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GMAT Quant Topic 1

General Arithmetic

Part A: Overlapping SETS

1. Of the films Empty Set Studios released last year, 60% were comedies and the rest were horror films. 75% of the comedies were profitable, but 75% of the horror moves were unprofitable. If the studio made a total of 40 films, and broke even on none of them, how many of their films were profitable?
18 19 20 21 22

2. At a certain hospital, 75% of the interns receive fewer than 6 hours of sleep and report feeling tired during their shifts. At the same time, 70% of the interns who receive 6 or more hours of sleep report no feelings of tiredness. If 80% of the interns receive fewer than 6 hours of sleep, what percent of the interns report no feelings of tiredness during their shifts?
6 14 19 20 81

3. All of the students of Music High School are in the band, the orchestra, or both. 80 percent of the students are in only one group. There are 119 students in the band. If 50 percent of the students are in the band only, how many students are in the orchestra only?
30 51 60 85 119

4. How many attendees are at a convention if 150 of the attendees are neither female nor students, one-sixth of the attendees are female students, two-thirds of the attendees are female, and one-third of the attendees are students?
300 450 600 800 900

5. Eighty percent of the lights at Hotel California are on at 8 p.m. a certain evening. However, forty percent of the lights that are supposed to be off are actually on and ten percent of the lights that are supposed to be on are actually off. What percent of the lights that are on are supposed to be off?
22(2/9)% 16(2/3)% 11(1/9)% 10% 5%

6. Of the 645 speckled trout in a certain fishery that contains only speckled and rainbow trout, the number of males is 45 more than twice the number of females. If the ratio of female speckled trout to male rainbow trout is 4:3 and the ratio of male rainbow trout to all trout is 3:20, how many female rainbow trout are there?
192 195 200 205 208

7. 30% of major airline companies equip their planes with wireless internet access. 70% of major airlines offer passengers free on-board snacks. What is the greatest possible percentage of major airline companies that offer both wireless internet and free on-board snacks?
21% 30% 40% 50% 70%

8. In country Z, 10% of the people do not have a university diploma but have the job of their choice, and 25% of the people who do not have the job of their choice have a university diploma. If 40% of the people have the job of their choice, what percent of the people have a university diploma?
35% 45% 55% 65% 75%

9. Seventy percent of the 800 students in School T are male. At least ten percent of the female students in School T participate in a sport. Fewer than thirty percent of the male students in School T do not participate in a sport. What is the maximum possible number of students in School T who do not participate in a sport?
216 383 384 416 417

10. 75% of the guestrooms at the Stagecoach Inn have a queen-sized bed, and each of the remaining rooms has a king-sized bed. Of the non-smoking rooms, 60% have a queen-sized bed. If 10% of the rooms at the Stagecoach Inn are non-smoking rooms with king-sized beds, what percentage of the rooms permit smoking?
25% 30% 50% 55% 75%

11. At the end of the day, February 14th, a florist had 120 roses left in his shop, all of which were red, white or pink in color and either long or short-stemmed. A third of the roses were short-stemmed, 20 of which were white and 15 of which were pink. The percentage of pink roses that were short-stemmed equalled the percentage of red roses that were short-stemmed. If none of the long-stemmed roses were white, what percentage of the long-stemmed roses were red?
20% 25% 50% 75% 80%

12. 3/8 of all students at Social High are in all three of the following clubs: Albanian, Bardic, and Checkmate. 1/2 of all students are in Albanian, 5/8 are in Bardic, and 3/4 are in Checkmate. If every student is in at least one club, what fraction of the student body is in exactly 2 clubs?
(A) 1/8 (B) 1/4 (C) 3/8 (D) 1/2 (E) 5/8

13. The waiter at an expensive restaurant has noticed that 60% of the couples order dessert and coffee. However, 20% of the couples who order dessert don't order coffee. What is the probability that the next couple the waiter seats will not order dessert?
20% 25% 40% 60% 75%

14. 50% of the apartments in a certain building have windows and hardwood floors. 25% of the apartments without windows have hardwood floors. If 40% of the apartments do not have hardwood floors, what percent of the apartments with windows have hardwood floors?
10% 16.66% 40% 50% 83.33%

15. A farmer has an apple orchard consisting of Fuji and Gala apple trees. Due to high winds this year 10% of his trees cross pollinated. The number of his trees that are pure Fuji plus the cross-pollinated ones totals 187, while 3/4 of all his trees are pure Fuji. How many of his trees are pure Gala?
22 33 55 77 88

16. In a group of 68 students, each student is registered for at least one of three classes – History, Math and English. Twenty-five students are registered for History, twenty-five students are registered for Math, and thirty-four students are registered for English. If only three students are registered for all three classes, how many students are registered for exactly two classes?
13 10 9 8 7

17. Each of the 59 members in a high school class is required to sign up for a minimum of one and a maximum of three academic clubs. The three clubs to choose from are the poetry club, the history club, and the writing club. A total of 22 students sign up for the poetry club, 27 students for the history club, and 28 students for the writing club. If 6 students sign up for exactly two clubs, how many students sign up for all three clubs?
2 5 6 8 9

18. Each of 435 bags contains at least one of the following three items: raisins, almonds, and peanuts. The number of bags that contain only raisins is 10 times the number of bags that contain only peanuts. The number of bags that contain only almonds is 20 times the number of bags that contain only raisins and peanuts. The number of bags that contain only peanuts is one-fifth the number of bags that contain only almonds. 210 bags contain almonds. How many bags contain only one kind of item?
256 260 316 320 350

19. What percent of the students at Jefferson High School study French but not Spanish?
(1) 30% of all students at Jefferson High School study French.
(2) 40% of all students at Jefferson High School do not study Spanish.

20. If none of the students are ambidextrous, what percentage of the 20 students in Mr. Henderson's class are left-handed?
(1) Of the 12 girls in the class, 25% are left-handed.
(2) 5 of the boys in the class are right-handed.

21. Guests at a recent party ate a total of fifteen hamburgers. Each guest who was neither a student nor a vegetarian ate exactly one hamburger. No hamburger was eaten by any guest who was a student, a vegetarian, or both. If half of the guests were vegetarians, how many guests attended the party?
(1) The vegetarians attended the party at a rate of 2 students to every 3 non-students, half the rate for non-vegetarians.
(2) 30% of the guests were vegetarian non-students.

22. To receive a driver license, sixteen year-olds at Culliver High School have to pass both a written and a practical driving test. Everyone has to take the tests, and no one failed both tests. If 30% of the 16 year-olds who passed the written test did not pass the practical, how many sixteen-year-olds at Culliver High School received their driver license?
(1) There are 188 sixteen year-olds at Culliver High School.
(2) 20% of the sixteen year-olds who passed the practical test failed the written test.

23. At a charity fundraiser, 180 of the guests had a house both in the Hamptons and in Palm Beach. If not everyone at the fundraiser had a house in either the Hamptons or Palm Beach, what is the ratio of the number of people who had a house in Palm Beach but not in the Hamptons to the number of people who had a house in the Hamptons but not in Palm Beach?
(1) One-half of the guests had a house in Palm Beach.
(2) Two-thirds of the guests had a house in the Hamptons

24. Recently Mary gave a birthday party for her daughter at which she served both chocolate and strawberry ice cream. There were 8 boys who had chocolate ice cream, and nine girls who had strawberry. Everybody there had some ice cream, but nobody tried both. What is the maximum possible number of girls who had some chocolate ice cream?
1) Exactly thirty children attended the party.
2) Fewer than half the children had strawberry ice cream.

25. Many of the students at the International School speak French or German or both. Among the students who speak French, four times as many speak German as don't. In addition, 1/6 of the students who don't speak German do speak French. What fraction of the students speak German?
(1) Exactly 60 students speak French and German.
(2) Exactly 75 students speak neither French nor German.

26. Each member of a pack of 55 wolves has either brown or blue eyes and either a white or a grey coat. If there are more than 3 blue-eyed wolves with white coats, are there more blue-eyed wolves than brown-eyed wolves?
(1) Among the blue-eyed wolves, the ratio of grey coats to white coats is 4 to 3.
(2) Among the brown-eyed wolves, the ratio of white coats to grey coats is 2 to 1.

27. What percentage of the current fourth graders at Liberation Elementary School dressed in costume for Halloween for the past two years in a row (both this year and last year)?
(1) 60% of the current fourth graders at Liberation Elementary School dressed in costume for Halloween this year.
(2) Of the current fourth graders at Liberation Elementary School who did not dress in costume for Halloween this year, 80% did not dress in costume last year.

28. Of all the houses on Kermit Lane, 20 have front porches, 20 have front yards, and 40 have back yards. How many houses are on Kermit Lane?
(1) No house on Kermit Lane is without a back yard.
(2) Each house on Kermit Lane that has a front porch does not have a front yard.

29. 55 people live in an apartment complex with three fitness clubs (A, B, and C). Of the 55 residents, 40 residents are members of exactly one of the three fitness clubs in the complex. Are any of the 55 residents members of both fitness clubs A and C but not members of fitness club B?
(1) 2 of the 55 residents are members of all three of the fitness clubs in the apartment complex.
(2) 8 of the 55 residents are members of fitness club B and exactly one other fitness club in the apartment complex.

30. At least 100 students at a certain high school study Japanese. If 4 percent of the students who study French also study Japanese, do more students at the school study French than Japanese?
(1) 16 students at the school study both French and Japanese.
(2) 10 percent of the students at the school who study Japanese also study French.

31. Set A, B, C have some elements in common. if 16 elements are in both A and B, 17 elements are in both A and C, and 18 elements are in both B and C, how many elements do all three of the sets A, B, and C have in common?
(1) Of the 16 elements that are in both A and B, 9 elements are also in C
2) A has 25 elements, B has 30 elements, and C has 35 elements.

32. Of the students who eat in a certain cafeteria, each student either likes or dislikes lima beans and each student either likes or dislikes Brussels sprouts. Of these students, 2/3 dislike lima beans; and of those who dislike lima beans, 3/5 also dislike Brussels sprouts. How many of the students like Brussels sprout but dislike lima beans?
(1) 120 students eat in the cafeteria.
(2) 40 of the students like lima beans.

Part B: Percentages

1. Two years ago, Arthur gave each of his five children 20 percent of his fortune to invest in any way they saw fit. In the first year, three of the children, Alice, Bob, and Carol, each earned a profit of 50 percent on their investments, while two of the children, Dave and Errol, lost 40 percent on their investments. In the second year, Alice and Bob each earned a 10 percent profit, Carol lost 60 percent, Dave earned 25 percent in profit, and Errol lost all the money he had remaining. What percentage of Arthur's fortune currently remains?
93% 97% 100% 107% 120%

2. A car dealership has 40 cars on the lot, 30% of which are silver. If the dealership receives a new shipment of 80 cars, 40% of which are not silver, what percent of the total number of cars are silver?
35% 37.5% 45% 47.5% 50%

3. Paul's income is 40% less than Rex's income, Quentin's income is 20% less than Paul's income, and Sam's income is 40% less than Paul's income. If Rex gave 60% of his income to Sam and 40% of his income to Quentin, Quentin's new income would be what fraction of Sam's new income?
11/12 13/17 13/19 12/19 11/19

4. A school's annual budget for the purchase of student computers increased by 60% this year over last year. If the price of student computers increased by 20% this year, then the number of computers it can purchase this year is what percent greater than the number of computers it purchased last year?
33.3% 40% 42% 48% 60%

5. Boomtown urban planners expect the city's population to increase by 10% per year over the next two years. If that projection were to come true, the population two years from now would be exactly double the population of one year ago. Which of the following is closest to the percent population increase in Boomtown over the last year?
20% 40% 50% 65% 75%

6. A retailer bought a shirt at wholesale and marked it up 80% to its initial retail price of $45. By how many more dollars does he need to increase the price to achieve a 100% markup?
1 2 3 4 5

7. A certain NYC taxi driver has decided to start charging a rate of r cents per person per mile. How much, in dollars, would it cost 3 people to travel x miles if he decides to give them a 50% discount?
3xr / 2 3x / 200r 3r / 200x 3xr / 200 xr / 600

8. Bob just filled his car's gas tank with 20 gallons of gasohol, a mixture consisting of 5% ethanol and 95% gasoline. If his car runs best on a mixture consisting of 10% ethanol and 90% gasoline, how many gallons of ethanol must he add into the gas tank for his car to achieve optimum performance?
9/10 1 10/9 20/19 2

9. Which of the following values is closest to 1/3 + 0.4 + 65%?
1.1 1.2 1.3 1.4 1.5

10. A certain tank is filled to one quarter of its capacity with a mixture consisting of water and sodium chloride. The proportion of sodium chloride in the tank is 40% by volume and the capacity of the tank is 24 gallons. If the water evaporates from the tank at the rate of 0.5 gallons per hour, and the amount of sodium chloride stays the same, what will be the concentration of water in the mixture in 2 hours?
43% 50% 52% 54% 56%

11. The useful life of a certain piece of equipment is determined by the following formula: $u = (8d)/h^2$, where u is the useful life of the equipment, in years, d is the density of the underlying material, in g/cm^3, and h is the number of hours of daily usage of the equipment. If the density of the underlying material is doubled and the daily usage of the equipment is halved, what will be the percentage increase in the useful life of the equipment?
300% 400% 600% 700% 800%

12. If m > 0, y > 0, and x is m percent of 2y, then, in terms of y, m is what percent of x?
y/200 2y 50y 50/y 5000/y

13. x% of y is increased by x%. What is the result in terms of x and y?
    A. 100xy + x
    B. xy + x/100
    C. 100xy + x/100
    D. 100xy + xy/100
    E. xy(x + 100)/10000

14. The manufacturer's suggested retail price (MSRP) of a certain item is $60. Store A sells the item for 20 percent more than the MSRP. The regular price of the item at Store B is 30 percent more than the MSRP, but the item is currently on sale for 10 percent less than the regular price. If sales tax is 5 percent of the purchase price at both stores, what is the result when the total cost of the item at Store B is subtracted from the total cost of the item at Store A?
    $0 $0.63 $1.80 $1.89 $2.10

15. Two years ago, Sam put $1,000 into a savings account. At the end of the first year, his account had accrued $100 in interest bringing his total balance to $1,100. The next year, his account balance increased by 10%. At the end of the two years, by what percent has Sam's account balance increased from his initial deposit of $1,000?
    19% 20% 21% 22% 25%

16. The price of a certain painting increased by 20% during the first year and decreased by 15% during the second year. The price of the painting at the end of the 2-year period was what percent of the original price?
    102% 105% 120% 135% 140%

17. If an item that originally sold for z dollars was marked up by x percent and then discounted by y percent, which of the following expressions represents the final price of the item?
    A. [10,000z + 100z(x – y) – xyz]/10000
    B. [10,000z + 100z(y – x) – xyz]/10000
    C. [100z(x – y) – xyz]/10000
    D. [100z(y – x) – xyz]/10000
    E. 10000 / [x – y]

18. A clock store sold a certain clock to a collector for 20 percent more than the store had originally paid for the clock. When the collector tried to resell the clock to the store, the store bought it back at 50 percent of what the collector had paid. The shop then sold the clock again at a profit of 80 percent on its buy-back price. If the difference between the clock's original cost to the shop and the clock's buy-back price was $100, for how much did the shop sell the clock the second time?
    $270 $250 $240 $220 $200

19. 90 students represent x percent of the boys at Jones Elementary School. If the boys at Jones Elementary make up 40% of the total school population of x students, what is x?
    125 150 225 250 500

20. Cindy has her eye on a sundress but thinks it is too expensive. It goes on sale for 15% less than the original price. Before Cindy can buy the dress, however, the store raises the new price by 25%. If the dress cost $68 after it went on sale for 15% off, what is the difference between the original price and the final price?
    $0.00 $1.00 $3.40 $5.00 $6.80

21. Jennifer has 60 dollars more than Brian. If she were to give Brian 1/5 of her money, Brian would have 25% less than the amount that Jennifer would then have. How much money does Jennifer have?
    40 100 120 140 180

22. The average computer price today is $700. If the average computer price three years ago was 80% of the average computer price today, what was the percentage increase in the average computer price over the past three years?
    15% 20% 25% 50% 80%

23. A small pool filled only with water will require an additional 300 gallons of water in order to be filled to 80% of its capacity. If pumping in these additional 300 gallons of water will increase the amount of water in the pool by 30%, what is the total capacity of the pool in gallons?
    1000 1250 1300 1600 1625

24. 0.2% of $(3/4)^2 \times (160 \div 10^{-2})$ =
    1.8×10^{-3} 1.8×10^{-2} 1.8 1.8×10 1.8×10^2

25. 0.35 represents what percent of 0.007?
0.05% 0.5% 5% 500% 5000%

26. The price of a certain property increased by 10% in the first year, decreased by 20% in the second year, and increased by 25% in the third year. What was the amount of the dollar decrease in the property price during the second year?
(1) The price of the property at the end of the third year was $22,000.
(2) The decrease in the property price over the first two years was $2,000 less than the increase in the property price during the third year.

27. A certain salesman's yearly income is determined by a base salary plus a commission on the sales he makes during the year. Did the salesman's base salary account for more than half of the salesman's yearly income last year?
(1) If the amount of the commission had been 30 percent higher, the salesman's income would have been 10 percent higher last year.
(2) The difference between the amount of the salesman's base salary and the amount of the commission was equal to 50 percent of the salesman's base salary last year.

28. In the month of June, a street vendor sold 10% more hot dogs than he sold in the month of May. How many total hot dogs did the vendor sell in May and June?
(1) The vendor sold 27 more hot dogs in June than in May.
(2) In July, the vendor sold 20% more hot dogs than he sold in May.

29. A sales associate earns a commission of 8% on her first $10,000 in sales revenue in a given week and a commission of 10% on any additional sales revenue that the associate generates that week. How much sales revenue did the associate generate last week?
(1) The sales associate earned a total of $1500 in commission last week.
(2) Last week, the sales associate was eligible for the 10% commission rate on $7000 worth of sales.

30. A certain football team played x games last season, of which the team won exactly y games. If tied games were not possible, how many games did the team win last season?
(1) If the team had lost two more of its games last season, it would have won 20 percent of its games for the season.
(2) If the team had won three more of its games last season, it would have lost 30 percent of its games for the season.

31. In 1994, Company X recorded profits that were 10% greater than in 1993, and in 1993 the company’s profits were 20% greater than they were in 1992. What were the company’s profits in 1992?
(1) In 1994, the company’s profits were $100,000 greater than in 1993.
(2) For every $3.00 in profits earned in 1992, Company X earned $3.96 in 1994.

32. In year x, it rained on 40% of all Mondays and 20% of all Tuesdays. On what percentage of all the weekdays in year x did it NOT rain?
(1) During year x, it rained on 10% of all Wednesdays.
(2) During year x, it did not rain on 70% of Thursdays and it did not rain on 95% of all Fridays.

33. The total cost of producing item X is equal to the sum of item X's fixed cost and variable cost. If the variable cost of producing X decreased by 5% in January, by what percent did the total cost of producing item X change in January?
(1) The fixed cost of producing item X increased by 13% in January.
(2) Before the changes in January, the fixed cost of producing item X was 5 times the variable cost of producing item X.

34. Of all the attendees at a dinner party, 40% were women. If each attendee arrived at the party either alone or with another attendee of the opposite sex, what percentage of the total number of attendees arrived at the party alone?
(1) 50% of the male attendees arrived with a woman.
(2) 25% of the attendees arriving alone were women.

35. What is 35 percent of a^b?
(1) b is 200 percent of a.
(2) 50 percent of b is a.

36. Three grades of milk are 1 percent, 2 percent, and 3 percent by volume. If x gallons of 1 percent grade, y gallons of 2 percent grade, z gallons of 3 percent grade are mixed to give x+y+z gallons of a 1.5 percent grade, what is x in terms of y and z?
A. y + 3z
B. (y + z)/4

C. $2y + 3z$
D. $3y + z$
E. $3y + 4.5z$

37. Whenever Martin has a restaurant bill with an amount between \$10 and \$99, he calculates the dollar amount of the tip as 2 times the tens digit of the amount of his bill. If the amount of Martin' most recent restaurant bill was between \$10 and \$99, was the tip calculated by Martin on this bill greater than 15 percent of the amount of the bill?
(1) The amount of the bill was between \$15 and \$30
(2) The tip calculated by Martin was \$8

38. Jack and Mark both received hourly wage increases of 6 percent. After the increases, Jack' hourly wage was how many dollars per hour more than Mark's?
(1) Before the wage increases, Jack's hourly wage is \$5 per hour more than Mark's
(2) Before the wage increases, the ratio of the Jack's hourly wage to Mark's hourly wage is 4 to 3.

39. A manufacture produced x percent more video cameras in 1994 than in 1993 and y percent more video cameras in 1995 than in 1994. If the manufacturer produced 1,000 video cameras in 1993, how many video cameras did the manufacturer produce in 1995?
(1) $xy=20$ (2) $x+y+xy/100 = 9.2$

40. What fraction of this year's graduation students at a certain college are males?
(1) Of this year's graduation students, 35% of male and 20% of female transferred from another college.
(2) Of this year's graduation students, 25% transferred from another college.

41. If y is greater than 110 percent of x, is y greater than 75?
(1) $x > 75$ (2) $y - x = 10$

42. At least 10 percent of the people in Country X who are 65 year old or older employed?
(1) In country X, 11.3 percent of the population is 65 year old or older
(2) In country X, of the population 65 year old or older, 20 percent of the men and 10 percent of the women are employed

43. In 1999 company X's gross profit was what percent of its revenue?
(1) In 1999 company X's gross profit was 1/3 of its expenses.
(2) In 1999 company X's expenses were 3/4 of its revenue.

44. Henry purchased 3 items during a sale. He received a 20 percent discount off the regular price of the most expensive item of a 10 percent discount off the regular price of each of the other 2 items. Was the total discount of these three items greater than 15 percent of the sum of the regular prices of the 3 items?
(1) The regular price of the most expensive item was \$50, and the regular price of the next most expensive item was \$20
(2) The regular price of the least expensive item was \$15

45. The rate of a certain chemical reaction is directly proportional to the square of the concentration of chemical A present and inversely proportional to the concentration of chemical B present. If the concentration of chemical B is increased by 100 percent, which of the following is closest to the percent change in the concentration of chemical A required to keep the reaction rate unchanged?
(A) 100% decrease
(B) 50% decrease
(C) 40% decrease
(D) 40% increase
(E) 50% increase

46. Of the 800 employees in a certain company, 70% have serviced more than 10 years. A number of y of those who have serviced more than 10 years will retire and no fresh employees join in. When is y if the 10 years employees become 60% of the total employees?
A. 200
B. 160
C. 112
D. 80
E. 56

47. Before being simplified, the instructions for computing income tax in Country R were to add 2 percent of one's annual income to the average (arithmetic mean) of 100 units of Country R's currency and 1 percent of one's annual income. Which of the following represents the simplified formula for computing the income tax, in Country R's currency, for a person in that country whose annual income is A?
50+A/200 50+3A/100 50+A/40 100+A/50 100+3A/100

48. A certain city with population of 132,000 is to be divided into 11 voting districts, and no district is to have population that is more than 10 percent greater than the population of any other district. What is the minimum possible population that the least population district could have?
10700 10800 10900 11000 11100

49. At the end of the first quarter, the share price of a certain mutual fund was 20 percent higher than it was at the beginning of the year. At the end of the second quarter, the share price was 50 percent higher than it was at the beginning of the year. What was the percent increase in the share price from the end of the first quarter to the end of the second quarter?
20% 25% 30% 33% 40%

50. A furniture dealer purchased a desk for $150 and then set the selling price equal to the purchase price plus a markup that was 40 percent of the selling price. If the dealer sold the desk at the selling price, what was the amount of the dealer's gross profit from the purchase and the sale of the desk?
$40 $60 $80 $90 $100

51. Bobby bought two shares of stock, which sold for $96 each. If he had a profit of 20 percent on the sale of one of the shares but a loss of 20 percent on the sale of the other share, then on the sale of both shares combined Bobby had:
a profit of $10 a profit of $8 a loss of 8 a loss of 10 neither profit nor loss

52. In May Mr. Lee's earnings were 60 percent of the Lee family's total income. In June Mr. Lee earned 20 percent more than in May. If the rest of the family's income was the same both months, then, in June, Mrs. Lee's earnings were approximately what percent of the Lee Family's total income?
(A) 64%
(B) 68%
(C) 72%
(D) 76%
(E) 80%

53. Amy's grade was the 90th percentile of the 80 grades for her class. Of the 100 grades from another class, 19 was higher than Amy's and the rest was lower. If no other grade is the same as Amy' grade, then Army's grade was what percentile of grades of two class combined.
72th 80th 81th 85th 92th

Part C: Work / Rate

1. Machine A and Machine B can produce 1 widget in 3 hours working together at their respective constant rates. If Machine A's speed were doubled, the two machines could produce 1 widget in 2 hours working together at their respective rates. How many hours does it currently take Machine A to produce 1 widget on its own?
   ½ 2 3 5 6

2. Adam and Brianna plan to install a new tile floor in a classroom. Adam works at a constant rate of 50 tiles per hour, and Brianna works at a constant rate of 55 tiles per hour. If the new floor consists of exactly 1400 tiles, how long will it take Adam and Brianna working together to complete the classroom floor?
   A. 26 hrs. 44 mins.
   B. 26 hrs. 40 mins.
   C. 13 hrs. 20 mins.
   D. 13 hrs. 18 mins.
   E. 12 hrs. 45 mins.

3. A copy machine, working at a constant rate, makes 35 copies per minute. A second copy machine, working at a constant rate, makes 55 copies per minute. Working together at their respective rates, how many copies do the two machines make in half an hour?
   90 2,700 4,500 5,400 324,000

4. Tom, working alone, can paint a room in 6 hours. Peter and John, working independently, can paint the same room in 3 hours and 2 hours, respectively. Tom starts painting the room and works on his own for one hour. He is then joined by Peter and they work together for an hour. Finally, John joins them and the three of them work together to finish the room, each one working at his respective rate. What fraction of the whole job was done by Peter?
   1/9 1/6 1/3 7/18 4/9

5. Machine A can complete a certain job in x hours. Machine B can complete the same job in y hours. If A and B work together at their respective rates to complete the job, which of the following represents the fraction of the job that B will not have to complete because of A's help?
   (x – y)/ (x + y) x / (y – x) (x + y) / xy y / (x – y) y / (x + y)

6. Lindsay can paint 1/x of a certain room in 20 minutes. What fraction of the same room can Joseph paint in 20 minutes if the two of them can paint the room in an hour, working together at their respective rates?
   1/3x 3x / (x – 3) (x – 3) / 3x x / (x – 3) (x – 3) / x

7. One smurf and one elf can build a treehouse together in two hours, but the smurf would need the help of two fairies in order to complete the same job in the same amount of time. If one elf and one fairy worked together, it would take them four hours to build the treehouse. Assuming that work rates for smurfs, elves, and fairies remain constant, how many hours would it take one smurf, one elf, and one fairy, working together, to build the treehouse?
   5/7 1 10/7 12/7 22/7

8. At Supersonic Corporation, the time required for a machine to complete a job is determined by the formula: $t = \sqrt{w} + \sqrt{(w-1)}$, where ***w*** = the weight of the machine in pounds and ***t*** = the hours required to complete the job. If machine A weighs 8 pounds, and machine B weighs 7 pounds, how many hours will it take the two machines to finish one job if they work together?

   A. $\frac{6}{7-\sqrt{3}}$

   B. $\frac{1}{2}(\sqrt{8}+\sqrt{6})$

   C. $\frac{1}{3}(6-\sqrt{3})$

   D. $3(\sqrt{3}+\sqrt{2})$

   E. $\sqrt{8}+2\sqrt{7}+\sqrt{6}$

9. A paint crew gets a rush order to paint 80 houses in a new development. They paint the first y houses at a rate of x houses per week. Realizing that they'll be late at this rate, they bring in some more painters and paint the rest of the houses at the rate of 1.25x houses per week. The total time it takes them to paint all the houses under this scenario is what fraction of the time it would have taken if they had painted all the houses at their original rate of x houses per week?
(A) $0.8(80 - y)$
(B) $0.8 + 0.0025y$
(C) $80/y - 1.25$
(D) $80/1.25y$
(E) $80 - 0.25y$

10. The third-place finisher of the Allen County hot dog eating contest, in which each contestant was given an equal amount of time to eat as many hot dogs as possible, required an average of 15 seconds to consume each hot dog. How many hot dogs did the winner eat?
(1) The winner consumed 24 more hot dogs than did the third-place finisher.
(2) The winner consumed hot dogs at double the rate of the third-place finisher.

11. On Sunday morning, a printing press printed its newspapers at a constant rate from 1:00 AM to 4:00 AM. How many newspapers did the printing press print on Sunday morning?
(1) The printing rate on Saturday morning was twice that of Sunday morning.
(2) On Saturday morning, the printing press ran at a constant rate from 1:00 AM to 3:00 AM, stopped for a half hour, and then ran at the same constant rate from 3:30 AM to 5:30 AM, printing a total of 4,000 newspapers.

12. Machine A can fill an order of widgets in a hours. Machine B can fill the same order of widgets in b hours. Machines A and B begin to fill an order of widgets at noon, working together at their respective rates. If a and b are even integers, is Machine A's rate the same as that of Machine B?
(1) Machines A and B finish the order at exactly 4:48 p.m.
(2) $(a + b)^2 = 400$

13. Reserve tank 1 is capable of holding z gallons of water. Water is pumped into tank 1, which starts off empty, at a rate of x gallons per minute. Tank 1 simultaneously leaks water at a rate of y gallons per minute (where $x > y$). The water that leaks out of tank 1 drips into tank 2, which also starts out empty. If the total capacity of tank 2 is twice the number of gallons that remains in tank 1 after one minute, does tank 1 fill up before tank 2?
(1) $zy < 2x^2 - 4xy + 2y^2$
(2) The total capacity of tank 2 is less than one-half that of tank 1.

14. Bill can dig a well in x! hours. Carlos can dig the same well in y! hours. If q is the number of hours that it takes Bill and Carlos to dig the well together, working at their respective rates, is q an integer?
(1) $x - y = 1$ (2) y is a nonprime even number.

15. Working alone at its own constant rate, a machine seals k cartons in 8 hours, and working alone at its own constant rate, a second machine seals k cartons in 4 hours. If the two machines, each working at its own constant rate and for the same period of time, together sealed a certain number of cartons, what percent of the cartons were sealed by the machine working at the faster rate?
A. 25%
B. $33\frac{1}{3}\%$
C. 50%
D. $66\frac{2}{3}\%$
E. 75%

Part D: SPEED and DISTANCE

1. Bob bikes to school every day at a steady rate of x miles per hour. On a particular day, Bob had a flat tire exactly halfway to school. He immediately started walking to school at a steady pace of y miles per hour. He arrived at school exactly t hours after leaving his home. How many miles is it from the school to Bob's home?
   A. (x + y) / t
   B. 2(x + t) / xy
   C. 2xyt / (x + y)
   D. 2(x + y + t) / xy
   E. x(y + t) + y(x + t)

2. Lexy walks 5 miles from point A to point B in one hour, then bicycles back to point A along the same route at 15 miles per hour. Ben makes the same round trip, but does so at half of Lexy's average speed. How many minutes does Ben spend on his round trip?
   40 80 120 160 180

3. Triathlete Dan runs along a 2-mile stretch of river and then swims back along the same route. If Dan runs at a rate of 10 miles per hour and swims at a rate of 6 miles per hour, what is his average rate for the entire trip in miles per minute?
   1/8 2/15 3/15 ¼ 3/8

4. Tom and Linda stand at point A. Linda begins to walk in a straight line away from Tom at a constant rate of 2 miles per hour. One hour later, Tom begins to jog in a straight line in the exact opposite direction at a constant rate of 6 miles per hour. If both Tom and Linda travel indefinitely, what is the positive difference, in minutes, between the amount of time it takes Tom to cover half of the distance that Linda has covered and the amount of time it takes Tom to cover twice the distance that Linda has covered?
   60 72 84 90 108

5. It takes the high-speed train x hours to travel the z miles from Town A to Town B at a constant rate, while it takes the regular train y hours to travel the same distance at a constant rate. If the high-speed train leaves Town A for Town B at the same time that the regular train leaves Town B for Town A, how many more miles will the high-speed train have travelled than the regular train when the two trains pass each other?
   $\frac{z(y-x)}{x+y}$ $\frac{z(x-y)}{x+y}$ $\frac{z(x+y)}{y-x}$ $\frac{xy(x+y)}{y-x}$ $\frac{xy(x+y)}{x-y}$

6. The 'moving walkway' is a 300-foot long conveyor belt that moves continuously at 3 feet per second. When Bill steps on the walkway, a group of people that are also on the walkway stands 120 feet in front of him. He walks toward the group at a combined rate (including both walkway and foot speed) of 6 feet per second, reaches the group of people, and then remains stationary until the walkway ends. What is Bill's average rate of movement for his trip along the moving walkway?
   A. 2 feet per second
   B. 2.5 feet per second
   C. 3 feet per second
   D. 4 feet per second
   E. 5 feet per second

7. John and Jacob set out together on bicycle traveling at 15 and 12 miles per hour, respectively. After 40 minutes, John stops to fix a flat tire. If it takes John one hour to fix the flat tire and Jacob continues to ride during this time, how many hours will it take John to catch up to Jacob assuming he resumes his ride at 15 miles per hour? (consider John's deceleration/acceleration before/after the flat to be negligible)
   3 3.33 3 ½ 4 4 ½

8. Stephanie, Regine, and Brian ran a 20 mile race. Stephanie and Regine's combined times exceeded Brian's time by exactly 2 hours. If nobody ran faster than 8 miles per hour, who could have won the race?
   I. Stephanie II. Regine III. Brian
   I only II only III only I or II only I, II, or III

9. A car travelled from Los Angeles to San Francisco in 6 hours at an average rate of x miles per hour. If the car returned along the same route at an average rate of y miles per hour, how long did it take for the car to make the entire round trip, in minutes?

A. (6 + 6x/y)*60

B. (6 + 6y/x)*60

C. 30(x + y)

D. 10(x + y)

E. (x + y)/360

10. Deb normally drives to work in 45 minutes at an average speed of 40 miles per hour. This week, however, she plans to bike to work along a route that decreases the total distance she usually travels when driving by 20%. If Deb averages between 12 and 16 miles per hour when biking, how many minutes earlier will she need to leave in the morning in order to ensure she arrives at work at the same time as when she drives?

135 105 95 75 45

11. Alex and Brenda both stand at point X. Alex begins to walk away from Brenda in a straight line at a rate of 4 miles per hour. One hour later, Brenda begins to ride a bicycle in a straight line in the opposite direction at a rate of R miles per hour. If R > 8, which of the following represents the amount of time, in terms of R, that Alex will have been walking when Brenda has covered twice as much distance as Alex?

R – 4 R / (R + 4) R / (R – 8) 8 / (R – 8)
$R^2 - 4$

12. On Monday, Lou drives his ford escort with 28-inch tires, averaging x miles per hour. On Tuesday, Lou switches the tires on his car to 32-inch tires yet drives to work at the same average speed as on Monday. What is the percent change from Monday to Tuesday in the average number of revolutions that Lou's tires make per second?

Decrease by 14.3% Decrease by 12.5% Increase by 14.3%
Increase by 12.5% cannot be determined with the given information.

13. Martha takes a road trip from point A to point B. She drives x percent of the distance at 60 miles per hour and the remainder at 50 miles per hour. If Martha's average speed for the entire trip is represented as a fraction in its reduced form, in terms of x, which of the following is the numerator?

110 300 1,100 3,000 30,000

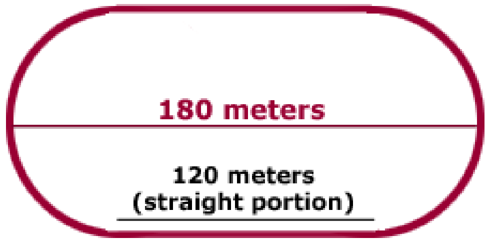
14. A not-so-good clockmaker has four clocks on display in the window. Clock #1 loses 15 minutes every hour. Clock #2 gains 15 minutes every hour relative to Clock #1 (i.e., as Clock #1 moves from 12:00 to 1:00, Clock #2 moves from 12:00 to 1:15). Clock #3 loses 20 minutes every hour relative to Clock #2. Finally, Clock #4 gains 20 minutes every hour relative to Clock #3. If the clockmaker resets all four clocks to the correct time at 12 noon, what time will Clock #4 display after 6 actual hours (when it is actually 6:00 pm that same day)?

5:00 5:34 5:42 6:00 6:24

15. At exactly what time past 7:00 will the minute and hour hands of an accurate working clock be precisely perpendicular to each other for the first time?

(A) 20 13/21 minutes past 7:00
(B) 20 13/17 minutes past 7:00
(C) 21 3/23 minutes past 7:00
(D) 21 9/11 minutes past 7:00
(E) 22 4/9 minutes past 7:00

16. The figure below represents a track with identical semi-circular ends used for a 4-lap relay race involving two 4-person teams (where each team member runs one complete lap around the track). The table below shows the lap times for each runner on Team A and Team B. Assuming that each runner runs at a constant rate, Team A win the race by how many meters?



Runner	Team A	Team B
1	42 sec	45 sec
2	46 sec	50 sec
3	49 sec	48 sec
4	41 sec	42 sec
Total	178 sec	185 sec

40 meters
$(40 + 10\pi)$ meters
$(40 + 20\pi)$ meters
$(20 + 10\pi)$ meters
$(20 + 20\pi)$ meters

17. What is the distance between Harry's home and his office?
(1) Harry's average speed on his commute to work this Monday was 30 miles per hour.
(2) If Harry's average speed on his commute to work this Monday had been twice as fast, his trip would have been 15 minutes shorter.

18. Bob and Wendy left home to walk together to a restaurant for dinner. They started out walking at a constant pace of 3 mph. At precisely the halfway point, Bob realized he had forgotten to lock the front door of their home. Wendy continued on to the restaurant at the same constant pace. Meanwhile, Bob, traveling at a new constant speed on the same route, returned home to lock the door and then went to the restaurant to join Wendy. How long did Wendy have to wait for Bob at the restaurant?
(1) Bob's average speed for the entire journey was 4 mph.
(2) On his journey, Bob spent 32 more minutes alone than he did walking with Wendy.

19. If a car travelled from Townsend to Smallville at an average speed of 40 mph and then returned to Town send later that evening, what was the average speed for the entire trip?
(1) The return trip took 50% longer than the trip there.
(2) The distance from Townsend to Smallville is 165 miles.

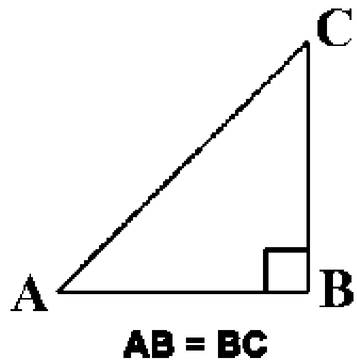
20. What was Bill's average speed on his trip of 250 miles from New York City to Boston?
(1) The trip took Bill 5 hours.
(2) At the midpoint of his trip, Bill was going exactly 50 miles per hour.

21. Train A leaves New York for Boston at 3 PM and travels at the constant speed of 100 mph. An hour later, it passes Train B, which is making the trip from Boston to New York at a constant speed. If Train B left Boston at 3:50 PM and if the combined travel time of the two trains is 2 hours, what time did Train B arrive in New York?
(1) Train B arrived in New York before Train A arrived in Boston.
(2) The distance between New York and Boston is greater than 140 miles.

22. Edwin is planning to drive from Boston to New Orleans. By what percent would his travel time be reduced if Edwin decides to split the driving time equally with his friend George, instead of making the trip alone?
(1) The driving distance from Boston to New Orleans is 1500 miles.
(2) George's driving speed is 1.5 times Edwin's driving speed.

23. Trains A and B travel at the same constant rate in opposite directions along the same route between Town G and Town H. If, after traveling for 2 hours, Train A passes Train B, how long does it take Train B to travel the entire distance between Town G and Town H?
(1) Train B started traveling between Town G and Town H 1 hour after Train A started traveling between Town H and Town G.
(2) Train B travels at the rate of 150 miles per hour.

24.



Greg and Brian are both at Point A (above). Starting at the same time, Greg drives to point B while Brian drives to point C. Who arrives at his destination first?

(1) Greg's average speed is 2/3 that of Brian's.

(2) Brian's average speed is 20 miles per hour greater than Greg's.

25. If it took Carol 1/2 hour to cycle from his house to the library yesterday, was the distance that he cycled greater than 6 miles? (1mile = 5,280feet)

(1) The average speed at which Carlos cycled from his house to the library yesterday was greater than 16 feet per second.

(2) The average speed at which Carlos cycled from his house to the library yesterday was less than 18 feet per second

26. How much time did it take a certain car to travel 400 kilometers?

(1) The car traveled the first 200 kilometers in 2.5 hours

(2) If the car's average speed had been 20 kilometers per hour greater than it was, it would have traveled the 400 kilometers in 1 hours less time than it did.

27. On his trip from Alba to Bento, Julio drove the first x miles at an average rate of 50 miles per hour and the remaining distant at an average rate of 60 miles per hour, how long did it take Julio to drive the x miles?

(1) On this trip, Julio drove for a total of 10 hours and drove a total of 530 miles

(2) On this trip, it took Julio 4 more hour to drive the first x miles than to drive the remaining distance

28. A hiker walking at a constant rate of 4 miles per hour is passed by a cyclist traveling in the same direction along the same path at a constant rate of 20 miles per hour. The cyclist stops to wait for the hiker 5 minutes after passing her, while the hiker continues to walk at her constant rate. How many minutes must the cyclist wait until the hiker catches up?

A. 6 2/3
B. 15
C. 20
D. 25
E. 26 2/3

29. A boat travelled upstream a distance of 90 miles at an average speed of (V-3) miles per hour and then travelled the same distance downstream at an average of (V+3) miles per hour. If the trip upstream took half an hour longer than the trip downstream, how many hours did it take the boat to travel downstream?

A. 2.5
B. 2.4
C. 2.3
D. 2.2
E. 2.1

Part E: SI / CI / Population Growth

1. Jolene entered an 18-month investment contract that guarantees to pay 2 percent interest at the end of 6 months, another 3 percent interest at the end of 12 months, and 4 percent interest at the end of the 18-month contract. If each interest payment is reinvested in the contract, and Jolene invested \$10,000 initially, what will be the total amount of interest paid during the 18-month contract?
\$506.00 \$726.24 \$900.00 \$920.24 \$926.24

2. Wes works at a science lab that conducts experiments on bacteria. The population of the bacteria multiplies at a constant rate, and his job is to notate the population of a certain group of bacteria each hour. At 1 p.m. on a certain day, he noted that the population was 2,000 and then he left the lab. He returned in time to take a reading at 4 p.m., by which point the population had grown to 250,000. Now he has to fill in the missing data for 2 p.m. and 3 p.m. What was the population at 3 p.m.?
50,000 62,500 65,000 86,666 125,000

3. The population of locusts in a certain swarm doubles every two hours. If 4 hours ago there were 1,000 locusts in the swarm, in approximately how many hours will the swarm population exceed 250,000 locusts?
6 8 10 12 14

4. An investor purchased a share of non-dividend-paying stock for p dollars on Monday. For a certain number of days, the value of the share increased by r percent per day. After this period of constant increase, the value of the share decreased the next day by q dollars and the investor decided to sell the share at the end of that day for v dollars, which was the value of the share at that time. How many working days after the investor

   bought the share was the share sold, if $r = 100(\sqrt{\frac{v+q}{p}} - 1)$

   (Options provided on next page)

A. Two working days later.
B. Three working days later.
C. Four working days later.
D. Five working days later.
E. Six working days later.

5. A certain investment grows at an annual interest rate of 8%, compounded quarterly. Which of the following equations can be solved to find the number of years, x, that it would take for the investment to increase by a factor of 16?
$16 = (1.02)^{x/4}$ $2 = (1.02)^{x}$ $16 = (1.08)^{4x}$ $2 = (1.02)^{x/4}$ $1/16 = (1.02)^{4x}$

6. A researcher has determined that she requires a minimum of n responses to a survey for the results to be valid. If p% of the surveyed individuals fail to respond to the survey, how many individuals, in terms of n and p, must the researcher survey to produce twice the minimum required number of responses?

A. $\frac{200n}{100-p}$

B. $\frac{2n}{100-p}$

C. $\frac{200n}{p}$

D. $\frac{2n(100+p)}{100}$

E. $\frac{2n+2np}{100}$

7. Louie takes out a three-month loan of $1000. The lender charges him 10% interest per month compounded monthly. The terms of the loan state that Louie must repay the loan in three equal monthly payments. To the nearest dollar, how much does Louie have to pay each month?
333 383 402 433 483

8. Donald plans to invest x dollars in a savings account that pays interest at an annual rate of 8% compounded quarterly. Approximately what amount is the minimum that Donald will need to invest to earn over $100 in interest within 6 months?
$1500 $1750 $2000 $2500 $3000

9. The number of antelope in a certain herd increases every year at a constant rate. If there are 500 antelope in the herd today, how many years will it take for the number of antelope to double?
(1) Ten years from now, there will be more than ten times the current number of antelope in the herd.
(2) If the herd were to grow in number at twice its current rate, there would be 980 antelope in the group in two years.

10. A scientist is studying bacteria whose cell population doubles at constant intervals, at which times each cell in the population divides simultaneously. Four hours from now, immediately after the population doubles, the scientist will destroy the entire sample. How many cells will the population contain when the bacteria is destroyed?
(1) Since the population divided two hours ago, the population has quadrupled, increasing by 3,750 cells.
(2) The population will double to 40,000 cells with one hour remaining until the scientist destroys the sample.

11. Grace makes an initial deposit of x dollars into a savings account with a z percent interest rate, compounded annually. On the same day, Georgia makes an initial deposit of y dollars into a savings account with a z percent annual interest rate, compounded quarterly. Assuming that neither Grace nor Georgia makes any other deposits or withdrawals and that x, y, and z are positive numbers no greater than 50, whose savings account will contain more money at the end of exactly one year?
(1) z = 4 (2) 100y = zx

12. **Poorly reported question (can be skipped) (additional)-->** A certain sum was invested in a high-interest bond for which the interest is compounded monthly. The bond was sold x number of months later, where x is an integer. If the value of the original investment doubled during this period, what was the approximate amount of the original investment in dollars?
(1) The interest rate during the period of investment was greater than 39 percent but less than 45 percent.
(2) If the period of investment had been one month longer, the final sale value of the bond would have been approximately $2,744.

13. If a certain culture of bacteria increases by a factor of x every y minutes, how long will it take for the culture to increase to ten-thousand times its original amount?
(1) $\sqrt[y]{x} = 10$ (2) In two minutes, the culture will increase to one-hundred times its original amount.

Part F: RATIOS

1. Which of the following fractions is at least twice as great as 11/50?
2/5 11/34 43/99 8/21 9/20

2. At the beginning of the year, the ratio of juniors to seniors in high school X was 3 to 4. During the year, 10 juniors and twice as many seniors transferred to another high school, while no new students joined high school X. If, at the end of the year, the ratio of juniors to seniors was 4 to 5, how many seniors were there in high school X at the beginning of the year?
80 90 100 110 120

3. 3/5 of a certain class left on a field trip. 1/3 of the students who stayed behind did **not** want to go on the field trip (all the others did want to go). When another vehicle was located, 1/2 of the students who **did** want to go on the field trip but had been left behind were able to join. What fraction of the class ended up going on the field trip?
½ 2/3 11/15 23/30 4/5

4. The ratio of boys to girls in Class A is 3 to 4. The ratio of boys to girls in Class B is 4 to 5. If the two classes were combined, the ratio of boys to girls in the combined class would be 17 to 22. If the number of boys in Class B is one less than the number of boys in Class A, and if the number of girls in Class B is two less than the number of girls in Class A, how many girls are in Class A?
8 9 10 11 12

5. John's front lawn is 1/3 the size of his back lawn. If John mows 1/2 of his front lawn and 2/3 of his back lawn, what fraction of his lawn is left unmowed?
1/6 1/3 3/8 ½ 5/8

6. At Jefferson Elementary School, the number of teachers and students (kindergarten through sixth grade) totals 510. The ratio of students to teachers is 16 to 1. Kindergarten students make up 1/5 of the student population and fifth and sixth graders account for 1/3 of the remainder. Students in first and second grades account for 1/4 of all the students. If there are an equal number of students in the third and fourth grades, then the number of students in third grade is how many greater or fewer than the number of students in kindergarten?
12 greater 17 fewer 28 fewer 36 fewer 44 fewer

7. A certain galaxy is known to comprise approximately 4×10^{11} stars. Of every 50 million of these stars, one is larger in mass than our sun. Approximately how many stars in this galaxy are larger than the sun?
800 1,250 8,000 12,000 80,000

8. A lemonade stand sold only small and large cups of lemonade on Tuesday. 3/5 of the cups sold were small and the rest were large. If the large cups were sold for 7/6 as much as the small cups, what fraction of Tuesday's total revenue was from the sale of large cups?
7/16 7/15 10/21 17/35 ½

9. Miguel is mixing up a salad dressing. Regardless of the number of servings, the recipe requires that 5/8 of the finished dressing mix be olive oil, 1/4 vinegar, and the remainder an even mixture of salt, pepper and sugar. If Miguel accidentally doubles the vinegar and forgets the sugar altogether, what proportion of the botched dressing will be olive oil?
15/29 5/8 5/16 ½ 13/27

10. Harold and Millicent are getting married and need to combine their already-full libraries. If Harold, who has 1/2 as many books as Millicent, brings 1/3 of his books to their new home, then Millicent will have enough room to bring 1/2 of her books to their new home. What fraction of Millicent's old library capacity is the new home's library capacity?
½ 2/3 ¾ 4/5 5/6

11. In a certain pet shop, the ratio of dogs to cats to bunnies in stock is 3 : 5 : 7. If the shop carries 48 cats and bunnies total in stock, how many dogs are there?
12 13 14 15 16

12. A foreign language club at Washington Middle School consists of n students, 2/5 of whom are boys. All of the students in the club study exactly one foreign language. 1/3 of the girls in the club study Spanish and 5/6 of the remaining girls study French. If the rest of the girls in the club study German, how many girls in the club, in terms of n, study German?
2n/5 n/3 n/5 2n/15 n/15

13. A certain ball team has an equal number of right- and left-handed players. On a certain day, two-thirds of the players were absent from practice. Of the players at practice that day, one-third were left-handed. What is the ratio of the number of right-handed players who were not at practice that day to the number of left- handed players who were not at practice?

1/3 2/3 5/7 7/5 3/2

14. Bag A contains red, white and blue marbles such that the red to white marble ratio is 1:3 and the white to blue marble ratio is 2:3. Bag B contains red and white marbles in the ratio of 1:4. Together, the two bags contain 30 white marbles. How many red marbles could be in bag A?

1 3 4 6 8

15. The ratio by weight, measured in pounds, of books to clothes to electronics in Jorge's suitcase initially stands at 8 to 5 to 3. Jorge then removes 4 pounds of clothing from his suitcase, thereby doubling the ratio of books to clothes. Approximately how much do the electronics in the suitcase weigh, to the nearest pound?

3 4 5 6 7

16. Joe, Bob and Dan worked in the ratio of 1:2:4 hours, respectively. How many hours did Bob work?

(1) Together, Joe, Bob and Dan worked a total of 49 hours.
(2) Dan worked 21 hours more than Joe.

17. In 2003 Acme Computer priced its computers five times higher than its printers. What is the ratio of its gross revenue for computers and printers respectively in the year 2003?

(1) In the first half of 2003 it sold computers and printers in the ratio of 3:2, respectively, and in the second half in the ratio of 2:1.
(2) It sold each computer for $1000.

18. If Pool Y currently contains more water than Pool X, and if Pool X is currently filled to 2/7 of its capacity, what percent of the water currently in Pool Y needs to be transferred to Pool X if Pool X and Pool Y are to have equal volumes of water?

(1) If all the water currently in Pool Y were transferred to Pool X, Pool X would be filled to 6/7 of its capacity.
(2) Pool X has a capacity of 14,000 gallons.

19. Three business partners shared all the proceeds from the sale of their privately held company. If the partner with the largest share received exactly 5/8 of the total proceeds, how much money did the partner with the smallest share receive from the sale?

(1) The partner with the smallest share received from the sale exactly 1/5 the amount received by the partner with the second largest share.
(2) The partner with the second largest share received from the sale exactly half of the two million dollars received by the partner with the largest share.

20. In a piggy bank filled with only pennies, nickels, and dimes, what is the ratio of pennies to dimes?

(1) The ratio of nickels to dimes is three to two.
(2) There is exactly $7 in the piggy bank.

21. In a certain solution consisting of only two chemicals, for every 3 milliliters of Chemical A, there are 7 milliliters of Chemical B. After 10 milliliters of Chemical C are added to this solution, what is the ratio of the quantities of Chemical A to Chemical C?

(1) Before Chemical C was added, there were 50 milliliters of solution.
(2) After Chemical C was added, there were 60 milliliters of solution.

22. On a certain sight-seeing tour, the ratio of the number of women to the number of children was 5 to 2. What was the number of men on the sight-seeing tour?

(1) On the sight-seeing tour, the ratio of the number of children to the number of men was 5 to 11.
(2) The number of women on the sight-seeing tour was less than 30.

23. Each employee of Company Z is an employee of either Division X or Division Y, but not both. If each division has some part-time employees, is the ratio of the number of full-time employees to the number of part-time employees greater for Division X than for Company Z?

(1) The ratio of the number of full-time employees to the number of part-time employees is less for Division Y than for Company Z.
(2) More than half the full-time employees of Company Z are employees of Division X, and more than half of the part-time employees of Company Z are employees of Division Y.

24. Of the 60 animals on a certain farm, 2/3 are either pigs or cows. How many of the animals are cows?
(1) the farm has more than twice as many cows at it has pigs.
(2) the farm has more than 12 pigs

25. Malik's recipe for 4 servings of a certain dish requires 3/2 cups of pasta. According to this recipe, what is the number of cups of pasta that Malik will use the next time he prepares this dish?
(1) The next time he prepares this dish, Malik will make half as many servings as he did the last time he prepared the dish.
(2) Malik used 6 cups of pasta the last time he prepared this dish.

Answer Key

GMAT Quant Topic 1: General Arithmetic

Part A: Overlapping Sets

1. E
2. C
3. B
4. E
5. D
6. D
7. B
8. B
9. B
10. E
11. B
12. A
13. B
14. E
15. B
16. B
17. C
18. D
19. E
20. C
21. A
22. C
23. E
24. A
25. E
26. C
27. E
28. A
29. E
30. B
31. A
32. D

Part B: Percentages

1. A
2. E
3. A
4. A
5. D
6. E
7. D
8. C
9. D
10. C
11. D
12. E

13. E
14. D
15. C
16. A
17. A
18. A
19. B
20. D
21. B
22. C
23. E
24. D
25. E
26. D
27. A
28. A
29. D
30. C
31. A
32. E
33. C
34. D
35. E
36. A
37. B
38. A
39. B
40. C
41. A
42. B
43. D
44. A
45. D
46. A
47. C
48. D
49. B
50. E
51. C
52. A
53. D

Part C: Work / Rate

1. E
2. C
3. B
4. E
5. E
6. C
7. D
8. B

9. B
10. C
11. C
12. A
13. A
14. C
15. D

Part D: SPEED and DISTANCE

1. C
2. D
3. A
4. E
5. A
6. E
7. B
8. D
9. A
10. D
11. C
12. B
13. E
14. A
15. D
16. B
17. C
18. B
19. A
20. A
21. D
22. B
23. A
24. A
25. E
26. B
27. A
28. C
29. A

Part E: SI / CI / Population Growth

1. E
2. A
3. D
4. B
5. B
6. A
7. C
8. D
9. B
10. A
11. B

12. C
13. D

Part F: RATIOS

1. E
2. E
3. C
4. E
5. C
6. C
7. C
8. A
9. A
10. B
11. A
12. E
13. C
14. D
15. C
16. D
17. E
18. A
19. B
20. E
21. D
22. C
23. D
24. C
25. C

GMAT Quant Topic 2

Statistics

Mean

1. The table below provides revenues of a certain company in 2002 and 2003. By what percent did the average quarterly revenue change from 2002 to 2003?

Quarter	**Quarterly revenues, MM USD**	
	2002	**2003**
1st	13	17
2nd	15	18
3rd	16	17
4th	16	20

(A) 20 (B) 30 (C) 25 (D) 15 (E) 28

2. During 2005, a company produced an average of 2,000 products per month. How many products will the company need to produce from 2006 through 2008 in order to increase its monthly average for the period from 2005 through 2008 by 200% over its 2005 average?
(A) 148,000 (B) 172,000 (C) 200,000 (D) 264,000 (E) 288,000

3. After his first semester in college, Thomas is applying for a scholarship that has a minimum Grade Point Average (GPA) requirement of 3.5. The point values of pertinent college grades are given in the table below. If Thomas took 5 courses, each with an equal weight for GPA calculations, and received two grades of A-, one grade of B+, and one grade of B, what is the lowest grade that Thomas could receive for his fifth class to qualify for the scholarship?
Point Values of Select Grades

Grade	A	A-	B+	B	B-	C+	C	C-
Value	4	3.7	3.3	3	2.7	2.3	2	1.7

(A) A (B) B+ (C) B (D) B- (E) C+

4. A certain portfolio consisted of 5 stocks, priced at \$20, \$35, \$40, \$45, and \$70, respectively. On a given day, the price of one stock increased by 15%, while the price of another stock decreased by 35% and the prices of the remaining three remained constant. If the average price of a stock in the portfolio rose by approximately 2%, which of the following could be the prices of the shares that remained constant?
(A) \$20, \$35, and \$70 (B) \$20, \$45, and \$70 (C) \$20, \$35, and \$40
(D) \$35, \$40, and \$70 (E) \$35, \$40, and \$45

5. If John makes a contribution to a charity fund at school, the average contribution size will increase by 50%, reaching \$75 per person. If there were 5 other contributions made before John's, what is the size of his donation?
(A) \$100 (B) \$150 (C) \$200 (D) \$250 (E) \$450

6. What is the minimum percentage increase in the mean of set X = {-4, -1, 0, 6, 9} if its two smallest elements are replaced with two different primes?
(A) 25% (B) 50% (C) 75% (D) 100% (E) 200%

7. If every member of set X = {-14, -12, 17, 28, 41, Z} is multiplied by number N, by what percent will the mean M of the set increase?
(1) Z = 60 (2) N = Z / M

8. Which of the following series of numbers, if added to the set {1, 6, 11, 16, 21}, will not change the set's mean?
I. 1.5, 7.11 and 16.89 II. 5.36, 10.7 and 13.24 III. -21.52, 23.3, 31.22
(A) I only (B) II only (C) III only (D) I and III only (E) None

9. If numbers N and K are added to set X {2, 8, 10, 12}, its mean will increase by 25%. What is the value of $N^2 + 2NK + K^2$?
(A) 28 (B) 32 (C) 64 (D) 784 (E) 3600

10. Set X consists of different positive numbers arranged in ascending order: K, L, M, 5, 7. If K, L and M are consecutive integers, what is the arithmetic mean of set X?

(1) The product K × L × M is a multiple of 6

(2) There are at least 2 prime numbers among K, L and M

11. A group of men and women gathered to compete in a marathon. Before the competition, each competitor was weighed and the average weight of the female competitors was found to be 120 lbs. What percentage of the competitors were women?

(1) The average weight of the men was 150 lb.

(2) The average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women.

12. The mean of $(54{,}820)^2$ and $(54{,}822)^2$ =

(A) $(54{,}821)^2$ (B) $(54{,}821.5)^2$ (C) $(54{,}820.5)^2$ (D) $(54{,}821)^2 + 1$ (E) $(54{,}821)^2 - 1$

13. Set S consists of integers 7, 8, 10, 12, and 13. If integer n is included in the set, the average (arithmetic mean) of set S will increase by 20%. What is the value of integer n?

10 12 16 22 24

14. A convenience store currently stocks 48 bottles of mineral water. The bottles have two sizes of either 20 or 40 ounces each. The average volume per bottle the store currently has in stock is 35 ounces. How many 40 ounce bottles must be sold for the average volume per bottle to be reduced to 25 ounces if no 20 ounce bottles are sold?

10 20 30 32 34

15. Last year, the five employees of Company X took an average of 16 vacation days each. What was the average number of vacation days taken by the same employees this year?

(1) Three employees had a 50% increase in their number of vacation days, and two employees had a 50% decrease.

(2) Three employees had 10 more vacation days each, and two employees had 5 fewer vacation days each.

16. In a room of men and women, the average weight of the women is 120 lbs, and the average weight of the men is 150 lbs. What is the average weight of a person in the room?

(1) There are twice as many men as women.

(2) There are a total of 120 people in the room.

17. If set R contains the consecutive integers from -5 to -1, what is the mean of set R?

-5 -3 0 3 5

18. Sarah is in a room with 6 other children. If the other children are 2, 4, 5, 8, 10, and 13 years old, is Sarah 7 years old?

(1) The age of the fourth oldest child is equal to the average (arithmetic mean) of the seven children's ages.

(2) Sarah is not the oldest child in the room.

19. x, y, and z are positive integers. The average (arithmetic mean) of x, y, and z is 11. If z is two greater than x, which of the following must be true?

I. x is even II. y is odd III. z is odd

I only II only III only I and II only I and III only

20. Set A contains the consecutive integers ranging from x to y, inclusive. If the number of integers in set A that are less than 75 is equal to the number of integers that are greater than 75, what is the value of 3x + 3y?

225 300 372 450 528

21. In a work force, the employees are either managers or directors. What is the percentage of directors?

(1) the average salary for manager is $5,000 less than the total average salary.

(2) the average salary for directors is $15,000 more than the total average salary.

22. In the first week of last month, Company X realized an average wholesale profit of $5304 per day from the sale of q units of Product Y. Which of the following CANNOT be the difference between Product Y's sale price and cost per unit?

$3 $4 $7 $11 $51

23. A certain bank has ten branches. What is the total amount of assets under management at the bank?

(1) There is an average of 400 customers per branch. When each branch's average assets under management per customer is computed, these values are added together and this sum is divided by 10. The result is $400,000 per customer.

(2) The bank has a total of 4,000 customers. When the total assets per branch are added up, each branch is found to manage, on average, 160 million dollars in assets.

24. Three baseball teams, A, B, and C, play in a seasonal league. The ratio of the number of players on the three teams is 2:5:3, respectively. Is the average number of runs scored per player across all three teams collectively greater than 22?

(1) The average number of runs scored per player for each of the three teams, A, B, and C, is 30, 17, and 25, respectively.

(2) The total number of runs scored across all three teams collectively is at least 220.

25. The average score of x number of exams is y. When an additional exam of score z is added in, does the average score of the exams increase by 50%?

(1) $3x = y$ (2) $2z - 3y = xy$

26. A new cell phone plan is offering pricing based on average monthly use. Brandon and Jodie are comparing their average use to determine the best plan for them. Brandon's average monthly usage in 2001 was q minutes. Was this less than, greater than, or equal to Jodie's 2001 average monthly usage, in minutes?

(1) From January to August 2001, Jodie's average monthly usage was 1.5q minutes.

(2) From April to December 2001, Jodie's average monthly usage was 1.5q minutes.

27. On Jane's credit card account, the average daily balance for a 30-day billing cycle is average (arithmetic mean) of the daily balances at the end of the 30 days. At the beginning of a certain 30-day billing cycle, Jane's credit card account had a balance of $600. Jane made a payment of $300 on the account during the billing cycle. If no other amounts were added to or subtracted from the account during the billing cycle, what was average daily balance on Jane's account for the billing cycle?

(1) Jane's payment was credited on the 21st day of the billing cycle.

(2) The average daily balance through the 25th day of the billing cycle was $540.

28. L spends total $6.00 for one kind of D and one kind of C. How many D did he buy?

(1) the price of 2D was $0.10 less than the price of 3C

(2) the average price of 1 D and 1 C was $0.35

29. x, y, and z are consecutive integers, and $x < y < z$. What is the average of x, y, and z?

(1) $x = 11$ (2) The average of y and z is 12.5.

Median

1. Set A consists of numbers {-2, 27.5, -6, 18.3, 9} and set B consists of numbers {-199, 0.355, 19.98, 10, 201, 16}. The median of set B is how much greater than the median of set A?
(A) 2 (B) 4 (C) 9 (D) 2.5 (E) 3

2. Which of the following could be the median of a set consisting of 6 different primes?
(A) 2 (B) 3 (C) 9.5 (D) 12.5 (E) 39

3. The median annual household income in a certain community of 21 households is $50,000. If the mean income of a household increases by 10% per year over the next 2 years, what will the median income in the community be in 2 years?
(A) $50,000 (B) $60,000 (C) $60,500 (D) $65,000 (E) Cannot get

4. What is the median of set A {-8, 15, -9, 4, N}?
(1) N is a prime and N^6 is even (2) $2N + 14 < 20$

5. T is a set of y integers, where $0 < y < 7$. If the average of Set T is the positive integer x, which of the following could NOT be the median of Set T?
(A) 0 (B) x (C) –x (D) y/3 (E) 2y/7

6. a, b, and c are integers and $a < b < c$. S is the set of all integers from a to b, inclusive. Q is the set of all integers from b to c, inclusive. The median of set S is (3/4) b. The median of set Q is (7/8) c. If R is the set of all integers from a to c, inclusive, what fraction of c is the median of set R?
(A) 3/8 (B) ½ (C) 11/16 (D) 5/7 (E) 3/4

7. Jim Broke's only source of income comes from his job as a question writer. In this capacity, Jim earns a flat salary of $200 per week plus a fee of $9 for every question that he writes. Every year, Jim takes exactly two weeks of unpaid vacation to visit his uncle, a monk in Tibet, and get inspired for the next year. If a regular year consists of 52 weeks and the number of questions that Jim wrote in each of the past 5 years was an odd number greater than 20, which of the following could be Jim's median annual income over the past 5 years?
(A) $22,474 (B) $25,673 (C) $27,318 (D) $28,423 (E) $31,227

8. Set A, Set B, and Set C each contain only positive integers. If Set A is composed entirely of all the members of Set B plus all the members of Set C, is the median of Set B greater than the median of Set A?
(1) The mean of Set A is greater than the median of Set B.
(2) The median of Set A is greater than the median of Set C.

9. If x and y are unknown positive integers, is the mean of the set {6, 7, 1, 5, x, y} greater than the median of the set?
(1) $x + y = 7$ (2) $x – y = 3$

10. Given the ascending set {x, x, y, y, y, y}. What is greater, the median or the mean?
A. The mean
B. The median
C. They are equal

11. There is a set of numbers in ascending order: {y - x, y, y, y, y, x, x, x, x + y}. If the mean is 9, and the median is 7, what is x?
(A) 13 (B) 14 (C) 16 (D) 7 (E) 4

12. During a behavioural experiment in a psychology class, each student is asked to compute his or her lucky number by raising 7 to the power of the student's favourite day of the week (numbered 1 through 7 for Monday through Sunday respectively), multiplying the result by 3, and adding this to the doubled age of the student in years, rounded to the nearest year. If a class consists of 28 students, what is the probability that the median lucky number in the class will be a non-integer?
(A) 0% (B) 10% (C) 20% (D) 30% (E) 40%

13. Given the ascending set of positive integers {a, b, c, d, e, f}, is the median greater than the mean?
(1) $a + e = (3/4)(c + d)$ (2) $b + f = (4/3)(c + d)$

14. For the set of terms [x, y, x + y, x – 4y, xy, 2y], if y > 6 and the mean of the set equals y + 3, then the median must be

(x + y) / 2   y + 3   y   3y/2   (x/2) + y

15. What is the median value of the set R, if for every term in the set, $R_n = R_{n-1} + 3$?

(1) The first term of set R is 15.   (2) The mean of set R is 36.

16. Peter, Paul, and Mary each received a passing score on his/her history midterm. The average (arithmetic mean) of the three scores was 78. What was the median of the three scores?

(1) Peter scored a 73 on his exam.   (2) Mary scored a 78 on her exam.

17. Set A: 3, x, 8, 10   Set B: 4, y, 9, 11.

The terms of each set above are given in ascending order. If the median of Set A is equal to the median of Set B, what is the value of y – x?

-2   -1   0   1   2

18. Set S includes elements {8, 2, 11, x, 3, y} and has a mean of 7 and a median of 5.5. If x < y, then which of the following is the maximum possible value of x?

0   1   2   3   4

19. If set S consist of the numbers 1, 5, -2, 8, and n, is 0<n<7?

(1) the median of the numbers in S is less than 5.

(2) the median of the numbers in S is greater than 1.

20. Set S consists of five consecutive integers, and set T consists of seven consecutive integers. Is the median of the numbers in set S equal to the median of the numbers in set T?

(1) The median of the numbers in set S is 0.

(2) The sum of the numbers in set S is equal to the sum of the numbers in set T.

21. The temperatures in Celsius recorder at 6 in the morning in various parts of a certain country were 10,5, -2, -1, -5 and 15. What is the median of these temperatures?

-2   -1   2   3   5

22. A student worked for 20 days. For each of the amounts shown in the first row of the table, the second row gives the number of days that the students earned that amount. What is the median amount of money that the student earned per day for the 20 days? (refer to table below)

\$84   \$90   \$85   \$95   \$105

Amount earned per day	\$96	\$84	\$80	\$70	\$48
Number of days	4	7	4	3	2

23.

Score	Number and Interval of Scores
50-59	2
60-69	10
70-79	16
80-89	27
90-99	18

The table above shows the distribution of test scores for a group of management trainees, which score interval contains the median of the 73 scores?

A. 60-69   B. 70-79   C. 80-89   D. 90-99   E. Can't get

24. Last month 15 homes were sold in Town X. The average (arithmetic mean) sale price of the homes was \$ 150,000 and the median sale price was \$130,000. Which of the following statement must be true?

I. at least one of the homes was sold for more than \$165,000

II. at least one of the homes was sold for more than \$130,000 and less than \$150,000

III. at least one of the homes was sold for less than \$130,000

A. I only   B. II only   C. III only   D. I and II   E. I and III

25. Five pieces of wood have an average (arithmetic mean) length of 124 centimeters and a median length of 140 centimeters. What is the maximum possible length in centimeters of the shortest piece of wood?

90   100   110   130   140

26. Amy's grade was the 90th percentile of the 80 grades for her class. Of the 100 grades from another class, 19 was higher than Amy's and the rest was lower. If no other grade is the same as Amy' grade, then Army's grade was what percentile of grades of two class combined.

72nd 80th 81st 85th 92nd

27.

Ann	$450,000
Bob	$360,000
Cal	$190,000
Dot	$210,000
Ed	$680,000

The table above shows the total sales recorded in July for the five salespeople. It was discovered that one of Cal's sales was incorrectly recorded as one of Ann's sales. After this error was corrected, Ann's total sales were still higher than Cal's total sales, and the median of 5 sales totals was $330,000. What was the value of the incorrectly recorded sale?

A. $30,000
B. $48,000
C. $90,000
D. $120,000
E. $140,000

Mode

1. Set A, B, and C consist of the following elements:

   A {0, 3, 4, 2, 0, 4, 7, 8, 4, 17}   B {20, 12, -7, -9, -5, -7, 11, -5, 68}   C {-1.5, 0, 1.5}.

   If Z is defined as the sum of modes of sets A, B, and C, what is the value of Z?

   -2   -1   -8   3   5

2. The mode of a set of integers is x. What is the difference between the median of this set of integers and x?

   (1) The difference between any two integers in the set is less than 3.

   (2) The average of the set of integers is x.

Range

1. If set X contains numbers {-21, 6, 19, 126, 1000} and set Y contains numbers {-21, 990, 993, 996.19, 997.05, 999, 1000}, what is the difference between the ranges of set X and set Y?
-2 -1 0 3 5

2. Set X consists of prime numbers {3, 11, 7, K, 17, 19}. If integer Y represents the product of all elements in set X and if 11Y is an even number, what is the range of set X?
(A) 14 (B) 16 (C) 17 (D) 20 (E) 26

3. What could be the range of a set consisting of odd multiples of 7?
(A) 21 (B) 24 (C) 35 (D) 62 (E) 70

4. What is the range of a set consisting of the first 100 multiples of 7 that are greater than 70?
(A) 693 (B) 700 (C) 707 (D) 777 (E) 847

5. Set X consists of all two-digit primes and set Y consists of all positive odd multiples of 5 less than 100. If the two sets are combined into one, what will be the range of the new set?
(A) 84 (B) 89 (C) 90 (D) 92 (E) 95

6. At a business school conference with 100 attendees, are there any students of the same age (rounded to the nearest year) who attend the same school?
(1) The range of ages of the participants is 22 to 30, inclusive
(2) Participants represent 10 business schools

7. Set S contains 100 consecutive integers. If the range of the negative elements of Set S equals 80, what is the average (arithmetic mean) of the positive numbers in the set?

A. 8
B. 8.5
C. 9
D. 9.5
E. 10

8. If a randomly selected non-negative single digit integer is added to set X {2, 3, 7, 8}, what is the probability that the median of the set will increase while its range will remain the same?
(A) 20% (B) 30% (C) 40% (D) 50% (E) 60%

9. Set A consists of all positive integers less than 100; Set B consists of 10 integers, the first four of which are 2, 3, 5, and 7. What is the difference between the median of Set A and the range of Set B?
(1) All numbers in Set B are prime numbers;
(2) Each element in Set B is divisible by exactly two factors.

10. Set A consists of 8 distinct prime numbers. If x is equal to the range of set A and y is equal to the median of set A, is the product xy even?
(1) The smallest integer in the set is 5. (2) The largest integer in the set is 101.

