Quant Session: Permutations and Combinations | Probability

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Definitions:

- Each of the different orders of arrangements, obtained by taking some, or all, of a number of things, is called a **Permutation.**

- Each of the different groups, or collections, that can be formed by taking some, or all, of a number of things, irrespective of the order in which the things appear in the group, is called a **Combination.**

Example:

Suppose, there are four quantities A, B, C, and D. The different orders of arrangements of these four quantities by taking three at a time, are:

ABC, ACB, BAC, BCA, CAB, CBA, ... (1) ABD, ADB, BAD, BDA, DAB, DBA, ... (2)
ACD, ADC, CAD, CDA, DAC, DCA, ... (3) BCD, BDC, CDB, CBD, DBC, DCB... (4)

Thus, each of the 24 arrangements, of the four quantities A, B, C, and D by taking three at a time, is called a permutation. Hence, it is clear that the number of permutations of four things taken three at a time is 24.

Again, it may be easily seen, from the above that out of these 24 permutations, the six, given in (1), are all formed of the same three quantities A, B, C in different orders; hence, they all belong to the same group. Similarly, the permutations, given in (2), all belong to a second group; those given in (3), belong to a third and those in (4), belong to a fourth. Hence, we see that **there are only four different groups** that can be formed of four quantities A, B, C, and D by taking three at a time. **Thus, the number of combinations of four things taken three at a time is only four.**

If there are **m** ways of doing a thing and **n** ways of doing a second thing and **p** ways of doing a third thing, then the total number of "distinct" ways of doing all these together is **m** × **n** × **p.**

Ex 1.

Suppose, there are five routes for going from a place A to another place B and six routes for going from the place B to a third place C. Find the numbers of different ways through which a person can go from A to C via B.

Sol.

Since there are five different routes from A to B, person can go from A to B in five different ways. After reaching B, he has six different ways of finishing the second part of his journey (i.e. going from B to C). Thus, for one way of going from A to B there are six different ways of completing the journey from A to C via B. Hence, the total number of different ways of finishing both parts of the journey (i.e. A to B and then from B to C) = 5 times six different ways = 5×6 = no. of ways from the first part to the second point
× number of ways from the second part to the third point

Ex 2.

Find the number of different ways in which four persons can be accommodated in three different chairs.

Sol.

Let's assume that the four persons are P, Q, R, and S. Since all the three different chairs are vacant, any one of the four persons can occupy the 1st chair. Thus, there are four ways of filling up the 1st chair. When the 1st chair has been filled up by any one of the four people, say P, the 2nd chair can be filled up by any one of the remaining three persons Q, R and S. Thus, for each way of filling up the 1st chair, the 2nd chair can be filled up in three different ways. Hence, total no. of ways in which the first two chairs can be filled up is equal to $4 \times 3 = 12$ ways. Again, when the 1st and 2nd chairs are filled up in any one way (i.e. the 1st by P and the 2nd by Q), the 3rd chair can be filled up by any one of the two remaining persons, R and S. Thus, for each way of filling up the first two chairs, there are $4 \times 3 \times 2$ i.e. 24 ways of filling up the third chair along with the first two chairs. Hence, the total no. of ways in which four persons can be accommodated in the three given chairs is equal to $4 \times 3 \times 2 = 24$. We therefore conclude that, the total number of different orders of arrangements of 4 different things, taken 3 at a time, is the same as the total number of different ways in which 3 places can be filled up by 4 different things.

PERMUTATIONS

- Permutations of n different things taken 'r' at a time is denoted by ${}^nP_r$ and is given by
  $\mathbf{{}^nP_r = n! / (n - r)!}$
- The total number of arrangements of n things taken r at a time, in which a particular thing always occurs $= \mathbf{r \times {}^{n-1}P_{r-1}}$
- The total number of permutations of n different things taken r at a time in which a particular thing never occurs $= \mathbf{{}^{n-1}P_r}$
- The total number of permutations of n dissimilar things taken r at a time with repetitions $= n^r$
- No. of circular permutations of n things taken all at a time $= (n - 1)!$
- No. of circular permutations of n different things taken r at a time $= {}^nP_r / r$
- The number of permutations when things are not all different: If there be n things, p of them of one kind, q of another kind, r of still another kind and so on, then the total number of permutations is given by

  $n! / (p!\ q!\ r!)$

COMBINATIONS

- Number of combinations of n dissimilar things taken 'r' at a time is denoted by ^nC_r and is given by $^nC_r = n! / [(n – r)!\, r!]$
- Number of combinations of n different things taken r at a time in which p particular things will always occur is $^{n-p}C_{r-p}$
- No. of combinations of n dissimilar things taken 'r' at a time in which 'p' particular things will never occur is $^{n-p}C_r$
- $^nC_r = {^nC_{n-r}}$

PROBABILITY

$$\text{Probability of an event occurring} = \frac{\text{Number of favorable outcomes}}{\text{Number of all possible outcomes}}$$

Note:

- If an event E is sure to occur, we say that the probability of the event E is equal to 1 and we write P (E) = 1.
- If an event E is sure not to occur, we say that the probability of the event E is equal to 0 and we write P (E) = 0.
- *Therefore, for any event E,* $0 \le P(E) \le 1$
- The probability of E not occurring, denoted by P (not E), is given by P (not E) = 1 – P(E)
- Odds in favor = No. of favorable cases / No. of unfavorable cases
- Odds against = No. of unfavorable cases / No. of favorable cases

Mutually Exclusive Events:

Two events are mutually exclusive if one happens, the other can't happen and vice versa. In other words, the events have no common outcomes. For example:

In rolling a die
E: – The event that the no. is odd
F: – The event that the no. is even
G: – The event that the no. is a multiple of three

In drawing a card from a deck of 52 cards
E: – The event that it is a spade
F: – The event that it is a club
G: – The event that it is a king

In the above 2 cases events E and F are mutually exclusive but the events E and G are not mutually exclusive or disjoint since they may have common outcomes.

ADDITION LAW OF PROBABILITY:

If E and F are two mutually exclusive events, then the probability that either event E or event F will occur in a single trial is given by:

P (E or F) = P(E) + P(F)

If the events are not mutually exclusive, then

P (E or F) = P(E) + P(F) – P (E and F together)

P (neither E nor F) = 1 – P (E or F).

Independent Events and Multiplication Law:

Two events are independent if the happening of one has no effect on the happening of the other. For

ex:

On rolling a die and tossing a coin together

E: – The event that no. 6 turns up.

F: – The event that head turns up.

In shooting a target

E: – Event that the first trial is missed.

F: – Event that the second trial is missed.

In both these cases events E and F are independent.

BUT, in drawing a card from a well shuffled pack

E: – Event that first card is drawn

F: – Event that second card is drawn without replacing the first

G: – Event that second card is drawn after replacing the first

In this case E and F are not Independent but E and G are independent → There is a concept in probability called sample space - it is the set of all possible outcomes (an event space is a subset that contains events to which we can assign probabilities). In mathematical terms, the reason E and F are not independent but E and G are, is that if F is done, then the sample space has changed fundamentally and is not the same any more as when E is done.

However, if E and G are done the sample space is restored to its original form, so they are independent. What does this mean in non-mathematical terms? Think intuitively - if you have a deck of cards and you pull one (event E). Whatever this card is, can you draw this one again without replacing it? No. Then event F fundamentally depends on what the outcome of event E was (what card was drawn; this card cannot be drawn again in event F). S events E and F are not independent - occurrence of one has an effect on the other.

MULTIPLICATION LAW OF PROBABILITY:

If the events E and F are independent then **P (E and F) = P (E) × P (F)**

and **P (not E and F) = 1 – P (E and F together).**

SUMMARY of concepts

Arrangements - keywords – seating, sitting, sequence, order, alphabets, schedule, ranking, itinerary, codes

Order important – gives unique arrangements

For e.g. A and B sitting on chair can be AB or BA so these are two distinct arrangements

It is basically selection followed by arrangement. So $^nP_r = {}^nC_r * r!$

$$^nP_r = \frac{n!}{(n-r)!}$$

Selection - keywords – team, committee, balls, handshakes, matches, picking

Order not important – For example choosing A and B from a group of 3 or four alphabets. The order does not matter. India playing a match against Australia is the same as Australia playing against India.

$$^nC_r = \frac{n!}{(n-r)! \times r!}$$

Different formulae

1. $$^nP_r = \frac{n!}{(n-r)!}$$

   When to use? When n distinct items present and r have to be selected and then arranged.

   E.g. – how many ways can you arrange 4 people in 5 chairs = $^5P_4$

2. n^r

   All n distinct selection of r but repetition is allowed.

   In how many ways can you wear three different rings on four fingers?
   $= 4^3$

3. $$\frac{n!}{p!\, q!\, r!}$$

   Arranging n things in which p are of one type, q of a second type and r of third type:

Ex: In how many ways can you arrange the letters of word Banana?

Ans $\dfrac{6!}{3!\, 2!}$

.

4. Special Cases

   5 people A, B, C, D, E to be arranges in which A and B are together.
   $4! \times 2!$

   5 people A, B, C, D, E to be arranged in which A and B are not together.
   $5! - 4! \times 2!$

5. **Block diagrams -** Some problems cannot be done with any formula but with a block diagram

Combinations

1. Select 5 people out of 10 Ans. ${}^{10}C_5$

Particular Cases – Select 5 out of 10 people such that A and B are always selected. This means only 3 of the remaining 8 are to be selected ${}^{8}C_3$

Select 5 out of 10 such that A and B are never selected. This means that out of remaining 8, 5 have to be selected so it is ${}^{8}C_5$

2. Select 5 out of 10 so that A and B are never together.
   = Total – Together = ${}^{10}C_5 - {}^{8}C_3$

AND denotes Multiplication

OR denotes Addition

Circular Permutations: $(n - 1)!$

Multiple trials of a single event: If multiple independent trials of a single event are performed, then the probability of r successes out of a total of n trials can be determined by ${}^{n}C_r \times p^r \times q^{n-r}$
Where
n = number of times the event is performed
r = number of successes
p = probability of success in one trial
q = probability of failure in one trial = 1 – p.

Solved examples for building key concepts:

1. Find the number of ways in which the letters of the word "machine" can be arranged such that the vowels may occupy only odd positions?
    A. 288
    B. 576
    C. 5040
    D. 48
    E. None of these

2. Sixteen jobs are vacant; how many different batches of men can be chosen out of twenty candidates? How often may any particular candidate be selected?

3. How many numbers greater than a million can be formed with the digits 2, 3, 0, 3, 4, 2, 3?
    A. 360
    B. 240
    C. 480
    D. 460
    E. 370

4. In how many ways can 3 letters be posted in four letter boxes in a village? If all the three letters are not posted in the same letter box, find the corresponding number of ways of posting.

5. In rolling two dice, find the probability that (1) there is at least one '6' (2) the sum is 5.

6. A single card is selected from a deck of 52 bridge cards. What is the probability that (1) it is not a heart, (2) it is an ace or a spade?

7. A box contains 2 red, 3 yellow and 4 blue balls. Three balls are drawn in succession with replacement. Find the probability that (1) all are yellow, (2) the first is red, the second is yellow, the third is blue, (3) none are yellow, (4) all three are of the same color.

8. With the data in Example 7, answer those questions when the balls are drawn in succession without replacement.

9. There are 7 Physics and 1 Chemistry book in shelf A. There are 5 Physics books in shelf B. One book is moved from shelf A to shelf B. A student picks up a book from shelf B. Find the probability that the Chemistry book: (1) is still in shelf A, (2) is in shelf B, (3) is taken by the student.

10. The ratios of number of boys and girls in X-A and X-B are 3: 1 and 2: 5 respectively. A student is selected to be the chairman of the students' association. The chance that the student is selected from X-A is 2/3. Find the probability that the chairman will be a boy.

11. The probability that a man will be alive in 25 years is 3/5 and the probability that his wife will be alive in 25 years is 2/3. Find the probability that: (1) both will be alive, (2) only the man will be alive, (1) only the wife will be alive, (4) at least one will be alive.

SOLUTIONS

1.
"Machine" consists of seven letters: four of them are consonants and three vowels. Let us mark out the position to be filled up as follows:

1	2	3	4	5	6	7
(a)	( )	(i)	( )	(e)	( )	( )

Since the vowels can be placed only in three out of the four positions marked 1,3,5,7, the total number of ways in which they can be made to occupy odd positions =

$^4P_3 = 4.3.2 = 24$ .... (1)

Suppose one arrangement of the vowels is as shown in the diagram; then for this particular arrangement of the vowel, the number of ways in which the 4 consonants can be made to occupy the remaining positions (marked 2,4,6,7) = $^4P_4 = 4.3\ 2.1 = 24$.

Hence, for each way of placing the vowels in odd positions there are 24 arrangements of the whole set. Consequently, the total number of arrangements of the given letters under the given condition = 24×24
= 576 Ans B is correct.

2
We have only to find out the number of different groups of 16 men that can be formed out of 20 without any reference to the appointment to be given to each.

Hence, the required number of ways = $^{20}C_{16} = {}^{20}C_4$

$= 20 \times 19 \times 18 \times 17 / (1 \times 2 \times 3 \times 4) = 5 \times 19 \times 3 \times 17 = 4845.$

Let us now find out how many times a particular candidate may be chosen.

Every time that a particular candidate is selected the other 15 candidates will have to be chosen from the remaining 19 candidates.

Hence a particular man may be selected as many times as we can select a group of 15 men out of the remaining 19. Hence, the required number of times = $^{19}C_{15} = {}^{19}C_4$

$= 19 \times 18 \times 17 \times 16 / (1 \times 2 \times 3 \times 4) = 19 \times 3 \times 17 \times 4 = 3876$

3. Since, each number is to consist of not less than 7 digits, we shall have to use all the digits in forming the numbers. Now, among these 7 digits there are 2 two's and 3 three's; hence the total number of ways of arranging the digits = $7! / (2!\ 3!) = 420$. But out of these arrangements we have to reject those that begin with zero, for they are six–digit numbers. Now, evidently there are as many such arrangements as there are ways of arranging the remaining 6 digits among themselves

Their no. = $6! /2!\ 3! = 60$ Hence, the required number = 420 – 60 = 360.
Option A is correct

4.

