \end{pmatrix}$

$$\therefore \quad R^1_0 = [V_1 \quad V_2 \quad V_3]$$

As R^1_0 is orthogonal

$$(R^1_0)^T (R^1_0) = I$$

$$\begin{bmatrix} V_1^T V_1 & V_1^T V_2 & V_1^T V_3 \\ V_2^T V_1 & V_2^T V_2 & V_2^T V_3 \\ V_3^T V_1 & V_3^T V_2 & V_3^T V_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing these 9 elements, it is proved that Columns of R^1_0 are orthogonal.
We can also infer that they are of unit length.

Q.2 To show $\det(R_0^1) = 1$

We know that R_0^1 is an orthogonal matrix

$$\therefore R_0^1 (R_0^1)^T = I$$

~~them~~ Property of determinants.

$$\Rightarrow \det(A) \times d(B) = \det(A \times B)$$

We know that $R_0^1 = (R_0^1)^T$

$$\therefore \det(R_0^1) = \det((R_0^1)^T)$$

$\therefore$ We can write

$$\det(R_0^1) \times \det((R_0^1)^T) = \det\left(R_0^1 (R_0^1)^T\right)$$

$$[\det(R_0^1)]^2 = \det(I)$$

$$[\det(R_0^1)]^2 = 1$$

$\therefore \det(R_0^1) = 1$ (in RHS system)

(-1 in LHS system or mirroring)

Q.5 Let R be Rotation matrix $R \in SO(3)$

$\therefore$ We know R is orthogonal

Let a, b be vectors in $\mathbb{R}^3$

We can write

$$R(a \times b) = Ra \times Rb \qquad (\because R \text{ is orthogonal})$$

$$LHS = R\,S(a)\,R^T b = R(a \times R^T b)$$

$$(\because S(a)p = a \times p)$$

$$= Ra \times RR^T b$$

$$= Ra \times b \qquad (\because RR^T = I)$$

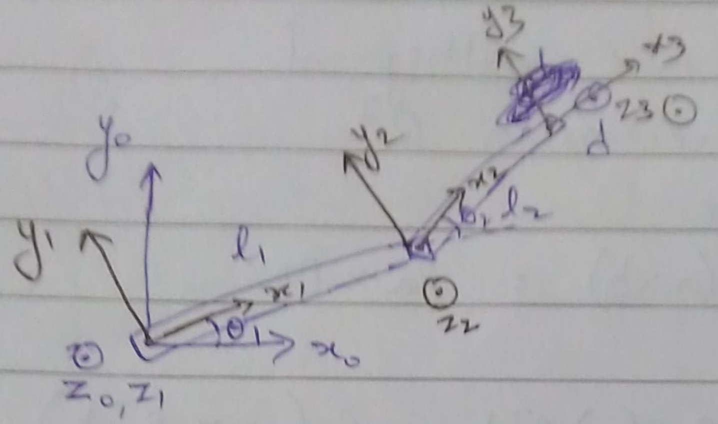
$$R\,S(a)\,R^T b = S(Ra) \times b$$

This equality holds for all $b \in \mathbb{R}^3$

$$R\,S(a)\,R^T = S(Ra)$$

Hence, proved.

Q.6



RRP SCARA

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

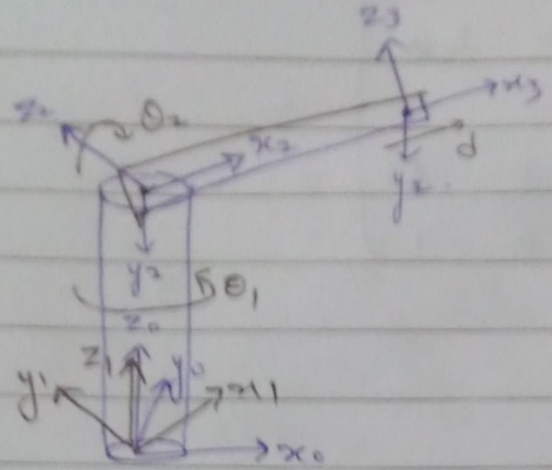
$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (c_1c_2 - s_1s_2) & (-s_2c_1 - s_1c_2) & 0 & c_1l_1 \\ (s_1c_2 + c_1s_2) & (-s_1s_2 + c_1c_2) & 0 & s_1l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} (c_1c_2 - s_1s_2) & (-s_2c_1 - s_1c_2) & 0 & l_2(c_1c_2 - s_1s_2) + c_1l_1 \\ (s_1c_2 + c_1s_2) & (-s_1s_2 + c_1c_2) & 0 & l_2(s_1c_2 + c_1s_2) + s_1l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} l_1(c_1c_2 - s_1s_2) + c_1l_1 \\ l_2(s_1c_2 + c_1s_2) + s_1l_1 \\ d \end{bmatrix}$$

Q.8



Stanford type RRP

$$R_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 \\ 0 & \sin\pi/2 & \cos\pi/2 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 1 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} 1 & -s_2 & 0 \\ 0 & 0 & -1 \\ s_2 & c_2 & 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 & -s_2 c_1 & s_1 & 0 \\ s_1 & -s_1 s_2 & -c_1 & 0 \\ s_2 & c_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 & -s_2 c_1 & s_1 & c_1 l_2 \\ s_1 & -s_1 s_2 & -c_1 & s_1 l_2 \\ s_2 & c_2 & 0 & s_2 l_2 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \; P_0 = \begin{bmatrix} c_1 d + c_1 l_2 \\ s_1 l_2 \\ s_2 l_2 + l_1 \end{bmatrix}$$

Q.10

Planetary Gear trains:

It is a compact and highly versatile in application. It is used in various powertrains. It has ability to produce high gear ratios. They can handle highest input speeds upto 8500 rpm, but their lost motion is also largest upto 4-6 Arcmin. They suffer from high losses derived from high virtual powers.

Harmonic Drives:

Initially used in aerospace carriers. It has very effective IP protection strategy. shape is characterized by larger diameters than lengths, while the weights are substantially lower than for other technologies and result in the best torque-to-weight ratios of the analyzed technologies.

Cycloid Drives:

It has applications in application mainly in boats, cranes, and some large equipment as steel strip rolling trains or CNC machines. It is very compact and difficult to manufacture. Its efficiency is highly dependent on the operating conditions. They have an inherent limitation to cope with high input speeds, caused by the presence of a large and relatively heavy planet (cam) wheel resulting in large inertias and imbalances.

Reference: Frontiers | Compact Gearboxes for Modern Robotics: A Review | Robotics and AI (frontiersin.org)