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## DS Traps, Tricks, and Techniques

Data Sufficiency at a glance:

### Topics covered:

- **General Algebra:** Inequalities, Absolute Values (modulus), Number Properties, Exponents, Polynomials, Equations, Quadratics, functions, progressions, and symbols
- **Statistics:** Mean, Median, Range, and Standard Deviation
- **Arithmetic:** Percents, Ratios, Mixtures, Work and Rate, Speed and Distance, Overlapping Sets, Simple and Compound Interest, Population Growth, and miscellaneous Word Problems
- **Geometry:** Plane figures, Solids (not in Focus Edition)
- Co-ordinate Geometry (present in Focus Edition)
- **Combinatorics:** Complex Counting, Permutations & Combinations, and Probability

## Data Sufficiency:

**Directions:** Each data sufficiency problem consists of a question and two statements, labelled (1) and (2), which contain certain data. Using these data and your knowledge of mathematics and everyday facts (such as the number of days in July or the meaning of the word counterclockwise), decide whether the data given are sufficient for answering the question and then indicate one of the following answer choices:

- A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
- B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
- C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- D. EACH statement ALONE is sufficient.
- E. Statements (1) and (2) TOGETHER are not sufficient.

**Note:** In data sufficiency problems that ask for the value of a quantity, the data given in the statements are sufficient only when it is possible to determine exactly one numerical value for the quantity.

**Numbers:** All numbers used are real numbers (this is true of the entire Quant section, PS and DS).

## Figures:

- (1) Figures conform to the information given in the question, but will not necessarily conform to the additional information given in statements (1) and (2).
- (2) Lines shown as straight are straight, and lines that appear jagged are also straight.
- (3) The positions of points, angles, regions, etc., exist in the order shown, and angle measures are greater than zero.
- (4) All figures lie in a plane unless otherwise indicated.

## DS Techniques to avoid falling in Traps:

- Simplify the question to the absolute basics... translate information as to what the question seeks to ask. Most of the DS questions can be simplified.
- Do not assume anything. For example, if a number is not mentioned to be an integer, don't assume it to be so.
- In geometrical figures, do not assume that a figure is what it looks like. If it is not mentioned that two lines are parallel, don't assume so. If a figure looks like a square but is not mentioned to be so, please do not assume it to be so. If an angle looks like an acute / right / obtuse, don't assume it to be so unless specified.
- While evaluating Statement (2), don't "mentally" carry forward the information from Statement (1) to Statement (2). Statement (2) is independent of Statement (1) and vice-versa.
- In "WHAT" questions, a unique numerical value is required. There should be NO AMBIGUITY.
- In "IS" or "Does" type of questions, you must get a unique YES or a unique NO. There should be NO AMBIGUITY.
  - An unambiguous "NO" is as acceptable as an unambiguous "YES".
  - **Intentionally try to create a yes / no situation: don't try to prove or disprove alone ... you should try both.**
- There is no need to calculate the answer in most cases. Avoid calculations, wherever possible.
- In a "WHAT" question, if two statements are not independently sufficient, but, on combining, result in a unique common value, then the common value will be the answer.
- The two statements never contradict each other.
- In questions involving the solving of two simultaneous equations, usually only one statement will be sufficient.

**There are two types of DS questions:**

**Type 1: Value question:**

- Questions like: "what is the value of  $x$ ?" | "How many days did Jack take to finish the work?" etc.
- In these questions, you are supposed to find a UNIQUE NUMERICAL VALUE.
- For example, for the question, "what is the value of  $x$ ", if a particular statement gives  $x = 2$  or  $3$  (this is not unique), it will be considered insufficient. If a particular statement gives  $x = a$ , it will be considered insufficient as 'a' is not a numerical value.

**Type 2: YES/NO questions:**

- Questions like "Is  $x$  an even number?" Or "Did Jack pass his exam?" fall in this category.
- In these questions, any statement that gives a CONFIRMED YES or a CONFIRMED NO will be considered sufficient, but any statement that gives both YES and NO possibilities will be considered insufficient. So, "sometimes yes, sometimes no" is not sufficient information.

## MOST IMPORTANT DS Questions to understand all types of DS traps

Note: Geometry questions have been redacted - Please ignore the blacked out portions while solving

1. If \$1,000 is deposited in a certain bank account and remains in the account along with any accrued interest, the dollar amount of interest,  $I$ , earned by deposit in the first  $n$  years is given by  $I = 1000[(1 + r/100)^n - 1]$ , where  $r$  percent is the annual interest rate paid by the bank. Is the annual interest rate paid by the bank greater than 8%?

- (1) The deposit earns a total of \$210 in interest in the first 2 years.  
(2)  $(1 + r/100)^2 > 1.15$

2. If  $y \geq 0$ , what is the value of  $x$ ? (1)  $|x - 3| \geq y$  (2)  $|x - 3| \leq -y$

3. If  $x$  and  $y$  are positive integers, is  $x$  a prime number?

- (1)  $|x - 2| < 2 - y$   
(2)  $x + y - 3 = |1 - y|$

4. On the number line shown, is zero halfway between  $r$  and  $s$ ? —r—s—t—

- (1)  $s$  is right to the zero  
(2) The distance between  $t$  and  $r$  is the same as the distance between  $t$  and  $(-s)$