We can post the first letter in 4 ways. Similarly, the second and third can be posted in 4 ways each. So, the total number of ways = $4 \times 4 \times 4 = 64$. Now all the three letters together can be posted in any letterbox. In this case there will be four ways and when all the letters are not posted together, the number of ways $= 64 - 4 = 60$.

Top 1% expert replies to student queries (can skip)

In this question, we first need to find the number of ways we can put the first letter in the four boxes, then the second letter and then the third letter and then multiply all of them to get the total number of ways. Now, for all letters to be not posted in the same letter box we need to subtract the ways in which all the letters are being posted in the same box from the total number of possible ways.

Complete step-by-step answer:
Now, given in the question that there are three letters and four letterboxes
Let us now place the first letter and check the number of ways possible
Now, this letter can be placed in any of the four boxes which gives
$\Rightarrow 4$ ways

Now, the second letter again to be posted has four boxes available which can be posted in
$\Rightarrow 4$ ways

Now, again the third letter also has four boxes available which has
$\Rightarrow 4$ ways

Now, the total number of ways in which these three letters can be posted is given by
$\Rightarrow 4 \times 4 \times 4$

Now, on further simplification we get,
$\Rightarrow 64$

Thus, three letters can be posted in four letterboxes in 64 ways.
Now, we need to place three letters in four boxes provided that all the letters not to be placed in one box
Here, for all the letters to be placed in the same box we have different possibilities
Now, for all the boxes to be placed in the first box we have 1 possible way
In the same way, for all the letters to be placed in the second box we have 1 way, to be placed in the third box we have 1 way and then in the fourth box we have 1 way.
Thus, for all the letters to be placed in the same box we have 4 ways.
Now, on subtracting these 4 ways of all the letters being placed in 1 box from the total number of ways we get the value for them to be not placed in 1 box
$\Rightarrow 64 - 4$

$\Rightarrow 60$

Thus, the number of ways for all three letters not posted in the same letterbox are 60.

Note: Instead of subtracting the ways for all letters to be placed in one box from total number of ways we can also solve it by finding the ways in which all the letters can be placed in different boxes and two letters in one box and the remaining one in the other and then all add these possibilities to get the result. It is important to note that while finding the total number of ways we need to consider 4 possible ways for all the three letters as there is no condition given in particular that there can't be more than one in one box.

5.
The total possible outcomes are 36 as shown below.
(1,1) (1,2) (1,3) (1,4) (1,5) (1,6);
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6);
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6);
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6);
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6);
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6);

The outcomes with at least one '6' are

(1,6), (2,6), ...(6,6). There are 11 such pairs.

(1)

P (at least one '6') = 11/36 Ans.

(2)

The pairs with a sum of 5 are (1,4), (2,3), (3,2), (4,1). P

(the sum is 5) = 4/36 = 1/9 Ans.

Top 1% expert replies to student queries (can skip)

There are 36 possible outcomes from rolling 2 dice because the first die has 6 possible outcomes and so does the second: (1,1), (1,2), (1,3).........(2,1), (2,2).......(6,5), (6,6)

If the first die comes out a 6, the second die can be from 1 to 6.

So, there are 6 ways and similarly 6 ways the second die can show a 6. But, we have counted the roll of (6,6) twice.

So, there are 11 ways to roll at least one 6, out of 36 possibilities, so the answer is 11/36.

OR

(1)
Probability of rolling at least one two = 1 — (Probability of rolling no sixes)

Probability of rolling no sixes = 5/6*5/6 = 25/36

Probability of rolling at least one six =. 1 — 25/36 = 11/36

(2)
The pairs with a sum of 5 are (1,4), (2,3), (3,2), (4,1). P (the sum is 5) = 4/36 = 1/9 **Ans**

6.

A deck of bridge cards has 4 suits – spade, heart, diamond and club. Each suit has 13 cards.

Ace, two, three, …., ten, jack, Queen, King.

(1) P (not a heart) = 1 – P (a heart) = 1 – 13/52 = 39/52 = 3/4 **Ans**.

(2) There are 4 aces and 12 spades besides the ace of spades …

P (an ace or a spade) = 16/52 = 4/13 **Ans**.

7.

(1)

In a draw, P (red) = 2/9, P (yellow) = 3/9, P (blue) = 4/9.

In 3 draws, Prob of all are yellow = (3/9). (3/9). (3/9) = 1/27 Ans.

(2)

Required probability = P (1st red). P (2nd yellow). P (3rd blue)

= 2/9. 3/9. 4/9 = 8/243 Ans.

(3)

Probability that none are yellow = P (1st not yellow). P (2nd not yellow). P (3rd not yellow)
= (1 – 3/9) × (1 – 3/9) × (1 – 3/9) = 8/27 Ans.

(4)

Probability that all three are of the same color

= P (all red) + P (all yellow) + P (all blue) {mutually exclusive}

= $(2/9)^3 + (3/9)^3 + (4/9)^3 = 11/81$ Ans.

8.

(1)

Prob of all yellow = P (1st yellow). P (2nd yellow). P (3rd yellow)

= 3/9. 2/8. 1/7 = 1/84 Ans.

Since when the first yellow ball has been drawn, there are 8 balls remaining in the bag of which 2 are yellow.

(2)

Required probability = P (1st red). P (2nd yellow). P (3rd blue)

= 2/9. 3/8. 4/7 = 1/21 Ans.

(3)

Probability that none are yellow.

= P (1st not yellow). P (2nd not yellow). P (3rd not yellow)

= (1 – 3/9) (1 – 3/8) (1 – 3/7) = 6/9. 5/8. 4/7 = 5/21 Ans.

Top 1% expert replies to student queries (can skip)

Probability that none are yellow means that P(first ball should not yellow)*P(second ball should not yellow)*P(third ball should not yellow)

P(first ball should not yellow) = 6/9 (red & blue balls/ total balls)
Since we are not replacing the balls, the number in numerator and denominator will decrease by 1 each
P(second ball should not yellow) = 5/8 (because one red or blue ball has already been picked in previous chance)
P(third ball should not yellow) = 4/7 (same logic as that for previous balls)

Thus total P = (6/9)*(5/8)*(4/7) Ans.

(4)

Probability that all three are of the same color

= P (all red) + (all yellow) + P (all blue)

= 2/9. 1/8. 0/7 + 3/9. 2/8. 1/7 + 4/9. 3/8. 2/7 = 5/84 Ans.

9.

(1)

The probability that it is in shelf A = 7/8 Ans. (this means that Physics book was picked up)

(2)
The probability that it is in shelf B = P (it is moved from A to B). P (it is not taken by the student)
= 1/8. 5/6 = 5/48 Ans.

(3)
The probability that is it taken by the student = P (it is moved from A to B). P (it is taken by the student)
= 1/8. 1/6 = 1/48 Ans.

10.

Probability that the boy comes from X-A = 2/3. 3/4 = 1/2
Probability that the boy comes from X-B = 1/3. 2/7 = 2/21

The required probability = 1/2 + 2/21 = 25/42

11.

(1)

P (both alive) = P (man alive) $\times$P (wife alive) = 3/5$\times$2/3 = 2/5 (2)

P (only man alive) = P (man alive) $\times$P (wife dead) = 3/5$\times$1/3 = 1/5 (3)

P (only wife alive) = P (man dead) $\times$P (wife alive) = 2/5$\times$2/3 = 4/15 Ans. (4)

P (at least one will be alive) = 1 – P (both dead) = 1 – (2/5$\times$1/3) = 13/15 Ans.

Questions for class discussion

1. A password contains at least 8 distinct digits. It takes 12 seconds to try one combination, what is the minimum amount of time required to guarantee access to the database?
   (A) 12 seconds
   (B) 24 seconds
   (C) 36 seconds
   (D) 48 seconds
   (E) None of these

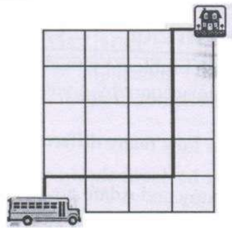
2. An engagement team consists of a project manager, team leader, and four consultants. There are 2 candidates for the position of project manager, 3 candidates for the position of team leader, and 7 candidates for the 4 consultant slots. If 2 out of 7 consultants refuse to be on the same team, how many different teams are possible?
   (A) 100
   (B) 120
   (C) 150
   (D) 200
   (E) None of these

3. A university cafeteria offers 4 flavors of pizza – pepperoni, chicken, Hawaiian and vegetarian. If a customer has an option to add, extra cheese, mushrooms, or both to any kind of pizza, how many different pizza varieties are available?
   (A) 12
   (B) 16
   (C) 20
   (D) 24
   (E) None of these

4. If 6 fair coins are tossed, how many different coin sequences will have exactly 3 tails, if all tails have to occur in a row?
   (A) 3
   (B) 4
   (C) 5
   (D) 6
   (E) 7

5. A telephone company needs to create a set of 3-digit area codes. The company is entitled to use only digits 2, 4 and 5, which can be repeated. If the product of the digits in the area code must be even, how many different codes can be created?
   (A) 25
   (B) 26
   (C) 27
   (D) 28
   (E) 30

6. Every morning, Casey walks from her house to the bus stop. She always travels exactly nine blocks from her house to the bus, but she varies the route she takes every day. (One sample route is shown.) How many days can Casey walk from her house to the bus stop without repeating the same route?



(A) 120
(B) 122
(C) 124
(D) 126
(E) 128

7. Anthony and Michael sit on the six-member board of directors for company X. If the board is to be split up into 2 three-person subcommittees, what percent of all the *possible* subcommittees that include Michael also include Anthony?
20% 30% 40% 50% 60%

8. Is the probability that Patty will answer all of the questions on her chemistry exam correctly greater than 50%?
(1) For each question on the chemistry exam, Patty has a 90% chance of answering the question correctly.
(2) There are fewer than 10 questions on Patty's chemistry exam.

9. There are 10 women and 3 men in room A. One person is picked at random from room A and moved to room B, where there are already 3 women and 5 men. If a single person is then to be picked from room B, what is the probability that a woman will be picked?
(A) 13/21 (B) 49/117 (C) 15/52 (D) 5/18 (E) 40/117

10. A telephone number contains 10 digits, including a 3-digit area code. Bob remembers the area code and the next 5 digits of the number. He also remembers that the remaining digits are not 0, 1, 2, 5, or 7. If Bob tries to find the number by guessing the remaining digits at random, the find probability that he will be able to find the correct number in at most 2 attempts.
(A) 1/5 (B) 2/5 (C) 1/25 (D) 2/25 (E) 14/89

11. A certain jar contains only B black marbles, W white marbles, and R red marbles, if one marble is to be chosen at random from the jar, is the probability that the marble chosen will be red greater than the probability that marble chosen will be white?
(1) $R / (B + W) > W / (B + R)$ (2) $B - W > R$

12. Tanya prepared 4 different letters to be sent to 4 addresses. For each letter she prepared an envelope with its correct address. If the 4 letters to be put in to 4 envelopes at random, what is the probability that only one letter will be put in to the envelope with the correct address?
(A) 1/3 (B) 2/5 (C) 1/25 (D) 2/25 (E) 14/89

13. In a certain group of 10 members, 4 members teach only French and the rest teach only Spanish or German. If the group is to choose a 3-person committee, which must have at least one member who teaches French, how many different committees can be chosen?
40 50 64 80 100

14. How many times will the digit 7 be written when listing the integers from 1 to 1000?
(A) 280 (B) 300 (C) 320 (D) 340 (E) 360

15. A committee of three people is to be chosen from four married couples. What is the number of different committees that can be chosen if two people who are married to each other cannot both serve on the committee?

16 24 26 30 32

16. What is the sum of all possible 5-digit numbers that can be constructed using the digits 1, 2, 3, 4, and 5, if each digit can be used only once in each number? Choose the closest answer:
(A) 1 million
(B) 2 million
(C) 3 million
(D) 4 million
(E) 5 million

17. Mary and Joe are to throw three dice each. The score is the sum of points on all three dice. If Mary scores 10 in her attempt what is the probability that Joe will outscore Mary in his?
(A) 1/5
(B) 1/4
(C) 1/3
(D) 1/2
(E) 1/6

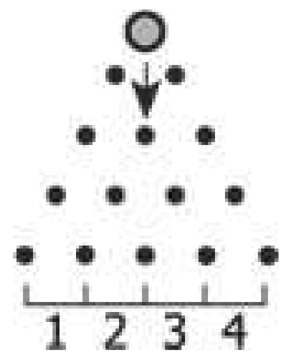
18. Each of the 25 balls in a certain box is red, blue or white and has a number from 1 to 10 painted on it. If one ball is to be selected at random from the box, what is the probability that the ball selected will either be white or have an even number painted on it?
(1) The probability that the ball will both be white and have an even number painted on it is 0.
(2) The probability that the ball will be white minus the probability that the ball will have an even number painted on it is 0.2.

19. If 2 different representatives are to be selected at random from a group of 10 employees and if p is the probability that both representatives selected will be women, is $p > ½$?
(1) More than ½ of the 10 employees are women.
(2) The probability that both representatives selected will be men is less than 1/10.

