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Quant Session: Inequalities + Mods (Absolute Values)

Inequalities Basics:

1. $a < b$
   Examples:
   - $2 < 3$
   - $0 < 3$
   - $-3 < 3$
   - $-3 < 0$

2. $a \leq b$
   Examples:
   - $2 \leq 3$
   - $3 \leq 3$
   - $0 \leq 3$
   - $-3 \leq -3$
   - $-3 \leq 0$

3. $a > b$
   Examples:
   - $3 > 2$
   - $3 > 0$
   - $3 > -3$
   - $0 > -3$

4. $a \geq b$
   Examples:
   - $3 \geq 2$
   - $3 \geq 3$
   - $3 \geq 0$
   - $-3 \geq -3$
   - $0 \geq -3$

5. So long as multiplication or division aren't involved, we can cancel or shift quantities just as we do in equations

    - For example, $x + y – 1 > x – y + 1$ means either we can transfer all terms from the RHS to the LHS and write $x + y – 1 – x + y – 1 > 0$ or $2y – 2 > 0$ or $2y > 2$ or $y > 1$ OR we can directly cancel x from both sides and write $y – 1 > –y + 1$ or $2y > 2$ or $y >1$.

6. If $xy > 0$, then x and y are of the same sign. Either both positive or both negative.

7. If $x/y > 0$, then x and y are of the same sign. Either both positive or both negative

    - So, $xy > 0$ means $x/y > 0$

Top 1% expert replies to student queries (can skip) (additional)

Query: Explain how we could divide LHS and RHS with y, when we do not know the sign of y?
Reply: We have $xy > 0$. So we know that x and y are of the same sign and that x and y are not 0.

If we divide this inequality by y^2, we get:

$xy/y^2 > 0$ [y^2 is positive and will not change the sign of the inequality]

$x/y > 0$

8. If $xy < 0$, then x and y are of the opposite sign. One positive and the other negative

9. If $x/y < 0$, then x and y are of the opposite sign. One positive and the other negative

    - So, $xy < 0$ means $x/y < 0$

10. If $a > b$, then $ax > bx$, if x is positive … this means the sign of the inequality doesn't change if we multiply both sides by a positive quantity

    - Similarly, if $ax > bx$, then $a > b$, if x is positive … this means the sign of the inequality doesn't change if we cancel a positive quantity from both sides

    Note: the same rule applies for division: $a > b$, then $a/x > b/x$ if x is positive and vice versa

11. If $a > b$, then $ax < bx$, if x is negative … this means the sign of the inequality changes if we multiply both sides by a negative quantity

    - Similarly, if $ax > bx$, then $a < b$, if x is negative … this means the sign of the inequality changes if we cancel a negative quantity from both sides

12. If $a/b > c/d$, then we can't just cross multiply to write $ad > bc$. Unless we know the sign of the quantities, we can't cross multiply.

    - On the other hand, if all a, b, c, and d are positive, then we can surely cross multiply and write $ad > bc$

13. The concept of number line is very useful in checking inequalities. The common values to check are $x = 0, 1, –1, >1$ (preferred value = 2), between 0 and 1 (preferred values = 1/2 and 0.9), between – 1 and 0 (preferred values = –1/2 and –0.9), and less than –1 (preferred value = –2). So, in short, there are 9 points: –2, –1, –0.9, –1/2, 0, 1/2, 0.9, 1, 2.

14. If $(x - a)(x - b) < 0$, then x lies between a and b. OR $a < x < b$.

- Here a is less than b.
- $(x - 3)(x - 5) < 0$, then x lies between 3 and 5
- $(x + 3)(x - 5) < 0$ we can write this as $[x - (-3)](x - 5) < 0$

  So, x lies between –3 and 5
- $(x + 5)(x + 3) < 0$ we can write this as $[x - (-5)][x - (-3)] < 0$

  So, x lies between –5 and –3

15. If $(x - a)(x - b) > 0$, then x lies outside a and b. OR $x < a$, $x > b$

- Here a is less than b.
- $(x - 3)(x - 5) > 0$, then x doesn't lie between 3 and 5. So either x is less than 3 or x is greater than 5.
- $(x + 3)(x - 5) > 0$ We can write this as $[x - (-3)](x - 5) > 0$

  So, x doesn't lie between –3 and 5. Either x is less than –3 or x is greater than 5.
- $(x + 5)(x + 3) > 0$ We can write this as $[x - (-5)][x - (-3)] > 0$

  So, x doesn't lie between –5 and –3. Either x is less than –5 or x is greater than –3.

16. If $x^2 > x$, then either $x > 1$ or x is negative $(x < 0)$.

17. If $x^2 < x$, then x lies between 0 and 1. $(0 < x < 1)$.

18. If $x^2 = x$, then $x = 0$ or $x = 1$.

19. If $x^3 > x$, then either $x > 1$ or x is between –1 and 0 (either $x > 1$ or $-1 < x < 0$).

Top 1% expert replies to student queries (can skip) (additional)

Proof for values of x calculated for inequality $x^3 > x$:

This can be simplified to: $x(x-1)(x+1) > 0$

Now, let us ignore the inequality sign for the moment.

What are the solutions to the equation $x(x-1)(x+1) = 0$? $x = -1, 0, 1$.

So we have 4 intervals to check.

First interval : $x < -1$
Second interval : $-1 < x < 0$
Third interval : $0 < x < 1$
Fourth interval : $x > 1$

For $x < -1$, $(x-1) < 0$, $x<0$ and $(x+1)<0$. So $x(x-1)(x+1) < 0$ [Product of three negative numbers will always be negative].
This cannot be a solution since we need $x(x-1)(x+1) > 0$

For $-1 < x < 0$, $(x-1) < 0$, $x<0$ and $(x+1)>0$. So $x(x-1)(x+1) > 0$ [Product of two negative numbers and one positive number will always be positive].
This can be a solution since we need $x(x-1)(x+1) > 0$

For $0 < x < 1$, $(x-1) < 0$, $x > 0$ and $(x+1)>0$. So $x(x-1)(x+1) < 0$ [Product of two positive numbers and one negative number will always be negative].
This cannot be a solution since we need $x(x-1)(x+1) > 0$

For $x > 1$, $(x-1) > 0$, $x> 0$ and $(x+1)>0$. So $x(x-1)(x+1) > 0$ [Product of three positive numbers will always be positive].
This can be a solution since we need $x(x-1)(x+1) > 0$

So our solution set is :($-1 < x < 0$) and ($x > 1$).

20. If $x^3 < x$, then either x lies between 0 and 1 or x is less than –1.
(Either $0 < x < 1$ or $x < -1$)

21. If $x^3 = x$, then $x = 0$ or $x = 1$ or $x = -1$.

22.

- If $1/x > 0$, then $x > 0$ Substitute x as –ve / 0 / +ve to verify.
- If $1/x < -x$, then x must be negative Substitute x as –ve / 0 / +ve to verify.

23. If $x^2 > y^2$, then $x > y$ and $x < y$ both results are possible, and x and y can be of the same sign and also of the opposite sign

- $5^2 > 3^2$ and $5 > 3$ $(-5)^2 > (-3)^2$ but $-5 < -3$ both same sign
- $5^2 > (-3)^2$ and $5 > -3$ $(-5)^2 > (3)^2$ but $-5 < 3$ opposite signs

24. If $x > y^2$, then $x > y$ and $x < y$ both results are possible

- $25 > 3^2$ and $25 > 3$ $1/3 > (1/2)^2$ but $1/3 < 1/2$

25. If $x > y^4$, then $x > y$ and $x < y$ both results are possible

- $100 > 3^4$ and $100 > 3$ $1/3 > (1/2)^4$ but $1/3 < 1/2$

26. If $x > y$, it is necessarily true that $x^3 > y^3$ or etc. So, odd powers and roots don't change sign.

27. Two inequalities with the same sign can be added just in the same way as two equations can be added

So, if

$a + b > c + d$
and
$e + f > g + h$

Then $a + b + e + f > c + d + g + h$

28. Two inequalities with different signs can be added after we change the sign of one of the inequalities by multiplying it by a negative sign.

So, if

$a + b > c + d$ and
$e + f < g + h$ **Check the less than sign**

Then we can write

$a + b > c + d$ and
$-(e + f) > -(g + h)$

Or

$a + b > c + d$ and
$-e - f > -g - h$

So, $a + b - e - f > c + d - g - h$

29. If X is positive, then
(1) $(a + X) / (b + X) > a/b$ if $a < b$
(2) $(a + X) / (b + X) < a/b$ if $a > b$

30. On the GMAT, any square number is always greater than or equal to 0. So $x^2 \geq 0$.
On the GMAT, a square number can't be negative.

Problems (Inequalities):

1. If x is not equal to –y, is $\frac{x-y}{x+y} > 1$?
   (1) x > 0 (2) y < 0

2. If x is positive which of the following could be correct ordering of 1/x, 2x, x^2?
   I. $x^2 < 2x < 1/x$ II. $x^2 < 1/x < 2x$ III. $2x < x^2 < 1/x$
   A. None B. I Only C. III Only D. I and II only E. I, II & III

3. If $x > y^2 > z^4$, which of the following statements could be true?
   I. x > y > z II. z > y > x III. x > z > y
   A. I only B. I and II only C. I and III only
   D. II and III only E. I, II, and III

4. Is M + Z > 0 (1) M – 3Z > 0 (2) 4Z – M > 0

5. If k is not equal to 0, 1, or –1, is 1/k > 0? (1) 1 / (k – 1) > 0 (2) 1 / (k + 1) > 0

6. The numbers x and y are not integers. The value of x is closest to which integer?
   (1) 4 is the integer that is closest to x + y (2) 1 is the integer that is closest to x – y

7. Are x and y both positive (1) 2x – 2y = 1 (2) x/y >1

8. If 500 is the multiple of 100 that is closest to X and 400 is the multiple of 100 closest to Y, then which multiple of 100 closest to X + Y? (1) X < 500 (2) Y < 400

9. Is $1 / p > r/(r^2 + 2)$ (1) p = r (2) r > 0

10. Is X + Y < 1? (1) X < 8 / 9 (2) Y < 1 / 8

11. Is x – y + 1 greater than x + y – 1? (1) x > 0 (2) y < 0

12. Is z the median of any 3 positive integers x, y and z? (1) x < y + z (2) y = z

13. On the number line, the distance between x and y is greater than the distance between x and z. Does z lie between x and y on the number line? (1) xyz < 0 (2) xy < 0