11. If set S = {7, y, 12, 8, x, 9}, is x + y less than 18?
(1) The range of set S is less than 9.
(2) The average of x and y is less than the average of set S.

12. *Question removed*

13 The GMAT is scored on a scale of 200 to 800 in 10 point increments. (Thus 410 and 760 are real GMAT scores but 412 and 765 are not). A first-year class at a certain business school consists of 478 students. Did any students of the same gender in the first-year class who were born in the same-named month have the same GMAT score?
(1) The range of GMAT scores in the first-year class is 600 to 780.
(2) 60% of the students in the first-year class are male.

14. S is a set of positive integers. The average of the terms in S is equal to the range of the terms in S. What is the sum of all the integers in S?
(1) The range of S is a prime number that is less than 11 and is not a factor of 10.
(2) S is composed of 5 different integers.

15. If S is a finite set of consecutive even numbers, is the median of S an odd number?

(1) The mean of set S is an even number.

(2) The range of set S is divisible by 6.

16. 10 students took a chemistry exam that was graded on a scale of 0 to 100. Five of the students were in Dr. Adams' class and the other five students were in Dr. Brown's class. Is the median score for Dr. Adams' students greater than the median score for Dr. Brown's students?

(1) The range of scores for students in Dr. Adams' class was 40 to 80, while the range of scores for students in Dr. Brown's class was 50 to 90.

(2) If the students are paired in study teams such that each student from Dr. Adams' class has a partner from Dr. Brown's class, there is a way to pair the 10 students such that the higher scorer in each pair is one of Dr. Brown's students.

17. x is an integer greater than 7. What is the median of the set of integers from 1 to x inclusive?

(1) The average of the set of integers from 1 to x inclusive is 11.

(2) The range of the set of integers from 1 to x inclusive is 20.

18.

Stock	number of shares
v	68
w	112
x	56
y	94
z	45

The table shows the number of shares of each of the 5 stocks owned by Mr. Sami. If Mr Sami was to sell 20 shares of Stock X and buy 24 shares of stock y, what would be the increase in range of the number of shares of the 5 stocks owned by Mr Sami?

4 6 9 15 20

19. The numbers of books read by 7 students last year were 10, 5, p, q, r, 29 and 20. What was the range of the numbers of books read by the 7 students last year?

(1) $5<p<q$ (2) $p<r<15$

20. A set of 15 different integers have a range of 25 and a median of 25. What is greatest possible integer that could be in this set?

32 37 40 43 50

Standard Deviation

1. Find the SD of 7, 8, 9 and 10.
   -2 -1.12 1.12 11.2 112

2. Set A consists of all prime numbers between 10 and 25; Set B consists of consecutive even integers, and set C consists of consecutive multiples of 7. If all the three sets have an equal number of terms, which of the following represents the ranking of these sets in an ascending order of the standard deviation?
   (A) C, A, B (B) A, B, C (C) C, B, A (D) B, C, A (E) B, A, C

3. Set A consists of all even integers between 2 and 100, inclusive. Set X is derived by reducing each term in set A by 50, set Y is derived by multiplying each term in set A by 1.5, and set Z is derived by dividing each term in set A by -4. Which of the following represents the ranking of the three sets in descending order of standard deviation?
   (A) X, Y, Z (B) X, Z, Y (C) Y, Z, X (D) Y, X, Z (E) Z, Y, X

4. If M is a negative integer and K is a positive integer, which of the following could be the standard deviation of a set {-7, -5, -3, M, 0, 1, 3, K, 7}?
   I. -1.5 II. -2 III. 0
   (A) I only (B) II only (C) III only (D) I and III only (E) None

5. Sets A, B and C are shown below. If number 100 is included in each of these sets, which of the following represents the correct ordering of the sets in terms of the absolute increase in their standard deviation, from largest to smallest?
   A {30, 50, 70, 90, 110}, B {-20, -10, 0, 10, 20}, C {30, 35, 40, 45, 50}
   (A) A, C, B (B) A, B, C (C) C, A, B (D) B, A, C (E) B, C, A

6. Is the standard deviation of the numbers X, Y and Z equal to the standard deviation of 10,15 and 20?
   (1) Z - X = 10
   (2) Z - Y = 5

7. The table below represents three sets of numbers with their respective medians, means and standard deviations. The third set, Set [A+B], denotes the set that is formed by combining Set A and Set B.

	Median	Mean	Standard Deviation
Set A	X	Y	Z
Set B	L	M	N
Set [A + B]	Q	R	S

   If X – Y > 0 and L – M = 0, then which of the following must be true?
   I. Z > N II. R > M III. Q > R
   (A) I only (B) II only (C) III only (D) I and II only (E) None

8. If the mean of a data set is 75 and the standard deviation is 10, what is the range of scores that fall within one standard deviation of the mean?
   65 to 85 95 to 105 60 to 70 35 to 65 50 to 60

9. The mean score of a class on a test was 60 and the standard deviation was 15. If Elena's score was within 2 standard deviations of the mean, what is the lowest score she could have received?
   25 10 20 30 50

10. If y = ax + b, and if the standard deviation of x series is ‘S’, what is the standard deviation of y series?
    -aS a/S S/a aS aS^2

11. If ax + by + c = 0, and if the standard deviation of x series is ‘S’, what is the standard deviation of y series?
    |a/b| x S ab/S a/bS a+bS a/S

12. For a certain exam, a score of 58 was 2 standard deviations below mean and a score of 98 was 3 standard deviations above mean. What was the mean score for the exam?
74 76 78 80 82

13. Which of the following has the same standard deviation as {s, r, t}?
I. {r - 2, s - 2, t - 2}
II. {0, s - t, s - r}
III. {|r|, |s|, |t|}
(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

14. Let Set T = {2, 4, 5, 7}. Which of the following values, if added to Set T, would most increase the standard deviation of Set T?
1 3 6 8 14

15. What is the standard deviation of Q, a set of consecutive integers?
(1) Q has 21 members.
(2) The median value of set Q is 20.

16. Does data set A = {1, 2, x} have a greater standard deviation than data set B = {1, 2, 3}?
(1) x is greater than 3.
(2) x is less than 1.

17. 9.4, 9.9, 9.9, 9.9, 10.0, 10.2, 10.2, 10.5
The mean and the standard deviation of the 8 numbers shown are 10 and 0.3, respectively. What percentage of the 8 number's are within 1 standard deviation?
A) 90% B) 85% C) 80% D) 75% E) 70%

18. 70, 75,80,85,90,105,105,130,130,130
The list shown consists of the times, in seconds, that I took each of 10 school children to run a distance of 400 on of meters. If the standard devastation of the 10 running times is 22.4 seconds, rounded to the nearest tenth of a second, how many of the 10 running times are more than 1 standard deviation below the mean of the 10 running times?
a) one b) two c) three d) four e) five

19. The residents of town x participated in a survey to determine the number of hours per week each resident spent watching television. The distribution of the result of the survey had a mean of 21 hours and a standard deviation of 6 hours. The number of hours of that participated, a resident of town x watching television last week was between 1 and 2 standard deviations below the mean. Which of the following could be the number of hours the participated watched television last week?
a.30 b.20 c.18 d.12 e.6

20. 7.51 8.22 7.86 8.36 8.09 7.83 8.30 8.01 7.73 8.25 7.96 8.53
A vending machine is designed to dispense 8 ounces of coffee into a cup. After a test that recorded the number of ounces of coffee in each of 1,000 cups dispensed by the vending machine, the 12 listed amounts, in ounces, were selected from the data. If the 1,000 recorded amounts have a mean of 8.1 ounces and a standard deviation of 0.3 ounce, how many of the 12 listed amounts are within 1.5 standard deviations of the mean?
A. Four
B. Six
C. Nine
D. Ten
E. Eleven

21. A certain list of 100 data has an average of 6 and a standard deviation of d, where d is positive. Which of the following pairs of data, when added to the list, must result in a list of 102 data with standard deviation less than d?
A. -6 and 0 B. 0 and 0 C. 0 and 6 D. 0 and 12 E. 6 and 6

22. The lifetime of all the batteries produced by a certain company in a year have a distribution that is symmetric about the mean m. If the distribution has a standard deviation of d, what percent of the distribution is greater than m+d?
1) 68% of the distribution ties in the interval from m-d to m+d, inclusive.
2) 16% of the distribution is less than m-d

Answer Key
GMAT Quant Topic 2: Statistics

Part A: Mean

1. A
2. D
3. A
4. E
5. C
6. D
7. C
8. C
9. D
10. E
11. B
12. D
13. D
14. D
15. B
16. A
17. B
18. C
19. B
20. D
21. C
22. D
23. B
24. A
25. B
26. B
27. D
28. E
29. D

Part B: Median

1. B
2. E
3. E
4. A
5. E
6. C
7. D
8. E
9. A
10. B
11. A
12. A
13. C
14. B
15. B

16. B
17. B
18. D
19. C
20. C
21. C
22. A
23. C
24. A
25. B
26. D
27. D

Part C: Mode

1. C
2. C

Part D: Range

1. C
2. C
3. E
4. A
5. D
6. C
7. D
8. B
9. E
10. A
11. B
12. Question removed
13. A
14. C
15. A
16. B
17. D
18. D
19. E
20. D

Part E: Standard Deviation

1. C
2. E
3. D
4. E
5. E
6. C
7. E
8. A
9. D
10. D
11. A

12. A
13. D
14. E
15. A
16. A
17. D
18. B
19. D
20. E
21. E
22. D

Quant Topic 3

Inequalities + Absolute Value (Modulus)

1. If $-1 < x < 0$, which of the following must be true?
   I. $x^3 < x^2$ II. $x^5 < 1 - x$ III. $x^4 < x^2$
   I only I and II only II and III only I and III only I, II and III

2. Is $x > 0$? (1) $|x + 3| < 4$ (2) $|x - 3| < 4$

3. If x and n are integers, is the sum of x and n less than zero?
   (1) $x + 3 < n - 1$ (2) $-2x > 2n$

4. Is $a > c$? (1) $b > d$ (2) $ab^2 - b > b^2c - d$

5. If x is an integer, what is the value of x? (1) $-5x > -3x + 10$ (2) $-11x - 10 < 67$

6. If $8x > 4 + 6x$, what is the value of the integer x?
   (1) $6 - 5x > -13$ (2) $3 - 2x < -x + 4 < 7.2 - 2x$

7. Is $a + b > c + d$? (1) $a > c$ (2) $d < b$

8. If $\sqrt{(xy)} = xy$, what is the value of $x + y$? (1) $x = -1/2$ (2) y is not equal to 0.

9. Is $x > y$? (1) $x^2 > y$ (2) $\sqrt{x} < y$

10. If $6xy = x^2y + 9y$, what is the value of xy? (1) $y - x = 3$ (2) $x^3 < 0$

11. What is the value of x? (1) $x^2 - 5x + 6 = 0$ (2) $x > 0$

12. What is x? (1) $x^2 + 3x + 2 = 0$ (2) $x < 0$

13. If $3|3 - x| = 7$, what is the product of all the possible values of x?
    1/9 1/3 2/3 16/9 32/9

14. Is $a/b < 0$?
    (1) $a^2 / b^3 > 0$ (2) $ab^4 < 0$

15. Is d negative? (1) $e + d = -12$ (2) $e - d < -12$

16. If $a - b > a + b$, where a and b are integers, which of the following must be true?
    I. $a < 0$ II. $b < 0$ III. $ab < 0$
    I only II only I and II only I and III only II and III only

17. If $|a| = 1/3$ and $|b| = 2/3$, which of the following CANNOT be the result of $a + b$?
    -1 -1/3 1/3 2/3 1

18. If $|a| = |b|$, which of the following must be true?
    I. $a = b$ II. $|a| = -b$ III. $-a = -b$
    I only II only III only I and III only None

19. Which of the following inequalities has a solution set that when graphed on the number line, is a single segment of finite length?
    A. $x^4 \geq 1$ B. $x^3 \leq 27$ C. $x^2 \geq 16$
    D. $2 \leq |x| \leq 5$ E. $2 \leq 3x+4 \leq 6$

20. If n is a nonzero integer, is $x^n < 1$? (1) $x > 1$ (2) $n > 0$

21. If x is an integer, is 3^x less than 500? (1) $4^{x-1} < 4^x - 120$ (2) $x^2 = 36$

22. Is $x^3 > 1$? (1) $x > -2$ (2) $2x - (b - c) < c - (b - 2)$

23. If $\sqrt{[(x + 4)^2]} = 3$, which of the following could be the value of $x - 4$?
-11 -7 -4 -3 5

24. Is $x > 10^{10}$? (1) $x > 2^{34}$ (2) $x = 2^{35}$

25. Is XY>0? 1). X-Y>-2 2). X-2Y<-6

26. If $|x - (9/2)| = 5/2$, and if y is the median of a set of p consecutive integers, where p is odd, which of the following must be true?
I. xyp is odd II. $xy(p^2 + p)$ is even III. $x^2y^2p^2$ is even
II only III only I and III II and III I, II, and III

27. If $|x| + |y| = -x - y$ and xy does not equal 0, which of the following must be true?
$x + y > 0$ $x + y < 0$ $x - y > 0$ $x - y < 0$ $x^2 - y^2 > 0$

28. If x and y are integers and xy does not equal 0, is $xy < 0$?
(1) $y = x^4 - x^3$ (2) $-12y^2 - y^2x + x^2y^2 > 0$

29. Is x a negative number?
(1) x^2 is a positive number. (2) $x . |y|$ is not a positive number.

30. If a, b, c, and d are integers and $ab^2c^3d^4 > 0$, which of the following must be positive?
I. a^2cd II. bc^4d III. $a^3c^3d^2$
I only II only III only I and III I, II, and III

31. Is $x|y| > y^2$? (1) $x > y$ (2) $y > 0$

32. What is x? (1) $|x| < 2$ (2) $|x| = 3x - 2$

33. Is $x > y$? (1) $\sqrt{x} > y$ (2) $x^3 > y$

34. If x is not equal to 0, is |x| less than 1? (1) $x / |x| < x$ (2) $|x| > x$

35. If $r + s > 2t$, is $r > t$? (1) $t > s$ (2) $r > s$

36. If a and b are integers, and $|a| > |b|$, is $a . |b| < a - b$?
(1) $a < 0$ (2) $ab \geq 0$

37. Is $a > c$? (1) $b > d$ (2) $ab^2 - b > b^2c - d$

38. If $p < q$ and $p < r$, is $(p)(q)(r) < p$? (1) $pq < 0$ (2) $pr < 0$

39. If $|x|*y+ 9 > 0$, and x and y are integers, is $x < 6$? (1) y is negative (2) $|y| \leq 1$

40. If n is not equal to 0, is $|n| < 4$? (1) $n^2 > 16$ (2) $1/|n| > n$

41. If x and y are non-zero integers and $|x| + |y| = 32$, what is xy?
(1) $-4x - 12y = 0$ (2) $|x| - |y| = 16$

42. What is the value of y? (1) $3|x^2 - 4| = y - 2$ (2) $|3 - y| = 11$

43. Is $x > 0$? (1) $|x + 3| = 4x - 3$ (2) $|x - 3| = |2x - 3|$

44. What is the value of |x|? (1) $|x^2 + 16| - 5 = 27$ (2) $x^2 = 8x - 16$

45. If $x > y$, $x^2 - 2xy + y^2 - 9 = 0$, and $x + y = 15$, what is x?
3 6 12 18 9

46. Is $|n| < 1$? (1) $n^x - n < 0$ (2) $x^{-1} = -2$

47. Is $5^n < 0.04$? (1) $(1/5)^n > 25$ (2) $n^3 < n^2$

48. What is the ratio of 2x to 3y?
(1) The ratio of x^2 to y^2 is equal to 36/25.
(2) The ratio of x^5 to y^5 is greater than 1.

49. If x and y are integers, does $x^y y^{-x} = 1$? (1) $x^x > y$ (2) $x > y^y$

50. If a is nonnegative, is $x^2 + y^2 > 4a$? (1) $(x + y)^2 = 9a$ (2) $(x - y)^2 = a$

51. If k is a positive constant and $y = |x - k| - |x + k|$, what is the maximum value of y?
(1) $x < 0$ (2) $k = 3$

52. If $x > 0$, what is the least possible value for $x + (1/x)$?
(A) 0.5 (B) 1 (C) 1.5 (D) 2 (E) 2.5

53. Is $(|x^{-1}y^{-1}|)^{-1} > xy$? (1) $xy > 1$ (2) $x^2 > y^2$

54. Is $xy + xy < xy$? (1) $x^2 / y < 0$ (2) $x^9 (y^3)^3 < (x^2)^4 y^8$

55. w, x, y, and z are positive integers. If $w/x < y/z < 1$, what is the proper order of magnitude, increasing from left to right, of the following quantities:
x/w, z/y, x^2/w^2, xz/wy, $(x + z) / (w + y)$, 1?

(A) 1, z/y, x/w, $(x+z)/(w+y)$, x^2/w^2, xz/wy
(B) 1, z/y, $(x+z)/(w+y)$, x/w, xz/wy, x^2/w^2
(C) 1, z/y, x/w, $(x+z)/(w+y)$, xz/wy, x^2/w^2
(D) 1, z/y, x/w, xz/wy, $(x+z)/(w+y)$, x^2/w^2
(E) 1, z/y, $(x+z)/(w+y)$, xz/wy, x^2/w^2, x/w

56. Two missiles are launched simultaneously. Missile 1 launches at a speed of x miles per hour, increasing its speed by a factor of $\sqrt{x}$ every 10 minutes (so that after 10 minutes its speed is $x\sqrt{x}$, after 20 minutes its speed is x^2, and so forth). Missile 2 launches at a speed of y miles per hour, doubling its speed every 10 minutes. After 1 hour, is the speed of Missile 1 greater than that of Missile 2?
1) $x = \sqrt{y}$ 2) $x > 8$

57. $8xy^3 + 8x^3y = 2x^2y^2 / 2^{-3}$, What is xy?
(1) $y > x$ (2) $x < 0$

58. If $(a - b)c < 0$, which of the following cannot be true?
$a < b$ $c < 0$ $|c| < 1$ $ac > bc$ $a^2 - b^2 > 0$

59. If $|ab| > ab$, which of the following must be true?
I. $a < 0$ II. $b < 0$ III. $ab < 0$
I only II only III only I and III II and III

60. If $b < c < d$ and $c > 0$, which of the following cannot be true if b, c and d are integers?
$bcd > 0$ $b + cd < 0$ $b - cd > 0$ $b/cd < 0$ $b^3cd < 0$

61. If $ab > cd$ and a, b, c and d are all greater than zero, which of the following CANNOT be true?
$c > b$ $d > a$ $b/c < d/a$ $a/c > d/b$ $(cd)^2 < (ab)^2$

62. Is $x + y > 0$?
(1) $x - y > 0$ (2) $x^2 - y^2 > 0$

63. Is $|x| < 1$?
(1) $|x + 1| = 2|x - 1|$ (2) $|x - 3| > 0$

64. Is $|a| > |b|$?
(1) $b < -a$ (2) $a < 0$

65. If r is not equal to 0, is $r^2 / |r| < 1$
(1) $r > -1$ (2) $r < 1$

66. Which of the following sets includes ALL of the solutions of x that will satisfy the equation: $|x-2| - |x-3| = |x-5|$?
   A. {-6, -5, 0, 1, 7, 8}
   B. {-4, -2, 0, 10/3, 4, 5}
   C. {-4, 0, 1, 4, 5, 6}
   D. {-1, 10/3, 3, 5, 6, 8}
   E. {-2, -1, 1, 3, 4, 5}

67. If $abc \neq 0$, what is the value of $(a^3 + b^3 + c^3) / abc$?
(1) $|a|=1, |b|=2, |c|=3$ (2) $a + b + c = 0$

68. Given that $w = |x|$ and $x = 2^b - (8^{30} + 8^5)$, which of the following values for b yields the lowest value for w?
(A) 35 (B) 90 (C) 91 (D) 95 (E) 105

69. If x is an integer, what is the value of x?
1) $|x - |x^2|| = 2$ 2) $|x^2 - |x|| = 2$

70. w, x, y, and z are integers. If $z > y > x > w$, is $|w| > x^2 > |y| > z^2$?
1) $wx > yz$ 2) $zx > wy$

71. If $ab \neq 0$, is $\left(\frac{a-b}{a^{-1}-b^{-1}}\right)^{-1} > a + b$?
(1) $|a| > |b|$ (2) $a < b$

72. Is $|a| + |b| > |a + b|$?
(1) $a^2 > b^2$ (2) $|a| \times b < 0$

73. Is $\sqrt{x}$ a prime number?
(1) $|3x - 7| = 2x + 2$ (2) $x^2 = 9x$

74. What is the average of x and |y| ?
(1) $x + y = 20$ (2) $|x + y| = 20$

75. If x and y are nonzero integers, is $(x^{-1} + y^{-1})^{-1} > [(x^{-1})(y^{-1})]^{-1}$?
(1) $x = 2y$ (2) $x + y > 0$

76. Is $p^2q > pq^2$? (1) $pq < 0$ $p < 0$

77. Is $m > n$? (1) $n - m + 2 > 0$ (2) $n - m - 2 > 0$

78. Is $3^p > 2^q$? (1) $q = 2p$ (2) $q > 0$

79. Is mp greater than m? (1) $m > p > 0$ (2) p is less than 1

80. Is w less than y?
(1) $1.3 < w < 1.3101$ (2) $1.3033 < y$

81. If a and b are integers and $a \neq b$, is $|a|b > 0$?
(1) $|a^b| > 0$ (2) $|a|^b$ is a non–zero integer

82. If 500 is the multiple of 100 that is closest to X and 400 is the multiple of 100 that is closest to Y, which multiple of 100 is closest to X+Y?
1). X<500 2). Y<400

83. Is the three-digit number n less than 550?
1). the product of the digits in n is 30 2). the sum of the digits in n is 10

84. If $X^4 + Y^4=100$, then the greatest possible value of X is between:
A. 0 and 3 B. 3 and 6 C. 6 and 9 D. 9 and 12 E. 12 and 15

85. Is $2X-3Y< X^2$? 1). $2X-3Y=-2$ 2). $X>2$ and $Y>0$

86. Is m+z>0 1). m-3z>0 2). 4z-m>0

87. If $X > Y^2 > Z^4$, which of the following statements could be true?

I. X >Y> Z   II. Z >Y >X   III.X >Z >Y

A. I only   B. I and II only   C. I and III only

D. II and III only   E. I , II , and III

88. Is X+Y <1   1). x <8/9   2).Y <1/8

89. If y is an integer and y=x+|x|, is y=0?   1). x<0   2). y<1

90. Is x-y+1 greater than x+y-1?   1) x>0   2) y<0

91. Is W greater than 1?   1). W + 2 > 0   2). W^2 > 1

92. If n and p are integers, is p>0?   1). n+1>0   2). np>0

93. The number x and y are not integers, the value of x is closest to which integer?

1). 4 is the integer that is closest to x+y

2). 1 is the integer that is closest to x-y

Answer Key

GMAT Quant Topic 3: Inequalities + Absolute Value (Modulus)

1. E
2. E
3. B
4. C
5. C
6. D
7. C
8. C
9. E
10. B
11. E
12. E
13. E
14. C
15. C
16. B
17. D
18. E
19. E
20. C
21. C
22. B
23. A
24. D
25. C
26. A
27. B
28. E
29. E
30. C
31. C
32. B
33. C
34. C
35. D
36. E
37. C
38. E
39. E
40. A
41. A
42. C
43. A
44. D
45. E
46. C
47. A

48. C
49. B
50. E
51. B
52. D
53. A
54. E
55. B
56. C
57. A
58. D
59. C
60. C
61. C
62. C
63. C
64. E
65. C
66. C
67. B
68. B
69. C
70. B
71. E
72. E
73. C
74. E
75. A
76. C
77. B
78. C
79. C
80. E
81. E
82. E
83. C
84. B
85. D
86. C
87. E
88. E
89. D
90. B
91. E
92. C
93. E

GMAT Quant Topic 4 (Numbers)

Types of Numbers

1. What is the sum of the digits of the positive integer n where n < 99?
(1) n is divisible by the square of the prime number y.
(2) y^4 is a two-digit odd integer.

2. If x is a positive integer, is x! + (x + 1) a prime number?
(1) x < 10 (2) x is even

3. Is √(x + y) an integer?
(1) $x^3 = 64$ (2) $x^2 = y - 3$

4. If x is a prime number, what is the value of x?
(1) 2x + 2 is the cube of a positive integer.
(2) The average of any x **consecutive integers** is an integer.

5. List K consists of 12 **consecutive integers**, if -4 is the least integer in list K, what is the range of the positive integers in the list K?
4 6 2 3 5

6. If m, r, x and y are positive, is the ratio of the m to r equal to the ratio of x to y?
1) the ratio of m to y is equal to the ratio of x to r
2) the ratio of m+x to r+y is equal to the ratio of x to y

7. If the integer a and n are greater than 1, and the product of the first 8 positive integers is a multiple of a^n, what is the value of a?
1). $a^n = 64$ 2). n = 6

8. If x is the sum of six **consecutive integers**, then x is divisible by which of the following:
I. 3 II. 4 III. 6
I only II only III only I and III I, II, and III

9. In a certain deck of cards, each card has a positive integer written on it. In a multiplication game, a child draws a card and multiplies the integer on the card by the next larger integer. If each possible product is between 15 and 200, then the least and greatest integers on the cards could be
3 and 15 3 and 20 4 and 13 4 and 14 5 and 14

10. If p is a positive integer, what is the value of p?
1). p/4 is a prime number 2). p is divisible by 3

11. The number 75 can be written as the sum of the squares of 3 different positive integers. What is the sum of these 3 integers?
17 16 15 14 13

12. An integer greater than 1 that is not prime is called composite. If the two-digit integer n is greater than 20, is n composite?
1). the tens digit of n is a factor of the units digit of n
2). the tens digit of n is 2.

13. If n is a multiple of 5 and n= p^2q, where p and q are prime numbers, which of the following must be a multiple of 25?
p^2 q^2 pq p^2q^2 p^3q

14. On the number line shown, is zero halfway between r and s?
-----r-----s-----t---
1). s is to the right of zero
2). the distance between t and r is the same as the distance between t and -s.

15. What is the sum of the first 10 prime numbers? 100
101 128 129 158

16. On the number line, the segment from 0 to 1 has been divided into fifths, as indicated by the large tick marks, and also into sevenths, as indicated by the small tick marks. What is the least possible distance between any two of the tick marks?

A. 1/70
B. 1/35
C. 2/35
D. 1/12
E. 1/7

17. For non-zero integers a, b, c and d, is ab/cd positive?
(1) ad + bc = 0 (2) abcd = -4

18. Is the positive integer J divisible by a greater number of different prime numbers than the positive integer k?
1). J is divisible by 30 2). k=1000

19. If n is a positive integer and the product of all the integers from 1 to n, inclusive, is a multiple of 990, what is the least possible value of n?
A. 10
B. 11
C. 12
D. 13
E. 14

20. For which of the following functions is f(a+b) = f(b) + f(a) for all positive numbers a and b?
A. $f(x)=x^2$ B. f(x)=x+1 C. $f(x)=\sqrt{x}$ D. f(x)=2/x E. f(x)=-3x

21. The point A, B, C, and D are on the number line, not necessarily in the order. If the distance between A and B is 18 and the distance between C and D is 8, what is the distance between B and D?
1). The distance between C and A is the same as the distance between C and B.
2). A is to the left of D on the number line.

22. A certain list consists of several different integers. Is the product of all the integers in the list positive?
1). the product of the greatest and the smallest of the integers in the list are positive.
3). There is even number of integers in the list.

23. The sum of positive integers x and y is 77. What is the value of xy?
1). x=y+1
2). x and y have the same tens' digit.

24. If there are more than two numbers in certain list, is each of the numbers in the list equal to 0?
1). The product of any two numbers in the list equal to 0.
2). The sum of any two numbers in the list equal to 0.

25. For which of the following values of x is $\{1-[2-(x^{1/2})]^{1/2}\}^{1/2}$ not defined as a real number?
1 2 3 4 5

26. For a finite sequence of nonzero numbers, the number of variations in sign is defined as the number of pairs of consecutive terms of the sequence for which the product of the two consecutive terms is negative. What is the number of variations is in sign for the sequence: 1, -3, 2, 5, -4, -6?
A. 1
B. 2
C. 3
D. 4
E. 5

27. If xy +z =x(y+z), which of the following must be true?
A. x=0 and z=0
B. x=1 and y=1
C. y=1 and z=0
D. x=1 or y=0
E. x=1 or z=0

28. Symbol * denote to be one of the operations add, subtract, multiply, or divide. Is (6*2)*4=6*(2*4)?
1). 3*2>3 2). 3*1=3

29. If m and r are two numbers on a number line, what is the value of r?
1). The distance between r and 0 is 3 time the distance between m ad 0.
2). 12 is halfway between m and r

30. As the table shows, m+n=?

+	x	Y	z
4	1	-5	m
E	7	N	10
F	2	-4	5

-2 -1 2 3 5

31. If w, y, and z are positive integers, and w = y – z, is w a perfect square?
(1) y + z is a perfect square. (2) z is even.

Odds and Evens

1. Is z even? (1) z/2 is even. (2) 3z is even.

2. If m, n, and p are integers, is m + n odd?
(1) $m = p^2 + 4p + 4$ (2) $n = p^2 + 2m + 1$

3. If a and b are both positive integers, is $b^{a+1} - ba^b$ odd?
(1) a + (a + 4) + (a – 8) + (a + 6) + (a – 10) is odd
(2) $b^3 + 3b^2 + 5b + 7$ is odd

4. Is the positive integer x odd?
(1) $x = y^2 + 4y + 6$, where y is a positive integer.
(2) $x = 9z^2 + 7z - 10$, where z is a positive integer.

5. If w, y, and z are positive integers, and w = y – z, is w a perfect square?
(1) y + z is a perfect square. (2) z is even.

6. If x and y are positive integers and 3x + 5 < x + 11, is x a prime number?
(1) The sum of x and y is even. (2) The product of x and y is odd.

7. Is the positive integer p even? (1) $p^2 + p$ is even. (2) 4p + 2 is even.

8. If p and q are integers and p + q + p is odd, which of the following must be odd?
p q p + q pq pq + p

9. If a , b, and c are integers and ab^2 / c is a positive even integer, which of the following must be true?
I. ab is even II. ab > 0 III. c is even
I only II only I and II I and III I, II, and III

10. If k and y are integers, and 10k + y is odd, which of the following must be true?
   A. k is odd
   B. k is even
   C. y is odd
   D. y is even
   E. both k and y are odd

11. Each digit in the two-digit number G is halved to form a new two-digit number H. Which of the following could be the sum of G and H?
153 150 137 129 89

12. If a is an even integer and b is an odd integer, which of the following cannot be an even integer?
ab a/b b/a a^b a^{2b+1}

13. If x and y are prime integers and x < y, which of the following cannot be true?
x is even x + y is odd xy is even y + xy is even 2x + y is even

14. If q, r, and s are **consecutive even integers** and q < r < s, which of the following CANNOT be the value of $s^2 - r^2 - q^2$?
(A) -20 (B) 0 (C) 8 (D) 12 (E) 16

15. n is an integer greater than or equal to 0. The sequence t_n for n > 0 is defined as $t_n = t_{n-1} + n$. Given that $t_0 = 3$, is t_n even?
(1) n + 1 is divisible by 3 (2) n – 1 is divisible by 4

16. y and z are nonzero integers, is the square of (y + z) even?
(1) y – z is odd. (2) yz is even.

17. If x and y are positive integers, is the product xy even?
1). 5x-4y is even 2). 6x+7y is even
.

18. If x and y are integers, is x (y+1) an even number?
1). x, and y are prime numbers. 2). y>7

19. For all positive integers m, (m) = 3m when m is odd and (m) = ½ m when m is even, which of the following is equivalent to (9)*(6)?

(81) (54) (36) (27) (18)

20. If m and n are integers, is m odd?

1). m+n is odd 2). m+n =n^2 + 5

21. If c and d are integers, is C even?

1). c(d+1) is even 2). (c+2)(d+4) is even

22. If x is an integer, is $(x^2+1)(x+5)$ an even number?

1). x is an odd number. 2). each prime factor of x^2 is greater than 7

23. If a is an even integer and b is an odd integer, which of the following cannot be an even integer?

ab a/b b/a a^b a^{2b+1}

24. If y and z are nonzero integers, is the square of (y + z) even?

(1) y – z is odd. (2) yz is even.

25. If x and y are prime integers and x < y, which of the following cannot be true?

A. x is even
B. x + y is odd
C. xy is even
D. y + xy is even
E. 2x + y is even

Unit's digits, factorial powers

1. 17^{27} has a units digit of:
   1 2 3 7 9

2. If r, s, and t are all positive integers, what is the remainder of $2^p / 10$, if $p = rst$?
   (1) s is even (2) $p = 4t$

3. $1^1+2^2+3^3+...+10^{10}$ is divided by 5. What is the remainder?
   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. Given that p is a positive even integer with a positive units digit, if the units digit of p^3 minus the units digit of p^2 is equal to 0, what is the units digit of p + 3?
   3 6 7 9 It cannot be determined from the given information.

5. If x is a positive integer, what is the units digit of $(24)^{(2x + 1)}(33)^{(x + 1)}(17)^{(x + 2)}(9)^{(2x)}$?
   (A) 4 (B) 6 (C) 7 (D) 8 (E) 9

6. If a and b are positive integers and $x = 4^a$ and $y = 9^b$, which of the following is a possible units digit of xy?
   1 4 5 7 8

7. If $x = 3^{21}$ and $y = 6^{55}$, what is the remainder when xy is divided by 10?
   (A) 2 (B) 3 (C) 4 (D) 6 (E) 8

8. If x is a positive integer, what is the remainder when $7^{12x+3} + 3$ is divided by 5?
   0 1 2 3 4

9. If x and y are positive integers and $n = 5^x + 7^{y + 15}$, what is the units digit of n?
   (1) $y = 2x – 15$ (2) $y^2 – 6y + 5 = 0$

10. What is the units digit of $(71)^5(46)^3(103)^4 + (57)(1088)^3$?
    0 1 2 3 4

11. If $\dfrac{(13!)^{16} - (13!)^8}{(13!)^8 + (13!)^4} = a$, what is the units digit of $\dfrac{a}{(13!)^4}$?

    (A) 0 (B) 1 (C) 3 (D) 5 (E) 9

12. What is the units digit of $177^{28} – 133^{23}$?

(A) 1 (B) 3 (C) 4 (D) 6 (E) 9

13. What is the greatest integer m for which the number $50! / 10^m$ is an integer?

(A) 5 (B) 8 (C) 10 (D) 11 (E) 12

14. How many terminating zeroes does 200! have?

(A) 40 (B) 48 (C) 49 (D) 55 (E) 64

15. If $(243)^x(463)^y = n$, where x and y are positive integers, what is the units digit of n?

(1) $x + y = 7$ (2) $x = 4$

16. If y is divisible by the square of an even prime number and x is the actual square of an even prime number, then what is the units digit of x^y?

0 2 4 6 8

17. If x is a positive integer, what is the units digit of x^2?

(1) The units digit of x^4 is 1. (2) The units digit of x is 3.

Decimals

1. In the number 1.4ab5, a and b represent single positive digits. If x = 1.4ab5, what is the value of 10 – x?
(1) If x is rounded to the nearest hundredth, then 10 – x = 8.56.
(2) If x is rounded to the nearest thousandth, then 10 – x = 8.564.

2. If a, b, c, d and e are integers and $p = 2^a 3^b$ and $q = 2^c 3^d 5^e$, is p/q a terminating decimal?
(1) a > c (2) b > d

3. If the fraction d were converted into a decimal, would there be more than 3 nonzero digits to the right of the decimal point?
(1) The denominator of d is exactly 8 times the numerator of d.
(2) If d were converted into a decimal, d would be a non-repeating decimal.

4. If x is an integer, can the number (5/28)(3.02)(90%)(x) be represented by a finite number of non-zero decimal digits?
(1) x is greater than 100 (2) x is divisible by 21

5. Given that a, b, c, and, d are non-negative integers, is the fraction $(ad) / (2^a 3^b 4^c 5^d)$ a terminating decimal?
(1) $d = (1 + a)(a^2 – 2a + 1) / (a – 1)(a^2 – 1)$ (2) $b = (1 + a)(a^2 – 2a + 1) – (a – 1)(a^2 – 1)$

6. If d represents the hundredths digit and e represents the thousandths digit in the decimal 0.4de, what is the value of this decimal rounded to the nearest tenth?
(1) d – e is equal to a positive perfect square. (2) $\sqrt{d} > e^2$

7. Is the hundredth digit of decimal d greater than 5?
1). The tenth digit of 10d is 7 2). The thousandth digit of d/10 is 7

8. The value of x is derived by summing a, b, and c and then rounding the result to the tenths place. The value of y is derived by first rounding a, b, and c to the tenths place and then summing the resulting values. If a = 5.45, b = 2.98, and c = 3.76, what is y – x?
-.1 0 .05 .1 .2

9. What is the value of the tenths digit of number x?
(1) The hundredths digit of x is 5
(2) Number x, rounded to the nearest tenth, is 54.5

10. If x and y each represent a single digit, does the number 8.3xy round to 8.3 when it is rounded to the nearest tenth?
(1) x = 5 (2) y = 9

11. If j and k each represent positive single digits, and y = 2.j3k, what is y rounded to the nearest tenth?
(1) j > k (2) If y is rounded to the nearest hundredth, the result is 2.74.

12. If the fraction d were converted into a decimal, would there be more than 3 nonzero digits to the right of the decimal point?
(1) The denominator of d is exactly 8 times the numerator of d.
(2) If d were converted into a decimal, d would be a non-repeating decimal.

13. **d = 83,521, y73/441, 682,36y**
In the expression above, the letter y represents a single digit from 0 to 9. Is d a decimal with exactly ten digits?
(1) The sum of all the digits in the numerator is not a multiple of 3.
(2) 33 is a factor of the denominator.

Sequences and Series

1. If integer k is equal to the sum of all even multiples of 15 between 295 and 615, what is the greatest prime factor of k?
5 7 11 13 17

2. If S is the infinite sequence $S_1 = 6$, $S_2 = 12$, ..., $S_n = S_{n-1} + 6$,..., what is the sum of all terms in the set $\{S_{13}, S_{14}, ..., S_{28}\}$?
1,800 1,845 1,890 1,968 2,016

3. In an increasing sequence of 5 **<u>consecutive even integers</u>**, the sum of the second, third, and fourth integer is 132. What is the sum of the first and last integers?
84 86 88 90 92

4. What is the sum of the multiples of 7 from 84 to 140, inclusive?
896 963 1008 1792 2016

5. In a sequence of terms in which each term is three times the previous term, what is the fourth term?
(1) The first term is 3. (2) The second to last term is 3^{10}.

6. If each term in the sum a1 + a2 + a3 + ... +an is either 7 or 77 and the sum is equal to 350, which of the following could equal to n?
38 39 40 41 42

7. $2+2+2^2+2^3+2^4+2^5+2^6+2^7+2^8=?$

A. 2^9

B. 2^{10}

C. 2^{16}

D. 2^{35}

E. 2^{37}

8. For any integer k from 1 to 10, inclusive, the kth of a certain sequence is given by $[(-1)^{(k+1)}] \times (1 / 2^k)$. If T is the sum of the first 10 terms of the sequence, then T is:
A. greater than 2 B. between 1 and 2 C. between 1/2 and 1
D. between 1/4 and ½ E. less than 1/4

9. Sequence A is defined by the equation $A_n = 3n + 7$, where n is an integer greater than or equal to 1. If set B is comprised of the first x terms of sequence A, what is the median of set B?
(1) The sum of the terms in set B is 275.
(2) The range of the terms in set B is 30

10. S is the infinite sequence $S_1 = 2$, $S_2 = 22$, $S_3 = 222$,… $S_k = S_{k-1} + 2(10^{k-1})$. If p is the sum of the first 30 terms of S, what is the eleventh digit of p, counting right to left from the units digit?
1 2 4 6 9

11. Sequence S is defined as $S_n = 2S_{n-1} - 2$. If $S_1 = 3$, then $S_{10} - S_9 = ?$
2 120 128 250 256

12. $S_n = 2S_{n-1} + 4$ and $Q_n = 4Q_{n-1} + 8$ for all n > 1. If $S_5 = Q_4$ and $S_7 = 316$, what is the first value of n for which Q_n is an integer?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

13. What is the sixtieth term in the following sequence? 1, 2, 4, 7, 11, 16, 22...
(A) 1,671 (B) 1,760 (C) 1,761 (D) 1,771 (E) 1,821

14. Sequence S is defined as $S_n = X + (1/X)$, where $X = S_{n-1} + 1$, for all n > 1.
If $S_1 = 201$, then which of the following must be true of Q, the sum of the first 50 terms of S?
(A) 13,000 < Q < 14,000 (B) 12,000 < Q < 13,000 (C) 11,000 < Q < 12,000
(D) 10,000 < Q < 11,000 (E) 9,000 < Q < 10,000

15. In a certain sequence, every term after the first is determined by multiplying the previous term by an integer constant greater than 1. If the fifth term of the sequence is less than 1000, what is the maximum number of positive integer values possible for the first term?
A) 60 B) 61 C) 62 D) 63 E) 64

16. The sum of the squares of the first 15 positive integers ($1^2 + 2^2 + 3^2 + \ldots + 15^2$) is equal to 1240. What is the sum of the squares of the second 15 positive integers ($16^2 + 17^2 + 18^2 + \ldots + 30^2$) ?
(A) 2480 (B) 3490 (C) 6785 (D) 8215 (E) 9255

17. Given a series of n consecutive positive integers, where $n > 1$, is the average value of this series an integer divisible by 3?
(1) n is odd (2) The sum of the first number of the series and $(n – 1) / 2$ is an integer divisible by 3

18. A certain series is defined by the following recursive rule: $S_n = k (S_{n-1})$, where k is a constant. If the 1st term of this series is 64 and the 25th term is 192, what is the 9th term?
-64 64 64^2 $64\sqrt[3]{3}$ $64\sqrt{2}$

19. The infinite sequence S_k is defined as $S_k = 10 S_{k-1} + k$, for all $k > 1$. The infinite sequence A_n is defined as $A_n = 10 A_{n-1} + (A_1 – (n - 1))$, for all $n > 1$. q is the sum of S_k and A_n. If $S_1 = 1$ and $A_1 = 9$, and if A_n is positive, what is the maximum value of k + n when the sum of the digits of q is equal to 9?
(A) 6 (B) 9 (C) 12 (D) 16 (E) 18

20. A certain club has exactly 5 new members at the end of its first week. Every subsequent week, each of the previous week's new members (and only these members) brings exactly x new members into the club. If y is the number of new members brought into the club during the twelfth week, which of the following could be y?
(A) $\sqrt[12]{5}$ (B) $3^{11}5^{11}$ (C) $3^{12}5^{12}$ (D) $3^{11}5^{12}$ (E) 60^{12}

21. $36^2 + 37^2 + 38^2 + 39^2 + 40^2 + 41^2 + 42^2 + 43^2 + 44^2 =$
(A) 14400 (B) 14440 (C) 14460 (D) 14500 (E) 14520

22. A certain established organization has exactly 4096 members. A certain new organization has exactly 4 members. Every 5 months the membership of the established organization increases by 100 percent. Every 10 months the membership of the new organization increases by 700 percent. New members join the organizations only on the last day of each 5- or 10-month period. Assuming that no member leaves the organizations, after how many months will the two groups have exactly the same number of members?
(A) 20 (B) 40 (C) 50 (D) 80 (E) 100

23. In the infinite sequence A, $A_n = x^{n-1} + x^n + x^{n+1} + x^{n+2} + x^{n+3}$, where x is a positive integer constant. For what value of n is the ratio of A_n to $x(1 + x(1 + x(1 + x(1 + x))))$ equal to x^5?
(A) 8 (B) 7 (C) 6 (D) 5 (E) 4

24. If the expression $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}}$ extends to an infinite number of roots and converges to a positive number x, what is x?

(A) $\sqrt{3}$ (B) 2 (C) $1 + \sqrt{2}$ (D) $1 + \sqrt{3}$ (E) $2\sqrt{3}$

25. What is the sum of the even integers between 200 and 400, inclusive?
29,700 30,000 30,300 60,000 60,300

26.

98	-200	310	-396	498
102	-202	290	-402	502
101	-198	305	-398	501
100	-204	295	-404	500
99	-196	300	-400	499

What is the sum of all of the integers in the chart above?
0 300 500 1,500 6,500

27. The sequence $f(n) = (2n)! \div n!$ is defined for all positive integer values of n. If x is defined as the product of the first 10 ten terms of this sequence, which of the following is the greatest factor of x?
(A) 2^{20} (B) 2^{30} (C) 2^{45} (D) 2^{52} (E) 2^{55}

Remainders, Divisibility

1. When the positive integer x is divided by 9, the remainder is 5. What is the remainder when 3x is divided by 9?
0 1 3 4 6

2. If (x # y) represents the remainder that results when the positive integer x is divided by the positive integer y, what is the sum of all the possible values of y such that (16 # y) = 1?
8 9 16 23 24

3. If k and x are positive integers and x is divisible by 6, which of the following CANNOT be the value of $\sqrt{288kx}$?
$24k\sqrt{3}$ $24\sqrt{k}$ $24\sqrt{(3k)}$ $24\sqrt{(6k)}$ $72\sqrt{k}$

4. $10^{25} – 560$ is divisible by all of the following EXCEPT:
11 8 5 4 3

5. x, y, a, and b are positive integers. When x is divided by y, the remainder is 6. When a is divided by b, the remainder is 9. Which of the following is NOT a possible value for y + b?
24 21 20 17 15

6. In order to play a certain game, 24 players must be split into n teams, with each team having an equal number of players. If there are more than two teams, and if each team has more than two players, how many teams are there?
(1) If thirteen new players join the game, one must sit out so that the rest can be split up evenly among the teams.
(2) If seven new players join the game, one must sit out so that the rest can be split up evenly among the teams.

7. When the positive integer x is divided by 4, is the remainder equal to 3?
(1) When x/3 is divided by 2, the remainder is 1. (2) x is divisible by 5.

8. Seven integers, x1, x2, x3, x4, x5, x6, and x7, are picked at random from the set of all integers between 10 and 110, inclusive. If each of these integers is divided by 7 and the 7 remainders are all added together, what would be the sum of the 7 remainders?
(1) The range of the remainders is 6. (2) The seven integers are **consecutive**.

9. When the integer x is divided by the integer y, the remainder is 60. Which of the following is a possible value of the quotient x/y?
I. 15.15 II. 18.16 III. 17.17
(A) I only (B) II only (C) III only (D) I and II only
(E) I and III only

10. If j and k are positive integers where k > j, what is the value of the remainder when k is divided by j?
(1) There exists a positive integer m such that k = jm + 5. (2) j > 5

11. Five **consecutive positive integers** are chosen at random. If the average of the five integers is odd, what is the remainder when the largest of the five integers is divided by 4?
(1) The third of the five integers is a prime number.
(2) The second of the five integers is the square of an integer.

12. Can a batch of identical cookies be split evenly between Laurel and Jean without leftovers and without breaking a cookie?
(1) If the batch of cookies were split among Laurel, Jean and Marc, there would be one cookie left over.
(2) If Peter eats three of the cookies before they are split, there will be no leftovers when the cookies are split evenly between Laurel and Jean.

13. Is n/18 an integer?
(1) 5n/18 is an integer. (2) 3n/18 is an integer.

14. If a and b are both single-digit positive integers, is a + b a multiple of 3?
(1) The two-digit number "ab" (where a is in the tens place and b is in the ones place) is a multiple of 3.
(2) a – 2b is a multiple of 3.

15. The ratio of cupcakes to children at a particular birthday party is 104 to 7. Each child at the birthday party eats exactly x cupcakes (where x is a positive integer) and the adults attending the birthday party do not eat anything. If the number of cupcakes that remain uneaten is less than the number of children at the birthday party, what must be true about the number of uneaten cupcakes?
I. It is a multiple of 2. II. It is a multiple of 3. III. It is a multiple of 7.
(A) I only (B) II only (C) III only (D) I and II only (E) I, II and III

16. When the positive integer x is divided by 11, the quotient is y and the remainder 3. When x is divided by 19, the remainder is also 3. What is the remainder when y is divided by 19?
0 1 2 3 4

17. x is a positive number. If $9^x + 9^{x+1} + 9^{x+2} + 9^{x+3} + 9^{x+4} + 9^{x+5} = y$, is y divisible by 5?
1) 5 is a factor of x. 2) x is an integer.

18. A group of n students can be divided into equal groups of 4 with 1 student left over or equal groups of 5 with 3 students left over. What is the sum of the two smallest possible values of n?
33 46 49 53 86

19. When x is divided by 4, the quotient is y and the remainder is 1. When x is divided by 7, the quotient is z and the remainder is 6. Which of the following is the value of y in terms of z?
(4z/7) + 5 (7z + 5)/6 (6z + 7)/4 (7z + 5)/4 (4z + 6)/7

20. If n is an integer and n^4 is divisible by 32, which of the following could be the remainder when n is divided by 32?
(A) 2 (B) 4 (C) 5 (D) 6 (E) 10

21. x_1 and x_2 are each positive integers. When x_1 is divided by 3, the remainder is 1, and when x_2 is divided by 12, the remainder is 4. If $y = 2x_1 + x_2$, then what must be true about y?
I. y is even II. y is odd III. y is divisible by 3
(A) I only (B) II only (C) III only
(D) I and III only (E) II and III only

22. Is x the square of an integer?
(1) x = 12k + 6, where k is a positive integer
(2) x = 3q + 9, where q is a positive integer

23. If r – s = 3p, is p an integer? (1) r is divisible by 735 (2) r + s is divisible by 3

24. If n is a positive integer, is $n^2 – 1$ divisible by 24?
(1) n is a prime number (2) n is greater than 191

25. The sum of all the digits of the positive integer q is equal to the three-digit number x13. If $q = 10^n – 49$, what is the value of n?
(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

26. Three consecutive integers are selected from the integers 1 to 50, inclusive. What is the sum of the remainders that result when each of the three integers is divided by x
(1) When the greatest of the consecutive integers is divided by x, the remainder is 0.
(2) When the least of the consecutive integers is divided by x, the remainder is 1.

27. Given that both x and y are positive integers, and that $y = 3^{(x-1)} – x$, is y divisible by 6?
(1) x is a multiple of 3 (2) x is a multiple of 4

28. If m and n are nonzero integers, is m/n an integer?
(1) 2m is divisible by n (2) m is divisible by 2n

29. If positive integer n is divisible by both 4 and 21, then n must be divisible by which of the following?
8 12 18 24 48

30. Susie can buy apples from two stores: a supermarket that sells apples only in bundles of 4, and a convenience store that sells single, unbundled apples. If Susie wants to ensure that the total number of apples she buys is a multiple of 5, what is the minimum number of apples she must buy from the convenience store?
0 1 2 3 4

31. Each of the following numbers has a remainder of 2 when divided by 11 except:
2 13 24 57 185

32. When positive integer n is divided by 3, the remainder is 2; and when positive integer t is divided by 5, the remainder is 3. What is the remainder when the product nt is divided by 15?
1). n-2 is divisible by 5 2). t is divisible by 3

33. If n is a positive integer and r is the remainder when (n-1)(n+1) is divided by 24, what is the value of r?
1). n is not divisible by 2 2). n is not divisible by 3

34. If n is a positive integer and r is the remainder when $n^2 - 1$ is divided by 8, what is the value of r?
1). n is odd 2). n is not divisible by 8

35. If n is a positive integer and r is the remainder when 4+7n is divided by 3, what is the value of r?
1). n+1 is divisible by 3 2). n>20

36. If r is the remainder when integer n is divided by 7, what is the value of r?
1). When n is divided by 21, the remainder is an odd number
2). When n is divided by 28, the remainder is 3

37. What is the remainder when the positive integer x is divided by 6?
1). When x is divided by 2, the remainder is 1; and when x is divided by 3, the remainder is 0
2). When x is divided by 12, the remainder is 3.

38. When the positive integer x is divided by 11, the quotient is y and the remainder 3. When x is divided by 19, the remainder is also 3. What is the remainder when y is divided by 19?
0 1 2 3 4

39. When x is divided by 4, the quotient is y and the remainder is 1. When x is divided by 7, the quotient is z and the remainder is 6. What is the value of y in terms of z?
A) 4z/7 + 5
B) (7z + 5) / 6
C) (6z + 7) / 4
D) (7z + 5) / 4
E) (4z + 6) / 7

Factors, Divisors, Multiples, LCM, HCF

1. If n is a non-negative integer such that 12^n is a divisor of 3,176,793, what is the value of $n^{12} - 12^n$?
 - 11   - 1   0   1   11

2. If the square root of p^2 is an integer, which of the following must be true?
 I. p^2 has an odd number of factors
 II. p^2 can be expressed as the product of an even number of prime factors
 III. p has an even number of factors
 I   II   III   I and II   II and III

3. The greatest common factor of 16 and the positive integer n is 4, and the greatest common factor of n and 45 is 3. Which of the following could be the value of n?
 6   8   9   12   15

4. If x is a positive integer, is x – 1 a factor of 104?
 (1) x is divisible by 3.   (2) 27 is divisible by x.

5. How many factors does 36^2 have?

 2   8   24   25   26

6. In a certain game, a large bag is filled with blue, green, purple and red chips worth 1, 5, x and 11 points each, respectively. The purple chips are worth more than the green chips, but less than the red chips. A certain number of chips are then selected from the bag. If the product of the point values of the selected chips is 88,000, how many purple chips were selected?
 1   2   3   4   5

7. For any integer k > 1, the term “length of an integer” refers to the number of positive prime factors, not necessarily distinct, whose product is equal to k. For example, if k = 24, the length of k is equal to 4, since 24 = 2 × 2 × 2 × 3. If x and y are positive integers such that x > 1, y > 1, and x + 3y < 1000, what is the maximum possible sum of the length of x and the length of y?
 5   6   15   16   18

8. *Question removed*

9. If a and b are positive integers divisible by 6, is 6 the greatest common divisor of a and b?
 (1) a = 2b + 6   (2) a = 3b

10. a, b, and c are positive integers. If a, b, and c are assembled into the six-digit number abcabc, which one of the following must be a factor of abcabc?
 (A) 16   (B) 13   (C) 5   (D) 3   (E) none of the above

11. If x and y are positive integers, which of the following CANNOT be the greatest common divisor of 35x and 20y?
 5   5(x – y)   20x   20y   35x

12. If P, Q, R, and S are positive integers, and P/Q = R/S, is R divisible by 5 ?
 (1) P is divisible by 140   (2) $Q = 7^x$, where x is a positive integer

13. For any four digit number, abcd, *abcd*= $(3^a)(5^b)(7^c)(11^d)$. What is the value of (n – m) if m and n are four- digit numbers for which *m* = $(3^r)(5^s)(7^t)(11^u)$ and *n* = (25)(*m*)?
 2000   200   25   20   2

14. w, x, y, and z are integers. If w > x > y > z > 0, is y a common divisor of w and x?

 (1) $\frac{w}{x} = z^{-1} + x^{-1}$ (2) $w^2 - wy - 2w = 0$

15. A restaurant pays a seafood distributor d dollars for 6 pounds of Maine lobster. Each pound can make v vats of lobster bisque, and each vat makes b bowls of lobster bisque. If the cost of the lobster per bowl is an integer, and if v and b are different prime integers, then which of the following is the smallest possible value of d?
(A) 15 (B) 24 (C) 36 (D) 54 (E) 90

16. a, b, c, and d are positive integers. If $(a + b)(c - d) = r$, where r is an integer, is $\sqrt{(c + d)}$ an integer?
(1) $(a + b)(c + d) = r^2$
(2) $(a + b) = x^4 y^6 z^2$, where x, y, and z are distinct prime numbers.

17. The function f(n) = the number of factors of n. If f(pq) = 4, what is the value of the integer p?
(1) $p + q$ is an odd integer (2) $q < p$

18. If x, y, and z are positive integers such that $x < y < z$, is x a factor of the odd integer z?
(1) x and y are prime numbers, whose sum is a factor of 57 (2) z is a factor of 57

19. What is the positive integer n?
(1) The sum of all of the positive factors of n that are less than n is equal to n
(2) $n < 30$

20. If p^3 is divisible by 80, then the positive integer p must have at least how many distinct factors?
(A) 2 (B) 3 (C) 6 (D) 8 (E) 10

21. Does the integer p have an odd number of distinct factors?
(1) $p = q^2$, where q is a nonzero integer. (2) $p = 2n + 1$, where n is a nonzero integer.

22. What is the positive integer n?
(1) For every positive integer m, the product $m(m + 1)(m + 2) \ldots (m + n)$ is divisible by 16
(2) $n^2 - 9n + 20 = 0$

23. What is the greatest common factor of positive integers a and b?
(1) $a = b + 4$ (2) b/4 is an integer

24. Which of the following is the lowest positive integer that is divisible by the first 7 positive integer multiples of 5?
140 210 1400 2100 3500

25. What is the value of the integer n?
(1) $n! = n \times (n - 1)!$ (2) $n^3 + 3n^2 + 2n$ is divisible by 3

26. $p^a q^b r^c s^d = x$, where x is a perfect square. If p, q, r, and s are prime integers, are they distinct?
(1) 18 is a factor of ab and cd (2) 4 is not a factor of ab and cd

27. K and L are each four-digit positive integers with thousands, hundreds, tens, and units digits defined as a, b, c, and d, respectively, for the number K, and p, q, r, and s, respectively, for the number L. For numbers K and L, the function W is defined as $5^a 2^b 7^c 3^d \div 5^p 2^q 7^r 3^s$. The function Z is defined as $(K - L) \div 10$. If W = 16, what is the value of Z?
(A) 16 (B) 20 (C) 25 (D) 40 (E) It cannot be determined from the information given.

28. How many numbers that are not divisible by 6 divide evenly into 264,600?
(A) 9 (B) 36 (C) 51 (D) 63 (E) 72

29. If n^2 / n yields an integer greater than 0, is n divisible by 30?
(1) n^2 is divisible by 20 (2) n^3 is divisible by 12

30. If a and b are **<u>consecutive positive integers</u>**, and ab = 30x is x a non-integer?
(1) a^2 is divisible by 21 (2) 35 is a factor of b^2

31. $\sqrt{ABC} = 504$. Is B divisible by 2? (1) C = 168 (2) A is a perfect square

32. If the prime factorization of the integer q can be expressed as $a^{2x}.b^x.c^{3x-1}$, where a, b, c, and x are distinct positive integers, which of the following could be the total number of factors of q?
(A) $3j + 4$, where j is a positive integer
(B) $5k + 5$, where k is a positive integer
(C) $6l + 2$, where l is a positive integer
(D) $9m + 7$, where m is a positive integer
(E) $10n + 1$, where n is a positive integer

33. Which of the following is the lowest positive integer that is divisible by 8, 9, 10, 11, and 12?
7,920 5,940 3,960 2,970 890

34. If x is a positive integer, is x prime?
(1) x has the same number of factors as y^2, where y is a positive integer greater than 2.
(2) x has the same number of factors as z, where z is a positive integer greater than 2.

35. h(n) is the product of the even numbers from 2 to n, inclusive, and p is the least prime factor of h(100)+1. What is the range of p?
< 40 < 30 > 40 < 10 Indeterminate

36. If d is positive integer, f is the product of the first 30 positive integers, what is the value of d?
1). 10^d is a factor of f 2). d>6

37. Does the integer k have a factor p such that $1<p<k$?
1). k>4! 2). 13!+2<= k<=13!+13

38. If x and y are integers greater than 1, is x a multiple of y?
1). $3y^2+7y=x$ 2). x^2-x is a multiple of y

39. The function f is defined for all positive integers n by the following rule. f(n) is the number of positive integers each of which is less than n and has no positive factor in common with n other than 1. If p is any prime number then f(p)=
p-1 p-2 (p+1)/2 (p-1)/2 2

40. In the fraction x/y, where x and y are positive integers, what is the value of y?
1). The least common denominator of x/y and 1/3 is 6 2). x=1

41. For any positive integer n, the length of n is defined as number of prime factors whose product is n, For example, the length of 75 is 3, since 75=3*5*5. How many two-digit positive integers have length 6?
0 1 2 3 4

42. If n and t are positive integers, what is the greatest prime factor of nt?
1. The greatest common factor of n and t is 5
2. The least common multiple of n and t is 105

43. If n is a positive integer less than 200 and 14n/60 is an integer, then n has how many different positive prime factors?
A. two B. three C. five D. six E. eight.

44. The positive integers x, y and z are such that x is a factor of y and y is a factor of z. Is z even?
1). xz is even 2). y is even

45. If k is a positive integer, then 20k is divisible by how many different positive integers?
1). K is prime. 2). K=7

46. x and y are positive integers such that x=8y+12, what is the greatest common divisor of x and y?
1). X=12u, where u is an integer. 2). Y=12z, where z is an integer.

47. What is the greatest prime factor of $4^{17} - 2^{28}$?
2 3 4 7 20

48. How many different prime numbers are factors of the positive integer n?
1). four different prime numbers are factors of 2n.
2). four different prime numbers are factors of n^2.

49. What is the greatest common factor of positive integers a and b?
(1) a = b + 4 (2) b/4 is an integer

50. Which of the following is the lowest positive integer that is divisible by the first 7 positive integer multiples of 5?
140 210 1400 2100 3500

Consecutive Integers

1. x is the sum of y consecutive integers. w is the sum of z consecutive integers. If $y = 2z$, and y and z are both positive integers, then each of the following could be true EXCEPT
   A. $x = w$
   B. $x > w$
   C. x/y is an integer
   D. w/z is an integer
   E. x/z is an integer

2. For positive integer k, is the expression $(k + 2)(k^2 + 4k + 3)$ divisible by 4?
   (1) k is divisible by 8. (2) $(k + 1) / 3$ is an odd integer.

3. If x is an integer, then $x(x – 1)(x – k)$ must be evenly divisible by three when k is any of the following values EXCEPT
   -4 -2 -1 2 5

4. The sum of n consecutive positive integers is 45. What is the value of n?
   (1) n is even (2) $n < 9$

5. Is positive integer $n – 1$ a multiple of 3?
   (1) $n^3 – n$ is a multiple of 3 (2) $n^3 + 2n^2 + n$ is a multiple of 3

6. a, b, c, and d are positive consecutive integers and $a < b < c < d$. If the product of b, c, and d is twice that of a, b, and c, then bc =
   2 6 12 20 30

7. *Question removed*

8. How many integers are there between 51 and 107, inclusive?
   51 55 56 57 58

9. If x, y, and z are 3 positive consecutive integers such that $x < y < z$, what is the remainder when the product of x, y, and z is divided by 8?
   (1) $(xz)^2$ is even (2) $5y^3$ is odd

10. If $x^3 – x = n$ and x is a positive integer greater than 1, is n divisible by 8?
   (1) When $3x$ is divided by 2, there is a remainder.
   (2) $x = 4y + 1$, where y is an integer.

11. a is the sum of x consecutive positive integers. b is the sum of y consecutive positive integers. For which of the following values of x and y is it impossible that $a = b$?
   (A) $x = 2; y = 6$ (B) $x = 3; y = 6$ (C) $x = 7; y = 9$
   (D) $x = 10; y = 4$ (E) $x = 10; y = 7$

12. Is x divisible by 30?
   (1) $x = k(m^3 - m)$, where m and k are both integers > 9
   (2) $x = n^5 - n$, where n is an integer > 9

13. If x, y, and z are positive integers, where $x > y$ and $z = \sqrt{x}$, are x and y consecutive perfect squares?
   (1) $x + y = 8z + 1$ (2) $x – y = 2z – 1$

Digits

1. Given that a, b, c, and d are different nonzero digits and that $10d + 11c < 100 - a$, which of the following could not be a solution to the addition problem below?

```
  abdc
+ dbca
```

(A) 3689 (B) 6887 (C) 8581 (D) 9459 (E) 16091

2.

```
  8k8
+ k88
1,6p6
```

If k and p represent non-zero digits within the integers above, what is p?

6 7 8 9 17

3. If x represents the sum of all the positive three-digit numbers that can be constructed using each of the distinct nonzero digits a, b, and c exactly once, what is the largest integer by which x must be divisible?

(A) 3 (B) 6 (C) 11 (D) 22 (E) 222

4. If the sum of the digits of the positive two-digit number x is 4, what is the value of x?

(1) x is odd. (2) Twice the value of x is less than 44.

5.

```
  2 2
  a 3
+ 4 b
  9 0
```

If a and b represent positive single digits in the correctly worked computation above, what is the value of the two digit integer ba?

10 15 25 51 52

6.

```
   a b c
+  d e f
  ------
   x y z
```

If, in the addition problem above, a, b, c, d, e, f, x, y, and z each represent different positive single digits, what is the value of z ?

(1) $3a = f = 6y$ (2) $f - c = 3$

7. For any four digit number, abcd, *abcd*= $(3^a)(5^b)(7^c)(11^d)$. What is the value of (n – m) if m and n are four- digit numbers for which *m* = $(3^r)(5^s)(7^t)(11^u)$ and *n* = (25)(*m*)?

2000 200 25 20 2

8. What is the three-digit number abc, given that a, b, and c are the positive single digits that make up the number?

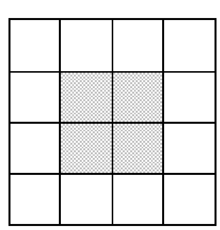
(1) $a = 1.5b$ and $b = 1.5c$

(2) $a = 1.5x + b$ and $b = x + c$, where x represents a positive single digit

9. What is the value of the three-digit number SSS if SSS is the sum of the three-digit numbers ABC and XYZ, where each letter represents a distinct digit from 0 to 9, inclusive?

1) $S = 1.75\ X$ 2) $S^2 = 49zx/8$

10. If the 4 x 4 grid pictured at right is filled with the consecutive integers from 37 to 52, inclusive, so that every row, column and major diagonal sums to the same value, which of the following is a possible value of the sum of the four center cells of the grid (indicated by the shaded area)?



(A) 124 (B) 153 (C) 178 (D) 192 (E) 214

Answer Key

GMAT Quant Topic 4: Numbers

Part A: Types of Numbers

1. C
2. E
3. C
4. E
5. B
6. B
7. B
8. A
9. C
10. C
11. E
12. A
13. D
14. C
15. D
16. B
17. D
18. C
19. B
20. E
21. E
22. C
23. D
24. B
25. E
26. C
27. E
28. A
29. E
30. E
31. E

Part B: Odds and Evens

1. A
2. C
3. D
4. B
5. E
6. B
7. E
8. B
9. A
10. C
11. D
12. C
13. E
14. C

15. B
16. A
17. D
18. C
19. D
20. B
21. C
22. D
23. C
24. A
25. E

Part C: Unit's digits, factorial powers

1. C
2. B
3. C
4. D
5. D
6. B
7. E
8. B
9. B
10. A
11. E
12. C
13. E
14. C
15. A
16. D
17. B

Part D: Decimals

1. B
2. B
3. A
4. B
5. B
6. E
7. D
8. D
9. C
10. A
11. B
12. A
13. B

Part E: Sequences and Series

1. C
2. D
3. C
4. C

5. A
6. C
7. A
8. D
9. D
10. C
11. E
12. C
13. D
14. C
15. D
16. D
17. B
18. D
19. E
20. D
21. C
22. E
23. B
24. B
25. C
26. D
27. E

Part F: Remainders, Divisibility

1. E
2. D
3. B
4. E
5. E
6. E
7. E
8. B
9. D
10. C
11. B
12. B
13. C
14. D
15. D
16. A
17. D
18. B
19. D
20. B
21. D
22. A
23. C
24. C
25. B
26. C

27. B
28. B
29. B
30. A
31. E
32. C
33. C
34. A
35. A
36. B
37. D
38. A
39. D

Part G: Factors, Divisors, Multiples, LCM, HCF

1. B
2. D
3. D
4. C
5. D
6. B
7. D
8. *Question removed*
9. A
10. B
11. C
12. C
13. B
14. D
15. A
16. C
17. E
18. A
19. E
20. C
21. A
22. C
23. C
24. D
25. E
26. B
27. D
28. D
29. C
30. C
31. C
32. B
33. C
34. A
35. C
36. C

37. B
38. A
39. A
40. E
41. C
42. B
43. B
44. D
45. B
46. B
47. D
48. B
49. C
50. D

Part H: Consecutive Integers

1. C
2. A
3. B
4. E
5. B
6. D
7. Question removed
8. D
9. D
10. D
11. D
12. B
13. B

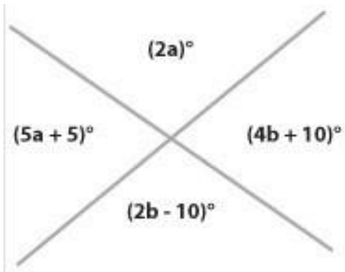
Part I: Digits

1. C
2. A
3. E
4. B
5. E
6. A
7. B
8. A
9. D
10. C

GMAT Quant Topic 5: Geometry

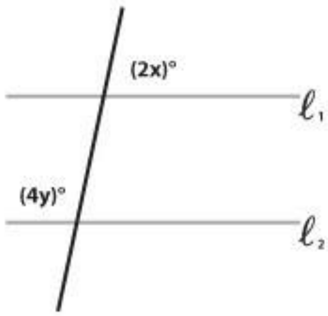
Part 1: Lines and Angles

1. What is the value of a + b?



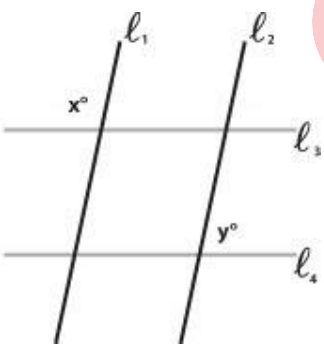
30 50 55 65 90

2. If l_1 is parallel to l_2, what is x + 2y?



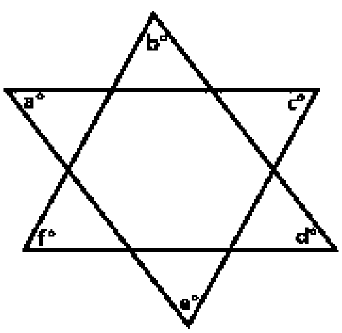
90 120 180 270 360

3. What is the value of x?



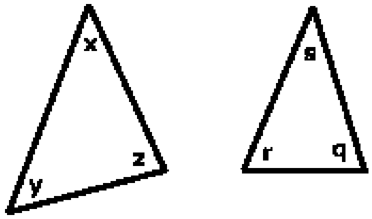
(1) l1 is parallel to l2 (2) y = 70

4. What is the value of a + b + c + d + e + f?



180 270 300 360 720

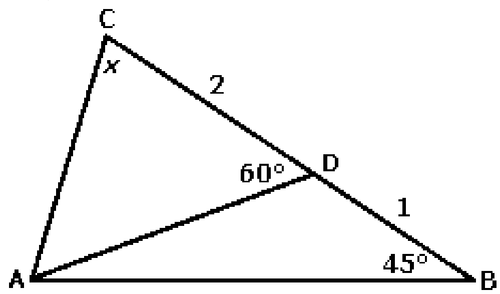
5. If x – q = s – y, what is the value of z?



Figures are not drawn to scale.

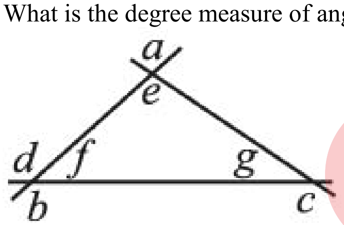
1) xq + sy + sx + yq = zr 2) zq – ry = rx – zs

6. In the figure, point D divides side BC of triangle ABC into segments BD and DC of lengths 1 and 2 units respectively. Given that ∠ADC = 60º and ∠ABD = 45º, what is the measure of angle x in degrees? (Note: Figure is not drawn to scale.)



(A) 55 (B) 60 (C) 70 (D) 75 (E) 90

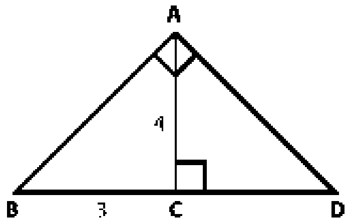
7. What is the degree measure of angle a?



(1) b + c = 287 degrees (2) d + e = 269 degrees

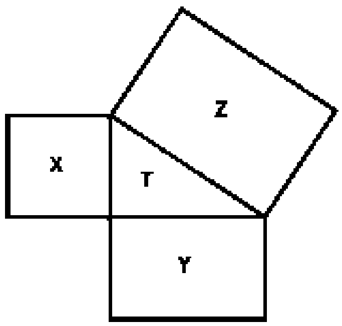
Part 2: Triangles

1. In triangle ABC, if BC = 3 and AC = 4, then what is the length of segment CD?



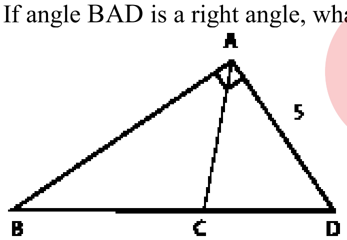
3 15/4 5 16/3 20/3

2. The figure is comprised of three squares and a triangle. If the areas marked X, Y and Z are 25, 144, and 169, respectively, what is the area of the triangle marked T?



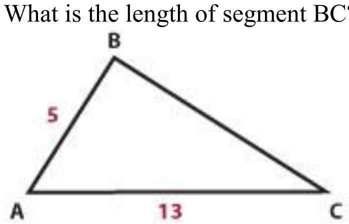
25 30 50 60 97

3. If angle BAD is a right angle, what is the length of side BD?



(1) AC is perpendicular to BD (2) BC = CD

4. What is the length of segment BC?



(1) Angle ABC is 90 degrees. (2) The area of the triangle is 30.