5. If  $y$  is an integer and  $y = x + |x|$ , is  $y = 0$ ? (1)  $x < 0$  (2)  $y < 1$

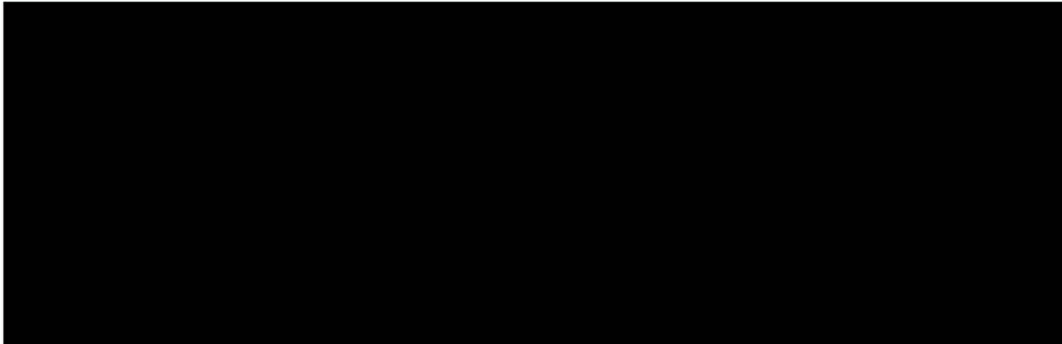
6. Is  $x^4 + y^4 > z^4$ ? (1)  $x^2 + y^2 > z^2$  (2)  $x + y > z$

7. The integers  $m$  and  $p$  are such that  $p > m > 2$ , and  $m$  is not a factor of  $p$ . If  $r$  is the remainder when  $p$  is divided by  $m$ , is  $r > 1$ ?

- (1) the greatest common factor of  $m$  and  $p$  is 2  
(2) the least common multiple of  $m$  and  $p$  is 30

8. If  $x$ ,  $y$ , and  $z$  are integers and  $xy + z$  is an odd integer, is  $x$  an even integer?

- (1)  $xy + xz$  is an even integer (2)  $y + xz$  is an odd integer



11. At least 100 students at a certain high school study Japanese. If 4 percent of the students at the school who study French also study Japanese, do more students at the school study French than Japanese?

- (1) 16 students at the school study both French and Japanese.  
(2) 10 percent of the students at the school who study Japanese also study French.

13. What is the value of  $x$ ?

- (1)  $X^3$  is a 2-digit positive odd integer.      (2)  $X^4$  is a 2-digit positive odd integer.

14. Is  $x$  negative?      (1)  $X^2$  is positive.      (2)  $X^3$  is non-positive.

15. If  $n$  is a positive integer, what is the greatest common factor of  $n$  and 64?

- (1) No two different factors of  $n$  sum to a prime number.  
(2) The greatest common factor of  $n$  and 2,310 is 165.

16. Is  $n/18$  an integer?      (1)  $5n/18$  is an integer.      (2)  $3n/18$  is an integer.

17. The sum of  $n$  consecutive positive integers is 45. What is the value of  $n$ ?

- (1)  $n$  is even      (2)  $n < 9$

18. Is  $x$  a negative number?

- (1)  $x^2$  is a positive number.      (2)  $x \cdot |y|$  is not a positive number.

19. What is  $x$ ?      (1)  $|x| < 2$       (2)  $|x| = 3x - 2$

20. What is the value of  $y$ ?      (1)  $3|x^2 - 4| = y - 2$       (2)  $|3 - y| = 11$

21. If the average of four distinct positive integers is 60, how many integers of these four are less than 50?

- (1) The median of the three largest integers is 51 and the sum of two largest integers is 190.  
(2) The median of the four integers is 50.

23. One kilogram of a certain coffee blend consists of  $X$  kilogram of type I and  $Y$  kilogram of type II. The cost of the blend is  $C$  dollars per kilogram, where  $C = 6.5X + 8.5Y$ . Is  $X < 0.8$ ?

- (1)  $Y > 0.15$       (2)  $C \geq 7.30$

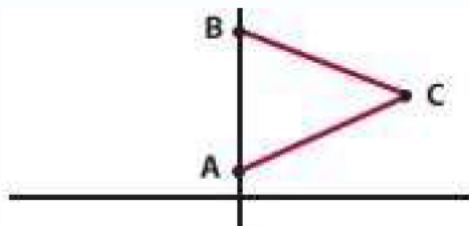
24. Marta bought several pencils. If each pencil was either a 23-cent pencil or a 21-cent pencil, how many 23-cent pencils did Marta buy?

- (1) Marta bought a total of 6 pencils.
- (2) The total value of the pencils Marta bought was 130 cents.

26. Is  $|x - 1| < 1$ ?

- (1)  $(x - 1)^2 \leq 1$
- (2)  $x^2 - 1 > 0$

27. If points A and B are on the y-axis in the figure, what is the area of equilateral triangle ABC?



- (1) Coordinates of point B are  $(0, 5\sqrt{3})$ .
- (2) Coordinates of point C are  $(6, 3\sqrt{3})$ .

28. What is the sum of the digits of the positive integer  $n$  where  $n < 99$ ?

- (1)  $n$  is divisible by the square of  $y$ .
- (2)  $y^4$  is a two-digit positive odd integer.

29. Joanna bought only \$0.15 stamps and \$0.29 stamps. How many \$0.15 stamps did she buy?

- (1) She bought an equal number of \$0.15 stamps and \$0.29 stamps.
- (2) She bought \$4.40 worth of stamps.

30. If  $x$  is non-negative integer, is  $x! + (x + 1)$  a prime number? (1)  $x < 10$

(2)  $x$  is even

**DS Traps Solutions – 30 questions**

1. A
2. B
3. D
4. C
5. D
6. E
7. A
8. A
9. B
10. A
11. B
12. C
13. C
14. C
15. D
16. C
17. E
18. E
19. B
20. C
21. D
22. A
23. B
24. B
25. E
26. E
27. B
28. E
29. B
30. E





## Solutions – DS Traps

1.

From (1), we will surely get a unique positive value of  $r$ , so sufficient. It does not matter what  $r$  is: if it is  $<8\%$ , we get a confirmed NO. If it is equal to  $8\%$ , we get a confirmed NO. If it is  $>8\%$ , we will get a confirmed YES.