20. A certain stock exchange designates each stock with a one, two or three letter code, where each letter is selected from the 26 letters of the alphabet. If the letters may be repeated and if the same letters used in a different order constitute a different code, how many different stocks is it possible to uniquely designate with these codes?

a) 2,951 b) 8,125 c) 15,600 d) 16,302 e) 18,278

21. The figure shown represents a board with four rows of pegs, and at the bottom of the board are four cells numbered 1 to 4. Whenever the ball shown passes through the opening between two adjacent pegs in the same row, it will hit the peg directly beneath the opening. The ball then has probability 1/2 of passing through the opening immediately to the left of that peg and probability 1/2 of passing through the opening immediately to the right. What is the probability that when the ball passes through the first two pegs at the top it will end up in cell 2?



a) 1/16 b) 1/8 c) ¼ d) 3/8 e) ½

22. A certain office supply store stocks 2 sizes of self-stick notepads, each in 4 colors: Blue, Green, Yellow or Pink. The store packs the notepads in packages that contain either 3 notepads of the same size and the same color or 3 notepads of the same size and of 3 different colors. If the order in which the colors are packed is not considered, how many different packages of the types described above are possible?
A) 6 B) 8 C) 16 D) 24 E) 32

23. A certain junior class has 1000 students and a certain senior class has 800 students. Among these students there are 60 sibling pairs, each consisting of 1 junior and 1 senior. If 1 student is to be selected at random from each class, what is the probability that 2 students selected will be sibling pair?
1) 3/40,000 2)1/3,600 3)9/2,000 4)1/60 5)1/15

24. How many integers between 324,700 and 458,600 have tens digit 1 and units digit 3?
(A) 10,300 (B) 10,030 (C) 1,353 (D) 1,352 (E) 1,339

25. On his drive to work, Leo listens to one of 3 radio stations, A, B, or C. He first turns to A. If A is playing a song he likes, he listens to it; if not, he turns to B. If B is playing a song that he likes, he listens to it; if not, he turns to C. If C is playing a song he likes, he listens; if not, he turns off the radio. For each station, the probability is 0.3 that at any given moment the station is playing a song Leo likes. On his drive to work, what is the probability that Leo ill hear a song he likes?
a. 0.027 b. 0.09 c. 0.417 d. 0.657 e. 0.9

26. A company that ships boxes to a total of 12 distribution centers uses color coding to identify each center. If either a single color or a pair of two different colors is chosen to represent each center and if each center is uniquely represented by that choice of one or two colors, what is the minimum number of colors needed for the coding? (Assume that the order of the colors in a pair does not matter.)
(A) 4 (B) 5 (C) 6 (D) 12 (E) 24

27. A contest consists of n questions, each answered either True or False. Anyone who answers all n correctly will be a winner. What is the least value of n for which the probability is Less than 1/ 1000 that a person who randomly guesses the answer to each will be a winner?

a. 8 b. 9 c. 10 d. 11 e. 12

28. There are 8 magazines lying on a table; 4 are fashion magazines and the other 4 are sports magazines. If 3 magazines are to be selected at random from 8 magazines, what is the probability that at least one of the fashion magazines will be selected?
a) 1/2 b) 2/3 c) 32/35 d) 11/12 e) 13/14

29. If a 3-digit integer is selected at random from the integers 100 thru 199, inclusive, what is the probability that the first digit and the last digit of the integer are each equal to one more than the middle digit?
A) 2/225 B) 1/111 C) 1/110 D) 1/100 E) 1/50

30. All of the stocks on the over-the-counter market are designated by either a 4-letter or a 5-letter code that is created by using the 26 letters of the alphabet. Which of the following gives the maximum number of different stocks that can be designated with these codes?
A. $2(26^5)$ B. $26(26^4)$ C. $27(26^4)$ D. $26(26^5)$ E. $27(26^5)$

31. A certain restaurant offers 6 kinds of cheese and 2 kinds of fruit for its dessert platter. If each dessert platter contains an equal number of kinds of cheese and kinds of fruit, how many different dessert platters could the restaurant offer?
a. 8 b. 12 c. 15 d. 21 e. 27

32. Meg, Bob and John are among the 8 participants in a cycling race. If each participant finishes the race and no two participants finish at the same time, in how many different possible orders can the participants finish the race so that Meg finishes ahead of Bob and Bob finishes ahead of John? Choose the closest answer.
(A) 4000
(B) 5000
(C) 6000
(D) 7000
(E) 8000

33. A box contains 10 light bulbs, fewer than half of which are defective. Two bulbs are to be drawn simultaneously from the box. If n of the bulbs in box are defective, what is the value of n?
(1) The probability that the two bulbs to be drawn will be defective is 1/15.
(2) The probability that one of the bulbs to be drawn will be defective and the other will not be defective is 7/15.

34. The probability that a visitor at the mall buys a pack of candy is 30%. If three visitors come to the mall today, what is the probability that exactly two visitors will buy a pack of candy? Choose the closest answer.
(A) 0.1
(B) 0.2
(C) 0.3
(D) 0.4
(E) 0.5

35. If a code word is defined to be a sequence of different letters chosen from the 10 letters A, B, C, D, E, F, G, H, I, and J, what is the ratio of the number of 5-letter code words to the number of 4 - letter code words?
A. 5 to 4 B. 3 to 2 C. 2 to 1 D. 5 to 1 E. 6 to 1

36. If an integer n is to be chosen at random from the integers 1 to 96, inclusive, what is the probability that n (n + 1) (n + 2) will be divisible by 8?
A. ¼ B. 3/8 C. ½ D. 5/8 E. ¾

37. What is the probability that a student randomly selected from a class of 60 students will be a male who has brown hair?
(1) One-half of the students have brown hair. (2) One-third of the students are males.

38. If Event A and Event B are independent, is the probability that both Event A and Event B will happen greater than 0.3?
(1) Probability that A will happen is 0.25
(2) Probability that B will NOT happen is 0.71

39. A gardener is going to plant 2 identical red rosebushes and 2 identical white rosebushes. If the gardener is to select each of the bushes at random, one at a time, and plant them in a row, what is the probability that the 2 rosebushes in the middle of the row will be the red rosebushes?
A. 1/12 B. 1/6 C. 1/5 D. 1/3 E. ½

40. A company has assigned a distinct 3-digit code number to each of its 330 employees. Each code number was formed from the digits 2, 3, 4, 5, 6, 7, 8, 9 and no digit appears more than once in any one code number. How many unassigned code numbers are there?
A. 6 B. 58 C. 174 D. 182 E. 399

41. On Saturday morning, Malachi will begin a camping vacation and he will return home at the end of the first day on which it rains. If on the first three days of the vacation the probability of rain on each day is 0.2, what is the probability that Malachi will return home at the end of the day on the following Monday?
A. 0.008 B. 0.128 C. 0.488 D. 0.512 E. 0.640

42. How many 4-digit positive integers are there in which all 4 digits are even?
A. 625 B. 600 C. 500 D. 400 E. 256

43. A basket contains only red and green chips. If two chips are drawn from the basket at random without replacement, what is the probability that both chips will be green?
(1) 20% of all chips in the basket are green.
(2) The ratio of the number of red chips to the number of green chips is 4:1.

44. A string of 10 lightbulbs is wired in such a way that if any individual lightbulb fails, the entire string fails. If for each individual lightbulb the probability of failing during time period T id 0.06, what is the probability that the string of lightbulbs will fail during time period T?
A.0.06 B. $(0.06)^{10}$ C. $1-(0.06)^{10}$ D. $(0.94)^{10}$ E. $1-(0.94)^{10}$

45. Of the three-digit positive integers that have no digits equal to zero, how many have two digits that are equal to each other and the remaining digit different from the other two?
A. 24 B. 36 C. 72 D. 144 E. 216

46. A three-digit code for certain logs uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 according to the following constraints. The first digit cannot be 0 or 1, the second digit must be 0 or 1, and the second and third digits cannot both be 0 in the same code. How many different codes are possible?
A. 144 B. 152 C. 160 D. 168 E. 176

47. In a meeting of 3 representatives from each of 6 different companies, each person shook hands with every person not from his or her own company. If the representatives did not shake hands with people from their own company, how many handshakes took place?
A. 45 B. 135 C. 144 D. 270 E. 288

48. Nine family members: 5 grandchildren (3 brothers and 2 sisters) and their 4 grandparents are to be seated around a circular table. How many different seating arrangements are possible so that 2 sisters are seated immediately between some pair of brothers?
(A) 1000
(B) 1100
(C) 1200
(D) 1300
(E) 1440

49. At a birthday party, 10 students are to be seated around a circular table. What is the probability that two of the students, Anna and Bill, do NOT sit next to each other?
(A) 3/9
(B) 4/9
(C) 5/9
(D) 6/9
(E) 7/9

50. There are 5 pairs of white, 3 pairs of black and 2 pairs of grey socks in a drawer. If four individual socks are picked at random what is the probability of getting at least two socks of the same color?
(A) 1/5
(B) 1/4
(C) 1/3
(D) 1/2
(E) 1

51. In a set of numbers from 100 to 1000 inclusive, how many integers are odd and do not contain the digit "5"?
(A) 288
(B) 292
(C) 296
(D) 300
(E) 312

52. How many five-digit numbers can be formed using the digits 0, 1, 2, 3, 4 and 5 which are divisible by 3, without repeating the digits?
(A) 120
(B) 150
(C) 180
(D) 216
(E) 320

53. How many odd three-digit integers greater than 800 are there such that all their digits are different?
(A) 32
(B) 40
(C) 72
(D) 90
(E) 105

54. How many three-digit integers greater than 710 are there such that all their digits are different?
(A) 200
(B) 205
(C) 206
(D) 207
(E) 216

55. A password on Mr. Wallace's briefcase consists of 5 digits. What is the probability that the password contains exactly three digits as 6?
(A) <0.1
(B) 0.1
(C) 0.2
(D) 0.3
(E) 0.4

56. If $x^2+2x-15=-m$, where x is an integer from -10 and 10, inclusive, what is the probability that m is greater than zero?
(A) 1/6
(B) 1/5
(C) 1/4
(D) 1/3
(E) 1/2

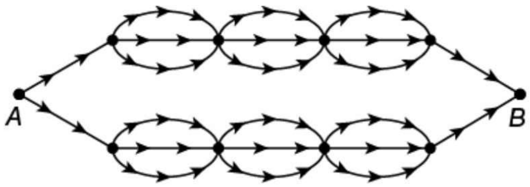
57. How many positive integers less than 10,000 are such that the product of their digits is 30?
(A) 10
(B) 20
(C) 30
(D) 40
(E) 50

58.

The letters C, I, R, C, L, and E can be used to form 6-letter strings such as CIRCLE or CCIRLE. Using these letters, how many different 6-letter strings can be formed in which the two occurrences of the letter C are separated by at least one other letter?

A. 96
B. 120
C. 144
D. 180
E. 240

59.



The map above shows the trails through a wilderness area. If travel is in the direction of the arrows, how many routes along the marked trails are possible from point *A* to point *B* ?

A. 11

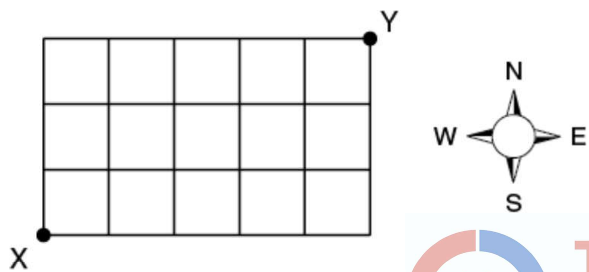
B. 18

C. 54

D. 108

E. 432

60.



In the figure above, X and Y represent locations in a district of a certain city where the streets form a rectangular grid. In traveling only north or east along the streets from X to Y, how many different paths are possible?

A. 720

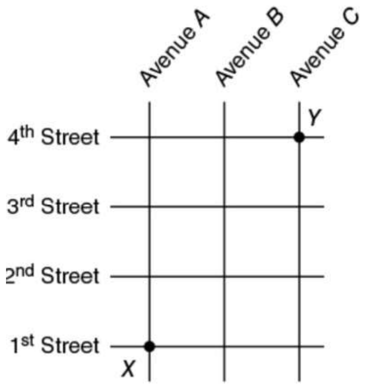
B. 512

C. 336

D. 256

E. 56

61.



Pat will walk from intersection *X* to intersection *Y* along a route that is confined to the square grid of four streets and three avenues shown in the map above. How many routes from *X* to *Y* can Pat take that have the minimum possible length?

A. Six

B. Eight

C. Ten

D. Fourteen

E. Sixteen

62.

Ben and Ann are among 7 contestants from which 4 semifinalists are to be selected. Of the different possible selections, how many contain neither Ben nor Ann?

A. 5

B. 6

C. 7

D. 14

E. 21

63.

Let *S* be a set of outcomes and let *A* and *B* be events with outcomes in *S*. Let $\sim B$ denote the set of all outcomes in *S* that are not in *B* and let $P(A)$ denote the probability that event *A* occurs. What is the value of $P(A)$?

1. $P(A \cup B) = 0.7$
2. $P(A \cup \sim B) = 0.9$

64.

A box of light bulbs contains exactly 3 light bulbs that are defective. What is the probability that a sample of light bulbs picked at random from this box will contain at least 1 defective light bulb?

1. The light bulbs in the sample will be picked 1 at a time without replacement.
2. The sample will contain exactly 20 light bulbs.

65.

If $n > 4$, what is the value of the integer n ?

1. $\dfrac{n!}{(n-3)!} = \dfrac{3!n!}{4!(n-4)!}$

2. $\dfrac{n!}{3!(n-3)!} + \dfrac{n!}{4!(n-4)!} = \dfrac{(n+1)!}{4!(n-3)!}$

Answer Key: Quant Session: Permutations and Combinations | Probability

1. E
2. C
3. B
4. B
5. B
6. D
7. C
8. E
9. B
10. D
11. A
12. A
13. E
14. B
15. E
16. D
17. D
18. E
19. E
20. E
21. D
22. C
23. A
24. E
25. D
26. B
27. C
28. E
29. D
30. C
31. E
32. D
33. D
34. B
35. B
36. D
37. E
38. D
39. B
40. A
41. B
42. C
43. E
44. E
45. E
46. B
47. B
48. E
49. E
50. E
51. A
52. D
53. C
54. D
55. A
56. D
57. E
58. E
59. C
60. E
61. C
62. A
63. C
64. E
65. A

Solutions – Permutations & Combinations, Probability

1. If a password contains at least 8 distinct digits, out of the 10 digits that are possible (0-9). More than 10 digits is not possible- as per question digits must be distinct. Therefore, total number of potential combinations that one could try are:

   1) Choose 8 out of the 10 possible digits and arrange them – resulting in ${}^{10}P_8$ arrangements.
   2) Choose 9 out of the possible 10 digits and arrange them – resulting in ${}^{10}P_9$ arrangements.
   3) Choose 10 out of the possible 10 digits and arrange them – resulting in ${}^{10}P_{10}$ arrangements.

   Therefore, the total number of potential combinations is a sum of the above

   three: $({}^{10}P_8 + {}^{10}P_9 + {}^{10}P_{10})$.

   Since time take for one combination is equal to 12 seconds, time taken to guarantee access to the database = total number of possible combinations * time taken for each combination = $({}^{10}P_8 + {}^{10}P_9 + {}^{10}P_{10})$ * 12 seconds

   Top 1% expert replies to student queries (can skip)

   The question is asking for passwords with distinct digits that has AT LEAST 8 digits: we can have 3 different scenarios: password with 8 digits or password with 9 digits or password with 10 digits. We can't have more, as we don't have more than 10 distinct digits.
   Total distinct digits in one password: Minimum 8 & Maximum 10.