5. What is the perimeter of isosceles triangle ABC?
(1) The length of side AB is 9 (2) The length of side BC is 4

6. The figure is made up of a series of inscribed equilateral triangles. If the pattern continues until the length of a side of the largest triangle (i.e. the entire figure) is exactly 128 times that of the smallest triangle, what fraction of the total figure will be shaded?

A. $\frac{1}{4}(2^0+2^{-4}+2^{-8}+2^{-12})$

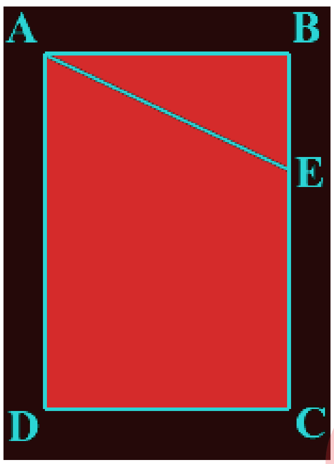
B. $\frac{1}{4}(2^0+2^{-2}+2^{-4}+2^{-6})$

C. $\frac{3}{4}(2^0+2^{-4}+2^{-8}+2^{-12})$

D. $\frac{3}{4}(2^0+2^{-2}+2^{-4}+2^{-6})$

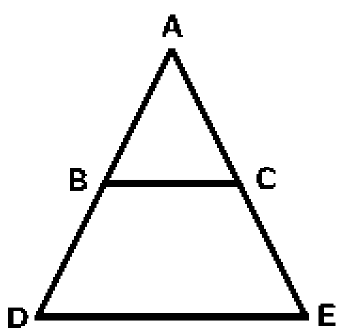
E. $\frac{3}{4}(2^0+2^{-1}+2^{-2}+2^{-3})$

7. Given that ABCD is a rectangle, is the area of triangle ABE > 25?
(Note: Figure above is not drawn to scale).



(1) AB = 6 (2) AE = 10

8. In the figure, AC = 3, CE = x, and BC is parallel to DE. If the area of triangle ABC is 1/12 of the area of triangle ADE, then x =?



A. $3\sqrt{3}-3$

B. 3

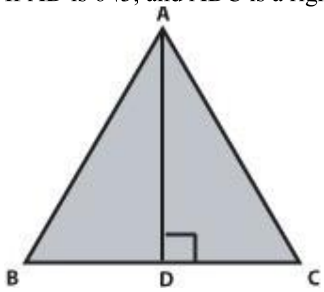
C. 6

D. $6\sqrt{3}-3$

E. $6\sqrt{3}$

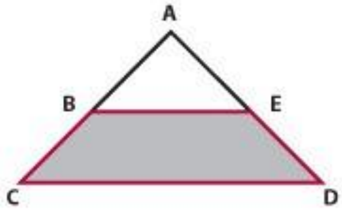
9. Triangle A has one side of length x. If $\sqrt{(x^8)} = 81$, what is the perimeter of Triangle A?
1) Triangle A has sides whose lengths are consecutive integers
2) Triangle A is NOT a right triangle

10. If AD is $6\sqrt{3}$, and ADC is a right angle, what is the area of triangular region ABC?



(1) Angle ABD = 60° (2) AC = 12

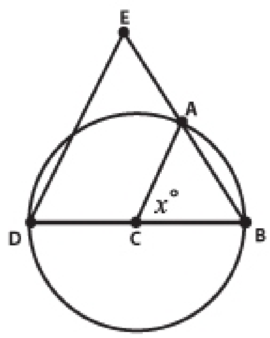
11. If BE || CD, and BC = AB = 3, AE = 4 and CD = 10, what is the area of trapezoid BEDC?



12 18 24 30 48

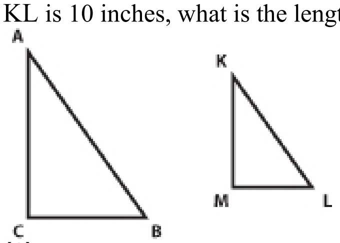
12. If the length of side AB is 17, is triangle ABC a right triangle?
(1) The length of side BC is 144. (2) The length of side AC is 145.

13. In the figure, if point C is the center of the circle and DB = 7, what is the length of DE?



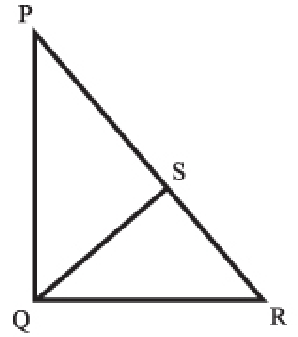
(1) x = 60° (2) DE || CA

14. The area of the right triangle ABC is 4 times greater than the area of the right triangle KLM. If the hypotenuse KL is 10 inches, what is the length of the hypotenuse AB?



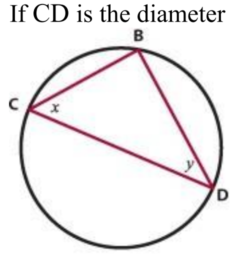
(1) Angles ABC and KLM are each equal to 55 degrees.
(2) LM is 6 inches.

15. In the diagram, triangle PQR has a right angle at Q and a perimeter of 60. Line segment QS is perpendicular to PR and has a length of 12. PQ > QR. What is the ratio of the area of triangle PQS to the area of triangle RQS?



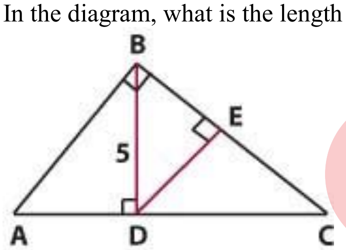
3/2 7/4 15/8 16/9 2

16. If CD is the diameter of the circle, does x equal 30?



(1) The length of CD is twice the length of BD. (2) y = 60

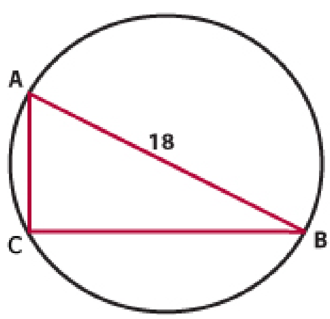
17. In the diagram, what is the length of AB?



(1) BE = 3 (2) DE = 4

18. Which of the following is a possible length for side AB of triangle ABC if AC = 6 and BC = 9?
I. 3 II. $9\sqrt{3}$ III. 13.5
I only II only III only II and III I, II and III

19. For the triangle shown, where A, B and C are all points on a circle, and line segment AB has length 18, what is the area of triangle ABC?



(1) Angle ABC measures 30°. (2) The circumference of the circle is 18π.

20. The perimeter of a certain isosceles right triangle is $16+16\sqrt{2}$, what is the length of the hypotenuse of the triangle?
(A) 8
(B) 16
(C) $4\sqrt{2}$
(D) $8\sqrt{2}$
(E) $16\sqrt{2}$

Part 3: Quadrilaterals

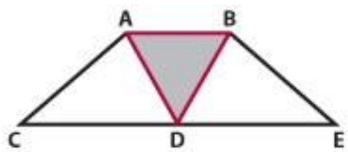
1. Is quadrilateral ABCD a rectangle?
(1) Line segments AC and BD bisect one another.
(2) Angle ABC is a right angle.

2. Is quadrilateral ABCD a rhombus?
(1) Line segments AC and BD are perpendicular bisectors of each other.
(2) AB = BC = CD = AD

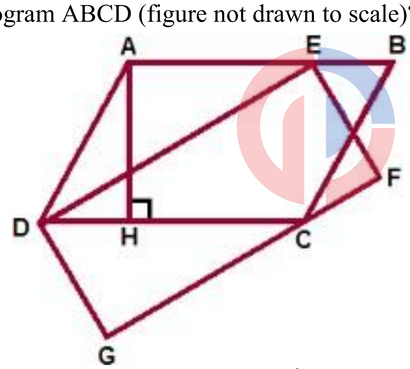
3. Is quadrilateral ABCD a square?
(1) ABCD is a rectangle. (2) AB = BC

4. Rectangle ABCD is inscribed in circle P. What is the area of circle P?
(1) The area of rectangle ABCD is 100. (2) Rectangle ABCD is a square.

5. If triangle ABD is an equilateral triangle and AB = 6 and CE = 18, what fraction of the trapezoid BACE is shaded?
A. 1/5
B. 1/4
C. 1/3
D. 3/8
E. 1/2

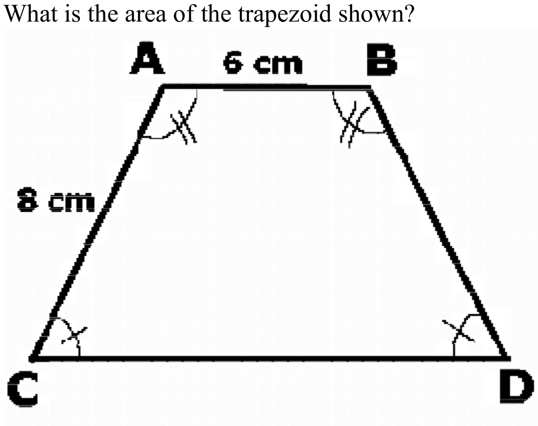


6. In the picture, quadrilateral ABCD is a parallelogram and quadrilateral DEFG is a rectangle. What is the area of parallelogram ABCD (figure not drawn to scale)?



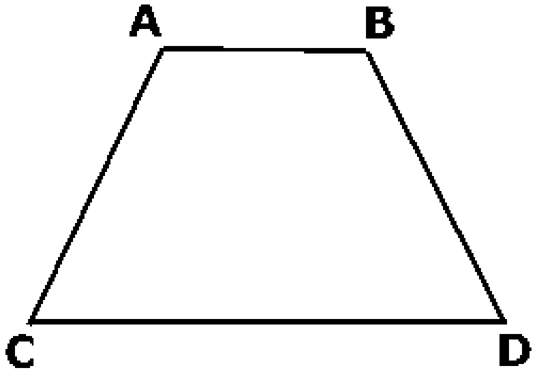
(1) The area of rectangle DEFG is $8\sqrt{5}$. (2) Line AH, the altitude of parallelogram ABCD, is 5.

7. What is the area of the trapezoid shown?



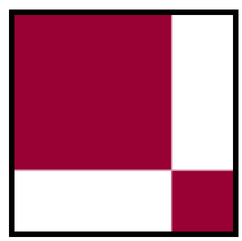
(1) Angle A = 120 degrees (2) The perimeter of trapezoid ABCD = 36.

8. The height of isosceles trapezoid ABDC is 12 units. The length of diagonal AD is 15 units. What is the area of trapezoid ABDC?

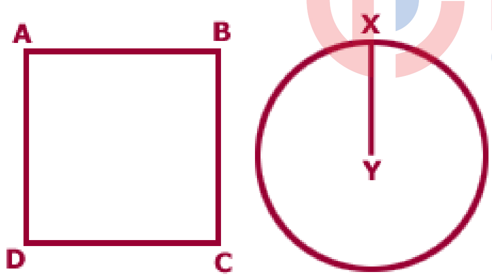


(A) 72 (B) 90 (C) 96 (D) 108 (E) 180

9. The combined area of the two black squares is equal to 1000 square units. A side of the larger black square is 8 units longer than a side of the smaller black square. What is the combined area of the two white rectangles in square units?
(A) 928 (B) 936 (C) 948 (D) 968 (E) 972



10. Jeff is painting two murals on the front of an old apartment building that he is renovating. One mural will cover the quadrilateral face ABCD while the other will cover the circular face (shown to the right, with radius XY). Assuming that the thickness of the coats of paint is negligible; will each mural require the same amount of paint? **Note:** Figures are not



drawn to scale.

(1) AB = BC = CD = DA, and AB = XY$\sqrt{\pi}$ (2) AC = BD and AC = XY$\sqrt{(2\pi)}$

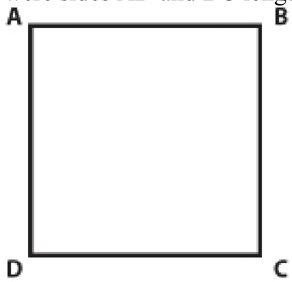
11. In the quadrilateral PQRS, side PS is parallel to side QR. Is PQRS a parallelogram?
(1) PS = QR (2) PQ = RS

12. E, F, G, and H are the vertices of a polygon. Is polygon EFGH a square?
(1) EFGH is a parallelogram.
(2) The diagonals of EFGH are perpendicular bisectors of one another.

13. What is the area of the quadrilateral with vertices A, B, C, and D?
(1) The perimeter of ABCD is equal to 16.
(2) Quadrilateral ABCD is a square.

14. The perimeter of a rectangular yard is completely surrounded by a fence that measures 40 meters. What is the length of the yard if the area of the yard is 64 meters squared?
8 10 12 14 16

15. Square ABCD has an area of 9 square inches. Sides AD and BC are lengthened to x inches each. By how many inches were sides AD and BC lengthened?



(1) The diagonal of the resulting rectangle measures 5 inches.
(2) The resulting rectangle can be cut into three rectangles of equal size.

16. In the rhombus ABCD, the length of diagonal BD is 6 and the length of diagonal AC is 8. What is the perimeter of ABCD?

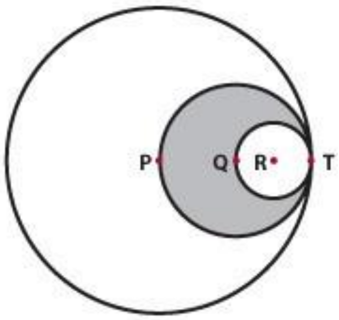
10 14 20 24 28

17. Is the measure of one of the interior angles of quadrilateral ABCD equal to 60 degrees?

1). two of the interior angles of ABCD are right angles
2). the degree measure of angle ABC is twice the degree measure of angle BCD

Part 4: Circles

1. If P, Q and R are the centers of circles P, Q, and R and the points P, Q, R and T all lie on the same line, what portion of circle P is shaded?



A. 3/16
B. 1/5
C. 6/25
D. 1/4
E. 3/8

2. If $1/a^2 + a^2$ represents the diameter of circle O and $1/a + a = 3$, which of the following best approximates the circumference of circle O?
28 22 20 16 12

3. A car is being driven on a road. Assuming that the car's wheels turn without slipping, how many full 360° rotations does each tire on the car make in 10 minutes?
(1) The car is traveling at 50 miles per hour.
(2) Each tire has a radius of 20 inches.

4. Two circular road signs are to be painted yellow. If the radius of the larger sign is twice that of the smaller sign, how much times paint is needed to paint the larger sign as compared to the smaller circle (assuming that a given amount of paint covers the same area on both signs)?
2 3 π 4 $3\pi/2$

5. The figure represents five concentric quarter-circles. The length of the radius of the largest quarter-circle is x. The length of the radius of each successively smaller quarter-circle is one less than that of the next larger quarter-circle. What is the combined area of the shaded regions (black), in terms of x?

A. $\pi(x^2 - 4x + 10)$

B. $\frac{\pi}{2}(x^2 - 4x + 10)$

C. $\frac{\pi}{4}(x^2 - 4x + 10)$

D. $\frac{\pi}{8}(x^2 - 4x + 10)$

E. $\frac{\pi}{16}(x^2 - 4x + 10)$

6. In the diagram (not drawn to scale), Sector PQ is a quarter-circle. The distance from A to P is half the distance from P to B. The distance from C to Q is 2/7 of the distance from Q to B. If the length of AC is 100, what is the length of the radius of the circle with center B?

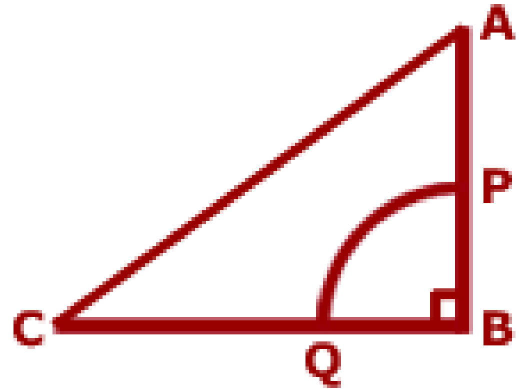
A. $\frac{280\sqrt{85}}{51}$

B. $\frac{240\sqrt{70}}{61}$

C. $\frac{240\sqrt{67}}{43}$

D. $\frac{230\sqrt{51}}{43}$

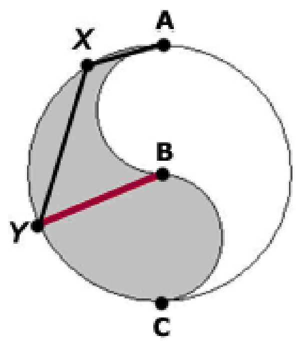
E. $\frac{220\sqrt{43}}{51}$



7. A circular gear with a diameter of 24 centimeters is mounted directly on another circular gear with a diameter of 96 centimeters. Both gears turn on the same axle at their exact centers and each gear has a single notch, at the 12 o'clock position. At the same moment, the gears begin to turn at the same rate, with the larger gear moving clockwise and the smaller gear counterclockwise. How far, in centimeters, will the notch on the larger gear have traveled the second time the notches pass each other?

(A) 32.2π
(B) 35.6π
(C) 38.4π
(D) 39.2π
(E) 40.8π

8. In the diagram, points A, B, and C are on the diameter of the circle with center B. Additionally, all arcs pictured are semicircles. Suppose angle YXA = 105 degrees. What is the ratio of the area of the shaded region above the line YB to the area of the shaded region below the line YB? (Note: Diagram is not drawn to scale and angles drawn are not accurate.)

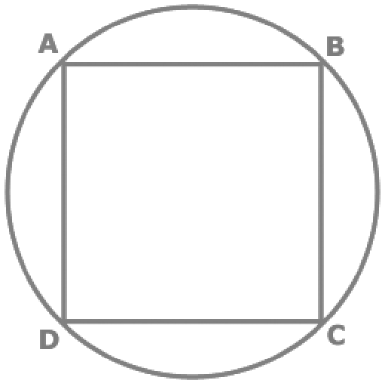


(A) ¾ (B) 5/6 (C) 1 (D) 7/5 (E) 9/7

9. For a circle with center point P, cord XY is the perpendicular bisector of radius AP (A is a point on the edge of the circle). What is the length of cord XY?

(1) The circumference of circle P is twice the area of circle P. (2) The length of Arc XAY = 2π/3.

10. ABCD is a square inscribed in a circle and arc ADC has a length of $\pi\sqrt{x}$. If a dart is thrown and lands somewhere in the circle, what is the probability that it will not fall within the inscribed square? (Assume that the point in the circle where the dart lands is completely random.)



(A) $2x$

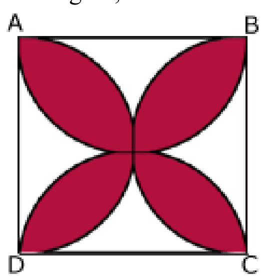
(B) $\pi(x) - 2x$

(C) $\pi(x) - \sqrt{2}(x)$

(D) $1 - \frac{2}{\pi}$

(E) $1 - \frac{2}{x}$

11. Figure ABCD is a square with sides of length x. Arcs AB, AD, BC, and DC are all semicircles. What is the area of the black region, in terms of x?



(A) $x^2 - \frac{\pi x^2}{4}$

(B) $\frac{\pi x^2}{2} - x^2$

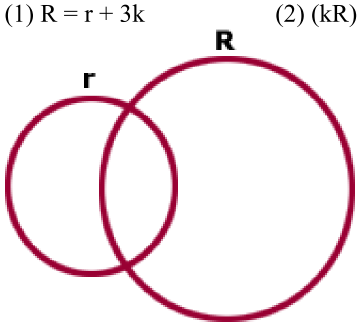
(C) $4\left(\frac{\pi x^2}{4} - 2x^2\right)$

(D) $4\left(\frac{\pi x^2}{2} + 2x^2\right)$

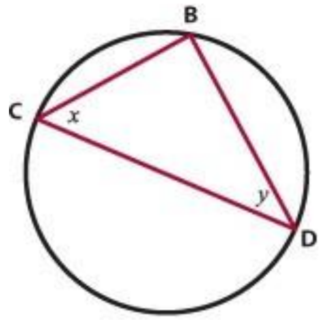
(E) $2x^2 - \frac{\pi x^2}{2}$

12. In the figure, a small circle with radius r intersects a larger circle with radius R (where R > r). If k > 0, what is the difference in the areas of the non-overlapping parts of the two circles?

(1) R = r + 3k (2) (kR) / (kr – 6) = -1

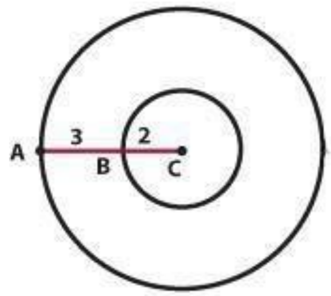


13. If CD is the diameter of the circle, does x equal 30?



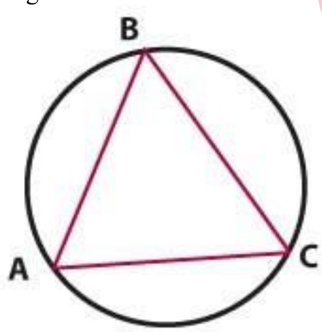
(1) The length of CD is twice the length of BD. (2) $y = 60$

14. Two circles share a center at point C, as shown. Segment AC is broken up into two shorter segments, AB and BC, with dimensions shown. What is the ratio of the area of the large circle to the area of the small circle?



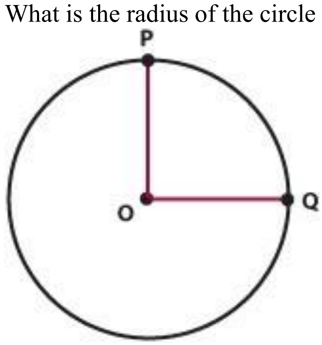
25/4 5/2 3/2 2/5 4/25

15. The length of minor arc AB is twice the length of minor arc BC and the length of minor arc AC is three times the length of AB. What is the measure of angle BCA?



20 40 60 80 120

16. What is the radius of the circle shown?



(1) The measure of arc PQ is 4π. (2) The center of the circle is at point O.

17. A cylindrical tank has a base with a circumference of $4\sqrt{\pi\sqrt{3}}$ meters and an equilateral triangle painted on the interior side of the base. A grain of sand is dropped into the tank, and has an equal probability of landing on any particular point on the base. If the probability of the grain of sand landing on the portion of the base outside the triangle is 3/4, what is the length of a side of the triangle?

A. $\sqrt{2\sqrt{6}}$

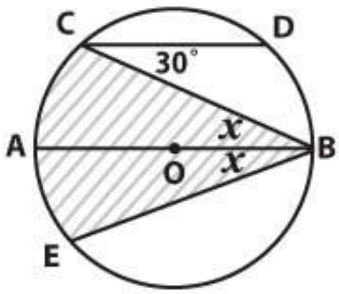
B. $\frac{\sqrt{6\sqrt{6}}}{2}$

C. $\sqrt{2\sqrt{3}}$

D. $\sqrt{3}$

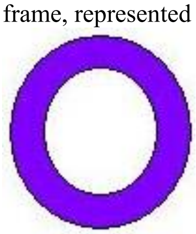
E. 2

18. In the figure, circle O has center O, diameter AB and a radius of 5. Line CD is parallel to the diameter. What is the perimeter of the shaded region?



A. $(5/3)\pi + 5\sqrt{3}$
B. $(5/3)\pi + 10\sqrt{3}$
C. $(10/3)\pi + 5\sqrt{3}$
D. $(10/3)\pi + 10\sqrt{3}$
E. $(10/3)\pi + 20\sqrt{3}$

19. The figure shows the top side of a circular medallion made of a circular piece of colored glass surrounded by a metal frame, represented by the shaded region.



If the radius of the medallion is r centimeter and width of the metal frame is s centimeter, then, in terms of r and s, what is the area of the metal frame, in square centimeter?

A. $\pi(r-s)^2$

B. $\pi(r^2-s^2)$

C. $2\pi(r-s)$

D. $\pi r(2r-s)$

E. $\pi s(2r-s)$

20. A thin piece of wire 40 meters long is cut into two pieces. One piece is used to form a circle with radius r, and the other is used to form a square. No wire is left over. Which of the following represents the total area, in square meters, of the circular and the square regions in terms of r?

A. $\pi * r^2$

B. $\pi * r^2 + 10$

C. $\pi * r^2 + \frac{1}{4} * \pi^2 * r^2$

D. $\pi * r^2 + (40 - 2\pi * r)^2$

E. $\pi * r^2 + (10 - \frac{1}{2}\pi * r)^2$

Part 5: Polygons

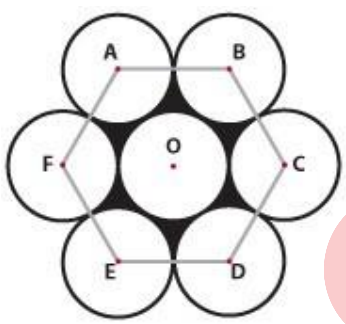
1. A certain game board is in the shape of a non-convex polygon, with spokes that extend from each vertex to the center of the board. If each spoke is 8 inches long, and spokes are used nowhere else on the board, what is the sum of the interior angles of the polygon?
(1) The sum of the exterior angles of the polygon is 360º.
(2) The sum of the exterior angles is equal to five times the total length of all of the spokes used.

2. The measures of the interior angles in a polygon are consecutive integers. The smallest angle measures 136 degrees. How many sides does this polygon have?
A) 8 B) 9 C) 10 D) 11 E) 13

3. If x represents the sum of the interior angles of a regular hexagon and y represents the sum of the interior angles of a regular pentagon, then the difference between x and y is equal to the sum of the interior angles of what geometric shape?
Triangle Square Rhombus Trapezoid Pentagon

4. If Polygon X has fewer than 9 sides, how many sides does Polygon X have?
(1) The sum of the interior angles of Polygon X is divisible by 16.
(2) The sum of the interior angles of Polygon X is divisible by 15.

5. Regular hexagon ABCDEF has a perimeter of 36. O is the center of the hexagon and of circle O. Circles A, B, C, D, E, and F have centers at A, B, C, D, E, and F, respectively. If each circle is tangent to the two circles adjacent to it and to circle O, what is the area of the shaded region (inside the hexagon but outside the circles)?



A. $108 - 18\pi$
B. $54\sqrt{3} - 9\pi$
X. $54\sqrt{3} - 18\pi$
D. $108 - 27\pi$
E. $54\sqrt{3} - 27\pi$

Part 6: General Solids (Cube, Box, Sphere)

1. Four spheres and three cubes are arranged in a line according to increasing volume, with no two solids of the same type adjacent to each other. The ratio of the volume of one solid to that of the next largest is constant. If the radius of the smallest sphere is ¼ that of the largest sphere, what is the radius of the smallest sphere?
1) The volume of the smallest cube is 72π.
2) The volume of the second largest sphere is 576π.

2. At 7:57 am, Flight 501 is at an altitude of 6 miles above the ground and is on a direct approach (i.e., flying in a direct line to the runway) towards The Airport, which is located exactly 8 miles due north of the plane's current position. Flight 501 is scheduled to land at The Airport at 8:00 am, but, at 7:57 am, the control tower radios the plane and changes the landing location to an airport 15 miles directly due east of The Airport. Assuming a direct approach (and negligible time to shift direction), by how many miles per hour does the pilot have to increase her speed in order to arrive at the new location on time?
A. $5\sqrt{13} - 10$ miles/hr

B. 100 miles/hr

C. $100\sqrt{13} - 200$ miles/hr

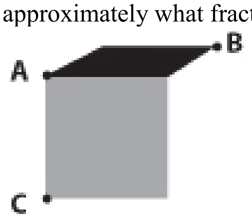
D. 200 miles/hr

E. $100\sqrt{13}$ miles/hr

3. What is the ratio of the surface area of a cube to the surface area of a rectangular solid identical to the cube in all ways except that its length has been doubled?
¼ 3/8 ½ 3/5 2

4. A sphere is inscribed in a cube with an edge of 10. What is the shortest possible distance from one of the vertices of the cube to the surface of the sphere?
$10(\sqrt{3} - 1)$ 5 $10(\sqrt{2} - 1)$ $5(\sqrt{3} - 1)$ $5(\sqrt{2} - 1)$

5. If the box shown is a cube, then the difference in length between line segment BC and line segment AB is approximately what fraction of the distance from A to C?



10% 20% 30% 40% 50%

Part 7: Cylinders

1. A cylindrical tank of radius R and height H must be redesigned to hold approximately twice as much liquid. Which of the following changes would be farthest from the new design requirements?
a 100% increase in R and a 50% decrease in H
a 30% decrease in R and a 300% increase in H
a 10% decrease in R and a 150% increase in H
a 40% increase in R and no change in H
a 50% increase in R and a 20% decrease in H

2. Cylinder A, which has a radius of x and a height of y, has a greater surface area than does Cylinder B, which has a radius of y and a height of x. How much greater is the surface area of Cylinder A than that of Cylinder of B?
A. $x^2 - y^2$

B. $2(x^2 - y^2)$

C. $\pi(x^2 - y^2)$

D. $2\pi(x^2 - y^2)$

E. $4\pi(x^2 - y^2)$

3. A right circular cylinder has a radius r and a height h. What is the surface area of the cylinder?
(1) r = 2h – 2/h
(2) h = 15/r – r

4. A cylindrical tank, with radius and height both of 10 feet, is to be redesigned as a cone, capable of holding twice the volume of the cylindrical tank. There are two proposed scenarios for the new cone: in scenario (1) the radius will remain the same as that of the original cylindrical tank, in scenario (2) the height will remain the same as that of the original cylindrical tank. What is the approximate difference in feet between the new height of the cone in scenario (1) and the new radius of the cone in scenario (2)?
(A) 13 (B) 25 (C) 30 (D) 35 (E) 40

5. The figure represents a deflated tire (6 inches wide as shown) with a hub (the center circle). The area of the hub surface shown in the picture is 1/3 the area of the tire surface shown in the picture. The thickness of the tire, when fully inflated is 3 inches. (Assume the tire material itself has negligible thickness.) Air is filled into the deflated tire at a rate of 4π inches^3 / second. How long (in seconds) will it take to inflate the tire?

24 27 48 81 108

6. The contents of one full cylindrical silo are to be transferred to another, larger cylindrical silo. The contents of the smaller silo will fill what portion of the larger silo?
(1) The larger silo has twice the base radius, and twice the height, of the smaller one.
(2) The smaller silo has a circular base with a radius of 10 feet.

7. When a cylindrical tank is filled with water at a rate of 22 cubic meters per hour, the level of water in the tank rises at a rate of 0.7 meters per hour. Which of the following best approximates the radius of the tank in meters?
√10/2 √10 4 5 10

8. A 10-by-6 inch piece of paper is used to form the lateral surface of a cylinder. If the entire piece of paper is used to make the cylinder, which of the following must be true of the two possible cylinders that can be formed?
A. The volume of the cylinder with height 10 is $60/\pi$ cubic inches greater than the volume of the cylinder with height 6.
B. The volume of the cylinder with height 6 is $60/\pi$ cubic inches greater than the volume of the cylinder with height 10.
C. The volume of the cylinder with height 10 is 60π cubic inches greater than the volume of the cylinder with height 6.
D. The volume of the cylinder with height 6 is 60π cubic inches greater than the volume of the cylinder with height 10.
E. The volume of the cylinder with height 6 is $240/\pi$ cubic inches greater than the volume of the cylinder with height 10.

Answer Key

GMAT Quant Topic 5: Geometry

Part A: Lines and Angles

1. C
2. A
3. E
4. D
5. A
6. D
7. A

Part B: Triangles

1. D
2. B
3. C
4. A
5. C
6. C
7. B
8. D
9. C
10. C
11. B
12. C
13. B
14. A
15. D
16. D
17. D
18. C
19. C
20. B

Part C: Quadrilaterals

1. C
2. D
3. C
4. C
5. B
6. A
7. D
8. D
9. B
10. C
11. A
12. E
13. C
14. E
15. A
16. C

17. E

Part D: Circles

1. A
2. B
3. C
4. D
5. C
6. A
7. C
8. D
9. D
10. D
11. B
12. C
13. D
14. A
15. B
16. E
17. E
18. D
19. E
20. E

Part E: Polygons

1. B
2. B
3. A
4. A
5. E

Part F: General Solids (Cube, Box, Sphere)

1. D
2. C
3. D
4. D
5. C

Part G: Cylinders

1. E
2. D
3. B
4. D
5. D
6. A
7. B
8. B

GMAT Quant Topic 6

Co-ordinate Geometry

1. If $ab \neq 0$ and points $(-a,b)$ and $(-b,a)$ are in the same quadrant of the xy–plane, is point $(-x,y)$ in this same quadrant?
(1) $xy > 0$ (2) $ax > 0$

2. In the xy–plane, at what two points does the graph of $y = (x+a)(x+b)$ intersect the x–axis?
(1) $a + b = -1$ (2) The graph intersects the y–axis at $(0, -6)$.

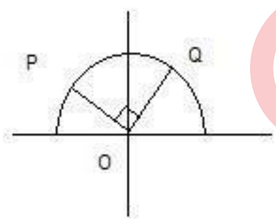
3. For any triangle T in the xy–coordinate plan, the center of T is defined to be the point whose x–coordinate is the average (arithmetic mean) of the x–coordinates of the vertices of T and whose y–coordinate is the average of the y–coordinates of the vertices of T. If a certain triangle has vertices at the points (0,0) and (6,0) and center at the point (3,2), what are the coordinates of the remaining vertex?
A. (3,4) B. (3,6) C. (4,9) D. (6,4) E. (9,6)

4. Circle C and line k lie in the xy–plane. if circle C is centered at the origin and has radius 1, does line k intersect circle C?
(1) the x–intercept of line k is greater than 1
(2) the slope of line k is $-1/10$

5. In the rectangular coordinate system, are the points (r,s) and (u,v) equidistant from the origin?
(1) $r + s = 1$ (2) $u = 1 - r$ and $v = 1 - s$

6. In the x–y plane, what is the y–intercept of the line l?
(1) The slope of the line l is 3 times its y intercept.
(2) The x–intercept of line l is $-1/3$

7. In the figure shown, point $P(-\sqrt{3}, 1)$ and $Q(s, t)$ lie on the circle with center O.



What is value of s?

A. $\frac{1}{2}$

B. 1

C. $\sqrt{2}$

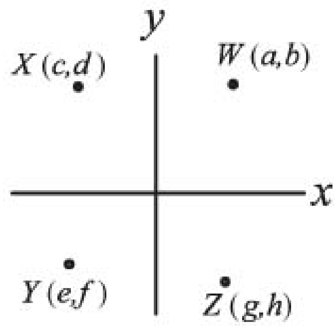
D. $\sqrt{3}$

E. $\frac{\sqrt{2}}{2}$

8. In the xy–plane, line k has positive slope and x–intercept 4. If the area of the triangle formed by line k and the two axes is 12, what of the y– intercept of line?
A. -6
B. -4
C. -3
D. 3
E. 6

9. Line l is defined by the equation $y - 5x = 4$ and line w is defined by the equation $10y + 2x + 20 = 0$. If line k does not intersect line l, what is the degree measure of the angle formed by line k and line w?
0 30 60 90 It cannot be determined from the information given.

10. In the rectangular coordinate plane points X and Z lie on the same line through the origin and points W and Y lie on the same line through the origin. If $a^2 + b^2 = c^2 + d^2$ and $e^2 + f^2 = g^2 + h^2$, what is the value of length XZ – length WY?



-2 -1 0 1 2

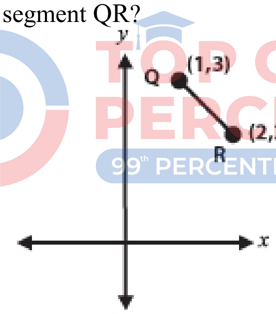
11. In the xy-coordinate system, what is the slope of the line that goes through the origin and is equidistant from the two points P = (1, 11) and Q = (7, 7)?
2 2.25 2.50 2.75 3

12. What is the slope of the line represented by the equation x + 2y = 1?
-3/2 -1 -1/2 0 ½

13. A certain square is to be drawn on a coordinate plane. One of the vertices must be on the origin, and the square is to have an area of 100. If all coordinates of the vertices must be integers, how many different ways can this square be drawn?
4 6 8 10 12

14. Does the equation y = (x – p)(x – q) intercept the x-axis at the point (2,0)?
(1) pq = -8 (2) -2 – p = q

15. Does line S (not pictured) intersect line segment QR?



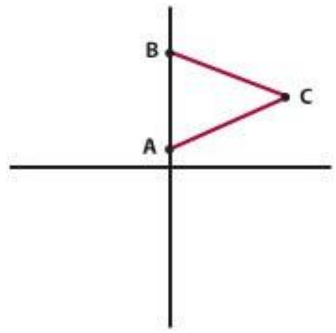
(1) The equation of line S is y = -x + 4. (2) The slope of line S is -1.

16. Line L contains the points (2,3) and (p,q). If q = 2, which of the following could be the equation of line m, which is perpendicular to line L?
(A) 2x + y = px + 7
(B) 2x + y = –px
(C) x + 2y = px + 7
(D) y – 7 = x ÷ (p – 2)
(E) 2x + y = 7 – px

17. Point K = (A,0), Point G = $\left(2A + 4,\ \sqrt{2A + 9}\right)$. Is the distance between point K and G prime?
(1) $A^2 - 5A - 6 = 0$ (2) A > 2

18. The (x, y) coordinates of points P and Q are (-2, 9) and (-7, -3), respectively. The height of equilateral triangle XYZ is the same as the length of line segment PQ. What is the area of triangle XYZ?
169/√3 84.5 75√3 169√3 /4 225√3 /4

19. If points A and B are on the y-axis in the figure, what is the area of equilateral triangle ABC?



(1) The coordinates of point B are (0, 5√3).
(2) The coordinates of point C are (6, 3√3).

20. The line 3x + 4y = 8 passes through all of the quadrants in the coordinate plane except:
I II III IV II and IV.

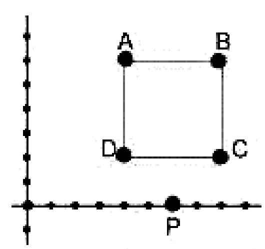
21. If p and q are nonzero numbers, and p is not equal to q, in which quadrant of the coordinate system does point (p, p – q) lie?
(1) (p, q) lies in quadrant IV. (2) (q, -p) lies in quadrant III.

22. The coordinates of points A and C are (0, -3) and (3, 3), respectively. If point B lies on line AC between points A and C, and if AB = 2BC, which of the following represents the coordinates of point B?
(1, -√5) (1, -1) (2, 1) (1.5, 0) (√5, √5)

23. In the xy-coordinate system, rectangle ABCD is inscribed within a circle having the equation $x^2 + y^2 = 25$. Line segment AC is a diagonal of the rectangle and lies on the x-axis. Vertex B lies in quadrant II and vertex D lies in quadrant IV. If side BC lies on line y = 3x + 15, what is the area of rectangle ABCD?
(A) 15 (B) 30 (C) 40 (D) 45 (E) 50

24. The line represented by the equation y = 4 – 2x is the perpendicular bisector of line segment RP. If R has the coordinates (4, 1), what are the coordinates of point P?
(A) (–4, 1) (B) (–2, 2) (C) (0, 1) (D) (0, –1) (E) (2, 0)

25. A certain computer program randomly generates equations of lines in the form y = mx + b. If point P is a point on a line generated by this program, what is the probability that the line does NOT pass through figure ABCD?



(A) ¾ (B) 3/5 (C) ½ (D) 2/5 (E) ¼

26. In the rectangular coordinate system, a line passes through the points (0,5) and (7,0). Which of the following points must the line also pass through?
(-14, 10) (-7, 5) (12, -4) (14, -5) (21, -9)

27. Which of the following equations represents a line that is perpendicular to the line described by the equation 3x + 4y = 8?
A. 3x + 4y = 18
B. 3x – 4y = 24
C. 4y – 3x = 26
D. 1.5y + 2x = 18
E. 8x – 6y = 24

28. How many units long is the straight line segment that connects the points (-1,1) and (2,6) on a rectangular coordinate plane?

4 $\sqrt{26}$ $\sqrt{34}$ 7 $\sqrt{58}$

29. In the rectangular coordinate system, lines m and n cross at the origin. Is line m perpendicular to line n?

(1) m has a slope of -1 and n passes through the point (-a, -a).

(2) If the slope of m is x and the slope of n is y, then -xy = 1.

30. Line A is drawn on a rectangular coordinate plane. If the coordinate pairs (3, 2) and (-1, -2) lie on line A, which of the following coordinate pairs does NOT lie on a line that is perpendicular to line A?

A. (5, 8) and (4, 9)
B. (3, -1) and (4, -2)
C. (-1, 6) and (-4, 9)
D. (2, 5) and (-3, 2)
E. (7, 1) and (6, 2)

31. Draw the following graphs (approximate shape)

a. $x^2 + 3x - 4 = 0$
b. $2x^2 - 4x - 3 = 0$
c. $x(x - 2) = 4$
d. $9x^2 + 12x + 4 = 0$
e. $3x^2 + 4x + 2 = 0$
f. $x^2 + 2x = 1$
g. $-2x^2 + 3x + 2$
h. $2x^2 + 3x + 2$

Answer Key

GMAT Quant Topic 6: Co-ordinate Geometry

1. C
2. C
3. B
4. E
5. C
6. E
7. B
8. A
9. D
10. C
11. B
12. C
13. E
14. C
15. A
16. A
17. C
18. A
19. B
20. C
21. D
22. C
23. B
24. D
25. C
26. D
27. E
28. C
29. D
30. D
31. *Check solution key*

GMAT Quant Topic 7

Permutations and Combinations

1. How many different anagrams can you make for the word GMAT? How many different anagrams can you make for the word MATHEMATICS?
A. 5 !, 11 ! / 2 ! * 2 ! * 2 !   B. 4 !, 11 ! / 3 ! * 2 ! * 2 !   C. 4 !, 11 ! / 2 ! * 2 ! * 2 !
D. 6 !, 11 ! / 2 ! * 2 ! * 2 !   E. 4 !, 11 ! / 2 ! * 3 ! * 3 !

2. If there are 7 people and only 4 chairs in a room, how many different seating arrangements are possible?

(A) 420   (B) 35   (C) 840   (D) 70   (E) 210

3. A man wants to visit at least two of the four cities A, B, C and D. How many travel itineraries can he make? All cities are connected to one another.
(A) 24
(B) 6
(C) 60
(D) 12
(E) None of the above

4. There are 2 black balls, one red ball and one green ball, identical in shape and size. How many different linear arrangements can be generated by arranging these balls?

(A) 12   (B) 30   (C) 60   (D) 45   (E) 50

5. From a list of 10 songs, a DJ has to play either 2 or 3 songs. What is the total number of song sequences that DJ can create?

(A) 10P1 + 10P2   (B) 100P2 + 10P3   (C) 10P2 + 10P3   (D) 10P2 + 100P3   (E) 10P4 + 10P3

6. A password contains at least 8 distinct digits. It takes 12 seconds to try one combination, what is the minimum amount of time required to guarantee access to the database?

A. 12 seconds
B. 24 seconds
C. 36 seconds
D. 48 seconds
E. None of these

7. Greg, Marcia, Peter, Jan, Bobby and Cindy go to a movie and sit next to each other in 6 adjacent seats in the front row of the theater. If Marcia and Jan will not sit next to each other, in how many ways different arrangements can the 6 people sit?

A. 240
B. 360
C. 480
D. 500
E. 730

8. In how many ways can a committee of 4 people be selected from a group of 7 people?

(A) 12   (B) 35   (C) 60   (D) 45   (E) 50

9. In a college, 8 students play at the State level and 10 at the National level. If 6 students play at both National and State levels, in how many ways can 9 students be selected from among these?

(A) 220   (B) 230   (C) 260   (D) 45   (E) 50

10. An engagement team consists of a project manager, team leader, and four consultants. There are 2 candidates for the position of project manager, 3 candidates for the position of team leader, and 7 candidates for the 4 consultant slots. If 2 out of 7 consultants refuse to be on the same team, how many different teams are possible?

A. 25
B. 35
C. 150
D. 210
E. 300

11. In how many ways can 3 letters out of five distinct 5 distinct letters A, B, C, D and E be arranged in a straight line so that A and B never come together?

A. 7
B. 23
C. 29
D. 37
E. 48

12. A nickel, a dime, and two identical quarters are arranged along a side of a table. If the quarters and the dime have to face heads up, while the nickel can face either heads up or tails up, how many different arrangements of coins are possible?

A. 12
B. 24
C. 48
D. 72
E. 96

13. At a certain laboratory, chemical substances are identified by an unordered combination of 3 colors. If no chemical may be assigned the same colors, what is the maximum number of substances that can be identified using 7 colors?

A) 21
B) 35
C) 105
D) 135
E) 210

14. An equity analyst needs to select 3 stocks for the upcoming year and rank these securities in terms of their investment potential. If the analyst has narrowed down the list of potential stocks to 7, in how many ways can she choose and rank her top 3 picks?

A. 21
B. 35
C. 210
D. 420
E. 840

15. How many different five-letter combinations can be created from the word TWIST?

A. 5
B. 24
C. 60
D. 120
E. 720

16. If an employee ID code must consist of 3 non-repeating digits and each digit in the code must be a prime number, how many ID codes can be created?

(a) 4
(b) 10
(c) 22
(d) 24
(e) 26

17. A university cafeteria offers 4 flavors of pizza – pepperoni, chicken, Hawaiian and vegetarian. If a customer has an option to add, extra cheese, mushrooms, or both to any kind of pizza, how many different pizza varieties are available?

(A) 4
(B) 8
(C) 12
(D) 16
(E) 32

18. Mario's Pizza has two choices of crust: deep dish and thin-and-crispy. The restaurant also has a choice of 5 toppings: tomatoes, sausage, peppers, onions, and pepperoni. Finally, Mario's offers every pizza in extra- cheese as well as 'regular'. If Linda's volleyball team decides to order a pizza with four toppings, how many different choices do the teammates have at Mario's Pizza?

A. 4 B. 5 C. 10 D. 15 E. 20

19. A book store has received 8 different books, of which 3/8 are novels, 25% are study guides and the remaining are textbooks. If all books must be placed on one shelf displaying new items and if books in the same category have to be shelved next to each other, how many different arrangements of books are possible?

A. 18
B. 36
C. 72
D. 216
E. 432

20. A group of 5 students bought movie tickets in one row next to each other. If Bob and Lisa are in this group, what is the number of ways of seating if both of them will sit next to only one other student from the group?

(A) 5
(B) 10
(C) 12
(D) 20
(E) 25

21. Mark's clothing store uses a bar-code system to identify every item. Each item is marked by a combination of 2 letters followed by 3 digits. Additionally, the three-digit number must be even for male products and odd for female products. If all apparel products start with the letter combination AP, how many male apparel items can be identified with the bar code?
A) 100
B) 405
C) 500
D) 729
E) 1000

22. Fernando purchased a university meal plan that allows him to have a total of 3 lunches and 3 dinners per week. If the cafeteria is closed on weekends and Fernando always goes home for a dinner on Friday nights, how many options does he have to allocate his meals?

(A) 20
(B) 24
(C) 40
(D) 100
(E) 120

23. If the President and the Vice President must sit next to each other in a row with 4 other members on the Board, how many different seating arrangements are possible?

(A) 120
(B) 240
(C) 300
(D) 360
(E) 720

24. To apply for the position of photographer at a local magazine, Veronica needs to include 3 or 4 of her pictures in an envelope accompanying her application. If she has pre-selected 5 photos representative of her work, how many choices does she have to provide the photos for the magazine?

(A) 5
(B) 10
(C) 12
(D) 15
(E) 50

25. A retail company needs to set up 5 additional distribution centers that can be located in three cities on the east coast (Boston, New York, and Washington D.C.), one city in the mid-west (Chicago), and three cities on the west coast (Seattle, San Francisco and Los Angeles). If the company must add 2 distribution centers on each coast and 1 in the mid-west, and only one center can be added in each city, in how many ways can the management allocate the distribution centers?

A. 3 B. 9 C. 18 D. 20 E. 36

26. Three couples need to be arranged in a row for a group photo. If the couples cannot be separated, how many different arrangements are possible?

A. 6
B. 12
C. 24
D. 48
E. 96

27. If 6 fair coins are tossed, how many different coin sequences will have exactly 3 tails, if all tails have to occur in a row?

A. 4
B. 8
C. 16
D. 20
E. 24

28. A telephone company needs to create a set of 3-digit area codes. The company is entitled to use only digits 2, 4 and 5, which can be repeated. If the product of the digits in the area code must be even, how many different codes can be created?

A. 20
B. 22
C. 24
D. 26
E. 30

29. Jake, Lena, Fred, John and Inna need to drive home from a corporate reception in an SUV that can seat 7 people. If only Inna or Jake can drive, how many seat allocations are possible?

A. 30
B. 42
C. 120
D. 360
E. 720

30. In how many ways can a teacher write an answer key for a mini-quiz that contains 3 true-false questions followed by 2 multiples-choice questions with 4 answer choices each, if the correct answers to all true-false questions cannot be the same?

A. 88
B. 90
C. 96
D. 98
E. 102

31. A student committee on academic integrity has 90 ways to select a president and vice-president from a group of candidates. The same person cannot be both president and vice-president. How many students are in the group?

A. 7
B. 8
C. 9
D. 10
E. 11

32. A pod of 6 dolphins always swims single file, with 3 females at the front and 3 males in the rear. In how many different arrangements can the dolphins swim?

A. 20
B. 36
C. 40
D. 18
E. 54

33. A British spy is trying to escape from his prison cell. The lock requires him to enter one number, from 1-9, and then push a pair of colored buttons simultaneously. He can make one attempt every 3 seconds. If there are 6 colored buttons, what is the longest possible time (in seconds) it could take the spy to escape from the prison cell?

(A) 405 (B) 430 (C) 460 (D) 545 (E) 450

34. Every morning, Casey walks from her house to the bus stop. She always travels exactly nine blocks from her house to the bus, but she varies the route she takes every day. (One sample route is shown.) How many days can Casey walk from her house to the bus stop without repeating the same route?

A. 120
B. 122
C. 124
D. 126
E. 128

35. Three dwarves and three elves sit down in a row of six chairs. If no dwarf will sit next to another dwarf and no elf will sit next to another elf, in how many different ways can the elves and dwarves sit?
A. 18
B. 36
C. 48
D. 72
E. 96

36. Gordon buys 5 dolls for his 5 nieces. The gifts include two identical Sun-and-Fun beach dolls, one Elegant Eddie dress-up doll, one G.I. Josie army doll, and one Tulip Troll doll. If the youngest niece doesn't want the G.I. Josie doll, in how many different ways can he give the gifts?
A. 12
B. 24
C. 36
D. 48
E. 60

37. How many different 5-person teams can be formed from a group of x individuals?

(1) If there had been x + 2 individuals in the group, exactly 126 different 5-person teams could have been formed.

(2) If there had been x + 1 individuals in the group, exactly 56 different 3-person teams could have been formed.

38. There are x people and y chairs in a room where x and y are positive prime numbers. How many ways can the x people be seated in the y chairs (assuming that each chair can seat exactly one person)?

(1) x + y = 12 (2) There are more chairs than people.

39. A gambler began playing blackjack with $110 in chips. After exactly 12 hands, he left the table with $320 in chips, having won some hands and lost others. Each win earned $100 and each loss cost $10. How many possible outcomes were there for the first 5 hands he played? (For example, won the first hand, lost the second, etc.)

(A) 10 (B) 18 (C) 26 (D) 32 (E) 64

40. In a 4-person race, medals are awarded to the fastest 3 runners. The first-place runner receives a gold medal, the second-place runner receives a silver medal, and the third-place runner receives a bronze medal. In the event of a tie, the tied runners receive the same color medal. (For example, if there is a two-way tie for first- place, the top two runners receive gold medals, the next-fastest runner receives a silver medal, and no bronze medal is awarded). Assuming that exactly three medals are awarded, and that the three medal winners stand together with their medals to form a victory circle, how many different victory circles are possible?

(A) 24 (B) 52 (C) 96 (D) 144 (E) 648

41. The organizers of a week-long fair have hired exactly five security guards to patrol the fairgrounds at night for the duration of the event. Exactly two guards are assigned to patrol the grounds every night, with no guard assigned consecutive nights. If the fair begins on a Monday, how many different pairs of guards will be available to patrol the fairgrounds on the following Saturday night?

(A) 9 (B) 7 (C) 5 (D) 3 (E) 2

42. Larry, Michael, and Doug have five donuts to share. If any one of the men can be given any whole number of donuts from 0 to 5, in how many different ways can the donuts be distributed?

(A) 21 (B) 42 (C) 120 (D) 504 (E) 5040

43. A woman has seven cookies—four chocolate chip and three oatmeal. She gives one cookie to each of her six children: Nicole, Ronit, Kim, Deborah, Mark, and Terrance. If Deborah will only eat the kind of cookie that Kim eats, in how many different ways can the cookies be distributed? (The leftover cookie will be given to the dog.)

(A) 5040 (B) 50 (C) 25 (D) 15 (E) 12

44. Sammy has x flavors of candies with which to make goody bags for Frank's birthday party. Sammy tosses out y flavors, because he doesn't like them. How many different 10-flavor bags can Sammy make from the remaining flavors? (It doesn't matter how many candies are in a bag, only how many flavors).

(1) If Sammy had thrown away 2 additional flavors of candy, he could have made exactly 3,003 different 10-flavor bags.

(2) $x = y + 17$

45. How many different combinations of outcomes can you make by rolling three standard (6-sided) dice if the order of the dice does not matter?

(A) 24 (B) 30 (C) 56 (D) 120 (E) 216

46. A certain league has four divisions. The respective divisions had 9, 10, 11, and 12 teams qualify for the playoffs. Each division held its own double-elimination tournament -- where a team is eliminated from the tournament upon losing two games -- in order to determine its champion. The four division champions then played in a single-elimination tournament -- where a team is eliminated upon losing one game -- in order to determine the overall league champion. Assuming that there were no ties and no forfeits, what is the maximum number of games that could have been played in order to determine the overall league champion?

(A) 79 (B) 83 (C) 85 (D) 87 (E) 88

47. You have a bag of 9 letters: 3 Xs, 3 Ys, and 3 Zs. You are given a box divided into 3 rows and 3 columns for a total of 9 areas. How many different ways can you place one letter into each area such that there are no rows or columns with 2 or more of the same letter? (Note: One such way is shown below.)

X	Y	Z
Y	Z	X
Z	X	Y

(A) 5 (B) 6 (C) 9 (D) 12 (E) 18

48. Eight women of eight different heights are to pose for a photo in two rows of four. Each woman in the second row must stand directly behind a shorter woman in the first row. In addition, all of the women in each row must be arranged in order of increasing height from left to right. Assuming that these restrictions are fully adhered to, in how many different ways can the women pose?

(A) 2 (B) 14 (C) 15 (D) 16 (E) 18

49. Company X has 6 regional offices. Each regional office must recommend two candidates, one male and one female, to serve on the corporate auditing committee. If each of the offices must be represented by exactly one member on the auditing committee and if the committee must consist of an equal number of male and female employees, how many different committees can be formed?

A. 5
B. 10
C. 15
D. 20
E. 40

50. You have a six-sided cube and six cans of paint, each a different color. You may not mix colors of paint. How many distinct ways can you paint the cube using a different color for each side? (If you can reorient a cube to look like another cube, then the two cubes are not distinct.)

(A) 24 (B) 30 (C) 48 (D) 60 (E) 120

51. A group of four women and three men have tickets for seven adjacent seats in one row of a theatre. If the three men will not sit in three adjacent seats, how many possible different seating arrangements are there for these 7 theatre-goers?

(A) 7! – 2!3!2! (B) 7! – 4!3! (C) 7! – 5!3! (D) 7 × 2!3!2! (E) 2!3!2!

52. Anthony and Michael sit on the six-member board of directors for company X. If the board is to be split up into 2 three-person subcommittees, what percent of all the possible subcommittees that include Michael also include Anthony?
20% 30% 40% 50% 60%

53. A family consisting of one mother, one father, two daughters and a son is taking a road trip in a sedan. The sedan has two front seats and three back seats. If one of the parents must drive and the two daughters refuse to sit next to each other, how many possible seating arrangements are there?
28 32 48 60 120

54. Six mobsters have arrived at the theater for the premiere of the film "Goodbuddies." One of the mobsters, Frankie, is an informer, and he's afraid that another member of his crew, Joey, is on to him. Frankie, wanting to keep Joey in his sights, insists upon standing behind Joey in line at the concession stand. How many ways can the six arrange themselves in line such that Frankie's requirement is satisfied?
6 24 120 360 720

55. A college admissions committee will grant a certain number of $10,000 scholarships, $5,000 scholarships, and $1,000 scholarships. If no student can receive more than one scholarship, how many different ways can the committee dole out the scholarships among the pool of 10 applicants?
(1) In total, six scholarships will be granted.
(2) An equal number of scholarships will be granted at each scholarship level.

56. A certain panel is to be composed of exactly three women and exactly two men, chosen from x women and y men. How many different panels can be formed with these constraints?
(1) If two more women were available for selection, exactly 56 different groups of three women could be selected.
(2) $x = y + 1$

57. A student committee that must consist of 5 members is to be formed from a pool of 8 candidates. How many different committees are possible?
5 8 40 56 336

58. How many ways are there to award a gold, silver and bronze medal to 10 contending teams?
$10 \times 9 \times 8$ $10! / 3!\,7!$ $10! / 3!$ 360 300

59. From a drawer containing black, blue and gray solid-color socks, including at least three socks of each color, how many matched pairs can be removed?
(1) The drawer contains 11 socks.
(2) The drawer contains an equal number of black and gray socks.

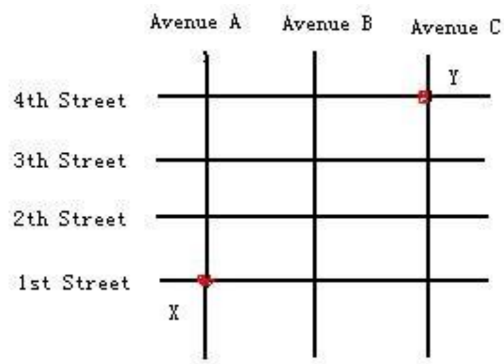
60. On Tuesday, Kramer purchases exactly 3 new shirts, 2 new sweaters, and 4 new hats, On the following day and each subsequent day thereafter, Kramer wears one of his new shirts together with one of his new sweaters and one of his new hats. Kramer avoids wearing the exact same combination of shirt, sweater, and hat for as long as possible. On which day is this no longer possible?
Tuesday Wednesday Thursday Friday Saturday

61. A certain stock exchange designates each stock with a one- , two-, or three-letter code, where each letter is selected from the 26 letters of the alphabet. If the letter may be repeated and if the same letters used in a different order constitute a different code, how many different stocks is it possible to uniquely designate with these codes?
2951 8125 15600 15302 18278

62. A certain law firm consists of 4 senior partners and 6 junior partners. How many different groups of 3 partners can be formed in which at least one member of the group is a senior partner? (Two groups are considered different if at least one group member is different.)
48 100 120 288 600

63. A company plans to assign identification numbers to its employees. Each number is to consist of four different digits from 0 to 9, inclusive, except that the first digit cannot be 0. How many different identification numbers are possible?
(A) 3,024
(B) 4,536
(C) 5,040
(D) 9,000
(E) 10,000

64. Pat will walk from intersection X to intersection y along route that is confined to the square grid of four streets and three avenues shown in the map above. How many routes from X to Y can Pat take that have the minimum possible length?



6 8 10 14 16

65. Tanya prepared four different letters to be sent to four different addresses. For each letter, she prepared an envelope with its correct address. If the 4 letters are to be put into the envelopes at random, what is the probability that only one letter will be put into the envelope with its correct address?
A. 1/24
B. 1/8
C. 1/4
D. 1/3
E. 3/8

66. 5 people are to be seated around a circular table. Two seating arrangement are considered different only when the positions of the people are different relative to each other. What is the total number of different possible seating arrangements for the group?

A. 5
B. 10
C. 24
D. 32
E. 12

Answer Key

GMAT Quant Topic 7: Permutations and Combinations

1	C
2	C
3	C
4	A
5	C
6	E
7	C
8	B
9	A
10	C
11	E
12	B
13	B
14	C
15	C
16	D
17	D
18	E
19	E
20	C
21	C
22	C
23	B
24	D
25	B
26	D
27	A
28	D
29	E
30	C
31	D
32	B
33	A
34	D
35	D
36	D
37	D
38	A
39	C
40	B
41	D
42	A
43	D
44	D
45	C
46	B

47	D
48	B
49	D
50	B
51	C
52	C
53	B
54	D
55	C
56	C
57	D
58	A
59	E
60	E
61	E
62	B
63	B
64	C
65	D
66	C

GMAT Quant Topic 8

Probability

1. A fair coin is flipped three times. What is the probability that the coin lands on heads exactly twice?
(A) 1/8 (B) 3/8 (C) 1/2 (D) 5/8 (E) 7/8

2. Is the probability that Patty will answer all of the questions on her chemistry exam correctly greater than 50%?
(1) For each question on the chemistry exam, Patty has a 90% chance of answering the question correctly.
(2) There are fewer than 10 questions on Patty's chemistry exam.

3. There are 10 women and 3 men in room A. One person is picked at random from room A and moved to room B, where there are already 3 women and 5 men. If a single person is then to be picked from room B, what is the probability that a woman will be picked?
(A) 13/21 (B) 49/117 (C) 15/52 (D) 5/18 (E) 40/117

4. If the probability of rain on any given day in Chicago during the summer is 50%, independent of what happens on any other day, what is the probability of having exactly 3 rainy days from July 4 through July 8, inclusive?
(A) 1/32 (B) 2/25 (C) 5/16 (D) 8/25 (E) ¾

5. In a shipment of 20 cars, 3 are found to be defective. If four cars are selected at random, what is the probability that exactly one of the four will be defective?
(A) 170/1615 (B) 3/20 (C) 8/19 (D) 3/5 (E) 4/5

6. A certain bag of gemstones is composed of two-thirds diamonds and one-third rubies. If the probability of randomly selecting two diamonds from the bag, without replacement, is 5/12, what is the probability of selecting two rubies from the bag, without replacement?
(A) 5/36 (B) 5/24 (C) 1/12 (D) 1/6 (E) ¼

7. Triplets Adam, Bruce, and Charlie enter a triathlon. If there are 9 competitors in the triathlon and medals are awarded for first, second, and third place, what is the probability that at least two of the triplets will win a medal?
(A) 3/14 (B) 19/84 (C) 11/42 (D) 15/28 (E) ¾

8. Set S is the set of all prime integers between 0 and 20. If three numbers are chosen randomly from set S and each number can be chosen only once, what is the positive difference between the probability that the product of these three numbers is a number less than 31 and the probability that the sum of these three numbers is odd?
(A) 1/336 (B) ½ (C) 17/28 (D) ¾ (E) 301/336

9. A random 10-letter code is to be formed using the letters A, B, C, D, E, F, G, H, I and I (only the "I" will be used twice). What is the probability that a code that has the two I's adjacent to one another will be formed?
(A) 1/10 (B) 1/8 (C) 1/5 (D) ¼ (E) ½

10. If $p^2 - 13p + 40 = q$, and p is a positive integer between 1 and 10, inclusive, what is the probability that $q < 0$?
(A) 1/10 (B) 1/5 (C) 2/5 (D) 3/5 (E) 3/10

11. A box contains three pairs of blue gloves and two pairs of green gloves. Each pair consists of a left-hand glove and a right-hand glove. Each of the gloves is separate from its mate and thoroughly mixed together with the others in the box. If three gloves are randomly selected from the box, what is the probability that a matched set (i.e., a left- and right-hand glove of the same color) will be among the three gloves selected?
(A) 3/10 (B) 23/60 (C) 7/12 (D) 41/60 (E) 5/6

12. A football team has 99 players. Each player has a uniform number from 1 to 99 and no two players share the same number. When football practice ends, all the players run off the field one-by-one in a completely random manner. What is the probability that the first four players off the field will leave in order of increasing uniform numbers (e.g., #2, then #6, then #67, then #72, etc)?
(A) 1/64 (B) 1/48 (C) 1/36 (D) 1/24 (E) 1/16

13. What is the probability that (u/v)/w and (x/y)/z are reciprocal fractions?
(1) v, w, y, and z are each randomly chosen from the first 100 positive integers.
(2) The product (u)(x) is the median of 100 consecutive integers.

14. Bill and Jane play a simple game involving two fair dice, each of which has six sides numbered from 1 to 6 (with an equal chance of landing on any side). Bill rolls the dice and his score is the total of the two dice. Jane then rolls the dice and her score is the total of her two dice. If Jane's score is higher than Bill's, she wins the game. What is the probability the Jane will win the game?
(A) 15/36 (B) 175/432 (C) 575/1296 (D) 583/1296 (E) ½

15. Kate and Danny each have \$10. Together, they flip a fair coin 5 times. Every time the coin lands on heads, Kate gives Danny \$1. Every time the coin lands on tails, Danny gives Kate \$1. After the five coin flips, what is the probability that Kate has more than \$10 but less than \$15?
(A) 5/16 (B) ½ (C) 12/30 (D) 15/32 (E) 3/8

16. There is a 10% chance that it won't snow all winter long. There is a 20% chance that schools will not be closed all winter long. What is the greatest possible probability that it will snow and schools will be closed during the winter?
(A) 55% (B) 60% (C) 70% (D) 72% (E) 80%

17. There are y different travelers who each have a choice of vacationing at one of n different destinations. What is the probability that all y travelers will end up vacationing at the same destination?
(A) 1/n! (B) n/n! (C) $1/n^y$ (D) $1/n^{y-1}$ (E) n/y^n

18. A small, experimental plane has three engines, one of which is redundant. That is, as long as two of the engines are working, the plane will stay in the air. Over the course of a typical flight, there is a 1/3 chance that engine one will fail. There is a 75% probability that engine two will work. The third engine works only half the time. What is the probability that the plane will crash in any given flight?
(A) 7/12 (B) ¼ (C) ½ (D) 7/24 (E) 17/24

19. Two pieces of fruit are selected out of a group of 8 pieces of fruit consisting only of apples and bananas. What is the probability of selecting exactly 2 bananas?
(1) The probability of selecting exactly 2 apples is greater than 1/2.
(2) The probability of selecting 1 apple and 1 banana in either order is greater than 1/3.

20. Ms. Barton has four children. You are told correctly that she has at least two girls but you are not told which two of her four children are those girls. What is the probability that she also has two boys? (Assume that the probability of having a boy is the same as the probability of having a girl.)
(A) ¼ (B) 3/8 (C) 5/11 (D) ½ (E) 6/11

21. Mike recently won a contest in which he will have the opportunity to shoot free throws in order to win \$10,000. In order to win the money Mike can either shoot 1 free throw and make it, or shoot 3 free throws and make at least 2 of them. Mike occasionally makes shots and occasionally misses shots. He knows that his probability of making a single free throw is p, and that this probability doesn't change. Would Mike have a better chance of winning if he chose to attempt 3 free throws?
(1) $p < 0.7$ (2) $p > 0.6$

22. Laura has a deck of standard playing cards with 13 of the 52 cards designated as a "heart." If Laura shuffles the deck thoroughly and then deals 10 cards off the top of the deck, what is the probability that the 10th card dealt is a heart?
(A) ¼ (B) 1/5 (C) 5/26 (D) 12/42 (E) 13/42

23. A license plate in the country Kerrania consists of four digits followed by two letters. The letters A, B, and C are used only by government vehicles while the letters D through Z are used by non-government vehicles. Kerrania's intelligence agency has recently captured a message from the country Gonzalia indicating that an electronic transmitter has been installed in a Kerrania government vehicle with a license plate starting with 79. If it takes the police 10 minutes to inspect each vehicle, what is the probability that the police will find the transmitter within three hours?
(A) 18/79 (B) 1/6 (C) 1/25 (D) 1/50 (E) 1/900

24. A grid of light bulbs measures x bulbs by x bulbs, where x > 2. If 4 light bulbs are illuminated at random, what is the probability, in terms of x, that the 4 bulbs form a 2 bulb by 2 square?
1. 4(x-1)/x^2

2. 24(x−1)/x^2∗(x+1)∗(x^2−2)∗(x^2−3)

3. 24(x+1)/ (x^2)(x^2-2)(x+1)

4. 4(x+1)/(x^2)(x2-2)(x-1)

5. 4(x-1)/(x^2)(x2-2).

25. Baseball's World Series matches 2 teams against each other in a best-of-seven series. The first team to win four games wins the series and no subsequent games are played. If you have no special information about either of the teams, what is the probability that the World Series will consist of fewer than 7 games?
(A) 12.5% (B) 25% (C) 31.25% (D) 68.75% (E) 75%

26. Harriet and Tran each have \$10. Together, they flip a fair coin 5 times. Every time the coin lands on heads, Tran gives Harriet \$1. Every time the coin lands on tails, Harriet gives Tran \$1. After the five coin flips, what is the probability that Harriet has more than \$10 but less than \$15?
(A) 5/16 (B) 1/2 (C) 12/30 (D) 15/32 (E) 3/8

27. If 40 percent of all students at College X have brown hair and 70 percent of all students at College X have blue eyes, what is the difference between the minimum and the maximum probability of picking a student from College X who has neither brown hair nor blue eyes?
(A) 0.2 (B) 0.3 (C) 0.4 (D) 0.6 (E) 0.7

28. In a room filled with 7 people, 4 people have exactly 1 friend in the room and 3 people have exactly 2 friends in the room (Assuming that friendship is a mutual relationship, i.e. if John is Peter's friend, Peter is John's friend). If two individuals are selected from the room at random, what is the probability that those two individuals are NOT friends?
5/21 3/7 4/7 5/7 16/21

29. Bill has a small deck of 12 playing cards made up of only 2 suits of 6 cards each. Each of the 6 cards within a suit has a different value from 1 to 6; thus, there are 2 cards in the deck that have the same value. Bill likes to play a game in which he shuffles the deck, turns over 4 cards, and looks for pairs of cards that have the same value. What is the chance that Bill finds at least one pair of cards that have the same value?
8/33 62/165 17/33 103/165 25/33

30. If a jury of 12 people is to be selected randomly from a pool of 15 potential jurors, and the jury pool consists of 2/3 men and 1/3 women, what is the probability that the jury will comprise at least 2/3 men?
24/91 45/91 2/3 67/91 84/91

31. John and Peter are among the nine players a basketball coach can choose from to field a five-player team. If all five players are chosen at random, what is the probability of choosing a team that includes John and Peter?
1/9 1/6 2/9 5/18 1/3

32. A small company employs 3 men and 5 women. If a team of 4 employees is to be randomly selected to organize the company retreat, what is the probability that the team will have exactly 2 women?
1/14 1/7 2/7 3/7 ½

33. A box contains one dozen donuts. Four of the donuts are chocolate, four are glazed, and four are jelly. If two donuts are randomly selected from the box, one after the other, what is the probability that both will be jelly donuts?
1/11 1/9 1/3 2/3 8/9

34. 8 cities, including Memphis, compete in a national contest to host a political convention. Exactly one city wins the competition. What is the probability that Memphis does not win the competition?
(1) The probability that any one of the 8 cities does not win the competition is 7/8.
(2) The probability that Memphis wins the competition is 1/8.

35. A hand purse contains 6 nickels, 5 pennies and 4 dimes. What is the probability of picking a coin other than a nickel twice in a row if the first coin picked is not put back?
8/25 12/35 13/35 9/25 17/25

36. Jim and Renee will play one game of Rock, Paper, Scissors. In this game, each will select and show a hand sign for one of the three items. Rock beats Scissors, Scissors beat Paper, and Paper beats Rock. Assuming that both Jim and Renee have an equal chance of choosing any one of the hand signs, what is the probability that Jim will win?
5/6 2/3 ½ 5/12 1/3

37. A certain box contains only red balls and green balls. If one ball is randomly selected from the box, what is the probability that it is red?
(1) Red balls comprise exactly two-thirds of all the balls in the box.
(2) The probability of selecting a green ball from the box is 1/3.