From (2), we get  $(1.08)^2 = 1.1664 \dots$  so for the rate to be more than  $8\%$ , the value of  $(1+r/100)^2$  must be more than  $1.1664$ . We are given  $(1+r/100)^2 > 1.15 \dots$  so (2) is not sufficient as  $>1.15$  may be  $1.155$  (which is less than  $1.1664$ ) or it may be  $1.17$  (which is more than  $1.1664$ ). So, Answer A.

### Alternate Solution from Gmatclub

Shortcut to multiply numbers of the form  $(100 + a)$  or  $(100 - a)$

Write  $a^2$  on the right hand side. Add  $a$  to the original number and write it on left side. The square is ready.

e.g.  $108^2 = (100 + 8)^2$  Write 64 on right hand side

\_\_\_\_\_ 64

Add 8 to 108 to get 116 and write that on left hand side

11664 - Square of 108

e.g.  $91^2 = (100 - 9)^2 \Rightarrow$  \_\_\_\_\_ 81  $\Rightarrow$  8281

(Here, subtract 9 from 91)

Note:  $a$  could be a two digit number as well.

e.g.  $112^2 = (100 + 12)^2 =$  \_\_\_\_\_ 44  $\Rightarrow$  12544

(Only last two digit of the square of 12 are written on the right hand side. The 1 of 144 is carried over and added to  $112 + 12$ )

This is Vedic Math though the trick uses basic algebra.

$$(100 + a)^2 = 10000 + 200a + a^2$$

$$(100 + 8)^2 = 10000 + 200 \times 8 + 64 = 10000 + 1600 + 64 = 11664$$



### Top 1% expert replies to student queries (can skip)

Statement 1 :

Deposits earns 210 dollars in interest in 2 years.

if the interest rate is  $8\%$ , then the interest earned in 2 years is :

$$I = 1000 * [(1 + r/100)^2 - 1] = 1000 * [(1.08)^2 - 1] = 1000 * [1.1664 - 1] = 1000 * 0.1664 = 166.4$$

Also, we know that  $I$  increases with  $r$ . Since  $166.4 < 210$ , we know that the  $r > 8\%$ . Sufficient!

Statement 2 :

$$(1 + r/100)^2 > 1.15$$

If  $r = 8\%$

$$(1 + r/100)^2 = 1.1664$$

Now, let's take 2 cases.

$$\text{Case 1 : } (1 + r/100)^2 = 1.20$$

Since  $1.2 > 1.1664$ , we can say that  $r > 8\%$

$$\text{Case 2 : } (1 + r/100)^2 = 1.155$$

Since  $1.155 < 1.1664$ , we can say that  $r < 8\%$

Since we cannot conclusively say that  $r > 8\%$ , this statement is insufficient! **Answer is A.**



We haven't done anything different here. If  $x > 1.15$ , we can't say that  $x$  will also be greater than 1.664. Therefore, statement 2 is insufficient!

2.

(1)  $y$  can have any values ... suppose  $y = 1, 2, 3 \dots$  then for each value of  $y$ , we will get various values of  $x$ . Imagine taking the equal to sign,  $|x - 3| = 1$  or 2 or 3 etc. we will not get a unique value of  $x$ .

There will be infinite possible values of  $x$ . **NS**

(2)  $|x - 3| \leq -y \dots$  we can't take  $y$  as positive as  $|x - 3|$  will become negative so the only value of  $y$  can be 0. When  $y = 0$ ,  $|x - 3| \leq 0$ . As  $|x - 3|$  can't be less than zero (by the definition of mods), so the only value of  $|x - 3|$  can be 0, so if  $|x - 3| = 0$ ,  $x$  will have a unique value as  $x = 3$ .

**Ans. B**

3.

(1)  $|x - 2| < 2 - y$ . The left-hand side of the inequality is an absolute value, so the least value of LHS is zero, thus  $0 < 2 - y$ ,

thus  $y < 2$  (if  $y$  is more than or equal to 2, then  $2 - y \leq 0$  and it cannot be greater than  $|x - 2|$ ).

Next, since given that  $y$  is a positive integer, then  $y = 1$ . So, we have that:  $|x - 2| < 1$ , which implies that  $-1 < x - 2 < 1$ , or  $1 < x < 3$

Thus  $x = 2 = \text{prime}$ . Sufficient.

(2)  $x + y - 3 = |1 - y|$ . Since  $y$  is a positive integer, then  $1 - y \leq 0$ , thus  $|1 - y| = -(1 - y)$ .

So, we have that  $x + y - 3 = -(1 - y)$ , which gives  $x = 2 = \text{prime}$ . Sufficient.

**The correct answer is D**

4.

**plug in numbers to the number line here:**

Statement (1)



if the line reads:  $r = -1$ , zero,  $s = 1$ ,  $t = 3$ , then zero is halfway between  $r$  and  $s$ . if the line reads:

zero,  $r = 1$ ,  $s = 2$ ,  $t = 3$ , then zero is not between  $r$  and  $s$ . Insufficient.

Statement (2)

a) if the line reads:  $-s = r = -2$ , zero,  $s = 2$ , and  $t = 4$ , then  $(t \text{ to } r) = (t \text{ to } -s) = 6$ .

So, zero is halfway in between  $r$  and  $s$ .

b) if the line reads:  $r = -4$ ,  $s = -2$ ,  $t = -1$ , zero, and  $-s = 2$ , then  $(t \text{ to } r) = (t \text{ to } -s) = 3$ .

zero is not halfway in between  $r$  and  $s$ .

Insufficient.

Together: Only case 2(a) is possible. Sufficient.

**Ans. C**

**Top 1% expert replies to student queries (can skip)**

Is zero halfway between  $r$  and  $s$ ?

Statement 1:-

Case I:

-----r--0--s----t---

0 is midway between  $r$  &  $s$ .

Case II:

--0--r----s-----t---

0 is not midway between r & s.  
Not Sufficient.

