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Quant Session

Arithmetic + Word Problems

PERCENTAGES

To express a% as a fraction divide it by 100

a% = a/100

To express a fraction as a percent multiply it by 100

a/b = [(a/b) × 100] %

To express percentage as a decimal we remove the symbol % and shift the decimal point by two places to the left. For example

10% can be expressed as 0.1.

6.5% = 0.065 etc.

To express decimal as a percentage we shift the decimal point by two places to the right and write the number obtained with the symbol % or simply we multiply the decimal with 100. Similarly, 0.7 = 70%.

Increase % = [Increase / Original value] × 100%

Decrease % = [Decrease / Original value] × 100%

Change % = [Change / Original value] × 100%

In increase %, the denominator is smaller, whereas in decrease %, the denominator is larger.

GENERAL CONCEPTS IN PERCENTAGES:

Let's start with a number X (= 1 X)

X increased by 10% would become X + 0.1 X = 1.1 X
X increased by 1% would become X + 0.01 X = 1.01 X
X increased by 0.1% would become X + 0.001 X = 1.001 X
X decreased by 10% would become X – 0.1 X = 0.9 X
X decreased by 1% would become X – 0.01 X = 0.99 X
X decreased by 0.1% would become X – 0.001 X = 0.999 X
X increased by 200% would become X + 2X = 3X
X decreased by 300% would become X – 3X = –2X

Also, let us remember that:

2 = 200% (or 100% increase)
3 = 300% (or 200% increase)
3.26 = 326% (means 226% increase)
Fourfold (4 times) = 400 % of original = 300% increase
10 times means 1000% of the original means 900% increase
0.6 means 60% of the original means 40% decrease
0.31 times means 31% of the original means 69% decrease etc.

1/2 = 50%

3/2 = 1 + 1/2 = 100 + 50 = 150%

5/2 = 2 + 1/2 = 200 + 50 = 250% etc.

2/3 = 1 – 1/3 = 100 – 33.33 = 66.66%

4/3 = 1 + 1/3 = 100 + 33.33 = 133.33%,

5/3 = 1 + 2/3 = 100 + 66.66 % = 166.66%

7/3 = 2 + 1/3 = 200 + 33.33 = 233.33%

8/3 = 2 + 2/3 = 200 + 66.66 = 3 – 1/3 = 300 – 33.33 = 266.66%

1/4 = 25%

3/4 = 75%

5/4 = (1 + 1/4) = 125% (= 25% increase)

7/4 = (1 + 3/4 = 2 – 1/4) = 175% (= 75% increase)

9/4 = (2 + 1/4) = 225% (= 125% increase)

11/4 = 275% = (175% increase)

1/5 = 20%

2/5 = 40%

3/5 = 60%

4/5 = 80%

6/5 = (1 + 1/5) = 120%

7/5 = (1 + 2/5) = 140% etc.

1/6 = 16.66%

5/6 = 83.33%

7/6 (1 + 1/6) = 116.66%

11/6 = 183.33%

1/8 = 12.5%

3/8 = 37.5%

5/8 = 62.5%

7/8 = 87.5%

9/8 = (1 + 1/8) = 112.5%

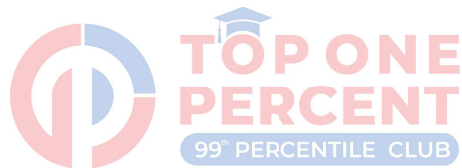11/8 = (1 + 3/8) = 137.5%

13/8 = 162.5%

15/8 = 187.5%

1/9 = 11.11%

2/9 = 22.22%

4/9 = 44.44%

5/9 = 55.55%

7/9 = 77.77%

8/9 = 88.88%

10/9 = (1 + 1/9) = 111.11%

11/9 = (1 + 2/9) = 122.22% etc.

If the present population of a town is p and if there is an increase of X% per annum. Then:

(i) Population after n years = $p [1 + (X/100)]^n$

Query: If the population is 100, it increases by 10%, it will be 110, then again it grows by 10% then it should be 10% on 110% and not 100%. Moreover, it should be added instead of multiplying in the formula for each year. Kindly clarify this part

Reply: $P + 10\%P \longrightarrow P(1+10\%) + 10\%*\{P(1+10\%)\} \longrightarrow P(1+10\%)(1+10\%)$

P (or 100) increased by 10%: P(1+10%) or (110)
Now this value is increased by 10%:
$P(1+10\%) + 10\%(P(1+10\%))$
$P(1+10\%)(1+10\%)$ Or (121)

When you write, you will understand that you have to take 10% of P (1+10%) which is not taken over here.

(ii) Population n years ago = $p / [1 + (X/100)]^n$

If the population of a town (or value of a machine) decreases at R% per annum, then:

   i.   population (or value of machine) after n years = $p [1 - (R/100)]^n$

   ii.   population (or value of machine) n years ago = $p / [1 - (R/100)]^n$

Profit % = (Profit /CP) x 100%

Loss % = (Loss/CP) x 100 %

In problems on **DISCOUNT,** remember the following:

Marked price is the price listed on the article (called list price).

Discount is calculated on Marked price and NOT on Cost price.

So, Marked Price – Discount = Sale Price. Also Cost Price + Profit = Sale Price.

Solved Examples

1. A child spends 30% of his pocket money, and has Rs 126 left. How much had he at first?

Sol.

Let the pocket money be X.
70% of pocket money = 126   Or   (70/100) * X = 126
X = 180 Rs.

2. When the cost of petroleum increases by 40%, a man reduces his annual consumption by 20%. Find the percentage change in his annual expenditure on petroleum.

Sol.

Expenditure = Consumption × Price

First Expenditure: Suppose 10 liters of petroleum at 10 units of money per liter, then total expenditure = 10 × 10 units of money = 100 units of money

Second Expenditure: Now, 8 liters of petroleum at 14 units of money per liter, total expenditure = 8 × 14 units of money = 112 units
So % change = (112 – 100)*100/100 = 12%

So, the expenditure increases by 12%

3. The number of students in a school increase at a certain rate per cent. The number at present is 1323 and the number two years ago was 1200; find the rate per cent of the increase.

Sol.

By formula, we have
$1200 \times (1 + R/100)^2 = 1323$
$(1 + R/100)^2 = 1323/1200 = 441/400$
Take square root
$(1 + R/100) = 21/20$
Subtract 1 on both sides
$R/100 = 1/20$
So,

$R = 100/20 = 5\%$

4. A trader marks his goods at 50 percent above cost price and allows discount of 20% percent for cash payment. What profit percent does he make?

Sol.

If the CP is 100, Marked Price = 150 But discount to the cash purchaser = 20% of Rs 150 = Rs 30

Now the reduced price he gets from the cash purchaser = 150 – 30 = 120

So, profit percent = 20%

5. A reduction of 20% in the price of apples could enable a man to get 120 more for Rs 1,440. Find the first price of one apple.

Sol.

We have 1440 = X × Y..............(1)

X = no. of apples, Y = price of one apple.

Now 1440 = (X + 120) × 0.8Y ..... (2)

Equate the two (as both are 1440)

X × Y = (X + 120) × 0.8Y

Y cancels   X = 480

Substitute X = 480 in (1)

Therefore, Y = Rs 3

6. A man's working hours a day were increased by 25%, and his wages per hour were increased by 20%. By how much percent were his daily earnings increased?