14. If $\sqrt{(x+4)^2} = 3$, which of the following could be the value of x – 4?
    –11 –7 –4 –3 5

15. If mv < pv < 0, is v > 0? (1) m < p (2) m < 0

16. Is $x < y$? (1) $x^2 > y$ (2) $\sqrt{x} < y$

17. If n is a nonzero integer, is $x^n < 1$? (1) $x > 1$ (2) $n > 0$

18. If x is an integer, is 3^x less than 500? (1) $4^{x-1} < 4^x - 120$ (2) $x^2 = 36$

19. Is $x > 10^{10}$? (1) $x > 2^{34}$ (2) $x = 2^{35}$

20. Is XY > 0? (1) X – Y > –2 (2) X – 2Y < –6

21. If x and y are integers and xy does not equal 0, is $xy < 0$? (1) $y = x^4 - x^3$ (2) $-12y^2 - y^2x + x^2y^2 > 0$

22. If $r + s > 2t$, is $r > t$? (1) $t > s$ (2) $r > s$

23. If $p < q$ and $p < r$, is $(p)(q)(r) < p$? (1) $pq < 0$ (2) $pr < 0$

24. Is $5^n < 0.04$? (1) $(1/5)^n > 25$ (2) $n^3 < n^2$

25. Is $p^2q > pq^2$? (1) $pq < 0$ (2) $p < 0$

26. Is $m > n$? (1) $n - m + 2 > 0$ (2) $n - m - 2 > 0$

27. Is $3^p > 2^q$? (1) $q = 2p$ (2) $q > 0$

28. Is *mp* greater than *m*? (1) $m > p > 0$ (2) *p* is less than 1

29. Is $2X - 3Y < X^2$? (1) $2X - 3Y = -2$ (2) $X > 2$ and $Y > 0$

30. Is $\frac{x+1}{y+1} > \frac{x}{y}$?

(1) $0 < x < y$

(2) $xy > 0$

Answer Key: Quant Session: Inequalities + Mods (Absolute Values)

Part 1: Inequalities

1. E
2. D
3. E
4. C
5. A
6. E
7. C
8. E
9. C
10. E
11. B
12. B
13. E
14. A
15. D
16. E
17. C
18. C
19. D
20. C
21. E
22. D
23. E
24. A
25. C
26. B
27. C
28. C
29. D
30. A

Absolute Values (Mods) – Concepts

1. $|x|$ is defined as the non–negative value of x and hence is never negative.
   - So $|x| \geq 0$, always (by definition)   AND $|x| < 0$ is impossible (by definition)
2. $|5| = 5$, $|-5| = 5$
   - $|x| = x$, if x is positive … If $x = 5$, then $|5| = 5$
   - $|x| = -x$, if x is negative … If x is -5, then $|-5| = -(-5) = 5$. Here x is negative and $-x$ is positive.
     - So, when $|x| = -x$, x is a negative number and $-x$ is a positive number
   - $|x| = -x$ (means x is negative)
     - This still means that $|x|$ is positive because in this case $-x$ is a positive number
3. $|x|$ is defined as the distance of point x from 0 on the number line. The point x can be anywhere on the line (positive or negative)
4. $|x - a|$ is defined as the distance of point x from a on the number line. The point x and a can be anywhere on the line (positive or negative).
5. We define $\sqrt{x^2} = |x|$ as both $\sqrt{x^2}$ and $|x|$ can't be negative.
   - $\sqrt{x^2} = |x|$ … squaring both sides, we get $x^2 = |x| \times |x|$

   Q. If $a^2 < a$, then is $|a| > a$? **Yes / No?**   Ans. **NO**

   If $a^2 < a$, then $0 < a < 1$, or a is positive. When a is positive, $|a| = a$

6. As square roots can't be negative, then on the GMAT (by definition)
   - $\sqrt{36} = 6$ and not -6
   - BUT if $x^2 = 36$, we have $\sqrt{x^2} = \sqrt{36}$ OR $|x| = 6$, which gives $x = 6$ or -6.

     Remember, we didn't take $\sqrt{36}$ to be both 6 or –6. $\sqrt{36}$ is 6 only.

     But because $\sqrt{x^2}$ is $|x|$, we wrote $|x| = 6$, which gave us $x = 6$ or -6.

     This is the most misunderstood concept on the GMAT.

     So $\sqrt{x^2} = |x| = x$ or $-x$ both are possible (Please note: $\sqrt{x^2} = |x|$ which is always a positive quantity. For $|x|$ to be positive, $|x| = x$ or $-x$ depending on whether x is positive or negative. Don't assume that x is positive and -x is negative. x doesn't have a sign of its own)
   - So $\sqrt{x^2} = x$ or $-x$ both are possible.

- If x is positive, then $\sqrt{x^2} = x$
    - Here x is positive and hence the square root is positive
- If x is negative, then $\sqrt{x^2} = -x$
    - Here x is negative, so –x is POSITIVE and hence the square root is positive
- The confusion arises because we assume x is positive and –x is negative. BUT x doesn't have a sign of its own, unless given. **Please don't assume anything.**

Let's see one real–GMAT question to understand the concept further:

If z is negative, then $\sqrt[4]{(4z-5)^4} + \sqrt{(2z-3)^2} + \sqrt{-z|z|}$ = ?

A. 5z – 8
B. 7z – 8
C. –8
D. 8 – 7z
E. 4z – 8

Sol. We can write this as
|4z – 5| + |2z – 3| + |z|

|4z – 5| = 4z – 5 OR –(4z – 5) = 5 – 4z, whichever is positive

Because z is negative, 4z – 5 will be negative and 5 – 4z will be positive

|2z – 3| = 2z – 3 OR –(2z – 3) = 3 – 2z, whichever is positive

Because z is negative, 2z – 3 will be negative and 3 – 2z will be positive

|z| = z or –z, whichever is positive

Because z is negative, –z will be positive

So, the answer will be: 5 – 4z + 3 – 2z + (–z) which gives 8 – 7z. **Ans. D**

7. $|x| = x \Rightarrow x \geq 0$ (substitute x as – / 0 / + and verify)

8. $|x| = -x \Rightarrow x < 0$ (substitute x as – / 0 / + and verify)

9. $|x| > x \Rightarrow x < 0$ (substitute x as – / 0 / + and verify)

10. $-x|x| > x \Rightarrow x < 0$ (substitute x as – / 0 / + and verify)

11. $-x|x| > 0 \Rightarrow x < 0$ (substitute x as – / 0 / + and verify)

12. $|x - a| > 0 \Rightarrow x \neq a$ Imagine |x – 3| > 0 … this expression is true for all values of x except x = 3.
Try to substitute x = –10, –5, 0, 1, 2, 4, 10, 100 … all of these will satisfy |x – 3| > 0.
So, |x – 3| > 0 means $x \neq 3$

13. $\frac{x}{|x|} = 1$ if x is positive Substitute any positive value of x and verify

14. $\frac{x}{|x|} = -1$ if x is negative. Substitute any negative value of x and verify

Q. If $x = y / |y|$, what is $|x|$?

Sol. $x = 1$ or -1, so $|x| = 1$

15. $|a| = |b| \Rightarrow a = b \; OR \; a = -b$ When we remove the mods, we substitute $\pm$

So, we will have $\pm a = \pm b$ which gives $+a = +b$, $-a = -b$, $+a = -b$, and $-a = +b$. So we get $a = b$ or $a = -b$

16. If $|x| = a$, then $x = a$ or $x = -a$.

a. If $|x| < a$, then $x < a$ or $x > -a$ so $-a < x < a$.

b. If $|x - a| < b$, then $-b < x - a < b$

c. If $|x| > a$, then $x > a$ or $x < -a$.

d. If $|x - a| > b$, then $x - a > b$ or $x - a < -b$.

Q. If $|7 - 3j| \leq 8$, what is the range for j?

$-8 \leq 7 - 3j \leq 8$

Subtract 7

$-15 \leq -3j \leq 1$

Divide by 3

$-5 \leq -j \leq 1/3$

Multiply by a negative sign

$-1/3 \leq j \leq 5$ **Ans.**

Q. If $|x|/|3| > 1$, which of the following must be true?

A. $x > 3$
B. $x < 3$
C. $x = 3$
D. $x \neq 3$
E. $x < -3$

Cross multiply: $|x| > 3$, which means either $x > 3$ or $x < -3$. In either case, $x \neq 3$. **Ans. D**

Problems (Mods / Absolute Values)

1. If $x < 0$, then $\sqrt{(-x|x|)}$ is
   A. $-x$ B. -1 C. 1 D. x E. $\sqrt{x}$

2. Is $|x| = y - z$? (1) $x + y = z$ (2) $x < 0$

3. $x^2 - 8x + 21 = |x - 4| + 5$. If the various values of x obtained from the equation above are the sides of a triangle, then the triangle must be:
   A. An acute–angled tringle
   B. An obtuse–angled tringle
   C. A right–angled tringle
   D. An isosceles triangle
   E. An equilateral triangle

4. Is $\sqrt{[(x-3)^2]} = 3-x$? (1) $x \neq 3$ (2) $-x|x| > 0$

5. Which of the following inequalities has a solution set that when graphed on the number line, is a single segment of finite length?
   A. $x^4 \geq 1$ B. $x^3 \leq 27$ C. $x^2 \geq 16$
   D. $2 \leq |x| \leq 5$ E. $2 \leq 3x + 4 \leq 6$

6. What is the average of x and $|y|$?
   (1) $x + y = 20$ (2) $|x + y| = 20$

7. Is $\frac{1}{(a-b)} < b - a$? (1) $a < b$ (2) $1 < |a-b|$

8. Is $|x| < 1$? (1) $|x + 1| = 2|x - 1|$ (2) $|x - 3| > 0$

9. Is $|a| + |b| > |a + b|$?
   (1) $a^2 > b^2$ (2) $|a| \times b < 0$

10. If $a < y < z < b$, is $|y - a| < |y - b|$? (1) $|z - a| < |z - b|$ (2) $|y - a| < |z - b|$

11. Is $|x - y| > |x| - |y|$? (1) $y < x$ (2) $xy < 0$

12. If $|ab| > ab$, which of the following must be true?
    I. $a < 0$ II. $b < 0$ III. $ab < 0$
    I only II only III only I and III II and III

13. Is $x \times |y| >$ (1) $x > y$ (2) $y > 0$

14. Is $(|x^{-1}y^{-1}|)^{-1} > xy$? (1) $xy > 1$ (2) $x^2 > y^2$

15. If x is not equal to 0, is $|x|$ less than 1? (1) $\frac{x}{|x|} < x$ (2) $|x| > x$

16. If n is not equal to 0, is $|n| < 4$? (1) $n^2 > 16$ (2) $1/|n| > n$

17. If $|x| + |y| = -x - y$ and xy does not equal 0, which of the following must be true?
    $x + y > 0$ $x + y < 0$ $x - y > 0$ $x - y < 0$ $x^2 - y^2 > 0$

18. Is $x > 0$? (1) $|x + 3| = 4x - 3$ (2) $|x - 3| = |2x - 3|$

19. What is the value of $|x|$? (1) $|x^2 + 16| - 5 = 27$ (2) $x^2 = 8x - 16$

20. If r is not equal to 0, is $r^2 / |r| < 1$
(1) $r > -1$ (2) $r < 1$

21. Is $x > 0$? (1) $|x + 3| < 4$ (2) $|x - 3| < 4$

22. Is $\sqrt{x}$ a prime number?
(1) $|3x - 7| = 2x + 2$ (2) $x^2 = 9x$

23. $|k|$ is a prime number.
x and k are both integers
$x > k$
$x^{-k} = 625$
What is the value of x? ______________

24. If $y = |x - 1|$ and $y = 3x + 3$, then the sum of all possible values of x is:

(A) $-5/2$ (B) -2 (C) $-1/2$ (D) $1/2$ (E) 2

25. If $a \neq b$, is $\frac{1}{a-b} > ab$?

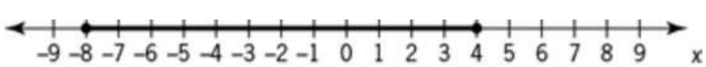
(1) $|a| > |b|$ (2) $a < b$

26. If q, s, and t are all different numbers, is $q < s < t$?

(1) $t - q = |t - s| + |s - q|$
(2) $t > q$

27. If $y = \frac{3x-5}{-x^2-3}$ for what value of x will the value of y be greatest?
A. -5
B. $-3/5$
C. 0
D. $3/5$
E. $5/3$

28.