Statement 2:-

Case I:

Let's say  $r = -s$ ;

$r = -2$ ;  $s = 2$   $t = 3$

-----r--0--s-----t---

$$|t-r| = |3-(-2)|=5$$

$$|t-s| = |3-(2)|=1$$

**0 is midway between r and s.**

Case II:

Let's say  $r = -s$ ;

$r = -4$ ;  $s = -2$   $t = -1$ ;  $-s = 2$

-----r--s--t--0----(-s)

$$|t-r| = |-1-(-4)|=3$$

$$|t-s| = |-1-(-2)|=1$$

**0 is not midway between r and s.**

Not Sufficient.

Combining 1 and 2:-

$r = -2$ ;  $s = 2$   $t = 3$

-----r--0--s-----t---

$$|t-r| = |3-(-2)|=5$$

$$|t-s| = |3-(2)|=1$$

0 is midway between r and s.

Sufficient.

**Ans: "C"**



**5.**

$y = x + |x|$  ... so y depends on x ... x can be -ve, 0, or +ve.

If x is -ve,  $y = 0$ , if x = 0,  $y = 0$ , if x = +ve,  $y = +ve$ . So y can't be negative.

Statement 1 is sufficient: If x is -ve,  $y = 0$ .

Statement 2 is  $y < 1$ , since y is an integer, and it is never negative, it can be only 0 if it is less than 1.

So, Statement 2 is sufficient too.

**Answer is D**

**Top 1% expert replies to student queries (can skip) (additional)**

Explanation for statement 2

y cannot assume negative values.

$$y = x + |x|.$$

If  $x \geq 0$ ,  $|x| = x$  and  $y = 2x > 0$

If  $x < 0$ ,  $|x| = -x$  and  $y = 0$

So, the minimum possible value of y is 0.

Statement 2 says that  $y < 1$ . So the only possible integer value of  $y$  is 0. Sufficient!

6.

Case 1:  $x = y = 100$ ,  $z = 0$ , satisfy both statements, and clearly give a YES answer.

Case 2:  $x = y = 3$  and  $z = 4$

In this case, then,  $x^4 + y^4 = 81 + 81 = 162$ , but  $z^4 = 256$ , giving a NO answer.

Insufficient

Answer = E

7.

Statement (1)

Let's just PICK A WHOLE BUNCH OF NUMBERS WHOSE GCF IS 2 and watch what happens.

4 and 6

6 and 8

8 and 10

10 and 12

4 and 10

6 and 14

6 and 16

8 and 18

8 and 22



In all of these examples, the remainders are greater than 1. In fact, there is an obvious pattern, which is that **they're all even**, since the numbers in question must be even. **In statement 1, both  $m$  and  $p$  are even. therefore, the remainder is even, so it's greater than 1.**

Sufficient.

Statement (2)

Just pick various numbers whose LCM is 30. Notice the numbers selected above:

5 and 6  $\rightarrow$  remainder = 1

10 and 15  $\rightarrow$  remainder = 5 > 1

insufficient.

**Ans. A**

8.

$xy + z$  is odd

$xy + xz$  is even

Subtract:  $xz - z = \text{odd}$  or  $z(x - 1) = \text{odd}$

We know that only Odd \* Odd is Odd.

Even \* Even, Even \* Odd, and Odd \* Even are all even.

So, both  $z$  and  $x - 1$  must be odd, so  $x$  must be even.

Sufficient.