Sol.

Wages per hour × Number of hours = Earnings
Let each value be 10, so that earnings = 100

10 (Wages per hour) increased by 20% is 12
10 (Number of hours) increased by 25% is 12.5

Initially, 10 × 10 = 100

Now: 12×12.5 = 150
From 100 to 150 → 50% increase

OR

Let initially X = number of hours and Y = wages/hour

Later these become 1.25 X and 1.2 X respectively.

Daily earnings initially = X × Y

Now Daily earnings = 1.25X × 1.2Y = 1.5 XY

So, % change = (1.5XY – XY)*100/XY = 50%

Hence 50% increase.

7. A shopkeeper allows a discount of 15% on the marked price. How much above the C.P. must he mark his goods to make a profit of 19%?

Sol.

Let CP = 100, Profit = 19, SP = 100 + 19 = 119

Now marked price should be such that Marked price reduced by 15% is equal to 119

So, 85% of M.P. = 119 or MP = 119 × 100/85 = Rs 140

Answer = 40% above the C.P.

8. The production of a firm increases from 340 MT to 500 MT. What is the percent increase?

Sol. 160/340 = 8/17 = 47.05%.

9. The production of a firm decreases from 500 MT to 340 MT. What is the percent decrease?

Sol. 160/500 = 32%.

Note the answers to the above 2 questions are different.

10. The production of a firm increases by 20%, 25% and 50% in 3 successive years over the previous year. If the production is 160 MT in the first year, find the production at the end of 3 years.

Sol. $160 \times 1.2 \times 1.25 \times 1.5 = 360$

Query: Can you explain why we multiplied with 1.2, 1.25 and 1.5?
Reply: We know that the initial value be 160

Increase in first year = 20%

Value in second year = 160 * (1 + 0.2) = 160 * 1.2 = 192

Increase in second year = 25%

Value in second year = 192 * (1 + 0.25) = 192 * 1.25 = (160*1.2)*1.25 = 240

Increase in third year = 50%

Value in third year = 240 * (1 + 0.5) = 240 * 1.5 = (192*1.25)*1.5 = (160*1.2*1.25)*1.5 = 360

11. The production of a firm decreases by 20% in the first year, then decreases by 25% in the next year and then increases by 50% the next year and then increases by 10% in the next year. All percentage changes being consecutive (over the previous year). If at the end of the changes, the value is 990 MT, what was the value initially?

Sol. A × 0.8 × 0.75 × 1.5 × 1.1 = 990 or A = 1000

12. Which is bigger: 0.004% of 25000 or 25000% of 0.004?

Sol. Both are equal. **A% of B = B % of A.**

13. If price decreases by 25%, by what % should consumption increase so that the expenditure does not increase?

Sol. Expenditure = price × consumption
Now, assume values

100 = 100 × 1
100 = 75 × x

x = 100/75 = 4/3 = 1.33

1 becomes 1.33 → 33.33% increase

14. If speed increases by 33.33%, what is the percent reduction in the time taken to travel the same distance?

Sol. Distance = speed × time
Now, assume values

100 = 100 × 1
100 = 133.33 × x

x = 100/133.33 = 3/4 = 0.75
1 becomes 0.75 → % decrease = 25%

15. The length of a rectangle increases by 25%. Find the percent drop in its width for area to remain same.

Sol.
Area = L × B
Assume values

100 = 1 × 100
100 = 1.25 × x

x = 100/1.25 = 80
x decreases from 100 to 80, → 20% decrease

16. If the length of a rectangle is decreased by 40% and the breadth is increased by 30%, then find the percentage change in the area of the rectangle.
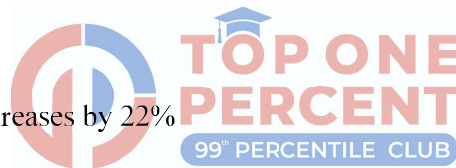
Sol.

Assume values

10 × 10 = 100

6 × 13 = 78

Hence the area of the rectangle decreases by 22%

17. The edge of a cube increases by 20%. Find the % increase in its surface area and volume.

Sol.

Remember that Surface area is proportional to the square of length and the volume is proportional to the cube of length.

Area ∝ **length²**

Volume ∝ **length³**

For area:

10 × 10 = 100

12 × 12 = 144

So, area increases by 44%.

 For volume:

10 × 10 × 10 = 1000

12 × 12 × 12 = 1728

So, volume increases by 72.8%.

For Area:
Conventional method:

Let the side of the square be a units

Then its surface area =6a² units²

If the length of the side is increased by 20% then the length of the side= (120/100)*a = (6/5)*a units

So, the new surface area =6*(6/5)(6/5)*a²

=6*(36/25)*a² units²

Increase in area=6× (36/25)a² – 6a²

= 6a²*{(36/25) - 1} = 6a²*(11/25)

Percentage of increase in area

=[{6a²*(11/25)}/(6a²)] ×100

= (11/25) ×100 = 44

So, percentage of increase in the area=44%

18. The side of a square decreases by 30%. Find the % decrease in its perimeter and area.

Sol. Perimeter changes by the same percent as length so 30% decrease.

For area: 10 × 10 = 100

7 × 7 = 49

So % Change = (100-49)*100/100 = 51%

So, area decreases by 51%.

Questions Based on Venn Diagrams (Overlapping Sets):

On your test, you are likely to see questions with 2 or 3 variables.

*In case of 2 variables, there are a maximum of four divisions possible:*

Imagine that at a B-school, applicants can choose Marketing and Finance among other specializations, where dual major is allowed. In this case, there can be only four types of sets of people possible:

1. Students taking Marketing Only
2. Students taking Finance Only
3. Students taking both Marketing and Finance
4. Students taking neither Marketing nor Finance

These types of questions are best solved by making a 2-way-matrix (table). Just remember that if one row has "Marketing", the other has to have "Not Marketing". If one column has "Finance", the other has to have "Not Finance".

	Finance	NOT FINANCE	TOTAL
Marketing	A	B	A + B
NOT Marketing	C	D	C + D
TOTAL	A + C	B + D	A + B + C + D

The table can be completed by putting all the given values in the question and thus the unknown can be found out by simple addition / subtraction of rows / columns. In case of percentage problems (where all values are mentioned in terms of percentage), it is better to take the total as 100.
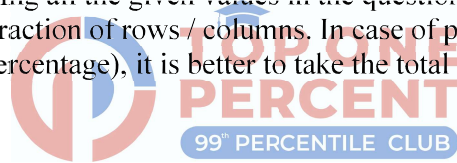
Let us solve a practical example:

50% of the apartments in a certain building have windows and hardwood floors. 25% of the apartments without windows have hardwood floors. If 40% of the apartments do not have hardwood floors, what percent of the apartments with windows have hardwood floors?

10%   16.66%   40%   50%   83.33%

This problem involves two sets:

Set 1: Apartments with windows / Apartments without windows

Set 2: Apartments with hardwood floors / Apartments without hardwood floors. It is easiest to organize

two-set problems by using a matrix as follows:

	Windows	NO Windows	TOTAL
Hardwood Floors			
NO Hardwood Floors			
TOTAL			

The problem is difficult for two reasons. First, it uses percentages instead of real numbers. Second, it involves complicated and subtle wording.