On the number line, the shaded interval is the graph of which of the following inequalities?
A. $|x| \leq 4$
B. $|x| \leq 8$
C. $|x - 2| \leq 4$
D. $|x - 2| \leq 6$
E. $|x + 2| \leq 6$

29. For how many pairs (x, y) that are solutions of the system of equations $2x + y = 12$ and $|y| \leq 12$ are x and y both integers?

A. 17
B. 10
C. 12
D. 13
E. 14

30. Is $|a| < |b|$?

(1) $\frac{1}{a-b} > \frac{1}{b-a}$

(2) $a + b < 0$

Answer Key: Quant Session: Inequalities + Mods (Absolute Values)

Part 2: Mods / Absolute Values

1. A
2. C
3. C
4. B
5. E
6. E
7. A
8. C
9. E
10. D
11. B
12. C
13. C
14. A
15. C
16. A
17. B
18. A
19. D
20. C
21. E
22. C
23. 25
24. C
25. E
26. A
27. E
28. E
29. D
30. C

Inequalities Solutions

1.
In this question, don't cross multiply as you don't know whether $x + y$ is positive or not.

So, we will have to combine:

If we take $x = 1$ and $y = -2$, we get

$$\frac{1-(-2)}{1-2} = -3 < 1$$

If we take $x = 2$ and $y = -1$, we get

$$\frac{2-(-1)}{2-1} = 3 > 1$$

Answer E.

2.

I.
$x^2 < 2x$ … cancel x from both sides (only because it is positive), we have $x < 2$

$2x < 1/x$
$2x^2 < 1$
$x^2 < 1/2$
$x < 1/\sqrt{2}$

$x < 0.707$

Combining, we have $x < 2$ and $x < 0.7$ … the common solution is $x < 0.7$ so statement I is possible.

II. $x^2 < 1/x$
$x^3 < 1$
$x < 1$

$1/x < 2x$
$2x^2 > 1$
$x^2 > 1/2$
$x > 1/\sqrt{2}$

$x > 0.707$

Combining, we have $0.707 < x < 1$ … so statement II is possible.

III. $2x < x^2$ … cancel x from both sides (only because x is positive) so we have

$x > 2$.

$x^2 < 1/x$ $x^3 < 1$ $x < 1$.

Combining: $x > 2$ and $x < 1$ … these 2 can't be true together … so statement III is impossible.
Ans. D

OR

you can try values: $-2, -1, -0.9, -1/2, 0, 1/2, 0.9, 1, 2$. Since x is positive, we may must try: $1/2, 0.9, 1, 2$.
I. is true for $x = 1/2$
II. is possible for $x = 0.9$
III. is not possible for any value of x.
So, D

3.
If $x > y^2$ we may have $x > y$ or $x < y$ both as valid. **Refer to the concepts given**
If $y^2 > z^4$, $y > z^2$ (square roots will be positive) … if $y > z^2$ we may have $y > z$ or $y < z$ both as valid.
Refer to the concepts given
If $x > z^4$, we may have $x > z$ or $x < z$ both as valid. **Refer to the concepts given**
Ans. E

4.

(1) $M - 3Z > 0$, we may have $M = 10$, and $Z = 1$, $M + Z = +ve$
$M - 3Z > 0$, we may have $M = 1$, and $Z = -2$, $M + Z = -ve$ **NS**

(2) $4Z - M > 0$, we may have $M = 1$, and $Z = 1$, $M + Z = +ve$
$4Z - M > 0$, we may have $M = -10$, and $Z = 1$, $M + Z = -ve$ **NS**

Combining… add the two statements…
$M - 3Z > 0$ Or $M > 3Z$
$4Z - M > 0$

$Z > 0$

Substitute $Z > 0$ in $M > 3Z$, so $M > 0$
So, $M + Z > 0$
Sufficient. C

5.

If k is not equal to 0, 1, or –1, is $1/k > 0$?

(1) $1 / (k - 1) > 0$ (2) $1 / (k + 1) > 0$

(1) tells that $(k - 1)$ must be positive so $(k - 1) > 0$ so $k > 1$ … so $1/k > 0$ always … sufficient.

(2) tells that $(k + 1)$ must be positive so $(k + 1) > 0$ so $k > -1$ … k can be $-1/2$ or 2 … so $1/k$ can be positive or negative. **Ans. A**

6.
This question deals with rounding–off.

$$3.5 \le x + y < 4.5$$
$$0.5 \le x - y < 1.5$$

$$4 \le 2x < 6$$

Therefore $2 \le x < 3$
But x could be 2.1 and the nearest integer will be 2

x could also be 2.9 and the nearest integer will be 3. NS

Ans. E

Top 1% expert replies to student queries (can skip)

(1) 4 is the integer that is closest to x+y
This should be $3.5<x+y<4.5$;
we shouldn't take $3.5<=x+y<=4.5$
4 is the integer that is closest to x+y i.e., there is a single integer that is closest to (x+y)
If (x+y) = 3.5, which integer is closest to it?
Both 3 and 4 are at equal distance i.e., they are both 0.5 away from (x+y).
But then, we cannot say that 4 is the integer closest to x+y.
Hence, x+y must be greater than 3.5. It must also be less than 4.5 due to the same reason.

Note: 3.5 is rounded up to 4 instead of 3 only because we generally follow the roundup convention. If we follow the 'round down' convention, 3.5 will be rounded off to 3.
3.5 is equidistant from both 3 and 4. **Ans. E**

7.

Statement (1) can be rephrased: $x - y = 1/2$.

As $x - y > 0$, $x > y$ $1/2 > 0$, always

Sample values: $x = 1$, $y = 1/2$ and x and y are both positive

Sample values: $x = -1/2$ and $y = -1$ and x and y are both negative **NS**

Statement (2)

Imagine $2/1 = 2 > 1$

In this case: x and y are both positive and x must be larger than y, so $x > y > 0$

Imagine $-2/-1 = 2 > 1$

In this case: x and y are both negative, then x is more negative than y
So, $x < y < 0$.

Because we don't know whether they are both positive or both negative, **(2) alone is insufficient.**

From (1), we know that $x > y$. The only option in (2) for this to be true is if they are both positive. (1) and (2) together are sufficient. **(C) is the answer.**

8.

This problem involves rounding–off.

"500 is the multiple of 100 that is closest to X"

This means that, of all multiples of 100, 500 comes closest to x.

In other words, 500 is closer to x than is 100, 200, 300, 400, or 600, 700, 800, ...

If you think about this for a sec, you'll realize that it means x has to be strictly between 450 and 550.

1. $450 < x < 500$

2. $350 < y < 400$

So, $800 < x + y < 900$ …
if $x + y = 810$, our answer will be 800.
if $x + y = 860$, our answer will be 900.

NS
Ans. E

Top 1% expert replies to student queries (can skip)

If 500 is the multiple of 100 that is closest to X and 400 is the multiple of 100 closest to Y, then which multiple of 100 is closest to X + Y ?

"500 is the multiple of 100 closest to X" --> 450<x<550;
"400 is the multiple of 100 closest to Y" --> 350<y<450.

(1) x<500 --> 450<x<500 --> add this inequality to inequality with y (350<y<450)
800<x+y<950
If x+y=810 then closest multiple of 100 is 800 BUT if x+y=860 then closest multiple of 100 is 900. Not sufficient.

(2) y<400 --> 350<y<400 --> add this inequality to inequality with x (450<x<550) --> 800<x+y<950. The same here: if x+y=810 then the closest multiple of 100 is 800 BUT if x+y=860 then the closest multiple of 100 is 900. Not sufficient.

Combine: (1)+(2) Sum 450<x<500 and 350<x<400 --> 800<x+y<900 --> and again if x+y=810 then closest multiple of 100 is 800 BUT if x+y=860 then closest multiple of 100 is 900. Not sufficient.

Answer: E.

Alternate Solution from Gmatclub

"500 is the multiple of 100 closest to X" --> $450 < x < 550$;
"400 is the multiple of 100 closest to Y" --> $350 < y < 450$.

(1) x<500 --> $450 < x < 500$ --> add this inequality to inequality with y --> $800 < x + y < 950$. If x+y=810 then closest multiple of 100 is 800 BUT if x+y=860 then closest multiple of 100 is 900. Not sufficient.

(2) y<400 --> $350 < y < 400$ --> add this inequality to inequality with x --> $800 < x + y < 950$. The same here: if x+y=810 then closest multiple of 100 is 800 BUT if x+y=860 then closest multiple of 100 is 900. Not sufficient.

(1)+(2) Sum $450 < x < 500$ and $350 < x < 400$ --> $800 < x + y < 900$ --> and again if x+y=810 then closest multiple of 100 is 800 BUT if x+y=860 then closest multiple of 100 is 900. Not sufficient.

Answer: E.

Top 1% expert replies to student queries (can skip)

450<X<550 and 350<Y<450
?<X+Y<?
(1) X < 500
Now, 450<X<500 and 350<Y<450
Examples of
Higher range => 490+440 = 930 (No closest: 900)
Lower range => 460+360 = 820 (No closest: 800)
Not sufficient

(2) Y < 400
Now,
450<X<550 and 350<Y<400
Examples of
Higher range =>540+390 = 930 (No closest: 900)
Lower range =>460+360 = 820 (No closest: 800)

Not sufficient

(1) + (2) combined.
Now,
$450<X<500$ and $350<Y<400$
Examples of
Higher range =>490+390 = 880 (No closest: 900)
Lower range =>460+360 = 820 (No closest: 800)
Not sufficient
Ans: E

9.
Is $1 / p > r/(r^2 + 2)$ (1) $p = r$ (2) $r > 0$

You shouldn't cross multiply because you don't know whether p is positive or not.

(1) imagine $p = r = 2$

$1 / p > r/(r^2 + 2)$ becomes $1/2 > 2 / (4 + 2)$ or $1/2 > 1/3$ … YES

Imagine $p = r = –2$

$1 / p > r/(r^2 + 2)$ becomes $–1/2 > –2 / (4 + 2)$ or $–1/2 > –1/3$ … NOOO

NS.