$xy + z$  is odd

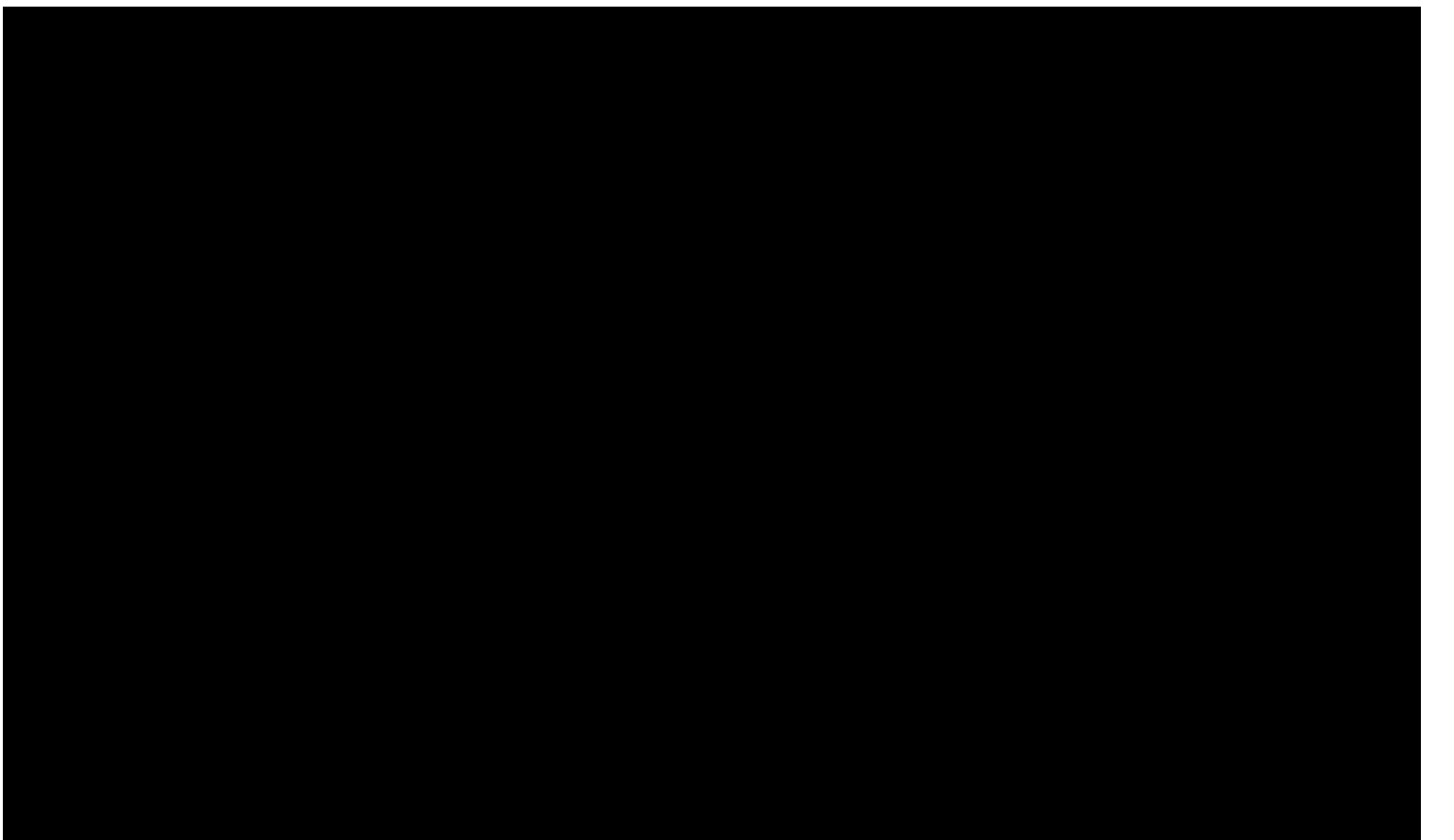
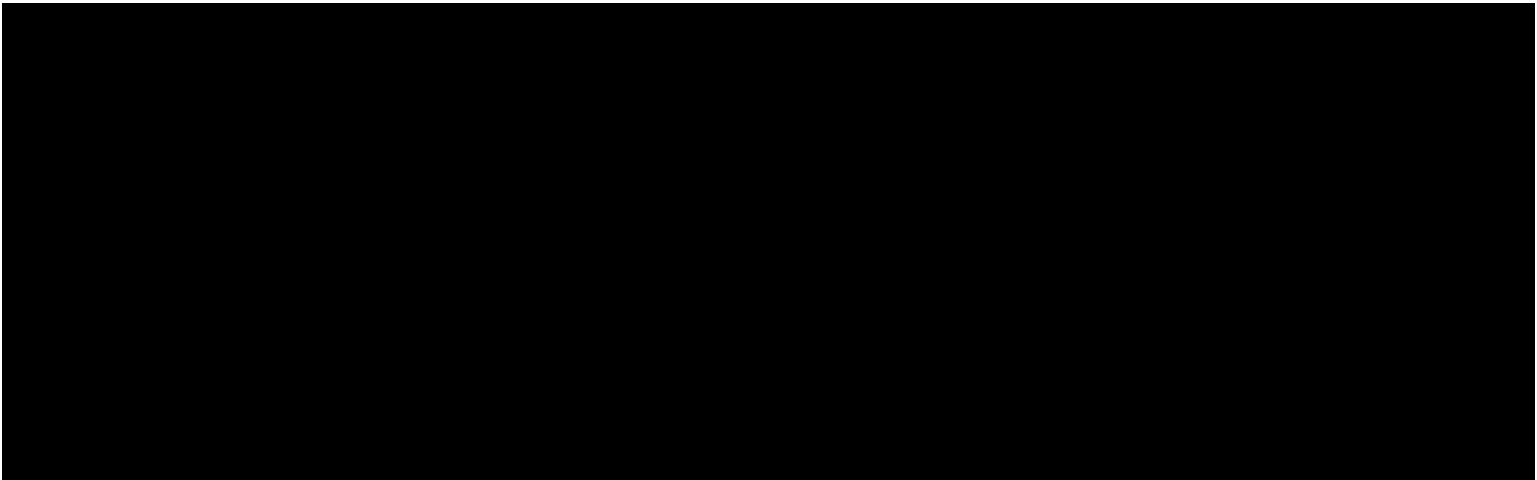
$y + xz$  is odd.

Add:  $(y + z)(x + 1) = \text{even}$

So,  $x + 1$  can be both even or odd. So  $x$  can be both even or odd.

Insufficient.

**Ans. A**



11.

(1)

	Japanese	NOT Japanese	TOTAL S
French	$0.04 F = 16$		F
Not French			
TOTAL	$J \geq 100$		

(1) gives  $0.04 F = 16$  so  $F = 400$  but we don't know how many students study Japanese. Insufficient.

(2)

	Japanese	NOT Japanese	TOTALS
French	$0.04 F = 0.1 J$		F
Not French			
TOTAL	$J \geq 100$		

(2) gives  $0.04 F = 0.1 J$  so  $F / J = 5 / 2$  so  $F > J$ . Sufficient.

**Answer B.**

**13.**

(1):  $x^3$  could be 11 or 27 or 97... so not a unique value

(2):  $x^4$  could be 11 or 27 or 97... so not a unique value (also,  $x$  could be positive or negative)

Combining: If  $x^3$  and  $x^4$  are both integers,  $x$  has to be an integer.

*Let's prove this:*

Imagine  $x^3 = 11$ , then  $x$  will be  $\sqrt[3]{11}$ , then  $x^4 = x^3 \times x = 11 \times \sqrt[3]{11}$ , which is not an integer.

This proves that if  $x^3$  and  $x^4$  are both integers,  $x$  has to be an integer.

So  $x^3$  can't be any other number except 27. So,  $x = 3$

**Ans. (C)**

**14.**

(1):  $x \neq 0$ , could be + or -ve.

(2)  $x = 0$  or -ve.

Comb...  $x$  is negative.

**Ans. (C)**

15.