38. At a certain car dealership, the 40 vehicles equipped with air conditioning represent 80% of all cars available for sale. Among all the cars, there are 15 convertibles, 14 of which are equipped with an air- conditioning system. If a customer is willing to purchase either a convertible or a car equipped with air conditioning, what is the probability that a randomly selected vehicle will fit customer specifications?
A. 41/50
B. 9/50
C. 6/50
D. 21/25
E. 8/25

39. In a card game, a combination of two aces beats all others. If Jose is the first to draw from a standard deck of 52 cards, what is the probability that he wins the game with the best possible combination?
A) 1/221
B) 13/221
C) 2/52
D) 3/51
E) 4/51

40. Derrick and Lena, a married couple attending the same business school, go to a corporate presentation that ends with two sequential drawings of a PDA (Personal Digital Assistant) among the 60 attending students. If each attendant is given one ticket participating in the lottery of the two PDAs and if each winning ticket is removed from the urn, what is the probability that both PDAs will go to the couple?
A. 1/1770
B. 1/120
C. 1/118
D. 1/60
E. 1/59

41. If two balls are randomly drawn from a green urn containing 5 black and 5 white balls and placed into a yellow urn initially containing 5 black and 3 white balls, what is the probability that the yellow urn will contain an equal number of black and white balls after this change?
A.2/9
B.4/9
C.5/9
D.1/3
E.1/9

42. In a certain game of dice, the player's score is determined as a sum of three throws of a single die. The player with the highest score wins the round. If more than one player has the highest score, the winnings of the round are divided equally among these players. If Jim plays this game against 21 other players, what is the probability of the minimum score that will guarantee Jim some monetary payoff?
A. 41/50
B. 1/221
C. 1/216
D. 1/84
E. 1/42

43. Mathematics, Physics and Chemistry books are stored on a library shelf that can accommodate 25 books. Currently 20% of the shelf spots remain empty. There are twice as many mathematics books as physics books and the number of physics books is 4 greater than that of the chemistry books. Ricardo selects one book at random from the shelf, reads it in the library, and then returns it to the shelf. Then he again chooses one book at random from the shelf and checks it out in order to read at home. What is the probability that Ricardo reads one book on mathematics and one on chemistry?
A. 1/10
B. 3/25
C. 1/5
D. 1/4
E. 9/20

44. Maria bought 4 black and a certain number of red and blue pencils at 15 cents each and carried them home in one bag. After Maria came home, she took out one pencil at random to write a note for her friend and then put this pencil back into the bag. Some time later, she needed to write another note and again took out a pencil from the bag. If the probability that Maria wrote both her notes in black is 1/36, how much did she spend on all pencils?
A. \$3
B. \$3.2
C. \$3.4
D. \$3.6
E. \$3.8

45. If Jessica tosses a coin 3 times, what is the probability that she will get heads at least once?

(A) 7/8 (B) 7/9 (C) 7/10 (D) 4/5 (E) 5/7

46. Set S consists of numbers 2, 3, 6, 48 and 164. Number K is computed by multiplying one random number from set S by one of the first 10 non-negative integers, also selected at random. If $Z = 6^k$, what is the probability that 678463 is not a multiple of Z?
A. 10%
B. 25%
C. 50%
D. 90%
E. 100%

47. Two identical urns—black and white—each contain 5 blue, 5 red and 10 green balls. Every ball selected from the black urn is immediately returned to the urn, while each ball selected from the white urn is removed and placed on a table. If Jenny receives a quarter for every blue ball, a dime for every red ball and a nickel for every green ball she selects, what is the probability that she will be able to buy a 25-cent candy bar with the proceeds from drawing four balls—two from each urn?

A) 143/152
B) 143/154
C) 121/180
D) 271/965
E) 152/1000

48. According to a recent student poll, 15 out of 21 members of the finance club are interested in a career in investment banking. If two students are chosen at random, what is the probability that at least one of them is interested in investment banking?
A. 1/14
B. 4/49
C. 2/7
D. 45/49
E. 13/14

49. If 4 fair dice are thrown simultaneously, what is the probability of getting at least one pair?
(A) 1/6
(B) 5/18
(C) 1/2
(D) 2/3
(E) 13/18

50. Operation '#" is defined as adding a randomly selected two-digit multiple of 6 to a randomly selected two-digit prime number and reducing the result by half. If operation '#' is repeated 10 times, what is the probability that it will yield at least two integers?
 A) 0%
 B) 10%
 C) 20%
 D) 30%
 E) 40%

51. Number N is randomly selected from a set of consecutive integers between 50 and 69, inclusive. What is the probability that N will have the same number of factors as 89?
a) 1/2
b 1/5
c) 0
d) 1/3
e) 1/4

52. Each year three space shuttles are launched, two in June and one in October. If each shuttle is known to occur without a delay in 90% of the cases and if the current month is January, what is the probability that at least one of the launches in the next 16 months will be delayed?

(A) 12/10000 (B) 230/1000 (C) 271/1000 (D) 271/10000 (E) 21/100

53. Rowan throws 3 dice and records the product of the numbers appearing at the top of each die as the result of the attempt. What is the probability that the result of any attempt is an odd integer divisible by 25?
A. 7/216
B. 5/91
C. 13/88
D. 1/5
E. 3/8

54. A telephone number contains 10 digits, including a 3-digit area code. Bob remembers the area code and the next 5 digits of the number. He also remembers that the remaining digits are not 0, 1, 2, 5, or 7. If Bob tries to find the number by guessing the remaining digits at random, the find probability that he will be able to find the correct number in at most 2 attempts.
A. 1/625
B. 2/625
C. 4/625
D. 25/625
E. 50/625

55. If number N is randomly drawn from a set of all non-negative single-digit integers, what is the probability that $5N^3/8$ is an integer?

(A) 1/2 (B) 3/4 (C) 6/7 (D) 4/5 (E) 5/11

56. The acceptance rate at a certain business school is 15% for the first time applicants and 20% for all re-applicants. If David is applying for admission for the first time this year, what is the probability that he will have to apply no more than twice before he is accepted?
A) 20%
B) 30%
C) 32%
D) 35%
E) 40%

57. If a randomly selected positive single digit multiple of 3 is multiplied by a randomly selected prime number less than 20, what is the probability that this product will be a multiple of 45?
A. 1/32
B. 1/28
C. 1/24
D. 1/16
E. 1/14

58. If a pencil is selected at random from a desk drawer, what is the probability that this pencil is red?
   (1) There are 6 black and 4 orange pencils among the pencils in the drawer.
   (2) There are three times as many red pencils in the drawer as pencils of all other colors combined.

59. What is the probability of selecting a white ball from an urn?
   (1) There are twice as many white balls as there are balls of any other color.
   (2) There are 30 more white balls as balls of all other colors combined.

60. Jonathan would like to visit one of the 12 gyms in his area. If he selects a gym at random, what is the probability that the gym will have both a swimming pool and a squash court?
   (1) All but 2 gyms in the area have a squash court.
   (2) Each of the 9 gyms with a pool has a squash court.

61. There were initially no black marbles in a jar. Subsequently, new marbles were added to the jar. If marbles are drawn at random and selected marbles are not returned to the jar, what is the probability of selecting 2 black marbles in a row?
   (1) After the new marbles are added, 50% of all marbles are black.
   (2) Among the 10 added marbles, 8 are black.

62. What is the probability that it will rain on each of the next 3 days if the probability of raining on any single day is the same in that period?
   (1) The probability of no rain throughout the first two days is 36%.
   (2) The probability of rain on the third day is 40%.

63. If a number is drawn at random from the first 1000 positive integers, what is the probability of selecting a refined number?
   (1) Any refined number must be divisible by 22.
   (2) A refined number is any even multiple of 11.

64. Number N is randomly selected from a set of all primes between 10 and 40, inclusive. Number K is selected from a set of all multiples of 5 between 10 and 40, inclusive. What is the probability that N + K is odd?
(A) 1/2
(B) 2/3
(C) 3/4
(D) 4/7
(E) 5/8

65. What is the probability of selecting a clean number from a set of integers containing all multiples of 3 between 1 and 99, inclusive?
   (1) A clean number is an integer divisible by only 2 factors, one of which is greater than 2.
   (2) A clean number must be odd.

66. On his drive to work, Leo listens to one of three radio stations A, B, or C. He first turns to A, if A is playing a song he likes, he listens to it; if not, he turns to B. If B is playing a song he likes, he listens to it; if not, he turns to C. If C is playing a song he likes, he listens to it; if not, he turns off the radio. For each station, the probability is 0.3 that at any given moment the station is playing a song Leo likes, on his drive to work, what is the probability that Leo will hear a song he likes?
A. 0.027
B. 0.090
C. 0.417
D. 0.657
E. 0.900

67. A certain junior class has 1000 students and a certain senior class has 800 students. Among these students, there are 60 siblings pairs each consisting of 1 junior and 1 senior. If 1 student is to be selected at random from each class, what is the probability that the 2 students selected will be a sibling pair?
3/40000 1/3600 9/2000 1/60 1/15

68. Each of the 25 balls in a certain box is either red, blue, or white and has a number from 1 to 10 painted on it. If one ball is to be selected at random from the box, what is the probability that the ball selected will either be white or have an even number painted on it?
1). The probability that the ball will both be white and have an even number painted on it is 0.
2). The probability that the ball will be white minus the probability that have an eve number painted on it is 0.2

69. A certain jar contains only B black marbles, W white marbles, and R red marbles, if one marble is to be chosen at random from the jar, is the probability that the marble chosen will be red greater than the probability that marble chosen will be white?

1). r/(B+W)> w/(B+R) 2). B-W > R

70. There are eight magazines, including 4 fashion books and 4 sports books. If three books are to be selected at random without replacement, what is the probability that at least one fashion book will be selected?

½ 2/3 32/35 11/12 13/14

71. What is the probability that a number selected from (-10, -6, -5, -4, -2.5, -1, 0, 2.5, 4, 6, 7, 10) can fulfill (x-5)(x+10)(2x-5)=0?

1/12 1/6 ¼ 1/3 1/2

Answer Key
GMAT Quant Topic 8: Probability

1. B
2. E
3. B
4. C
5. C
6. C
7. B
8. C
9. C
10. B
11. D
12. D
13. C
14. C
15. D
16. E
17. D
18. D
19. C
20. E
21. B
22. A
23. D
24. B
25. D
26. D
27. B
28. E
29. C
30. D
31. D
32. D
33. A
34. D
35. B
36. E
37. D
38. A
39. A
40. A
41. A
42. C
43. B
44. D
45. A
46. D
47. A

48. E
49. E
50. A
51. B
52. C
53. A
54. E
55. A
56. C
57. C
58. B
59. A
60. B
61. C
62. D
63. B
64. D
65. A
66. D
67. A
68. E
69. A
70. E
71. B

MISCELLANEOUS QUESTIONS

Part A: Word Problems

1. A political candidate collected \$1,749 from a fund-raising dinner. If each supporter contributed at least \$50, what is the greatest possible number of contributors at the dinner?
   33 34 35 36 37

2. Joan, Kylie, Lillian, and Miriam all celebrate their birthdays today. Joan is 2 years younger than Kylie, Kylie is 3 years older than Lillian, and Miriam is one year older than Joan. Which of the following could be the combined age of all four women today?
   51 52 53 54 55

3. Janet is now 25 years younger than her mother Carol. If in 6 years Janet's age will be half Carol's age, how old was Janet 5 years ago?
   10 14 16 19 25

4. A certain company has budgeted \$1,440 for entertainment expenses for the year, divided into 12 equal monthly allocations. If by the end of the third month, the total amount spent on entertainment was \$300, how much was the company under budget or over budget?
   A. \$60 under budget
   B. \$30 under budget
   C. \$30 over budget
   D. \$60 over budget
   E. \$180 over budget

5. The ACME company manufactured x brooms per month from January to April, inclusive. On the first of each month, during the following May to December, inclusive, it sold x/2 brooms. At the beginning of production on January 1st, the ACME company had no brooms in its inventory. If storage costs were \$1 per month per broom, approximately how much, in terms of x, did the ACME company pay for storage from May 2nd to December 31st, inclusive?
   \$x \$3x \$4x \$5x \$14x

6. The number of passengers on a certain bus at any given time is given by the equation $P = -2(S - 4)^2 + 32$, where P is the number of passengers and S is the number of stops the bus has made since beginning its route. If the bus begins its route with no passengers, how many passengers will be on the bus two stops after the stop where it has its greatest number of passengers?
   32 30 24 14 0

7. John was 27 years old when he married Betty. They just celebrated their fifth wedding anniversary, and Betty's age is now 7/8 of John's. How old is Betty?
   24 26 28 30 32

8. Joe needs to paint all the airplane hangars at the airport, so he buys 360 gallons of paint to do the job. During the first week, he uses 1/4 of all the paint. During the second week, he uses 1/5 of the remaining paint. How many gallons of paint has Joe used?
   18 144 175 216 250

9. A certain movie star's salary for each film she makes consists of a fixed amount, along with a percentage of the gross revenue the film generates. In her last two roles, the star made \$32 million on a film that grossed \$100 million, and \$24 million on a film that grossed \$60 million. If the star wants to make at least \$40 million on her next film, what is the minimum amount of gross revenue the film must generate?
   \$110 million \$120 million \$130 million \$140 million \$150 million

10. As a bicycle salesperson, Norman earns a fixed salary of \$20 per week plus \$6 per bicycle for the first six bicycles he sells, \$12 per bicycle for the next six bicycles he sells, and \$18 per bicycle for every bicycle sold after the first 12. This week, Norman earned more than twice as much as he did last week. If he sold x bicycles last week and y bicycles this week, which of the following statements must be true?
    I. $y > 2x$ II. $y > x$ III. $y > 3$
    I only II only I and II II and III I, II, and III

11. A basketball team composed of 12 players scored 100 points in a particular contest. If none of the individual players scored fewer than 7 points, what is the greatest number of points that an individual player might have scored?
    7 13 16 21 23

12. Sally has a gold credit card with a certain spending limit, and a platinum card with twice the spending limit of the gold card. Currently, she has a balance on her gold card that is 1/3 of the spending limit on that card, and she has a balance on her platinum card that is 1/5 of the spending limit on that card. If Sally transfers the entire balance on her gold card to her platinum card, what portion of her limit on the platinum card will remain unspent?
11/30 29/60 17/30 19/30 11/15

13. Martina earns one-sixth of her annual income during the month of June and one-eighth in August. Pam earns one-third of her annual income in June and one-fourth in August. Martina's earnings for June and August equal Pam's earnings for the same period. What portion of their combined annual income do the two girls earn during the ten months NOT including June and August?
1/8 7/24 7/18 11/18 7/8

14. On January 1, 2076, Lake Loser contains x liters of water. By Dec 31 of that same year, 2/7 of the x liters have evaporated. This pattern continues such that by the end of each subsequent year the lake has lost 2/7 of the water that it contained at the beginning of that year. During which year will the water in the lake be reduced to less than 1/4 of the original x liters?
2077 2078 2079 2080 2081

15. 3/4 of all married couples have more than one child. 2/5 of all married couples have more than 3 children. What fraction of all married couples have 2 or 3 children?
1/5 ¼ 7/20 3/5 3/22

16. Billy has an unlimited supply of the following coins: pennies (1¢), nickels (5¢), dimes (10¢), quarters (25¢), and half-dollars (50¢). On Monday, Billy bought one candy for less than a dollar and paid for it with exactly four coins (i.e., he received no change). On Tuesday, he bought two of the same candy and again paid with exactly four coins. On Wednesday, he bought three of the candies, on Thursday four of the candies, and on Friday five of the candies; each day he was able to pay with exactly four coins. Which of the following could be the price of one candy in cents?
8¢ 13¢ 40¢ 53¢ 66¢

17. A certain violet paint contains 30 percent blue pigment and 70 percent red pigment by weight. A certain green paint contains 50 percent blue pigment and 50 percent yellow pigment. When these paints are mixed to produce a brown paint, the brown paint contains 40 percent blue pigment. If the brown paint weighs 10 grams, then the red pigment contributes how many grams of that weight?
2.8 3.5 4.2 5 7

18. The workforce of a certain company comprised exactly 10,500 employees after a four-year period during which it increased every year. During this four-year period, the ratio of the number of workers from one year to the next was always an integer. The ratio of the number of workers after the fourth year to the number of workers after the second year is 6 to 1. The ratio of the number of workers after the third year to the number of workers after the first year is 14 to 1. The ratio of the number of workers after the third year to the number of workers before the four-year period began is 70 to 1. How many employees did the company have after the first year?
50 70 250 350 750

19. A certain farm has a group of sheep, some of which are rams (males) and the rest ewes (females). The ratio of rams to ewes on the farm is 4 to 5. The sheep are divided into three pens, each of which contains the same number of sheep. If the ratio of rams to ewes in the first pen is 4 to 11, and if the ratio of rams to ewes in the second pen is the same as that of rams to ewes in the third, which of the following is the ratio of rams to ewes in the third pen?
8/7 2/3 ½ 3/12 1/6

20. The price of a bushel of corn is currently \$3.20, and the price of a peck of wheat is \$5.80. The price of corn is increasing at a constant rate of 5x cents per day while the price of wheat is decreasing at a constant rate of $(x\sqrt{2} - x)$ cents per day. What is the approximate price when a bushel of corn costs the same amount as a peck of wheat?
\$4.50 \$5.10 \$5.30 \$5.50 \$5.60

21. At a certain college, students can major in science, math, history, or linguistics. If there are 1/3 as many science majors as there are history majors, and 2/3 as many math majors as there are history majors, how many of the 2000 students major in linguistics?
(1) There are as many linguistics majors as there are math majors.
(2) There are 250 more math majors than there are science majors.

22. A green bucket and a blue bucket are each filled to capacity with several liquids, none of which combine with one another. Liquid A and liquid B each compose exactly 10% of the total liquid contained in the green bucket. Liquid C composes exactly 10% of the total liquid contained in the blue bucket. The entire contents of the green and blue buckets are poured into an empty red bucket, completely filling it with liquid (and with no liquid overflowing). What percent of the liquid now in the red bucket is not liquids A, B, or C?
(1) The total amount of liquids A, B, and C now in the red bucket is equal to 1.25 times the total amount of liquids A and B initially contained in the green bucket.
(2) The green and blue buckets did not contain any of the same liquids.

23. At a certain bookstore, each notepad costs x dollars and each markers costs y dollars. If $10 is enough to buy 5 notepads and 3 markers, is $10 enough to buy 4 notepads and 4 markers instead?
(1) each notepad cost less than $1 (2) $10 is enough to buy 11 notepads

24. If Jim earns x dollars per hour, it will take him 4 hours to earn exactly enough money to purchase a particular jacket. If Tom earns y dollars per hour, it will take him exactly 5 hours to earn enough money to purchase the same jacket. How much does the jacket cost?
(1) Tom makes 20% less per hour than Jim does. (2) x + y = $43.75

25. Bill runs a hot dog stand, and at the end of the day he has collected an assortment of $1, $5, and $10 bills. He discovers that the number of $1, $5, and $10 bills that he has is in the ratio of 10 : 5 : 1, respectively. How many $10 bills does he have?
(1) The dollar value of his $1 bills equals the dollar value of his $10 bills.
(2) Bill has a total of $225.

26. In a single row of yellow, green and red colored tiles, every red tile is preceded immediately by a yellow tile and every yellow tile is preceded immediately by a green tile. What color is the 24th tile in the row?
(1) The 18th tile in the row is not yellow.
(2) The 19th tile in the row is not green.

27. A number of apples and oranges are to be distributed evenly among a number of baskets. Each basket will contain at least one of each type of fruit. If there are 20 oranges to be distributed, what is the minimum number of apples needed so that every basket contains less than twice as many apples as oranges?
(1) If the number of baskets were halved and all other conditions remained the same, there would be twice as many oranges in every remaining basket.
(2) If the number of oranges were halved, it would no longer be possible to place an orange in every basket.

28. A store purchases 20 coats that each cost an equal amount and then sold each of the 20 coats at an equal price, what was the store's gross profit on the 20 coats?
(1) If the selling price per coat had been twice as much, the store's gross profit on the 20 coats would have been 2400
(2) If the store selling price per coat had been $2 more, the store's gross profit on the 20 coats would have been 440

29. Six countries in a certain region sent 75 representatives to an international congress, and no two countries sent the same number of representatives. Of the six countries, if country A sent the second greatest number of representatives, did country A send at least 10 representatives?
(1) One of the six countries sent 41 representatives to the congress.
(2) Country A sent fewer than 12 representatives to the congress.

30. In a certain conference room each row of chairs has the same number of chairs, and the number of rows is 1 less than the number of chairs in a row. How many chairs are in a row?
(1) There is a total of 72 chairs.
(2) After 1 chair is removed from the last row, there are a total of 17 chairs in the last 2 rows.

31. Store S sold a total of 90 copies of a certain book during the seven days of last week, and it sold different numbers of copies on any two of the days. If for the seven days Store S sold the greatest number of copies on Saturday and the second greatest number of the copies on Friday, did Store S sell more than 11 copies on Friday?
(1) Last week store S sold 8 copies of the book on Thursday.
(2) Last week store S sold 38 copies of the book on Saturday.

32. Each person attending a fund-raising party for a certain club was charged the same admission fee, how many people attended the party?
(1) If the admission fee had been $0.75 less and 100 more people had attended, the club would have received the same amount in admission fees.
(2) If the admission fee had been $1.50 more and 100 fewer people had attended, the club would have received the same amount in admission fees.

33. If Bob produces 36 or fewer in a week, he is paid X dollars per item. If Bob produces more than 36 items, he is paid X dollars per item for the first 36 items, and 3/2 times that amount for each additional item. How many items did Bob produce last week?
(1) Last week Bob was paid total of $480 for the items that he produced that week.
(2) This week produced 2 items more than last week and was paid a total of $510 for the item that he produced this week.

34. Did one of three members of a certain team sell at least 2 raffle tickets yesterday?
(1) The three members sold a total of 6 raffle tickets yesterday.
(2) No two of the three members sold same number of raffle tickets yesterday.

35. One kilogram of a certain coffee blend consists of X kilogram of type I and Y kilogram of type II. The cost of the blend is C dollars per kilogram, where $C=6.5X + 8.5Y$. Is $X < 0.8$?
(1) $Y > 0.15$ (2) $C \geq 7.30$

36. Marta bought several pencils. If each pencil was either a 23-cent pencil or a 21-cent pencil, how many 23- cent pencils did Marta buy?
(1) Marta bought a total of 6 pencils.
(2) The total value of the pencils Marta bought was 130 cents.

37. Juan bought some paperback books that cost $8 each and hardcover books that $25 each. If Juan bought more than 10 paperback books, how many hardcover books did he buy?
(1) The total cost of hardcover books that Juan bought was at least $150.
(2) The total cost of all books that Juan bought was less than $260.

38. For Manufacturer M, the cost C of producing X Units of its product per month is given by $c=kx+t$, where c is in dollars and k and t are constants. Last month if Manufacturer M produced 1,000 units of its product and sold all the units for k+60 dollars each, what was Manufacturer M's gross profit on the 1,000 units?
(1) Last month, Manufacturer M's revenue from the sale of the 1,000 units was 150,000.
(2) Manufacturer M's cost of producing 500 Units in a month is 45,000 less than its cost of producing 1,000 units in a month.

39. A computer chip manufacturer expects the ratio of the number of defective chips to the total number of chips in all future shipments to equal the corresponding ratio for shipments S1, S2, S3 and S4 combined, as shown in the following table. What is the expected number of defective chips in a shipment of 60,000 chips?

Shipment	Number of defective chips in the shipment	Total number of chips in the shipment
S1	2	5,000
S2	5	12,000
S3	6	18,000
S4	4	16,000

14 20 22 24 25

40. A certain library assesses fines for overdue books as follows. On the first day that a book is overdue, the total fine is $0.10. For each additional day that the book is overdue the total fine is either increased by $0.30 or double, whichever results in the lesser amount. What is the total fine for a book on the fourth day it is overdue?
$0.60 $0.70 $0.80 $0.90 $1.00

41. When a certain tree was first planted, it was 4 feet tall, and the height of the tree increased by a constant amount each year for the next 6 years. At the end of the 6th year, the tree was 1/5 taller than it was at the end of 4th year. By how many feet did the height of the tree increase each year?
3/10 2/5 ½ 2/3 6/5

42. To celebrate a colleague's retirement, the T coworkers in an office agreed to share equally the cost of a catered lunch. If the lunch cost a total of x dollars and S of the coworkers fail to pay their share, which of the following represents the additional amount, in dollars, that each of the remaining coworkers would have to contribute so that the cost of the lunch is completely paid?
A. x/T
B. x/(T-S)
C. Sx/(T-S)
D. Sx/T(T-S)
E. x(T-S)/T

43. A certain business company produced x rakes each month from November through February and shipped x/2 at the beginning of each month from March through October. The business paid no storage cost for the rakes from November through February, but it paid storage costs of $0.10 per rake each month from March through October for the rakes had not been shipped. In terms of x, what was the total storage cost, in dollars, that the business will paid for the rakes for the 12 months from November through October?
A. 0.40x
B. 1.20x
C. 1.40x
D. 1.60x
E. 3.20x

44. A certain company plans to sell Product X for p dollars per unit, where p is randomly chosen from all possible positive values not greater than 100. The monthly manufacturing cost for Product X (in thousands of dollars) is 12 - p, and the projected monthly revenue from Product X (in thousands of dollars) is p(6 - p). If the projected revenue is realized, what is the probability that the company will NOT see a profit on sales of Product X in the first month of sales?
(A) 0 (B) 1/100 (C) 1/25 (D) 99/100 (E) 1

45. Every day a certain bank calculates its average daily deposit for that calendar month up to and including that day. If on a randomly chosen day in June the sum of all deposits up to and including that day is a prime integer greater than 100, what is the probability that the average daily deposit up to and including that day contains fewer than 5 decimal places?
(A) 1/10 (B) 2/15 (C) 4/15 (D) 3/10 (E) 11/30

46. Three completely unmarked containers are used for measuring water. Water may be poured from one container to another, but no water may be poured outside the containers. Using nothing but the three containers and an unlimited supply of water, is it possible to measure exactly 4 gallons of water?
(1) The capacity of the first container is 2 gallons more than the capacity of the second container.
(2) The capacity of the second container is 2 gallons more than the capacity of the third container.

47. A certain cube is composed of 1000 smaller cubes, arranged 10 by 10 by 10. The top layer of cubes is removed from a face, then from the adjacent face above it, then from the adjacent face to the right of the first. The process is repeated on the same three faces in reverse order. Finally, a last layer is taken from the first face. How many smaller cubes have been removed from the larger cube?
(A) 488 (B) 552 (C) 612 (D) 722 (E) 900

48. Chandra and Ken are waiting in line for concert tickets. If each person takes up 2 feet of space in the line, how long is the line?
1) There are three people in front of Chandra and three people behind Ken
2) Two people are standing between Chandra and Ken

49. Nina and Teri are playing a dice game. Each girl rolls a pair of 12-sided dice, numbered with the integers from -6 through 5, and receives a score that is equal to the negative of the sum of the two die. (E.g., If Nina rolls a 3 and a 1, her sum is 4, and her score is -4.) If the player who gets the highest score wins, who won the game?
(1) The value of the first die Nina rolls is greater than the sum of both Teri's rolls.
(2) The value of the second die Nina rolls is greater than the sum of both Teri's rolls.

50. In the game Cako, a player is awarded one tick for every third Alb captured, and one click for every fourth Berk captured. The total score is equal to the product of clicks and ticks. If a player has a score of 77, how many Albs did he capture?
(1) The difference between Albs captured and Berks captured is 7.
(2) The number of Albs captured is divisible by 4.

51. The vertical position of an object can be approximated at any given time by the function: $p(t) = rt - 5t^2 + b$, where p(t) is the vertical position in meters, t is the time in seconds, and r and b are constants. After 2 seconds, the position of an object is 41 meters, and after 5 seconds the position is 26 meters. What is the position of the object, in meters, after 4 seconds?
(A) 24 (B) 26 (C) 39 (D) 41 (E) 45

52. A Trussian's weight, in keils, can be calculated by taking the square root of his age in years. A Trussian teenager now weighs three keils less than he will seventeen years after he is twice as old as he is now. How old is he now?
(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

53. There are x high-level officials (where x is a positive integer). Each high-level official supervises x^2 mid-level officials, each of whom, in turn, supervises x^3 low-level officials. How many high-level officials are there?
(1) There are fewer than 60 low-level officials.
(2) No official is supervised by more than one person.

54. Jim went to the bakery to buy donuts for his office mates. He chose a quantity of similar donuts, for which he was charged a total of $15. As the donuts were being boxed, Jim noticed that a few of them were slightly ragged-looking so he complained to the clerk. The clerk immediately apologized and then gave Jim 3 extra donuts for free to make up for the damaged goods. As Jim left the shop, he realized that due to the addition of the 3 free donuts, the effective price of the donuts was reduced by $2 per dozen. How many donuts did Jim receive in the end?
(A) 18 (B) 21 (C) 24 (D) 28 (E) 33

55. Bobby and his younger brother Johnny have the same birthday. Johnny's age now is the same as Bobby's age was when Johnny was half as old as Bobby is now. What is Bobby's age now?
(1) Bobby is currently four times as old as he was when Johnny was born.
(2) Bobby was six years old when Johnny was born.

56. A certain clothing manufacturer makes only two types of men's blazer: cashmere and mohair. Each cashmere blazer requires 4 hours of cutting and 6 hours of sewing. Each mohair blazer requires 4 hours of cutting and 2 hours of sewing. The profit on each cashmere blazer is $40 and the profit on each mohair blazer is $35. How many of each type of blazer should the manufacturer produce each week in order to maximize its potential weekly profit on blazers?
1) The company can afford a maximum of 200 hours of cutting per week and 200 hours of sewing per week.
2) The wholesale price of cashmere cloth is twice that of mohair cloth.

57. Roberto has three children: two girls and a boy. All were born on the same date in different years. The sum of the ages of the two girls today is smaller than the age of the boy today, but a year from now the sum of the ages of the girls will equal the age of the boy. Three years from today, the difference between the age of the boy and the combined ages of the girls will be
A) 1 B) 2 C) 3 D) –2 E) –1

58. x years ago, Cory was one fifth as old as Tania. In x years, Tania will be twice as old as Cory. What is the ratio of Cory's current age to Tania's current age?
(A) 7:23 (B) 9:17 (C) 5:13 (D) 3:7 (E) 11:15

59. Ten years ago, scientists predicted that the animal z would become extinct in t years. What is t?
(1) Animal z became extinct 4 years ago.
(2) If the scientists had extended their extinction prediction for animal z by 3 years, their prediction would have been incorrect by 2 years.

60. The longevity of a certain metal construction is determined by the following formula: $l = (7.5 - x)^4 + 8.97^c$, where l is the longevity of the construction, in years, x is the density of the underlying material, in g/cm^3, and c is a positive constant equal to 1.05 for this type of metal constructions. For what value of density, x, expressed in g/cm^3, will the metal construction have minimal longevity?
-7.5 0 7.5 15 75

Part B: Calculations, Exponents, Basic Algebra

1. $\sqrt{\sqrt{96}+\dfrac{2}{5+2\sqrt{6}}} = ?$ lies between:

1 & 2 2 & 3 3 & 4 4 & 5 5 & 6

2. List the following in increasing order from left to right: $\sqrt[3]{2}, \sqrt[6]{5}, \sqrt[10]{10}, \sqrt[15]{30}$?

3. $\sqrt{24+5\sqrt{23}} + \sqrt{24-5\sqrt{23}}$ lies between:

4 & 5 5 & 6 6 & 7 7 & 8 8 & 9

4. $8^a(1/4)^b = ?$ (1) $b = 1.5a$ (2) $a = 2$

5. A, B, C, D, E, F, G, and H are all integers, listed in order of increasing size. When these numbers are arranged on a number line, the distance between any two consecutive numbers is constant. If G and H are equal to 5^{12} and 5^{13}, respectively, what is the value of A?

$-24(5^{12})$ $-23(5^{12})$ $-24(5^6)$ $23(5^{12})$ $24(5^{12})$

6. $(3^{5x} + 3^{5x} + 3^{5x})(4^{5x} + 4^{5x} + 4^{5x} + 4^{5x}) =$

12^{5x+1} $3^{15x} + 4^{20x}$ 25^{5x} 7^{35x} 25^{5x+1}

7. The three-digit positive integer x has the hundreds, tens, and units digits of a, b, and c, respectively. The three-digit positive integer y has the hundreds, tens, and units digits of k, l, and m, respectively. If $(2^a)(3^b)(5^c) = 12(2^k)(3^l)(5^m)$, what is the value of $x - y$?

21 200 210 300 310

8. Is $x > 10^{10}$? (1) $x > 2^{34}$ (2) $x = 2^{35}$

9. $\sqrt{3\sqrt{80}+\dfrac{3}{9+4\sqrt{5}}} = ?$

$2\sqrt{3\sqrt{5}}$ 3 $3\sqrt{3}$ $9+4\sqrt{5}$ $3+2\sqrt{5}$

10. What is the value of 2^a4^b?
(1) $a = -2b$ (2) $b = 4$

11. If $27^{4x+2} \times 162^{-2x} \times 36^x \times 9^{6-2x} = 1$, then what is the value of x?

-9 -6 3 6 9

12. If $(2^{2x+1})(3^{2y-1}) = 8^x27^y$, then $x + y =$

-3 -1 0 1 3

13. If $(6^2)(44)(5^x)(20) / (8^2)(9) = 1375$, what is the value of x?

-1 0 1 2 3

14. If $5^x = y$, what is x? (1) $y^2 = 625$ (2) $y^3 = 15{,}625$

15. Wendy, Jim, and Pedro are golfing. Collectively, they have 24 golf balls. How many golf balls does Jim have?
(1) Jim has 1/3 of the number of golf balls that Wendy has.
(2) Pedro has 1/2 of the total number of golf balls.

16. If x, y, and z are integers greater than 1, and $(3^{27})(35^{10})(z) = (5^8)(7^{10})(9^{14})(x^y)$, then what is the value of x?
(1) z is prime (2) x is prime

17. If $4^{4x} = 1600$, what is the value of $(4^{x-1})^2$? 40

20 10 5/2 5/4

18. If x and y are integers and $(15^x + 15^{x+1}) / 4^y = 15^y$, what is the value of x?

2 3 4 5 Cannot be determined

19. If $3^m3^m3^m = 9^n$, then m/n =

1/3 2/3 1 3/2 3

20. If $a = 3^{b-1}$, what is the value of a + b? (1) $3^{b+2} = 243$ (2) $a = 3^{2b-4}$

21. What is the value of $\left(\sqrt{7+\sqrt{29}} - \sqrt{7-\sqrt{29}}\right)^2$?

-26 2√29 14 – 4√5 14 14 + 4√5

22. If $x^2/9 - 4/y^2 = 12$, what is the value of x?

(1) $x/3 + 2/y = 6$ (2) $x/3 - 2/y = 2$

23. What is the value of $(a + b)^2$? (1) a = 15/b (2) $(a - b)^2 = 4$

24. If x and y are positive and $x^2y^2 = 18 - 3xy$, then x^2 =

$18 - 3y / y^3$ $18 / y^2$ $18 / y^2 + 3y$ $9 / y^2$ $36 / y^2$

25. If y = √(3y + 4), then the product of all possible solutions for y is

-4 -2 0 3 6

26. If the sum of the cubes of a and b is 8 and $a^6 - b^6 = 14$, what is the value of $a^3 - b^3$?

¼ ½ 5/4 7/4 2

27. If x does not equal y, and xy does not equal 0, then when x is replaced by 1/x and y is replaced by 1/y everywhere in the expression (x + y) / (x – y), the resulting expression is equivalent to

- (x + y) / (x – y) (x - y) / (x + y) (x + y) / (x – y) (x + y) (x – y)

28. If x and y are non-zero integers, and $9x^4 - 4y^4 = 3x^2 + 2y^2$, which of the following could be the value of x^2 in terms of y?

$-4y^2/3$ $-2y^2$ $(2y^2+1)/3$ $2y^2$ $6y^2/3$

29. What is the ratio of r to s? (1) r + s = 7 (2) $r^2 - s^2 = 7$

30. If there are x men and y women in a choir, and there are z more men than there are women in that choir, what is z?

(1) $x^2 - 2xy + y^2 - 9 = 0$ (2) $x^2 + 2xy + y^2 - 225 = 0$

31. The value of x is one quarter of z. The sum of x, y, and z is equal to 26. If the value of y is twice the value of z, what is the largest factor of the sum of y and z?

2 3 8 12 24

32. If 2 + 5a – b/2 = 3c, what is the value of b?

(1) a + c = 13 (2) -12c = -20a + 4

33. If xy does not equal zero, what is the value of xy? (1) 2/x + 2/y = 3 (2) $x^3 - (2/y)^3 = 0$

34. The expression 3 / (2 + √3) is equal to:

6 + 3√3 6 - 3√3 (6 + 3√3) / 7 (6 - 3√3) / 7 1.5 + √3

35. If $(1/5)^m \times (1/4)^{18} = 1/(2 \times (10)^{35})$, then m=?

A. 17
B. 18
C. 34
D. 35
E. 36

36. If $5^{21} \times 4^{11} = 2 \times 10^n$, what is the value of n?

11 21 22 23 32

37. Which of the following best approximates the value of q if $5^{28}+3^{11}=5^q$?

39 30 28 27 17

38. What is the value of $(2^x + 2^x) / 2^y$? (1) x – y = 8 (2) x/y = -3

39. If a number N is decreased by p percent and then the resulting value is increased by q percent, the final result is equal to N. If both p and q are positive integers, what is the value of p?
(1) p is not a multiple of 10.
(2) q is not a multiple of 10.

40. Is Y greater than 7/11? (1) $1/5 < Y < 11/12$ (2) $2/9 < Y < 8/13$

41. If $(\sqrt{x} + \sqrt{y}) / (x - y) = (2\sqrt{x} + 2\sqrt{y}) / [x + 2\sqrt{(xy)} + y]$, what is the ratio of x to y?
½ 2 4 7 9

42. Is pq = 1? (1) pqp = p (2) qpq = q

43. $(16x^4 - 81y^4) / (2x + 3y) = 12x^2 + 27y^2$ and $4x + 3y = 9$, what is x?
1 1.5 2 3 4

44. If $f(x) = ax^4 - 4x^2 + ax - 3$, then $f(b) - f(-b)$ will equal:
0 2ab $2ab^4 - 8b^2 - 6$ $-2ab^4 + 8b^2 + 6$ $2ab^4 - 8b^2 + 2ab - 6$

45. If $p\&q = p^2 + q^2 - 2pq$, for what value of q is p&q equal to p^2 for all values of p?
-2 -1 0 1 2

46. If t and u are positive integers, what is the value of $t^{-2} * u^{-3}$?
(1) $t^{-3u} * u^{-2} = 1/36$ (2) $t * u^{-1} = 1/6$

47. If a and b are different values and $a - b = \sqrt{a} - \sqrt{b}$, then in terms of b, a equals:
$\sqrt{b}$ b $b - 2\sqrt{b} + 1$ $b + 2\sqrt{b} + 1$ $b^2 - 2b\sqrt{b} + b$

48. What is the value of (a! + b!)(c! + d!)?
(1) b!d! = 4(a!d!)
(2) 60(b!c!) = (b!d!)

49. If $f(x) = 125/x^3$, what is the value of $f(5x) / f(x/5)$ in terms of f(x)?
(A) $(f(x))^2$ (B) $f(x^2)$ (C) $(f(x))^3$ (D) $f(x^3)$ (E) f(125x)

50. If $[3(ab)^3 + 9(ab)^2 - 54ab] / [(a-1)(a + 2)] = 0$, and a and b are both non-zero integers, which of the following could be the value of b?
I. 2 II. 3 III. 4
(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

51. If the reciprocals of two consecutive integers are added to one another, what is the sum in terms of the greater integer x?
A. x/3
B. x^2 - x
C. 2x - 1
D. (2x - 1)/(x^2 + x)
E. (2x - 1)/(x^2 - x)

52. What is the value of $y + x^3 + x$? (1) $y = x (x - 3) (x + 3)$ (2) $y = -5x$

53. If $3x - 2y - z = 32 + z$ and $\sqrt{(3x)} - \sqrt{(2y + 2z)} = 4$, what is the value of x + y + z? (A) 3
(B) 9 (C) 10 (D) 12 (E) 14

54. If z is not equal to zero, and $z = \sqrt{6zs - 9s^2}$, then z equals:
s 3s 4s -3s -4s

55. If $3^k + 3^k = (3^9)^{3^9} - 3^k$, then k = ?

(A) 11/3
(B) 11/2
(C) 242
(D) 3^10
(E) 3^11 – 1

56.

If $a^{\frac{2}{3}} - b^{\frac{2}{3}} = 12$, then $\sqrt[3]{a} + \sqrt[3]{b}$ = ?

(1) $\sqrt[3]{a} = \sqrt[3]{b} + 2$ (2) $a = 64$

57. If a, b, x and y are positive integers, what is the value of a - b?

(1) $x^a = x^b + x^b + x^b$ (2) $y^a = y^b + y^b + y^b + y^b$

58. If $(a+b)^x = a^x + y(a^{x-1}b^{x-4}) + z(a^{x-2}b^{x-3}) + z(a^{x-3}b^{x-2}) + y(a^{x-4}b^{x-1}) + b^x$, what is the value of yz?
(A) 24 (B) 30 (C) 36 (D) 42 (E) 50

59. x and y are positive integers. If $5^x - 5^y = (2^{y-1})(5^{x-1})$, what is the value of xy?
(A) 48 (B) 36 (C) 24 (D) 18 (E) 12

60. For a three-digit number ***xyz***, where x, y, and z are the digits of the number, $f(xyz) = 5^x 2^y 3^z$. If $f(abc) = 3 * f(def)$, what is the value of $abc - def$?
(A) 1 (B) 2 (C) 3 (D) 9 (E) 27

61. If x, y, and z are integers and $2^x 5^y z = 0.00064$, what is the value of xy?
(1) z = 20 (2) x = –1

62. If x is a non-zero integer, what is the value of x^y?
(1) x = 2 (2) $(128^x)(6^{x+y}) = (48^{2x})(3^{-x})$

63. If n is an integer and f(n) = f(n – 1) – n, what is the value of f (4)?
(1) f(3) = 14 (2) f(6) = -1

64. If #p# = ap^3+ bp – 1 where a and b are constants, and #-7# = 3, what is the value of #7#?
5 0 -2 -3 -5

65. Let f(x) = x^2 + bx + c. If f(6) = 0 and f(-3) = 0, then b + c =
18 15 -15 -21 -24

66. If $\sqrt{4 + x^{1/2}} = \sqrt{x+2}$, then x could be equal to which of the following?
-1 0 1 4 cannot be determined.

67. If 6xy = x^2y + 9y, what is the value of xy?
(1) y – x = 3 (2) $x^3 < 0$

Answer Key
MISCELLANEOUS QUESTIONS

Part A: Word Problems

1. B
2. D
3. B
4. A
5. E
6. C
7. C
8. B
9. D
10. D
11. E
12. D
13. D
14. D
15. C
16. C
17. B
18. C
19. A
20. E
21. D
22. C
23. E
24. B
25. B
26. E
27. B
28. B
29. E
30. D
31. B
32. C
33. E
34. D
35. B
36. B
37. C
38. E
39. B
40. B
41. D
42. D
43. C
44. D
45. D
46. C

47. B
48. E
49. E
50. A
51. D
52. C
53. C
54. A
55. B
56. A
57. D
58. C
59. E
60. C

Part B: Calculations, Exponents, Basic Algebra

1. C
2. $\sqrt[15]{30}$ $\sqrt[10]{10}$ $\sqrt[3]{2}$ $\sqrt[6]{5}$
3. D
4. A
5. B
6. A
7. C
8. D
9. C
10. A
11. A
12. C
13. D
14. D
15. C
16. D
17. D
18. A
19. B
20. D
21. C
22. D
23. C
24. D
25. D
26. D
27. A
28. C
29. C
30. A
31. E
32. B
33. B
34. B
35. D

36. B
37. C
38. A
39. C
40. B
41. E
42. E
43. C
44. B
45. C
46. A
47. C
48. E
49. A
50. A
51. E
52. C
53. E
54. B
55. E
56. A
57. A
58. E
59. E
60. A
61. A
62. B
63. D
64. E
65. D
66. D
67. B

SOLUTIONS

GMAT Quant Topic 1: General Arithmetic
Part A: Overlapping SETS

1.
For an overlapping sets problem it is best to use a double set matrix to organize the information and solve. Fill in the information in the order in which it is given.
Of the films Empty Set Studios released last year, 60% were comedies and the rest were horror films.

	Comedies	**Horror Films**	**Total**
Profitable			
Unprofitable			
Total	**0.6x**	**0.4x**	**x**

75% of the comedies were profitable, but 75% of the horror moves were unprofitable.

	Comedies	**Horror Films**	**Total**
Profitable	**0.75(0.6x)**		
Unprofitable		**0.75(0.4x)**	
Total	0.6x	0.4x	x

If the studio made a total of 40 films...

	Comedies	**Horror Films**	**Total**
Profitable	0.75(24) =**18**		
Unprofitable		0.75(16)= **12**	
Total	0.6(40) =**24**	0.4(40) =**16**	**x = 40**

Since each row and each column must sum up to the Total value, we can fill in the remaining boxes

	Comedies	**Horror Films**	**Total**
Profitable	18	**4**	**22**
Unprofitable	**6**	12	**18**
Total	24	16	40

The problem seeks the total number of profitable films, which is 22.

The correct answer is E.

2.
For an overlapping-sets problem we can use a double-set matrix to organize our information and solve. Because the values are in percents, we can assign a value of 100 for the total number of interns at the hospital. Then, carefully fill in the matrix based on the information provided in the problem. The matrix below details this information. Notice that the variable *x* is used to detail the number of interns who receive 6 or more hours of sleep, 70% of whom reported no feelings of tiredness.

	Tired	Not Tired	TOTAL
6 or more hours	$.3x$	$.7x$	x
Fewer than 6 hours	75		80
TOTAL			100

In a double-set matrix, the sum of the first two rows equals the third and the sum of the first two columns equals the third. Thus, the boldfaced entries below were derived using the above matrix.

	Tired	Not Tired	TOTAL
6 or more hours	**6**	**14**	**20**
Fewer than 6 hours	75	**5**	80
TOTAL	**81**	**19**	100

We were asked to find the percentage of interns who reported no feelings of tiredness, or 19% of the interns.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

Have a look at the figure below.

	tired	Not tired
Less than 6 hours	3x/4	z
6 or more than 6 hours	3y/10	7y/10

Let the total number of interns be x.

Number of interns who receive fewer than 6 hours of sleep and report feeling tired = 75% of x = 3x/4

Let the number of interns who receive 6 or more hours of sleep be y.

Number of interns who receive 6 or more hours of sleep and report no tiredness = 70% of y = 7y/10
Number of interns who receive 6 or more hours of sleep and report tiredness = 30% of y = 3y/10

Let the number of people who receive less than 6 hours of sleep and do not report tiredness = z [Look at the figure]

Interns who receive fewer than 6 hours of sleep = 8x/10

From the figure, we can see that:

$3x/4 + z = 8x/10$

$z = x/20$

Also,

$3x/4 + z + 3y/10 + 7y/10 = x$ [Total number of interns = x]

$8x/10 + y = x$

$y = x/5$

How many interns do not feel tired after their shifts?
$z + 7y/10 = x/20 + 7x/50 = 19x/100$

Percent interns = (19x/100)/x * 100 = 19%
The correct answer is C.

3.

This is an overlapping sets problem concerning two groups (students in either band or orchestra) and the overlap between them (students in both band and orchestra). If the problem gave information about the students only in terms of percents, then a smart number to use for the total number of students would be 100. However, this problem gives an actual number of students ("there are 119 students in the band") in addition to the percentages given. Therefore, we cannot assume that the total number of students is 100.

Instead, first do the problem in terms of percents. There are three types of students: those in band, those in orchestra, and those in both. 80% of the students are in only one group. Thus, 20% of the students are in both groups. 50% of the students are in the band only. We can use those two figures to determine the percentage of students left over: 100% - 20% - 50% = 30% of the students are in the orchestra only.

Great - so 30% of the students are in the orchestra only. But although 30 is an answer choice, watch out! The question doesn't ask for the percentage of students in the orchestra only, it asks for the number of students in the orchestra only. We must figure out how many students are in Music High School altogether.

The question tells us that 119 students are in the band. We know that 70% of the students are in the band: 50% in band only, plus 20% in both band and orchestra. If we let x be the total number of students, then 119 students are 70% of x, or 119 = .7x. Therefore, x = 119 / .7 = 170 students total.

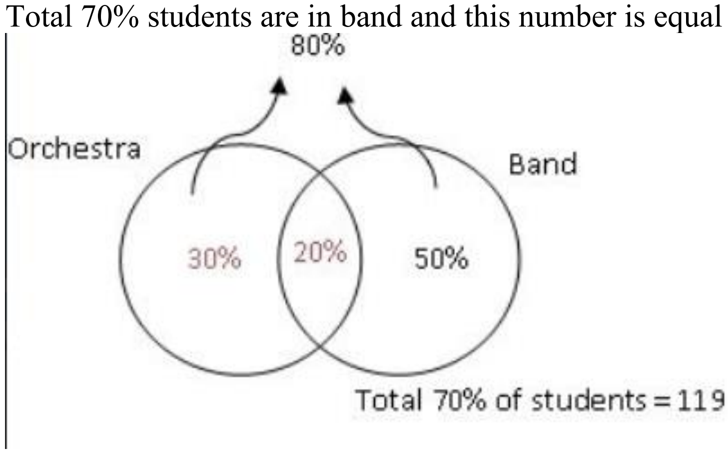
The number of students in the orchestra only is 30% of 170, or .3 × 170 = 51.

The correct answer is B.

Top 1% expert replies to student queries (can skip)
The question says 80% students are in only one group:
Does that mean band only + orchestra only = 80%? YES!
The table would look like this:

	Orchestra	No Orchestra	Total
Band	0.2x	**0.5x**	119
No Band	**0.8x – 0.5x = 0.3x**	0	0.3x
Total	0.5x	0.5x	**x**

We can also use Venn- diagram:
Look at the diagram below:
If a total of 80% students are in one group, and 50% are in band only then 30% must be in orchestra only. Remaining 20% must be in both.
Total 70% students are in band and this number is equal to 119.



70% of total students = 119 so total students = 170
No of students in orchestra only = 30% of 170 = 51
The correct answer is B.

4.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. Let's call P the number of people at the convention. The **boldface** entries in the matrix below were given in the question. For example, we are told that one sixth of the attendees are female students, so we put a value of $P/6$ in the female students cell.

	FEMALE	NOT FEMALE	TOTALS
STUDENTS	***P*/6**	$P/6$	***P*/3**
NON STUDENTS	$P/2$	**150**	$2P/3$
TOTALS	**2*P*/3**	$P/3$	***P***

The non-boldfaced entries can be derived using simple equations that involve the numbers in one of the "total" cells. Let's look at the "Female" column as an example. Since we know the number of female students ($P/6$) and we know the total number of females ($2P/3$), we can set up an equation to find the value of female non-students:

$P/6 + \textit{Female Non Students} = 2P/3$.
Solving this equation yields: $\textit{Female Non Students} = 2P/3 - P/6 = P/2$.

By solving the equation derived from the "NOT FEMALE" column, we can determine a value for P.
$P/6+150 = P/3$
$P + 900 = 2P$
$P=900$

The correct answer is E.

5.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. Because the values here are percents, we can assign a value of 100 to the total number of lights at Hotel California. The information given to us in the question is shown in the matrix in boldface. An x was assigned to the lights that were "Supposed To Be Off" since the values given in the problem reference that amount. The other values were filled in using the fact that in a double-set matrix the sum of the first two rows equals the third and the sum of the first two columns equals the third.

	Supposed To Be On	Supposed To Be Off	TOTAL
Actually on		**0.4*x***	**80**
Actually off	**0.1(100 –*x*)**	$0.6x$	20
TOTAL	$100 - x$	***x***	**100**

Using the relationships inherent in the matrix, we see that:
$0.1(100 - x) + 0.6x = 20$
$10 - 0.1x + 0.6x = 20$
$0.5x = 10$ so $x = 20$

We can now fill in the matrix with values:

	Supposed To Be On	Supposed To Be Off	TOTAL
Actually on	72	8	80
Actually off	8	12	20
TOTAL	80	20	100

Of the 80 lights that are actually on, 8, or 10% percent, are supposed to be off.

The correct answer is D.

Alternate sol from gmatclub (additional)

You can do it using algebra to get an equation with a single variable:

Say, total 100 lights. 80 are ON.

Say L are supposed to be on and 100-L are supposed to be off.

Lights that are on = 40% of (100 - L) + 90% of L = 80
L = 80 = Number of lights supposed to be on.
20 = Number of lights supposed to be off. 40% of these are on so should be switched off. 40% of 20 = 8

Of the lights that are on, 8/80 = 10% should be switched off.

Answer (D)

6.

This question involves overlapping sets so we can employ a double-set matrix to help us. The two sets are speckled/rainbow and male/female. We can fill in 645 for the total number of total speckled trout based on the first sentence. Also, we can assign a variable, x, for female speckled trout and the expression $2x + 45$ for male speckled trout, also based on the first sentence.

	Male	Female	Total
Speckled	$2x + 45$	x	645
Rainbow			
Total			

We can solve for x with the following equation: $3x + 45 = 645$. Therefore, $x = 200$.

	Male	Female	Total
Speckled	445	200	645
Rainbow			
Total			

If the ratio of female speckled trout to male rainbow trout is 4:3, then there must be 150 male rainbow trout. We can easily solve for this with the below proportion where *y* represents male rainbow trout:
$4/3 = 200/y$
Therefore, $y = 150$. Also, if the ratio of male rainbow trout to all trout is 3:20, then there must be 1000 total trout using the below proportion, where *z* represents all trout:
$3/20 = 150/z$

	Male	Female	Total
Speckled	445	200	645
Rainbow	150		
Total			1000

Now we can just fill in the empty boxes to get the number of female rainbow trout.

	Male	Female	Total
Speckled	445	200	645
Rainbow	150	205	355
Total			1000

The correct answer is D.

Alternate sol from gmatclub (additional)

{All} = {speckled} + {rainbow}

The ratio of male rainbow to all is 3x:20x:
20x = {speckled} + {female rainbow} + 3x

Since given that there are 645 speckled, then:
20x = 645 + {female rainbow} + 3x
{female rainbow} + 645 = 17x = {multiple of 17}

So, we have that {correct answer} + 645 = {multiple of 17}. Only option D works.

Answer: D.

7.

Begin by constructing a double-set matrix and filling in the information given in the problem. Assume there are 100 major airline companies in total since this is an easy number to work with when dealing with percent problems.

	Wireless	No Wireless	TOTAL
Snacks	**MAX**		70
NO Snacks			30
TOTAL	30	70	100

Notice that we are trying to maximize the cell where *wireless* intersects with *snacks*. What is the maximum possible value we could put in this cell. Since the total of the snacks row is 70 and the total of the wireless column is 30, it is clear that 30 is the limiting number. The maximum value we can put in the wireless-snacks cell is therefore 30. We can put 30 in this cell and then complete the rest of the matrix to ensure that all the sums will work correctly.

	Wireless	No Wireless	TOTAL
Snacks	**30**	40	70
NO Snacks	0	30	30
TOTAL	30	70	100

The correct answer is B.

8.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. Because the given values are all percentages, we can assign a value of 100 to the total number of people in country Z. The matrix is filled out below based on the information provided in the question.

The first sentence tells us that 10% *of all of the people* do have their job of choice but do not have a diploma, so we can enter a 10 into the relevant box, below. The second sentence tells us that 25% *of those who do not have their job of choice* have a diploma. We don't know how many people do not have their job of choice, so we enter a variable (in this case, *x*) into that box. Now we can enter 25% of those people, or 0.25x, into the relevant box, below. Finally, we're told that 40% of all of the people have their job of choice.

	University Diploma	NO University Diploma	TOTAL
Job of Choice		10	40
NOT Job of Choice	0.25x		x
TOTAL			100

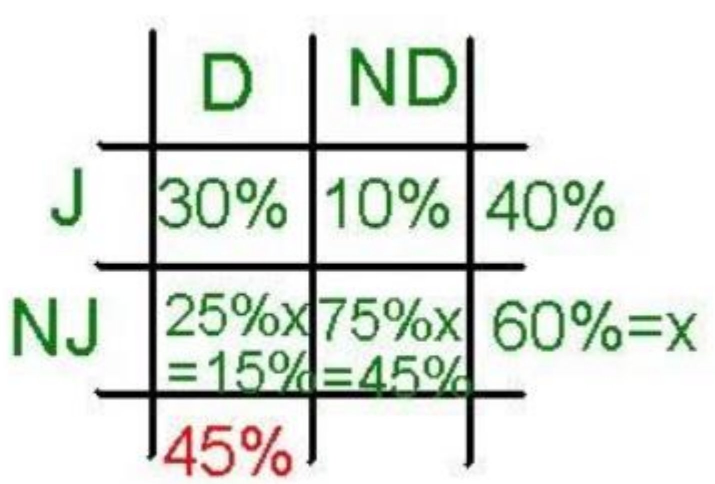
In a double-set matrix, the sum of the first two rows equals the third and the sum of the first two columns equals the third. Thus, the boldfaced entries below were derived using relationships (for example: $40 + x = 100$, therefore $x = 60$. $0.25 \times 60 = 15$. And so on.).

	University Diploma	NO University Diploma	TOTA L
Job of Choice	**30**	10	40
NOT Job of Choice	**15**	**45**	**60**
TOTAL	***45***	**55**	100

We were asked to find the percent of the people who have a university diploma, or 45%.

The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)



Step 1: No Diploma (ND) and Job(J) = 10% of People
Step 2: (Pay attention to the wordings) People who don't have a job of their choice = x,
So, 25% of x have a Diploma (D).
So, 75%of x will not have a Diploma. (NJ and ND) (2nd Row)

Step 3: 40% of People have a Job of Choice(J) = 40% of People
So, 60% of People don't have a Job of Choice (NJ) = 60% of People
So, x = 60% of People
So, NJ and ND = 75% of (x) = 75% of (60% of People) = 45% of People
So, NJ and D = 15% of People (2nd Row)

Given, ND and J = 10% of People, So D and J = 30% (1st Row)

So, Diploma / Total = 45% of People/100%of People = 45%

The correct answer is B.

9.

This is a problem that involves two overlapping sets so it can be solved using a double-set matrix. The problem tells us that there are 800 total students of whom 70% or 560 are male. This means that 240 are female and we can begin filling in the matrix as follows:

	Male	Female	TOTAL
Sport			
No Sport			*maximize*
TOTAL	560	240	800

The question asks us to MAXIMIZE the total number of students who do NOT participate in a sport. In order to maximize this total, we will need to maximize the number of females who do NOT participate in and the number of males who do NOT participate in a sport.

The problem states that at least 10% of the female students, or 24 female students, participate in a sport. This leaves 216 female students who may or may not participate in a sport. Since we want to maximize the number of female students who do NOT participate in a sport, we will assume that all 216 of these remaining female students do not participate in a sport.

The problem states that fewer than 30% of the male students do NOT participate in a sport. Thus, fewer than 168 male students (30% of 560) do NOT participate in a sport. Thus anywhere from 0 to 167 male students do NOT participate in a sport. Since we want to maximize the number of male students who do NOT participate in a sport, we will assume that 167 male students do NOT participate in a sport. This leaves 393 male students who do participate in a sport.

Thus, our matrix can now be completed as follows:

	Male	Female	TOTAL
Sport	393	24	417
No Sport	167	216	**383**

TOTAL	560	240	800

Therefore, the maximum possible number of students in School T who do not participate in a sport is 383.

The correct answer is B.

10.

This is an overlapping sets problem, which can be solved most efficiently by using a double set matrix. Our first step in using the double set matrix is to fill in the information given in the question. Because there are no real values given in the question, the problem can be solved more easily using 'smart numbers'; in this case, we can assume the total number of rooms to be 100 since we are dealing with percentages. With this assumption, we can fill the following information into our matrix:

There are 100 rooms total at the Stagecoach Inn.

Of those 100 rooms, 75 have a queen-sized bed, while 25 have a king-sized bed.

Of the non-smoking rooms (let's call this unknown n), 60% or $.6n$ have queen-sized beds.

10 rooms are non-smoking with king-sized beds.

Let's fill this information into the double set matrix, including the variable n for the value we need to solve the problem:

	SMOKING	NON-SMOKING	TOTALS
KING BED		10	25
QUEEN BED		$.6n$	75
TOTALS		n	100

In a double-set matrix, the first two rows sum to the third, and the first two columns sum to the third. We can therefore solve for n using basic algebra:

$10 + .6n = n$
$10 = .4n$
$n = 25$

We could solve for the remaining empty fields, but this is unnecessary work. Observe that the total number of smoking rooms equals $100 - n = 100 - 25 = 75$. Recall that we are working with smart numbers that represent percentages, so 75% of the rooms at the Stagecoach Inn permit smoking.

The correct answer is E.

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The difference is between non-smoking rooms and ALL the rooms:
Let the total number of rooms at the Stagecoach Inn be 100 rooms.
60% of the non-smoking rooms (let's call this unknown n) have queen-sized beds: 60% of 'n' or 0.6n
10% of ALL the rooms (100) at the Stagecoach Inn are non-smoking rooms with king-sized beds: 10 rooms are non-smoking with king-sized beds (i.e. 0.4 n)

Refer diagram below:

Say total rooms = 100

We require to find value of "x" (Shaded in Green)

Setting up equation from values calculated in table

$$\frac{60}{100}(100-x)+10=100-x$$

x = 75%

Answer = E

	Queen Size	King Size	Total
Smoking			x
Non- Smoking	60% of (100-x)	10% of 100=10	100-x
Total	75	25	100

Top 1% expert replies to student queries (can skip) (additional)

We are given that 75% of the guestrooms have a queen-size bed.
25% of the rooms have a king-sized bed.
Of the non-smoking rooms, 60% are queen-sized. But we don't know the number of non-smoking rooms. So doing 60% of 75 would be incorrect.

As for how we should be approaching this problem, we should use the information given about a specific section of rooms. In this question, we have information about non-smoking rooms.

Let the number of non-smoking rooms be 'n'.
So, the number of non-smoking rooms with a queen-sized bed = 0.6n
We are given that the number of non-smoking rooms with a king-sized bed = 10

Number of non-smoking rooms with a queen-sized bed + Number of non-smoking rooms with a king-sized bed = Number of non-smoking rooms
0.6n + 10 = n
n = 25
The correct answer is E.

11.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. The boldfaced values were given in the question. The non-boldfaced values were derived using the fact that in a double-set matrix, the sum of the first two rows equals the third and the sum of the first two columns equals the third. The variable *p* was used for the total number of pink roses, so that the total number of pink and red roses could be solved using the additional information given in the question.

	Red	Pink	White	TOTAL
Long-stemmed			**0**	80
Short-stemmed	5	**15**	**20**	**40**
TOTAL	100 - *p*	*p*	20	**120**

The question states that the percentage of red roses that are short-stemmed is equal to the percentage of pink roses that are short stemmed, so we can set up the following proportion:

$5/(100-p) = 15/p$
$5p = 1500 - 15p$
$p = 75$

This means that there are a total of 75 pink roses and 25 red roses. Now we can fill out the rest of the double-set matrix:

	Red	Pink	White	TOTAL
Long-stemmed	20	60	**0**	80
Short-stemmed	5	**15**	**20**	**40**
TOTAL	25	75	20	**120**

Now we can answer the question. 20 of the 80 long-stemmed roses are red, or 20/80 = 25%.

The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)

First create this table-

	Red	White	Pink	Total
Short Stemmed	5	20	15	40
Long Stemmed		0		80
	100-x	20	X	120

Total roses=120

Out of which 1/3rd which is 40 are short stemmed (it is mentioned 20 are white and 15 are pink so remaining 5 are red)
So, rest 120-40=80 are long stemmed

Percentage of Pink roses short stemmed=15/X
Percentage of red roses short stemmed=5/100-x(Percentage of pink roses short stemmed means short stemmed pink roses/Total pink roses) not total roses in denominator as it is taking a percentage of pink roses (mentioned)---out of which those are short stemmed----not out of the total roses.
On solving u will get x=75
So Total red roses=100-x=25
Long stemmed roses that are red are=25-5=20

Percentage of long stemmed roses that are red=20/80*100=25%

The correct answer is B.

12.
Let's # of students at Social High be 8 (I picked 8 as in this case 3/8 of total and 5/8 of total will be an integer).

3/8 of all students at Social High are in all three clubs --> 3/8*8=3 people are in exactly 3 clubs;

1/2 of all students are in Albanian club --> 1/2*8=4 people are in Albanian club;
5/8 of all students are in Bardic club --> 5/8*8=5 people are in Bardic club;
3/4 of all students are in Checkmate club --> 3/4*8=6 people are in Checkmate club;

Also as every student is in at least one club then # of students in neither of clubs is 0;

Total=A+B+C-{# of students in exactly 2 clubs}-2*{# of students in exactly 3 clubs}+{# of students in neither of clubs};

8=4+5+6-{# of students in exactly 2 clubs}-2*3+0 --> {# of students in exactly 2 clubs}=1, so fraction is 1/8.

The correct answer is A.

13.
You can solve this problem with a matrix. Since the total number of diners is unknown and not important in solving the problem, work with a hypothetical total of 100 couples. Since you are dealing with percentages, 100 will make the math easier.

Set up the matrix as shown below:

	Dessert	**NO dessert**	**TOTAL**
Coffee			

NO coffee			
TOTAL			**100**

Since you know that 60% of the couples order BOTH dessert and coffee, you can enter that number into the matrix in the upper left cell.

	Dessert	**NO dessert**	**TOTAL**
Coffee	**60**		
NO coffee			
TOTAL			100

The next useful piece of information is that 20% of the couples who order dessert don't order coffee. **But be careful!** The problem does not say that 20% of the *total* dinners order dessert and don't order coffee, so you CANNOT fill in 40 under dessert, no coffee" (first column, middle row). Instead, you are told that 20% **of the couples who order dessert** don't order coffee.

Let ***x*** = total number of couples who order dessert. Therefore, you can fill in ***.2x*** for the number of couples who order dessert but no coffee.

	Dessert	**NO dessert**	**TOTAL**
Coffee	60		
NO coffee	***.2x***		
TOTAL	***x***		100

Set up an equation to represent the couples that order dessert and solve:

$60 + .2x = x$
$60 = .8x$
$X = 75$

75% of all couples order dessert. Therefore, there is only a 25% chance that the next couple will *not* order dessert.

The correct answer is B.

14.

This problem involves two sets:
Set 1: Apartments with windows / Apartments without windows
Set 2: Apartments with hardwood floors / Apartments without hardwood floors.

It is easiest to organize two-set problems by using a matrix as follows:

	Windows	NO Windows	TOTAL
Hardwood Floors			
NO Hardwood Floors			
TOTAL			

The problem is difficult for two reasons. First, it uses percents instead of real numbers. Second, it involves complicated and subtle wording.

Let's attack the first difficulty by converting all of the percentages into REAL numbers. To do this, let's say that there are 100 total apartments in the building. This is the first number we can put into our matrix. The absolute total is placed in the lower right hand corner of the matrix as follows:

	Windows	NO Windows	TOTAL
Hardwood Floors			
NO Hardwood Floors			
TOTAL			**100**

Next, we will attack the complex wording by reading each piece of information separately, and filling in the matrix accordingly.

Information: **<u>50% of the apartments in a certain building have windows and hardwood floors.</u>** Thus, 50 of the 100 apartments have BOTH windows and hardwood floors. This number is now added to the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**		
NO Hardwood Floors			
TOTAL			**100**

Information: **<u>25% of the apartments without windows have hardwood floors.</u>** Here's where the subtlety of the wording is very important. This does NOT say that 25% of ALL the apartments have no windows and have hardwood floors. Instead it says that OF the apartments without windows, 25% have hardwood floors. Since we do not yet know the number of apartments without windows, let's call this number **x**. Thus the number of apartments without windows and with hardwood floors is **.25x**. These figures are now added to the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**.25x**	
NO Hardwood			
Floors			
TOTAL		**x**	**100**

Information: **<u>40% of the apartments do not have hardwood floors.</u>** Thus, 40 of the 100 apartments do not have hardwood floors. This number is put in the Total box at the end of the "No Hardwood Floors" row of the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**.25x**	
NO Hardwood Floors			**40**
TOTAL		**x**	**100**

Before answering the question, we must complete the matrix. To do this, fill in the numbers that yield the given totals. First, we see that there must be be 60 total apartments with Hardwood Floors (since 60 + 40 = 100) Using this information, we can solve for **x** by creating an equation for the first row of the matrix:

$50 + 0.25x = 60$
$0.25x = 10$
$x= 40$

Now we put these numbers in the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**10**	**60**
NO Hardwood Floors			**40**
TOTAL		**40**	**100**

Finally, we can fill in the rest of the matrix:

	Windows	NO Windows	TOTAL
Hardwood Floors	**50**	**10**	**60**
NO Hardwood Floors	**10**	**30**	**40**
TOTAL	**60**	**40**	**100**

We now return to the question: What percent of the apartments with windows have hardwood floors?

Again, pay very careful attention to the subtle wording. The question does NOT ask for the percentage of TOTAL apartments that have windows and hardwood floors. It asks what percent OF the apartments with windows have hardwood floors. Since there are 60 apartments with windows, and 50 of these have hardwood floors, the percentage is calculated as follows:

$$\frac{50}{60} = .8\overline{3} = 83\frac{1}{3}\%$$

Thus, the correct answer is E.

15.
This problem can be solved using a set of three equations with three unknowns. We'll use the following definitions:

Let F = the number of Fuji trees
Let G = the number of Gala trees
Let C = the number of cross pollinated trees

10% of his trees cross pollinated

$C = 0.1(F + G + C)$
$10C = F + G + C$
$9C = F + G$

The pure Fujis plus the cross pollinated ones total 187
(4) $F + C = 187$

3/4 of his trees are pure Fuji
(5) $F = (3/4)(F + G + C)$
(6) $4F = 3F + 3G + 3C$
(7) $F = 3G + 3C$

Substituting the value of F from equation (7) into equation (3) gives us:

(8) $9C = (3G + 3C) + G$
(9) $6C = 4G$
(10) $12C = 8G$

Substituting the value of F from equation (7) into equation (4) gives us:
(11) $(3G + 3C) + C = 187$
(12) $3G + 4C = 187$
(13) $9G + 12C = 561$

Substituting equation (10) into (13) gives:
(14) $9G + 8G = 561$
(15) $17G = 561$
(16) $G = 33$

So the farmer has 33 trees that are pure

Gala. **The correct answer is B.**

Top 1% expert replies to student queries (can skip)
Given:
Let,
Gala apples - G
Fuji apples - F
C - Cross Pollinated apples
Total apples - X
Given:
F = 3/4X
X = G + F + cross pollinated
Fuji + Cross pollinated (10 % of all apples) = 187

Solution:
3/4X + 1/10X = 187
Hence X = 220
X = G + (F + cross pollinated)
220 = G + (187)
Hence G = 33.

The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)

Query: How do we know that no. of trees not cross pollinated is 0?

Reply: That is because there are cross-pollinated trees too, not just Fuji and Gala. In that case, if Fuji were to be 3/4 of the total number of trees, then Gala would have been 1/4 of the total number of trees. See the following solution:

Consider two overlapping circles. Left-most part is only Fuji, the right-most part is only Gala, and the overlapped part is cross-pollination. We are not treating them as a different category, we are simply denoting them by a variable and setting up equations with the variables to set up relationships among them.

You can also think this way:

Total trees in the orchard = 100x [for some positive number x]

Overlapped portion = 10x

Say left-most part is y [for some positive number y]

Then y + 10x = 187 ... (i)

y = 300x/4 = 75x ... (ii)

Putting (ii) in (i) we get

85x = 187

or x = 187 / 85 ... (iii)

75% of all trees are pure Fuji, 10% are cross-pollinated. Then pure Gala = 15% of all trees = 15x = (15)(187)/85 = 33

OR

Total number of trees = 100x = (100)(187)/85 = 220

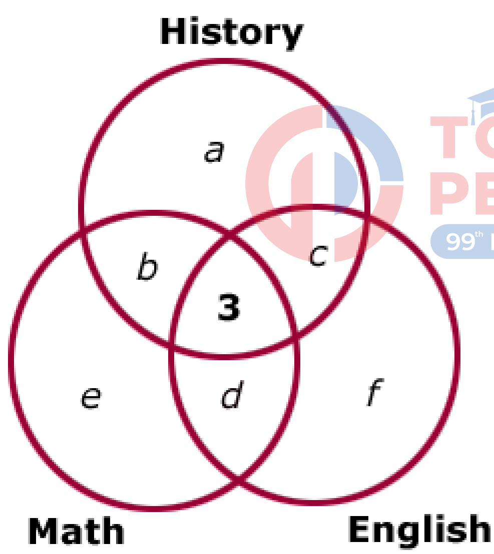
Number of cross-pollinated trees = 22

Number of pure Fuji trees = (3/4)(220) = 165

Number of pure Gala trees = 220 - 22 - 165 = 33

The correct answer is B.

16.
For an overlapping set problem with three subsets, we can use a Venn diagram to solve.



Each circle represents the number of students enrolled in the History, English and Math classes, respectively. Notice that each circle is subdivided into different groups of students. Groups *a*, *e*, and *f* are comprised of students taking only 1 class. Groups *b*, *c,* and *d* are comprised of students taking 2 classes. In addition, the diagram shows us that 3 students are taking all 3 classes. We can use the diagram and the information in the question to write several equations:

History students: $a + b + c + 3 = 25$
Math students: $e + b + d + 3 = 25$
English students: $f + c + d + 3 = 34$
TOTAL students: $a + e + f + b + c + d + 3 = 68$

The question asks for the total number of students taking exactly 2 classes. This can be represented as $b + c + d$.

If we sum the first 3 equations (History, Math and English) we get:

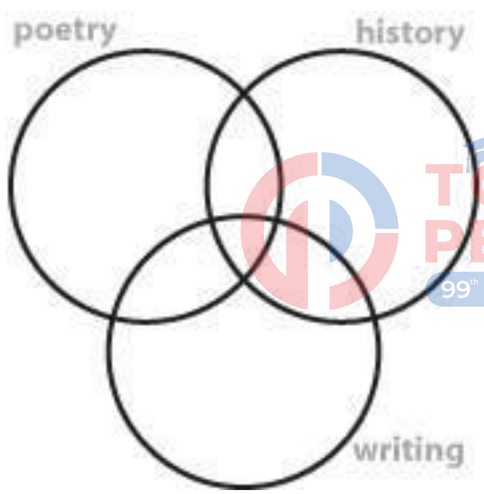
$a + e + f + 2b + 2c + 2d + 9 = 84.$

Taking this equation and subtracting the 4th equation (Total students) yields the following:

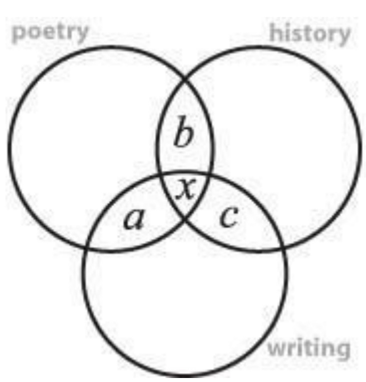
$$\begin{array}{r} a + e + f + 2b + 2c + 2d + 9 = 84 \\ \underline{-[a + e + f + b + c + d + 3 = 68]} \\ b + c + d = 10 \end{array}$$

The correct answer is B.