(2) doesn't talk about p … NS

Combining: $p = r$ and both p and r are positive.

The question becomes: "Is $1/p > p / (p^2 + 2)$ … we can cross multiply here (all values positive)

$\Rightarrow p^2 + 2 > p^2$ or $2 > 0$, which is always true. We get a confirmed YES answer.

Ans. C

Top 1% expert replies to student queries (can skip) (additional)

When we combine, we know that p=r and r is positive
Question:
Is $1/p > r/(r^2 +2)$?
Is $1/r > r/(r^2 + 2)$?
Since r is positive, we can multiply r to the right and $(r^2 +2)$ to the left
Is $(r^2 + 2) > r^2$?
Clearly Left Hand Side is greater than Right Hand Side.
So always YES answer to the question stem.

Ans. C

10.
Is $X + Y < 1$? (1) $X < 8 / 9$ (2) $Y < 1 / 8$

Combining (1) and (2) $x + y < 73/72$ … $x + y$ can be $72.5 / 72$ … > 1 or can be $71 / 72$ … < 1. NS.
Ans. E

11.

Simplify the question to   is $y < 1$?

(1) NS
(2) Sufficient … if $y < 0$, y will surely be less than 1.

Ans. B

12. Is z the median of any 3 positive integers x, y and z? (1) $x < y + z$ (2) $y = z$

This is the same as asking: **is *z* equal to the middle number of the three numbers?**

Statement (1)
$x < y + z$
Imagine the numbers 1, 2 and 3
We know that 2 is the median
$1 < 2 + 3$
$2 < 1 + 3$
$1 < 3 + 2$

We see that 2 can come in place of x or y or z. So any of the three values can be the median. NS

Statement (2)
If y and z are equal, there are three possibilities:

$x = y = z$
We can write the three numbers as z, z, z Median = z

$x < y = z$
We can write the three numbers as x, z, z Median = z

$x > y = z$
We can write the three numbers as z, z, x Median = z

The median is z in all the cases. **Sufficient. B**

Top 1% expert replies to student queries (can skip) (additional)

For Statement:2

We know that $y = z$.

Case 1 : $y = z = 5$ and $x = 5$. In this case, median = 5 = z

Case 2 : $y = z = 5$ and $x = 4$. In this case, median = 5 = z

Case 3 : $y = z = 5$ and $x = 6$. In this case, median = 5 = z

So sufficient!
Answer is B

13.

Even if you satisfy all the conditions from both (1) and (2), we have these 2 possibilities

__–1______0_______1______2__________
x   z   y

z lies between x and y.

OR

__–1______0_______1______2__________
y   x   z

z doesn't lie between x and y.

Ans. E

Top 1% expert replies to student queries (can skip)

The distance between x and y is greater than the distance between x and z, means that we can have one of the following four scenarios:
A. y--------z--x (YES case);
B. x--z--------y (YES case);
C. y--------x--z (NO case);
D. z--x--------y (NO case);

The question asks whether we have scenarios A or B (z lies between x and y).
(1) $xyz<0$ --> either all three are negative or any two are positive and the third one is negative.
If we place zero between y and z in case A (making y negative and x, z positive), then the answer would be YES but if we place zero between y and x in case C, then the answer would be NO. **Not sufficient**.

(2) $xy<0$ --> x and y have opposite signs. The same here: We can place zero between y and x in case A and the answer would be YES but we can also place zero between y and x in case C and the answer would be NO. **Not sufficient**.

Statements (1)+(2) Both case A (answer YES) and case C (answer NO) satisfy the statements. **Not sufficient**.
Ans. E

14.

The expression $\sqrt{(x+4)^2}$ can be simplified to $|x+4|$, and the original equation can be solved accordingly.
If $|x + 4| = 3$

$(x + 4) = 3$ or $(x + 4) = -3$

$x = -1$ or $x = -7$

Watch out! Although –7 is an answer choice, it is not correct. The question does not ask for the value of x, but rather for the value of $x - 4 = -7 - 4 = -11$.

The correct answer is A.

15.

$mv < 0$ means m and v are of opposite signs
$pv < 0$ means p and v are of opposite signs

$mv < pv$

If v is positive, we can cancel v without changing the sign, so $m < p$

If v is negative, we can cancel v but we have to change the sign, so $m > p$

(1) gives $m < p$, so v has to be positive.

(2) gives m is negative so v has to be positive.

Ans. D

16.

(1) INSUFFICIENT: The fact that x^2 is greater than y does not tell us whether x is greater than y. For example, if $x = 3$ and $y = 4$, then $x^2 = 9$, which is greater than y although x itself is less than y. But if $x = 5$ and $y = 4$, then $x^2 = 25$, which is greater than y and x itself is also greater than y.

(2) INSUFFICIENT: We can square both sides to obtain $x < y^2$. As we saw in the examples above, it is possible for this statement to be true whether y is less than or greater than x (just substitute x for y and vice–versa in the examples above).

(1) AND (2) INSUFFICIENT: Taking the statements together, we know that $x < y^2$ and $y < x^2$, but we do not know whether $x > y$. For example, if $x = 3$ and $y = 4$, both of these inequalities hold ($3 < 16$ and $4 < 9$) and $x < y$. But if $x = 4$ and $y = 3$, both of these inequalities still hold ($4 < 9$ and $3 < 16$) but now $x > y$.

The correct answer is E.

17.

(1) INSUFFICIENT: If $x = 2$ and $n = 2$, $x^n = 2^2 = 4$. If $x = 2$ and $n = –2$, $x^n = 2^{-2} = 1/(2^2) = 1/4$.

(2) INSUFFICIENT: If $x = 2$ and $n = 2$, $x^n = 2^2 = 4$. If $x = 1/2$ and $n = 2$, $x^n = (1/2)^2 = 1/4$.

(1) AND (2) SUFFICIENT: Taken together, the statements tell us that x is greater than 1 and n is positive. Therefore, for any value of x and for any value of n, x^n will be greater than 1 and we can answer definitively "no" to the question.

The correct answer is C.

18.
Since $3^5 = 243$ and $3^6 = 729$, 3^x will be less than 500 only if the integer x is less than 6. So, we can rephrase the question as follows: "Is $x < 6$?"

(1) INSUFFICIENT: We can solve the inequality for x.
$4^{x-1} < 4^x – 120$
$4^{x-1} – 4^x < –120$
$4^x(4^{-1}) – 4^x < –120$
$4^x(1/4) – 4^x < –120$
$4^x[(1/4) – 1] < –120$
$4^x(–3/4) < –120$

$4^x > 160$

Since $4^3 = 64$ and $4^4 = 256$, x must be greater than 3. However, this is not enough to determine if $x < 6$.

x could be 4, 5, 6, 7 … anything
(2) INSUFFICIENT: If $x^2 = 36$, then $x = 6$ or -6. Again, this is not enough to determine if $x < 6$.

(1) AND (2) SUFFICIENT: Statement (1) tells us that $x > 3$ and statement (2) tells us that $x = 6$ or -6. Therefore, we can conclude that $x = 6$.
This is sufficient to answer the question "Is $x < 6$?" (Recall that the answer "no" is sufficient.)

The correct answer is C

19.
(1) SUFFICIENT: Statement (1) tells us that $x > 2^{34}$, so we want to prove that $2^{34} > 10^{10}$. We'll prove this by manipulating the expression 2^{34}.
$2^{34} = (2^4)(2^{30})$ $\qquad$ $2^{34} = 16(2^{10})^3$

Now $2^{10} = 1024$, and 1024 is greater than 10^3. Therefore:

$2^{34} > 16(10^3)^3$ $\qquad$ $2^{34} > 16(10^9)$ $\qquad$ $2^{34} > 1.6(10^{10})$.

Since $2^{34} > 1.6(10^{10})$ and $1.6(10^{10}) > 10^{10}$, then $2^{34} > 10^{10}$.

(2) SUFFICIENT: Statement (2) tells us that that $x = 2^{35}$, so we need to determine if $2^{35} > 10^{10}$. Statement (1) showed that $2^{34} > 10^{10}$, therefore $2^{35} > 10^{10}$.

The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

* $x > 10^{10}$

$x > (10)^{10} \rightarrow$ Is $x > ((10)^3)^3 (10)^1$? $\rightarrow$ Is $x > (1000)^3 (10)$?

St 1:
$x > 2^{34}$
$x > (2^{10})^3 (2)^4$
$x > (1024)^3 (16)$

Between $(1000)^3 (10)$ [A] and $(1024)^3 (16)$ [B]
$(1024)^3 (16)$ will be greater.

So on the number line:

A ——— B

The Question stem asks
Is $x > A$
St1 says $\underline{x > B}$; hence a Definite (YES)

St 2:
$x = 2^{35}$
Since this value lies to the right of B, therefore it will be greater than A. (YES, answer)

$\boxed{\text{Hence (D)}}$

20.

X–2Y < –6

Multiply by a –ve sign to change the sign

–X + 2Y > 6

Combined with X – Y > –2, we get

X – Y > –2 means X > Y – 2
–X + 2Y > 6

---

we get Y > 4

Substitute Y > 4 in the first, we get X > 2

Therefore, XY > 0

Answer is C

Alternate Solution from GMATCLUB

Note that question basically asks whether x and y have the same sign.

(1) x-y > -2 --> we can have an YES answer, if for example x and y are both positive ($x = 10$ and $y = 1$) as well as a NO answer, if for example x is positive and y is negative ($x = 10$ and $y = -10$). Not sufficient.

(2) x-2y <-6 --> again it' easy to get an YES answer, if for example x and y are both positive ($x = 1$ and $y = 10$) as well as a NO answer, if for example x is negative and y is positive ($x = -1$ and $y = 10$). Not sufficient.

You can get that the the two statement individually are not sufficient in another way too: we have (1) $y < x + 2$ and (2) $y > \frac{x}{2} + 3$. We are asked whether x and y have the same sign or whether the points (x,y) are in the I or III quadrant ONLY. But all (x,y) points below the line $y = x + 2$ (for 1) and all (x, y) points above the line $y = \frac{x}{2} + 3$ cannot lie **only** in I or III quadrant: points above or below some line (not parallel to axis) lie at least in 3 quadrants.

(1)+(2) Now, remember that we can subtract inequalities with the signs in opposite direction --> subtract (2) from (1): $x - y - (x - 2y) > -2 - (-6)$ --> $y > 4$. As $y > 4$ and (from 1) $x > y - 2$ then $x > 2$ (because we can add inequalities when their signs are in the same direction, so: $y + x > 4 + (y - 2)$ --> $x > 2$) --> we have that $y > 4$ and $x > 2$: both x and y are positive. Sufficient.

Answer: C.