(1) SUFFICIENT: Every number has 1 as a factor. If  $n$  were an even integer, then 1 and 2 would both be factors of  $n$ . The sum of 1 and 2 is 3, though, which is prime. Therefore, because 1 has to be a factor, 2 cannot also be a factor. Therefore,  $n$  is odd, as are all factors of  $n$  (since an odd number can't have an even factor). The prime factorization of 64 is  $2^6$ , so 64 has no odd factors other than 1. All factors of  $n$  are odd, and all factors of 64 are even except 1. The greatest common factor of  $n$  and 64 is therefore 1. The statement is sufficient.

(2) SUFFICIENT: 2,310 is an even integer. If  $n$  were an even integer, then the greatest common factor of  $n$  and 2,310 would be even (since  $n$  and 2,310 would have at least the factor 2 in common). Since the greatest common factor, 165, is odd, it follows that  $n$  cannot be even. Thus all factors of  $n$  are odd, and, as mentioned above, all factors of 64 are even except 1. The greatest common factor of  $n$  and 64 is therefore 1. The statement is sufficient. **The correct answer is (D).**

16.

(1) INSUFFICIENT:

If  $5n/18 = x$ , then the question changes to *is  $x$  an integer?*

(1)  $5x$  is an integer

(2)  $3x$  is an integer

(1) If  $5x = 1$ ,  $x = 1/5$

If  $5x = 5$ ,  $x = 1$

(2) If  $3x = 1$ ,  $x = 1/3$

If  $3x = 3$ ,  $x = 1$



Combining

$5x - 3x = \text{Integer} - \text{Integer}$  or  $2x = \text{integer}$

$3x - 2x = \text{Integer} - \text{Integer}$  or  $x = \text{integer}$

**Ans. C**



## Alternate Solution from GMATCLUB

Is  $n/18$  an integer?

Notice that we are NOT told that  $n$  is an integer.

(1)  $5n/18$  is an integer:

If  $\frac{5n}{18} = 0$ , then  $n = 0$  and  $\frac{n}{18} = 0 = \text{integer}$ ;

If  $\frac{5n}{18} = 1$ , then  $n = \frac{18}{5}$  and  $\frac{n}{18} = \frac{1}{5} \neq \text{integer}$ .

Two different answers. Not sufficient.

(2)  $3n/18$  is an integer  $\rightarrow \frac{3n}{18} = \frac{n}{6} = \text{integer} \rightarrow n = 6 * \text{integer} = \text{integer}$ . So, this statement implies that  $n$  is a multiple of 6.

If  $n = 0$ , then  $\frac{n}{18} = 0 = \text{integer}$

If  $n = 6$ , then  $\frac{n}{18} = \frac{1}{3} \neq \text{integer}$ .

Two different answers. Not sufficient.

(1)+(2) Since from (2) we have that  $n$  is an integer, then from (1) it follows that it must be a multiple of 18. Sufficient.

Answer: C.



17.

$$22 + 23 = 45 \dots n \text{ could be } 2$$

$$5 + 6 + 7 + 8 + 9 + 10 = 45 \dots n \text{ could be } 6.$$

(1) INSUFFICIENT: If  $n$  is even,  $n$  could be either 2 or 6. Statement (1) is NOT sufficient.

(2) INSUFFICIENT: If  $n < 9$ ,  $n$  could again take on either of the values 2 or 6

(1) and (2) INSUFFICIENT: if we combine the two statements,  $n$  must be even and less than 9, so  $n$  could still be either of the values: 2 or 6.

**The correct answer is E.**

**Alternate sol from gmatclub (additional)**

(1)  $n$  is even:

$$n \text{ can be } 2: 22 + 23 = 45.$$

$$\text{But it also can be } 6: x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5) = 45, \text{ which gives } x=5.$$

At least two values of  $n$  are possible, 2 and 6. Not sufficient.

(2)  $n < 9$ :

The above example is also valid for this statement, hence not sufficient.

(1)+(2) Still at least two values of  $n$  are possible. Not sufficient.

**Answer: E.**

18.

(1) gives  $x$  is either positive or negative but not 0. NS

(2)  $x \cdot |y|$  is either 0 or negative (not positive means 0 or negative)

So, we can have

$$x \text{ -ve, } y \text{ -ve} \quad x \cdot |y| \text{ is -ve}$$

$$x \text{ -ve, } y \text{ +ve} \quad x \cdot |y| \text{ is -ve}$$

$$x = 0, y = 0 \quad x \cdot |y| \text{ is } 0$$

$$\text{or } x = +, y = 0 \quad x \cdot |y| \text{ is } 0$$

$$\text{or } x = -, y = 0 \quad x \cdot |y| \text{ is } 0$$

So  $x$  can be 0, -ve or +ve.

(1) AND (2) INSUFFICIENT: We know from statement 1 that  $x$  cannot be zero, however, there are still two possibilities for  $x$ :  $x$  could be positive ( $y$  is zero), or  $x$  could be negative ( $y$  is anything).

**The correct answer is E.**

19.

(1) INSUFFICIENT: This expression provides only a range of possible values for  $x$ .

(2) For the case where  $x > 0$ :

$$x = 3x - 2$$

$$-2x = -2$$

$$x = 1$$

For the case when  $x < 0$ :

$$x = -1(3x - 2) \text{ We multiply by } -1 \text{ to make } x \text{ equal a negative quantity.}$$

$$x = 2 - 3x$$

$$4x = 2$$

$$x = 1/2$$

Note however, that the second solution  $x = 1/2$  contradicts the stipulation that  $x < 0$ , hence there is no solution for  $x$  where  $x < 0$ . Therefore,  $x = 1$  is the only valid solution for (2).

Or you can try to substitute  $x = 1/2$  in the original equation, and you will see that  $x = 1/2$  doesn't satisfy.

Or any mod must be  $\geq 0$ , so  $3x - 2 \geq 0$  or  $x \geq 2/3$  so  $x = 1/2$  is an invalid root.

**The correct answer is B.**

20.