   For 8 digits password, no. of distinct combinations possible= 8!*10C8= 10!/2!= 10!/2
   I.e. 10*9*8*7*6*5*4*3
   For 9 digits password, no. of distinct combinations possible= 9!*10C9= 10!/1!= 10!
   I.e. 10*9*8*7*6*5*4*3*2
   For 10 digits password, no. of distinct combinations possible= 10!*10C10= 10!/0!= 10!
   I.e.10*9*8*7*6*5*4*3*2*1
   Total possible passwords= 10!*0.5 + 10! + 10! = 10!*2.5

   Time required to try one password= 12sec= 1/5 min.

   Therefore, Time required to try 10!*2.5(=10! +10! + 10!/2) password= 10!*2.5/5 min= 10!/2 mins.

   Hence, **Ans E**

2. Project Manager ______- 1 slot
   Team Leader ______- 1 slot
   Consultants ________- 4 slots

   Number of ways in which a project manager can be chosen = ${}^{2}C_1$
   Number of ways in which a team leader can be chosen = ${}^{3}C_1$

Number of ways in which 2 of the consultants can be in the same team = number of ways in which a consultant can be chosen – Number of ways in which 2 consultants can always be together

Number of ways in which a consultant can be chosen = ${}^{7}C_{4}$
Number of ways in which 2 consultants can always be together:
4 consultant slots and 2 people always together.

If two people always have to be on the team, the remaining 2 people can be picked out of the 5 available people in ${}^{5}C_{2}$ ways.

Therefore, Number of ways in which 2 of the consultants can be in the same team = number of ways in which a consultant can be chosen – Number of ways in which consultants can always be together
$= {}^{7}C_{4} - {}^{5}C_{2}$.

Therefore, possible ways of picking the entire engagement team
$= {}^{2}C_{1} * {}^{3}C_{1} * ({}^{7}C_{4} - {}^{5}C_{2})$
= 150 possible teams.
Answer is C

Top 1% expert replies to student queries (can skip)
Those 2 consultants don't want to be in same team with whom with each other or with other 2 out of 5 consultants? -> With each other

The way to solve this question is as below:
Constraint: 2 consultants don't want to be on the same team
Answer= Total number combinations - Total number of combinations when those 2 consultants are on the team together

Total number of combinations = ${}^{2}C_{1} * {}^{3}C_{1} * {}^{7}C_{4} = 210$
Total number of combinations with constraints = ${}^{2}C_{1} * {}^{3}C_{1} * {}^{5}C_{2} = 60$
(${}^{5}C_{2}$ because those 2 consultants are already chosen. Now, we need to pick remaining 2 consultants out of 5 consultants)
Answer = 210-60 = 150
Answer is C

Alternate Solution from Top 1% expert replies and Gmatclub (additional)

a) No of ways to select 1 Manager = ${}^{2}C_{1} = 2$
b) No of ways to select 1 Team leader = ${}^{3}C_{1} = 3$
c) No of ways to select 4 Consultants = ${}^{7}C_{4} = 35$
Therefore, possible teams without any constraint = 2x3x35 = 210
No of ways to select 4 Consultants out of 7 when 2 of them are always together = ${}^{6}C_{4}$ x2! = 60
Therefore, possible teams with given constraint = 210 - 60 = 150
Answer is C

3. Number of flavours of pizza available = 4
Number of options available to the customer = extra cheese, extra mushrooms, both, neither (4 options).

Therefore, number of pizza varieties available = 4 * 4 = 16 varieties.

Answer is B

Top 1% expert replies to student queries (can skip)

Take the task of building a pizza and break it into stages:

Stage 1: Choose one of the flavors
There are 4 flavors of pizza (pepperoni, chicken, Hawaiian and vegetarian), so we can complete stage 1 in 4 ways

Stage 2: Choose whether to add extra cheese
We can either add extra cheese or not add extra cheese, so we can complete stage 2 in 2 ways.

Stage 3: Choose whether to add mushrooms
We can either add mushrooms or not mushrooms, so we can complete stage 3 in 2 ways.

By the Fundamental Counting Principle (FCP) we can complete all 3 stages (and thus build a pizza) in (4)*(2)*(2) ways (= 16 ways)

Answer is B

4. Let us list down all the possible outcomes where all three tails occur in a row:

   TTTHHH
   HTTTHH
   HHTTTH
   HHHTTT
   (Where T represents Tails and H represents Heads)
   Thus, there are only 4 possible ways in which this can happen.
   Answer is B

5. Possible digits that can be used = 2, 4 and 5.

   The product of the digits in the area code will be even every time 2 or 4 are chosen as a digit in the area code. The only time they will not be chosen is when all three digits of the area code are 5.

   The number of ways in which the product will be even = total number of possible combinations (with repetition) – 1 (the combination '555' which is the only combination with an odd product of digits)

   = (3*3*3) – 1 = 26 ways.
   Answer is B

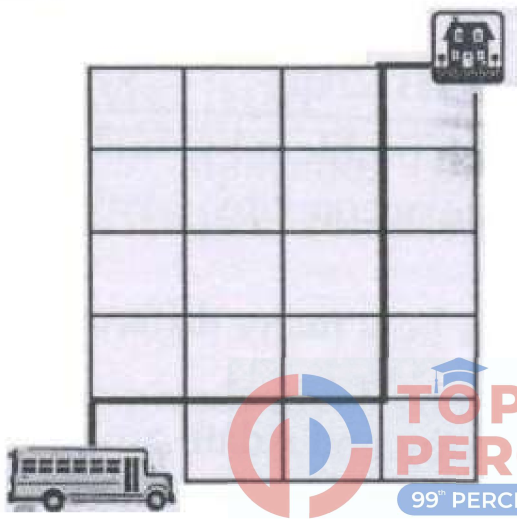
Top 1% expert replies to student queries (can skip)

Any product with just one even number in it is always even. Any product with two even numbers in it is also even. In this case, therefore, the ONLY case where the product will not be even is when the code is 555 (because one of 2 or 4 or both being in the product will make the product even)

As the digits can be repeated the three places can be filled up in 3 x 3 x 3 ways. As the product only needs to be even, we have to take out the one case of 555.

Then the total number of ways is 3x3x3 -1 = 27 -1 = 26
Answer is B

6. Whatever route Casey takes to the bus stop, she will cover a total of 9 paths – 4 horizontal and 5 vertical.



The 9 paths can be taken in 9! Ways.
But those nine paths will always consist of 4 horizontal and 5 vertical paths.
If the horizontal path is represented by H and the vertical path is represented by V,

We will basically have a combination of 4H's and 5 V's to get Casey from her house to the bus stop.

Therefore, total number of possible ways = 9!/4! * 5! = 126 different ways.
Answer is D

Top 1% expert replies to student queries (can skip) (additional)

To reach the bus stop, Casey will have to take 5 steps in the y-direction and 4 steps in the x-direction.

In other words, Casey has to take a total of 9 steps, 5 of which are of one kind and 4 are of another.

Number of ways = 9!/(5! * 4!)

[Number of ways of arranging N items, n1 of which are of the first kind, n2 are of the second kind, n3… so on and so forth = N! / (n1! * n2! * n3! ...)]
Answer is D

7. The first thing that we have to find conclusively is the number of sub-committees that include Michael.

If Michael must be on each of the three-person committees that we are considering, we are essentially choosing people to fill the two remaining spots of the committee. This can be done in $^5C_2$ ways.

If Michael and Anthony both have to be a part of the selected Sub-committee, we have one seat remaining and 4 board members vying for it. This seat can therefore be filled in $^4C_1$ ways.

Therefore, % of sub-committees that include Michael and Anthony = Number of sub-committees that have both Michael and Anthony/ Number of committee that have Michael
$= {^4C_1}/{^5C_2}$
$= 4/10$
$= 40$ %.

Hence, Option (C) is the right answer choice.

Top 1% expert replies to student queries (can skip)

Let's take the group with Michael: there is a place for two other members and one of them should be taken by Anthony

# of selections of 2 out of 5 = total # of outcomes.

Select Michael = $^1C_1$, Select Anthony = $^1C_1$, select any one member out of 4 = $^4C_1$

Winning outcome = $^1C_1 * {^1C_1} * {^4C_1}$

Required probability is winning outcome/total outcome= $({^1C_1} * {^1C_1} * {^4C_1}) / {^5C_2} = 4/10 = 2/5 = 40\%$. **C is correct**.

Alternate sol from gmatclub (additional)

First approach:

Let's take the group with Michael: there is a place for two other members and one of them should be taken by Anthony, as there are total of 5 people left, hence there is probability of $\frac{2}{5} = 40\%$.

Second approach:

Again in Michael's group 2 places are left, # of selections of 2 out of 5 is $C_5^2 = 10$ = total # of outcomes.

Select Anthony - $C_1^1 = 1$, select any third member out of 4 - $C_4^1 = 4$, total # $= C_1^1 * C_4^1 = 4$ - total # of winning outcomes.

$$P = \frac{\#\ of\ winning\ outcomes}{total\ \#\ of\ outcomes} = \frac{4}{10} = 40\%$$

Third approach:

Michael's group:

Select Anthony as a second member out of 5 - 1/5 and any other as a third one out of 4 left 4/4, total $= \frac{1}{5} * \frac{4}{4} = \frac{1}{5}$;

Select any member but Anthony as second member out of 5 - 4/5 and Anthony as a third out of 4 left 1/4, $total = \frac{4}{5} * \frac{1}{4} = \frac{1}{5}$;

$$Sum = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} = 40\%$$

Fourth approach:

Total # of splitting group of 6 into two groups of 3: $\frac{C_6^3 * C_3}{2!} = 10$;

# of groups with Michael and Anthony: $C_1^1 * C_1^1 * C_4^1 = 4$.

$$P = \frac{4}{10} = 40\%$$

Answer: C.

8. Let us say that there are n questions on the exam. Let us also say that p_1 is the probability that Patty will get the first problem right, and p_2 is the probability that Patty will get the second problem right, and so on until p_n, which is the probability of getting the last problem right. Then the probability that Patty will get all the questions right is just $p_1 \times p_2 \times \ldots \times p_n$. We are being asked whether $p_1 \times p_2 \times \ldots \times p_n$ is greater than 50%.

   Statement (1) INSUFFICIENT: This tells us that for each question, Patty has a 90% probability of answering correctly. However, without knowing the number of questions, we cannot determine the probability that Patty will get all the questions correct.

   Statement (2) INSUFFICIENT: This gives us some information about the number of questions on the exam but no information about the probability that Patty will answer any one question correctly.

   (1) AND (2) INSUFFICIENT: Taken together, the statements still do not provide a definitive "yes" or "no" answer to the question. For example, if there are only 2 questions on the exam, Patty's probability of answering all the questions correctly is equal to $.90 \times .90 = .81 = 81\%$. On the other hand if there are 7 questions on the exam, Patty's probability of answering all the questions correctly is equal to $.90 \times$

$.90 \times .90 \times .90 \times .90 \times .90 \times .90 \approx 48\%$. We cannot determine whether Patty's chance of getting a perfect score on the exam is greater than 50%.

The correct answer is E.

9. In order to solve this problem, we have to consider two different scenarios. In the first scenario, a woman is picked from room A and a woman is picked from room B. In the second scenario, a man is picked from room A and a woman is picked from room B.

   The probability that a woman is picked from room A is 10/13. If that woman is then added to room B, this means that there are 4 women and 5 men in room B (Originally there were 3 women and 5 men).
   So, the probability that a woman is picked from room B is 4/9.
   Because we are calculating the probability of picking a woman from room A AND then from room B, we need to multiply these two probabilities:
   10/13 x 4/9 = 40/117

   The probability that a man is picked from room A is 3/13. If that man is then added to room B, this means that there are 3 women and 6 men in room B.
   So, the probability that a woman is picked from room B is 3/9.
   Again, we multiply these two probabilities:
   3/13 x 3/9 = 9/117

   To find the total probability that a woman will be picked from room B, we need to take both scenarios into account. In other words, we need to consider the probability of picking a woman and a woman OR a man and a woman. In probabilities, OR means addition. If we add the two probabilities, we get:
   40/117 + 9/117 = 49/117
   The correct answer is B.

10. D
    The last two digits of the telephone number can be one of the following: 3,4,6,8 and 9.
    Total number of possible combinations of the digits = 25 (33,34,36,38,39,43, 44… etc)

    Probability of Bob getting the last two digits right in at most two attempts =
    Probability of Bob getting it right in the first attempt + Probability of Bob getting it right in the second attempt
    = 1/25 + (probability of Bob getting it wrong in the first attempt * probability of Bob getting it right the second time)
    = 1/25 + (24/25 * 1/24)
    = 1/25 +1/25
    = 2/25.

    Top 1% expert replies to student queries (can skip) (additional)

    How many total phone numbers can Bob form under these circumstances?

    He knows 8 digits, and the 9th and 10th digits cannot be 0, 1, 2, 5, or 7. So the 9th and 10th digits can be 3, 4, 6, 8, or 9 (5 digits total). Also, the digits can repeat.

So he can form a total of 5 x 5 = 25 numbers
[5 digits for the 9th place, 5 digits for the 10th place]

Out of these, only one is the correct number.

Bob has to find this correct number in at most 2 attempts - so he got it right in the first attempt OR in the second attempt (these two cases will be added because of the OR condition)

Probability Bob got it right in the first attempt = 1/25

Probability Bob got it right in the second attempt = Probability he got it wrong in the first attempt AND Probability he got it right in the second attempt

Probability of getting the number wrong in the first attempt = 24/25

Now understand that 1 number has been tried for the first attempt and found to be incorrect. Then there are 24 possible numbers remaining, out of which 1 is correct

Probability of getting it right in the second attempt = 1/24

Then probability of Bob getting the number wrong in the first attempt AND him getting it right in the second attempt = 24/25 x 1/24 = 1/25

Total required probability = 1/25 + 1/25 = 2/25

11. P(r)=r/(r+b+w)
P(w)=w/(r+b+w)

simplifying the question stem,
Q. P(r)>P(w)?
=>Is r>w?