21.
Question: Do x and y have the opposite signs?

(1) INSUFFICIENT:
$y = x^4 - x^3$

If x = 2, y is +ve, and xy is +ve
If x = –2, y is positive, and xy is –ve
NS

(2) INSUFFICIENT:
$-12y^2 - y^2x + x^2y^2 > 0$
$y^2(-12 - x + x^2) > 0$
$y^2(x^2 - x - 12) > 0$
$y^2(x + 3)(x - 4) > 0$

$y^2(x + 3)(x - 4) > 0$ is satisfied if x = 5, and y +ve … xy = +ve
$y^2(x + 3)(x - 4) > 0$ is also satisfied if x = 5, and y –ve … xy = –ve
NS
The correct answer is E.

Top 1% expert replies to student queries (can skip):
Why is Statement 2 insufficient?

12y^2 – y^2x + x^2y^2 > 0

= y^2 (-12 -X + X^2) > 0
= y^2 (X^2 -X -12) > 0
= y^2 { (X-4) (X+3) } > 0

When X=4 then XY = 0.
Therefore, X cannot be 4
Plug-in values depend on the conditions of the question as well (not just the values that you mentioned), here we have to plug-in x=4, x>4 (so consider x=5) and x<0.
When X = 5 then y^2 { (X-4) (X+3) } = y^2 * 8 ; X is +ve and Y can be +ve or - ve
When X = -4 then y^2 { (X-4) (X+3) } = y^2 * 8 ; X is -ve and Y can be +ve or - ve

Statement 2 is insufficient.

$y2(x + 3)(x - 4) > 0$ or $(x + 3)(x - 4) > 0$ or $[(x - (-3)][(x - 4)] > 0$. $x > 4$ OR $x < -3$. This is obviously not enough to determine the sign of x. The sign of y is anyway not determinable.

The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

Query: How did we get values of x for which (x+3)(x+4) > 0 inequality is satisfied ?
Reply: We have (x+3)(x-4) > 0

For now, forget the inequality sign. What are the roots of the equation: (x+3)(x-4) = 0? Roots are x = -3 and x = 4. So the sign changes in the expression (x+3)(x-4) will happen around these points.

If x < -3,

(x+3) < 0 and (x-4) < 0. so (x+3)(x-4) > 0 [two negative numbers when multiplied give a positive result]

If -3 < x < 4,

(x+3) > 0 and (x-4) < 0. so (x+3)(x-4) < 0 [one positive and one negative number when multiplied give a negative result]

If x > 4,

(x+3) > 0 and (x-4) > 0. so (x+3)(x-4) > 0 [two positive numbers when multiplied give a positive result]

So the solution to the inequality is: x<-3 and x > 4.

22.
(1) SUFFICIENT: We can combine the given inequality $r + s > 2t$ with the first statement by adding the two inequalities:

$$\begin{array}{c} r + s > 2t \\ \underline{\quad t > s \quad} \\ r + s + t > 2t + s \\ r > t \end{array}$$

(2) SUFFICIENT: We can combine the given inequality $r + s > 2t$ with the second statement by adding the two inequalities:

$$\begin{array}{c} r + s > 2t \\ \underline{\quad r > s \quad} \\ 2r + s > 2t + s \\ 2r > 2t \\ r > t \end{array}$$

The correct answer is D.

Alternate Solution from GMATCLUB

(1) t > s --> since the signs of two equations (t > s and r + s > 2t) are the same direction we can sum them: $t + (r + s) > s + 2t$ --> $r > t$. Sufficient.

(2) r > s --> the same here: since the signs of two equations (r > s and r + s > 2t) are the same direction we can sum them: $r + (r + s) > s + 2t$ --> $2r > 2t$ --> $r > t$. Sufficient.

Answer: D.

'23.

From (1) or (2), we get p is –ve and q and r are positive.

Q. Is pqr < p?
We know that p is negative
Cancel but also change the sign
Q. Is qr > 1?
We know q and r are positive.
Take q = r = 2, we get qr = 4 > 1
Take q = r = 1/2, we get qr = 1/4 < 1
Answer. E

24.
$0.04 = 1/25 = 5^{-2}$
We can rewrite the question in the following way: "Is $5^n < 5^{-2}$?"

OR "Is $n < -2$"?

(1) SUFFICIENT: Let's simplify (or rephrase) the inequality given in this statement.

$(1/5)^n > 25$
$(1/5)^n > 5^2$
$5^{-n} > 5^2$

$-n > 2$
$n < -2$ (recall that the inequality sign flips when dividing by a negative number)

This is sufficient to answer our rephrased question.

(2) INSUFFICIENT: n^3 will be smaller than n^2 if n is either a negative number or a fraction between 0 and 1. We cannot tell if n is smaller than -2.

The correct answer is A.

Top 1% expert replies to student queries (can skip)

In an inequality equation, the base has to be a positive integer to strike out bases and reduce the equation to their powers. e.g 2^x > 2^3, therefore x>3(strike out the bases 2^x > 2^3).
Let's consider x=4 (because x>3).
This will satisfy the equation 2^x > 2^3
2^4 > 2^3
16>8 **Correct**.

In an inequality equation, if the base is a fraction between 0 and 1, then we cannot strike out the bases and reduce the equation to their powers. e.g. we cannot write (1/2)^x > (1/2)^3 as x>3 (we can't strike out the bases (1/2)^x > (1/2)^3

Because, this will not satisfy the equation: Let's consider x=4 (because x>3).
This will not satisfy the equation (1/2)^x > (1/2)^3 --> (1/2)^x > (1/2)^3 -->(1/16)>(1/8)--> Incorrect

Lets consider n=1 (because (n>-2)), then the equation becomes (1/5)^1 > (1/5)^-2 --> (1/5) >25--> Incorrect.
The correct answer is A.

Summary: In an inequality equation, you have to convert the base to positive integers to strike out the base and reduce the equation to their powers.

25.

The question can first be rewritten as "Is $p(pq) > q(pq)$?"

If pq is negative, and p is negative, we know that q is positive.

We can cancel pq and change the direction of the inequality sign and the question becomes:

"Is $p < q$?"

If p is –ve and q is +ve, so p < q

The correct answer is C.

26.
We can rephrase the question: "Is $m - n > 0$?"

(1) INSUFFICIENT: If we solve this inequality for $m - n$, we get $m - n < 2$. This does not answer the question "Is $m - n > 0$?".

(2) SUFFICIENT: If we solve this inequality for $m - n$, we get $m - n < -2$. This answers the question "Is

$m - n > 0$?" with an absolute NO.

The correct answer is B.

Alternate sol from gmatclub (additional)

Question is m > n or is m -n > 0

STAT1

n-m +2 > 0

m - n < 2, So, m-n can be > 0 or can be < 0 also

So, INSUFFICIENT

STAT2

n-m-2 > 0

=> m - n < -2

=> m -n < 0 for sure

=> m - n is NOT > 0

So, SUFFICIENT

So, Answer will be B

27.

(1) INSUFFICIENT: We can substitute $2p$ for q in the inequality in the question: $3^p > 2^{2p}$.
This can be simplified to $3^p > (2^2)^p$ or $3^p > 4^p$.

If $p > 0$, $3^p < 4^p$ (for example $3^2 < 4^2$ and $3^{0.5} < 4^{0.5}$)
If $p < 0$, $3^p > 4^p$ (for example $3^{-1} > 4^{-1}$)

So the question becomes: is $p < 0$?

(2) INSUFFICIENT: This tells us nothing about p.

(1) AND (2) SUFFICIENT: If $q > 0$, then p is also greater than zero since $p = q/2$ = +ve / 2 = +ve.

If $p > 0$, then $3^p < 4^p$. The answer to the question is a definite NO.

The correct answer is C.

28.

Simplify the question

Cancel m (as m and p are both positive)

The question becomes: is $p > 1$?

If we combine, we get $0 < p < 1$, So we get a confirmed NO answer.
Ans. C

Top 1% expert replies to student queries (can skip)

Plug-in Method:
Solved by plugging in some values -
If $m > p > 0$
Let, $2 > 1 > 0$
So, $mp = 2$ and $m = 2$
Now, mp is not greater than m
Again -
If $m > p > 0$
Let, $3 > 2 > 0$
So, $mp = 6$ and $m = 3$
Now, mp is greater than m
Thus option (1) alone can not be used...
From (2) we get nothing -
Using (1) and (2)
If If $m > p > 0$ and $p < 1$
Let, $1/3 > 1/2 > 0$
So, $mp = 1/6$ and $m = 1/3$
Thus , mp will always be greater than m
So, BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient to answer the question asked to solve this question..
Answer will be (C)

Conceptual method:
There are some great takeaways on number properties in this question. Let's look at them:
Question: Is mp greater than m?
Forget greater, think less because it is less intuitive so there will be fewer cases to worry about. When will the product of 2 numbers be less than one of them? Two simple cases we can think of are $6*(1/2) = 3$ or $6*(-3) = -18$.
(One number is greater than 1 and the other is less than 1, one number is positive and the other is negative)
Numbers between 0 to 1 when multiplied to positive numbers, make the product smaller.
Numbers between 0 to 1 when multiplied to negative numbers, make the product greater because the product becomes 'less negative'.
Negative numbers when multiplied to positive numbers make the product smaller (negative).
Now go on to the statements:
(1) $m > p > 0$
This only tells us that both the numbers are positive. We don't know whether p is less than 1 or greater than 1. Not sufficient.
(2) p is less than 1
If p is less than 1, it will make the product mp less than m if m is positive. But if m is negative, the product will become greater. Not sufficient.
Using both, given that m is positive and p is less than 1, we can say that the product mp will be less than m. Hence, together both the statements are sufficient.
Answer (C)

Algebraic method:
The question ($mp>m$?) can be rephrased as "Is $mp-m>0$ or $m(p-1)>0$, which is the same as asking whether m and $(p-1)$ have the same sign?"
(1) Not sufficient. In order to know the sign of $p-1$ we have to know whether p is greater or less than 1.
(2) Not sufficient. We know nothing about m.
(1) and (2): Now we know for sure that m is positive and $(p-1)$ is negative. Hence, $m(p-1)$ is not >0 or (mp) is not $>m$. This gives a definite NO.
Sufficient.
Answer C

Top 1% expert replies to student queries (can skip) (additional)

So we need to check if $m(p-1) > 0$

Statement 1:

$m > p > 0$

Case 1: $m = 5$, $p = 2$. In this case, $m(p-1) = 5 * 1 = 5 > 0$
Case 2: $m = 5$, $p = 0.5$. In this case, $m(p-1) = 5 * (-0.5) = -2.5 < 0$

Since we do not have a conclusive answer, statement 1 is insufficient.

Statement 2 :

$p < 1$.

So $(p-1) < 0$

But we do not know m's sign. If $m > 0$, then $m(p-1) < 0$ and if $m < 0$, then $m(p-1) > 0$. Insufficient!

Combining the two,

$m > p > 0$ and $p < 1$.

So, $1 > p > 0$ OR p lies between 0 and 1. So $(p-1) < 0$.