Let's attack the first difficulty by converting all of the percentages into REAL numbers. To do this, let's say that there are 100 total apartments in the building. This is the first number we can put into our matrix. The absolute total is placed in the lower right-hand corner of the matrix as follows:

	Windows	NO Windows	TOTAL
Hardwood Floors			
NO Hardwood Floors			
TOTAL			100

Next, we will attack the complex wording by reading each piece of information separately, and filling in the matrix accordingly.

Information: **50% of the apartments in a certain building have windows and hardwood floors.** Thus, 50 of the 100 apartments have BOTH windows and hardwood floors. This number is now added to the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	50		
NO Hardwood Floors			
TOTAL			100

Information: **25% of the apartments without windows have hardwood floors.** Here's where the subtlety of the wording is very important. This does NOT say that 25% of ALL the apartments have no windows and have hardwood floors. Instead, it says that OF the apartments without windows, 25% have hardwood floors. Since we do not yet know the number of apartments without windows, let's call this number **x**. Thus, the number of apartments without windows and with hardwood floors is **.25x**. These figures are now added to the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**.25x**	
NO Hardwood Floors			
TOTAL		**X**	**100**

Information: **40% of the apartments do not have hardwood floors.** Thus, 40 of the 100 apartments do not have hardwood floors. This number is put in the Total box at the end of the "No Hardwood Floors" row of the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**.25x**	
NO Hardwood Floors			**40**
TOTAL		**x**	**100**

Before answering the question, we must complete the matrix. To do this, fill in the numbers that yield the given totals. First, we see that there must be 60 total apartments with Hardwood Floors (since 60 + 40 = 100) Using this information, we can solve for x by creating an equation for the first row of the matrix:

$50 + 0.25x = 60$ or $x = 40$

Now we put these numbers in the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	50	10	60
NO Hardwood Floors			40
TOTAL		40	100

Finally, we can fill in the rest of the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	50	10	60
NO Hardwood Floors	10	30	40
TOTAL	60	40	100

We now return to the question: What percent of the apartments with windows have hardwood floors? Again, pay very careful attention to the subtle wording. The question does NOT ask for the percentage of TOTAL apartments that have windows and hardwood floors. It asks what percent OF the apartments with windows have hardwood floors. Since there are 60 apartments with windows, and 50 of these have hardwood floors, the percentage is calculated as follows:
$50/60 = 83.33\%$

Thus, the correct answer is E.

In case of 3 variables, there are a maximum of eight divisions possible. So, a table will become very complicated. So, we will deal with such questions by drawing 3 overlapping circles. Focus on the example below:

Each of the 59 members in a high school class is required to sign up for a minimum of one and a maximum of three academic clubs. The three clubs to choose from are the poetry club, the history club, and the writing club. A total of 22 students sign up for the poetry club, 27 students for the history club, and 28 students for the writing club. If 6 students sign up for exactly two clubs, how many students sign up for all three clubs?

2   5   6   8   9

This is a three-set overlapping sets problem. When given three sets, a Venn diagram can be used. The first step in constructing a Venn diagram is to identify the three sets given. In this case, we have students signing up for the poetry club, the history club, and the writing club. The shell of the Venn diagram will look like this:

When filling in the regions of a Venn diagram, it is important to work from inside out. If we let x represent the number of students who sign up for all three clubs, a represent the number of students who sign up for poetry and writing, b represent the number of students who sign up for poetry and history, and c represent the number of students who sign up for history and writing, the Venn diagram will look like this:
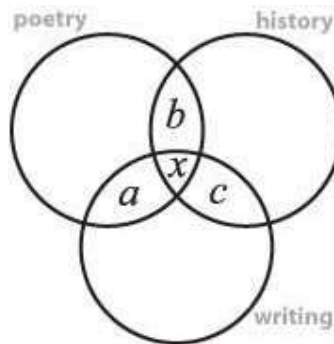
We are told that the total number of poetry club members is 22, the total number of history club members is 27, and the total number of writing club members is 28. We can use this information to fill in the rest of the diagram:
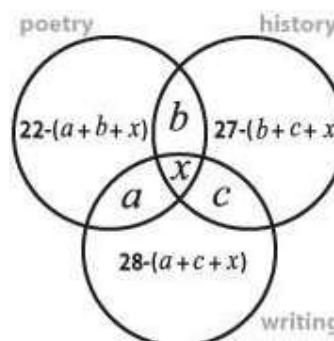
We can now derive an expression for the total number of students by adding up all the individual segments of the diagram. The first bracketed item represents the students taking two or three courses. The second bracketed item represents the number of students in only the poetry club, since it's derived by adding in the total number of poetry students and subtracting out the poetry students in multiple clubs. The third and fourth bracketed items represent the students in only the history or writing clubs respectively.

$59 = [a + b + c + x] + [22 - (a + b + x)] + [27 - (b + c + x)] + [28 - (a + c + x)]$

$59 = a + b + c + x + 22 - a - b - x + 27 - b - c - x + 28 - a - c - x$

$59 = 77 - 2x - a - b - c$

$59 = 77 - 2x - (a + b + c)$

By examining the diagram, we can see that $(a + b + c)$ represents the total number of students who sign up for two clubs. We are told that 6 students sign up for exactly two clubs. Consequently:

$59 = 77 - 2x - 6$

$2x = 12$

$x = 6$

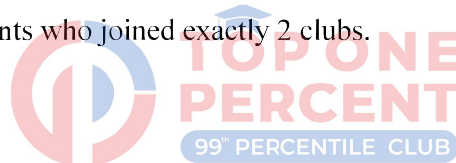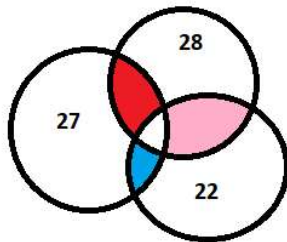**So, the number of students who sign up for all three clubs is 6.**

The correct answer is C.

Firstly, the number of students cannot be negative, so x can't be -6.
Secondly, the number of students in only 2 clubs are not 6+6+6, but it is only 6 altogether.

The shaded areas cover the 6 students who joined exactly 2 clubs.

Total number of students = (3club) + (2club) + (1club).
Number of students in all the 3 clubs: X
Number of students in only 2 clubs: **6 (includes all the three shaded area)**
Neither=0 (as members are required to sign up for a **minimum of one**);
"22 students sign up for the poetry club": P=22;
"27 students for the history club": H=27;
"28 students for the writing club": W=28;
Total = P+ H+ W - {Sum of Exactly 2 groups members} - [2*(all the 3 groups members)] + Neither
59= (22+27+28- (6) - 2X)
2X=6+77-12-59
2X=12
X=6.

The correct answer is C.

RATIOS:

The comparison between two quantities of the same kind of unit is the Ratio of one quantity to another.

The ratio of a and b is usually written as a:b or a/b, where a is called the antecedent (numerator) and b the consequent (denominator).