1) r/(b + w) > w/(b + r)
=>r/(b + w)-w/(b + r)>0
=>r(b+r)-w(b+w)/{(b+w)(b+r)}>0
=>rb+r^2-wb-w^2>0
=>b(r-w)+r^2-w^2>0
=>b(r-w)+(r+w)(r-w)>0
=>(r-w)(b+r+w)>0
=>(r-w)>0 => r>w sufficient

2) b-w > r
From this statement we can't say whether r>w .
{When, b= 100, w =10, r= 90 – in this case r>w…
B = 100 w = 90, r = 10 – in this case r <w...}

Hence, not sufficient

Option (A) is therefore the right answer choice.

Top 1% expert replies to student queries (can skip)

You don't have to assume values to prove that statement 1 is sufficient. You can simply solve the inequality given.

we are that R/(B+W) > W/(B+R)

If we multiply both sides with (B+R)*(B+W), we would get :   R * (B+R) > W * (B+W)

This can also be written as: BR + R^2 > WB + W^2

Further, this can be written as BR - WB + (R^2 - W^2) > 0 (We have simple rearranged the terms here)

This can further be factored into :   B*(R-W) + (R-W)*(R+W) > 0

Taking (R-W) common in both the terms. we get

(R-W) * (B + R + W) > 0 ------Equation 1

Once you arrive at this inequality, it becomes very easy to check for statement 1.

We know that B + R + W > 0 (since this expression is the total number of balls in the jar)

For equation 1 to be true then, we must have R-W > 0.

So we know that R>W.

P (Red marble will be chosen) = R/(B+R+W)

P (White marble will be chosen) = W/(B+R+W)

For P(Red) > P(White), we must have R>W. Which we have proven using the given inequality.

So, Statement 1 is sufficient.

Option (A) is therefore the right answer choice.

12. A

Let R denote the letter in the right envelope and W denote the letter in the wrong envelope. We are trying to find the probability of 1R3W.

Probability = number of ways to get 1R3W/number of ways total

number of ways total is 4! = 24. Imagine stuffing envelopes randomly. Stacy can put any of 4 letters into the first envelope, any of the remaining 3 into the next, either of the remaining 2 into the next, and has no choice to make on the last, or 4*3*2*1.

number of ways to get 1R3W: She could fill the first envelope with the right letter (1 way), then put either of the 2 wrong remaining letters in the next (2 ways), then put a wrong letter in the next (1 way). That's 1*2*1*1 = 2.

But since it doesn't have to be the first envelope that has the Right letter, it could be any of the 4 envelopes (i.e., we could have RWWW, WRWW, WWRW, WWWR), the total ways to get 1R3W is 4*2 = 8.

Probability is 8/24 = 1/3.

Alternate Solution from Gmatclub

Total # of ways of assigning 4 letters to 4 envelopes is $4! = 24$.

Only one letter in the right envelope: 4(# of envelopes)*2(# of ways possible to arrange 3 letters incorrectly in the envelopes, when one is correct).

ABCD(envelopes)
ACDB(letters)
ADBC(letters)
(When A is in the right envelope other three have only 2 possible incorrect arrangements)
As we have 4 letters, total # of ways 4*2=8

$P(C=1) = \frac{8}{24} = \frac{1}{3}$

13. Total number of ways in which a committee of 3 can be picked from 10 people = 10C3 = 120

    Since there are 6 non-French teachers, we can calculate the probability that three non- French teachers will be selected in a row = (6/10) x (5/9) x (4/8) = 1/6.

    So, 1/6 of the 120 arrangements - or 20 - contain no French teachers. If 20 don't, then the other 100 arrangements do contain French teachers.

    Option (E) is the right answer choice.

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There are 3 cases:

Consider the following: All cases:
Case 1
1french 2either German or Spanish: 4C1 * 6C2 = 4 * 15 = 60
Case 2
2french 1either German or Spanish: 4C2 * 6C1 = 6 * 6 = 36
Case 3
3french: 4C3 = 4
Answer is E = 60+36+4= 100
Option (E) is the right answer choice.

Top 1% expert replies to student queries (can skip)

Or use the alternative approach: (all - none) = at least one:
Without any restrictions, the number of ways to choose 3 people from 10 is 10C3 = (10 x 9 x 8) / (3 x 2) = 720/6 = 120.
Let's assume a committee can be picked without any member who teaches French; then there are 6C3 = (6 x 5 x 4) / (3 x 2) = 120/6 = 20 ways.
So there are a total of 120 different committees (if there are no restrictions) and 20 of them consist of no members who can teach French.
Therefore, there must be 120 - 20 = 100 different committees with at least one member who teaches French.

Top 1% expert replies to student queries (can skip) (additional)

Let the 4 French teachers be F1, F2, F3 and F4.

Case 1:

You're choosing 1 French teacher (4C1). Let us say that we choose F1.
Now, you choose 1 teacher from 9 teachers. Say you choose F2
Now, you choose 1 teacher from 8 teachers. Say you choose F3

So, you have a committee comprising F1, F2 and F3

Case 2:

You're choosing 1 French teacher (4C1). Let us say that we choose F2.
Now, you choose 1 teacher from 9 teachers. Say you choose F1

Now, you choose 1 teacher from 8 teachers. Say you choose F3

So, you again have a committee comprising F1, F2 and F3.

We're counting the two committees as separate committees. But these are the same committees and should only be counted once. Therefore, this method is inefficient and will lead to double-counting.

This is what you should do. We want the number of committees with at least 1 French teacher.

Number of committees with at least 1 French teacher = Total number of committees possible - Number of committees with no French Teacher.

Total number of committees possible = 10C3 = 120
Number of committees with no French Teacher = Number of committees formed using the other 6 teachers = 6C3 = 20

Therefore,

Number of committees with at least 1 French teacher = 120 - 20 = 100

14.

Consider numbers from 0 to 999 written as follows:

1. 000

2. 001

3. 002

4. 003

...

1000. 999

We have 1000 numbers. We used 3 digits per number, hence used total of $3*1000=3000$ digits.
Now, why should ANY digit have preferences over another?
We use each of 10 digits equal # of times, thus we used each digit (including 7) $3000/10=300$ times.

Answer B

15. Choose 3 people from 8 people (where order does not matter)

8C3 = (8 * 7 * 6 * 5!)/(3!)*(5!) = 56

56 includes all combinations including those in which husband and wife are on the committee.

Since we have 4 married couples we can have each of them serve on a committee and third place could be filled by one of the other 6 remaining people. For example, if A1 and A2 are husband and wife, the arrangement would look like:

A1 A2 (3rd slot - any of the other six)

Similarly,

B1 B2 (3rd slot - any of the other six, which in this case includes A1 and A2)

Therefore, we have 4 possibilities (for husband and wife combo) * 6 of the remaining 8

=> 4*6 = 24 combinations have husband-wife on committee.

Therefore, 56 - 24 = 32 will satisfy the requirement that no husband-wife combo is together on a committee.

Top 1% expert replies to student queries (can skip)

1st place -8 options
2nd place- 6 options as the husband or wife of 1st can't be selected
3rd place- 4 options as excluding the husband or wife of 1st and 2nd place
total= 8*6*4
Total= 8*6*4:- this is partially correct.
So the total no. of ways we can choose the people will be 8*6*4 ways (= will contain duplication and to get rid of them you should divide this number by the factorial of the # of people - 3!)
Since order is not important (i.e A,B,C is the same as B,A,C) so we divide the total ways by 3!.
Hence, it becomes be {8*6*4 / 3!} =32.

Top 1% expert replies to student queries (can skip) (additional)

8*6*4=192 will contain duplication and to get rid of them you should divide this number by the factorial of the # of people - 3! --> 192/3!=32.
Consider this: there are two couples and we want to choose 2 people not married to each other.
Couples: A1, A2 and B1, B2. Committees possible:
A1,B1;
A1,B2;
A2,B1;
A2,B2.
Only 4 such committees are possible.

If we do the way you are doing we'll get: 4*2=8. And to get the right answer we should divide 8 by 2! --> 8/2! = 4.

Explanation:
Each couple can send only one "representative" to the committee. Let's see in how many ways we can choose 3 couples (as there should be 3 members) out of 4 to send only one "representative" to the committee: 4C3=4.
But each of these 3 couples can send two persons (husband or wife): 2*2*2=2^3=8.
Total # of ways: 4C3*2^3=32.
Answer: E.
Or logically:
Since there are 4 couples, we have 8 people involved.
The First person can be selected from the 8 people in 8 ways
The second person should not be a spouse of the first and hence we have 6 ways to choose him/her
The Third person should not be a spouse of either of the 2, so we can choose him in 4 ways.
So the total no. of ways we can choose the people will be 8*6*4 ways.
However since order is not important (i.e A,B,C is the same as B,A,C) so we divide the total ways by 3!
For Example: If we select M1 F2 F3
It may be the case when we select
F3 first, then F2 and then M1
F3 F2 M1

Both are the same; the order is different but it won't matter as we are talking about the number of committees (or groups) that can be formed.

So, the correct answer will come when you exclude all the extra cases.
Since you counted each case 6 times.
(M1 F2 F3 can be arranged in 3! Ways)
So, divide your answer by 3! Or 6.

Therefore,
(8*6*4) / 3! =32
Hence the total number of groups is 32.
Answer E

16. S = n/2 (a + l).
= (5!/2) (12345+54321)

Top 1% expert replies to student queries (can skip)

This type of question does not follow directly the Arithmetic properties but indirectly we can apply, i.e. The difference of each side of the AP sequence has the same difference from the mean. e.g. 123, 132, 213, 231, 312, 321 has the mean as 222. Now if you find the difference between each term with respect to mean (123, 132, 213, 222, 231, 312, 321), the first and last must have the same difference.
Second last of each side must have the same difference.
The same pattern follows here also. We need to take difference with respect to mean, i.e. 222-123 = 99, 321-222 = 99, 222-132 = 90, 312-222 = 90, So, the difference between first and last will be the same, difference between second and second last will

be the same and so on. WE CAN APPLY THE SAME PROPERTY TO N NUMBER OF DIGITS. So, we can use the formula to find the sum of the A.P. sequence.

Sum = n/2 (a+l)
Now, n = Total possible no. of numbers = 5p5 = n! = 5!
a= first number, i.e. smallest one = 12345
l= last number, i.e. largest one = 54321,

= (5!/2) (12345+54321)
= 5!/2 * 66666
= 3,99,960
Ans. D.

17. To outscore Mary, Joe has to score in the range of 11-18. The probability to score 3 is the same as the probability to score 18 (1-1-1 combination against 6-6-6, if 1-1-1 is on the tops of the dice the 6-6-6 is on the bottoms). By the same logic, the probability to score x is the same as the probability to score 21−x. Therefore, the probability to score in the range 11-18 equals the probability to score in the range of 3-10. As 3-18 covers all possible outcomes the probability to score in the range 11-18
is 1/2 or 32/64.
Answer D

Top 1% expert replies to student queries (can skip) (additional)

The minimum sum possible = 3
The maximum sum possible = 18

For sum = 3, how many possibilities do we have? (1,1,1) - Only 1
For sum = 18, how many possibilities do we have? (6,6,6) - Only 1

For sum = 4, how many possibilities do we have? (1,1,2) , (1,2,1) , (2,1,1) - 3
For sum = 17, how many possibilities do we have? (6,6,5) , (6,5,6), (5,6,6) - 3

For sum = 5, how many possibilities do we have? (1,2,2) , (2,2,1) , (2,1,2),(3,1,1) , (1,3,1) ,(1, 1, 3) - 6
For sum = 16, how many possibilities do we have? By symmetry, we have only 6 possibilities.

As can be seen from the above analysis, the number of possibilities of getting a number (x) is the same as the number of possibilities of getting the number (21-x)

So

P(sum = 3) = P(sum = 18)
P(sum = 4) = P(sum = 17)
P(sum = 5) = P(sum = 16)
.....
......
P(sum = 10) = P(sum = 11)

Adding all these equations, we get :

p(1) + p(2) + p(3) + ... + p(10) = p(11) + p(12) + ...p(18)

p(sum is less than or equal to 10) = p(sum > 10)

But we know that :

p(sum is less than or equal to 10) + p(sum > 10) = 1 [Total probability = 1]

So p(sum > 10) = ½
Answer D

Top 1% expert replies to student queries (can skip)

When 3 dice are rolled, the minimum one can get is 3 (1, 1, 1) and the maximum one can get is 18 (6, 6, 6).
The total Score one can get can range from 3 to 18.
One can score 3 by getting 1 rolling first dice, by getting 1 rolling second dice, and by getting 1 rolling third dice.
One can score 4 by getting 1, 1 and 2 (in any order).
One can score 5 by getting 1, 2 and 2 (in any order).
.
.
.
.
One can score 17 by getting 5, 6 and 6 (in any order).
And finally18 by getting 6, 6 and 6.

Mary scores a 10. To outscore Mary, Joe cannot score 3,4,5,6,7,8,9,10.
To outscore Mary, Joe has to get a score between 11 and 18.
Hence, Desired outcome is 8 (i.e.11, 12, 13, 14, 15, 16, 17 and 18)
Total outcome is 16 (i.e 3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18)
Probability to outscore Mary = Desired outcome/ Total outcome = 8/16 = ½

Hence Ans D is correct.

Top 1% expert replies to student queries (can skip)

The entire solution is mentioned below:
The expected value of one die is 1/6*(1+2+3+4+5+6)=3.5.
The Expected value of three dice is 3*3.5=10.5.
Mary scored 10 so the probability to get the sum more than 10 (11, 12, 13, ..., 18), or more than the average, is the same as to get the sum less than average (10, 9, 8, ..., 3) = 1/2 = 32/64.
That's because the probability distribution is symmetrical for this case:
The probability of getting the sum of 3 (min possible sum) = the probability of getting the sum of 18 (max possible sum);
The probability of getting the sum of 4 = the probability of getting the sum of 17;
The probability of getting the sum of 5 = the probability of getting the sum of 16;
...
The probability of getting the sum of 10 = the probability of getting the sum of 11;
Thus the probability of getting the sum from 3 to 10 = the probability of getting the sum from

11 to 18 = 1/2.
Answer: D.

Top 1% expert replies to student queries (can skip)

Expected value of one die is 1/6*(1+2+3+4+5+6)=3.5.
Expected value of three dice is 3*3.5=10.5.