We also know that $m > 0$.
So $m(p-1)$ = positive * negative < 0. Sufficient!

The answer is C.

29.

SQUARE is never negative… square ≥ 0 always.

Given equation can be written as: is $x^2 – 2x + 3y > 0$?

(1) $2x–3y = –2$

The given equation becomes $x^2 – (2x – 3y) = x^2 + 2$, which is always > 0 … (SUFF)

(2) $x > 2$ and $y > 0$

Given equation can be written as $x(x – 2) + 3y$, which is also always > 0, if $x > 2$ and $y > 0$.

Sufficient

Answer D.

30.

1. Given that $0 < x < y$, the following steps show how to obtain $\frac{x+1}{y+1} > \frac{x}{y}$.

$y > x$	given
$xy + y > xy + x$	add xy to both sides
$y(x + 1) > x(y + 1)$	factor
$\frac{x+1}{y+1} > \frac{x}{y}$	divide both sides by $y(y + 1)$

In the last step, the direction of the inequality is not changed because both y and $y + 1$ are positive, and hence the product $y(y + 1)$ is positive. These steps can be discovered by performing standard algebraic manipulations that transform $\frac{x+1}{y+1} > \frac{x}{y}$ into $y > x$, and then verifying that it is mathematically valid to reverse the steps; SUFFICIENT.

2. Given that $xy > 0$, it is possible that $\frac{x+1}{y+1} > \frac{x}{y}$ can be true (choosing $x = 1$ and $y = 2$, the inequality becomes $\frac{2}{3} > \frac{1}{2}$) and it is possible that $\frac{x+1}{y+1} > \frac{x}{y}$ can be false (choosing $x = -1$ and $y = -2$, the inequality becomes $\frac{0}{-1} > \frac{-1}{-2}$ or $0 > \frac{1}{2}$); NOT sufficient.

Ans. A

(Mods / Absolute Values) – Solutions

1.
The quick way to approach will be pick a number $x < 0$.
Let's pick –5. So, we know $x = -5$.
$\sqrt{(-x|x|)} = \sqrt{(-(-5)|-5|)} = \sqrt{(5*5)} = \sqrt{(25)} = 5 = -(-5) = -x$ so **Answer A.**

Shortcut: Seemingly the answer can be only x or –x … but the square root can't be negative … if x is given to be negative, –x will be positive. So, the answer has to be –x.

2.

As the value of |x| is never negative, we may rephrase the question as: "Is $y - z > 0$?".
(1) gives $y - z = -x$ NS
(2) gives $x < 0$ NS
Combine $y - z > 0$ Ans.
YES
Ans. C

3.
$x^2 - 8x + 21 = |x - 4| + 5$
$x^2 - 8x + 16 = |x - 4|$
$(x - 4)(x - 4) = |x - 4|$
$(x - 4)^2 = |x - 4|$
There are only three solutions to $y2 = |y|$ … $y = 0$, 1 and –1
So, $x - 4 = -1$, 0, or 1 $x = 3$, 4, or 5
These make a right–angled triangle. Ans. C

4.

As square roots cannot be negative, the question reduces to: "Is $3 - x > 0$?"
(1) $x \neq 3$ can be $x = 2$ or $x = 4$.
If $x = 2$, $3 - x$ is positive
If $x = 4$, $3 - x$ is negative
So, NS
(2) $-x\,|x| > 0$ means x is negative so $3 - x$ is positive … Suff.
Ans. B

OR if we take $x - 3$ as y, the question is asking, Is $\sqrt{y^2} = -y$? Or Is $|y| = -y$ Or is y negative?

(1) If $x = 2$, $3 - x$ is positive means y is negative
If $x = 4$, $3 - x$ is negative means y is positive
So, NS

(2) $-x\,|x| > 0$ means x is negative so $3 - x$ is positive so y is negative. … **Suff**.

5. A, B, C will give infinite graphs. D will give a graph that will consist of two line segments …

__–5__________–2___0_______2__________5_

The highlighted portion above gives the graph for D.

E gives $2 \le 3x+4 \le 6$ … subtract 4 from each … $-2 \le 3x \le 2$ or $-2/3 \le x \le 2/3$ … this will be a single finite line (line segment).

________–2/3_______**0**________+2/3_________

Top 1% expert replies to student queries (can skip)

x^3≤27 gives x≤3, not x = 3.

This images could help:

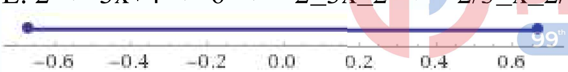
A. x^4 >= 1 --> x≤−1 or x≥1: two infinite ranges;

B. x^3 <= 27 --> x≤3: one infinite range;

C. x^2 >= 16 --> x≤−4 or x≥4: two infinite ranges;

D. 2 <= |x| <= 5 --> −5≤x≤−2 or 2≤x≤5: two finite ranges;

E. 2 <= 3x+4 <= 6 --> −2≤3x≤2 --> −2/3≤x≤2/3: one finite range.



OR

The key words in the stem are: "a single line segment of finite length"

Now, answer choices A, B, and C can not be correct answers as solutions sets for these exponential functions are not limited at all (>= for even powers and <= for odd power) and thus cannot be finite (x can go to + or -infinity for A and C and x can got to -infinity for B). As for D: We have that absolute value of x is between two positive values, thus the solution set for x (because of absolute value) will be two line segments which will be mirror images of each other.

Answer: E.

6.

(1) INSUFFICIENT: We know that the sum of x and y is 20. Here are two possible scenarios, yielding different answers to the question:

X	y	Sum	Average of x and $\|y\|$
10	10	20	10
25	–5	20	15

(2) INSUFFICIENT: We know that $|x + y| = 20$. The same scenarios listed for statement (1) still apply here. There is more than one possible value for the average of x and $|y|$,

(1) AND (2) INSUFFICIENT: We *still* have the same scenarios listed above. Since there is more than one possible value for the average of x and $|y|$, both statements taken together are NOT sufficient.
The correct answer is E.

Top 1% expert replies to student queries (can skip)

1. if x=30 and y=-10, then average of x and |y| is (x + |y|)/2, i.e {30 +|-10|} / 2 =30+10 /2 =20

2. On the GMAT, the radical sign always indicates the positive square root, but a squared variable has both positive and negative roots.
E.g.
√25 is always 5(only the positive value), i.e. √25 = 5 (not -5)
but
if (x)^2 = 25, then x can be both 5 and -5.
The correct answer is E.

7.
If $a - b = x$, the question becomes
Is $1/x < -x$
This is valid only if x is negative.
Substitute x as –ve / 0 / +ve to verify.
So, the question becomes: is x negative?

(1) if $a < b$, $a - b < 0$ or $x < 0$, sufficient.

(2) $|a - b| > 1$ means $|x| > 1$, so x can be 2 or –2, so x can be positive or negative. **NS**
Ans. A

8.
We can rephrase the question: "Is $-1 < x < 1$?"

(1) $|a| = |b|$ means $a = b$ or $a = -b$. So, we have

$x + 1 = 2(x - 1)$ or $x = 3$

OR

$x + 1 = 2[-(x - 1)]$ or $x = 1/3$.

If $x = 1/3$, $|x| < 1$, but if $x = 3$, $|x| > 1$. Thus, we cannot answer the question.

(2) INSUFFICIENT: $|x - 3| > 0$ means $x \neq 3$. **Refer to the concepts.**

This does not answer the question as to whether x is between –1 and 1. x could be ½ and $|x| < 1$ or x could be 10 and $|x| > 1$.

(1) AND (2) SUFFICIENT: According to statement (1), x can be 3 or 1/3.
According to statement (2), x cannot be 3. Thus, using both statements, we know that $x = 1/3$ which IS between –1 and 1.
Ans. C

Alternate sol from gmatclub (additional)

Is $|x| < 1$?

Is $|x| < 1$, means is x in the range (-1,1) or is $-1 < x < 1$ true?

(1) $|x+1| = 2|x-1|$

Two key points: $x = -1$ and $x = 1$ (key points are the values of x when absolute values equal to zero), thus three ranges to check:

---------{-1}--------{1}---------

A. $x < -1$ (blue range) --> $|x+1| = 2|x-1|$ becomes: $-x-1 = 2(-x+1)$ --> $x = 3$, not OK, as this value is not in the range we are checking ($x < -1$);

B. $-1 \le x \le 1$ (green range) --> $|x+1| = 2|x-1|$ becomes: $x+1 = 2(-x+1)$ --> $x = \frac{1}{3}$. OK, as this value is in the range we are checking ($-1 \le x \le 1$);

C. $x > 1$ (red range) --> $|x+1| = 2|x-1|$ becomes: $x+1 = 2(x-1)$ --> $x = 3$. OK, as this value is in the range we are checking ($x > 1$).

So we got TWO values of x (two solutions): $\frac{1}{3}$ and 3, first is in the range (-1,1) but second is out of the range. Not sufficient.

(2) $|x-3| \ne 0$

Just says that $x \ne 3$. But we don't know whether x is in the range (-1,1) or not.

(1)+(2) $x = \frac{1}{3}$ or $x = 3$ AND $x \ne 3$ --> means x can have only value $\frac{1}{3}$, which is in the range (-1,1). Sufficient.

Answer: C.

9.

You may try (a, b) as (2, 1), (–2, –1) and (–2, 1) … you will see that $|a| + |b| > |a + b|$ is true only for (–2, 1).

So, for $|a| + |b| > |a + b|$ to be true, a and b must have opposite signs. If a and b have the same signs (i.e. both positive or both negative), the expressions on either side of the inequality will be the same. The question is really asking if a and b have opposite signs.

(1) INSUFFICIENT: This tells us that $|a| > |b|$. This implies nothing about the signs of a and b.

(2) INSUFFICIENT: Since the absolute value of a is always positive, this tells us that $b < 0$. Since we don't know the sign of a, we can't answer the question.

(1) AND (2) INSUFFICIENT: We know the sign of b from statement 2 but statement 1 does not tell us the sign of a. For example, if $b = –4$, a could be 5 or –5.

The correct answer is E

10. Assume a number line with a, y, z, b in the increasing order as plotted below:

__a____________y_________z_____________b_________

D1 D2 D3

$|y - a|$ = distance between y and a = D1
$|z - y|$ = distance between z and y = D2
$|z - b|$ = distance between z and b = D3

We also know that D1, D2, D3 are all positive.

So, the question translates to: "Is D1 < D2 + D3?"

(1) gives D1 + D2 < D3 so D1 < D3 – D2 … this automatically means D1 < D3 + D2 … sufficient.
(imagine 1 < 5 – 3 … so obviously 1 < 5 + 3)

(2) gives D1 < D3 … this automatically means D1 < D2 + D3 … sufficient.
(imagine 1 < 2 … so obviously 1 < 2 + 3)

Ans. D

11.

(1)
Let's take (x, y) as (4, 2) … we get 2 = 2 … so we get NO for the main question.

Let's take (x, y) as (4, –2) … we get 6 > 2 … so we get YES for the main question.

