$$\lim_{n \to \infty} \frac{f(n)}{g(n)}$$

$f(n) = 1000n$

$g(n) = n^2$

We can't use limits!

$f \le g$ ✗

$f \ge g$ ✓

$f = g$ ✗

g is "worse" than f.

For algorithm A and input x

$t_A^*(x) :=$ # of steps taken by A on i/p x.

worst-case time of A

$$t_A(n) = \max_{x : |x| = n} t_A^*(x)$$

$t_A : \mathbb{N} \to \mathbb{N}$ (No definition of continuity!)

1) Consider behavior only on large inputs.

We say $f \leq g$ if there is some $n_0 \in \mathbb{N}$ s.t for all $n \geq n_0$

$$f(n) \leq g(n)$$

Not enough!

Note: $1000n \leq n^2$ why?

Pick $n_0 = 1000$ and satisfy the definition.

For any constant k

$$kn \leq n^2 \quad (\text{Pick } n_0 = k)$$

Also, observe $\lim_{n \to \infty} \frac{kn}{n^2} = 0$

2) Constant factors do not matter

$2n^2 \leq n^2$ ✓

Compare $2n^2$ and n^2 $\quad n^2 \leq 2n^2$

$$f(n) = \begin{cases} n^2, & \text{if } n \text{ odd} \\ 2n^2, & n \text{ even} \end{cases} \qquad g(n) = \begin{cases} 2n^2, & n \text{ odd} \\ n^2, & n \text{ even} \end{cases}$$

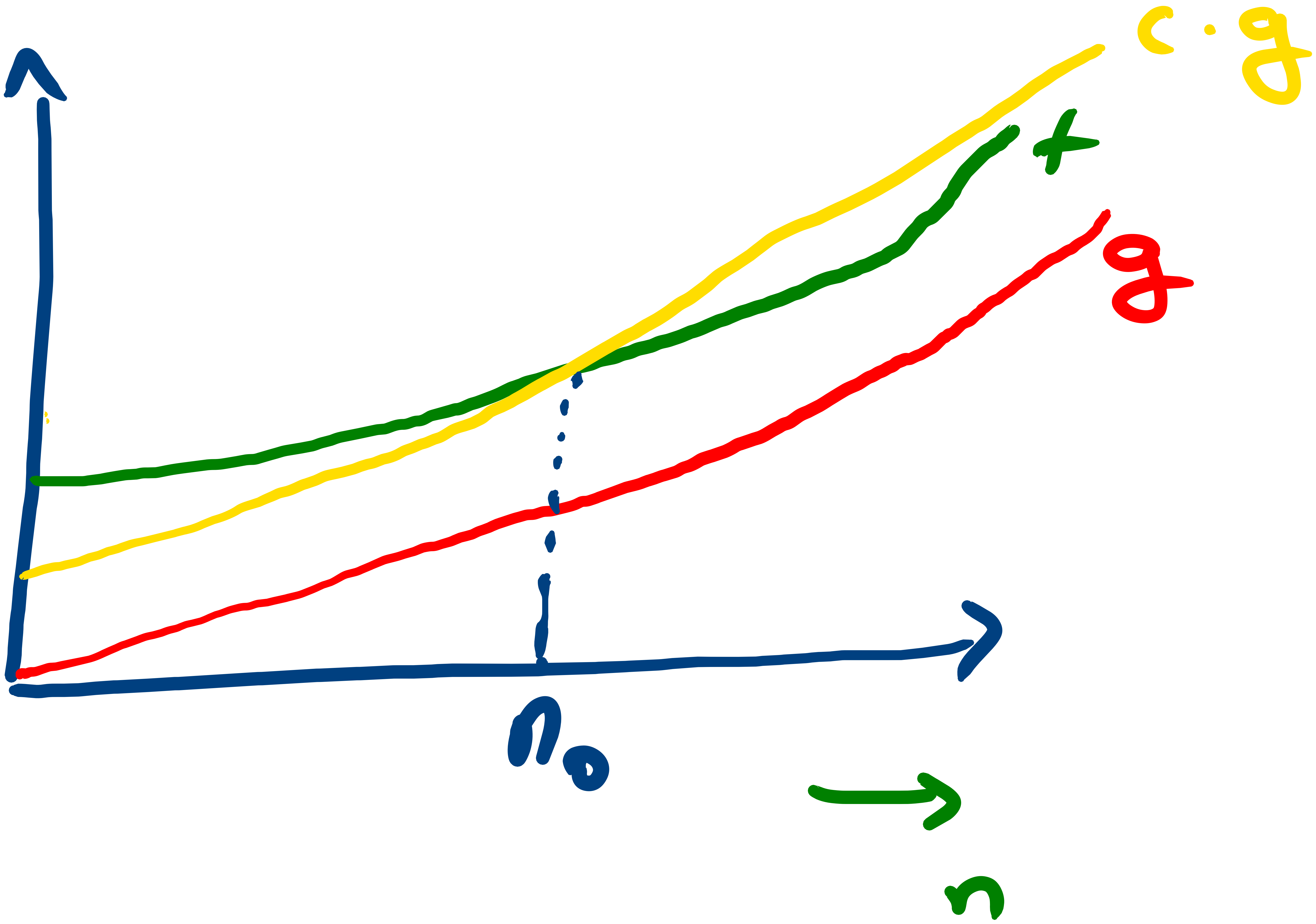
$f(n) = n^2$ $\qquad$ $g(n) = 2n^2$

$$\lim_{n \to \infty} \frac{f(2n)}{f(n)} = 4 \qquad \lim_{n \to \infty} \frac{g(2n)}{g(n)} = 4$$

$f \leq g$ if there exists $n_0 \in \mathbb{N}$, $c > 0$ s.t for all $n \geq n_0$ $f(n) \leq c \cdot g(n)$

$O(g) = \{ f \mid f \leq g \text{ as above} \}$

"big-Oh" of g


c · g
f
g
n₀
n

For any fn. g: $O(g)$ is a set of functions.

$f \in O(g) \sim f = O(g)$

$f \notin O(g) \qquad f \neq O(g)$

$$O(n^2) \leq O(n^3) \leq O(2^n)$$

$$f(n) = 2^n, \quad n \leq 1000$$

$$2^{1000} + n^2, \quad n > 1000$$

$$n_0 = 1001, \quad c = 2^{1000} \left\{ \begin{array}{l} f \leq c \cdot n^2 \\ \text{for } n \geq n_0 \end{array} \right\}$$

To Prove $f \notin O(n)$ you have to show for all constants $c > 0$ there are infinitely many n' s.t

$$f(n') > c \cdot n'$$

"big-omega" of g

$$\Omega(g) = \left\{ f \;\middle|\; \begin{array}{l} \text{there are constants } n_0, c > 0, \\ \text{s.t. } \quad c \cdot g(n) \leq f(n) \\ \qquad \text{for all } n \geq n_0 \end{array} \right\}$$

Observe $\quad \dfrac{n^2}{1000} = \Omega(n) \quad \Big| \quad \dfrac{n}{1000} \stackrel{?}{=} \Omega(n)$

$$f(n) = \begin{cases} n, & n \text{ even} \\ n^2, & n \text{ odd} \end{cases} \qquad g(n) = \begin{cases} n^2, & n \text{ even} \\ n, & n \text{ odd} \end{cases}$$