1. a : b = ka : kb where k is a constant
2. a : b = a/k : b/k
3. a : b : c = X : Y : Z is equivalent to a/X = b/Y = c/Z
4. If a/b > 1 or a > b then (a + X) / (b + X) < a/b a, b, X are natural numbers
5. If a/b < 1 or a < b then (a + X) / (b + X) > a/b a, b, X are natural numbers

VARIATION:

Direct proportion: If two quantities X & Y are related such that any increase or decrease in 'Y' produces a proportionate increase or decrease in 'X' or vice versa, then the two quantities are said to be in direct proportion.
In other words,
X : Y = X/Y = k (a constant)   or   X = KY or Y = K'X (where K and K' are constants)

X is directly proportional to Y is written as X α Y or X = K Y

Inverse proportion: Here two quantities X & Y are related such that, any increase in X would lead to a decrease in Y or any decrease in X would lead to an increase in Y. Thus, the quantities X & Y are said to be inversely related and X is inversely proportional to Y is written as X α 1/Y or X = k/Y or XY = k (constant) or the product of two quantities remains constant.

Solved Examples:

1. In what ratio should tea @ 35/kg be mixed with tea @ 27/kg so that mixture may cost Rs. 30/kg?

   Sol.
   The average cost is 30
   Let x kg of tea at Rs. 35/kg be mixed with y kg of tea at Rs. 27/kg
   Total amount of tea = x + y kg
   Average price = Rs. 30/kg

   So, 35x + 27y = 30 (x + y)
   => 5x = 3y
   => x/y = 3/5

2. Find a : b : c if 6a = 9b = 10c.

   Sol.

   a/b = 9/6 = 3 : 2 = 15 : 10, b/c = 10/9 = 10 : 9.
   Hence a : b : c = 15 : 10 : 9.

3. A's present age is to be B's as 8:5; and 20 years ago, it was as 12 : 5. Find the present age of each.

Sol.

Let the ages be 8X and 5X

$(8X – 20) / (5X – 20) = 12 / 5$
Solving this, we get: $X = 7$

A's age = 8 X = 56 years
B's age = 5 X = 35 years

4. An alloy contains 24% of tin by weight. How much more tin to the nearest kg must be added to 100 kg of the alloy so that the % of tin may be doubled?

Sol.

Let X kg of tin be added to the alloy.

Tin (kg)	Alloy (kg)
24	100
24 + X	100 + X

Now, Tin / Total must be 48%

So, $(24 + X) / (100 + X) = 48/100$

So, $X = 46$

Hence 46 kg of tin must be added to the alloy. **Ans.**

5. The expenses of a hotel consist of two parts. One part varies with the number of inmates, while the other is constant. When the number of inmates is 200 & 250, the expenses are respectively Rs. 1300 & Rs. 1600. Then find the expenses for 300 inmates.

Sol.

Let $E = K_1X + K_2$

E stands for expenses, X for the number of inmates.

When $X = 200$ & $E = 1300$ we have $[200 K_1 + K_2 = 1300]$.

When $X = 250$ & $E = 1600$ we have $[250 K_1 + K_2 = 1600]$.

Solving the equations, we have

$K_1 = 6$ & $K_2 = 100$

So, $E = 6X + 100$. Now when $X = 300$, $E = 6 \times 300 + 100 = $ Rs 1900 **Ans.**

Query: What do K_1 and K_2 Stand for?
Reply: (K_1X) is the part of the expenses that varies with the number of inmates.

K_1 = variable expenses
K_1X = variable expenses/K_1 = variable expenses per inmate

K_2 on the other hand is the fixed expense.

So Total Expenses = Variable + constant = $K_1X + K_2$

6. Two tins A and B contain mixtures of wheat and rice. In A, the weights of wheat and rice are in the ratio 2 : 3 and in B they are in the ratio 3 : 7. What quantities must be taken from A and B to form a mixture containing 5 kg of wheat and 11 kg of rice?

   Sol.

   Let X kg of mixture be taken from A, then $(16 - X)$ kg is taken from B

   So, $2X/5$ kg of wheat from A and $3(16 - X)/10$ kg of wheat from B is to be taken.

   Equate wheat from both sides:

   We have, $2X/5 + 3(16 - X)/10 = 5$ or $X = 2$ kg

   So, 2 kg from A and 14 kg from B.

Tin A contains wheat and rice in ratio 2:3.
Let's suppose concrete values are 2x and 3x for wheat and rice respectively.
Similarly, Tin B has ratios 3:7. Therefore, concrete values are 3y and 7y.

Since we need to have 5kg wheat and 11kg rice:
$2x+3y = 5$
$3x+7y = 11$
Solving for x & y we get $x=2/5$ and $y=7/5$

Mixture to be taken from Tin A = $2x+3x = 5x = 2$kgs
Mixture to be taken from Tin B = $3y+7y = 10y = 14$kgs

OR

For alligation, you can use the following method:

Let X be the quantity of mixture taken from tin A.
Then automatically $16 - X$ will be the quantity taken from Tin B.
(As 5:11 has a total of 16 parts)

Now we know that we need 5kg of wheat and 11 kg of rice.

So in the first tin A --> Wheat is 2/5. So we shall take $2/5X$ of mixture from tin A
In Tin B, wheat is 3/10. So lets take $3/10 (16 - X)$

Totally, we get 2/5X = 3(16-X)/10 = 5.
16-X will be 14
Solve, we get the answer of X as 2.

Tins A and B contain both wheat and rice. We're taking some quantity from both the tins and mixing them together.

We know that the final mixture contains 5 kg of wheat and 11kg of rice. Meaning that the final mixture has a weight of 16 kg (11+5).

Since the final mixture was formed after mixing some quantity from Tin A and some from Tin B,

Weight from Tin A + Weight from Tin B = Weight of the final mixture = 16 kg.

In the solution, we have assumed that weight from Tin A = x kg and therefore, weight from Tin B = 16-x kg.

We know that in Tin A, the weight of wheat and rice are in the ratio 2:3. Meaning, for every 2 kgs of wheat, we have 3 kgs of rice. Or in every 1 kg of mixture in Tin A, we have 0.4 kg of wheat and 0.6 kg of rice. So in x kg taken from Tin A, we will have 0.4x kg wheat and 0.6kg rice.

Similarly, we know that in Tin B, the weight of wheat and rice are in the ratio 3:7. Meaning, for every 3 kgs of wheat, we have 7 kgs of rice. Or in every 1 kg of mixture in Tin B, we have 0.3 kg of wheat and 0.7 kg of rice. So in (16-x) kg taken from Tin B, we will have 0.3(x-16) kg wheat and 0.7(x-16) kg rice.

Now,

Wheat from the weight taken from A + wheat from the weight taken from B = Wheat in the final mixture [Intuitive]

0.4x + 0.3(16-x) = 5

Solving for x, we get x = 2

We could also have equated the quantity of rice

Rice from the weight taken from A + rice from the weight taken from B = Rice in the final mixture

0.6x + 0.7(16-x) = 11

This equation will also give us x = 2.

The entire solution is based on conservation of mass.

7. Two vessels contain mixtures of spirit and water. In the first vessel the ratio of spirit to water is 8 : 3 and in the second vessel the ratio is 5 : 1. A 35–liter cask is filled from these vessels so as to contain a mixture of spirit and water in the ratio of 4 : 1. How many liters are taken from the first vessel?

Sol.

Let X liters be taken from the first vessel; then (35 – X) liters are taken from the second.
In the first vessel 8/11 of the mixture, and in the second vessel 5/6 of the mixture, is spirit
So, the spirit in the 35–liter cask is 4/5 of the mixture.