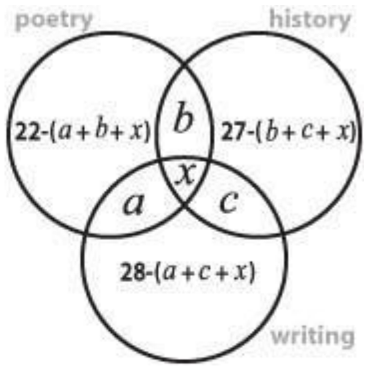
17. This is a three-set overlapping sets problem. When given three sets, a Venn diagram can be used. The first step in constructing a Venn diagram is to identify the three sets given. In this case, we have students signing up for the poetry club, the history club, and the writing club. The shell of the Venn diagram will look like this:



When filling in the regions of a Venn diagram, it is important to work from inside out. If we let x represent the number of students who sign up for all three clubs, a represent the number of students who sign up for poetry and writing, b represent the number of students who sign up for poetry and history, and c represent the number of students who sign up for history and writing, the Venn diagram will look like this:



We are told that the total number of poetry club members is 22, the total number of history club members is 27, and the total number of writing club members is 28. We can use this information to fill in the rest of the diagram:



We can now derive an expression for the total number of students by adding up all the individual segments of the diagram. The first bracketed item represents the students taking two or three courses. The second bracketed item represents the number of students in only the poetry club, since it's derived by adding in the total number of poetry students and subtracting out the poetry students in multiple clubs. The third and fourth bracketed items represent the students in only the history or writing clubs respectively.

$59 = [a + b + c + x] + [22 – (a + b + x)] + [27 – (b + c + x)] + [28 – (a + c + x)]$
$59 = a + b + c + x + 22 – a – b – x + 27 – b – c – x + 28 – a – c – x$
$59 = 77 – 2x – a – b – c$
$59 = 77 – 2x – (a + b + c)$

By examining the diagram, we can see that $(a + b + c)$ represents the total number of students who sign up for two clubs. We are told that 6 students sign up for exactly two clubs. Consequently:

$59 = 77 – 2x – 6$
$2x = 12$
$x = 6$
So, the number of students who sign up for all three clubs is 6.

Alternatively, we can use a more intuitive approach to solve this problem. If we add up the total number of club sign-ups, or registrations, we get 22+27+28 = 77 We must remember that this number includes overlapping registrations (some students sign up for two clubs, others for three). So, there are 77 registrations and 59 total students. Therefore, there must be 77 – 59 = 18 duplicate registrations.

We know that 6 of these duplicates come from those 6 students who sign up for exactly two clubs. Each of these 6, then, adds one extra registration, for a total of 6 duplicates. We are then left with 18 – 6 = 12 duplicate registrations. These 12 duplicates must come from those students who sign up for all three clubs.

For each student who signs up for three clubs, there are two extra sign-ups. Therefore, there must be 6 students who sign up for three clubs:

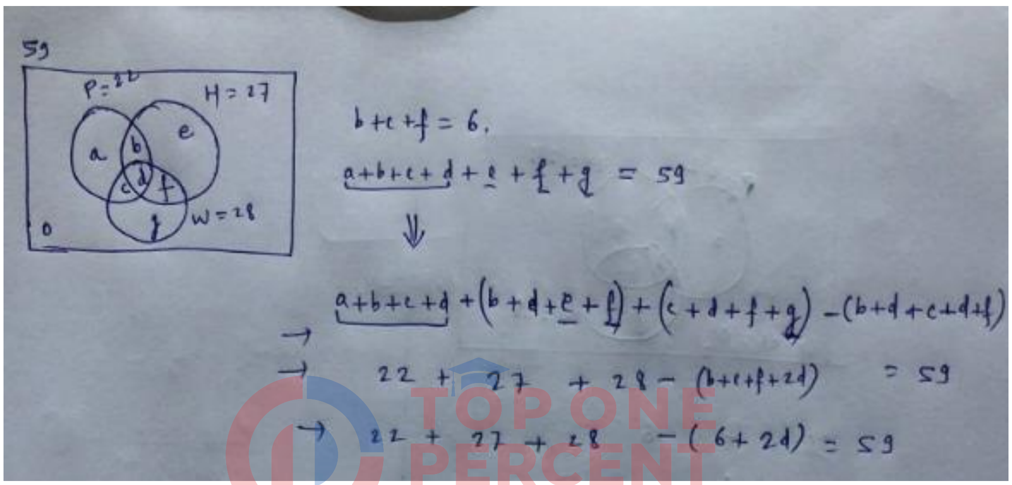
12 duplicates / (2 duplicates/student) = 6 students

Between the 6 students who sign up for two clubs and the 6 students who sign up for all three, we have accounted for all 18 duplicate registrations.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

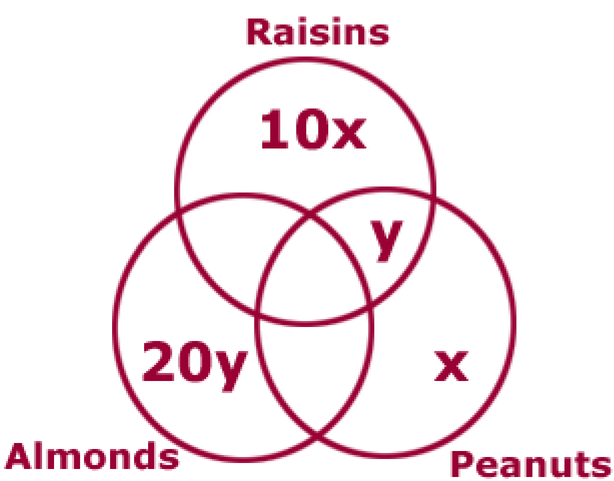
Query: Explain why when calculating the value of d we are dividing by 2?
Reply:



18.

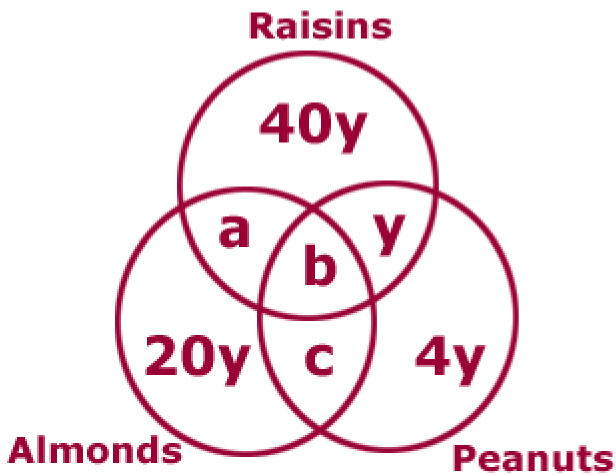
This problem involves 3 overlapping sets. To visualize a 3-set problem, it is best to draw a Venn Diagram.

We can begin filling in our Venn Diagram utilizing the following 2 facts: (1) The number of bags that contain only raisins is 10 times the number of bags that contain only peanuts. (2) The number of bags that contain only almonds is 20 times the number of bags that contain only raisins and peanuts.



Next, we are told that the number of bags that contain only peanuts (which we have represented as x) is one-fifth the number of bags that contain only almonds (which we have represented as 20y).

This yields the following equation: $x = (1/5)20y$ which simplifies to $x = 4y$. We can use this information to revise our Venn Diagram by substituting any x in our original diagram with 4y as follows:



Notice that, in addition to performing this substitution, we have also filled in the remaining open spaces in the diagram with the variable a, b, and c.

Now we can use the numbers given in the problem to write 2 equations. First, the sum of all the expressions in the diagram equals 435 since we are told that there are 435 bags in total. Second, the sum of all the expressions in the almonds circle equals 210 since we are told that 210 bags contain almonds.

$435 = 20y + a + b + c + 40y + y + 4y$
$210 = 20y + a + b + c$

Subtracting the second equation from the first equation, yields the following:

$225 = 40y + y + 4y$
$225 = 45y$
$5 = y$

Given that $y = 5$, we can determine the number of bags that contain only one kind of item as follows:

The number of bags that contain only raisins = 40y = 200
The number of bags that contain only almonds = 20y =
100 The number of bags that contain only peanuts = 4y =
20

Thus there are 320 bags that contain only one kind of item.

The correct answer is D.

Top 1% expert replies to student queries (can skip)
Fill the diagram step by step:

PFA the attached diagram-

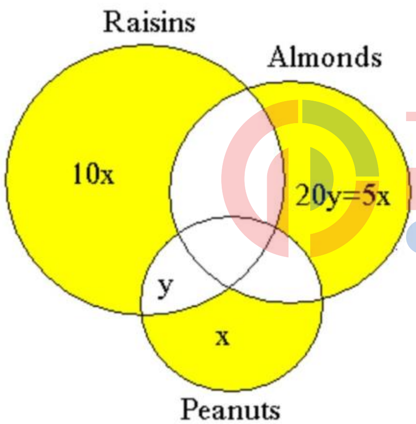
Also given that there are total of 435 bags and 210 bags contain almonds.

From the diagram $20y=5x$;
$y = x/4$.

Now, Total = 435 = {Almonds} + 10x + y + x;
$435 = 210 + 10x + x/4 + x$;
$x = 20$.

The number of bags that contain only one kind of item is the sum of yellow segments: $10x + x + 5x = 16x = 320$.

Answer: D.



19.
This is an overlapping sets problem. This question can be effectively solved with a double-set matrix composed of two overlapping sets: [Spanish/Not Spanish] and [French/Not French]. When constructing a double-set matrix, remember that the two categories adjacent to each other must be mutually exclusive, i.e. [French/not French] are mutually exclusive, but [French/not Spanish] are not mutually exclusive. Following these rules, lets construct and fill in a double-set matrix for each statement. To simplify our work with percentages, we will also pick 100 for the total number of students at Jefferson High School.

INSUFFICIENT: While we know the percentage of students who take French and, from that information, the percentage of students who do not take French, we do not know anything about the students taking Spanish. Therefore we don't know the percentage of students who study French but not Spanish, i.e. the number in the target cell denoted with *x*.

	FRENCH	NOT FRENCH	TOTALS
SPANISH			
NOT SPANISH	*x*		
TOTALS	30	70	100

(2) INSUFFICIENT: While we know the percentage of students who do not take Spanish and, from that information, the percentage of students who do take Spanish, we do not know anything about the students taking French. Therefore we don't know the percentage of students who study French but not Spanish, i.e. the number in the target cell denoted with *x*.

	FRENCH	NOT FRENCH	TOTALS
SPANISH			60
NOT SPANISH	*x*		40
TOTALS			100

(1) AND (2) INSUFFICIENT: Even after we combine the two statements, we do not have sufficient information to find the percentage of students who study French but not Spanish, i.e. to fill in the target cell denoted with *x*.

	FRENCH	NOT FRENCH	TOTALS
SPANISH			60
NOT SPANISH	*x*		40
TOTALS	30	70	100

The correct answer is E.

20.
For this overlapping sets problem, we want to set up a double-set matrix.
The first set is boys vs. girls; the second set is left-handers vs. right-handers.

The only number currently in our chart is that given in the question: 20, the total number of students.

	GIRLS	BOYS	TOTALS
LEFT-HANDED			
RIGHT-HANDED			
TOTALS			20

INSUFFICIENT: We can figure out that three girls are left-handed, but we know nothing about the boys.

	GIRLS	BOYS	TOTALS
LEFT-HANDED	(0.25)(12) = 3		
RIGHT-HANDED			
TOTALS	12		20

(2) INSUFFICIENT: We can't figure out the number of left-handed boys, and we know nothing about the girls.

	GIRLS	BOYS	TOTALS
LEFT-HANDED			
RIGHT-HANDED		5	
TOTALS			20

(1) AND (2) SUFFICIENT: If we combine both statements, we can get the missing pieces we need to solve the problem. Since we have 12 girls, we know that there are 8 boys. If five of them are right-handed, then three of them must be left-handed. Add that to the three left-handed girls, and we know that a total of 6 students are left- handed.

	GIRLS	BOYS	TOTALS
LEFT-HANDED	3	**3**	**6**
RIGHT-HANDED		5	
TOTALS	12	**8**	20

The correct answer is C.

21.

For this overlapping set problem, we want to set up a two-set table to test our possibilities. Our first set is vegetarians vs. non-vegetarians; our second set is students vs. non-students.

	VEGETARIAN	NON-VEGETARIAN	TOTAL
STUDENT			
NON-STUDENT		15	
TOTAL	x	x	?

We are told that each non-vegetarian non-student ate exactly one of the 15 hamburgers, and that nobody else ate any of the 15 hamburgers. This means that there were exactly 15 people in the non-vegetarian non-student category. We are also told that the total number of vegetarians was equal to the total number of non-vegetarians; we represent this by putting the same variable in both boxes of the chart.

The question is asking us how many people attended the party; in other words, we are being asked for the number that belongs in the bottom-right box, where we have placed a question mark.

The second statement is easier than the first statement, so we'll start with statement (2).

(2) INSUFFICIENT: This statement gives us information only about the cell labelled "vegetarian non-student"; further it only tells us the number of these guests as a *percentage* of the total guests. The 30% figure does not allow us to calculate the actual number of any of the categories.

SUFFICIENT: This statement provides two pieces of information. First, the vegetarians attended at the rate, or in the ratio, of 2:3 students to non- students. We're also told that this 2:3 rate is half the rate for non-vegetarians. In order to double a rate, we double the first number; the rate for non- vegetarians is 4:3 We can represent the actual numbers of non-vegetarians as $4a$ and $3a$ and add this to the chart below. Since we know that there were 15 non-vegetarian non-students, we know the missing common multiple, a, is 15/3 = 5. Therefore, there were (4)(5) = 20 non-vegetarian students and 20 + 15 = 35 total non-vegetarians (see the chart below). Since the same number of vegetarians and non-vegetarians attended the party, there were also 35 vegetarians, for a total of 70 guests.

	VEGETARIAN	NON-VEGETARIAN	TOTAL
STUDENT		$4a$ or 20	
NON-STUDENT		$3a$ or 15	
TOTAL	x or 35	x or 35	? or 70

The correct answer is A.

Alternate sol from gmatclub (additional)

Now, as guests ate a total of 15 hamburgers and each guest who was **neither a student nor a vegetarian** (group #4) ate exactly one hamburger and also as **no hamburger was eaten by any guest who was a student, a vegetarian, or both** (groups #1, #2 and #3) then this simply tells us that there were **15 non-vegetarian non-students** at the party (group #4 = 15).

Make a matrix:

	Students	Non-students	TOTAL
Vegetarians			X/2
Non-vegetarians		15	X/2
TOTAL			X

Note that we denoted total # of guests by x so both vegetarians and non-vegetarians equal to $\frac{x}{2}$.

(1) The vegetarians attended the party at a rate of 2 students to every 3 non-students, half the rate for non-vegetarians --> $\frac{vegetarian\ students}{vegetarian\ non-students}=\frac{2}{3}$ --> if the rate X (some fraction) is half of the rate Y (another fraction), then Y = 2*X --> $\frac{non-vegetarian\ students}{non-vegetarian\ non-students}=2*\frac{2}{3}=\frac{4}{3}$ --> so, **non-vegetarian non-students** compose 3/7 of all non vegetarians:
$non-vegetarian\ non-students=15=\frac{3}{7}*\frac{x}{2}$ --> $x=70$. Sufficient.

	Students	Non-students	TOTAL
Vegetarians			X/2
Non-vegetarians	(4/7)*X/2	(3/7)*X/2=15	X/2
TOTAL			X

(2) 30% of the guests were vegetarian non-students --> just says that # of $vegetariannon-students$ equal to $0.3x$ --> insufficeint, to calculate x.

	Students	Non-students	TOTAL
Vegetarians		0.3*X	X/2
Non-vegetarians		15	X/2
TOTAL			X

22.

For an overlapping set question, we can use a double-set matrix to organize the information and solve. The two sets in this question are the practical test (pass/fail) and the written test (pass/fail).

From the question we can fill in the matrix as follows. In a double-set matrix, the sum of the first two rows equals the third and the sum of the first two columns equals the third. The bolded value was derived from the other given values. The question asks us to find the value of $.7x$

	PRACTICAL - PASS	PRACTICAL - FAIL	TOTALS
WRITTEN - PASS	$\mathbf{.7x}$	$.3x$	x
WRITTEN - FAIL		0	
TOTALS		$.3x$	

(1) INSUFFICIENT: If we add the total number of students to the information from the question, we do not have enough to solve for $.7x$.

	PRACTICAL - PASS	PRACTICAL - FAIL	TOTALS
WRITTEN - PASS	$\mathbf{.7x}$	$.3x$	x
WRITTEN - FAIL		0	
TOTALS		$.3x$	188

(2) INSUFFICIENT: If we add the fact that 20% of the *sixteen year-olds who passed the practical test* failed the written test to the original matrix from the question, we can come up with the relationship $.7x = .8y$. However, that is not enough to solve for $.7x$.

	PRACTICAL - PASS	PRACTICAL - FAIL	TOTALS
WRITTEN - PASS	$\mathbf{.7x = .8y}$	$.3x$	x
WRITTEN - FAIL	$.2y$	0	$.2y$
TOTALS	y	$.3x$	

(1) AND (2) SUFFICIENT: If we combine the two statements we get a matrix that can be used to form two relationships between x and y:

	PRACTICAL - PASS	PRACTICAL - FAIL	TOTALS
WRITTEN - PASS	**$.7x = .8y$**	$.3x$	x
WRITTEN - FAIL	$.2y$	0	$.2y$
TOTALS	y	$.3x$	188

$.7x = .8y$
$y + .3x = 188$

This would allow us to solve for x and in turn find the value of $.7x$, the number of sixteen year-olds who received a driver license.

The correct answer is C.

Alternate sol from gmatclub (additional)

There are four outcomes. You can:
A) Pass Written and Pass Practical (which implies you get a license)
B) Pass Written and Fail Practical
C) Fail Written and Pass Practical
D) Fail Written and Fail Practical.

We are given that no one (0%) has failed both (outcome D) and that 30% Pass written, Fail practical (outcome B).
We need to find the number corresponding to outcome A (not the percentage!)

(1) There are 188 sixteen year-olds at Culliver High School.
Great, this tells you that 0 kids fall into outcome D and 57 kids fall into outcome B. But this doesn't tell you how many fall into outcome A. It tells you that the total number of kids in outcome A and outcome C is 188-57=131.

(2) 20% of the sixteen year-olds who passed the practical test failed the written test.
We now know that 20% are in outcome C. Since we know the % of kids in outcomes B, C, and D, we can figure out the % of kids in outcome A. There are 50% of kids in outcome A, since outcomes B+C+D = 50%. But, this still doesn't tell how many students are in outcome A, just the percentage.

Given both (1) and (2), we now know that 50% of 188 students (or 94 students) passed both the written and practical exam.

So the answer is C.

23.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. We are told in the question stem that 180 guests have a house in the Hamptons and a house in Palm Beach. We can insert this into our matrix as follows:

	House in Hamptons	No House in Hamptons	TOTALS
House in Palm Beach	180		
No House in Palm Beach			
TOTALS			T

The question is asking us for the ratio of the darkly shaded box to the lightly shaded box.

INSUFFICIENT: Since one-half of all the guests had a house in Palm Beach, we can fill in the matrix as follows:

	House in Hamptons	No House in Hamptons	TOTALS
House in Palm Beach	180	$(1/2)T - 180$	$(1/2)T$
No House in Palm Beach			
TOTALS			T

We cannot find the ratio of the dark box to the light box from this information alone.

(2) INSUFFICIENT: Statement 2 tells us that two-thirds of all the guests had a house in the Hamptons. We can insert this into our matrix as follows:

	House in Hamptons	No House in Hamptons	TOTALS
House in Palm Beach	180		
No House in Palm Beach	$(2/3)T - 180$		
TOTALS	$(2/3)T$		T

We cannot find the ratio of the dark box to the light box from this information alone.

(1) AND (2) INSUFFICIENT: we can fill in our matrix as follows.

	House in Hamptons	No House in Hamptons	TOTALS
House in Palm Beach	180	$(1/2)T - 180$	$(1/2)T$
No House in Palm Beach	$(2/3)T - 180$	**180 – (1/6)*T***	**(1/2)*T***
TOTALS	$(2/3)T$	**(1/3)*T***	T

The ratio of the number of people who had a house in Palm Beach but not in the Hamptons to the number of people who had a house in the Hamptons but not in Palm Beach (i.e. dark to light) will be: $\frac{\left(\frac{1}{2}\right)T-180}{\left(\frac{2}{3}\right)T-180}$

This ratio doesn't have a constant value; it depends on the value of *T*. We can try to solve for *T* by filling out the rest of the values in the matrix (see the **bold** entries above); however, any equation that we would build using these values reduces to a redundant statement of $T = T$. This means there isn't enough unique information to solve for T.

The correct answer is E.

24.

Since there are two different classes into which we can divide the participants, we can solve this using a double-set matrix. The two classes into which we'll divide the participants are Boys/Girls along the top (as column labels), and Chocolate/Strawberry down the left (as row labels).

The problem gives us the following data to fill in the initial double-set matrix. We want to know if we can determine the maximum value of *a*, which represents the number of girls who ate chocolate ice cream.

	BOYS	GIRLS	TOTALS
CHOCOLATE	8	a	
STRAWBERRY		9	
TOTALS			

(1) SUFFICIENT: Statement (1) tells us that exactly 30 children came to the party, so we'll fill in 30 for the grand total. Remember that we're trying to maximize a.

	BOYS	GIRLS	TOTALS
CHOCOLATE	8	a	b
STRAWBERRY	c	9	d
TOTALS			30

In order to maximize a, we must maximize b, the total number of chocolate eaters. Since $b + d = 30$, implying $b = 30 - d$, we must minimize d to maximize b. To minimize d we must minimize c. The minimum value for c is 0, since the question doesn't say that there were necessarily boys who had strawberry ice cream.

Now that we have an actual value for c, we can calculate forward to get the maximum possible value for a. If $c = 0$, since we know that $c + 9 = d$, then $d = 9$. Since $b + d = 30$, then $b = 21$. Given that $8 + a = b$ and $b = 21$, then $a = 13$, the maximum value we were looking for. Therefore statement (1) is sufficient to find the maximum number of girls who ate chocolate.

(2) INSUFFICIENT: Knowing only that fewer than half of the people ate strawberry ice cream doesn't allow us to fill in any of the boxes with any concrete numbers. Therefore statement (2) is insufficient.

The correct answer is A.

25.

Since we are dealing with overlapping sets and there are two independent criteria, the best way to solve this problem is with a double-set matrix.

The first statement in the question stem tells us that of the students who speak French (represented by the first column), four times as many speak German as don't. This information yields the following entries in the double-set matrix:

	FRENCH	NO FRENCH	TOTALS
GERMAN	$4x$		
NO GERMAN	x		
TOTALS			

The second statement in the question stem tells us that 1/6 of the of the students who don't speak German do speak French. This is fact represented in the double-set matrix as follows:

	FRENCH	NO FRENCH	TOTALS
GERMAN	$4x$		
NO GERMAN	$x = y/6$		y
TOTALS			

Now since $x = y/6$, we can get rid of the new variable y and keep all the expressions in terms of x.

	FRENCH	NO FRENCH	TOTALS
GERMAN	$4x$		
NO GERMAN	x		$6x$
TOTALS			

Now we can fill in a few more boxes using the addition rules for the double-set matrix.

	FRENCH	NO FRENCH	TOTALS
GERMAN	$4x$		
NO GERMAN	x	$5x$	$6x$
TOTALS	$5x$		

The main question to be answered is what fraction of the students speak German, a fraction represented by A/B in the final double-set matrix. So, if statements (1) and/or (2) allow us to calculate a numerical value for A/B, we will be able to answer the question.

	FRENCH	NO FRENCH	TOTALS
GERMAN	$4x$		A
NO GERMAN	x	$5x$	$6x$
TOTALS	$5x$		B

(1) INSUFFICIENT: Statement (1) tells us that 60 students speak French and German, so $4x = 60$ and $x = 15$. We can now calculate any box labeled with an x, but this is still insufficient to calculate A, B, or A/B.

(2) INSUFFICIENT: Statement (2) tells us that 75 students speak neither French nor German, so $5x = 75$ and $x = 15$. Just as with Statement (1), we can now calculate any box labelled with an x, but this is still insufficient to calculate A, B, or A/B.

(1) AND (2) INSUFFICIENT: Since both statements give us the same information (namely, that $x = 15$), putting the two statements together does not tell us anything new. Therefore (1) and (2) together are insufficient.

The correct answer is E.

26.
In an overlapping set problem, we can use a double set matrix to organize the information and solve.
From information given in the question, we can fill in the matrix as follows:

	GREY	WHITE	TOTALS
BLUE		> 3	
BROWN			
TOTALS			55

The question is asking us if the total number of blue-eyed wolves (fourth column, second row) is greater than the total number of brown-eyed wolves (fourth column, third row).

(1) INSUFFICIENT. This statement allows us to fill in the matrix as below. We have no information about the total number of brown-eyed wolves.

	GREY	WHITE	TOTALS
BLUE	$4x$	$3x$	**7x**
BROWN			
TOTALS			55

(2) INSUFFICIENT. This statement allows us to fill in the matrix as below. We have no information about the total number of blue-eyed wolves.

	GREY	WHITE	TOTALS
BLUE			
BROWN	$1y$	$2y$	$\mathbf{3y}$
TOTALS			55

TOGETHER, statements (1) + (2) are SUFFICIENT. Combining both statements, we can fill in the matrix as follows:

	GREY	WHITE	TOTALS
BLUE	$4x$	$3x$	**7x**
BROWN	$1y$	$2y$	$\mathbf{3y}$
TOTALS	$\mathbf{4x + y}$	$\mathbf{3x + 2y}$	55

Using the additive relationships in the matrix, we can derive the equation $7x + 3y = 55$ (notice that adding the grey and white totals yields the same equation as adding the blue and brown totals).

The original question can be rephrased as "Is $7x > 3y$?"

On the surface, there *seems* to NOT be enough information to solve this question. However, we must consider some of the restrictions that are placed on the values of x and y:

(1) ***x* and *y* must be integers** (we are talking about numbers of wolves here and looking at the table, y, $3x$ and $4x$ must be integers so x and y must be integers)

(2) **x must be greater than 1** (the problem says there are more than 3 blue-eyed wolves with white coats so 3x must be greater than 3 or x > 1)

Since x and y must be integers, there are only a few x,y values that satisfy the equation $7x + 3y = 55$. By trying all integer values for x from 1 to 7, we can see that the only possible x,y pairs are:

x	y	7x	3y
1	16	7	48
4	**9**	**28**	**27**
7	**2**	**49**	**6**

Since x cannot be 1, the only two pairs yield $7x$ values that are greater than the corresponding $3y$ values (28 > 27 and 49 > 6).

The correct answer is C.

Alternate Solution from Gmatclub

Look at the matrix below:

	Brown eyes	Blue eyes	TOTAL
White coats		>3	
Grey coats			
TOTAL			55

"There are more than 3 blue-eyed wolves with white coats" means that # of wolves which have blue eyes AND white coats is more than 3. The question asks whether there are more blue-eyed wolves (blue box) than brown-eyed wolves (brown box).

(1) Among the blue-eyed wolves, the ratio of grey coats to white coats is 4 to 3. Not sufficient on its own.

(2) Among the brown-eyed wolves, the ratio of white coats to grey coats is 2 to 1. Not sufficient on its own.

(1)+(2) When taken together we get the flowing matrix:

	Brown eyes	Blue eyes	TOTAL
White coats	2y	3x>3	
Grey coats	y	4x	
TOTAL	**3y**	**7x**	**55**

Notice that x and y must be integers (they represent some positive multiples for the ratios given in the statements).

So, we have that $3y+7x=55$. After some trial and error we can find that this equation has only 3 positive integers solutions:
$y=2$ and $x=7$ --> $3y+7x=6+49=55$;
$y=9$ and $x=4$ --> $3y+7x=27+28=55$;
$y=16$ and $x=1$ --> $3y+7x=48+7=55$;

Now, the third solution ($x=1$) is not valid, since in this case # of wolves which have blue eyes AND white coats becomes $3x=3$, so not more than 3 as given in the stem. As for the first two cases, in both of them 7x is more than 3y ($49>6$ and $28>27$), so we can answer definite YES, to the question whether there are more blue-eyed wolves (blue box) than brown-eyed wolves (brown box).

The correct answer is C.

27.

We can divide the current fourth graders into 4 categories:
(1) The percentage that dressed in costume this year ONLY.
(2) The percentage that dressed in costume last year ONLY.
(3) The percentage that did NOT dress in costume either this year or last year.
(4) The percentage that dressed in costume BOTH years.

We need to determine the last category (category 4) in order to answer the question.

INSUFFICIENT: Let's assume there are 100 current fourth graders (this simply helps to make this percentage question more concrete). 60 of them dressed in costume this year, while 40 did not. However, we don't know how many of these 60 dressed in costume last year, so we can't divide this 60 up into categories 1 and 2.

INSUFFICIENT: This provides little relevant information on its own because we don't know how many of the students didn't dress up in costumes this year and the statement references that value.

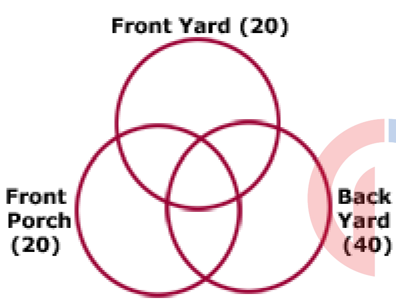
(1) AND (2) INSUFFICIENT: From statement 1 we know that 60 dressed up in costumes this year, but 40 did not. Statement 2 tells us that 80% of these 40, or 32, didn't dress up in costumes this year either. This provides us with a value for category 3, from which we can derive a value for category 2 (8). However, we still don't know how many of the 60 costume bearers from this year wore costumes last year.

Since this is an overlapping set problem, we could also have used a double-set matrix to organize our information and solve. Even with both statements together, we can not find the value for the Costume Last Year / Costume This Year cell.

	Costume This Year	No Costume This Year	TOTALS
Costume Last Year		8	
No Costume Last Year		32	
TOTALS	60	40	100

The correct answer is E.

28.



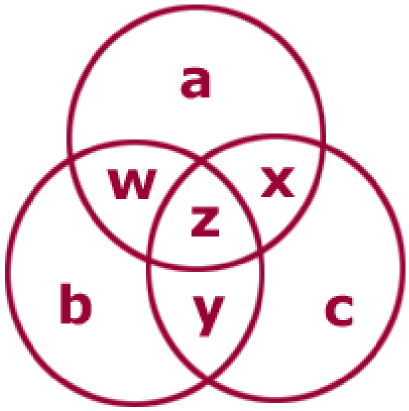
A Venn-Diagram is useful to visualize this problem.
Notice that the Venn diagram allows us to see the 7 different types of houses on Kermit lane. Each part of the diagram represents one type of house. For example, the center section of the diagram represents the houses that contain all three amenities (front yard, front porch, *and* back yard). Keep in mind that there may also be some houses on Kermit Lane that have none of the 3 amenities and so these houses would be outside the diagram.

SUFFICIENT: This tells us that no house on Kermit Lane is without a backyard. Essentially this means that there are 0 houses in the three sections of the diagram that are NOT contained in the Back Yard circle. It also means that there are 0 houses outside of the diagram. Since we know that 40 houses on Kermit Lane contain a back yard, there must be exactly 40 houses on Kermit Lane.

INSUFFICIENT: This tells us that each house on Kermit Lane that has a front porch does not have a front yard. This means that there are 0 houses in the two sections of the diagram in which Front Yard overlaps with Front Porch. However, this does not give us information about the other sections of the diagram. Statement (2) ALONE is not sufficient.

The correct answer is A.

29.
This is a problem that involves three overlapping sets. A helpful way to visualize this is to draw a Venn diagram as follows:



Each section of the diagram represents a different group of people. Section *a* represents those residents who are members of only club *a*. Section *b* represents those residents who are members of only club *b*. Section *c* represents those residents who are members of only club *c*. Section *w* represents those residents who are members of only clubs *a* and *b*. Section *x* represents those residents who are members of only clubs *a* and *c*. Section *y* represents those residents who are members of only clubs *b* and *c*. Section *z* represents those residents who are members of all three clubs.

The information given tells us that $a + b + c = 40$. One way of rephrasing the question is as follows: Is $x > 0$? (Recall that *x* represents those residents who are member of fitness clubs A and C but not B).

Statement (1) tells us that $z = 2$. Alone, this does not tell us anything about *x*, which could, for example, be 0 or 10, among many other possibilities. This is clearly not sufficient to answer the question.

Statement (2) tells us that $w + y = 8$. This alone does not give us any information about *x,* which, again could be 0 or a number of other values.

In combining both statements, it is *tempting* to assert the following.

We know from the question stem that $a + b + c = 40$. We also know from statement one that $z = 2$. Finally, we know from statement two that $w + y = 8$. We can use these three pieces of information to write an equation for all 55 residents as follows:

$a + b + c + w + x + y + z = 55$.
$(a + b + c) + x + (w + y) + (z) = 55$.
$40 + x + 8 + 2 = 55$
$x = 5$

This would suggest that there are 5 residents who are members of both fitness clubs A and C but not B.

However, this assumes that all 55 residents belong to at least one fitness club. Yet, this fact is not stated in the problem. It is possible then, that 5 of the residents are not members of *any* fitness club. This would mean that 0 residents are members of fitness clubs A and C but not B.

Without knowing how many residents are not members of *any* fitness club, we do not have sufficient information to answer this question.

Therefore, Statements (1) and (2) TOGETHER are NOT sufficient.

The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

Why are we not accepting the conclusion that there are 5 residents who are members of both fitness clubs A and C but not B?
Total = none + (at least 1)
Total = none + (A or B or C)
55 = none + (A or B or C)
We have no information whether these 55 members are all members of at least 1 club.
If they are, then your answer is correct (x=5)
Let's say there are 5 people who are not members of these clubs (none =5),then the value of x will be 0.
Hence, **Option E**

30.

From 1, 16 students study both French and Japanese, so 16/0.04=400 students study French, combine "at least 100 students study Japanese", insufficient.
From 2, we can know that, 10% Japanese studying students=4% French studying students.
Apparently, more students at the school study French than study Japanese.

The correct answer is B.

31.
Statement 1 is sufficient.
For 2, I=A+B+C-AB-AC-BC+ABC, we know A, B ,C, AB, AC, BC, but we don't know I, so, ABC cannot be resolve out.

The correct answer is A.

32.

1). The total number is 120, then the number is: 120*2/3*(1-3/5)=32
2). 40 students like beans, then total number is 40/1/3=120, we can get the same result.
The correct answer is D.

GMAT Quant Topic 1: General Arithmetic
Part B: Percentages

1. Percentage problems involving unspecified amounts can usually be solved more easily by using the number 100. If Arthur's fortune was originally \$100, each of his children received \$20.

   Let's see what happened to each \$20 investment in the first year:

   Alice: \$20 + \$10 profit = \$30
   Bob: \$20 + \$10 profit = \$30
   Carol: \$20 + \$10 profit = \$30
   Dave: \$20 - \$8 loss = \$12
   Errol: \$20 - \$8 loss = \$12

   We continue on with our new amounts in the second year:

   Alice: \$30 + \$3 profit = \$33
   Bob: \$30 + \$3 profit = \$33
   Carol: \$30 - \$18 loss = \$12
   Dave: \$12 + \$3 profit = \$15
   Errol: \$12 - \$12 = 0

   At the end of two years, \$33 + \$33 + \$12 + \$15 = \$93 of the original \$100 remains.

   The correct answer is A.

2. This is a weighted average problem; we cannot simply average the percentage of silver cars for the two batches because each batch has a different number of cars. The car dealership currently has 40 cars, 30% of which are silver. It receives 80 new cars, 60% of which are silver (the 40% figure given in the problem refers to cars which are *not* silver). Note that the first batch represents 1/3 of the total cars and the second batch represents 2/3 of the total cars. Put differently, in the new total group there is 1 *first-batch* car for every 2 *second-batch* cars.

We can calculate the weighted average, weighting each percent according to the ratio of the number of cars represented by that percent:

Weighted average = $\frac{1(30\%)+2(60\%)}{3}$ = 50

Alternatively, you can calculate the actual number of silver cars and divide by the total number of cars. 40(0.3) + 80(0.6) = 12 + 48 = 60. 60/120 = 50%.

The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

Conventional:
Silver: 30%(40) + 60%(80)= 12+48 = 60
(Out of 80, 40% are not silver so 60% will be silver)
Total : 40 + 80 : 120

Hence % of silver: 60/120 : 50%

Weighted:
Sum of all samples = (total samples) * Average
=> 30%(40) + 60%(80) = (80+40)* Average
=> 12 + 48 = 120* Average
=> 60 = 120* Average
=> Average = 1/2 = 50%
The correct answer is E.

3. Notice that Paul's income is expressed as a percentage of Rex's and that the other two incomes are expressed as a percent of Paul's.

   Lets assign a value of \$100 to Rex's income. Paul's income is 40% less than Rex's income, so (0.6)(\$100) =\$60.

   Quentin's income is 20% less than Paul's income, so (0.8)(\$60) = \$48.

   Sam's income is 40% less than Paul's income, so (0.6)(\$60) = \$36.

   If Rex gives 60% of his income, or \$60, to Sam, and 40% of his income, or \$40, to Quentin, then: Sam would have \$36 + \$60 = \$96 and Quentin would have \$48 + \$40 = \$88.

   Quentin's income would now be \$88/\$96 = 11/12 that of Sam's.

   The correct answer is A.

4.
Let's denote the formula for the money spent on computers as $pq = b$, where
p = price of computers
q = quantity of computers
b = budget

We can solve a percent question that doesn't involve actual values by using smart numbers. Let's assign a smart number of 1000 to last year's computer budget (b) and a smart number 100 to last year’s computer price (p). 1000 and 100 are easy numbers to take a percent of.

This year’s budget will equal $1000 \times 1.6 = 1600$
This year’s computer price will equal $100 \times 1.2 = 120$

Now we can calculate the number of computers purchased each year, $q = b/p$
Number of computers purchased last year = 1000/100 = 10
Number of computers purchased this year = 1600/120 = 13 1/3 (while 1/3 of a computer doesn’t make sense it won’t affect the calculation)

	p	q	b
This Year	100	10	1000
Last Year	120	13 1/3	1600

The question is asking for the percent increase in quantity from last year to this year

$$= \frac{\text{new}-\text{old}}{\text{old}} \text{ x } 100\ \% = \frac{\frac{40}{3}-10}{10} \text{ x } 100\% = 33\frac{1}{3}\%$$

This question could also have been solved algebraically by converting the percent increases into fractions.

Last year: $pq = b$, so $q = b/p$
This year: $(6/5)(p)(x) = (8/5)b$
If we solve for x (this year's quantity), we get $x = (8/5)(5/6)b/p$ or $(4/3)b/p$
If this year's quantity is 4/3 of last year's quantity (b/p), this represents a 33 1/3% increase.

The correct answer is A.

5. This problem can be solved most easily with the help of smart numbers. With problems involving percentages, 100 is typically the “smartest” of the smart numbers.

   If we assume that today’s population is 100, next year it would be $1.1 \times 100 = 110$, and the following year it would be $1.1 \times 110 = 121$. If this is double the population of one year ago, the population at that time must have been $0.5 \times 121 = 60.5$. Because the problem seeks the “closest” answer choice, we can round 60.5 to 60.

   In this scenario, the population has increased from 60 to 100 over the last year, a net increase of 40 residents. To determine the percentage increase over the last year, divide the net increase by the initial population: 40/60 = 4/6 = 2/3, or roughly 67%.

   For those who prefer the algebraic approach: let the current population equal p. Next year the population will equal $1.1p$, and the following year it will equal

$1.1 \times 1.1p = 1.21p$. Because the question asks for the closest answer choice, we can simplify our algebra by rounding $1.21p$ to $1.2p$. Half of $1.2p$ equals $0.6p$. The population increase would be equal to $0.4p/0.6p = 0.4/0.6 = 2/3$, or roughly 67%.

The correct answer is D.

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Let the population this year be x

Population one year later = 1.1x

Population two years later = 1.21x

Let the population one year ago be y

According to the question, 1.21x = 2y

So, y = 0.605x

We want to know the population increase in the past year. In the past year, the population increased from y = 0.605x to x.

Population increase % = (1 - 0.605)/0.605 = 0.395/0.605 = 65%

The correct answer is D.

6.

To solve this problem, first find the wholesale price of the shirt, then compute the price required for a 100% markup, then subtract the $45 initial retail price to get the required increase.

Let x equal the wholesale price of the shirt. The retailer marked up the wholesale price by 80% so the initial retail price is x + (80% of x). The following equation expresses the relationship mathematically:

$x + 0.80x = 45$
$1.8x = 45$
$x = 45/1.8$
$x = 450/18$
$x = 25$

Since the wholesale price is $25, the price for a 100% markup is $50. Therefore, the retailer needs to increase the $45 initial retail price by $5 to achieve a 100% markup.

The correct answer is E.

7.
We can solve this as a VIC (Variable In answer Choices) and plug in values for *x* and *r*.

R	cents per person per mile	10
X	# of miles	20

Since there are 3 people, the taxi driver will charge them 30 cents per mile.
Since they want to travel 20 miles, the total charge (no discount) would be (30)(20) = 600.
With a 50% discount, the total charge will be 300 cents or 3 dollars.

If we plug $r = 10$ and $x = 20$ into the answer choices, the only answer that yields 3 dollars is D.

The correct answer is D.

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Actual charge = r
After discount = r - 50% r = r/2

r/2 cents per person per mile
3r/2 cents per 3 person per mile
3rx/2 cents per 3 person per x miles

So 3rx/200 dollars.

The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

The driver charges people r cents per person per mile. Meaning, to drive 1 person for 1 mile, the driver r cents

Now, for the trip mentioned, there are 3 people who are travelling a distance of x miles. Therefore, the driver would charge:

(r cents per person per mile) * (3 people) * (x miles) = 3rx cents

But it is given that the driver gives a 50% discount.
Therefore, fare = 1/2 * 3rx = 3rx/2 cents

In dollars, fare = (3rx/2)/100 = 3rx/200 dollars.
The correct answer is D.

8.
Bob put 20 gallons of gasohol into his car's gas tank, consisting of 5% ethanol and 95% gasoline. Chemical Mixture questions can be solved by using a mixture chart.

SUBSTANCES	AMOUNT	PERCENTAGE
ETHANOL	1	5%
GASOLINE	19	95%
TOTALS	20	100%

This chart indicates that there is 1 gallon of ethanol out of the full 20 gallons, since 5% of 20 gallons is 1 gallon.

Now we want to add *x* gallons of ethanol to raise it to a 10% ethanol mixture. We can use another mixture chart to represent the altered solution.

SUBSTANCES	AMOUNT	PERCENTAGE
ETHANOL	$1 + x$	10%
GASOLINE	19	90%
TOTALS	$20 + x$	100%

Therefore, the following equation can be used to solve for *x*:

$$\frac{1+x}{20+x} = 10\%$$

$1 + x = 2 + 0.1x$

$0.9x = 1$

$x = 10/9$

The correct answer is C.

9. Noting that 65% is very close to 2/3, we may approximate the original expression as follows:

1/3 + 0.4 + 65%	Original expression
1/3 + 0.4 + 2/3	Close approximation
1 + 0.4	
1.4	

The correct answer is D.

10.
First, let's find the initial amount of water in the tank:
Total mixture in the tank =1/4 × (capacity of the tank) = (1/4) × 24 = 6 gallons
Concentration of water in the mixture = 100% – (concentration of sodium chloride) = 100% – 40% = 60%

Initial amount of water in the tank = 60% × (total mixture) = 0.6 × 6 = 3.6 gallons

Next, let's find the amount and concentration of water after 2 hours:
Amount of water that will evaporate in 2 hours = (rate of evaporation) (time) = 0.5(2) = 1 gallon
Remaining amount of water = initial amount – evaporated water = 3.6 – 1 = 2.6 gallons
Remaining amount of mixture = initial amount – evaporated water = 6 – 1 = 5 gallons
Concentration of water in the mixture in 2 hours =$\frac{remaining\ water}{remaining\ mixture}$×100%
which equals: $\frac{2.6}{5}$× 100% = 52%

The correct answer is C.

11.
One of the most effective ways to solve problems involving formulas is to pick numbers. Note that since we are not given actual values but are asked to compute only the relative change in the useful life, we can select easy numbers and plug them into the formula to compute the percentage increase. Lets pick $d = 3$ and $h = 2$ to simplify our computations:

Before the change: $d = 3$, $h = 2$; $u = (8)(3)/2^2 = 24/4 = 6$
After the change: $d = (2)(3)= 6$, $h =2/2 =1$; $u = (8)(6)/1^2 = 48$

Finally, percent increase is found by first calculating the change in value divided by the original value and then multiplying by 100:
(48 – 6)/6 = (42/6) = 7
(7)(100) = 700%

The correct answer is D.

12. Since there are variables in the answer choices, as well as in the question stem, one way to approach this problem is to pick numbers and test each answer choice. We know that x is m percent of $2y$, so pick values for m and y, then solve for x.
$y = 100$
$m = 40$
x is m percent of $2y$, or x is 40 percent of 200, so $x = (0.40)(200) = 80$.

So, for the numbers we are using, m is what percent of x? Well, $m = 40$, which is half of $x = 80$. Thus, m is 50 percent of x. The answer choice that equals 50 will be the correct choice.

(A) $y/200 = 100/200 = 0.5$ WRONG
(B) $2y = (2)(100) = 200$ WRONG
(C) $50y = (50)(100) = 5000$ WRONG
(D) $50/y = 50/100 = 0.5$ WRONG
(E) $5000/y = 5000/100 = 50$ CORRECT

Alternatively, we can pursue an algebraic solution.

We are given the fact that x is m percent of $2y$, or $x = (m/100)(2y) = my/50$. Since the question asks about m ("m is what percent of x?"), we should solve this equation for m to get $m = (50/y)(x)$.

Putting the question "m is what percent of x?" into equation form, with the word "*Answer*" as a placeholder, $m = (Answer/100)(x)$.

Now we have two equations for m. If we set them equal, we can solve for the "*Answer*."
$(Answer/100)(x) = (50/y)(x)$
$(Answer/100) = (50/y)$
$Answer = 5000/y$

The correct answer is E.

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Lets look at the following solution

m,y > 0.

x is m% of 2y

x = m/100 * 2y

m = (50/y) * x [Equation 1]

We want to find "m is what percent of x?"

m = z% of x   [Equation 2]

From 1 and 2,

z% = (50/y)
z = 5000/y

The correct answer is E.

13. The easiest way to solve this problem is to use the VIC method of substituting actual numbers for x and y. The problem asks us to take x% of y and increase it by x%. Since we are dealing with percentages, and the whole (y) is neither given to us nor asked of us, let's set $y = 100$ and $x = 10$. Note that this is a variation on the typical method of picking small numbers in VIC problems.

10% of 100 is 10. Increasing that 10 by 10% gives us 10 + 1 = 11. Therefore 11 is our target number. Let's test each answer choice in turn to see which of them matches our target number.

(A) $100xy + x$ = 100(10)(100) + 10 which doesn't equal 11.
(B) $xy + x/100$ = 10(100) + 10/100 which doesn't equal 11.
(C) $100xy + x/100$ = 100(10)(100) + 10/100 which doesn't equal 11.
(D) $100xy$ +xy/100 = 100(10)(100)+ 1000/100 which doesn't equal 11.
(E) xy(x + 100)/10000 = 10(100)(10+100)/10000 which is equal to 11

The correct answer is E.

14.
First, determine the total cost of the item at each store.

Store A:
$60 (MSRP)
+ $12 (+ 20% mark-up = 0.20 × $60)
$72.00 (purchase price)
+ $3.60 (+ 5% sales tax = 0.05 × $72)
$75.60 (total cost)

Store B:
$60 (MSRP)
+ $18 (+ 30% mark-up = 0.30 × $60)
$78.00 (regular price)
– $7.80 (–10% sale = –0.10 × $78)
$70.20 (current purchase price)
+ $3.51 (5% sales tax = 0.05 × $70.20)
$73.71 (total cost)

The difference in total cost, subtracting the Store B cost from the Store A cost, is thus $75.60 - $73.71 = $1.89.

The correct answer is D.

15.
Given an initial deposit of $1,000, we must figure out the ending balance to calculate the total percent change.
After the first year, Sam's account has increased by $100 to $1,100.
After the second year, Sam's account again increased by 10%, but we must take 10% of $1,100, or $110. Thus the ending balance is $1,210 ($1,100 + $110).

To calculate the percent change, we first calculate the difference between the ending balance and the initial balance: $1,210 – $1,000 = $210. We divide this difference by the initial balance of $1,000 and we get $210/$1,000 = .21 = 21%.

The correct answer is C.

16.
Problems that involve successive price changes are best approached by selecting a smart number. When the problem deals with percentages, the most convenient smart number to select is 100. Lets assign the value of 100 to the initial price of the painting and perform the computations:

Original price of the painting = 100.
Price increase during the first year = 20% of 100 = 20.
Price after the first year = 100 + 20 = 120.
Price decrease during the second year = 15% of 120 = 18.
Price after the second year = 120 – 18 = 102.
Final price as a percent of the initial price = (102/100) = 1.02 = 102%.

The correct answer is A.

17. We can solve this question as a VIC (Variable in answer choices) by plugging in values for *x*, *y* and *z*:

x	percent mark-up (1st)	10
y	percent discount (2nd)	20
z	original price	100

If a \$100 item is marked up 10% the price becomes \$110. If that same item is then reduced by 20% the new price is \$88.
If we plug $x = 10$, $y = 20$, $z = 100$ into the answer choices, only answer choice (A) gives us:

$$\frac{10000(100) + 100(100)(10 - 20) - (10)(20)(100)}{10000} = 88$$

Alternatively, we could have solved this algebraically:

A price markup of *x* percent is the same as multiplying the price *z* by (1 + x/100)

A price discount of *y* percent is the same as by multiplying by (1-y/100)

We can combine these as: $z(1 + x/100)(1 – y/100)$.
This can be simplified to: (10,000z + 100z(x – y) – xyz)/10000

The correct answer is A.

18. If p is the price that the shop originally paid for the clock, then the price that the collector paid was $1.2p$ (to yield a profit of 20%). When the shop bought back the clock, it paid 50% of the sale price, or $(.5)(1.2)p = .6p$. When the shop sold the clock again, it made a profit of 80% on $.6p$ or $(1.8)(.6)p = 1.08p$.

The difference between the original cost to the shop (p) and the buy-back price ($.6p$) is \$100.
Therefore, $p - .6p = \$100$. So, $.4p = \$100$ and $p = \$250$.

If the second sale price is $1.08p$, then $1.08(\$250) = \$270$. (Note: at this point, if you recognize that $1.08p$ is greater than \$250 and only one answer choice is greater than \$250, you may choose not to complete the final calculation if you are pressed for time.)

The correct answer is A.

19. We are told that the boys of Jones Elementary make up 40% of the total of x students.
Therefore: # *of boys* = $.4x$
We are also told that x% of the # *of boys* is 90.
Thus, using $x/100$ as x%:
$(x/100) \times$ (# *of boys*) = 90

Substituting for # *of boys* from the first equation, we get:
$(x/100) \times .4x = 90$
$(.4x^2) / 100 = 90$
$.4x^2 = 9{,}000$
$x^2 = 22{,}500$
$x = 150$

Alternatively, we could have plugged in each answer choice until we found the correct value of x. Because the answer choices are ordered in ascending order, we can start with answer choice C. That way, if we get a number too high, we can move to answer choice B and if we get a number too low, we can move to answer choice D.

Given an x of 225 in answer choice C, we first need to take 40%. We do this by multiplying by .4.
$.4 \times 225 = 90$

Now, we need to take x% of this result. Again, x% is just $x/100$, in this case 225/100 or 2.25.
Thus x% of our result is: $2.25 \times 90 = 202.5$

This is too high so we try answer choice B. Following the same series of calculations we get:
$.4 \times 150 = 60$

$x\% = 150/100 = 1.5$
$1.5 \times 60 = 90$
This is the result we are looking for, so we are done.

The correct answer is B.

20. The dress has three different prices throughout the course of the problem: the original price (which we will call x), the initial sales price (\$68) and the final selling price (which we will call y). In order to answer the question, we must find the other two prices x and y.
According to the problem, (the original price) × 85% = initial sales price = \$68, therefore $x = 68 / 0.85$. How can we do this arithmetic efficiently? 0.85 is the same as 85/100 and this simplifies to 17/20. $68 / (17/20) = 68 \times (20/17)$. 17 goes into 68 four times, so the equation further simplifies to $4 \times 20 = 80$. The original price was therefore \$80.
According to the problem, the initial sales price × 125% = final selling price, therefore $68 \times 125\% = y$. Multiplying by 125% is the same thing as finding 25% of 68 and adding this figure to 68. 25% of 68 is 17, so the final selling price was \$68 + \$17 = \$85.
The difference between the original and final prices is \$85 – \$80 = \$5.

The correct answer is D.

21.
If we denote the amount of money owned by Jennifer as j and that owned by Brian as b, we can create two equations based on the information in the problem.
First, Jennifer has 60 dollars more than Brian: $j = b + 60$.

Second, if she were to give Brian 1/5 of her money, she would have $j - (1/5)j = (4/5)j$ dollars. Brian would then have $b + (1/5)j$ dollars. Therefore, since Brian's amount of money would be 75% of Jennifer's, we can create another equation:
$b + (1/5)j = (0.75)(4/5)j$, which can be simplified as follows:
$b + (1/5)j = (0.75)(4/5)j$
$b + (1/5)j = (3/4)(4/5j)$
$b + (1/5)j = (3/5)j$
$b = (3/5)j - (1/5)j$
$b = (2/5)j$

Substitute this expression for b back into the first equation, then solve for j:
$j = b + 60$
$j = (2/5)j + 60$
$j - (2/5)j = 60$
$(3/5)j = 60$
$j = (60)(5/3) = 100$
Therefore, Jennifer has 100 dollars.

The correct answer is B.

22.
In this case, the average computer price three years ago represents the original amount.
The original amount = 80% of \$700 or \$560.
The *change* is the difference between the original and new prices
=> \$700 – \$560 = \$140.
% change = 25%.

The correct answer is C.

23. To determine the total capacity of the pool, we need to first determine the amount of water in the pool *before* the additional water is added. Let's call this amount b. Adding 300 gallons represents a 30% increase over the original amount of water in the pool. Thus, $300 = 0.30b$. Solving this equation, yields $b = 1000$. There are 1000 gallons of water originally in the pool. After the 300 gallons are added, there are 1300 gallons of water in the pool. This represents 80% of the pool's total capacity, T.

$1300 = .80T$

$1300 = (4/5)T$

$1300(5/4) = T$

$T = 1625$

The correct answer is E.

24. Instead of performing the computation, set up a fraction and see if terms in the numerator cancel out terms in the denominator: Notice that the 16 cancels out the two 4s and that the 1000 in the numerator cancels the 1000 in the denominator. Thus, we are left with $2 \times 3 \times 3 = 18$. This is the equivalent of 1.8×10.

The correct answer is D.

25. 0.35 is greater than 0.007 so it must represent more than 100% of .007.
This eliminates answer choices A, B, and C.
Use benchmarks values to help you arrive at the final answer:
100% of 0.007 = 0.007 (Taking 100% of a number is the equivalent of multiplying by 1.)
500% of 0.007 = 0.035 (Taking 500% of a number is the equivalent of multiplying by 5.)
5000% of 0.007 = 0.35 (Taking 5000% of a number is the equivalent of multiplying by 50.)
The correct answer is E.

26.
We are asked to find the dollar change in the price of the property during the second year. Since we know the percent changes in each year, we will be able to answer the question if we know the price of the property in any of these years, or, alternatively, if we know the dollar change in the property price in any particular year.

(1) SUFFICIENT: Since we know the price of the property at the end of the three- year period, we can find the original price and determine the price increase during the second year. Let p denote the original price of the property:

$p(1.1)(0.8)(1.25) = 22{,}000$
$1.1p = 22{,}000$
$p = 20{,}000$

Price of the property after the first year: $20{,}000(1.1) = 22{,}000$
Price of the property after the second year: $22{,}000(0.8) = 17{,}600$
Decrease in the property price = $22{,}000 - 17{,}600 = 4{,}400$

(2) SUFFICIENT: This information is sufficient to create an equation to find the original price and determine the dollar change during the second year:

Price at the end of the first two years: $p(1.1)(0.8) = 0.88p$
Price decrease over the two-year period = original price – ending price = $p - 0.88p = 0.12p$
Price increase in the third year = (price at the end of two years)(25%) = $0.88p(0.25) = 0.22p$

Thus, since we know that the difference between the price decrease over the first two years and the price increase over the third year was \$2,000, we can create the following equation and solve for p:

$0.22p - 0.12p = 2{,}000$
$0.1p = 2{,}000$
$p = 20{,}000$

Price of the property after the first year: $20{,}000(1.1) = 22{,}000$
Price of the property after the second year: $22{,}000(0.8) = 17{,}600$
Decrease in the property price = $4{,}400$

The correct answer is D.

27.
Let i be the salesman's income, let s be the salary, and let c be the commission.
From the question stem we can construct the following equation:
$i = s + c$
We are asked whether s accounts for more than half of i. We can thus rephrase the question as "Is s greater than c?"
SUFFICIENT: This allows us to construct the following equation:
$1.1i = s + 1.3c$
Since we already have the equation $i = s + c$, we can subtract this equation from the one above:
$.1i = .3c$
Notice that the s's cancel each other out when we subtract. We can isolate the c by multiplying both sides by 10/3 (the reciprocal of .3 or 3/10):
$(1/10)i = (3/10)c$
$(1/10)i \times (10/3) = (3/10)c \times (10/3)$
$(1/3)i = c$
Therefore c is one-third of the salesman's income. This implies that the salary must account for two-thirds of the income. Thus, we can answer definitively that the salary accounts for more than half of the income.
INSUFFICIENT: Either $s - c = .5s$ or $c - s = .5s$. Coupled with our knowledge that s and c must add to 100% of the salesman's income, we can say that one of the two is worth 75% of the income and the other is worth 25%.
However, we don't know which is the bigger number: s or c.

The correct answer is A.

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Given: {Income} = {salary} + {commission}. Question basically asks: is {salary} > {commission}?

(Statement 2) The difference between the amount of the salesman's base salary and the amount of the commission was equal to 50 percent of the salesman's base salary last year:

|{salary} - {commission}| = 0.5{salary}, notice that {salary} - {commission} is in absolute value sign ||, meaning that we can have two cases:

A. {salary} - {commission} = 0.5{salary};

0.5{salary} = {commission};

{salary} > {commission}, thus the answer would be YES;

Or:
B. {commission} - {salary} = 0.5{salary};

1.5{salary} = {commission};

{salary} < {commission}, thus the answer would be No.

Two different answers. So, not sufficient.

The correct answer is A.

28. Let's assume m is the number of hot dogs sold in May, and j is the number sold in June. We know that the vendor sold 10% more hot dogs in June than in May, so we can set up the following relationship: $j = 1.1m$. If we can find the value for one of these variables, we will be able to calculate the other and will, therefore, be able to determine the value of $m + j$.

(1) SUFFICIENT: If the vendor sold 27 more hot dogs in June than in May, we can say $j = 27 + m$. Now we can use the two equations to solve for j and m:

$j = 1.1m$
$j = 27 + m$

Substituting 1.1m in for j gives:

$1.1m = 27 + m$
$.1m = 27$
$m = 270$
$j = m + 27 = 297$

So the total number of hot dogs sold is $m + j = 270 + 297 = 567$.

(2) INSUFFICIENT: While knowing the percent increase from May to July gives us enough information to see that the number of hot dogs sold each month increased, it does not allow us to calculate the actual number of hot dogs sold in May and June. For example, if the number of hot dogs sold in May were 100, then the number sold in June would be $1.1(100) = 110$, and the number sold in July would be $1.2(100) = 120$. The total number sold in May and June would be $100 + 110 = 210$. However, the number sold in May could just as easily be 200, in which case the number sold in June would be $1.1(200) = 220$, and the number sold in July $1.2(200) = 240$. The total number for May and June in this case would be $200 + 220 = 420$.

The correct answer is A.

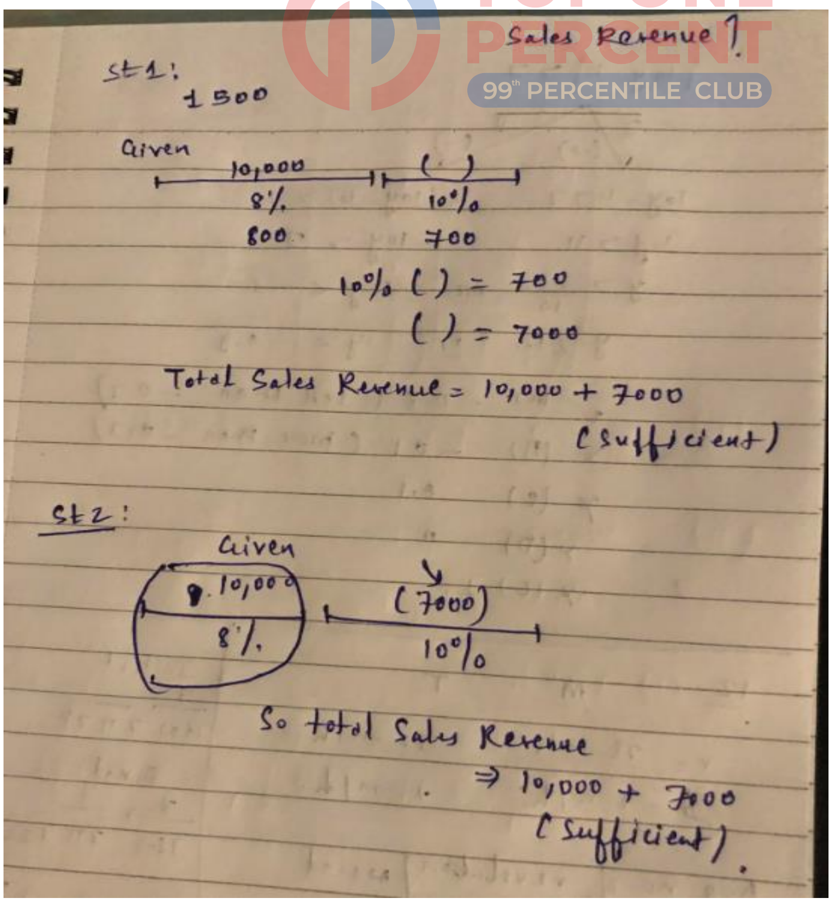
29. We can determine the sales revenue that the sales associate generated by analysing her commission earnings for the week.

SUFFICIENT: The sales associate earned a total of \$1500 in commission last week. We know that on the first \$10,000 in sales revenue, the associate earns 8% or \$800 in commission. This means that the associate earned \$700 in additional commission. Since this additional commission is calculated based on a 10% rate, the sales associate must have generated an additional \$7000 worth of sales revenue. Thus, we know from statement 1 that the sales associate generated \$10,000 + \$7000 = \$17,000 in sales revenue last week.
Statement 1 alone is sufficient.

SUFFICIENT: The sales associate was eligible for the 10% commission rate on \$7000 worth of sales. Since the 10% rate only kicks in after the first \$10,000 in sales, this means that the sales associate generated \$7000 in sales revenue *above* the \$10,000 threshold. Thus, we know from statement 2 that the sales associate generated \$10,000 + \$7000 = \$17,000 in sales revenue last week.
Statement 2 alone is sufficient.

The correct answer is D.

Top 1% expert replies to student queries (can skip)



We do know that he got 10% on 7000 (from given info in the question we also know that he got 8% commission on the first 10000 sales revenue)
So we know the total sales revenue (10000 + 7000)

8% on 10,000 and then 10% on additional (7000)

The correct answer is D.

30.
We are told that the team won y games out of a total of x games. Then we are asked for the value of y. We cannot rephrase the question in any useful way, so we must proceed to the statements.

(1) INSUFFICIENT: We are told that if the team had lost two more games, it would have won 20% of its games for the season. This implies that it would have lost 80% of its games under this condition. The number of games that the team lost is $x - y$. So, we can construct the following equation:

$$\frac{x - y + 2}{x} = \frac{80}{100}$$

$100x - 100y + 200 = 80x$
$20x + 200 = 100y$
$x + 10 = 5y$
This is not sufficient to tell us the value of y.

(1) INSUFFICIENT: We are told that if the team had won three more games, it would have lost 30% of its games for the season. This implies that it would have won 70% of its games under this condition. So, we can construct the following equation:

$$\frac{y + 3}{x} = \frac{70}{100}$$

$100y + 300 = 70x$
$10y + 30 = 7x$
This is not sufficient to tell us the value of y.

(1) AND (2) SUFFICIENT: We now have two different equations containing only the same two unknowns. We can use these equations to solve for y (though recall that you should only take the calculation far enough that you know you can finish, since this is data sufficiency):
$7x - 30 = 10y$
$x + 10 = 5y$
$7x - 30 = 10y$
$2(x + 10 = 5y)$

$7x - 30 = 10y$
$2x + 20 = 10y$

Subtract bottom equation from top:
$5x - 50 = 0$
$5x = 50$
$x = 10$

If $x = 10$, then all we need to do is plug 10 in for x in one of our equations to find the value of y:
$x + 10 = 5y$
$10 + 10 = 5y$
$20 = 5y$
$4 = y$

The correct answer is C.

31.
Let x represent the company's profits in 1992, y represent the profits in 1993, and z represent the profits in 1994. Since the profits in 1993 were 20% greater than in 1992, $y = 1.2x$, or $y/x = 1.2$. Similarly, since the profits in 1994 were 10% greater than in 1993, $z = 1.1y$, or $z/y = 1.1$. Since we have ratios relating the three variables, knowing the profits from any of the three years will allow us to calculate the profits in 1992.
So, the rephrased question is: "What is x, y, or z?"

(1) SUFFICIENT: This statement tells us that $z = 100{,}000 + y$. We also know the ratio of z to y: $z/y = 1.1$. Combining the two equations and substituting for z gives:

$z/y = 1.1$
$(100{,}000 + y)/y = 1.1$
$100{,}000 + y = 1.1y$
$100{,}000 = .1y$
$y = 1{,}000{,}000$

The profits in 1993 were $1,000,000. Since we know $y = 1.2x$, this information is sufficient to determine the profits in 1992.

(2) INSUFFICIENT: This tells us that the ratio of z to x is: $z/x = 3.96/3 = 1.32$.
However, we already know from information given in the question that:

$y = 1.2x$ and $z = 1.1y$
$z = 1.1(1.2x)$
$z = 1.32x$
$z/x = 1.32$

So, statement (2) gives no new information.

The correct answer is A.

32.
In order to answer the question, we must know the annual rain percentages for each weekday and the proportion each weekday represents relative to the total number of weekdays in year x. (Since we don't know the specific day that year x starts, we cannot assume that Mondays represent exactly 1/5 of the total weekdays in year x, Tuesdays represent 1/5 of the total weekdays in year x, etc.)

(1) INSUFFICIENT: This provides information about the percentage of Wednesday that it rained but this ALONE is not sufficient.

(2) INSUFFICIENT: This provides information about the percentage of Thursdays and the percentage of Fridays that it rained, but this ALONE is not sufficient.

(1) AND (2) INSUFFICIENT: Both statements together, in conjunction with the information given in the question, provide the annual rain percentages for each weekday during year x. However, because we do not know the proportion each weekday represents relative to the total number of weekdays in year x, we still do not have sufficient information to answer the question.

The correct answer is E.

33. According to the question stem,
total cost = fixed cost + variable
cost $C_t = C_f + C_v$

The question is asking for the percent change of the total cost of production of item X in January. Clearly if we knew the total cost of producing X before January and then in January, we could calculate the percent change. From the question, however, it doesn't seem like we will be provided with this information.

(1) INSUFFICIENT: Since the total cost of production is also the sum of the fixed and variable costs, it would stand to reason that we should be able to calculate the percent change to the total cost if we knew the percent change of the fixed and variable costs.

However, it is not that simple. We cannot simply average the percent change of the fixed and variable costs to find the percent change of the total cost of production. Two percents cannot be averaged unless we know what relative portions they represent.

Let's use numbers to illustrate this point. In the first set of numbers in the table below, the fixed cost is 100 times the size of the variable cost. Notice that the percent change of the total cost of production is almost identical to the percent change of the fixed cost. In the second set of numbers, the fixed and variable costs are identical.
Notice that the percent change of the total cost of production is exactly the average of the percent change of the variable cost and the fixed cost (4% is the average of 13% and -5%).

	Before	In January	TOTALS
C_f	100	113	+ 13%
C_v	1	.95	- 5%
C_t	101	113.95	≈ +13%
C_f	100	113	+ 13%
C_v	100	95	- 5%
C_t	200	208	+ 4%

(2) INSUFFICIENT: Lacking information about the percent change of the fixed cost, we cannot solve.

(1) AND (2) SUFFICIENT: Using the two statements, we not only have the percent changes of the fixed and variable percents, but we also know the relative portions they represent.

If the fixed cost before January was five times that of the variable cost, we can calculate the percent change to the cost of production using a weighted average:
Percent change of C_t = (5 x percent change of C_f + (1 x percent change of C_y))/6
Percent change of C_t = (5 × 13%) + (1 × -5%)/6 = 10%
Alternatively, if we try different values for C_f and C_v that each maintain the 5:1 ratio, we will come up with the same result. The cost of production increased in January by 10%.

The correct answer is C.

Top 1% expert replies to student queries (can skip)

Let the total cost in January be C_2 and the total cost before be C_1.

Given: $C_2 = F_2 + V_2$ and $C_1 = F_1 + V_1$, also $V_2 = 0.95V_1$.
Question: $\frac{C_2}{C_1} = \frac{F_2+V_2}{F_1+V_1} = \frac{F_2+0.95V_1}{F_1+V_1} = ?$

(1) The fixed cost of producing item X increased by 13% in January --> $F_2 = 1.13F_1$ --> $\frac{1.13F_1+0.95V_1}{F_1+V_1} = ?$. Not sufficient to get the exact fraction.

(2) Before the changes in January, the fixed cost of producing item X was 5 times the variable cost of producing item X --> $F_1 = 5V_1$ --> $\frac{F_2+0.95V_1}{5V_1+V_1} = ?$. Not sufficient.

(1)+(2) $F_2 = 1.13F_1$ and $F_1 = 5V_1$ --> $F_2 = 1.13F_1 = 5.65V_1$ --> from (2) $\frac{F_2+0.95V_1}{F_1+V_1} = ?$ --> substituting F_2 and F_1 --> $\frac{5.65V_1+0.95V_1}{5V_1+V_1} = \frac{6.6}{6} = 1.1$ --> in January total cost increased by 10%. Sufficient. (Actually no calculations are needed: stem and statement provide us with such relationships of 4 unknowns that 3 of them can be written with help of the 4th one and when we put them in fraction, which we want to calculate, then this last unknown is reduced, leaving us with numerical value).

Answer: C.

34.

This problem can be conceptualized with the help of smart numbers and some simple algebra. Because we are working with percentages and are given no real values, it is sensible to begin with the assumption that there are 100 attendees at the party. Therefore, there must be 40 females and 60 males.

Let m equal the number of men who arrived solo, w equals the number of women who arrived solo, and p equal the number of attendees who arrived with a companion ($p/2$ would equal the number of pairs). Using our smart numbers assumption, $m + w + p = 100$. This question might therefore be rephrased, "What is $m + w$?" or "What is $100 - p$?"

(1) SUFFICIENT: Given 60 male guests, Statement (1) tells us that 30 arrived with a companion. Therefore, 30 men and 30 women arrived in pairs. Recall that p equals the total number of guests arriving in pairs, so $p = 60$. Given that $100 - p$ is sufficient to solve our problem, Statement (1) is sufficient: 40 individuals (40% of the total number of guests) arrived at the party alone.

(2) SUFFICIENT: This statement tells us that

$.25(m + w) = w$
$.25m + .25w = w$
$.25m = .75w$
$m = 3w$

Further, observe that the total number of women at the party would equal the number arriving solo plus the number arriving with a companion:

$40 = w + p/2$
$80 = 2w + p$

Finally, recall that $m + w + p = 100$.

We now have three equations with three unknowns and are able solve for m, w and p, so Statement (2) is sufficient. While it is unnecessary to complete the algebra for this data sufficiency problem, witness:

Substituting $3w$ for m in the equation $m + w + p = 100$ yields $4w + p = 100$.

$2w + p = 80$
$4w + p = 100$

Subtracting the first equation from the second yields

$2w = 20$

$w = 10$
$p = 60$
$m = 30$

The correct answer is D.

Top 1% expert replies to student queries (can skip)

We can let the number of attendees be 100. Thus, there were 40 women and 60 men.

Statement One Alone:
50% of the male attendees arrived with a woman.

We see that there are 30 men arriving with their 30 female companions. That gives us 60 - 30 = 30 men and 40 - 30 = 10 women arriving alone. Thus, the percentage of people who arrived at the party alone is (30 + 10)/100 = 40/100 = 40%.

Statement one alone is sufficient.

Statement Two Alone:
25% of the attendees arriving alone were women.

Let the total number of people who arrived alone be n. Since 25% of the guests who arrived alone were women, n/4 women arrived alone, and 3n/4 men arrived alone. Since the number of women who arrived with a man must equal the number of men who arrived with a woman, we must have:

40 - n/4 = 60 - 3n/4

160 - n = 240 - 3n
2n = 80
n = 40
Thus, the percentage of people who arrived alone is 40/100 = 40%.
Statement two alone is sufficient.