Not sufficient.

(2) means x and y are of opposite signs.

In this case, $|x - y|$ will result in addition of x and y (overall positive sign) but $|x| - |y|$ will result in a subtraction of 2 positive quantities … hence LHS will always be bigger than RHS.

Take all possible cases
(1, –2) we get 3 > –1
(2, –1) we get 3 > 1
(–1, 2) we get 3 > –1
(–2, 1) we get 3 > 1.

Sufficient … Ans. B

Top 1% expert replies to student queries (can skip)

Statement 1 is Not sufficient, but statement 2 is sufficient.

LHS = |x-y|; RHS = |x| - |y|
1) if y<x
Case I: x<0 => y<0 --> LHS = RHS
Case II: x>0, y>0 but <x --> LHS = RHS
Case III: x>0, y<0 --> LHS > RHS
Hence 1) alone is not sufficient.
2) if x*y <0 => either x or y <0 and other has to be >0
Case I: x<0, y>0 --> LHS > RHS
Cae II: x>0, y<0 --> LHS > RHS
No other case.
Hence, 2) alone is sufficient.
Answer: B

12. Assume ab = x, so |x| > x, means x is negative.

So, |ab| > ab, *ab* must be negative.

We can rephrase this question: "Is *ab* < 0?"

I. UNCERTAIN: We know nothing about the sign of *b*.

II. UNCERTAIN: We know nothing about the sign of *a*.

III. TRUE: This answers the question directly.

The correct answer is C.

13.

It is extremely tempting to divide both sides of this inequality by y or by the |y|, to comeup with a rephrased question of –is $x > y$?‖ However, we do not know the sign of y, so this cannot be done.

(1) INSUFFICIENT: On a yes/no data sufficiency question that deals with number properties (positive/negatives), it is often easier to plug numbers. There are two good reasons why we should try both positive and negative values for y: (1) the question contains the expression |y|, (2) statement 2 hints that the sign of y might be significant.If we do that we come up with both a yes and a no to the question.

x	y	$x \cdot \lvert y \rvert > y^2$	?
-2	-4	$-2(4) > (-4)^2$	N
4	2	$4(2) > 2^2$	Y

(2) INSUFFICIENT: Using the logic from above, when trying numbers here we should take careto pick x values that are both greater than y and less than y.

x	y	$x \cdot \lvert y \rvert > y^2$	?
2	4	$2(4) > 4^2$	N
4	2	$4(2) > 2^2$	Y

(1) AND (2) SUFFICIENT: If we combine the two statements, we must choose positive x and y values for which $x > y$.

x	y	$x \cdot \lvert y \rvert > y^2$	?
3	1	$3(1) > 1^2$	Y
4	2	$4(2) > 2^2$	Y
5	3	$5(3) > 3^2$	Y

14.

We can rephrase the question by manipulating it algebraically:

$(|x^{-1} * y^{-1}|)_{-1} > xy$

$(|1/x * 1/y|)^{-1} > xy$

$(|1/xy|)^{-1} > xy$

$1/(|1/(xy)|) > xy$

Is $|xy| > xy$? If xy = z, then this is asking, is $|z| > z$? Or is z negative?

Or is $xy < 0$? Or do x and y have opposite signs?

(1) SUFFICIENT: If $xy > 1$, xy is definitely positive. **Confirmed NO**

(2) INSUFFICIENT: $x^2 > y^2$

Algebraically, this inequality reduces to $|x| > |y|$. This tells us nothing about the sign of x and y. They could have the same signs or opposite signs.

The correct answer is A

15.

$x / |x| = 1$ if x is positive and $x/ |x| = –1$ if x is negative.

$x > x / |x|$ means

$x > 1$ if x is positive … let's say x = 2, 3, 4 …

and

$x > –1$ if x is negative … so $–1 < x < 0$, if x is negative. For example, $x = –1/2$. **Note the range can't be beyond 0.**

(1) $x = – ½$ and $x = 2$ both satisfy (1) but with $x = – ½$, $|x| < 1$ and with $x = 2$, $|x| > 1$ so insufficient.

(2) means x is negative. With $x = – ½$, $|x| < 1$ and with $x = –2$, $|x| > 1$ so insufficient.

Combining, x has to be negative and any value less than –1 (such as –2, –3 etc.) don't satisfy (1), so the combination will mean that x can lie only between –1 and 0.

$–1 < x < 0$. In this case, $|x| < 1$ always.
Ans. C

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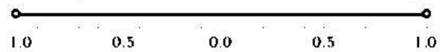
Summary:
Range from (1): -----(-1)----(0)----(1)---- $-1<x<0$ or $x>1$, green area;
Range from (2): -----(-1)----(0)----(1)---- $x<0$, blue area;
From (1) and (2): ----(-1)----(0)----(1)---- $-1<x<0$, common range of x from (1) and (2) (intersection of ranges from (1) and (2)), red area.

Detailed explanation:

Is $|x| < 1$?

Is $-1 < x < 1$? ($x \neq 0$)

So, the question asks whether x is in the range shown below:



(1) $\frac{x}{|x|} < x$

Two cases:

A. $x < 0$ --> $\frac{x}{-x} < x$ --> $-1 < x$. But remember that $x < 0$, so $-1 < x < 0$

B. $x > 0$ --> $\frac{x}{x} < x$ --> $1 < x$.

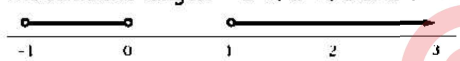
Two ranges $-1 < x < 0$ or $x > 1$. Which says that x either in the first range or in the second. Not sufficient to answer whether $-1 < x < 1$. (For instance x can be -0.5 or 3)

Second approach: look at the fraction $\frac{x}{|x|}$ it can take only two values:

1 for $x > 0$ --> so we would have: $1 < x$;

Or -1 for $x < 0$ --> so we would have: $-1 < x$ and as we considering the range for which $x < 0$ then completer range would be: $-1 < x < 0$.

The same two ranges: $-1 < x < 0$ or $x > 1$:

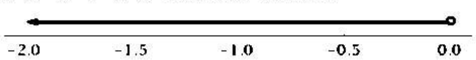


(2) $|x| > x$. Well this basically tells that x is negative, as if x were positive or zero then $|x|$ would be equal to x. Only one range: $x < 0$, but still insufficient to say whether $-1 < x < 1$. (For instance x can be -0.5 or -10)

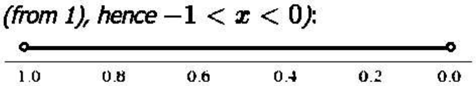
Or consider two cases again:

$x < 0$--> $-x > x$--> $x < 0$.

$x > 0$ --> $x > x$: never correct.



(1)+(2) Intersection of the ranges from (1) and (2) is the range $-1 < x < 0$ *($x < 0$ (from 2) and $-1 < x < 0$ or $x > 1$ (from 1), hence $-1 < x < 0$)*:



Every x from this range is definitely in the range $-1 < x < 1$. So, we have a definite YES answer to the question. Sufficient.

Answer: C.

16.

The question is: "Is –4 < *n* < 4?" (*n* is not equal to 0)

(1) SUFFICIENT: The solution to this inequality is *n* > 4 (if *n* > 0) or *n* < –4 (if *n* < 0). This provides us with enough information to guarantee that *n* is definitely NOT between –4 and 4. Remember that an absolute no is sufficient!

(2) INSUFFICIENT: n can be any negative value. This is already enough to show that the statement is insufficient because n might not be between –4 and 4.

The correct answer is A.

17.
$|x| = -x$, means x is negative, $|y| = -y$ means y is negative. So, x + y will also be negative.
Ans. B

18.

(1) Sufficient

$x + 3 = 4x - 3$ or $x = 2$ … valid solution

$-(x + 3) = 4x - 3$ or $x = 0$ … invalid solution.

We know that 2 is the only solution possible and we can say that x is definitely positive. OR

$4x - 3 \geq 0$, so $x \geq 3/4$, so $x = 0$ is invalid.

(2) INSUFFICIENT:

$x - 3 = 2x - 3$ so $x = 0$ (valid solution)

$x - 3 = -(2x - 3)$ … $x = 2$ (valid solution)

Therefore, both 2 and 0 are valid solutions and we cannot determine whether x is positive, since one value of x is zero, which is not positive, and one is positive.
The correct answer is A.

19.

Note that the question is asking for the absolute value of x rather than just the value of x. Keep this in mind when you analyse each statement.

(1) SUFFICIENT: Since the value of x^2 must be non–negative, the value of $(x^2 + 16)$ is always positive, therefore $|x^2 + 16|$ can be written $x^2 + 16$. Using this information, we can solve for x:
$|x^2 + 16| - 5 = 27$
$x^2 + 16 - 5 = 27$
$x^2 + 11 = 27$
$x^2 = 16$
$x = 4$ or $x = -4$

Since $|-4| = |4| = 4$, we know that $|x| = 4$; this statement is sufficient.

(2) SUFFICIENT:
$x^2 = 8x - 16$
$x^2 - 8x + 16 = 0$
$(x - 4)^2 = 0$
$(x - 4)(x - 4) = 0$
$x = 4$

Therefore, $|x| = 4$; this statement is sufficient. The correct answer is D.

20.

We may write $r^2 = |r| * |r|$
And as r isn't 0, |r| will always be positive
So, we can cancel |r| from numerator and denominator
So, the question becomes: Is $|r| < 1$?

(1) r can be –1/2 or 10, so |r| can be 1/2 and 10 … NS
(2) r can be 1/2 or –10, so |r| can be 1/2 and 10 … NS

(1) AND (2) SUFFICIENT: $-1 < r < 1$, so $|r| < 1$ always
The correct answer is C.

Alternate sol from gmatclub (additional)

Since $|r|$ is always positive, we can multiply both sides of the inequality by $|r|$ and rephrase the question as: Is $r^2 < |r|$? The only way for this to be the case is if r is a nonzero fraction between -1 and 1.

(1) **INSUFFICIENT:** This does not tell us whether r is between -1 and 1. If $r = -\frac{1}{2}$, $|r| = \frac{1}{2}$ and $r^2 = \frac{1}{4}$, and the answer to the rephrased question is **YES**. However, if $r = 4$, , $|r| = 4$and $r^2 = 16$, and the answer to the question is **NO.**

(2) **INSUFFICIENT:** This does not tell us whether r is between -1 and 1. If $r = \frac{1}{2}$, $|r| = \frac{1}{2}$ ans $r^2 = \frac{1}{4}$, and the answer to the rephrased question is **YES**. However, if $r = -4$, $|r| = 4$ and $r^2 = 16$, and the answer to the question is **NO.**

(1) AND (2) **SUFFICIENT:** Together, the statements tell us that r is between -1 and 1. The square of a proper fraction (positive or negative) will always be smaller than the absolute value of that proper fraction.

The correct answer is C.