Mary scored 10 so the probability to get the sum more then 10 (11, 12, 13, ..., 18), or more then the average, is the same as to get the sum less than average (10, 9, 8, ..., 3) = 1/2 = 32/64.

That's because the probability distribution is symmetrical for this case:
The probability of getting the sum of 3 (min possible sum) = the probability of getting the sum of 18 (max possible sum);
The probability of getting the sum of 4 = the probability of getting the sum of 17;
The probability of getting the sum of 5 = the probability of getting the sum of 16;
...
The probability of getting the sum of 10 = the probability of getting the sum of 11;

Thus the probability of getting the sum from 3 to 10 = the probability of getting the sum from 11 to 18 = 1/2.
Answer D.

18. 25 balls
each one is red, white, or blue
each one has a number from 1 to 10

Requirement: white OR even (note that we DON'T want white AND even - we have to be able to strip out those that fall into both categories). So our equation will be: probability of white + probability of even - probability of white & even

Statement (1):
Translated, this means there aren't any that are both white and even. This doesn't tell us how many are white or how many are even. Hence Insufficient.

Statement (2):
Pwhite - Peven = 0.2. So, Pwhite could be 0.4 which would make Peven 0.2. Or Pwhite could be 0.3 which would make Peven 0.1. And (by itself) it doesn't tell us Prob of even & white, which I'd need to subtract, so... insufficient in many ways.

Combining statement (1) AND statement (2):
Now we know that Peven+white = 0. BUT, we still have multiple possibilities for Pwhite and Peven (see above). 0.4+0.2-0=0.6. 0.3+0.1-0=0.4. ?? Still insufficient.
Hence Option (E) is the right answer choice in this case.

19. Let's first Rephrase the question:
What is the probability of selecting 2 women from a group of 10. Let's assume there are n women.

P(2w) = ${}^{n}C_2$ / ${}^{10}C_2$ = n*(n-1) / 10*9

Question is asking is P(2W) >1/2
or n*n-1 / 90 >1/2
or n* n-1 > 45

Now what value of 'n' could satisfy the above equation, when we know
n<=10 n=10 10*9 = 90 > 45
n=9 9*8 = 72 > 45
n=8 8*7 = 56 > 45
n=7 7*6 =42 which is not greater than 45
So we know for the probability of selecting two women to be more than 1/2, we need n>=8 women in the group of 10 people.

Statement (1):
Number of women > 5 ,
doesn't tell us whether >=8 . Hence Insufficient.

Statement (2):
P(2m) <1/10
Going by above
method m*m-1 <9
m= 0 , 0 < 9
m= 1 , 0 <9
m=2 2*1 <9
m=3 3*2 = 6 <9
m=4 4*3 , which is greater than 9.
So we know that total men in the group are <=3 , which means women are >=7.
But it still doesn't confirm whether women are >=8. Hence Insufficient.

Now combining both the statements, we get:
we know m <=3 and w>=5
But still doesn't tell us that whether we have more than 8 women to have probability of selecting two women to be more than 1/2.

Option (E) is therefore the best answer choice.

Alternate Solution from Gmatclub

The probability of selecting 2 women out of 10 people is $\frac{w}{10} * \frac{w-1}{9}$.

The question asks whether $\frac{w}{10} * \frac{w-1}{9} > \frac{1}{2}$ --> is $w(w-1) > 45$? --> is $w > 7$?

(1) More than 1/2 of the 10 employees are women --> $w > 5$. Not sufficient.

(2) The probability that both representatives selected will be men is less than 1/10 --> $\frac{10-w}{10} * \frac{10-w-1}{9} < \frac{1}{10}$ --> $(10-w)(9-w) < 9$ --> $w > 6$. Not sufficient

(1)+(2) $w > 5$ and $w > 6$: w can be 7, answer NO or more than 7, answer YES. Not sufficient.

Answer E.

You can use Combinations, to solve as well:

C_w^2 the number of selections of 2 women out of w employees;

C_{10}^2 the total number of selections of 2 representatives out of 10 employees.

The question asks whether $\frac{C_w^2}{C_{10}^2} > \frac{1}{2}$ --> is $\frac{\frac{w(w-1)}{2}}{45} > \frac{1}{2}$? --> is $w(w-1) > 45$? --> is $w > 7$?

(1) More than 1/2 of the 10 employees are women --> $w > 5$, not sufficient.

(2) The probability that both representatives selected will be men is less than 1/10 --> $C_{(10-w)}^2$ # of selections of 2 men out of $10 - w = m$ employees --> $\frac{C_{(10-w)}^2}{C_{10}^2} < \frac{1}{10}$ --> $\frac{\frac{(10-w)(10-w-1)}{2}}{45} < \frac{1}{10}$ --> $(10-w)(9-w) < 9$ --> $w > 6$, not sufficient

(1)+(2) $w > 5$ and $w > 6$: w can be 7, answer NO or more than 7, answer YES. Not sufficient.

Answer E.

20. The key to this problem is to remember that repetition is allowed.
    A 1 letter code can be obtained in 26 different ways.
    A 2 letter code can be obtained in 26*26 ways
    A 3 letter code can be obtained in 26*26*26 ways.

    Number of codes possible to be generated with this code = 26 + 26*26 + 26*26*26 = 18278.
    Hence option (E) is the right answer choice.

Top 1% expert replies to student queries (can skip) (additional)

1 letter codes = 26
2 letter codes = 26^2
3 letter codes = 26^3

Total = 26 + 26^2 + 26^3

The problem we are faced now is how to get the answer quickly. Note that the units digit of 26+26^2+26^3 would be (6+6+6=18) 8. Only one answer choice has 8 as unit digit: E (18,278). So I believe, even not calculating 26+26^2+26^3, that answer is E.

21. The ball drops between the top two pegs and hits the peg in the middle of row 2. To figure out the probability for its final location, we should look at the possible routes it could travel from row 2. There are 8 possibilities, with L meaning the ball goes left, and R meaning it goes right:

    It could go LLL--this puts it into cell 1. It could go LLR--this puts it into cell 2. It could go LRL--this puts it into cell 2. It could go LRR--this puts it into cell 3. It could go RLL--this puts it into cell 2. It could go RLR--this puts it into cell 3. It could go RRL--this puts it into cell 3. Or, it could go RRR--this puts it into cell 4.

    There are 8 total possibilities, and 3 of them give us a result of cell 2. So the probability of cell 2 is 3/8.

22. Part I: Let's consider all the possibilities that will give us the same size (two possible ways) and the same color (4 possible ways).

    Since there are two sizes and 4 colors, we can make a possible number of 8 **DIFFERENT** packages. (Remember that different packages means unique packages and with the same colour, say green, we cannot consider GGG different from another GGG).
    Part II : Consider all the possibilities for a same size but three different color package. Since we have four colors to choose from, we can use the combination formula to find how many ways to choose 3 from 4 colors($^4C_3$). This will give us 4 options, but since we have two sizes, we have a total of 8 ways to package the notes in this category.

    Therefore the total number of packages available will be a sum of part I and part II = 8 + 8 = 16.
    Hence option (C) is the right answer choice.

    Top 1% expert replies to student queries (can skip)

    Notepads of the different colors = $^4C_3$=4 (we should choose 3 different colors out of 4) As we have two sizes then total for the different color=4*2=8.
    Notepads of the same color = 4 (we have 4 colors). As we have two sizes then total for the same color=4*2=8

    Total=8+8=16

    Answer: C.

23. This is an "AND" probability question because both individuals must be a part of the sibling pair for the winning outcome to occur.

Since this is an "AND" question, it will involve multiplication. Also keep in mind that 60 "sibling pairs" is really 120 people.

Probability of selecting one sibling pair from class of

juniors = 60 / 1,000

Probability of selecting a sibling pair from the senior class that is THE match to the one we selected from the junior class?
There is only one person that would be the match, so winning outcomes / total possibilities = 1/800
Therefore: Answer = 60/1000 * 1/800 = 60/800,000 = 6/80,000 = 3/40,000.
Hence option (A) is the right answer choice.

24. Upper Limit- 458600
Lower Limit - 324700

Diff - 133900

The case of the last two digits ending in ‘13’ will happen in exactly every hundredth integer. And the total pool of integers under consideration is a multiple of 100, so there won't be any pattern interrupts. Therefore, we can just divide the total number of integers by 100 and we will arrive at our answer.

So in this case, the right answer is 1339 integers that end in ‘13’.
Hence Option (E) is the right answer choice.

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13 will appear as last two digits once every 100.
Eg. From 1 to 100 -> 13
From 101 to 200 -> 113
And so on

We subtract 324700 from 458600 to count how many total numbers are present between both these figures. Once we get that there are 458699 - 324700 = 133900 numbers between both. We divide it by 100 because we know that 13 occurs as the last 2 digits once every 100. Thus how many integers occur with last 2 digits as 13 => 133900/100 = 1339

Hence Option (E) is the right answer choice.

25. Probability that Leo will hear a song that he likes = 1- probability that Leo will not hear a song that he likes.

    Individual probability of Leo hearing a song he likes = 0.3
    Individual probability of Leo not hearing a song that he likes = 0.7

    Therefore the probability that Leo will hear a song that he likes = 1 – 0.7*0.7*0.7 = 1-0.343= 0.657.

    Hence Option (D) is the right answer choice.

26. The best way to answer this question is to work backward from the options in hand.

    Option (A) : With four colors we can code a total of : 4 + ${}^4C_2$ DC's = 4 + 6 = 10 DC's. Not sufficient.

    Option (B) : With 5 colours we can code a total of : 5 + ${}^5C_2$= 5 + 10 = 15 DC's. Sufficient.

    There is no point proceeding to the next few options as the numbers will just get larger and we are only concerned with the minimum number of colors needed for coding. Hence Option (B) is the right answer choice.

27. C
    Let us rephrase the questions first:
    'what is the least value of n for which there is less than a 1/1000 chance of guessing n questions in a row correctly?'
    This should be the thought process:
    * there is a 1/2 chance of guessing each question correctly
    * each question is independent of the other questions, so the chance of guessing n questions correctly is (1/2)(1/2)(1/2)...(1/2), where there are n (1/2)'s
    * this is (1/2)^n, or 1/(2^n)
    so:
    1/2^n < 1/1000
    take reciprocals:
    2^n > 1000
    n $\geq$ 10 (because 2^10 = 1024)

28. Probability that at least one of the fashion magazines will be selected = 1- probability that only sports magazines are selected.

    Probability that only sports magazines are selected:
    Probability that the first magazine is a sports magazine =4/8
    Probability that the second magazine is a sports magazine
    = 3/7 * Probability that the third magazine is a sports magazine = 2/6
    Therefore, probability that all 3 magazines are sports magazines =
    4/8*3/7*2/6 = 1/14
    Hence, probability that at least one of the fashion magazines will be
    selected = 1- 1/14 = 13/14.

Option (E) is therefore the right answer choice.

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We can do by 1 - all sports = 13/14 This is correct.
OR
Total no. of ways = ${}^8C_3$= 56
Favourable outcomes = 1 fashion & 2 sports OR 2 fashion & 1 sports OR 3 fashion
= $({}^4C_1 * {}^4C_2)+({}^4C_2 * {}^4C_1)+({}^4C_3)$
= 24 + 24 + 4
= 52
Probability = 52/56 = 13/14
Option (E) is therefore the right answer choice

29. The question indicates that the first and last digit are equal to 1 more than the middle digit indicating that the first and last digits are equal.
Since the range is less than 199, the first digit cannot be greater than 1.
This implies that with 0 as the middle digit and 1 as the first and last digits, 101 is the only integer possible out of the 100 integers.
Hence probability = 1/100.
Hence option (D) is the right answer choice.

30. 4 letter possibilities = 26^4
5 letter possibilities = 26^5.
Adding them, we get,
$26^4 + 26^5 = (26^4) * (1+26) = 27*(26^4)$.
Option (C) is the right answer choice.

31. Equal kinds of cheese and fruits indicate that we can have platters with a maximum of 2 fruits since there are only two different kinds of fruits available.

Remember that we can even have a 1 fruit, 1 cheese option.
Hence:
2C-2F = ${}^6C_2 * {}^2C_2$=15*1=15
1C-1F=${}^6C_1 * {}^2C_1$=12
Therefore, total number of options available = 15 + 12 = 27.
Option (E) is the right answer choice.

Top 1% expert replies to student queries (can skip)

As dessert platter should contain equal number of kinds of cheese and fruits, desert can contain:
A. 2 kinds of cheese and 2 kinds of fruits --> 6C2*2C2=15*1=15
B. 1 kind of cheese and 1 kind of fruit --> 6C1*2C1=6*2=12
A+B=15+12=27
Answer: E.

32. Meg, Bob, and John can arrange themselves in 3! = 6 ways out of which only one way will give Meg > Bob > John.
So the total number of favourable ways = (1/6)*8!
Answer :D

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Number of ways of arranging the 8 participants = 8!

We need Meg > Bob > John.

Now, Meg, Bob and John can be arranged in 3! = 6 ways. Each combination is equally likely and has a probability of 1/6.

Therefore, number of ways of arranging the participants such that M > B > J = 8!/6

Answer: D

33.

Total number of ways of picking two bulbs out 10 bulbs = $^{10}C_2$
Total number of ways of picking 2 bulbs out of 'n' defective bulbs = $^{n}C_2$

Statement (1) : It is given that $^{n}C_2/^{10}C_2 = 1/15$ i.e $^{n}C_2 = 3$. Hence n = 3.
This statement is therefore sufficient.

Statement (2): Since the two bulbs are drawn simultaneously, probability that the first will be defective and the second will not be defective = $n(10-n)/^{10}C_2 = 7/15$
From the above, we get, n(10-n) = 21. Solving for n, we get n = 3. Hence this statement is sufficient.

Option (D) is therefore the right answer choice

34. The event when 2 out of 3 visitors Buy a pack of candy can occur in 3!/2!=3 ways: BBN, BNB, NBB (3!/2!=3 is basically the # of permutations of 3 letters out of which 2 B's are identical).
Now, each B has the probability of 0.3 and N has the probability of 1−0.3=0.7, so $P(B=2)=3!/2!*0.3^2*0.7=0.189$

The correct answer is B

Top 1% expert replies to student queries (can skip)

The point is to find number of ways favorable scenario to occur: in our case we are asked to find the probability of 2 out of 3 visitors to buy the candy. In such cases, 'order' has to be considered.