Equate spirit on both sides:

So, $8/11\ X + 5/6\ (35 – X) = 4/5 × 35$
So, $X = 11$

So, 11 liters are taken from the first vessel **Ans**.

==Alternate sol from gmatclub (additional)==

```
Ratio-----------Spirit------Water------Total
1st vessel--------8----------3---------11
2nd vessel--------5----------1----------6
Final------------4----------1---------5
```

Converting these ratios to same total,

```
Ratio-----------Spirit------Water------Total
1st vessel--------240-------90-------330
2nd vessel-------275--------55-------330
Final----------- 264-------66-------330
```

Now, final ratios of two solutions will be
1st vessel : 2nd vessel
(275 - 264) : (264 - 240)
=> 11 : 24

As final solution is 35 litre, first solution will be $\dfrac{11}{(11+24)} * 35 = 11$ litre

8. A bag contains $600 in the form of one–dollar, 50 cents & 25 cents coins in the ratio 3 : 4 : 12. Find the number of 25 cents coins.

Sol.
Ratio of values of coins = 3/1 : 4/2 : 12/4 = 3 : 2 : 3.
Value of 25 cents coins = Rs 600 × 3/ (3 + 2 + 3) = 225.
No. of 25 cents coins = 225 x 4 = 900 **Ans**.

Alternate method:

Assume that the number of 1 $ coins is 3X.

Then the value equation would be 3 X + 4 X (0.50) + 12 X (0.25) = 600.

So,

3X + 2X + 3X = 600

8X = 600

X = 75

Now, since we need 25 cent coins, we need to find 12X.

So,

12X = 75*12 = 900 **Ans.**

9. A mixture contains milk & water in the ratio 5 : 1. On adding 5 liters of water, the ratio of milk and water becomes 5 : 2. Find the quantity of milk in the original mixture.

   Sol.

   Let the quantity of milk be 5X & that of water be X.

   Then 5X / (X + 5) = 5/2 or X = 5.

   So, quantity of milk = 5X = 25 liters

10. The ratio of the number of boys to the number of girls in a school of 546 is 4 : 3. If the number of girls increases by 6, what must be the increase in the number of boys to make the new ratio of boys to girls 3 : 2?

    Sol.

    Original no. of boys = 546 × 4/7 = 312.

    Original no. of girls = 78 × 3 = 234.

    Final no. of girls = 234 + 6 = 24

    So, number of boys required to make the new ratio = 240 × 3/2 = 360

    So, The required. increase in the number of boys = 360 – 312 = 48 **Ans**.

11. Two numbers are in the ratio of 3 : 4. If 5 is subtracted from each, the resulting numbers are in the ratio 2 : 3. Find the numbers.

    Sol.

    Let 3X and 4X be the numbers.

    So, (3X – 5) / (4X – 5) = 2/3

    So, 9X – 15 = 8X – 10

    => X = 5

    So, the required numbers are 15 and 20 **Ans**.

Work / Rate:

FUNDAMENTAL CONCEPTS:

- If A alone takes X hours and B alone takes Y hours to do a piece of work, and if T is the total time taken when they work together, then we have: $1/X + 1/Y = 1/T$ or $T = XY/(X + Y)$

  So, If A and B can do a piece of work in X & Y days respectively while working alone, they will together take $XY / (X + Y)$ days to complete it.

  It is best to take the LCM of times taken to avoid using fractions in such questions.

- If A is twice as good a workman as B, then A will take half the time B takes to finish a piece of work.

Solved examples:

1. If A and B together finish a piece of work in 10 days & B alone can finish it in 20 days. In how many days can A alone finish the work?

2. Four men working together all day, can finish a piece of work in 11 days; but two of them having other engagements can work only one half–time and quarter time respectively. How long will it take them to complete the work?

3. 20 men can complete a piece of work in 10 days, but after every 4 days 5 men are called off, in what time will the work be finished?

4. A vessel can be filled by one pipe A in 10 minutes, by a second pipe B in 15 minutes. It can be emptied by a waste pipe C in 9 minutes. In what time will the vessel be filled if all the three were turned on at once?

5. Three pipes A, B and C can fill a tank in 15, 20 and 30 min respectively. They were all turned on at the same time. After 5 minutes the first two pipes were turned off. In what time will the tank be filled?

6. A tank can be filled by two taps A and B in 12 minutes and 14 minutes respectively and can be emptied by a third in 8 minutes. If all the taps are turned on at the same moment, what part of the tank will remain unfilled at the end of 7 minutes?

Solutions:

1. Let X and Y be the number of days required by A and B respectively.

   By the standard formula,

   $1/X + 1/Y = 1/10$

   $\Rightarrow XY / (X + Y) = 10$

   If Y = 20
   Then, $X * 20 / (X + 20) = 10$
   X = 20 days **Ans**.

2. Each man will take $11 \times 4 = 44$ days to complete the work.
   If one man works half day/day he will take $44 \times 2 = 88$ days to finish the work.
   Similarly, a man working quarter day/day will take $44 \times 4 = 176$ days to finish the work.
   When these people work together, they will require
   We have: $1/44 + 1/44 + 1/88 + 1/176 = 1/T$

   T = 16 days. **Ans**

   Assuming the 4 men are equally efficient, if 4 men can complete a piece of work in 11 days, 1 man can complete the work in 44 days. In 1 day, 1 man can complete 1/44 of the work.

   Let the new time taken by the 4 men be T days.

   Now, 2 of the 4 men work for T days. In T days, 1 man can complete (T/44) of the work. So 2 men can complete (T/22) of the work.

   One of the other 2 men worked for only T/2 days. In T/2 days, the man will complete (T/88) of the work.

   The last man worked for only T/4 days. In T/4 days, the man will complete (T/176) of the work.

   Total work completed by the 4 men = T/22 + T/88 + T/176 = 11T/176 = 1 [because in T days, the total work is completed]

   T = 16 days. **Ans**

3. Total work = 20 x 10 = 200 man-days
   First 4 days' work = 20 x 4 = 80 man-days
   Next 4 days' work = 15 x 4 = 60 man-days
   Next 4 days' work = 10 x 4 = 40 man-days
   Next 4 days' work = 5 x 4 = 20 man-days

   Total 200 man-days … Hence, days required = 4 + 4 + 4 + 4 = 16 **Ans**

4. $1/10 + 1/15 – 1/9 = 1/T$ or T = 18 … so the vessel will be filled in 18 minutes **Ans**.

5. A, B and C can fill $(1/15 + 1/20 + 1/30)$ or 3/20 of the tank in 1 minute

   A, B and C filled $(3/20 \times 5)$ or 3/4 of the tank in 5 min.