21.
(1) INSUFFICIENT:

$-4 < x + 3 < 4$

Subtract 3

$-7 < x < 1$

(2) INSUFFICIENT:

$-4 < x - 3 < 4$

Add 3

$-1 < x < 7$

If we combine the solutions from statements (1) and (2) we get an overlapping range of $-1 < x < 1$.
We still can't tell whether x is positive.
The correct answer is E.

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Question: Is x positive?

When $|x| <1$
$\Rightarrow -1<x<1$ (explained in the basic videos)
Statement 1: so, when $|x+3|<4$
$\Rightarrow -4 < (x+3) <4$

=> $-7 < x < 1$
(x can be positive or negative)
Not Sufficient

Similarly for Statement 2: $-4 < (x-3) < 4$
=> $-1 < x < 7$
(x can be positive or negative)
Not sufficient

Combining (we want those values of x which satisfies both inequalities; hence we need intersection of both the solutions)

-7—————— {(-1) ————— 1} —————-7

The common solution would be $-1 < x < 1$
Again, x can be positive or negative. (We are combining 2 ranges not 4)
The correct answer is E.

22.
(1) $|3x – 7| = 2x + 2$

So, either $3x – 7 = 2x + 2$ or $3x – 7 = – (2x+2)$, so x may be 1 or 9.

$\sqrt{x} = 1$ or 3 … Since 1 is not a prime number, but 3 is a prime number, it is NOT possible to answer the original question using statement one, alone.

(2) $x^2 = 9x$ so $x = 0$ or 9. So $\sqrt{x} = 0$ or 3. Since 0 is not a prime number, but 3 is a prime number, it is NOT possible to answer the original question using statement two, alone.

(1) and (2)
$\sqrt{x} = 3$
Ans. C

23.
$625 = 25^2$, so you immediately know one of the possibilities. x could be 25, and k could be -2.
Also, notice that 25 can be factored down more. $25^2 = 5^4$. So, finally, x could be 5, and k could be -4.

List all the possibilities on your paper (Note: cases like x=-5, k=-4 aren't possible since we are given that x>k):
$x = 25, k = -2$
$x = 625, k = -1$
$x = 5, k = -4$
2 is the only prime value for k in our list. So, if we know that |k| is prime, then the first possibility is the only one that works.
So, $x = 25$

24.
$(x − 1) = 3x + 3$ or $x = −2$ (invalid root)

$−(x − 1) = 3x + 3$ or $x = −1/2$ (valid root)

Ans. C

Top 1% expert replies to student queries (can skip)

(x − 1) = 3x + 3 or x = −2 (invalid root)
After getting x = -2, Plug in the value of x in the equation to cross-verify.
$|x-1|$= 3x + 3
$|-2-1|$=(3*-2) + 3
$|-3|$= -6 +3
$|-3|$= -3
'Mod' of any value cannot be zero. Hence, it is invalid.
$|-3|$= -3
3 = -3 (therefore, x cannot be -2)
−(x − 1) = 3x + 3 or x = −1/2 (valid root)

Ans. C

25.

	a	*b*		*Is 1/(a–b)>ab?*
Case 1	–2	1	✓	$\frac{1}{-3} > -2$? YES
Case 2	–2	–1	✓	$\frac{1}{-1} > 2$? NO

Ans. E

Top 1% expert replies to student queries (can skip)

For these kinds of problems, use number plugging at one point or another.
If a is not equal to b, is 1/(a-b) > ab?
(1) |a| > |b|. This statement implies that a is further from 0 then b. We can have 4 cases:
--------0--b--a--
-----b--0-----a--
--a-----0--b-----
--a--b--0--------
For the second case the LHS is positive, while RHS is negative: 1/(a-b) > ab;
For the fourth case the LHS is negative, while RHS is positive: 1/(a-b) < ab.
Two different answers. Not sufficient.
(2) a < b --> a - b < 0. The LHS is negative:
If a=-2 and b=1, then (1/(a-b)=-1/3) > (ab=-2);
If a=-2 and b=-1, then (1/(a-b)=-1) < (ab=2).
Two different answers. Not sufficient.
(1)+(2) We can have only the third or fourth cases from (1):
--a-----0--b-----
--a--b--0--------
We can use the same example as for (2):
If a=-2 and b=1, then (1/(a-b)=-1/3) > (ab=-2);
If a=-2 and b=-1, then (1/(a-b)=-1) < (ab=2).
Two different answers. Not sufficient.
Answer: E.

26.

(1)

It is given that $t - q = |t - s| + |s - q|$, which can be rewritten without absolute values in four mutually exclusive and collectively exhaustive cases by making use of the algebraic definition of absolute value. Recall that $|x| = x$ if $x > 0$, and $|x| = -x$ if $x < 0$. Thus, for example, if $t - s < 0$, then $|t - s| = -(t - s)$.

Case 1: $t > s$ and $s > q$. In this case, $t - s > 0$ and $s - q > 0$, and so $t - q = |t - s| + |s - q|$ is equivalent to $t - q = (t - s) + (s - q)$, which is an identity. Therefore, the case for which $t > s$ and $s > q$ is consistent with the given information and the assumption $t - q = |t - s| + |s - q|$.

Case 2: $t > s$ and $s < q$. In this case, $t - s > 0$ and $s - q < 0$, and so $t - q = |t - s| + |s - q|$ is equivalent to $t - q = (t - s) - (s - q)$, or $s = q$, which is not consistent with the assumption that q, s, and t are all different numbers. Therefore, the case for which $t > s$ and $s < q$ is not consistent with the given information and the assumption $t - q = |t - s| + |s - q|$.

Case 3: $t < s$ and $s > q$. In this case, $t - s < 0$ and $s - q > 0$, and so $t - q = |t - s| + |s - q|$ is equivalent to $t - q = -(t - s) + (s - q)$, or $t = s$, which is not consistent with the assumption that q, s, and t are all different numbers. Therefore, the case for which $t < s$ and $s > q$ is not consistent with the given information and the assumption $t - q = |t - s| + |s - q|$.

Case 4: $t < s$ and $s < q$. In this case, $t - s < 0$ and $s - q < 0$, and so $t - q = |t - s| + |s - q|$ is equivalent to $t - q = -(t - s) - (s - q)$, or $t = q$, which is not consistent with the assumption that q, s, and t are all different numbers. Therefore, the case for which $t < s$ and $s < q$ is not consistent with the given information and the assumption $t - q = |t - s| + |s - q|$. The only case that is consistent with the given information and the assumption $t - q = |t - s| + |s - q|$ is Case 1. Therefore, it follows that $t > s$ and $s > q$, and this implies $q < s < t$; SUFFICIENT.

(2)

Given that $t > q$, it is possible that $q < s < t$ is true (for example, when s is between t and q) and it is possible that $q < s < t$ is false (for example, when s is greater than t); NOT sufficient. **Ans. A**

27.

Square of any number is positive (if x is positive or negative) or zero (if x is zero).
This means x^2 is positive or zero.
Denominator: -(x^2) -3 = -(positive) -3 OR -(zero) -3 = -(Positive+3) OR -(zero+3) = -positive OR -positive = negative OR negative
Irrespective of the value of x, [-(x^2) -3] will always be negative.

Since the absolute value of any real number is greater than or equal to zero, it follows that $|3x - 5| \geq 0$. Also, for any real number x we have $x^2 \geq 0$, and hence $-x^2 \leq 0$. Subtracting 3 from both sides of the last inequality gives $-x^2 - 3 \leq -3$. Therefore, the numerator of the expression for y is greater than or equal to zero and the denominator of the expression for y is negative. It follows that the value of y cannot be greater than 0. However, the value of y is equal to 0 when $|3x - 5| = 0$, or $3x - 5 = 0$, or $x = \frac{5}{3}$. Therefore, the value of x for which the value of y is greatest (i.e., when $y = 0$) is $x = \frac{5}{3}$.

The correct answer is E.

28.
The midpoint of the interval from -8 to 4, inclusive, is -2 and the length of the interval from -8 to 4, inclusive, is $4 - (-8) = 12$, so the interval consists of all numbers within a distance of $12/2 = 6$ from -2. Using an inequality involving absolute values, this can be described by $|x - (-2)| \le 6$, or $|x + 2| \le 6$. Alternatively, the inequality $-8 \le x \le 4$ can be written as the conjunction $-8 \le x$ and $x \le 4$. Rewrite this conjunction so that the lower value, -8, and the upper value, 4, are shifted to values that have the same magnitude. This can be done by adding 2 to each side of each inequality, which gives $-6 \le x + 2$ and $x + 2 \le 6$. Thus, $x + 2$ lies between -6 and 6, inclusive, and it follows that $|x + 2| \le 6$.
The correct answer is E

29.

From $|y| \le 12$, if y must be an integer, then y must be in the set $S = \{\pm12, \pm11, \pm10, \ldots, \pm3, \pm2, \pm1, 0\}$. Since $2x + y = 12$, then $x = (12 - y) / 2$. If x must be an integer, then $12 - y$ must be divisible by 2; that is, $12 - y$ must be even. Since 12 is even, $12 - y$ is even if and only if y is even. This eliminates all odd integers from S, leaving only the even integers $\pm12, \pm10, \pm8, \pm6, \pm4, \pm2$, and 0. Thus, there are 13 possible integer y–values, each with a corresponding integer x–value and, therefore, there are 13 ordered pairs (x, y), where x and y are both integers, that solve the system.
The correct answer is D.

30. C
(1)
$a = 4, b = 2$, then $|a| > |b|$
$a = 2, b = -10$, then $|a| < |b|$
NS
(2)
$a = -3, b = -4$, then $|a| < |b|$ a
$= -4, b = -3$, then $|a| > |b|$
NS

Combined:
$a > b$ and $a + b < 0$
Or $a - b > 0$ and $a + b < 0$
Multiplying
$a^2 - b^2 < 0$ or $a^2 < b^2$ or $|a| < |b|$
SUFF

OR

Both a and b can be negative or a can be positive and b can be negative.
If a and b are both negative
We have to satisfy $a > b$ and $a + b < 0$
imagine $a = -3$ and $b = -4$
then $|a| < |b|$
But
If a is positive and b is negative, we have to satisfy $a > b$ and $a + b < 0$
Imagine $a = 2$ and $b = -3$
then also $|a| < |b|$
Sufficient
Ans. C

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Statement 1:
"Multiply both sides by (a - b) then (b - a)" but we don't know if these two terms are positive or negative. We wouldn't know if the inequality should result in ">" or "<", thus we wouldn't be allowed to

"move" these terms without knowing the signs.
However, we do know (a - b) and (b - a) have OPPOSITE signs so no matter what the result is $b-a<a-b$ for this expression and we get $b<a$.

Statement 2:
$a + b < 0$ --> $a<-b$

Combine 1 and 2:-
We may write statement 2 as $a<-b$. Combining $b<a$ from statement 1 we have $b<a<-b$ which means b must be negative.
Since a is between both b's, a's magnitude must be less than that of b, which we can translate to $|a|<|b|$.
Sufficient.
Ans: C