ES242 Tutorial 1

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1. Define LCM.
2. Prove that $lcm(a, b) = (a \times b)/gcd(a, b)$ using a direct proof. Did you have to use all properties of lcm and gcd stated in their definitions?
3. Is the above a good way to compute the LCM in the real-world? Why or why not? If not, what is a good way?
4. Suppose I want to compute the GCD of three numbers. I do it as gcd(a, b, c) = gcd(gcd(a, b), c). I compute the GCD of two numbers using Euclid's algorithm. The time taken is the sum of the times taken by two gcds. Can you do better?
5. What about computing the gcd of a given sequence of numbers?
6. The *Fibonacci sequence* is defined as: $F_0 = 0, F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$. Prove that $F_n \leq (1.7)^n$ *for all* n using induction.
7. Prove that $F_n \geq (1.6)^n$ for large n.
8. Solve the recurrence $t_1(n) = t_1(n-1) + 3$ with the base case $t_1(0) = 5$ to derive a closed form for $t_1(n)$, a function on natural numbers.
9. Derive closed form for $t_2(n) = t_2(n-a) + b$ where $a, b \geq 1$ are constants (which means they do not depend on n). What are the base cases that need to be defined? Define in the most general way and solve.
10. Derive closed form for $t_3(n) = t_3(\alpha n) + \beta$ where $0 < \alpha < 1$ and β is a constant. What base cases should be defined?
11. Derive closed form for $t_4(n) = 2t_4(\lceil 4n/5 \rceil) + n$ with suitable base cases.