Now A and B are turned off

(1 – 3/4) or 1/4 of the tank will be filled by C

So, C will fill 1/4 of the tank in (30 x 1/4) or 7.5 minutes

So, the tank will be filled in 7.5 + 5 or 12.5 min. **Ans**

6. We have (7/12) + (7/14) – 7/8 = 5/24 part filled in 7 minutes.
   Hence 1 – 5/24 = 19/24th of the tank   is unfilled.

Time taken by tap A to fill the tank = 12 minutes
Portion of the tank filled by tap A in 1 minute = 1/12

Time taken by tap B to fill the tank = 14 minutes
Portion of the tank filled by tap B in 1 minute = 1/14

Time taken by tap C to empty the tank = 8 minutes
Portion of the tank emptied by tap C in 1 minute = 1/8

Therefore, in 1 minute, net portion of the tank filled = Portion of the tank filled by tap A in 1 minute + Portion of the tank filled by tap B in 1 minute - Portion of the tank emptied by tap C in 1 minute

Net portion filled in 1 minute = 1/12 + 1/14 - 1/8 = 5/168.

Using this, net portion filled in 7 minutes = Net portion filled in 1 minute * 7
Net portion filled in 7 minutes = 35/168

Therefore, unfilled portion of tank in 7 minus = 1 - 35/168 = 133/168 = 19/24

Time / Speed / Distance

- If A goes from X to Y at U km/hour and comes back from Y to X at V km/hour, then Average speed during the whole journey = 2UV / (U + V) km/hr.

- If a man changes his speed in the ratio m : n then the ratio of times taken becomes n : m.

- When two objects travel in the same direction, relative speed = difference of speeds and time to catch / overtake = lead distance / difference of speeds

  So … Time to overtake (same directions) = Gap distance / difference of speeds

- When two objects travel in the Opposite directions, relative speed = sum of speeds and time to meet = lead distance / sum of speeds.

  SO … Time to meet (opposite directions: towards each other) = Gap distance / sum of speeds

- Average Speed = Total Distance / Total Time

- If the speed of a boat (or man) in still water be X km/hour, and the speed of the stream (or current) be Y km/hour, then
  (a) Speed of boat with the stream (or Downstream or D/S) = (X + Y) km/hour
  (b) Speed of boat against the stream (or upstream or U/S) = (X – Y) km/hour

Solved Examples:

1. A policeman goes after a thief who is 176 m ahead of him. When and where will the policeman catch the thief when they run at the rates of 11440 and 10560 meters per hour respectively?

2. If I walk at the rate of 4 kms an hour, I reach my destination 30 min too late; If I walk at the rate of 5 kms an hour I reach 30 minutes too soon. How far is my destination?

3. A man rows 18 kms down a river in 4 hours with the stream and returns while traveling upstream in 12 hours; find his speed and also the velocity of the stream.

4. A, B and C can walk at the rates of 3, 4, 5 kms an hour. They start from X at 1, 2, 3 o'clock respectively; when B catches up with A, B sends him back with a message to C; when will C get the message?

5. A student walks to school at the rate of 2.5 kms an hour and reaches 6 minutes too late. Next day he increases his speed by 2 kms an hour and then reaches there 10 minutes too soon. Find the distance of the school from his home.

6. A man can row in still water a distance of 4 kms in 20 minutes and 4 kms with the current in 16 min. How long will it take him to row the same distance against the current?

Solutions:

1. Time to catch / overtake = lead distance / difference of speeds
   = 176 / (11440 – 10560)
   = 176 / 880
   = 1/5 hours
   = 12 minutes

   So, the time required to overtake the thief = **12 min**.
   The distance from the starting point = 11440 × 12/60 kms = **2288 meters Ans**

2. Let time taken be T hours for the distance to be covered at the normal speed (neither fast nor slow).
   Then we have 4 (T + 0.5) = 5 (T – 0.5) {Note: 0.5 here is 30 min}
   Therefore, T = 4.5 hours
   Hence, Distance = 4 (T + 0.5) = 4 x 5 = 20 kms. **Ans**

3. Speed with the stream = 18/4 = 4.5 kms an hour.
   Speed against the stream = 18/12 = 1.5 kms an hour.
   Now,
   If the speed of a boat (or man) in still water be X km/hour, and
   the speed of the stream (or current) be Y km/hour, then
   x + y = 4.5
   x – y = 1.5
   Solving both the equations, we get x = 3 and y = 1.5 **Ans**

4. A starts at 1 o clock and B starts at 2 o clock.

   If I consider time 0 hr when A started, after 1 hr, A is 3 km away and B has just started.

   After 2 hrs, A is 6 km away and B is 4 km away.

   After 3 hrs, A is 9 km and B is 8 km.

   Next hr, they both meet at 12 km.

   You can also, get this by taking the LCM of 3 and 4 that is 12 km.

   So, after 4hr, A gets the message. So, at 5 o clock, he got it. Since A started at 1 o clock.

   Now, C started at 3 o clock.

   That means it has walked for 2 hrs. So, he walked 10km.

   Thus now, the distance between A and C is 12–10 = 2 km.

   Now they move in opposite directions at speed 3 km/hr and 5 km/hr

   Distance / sum of speeds = time to meet
   2/ (5 + 3) = 2/8 = 1/4 hours = 15 min

   Thus, C gets the letter at 5:15.

   OR
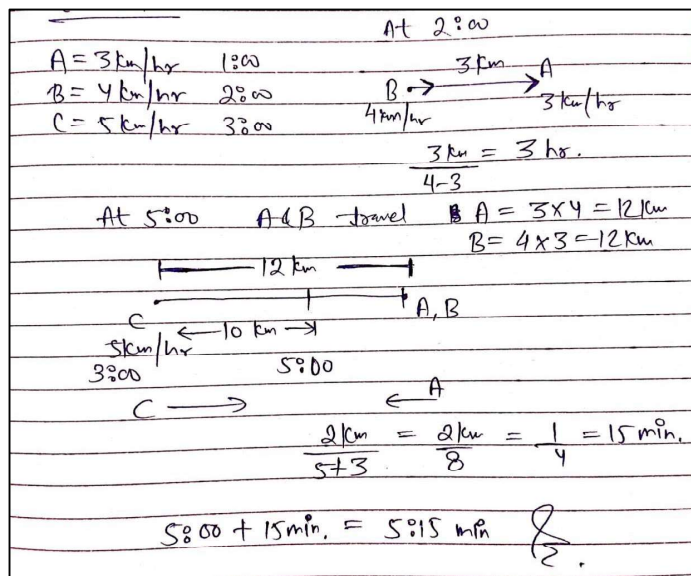
   In 15 mins, A walks 3/4km and C walks 5/4km

   Total distance walked = 3/4+5/4 = 8/4 = 2 kms

   Time to meet = Distance / sum of speeds = 2/ (5 + 3) = 2/8 = 1/4 hours = 15 min

   Thus, they meet after 15 mins.

   Thus, C gets the letter at 5:15.

5. Let t be the usual time.

`   We have 2.5 × (t + 1/10) = 4.5 (t – 1/6), or t = 1/2 hours.

                 Hence distance = 2.5 (1/2 + 1/10) = 2.5 x 6 /10 = 1.5 kms **Ans**

6. X = 4 / (20/60) = 12,
    X + Y = 4 × 60 /16 = 15
    Y = 3.
    Therefore, Time = 4 / (X – Y) = 4 x 60 /9 = 80/3 minutes **Ans**

==Top 1% expert replies to student queries (can skip)==

Speed in still water (4 km in 20 mins) = 12 km/hr (lets say 'x' km/hr)
Speed with current (4 km in 16 mins) = 15 km/hr (lets say 'a' km/hr)
Speed against the current = 'b' km/hr

x = (a+b)/2
12 = (15+b)/2
b = 9 km/hr
So , to travel 4 km against the current, the man will take [(4/9)*60] mins = 26.7 mins **Ans**

==**Top 1% expert replies to student queries (can skip) (additional)**==

So we know the speed of the person in still water to be 4 km in 20 mins or 1/3rd of an hour.
So the speed in still water is 12 km / hr. Let's say the speed of the current is c km / hr.
If the man is rowing with the current (in the direction the water is flowing), the total speed will be his still-water speed + the speed of the current (the current is helping him basically).
If the man is rowing against the current, now the current is hampering him, and his speed will be speed in still water - the speed of the current.