(1) INSUFFICIENT: Since this equation contains two variables, we cannot determine the value of  $y$ .

We can, however, note that the absolute value expression  $|x^2 - 4|$  must be greater than or equal to 0.

Therefore,  $3|x^2 - 4|$  must be greater than or equal to 0, which in turn means that  $y - 2$  must be greater than or equal to 0. If  $y - 2 \geq 0$ , then  $y \geq 2$ .

(2) INSUFFICIENT: To solve this equation for  $y$ , we must consider both the positive and negative values of the absolute value expression:

$$\text{If } 3 - y > 0, \text{ then } 3 - y = 11$$

$$y = -8$$

$$\text{If } 3 - y < 0, \text{ then } 3 - y = -11$$

$$y = 14$$

Since there are two possible values for  $y$ , this statement is insufficient.

(1) AND (2) SUFFICIENT: Statement (1) tells us that  $y$  is greater than or equal to 2, and statement (2) tells us that  $y = -8$  or 14. Of the two possible values, only 14 is greater than or equal to 2.

Therefore, the two statements together tell us that  $y$  must equal 14.

The correct answer is C.

21.

It's almost always better to express the average in terms of the sum: the average of four distinct positive integers is 60, means that the sum of four **distinct positive** integers is

$4 * 60 = 240$ . Say four integers are  $a, b, c$  and  $d$  so that  $0 < a < b < c < d$ .

So, we have that  $a + b + c + d = 240$ .

(1) The median of the three largest integers is 51 and the sum of two largest integers is 190. The median of  $\{b, c, d\}$  is 51 means that  $c = 51$ . Now, if  $b = 50$ , then only  $a$ , will be less than 50, but if  $b < 50$ , then both  $a$  and  $b$ , will be less than 50. But we are also given that  $c + d = 190$ . Substitute this value in the above equation:  $a + b + 190 = 240$ , which boils down to  $a + b = 50$ . Now, since given that all integers are positive then both  $a$  and  $b$  must be less than 50. Sufficient.

(2) The median of the four integers is 50. The median of a set with even number of terms is the average of two middle terms, so median  $= (b + c) / 2 = 50$ .

Since given that  $b < c$  then  $b < 50 < c$ , so both  $a$  and  $b$  are less than 50. Sufficient.

**The correct answer is D**

23.

"one kilogram of a certain coffee blend consists of  $X$  kilogram of type I and  $Y$  kilogram of type II" means that  $X + Y = 1$

Combined  $C = 6.5X + 8.5Y$ , we get:

$$X = (8.5 - C) / 2, Y = (C - 6.5) / 2$$

Combined  $C \geq 7.3$

$$\text{So, } X \leq (8.5 - C) / 2$$

$$\text{So, } X \leq 1.2/2$$

$$\text{So, } X \leq 0.6$$

Answer is B



24.

$21x + 23y = 130$ , we try  $x = 1, 2, 3, 4, 5...$  and find that only  $x=4, y=2$  can fulfill the requirements.

**Unique solution using (2) alone.**

Answer is B.

26.

(1)  $(x - 1)^2 \leq 1$ . Since both sides of the inequality are non-negative then we can take square root from both parts:  $|x - 1| \leq 1$ , so  $|x - 1|$  can be less than 1 (answer YES), as well as equal to 1, for  $x = 2$  or  $x = 0$  (answer NO). Not sufficient. Notice that  $|x - 1| \leq 1$ , means  $0 \leq x \leq 2$ .

(2)  $x^2 - 1 > 0$ . Rearrange:  $x^2 > 1$ . Again, since both sides of the inequality are non-negative then we can take square root from both parts:  $|x| > 1$ . If  $x = 1.5$  then the answer is YES but if  $x = 2$  then the answer is NO. Not sufficient.

(1) + (2)  $x = 1.5$  and  $x = 2$  satisfy both statements and give different answer to the question. Not sufficient.

**The correct answer is E**

27.

In an equilateral triangle

$$h = \frac{\sqrt{3}}{2}a \text{ and } A = \frac{\sqrt{3}}{4}a^2 \text{ where 'a' is the side of the triangle.}$$

(1) INSUFFICIENT: This does not give us the length of a side or the height of the equilateral triangle since we don't have the coordinates of point  $A$ .

(2) SUFFICIENT: Since  $C$  has an  $x$ -coordinate of 6, the height of the equilateral triangle must be 6. We can determine the side and hence the area.

The correct answer is B.

28.

It seems that (1) and (2) combined are enough to solve this question:  $y$  could be only equal to 3 or  $-3$  so  $n$  will be divisible by 9 ... and 9, 18, 27, 36, 45, 54, 63, 72, 81, 90 ... all have a sum of digits as 9 ... so we have a unique answer.

**WRONG!!!**

Imagine  $y = \sqrt{5}$ ,  $\sqrt{7}$ , 3, etc. So  $y^4 = 25$  or 49 or 81 and  $y^2 = 5$  or 7 or 9 ... so  $n$  may be divisible by 5 or 7 or 9 ... we are not sure what the sum of the digits of  $n$  will be ... suppose  $n = 21$  or 25 ... we get different answers for the sum of digits as 3 or 7 ... Not sufficient. **Answer E.**

**Top 1% expert replies to student queries (can skip)**

(1)  $n$  is divisible by the square of  $y$  --> clearly insufficient, as no info about  $y$ .

(2)  $y^4$  is a two-digit odd integer --> also insufficient, as no info about  $n$ , but from this statement we know that if  $y^2$  is an integer, Imagine  $y = \sqrt{3}, \sqrt{5}, \sqrt{7}, 3$ , etc. So  $y^4 = 9$  or  $25$  or  $49$  or  $81$  and  $y^2 = 3$  or  $5$  or  $7$  or  $9$  ... so  $n$  may be divisible by  $3$  or  $5$  or  $7$  or  $9$  ... we are not sure what the sum of the digits of  $n$  will be ... suppose  $n = 21$  or  $25$  ... we get different answers for the sum of digits as  $3$  or  $7$ .