The correct answer is D.

35. In order to answer this question, we must determine the value of a^b.
INSUFFICIENT: This tells us that $b = 2a$. However, this does not allow us to solve for a^b.
INSUFFICIENT: This tells us that $.5b = a$. This can be rewritten as $b = 2a$. However, this does not allow us to solve for a^b.
(1) AND (2) INSUFFICIENT: Since both statements provide the exact same information, taking the two together still does not allow us to solve for a^b.

The correct answer is E.

36.
Fat in milk is x*1%, y*2% and z*3%, respectively.
So we have the equation: x*1%+y*2%+z*3%=(x+y+z)*1.5%
Simplify the equation, we can obtain that x=y+3z

The correct answer is A.

Top 1% expert replies to student queries (can skip)

We are given that x gallons of the 1 percent grade, y gallons of the 2 percent grade, and z gallons of the 3 percent grade are mixed to give (x + y + z) gallons of a 1.5 percent grade. We can use this information to create the following equation:

$0.01x + 0.02y + 0.03z = 0.015(x + y + z)$

Multiplying the entire equation by 1000, we have:

$10x + 20y + 30z = 15x + 15y + 15z$

We must now solve for x in terms of y and z:

$5y + 15z = 5x$

We can divide the entire equation by 5:

$y + 3z = x$

The correct answer is A.

37.

For Statement 1, the tip for a \$15 bill will be \$2, which is less than \$15*15%=2.25; the tip for a \$20 will be \$4, which is greater than \$15*15%=2.25. **Insufficient**.

For Statement 2, tips is \$8, means the tens digit of the bill is 4, and the largest possible value of the bill is \$49. \$8>49*15%=7.35. **Sufficient alone**.

The correct answer is B.

38.

Let their hourly wage are x and y.

Therefore, after the increases, the difference between their wages is 1.06x-1.06y

From 1, x-y=5, we can solve out 1.06x-1.06y

From 2, x/y=4/3, insufficient.

The correct answer is A.

39.

Let M be the number of the cameras produced in 1995.

[M/(1+y%)]/(1+x%)=1000

M=1000+10x+10y+xy/10

Knowing that x+y+ xy/100 = 9.2, M can be solve out.

The correct answer is B.

40.

Let x and y be the numbers of the male and female students.

Combined 1 and 2,

35%X+20%Y=25%(X+Y)

10%X=5%Y

Y=2X

Now, X/(X+Y) = X/(X+2X) =1/3

The correct answer is C.

41.

For statement 1, x>75, then y>1.1*75>75

For statement 2, x=10, y=20 can fulfil the requirement, but the y<75

The correct answer is A.

42.
Obviously, 20%men+10%women>10%*(men+women).
The correct answer is B.

Top 1% expert replies to student queries (can skip)

Let the number of people who are 65 or older be x
We need to check if the number of employed people > x/10

Statement 1 : This is insufficient. This does not tell us anything about what fraction of those people are employed.

Statement 2 : Of the population 65 years old or above, 20 percent of the men and 10 percent of the women are employed.

We have assumed that the number of people who are 65 or older is x

Let us assume that the number of men 65 years old or above = y
Then the number of women 65 years old or above = x - y

20% of men = y/5
10% of women = (x-y)/10

Number of employed people 65 years or above = y/5 + (x - y)/10

We want the number of employed people to be greater than x/10
is y/5 + (x - y)/10 > x/10

is x/10 - y/10 > x/10. The answer is NO! Since y > 0, we know that the x/10 - y/10 < x/10, and so the number of employed people is less than 10%.

The correct answer is B.

43.
We know that: revenue=gross profit + expense
1). Revenue=(1/3)*expense + expense=(4/3)*expense, gross profit is 1/4 of its revenue.
2). Gross profit=revenue - expense=(1/4)*revenue, gross profit is 1/4 of its revenue.
The correct answer is D.

44.
Statement 1 says that the regular price of the most expensive item is 50 and the price of the next most expensive item is 20. Let the price of the least expensive item be x.

Sum of regular prices = (70+x)
Sum of discounts = (0.2)(50) + (0.1)(20) + 0.1x = (12+0.1x)

We need to check if the sum of discounts > 0.15 * Sum of regular prices.

12+0.1x > 0.15 * (70+x)
12+0.1x > 10.5 + 0.15x
1.5 > 0.05x
x < 30

So, for the above condition to be satisfied, x should be less than 30. But we already know that x<=20 since x is the price of the least expensive item. So, statement 1 is sufficient

Statement 2: The regular price of the least expensive item was 15

Let the price of the other 2 items be x and y, where x > y.

Discount on the least expensive item = 10% of 15 = 1.5
Discount on the middle item = 10% of y = y/10
Discount on the most expensive item = 20% of x = x/5

total amount of the 3 discounts = 1.5 + x/5 + y/10

Sum of regular prices of the 3 items = x + y + 15

We need to check if :

1.5 + x/5 + y/10 > 15/100 * [x + y + 15]
1.5 + x/5 + y/10 > 3x/20 + 3y/20 + 2.25
x/20 - y/20 > 0.75
x - y > 15

Therefore, for the condition to be satisfied, we need the above inequality to hold.

So if x = 40 and y = 20, then the condition is satisfied.
But if x = 50 and y = 40, then the condition will not be satisfied. Therefore, statement 2 is insufficient

The correct answer is A.

Alternate sol from gmatclub (additional)

Let the regular prices be a, b, and c, so that a > b > c.

Basically the questions: is $0.2a+0.1b+0.1c>0.15(a+b+c)$? --> is $a>b+c$?

(1) The regular price of the most expensive item was $50 and the regular price of the next most expensive item was $20 --> $a=50$, $b=20$, $c\leq 20$ (as the second most expensive item was $20 then the least expansive item, the third one, must be less than or equal to 20). So the question becomes: is $50>20+c$ --> is $c<30$? As we got that $c\leq 20$, hence the above is always true. Sufficient.

(2) The regular price of the least expensive item was $15. Clearly insufficient.

Answer: A.

45.

Rate = A^2/B, the question asks how shall A change to cope with a 100% increase of B to make the rate constant.

$(xA)^2/(2B) = A^2/B \Rightarrow x^2=2 \Rightarrow x=1.414 \Rightarrow$ A has to increase to 1.414A, equivalent to say an increase of approximate 40%

The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Let the rate be r (whatever units - if I remember any chemistry the rate of a reaction is measured in something called moles / unit volume / unit time, I may be wrong).

Let the concentration of Chemical A be a (again whatever units, but say moles / unit volume, again this is the actual unit of concentration usually I think)

Let the concentration of Chemical B be b

$r = ka^2$ [k is any real non-zero constant]

$r = d/b$ [d is any real mom-zero constant]

So combining the two we get that $r = (kd)\, a^2 / b$

If b becomes 2b [100% increase], the rate would become half, if the numerator didn't change. But it is changing. (kd) cannot change, and so say a becomes some quantity xa, where x is a number > 1

Now $(xa)^2$ will have to be equal to $2a^2$ (for the numerator and denominator 2 to cancel each other out and for r to remain what it is)

or, $x^2 . a^2 = 2a^2$

or, $x^2 = 2$ [a can cancel out because it is non-zero]

or, $x = \sqrt{2}$

or, $x \sim 1.414$

So a has to become 1.414a, which is roughly a 41.4% increase
equivalent to say an increase of approximate 40%

The correct answer is D.

46.

let y=x, the number of people working more than 10 years=560 (560-x)/(800-x)=60%=>x=200

The correct answer is A.

Top 1% expert replies to student queries (can skip) (additional)

We have 800 employees. Of these, 70%, that is 560 have worked for more than 10 years. Of these 560 employees, 'y' employees will retire and no new employees will join.

Therefore, total number of employees = 800-y
Number of employees who have worked for more than 10 years = 560-y

We need to find the value of y for which these employees comprise 60% of the total number of employees.

Therefore,

(560-y)/(800-y) = 0.6
560-y = 480-0.6y
0.4y=80
y=200

47.

Tax is the sum of the following:

2 percent of one's annual income - $0.02I$;

The average (arithmetic mean) of 100 units of country R's currency and 1 percent of one's annual income - $\frac{100+0.01I}{2}$.

$$Tax = 0.02 * I + \frac{100+0.01*I}{2} = 50 + \frac{0.04*I+0.01*I}{2} = 50 + \frac{0.05*I}{2} = 50 + \frac{I}{40}.$$

The correct answer is C.

48.
Let the least one is x.
When other 10 populations have the greatest value, x will have the minimum value.
X+10*1.1X=132000
X=11000
The correct answer is D.

49.
Let the price at the beginning is 1, then at the end of the first quarter, it was 1.2, at the end of the second quarter, it was 1.5.
(1.5-1.2)/1.2=25%
The correct answer is B.

50.
(150+profit)*40%=profit
So, the profit is $100
The correct answer is E.

Top 1% expert replies to student queries (can skip)

40% is only on x (selling price)
Purchase price = 150
Selling price = x
150 + 0.4*x = x
0.6*x = 150
x = 250
Profit = 250 - 150 = 100
The correct answer is E.

51.
The profit is sale price-cost
For one stock, (sale price-cost)/cost=20%, its cost is 96/1.2=80
For the other stock, (sale price-cost)/cost=-20%, cost is 96/0.8=120
Then, the total profit for the two stocks is:
96-80+96-120=-8
The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

So let's say Bobby bought the profit-making share at $x.
Then x + 0.2x = 96 or 1.2x = 96 or x = 80.
He bought the share at $80 and sold it at $96 to make a 20% profit

Let's say Bobby bought the loss-making share at $y.
Then y - 0.2y = 96 or 0.8y = 96 or y = 120.
He bought the share at $120 and sold it for $96 and took a 20% loss

So his total cost to purchase two shares was $200.
His total selling price of the two shares was $96 + $96 = $192.
So overall, he took a $8 loss.
The correct answer is C.

52.

Let total income of the family be 100$. In May Mrs Lee's earnings were 60 percent = 60$. Rest of the family's earnings = 40$.

In June family's earnings did not change = 40$, Mrs Lee's earnings = 60$*1.2 = 72$. Total = 40 + 72 = 112$. Mrs Lees share = $\frac{72}{112} \approx 64\%$.

Answer: A.

The correct answer is A.

53.
Amy was the 90th percentile of the 80 grades for her class, therefore, 10% are higher than Amy's, 10%*80=8.
19 of the other class was higher than Amy. Totally, 8+19=27
Then, the percentile is: (180-27)/180=85/100
The correct answer is D.

GMAT Quant Topic 1: General Arithmetic
Part C: Work / Rate

1. Let a be the number of hours it takes Machine A to produce 1 widget on its own. Let b be the number of hours it takes Machine B to produce 1 widget on its own.

   The question tells us that Machines A and B together can produce 1 widget in 3 hours. Therefore, in 1 hour, the two machines can produce 1/3 of a widget. In 1 hour, Machine A can produce $1/a$ widgets and Machine B can produce $1/b$ widgets. Together in 1 hour, they produce $1/a + 1/b = 1/3$ widgets.

   If Machine A's speed were doubled it would take the two machines 2 hours to produce 1 widget. When one doubles the speed, one cuts the amount of time it takes in half. Therefore, the amount of time it would take Machine A to produce 1 widget would be $a/2$. Under these new conditions, in 1 hour Machine A and B could produce $1/(a/2) + 1/b = 1/2$ widgets. We now have two unknowns and two different equations. We can solve for a.

   The two equations:
   $2/a + 1/b = 1/2$ (Remember, $1/(a/2) = 2/a$)
   $1/a + 1/b = 1/3$

   Subtract the bottom equation from the top:
   $2/a - 1/a = 1/2 - 1/3$
   $1/a = 3/6 - 2/6$
   $1/a = 1/6$
   Therefore, $a = 6$.
   The correct answer is E.

2.
Because Adam and Brianna are working together, add their individual rates to find their combined rate:
50 + 55 = 105 tiles per hour
The question asks how long it will take them to set 1400 tiles.
Time = Work / Rate = 1400 tiles / (105 tiles / hour) = 40/3 hours = 13 and 1/3 hours = 13 hours and 20 minutes
The correct answer is C.

3.
To find the combined rate of the two machines, add their individual rates:
35 copies/minute + 55 copies/minute = 90 copies/minute.
The question asks how many copies the machines make in half an hour, or 30 minutes.
90 copies/minute × 30 minutes = 2,700 copies.
The correct answer is B.

4. Tom's individual rate is 1 job / 6 hours or 1/6.
   During the hour that Tom works alone, he completes 1/6 of the job (using $rt = w$).
   Peter's individual rate is 1 job / 3 hours.
   Peter joins Tom and they work together for another hour;
   Peter and Tom's respective individual rates can be added together to calculate their combined rate: 1/6 + 1/3 = 1/2.
   Working together then they will complete 1/2 of the job in the 1 hour they work together.

   At this point, 2/3 of the job has been completed (1/6 by Peter alone + 1/2 by Peter and Tom), and 1/3 remains.

   When John joins Tom and Peter, the new combined rate for all three is:
   1/6 + 1/3 + 1/2 = 1.
   The time that it will take them to finish the remaining 1/3 of the job can be solved:
   $rt = w \longrightarrow (1)(t) = 1/3 \longrightarrow t = 1/3.$

   The question asks us for the fraction of the job that Peter completed. In the hour that Peter worked with Tom he alone completed:
   $rt = w \rightarrow w = (1/3)(1) = 1/3$ of the job.
   In the last 1/3 of an hour that all three worked together, Peter alone completed:
   (1/3)(1/3) = 1/9 of the job.
   Adding these two values together, we get 1/3 + 1/9 of the job = 4/9 of the job.

   The correct answer is E.

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Say Tom does t units of work in 1 hour. He does 6*t units of work in 6 hours
Exactly similarly, Peter does 3p in 3 hours and John does 2j in 2 hours
Each of these is individually equal to the total work
Then total work is 6t = 3p = 2j = w (say; any random variable to make the ratios uniform)
Then t = w/6; p = w/3; j = w/2

Tom works for 1 hour and hence does t units of work = w/6 units. He is then joined by Peter and they work together for 1 hour. Tom does t units more (for a total of w/3), and Peter does p units = w/3

Total work that has been done so far is 2w/3. Amount left is w/3

When John joins them, they (Tom, Peter and John) each are working at w/6, w/3 and w/2 per hour respectively. Say they take y hours more to complete the work.
Then y(w/6 + w/3 + w/2) = w/3

Or y(w) = w/3
Or y = 1/3 (w is the total amount of work and is not equal to 0)
In 1/3 hours, Peter would have done w/9 (= 1/3 x w/3) units of work

So Peter did a total of 4w/9 work (sum of the bolded parts above). Total work that was done was w

Then Peter's fraction of the full work is 4/9
The correct answer is E.

5. We can solve this problem as a VIC (Variable In Answer Choice) and plug in values for the two variables, x and y. Let's say $x = 2$ and $y = 3$.

Machine A can complete one job in 2 hours. Thus, the rate of Machine A is 1/2.

Machine B can complete one job in 3 hours. Thus, the rate of Machine B is 1/3.

The combined rate for Machine A and Machine B working together is: 1/2 + 1/3 = 5/6.

Using the equation (Rate)(Time) = Work, we can plug 5/6 in for the combined rate, plug 1 in for the total work (since they work together to complete 1 job), and calculate the total time as 6/5 hours.

The question asks us what fraction of the job machine B will NOT have to complete because of A's help. In other words we need to know what portion of the job machine A alone completes in that 6/5 hours.

A's rate is 1/2, and it spends 6/5 hours working. By plugging these into the RT=W formula, we calculate that, A completes (1/2)(6/5) = 3/5 of the job. Thus, machine B is saved from having to complete 3/5 of the job.

If we plug our values of $x = 2$ and $y = 3$ into the answer choices, we see that only answer choice E yields the correct value of 3/5.
The correct answer is E.

6. We can solve this problem as a VIC (Variable In answer Choice) and plug in values for the variable x. Let's say $x = 6$. (Note that there is a logical restriction here in terms of the value of x. Lindsay has to have a rate of less than less than 1 room per hour if she needs Joseph's help to finish in an hour).

If Lindsay can paint 1/6 of the room in 20 minutes (1/3 of an hour), her rate is 1/2.
$rt = w$
$r(1/3) = 1/6$
$r = 1/2$

Let J be the number of hours it takes Joseph to paint the entire room. Joseph's rate then is $1/J$. Joseph and Lindsay's combined rate is $1/2 + 1/J$, which can be simplified:

$$1/2 + 1/J \longrightarrow J/2J + 2/2J \longrightarrow (J+2)/2J$$

If the two of them finish the room in one hour, using the formula of $rt = w$, we can solve for J.

$rt = w$ and $t = 1$ (hour), $w = 1$ (job)

$((J + 2) / 2J)(1) = 1 \longrightarrow J + 2 = 2J \longrightarrow J = 2$

That means that Joseph's rate is 1/2, the same as Lindsay's. The question though asks us what fraction of the room Joseph would complete in 20 minutes, or 1/3 of an hour.
$rt = w$
$(1/2)(1/3) = w$
$w = 1/6$

Now we must look at the answer choices to see which one is equal to 1/6 when we plug in $x = 6$. Only C works: $(6 – 3) / 18 = 1/6$.

The correct answer is C.

7.
The combined rate of individuals working together is equal to the sum of all the individual working rates.

Let s = rate of a smurf, e = rate of an elf, and f = rate of a fairy. A rate is expressed in terms of treehouses/hour. So, for instance, the first equation below says that a smurf and an elf working together can build 1 treehouse per 2 hours, for a rate of 1/2 treehouse per hour.
$s + e = 1/2$
$s + 2f = 1/2$
$e + f = 1/4$

The three equations can be combined by solving the first one for s in terms of e, and the third equation for f in terms of e, and then by substituting both new equations into the middle equation.

1) $s = 1/2 – e$

2) $(1/2 – e) + 2 (1/4 – e) = 1/2$

3) $f = 1/4 – e$

Now, we simply solve equation 2 for e: $(1/2 – e) + 2 (1/4 – e) = 1/2$
$2/4 – e + 2/4 – 2e = 2/4$
$4/4 – 3e = 2/4$
$-3e = -2/4$
$e = 2/12$
$e = 1/6$
Once we know e, we can solve for s and f:

$s = 1/2 – e$
$s = 1/2 – 1/6$ $s = 3/6 – 1/6$ $s = 2/6$
$s = 1/3$
$f = 1/4 – e$
$f = 1/4 – 1/6$

$f = 3/12 – 2/12$
$f = 1/12$
We add up their individual rates to get a combined rate:
$e + s + f = 1/6 + 1/3 + 1/12 = 2/12 + 4/12 + 1/12 = 7/12$

Remembering that a rate is expressed in terms of treehouses/hour, this indicates that a smurf, an elf, and a fairy, working together, can produce 7 treehouses per 12 hours. Since we want to know the number of hours per treehouse, we must take the reciprocal of the rate. Therefore, we conclude that it takes them 12 hours per 7 treehouses, which is equivalent to 12/7 of an hour per treehouse.

The correct answer is D.

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s,e,f - speed of smurf, elf, and fairy
t- time to build the treehouse with one smurf, one elf, and one fairy, working together

s+e=1/2
s+2f=1/2
e+f=1/4
s+e+f=1/t

fast way to solve: multiply first equation by 2 and sum three equations:

3*(s+e+f)=1+1/2+1/4
3*1/t=7/4
t=12/7

1/s+1/e=1/2
1/s+2/f=1/2
1/e+1/f=1/4

Make 1/s+1/e=1/s+2/f

1/e=2/f --> 2e=f
1/e+1/2e=1/4 --> 3/2e=1/4 --> e=6
1/s+1/6=1/2 --> 1/s= 2/6 --> s=3
1/3+2/f=1/2 --> 2f+12=3f --> f=12

Now its just
1/6+1/12+1/3 --> 1/12+2/12+4/12 --> 7/12 equals combined rate. we need t=1/7/12 --> 12/7hrs or ~1.71hrs

The correct answer is D.

Smurf: S work/hour
Elf: E work/hour
Fairy: F work/hour

2S + 2E = 1; 2S = 1-2E --- A
2S + 4F = 1; 1-2E+4F=1; 2E-4F=0 ---- 1
4E + 4F = 1 ----------2

Solving 1 and 2
E = 1/6; F=1/12;
Substituting them in A
S = 1/3

Now;
1/6+1/12+1/3 = 1/t
t = 12/7

The correct answer is D.

8.
Rate is defined as distance divided by time. Therefore:

The RATE of machine A = $\frac{\text{Distance}}{\text{Time}} = \frac{\mathbf{1}\text{ job}}{\sqrt{w}+\sqrt{w-1}} = \frac{1}{\sqrt{8}+\sqrt{7}}$

The RATE of machine B = $\frac{\text{Distance}}{\text{Time}} = \frac{\mathbf{1}\text{ job}}{\sqrt{w}+\sqrt{w-1}} = \frac{1}{\sqrt{7}+\sqrt{6}}$

The COMBINED RATE of machine A and machine B = $\frac{1}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{6}}$

This expression can be simplified by eliminating the roots in the denominators as follows:

$$\frac{1}{\left(\sqrt{8}+\sqrt{7}\right)}\frac{\left(\sqrt{8}-\sqrt{7}\right)}{\left(\sqrt{8}-\sqrt{7}\right)} + \frac{1}{\sqrt{7}+\sqrt{6}}\frac{\left(\sqrt{7}-\sqrt{6}\right)}{\left(\sqrt{7}-\sqrt{6}\right)} = \frac{\sqrt{8}-\sqrt{7}}{1} + \frac{\sqrt{7}-\sqrt{6}}{1} = \sqrt{8}-\sqrt{6}$$

The question asks us for the time, ***t***, that it will take both machines working together to finish one job.

Using the combined rate above and a distance of **1** job, we can solve for ***t*** as follows:

$$t = \frac{d}{r} = \frac{1}{\sqrt{8}-\sqrt{6}} = \frac{1}{\left(\sqrt{8}-\sqrt{6}\right)}\frac{\left(\sqrt{8}+\sqrt{6}\right)}{\left(\sqrt{8}+\sqrt{6}\right)} = \frac{\sqrt{8}+\sqrt{6}}{2} = \frac{1}{2}\left(\sqrt{8}+\sqrt{6}\right)$$

The correct answer is B.

9.
Since this is a work rate problem, we'll use the formula *rate* × *time* = *work*. Since we'll be calculating times, we'll use it in the form *time* = *work* / *rate*.

First let T_0 equal the time it takes to paint the houses under the speedup scenario. T_0 will equal the sum of the following two values:
1. the time it takes to paint y houses at a rate of x
2. the time it takes to paint $(80 – y)$ houses at a rate of $1.25x$.

$T_o = y/x + (80-y)/1.25x$
$T_o = 1.25y/1.25x + (80-y)/1.25x$
$T_o = (80+0.25y)/1.25x$

(Continued on next page)

Then let T_1 equal the time it takes to paint all 80 houses at the steady rate of x.
$T_1 = 80/x$
The desired ratio is T_0 / T_1
This equals T_0 times the reciprocal of T_1.
$T_0/T_1 = (80+0.25y)/100$

As a quick check, note that if $y = 80$, meaning they paint ALL the houses at rate x before bringing in the extra help, then $T_0/T_1 = 1$ as expected.

The correct answer is B.

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We're told that a paint crew gets a rush order to paint 80 houses in a new development. They paint the first Y houses at a rate of X houses per week. Realizing that they'll be late at this rate, they bring in some more painters and paint the rest of the houses at the rate of 1.25X houses per week. We're asked to find the total time it takes them to paint all the houses under this scenario as a fraction of the time it would have taken if they had painted all the houses at their original rate of X houses per week. This question can be solved in a number of different ways, including by TESTING VALUES (and the answer choices are written in such a way that you don't have to do too much math overall to answer the question).

To start, we should choose a value for X that will work well with 1.25X. Let's choose X = 4 (so 1.25X = 5). In addition, we should look to choose a value for Y that will leave a remaining number of house that will be a multiple of 5...

Let's TEST...
X = 4
Y = 20

For the first 20 houses, painting 4 houses/week will take 20/4 = 5 weeks.
For the remaining 60 houses, painting 5 houses/week will take 60/5 = 12 weeks.
Total time = 5 + 12 = 17 weeks

At the original rate, the 80 houses would take 80/4 = 20 weeks.

Thus, we're looking for a fraction that equals 17/20 = .85.... notice how that answer is a number that is LESS than 1. Considering how the answer choices are written, and that our Y = 20, you should be able to eliminate all of the wrong answers without doing too much math...

Answer A: (.8)(60) --> greater than 1
Answer B: (.8) + a tiny decimal --> exactly what we're looking for!
Answer C: 80/20 - 1.25 --> a little less than 3
Answer D: (80)/(1.25)(20) = 80/25 --> greater than 1
Answer E: (80) - (.25)(20) --> greater than 1

The correct answer is B.

10. There are several ways to achieve sufficiency in solving this rate problem, so the question cannot be rephrased in a useful manner.

(1) INSUFFICIENT: This statement provides the difference between the number of hot dogs consumed by the third-place finisher (lets call this t) and the number of hot dogs consumed by the winner (lets call this w). We now know that $w = t + 24$, but this does not provide sufficient information to solve for w.

(2) INSUFFICIENT: The third-place finisher consumed one hot dog per 15 seconds. To simplify the units of measure in this problem, lets restate this rate as 4 hot dogs per minute. Statement (2) tells us that the winner consumed 8 hot dogs per minute. This does not provide sufficient information to solve for w.

(1) AND (2) SUFFICIENT: The rate of consumption multiplied by elapsed time equals the number of hot dogs consumed. This equation can be restated as time = hot dogs/rate. Because the elapsed time is equal for both contestants, we can set the hot dogs/rate for each contestant equal to one another:

$w/8 = t/4$
$w = 2t$

Substituting $w - 24$ for t yields

$w = 2(w - 24)$
$w = 2w - 48$
$48 = w$

The correct answer is C.

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Information about 15 seconds is quite redundant. Say the third-place finisher ate T hot dogs and the winner ate W hot dogs.

(1) The winner consumed 24 more hot dogs than did the third-place finisher --> W=T+24 --> one equation two unknowns. Not sufficient.

(2) The winner consumed hot dogs at double the rate of the third-place finisher --> at double rate the winner would consume twice as many hot dogs, so W=2T --> one equation two unknowns. Not sufficient.

(1)+(2) We have two distinct linear equations with two unknowns, so we can get the single numerical values of T and W: T+24=2T --> T=24. Sufficient.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

Let the total time given to each participant be "T" seconds.

The third-place finisher took an average of 15 seconds to consume each hot dog. So total number of hot dogs eaten in T seconds = T/15 hot dogs

We want to find out the number of hot dogs eaten by the winner.

Combining statements 1 and 2,

Using statement 1, number of hot dogs eaten by the winner = (T/15+24)

Using statement 2, the winner ate the hot dogs at double the rate of the third-place finisher.

Meaning, that the time in which the third place finisher ate x hot dogs, the winner ate 2x hot dogs.

So if the third place finisher took 15 seconds on average to each a hot dog, the winner took 7.5 seconds.

So in T seconds, the winner ate (T/7.5) hot dogs

From the 2 statements,

T/15 + 24 = T/7.5 = 2T/15

T/15 = 24

T = 360 seconds

So, number of hot dogs eaten by the winner = T/7.5 = 360/7.5 = 48

Sufficient

11.
This is a work problem. We can use the equation Work = Rate × Time ($W = R \times T$) to relate the three variables Work, Rate, and Time. The question asks us to find the number of newspapers printed on Sunday morning. We can think of the "number of newspapers printed" as the "work done" by the printing press. So, the question is asking us to find the work done on Sunday morning, or W_{sunday}.

The printing press runs from 1:00 AM to 4:00 AM on Sunday morning, so $T_{sunday} = 3$ hours. Since $W_{sunday} = R_{sunday} \times T_{sunday}$, or in this case $W_m = R_{sunday} \times 3$, knowing the rate of printing, R_{sunday}, will allow us to calculate W_{sunday} (the number of newspapers printed on Sunday morning). Therefore, the rephrased question becomes: What is R_{sunday}?

(1) INSUFFICIENT: This statement tells us that $R_{saturday} = 2R_{sunday}$. While this relates Saturday's printing rate to Sunday's printing rate, it gives no information about the value of either rate.

(2) INSUFFICIENT: For Saturday morning, $W_{saturday} = 4{,}000$ and $T_{saturday} = 4$ hours. We can set up the following equation:

$W_{saturday} = R_{saturday} \times T_{saturday}$

$4{,}000 = R_{saturday} \times 4$

$R_{saturday} = 1{,}000$

This gives the rate of printing on Saturday morning, but fails to give any information about Sunday‟s rate.

(1) AND (2) SUFFICIENT: Statement (1) tells us that $R_{saturday} = 2R_{sunday}$ and statement (2) tells us that $R_{saturday} = 1{,}000$. Putting this information together yields:

$R_{saturday} = 2R_{sunday}$

$1{,}000 = 2R_{sunday}$

$500 = R_{sunday}$

The correct answer is C.

12.
To find the combined rate of Machines A and B, we combine their individual rates. If

Machine A can fill an order of widgets in *a* hours, then in 1 hour it can fill 1/a of the order. By the same token, if Machine B can fill the order of widgets in *b* hours, then in 1 hour, it can fill 1/b of the order.

So together in 1 hour, Machines A and B can 1/a + 1/b fill of the order:

$$\frac{1}{a} + \frac{1}{b} = \frac{(b)1}{(b)(a)} + \frac{(a)1}{(a)(b)} = \frac{b}{ab} + \frac{a}{ab} = \frac{a+b}{ab}$$

So, in 1 hour, Machines A and B can complete $\frac{a+b}{ab}$ of the order. To find the number of hours the machines need to complete the *entire* order, we can set up the following equation:

(Fraction of order completed in 1 hour) x (number of hours needed to complete entire order) = 1 order.

If we substitute $\frac{a+b}{ab}$ for the fraction of the order completed in 1 hour, we get:

$\frac{a+b}{ab}x = 1$ where x is the number of hours needed to complete the entire order.
If we divide both sides by $\frac{a+b}{ab}$, we get: $x = \frac{ab}{a+b}$

In other words, it will take Machines A and B $\frac{ab}{a+b}$ hours to complete the entire order working together at their respective rates.

The question stem tells us that a and b are both even integers. We are then asked whether a and b are equal. If they are equal, we can express each as $2z$, where z is a non-zero integer, because they are even. If we replace a and b with $2z$ in the combined rate, we get:

$$\frac{(2z)(2z)}{2z+2z} = \frac{4z^2}{4z} = z$$

So if a and b are equal, the combined rate of Machines A and B must be an integer (since z is an integer). We can rephrase the question as:

Is the combined rate of Machines A and B an integer?

Statement 1 tells us that it took 4 hours and 48 minutes for the two machines to fill the order (remember, they began at noon). This shows that the combined rate of Machines A and B is NOT an integer (otherwise, it would have taken the machines a whole number of hours to complete the order). So we know that a and b cannot be the same. Sufficient.

Statement 2 tells us that $(a + b)^2 = 400$. Since both a and b must be positive (because they represent a number of hours), we can take the square root of both sides of the equation without having to worry about negative roots. Therefore, it must be true that $a + b = 20$. So it is possible that $a = 10$ and that $b = 10$, which would allow us to answer "yes" to the question. But it is also possible that $a = 12$ and $b = 8$ (or any other combination of positive even integers that sum to 20), which would give us a "no". Insufficient.

The correct answer is A: Statement 1 alone is sufficient, but statement 2 alone is not.

The correct answer is A.

13. If water is rushing into tank 1 at x gallons per minute while leaking out at y gallons per minute, the net rate of fill of tank 1 is $x - y$. To find the time it takes to fill tank 1, divide the capacity of tank 1 by the rate of fill: $z / (x - y)$.

We know that the rate of fill of tank 2 is y and that the total capacity of tank 2 is twice the number of gallons remaining in tank 1 after one minute. After one minute, there are $x - y$ gallons in tank 1, since the net fill rate is $x - y$ gallons per minute. Thus, the total capacity of tank 2 must be $2(x - y)$.

The time it takes to fill tank two then is 2(x-y)/y

The question asks us if tank 1 fills up before tank 2

We can restate the question: Is [z/(x-y)] <[2(x-y)/y] ?
SUFFICIENT: We can manipulate $zy < 2x^2 - 4xy + 2y^2$:

$zy < 2x^2 - 4xy + 2y^2$
$zy < 2(x^2 - 2xy + y^2)$
$zy < 2(x - y)(x - y)$ (dividing by $x - y$ is okay since $x - y > 0$)
[z/(x-y)] < [2(x-y)/y] (dividing by y is okay since $y > 0$)

This manipulation shows us that the time it takes to fill tank 1 is definitely longer than the time it takes to fill tank 2.

INSUFFICIENT: We can express this statement algebraically as: 1/2(z) > 2($x - y$). We cannot use this expression to provide us meaningful information about the question.

The correct answer is A.

14.
From the question stem, we know that Bill's rate is 1 well per x! hours and Carlos's rate is 1 well per y! hours.

Therefore, their combined rate is $\frac{1}{x!} + \frac{1}{y!} = \frac{(x!) + (y!)}{x!y!}$.

Since q is the amount of time it takes Bill and Carlos to dig a well together, we can

use the rate formula R = D/T to find q in terms of x and y.

We can rearrange the formula to isolate T: T = D/R .

Since q is the amount of time (T) it takes the two men to dig 1 well together, the

"distance" (D) here is 1 well. Therefore, $T = \frac{1}{\frac{(x!) + (y!)}{x!y!}} = \frac{x!y!}{(x!) + (y!)}$. So we know that $q = \frac{x!y!}{(x!) + (y!)}$

The question then becomes: Is $\frac{x!y!}{(x!) + (y!)}$ an integer?

Statement (1) tells us that x – y =1. We now know that y = x - 1. We can substitute for y and simplify:

$q = \frac{x!(x-1)!}{(x!) + (x-1)!} \rightarrow$

$q = \frac{x!(x-1)!}{x(x-1)! + (x-1)!} \rightarrow$

$q = \frac{x!(x-1)!}{(x-1)!(x+1)} \rightarrow$

$q = \frac{x!\cancel{(x-1)!}}{\cancel{(x-1)!}(x+1)} \rightarrow$

$q = \frac{x!}{x+1}$

Is this sufficient to tell us whether q is an integer? Let's try some numbers. If $x = 5$, then $\frac{5!}{6} = \frac{120}{6} = 20$. But if $x = 2$, then $\frac{2!}{3} = \frac{2}{3}$. So, in one case we get an integer, in another case we get a fraction. Statement (1) alone is insufficient to answer the question.

Statement (2) tells us that y is a nonprime even number. This means y can be any even number other than 2. We cannot tell from this whether q is an integer. For example, if $y = 4$ and $x = 2$, then $\frac{2!4!}{(2!)+(4!)} = \frac{24}{13}$. But if $y = 4$ and $x = 5$, then $\frac{5!4!}{(5!)+(4!)} = 20$. So, in one case we get a fraction, in another we get an integer. Statement (2) alone is insufficient to answer the question.

If we take the statements together, we know that $q = \frac{x!}{x+1}$ and that y is an even number greater than 2 (because we are dealing with rates here, we do not have to worry about zero or negative evens).

Let's begin by analysing the denominator of q, the expression $x + 1$. Since y is even and $y = x - 1$, x must be odd. Therefore $x + 1$ must be even. If $x + 1$ is even, it must be the product of 2 and some integer (call it z) that is less than x.

Now let's analyse the numerator of q, the expression $x!$. Since x is greater than y, it must be greater than 2. This means $x!$ will have both 2 and z as factors (remember, z is less than x).

Therefore, both the 2 and z in $x + 1$ (the denominator of q) will cancel out with the 2 and z in $x!$ (the numerator of q), leaving only the product of integers.

For example, if $x = 5$ and $y = 4$,

$$\frac{5!4!}{(5!)+(4!)} \rightarrow$$

$$\frac{5!4!}{5(4!)+1(4!)} \rightarrow$$

$$\frac{5!4!}{(4!)(5+1)} \rightarrow$$

$$\frac{5!\cancel{4!}}{(\cancel{4!})(6)} \rightarrow$$

$$\frac{5!}{6} \rightarrow$$

$$\frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{3} \cdot \cancel{2}} \rightarrow$$

$$5 \cdot 4 \cdot 1 = 20$$

Therefore, if $x - y = 1$ and if y is an even number greater than 2, then q will always be an integer.

The correct answer is C: BOTH statements TOGETHER are sufficient but NEITHER statement ALONE is sufficient.

The correct answer is C.

Top 1% expert replies to student queries (can skip)

As per the question:
Rate of Bill+ Rate of Carlos = Combined rate
1/x!+1/y!=1/q
=> q=(x!∗y!)/(x!+y!)
Statement 2: no condition about X, thus not sufficient.
Statement 1: x -y=1 => x = y+1 => x!=(y+1)y!
q=(x!∗y!) / (x!+y!)=y!∗y!∗{(y+1)/[y!∗(y+2]}
=>q=(y+1)! / (y+2)
if y = 2 q = 3!/4 = 3/2 not an integer
if y = 4 Q= 5!/6 = 20 = integer
Thus not sufficient.
(1+2) :-
Considering both the statement y cannot be 2, for all even values of y except 2, q is an integer.
=> for all even values of y(except 2), q = integer.
if y = 4 Q= 5!/6 = 20 = integer
if y = 6 Q= 7!/8 = 630 = integer

The correct answer is C.

Alternate Solution from Gmatclub

$$\frac{1}{x!} + \frac{1}{y!} = \frac{1}{q} => q = \frac{(x!*y!)}{(x!+y!)}$$

Statement 2: no condition about X, thus not sufficient.

Statement 1: x -y=1 => x = y+1 => $x! = (y+1)y!$

$$q = \frac{(x!*y!)}{(x!+y!)} = y! * y! * \frac{(y+1)}{(y!*(y+2))}$$

$$=> q = \frac{(y+1)!}{(y+2)}$$

if y = 2 q = 3!/4 = 3/2 not an integer
if y = 4 Q= 5!/6 = 20 = integer

Thus not sufficient.

Considering both the statement y cannot be 2, for all even values of y except 2, q is an integer.

The correct answer is C.

15. 1/4/(1/4+1/8)=2/3
    The correct answer is D.

Top 1% expert replies to student queries (can skip)

Method-1: Plug-in
Machine A can produce K cartons in 8 hours
Machine B can produce K cartons in 4 hours
Let's Plug-in K = 2
So....
Machine A = 2 cartons every 8 hours
Machine B = 2 cartons every 4 hours
We're told that each machine works on its own for the SAME amount of time.
Let's say they both work for 8 hours. This means...
Machine A seals 2 cartons
Machine B seals 4 cartons
Total = 6 cartons
The question asks what ratio of the cartons the faster machine sealed. Machine B is the faster machine, and it sealed 4/6 of the cartons.
4/6 = 2/3 = 66 2/3%.
The correct answer is D.

Method:2- logic of ratios:
The First machine seals k cartons in 8 hours.
The Second machine seals k cartons in 4 hours i.e. 2k cartons in 8 hours.
If they both are working for the same period of time i.e. 8 hours, they together seal 3k cartons. Faster machine (second one) seals 2k out of these 3k so it seals 2/3 = 66.66% of the total cartons.
The correct answer is D.

GMAT Quant Topic 1: General Arithmetic
Part D: SPEED and DISTANCE

1.

Let b be the number of hours Bob spends biking. Then $(t - b)$ is the number of hours he spends walking. Let d be the distance in miles from his home to school. Since he had the flat tire halfway to school, he biked $d/2$ miles and he walked $d/2$ miles. Now we can set up the equations using the formula *rate* x *time* = *distance*. Remember that we want to solve for d, the total distance from Bob's home to school.

1) $xb = d/2$
2) $y(t - b) = d/2$

Solving equation 1) for b gives us:

3) $b = d/2x$ Substituting this value of b into equation 2 gives:

4) $y(t - d/2x) = d/2$ Multiply both sides by $2x$:

5) $2xy(t - d/2x) = dx$ Distribute the $2xy$

6) $2xyt - dy = dx$ Add dy to both sides to collect the d's on one side.

7) $2xyt = dx + dy$ Factor out the d

8) $2xyt = d(x + y)$ Divide both sides by $(x + y)$ to solve for d

9) $2xyt / (x + y) = d$

The correct answer is C.

2. We begin by figuring out Lexy's average speed. On her way from A to B, she travels 5 miles in one hour, so her speed is 5 miles per hour. On her way back from B to A, she travels the same 5 miles at 15 miles per hour. Her average speed for the round trip is NOT simply the average of these two speeds. Rather, her average speed must be computed using the formula $RT = D$, where R is rate, T is time and D is distance. Her average speed for the **whole** trip is the **total** distance of her trip divided by the **total** time of her trip.

   We already know that she spends 1 hour going from A to B. When she returns from B to A, Lexy travels 5 miles at a rate of 15 miles per hour, so our formula tells us that $15T = 5$, or $T = 1/3$. In other words, it only takes Lexy 1/3 of an hour, or 20 minutes, to return from B to A. Her total distance travelled for the round trip is 5+5=10 miles and her total time is 1+1/3=4/3 of an hour, or 80 minutes.

   We have to give our final answer in minutes, so it makes sense to find Lexy's average rate in miles per minute, rather than miles per hour. 10 miles / 80 minutes = 1/8 miles per minute. This is Lexy's average rate.

We are told that Ben's rate is half of Lexy's, so he must be traveling at 1/16 miles per minute. He also travels a total of 10 miles, so $(1/16)T = 10$, or $T = 160$. Ben's round trip takes 160 minutes.

Alternatively, we could use a shortcut for the last part of this problem. We know that Ben's rate is half of Lexy's average rate. This means that, for the entire trip, Ben will take twice as long as Lexy to travel the same distance. Once we determine that Lexy will take 80 minutes to complete the round trip, we can double the figure to get Ben's time. $80 \times 2 = 160$.

The correct answer is D.

3. There is an important key to answering this question correctly: this is not a simple average problem but a weighted average problem. A weighted average is one in which the different parts to be averaged are not equally balanced. One is "worth more" than the other and skews the "simple" average in one direction. In addition, we must note a unit change in this problem: we are given rates in miles per hour but asked to solve for rates in miles per minute.

Average rate uses the same D = RT formula we use for rate problems but we have to figure out the different lengths of time it takes Dan to run and swim along the total 4- mile route. Then we have to take the 4 miles and divide by that total time. First, Dan runs 2 miles at the rate of 10 miles per hour. 10 miles per hour is equivalent to 1 mile every 6 minutes, so Dan takes 12 minutes to run the 2 miles. Next, Dan swims 2 miles at the rate of 6 miles per hour. 6 miles per hour is equivalent to 1 mile every 10 minutes, so Dan takes 20 minutes to swim the two miles.

Dan's total time is 12 + 20 = 32 minutes. Dan's total distance is 4 miles. Distance / time = 4 miles / 32 minutes = 1/8 miles per minute.

Note that if you do not weight the averages but merely take a simple average, you will get 2/15, which corresponds to incorrect answer choice B. 6 mph and 10 mph average to 8mph. (8mph)(1h/60min) = 8/60 miles/minute or 2/15 miles per minute.

The correct answer is A.

4. The formula to calculate distance is Distance = (Rate)(Time). So at any given moment Tom's distance (let's call it D_T) can be expressed as $D_T = 6T$. So, at any given moment, Linda's distance (let's call it D_L) can be expressed as $D_L = 2(T + 1)$ (remember, Linda's time is one hour more than Tom's). The question asks us to find the positive difference between the amount of time it takes Tom to cover half of Linda's distance and the time it takes him to cover twice her distance. Let's find each time separately first.

When Tom has covered half of Linda's distance, the following equation will hold: $6T = (2(T + 1))/2$. We can solve for T:
$6T = (2(T + 1))/2$
$6T = (2T + 2)/2$
$6T = T + 1$
$5T = 1$
$T = 1/5$

So it will take Tom 1/5 hours, or 12 minutes, to cover half of Linda's distance.
When Tom has covered twice Linda's distance, the following equation will hold: $6T = 2(2(T + 1)$. We can solve for T:
$6T = 2(2(T + 1)$
$6T = 2(2T + 2)$
$6T = 4T + 4$
$2T = 4$
$T = 2$

So it will take Tom 2 hours, or 120 minutes, to cover twice Linda's distance.
We need to find the positive difference between these times: 120 – 12 = 108.

The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

Linda's speed = 2 miles/hr
Tom's speed = 6 miles/hr

Distance covered by Linda in 1 hour = 2 miles. After 1 hour, Tom starts moving in the opposite direction.

From the time when Tom starts moving, the relative speed of Tom wrt Linda = 8 mph [6+2, since they're moving in opposite directions.]

Now, at any time 'T' hours after Tom starts jogging, the distance between Tom and Linda = 2 + 8T [The initial distance between Tom and Linda is 2 miles. Once Tom starts moving, the relative distance between Tom and Linda is 8 mph. Therefore, distance increases by 8T every T hours]

Distance covered by Tom + Distance covered by Linda = Total distance between Tom and Linda = (8T + 2)

Now, we need to find the time that it takes Tom to travel half the distance travelled by Linda.

At any time T, distance covered by Tom = 6T; distance covered by Linda = (2T + 2)

6T = 1/2 * (2T+2)

T = 1/5 hours = 12 minutes

We also need to find the time that it takes Tom to travel twice the distance travelled by Linda.

At any time T, distance covered by Tom = 6T ; distance covered by Linda = (2T + 2)

6T = 2 * (2T+2)

T = 2 hours = 120 minutes.

Difference = 120 - 12 = 108 minutes.
The correct answer is E.

5. A question with variables in the answer choices (VIC) can be solved by picking values for the variables.

   Let's pick the following values for x, y and z:

x	4	time for high speed travel
y	6	time regular travel
z	12	distance from A to B

When picking values for a VIC question, it is best to pick numbers that are easy to work with (i.e., 12 is divisible by 4 and 6 here), but that don't have any extraneous relationships between them. For example, $x = 4$, $y = 3$, $z = 12$ would be a less favorable set of numbers because xy would equal z in that case and there is no need for the product of the two times to equal the distance. Picking variables with extraneous relationships can lead to false positives when checking the answer choices.

Now let's solve the question according to the values we selected.
If the high-speed train travels the 12 miles from A to B in 4 hours, it is traveling at 3 mph.
If the regular train travels the 12 miles from A to B in 6 hours, it is traveling at 2 mph.

To evaluate how far each train travels when they move toward each other starting at opposite ends, let's set up an RTD chart.

	High-speed	Regular	Total
R	3	2	
T	t	t	
D	d	$12 - d$	12

We can set-up two equations with two unknowns and solve.

$3t = d$
(+) $2t = 12 - d$
- - - - - - - - - - -
$5t = 12$, so $t = 2.4$

In the 2.4 hours it takes for the two trains to meet,
the high speed train will have traveled 3(2.4) = 7.2 miles,
and the regular train will have traveled 2(2.4) = 4.8 miles.
Therefore the high speed train will have traveled 7.2 – 4.8 = 2.4 miles farther than the regular train.

2.4 is our target number.

Let's see which of the five answer choices give us 2.4 when we plug in our values for x, y and z:

	Plug	Result	Match Target?
(A)	$\frac{12(6-4)}{4+6}$	2.4	Yes
(B)	$\frac{12(4-6)}{4+6}$	-2.4	No
(C)	$\frac{12(4+6)}{6-4}$	60	No
(D)	$\frac{4(6)(4-6)}{4+6}$	-4.8	No
(E)	$\frac{4(6)(6-4)}{4+6}$	4.8	No

Only A matches the target.

This question can also be solved algebraically.
Since the trains travelled the z miles in x and y hours, their speeds can be represented as z/x and z/y respectively.

We can again use an RTD chart to evaluate how far each train travels when they move toward each other starting at opposite ends. Instead of using another variable d here, let's express the two distances in terms of their respective rates and times.

	High-speed	Regular	Total
R	z/x	z/y	
T	t	t	
D	zt/x	zt/y	z

Since the two distances sum to the total when the two trains meet, we can set up the following equation:

$zt/x + zt/y = z$ divide both sides of the equation by z
$t/x + t/y = 1$ multiply both sides of the equation by xy
$ty + tx = xy$ factor out a t on the left side
$t(x + y) = xy$ divide both sides by $x + y$

$t = xy/(x+y)$

To find how much further the high-speed train went in this time:
($rate_{high}$ × time) – ($rate_{reg}$ × time)
($rate_{high}$ – $rate_{reg}$) × time
(z/x-z/y) * (xy/(x+y))
((zy-zx)/xy)* (xy/(x+y))
(z(y-x))/(x+y)

The correct answer is A.

Alternate Solution from GMATCLUB

It takes the high-speed train x hours to travel the z miles --> rate of high-speed train is $rate_{high-speed} = \frac{distance}{time} = \frac{z}{x}$;

It takes the regular train y hours to travel the same distance --> rate of regular train is $rate_{regular} = \frac{distance}{time} = \frac{z}{y}$;

Time in which they meet is $time = \frac{distance}{combined-rate} = \frac{z}{\frac{z}{x}+\frac{z}{y}} = \frac{xy}{x+y}$.

Difference in distances covered: {Time}*{Rate of high-speed train} - {Time}{Rate of regular train} --> $\frac{xy}{x+y} * \frac{z}{x} - \frac{xy}{x+y} * \frac{z}{y} = \frac{z(y-x)}{x+y}$.

Answer: A.

The correct answer is A.

6. To determine Bill's average rate of movement, first recall that Rate × Time = Distance. We are given that the moving walkway is 300 feet long, so we need only determine the time elapsed during Bill's journey to determine his average rate.

   There are two ways to find the time of Bill's journey. First, we can break down Bill's journey into two legs: walking and standing. While walking, Bill moves at 6 feet per second. Because the walkway moves at 3 feet per second, Bill's foot speed along the walkway is 6 – 3 = 3 feet per second. Therefore, he covers the 120 feet between himself and the bottleneck in (120 feet)/(3 feet per second) = 40 seconds.

   Now, how far along is Bill when he stops walking? While that 40 seconds elapsed, the crowd would have moved (40 seconds)(3 feet per second) = 120 feet.

   Because the crowd already had a 120 foot head start, Bill catches up to them at 120 + 120 = 240 feet. The final 60 feet are covered at the rate of the moving walkway, 3 feet per second, and therefore require (60 feet)/(3 feet per second) = 20 seconds. The total journey requires 40 + 20 = 60 seconds, and Bill's rate of movement is (300 feet)/(60 seconds) = 5 feet per second.

   This problem may also be solved with a shortcut. Consider that Bill's journey will end when the crowd reaches the end of the walkway (as long as he catches up with the crowd before the walkway ends). When he steps on the walkway, the crowd is 180 feet from the end. The walkway travels this distance in (180 feet)/(3 feet per second) = 60 seconds, and Bill's average rate of movement is (300 feet)/(60 seconds) = 5 feet per second.

The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

Average speed = Total distance/Total time spent = 300/(120/3 + 40/3) = 300/(40+20) = 5 fps.

The correct answer is E.

7. It is easier to break this motion up into different segments. Let's first consider the 40 minutes up until John stops to fix his flat.

   40 minutes is 2/3 of an hour.
   In 2/3 of an hour, John traveled 15 × 2/3 = 10 miles ($rt = d$)
   In that same 2/3 of an hour, Jacob traveled 12 × 2/3 = 8 miles
   John therefore had a two-mile lead when he stopped to fix his tire.

   It took John 1 hour to fix his tire, during which time Jacob traveled 12 miles. Since John began this 1-hour period 2 miles ahead, at the end of the period he is 12 – 2 = 10 miles behind Jacob.
   The question now becomes "how long does it take John to bridge the 10-mile gap between him and Jacob, plus whatever additional distance Jacob has covered, while traveling at 15 miles per hour while Jacob is traveling at 12 miles per hour?" We can set up an $rt = d$ chart to solve this.

	John	Jacob
R	15	12
T	t	t
D	$d + 10$	d

John's travel during this "catch-up period" can be represented as $15t = d + 10$
Jacob's travel during this "catch-up period" can be represented as $12t = d$

If we solve these two simultaneous equations, we get: $15t = 12t + 10$

$3t = 10$
$t = 3\ 1/3$ hours

Another way to approach this question is to note that when John begins to ride again, Jacob is 10 miles ahead. So John must make up those first 10 miles plus whatever additional distance Jacob has covered while both are riding. Since Jacob's additional distance at any given moment is $12t$ (measuring from the moment when John begins riding again) we can represent the distance that John has to make up as $12t + 10$. We can also represent John's distance at any given moment as $15t$. Therefore, $15t = 12t + 10$, when John catches up to Jacob. We can solve this question as outlined above.

The correct answer is B.

8.

Use S, R and B to represent the individual race times of Stephanie, Regine, and Brian respectively. The problem tells us that Stephanie and Regine's combined times exceed Brian's time by 2 hours. Therefore:

$$S+R=B+2$$

In order to win the race, an individual's time must be less than one-third of the the combined times of all the runners. Thus, in order for Brian to win the race (meaning that Brain would have the lowest time), his time would need to be less than one-third of the combined times for all the runners. This can be expressed as follows:

$$B<\frac{1}{3}(S+R+B)$$

This inequality can be simplified as follows:

$$B<\frac{1}{3}(S+R+B)$$
$$3B<S+R+B$$
$$2B<S+R$$

Using the fact that $S+R=B+2$, the inequality can be simplified even further:

$$2B<S+R$$
$$2B<B+2$$
$$B<2$$

This tells us that in order for Brian to win the race, his time must be less than 2 hours. However, this is impossible! We know that the fastest Brian runs is 8 miles per hour, which means that the shortest amount of time in which he could complete the 20 mile race is 2.5 hours.

This leaves us with Stephanie and Regine as possible winners. Since the problem gives us identical information about Stephanie and Regine, we cannot eliminate either one as a possible winner. Thus, the correct answer is D: Stephanie or Regine could have won the race

The correct answer is D.

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8 mph implies that each person took at least 20/8 = 2.5 hrs. They could have taken more time too.
So Stephanie and Regina's combined time is at least 2.5*2 = 5 hrs. So Brian's time taken is at least 3 hrs. Can Brian win? No. The difference between S and R's combined time and Brian's time is 2 hrs but each person takes more than 2.5 hrs.
If S and R together took 5 hrs, B took 3 hrs. Both S and R must have taken 2.5 hrs each.
If S and R together took 6 hrs, B took 4 hrs. Both S and R must have taken less than 4 hrs since each person takes at least 2.5 hrs.
If S and R together took 10 hrs, B took 8 hrs. Both S and R must have taken less

than 8 hrs since each person takes at least 2.5 hrs.
Brian could never win.

OR

Given that S+R=B+2, where S, R, and B are times in which Stephanie, Regine, and Brian completed the race.
The Minimum time one could complete the race is 20/8=2.5 hours. Let's see if Brian could have won the race: if he ran at the fastest rate, he would complete the race in 2.5 hours, so combined time needed for Stephanie and Regine would be S+R=B+2=4.5 hours, which is not possible as sum of two must be more than or equal the twice the least time: 2*2.5=5. So Brian could not have won the race.
There is no reason to distinguish Stephanie and Regine so if one could have won the race, another also could. So both could have won the race.
The correct answer is D.

9. One way to approach this problem is to pick numbers for the variables. So let's say that

x = 60 miles per hour
y = 30 miles per hour

On the initial trip, the car traveled for 6 hours at 60 miles per hour.
Since distance = rate × time, the distance for this initial trip is 60 x 6 = 360 miles.
The return trip went along the same 360-mile route, but at only 30 miles per hour.
This means that for the return trip, 360 = 30 x time, so the duration of the return trip was 360/30 = 12 hours.

The entire trip took 6 + 12 = 18 hours which is equal to 18(60) minutes.

Plug our chosen values for x and y (60 and 30 respectively) into the answer choices and see which one yields the value 18(60). The only one that does this is answer choice A

Alternatively, we can solve this problem using only algebra. Let us call t the time in hours for the return trip. Then, using the formula distance = rate × time, we can say that

distance for initial trip = $x \times 6$, and distance for return trip = $t \times y$.

Since the distance for the initial trip equals the distance for the return trip, we can combine the two equations to say

$6x = ty$

Solving for t, we get,
$6x/y = y$
The total time for the round trip will be the time for the initial trip (6 hours) plus the time for the return trip. Expressed in minutes, this is 60(6+6x/y) minutes

The correct answer is A.

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Since distance = rate x time and we are given that the car traveled from Los Angeles (LA) to San Francisco (SF) in 6 hours at an average rate of x miles per hour, we can express the distance from LA to SF as 6x. We are also given that the car returned along the same route (i.e., traveling the same distance) from SF to LA at an average rate of y miles per hour. If we let t = the time returning from SF to LA, in hours, we have:

6x = yt

t = 6x/y hours = time from SF to LA

Thus, the round trip time = time from LA to SF + time from SF to LA = 6 + 6x/y hours

Finally, we convert our time from number of hours to number of minutes by multiplying the number of hours by 60, since 1 hour = 60 minutes:

(6 + 6x/y) hours = 60(6 + 6x/y) minutes

The correct answer is A.

10.

This standard rate problem will rely heavily on the formula *RT*=*D*, where *R* is the rate, *T* is the time and *D* is the distance travelled.

First, we should find the driving and biking distances:
If Deb drives for 45 minutes, or 0.75 hours, at a rate of 40mph, she drives a total distance of (0.75)*(40) = 30 miles.
If the bike route is 20% shorter than the driving route, the bike route is 30 – 30(0.2) = 30 – 6 = 24 miles.

Next, we need to determine how long it will take Deb to travel the route by bike. She wants to ensure that she'll get to work by a particular time, so we want to calculate the longest possible time it could take her; therefore, we have to assume she will bike at the slowest end of the range of the speeds given: 12mph. If she travels 24 miles at 12mph, it will take her 24/12 = 2 hours or 120 minutes.

If Deb normally takes 45 minutes to drive to work but could take up to 120 minutes to bike to work, then she must leave 120 – 45 = 75 minutes earlier than she normally does to ensure that she will arrive at work at the same time.

The correct answer is D.

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Distance to work decreases = 30 * 80/100 = 24 miles
If Deb averages between 12 and 16 miles per hour when biking, how many minutes earlier will she need to leave in the morning in order to ensure she arrives at work at

the same time as when she drives? (To ensure that this happens, we need to consider the longest time taken and hence her slowest speed, as time is inversely proportional to speed)
To ENSURE that she arrives BEFORE or at the same time as usual, she must consider her slowest average speed and calculate the maximum time she could take. So, we should consider the slowest speed and not the higher speed or average speed. She should start with that margin. If her speed is 12 mph (slowest), she will take 24/12 = 2 hours to bike to work. Normally she takes 45 mins so she must start 1 hour 15 mins before to ensure that she reaches on or before time (depending on her speed).
1 hr 15 mins = 60 + 15 = 75 mins.
The correct answer is D.

11.
If we want Brenda's distance to be twice as great as Alex's distance, we can set up the following equation:

$2(4T) = R(T-1)$, where $4T$ is Alex's distance (rate × time) and $R(T-1)$ is Brenda's distance (since Brenda has been traveling for one hour less).

If we simplify this equation to isolate the T (which represents Alex's total time), we get:

$2(4T) = R(T-1)$
$8T = RT - R$
$R = RT - 8T$
$R = T(R-8)$
$R/(R-8) = T$

The correct answer is C.

Top 1% expert replies to student queries (can skip)

If you take T for Alex, then take T-1 for Brenda OR If you take T+1 for Alex, then take T for Brenda.
Consider both cases:
Let T be the time that Alex will have been walking when Brenda has covered twice as much distance as Alex.
In T hours Alex will cover 4T miles;
Since Brenda begins her journey 1 hour later than Alex then total time for her will be T-1 hours, and the distance covered in that time will be R(T-1);
We want the distance covered by Brenda to be twice as much as that of Alex:
2*4T=R(T-1) --> 8T=RT-R --> T=R/(R-8).
The correct answer is C.

OR
We are given that Alex has a rate of 4 mph and Brenda has a rate of R mph. We are also given that Alex begins to walk away from Brenda in a straight line at a rate of 4 mph. One hour later, Brenda begins to ride a bicycle in a straight line in the opposite direction. Thus, we can let Alex's time = T + 1 and Brenda's time = T. Finally, since distance = rate x time, Alex's distance = 4(T + 1) = 4T + 4 and

Brenda's distance = RT. We need to determine the amount of time it takes Brenda to cover twice the distance Alex has gone. So, we can create the following equation and determine T:

2(4T + 4) = RT

8T + 8 = RT

8 = RT - 8T

8 = T(R - 8)

8/(R - 8) = T

Since we are asked for Alex's time, T + 1 = 8/(R - 8) + (R - 8)/(R - 8) = R/(R - 8).

The correct answer is C.

12.

The key to solving this question lies in understanding the mathematical relationship that exists between the speed (s), the circumference of the tires (c) and the number of revolutions the tires make per second (r). It makes sense that if you keep the speed of the car the same but increase the size of the tires, the number of revolutions that the new tires make per second should go down. What, however, is the exact relationship?

Sometimes the best way to come up with a formula expressing the relationship between different variables is to analyze the labels (or units) that are associated with those variables. Let's use the following units for the variables in this question (even though they are slightly different in the question):

c: inches/revolution

s: inches/sec

r: revolutions/sec

The labels suggest: (rev/sec) x (inches/rev) = (inches/sec), which means that $rc = s$.

When the speed is held constant, as it is in this question, the relationship $rc = s$ becomes $rc = k$. r and c are inversely proportional to one another. When two variables are inversely proportional, it means that whatever factor you multiply one of the variables by, the other one changes by the inverse of that factor. For example, if you keep the speed constant and you double the circumference of the tires, the rev/sec will be halved. In this way the product of c and r is kept constant.

In this question the circumferences of the tires are given in inches, the speed in miles per hour, and the rotational speed in revolutions per second. However, the discrepancies here don't affect the fundamental mathematical relationship of inverse proportionality: if the speed is kept constant, the rev/sec of the tires will change in an inverse manner to the circumference of the tires.

Let's assign c_1 = initial circumference; c_2 = new circumference
r_1 = initial rev/sec; r_2 = new rev/sec

Since the speeds are held constant:

$c_1r_1 = c_2r_2$

$r_2 = (c_1/c_2)r_1$

$r_2 = (28/32)r_1$

$r_2 = (7/8)r_1$

If the new rev/sec is 7/8 of the previous rev/sec, this represents a 1/8 or 12.5% decrease and the correct answer is (B). Relationships of inverse proportionality are important in any word problem on the GMAT involving a formula in the form of $xy = z$ and in which z is held constant.

The correct answer is B.

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→ average no. of revolution/sec.

Monday

x miles → 3600 sec.

In 1 revolution,
distance travelled → $2\pi(28)$ inch
(No need to convert into miles, since question asks for % change)

1 revolution → $2\pi(28)$ inches
? → x

Total Revolution → $\frac{x}{2\pi(28)}$

Total Revolution/sec → $\frac{x}{2\pi(28)(3600)}$

For ~~Monday~~ Tue

~~x 28~~

Total Revolution/sec. → $\frac{x}{2\pi(32)(3600)}$

% change

Mon ⟶ Tue.

$\frac{x}{2\pi(28)(3600)}$ $\frac{x}{2\pi(32)(3600)}$

change ⇒ $\frac{\frac{x}{2\pi(3600)}\left(\frac{1}{32} - \frac{1}{28}\right)}{\frac{x}{2\pi(3600)(28)}} \times 100$

⇒ $28\left(\frac{1}{32} - \frac{1}{28}\right) \times 100$

⇒ $\cancel{28}\left(\frac{-4}{\cancel{28}\cdot 32}\right) \times 100$

⇒ $-\frac{1}{8} \times 100$

⇒ -12.5%.

The correct answer is B.

13.
The crux of this problem is recalling the average speed formula:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

In this particular case, since Martha drove at one speed for some time and at another speed for the remainder of the trip, the total time will be the sum of the times spent at the two speeds. Let T_1 be the time spent traveling at the first speed and let T_2 be the time spent traveling at the second speed. Martha's average speed can then be expressed as:

$$\text{average speed} = \frac{\text{total distance}}{T_1 + T_2}$$

Since we do not know the total distance, we can call it *d*. We do not know either T_1 or T_2, but we can express them in terms of *d* by recalling that T = D/R, where *D* is the distance and *R* is the rate.

Let's find T_1 first.

Since Martha travelled the first *x* percent of the journey at 60 miles per hour, *D* for that portion of the trip will be equal to $\frac{dx}{100}$ and T_1 will therefore be equal to $\frac{\frac{dx}{100}}{60}$

Now let's find T_2. The remaining distance in Martha's trip can be expressed as $d - \frac{dx}{100}$.
Therefore, T_2 will be equal to $\frac{d-\frac{dx}{100}}{50}$.
We can plug these into our average rate formula and simplify:

$$\frac{d}{\frac{\frac{dx}{100}}{60} + \frac{\left(d - \frac{dx}{100}\right)}{50}} \rightarrow$$

$$\frac{\cancel{d}}{\frac{\cancel{d}x}{6000} + \frac{\cancel{d}}{50} - \frac{\cancel{d}x}{5000}} \rightarrow$$

$$\frac{1}{\frac{x}{6000} + \frac{1}{50} - \frac{x}{5000}} \rightarrow$$

$$\frac{1}{\frac{5x}{30000} + \frac{600}{30000} - \frac{6x}{30000}} \rightarrow$$

$$\frac{1}{\frac{600 - x}{30000}} \rightarrow$$

$$\frac{30000}{600 - x}$$

We cannot reduce this fraction any further.

Therefore, the numerator of Martha’s average speed is 30,000.
The correct answer is E.

14.

One quick way to solve this problem is to create a chart relating each clock to the next:

Real Time:	6 PM
Clock #1 Displays:	6 PM – 15 min (6) = 6 PM – 90 min = 4:30 PM
Clock #2 Displays:	4:30 PM + 15 min (4.5) = 4:30 PM + 67.5 min = 5:375 PM
Clock #3 Displays:	5:375 PM – 20 min (5 5/8) = 5:375 PM – 112.5 min = 3:45 PM
Clock #4 Displays:	3:45 PM + 20 min (3 3/4) = 3:45 PM + 75 min = 5:00 PM

Notice that each clock runs relative to the previous clock. For example, when Clock #2 gains 15 minutes an hour, it does so for only 4.5 hours, since Clock #1 progressed only
4.5 hours.

The correct answer is A: At 6 PM real time, Clock #4 displays 5:00 PM

The correct answer is A.

Top 1% expert replies to student queries (can skip) (additional)

There are two things essentially here - actual time passing, and how each clock is moving. Finally 6 actual hours have passed, so let's bring every clock's 'movement' on to the same terms (in terms of actual time having passed)

When actual 60 minutes have passed, Clock 1 has 'moved' 45 minutes. So Clock 1 moves 45/60 = 3/4 as fast as actual time

When Clock 1 has moved 60 minutes, Clock 2 has moved 75 minutes. So Clock 2 moves 75/60 = 5/4 as fast as Clock 1 = (5/4)(3/4) = 15/16 as fast as actual time

Similarly, when Clock 2 has moved 60 minutes, Clock 3 has moved 40 minutes. So Clock 3 moves 40/60 = 2/3 as fast as Clock 2 = 5/8 as fast as actual time

Finally, when Clock 3 has moved 60 minutes, Clock 4 has moved 80 minutes. So Clock 4 moves 80/60 = 4/3 as fast as Clock 3 = 5/6 as fast as actual time

Now 6 hours have passed in actual time. Putting this in the ratio, 5 hours would have passed in Clock 4. So when it is actually 6 PM that day, Clock 4 will show 5 PM

The correct answer is A.

15.

The hands of the clock will be perpendicular when the angle of the hour hand minus the angle of the minute hand (both relative clockwise to the very top of the clock, or the “12” position) is exactly 90 degrees.

At 7:00 exactly, the minute hand is exactly at the “12” position, so it is at 0 degrees. A clock face is 360 degrees around and there are 60 minutes in an hour so each minute elapsed will result in the minute hand moving 360/60 = 6 degrees clockwise. Therefore, at x minutes past 7:00, the minute hand is at $6x$ degrees.

If the hour hand moves 30 degrees during the course of an hour, it moves 1/2 a degree every minute (since there are 60 minutes in an hour). Therefore, at x minutes past 7:00, the hour hand will be at $210 + 1/2x$ degrees.

We want to solve for x (which is the number of minutes past 7:00) such that the following holds true align=center > (angle of hour hand) – (angle of minute hand) = 90 degrees

Minute hand of the clock moves with 6 degree per minute speed and the hour hand moves at 1/2 degree per minute.
Therefore in x min hour hand will travel (1/2)*x degree further 210 degree from initial position; meanwhile minute hand must cover 6x degrees.
The Difference between a minute and hour hand is 90 degrees. Formed the following equation and solved for x.
((1/2)x+210)−6x=90
Solve: x=240/11=21 9/11

This can be rewritten mathematically as follows:

$$(210+\frac{1}{2}x)-(6x)=90$$
$$(420+x)-(12x)=180$$
$$-11x=-240$$
$$x=\frac{240}{11}=21\frac{9}{11}$$

The exact time that the hour and minutes hands are perpendicular is 21 9/11 minutes past 7:00
The correct answer is D.

Top 1% expert replies to student queries (can skip)

The clock face is 360 deg/60 min (1 min = 6 deg)

The minute hand travels 360 deg / hr (M_sp)
The hour hand travels 30 deg / hrs (H_sp) (For every 5 mins, an hour hand moves from one number to other: for e.g. 7o clock to 8o clock will take 5 mins or 360/12= 30 degrees)

Original location of minute hand = 0 deg

Original location of hour hand = (360/12) ×7= 210 deg

After time T, location of hour hand would be 210 + T.H_sp = 210 + 30T (H_loc), and location of minute hand = 0 + T.M_sp = 0 + 360T (M_loc)

After T hours: H_loc - M_local = 90

210 + 30T - 360T = 90

120 = 330T

T = 4/11 hrs

(Note that T converted to minutes is also the final location of the minute hand and therefore the answer to the question)

Convert to minutes: 240/11 = 21 9/11.
The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Speed of Minute hand = 6 degree/ min
Speed of Hour hand = 0.5 degree / min

Initial distance between them (clockwise: 210 degree)

Final distance between them (clockwise: 90 degree)

This means that "overall" they are travelling 120 degrees
in 1 min ,Minute hand travels 6 degree and hour hand travels 0.5 degree in the same direction, so overall they travel 5.5 degrees or the gap/ distance between them is 5.5 degrees less.
(Example: (7:00pm reference)after 10 min , minute hand will travel 60 degrees and hours hand will travel 5 degrees; so overall their gap is 55 degrees lesser. Initially the gap was 210 degrees, now the gap will be (210-55=> 155 degrees)
(try to draw 7:00 pm scenario with minute and hour hands on the clock)

1 min 5.5 degrees
Xmin 120 degrees

1*120 = 5.5 x
=> x = 240/11 min
Both hands will travel for this many minutes so that the distance between them is 99 degrees.
The correct answer is D.

16.

We know that Team A wins the race by 7 seconds, which means that Runner 4 on Team B will cross the finish line 7 seconds after Runner 4 on Team A crosses the finish line. Thus, the question can be rephrased as follows: How far does Runner 4 on Team B run in 7 seconds? Since his lap time is 42 seconds, he covers 7/42, or 1/6, of the track in 7 seconds.

Therefore, we must determine the length of the track. The track is formed by a rectangle with two adjoining semicircles. The length of the track is equal to 2 times the length of the rectangle plus the circumference of the circle (the two semi-circles combined).

The diameter of the circle is: 180 meters – 120 meters = 60 meters. Thus, the radius of the circle is 30 meters and the circumference is $2\pi r = 60\pi$ meters. Finally, the length of the track is: $(2 \times 120 + 60\pi)$ meters = $(240 + 60\pi)$ meters.

Remember, Runner 4 on Team B still has 1/6 of the lap to run when Runner 4 on Team A finishes the race. So, Team B loses the race by: $(240 + 60\pi) / 6 = (40 + 10\pi)$ meters.

The correct answer is B.

17.

Distance = Rate × Time, or $D = RT$.

(1) INSUFFICIENT: This statement tells us Harry's rate, 30 mph. This is not enough to calculate the distance from his home to his office, since we don't know anything about the time required for his commute.

$D = RT$ = (30 mph) (T)

D cannot be calculated because T is unknown.

(2) INSUFFICIENT: If Harry had travelled twice as fast, he would have gotten to work in half the time, which according to this statement would have saved him 15 minutes. Therefore, his actual commute took 30 minutes. So we learn his commute time from this statement, but don't know anything about his actual speed.

$D = RT$ = (R) (1/2 hour)

D cannot be calculated because R is unknown.

(1) AND (2) SUFFICIENT: From statement (1) we learned that Harry's rate was 30 mph. From Statement (2) we learned that Harry's commute time was 30 minutes. Therefore, we can use the rate formula to determine the distance Harry travelled.

$D = RT$ = (30 mph) (1/2 hour) = 15 miles

The correct answer is C.

Alternate sol from gmatclub (additional)

Distance between home and office = d

I) Speed s= 30mph.
No other info provided, hence Insufficient

II) Original Speed = s
Original Time = t
New Speed = 2s
New Time = (t-0.25) [15 mins = 0.25hrs)

Distance is same, hence

s*t = 2s(t-0.25)
Solving results in t=0.5 hrs.

No info about speed hence, Insufficient.

I&II) s= 30mph
t= 0.5hrs
d= s*t => 15miles. Ans: C

18.