The units are not hard and fast. You should change them in a way that aids calculation the most. It is not necessary that you have to keep the figures in certain units; change them wherever the numbers become easiest to deal with.
As an example, converting 4 km in 16 minutes to km / hour will make the number difficult to deal with. However, if we change the speed to km / min, we get the speed as 1/4 km / min - an arguably easier number to deal with. Just ensure that all units in the solution are the same.

So, the speed in still water becomes 1/5 km/min

Let the speed of the current be c (very important: in km/min)

Then speed in still water + speed of current = speed with the current

1/5 + c = 1/4

c = 1/4 - 1/5 = 1/20 km/min

What will be the speed against the current?
Speed in still water - speed of current = 1/5 - 1/20 = 3/20 km/min

How long will the man take to row 4 km with this speed?
(20/3) x 4 = **80/3 minutes = 26 and 2/3 minutes   Ans**

Solved examples on translating Word Problems into equations:

1. Find two consecutive odd numbers the difference of whose squares is 296.

2. A is 29 years older than B; B is 3 years older than C and D is 2 years younger than C. Two years hence A's age will be twice the united ages of B, C and D. Find their present ages.

3. A number consists of three consecutive digits, that in the unit's place being the greatest of the three. The number formed by reversing the digits exceeds the original number by 22 times the sum of the digit. Find the number.

    (A) 234   (B) 345   (C) 456   (D) 567   (E) 678

4. The crew of a boat can row at the rate of 5 miles an hour in still water. If to row 24 miles, they take 4 times as long upstream as to row the same distance down the river, find the speed at which the river flows.

5. The area of a rectangle remains the same if the length is increased by 7 meters and the breadth is decreased by 3 meters. The area remains unaffected if the length is decreased by 7 m and breadth is increased by 5 m. Find the dimensions of the rectangle.

6. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves Rs. 200 per month, find their monthly incomes.

7. Find two consecutive even numbers such that 1/6th of the greater exceeds 1/10th of the smaller by 29.

8. A number consists of two digits whose sum is 12. The ten's digit is three times the unit's digit. What is the number?

9. A train travelled a certain distance at a uniform rate. Had the speed been 6 miles an hour more, the journey would have occupied 4 hours less; and had the speed been 6 miles an hour less, the journey would have occupied 6 hours more. Find the distance.

10. A sum of money was divided equally among a certain number of persons; had there been six more, each would have received a rupee less, and had there been four fewer, each would have received a rupee more than he did; find the sum of money and the number of men.

Solutions:

1. Let the numbers be 2X + 1 and 2X + 3

   Then $(2X + 3)^2 - (2X + 1)^2 = 296$

   => X = 36

   Hence 2X + 1 = 2*36 + 1 = 73 and 2X + 3 = 75

   The required numbers are 73 and 75.

   [Verification. $(75)^2 - (73)^2 = 5625 - 5329 = 296$]

   The equation we have to solve is :

   $(2x + 3)^2 - (2x + 1)^2 = 296$

   $(4x^2 + 12x + 9) - (4x^2 + 4x + 1) = 296$

   $8x + 8 = 296$

   $8x = 288$

   $x = 36$

   Therefore,

   2x + 1 = 73
   2x + 3 = 75

2. Let D's age be = X years
   Then C's age = (X + 2) years
   B's age = (X + 5) years
   A's age = (X + 34) years.

   Two years hence A's, B's, C's and D's ages will be X + 36, X + 7, X + 4 and X + 2 years respectively.
   So, we have: 2 (X + 2 + X + 4 + X + 7) = X + 36
   =>X = 2
   So, A's age = 36 years; B's age = 7 years, C's age = 4 years; D's age = 2 years. **Ans**

3. **Try option A**
   234 ... 2 + 3 + 4 = 9
   432 – 234 = 198 = 22*9
   All verified. So, **A** is correct

   Detailed Solution:

   Let the hundred's digit be X. Then the ten's digit = X + 1 and the unit's digit = X + 2.
   The number = 100*X + 10(X + 1) + X + 2 = 111*X + 12.
   The number formed by reversing the digits = 100(X + 2) + 10(X + 1) + X = 111*X + 210

   We have 111*X + 210 – 111*X – 12 = 22 (X + 2 + X + 1 + X).
   So, we get, X = 2. Hence the required number = 234. **Ans**

4. Let X miles per hour be the speed of the river.

Hence, on equating the times, we get:

Speed upstream = 5 – X
Speed downstream = 5 + X

Time taken upstream = Distance / speed = 24 / (5 – X)
Time taken downstream = Distance / speed = 24 / (5 + X)
24 / (5 – X) = 4 × 24/ (5 + X)
X = 3.
Thus, the river flows at the rate of 3 miles an hour.

5. (X + 7) (Y – 3) = XY or – 3X + 7Y – 21 = 0 ...(1)

(X – 7) (Y + 5) = XY or 5X – 7Y – 35 = 0..(2)

Solve the two simultaneous equations to get

Y = 15 and X = 28
Hence length = 28 m and breadth = 15 m **Ans**

6. Let the monthly income of first person be Rs 9X and the monthly income of second person be Rs 7X.
Let the expenditure of first person be 4Y and the expenditure of second person be 3Y.
Saving of the first person = Rs (9X – 4Y) and solving of second person = Rs (7X – 3Y).
Using the given information, we have:
9X – 4Y = 200 .... (1)
7X – 3Y = 200 …(2)
So, X = 200.
Hence, the monthly income of first person = Rs. 9 × 200 = Rs. 1800 and the monthly income of second person = Rs. 7 × 200 = Rs. 1400 **Ans**.

7. Let the numbers be 2X and 2X + 2

   Then $(2X + 2)/6 - 2X/10 = 29$.

   So, X = 215

   Hence 2X = 430 and 2X + 2 = 432.

   The required numbers are 430 and 432. [Verification. $432/6 - 430/10 = 72 - 43 = 29$]

8. Let the unit's digit be X, Then the ten's digit is 12 – X.

   We have $3X = 12 - X$ or $X = 3$

   Hence the number is 93. [Verification. $9 = 3 \times 3$; and $9 = 3 = 12$]

9. Let us suppose that X miles per hour is the speed of the train and Y hours is the time taken for the journey.

   Distance travelled $= XY = (X + 6)(Y - 4) = (X - 6)(Y + 6)$

   Solve exactly like Q. 5 above

   Generate two simultaneous equations. You will get: X = 30, Y = 24

   Distance $= XY = 720$ miles **Ans**.

10. Let X be the number of persons and Rs Y be the share of each. Then by the conditions of the problem, we have

    $(X + 6)(Y - 1) = XY$ ... (1)
    $(X - 4)(Y + 1) = XY$ ... (2).