Not sufficient.

**Answer E.**

29.

(1) Any number of stamps could be purchased  $(5, 5), (10, 10), (100, 100)$  etc. INSUFFICIENT.

(2) The total value of the  $\$0.15$  stamps must be a dollar amount that ends in  $5$  or  $0$ . In order for the total value of both stamps to equal  $\$4.40$ , therefore, the total value of the  $\$0.29$  stamps must also be a dollar amount that ends in  $5$  or  $0$ .

$$15x + 29y = 440$$

The only unique solution is  $(10, 10)$ .

**Ans. B**

**Top 1% expert replies to student queries (can skip)**

Let  $x$  be the # of  $\$0.15$  stamps and  $y$  the # of  $\$0.29$  stamps. Note that  $x$  and  $y$  must be integers. Question:  $x=?$

**Clearly Statement 1 is insufficient.**

Statement (2) She bought  $\$4.40$  worth of stamps -->  $15x + 29y = 440$ . Only one integer combination of  $x$  and  $y$  is possible to satisfy  $15x + 29y = 440$ :  $x=10$  and  $y=10$ . **Sufficient.**

NOTE: So when we have an equation of a type  $ax + by = c$  and we know that  $x$  and  $y$  are non-negative integers, there can be multiple solutions possible for  $x$  and  $y$  (eg  $5x + 6y = 60$ ) OR just one combination (eg  $15x + 29y = 440$ ). Hence in some cases  $ax + by = c$  is NOT sufficient and in some cases it is sufficient.

**Ans. B**

30.

The best approach will be to test numbers. Note that since the question is Yes/No, all you need to do to prove insufficiency is to find one Yes and one No.

(1) INSUFFICIENT: Statement (1) says that  $x < 10$

$$0! + 0 + 1 = 2 \text{ (prime)}$$

$$2! + (2 + 1) = 5, \text{ which is prime.}$$

$$3! + (3 + 1) = 6 + (3 + 1) = 10, \text{ which is not prime.}$$

NOT sufficient.

(2) INSUFFICIENT: Statement (2) says that  $x$  is even

$$0! + 0 + 1 = 2 \text{ (prime)}$$

$$2! + (2 + 1) = 5, \text{ which is prime.}$$

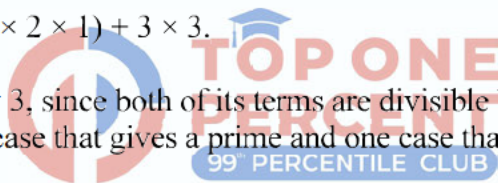
Now consider  $x = 8$

$$8! + (8 + 1) = (8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3 \times 3.$$

This expression must be divisible by 3, since both of its terms are divisible by 3. Therefore, it is not a prime number. Since we found one case that gives a prime and one case that gives a non-prime, statement (2) is NOT sufficient.

(1) and (2) INSUFFICIENT: since the number 2 gives a prime, and the number 8 gives a non-prime, both statements taken together are still insufficient.

The correct answer is E.





### Alternate Solution from GMATCLUB

**Given:**  $x$  is a positive integer. So, possible values of  $x$ : 1, 2, 3, 4 . . .

**To find:** If  $x! + x + 1$  is Prime

**Analysis:**

For  $x = 1$ ,  $x! = 1$

And,  $x! + x + 1 = 1 + 1 + 1 = 3$ , which is a Prime number

**For  $x > 1$ ,**

$x!$  will always be an even number (Because,  $x!$  contains the product of consecutive integers. So, this product will have Even AND odd terms. We know that when an even number is multiplied with anything, the product is always even)

So, the sum  $(x! + x + 1) = (\text{An even number} + x + \text{Odd number}) = (\text{Odd number} + X)$

Even for the lowest possible value of  $x$  ( $x = 1$ ), the value of the sum  $(x! + x + 1)$  was equal to 3.

So, for  $x > 1$ , the value of this sum is definitely going to be greater than 3.

And, we know that all Prime numbers greater than 2 are odd.

So, the sum  $(\text{Odd number} + X)$  can be prime only if first, this sum is an odd number.

That is, if,  $X$  is an even number.

So, for  $x > 1$ ,  $x$  being an even number is a **NECESSARY** condition for the sum  $(x! + x + 1)$  to be prime.

But is it a **SUFFICIENT** condition? That is, can you say that if  $x$  is even, that must mean that the sum  $(x! + x + 1)$  will be prime?



Let's see:

$$x x! + x + 1$$

$$2 2! + 2 + 1 = 7 \text{ (Prime)}$$

$$4 4! + 4 + 1 = 29 \text{ (Prime)}$$

$$6 6! + 6 + 1 = 727 \text{ (Prime)}$$

$$8 8! + 9 \text{ (both terms in this sum are divisible by 3) (NOT Prime)}$$

Thus, we see that for some even values of  $x$ , the sum  $(x! + x + 1)$  will be Prime and for others, it will not be.

With this understanding, let's now look at the two statements:

Statement 1:  $x < 10$

As per this statement,  $x$  can be even ( $\Rightarrow$  some possibility of the sum  $(x! + x + 1)$  to be prime)  
or  $x$  can be odd ( $\Rightarrow$  NO possibility of the sum  $(x! + x + 1)$  to be prime)

Clearly not sufficient.

Statement 2:  $x$  is even

As illustrated in our analysis of the question statement,  $x$  being Even is not a sufficient condition for the sum to be prime.

Not Sufficient.

Statement 1 + 2

$\Rightarrow x$  is an even number  $< 10$



As we saw in our analysis above, for  $x = 8$ , the sum is not prime. For all other possible values of  $x$ , the sum is prime.

Not Sufficient.

Therefore, **correct answer: Option E**