0.3*0.3*0.7 is equivalent to saying the first person picks candy and second person picks candy and third person doesn't pick candy.

However, this is a different case compared to
0.3*0.7*0.3 is equivalent to saying that the first person picks candy and second

person doesn't pick candy and third person picks candy.

which in turn is a different case compared to 0.7*0.3*0.3
Hence the answer is 3*(0.3*0.3*0.7)=0.189

The correct answer is B

35. The five letter code:

The first position can be occupied by any one of 10 letters, the second by any one of 9, the third by any one of 8 and so on…
Number of possible 5-letter code words = 10*9*8*7*6
Similarly, the number of possible 4-letter code words = 10*9*8*7
Ratio of 5-letter code words to 4-letter code words = 10*9*8*7*6/ 10*9*8*7

= 6/1. Hence option (E) is the right answer choice.

36. n(n + 1)(n + 2) is a product of 3 consecutive integers.

If n is even, n(n + 1)(n + 2), will be divisible by 8.
Even integers from 1-96 inclusive = (96-2)/ 2 = 94/2 = 47, 47+1 = 48.
Also if n is odd and 1 less than multiple 8, n(n + 1)(n + 2) will be divisible by 8, because this will have at least 1 multiple of 8.
Multiples of 8 from 1-96 = (96-8)/8 = 11, 11+1 = 12.
Total number of favourable outcomes = 48 + 12 = 60.
Total number of possible outcomes = 96.
Probability = Number of favourable outcomes / Number of possible outcomes
= 60/96
= 5/8.
Hence Option (D) is the right answer choice.

37. Statement (1):

Since we have no information about the number of students who are male, we cannot answer the question prompt. Hence this statement alone is insufficient.
Statement (2):
Since we have no information about the number of students who are brown haired, we cannot answer the question prompt. Hence this statement alone is insufficient.

Combining statements (1) and (2):
We have 20 males and 40 females. We have 30 students with brown hair.
So we could have 20 males and all of them brown haired (probability would be 1/3), or 20 males non brown haired (30 females brown haired) and then probability would be 0.

Hence the combination of the statements is insufficient as well.
Option (E) is therefore the right answer choice.

38. Notice that since events A and B are independent, then the probability that both occur, equals to the product of their individual probability, so $P(A \text{ and } B)=P(A)*P(B)$. Also notice that $0 \le P(A) \le 1$ and $0 \le P(B) \le 1$.

(1) Probability that A will happen is 0.25. Now, since $P(A)=0.25$,
then $P(A \text{ and } B)=P(A)*P(B) \le 0.25 < 0.3$.
(2) Probability that B will NOT happen is 0.71. The same here:
since $P(B)=1-0.71=0.29$
then $P(A \text{ and } B)=P(A)*P(B) \le 0.29 < 0.3$. Sufficient.

The correct answer is D

39. The number of ways to arrange the red bushes in the desired fashion is as follows :
W1R1R2W2, W2R1R2W1, W1R2R1W2, W2R2R1W1.
Total number of ways in which 4 bushes can be arranged = 4! = 24.
Hence, probability that of the event occurring = 4/ 24 = 1/6.
Option (B) is the right answer choice.

40. To make a code number, we have 8 choices for the first digit, 7 choices for the second digit, and 6 choices for the third digit (subtracting one each time, since we cannot use the same digit more than once), and therefore 8*7*6 = 336 code numbers are possible in total.
Since 330 code numbers have been used already, there are 336-330 = 6 unused code numbers.
Option (A) is the right answer choice.

41. Let's interpret the question in the right fashion :
The question is basically: "what's the probability that it will rain on Monday and not on first two days."
Probability that it will not rain on the first two days = 0.8*0.8
Probability that it will rain on Monday = 0.2.
Hence overall probability = 0.8*0.8*0.2 = 0.128.
Option (B) is the right answer choice.

42. For all four digits of the number to be even, we have to only consider the digits {0,2,4,6,8} in our calculations.
The first digit can be picked in 4 ways (we cannot consider 0).
The second digit can be picked in 5 ways
The third digit can be picked in 5 ways
The fourth digit can be picked in 5 ways.
Hence, total number of 4 digit positive integers = 4*5*5*5 = 500.
Option (C) is the right answer choice.

43.

(1) 20% of all chips in the basket are green. Therefore 80% of all chips in the basket are red, which means that the ratio of the number of red chips to the number of green chips is 4:1 (80:20). Now, if there are total of 5 chips in the basket (4 red + 1 green) then the probability that both chips will be green will be 0 (since there are NOT two chips in a basket) but if there are total of 10 chips in the basket (8 red + 2 green) then the probability that both chips will be green will be more than 0. Not sufficient.

(2) The ratio of the number of red chips to the number of green chips is 4:1. The same info as above. Not sufficient.

(1)+(2) Both statements tell the same thing, so we have no new info. Not sufficient.

The correct answer is E

44. E. Probability of one light bulb failing during time interval T = 0.06.
Hence, probability of not failing is 0.94. We have 10 light bulbs in the string.
Even if one light bulb fails, the entire string fails.
Hence in order for the string to be successful, all the light bulbs need to pass.
Lets find out the probability of not failing.
P(10 light bulbs do not fail) = $(0.94)^{10}$.
P(string of light bulbs failing) = 1-P(10 light bulbs not fail) = $1-(0.94)^{10}$

Alternate Solution from GMATCLUB

Aside: If P(bulb fails) = 0.06, then P(bulb doesn't fail) = 0.94

Okay, the entire string of lightbulbs will fail if 1 or more lightbulbs fail.
So, we want P(**at least** 1 lightbulb fails)

When it comes to probability questions involving "at least," it's best to try using the complement.
That is, P(Event A happening) = 1 - P(Event A not happening)
P(at least 1 lightbulb fails) = 1 - P(zero lightbulbs fail)

P(zero lightbulbs fail)
P(zero lightbulbs fail) = P(1st bulb doesn't fail AND 2nd bulb doesn't fail AND 3rd bulb doesn't fail AND . . . AND 9th bulb doesn't fail AND 10th bulb doesn't fail)
= P(1st bulb doesn't fail) x P(2nd bulb doesn't fail) x P(3rd bulb doesn't fail) x . . . x P(9th bulb doesn't fail) x P(10th bulb doesn't fail)
= (0.94) x (0.94) x (0.94) x . . . x(0.94) x (0.94)
= (0.96)^10

So, P(at least 1 lightbulb fails) = 1 - P(zero lightbulbs fail)
= 1 - (0.94)^10

Top 1% expert replies to student queries (can skip) (additional)

Query: Why can't we do 0.06^10 to get the probability of string of light bulb failing?
Reply: The string of bulbs fails if one light bulb fails.

P(bulb fail)=0.06

P(bulb doesn't fail)=0.94

P(atleast one fail)=1-P(none fails)=1-(0.94)^10

Please note that string fails even if one fails.

So, this logic does not stand.
p(10 light bulbs fail)=0.06^10
but not one...we are considering the failure of all the light bulbs.

We need to understand that if one fails, the entire string of bulb fails.

So failure can be of one bulb also.(failure can be of 2 bulbs also and so on)(so basically 1-p(all bulbs working)

45. We have to consider three cases here.
Two digits equal & 1 digit different:
Case I [ABB] 8*9*1 = 72.
Case II [BAB] 8*9*1 = 72.
Case II [BBA]. 8*1*9 = 72.
Total integers or permutations = 72+72+72 = 216
Answer E

Top 1% expert replies to student queries (can skip)

Assume: a,b,c is the digit and a,b,c not= 0
Thus a,b,c could be 1, 2, 3, 4, 5, 6, 7, 8, and 9
Count the possible way to get a three digit number abc.
a = b, and c must be different from a, b
Thus, there are 3 possible ways of digit arrangement: aac, aca, caa
The solution is (9*1*8)*3 as we are multiplying by 3 because there are # such cases:
Case I: aac
=> (digit 1st) x (digit 2nd) x (digit 3rd)
=> 9 x 1 x 8 {pick any number from group = 9 possible ways} x {pick number the same as the first pick = 1 way} x {pick any number from the rest = 8 possible ways}
= 9 x 1 x 8 = 72 possible ways
Case II: aca
=> same as case I you have 72 possible ways
Case III: caa
=> same as case I you have 72 possible ways
total of this set of number = 72 + 72 + 72 = 216
E) is the answer

46. Let the code be XYZ, X can take 8 values-2,3,4,5,6,7,8,9.
Y can take 2 values- 0,1.
Z can take 9 if y = 0 or if Y = 1 then 10 values, if y = 0 : we have 8*1*9 = 72 options; if y = 1 : we have 8*1*10 = 80 options;

Total = 72+80 = 152.
Hence option (B) is the right answer choice.

47. B

The easiest formula to remember for handshakes among n people is = nC2 = n(n-1)/2.
The logic for the formula is that n people will shake hands with n-1 people. (Because a person won't shake hands with himself, therefore n-1 is used).

So the total no. of handshakes is n(n-1). BUT we just double counted the handshakes, because we counted that person A shake hands with person B and ALSO counted person B shaking hands with A.

We have to correct for this double counting by dividing by 2.

Therefore the number of handshakes among n people = n(n-1)/2.

There are a total of 18 reps.

Total no. of handshakes among all reps (including own company) = (18 X 17)/2 = 153.
No. of handshakes among one company's own reps = (3 X 2)/2 = 3.
No. of handshakes among 6 company's own reps = 3 X 6 = 18.
Total no. of handshakes among all reps excluding own company's reps = 153-18 = 135

48. E

Consider two brothers and two sisters between them as one unit: {BSSB}.

So, now we have 6 units: {G}, {G}, {G}, {G}, {B}, and {BSSB}.

These 6 units can be arranged around a circular table in $(6-1)! = 5!$ ways.
Next, analyze {BSSB} unit:

We can choose 2 brothers out of 3 for the unit in ${}^{3}C_{2}=3$ ways;
These brothers, within the unit, can be arranged in 2! ways: {B1,S,S,B2} or {B2,S,S,B1}.
The sisters, within the unit, also can be arranged in 2! ways: {B,S1,S2,B}or {B,S2,S1,B}.
Therefore, the final answer is 5!*3*2*2=1440.

Top 1% expert replies to student queries (can skip)

Query : In this Question, is this case also possible?
B1 S1 G1 S2 B2 G2 G3 G4 B3 The question never says that only sisters need to be in between the brothers.
Reply : The question says between any 2 brothers, 2 sisters are to be seated. So these **sisters need to be together {BSSB}**

49.E

10 students around a circular table can be arranged in $(10-1)!=9!$ ways.
Now, consider Anna and Bill as one unit - {Anna, Bill}.
We will have 9 units to arrange: 8 students and {Anna, Bill}.
Those 9 units can be arranged around a circular table in $(9-1)!=8!$ways.
Anna and Bill within their unit can be arranged in two ways {Anna, Bill} or {Bill, Anna}.
Thus the number of ways to arrange 10 students around a circular table so that two ofthem, Ana and Bill, sit next to each other is $8!*2$.
Therefore, the number of ways to arrange 10 students around a circular table so that two of them, Ana and Bill, do NOT sit next to each other is $9!-8!*2$
The probability = favourable/total=$(9!-8!*2)/9! = 1-2/9=7/9$

50.E

No formula is needed to answer this question. The trick here is that we have only 3 different color socks but we pick 4 socks, which ensures that in ANY case we'll have at least one pair of the same color (if 3 socks we pick are of the different color, then the 4th sock must match with either of previously picked one). P=1.

51.A

Examine what digits these set members can contain:

(A) First digit (hundreds): 8 choices (1, 2, 3, 4, 6, 7, 8, 9 - cannot be 0 or 5)
(B) Second digit (tens): 9 choices (0, 1, 2, 3, 4, 6, 7, 8, 9 - cannot be 5)
(C) Last digit (units): 4 choices (1, 3, 7, 9 - cannot be 0, 2, 4, 5, 6, 8)
The answer is $8*9*4=32*9=288$.

Top 1% expert replies to student queries (can skip)

Let the number be= XYZ
Z= units digit
Y= tenths digit
X= Hundredths digit

For any 3 digit number to be odd, Unit digit must be odd
So Z can be filled with 1,3,7,9 (we are excluding digit 5) = 4 ways.
Y can be filled with 0,1,2,3,4,6,7,8,9 (we are excluding digit 5) = 9 ways
X can be filled with 1,2,3,4,6,7,8,9 (we are excluding digits 0 and 5 ..) = 8 ways
No of ways= 4*9*8= 288. option A is correct.

52.D

First step:

We should determine which 5 digits from given 6, would form the 5 digit number divisible by 3.

We have six digits: 0, 1, 2, 3, 4, 5. Their sum is 15.

For a number to be divisible by 3 the sum of the digits must be divisible by 3. As the sum of the six given numbers is 15 (divisible by 3) only 5 digits good to form our 5 digit number would be 15-0={1, 2, 3, 4, 5} and 15-3={0, 1, 2, 4, 5}. Meaning that no other 5 from given six will total the number divisible by 3.

Second step:
We have two set of numbers:

{1, 2, 3, 4, 5} and {0, 1, 2, 4, 5}. How many 5 digit numbers can be formed using these two sets:

{1, 2, 3, 4, 5} This set gives 5! numbers, as any combination of these digits would give us 5 digit number divisible by 3. 5!=120
{0, 1, 2, 4, 5}. Now, here we cannot use 0 as the first digit, otherwise number won't be any more 5 digit and become 4 digit. So, desired # would be total combinations 5!, minus combinations with 0 as the first digit (combination of 4) $5!-4!=96$.
Total 120+96=216

53. C

In the range 800 - 900:

1 choice for the first digit: 8;

5 choices for the third digit: 1, 3, 5, 7, 9 (since integer must be odd);

8 choices for the second digit: 10 digits - first digit - third digit = 8 digits.

$$1*5*8=40$$

In the range 900 - 999:
1 choice for the first digit: 9;

4 choices for the third digit: 1, 3, 5, 7 (9 is out as it's used as the first digit);

8 choices for the second digit: 10 digits - first digit - third digit = 8 digits.

$$1*4*8=32$$

Total: 40+32=72.

54.D

First find how many integers between 700 and 999 are such that all their digits are different.