    Solve exactly like Q. 5 above

    Generate two simultaneous equations. You will get: X = 24 and Y = 5

    Thus, the number of people is X = 24 and the share of each is Y = Rs 5.

    The sum of money $= 5 \times Rs\ 24 = Rs\ 120$

Questions based on compound interest / Rate of Increase

**Compound Interest formula:** $A = P\left(1 + \dfrac{R}{100}\right)^{n}$ CI = A – P.

For half yearly calculation of the interest: $A = P\left(1 + \dfrac{R}{200}\right)^{2n}$

For quarterly calculation of the interest: $A = P\left(1 + \dfrac{R}{400}\right)^{4n}$

In all these results: A = Final Amount, P = Principal (Initial Amount), R = Rate per annum, n = number of years.

Population increase formula:

$$Final = Initial\ [Factor\ of\ Multiplication]^{\frac{Total\ Time\ Available}{Time\ interval\ required\ for\ one\ Multiplication}}$$
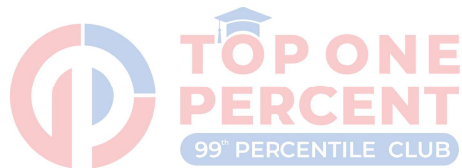
Solved examples:

1. Donald plans to invest x dollars in a savings account that pays interest at an annual rate of 8% compounded quarterly. Approximately what amount is the minimum that Donald will need to invest to earn over $100 in interest within 6 months?

    $1500 $1750 $2000 $2500 $3000

2. Wes works at a science lab that conducts experiments on bacteria. The population of the bacteria multiplies at a constant rate, and his job is to notate the population of a certain group of bacteria each hour. At 1 p.m. on a certain day, he noted that the population was 2,000 and then he left the lab. He returned in time to take a reading at 4 p.m., by which point the population had grown to 250,000. Now he has to fill in the missing data for 2 p.m. and 3 p.m. What was the population at 3 p.m.?

    50,000   62,500   65,000   86,666   125,000

3. The population of locusts in a certain swarm doubles every two hours. If 4 hours ago there were 1,000 locusts in the swarm, in approximately how many hours will the swarm population exceed 250,000 locusts?

    6   8   10   12   14

4. A scientist is studying bacteria whose cell population doubles at constant intervals. In the last 2 hours, the population has quadrupled, increasing by 3,750 cells. How many cells will the population contain four hours from now?

Solutions:

1. The formula for calculating compound interest is $A = P*(1 + r/n)^{nt}$ where the variables represent the   following:

    A = amount of money accumulated after t years (principal + interest)
    P = principal investment
    r = interest rate (annual)
    n = number of times per year interest is compounded
    t = number of years

    In this case, x represents the unknown principal, r = 8%, n = 4 since the compounding is done quarterly, and t = .5 since the time frame in question is half a year (6 months).

    r = 8% = 8/100

    So, we have:

    $P (1 + 8/400)^2$

    From the options: $2500 (1 + 8/400)^2 = 2601$ … so the interest = 101, which is close to 100.

    OR … by approximation:

    8% interest over half a year, however that interest is compounded, is approximately 4% interest. So, to compute the principal, it's actually a very simple calculation:

    $100 = 0.04x$

    $2500 = x$

    The correct answer is D.

Easy Approach-
8% compounded quarterly = 2% per quarter = 4% for half year
If 4% is 100 then 100% would be 2500
Answer D.

Conventional approach-
Compound interest formula
$A = P (1+r/n)^{nt}$
given, n= 4 (quarterly); r =.08
The approach is substitution,
Our interest requirement is 100$ after 6 months, 2 compounding periods. interest per compounding period is 2%
Let's take 1500, after 3 months interest accumulated is 30$, total amount is 1530$
after 6 months, interest is 30.6$ and total is 1560.6$, so not 1500$
1500$ & 1750$ have a difference of 250$ only , but the expected interest different is around 40$. Hence you can straightaway rule out 1750
2000 is again can be ruled out as approx. 4% interest yields only 80$
2500$ is a good bet, first 3 months it earns 50$ as interest, next 3 months it will earn 51$ as interest.
hence, answer is D

2. If we decide to find a constant multiple by the hour, then we can say that the population was multiplied by a certain number three times from 1 p.m. to 4 p.m.: once from 1 to 2 p.m., again from 2 to 3 p.m., and finally from 3 to 4 p.m.

   Let's call the constant multiple x.

   $2,000(x)(x)(x) = 250,000$
   $2,000(x^3) = 250,000$
   $x^3 = 250,000/2,000 = 125$
   $x = 5$

   Therefore, the population gets five times bigger each hour.

   At 3 p.m., there were $2,000(5)(5) = 50,000$ bacteria.

   The correct answer is A.

3. A population problem is best solved with a population chart that illustrates the swarm population at each unit of time. An example of a population chart is shown below:

Time	Population
4 hours ago	1,000
2 hours ago	2,000
NOW	4,000
in 2 hours	8,000
in 4 hours	16,000
in 6 hours	32,000
in 8 hours	64,000
in 10 hours	128,000
in 12 hours	256,000

As can be seen from the chart, in 12 hours the swarm population will be equal to 256,000 locusts. Thus, we can infer that the number of locusts will exceed 250,000 in slightly less than 12 hours.

Since we are asked for an approximate value, 12 hours provides a sufficiently close approximation and is therefore the correct answer.

OR

2^8 =256 (more than 250 times) so 8 intervals are needed, means 16 hours. 4 hours are already past, so 12 more hours are needed.

The correct answer is D

Alternate Solution from Gmatclub
4 hours ago = 1000
2 hours ago = 2000
right now = 4000

now the given equation says

4000*2n>25000

=> 2n>62.5 since n must be an integer... thus take

2n=64=2^6 thus it got doubled 6 times thus 12 hours

Thus D

4. Let x equal the population immediately after that division

$4x = x + 3,750$
$3x = 3,750$
$x = 1,250$

So according to the data given, if we extrapolate to the value at 4 hours from now,

(2 hours earlier) 1250(= x), 2500, 5000 (now), 10000, 20000, 40000, 80000 (4 hours from now)

Ans. 80,000

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The population is just divided and, since the population divided two hours ago, the population has quadrupled, increasing by 3,750 cells.
When the population increases by a factor of 4, the result is 3750 more cells:
$4p = p + 3750$
$p = 1250$.
Thus:
2 hours ago, p=1250.
Now, p = 1250+3750 = 5000.
Since the population quadruples every 2 hours, over the next 4 hours it will quadruple twice:
p = 4*4*5000 = 80,000.

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If the population quadrupled during the last two hours, it doubled twice during that interval, meaning that the population doubled at 60 minute intervals (Since the division two hours ago and the division just now, the population quadrupled -- that's two doubling periods, so each doubling period is about an hour.).

Since it has increased by 3,750 bacteria, we have:

Population (Now) – Population (2 hours ago) = 3,750

Population (Now) = 4·Population (2 hours ago)

Substituting, we get 4·Population (2 hours ago) – Population (2 hours ago) = 3,750

Population (2 hours ago) = 1,250.

The population will double 6 times from that point to 4 hours from now

Population (4 hours from now) = $(2^6)(1,250) = 80,000$.