This question cannot necessarily be rephrased, but it is important to recognize that we need not necessarily calculate Wendy's or Bob's travel time individually. Determining the difference between Wendy's and Bob's total travel times would be sufficient. This difference might be expressed as $t_b - t_w$.

(1) INSUFFICIENT: Calculating Bob's rate of speed for any leg of the trip will not give us sufficient information to determine the time or distance of his journey, at least one of which would be necessary to determine how quickly Wendy reaches the restaurant.

(2) SUFFICIENT: To see why this statement is sufficient, it is helpful to think of Bob's journey in two legs: the first leg walking together with Wendy (t_1), and the second walking alone (t_2). Bob's total travel time $t_b = t_1 + t_2$. Because Wendy travelled halfway to the restaurant with Bob, her total travel time $t_w = 2t_1$. Substituting these expressions for $t_b - t_w$,

$t_1 + t_2 - 2t_1 = t_2 - t_1$

$t_b - t_w = t_2 - t_1$

Statement (2) tells us that Bob spent 32 more minutes traveling alone than with Wendy. In other words, $t_2 - t_1 = 32$. Wendy waited at the restaurant for 32 minutes for Bob to arrive.

The correct answer is B.

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(1) Bob's average speed for the entire journey was 4 mph.
While we know that his average speed was 4 miles/hour, the answer to the question depends on the distance from home to diner.
Let's say that the distance was 8. Furthermore, we can break it up into two, 4 mile segments.
Time = Distance/Rate
Wendy's Time = 8/3 hours or roughly 160 minutes.
Now Bob traveling at his faster rate would cover 4 miles in an hour. However, Bob would need to travel the 4 miles with Wendy, 4 miles back home, then 8 miles to the diner meaning he covered 16 miles in total. He would have spent 4 hours (240 minutes) walking so she would have spent 240-160 = 80 minutes waiting.
Now lets pretend the diner was 4 miles away.
Time = Distance/Rate
Wendy Time = 4/3 hours or roughly 75 minutes.
Bob would travel 2 miles (the half way point), two miles back home then another 4 miles from home to the diner for a total of 8 miles. At 4 MPH average this would take 2 hours (120 minutes) meaning she waited 120-75 = 35 minutes.
INSUFFICIENT

(2) On his journey, Bob spent 32 more minutes alone than he did walking with Wendy.
Bob and Wendy left at a constant rate. Let's say the distance from home to the Diner was 4 miles. This means that T=d/r and Wendy took 4/3rds hours (75 minutes) to get from home to the diner. Bob would have traveled 37.5 minutes with Wendy before he turned around to go lock the door. If he spent 37.5 minutes with Wendy then he would have spent 37.5+32 = 69.5 minutes traveling from midpoint to home to the restaurant. During this time Wendy would have walked an additional 37.5 minutes to get to the diner. The time she spent alone would have been 69.5-37.5 = 32 minutes alone.
SUFFICIENT
The correct answer is B.

OR

Let the distance be 'x'.
time taken to travel half the distance = x/6 hours. Wendy reaches the end in x/3 and waits.
let bob's new speed k. total time travelled by bob upto reaching end of journey-
x/6+ 3x/2k
Hence Wendy waited x/6 +3x/2k - x/3 = 3x/2k - x/6 hours.
statement 1. gives k but not x hence insufficient.
statement 2. 3x/2k - x/6= 32 minutes exactly what we want. hence sufficient
The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)

Explanation for Statement 2:

Let the distance between home and restaurant be x miles
Let the changed speed of Bob be v mph

In statement 2,

We have been given that Bob spends 32 more minutes walking alone than he did walking with Wendy.

Time Bob spent walking with Wendy = $(x/2)/3 = x/6$ h

Time spent by Bob spends walking alone = $(3x/2)/v = 3x/2v$ [Distance travelled by Bob alone = $(x/2 + x)$ miles, Constant speed at which Bob walks = v mph]

We have been given that :

$3x/2v = x/6 + 32/60$

$3x/2v - x/6 = 32/60$

Now, time taken by Bob to reach the restaurant from the middle of the path = $3x/2v$

Time taken by Wendy to reach the restaurant from the middle of the path = $x/6$

Therefore,

time that Wendy had to wait at the restaurant = time taken by Bob to reach the restaurant from the middle of the path - time taken by Wendy to reach the restaurant from the middle of the path

time that Wendy had to wait at the restaurant = $3x/2v - x/6 = 32/60$ (32 minutes).

Hence, statement 2 is sufficient.

19. To determine the average speed for the trip from Townsend to Smallville and back again, we need to know the average speed in each direction. Because the distance in each direction is the same, if we have the average speed in each direction we will be able to find the average speed of the entire trip by taking the total distance and dividing it by the total time.

SUFFICIENT: This allows us to figure out the average speed for the return trip. If the return time was 3/2 the outgoing time, the return speed must have been 2/3 that of the outgoing. Whenever the distance is fixed, the ratio of the times will be the *inverse* of the ratio of the speeds.

We can see this by looking at an example. Let's say the distance between the two towns was 80 miles.

	Going	Returning
R	40	

T		
D	80	80

We can calculate the "going" time as 2 hours. Since, the return trip took 50% longer, the "returning time" is 3 hours. Thus, the average rate for the return trip is Distance/Time or 80/3 miles per hour.

	Going	Returning
R	40	**80/3**
T	2	3
D	80	80

We can use this table to calculate the average speed for the entire trip: take the total distance, 160, and divide by the total time, 5.

	Going	Returning	TOTAL
R	40	**80/3**	**---**
T	2	3	5
D	80	80	160

This results in an average speed of 32 miles per hour.

It does not matter that we chose a random distance of 80; we would able to solve using any distance or even using a variable x as the distance. The times would adjust accordingly based on the distance we used and the same average speed of 32 would result.

INSUFFICIENT: If all we know is the distance from Riverdale to Smallville, we will be able to find the time travelled on the way there but we will have no indication of how fast the car travelled on the way back and therefore no way of knowing what the average overall speed was.

The correct answer is A.

20.
The average speed is defined to be the total distance of the trip, divided by the total time of the trip. The question stem tells us the distance from New York to Boston is 250 miles, so we can rephrase the question as "How long did it take Bill to drive from New York to Boston?"
SUFFICIENT: Statement (1) tells us it took Bill 5 hours to drive from New York to Boston, answering the rephrased question. In fact, his average rate of speed equals 250/5 = 50 miles per hour.
INSUFFICIENT: Statement (2) tells us that at the midpoint of the trip Bill was going exactly 50 miles per hour, but we can't figure out how long the trip took from this information. Bill *may* have travelled at a constant rate of 50 mph throughout the whole trip, but he might also have been going faster or slower at different times.

The correct answer is A.

21.

We can attack this problem by first setting up a rate chart, identifying what we already know, and using variables for any unknown values:

	Train A	Train B
Rate	100 mph	*r*
Time	2 - *t*	*t*
Distance	*d*	*d*

The chart yields two equations:

(A) $200 - 100t = d$ (B) $rt = d$

We also know that when the trains pass each other (going in opposite directions), Train A has been traveling for 1 hour and Train B has been traveling for 10 minutes or 1/6 of an hour.

This means that Train A has travelled 100 miles and Train B has travelled r/6 miles.

Thus, the total distance from New York to Boston can be expressed using the

equation, $d = 100 + \frac{r}{6}$.

We now have three equations and three variables:

(A) $200 - 100t = d$ (B) $rt = d$ (C) $d = 100 + \frac{r}{6}$

Setting equation (A) and equation (B) to be equal, we can solve for *t*, as follows:

$$200 - 100t = rt$$
$$t = \frac{200}{100 + r}$$

Then we can set equation (B) and equation (C) to be equal, substitute for *t*, and solve for *r* as follows:

$$rt = 100 + \frac{r}{6}$$
$$r\left(\frac{200}{100 + r}\right) = 100 + \frac{r}{6} \quad \rightarrow \quad (r - 300)(r - 200) = 0$$

Thus, the rate, *r*, of Train B is either 300 mph or 200 mph. Using this information we can chart out the two possible scenarios.

Scenario 1: Train B has a rate of 300 mph. It travels 50 miles in 1/6 hour, at which point it meets Train A which has already traveled 100 miles. Therefore, the total distance from Boston to New York must be 150 miles. Thus, Train B's total traveling time was 1/2 hour, and Train A's total traveling time was 1 1/2 hours. Train B arrived in New York at 4:20 PM and Train A arrived in Boston at 4:30 PM.

Scenario 2: Train B has a rate of 200 mph. It travels 33 1/3 miles in 1/6 hour, at which point it meets Train A which has already traveled 100 miles. Therefore, the total distance from Boston to New York must be 133 1/3 miles. Thus Train B's total traveling time was 2/3 hour, and Train A's total traveling time was 1 1/3 hours. Train B arrived in New York at 4:30 PM and Train A arrived in Boston at 4:20 PM.

Statement (1) tells us that Train B arrived in New York before Train A arrived in Boston. From this, we know that **Scenario 1** must have occurred and Train B arrived in New York at 4:20 PM. We have sufficient information to answer the question.

Statement (2) tells us that the distance between New York and Boston is greater than 140 miles. This means that **Scenario 2** is not possible so **Scenario 1** must have occurred: Train B arrived in New York at 4:20 PM. Again, we have sufficient information to answer the question.

The correct answer is (D): Each statement ALONE is sufficient.
The correct answer is D.

Top 1% expert replies to student queries (can skip)

We can assume that B leaves Los Angeles not at noon but at 2pm Eastern Time, so 5 hours later than A leaves New York.

In 5 hours A covers 5*40=200 miles. So, A and B together should cover 3,000-200=2,800 miles.
Combined rate = 40 + 60 = 100 miles per hour.
To cover 2,800 miles they'll need 2,800/100=28 hours.
2pm Eastern Time on Monday + 28 hours = 6pm Eastern Time on Tuesday.
The correct answer is (D): Each statement ALONE is sufficient.

OR

We need to find at what time they meet. Say they meet t hrs after train A starts at 9 AM EST.
Train A runs for t hrs. Train B runs for (t - 5) hrs since it starts 3 hrs late and takes a break of 2 hrs in between (It doesn't matter whether the break was taken 10 mins into the journey, 1 hr or 2 hrs as long as it was early enough so that the break was over before they met).
Together they covered 3000 miles.

$40 * t + 60*(t - 5) = 3000$
$t = 33$ hrs
That gives us 6 pm Tuesday.
The correct answer is (D): Each statement ALONE is sufficient.
The correct answer is D.

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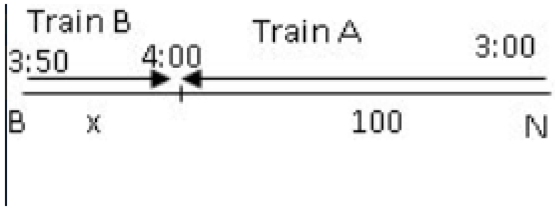
The diagram below: incorporates the data given in the question stem. Let x be the distance from meeting point to Boston.



Speed of train A = 100 mph
Speed of train B = x/(10 min) = 6x mph (converted min to hr)
Total time taken by both is 2 hrs. Already accounted for is 1hr + (1/6) hr
The remaining (5/6) hrs is the time needed by both together to reach their respective destinations.

Time taken by train A to reach B + time taken by train B to reach NY = 5/6
x/100 + 100/6x = 5/6
3x^2 - 250x + 5000 = 0 (Painful part of the question)
x = 50, 33.33

(1) Train B arrived in New York before Train A arrived in Boston.
If x = 50, time taken by train A to reach B = 1/2 hr, time taken by train B to reach NY = 1/3 hr
If x = 33.33, time taken by train A to reach B = 1/3 hr, time taken by train B to reach NY = 1/2 hr
Since train B arrived first, x must be 50 and B must have arrived at 4:20. Sufficient.

(2) The distance between New York and Boston is greater than 140 miles.
x must be 50 to make the total distance more than 140. Time taken by train B must be 1/3 hr and it must have arrived at 4:20. Sufficient.

The correct answer is (D): Each statement ALONE is sufficient.
The correct answer is D.

22. The question asks for the percent decrease in Edwin's travel time. To determine this, we need to be able to find the ratio between, T_1 (the travel time if Edwin drives alone) and T_2 (the travel time if Edwin and George drive together). Note that we do NOT need to determine specific values for T_1 and T_2; we only need to find the ratio between them.

Why? Percentage change is defined as follows: $\frac{\text{Difference}}{\text{Original}} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$

Ultimately, we can solve the percentage change equation above by simply

determining the value of $\frac{T_2}{T_1}$.
Using the formula Rate × Time = Distance, we can write equations for each of the 2 possible trips.

T_1 = Travel time if Edwin drives alone
T_2 = Travel time if Edwin and George drive
together E = Edwin's Rate

G = George's Rate
D = Distance of the trip

If Edwin travels alone: $ET_1 = D$

If Edwin and George travel together: $5(E+G)T_2 = D$
(Since Edwin and George split the driving equally, the rate for the trip is equal to the average of Edwin and George's individual rates).

Since both trips cover the same distance (D), we can combine the 2 equations as follows:

$$ET_1 = .5(E+G)T_2$$

Then, we can isolate the ratio of the times (T_2/T_1) as follows:

$$\frac{E}{.5(E+G)} = \frac{T_2}{T_1}$$

Now we look at the statements to see if they can help us to solve for the ratio of the times.

Statement (1) gives us a value for D, the distance, which does not help us since D is not a variable in the ratio equation above.

Statement (2) tells us that George's rate is 1.5 times Edwin's rate.
Thus, G = 1.5E. We can substitute this information into the ratio equation above:

$$\frac{E}{.5(E+G)} = \frac{T_2}{T_1} \rightarrow \frac{E}{.5(E+1.5E)} = \frac{T_2}{T_1} \rightarrow \frac{E}{.5E+.75E} = \frac{T_2}{T_1}$$

$$\rightarrow \frac{E}{1.25E} = \frac{T_2}{T_1} \rightarrow \frac{1}{1.25} = \frac{T_2}{T_1} \rightarrow .8 = \frac{T_2}{T_1}$$

Thus, using this ratio we can see that Edwin's travel time for the trip will be reduced as follows:

$$1 - \frac{T_2}{T_1} = 1 - .8 = .2 \rightarrow 20\%$$

Statement (2) alone is sufficient to answer the question.
The correct answer is B.

Top 1% expert replies to student queries (can skip)

1) Statement-1: D = 1500
Not sufficient

2) Statement-2: Use formula: R1T1 + R2T2 = D
T_old= D/S
T2 = T1 (Given: "split the driving time equally")
R1 = S
R2 = 1.5S

ST1 + 1.5ST1 = D
T1 (2.5S) = D
T1 = D/(2.5S)
T_new = T1 + T2 = 2*T1 = 2D/(2.5S)
Now we have both T_new and T_old so we can stop here =>
Sufficient ANSWER: B
Just to show the calculations:
(D/S - 2D/2.5S)/(D/S) = ((2.5D - 2D)/(2.5S))/(D/S) = (0.5D/2.5S)/(D/S) =
0.5/2.5 = 0.2 = 20% => Edwin's travel time decreases by 20%
The correct answer is B.

23. We are asked to find the time that it takes Train B to travel the entire distance between the two towns.

SUFFICIENT: This tells us that B started traveling 1 hour after Train A started traveling. From the question we know that Train A had been traveling for 2 hours when the trains passed each other. Thus, train B, which started 1 hour later, must have been traveling for 2 - 1 = 1 hour when the trains passed each other.

Let's call the point at which the two trains pass each other Point P. Train A travels from Town H to Point P in 2 hours, while Train B travels from Town G to Point P in 1 hour. Adding up these distances and times, we have it that the two trains covered the entire distance between the towns in 3 (i.e. 2 + 1) hours of combined travel time. Since both trains travel at the same rate, it will take 3 hours for either train to cover the entire distance alone. Thus, from Statement (1) we know that it will take Train B 3 hours to travel between Town G and Town H.

INSUFFICIENT: This provides the rate for Train B. Since both trains travel at the same rate, this is also the rate for Train A. However, we have no information about when Train B started traveling (relative to when Train A started traveling) and we have no information about the distance between Town G and Town H. Thus, we cannot calculate any information about time.
The correct answer is A.

Top 1% expert replies to student queries (can skip)

Statement 1 along with the stem of the question helps us pin down all variables required - we know the speeds are the same, we know A travelled for 2 hours and B travelled for 1 hour, and hence after they meet (because speeds are same), B will take 2 hours to travel the distance A did and so will take a total of 3 hours. The way Statement 1 is worded also makes it clear that B started from G and A started from H.

If you look at Statement 2 and assume both trains started at the same time, we are looking at a situation where A and B both travelled 2 hours each, for a total of 600 miles distance. We also don't know their starting points, so the distance between G and H is at least 600 miles, and can be more (if one or both started from somewhere within G and H). Since this is DS, the more important thing is if we assume this we can uniquely find an answer. There is however, nothing to indicate they did start at the same time. plus the fact that Statement 1 explicitly mentions they did not start at the same time, should alert you to the fact that the question setters do not want you to assume that the starting times are the same (and that the starting points are towns G and H). So to continue our thought process, if in Statement 2 we say the two trains did not start at the same time, we are unable to find the answer required, as we don't have any idea about the staggered start times, and hence the distances covered by the trains (and the corresponding times required, which is our required answer).

Basically, in Statement 2 if you assume they only started at the same time, you are minimizing yourself to a subset of all possibilities, and you are not considering the full set, which will make your answer incorrect.
The correct answer is A.

24.
Since AB = BC, triangle ABC is a 45-45-90 triangle. Such triangles have fixed side ratios as follows:
$AB : BC : AC \rightarrow 1 : 1 : \sqrt{2}$
Thus, we can call Greg's distance (AB) x, while Brian's distance (AC) is
$\sqrt{2}x$ or $\sim 1.4x$. Brian has a greater distance to travel.
Let's first analyse Statement (1) alone: Greg's average speed is 2/3 that of Brian's. This indicates that Brian is traveling 1.5 times faster than Greg. If Greg's rate is r, than Brian's rate is 1.5r. However, recall that Brian also has a greater distance to travel.
To determine who will arrive first, we use the distance formula:
$\text{Rate} \times \text{Time} = \text{Distance}$. Whoever has a shorter **TIME** will arrive first.

Greg's time $= \frac{\text{Distance}}{\text{Rate}} = \frac{x}{r}$	Brian's time $= \frac{\text{Distance}}{\text{Rate}} = \frac{1.4x}{1.5r} = .93\left(\frac{x}{r}\right)$

Since Brian is traveling for less time, he will arrive first.
Statement (1) alone is sufficient.
Let's now analyse Statement (2) alone: Brian's average speed is 20 miles per hour greater than Greg's.
This gives us no information about the ratio of Brain's average speed to Greg's average speed. Thus, although we know that Brian's distance is approximately 1.4 times Greg's speed, we do not know the ratio of their speeds, so we cannot determine who will arrive first.
For example, if Brian travels at 25 mph, Greg travels at 5 mph. In this case Brian arrives first. However, if Brian travels at 100 mph, Greg travels at 80 mph. In this case Greg arrives first.
Therefore, Statement (2) alone is not sufficient.
Since statement (1) alone is sufficient, but statement (2) alone is not sufficient.
The correct answer is A.

25.
From 1, V>16 feet/second= (16/5280)/(1/3600)=10.9 mile/hour, the distance that he cycled is greater than 10.9*1/2=5.45. We cannot know whether it greater than 6.
From 2, V<18 feel/second= (18/5280)/(1/3600)=12.27mile/hour, the distance that he cycled is less than 12.27*1/2=6.135.
Combine 1 and 2, 5.45<the distance<6.135,
insufficient. **The correct answer is E.**

Alternate sol from gmatclub (additional)

First of all from 16<x<18 you cannot say that x=17. You have the range for x, you cannot take an average and say that x equals to it.

If it took Carlos 1/2 hour to cycle from his house to the library yesterday, was the distance that he cycled greater than 6 miles? (Note: 1 mile = 5280 ft)

The question asks:

Is $d > 6$?

Since $rt = d$ (where r is the rate in miles per hour and t = 1/2 hour) then question becomes:

Is $(rt = d) > 6$?

Is $r * \frac{1}{2} > 6$

Is $r > 12$ miles/hour?

$12\ miles/hour = \frac{12*5280}{60*60}\ feet/second = 17.6\ feet/sec.$

Is $r > 17.6$ feet/sec?

(1) The average speed at which Carlos cycles from his house to the library yesterday was greater than 16 feet per second --> $r > 16$ feet/sec. Not sufficient.

(2) The average speed at which Carlos cycles from his house to the library yesterday was less than 18 feet per second --> $r < 18$ feet/sec. Not sufficient.

(1)+(2) $16 < r < 18$ still not sufficient to say whether $r > 17.6$.

Answer: E.

26.
1). Insufficient, no idea about the car's speed for the next 200 kms
2). (Actual speed + 20) * (actual time - 1 hour) = 400 kms = actual speed * actual time Can solve for actual time.
The correct answer is B.

27.
1). x/50+(530-x)/60=10, so, x can be solve out and x/50 will be the answer
-2). The time cost on two distances could be 5h, 1h; 6h, 2h;... insufficient.
The correct answer is A.

Alternate sol from gmatclub (additional)

(1) On this trip, Julio drove for a total of 10 hours and a total of 530 miles --> $total\ time = 10 = \frac{x}{50} + \frac{530-x}{60}$ --> we have the linear equation with one unknown, so we can solve for x. Sufficient.

(2) On this trip, it took Julio 4 more hours to drive the first x miles than to drive the remaining distance --> $\frac{x}{50} = \frac{y}{60} + 4$, where y is the remaining distance --> we have the linear equation with two unknowns, so we cannot solve for x. Not sufficient.

Answer: A.

Top 1% expert replies to student queries (can skip) (additional)

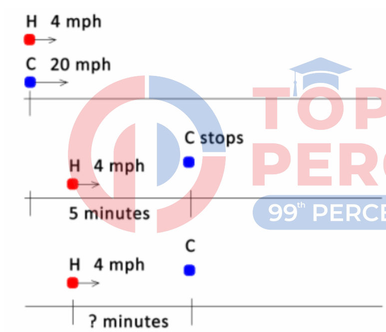
Explanation for Statement 2:

Let remaining distance = y
Let time taken for remaining distance = t
Then, time taken for x miles = t + 4
Let's create the equations:
For first x miles -> 50*(t+4) = x
For next y miles -> 60*t = y

We have 3 variables but 2 equations. Hence, we can't find 1 unique value for t. Thus, insufficient

28. **Top 1% expert replies to student queries**

Distance between them should have been 20*(5/60)- 4*(5/60)= 4/3 miles.
The Cyclist should wait till the hiker walks 4/3 miles=(4/3)/4=1/3 hours = 20 minutes.
Consider this:



In 1/12 hours (5 minutes) after the hiker is passed by the cyclist the distance between them will comprise $(20-4)*1/12=4/3$ miles (note that during these 5 minute hiker walks too, so their relative rate is 20-4 miles per hour). The hiker thus will need (4/3)/4=1/3 hours, or 20 minutes to catch up.
The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

Speed of hiker = 4 mph
Speed of cyclist = 20 mph.

The cyclist overtakes the hiker. After travelling for 5 minutes after passing the hiker, the cyclist stops and waits for the hiker. Now, bear in mind that in these 5 minutes, both the hiker and the cyclist were moving.
The cyclist was cycling at 20 mph and the hiker was walking at 5 pmh.

In 5 minutes, how much distance did the cyclist cover = 20mph * 5 minutes = 20mph * 5/60 hours = 5/3 miles
In 5 minutes, how much distance did the hiker cover (Remember, the hiker was also walking in these 5 minutes) = 4mph * 5 minutes = 4mph * 5/60 hours = 1/3 miles

So, in the 5 minutes following the overtake, the cyclist covered 5/3 miles and the hiker covered 1/3 miles.

How far ahead is the cyclist? 5/3 - 1/3 = 4/3 miles.

So the hiker needs to cover 4/3 miles to catch up to the cyclist.

Distance to be covered = 4/3 miles
Speed of the hiker = 4 mph

Time taken = Distance/Speed = (4/3)/4 = 1/3 hours = 20 minutes.

So the hiker will take 20 minutes to catch up. And that is how long the cyclist will have to wait.

The correct answer is C.

29.

$90/(v-3) - 90/(v+3)=0.5$
$v=33$
So, $t=90/36=2.5$
The correct answer is A.

GMAT Quant Topic 1: General Arithmetic
Part E: SI / CI / Population Growth

1.
The investment contract guarantees to make three interest payments:

$10,000 (initial investment)
+ $200 (1% interest on $10,000 principal = $100, so 2% = 2 × $100)
$10,200

+ $306 (1% interest on $10,200 principal = $102, so 3% = 3 × $102)
$10,506

+ $420.24 (1% interest on $10,506 principal = $105.06, so 4% = 4 × $105.06)
$10,926.24

The final value is $10,926.24 after an initial investment of $10,000. Thus, the total amount of interest paid is $926.24 (the difference between the final value and the amount invested).

The correct answer is E.

Top 1% expert replies to student queries (can skip)

In this question, interest is accruing on previous interest (because interest payments are being reinvested into the contract). We are not being able to use a standard CI amount formula here because we don't have one fixed rate to compound

Initial investment = 10,000
Interest paid at the end of 6 months = 200
Amount at that point of time becomes 10,200
At the end of 12 months, interest will be calculated at 3% of this previous amount. Hence interest = 306
Amount at that point in time becomes 10,506 and this is the amount on which 4% interest will be paid at the end of 18 months
Then interest paid at the end of 18 months = 420.24 and amount becomes 10,906.24

Initial investment was 10,000, then total interest paid out was 926.24

The correct answer is E.

2.
If we decide to find a constant multiple by the hour, then we can say that the population was multiplied by a certain number three times from 1 p.m. to 4 p.m.: once from 1 to 2 p.m., again from 2 to 3 p.m., and finally from 3 to 4 p.m.

Let's call the constant multiple x.

$2{,}000(x)(x)(x) = 250{,}000$
$2{,}000(x^3) = 250{,}000$
$x^3 = 250{,}000/2{,}000 = 125$
$x = 5$

Therefore, the population gets five times bigger each hour.
At 3 p.m., there were 2,000(5)(5) = 50,000 bacteria.
The correct answer is A.

3.
A population problem on the GMAT is best solved with a population chart that illustrates the swarm population at each unit of time. An example of a population chart is shown below:

Time	Population
4 hours ago	1,000
2 hours ago	2,000
NOW	4,000
in 2 hours	8,000
in 4 hours	16,000
in 6 hours	32,000
in 8 hours	64,000
in 10 hours	128,000
in 12 hours	256,000

As can be seen from the chart, in 12 hours the swarm population will be equal to 256,000 locusts. Thus, we can infer that the number of locusts will exceed 250,000 in slightly less than 12 hours. Since we are asked for an approximate value, 12 hours provides a sufficiently close approximation and is therefore the correct answer.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

Formula:
Nth term of GP is given by = (first term) ((Ratio^n)-1) or an=a1R(n−1)
Pure calculation:
The stem says that it doubles every 2 hours. We know that 4 hours ago their number was 1000. So, we add - 4 hours and the number 1000. In 2 hours, their number doubles, so we add - 2 hours and the number 2000. Using the same logic, their number is now 2000*2= 4000. We continue by adding hours to now, so now+2, now+4, now+6..... When we reach to + 12 their number is 256000, which is more than 250000, and we can stop!

4 hours ago: 1,000
2 hours ago: 2,000
Now: 4,000
In 2 hours: 8,000
in 4 hours: 16,000
in 6 hours: 32,000
in 8 hours: 64,000
in 10 hours: 128,000
in 12 hours: 256,000

Or using formula:
a1 is the first term and then an=a1R(n−1), which is the term in the nth place.
Between the first term and the nth term, n−1 multiplications by the ratio R take place, and this is reflected in the exponent of n−1.
Using the formula, you deduced that if a1=4000 is the first term, then the 7th term will be greater than 250,000. Between the first population and the 7th one, 6 cycles of 2 hours passed, a total of 12 hours, which is the correct answer**.**

The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Let the population 4 hours ago be x

Population 2 hours ago (2 hours after '4 hours ago') = 2x

Population right now = 4x

We're given that x = 1000

So current population = 4x = 4000

Let the population exceed 250000 after n '2 hour' periods.

So, population = 4000 * (2)^n > 250000

2^n > 250/4

2^n > 62.5

n = 6 (2^6 = 64)

So 6 '2 hour periods' = 2 * 6 = 12 hours. So our answer is 12 hours.
The correct answer is D.

4.
We need to consider the formula for compound interest for this problem:
$F = P(1 + r)^x$, where F is the final value of the investment, P is the principal, r is the interest rate per compounding period as a decimal, and x is the number of compounding periods (NOTE: sometimes the formula is written in terms of the annual interest rate, the number of compounding periods per year and the number of years). Let's start by manipulating the given expression for r:

$$r = 100\left(\sqrt{\frac{v+q}{p}} - 1\right) \rightarrow \frac{r}{100} = \sqrt{\frac{v+q}{p}} - 1 \rightarrow 1 + \frac{r}{100} = \sqrt{\frac{v+q}{p}} \rightarrow$$

$$\left(1 + \frac{r}{100}\right)^2 = \left(\sqrt{\frac{v+q}{p}}\right)^2 \rightarrow \left(1 + \frac{r}{100}\right)^2 = \frac{v+q}{p} \rightarrow p\left(1 + \frac{r}{100}\right)^2 = v + q \rightarrow$$

$$v = p\left(1 + \frac{r}{100}\right)^2 - q$$

Let's compare this simplified equation to the compound interest formula. Notice that r in this simplified equation (and in the question) is not the same as the r in the compound interest formula. In the formula, the r is already expressed as a decimal equivalent of a percent, in the question the interest is r percent. The simplified equation, however, deals with this discrepancy by dividing r by 100.
In our simplified equation, the cost of the share of stock (p), corresponds to the principal (P) in the formula, and the final share price (v) corresponds to the final value (F) in the formula. Notice also that the exponent 2 corresponds to the x in the formula, which is the number of compounding periods. By comparing the simplified equation to the compound interest formula, we see that the equation tells us that the share rose at the daily interest rate of p percent for TWO days. Then the share lost a value of q dollars on the third day, i.e. the "– q" portion of the expression. If the investor bought the share on Monday, she sold it three days later on Thursday.

The correct answer is B.

5.
Compound interest is computed using the following formula:

$F = P\,(1 + r/n)^{nt}$, where
F = Final value
P = Principal
r = annual interest rate
n = number of compounding periods per year
t = number of years

From the question, we can deduce the following information about the growth during this period:

At the end of the x years, the final value, F, will be equal to 16 times the principal (the money is growing by a factor of 16).
Therefore, $F = 16P$.
$r = .08$ (8% annual interest rate)
$n = 4$ (compounded quarterly)
$t = x$ (the question is asking us to express the time in terms of x number of years)

We can write the equation
$16P = P\,(1 + .08/4)^{4x}$
$16 = (1.02)^{4x}$

Now we can take the fourth root of both sides of the equation. (i.e.the equivalent of taking the square root twice) We will only consider the positive root because a negative 2 doesn't make sense here.
$16^{1/4} = [(1.02)^{4x}]^{1/4}$
$2 = (1.02)^{x}$

The correct answer is B.

6.

Say x individuals must be surveyed.

p% of the surveyed individuals fail to respond --> the number of individuals who did NOT fail to respond = $x - x * \frac{p}{100}$.

The above must be equal to 2n, so:

$x - x * \frac{p}{100} = 2n$ --> $x(1 - \frac{p}{100}) = 2n$ --> $x = \frac{200n}{100-p}$.

Answer: A.

7.
The question asks us to find the monthly payment on a \$1000 loan at 10% monthly interest compounded monthly for three months. Let's define the following variables:

P = Principal = \$1000
i = monthly interest rate = 10% = 0.1
c = compound growth rate = 1 + i = 1.1
x = monthly payment (to be calculated)

At the start, Louie's outstanding balance is P. During the next month, the balance grows by a factor of c as it accumulates interest, then decreases by x when Louie makes his monthly payment. Therefore, the balance after month 1 is $Pc - x$. Each month, you must multiply the previous balance by c to accumulate the interest, and then subtract x to account for Louie's monthly payment. In chart form:

Balance at start: P
Balance after month 1: $Pc - x$
Balance after month 2: $[Pc - x]c - x = Pc^2 - x(c+1)$
Balance after month 3: $[Pc^2 - x(c+1)]c - x = Pc^3 - x(c^2+c+1)$

Finally, the loan should be paid off after the third month, so the last loan balance must equal 0. Therefore:

$0 = Pc^3 - x(c^2+c+1)$
$x(c^2+c+1) = Pc^3$
$x = (Pc^3) / (c^2+c+1)$ Note that $c = 1.1$; $c^2 = 1.21$; $c^3 = 1.331$

$x = 1000(1.331) / (1.21+1.1+1)$
$x = 1331 / 3.31$

Rounded to the nearest dollar, $x = 402$.

The correct answer is C.

Top 1% expert replies to student queries (can skip)

Approximation:
Louie pays EMI every month from the month he receives the loan. So, we can not consider the interest on loan amount for 3 months, it should be for 2 months.
basically he is getting 10% interest per month for TWO month since he pays off in 3 months.
so 1000*1.1*1.1 = 1210
now divide by 3 = ~403.333. C is correct.
Accurate calculation:
Since he pays after each month, then after the first month (after the first payment) the interest is calculated on reduced balance
The interest has to be calculated on a reducing balance.
If monthly repayment = x
At the end of the 3 month period,
1.1*[1.1*{1.1*(1000)-x}-x]-x = 0
=> 3.31x = 1331
=> x ~ 402
The correct answer is C.

Top 1% expert replies to student queries (can skip)

Let the monthly payment be x.
After the 1st month there will be 1,000∗1.1−x dollars left to repay;
After the 2nd month there will be (1,000∗1.1−x)∗1.1−x=1,210−2.1x dollars left to repay;
After the 3rd month there should be 0 dollars left to repay:
(1,210−2.1x)∗1.1−x=0 --> 1331=3.31x --> x≈402
The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

TVM essentially says a dollar today is worth more than a dollar at a later date. So, if you receive a dollar at a later date, it is worth lesser than a dollar today. Why is that the case? because you can take something lesser than a dollar today, invest it at y%, and convert it to a dollar at that later date. This y% will obviously depend on a whole bunch of factors - the time you have in hand, how much lesser than a dollar you are starting with, what compounding you are looking at, and so on.

Now think of this from the lender's perspective. They have a cash outlay of $1,000 today. They will receive $x, $x, and $x in the future (one cash inflow every month from now). Because he can lend it out at 10% per month, his hurdle rate (the minimum rate of return he wants / is able to get) is 10% per month. Now, as we saw in TVM, each $x in the future is worth lesser than $x to the lender right now (why? because he could have taken the $x today, invested it at 10% per month, and converted it to 1.1x in one month, (1.1)^2x in 2 months, and (1.1)^3 in 3 months). So now think of the x he receives 1 month from now. How much is it worth to him today? x/1.1 Why? Because he can take x/1.1, invest it at 10%, and convert it to x in 1 month. Similarly, how much is the x he receives 2 months from now worth today? x/(1.1)^2. Because he can take x/(1.1)^2, invest it at 10% per month compounded monthly, and convert it to x at the end of two months from now. And similarly the x he receives 3 months from now is worth x/(1.1)^3. So, what is the present value of all the 3 $x he will receive in the future? (x/1.1) + (x/(1/1)^2) + (x/(1.1)^3) = Z (say). So receiving 3 payments of $X each in three months is equivalent to receiving this much right now. He will receive this much, but in the present he is also spending $1,000. So the net present value of his investment right now is Z - 1000

Now think of this - will the lender receive free money from Louie in the present? Or will the lender give out free money to Louie in the present? The answer to both questions is no. So what the lender gives out in the present is the value that he will receive in the present.

So Z = 1000 (in the language of finance we say NPV = Z - 1000 = 0; a project is only undertaken if the NPV is >= 0)

or, (x/1.1) + (x/(1/1)^2) + (x/(1.1)^3) = 1000

or, [(1.1)^2 + (1.1) + (1)]x/(1.1)^3 = 1000

Not the most comfortable arithmetic for sure, but very solvable

If you go through with it (isn't that long at all), you will get this:

(1.21 + 1.1 +1)x = 1000 * 1.331

or, 3.31x = 1331

or, x = 1331 / 3.31 = 402.11

Option (C) is the answer

8. The formula for calculating compound interest is $A = P(1 + r/n)^{nt}$ where the variables represent the following:

A = amount of money accumulated after t years (principal + interest)
P = principal investment
r = interest rate (annual)
n = number of times per year interest is compounded
t = number of years

In this case, x represents the unknown principal, r = 8%, n = 4 since the compounding is done quarterly, and t = .5 since the time frame in question is half a year (6 months). You can solve this problem without using compound interest. 8% interest over half a year, however that interest is compounded, is approximately 4% interest. So, to compute the principal, it's actually a very simple calculation:

$100 = .04x$
$2500 = x$

The correct answer is D.

9.

To solve a population growth question, we can use a population chart to track the growth. The annual growth rate in this question is unknown, so we will represent it as x. For example, if the population doubles each year, $x = 2$; if it grows by 50% each year, $x = 1.5$. Each year the population is multiplied by this factor of x.

Time	Population
Now	500
in 1 year	$500x$
in 2 years	$500x^2$
:	:
in n years	$500x^n$

The question is asking us to find the minimum number of years it will take for the herd to double in number. In other words, we need to find the minimum value of n that would yield a population of 1000 or more.

We can represent this as an inequality:
$500x^n > 1000$
$x^n > 2$

In other words, we need to find what integer value of n would cause x^n to be greater than 2. To solve this, we need to know the value of x. Therefore, we can rephrase this question as: "What is x, the annual growth factor of the herd?"

(1) INSUFFICIENT: This tells us that in ten years the following inequality will hold: $500x^{10} > 5000$
$x^{10} > 10$

There are an infinite number of growth factors, x, that satisfy this inequality.
For example, $x = 1.5$ and $x = 2$ both satisfy this inequality.
If $x = 2$, the herd of antelope doubles after one year.
If $x = 1.5$, the herd of antelope will be more than double after two years $500(1.5)(1.5) = 500(2.25)$.

(2) SUFFICIENT: This will allow us to find the growth factor of the herd. We can represent the growth factor from the statement as y. (NOTE y does not necessarily equal $2x$ because x is a growth factor. For example, if the herd actually grows at a rate of 10% each year, $x = 1.1$, but $y = 1.2$, i.e. 20%)

Time	Population
Now	500
in 1 year	$500y$
in 2 years	$500y^2$

According to the statement, $500y^2 = 980$
$y^2 = 980/500$
$y^2 = 49/25$
$y = 7/5$ OR 1.4 (y can"t be negative because we know the herd is growing)
This means that the hypothetical double rate from the statement represents an annual growth rate of 40%.
The actual growth rate is therefore 20%, so $x = 1.2$.

The correct answer is B.

10.

Top 1% expert replies to student queries (can skip) (additional)

Say we start with x cells. After one doubling, each cell splits into 2. So now we have 2x total cells. After another doubling, each of the 2x cells will split into two, leading to a total of 4x cells...and so on.

Then initially there are (2^0)x cells, after the first doubling there are (2^1)x cells, after the second doubling, there are (2^2)x cells and so on. The total population after each doubling forms a GP with common ratio 2. Not super useful, but important to point out for your mathematical sense.

Statement 1 - The population divided 2 hours ago, say was y immediately after the division. Post that the population has doubled twice, so now it has become 4y. 4y-y = 3750. Don't need to calculate anything, but this gives us two things:
i. We can find y
ii. We know for sure, because there have been two doublings, and each doubling happens after a constant period of time, that each doubling period is 1 hour. Then from now (when total population is 4y and we know y), till the time when the scientist destroys the entire population, there will be four GP terms corresponding to the total population Starting with 4y, we need to find the fourth term of the GP as the answer. We can. **So this statement by itself is sufficient**

Statement 2 - We know the population will double to 40,000 total cells after 3 hours from now. We also know the population will again double one hour from then (four hours from now), at which time the scientist will destroy it. But does this give us the doubling period? No. It can be 1 hour, it can also be anything less than that. So we don't know the doubling period from this information, and so don't know the number of times the population will double after it becomes 40,000, and so we don't know what the total population will become when it doubles at the four hour mark. **This statement by itself is not sufficient**

The correct answer is A.

11.
In order to answer this question, we need to know the formula for compound interest:

$$FV = P\left(1 + \frac{r}{100n}\right)^{nt}$$

FV is the future value.
P is the present value (or the principle).
r is the rate of interest.
n is the number of compounding periods per year.
t is the number of years.

Since Grace deposited *x* dollars at a rate of *z* percent, compounded annually:

$$\text{Grace's } FV = x\left(1 + \frac{z}{100}\right)^{(1)(1)}$$

And since Georgia deposited *y* dollars at a rate of *z* percent, compounded quarterly (four times per year):

$$\text{Georgia's } FV = y\left(1 + \frac{z}{(4)(100)}\right)^{(4)(1)}$$

So the question becomes:

Is $x\left(1 + \frac{z}{100}\right)^{(1)(1)} > y\left(1 + \frac{z}{(4)(100)}\right)^{(4)(1)}$?

Statement 1 tells us that $z = 4$. This tells us nothing about *x* or *y*. Insufficient.

Statement 2 tells us that $100y = zx$. Therefore, it must be true that $y = zx/100$. We can use this information to simplify the question:

$$x\left(1 + \frac{z}{100}\right)^{(1)(1)} > y\left(1 + \frac{z}{(4)(100)}\right)^{(4)(1)} \rightarrow$$

$$x\left(1 + \frac{z}{100}\right) > \frac{zx}{100}\left(1 + \frac{z}{400}\right)^{4} \rightarrow$$

$$\left(\frac{100}{x}\right)(x)\left(1 + \frac{z}{100}\right) > \left(\frac{100}{x}\right)\left(\frac{zx}{100}\right)\left(1 + \frac{z}{400}\right)^{4} \rightarrow$$

$$100\left(1 + \frac{z}{100}\right) > z\left(1 + \frac{z}{400}\right)^{4} \rightarrow$$

$$100 + z > z\left(1 + \frac{z}{400}\right)^{4}$$

The question is now:

Is $100 + z > z\left(1 + \frac{z}{400}\right)^{4}$?

We know from the question stem that *z* has a maximum value of 50. If we substitute that maximum value for *z*, we get:

$$100 + 50 > 50\left(1 + \frac{50}{400}\right)^4 \rightarrow$$
$$150 > 50(1 + .125)^4 \rightarrow$$
$$3 > (1.125)^4$$

So, the question is now:

Is $3 > (1.125)^4$?

Using estimation, we can see that this inequality is true. Since the maximum value of *z* makes this inequality true, all smaller values of *z* will do so as well. Therefore, we can answer "yes" to the rephrased question. Sufficient.

The correct answer is B: Statement 2 alone is sufficient, but statement 1 alone is not.
The correct answer is B.

Top 1% expert replies to student queries (can skip)

Quarterly compounding yields more than annual compounding but the difference is minuscule in % terms.
e.g. if you invest \$10 at 10% annual compounding, you get \$11 at the end of the year.
but if you invest \$10 at 10% quarterly compounding, you get \$11.038 at the end of the year.
You get a small fraction of interest extra.
So x is invested at annual compounding and y at quarterly compounding. If x=y, the amount received from y will be a little more.
Statement 1 tells us z = 4. We need to compare x with y so this is not sufficient.
Statement 2 tells us 100y = zx
x/y = 100/z
Since the maximum value of z is 50, x is at least twice of y.
If z% = 50%, the amount obtained from x is 1.5x (= 3y) and that obtained from y is a little more than 1.5y.
Definitely an investment of \$x results in a higher amount at the end of the year.
The correct answer is B.

12.
In order to answer this question, we need to recall the compound interest formula:
$FV = PV\left(1 + \frac{r}{100}\right)^n$, where *FV* is the future value of the investment, *PV* is the present value, *r* is the interest rate, and *n* is the number of compounding time periods.

In this case, we do not know the value of any of the unknowns and are asked to find *PV*. We do, however, know that the value of *PV* doubled. Therefore, *FV* = 2*PV*.

We can use this to construct and simplify the following equation:

$$2PV = PV\left(1+\frac{r}{100}\right)^{n} \rightarrow$$

$$\frac{2PV}{PV} = \frac{PV\left(1+\frac{r}{100}\right)^{n}}{PV} \rightarrow$$

$$2 = \left(1+\frac{r}{100}\right)^{n} \rightarrow$$

$$\sqrt[n]{2} = \sqrt[n]{\left(1+\frac{r}{100}\right)^{n}} \rightarrow$$

$$\sqrt[n]{2} = \left(1+\frac{r}{100}\right) \rightarrow$$

$$(100)\sqrt[n]{2} = (100)\left(1+\frac{r}{100}\right) \rightarrow$$

$$100\sqrt[n]{2} = 100 + r \rightarrow$$

$$100\sqrt[n]{2} - 100 = r \rightarrow$$

$$100(\sqrt[n]{2} - 1) = r$$

Therefore, the interest rate $r = 100(\sqrt[n]{2} - 1)$. Now we can look at the statements.

Statement (1) tells us that the interest rate was between 39% and 45%. Therefore, the value of $100(\sqrt[n]{2} - 1)$ is between 39 and 45.

If $n = 1$, then $r = 100(\sqrt[1]{2} - 1) = 100(2 - 1) = 100(1) = 100$. This value is not between 39 and 45. Therefore, n does not equal 1.

If $n = 2$, then $r = 100(\sqrt[2]{2} - 1) = 100(1.4 - 1) = 100(.4) = 40$. (Note that the square root of 2 is approximately 1.4.) 40 is between 39 and 45, so 2 is a possible value of n.

Can n be greater than 2? Since the value of r (40) is almost at the lower limit of the given range (39 to 45) when $n = 2$, it is not possible that increasing the value of n to 3 (resulting in our taking the cube root of 2, which is approximately 1.26) would yield a value of r that is above 39.

So, n must equal 2 and r must be approximately 40. But this does not tell us the value of PV.

Statement (2) tells us that the sale value of the bond would have been approximately 2,744 if the period of investment had been one month longer. We can set up the following equation:

$$2{,}744 = PV\left(1+\frac{r}{100}\right)^{n+1}$$

This does not allow us to find a value for PV. Statement (2) is insufficient.

If we take the statements together, we can substitute the values of *r* and *n* derived from statement (1):

$$2,744 = PV\left(1+\frac{40}{100}\right)^{2+1} \rightarrow$$
$$2,744 = PV(1+.4)^3 \rightarrow$$
$$2,744 = PV(1.4)^3 \rightarrow$$
$$2,744 = PV(2.744) \rightarrow$$
$$1000 = PV$$

Therefore, the approximate value of the original investment is $1,000.

The correct answer is C, both statements together are sufficient, but neither statement alone is sufficient.

The correct answer is C.

13.

Let's say:
I = the original amount of bacteria
F = the final amount of bacteria
t = the time bacteria grows
If the bacteria increase by a factor of x every y minutes, we can represent the growth of the bacteria with the equation:
$F = I(x)^{t/y}$

To understand why, let's assign some values to I, x and y:

I =	100
x =	2
y =	3

If the bacteria start off 100 in number and they double every 3 minutes, after 3 minutes there will be 100(2) bacteria. Let's construct a table to track the growth of the bacteria:

t (time)	F (final count)
3	$100(2) = 100(2)^1$
6	$100(2)(2) = 100(2)^2$
9	$100(2)(2)(2) = 100(2)^3$
12	$100(2)(2)(2)(2) = 100(2)^4$

We can generalize the F values in the table as
100(2)n. The 100 represents the initial count,
I.
The 2 represents the factor of growth (in this problem x).
The n represents the number of growth periods. The number of growth periods is found by dividing the time, t, by the amount of time it takes to complete a period, y.
From this example, we can extrapolate the general formula for exponential growth:
$F = I(x)^{t/y}$
This question asks us how long it will take for the bacteria to grow to 10,000 times their original amount. The bacteria will have grown to 10,000 times their original amount when F = 10,000I.

If we plug this into the general formula for exponential growth, we get: $10{,}000I = I(x)^{t/y}$ or $10{,}000 = (x)^{t/y}$.
The question is asking us to solve for t.
(1) SUFFICIENT: This statement tells us that $x1/y=10$. If we plug this value into the equation we can solve for t.

$10{,}000 = (x)^{t/y}$
$10{,}000 =[(x)^{1/y}]^t$
$10{,}000 =(10)^t$
$t = 4$

(2) SUFFICIENT: The bacteria grow one hundredfold in 2 minutes, that is to say they grow by a factor of 10^2. Since exponential growth is characterize by a constant factor of growth (i.e. by x every y minutes), for the bacteria to grow 10,000 fold (i.e. a factor of 10^4), they will need to grow another 2 minutes, for a total of four minutes ($10^2 \times 10^2 = 10^4$).
The correct answer is D, EACH statement ALONE is sufficient to answer the question.

The correct answer is D.

OR

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Statement:2
The culture grows one-hundredfold in 2 minutes. In other words, the sample grows by a factor of 10^2. Since exponential growth is characterized by a constant factor of growth (i.e. by a factor of x every y minutes), in another 2 minutes, the culture will grow by another factor of 10^2. Therefore, after a total of 4 minutes, the culture will have grown by a factor of 10^2 × 10^2 = 10^4, or 10,000. Therefore, the time taken would be 4 minutes. Mathematically, If in 2 mins 'a' becomes 100 times, in 4 mins(2mins*2) 'a' will become 100*100=10,000 times.
The Keyword is "by a factor of ", and is used commonly to mean the same as "multiplied by" or "divided by." If x is INCREASED by a factor of 4, it becomes 4x. If x is DECREASED by a factor of 4, it becomes x/4. The key word is the direction of change i.e. (increased / decreased) by a factor of. so here from both the points, we can derive that every y minute(s) (the number increases/multiplies by a factor of 10) or in case of (2) we straightaway know that at 2 minutes we have 100, so 10^2 = 100 means that the number became 10 at 1 minute and 100 at 2 minutes.

GMAT Quant Topic 1: General Arithmetic
Part F: RATIOS

1. First, let us rephrase the question. Since we need to find the fraction that is at least twice greater than 11/50, we are looking for a fraction that is equal to or greater than 22/50. Further, to facilitate our analysis, note that we can come up with an easy benchmark value for this fraction by doubling both the numerator and the denominator and thus expressing it as a percent: 22/50 = 44/100 = 44%. Thus, we can rephrase the question: "Which of the following is greater than or equal to 44%?"

   Now, let's analyse each of the fractions in the answer choices using benchmark values:
   2/5: This fraction can be represented as 40%, which is less than 44%.
   11/34: This value is slightly less than 11/33 or 1/3. Therefore, it is smaller than 44%.
   43/99: Note that the fraction 43/99 is smaller than 44/100, since fractions get smaller if the same number (in this case integer 1) is subtracted from both the numerator and the denominator.
   8/21: We know that 8/21 is a little less than 8/20 or 2/5. Thus, 8/21 is less than 44%.
   9/20: Finally, note that by multiplying the numerator and the denominator by 5, we can represent this fraction as 45/100, thus concluding that this fraction is greater than 44%.

   The correct answer is E.

2. Let's denote the number of juniors and seniors at the beginning of the year as j and s, respectively. At the beginning of the year, the ratio of juniors to seniors was 3 to 4: $j/s = 3/4$.
   Therefore, $j = 0.75s$
   At the end of the year, there were (j - 10) juniors and (s - 20) seniors.
   Additionally, we know that the ratio of juniors to seniors at the end of the year was 4 to 5.
   Therefore, we can create the following equation:

   $(j-10)/(s-20) = 4/5$

   Let's solve this equation by substituting $j = 0.75s$:
   $(j - 10) = 0.8(s - 20)$
   $(0.75s - 10) = 0.8s - 16$
   $0.8s - 0.75s = 16 - 10$
   $0.05s = 6$
   $s = 120$

   Thus, there were 120 seniors at the beginning of the year.

   The correct answer is E.

3. For a fraction question that makes no reference to specific values, it is best to assign a smart number as the "whole value" in the problem. In this case we'll use 30 since that is the least common denominator of all the fractions mentioned in the problem.

   If there are 30 students in the class, 3/5 or 18, left for the field trip. This means that 12 students were left behind.

   1/3 of the 12 students who stayed behind, or 4 students, didn't want to go on the field trip.

   This means that 8 of the 12 who stayed behind *did* want to go on the field trip.

   When the second vehicle was located, half of these 8 students or 4, were able to join the other 18 who had left already.

   That means that 22 of the 30 students ended up going on the trip. 22/30 reduces to 11/15.

   The correct answer is C.

4. The ratio of boys to girls in Class A is 3 to 4. We can represent this as an equation: $b/g = 3/4$. We can isolate the boys:
   $4b = 3g$
   $b = (3/4)g$
   Let's call the number of boys in Class B x, and the number of girls in Class B y. We know that the number of boys in Class B is one less than the number of boys in Class A. Therefore, $x = b - 1$. We also know that the number of girls in Class B is two less than the number of girls in Class A. Therefore, $y = g - 2$.
   We can substitute these in the combined class equation:
   The combined class has a boy/girl ratio of 17 to 22:
   $(b + x)/(g + y) = 17/22$. $(b + b - 1)/(g + g - 2) = 17/22$
   $(2b - 1)/(2g - 2) = 17/22$

Cross-multiplying yields:
$44b - 22 = 34g - 34$
Since we know that $b = (3/4)g$, we can replace the b:
$44(3/4)g - 22 = 34g - 34$
$33g - 22 = 34g - 34$
$12 = g$
Alternatively, because the numbers in the ratios and the answer choices are so low, we can try some real numbers. The ratio of boys to girls in Class A is 3:4, so here are some possible numbers of boys and girls in Class A:
B:G
3:4
6:8
9:12

The ratio of boys to girls in Class B is 4:5, so here are some possible numbers of boys and girls in Class A:
B:G
4:5
8:10
12:15
We were told that there is one more boy in Class A than Class B, and two more girls in Class A than Class B. If we look at our possibilities above, we see that this information matches the case when we have 9 boys and 12 girls in Class A and 8 boys and 10 girls in Class B. Further, we see we would have 9 + 8 = 17 boys and 12 + 10 = 22 girls in a combined class, so we have the correct 17:22 ratio for a combined class. We know now there are 12 girls in Class A.

The correct answer is E.

5.
We can solve this problem by choosing a smart number to represent the size of the back lawn. In this case, we want to choose a number that is a multiple of 2 and 3 (the denominators of the fractions given in the problem). This way, it will be easy to split the lawn into halves and thirds. Let's assume the size of the back lawn is 6.
Now we can use these numbers to calculate how much of each lawn has been mowed:

(size back lawn) = 6 units
(size front lawn) = (1/3)(size back lawn) = 2 units
(size total lawn) = (size back lawn) + (size front lawn) = 8 units

Front lawn: (1/2)(2) = 1 unit
Back lawn: (2/3)(6) = 4 unit
So, in total, 5 units of lawn have been mowed. This represents 5/8 of the total, meaning 3/8 of the lawn is left unmowed.

Alternatively, this problem can be solved using an algebraic approach. Let's assume the size of the front lawn is x and size of the back lawn is y. So, John has mowed $(1/2)x$ and $(2/3)y$, for a total of $(1/2)x + (2/3)y$. We also know that $x = (1/3)y$.
Substituting for x gives:

$(1/2)x + (2/3)y$
$(1/2)(1/3)y + (2/3)y$
$(1/6)y + (2/3)y$
$(5/6)y$ = lawn mowed

The total lawn is the sum of the front and back, $x + y$. Again, substituting for x gives $(1/3)y + y = (4/3)y$.
So, the fraction of the total lawn mowed is: lawn mowed/total lawn
$=\frac{(5/6)y}{(4/3)y}$
= (5/6) * (3/4)
= 15/24
=5/8
This leaves 3/8 unmowed.
The correct answer is C.

6. We know that the student to teacher ratio at the school is 16 to 1, and the total number of people is 510. Therefore:
   Number of students = (16/17)(510) = 480
   Number of teachers = (1/17)(510) = 30

   Kindergarten students make up 1/5 of the student population, so:
   Number of kindergarten students = (1/5)(480) = 96

   Fifth and sixth graders account for 1/3 of the remainder (after kindergarten students are subtracted from the total), therefore:
   Number of 5th and 6th grade students = (1/3)(480 – 96) = (1/3)(384) = 128

   Students in first and second grades account for 1/4 of all the students, so:
   Number of 1st and 2nd grade students = (1/4)(480) = 120
   So far, we have accounted for every grade but the 3rd and 4th grades, so they must consist of the students left over:
   Number of 3rd and 4th grade students = Total students – students in other grades
   Number of 3rd and 4th grade students = 480 – 96 – 128 – 120 = 136

   If there are an equal number of students in the third and fourth grades, then:
   Number of 3rd grade students = 136/2 = 68

   The number of students in third grade is 68, which is fewer than 96, the number of students in kindergarten. The number of students in 3rd grade is thus 96 – 68 = 28 fewer than the number of kindergarten students.

   The correct answer is C.

7. 50 million can be represented in scientific notation as 5×10^7. Restating this figure in scientific notation will enable us to simplify the division required to solve the problem. If one out of every 5×10^7 stars is larger than the sun, we must divide the total number of stars by this figure to find the solution:

   $$= \frac{4 \times 10^{11}}{5 \times 10^{7}}$$

   $= 4/5 \times 10^{(11-7)}$
   $= 0.8 \times 10^4$

   The final step is to move the decimal point of 0.8 four places to the right, with a result of 8,000.
   The correct answer is C.

8. For a fraction word problem with no actual values for the total, it is best to plug numbers to solve.

   Since 3/5 of the total cups sold were small and 2/5 were large, we can arbitrarily assign 5 as the number of cups sold.

Total cups sold = 5
Small cups sold = 3
Large cups sold = 2

Since the large cups were sold at 7/6 as much per cup as the small cups, we know:
$Price_{large} = (7/6)Price_{small}$
Let's assign a price of 6 cents per cup to the small cup.
Price of small cup = 6 cents
Price of large cup = 7 cents

Now we can calculate revenue per cup type:
Large cup sales = quantity × cost = 2 × 7 = 14 cents
Small cup sales = quantity × cost = 3 × 6 = 18 cents
Total sales = 32 cents

The fraction of total revenue from large cup sales = 14/32 = 7/16.

The correct answer is A.

9. This problem can be solved most easily by picking smart numbers and assigning values to the portion of each ingredient in the dressing. A smart number in this case would be one that enables you to add and subtract ingredients without having to deal with fractions or decimals. In a fraction problem, the "smart number" is typically based on the least common denominator among the given fractions.

   The two fractions given, 5/8 and 1/4, have a least common denominator of 8. However, we must also consider the equal parts salt, pepper and sugar.
   Because 1/4 = 2/8, the total proportion of oil and vinegar combined is 5/8 + 2/8 = 7/8. The remaining 1/8 of the recipe is split three ways: 1/24 each of salt, pepper, and sugar. 24 is therefore our least common denominator, suggesting that we should regard the salad dressing as consisting of 24 units. Let's call them cups for simplicity, but any unit of measure would do. If properly mixed, the dressing would consist of
   5/8 × 24 = 15 cups of olive oil
   1/4 × 24 = 6 cups of vinegar
   1/24 × 24 = 1 cup of salt
   1/24 × 24 = 1 cup of sugar
   1/24 × 24 = 1 cup of pepper

   Miguel accidentally doubled the vinegar and omitted the sugar. The composition of his bad salad dressing would therefore be

   15 cups of olive oil
   12 cups of vinegar
   1 cup of salt
   1 cup of pepper

   The total number of cups in the bad dressing equals 29. Olive oil comprises 15/29 of the final mix.

   The correct answer is A.

10.
This problem never tells us how many books there are in any of the libraries. We can, therefore, pick numbers to represent the quantities in this problem. It is a good idea to pick Smart Numbers, i.e. numbers that are multiples of the common denominator of the fractions given in the problem.
In this problem, Harold brings 1/3 of his books while Millicent brings 1/2. The denominators, 2 and 3, multiply to 6, so let's set Harold's library capacity to 6 books. The problem tells us Millicent has twice as many books, so her library capacity is 12 books. We use these numbers to calculate the size of the new home's library capacity. 1/3 of Harold's 6-book library equals 2 books. 1/2 of Millicent's 12-book library equals 6 books. Together, they bring a combined 8 books to fill their new library.

The fraction we are asked for, (new home's library capacity) / (Millicent's old library capacity), therefore, is 8/12, which simplifies to 2/3.

The correct answer is B.

11.
The ratio of dogs to cats to bunnies (Dogs: Cats: Bunnies) can be expressed as 3x: 5x : 7x. Here, x represents an "unknown multiplier." In order to solve the problem, we must determine the value of the unknown multiplier.

Cats + Bunnies = 48
5x + 7x = 48
12x = 48
x = 4

Now that we know that the value of x (the unknown multiplier) is 4, we can determine the number of dogs.

Dogs = 3x = 3(4) = 12

The correct answer is A.

12.
Boys = 2n/5, girls = 3n/5
Girls studying Spanish = $\frac{3n}{5} \times \frac{1}{3} = \frac{n}{5}$
Girls not studying Spanish = $\frac{3n}{5} - \frac{n}{5} = \frac{2n}{5}$
Girls studying French = $\frac{2n}{5} \times \frac{5}{6} = \frac{n}{3}$
Girls studying German = (all girls) – (girls studying Spanish) – (girls studying French)
$= \frac{3n}{5} - \frac{n}{5} - \frac{n}{3} = \frac{n}{15}$

The correct answer is E.

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2/5 of the students are boys, thus 3/5 of the students are girls.
1/3 of the girls in the club study Spanish and 5/6 of the remaining girls study French. Thus 5/6 of 2/3, or 10/18, of the girls study French.
The rest of the girls study German, thus 1-(1/3+10/18)=1/9 of the girls study German.
Since girls comprise 3/5 of the students, then 3/5*1/9=1/15 of all students study German.
The correct answer is E.

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There are a total of n students in the class.

Number of boys = 2n/5
Number of girls = 3n/5
Number of girls taking Spanish = (1/3)(3n/5) = n/5
Remaining number of girls = 3n/5 - n/5 = 2n/5
Number of girls studying French = (5/6)(2n/5) = n/3
Rest every girl studies German, then
Number of girls studying German = Total number of girls - Number of girls taking Spanish - Number of girls taking French

Number of girls studying German = 3n/5 - n/5 - n/3 = (9n - 3n - 5n)/15 = n/15
The correct answer is E.

13.
Since the problem deals with fractions, it would be best to pick a smart number to represent the number of ball players. The question involves thirds, so the number we pick should be divisible by 3. Let's say that we have 9 right-handed players and 9 left-handed players (remember, the question states that there are equal numbers of righties and lefties).
Two-thirds of the players are absent from practice, so that is (2/3)(18) = 12. This leaves 6 players at practice. Of these 6 players, one-third were left-handed. This yields (1/3)(6) = 2 left-handed players at practice and 9 – 2 = 7 left-handed players NOT at practice. Since 2 of the 6 players at practice are lefties, 6 – 2 = 4 players at practice must be righties, leaving 9 – 4 = 5 righties NOT at practice.
The question asks us for the ratio of the number of righties not at practice to the number of lefties not at practice. This must be 5 : 7 or 5/7.

The correct answer is C.

14.
We are told that bag B contains red and white marbles in the ration 1:4. This implies that W_B, the number of white marbles in bag B, must be a multiple of 4.

What can we say about W_A, the number of white marbles in bag A? We are given *two* ratios involving the white marbles in bag A. The fact that the ratio of red to white marbles in bag A is 1:3 implies that W_A must be a multiple of 3. The fact that the ratio of white to blue marbles in bag A is 2:3 implies that W_A must be a multiple of 2. Since W_A is both a multiple of 2 *and* a multiple of 3, it must be a multiple of 6.

We are told that $W_A + W_B = 30$. We have already figured out that W_A must be a multiple of 6 and that W_B must be a multiple of 4. So all we need to do now is to test each candidate value of W_A (i.e. 6, 12, 18, and 24) to see whether, when plugged into $W_A + W_B = 30$, it yields a value for W_B that is a multiple of 4. It turns out that $W_A = 6$ and $W_A = 18$ are the only values that meet this criterion.

Recall that the ratio of red to white marbles in bag A is 1:3. If there are 6 white marbles in bag A, there are 2 red marbles. If there are 18 white marbles in bag A, there are 6 red marbles. Thus, the number of red marbles in bag A is either 2 or 6. Only one answer choice matches either of these numbers.

The correct answer is D.

15. Initially the ratio of B: C: E can be written as $8x$: $5x$: $3x$. (Recall that ratios always employ a common multiplier to calculate the actual numbers.)
    After removing 4 pounds of clothing, the ratio of books to clothes is doubled. To double a ratio, we double just the first number; in this case, doubling 8 to 5 yields a new ratio of 16 to 5. This can be expressed as follows:

   Books/clothing = 8x/(5x-4) = 16/5 [Cross multiply to solve for x]

$40x = 80x – 64$
$40x = 64$
$x = 8/5$
The question asks for the approximate weight of the electronics in the suitcase. Since there are $3x$ pounds of electronics there are $3 \times (8/5) = 24/5$ or approximately 5 pounds of electronics in the suitcase.

The correct answer is C.

16.
It is useful to think of the ratio as $1x : 2x : 4x$, where x is the "missing multiplier" that you use to find the actual numbers involved. For example, if $x = 1$, then the numbers of hours worked by the three men are 1, 2, and 4. If $x = 2$, then the numbers are 2, 4, and 8. If $x = 11$, then the numbers are 11, 22, and 44. Notice that these numbers all retain the original ratio. If we knew the multiplier, we could figure out the number of hours any of the men worked. So we can rephrase the question as, "What is the missing multiplier?"
SUFFICIENT: Since the three men worked a total of 49 hours and since $1x + 2x + 4x = 7x$, we know that $7x = 49$. Therefore, $x = 7$. Since Bob worked $2x$ hours, we know he worked $2(7) = 14$ hours.
SUFFICIENT: This statement tells us that $4x = 1x + 21$. Therefore, $3x = 21$ and $x = 7$. Since Bob worked $2x$ hours, we know he worked $2(7) = 14$ hours.

The correct answer is D.

17.

The question asks us to find the ratio of gross revenue of computers to printers, given that the price of a computer is five times the price of a printer. We will prove that the statements are insufficient either singly or together by finding two examples that satisfy all the criteria but give two different ratios for the gross revenue of computers to printers.

(1) INSUFFICIENT: Statement (1) says that the ratio of computers to printers sold in the first half of 2003 was in the ratio of 3 to 2, so let's assume they sold 3 computers and 2 printers. Using an example price of $5 and $1 indicates that the computer gross was $15 and the printer gross was $2.

During the second half of 2003, the ratio of computers to printers sold was 2 to 1. For example, they may have sold 2 computers and 1 printer grossing $10 and $1 respectively. Adding in the first half revenue, we can calculate that they would have grossed $25 and $3 respectively for the full year.

Alternatively for the second half of 2003 they may have sold 4 computers and 2 printers, which is still in the ratio of 2 to 1. In this case they would have grossed $20 and $2 respectively. Now adding in the first half revenue indicates they would have grossed $35 and $4 respectively for the full year, which is a different ratio. Therefore statement (1) is insufficient to give us a definitive answer.

(2) INSUFFICIENT: Statement (2) tells us that a computer costs $1,000, but it tells us nothing about the ratio or numbers of computers or printers sold.

(1) and (2) INSUFFICIENT: Statement (2) fixes the price of a computer at $1000, but the counterexample given in the explanation of statement (1) still holds, so statements (1) and (2) together are still insufficient.

The correct answer is E.

18. Let x represent the amount of water in Pool X, and y represent the amount of water in Pool Y. If we let z represent the proportion of Pool Y's current volume that needs to be transferred to Pool X, we can set up the following equation and solve for z: (water currently in Pool X) + (water transferred) = (water currently in Pool Y) – (water transferred)

$x + zy = y - zy$

$x + 2zy = y$

$2zy = y - x$

$z = y/2y - x/2y$

$z = \frac{1}{2}(1 - x/y)$

So, the value of z depends only on the ratio of the water currently in Pool X to the water currently in Pool Y, or x/y, The rephrased question is: "What is x/y ?"

Remember that x and y do NOT represent the capacities of either pool, but rather the ACTUAL AMOUNTS of water in each pool.

(1) SUFFICIENT: if we let X represent the capacity of Pool X, then the amount of water in Pool X is (2/7)X. So, $x = (2/7)X$. We can calculate the total amount of water in Pool Y, or y, as follows: $y = (6/7)X – (2/7)X = (4/7)X$. We can see that Pool Y has twice as much water as Pool X, or $2x = y$, or $x/y = 1/2$

(2) INSUFFICIENT: This gives no information about the amount of water in

Pool Y.

The correct answer is A.

19. We can rewrite the information in the question as an equation representing the T, the total dollar value of the sale:

L + M + S = T

L = the dollar amount received by the partner with the largest share

M = the dollar amount received by the partner with the middle (second largest) share

S = the dollar amount received by the partner with the smallest share

We are also told in the question that L = (5/8)T. Thus we can rewrite the equation as follows:(5/8)T + M + S = T.

Since the question asks us the value of S, we can simplify the equation again as follows: S = M + (3/8)T

Thus, in order to solve for S, we will need to determine the value of both M and T.

The question can be rephrased as, what is the value of M + (3/8)T?

NOT SUFFICIENT: The first statement tells us that S = (1/5)M. This gives us no information about T so statement one alone is not sufficient.

SUFFICIENT: The second statement tells us that M = (1/2)L = \$1 million. Additionally, since we know from the question that L = (5/8)T, then M must be equal to 1/2 of 5/8(T) or 5/16(T). We can therefore solve for T as follows:

M = \$1,000,000 = 5/16T

\$3,200,000 = T

We can now easily solve for S:

L + M + S = T

2 million + 1 million + S = \$3.2 million

S = .2 million

The correct answer is B.

20. The question asks us to solve for the ratio of pennies (p) to dimes (d). INSUFFICIENT: This tells us that the ratio of nickels (n) to dimes (d) is 3:2. This gives us no information about the ratio of pennies to dimes. INSUFFICIENT: This tells us that there is \$7, or 700 cents in the piggy bank. We can write an equation for this as follows, using the value of each type of coin: $10d + 5n + p = 700$. This is not enough information for us to figure out the ratio of p to d.

(1) AND (2) INSUFFICIENT: Taken together, both statements still do not provide enough information for us to figure out the ratio of p to d. For example, there may be 3 nickels, 2 dimes, and 665 pennies in the piggy bank (this keeps the ratio of nickels to dimes at 3:2 and totals to \$7). Alternatively, there may be 30

nickels, 20 dimes, and 350 pennies (this also keeps the ratio of nickels to dimes at 3:2 and totals to $7). In these 2 cases the ratio of pennies to dimes is not the same.

The correct answer is E.

21. To determine the ratio of Chemical A to Chemical C, we need to find the amount of each in the solution. The question stem already tells us that there are 10 milliliters of Chemical C in the final solution. We also know that the original solution consists of only Chemicals A and B in the ratio of 3 to 7. Thus, we simply need the original volume of the solution to determine the amount of Chemical A contained in it.

SUFFICIENT: This tells us that original solution was 50 milliliters. Thus, there must have been 15 milliliters of Chemical A (to 35 milliliters of Chemical B). The ratio of A to C is 15 to 10 (or 3 to 2).
SUFFICIENT: This tells us that the final solution was 60 milliliters. We know that this includes 10 milliliters of Chemical C. This means the original solution contained 50 milliliters. Thus, there must have been 15 milliliters of Chemical A (to 35 milliliter of Chemical B). The ratio of A to C is 15 to 10 (or 3 to 2).

The correct answer is D.

22. Given woman: children=5:2
1). children: man=5:11, you agree it is
insufficient 2). W<30, you also agree it alone is
insufficient
Together, w:c:m = 25:10:22 (all have to be integers!) thus w=25 and m=22.
The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

Explanation for Statement 1:

The information given in the question and statement 1, we get the ratio of the number of children to men to women. But what we have is the ratio and not the actual number of children, men and women.

if the ratio is 25 : 10 : 22, then :

Case 1 : Number of women = 50, number of children = 20, number of men = 44 satisfy the ratio.

Case 2 : Number of women = 75, number of children = 30, number of men = 66 also satisfy the ratio.

There are infinitely many values that will satisfy this ratio. Since we do not have a unique answer to the number of men, statement 1 is insufficient

23.
(1) Remember that ratios are the same as fractions in this sort of context.
this statement means that the FT employees are a smaller fraction of division Y than of the company as a whole. this means that they must be a bigger fraction of division X than of the company as a whole, because the fraction of the whole company that's employed FT must be between the two divisions' fractions. --Sufficient.

(Analogy: if i mix two powders together to make a shake that's 5% fat, and the first powder is 3% fat, then the second powder must be more than 5% fat)

(2)
The first part means that (FT in div. X) > (FT in div. Y), and the second part means that (PT in div. X) < (PT in div. Y).
Therefore, considering the ratio of FT : PT for each division, we have that FT/PT for div. X must be greater than FT/PT for div. Y. (this is the case because of either the numerator or the denominator: the numerator of X is greater, and the denominator is smaller.)
since the FT/PT fraction is bigger for div. X than for div. Y, it must be bigger for div. X than for the company as a whole (see the reasoning above under statement (1) for why this is true).

The correct answer is D.