We have: (3 options for the first digit)∗(9 options for the second digit)∗(8 options for the third digit)=216(3 options for the first digit)∗(9 options for the second digit)∗(8 options for the third digit)=216 numbers.
Among these 216 numbers, 9 (701, 702, 703, 704, 705, 706, 708, 709, 710) are not bigger than 710. The answer to the question is therefore $216-9=207$.

55.

Total # of 5-digit codes is 10^5, notice that it's not $9*10^4$, since in a code we can have zero as the first digit.
# of passwords with three digits 6 is $9*9*5C3=810$: each out of two other digits (not 6) has 9 choices, thus we have $9*9$ and 5C3 is ways to choose which 3 digits will be 6's out of 5 digits we have.
P=favourable/total=$810/10^5$.

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Number of ways in which 3 places are occupied by the digit 6 and the remaining 2 places are occupied by two different digits, say 1 and 2

Now, let us see what happens when we calculate 9*8*5!/(3!) = 1440 ways

-> 9*8 implies we are considering all the arrangements pertaining to two digits. As we took digits 1 and 2 as our examples, we happened to consider the

arrangements 1 2 and 2 1 separately

Now let us consider the arrangement 1 2 followed by three 6's. when you multiply this arrangement of 1 2 6 6 6 with 5!/(3!), you consider the cases including the ones in which 2 comes before 1 . For example 2 1 6 6 6 is also considered as 5!/(3!) includes all the possible arrangements.

But as explained earlier, since 9*8 considers the arrangement 2 1 as distinct from 1 2 , you therefore multiply the arrangement 2 1 6 6 6 with 5!/(3!) separately, resulting in the double counting since all the arrangements pertaining to three 6's and 1,2 are already counted.

Therefore you should divide the computation 9*8*5!/(3!) by 2! in order to avoid double counting.

So the answer is 720+ 90 (Number of ways in which two other digits are same i.e

9*1*5!/(3!*2!)) = 810

Shortest approach:
Total # of 5-digit codes is 10^5
# of passwords with three digits 6 = 9*9*5C3 = 810.
5C3 is the number of ways to choose which 3 digits will be 6's out of 5 digits we have. The remaining 2 places will be taken by non-sixes (9*9 combination).
P= $810/ 10^5$

Another approach:
We'll consider two cases:
case i: We have three 6's and two DIFFERENT digits (e.g., 66612)
case ii: We have three 6's and two IDENTICAL digits (e.g., 66677)
case i: We have three 6's and two DIFFERENT digits (e.g., 66612)
We already have three 6's. So, we must select 2 different digits from (0,1,2,3,4,5,7,8 and 9)
We can do this in 9C2 ways (=36 ways)
Now that we've selected our 5 digits, we must ARRANGE them, which means we can use the MISSISSIPPI rule.
We can arrange 3 identical digits and 2 different digits in 5!/3! ways = 20 ways
So, we can have three 6's and two DIFFERENT digits in (36)(20) ways (= 720 ways)
case ii: We have three 6's and two IDENTICAL digits (e.g., 66677)
We already have three 6's. So, we must select 1 digit (which we'll duplicate)
Since we're selecting 1 digit from (0,1,2,3,4,5,7,8,9) we can do so in 9 ways
Now that we've selected our 5 digits, we must ARRANGE them, which means we can use the MISSISSIPPI rule.
We can arrange 3 identical 6's and 2 other identical digits in 5!/3!2! ways = 10 ways
So, we can have three 6's and two IDENTICAL digits in (9)(10) ways (= 90 ways)
So, TOTAL number of ways to have three 6's = 720 + 90 = 810
Since there are 100,000 possible 5-digit codes, P(having exactly three 6's) = 810/100,000
Answer: A

Top 1% expert replies to student queries (can skip) (additional)

The password consists of 5 digits. We want the probability that the password contains exactly three digits as 6.

So first of all, we have to choose which three digits out of five will be 6. Number of ways of choosing three places out of five = 5C3 = 10

Now that we have our chosen three digits, what is the probability that all three digits are 6? (1/10)^3. Why? Because we have a total of 10 possibilities (0,1,2,3,4,5,6,7,8,9) for each of the three digits. Out of these 10 possibilities, we need only 1 favorable event, that is when each of the 3 digits is 6. So there is a 1/10 probability that one of these digits will be 6. So, the probability that all three digits will be 6 is 1/10 * 1/10 * 1/10 = 1/1000 [Independent events]

Now, for the other two places, we have 9 favourable possibilities and 10 total possibilities. Note that none of the remaining digits can be 6, since we need exactly three 6s, and we've already budgeted for them. The probability that a remaining digit will not be 6 = 9/10. So, the probability that both remaining digits will not be 6 = $(9/10)^2$ = 81/100

Therefore, probability that only three digits are 6 = 10 * 1/1000 * 81/100 = 810/100000
Answer: A

56.D

Re-arrange the given equation: $-x^2-2x+15=m$.
Given that x is an integer from -10 and 10, inclusive (21 values) we need to find the probability that $-x^2-2x+15$ is greater than zero, so the probability that $-x^2-2x+15>0$.
Factorize: $(x+5)(3-x)>0$. This equation holds true for $-5<x<3$.
Since x is an integer then it can take the following 7 values: -4, -3, -2, -1, 0, 1, and 2.
So, the probability is 7/21=1/3

57.E

30=2∗3∗5=6∗5 (only 2∗3 gives single digit number 6). So, we should count the number of positive integers less than 10,000 with the digits {2, 3, 5} and {5, 6} and any number of 1's with each set.

2-digit numbers:

{5, 6} - the number of combinations = 2: 56 or 65.

3-digit numbers:

{1, 5, 6} - the number of combinations = 3! = 6: 156, 165, 516, 561, 615, or 651.

{2, 3, 5} - the number of combinations = 3! = 6.

4-digit numbers:

{1, 1, 5, 6} - the number of combinations = 4!/2! = 12.

{1, 2, 3, 5} - the number of combinations = 4! = 24.

Total = 2 + 6 + 6 + 12 + 24 = 50.

58.CIRCLE
If there is no restriction, the number of ways = 6!/2! = 360
If the two Cs always come together, the number of ways (consider the two Cs one item) = 5! = 120
Required answer = 360 – 120 = 240. **Ans. E**

59.

To determine the number of routes that begin by going up from point A, we can apply the Multiplication Principle. There are 3 locations at which branches occur. Moreover, at each of these locations, there are 3 different trails that can be taken. Finally, the choices of which trail to take at each location can be made independently. Therefore, the Multiplication Principle applies and we get (3)(3)(3) = 27 for the number of routes that begin by going up from point A. Hence, the number of routes from point A to point B is 2(27) = 54.

The correct answer is C.

60.E

Each possible path will consist of traveling a total of 3 grid segments north and 5 grid segments east. Thus, letting 'N' represent traveling north by one grid segment and 'E' represent traveling east by one grid segment, each path can be uniquely represented by an appropriate 8-character string of N's and E's. For example, as shown in the figure below, NEENEENE represents grid segments traveled in the order north, east, east, north, east, east, north, and east.

Therefore, the number of possible paths is equal to the number of appropriate 8-character strings of N's and E's, which is $\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{(5!)(6)(7)(8)}{(2)(3)(5!)}$ = (7)(8) = 56, since each appropriate string is determined when a specification is made for the 3 positions in the string at which the N's are to be placed. Alternatively, the number of possible paths is equal to the number of permutations of 8 objects in which 3 are identical (the N's) and the remaining 5 are identical (the E's), and thus equal to $\frac{8!}{(3!)(5!)}$.

61.C

Each minimum-length route will consist of traveling a total of 3 grid segments up and 2 grid segments right. Thus, letting 'U' represent traveling up by one grid segment and 'R' represent traveling right by one grid segment, each minimum-length route can be uniquely represented by an appropriate 5-character string of U's and R's. For example, URUUR represents grid segments traveled in the order up, right, up, up, and right. Therefore, the number of possible minimum-length routes is equal to the number of appropriate 5-character strings of U's and R's, which is $\binom{5}{3} = \frac{5!}{3!(5-3)!}$ = 10, since each appropriate string is determined when a specification is made for the 3 positions in the string at which the U's are to be placed.

62.

The number of possible selections of 4 semifinalists that do not contain Ben or Ann is equal to the number of possible selections of 4 semifinalists from the remaining 7 – 2 = 5 contestants, which is equal to $\binom{5}{4} = \frac{5!}{4!(5-4)!}$ = 5. Alternatively, the number of possible selections of 4 semifinalists from the remaining 5 contestants is equal to the number of possible selections of exactly 1 non-semifinalist, which is equal to 5.

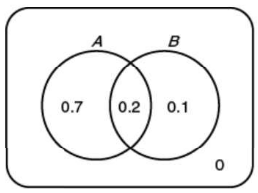
The correct answer is A.

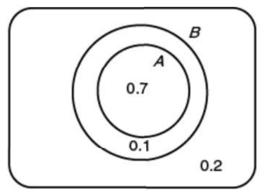
63.

The general addition rule for sets applied to probability gives the basic probability equation

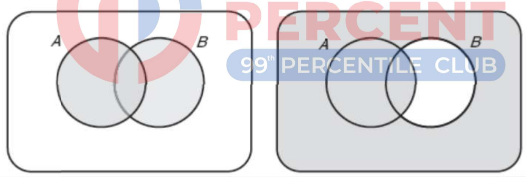
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

1. Given that $P(A \cup B) = 0.7$, it is not possible to determine the value of $P(A)$ because nothing is known about the relation of event A to event B. For example, if every outcome in event B is an outcome in event A, then $A \cup B = A$ and we have $P(A \cup B) = P(A) = 0.7$. However, if events A and B are mutually exclusive (i.e., $P(A \cap B) = 0$) and $P(B) = 0.2$, then the basic probability equation above becomes $0.7 = P(A) + 0.2 - 0$, and we have $P(A) = 0.5$; NOT sufficient.
2. Given that $P(A \cup \sim B) = 0.9$, it is not possible to determine the value of $P(A)$ because nothing is known about the relation of event A to event $\sim B$. For example, as indicated in the first figure below, if every outcome in event $\sim B$ is an outcome in event A, then $A \cup \sim B = A$ and we have $P(A \cup \sim B) = P(A) = 0.9$. However, as indicated in the second figure below, if events A and $\sim B$ are mutually exclusive (i.e., $P(A \cap \sim B) = 0$) and $P(\sim B) = 0.2$, then the basic probability equation above, with $\sim B$ in place of B, becomes $0.9 = P(A) + 0.2 - 0$, and we have $P(A) = 0.7$; NOT sufficient.





Given (1) and (2), if we can express event A as a union or intersection of events $A \cup B$ and $A \cup \sim B$, then the basic probability equation above can be used to determine the value of $P(A)$. The figure below shows Venn diagram representations of events $A \cup B$ and $A \cup \sim B$ by the shading of appropriate regions.



Inspection of the figure shows that the only portion shaded in both Venn diagrams is the region representing event A. Thus, A is equal to the intersection of $A \cup B$ and $A \cup \sim B$, and hence we can apply the basic probability equation with event $A \cup B$ in place of event A and event $A \cup \sim B$ in place of event B. That is, we can apply the equation

$P(C \cup D) = P(C) + P(D) - P(C \cap D)$

with $C = A \cup B$ and $D = A \cup \sim B$. We first note that $P(C) = 0.7$ from (1), $P(D) = 0.9$ from (2), and $P(C \cap D) = P(A)$. As for $P(C \cup D)$, inspection of the figure above shows that $C \cup D$ encompasses all possible outcomes, and thus $P(C \cup D) = 1$. Therefore, the equation above involving events C and D becomes $1 = 0.7 + 0.9 - P(A)$, and hence $P(A) = 0.6$.

The correct answer is C;

64.

It is clear that neither (1) alone nor (2) alone is sufficient.

Given (1) and (2), if the box contains 22 light bulbs, then a sample of 20 light bulbs must contain at least one defective light bulb, and hence the desired probability is equal to 1. However, if the box contains 22,000 light bulbs, then it is clear that the probability that a sample of 20 light bulbs contains at least one defective light bulb is less than 1.

The correct answer is E;

65.

1. Because the numerators of the two fractions have several common factors, and similarly for the denominators, a reasonable strategy is to begin by appropriately canceling these common factors.

$$
\begin{array}{rcll}
\dfrac{n!}{(n-3)!} & = & \dfrac{3!n!}{4!(n-4)!} & \text{given} \\[2ex]
\dfrac{1}{(n-3)!} & = & \dfrac{3!}{4!(n-4)!} & \text{divide both sides by } n! \\[2ex]
\dfrac{1}{(n-3)!} & = & \dfrac{1}{4(n-4)!} & 4! = 3! \times 4 \\[2ex]
4(n-4)! & = & (n-3)! & \text{cross-multiply} \\
4(n-4)! & = & (n-4)! \times (n-3) & \\
4 & = & n-3 & \text{divide both sides by } (n-4)! \\
n & = & 7 &
\end{array}
$$

Alternatively, we could begin by reducing each of the fractions to lowest terms by using identities such as $n! = (n-3)! \times (n-2)(n-1)(n)$, and then performing operations on the resulting equation; SUFFICIENT.

2. For the same reason given in (1) above, we begin by canceling factors that are common on the left and right sides of the equality.

$$
\begin{array}{rcll}
\dfrac{n!}{3!(n-3)!} + \dfrac{n!}{4!(n-4)!} & = & \dfrac{(n+1)!}{4!(n-3)!} & \text{given} \\[2ex]
\dfrac{1}{3!(n-3)!} + \dfrac{1}{4!(n-4)!} & = & \dfrac{n+1}{4!(n-3)!} & \text{divide both sides by } n! \\[2ex]
\dfrac{4}{(n-3)!} + \dfrac{1}{(n-4)!} & = & \dfrac{n+1}{(n-3)!} & \text{multiply both sides by } 4! = 3! \times 4 \\[2ex]
4 + (n-3) & = & n+1 & \text{multiply both sides by } (n-3)! =
\end{array}
$$

The manipulations above show that the original equation is identically true for all integers greater than 4, and thus n can be any integer greater than 4.

Alternatively, we could begin by reducing each of the fractions to lowest terms by using identities such as $n! = (n-3)! \times (n-2)(n-1)(n)$, and then performing operations on the resulting equation; NOT sufficient.

The correct answer is A;