Recursion = Self-reference

$\min(\{1,2,5,-7\}) := \min(\{1,2,5,-7\})$

Correct. But useless.

$$\min(A) = \begin{cases} \infty & \text{if } A = \emptyset \\ a_0 & \text{if } a_0 < a^* \\ a^* & \text{o/w} \end{cases} \quad \Big| \; a^* = \min(A \setminus \{a_0\})$$

Correct & useful.

$\min(\{1, 2, -1\})$

Pick $a_0 = 1$

$a^* = \min(\underline{\{2, -1\}}) = -1$

$a_0 > a^*$ so -1 is the min.

Programs that we can write using <u>recursion</u>

writing small board games like tic-tac-toe.

best_move(B, Player)

Return : move & best possible score in worst-case

score

for each move α for Player

for each move α for player
let B_α be the position after
playing α
compute score of
best_move(B_α, other player)
keep track of α that minimizes
the score above.

To Solve towers of Hanoi on n disks from 1 to 3 with 2 as intermediate.

1 ... 3 for $n=1$

1. Move disks $1 \ldots (n-1)$ from 1 to 2 with 3 as intermediate
2. Shift disk n from 1 to 3
3. move $1 \ldots (n-1)$ from 2 to 3 with 1 as intermediate.

Permutations

A recursive definition for Permutations.

(0, 1, 2)

- Pick **each** element.
- Permute remaining.
- Prepend element picked.

- Pick 0

  0, 1, 2
  0, 2, 1

- Pick 1

  1, 0, 2
  1, 2, 0

- Pick 2

  2, 0, 1
  2, 1, 0

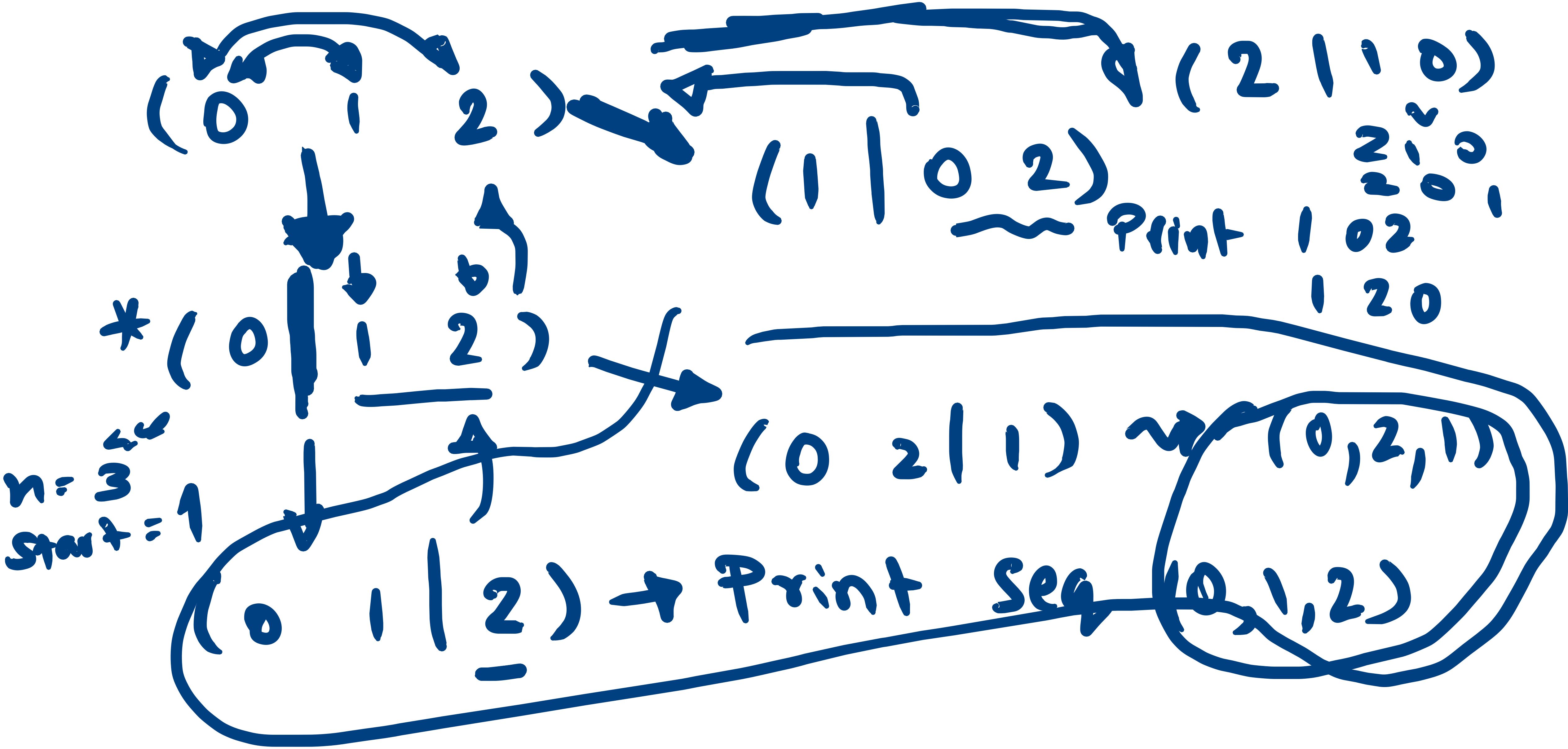
Say I want to generate all permutations of an n-element set.

If an elt takes 1 byte of storage

Total storage for all permutations.

$$\underbrace{n! \times n} \qquad \underbrace{2}^{32} \text{ bytes}$$

What is the smallest n that is bigger than 2^{32}?


(0 1 2)
(1 | 0 2)
Print 1 0 2
1 2 0
(2 | 1 0)
2 1 0
2 0 1
* (0 | 1 2)
n = 3
start = 1
(0 2 | 1)
(0, 2, 1)
(0 1 | 2) → Print Seq
(0, 1, 2)