24.
The total number of pigs and cows is 40.
For1, C>2P
For 2, P>12
Combine 1 and 2, if P=13, C is 14;if P=14, C is 12, it is impossible.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

Given Info:
2/3 of animals are (cows or pigs)
2/3 of 60 = 40 = Cows + Pigs = C + P
St 1 : C > 2 P (the statement says more than twice as many cows ; it means that the no. of cows is more)
Using Given info :   P = 40 - C
C > 2 (40 - C)
3C > 80
C > 26.66 (can be 27,28 and hence NOT SUFFICIENT)

St 2 : P > 12
P = 13, C = 27
P = 14 , C = 26

NOT SUFFICIENT

St 1 + 2,
P> 12 and C > 2P
P= 13 , C = 27 (Accepted)
P = 14 , C =26 (Not our solution as it doesn't satisfy C > 2 P)
Hence, the only solution is C = 27 (Unique value)
The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

Say number of pigs is p, number of cows is c

c + p = 40 ... (i)

Statement 1: Number of cows is >2p

Then number of cows + pigs = >3p

So something >3p = 40

So 3.xx p = 40

p = 40 / 3.xx

So p < 13.xx

But we don't uniquely know what the value of p has to be. And so we definitely don't know how many cows are there on the farm (and anyway, right from the start of this statement as it says more than twice without giving a fixed number)

Max p = 13, so min C = 27 (whole number greater than 2p)

Statement 2: Again, doesn't tell us anything about the number of cows

Combining the two,

We know that Max p = 13. But statement 2 tells us that p > 12. So the only possible value of p = 13.

If p = 13, then c = 27

Sufficient!
The correct answer is C.

25. Premise: one serving includes a certain number of dishes. (we don't know the exact number),and a dish requires 3/2 cups of pasta.(it means 4Y=mX, and X=3/2pasta.)
Question: nY require how many cups of pasta?
1). if Malik make X servings next time. He did prepare 2X dishes last time.
2). Malik used 6 cups of pasta the last time he prepared this dish.(it means 2X=6).
In this case, either condition one or condition two cannot deduce the final answer in that the decisive factors m, n are unknown.
As a result, the correct answer is C.

Top 1% expert replies to student queries (can skip)

Given: 4 servings of a certain dish requires 1 1/2 cups of pasta.
So, if we can determine the NUMBER OF DISH SERVINGS Malik prepares, we can determine how much past is required.

Statement 1: The next time he prepare this dish, Malik will make half as many servings as he did the last time he prepared the dish
We have no idea how many servings Malik made last time. So, we cannot determine the number of servings he'll make next time.
So, statement 1 is NOT SUFFICIENT

Statement 2: Malik used 6 cups of pasta the last time he prepared this dish
This information does not help us determine how much pasta he'll need NEXT TIME.
So, statement 2 is NOT SUFFICIENT

Statements 1 and 2 combined
Statement 2: Malik used 6 cups of pasta the last time
Statement 1: Next time, Malik will make half as many servings as he did the last time. If we makes half as many servings, Malik will need half as much pasta as he needed last time.
So, Malik will need 3 cups of pasta next time
Since we can answer the target question with certainty, the combined statements are SUFFICIENT

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

4 servings require 3/2 cups of pasta.

We want to know the number of cups of pasta that Malik will use the next time he prepares this dish. Please focus on the words " the next time".

Statement 1 :

We don't know the number of servings Malik makes the LAST TIME. So, there is no way for us to know the number of servings being made THE NEXT TIME. Insufficient!

Statement 2 :

We know that Malik used 6 cups of pasta the last time he prepared the dish. This does not tell us anything about the number of cups used the next time. Insufficient!

Taking the 2 statements together,

number of cups of pasta used the last time = 6
number of cups of pasta used the next time = 3

Sufficient!

The correct answer is C.

GMAT Quant Topic 2: Statistics
Part A: Mean

1. 2002 total = 60, mean = 15, in 2003, total = 72, mean = 18, from 15 to 18, increase = 20%

   Answer is 20%

2. A 200% increase over 2,000 products per month would be 6,000 products per month. (Recall that 100% = 2,000, 200% = 4,000, and "200% over" means 4,000 + 2,000 = 6,000.) In order to average 6,000 products per month over the 4-year period from 2005 through 2008, the company would need to produce 6,000 products per month × 12 months × 4 years = 288,000 total products during that period. We are told that during 2005 the company averaged 2,000 products per month. Thus, it produced 2,000 × 12 = 24,000 products during 2005. This means that from 2006 to 2008, the company will need to produce an additional 264,000 products (288,000 – 24,000).

   The correct answer is D.

3. **Top 1% expert replies to student queries**

   Required total points = 3.5*5=17.5
   Actual total points (4 subjects) = 2*3.7 + 3.3 + 3 = 13.7
   Required total points for the last subject = 17.5 - 13.7 = 3.8
   He has to get a minimum of 3.8 points or Grade A (that is 4 points). (Getting A-, 3.7 GPA, won't be enough to get an overall minimum GPA of 3.5 for the 5 subjects)
   The correct answer is A.

4. **Top 1% expert replies to student queries**

   If the average price of the stocks rose by approximately 2%, then a stock with a higher price (for example, $45 or $70) must have increased by 15%, while a stock with a lower price (for example, $20 or $35) must have decreased by 35%. So let's guess that the stock with the highest price has increased by 15%, and the stock with the lowest price has decreased by 35%. We need to verify that this is indeed the case.
   Old average price = (20 + 35 + 40 + 45 + 70)/5 = 210/5 = $42
   New average price = (20 x 0.65 + 35 + 40 + 45 + 70 x 1.15) = 213.5/5 = $42.7
   Now let's calculate the percent change:
   (42.7 - 42)/42 x 100 = 0.7/42 x 100 = 1.67% ≈ 2%
   Therefore, we do see that the stock with the highest price has increased by 15%, and the stock with the lowest price has decreased by 35%. That is, the three stocks whose prices remain constant are $35, $40, and $45.
   The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

4. 20, 35, 40, 45, 70.

Average Price ⇒ $\frac{210}{5}$ ⇒ 42.

2% increase in the Average Price

→ 2% (42)

→ $\underbrace{0.84}$

(This means there's an increase of $\underbrace{0.84 \times 5}$ (in sum) ⇒ $\boxed{4.2}$)

Avg increase → 0.84; no. of samples → 5

How did we get 4.2 ;

there's an increase in one stock (15%)

15% (S_1)

there's also a decrease of 35% in stock (2).

35% (S_2)

[We don't know what is S_1 and S_2 yet]

→ 15% (S_1) − 35% (S_2) ⇒ $\underbrace{4.2}_{}$ overall increase

→ (use options to your advantage now; don't calculate)

Stocks which increased and decreased.

(A) 40, 45 — $S_1 = 40, S_2 = 45$; $S_1 = 45, S_2 = 40$] won't satisfy.

(B) 35, 40 → won't satisfy similarly.

(C) 45, 70 → " " " .

(D) 20, 45 → " " " .

(E) 20, 70 → $S_1 = 70$, $S_2 = 20$ (This satisfies).

* $\boxed{\text{Use this approach.}}$

The correct answer is E.

5.

Let a be the average contribution size before John makes his contribution.
Let c be the total contribution size before John makes his contribution.
Let x be John's contribution.

$1.5a = 75$ --> $a = 50$

$a = 50 = \frac{c}{5}$ --> $c = 250$

$\frac{c+x}{6} = 75$ --> $\frac{250+x}{6} = 75$ --> $x = 200$

Answer: C.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

After John's contribution, the average of all contributions reached 75.
There were 6 people in total. Therefore, total contribution after John's contribution= 75*6 = 450

The initial average (before John's contribution) increased by 50% to 75 after John's contribution.

Let the initial average be x

x(1.5) = 75
x = 50 (Initial average)

Before John's contribution, the average was 50 and the number of people was 5. Therefore, total contribution before John's contribution = 50*5 = 250

John's contribution = total contribution after John's contribution - total contribution before John's contribution = 450 - 250 = 200.

The correct answer is C.

6. Mean of X is (-4-1+0+6+9)/5=2;

In order the increase to be minimal we should replace two smallest elements of X, which are -4 and -1, with two smallest primes, which are 2 and 3. Hence our new set will be {2, 3, 0, 6, 9} --> new mean is (2+3+0+6+9)/5=4.

Percent increase=(4-2)/2*100=100%.

The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Let us first calculate the current mean=(-4-1+0+6+9)/5=2

Since we have to consider min increase. We will select 2 smallest prime 2,3
We are replacing -4,-1 with 2,3

So now mean=4

Percentage increase=((4-2)/2*100)=100%

The correct answer is D.

7. 1. Z=60

We get a mean of 20 here initially.
If the set is multiplied by 10, the new mean becomes 200, increasing the mean by 900%
If the set is multiplied by 2, the new mean becomes 40, increasing the mean by 100%

We must find an exact value of multiplier or mean to find the percent increase in mean.
Not Sufficient.

2. N = Z / M

(-14, -12, 17, 28, 41 + Z)/6 = M

60+Z=6M
N = Z / M; Z=NM

60+NM=6M.
Not Sufficient to find an exact mean or multiplier.
Not Sufficient.

Combining both;
60+NM=6M
NM=Z=60
60+60=6M
M = 120/6= 20
And
N = Z/M = 60/20=3

We know the multiplier.
Sufficient.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

Explanation for Statement 2:

Let us look at statement 2.

Mean of the set = M = (-14 - 12 + 17 + 28 + 41 + Z)/6 = (Z+60)/6

Z + 60 = 6M

We're given that N = Z/M OR Z = MN

Therefore,

MN + 60 = 6M

N = 6 - 60/M

Mean of the set if every element is multiplied by N = MN

We need to find the percentage change in mean. Percent change = (MN - M)/M = N - 1 = 5 - 60/M.

This is clearly insufficient, since we need the value of M to arrive at a unique value.
So the answer cannot be B.

8.

Mean of the given set is (1+6+11+16+21)/5=11.

Now, in order the mean not to change, the mean of the new set we add to the old one should also be equal to 11 (or as in all 3 new sets there are 3 numbers, then their sum must be 3*11=33). Let's check:

I. 1.5, 7.11 and 16.89 --> will end with 0.5 sum not 33. Discard.
II. 5.36, 10.7 and 13.24 --> will end with 0.3 sum not 33. Discard.
III. -21.52, 23.3, 31.22 --> -21.52+23.3+31.22=-21.52+54.52=33. Correct.

Answer: C (III only).
The correct answer is C.

9. D

Mean of {2,8,10,12} is $mean=\frac{2+8+10+12}{4}=\frac{32}{4}=8$ --> new mean thus should equal to $8*1.25=10$, so $\frac{2+8+10+12+n+k}{6}=10$ (note that now we have the set of 6 terms not 4) --> $n+k=60-32=28$ --> $n^2+2nk+k^2=(n+k)^2=28^2=784$.

Answer: D.

10. Statement 1 says - KxLxM is a multiple of 6. This could be either 1,2,3 which equals 6 (6 is a multiple of itself) or 2,3,4 (which equals 12 also a multiple of 6) - so insuff.
Statement 2 says - There are at least 2 prime numbers among K,L, and M - this since K,L,M has to be 1-4 and consecutive this gives us no new information - so insuff.

Taken together doesn't help us either.

The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

The numbers arranged in ascending order are : K L M 5 7.

We also know that K, L, M are consecutive integers. We want to calculate the mean of set X. Meaning, we need a unique set of values for K, L, M.

Statement 1 :

KLM is a multiple of 6

Case 1: K = 1, L = 2, M = 3. KLM = 6, which is a multiple of 6
Case 2: K = 2, L = 3, M = 4. KLM = 24, which is a multiple of 6

Since we do not have a unique set of values for K,L,M, statement 1 is insufficient

Statement 2 :

There are at least 2 prime numbers among K, L, M

Case 1 : K = 1, L = 2, M = 3. Here, L and M are prime (Valid case)
Case 2 : K = 2, L = 3, M = 4. Here, K and L are prime (Valid case)

Again, since we do not have a unique set of values for K,L,M, statement 2 is insufficient.

Combining the two statements,

KLM is a multiple of 6 AND There are at least 2 prime numbers among K, L, M

Case 1 : K = 1, L = 2, M = 3. (Valid case)
Case 2 : K = 2, L = 3, M = 4. (Valid case)

Again, since we do not have a unique set of values for K,L,M, the two statements combined are insufficient.
The answer is E

11. This question deals with weighted averages. A weighted average is used to combine the averages of two or more subgroups and to compute the overall average of a group. The two subgroups in this question are the men and women. Each subgroup has an average weight (the women's is given in the question; the men's is given in the first statement). To calculate the overall average weight of the group, we would need the averages of each subgroup along with the ratio of men to women. The ratio of men to women would determine the weight to give to each subgroup's average. However, this question is not asking for the weighted average, but is simply asking for the ratio of women to men (i.e. what percentage of the competitors were women).
(1) INSUFFICIENT: This statement merely provides us with the average of the other subgroup – the men. We don't know what weight to give to either subgroup; therefore we don't know the ratio of the women to men.
(2) SUFFICIENT: If the average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women, there must be twice as many men as women. With a 2:1 ratio of men to women of, 33 1/3% (i.e. 1/3) of the competitors must have been women. Consider the following rule and its proof. RULE: The ratio that determines how to weight the averages of two or more subgroups in a weighted average ALSO REFLECTS the ratio of the distances from the weighted average to each subgroup's average.
Let's use this question to understand what this rule means. If we start from the solution, we will see why this rule holds true. The average weight of the men here is 150 lbs, and the average weight of the women is 120 lbs. There are twice as many men as women in the group (from the solution) so to calculate the weighted average, we would use the formula [1(120) + 2(150)] / 3. If we do the math, the overall weighted average comes to 140.
Now let's look at the distance from the weighted average to the average of each subgroup.
Distance from the weighted avg. to the avg. weight of the men is 150 – 140 = 10. Distance from the weighted avg. to the avg. weight of the women is 140 – 120 = 20. Notice that the weighted average is twice as close to the men's average as it is to the women's average, and notice that this reflects the fact that there were twice as many men as women. In general, the ratio of these distances will always reflect the relative ratio of the subgroups.
The correct answer is (B), Statement (2) ALONE is sufficient to answer the question, but statement (1) alone is not.

The correct answer is B.

Top 1% expert replies to student queries (can skip)

If the average weight of the entire group were twice as close to the average weight of the men as it were to the average weight of the women, then there must be twice as many men as women. With a 2:1 ratio of men to women of, 33 1/3% (i.e. 1/3) of the competitors must have been women and 66 2/3% (i.e. 2/3) of the competitors must have been men.

Explanation:
Statement 2 is a direct reference to the number line method. Or you can say that it refers to the core concept of average weight.
It says that the group avg is twice as close to the men's average as the women's average. We know that group average lies between the two averages. If men are more, the group average will be closer to men's average. If women are more, the group average will be closer to women's average

....................|......................................|..........................|
.........Women's avg.....................Group Avg...................Men's Avg

The statement says that group average is twice as close to men's avg so it means that number of men is twice of number of women. If you look at the scale method discussed in the link, you will understand it even better. We are given that the distance between group avg and men's avg is half the distance between group avg and women's avg. So men must be twice as much as women.
The correct answer is (B), Statement (2) ALONE is sufficient to answer the question, but statement (1) alone is not.
The correct answer is B.

12. We can simplify this problem by using variables instead of numbers. x = 54,820, x + 2 = 54,822. The average of $(54,820)^2$ and $(54,822)^2$ =

$$\frac{(54,820)^2 + (54,822)^2}{2} = \frac{x^2 + (x+2)^2}{2}$$

$$= \frac{x^2 + (x^2 + 4x + 4)}{2} = \frac{2x^2 + 4x + 4}{2} = x^2 + 2x + 2$$

Now, factor $x^2 + 2x + 2$. This equals $x^2 + 2x + 1 + 1$, which equals $(x + 1)^2 + 1$.

Substitute our original number back in for x as follows:

$(x + 1)^2 + 1 = (54,820 + 1)^2 + 1 = (54,821)^2 + 1$.

The correct answer is D.

13. First, lets use the average formula to find the current mean of set S: Current mean of set S = (sum of the terms)/(number of terms): (sum of the terms) = (7 + 8 + 10 + 12 + 13) = 50
(number of terms) = 5
=50/5
= 10
Mean of set S after integer n is added = 10 × 1.2 = 12
Next, we can use the new average to find the sum of the elements in the new set and compute the value of integer n. Just make sure that you remember that after integer n is added to the set, it will contain 6 rather than 5 elements. Sum of all elements in the new set = (average) × (number of terms) = 12 × 6 = 72

Value of integer n = sum of all elements in the new set – sum of all elements in the original set = 72 – 50 = 22

The correct answer is D.

14. Let x = the number of 20 oz. bottles
48 – x = the number of 40 oz. bottles
The average volume of the 48 bottles in stock can be calculated as a weighted average:
20x + (48-x)40 = 35
x = 12

Therefore there are 12 twenty oz. bottles and 48 – 12 = 36 forty oz. bottles in stock. If no twenty oz. bottles are to be sold, we can calculate the number of forty oz. bottles it would take to yield an average volume of 25 oz:
Let n = number of 40 oz. bottles
(12*20+n*40)/(n+12) = 25
(12)(20) + 40n = 25n + (12)(25)
15n = (12)(25) – (12)(20)
15n = (12)(25 – 20)
15n = (12)(5)
15n = 60
n = 4

Since it would take 4 forty oz. bottles along with 12 twenty oz. bottles to yield an average volume of 25 oz, 36 – 4 = 32 forty oz. bottles must be sold.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

The above approach involved weighted average:
Weighted average-

20 40
\ /
35
/ \
15 5

Ratio: 3: 1 or we can say that 40 ounce bottles are three time as many as 20 ounce
So number of 40 ounce bottles are 36 and 20 ounce bottles are 12
next (20(12) + 40x)/12+x = 25 -->where x is the remaining 40 ounce bottles
x= 4 i.e. 36-4=> 32 of 40 ounce bottles have been sold.
OR
Total bottles = 48
Present ratio of 20 to 40 ounces bottles to make average 35 ounces = 1:3
20 ounces bottles = 12
40 ounces bottles = 36
To make average 25 ounces , ratio of 20 to 40 ounces bottles = 3:1
Since no 20 ounces bottles are sold, number of 40 ounces bottles = 12/3 = 4
Number of 40 ounces bottles to be sold = 36-4 = 32 bottles

OR

We can also use algebraic approach (may consume a little more time):
Let 'x' be number of 20 ounce bottles , 'y' be 40 ounce bottles
x+y=48
20x+40y=1680

x=12,y=36
let's get to the remaining part of the question
Value of 20 ounce bottles will be 0 , if we consider the bottles sold.
20x+40z=25(x+z);
z = remaining bottles
x=12 , no removal here
20*12+40(z)=25(12+z);z=4
If remaining is 4 ,then (36-4)were sold=32.

The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Essentially there are 12 20-ounce bottles and 36 40-ounce bottles in the store currently. I believe you have got to this stage

Now say m 40-ounce bottles are sold. No 20-ounce bottles are sold. Then the final number of bottles in the store will be 48-m, with 12 20-ounce bottles and 36-m 40-ounce bottles remaining

Then [12*20 + (36-m)*40]/(48-m) = 25

Simply solve for m

240 + 36.40 - 40m = 48.25 - 25m

or, 15m = 240 + 36.40 - 48.25

or, 15m = 240 + 12.3.5.8 - 12.4.5.5

or, 15m = 240 + 12.5 (24 - 20)

or, 15m = 240 +12.5.4

or, 15m = 240 + 240

or, 15m = 480

or, m = 32

15. The average number of vacation days taken this year can be calculated by dividing the total number of vacation days by the number of employees. Since we know the total number of employees, we can rephrase the question as: How many total vacation days did the employees of Company X take this year?
(1) INSUFFICIENT: Since we don't know the specific details of how many vacation days each employee took the year before, we cannot determine the actual numbers that a 50% increase or a 50% decrease represent. For example, a 50% increase for someone who took 40 vacation days last year is going to affect the overall average more than the same percentage increase for someone who took only 4 days of vacation last year.
(2) SUFFICIENT: If three employees took 10 more vacation days each, and two employees took 5 fewer vacation days each, then we can calculate how the number of vacation days taken this year differs from the number taken last year:
(10 more days/employee)(3 employees) – (5 fewer days/employee)(2 employees) = 30 days – 10 days = 20 days
20 additional vacation days were taken this year.
In order to determine the total number of vacation days taken this year (i.e., in order to answer the rephrased question), we need to determine the number of vacation days taken last year. The 5 employees took an average of 16 vacation days each last year, so the total number of vacation days taken last year can be determine by taking the product of the two:
(5 employees)(16 days/employee) = 80 days

80 vacation days were taken last year. Hence, the total number of vacation days taken this year was 100 days. Note: It is not necessary to make the above calculations -- it is simply enough to know that you have enough information in order to do so (i.e., the information given is sufficient)!.
The correct answer is B.

16. The question is asking us for the weighted average of the set of men and the set of women. To find the weighted average of two or more sets, you need to know the average of each set and the ratio of the number of members in each set. Since we are told the average of each set, this question is really asking for the ratio of the number of members in each set.

(1) SUFFICIENT: This tells us that there are twice as many men as women. If m represents the number of men and w represents the number of women, this statement tells us that m = 2f. To find the weighted average, we can sum the total weight of all the men and the total weight of all the women, and divide by the total number of people. We have an equation as follows:
M * 150 + F * 120 / M + F
Since this statement tells us that m = 2f, we can substitute for m in the average equation and average now = 140. Notice that we don't need the actual number of men and women in each set but just the ratio of the quantities of men to women.
(2) INSUFFICIENT: This tells us that there are a total of 120 people in the room but we have no idea how many men and women. This gives us no indication of how to weight the averages.

The correct answer is A.

17. The mean or average of a set of consecutive integers can be found by taking the average of the first and last members of the set. Mean = (-5) + (-1) / 2 = -3.

The correct answer is B.

18.

(1) The sum of the ages of the six other children is 42.
If x is the age of Sarah and $x \geq 8$ then $(42+x)/7 = 8$, as the fourth age in the sequence is 8. Then $x = 14$.
If $5 \leq x < 8$ then $(42+x)/7 = x$, as now the fourth age in the sequence is x. This gives $x = 7$.
If $x < 5$ then $(42+x)/7 = 5$, as now the fourth age in the sequence is 5. This gives $x = -7$, impossible.
Not sufficient.

(2) Obviously, not sufficient.

(1) and (2):
Sufficient, because we can now choose between 7 and 14, and Sarah must be of age 7, as she cannot be the eldest.

Answer C.

19. We know that the average of x, y, and z is 11. We can therefore set the up the following equation:
(x + y + z)/3 = 11 Cross multiplying yields
x + y + z = 33
Since z is two more than x, we can replace z:
x + y + x + 2 = 33
2x + y + 2 = 33
2x + y = 31
Since 2x must be even and 31 is odd, y must also be odd (only odd + even = odd). x and z can be either odd or even. Therefore, only statement II (y is odd) must be true.

The correct answer is B.

20. It helps to recognize this problem as a consecutive integers question. The median of a set of consecutive integers is equidistant from the extreme values of the set. For example, in the set {1, 2, 3, 4, 5}, the median is 3, which is 2 away from 1 (the smallest value) and 2 away from 5 (the largest value). Therefore, the median of Set A must be equidistant from the extreme values of that set, which are x and y. So the distance from x to 75 must be the same as the distance from 75 to y. We can express this algebraically:

$75 - x = y - 75$

$150 - x = y$

$150 = y + x$

We are asked to find the value of $3x + 3y$. This is equivalent to $3(x + y)$. Since $x + y = 150$, We know that $3(x + y) = 3(150) = 450$.

Alternatively, the median of a set of consecutive integers is equal to the average of the extreme values of the set. For example, in the set {1, 2, 3, 4, 5}, the median is 3, which is also the average of 1 and 5. Therefore, the median of set A will be the average of x and y.

We can express this algebraically:

$(x + y)/2 = 75$ $x + y = 150$

$3(x + y) = 3(150)$

$3x + 3y = 450$

The correct answer is D.

21.

Let the total average be t, percentage of director is d.

Then, $t*100=(t-5000)(100-d)+(t+15000)d$

d can be solved out.

The correct answer is C.

22.
This question takes profit analysis down to the level of per unit analysis.

Let P = profit
R = revenue
C = cost
q = quantity
s = sale price per unit
m = cost per unit

Generally we can express profit as P = R – C
In this problem we can express profit as P = qs – qm

We are told that the average daily profit for a 7 day week is $5304, so
(qs – qm) / 7 = 5304 →q(s – m) / 7 = 5304 → q(s – m) = (7)(5304).

To consider possible value for the difference between the sale price and the cost per unit,
s – m, lets look at the prime factorization of (7)(5304):
(7)(5304) = 7 × 2 × 2 × 2 × 3 × 13 × 17

Since q and (s – m) must be multiplied together to get this number and q is an integer (i.e. # of units),
s – m must be a multiple of the prime factors listed above.
From the answer choices, only 11 cannot be formed using the prime factors above.

The Correct answer is D.

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5304 is not divisible by even 7. Then even C can be the answer. But, it's not. We need to see whether (7*5304) is divided by the given options.
Notice that the difference between sale price and cost per unit is the profit per unit. So, we are asked to find which of the options cannot be the profit per unit.
Now, since the average daily profit was $5304, then the profit per week was $7*5304=7*(2^3*3*13*17).
We also know that this profit was generated from the sale of q units, thus q=(profit per week)/(profit per unit)=7*(2^3*3*13*17)/(profit per unit).
q=(# of units sold)=integer, thus from the options presented profit per unit cannot be 11 because in this case q won't be an integer.
The Correct answer is D.

23.
Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.

(1) INSUFFICIENT: When the average assets under management (AUM) per customer of each of the 10 branches are added up and the result is divided by 10, the value that is obtained is the simple average of the 10 branches average AUM per customer. Multiplying this number by the total number of customers will not give us the total amount of assets under management. The reason is that what is needed here is a weighted average of the average AUM per customer for the 10 banks. Each branch's average AUM per customer needs to be weighted according to the number of customers at that branch when computing the overall average AUM per customer for the whole bank.

Lets look at a simple example to illustrate:

	Apples	People	Avg # of Apples per Person
Room A	8	4	8/4 = 2 apples/person
Room B	18	6	18/6 = 3 apples/person
Total	26	10	26/10 = 2.6 apples/person

If we take a simple average of the average number of apples per person from the two rooms, we will come up with (2 + 3) / 2 = 2.5 apples/person. This value has no relationship to the actual total average of the two rooms, which in this case is 2.6 apples. If we took the simple average (2.5) and multiplied it by the number of people in the room (10) we would NOT come up with the number of apples in the two rooms. The only way to calculate the actual total average (short of knowing the total number of apples and people) is to weight the two averages in the following manner: 4(2) + 6(3) / 10.

(2)SUFFICIENT: The average of $160 million in assets under management per branch spoken about here was NOT calculated as a simple average of the 10 branches‘ average AUM per customer as in statement 1. This average was found by adding up the assets in each bank and dividing by 10, the number of branches (―the total assets per branch were added up…‖). To regenerate that original total, we simply need to multiply the $160 million by the number of branches, 10. (This is according to the simple average formula: average = sum / number of terms)

The correct answer is B.

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A certain bank has ten branches. What is the total amount of assets under management at the bank?
(1) There are an average of 400 customers per branch. When each branch's average assets under management per customer is computed, these values are added together and this sum is divided by 10. The result is $400,000.
(2) The bank has a total of 4,000 customers. When the total assets per branch are added up, each branch is found to manage, on average, 160 million dollars in assets.
Solution:
Total Asset at Bank = Average asset at bank per customer*Total Customer
OR
Total Asset at Bank = Average asset per branch (if all branches have the same average)*Total number of branches
Question: Total Asset under Management at Bank = ?
Statement 1: There is an average of 400 customers per branch. When each branch's average assets under management per customer is computed, these values are added together and this sum is divided by 10. The result is $400,000 per customer.
It is unknown whether all branches have the same number of customers or the same average across all branches hence we can not find 'Average asset per branch' or ' Average asset at bank per customer'.
NOT SUFFICIENT
Statement 2: The bank has a total of 4,000 customers. When the total assets per branch are added up, each branch is found to manage, on average, 160 million dollars in assets.
Total Asset of all 10 branches = 160*10 million
SUFFICIENT
The correct answer is B.

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(1) INSUFFICIENT: When the average assets under management (AUM) per customer of each of the 10 branches are added up and the result is divided by 10, the value that is obtained is the simple average of the 10 branches' average AUM per customer. Multiplying this number by the total number of customers will not give us the total amount of assets under management. The reason is that what is needed here is a weighted average of the average AUM per customer for the 10 branches. Each branch's average AUM per customer needs to be weighted according to the number of customers at that branch when computing the overall average AUM per customer for the whole bank.

(2) SUFFICIENT: The average of $160 million in assets under management per branch spoken about here was NOT calculated as a simple average of the 10 branches' average AUM per customer as in statement 1. This average was found by adding up the assets in each bank and dividing by 10, the number of branches ("the total assets per branch were added up…"). To regenerate that original total, we simply need to multiply the $160 million by the number of branches, 10. (This is according to the simple average formula: average = sum / number of terms)
The correct answer is B.

24.
We're asked to determine whether the average number of runs, per player, is greater than 22. We are given one piece of information in the question stem: the ratio of the number of players on the three teams.
The simple average formula is just A = S/N where A is the average, S is the total number of runs and N is the total number of players. We have some information about N: the ratio of the number of players. We have no information about S.
SUFFICIENT. Because we are given the individual averages for the team, we do not need to know the actual number of members on each team. Instead, we can use the ratio as a proxy for the actual number of players. (In other words, we don't need the actual number; the ratio is sufficient because it is in the same proportion as the actual numbers.) If we know both the average number of runs scored and the ratio of the number of players, we can use the data to calculate:

# RUNS	RELATIVE # PLAYERS	R*P
30	2	60
17	5	85
25	3	75

The S, or total number of runs, is 60 + 85 + 75 = 220. The N, or number of players, is 2 + 5 + 3 = 10. A = 220/10 = 22. The collective, or weighted, average is 22, so we can definitively answer the question: No. (Remember that "no" is a sufficient answer. Only "maybe" is insufficient.)
INSUFFICIENT. This statement provides us with partial information about S, the sum, but we need to determine whether it is sufficient to answer the question definitively. "Is at least" means S is greater than or equal to 220. We know that the minimum number of players, or N, is 10 (since we can't have half a player). If N is 10 and S is 220, then A is 220/10 = 22 and we can answer the question No: 22 is not greater than 22. If N is 10 and S is 221, then A is 221/10 = 22.1 and we can answer the question Yes: 22.1 is greater than 22. We cannot answer the question definitively with this information.

The correct answer is A.

25.

We can rephrase this question by representing it in mathematical terms. If x number of exams have an average of y, the sum of the exams must be xy (average = sum / number of items). When an additional exam of score z is added in, the new sum will be xy + z.
The new average can be expressed as the new sum divided by x + 1, since there is now one more exam in the lot. New average = (xy + z)/(x + 1).
The question asks us if the new average represents an increase in 50% over the old average, y. We can rewrite this question as: Does (xy + z)/(x + 1) = 1.5y ?
If we multiply both sides of the equation by 2(x + 1), both to get rid of the denominator expression (x + 1) and the decimal (1.5), we get: 2xy + 2z = 3y(x + 1)
Further simplified, 2xy + 2z = 3xy +3y OR 2z = xy + 3y ?
Statement (1) provides us with a ratio of x to y, but gives us no information about z. It is INSUFFICIENT.

Statement (2) can be rearranged to provide us with the same information needed in the simplified question, in fact 2z = xy + 3y. Statement (2) is SUFFICIENT and the correct answer is (B).

We can solve this question with a slightly more sophisticated method, involving an understanding of how averages change. An average can be thought of as the collective identity of a group. Take for example a group of 5 members with an average of 5. The identity of the group is 5. For all intents and purposes each member of the group can actually be considered 5, even though there is likely variance in the group members. How does the average —identity of the group then change when an additional sixth member joins the group? This change in the average can be looked out WITHOUT thinking of a change to the sum of the group. For a sixth member to join the group and there to be no change to the

average of the group, that sixth member would have to have a value identical to the existing average, in this case 5. If it has a value of let's say 17 though, the average changes. By how much though?
5 of the 17 satisfy the needs of the group, like a poker ante if you will. The spoils that are left over are 12, which is the difference between the value of the sixth term and the average. What happens to these spoils? They get divided up equally among the now six members of the group and the amount that each member receives will be equal to the net change in the overall average. In this case the extra 12 will increase the average by 12/6 = 2.
Put mathematically, change in average = (the new term – existing average) / (the new # of terms)
We could have used this formula to rephrase the question above: (z – y) / (x + 1) = 0.5y Again if we multiply both sides of the expression by 2(x + 1), we get 2z – 2y = xy + y
OR 2z = xy + 3y. Sometimes this method of dealing with average changes is more useful than dealing with sums, especially when the sum is difficult or cumbersome to find.

The correct answer is B.

26.
To solve this problem, use what you know about averages. If we are to compare Jodie's average monthly usage to Brandon's, we can simplify the problem by dealing with each person's total usage for the year. Since Brandon's average monthly usage in 2001 was q minutes, his total usage in 2001 was 12q minutes. Therefore, we can rephrase the problem as follows:
Was Jodie's total usage for the year less than, greater than, or equal to 12q?
Statement (1) is insufficient. If Jodie's average monthly usage from January to August was 1.5q minutes, her total yearly usage must have been at least 12q. However, it certainly could have been more. Therefore, we cannot determine whether Jodie's total yearly use was equal to or more than Brandon's.

Statement (2) is sufficient. If Jodie's average monthly usage from April to December was 1.5q minutes, her total yearly usage must have been at least 13.5q. Therefore, her total yearly usage was greater than Brandon's.
The correct answer is B: Statement (2) alone is sufficient, but statement (1) alone is not sufficient.

The correct answer is B.

Top 1% expert replies to student queries (can skip)

Brandon's total usage in 2001 = (12 months)(average monthly usage) = 12q.
For Jodie's average monthly usage to be greater than Brandon's, Jodie's total usage in 2001 must be greater than Brandon's total usage in 2001.
Questions stem, rephrased:
Was Jodie's total usage in 2001 > 12q?
Statement 1:
Jodie's total usage for the 8 months January through August = (8 months)(average monthly usage) = (8)(1.5)q = 12q.
If Jodie has no usage during the remaining 4 months, then her total usage = 12q, so the answer to the question stem is NO.
If Jodie has some usage during the remaining 4 months, then her total usage > 12q, so the answer to the question stem is YES.
Since the answer is NO in the first case but YES in the second case, INSUFFICIENT.
Statement 2:
Jodie's total usage for the 9 months April through December = (9 months)(average monthly usage) = (9)(1.5)q = 13.5q.
Thus, the answer to the question stem is YES.
SUFFICIENT.

The correct answer is B: Statement (2) alone is sufficient, but statement (1) alone is not sufficient

The correct answer is B.

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1) from Jan to august we have 8 months, and average of these 8 months = 1.5q, thus total usage for these 8 months = 12q { (total usage/ 8)=1.5)}

now lets assume she didn't call at the remember 4 months. i.e. total usage for the last 4 months=0. then total average usage of jodi will be 12q/12 = q. now, if she continue to have the same average of 1.5q minutes, then her total cell phone usage for the last 4 months will be 1.5*4= 6. and the average for the whole year will be 12q (for the first eight months) + 6q (last 4 months) /12 = 1.5q, which is more than q. Thus final usage could be either equal or greater than q. Thus statement 1 alone is not sufficient.

2)

from April to December we have 9 months, and average of these 9 months =1.5q, thus total usage for these 9 months = 1.5q*9 =13.5q

now even if we assume. she didn't use her phone for the initial 3 months. then also her average phone usage for the whole year becomes more than q. (13.5q/12 >q) thus statement 2 alone is sufficient to answer.

hence B

(2) For the months April-December, J=1.5Q, which means that if Q=2, J=3 for 9 months, therefore Q=18 and J=27 for these 9 months. Whatever happens to J, on the first 3 months, it will still be J>Q at the end of the year.

So statement (2) is sufficient, and Answer is B.

From April to December 2001, Jodie's average monthly usage was 1.5q minutes.

The total usage from April to December is 1.5q x 9 = 13.5q minutes, which means her average monthly average is at least 13.5q/12 = 1.125q minutes. Even if she didn't use any minutes from January to March, her average monthly usage would still be greater than that of Brandon. Statement two alone is sufficient.

The correct answer is B.

27.
Before she made the payment, the average daily balance was $600, from the day, balance was $300.
When we find in which day she made the payment, we can get it.
Statement 1 is sufficient.
For statement 2, let the balance in x days is $600, in y days is $300.
X+Y=25
(600X+300Y)/25=540
x=20, y=5 can be solved out.
We know that on the 21 day, she made the payment.
The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Jane's credit card's average daily balance is the mean of daily balance at the end of the 30 days.

For example, if for the first 10 days of the month, the balance is 100, for the next 10 days, the balance is 200 and for the last 10 days, the balance is 300, then the average daily balance is :

Average daily balance = [(10)(100) + (10)(200) + (10)(300)]/30 = (1000 + 2000 + 3000)/30 = 6000/30 = 200

In this question, we know that Jane's balance at the beginning of the 30-day cycle = 600 dollars

We also know that during the 30-day cycle, Jane makes a payment of 300 dollars. So after the payment, the balance will be 300 dollars

Let us assume that the payment was made x days into the payment cycle.

So for x days, the balance was 600 and for the remaining (30-x) days, the balance was 300.

So, average for the 30-day cycle = [600x + 300(30-x)]/30 = 20x + 10(30-x) = 300 + 10x

Statement 1 : x = 20 (since payment was made on the 21st day)

average = 300 + 10x = 500 (sufficient!)

Statement 2 : The average daily balance through the 25th day is 540. Meaning, the average daily balance for 25 days is 540.

Using the same logic as before, and assuming that the payment of 300 dollars was made x days into the billing cycle, where x < 25.

Average of 25 days = [600x + 300(25-x)]/25 = 24x + 12(25-x) = 24x + 300 - 12x = 300 + 12x = 540

12x = 240
x = 12

Now that we know the value of x, we can calculate the daily average balance for 30 days.

Average = 300 + 10x = 420 (sufficient!)

So the answer is D.

28.

Combine 1 and 2, we can solve out price for C and D, C=$0.3, D=$0.4
To fulfil the total cost $6.00, number of C and D have more than one combination, for example: 4C and 12D, 8C and 9D…
The correct answer is E.

Top 1% expert replies to student queries (can skip)

Let D = the NUMBER of donuts purchased.
Let C = the NUMBER of cupcakes purchased.
Let X = the PRICE per donut (in CENTS)
Let Y = the PRICE per cupcake (in CENTS)
Question: How many doughnuts did Lew buy? D=?
NOTE: Given that we have 4 different variables, we will likely need 4 equations to answer the target question.

Given: Lew spent a total of $6.00 for one kind of cupcake and one kind of doughnut.
In other words, Lew spent 600 CENTS
We can write: **DX + CY = 600**

Okay that's 1 equation. When I SCAN the two statements, I can see that I will be able to create one equation for each statement.
This means we will have a total of 3 equations, which likely means the combined statements are insufficient.
Given this let's jump to ......

Statements 1 and 2 combined

From statement 1, we can write: 2X = 3Y - 10
From statement 2, we can write: 1X + 1Y = 70 (CENTS)

We can solve this system to get, **X = 40** and **Y = 30**
When we can plug these values into our first equation, DX + CY = 600, we get: D(40) + C(30) = 600
Rewrite as: 40D + 30C = 600
Divide both sides by 10 to get: **4D + 3C = 60**

There are several solutions to this equation. Here are two:
Case a: D = 3 and C = 16. In this case, the answer to the target question is **Lew bought 3 donuts**
Case b: D = 6 and C = 12. In this case, the answer to the target question is **Lew bought 6 donuts**

Since we cannot answer the question with certainty, the combined statements are NOT SUFFICIENT.

The correct answer is E.

Top 1% expert replies to student queries (can skip)

Firstly, why is St 2 not possible? The stem says one kind of D and one kind of C. Let's say D is banana and C is apple. L bought one kind of banana (but we don't know the number of that one kind of banana) and one kind of apple (again we don't know how many of that one kind of apple). St 2 says 1 banana and 1 apple together cost $0.35

Now to solve the question, let's say they bought d bananas (D is just a label for bananas, D doesn't stand for anything numerically; d is the number of the product D purchased) and c apples (same explanation for c and C as for d and D), at $x each and $y each respectively

Stem: xd + yc = 6 —> We need d here as the solution of the question / any information that can help us get to d
St1: 2x = 3y - 0.1 —> two variables, one equation - NS
St2: (x + y)/2 = 0.35 —> again NS by itself

Combining St 1 and St 2, we can solve for x and y, but without any more information, we cannot conclusively find d and / or c

The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

Let the number of C be x
Let the number of D be y
Let the price of 1 unit C be p
Let the price of 1 unit D be q

px + qy = 6

Statement 1 : 2px = 3qy - 0.1

This is insufficient!

Statement 2 :

(p+q)/2 = 0.35

p+q = 0.7

This is also insufficient!

Combining the two,

2px = 3qy - 0.1
p+q = 0.7

We want to find the value of 'y'. This again is insufficient, since we have 4 variables and 3 equations. We therefore cannot find the value of y.

Answer is E.

29.

The average of x, y and z is (x+y+z)/3. In order to answer the question, we need to know what x, y, and z equal. However, the question stem also tells us that x, y and z are consecutive integers, with x as the smallest of the three, y as the middle value, and z as the largest of the three. So, if we can determine the value of x, y, or z, we will know the value of all three. Thus a suitable rephrase of this question is ―what is the value of x, y, or z?

(1) SUFFICIENT: This statement tells us that x is 11. This definitively answers the rephrased question ―what is the value of x, y, or z? To illustrate that this sufficiently answers the original question: since x, y and z are consecutive integers, and x is the smallest of the three, then x, y and z must be 11, 12 and 13, respectively. Thus the average of x, y, and z is (11+12+13)/3=12

(2) SUFFICIENT: This statement tells us that the average of y and z is 12.5, or (y+z)/2 = 12.5.

Multiply both sides of the equation by 2 to find that y + z = 25. Since y and z are consecutive integers, and y < z, we can express z in terms of y: z = y + 1.
So y + z = y + (y + 1) = 2y + 1 = 25, or y = 12.
This definitively answers the rephrased question ―what is the value of x, y, or z? To illustrate that this sufficiently answers the original question: since x, y and z are consecutive integers, and y is the middle value, then x, y and z must be 11, 12 and 13, respectively. Thus the average of x, y, and z is 11 + 12 + 13 / 3 = 36/3 = 12.

The correct answer is D

GMAT Quant Topic 2: Statistics

Part B: Median

1. Series A in Ascending order : -6, -2, 9, 18.3, 27,5 --> median = 9
Series B in Ascending order : -199, 0.355, 10, 16, 19.98, 201 --> Median = (10+16)/2=13

Hence, 13-9 = 4

Answer is 4.

2. Median of 6 primes : (odd+odd)/2 = Integer. Options C and D are discarded immediately.
   Options A and B can never be the median of 6 primes.
   Only option E remains.
   The correct answer is E.

3. If the mean increases by a certain percent it doesn't necessarily mean that each term increases by the same percent.

   Consider the following set: {1, 2, 3} --> mean=median=2 and sum=6. Now, if we increase the mean by 100%, we increase the sum by 100%, so it'll become 12. But new set can be {0, 0, 12}: the third element increased and the first two elements decreased or {2, 2, 8}: the first and the third elements increased and the second remained the same...

   You can apply the same logic to the question at hand.

   The correct answer is E.

4.
1) N is prime and N^6 is even implies N is 2 (as all other primes are odd and odd^6 = odd)
hence A is sufficient.

2) 2N + 14 < 20
N < 3
clearly insufficient.
The correct answer is A.

5.
One approach to this problem is to try to create a Set T that consists of up to 6 integers and has a median equal to a particular answer choice. The set {–1, 0, 4) yields a median of 0. Answer choice A can be eliminated.
The set {1, 2, 3} has an average of 2. Thus, x = 2. The median of this set is also 2.
So the median = x. Answer choice B can be eliminated.
The set {–4, –2, 12} has an average of 2. Thus, x = 2. The median of this set is –2.
So the median = –x. Answer choice C can be eliminated.
The set {0, 1, 2} has 3 integers. Thus, y = 3. The median of this set is 1.
So the median of the set is (1/3)y. Answer choice D can be eliminated.
As for answer choice E, there is no possible way to create Set T with a median of (2/7)y. Why? We know that y is either 1, 2, 3, 4, 5, or 6. Thus, (2/7)y will yield a value that is some fraction with denominator of 7.

The possible values of (2/7)y are as follows: $\frac{2}{7}, \frac{4}{7}, \frac{6}{7}, \frac{8}{7}, \frac{10}{7}, \frac{12}{7}$

However, the median of a set of integers must always be either an integer or a fraction with a denominator of 2 (e.g. 2.5, or 5/2). So (2/7)y cannot be the median of Set T.

The correct answer is E.

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For this question, you can TEST VALUES to eliminate the 4 answers that COULD be the median of Set T.

We're told that Set T consists of Y integers (where 0 < Y < 7) and that the AVERAGE = X = a POSITIVE INTEGER. We're asked which of the answers could NOT be the median.

Answer A: 0... If Set T is {0,0,3} then the average = 1 and the median = 0. Eliminate A.

Answer B: X... If Set T is {1,1,1} then the average = 1 and the median = 1 = X. Eliminate B.

Answer C: -X... If Set T is {-1,-1,5} then the average = 1 and the median = -1 = -X. Eliminate C.

Answer D: Y/3... If Set T is {1,1,1} then the average = 1 and the median = 1 = Y/3. Eliminate D.

There's only one answer left... Option E. It has to be correct.

The correct answer is E.

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Average of two integers will always be either an integer (if both are odd or both are even) or Integer/2 in lowest form (if one is odd and the other is even). The average can never be integer/7 in lowest terms. Since y is between 0 and 7 exclusive, y is not divisible by 7 and hence, (2/7)y will be (integer/7) in lowest terms.
Median of a set of odd number of integers will be one of the integers.
Median of a set of even number of integers will be the average of middle two integers.
Hence, median cannot be of the form (2/7)y.
Answer (E)
Basically,
Number of integers in T could be anything from 1 to 6.
Avg of T is x.
We need the median of T. The median of T will be either the middle integer or the avg of two middle integers. So median will always be either an Integer or Integer/2. When an integer is divided by 2, it gives either an integer or something.5.
Can an Integer/2 give you 3.1428? No.
Even Integer/ 2 = Integer
Odd Integer/2 = a.5
Now look at the given options.
(E) (2/7)y
You know that y will be 1/2/3/4/5/6. It cannot be 7 or a multiple of 7. So you will need to divide by 7.
Upon division by 7, you get a non terminating decimal. This cannot be the median.
The correct answer is E.

6. Since S contains only consecutive integers, its median is the average of the extreme values a and b. We also know that the median of S is (¾)b We can set up and simplify the following equation:

$$\frac{a+b}{2} = \frac{3b}{4} \rightarrow$$
$$4a + 4b = 6b \rightarrow$$
$$4a = 2b \rightarrow$$
$$2a = b$$

Since set Q contains only consecutive integers, its median is also the average of the extreme values, in this case b and c. We also know that the median of Q is (7/8)c. We can set up and simplify the following equation:

$$\frac{b+c}{2} = \frac{7c}{8} \rightarrow$$
$$8b + 8c = 14c \rightarrow$$
$$8b = 6c \rightarrow$$
$$4b = 3c$$

We can find the ratio of a to c as follows: Taking the first equation, $2a = b \rightarrow 8a = 4b$

and the second equation, 4b= 3c and setting them equal to each other, yields the following:

$8a = 3c \rightarrow \frac{a}{c} = \frac{3}{8}$. Since set R contains only consecutive integers, its median is the average of the extreme values a and c: (a+c)/2. We can use the ratio a/c = 3/8 to substitute 3c/8 for a:

$$\frac{\frac{3c}{8} + c}{2} \rightarrow$$
$$\frac{\frac{11c}{8}}{2} \rightarrow$$
$$\frac{11}{16}c$$

Thus, the median of set R is 11c/16.
The correct answer is C.

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Since S runs from a to b, its mean/median = (a+b)/2

Since Q runs from b to c, its mean/median=(b+c)/2

So now we can use the provided information about the medians to create two equations:

(a+b)/2=(3/4)b
(b+c)/2=(7/8)c

From here we just need to do some manipulations to get to our answer:

(a+b)/2=(3/4)b
(a+b)=(3/2)b
a=b/2

(b+c)/2=(7/8)c
(b+c)=(7/4)c
b=(3/4)c

Since R runs from a to c, let's combine these results to find a in terms of c:

$a=(3c/4)/2$
$a=3c/8$

Now we can plug these values in to find the median of R.
The mean/median of $R=(a+c)/2=(3c/8 + c)/2=(11c/8)/2=11c/16$

The correct answer is C.

7. Since a regular year consists of 52 weeks and Jim takes exactly two weeks of unpaid vacation, he works for a total of 50 weeks per year. His flat salary for a 50-week period equals 50 × \$200 = \$10,000 per year. Because the number of years in a 5-year period is odd, Jim's median income will coincide with his annual income in one of the 5 years.
Since in each of the past 5 years the number of questions Jim wrote was an odd number greater than 20, his commission compensation above the flat salary must be an odd multiple of 9. Subtracting the \$10,000 flat salary from each of the answer choices, will result in the amount of commission. The only odd values are \$15,673, \$18,423 and \$21,227 for answer choices B, D, and E, respectively.
Since the total amount of commission must be divisible by 9, we can analyze each of these commission amounts for divisibility by 9. One easy way to determine whether a number is divisible by 9 is to sum the digits of the number and see if this sum is divisible by 9. This analysis yields that only \$18,423 (sum of the digits = 18) is divisible by 9 and can be Jim's commission. Hence, \$28,423 could be Jim's median annual income.
The correct answer is D.

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Query: Why and how did we conclude that we need to check whether the amount earned by new questions is divisible by 9?
Reply: The question says Jim earns \$9 per new question he writes. Say he writes x new questions. He earns \$9x. 9x is divisible by, surprise, 9!

If you want to understand how to solve this problem, I would use the answer choices here rather than solving straight. His flat salary is \$10,000 per year. So Option (B), Option (C), and Option (E) are ruled out (because the amount - 10,000, i.e. what he earned from writing new questions, is not a multiple of 9).

We have Option (A) and Option (D) remaining. In the last 5 years, his middle income will be the answer (when all his earnings are arranged in increasing order). We don't know which year that came in, but it doesn't matter either.

Option (A) - His flat salary is \$10,000, his earnings from new questions is \$12,474. 12,474 / 9 = 1,386. So he wrote 1,386 new questions. This cannot be, as he wrote an odd number of questions

Option (D) - Again, his flat salary is \$10,000, his earnings from new questions is \$18,423. 18,423 / 9 = 2,047. 2047 is an odd number > 20. So this is the only possible answer
The correct answer is D.

8. From the question stem, we know that Set A is composed entirely of all the members of Set B plus all the members of Set C. The question asks us to compare the median of Set A (the combined set) and the median of Set B (one of the smaller sets). Statement (1) tells us that **the mean of Set A is greater than the median of Set B.** This gives us no useful information to compare the medians of the two sets. To see this, consider the following: Set B: **{** 1, 1, 2 **},** Set C: **{** 4, 7 **},** Set A: **{** 1, 1, 2, 4, 7 **}.** In the example above, the mean of Set A (3) is greater than the median of Set B (1) and the median of Set A (2) is **GREATER** than the median of Set B (1). However, consider the following example: Set B: **{** 4, 5, 6 **},** Set C: **{** 1, 2, 3, 21 **},** Set A: **{** 1, 2, 3, 4, 5, 6, 21 **},** Here

the mean of Set A (6) is greater than the median of Set B (5) and the median of Set A (4) is **LESS** than the median of Set B (5). This demonstrates that Statement (1) alone does is not sufficient to answer the question. Let's consider Statement (2) alone: **The median of Set A is greater than the median of Set C.** By definition, the median of the combined set (A) must be any value at or between the medians of the two smaller sets (B and C). Test this out and you'll see that it is always true. Thus, before considering Statement (2), we have three possibilities: Possibility 1: The median of Set A is greater than the median of Set B but less than the median of Set C.

Possibility 2: The median of Set A is greater than the median of Set C but less than the median of Set B.

Possibility 3: The median of Set A is equal to the median of Set B or the median of Set C. Statement (2) tells us that the median of Set A is greater than the median of Set C. This eliminates Possibility 1, but we are still left with Possibility 2 and Possibility 3. The median of Set B may be greater than OR equal to the median of Set A. Thus, using Statement (2) we cannot determine whether the median of Set B is greater than the median of Set A. Combining Statements (1) and (2) still does not yield an answer to the question, since Statement (1) gives no relevant information that compares the two medians and Statement (2) leaves open more than one possibility. Therefore, the correct answer is Choice **(E): Statements (1) and (2) TOGETHER are NOT sufficient.**
The correct answer is E.

9. To find the mean of the set {6, 7, 1, 5, x, y}, use the average formula:
A = S/n
,where A = the average, S = the sum of the terms, and n = the number of terms in the set.
Using the information given in statement (1) that x + y = 7, we can find the mean:

$$\frac{6+7+1+5+(x+y)}{6} = \frac{6+7+1+5+7}{6} = 4\frac{1}{3}$$
.

Regardless of the values of x and y, the mean of the set is $4\frac{1}{3}$ because the sum of x and y does not change. To find the median, list the possible values for x and y such that x + y = 7. For each case, we can calculate the median.

x	y	DATA SET	MEDIAN
1	6	1, 1, 5, 6, 6, 7	5.5
2	5	1, 2, 5, 5, 6, 7	5
3	4	1, 3, 4, 5, 6, 7	4.5
4	3	1, 3, 4, 5, 6, 7	4.5
5	2	1, 2, 5, 5, 6, 7	5
6	1	1, 1, 5, 6, 6, 7	5.5

Regardless of the values of x and y, the median (4.5, 5, or 5.5) is always greater than the mean ($4\frac{1}{3}$). Therefore, statement (1) alone is sufficient to answer the question. Now consider statement (2). Because the sum of x and y is not fixed, the mean of the set will vary. Additionally, since there are many possible values for x and y, there are numerous possible medians. The following table illustrates that we can construct a data set for which x – y = 3 and the mean is greater than the median. The table ALSO shows that we can construct a data set for which x – y = 3 and the median is greater than the mean.

x	y	DATA SET	MEDIAN	MEAN
22	19	1, 5, 6, 7, 19, 22	6.5	10
4	1	1, 1, 4, 5, 6, 7	4.5	4

Thus, statement (2) alone is not sufficient to determine whether the mean is greater than the median. The correct answer is (A): Statement (1) alone is sufficient, but statement (2) alone is not sufficient.

The correct answer is A.

10. Median > Mean
Clearly since the numbers are in ascending order "y" is the median. Now Mean is nothing but a number that can represent all the numbers given. If all the numbers were y then mean would y but since two numbers are less than y, mean can never be greater than y. Hence median is greater than mean.
The correct answer is B.

11. Given: There is a set of numbers in ascending order: {y - x, y, y, y, y, x, x, x, x + y}.

Asked: If the mean is 9, and the median is 7, what is x?

the mean is 9
y - x + 4y + 3x + x + y = 9*9 = 81
3x +6y = 81

the median is 7
y = 7
3x + 42 = 81
3x = 39
x = 13

Answer is 13

12.
Since any power of 7 is odd, the product of this power and 3 will always be odd. Adding this odd number to the doubled age of the student (an even number, since it is the product of 2 and some integer) will always yield an odd integer. Therefore, all lucky numbers in the class will be odd.

The results of the experiment will yield a set of 28 odd integers, whose median will be the average of the 14th and 15th greatest integers in the set. Since both of these integers will be odd, their sum will always be even and their average will always be an integer. Therefore, the probability that the median lucky number will be a non-integer is 0%.
The correct answer is A.

13. Since the set {a, b, c, d, e, f} has an even number of terms, there is no one middle term, and thus the median is the average of the two middle terms, c and d. Therefore, the question can be rephrased in the following manner:
Is (c + d)/2 > (a + b + c + d + e + f)/6 ?
Is 3(c + d) > a + b + c + d + e + f ?
Is 3c + 3d > a + b + c + d + e + f ?
Is 2c + 2d > a + b + e + f ?

(1) INSUFFICIENT: We can substitute the statement into the question and continue rephrasing as follows:
Is 2c + 2d > (3/4)(c + d) + b + f ?
Is (5/4)(c + d) > b + f ?
From the question stem, we know c > b and d < f; however, since these inequalities do not point the same way as in the question (and since we have a coefficient of 5/4 on the left side of the question), we cannot answer the question. We can make the answer to the question "Yes" by relatively picking small b and f (compared to c and d) -- for instance, b = 2, c = 7, d = 9 and f= 12 (still leaving room for a and e, which in this case would equal 1 and 11, respectively). On the other hand, we can make the answer "No" by changing f to a very large number, such as 1000.
(2) INSUFFICIENT: Going through the same argument as above, we can substitute the statement into the question:
Is 2c + 2d > a + e + (4/3)(c + d) ?
Is (2/3)(c + d) > a + e ?
This is also insufficient. It is true that we know that a + e < (4/3)(c + d). The reason we know this is that the set of integers is ascending, so a < b and e < f. Therefore a + e < b + f, and b + f = (4/3)(c + d) according to this statement. However, we don't know whether a + e < (2/3)(c+ d).
(1) AND (2) SUFFICIENT: If we substitute both statements into the rephrased inequality, we get a definitive answer.
Is 2c + 2d > a + b + e + f ?
Is 2(c + d) > (3/4)(c + d) + (4/3)(c + d)?
Is 2(c + d) > (25/24)(c + d)?
Now, we can divide by c + d, a quantity we know to be positive, so the direction of the inequality symbol does not change.
Is 2 > 25/24 ?
2 is NOT greater than 13/8, so the answer is a definite "No." Recall that a definite "No" is sufficient.

The correct answer is C.

14. The mean of a set is equal to the sum of terms divided by the number of terms in the set. Therefore,

$$\frac{x+y+x+y+x-4y+xy+2y}{6} = y+3$$

x (y + 3) = 6 (y + 3)
x = 6. Given that y > 6 and substituting x = 6, the terms of the set can now be ordered from least to greatest:
6 – 4y, 6, y, y + 6, 2y, 6y.
The median of a set of six terms is the mean of the third and fourth terms (the two middle terms). The mean of the terms y and y + 6 is

$$\frac{2y+6}{2} = y+3$$

The correct answer is B.

15. The set $R_n = R_{n-1} + 3$ describes an evenly spaced set: each value is three more than the previous. For example, the set could be 3, 6, 9, 12 . . . For any evenly spaced set, the mean of the set is always equal to the median. A set of consecutive integers is an example of an evenly spaced set. If we find the mean of this set, we will be able to find the median because they are the same.

(1) INSUFFICIENT: This does not give us any information about the value of the mean. The only other way to find the median of a set is to know every term of the set.

(2) SUFFICIENT: The mean must be the median of the set since this is an evenly spaced set. This statement tells us that mean is 36. Therefore, the median must be 36.

The correct answer is B.

16. This question is asking us to find the median of the three scores. It may seem that the only way to do this is to find the value of each of the three scores, with the middle value taken as the median. Using both statements, we would have two of the three scores, along with the mean given in the question, so we would be able to find the value of the third score. It would seem then that the answer is C. On GMAT data sufficiency, always be suspicious, however, of such an obvious C. In such cases, one or both of the statements is often sufficient.

(1) INSUFFICIENT: With an arithmetic mean of 78, the sum of the three scores is 3 × 78 = 234. If Peter scored 73, the other two scores must sum to 234 – 73 = 161. We could come up with hundreds of sets of scores that fit these conditions and that have different medians. An example of just two sets are: 73, 80, 81 median = 80 and 61, 73, 100 median = 73

(2) SUFFICIENT: On the surface, this statement seems parallel to statement (1) and should therefore also be insufficient. However, we arent just given one of the three scores in this statement. We are given a score with a value that is THE SAME AS THE MEAN. Conceptually, the mean is the point where the deviations of all the data net zero. This means that the sum of the differences from the mean to each of the points of data must net to zero. For a simple example, consider 11, which is the mean of 7, 10 and 16.
7 – 11 = -4 (defined as negative because it is left of the mean on the number line)10 – 11 = -1
16 – 11 = +5 (defined as positive because it is right of the mean on the number line)
The positive and negative deviations (differences from the mean) net to zero. In the question, we are told that the mean score is 78 and that Mary scored a 78. Mary's deviation then is 78 – 78 = 0. For the deviations to net to zero, Peter and Paul's deviations must be -x and +x (not necessarily in that order).
Mary's deviation = 78 – 78 = 0 Peter's (or Paul's) deviation = -x
Paul's (or Peter's) deviation = +x
We can then list the data in order: 78 – x, 78, 78 + x
This means that the median must be 78. NOTE: x could be 0, which would simply mean that all three students scored a 78. However, the median would remain 78.

The correct answer is B.

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Since the average of 3 score is 78 so the total score is 234.

Statement 1 says peter's score is 73, so the sum of the other 2 score is 161, it could be that one of the score is 72 and the other is 89 so the median is 73, or it could be that one of the other score is 75 and 86, so the median would be 75. So this is insufficient.

Statement 2 says Mary's score is 78, you know that since the average is 78, the other two score cannot be all lower than 78 or all higher than 78. It could be that one other score is lower than 78 and the other one higher than 78. Or the other two score are 78 as well. Anyway, the median will be 78. So this is sufficient.

Answer B

For statement 2, You want sum to be 234 and avg is 78 and one number is also 78. So the sum of the other two is 156.
156= 70,86. (70,78,86)
78,78. (78,78,78)

Don't make any rules. Follow the questions logic. You just have to conclude that either the numbers are equal or one is greater than 78 and the other is lesser.
In both the cases, median will be 78.
The correct answer is B.

17. Since each set has an even number of terms, the median of each set will be equal to the average of the two middle terms of that set. So, the median of Set A will be equal to the average of x and 8. The median of Set B will be equal to the average of y and 9. The question tells us that the median of Set A is equal to the median of Set B. We can express this algebraically as
$\frac{x+8}{2} = \frac{y+2}{2}$
We can multiply both sides by 2:
x + 8 = y + 9
We can subtract x from both sides (remember, we are looking for y – x): 8 = y – x + 9

We can subtract 9 from both sides to isolate y – x: y – x = 8 – 9 = -1
The correct answer is B.

18. To find the maximum possible value of x, we will first consider that the set's mean is 7, and then that its median is 5.5.

For any set, the sum of the elements equals the mean times the number of elements. In this case, the mean is 7 and the number of elements is 6, so the sum of the elements equals 42.

42 = 8 + 2 + 11 + x + 3 + y
42 = 24 + x + y
18 = x + y

Now consider that the median is 5.5. Letting x = 1 and y = 17 such that they sum to 18, we can arrange the values in increasing order as follows:

x, 2, 3, 8, 11, y

Since 3 and 8 are the middle values, the median equals 5.5 as required. The question asks for the maximal value of x, so let's increase x as far as possible without changing the median. As x increases to 3 (and y decreases to 15), the middle values of 3 and 8 don't change, so the median remains at 5.5. However, as x increases beyond 3, the median also increases, so the maximal value of x that leaves the median at 5.5 is 3.

The correct answer is D.

19.

If set S consists of the numbers 1, 5,-2, 8 and n, is 0 < n < 7?

Note that:
If a set has odd number of terms the median of a set is the middle number when arranged in ascending or descending order;
If a set has even number of terms the median of a set is the average of the two middle terms when arranged in ascending or descending order.

So the median of our set of 5 numbers: **{-2, 1, 5, 8, n}** must be the middle number, so it can be:
1 if $n \leq 1$;
5 if $n \geq 5$;
n itself if $1 \leq n \leq 5$.

(1) The median of the numbers in S is less than 5 --> so either the median=1 and in this case $n \leq 1$ so not necessarily in the range $0 < n < 7$ or median=n and in this case $1 \leq n < 5$ and in this case $0 < n < 7$ is always true. Not sufficient.

(2) The median of the numbers in S is greater than 1 --> again either the median=5 and in this case $n \geq 5$ so not necessarily in the range $0 < n < 7$ or median=n and in this case $1 < n \leq 5$ and in this case $0 < n < 7$ is always true. Not sufficient.

(1)+(2) $1 < median < 5$ --> $median = n$ --> $1 < n < 5$, so $0 < n < 7$ is true. Sufficient.

Answer: C.

20. .
Set S: s-2;s-1;s;s+1;s+2, set T: t-3;t-2;t-1;t;t+1;t+2;t+3
According to 2, 5s=7t, insufficient. S could be 7, t could be 5.
According to 1, s=0.
Combining 1 and 2, s=t=0
The correct answer is C.

21. First, we arrange the 10,5,-2,-1,-5 and 15 at sequence: -5, -2, -1, 5, 10, 15.
So, the median is (-1+5)/2=2
Answer is 2

22.
Median: the middle measurement after the measurements are ordered by size (or the average of the two middle measurements if the number of measurements is even)
In this question, the median is the average of the amount in 10th and 11th day after ordered by size.
Both the 10th and 11th amounts are $84, so, the median is $84
Answer is $84.

23.
There are 73 scores, so, (73+1)/2=37, the 37th number is the median.
It is contained by interval 80-89.
The correct answer is C.

24.
In order to solve the question easier, we simplify the numbers such as 150, 000 to 15, 130,000 to 13, and so on.

I. Median is 13, so, the greatest possible value of sum of eight prices that no more than median is 13*8=104. Therefore, the least value of sum of other seven homes that greater than median is (15*15-104)/7=17.3>16.5. It's true.
II. According the analysis above, the price could be, 13, 13, 13, 13, 13,13,13,13, 17.3, 17.3, 17.3... So, II is false.
III. Also, false.
Answer: only I must be true.
The correct answer is A.

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The question asks "Which of the following statements must be true?" It is not asking which could be true? The example which you have given is a possibility, but need not be true in all the cases.
So, if it's possible to create a scenario in which the statement is not true, we can eliminate it.
So, let's create a possible scenario and see which answer choices we can eliminate.
So, one possible scenario is:
130, 130, 130, 130, 130, 130, 130, 130, 130, 130, 130, 130, 130, 130, 430
II. At least one of the homes was sold for more than $130,000 and less than $150,000
This statement means at least 1 house' price is between $130,000 and $150,000. We don't have any number(x), such that 130<x<150.
Notice that this scenario tells us that statements II and III need not be true.
Since answer choices B, C, D and E all include either II or III, we can eliminate them.
This leaves us with A, which must be the correct answer.
The correct answer is A.

25.
To get the maximum length of the shortest piece, we must let other values as little as possible. That is, the values after the median should equal the median, and the value before the median should be equal to each other.
Let the shortest one be x:
x+x+140*3=124*5
x=100
The correct answer is B.

26.
Amy was the 90th percentile of the 80 grades for her class, therefore, 10% are higher than Amy's, 10%*80=8.
19 of the other class was higher than Amy. Totally, 8+19=27
Then, the percentile is: (180-27)/180=85/100
The correct answer is D.

27.
If median of a list of 5 values is 330, it means that 330 must be one of the 5 values. 2 values must be greater than 330 and 2 must be smaller than 330.
Ann's actual sale is 450-x, Cal's 190+x, after corrected, Ann still higher than Cal, so Ann is the median.
Or we can explain it in another way:
450-x=330, so x=120
Ann's actual sale is 450-x Cal's 190+x,
Suppose that either Ann or Cal can be the median, if Ann is the median, than we get the previous answer; however, if Cal is median (330), we will have 190+x=330, x=140, then Ann (450- 140=310) will less than Cal (330), that is incorrect.
This can explain why Cal cannot be the median and Ann must.

The correct Answer is D.

Alternate solution from Gmatclub:

Old sales: {Cal=190, 210, 360, Ann=450, 680}.

Now, the median of a set with odd # of terms is just the middle term (when ordered in ascending /descending order). So, old median=360.

We know that new median=330 and Cal's sales are still less than Ann's, hence Ann's new sales must be 330 (Ann's sales # must be reduced so that it becomes the middle term: 360 and Ann must switch places): {210, Cal, Ann=330, 360, 680}.

The value of the incorrectly recorded sale is Ann's old sales - Ann's new sales = 450 - 330 = 120.

The correct Answer is D.

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5 nos- 680.450.360.210,190
3 nos will remain unchanged.
Only 450 and 190 can change as per the question (i.e. Cal's sale and Ann's sale)

Now I know 330 is the median

So obviously 3rd no is 330

So I know 680,x,330,210,y

Now the point to find out is whose sale is 330

If Ann's sale decreased to 330 from 450
then Cal's sales rose from 190 to 330.(As median is 330)

So the incorrect value is 120.

The correct Answer is D.

GMAT Quant Topic 2: Statistics

Part C: Mode

1. A {0, 3, 4, 2, 0, 4, 7, 8, 4, 17} - Mode is 4 (repeated 3 times)
B {20, 12, -7, -9, -5, -7, 11, -5, 68} - Mode is -5 and -7 (each repeated twice)
C {-1.5, 0, 1.5}. - No repetition so Mode is zero

Sum of modes = 4-12=-8
Answer is -8

2.
Statement 1 tells us that the difference between any two integers in the set is less than 3. This information alone yields a variety of possible sets. For example, one possible set (in which the difference between any two integers is less than 3) might be: (x, x, x, x + 1, x + 1, x + 2, x + 2). Mode = x (as stated in question stem). Median = x + 1. Difference between median and mode = 1.
Alternately, another set (in which the difference between any two integers is less than 3) might look like this: (x – 1, x, x, x + 1). Mode = x (as stated in the question stem). Median = x. Difference between median and mode = 0. We can see that statement (1) is not sufficient to determine the difference between the median and the mode.
Statement (2) tells us that the average of the set of integers is x. This information alone also yields a variety of possible sets. For example, one possible set (with an average of x) might be: (x – 10, x, x, x + 1, x + 2, x + 3, x + 4). Mode = x (as stated in the question stem). Median = x + 1. Difference between median and mode = 1.
Alternately, another set (with an average of x) might look like this: (x – 90, x, x, x + 15, x + 20, x + 25, x + 30). Mode = x (as stated in the question stem). Median = x + 15. Difference between median and mode = 15. We can see that statement (2) is not sufficient to determine the difference between the median and the mode. Both statements taken together imply that the only possible members of the set are x – 1, x, and x + 1 (from the fact that the difference between any two integers in the set is less than 3) and that every x – 1 will be balanced by an x + 1 (from the fact that the average of the set is x). Thus, x will lie in the middle of any such set and therefore x will be the median of any such set. If x is the mode and x is also the median, the difference between these two measures will always be 0. The correct answer is C: BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

The correct answer is C.

GMAT Quant Topic 2: Statistics

Part D: Range

1. 0
Range = Highest element – Lowest element
Arrange each set in ascending order to find the highest & lowest elements
Range(X) = 1000 – (-21) = 1021
Range(Y) = 1000 – (-21) = 1021
Difference between ranges of set X & set Y = 1021 – 1021 = 0

2.

y = product of all elements in set X. now, y will be even if 2 is part of this set (k=2), else y will be odd. since we have, 11y as an even integer. therefore, y must be even. (Because 11 is odd, therefore the only way in which 11y can be even is when y is even.)
Thus, elements in the set X are {2,3,11,7,17,19} and thus its range is 19-2=17
The correct answer is C.

3.

The *range* of a set is the difference between the largest and smallest elements of a set.

Set consists of odd multiples of 7 (numbers of a type $7 * odd$), so there can be following elements: ..., -7, 7, 21, 35, ...

The range of this set would be: $7 * odd_{max} - 7 * odd_{min} = 7(odd_{max} - odd_{min}) = 7 * even$, (as odd-odd=even). So the range is even multiple of 7.

Answer: E.

The correct answer is E.

4. Top 1% expert replies to student queries
First multiple of 7 after 70= 70+7= 77
100th multiple of 7 after 70= 110th multiple of 7= 70+700= 770
Range= 100th multiple-first multiple= 770-77= 693.
The correct answer is A.

5.

Set X = {11, 13, 17,............., 83, 89, 97}
Set Y = {5, 15, 25,..............., 75, 85, 95}

Combining two sets, say Set Z

Set Z = {5, 11, 13, 15, 17, 25,.................., 75, 83, 85, 89, 95, 97}

Range = Max Value - Min Value
Range (Z) = 97 - 5 = 92
The correct answer is D.

6. Top 1% expert replies to student queries
(1) The range of ages of the participants is 22 to 30, inclusive
There could be 100 schools represented by 100 students so no two students will have the same age-school combination.
All students could be from the same school so there would be multiple same age-school combinations.
Not sufficient.

(2) Participants represent 10 business schools.
The age of the students could range from 20 to 80 so we may or may not have the same age-school combinations. Not sufficient.

Now let's consider both statements (1+2):
Ages are 22, 23 ...30 - 9 different figures
Schools are A, B, C..., J - 10 different schools

How many unique age school combinations can we make? A22, A23, ... A30, B22, B23, ..., J22, J23, ...J30
A total of 9*10 = 90 combinations. So we can have 90 unique age-school combinations for 90 students. Now what about the remaining 10? They must also have age between 22 to 30 and must represent schools A to J. So say for the 91st student, we pick age 25 and school C. But note that we already have a student C25 since we accounted for all combinations in our 90 combinations. So the rest of the 10 students will need to repeat the age-school combination. Hence there must be students (at least 10) who have the same age and represent the same school.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

We have 100 students. We need to check if there are any students of the same age who attend the same school.

Statement 1: We have the range of ages, but we don't know the number of schools. So this is insufficient.

Statement 2: We have the number of schools, but we don't know the distribution of ages. So this is insufficient.

Combining 1 and 2,

We have 100 students, 10 schools and the range of ages is 22-30 (9 ages).

If we assume that each school has 10 students, then let us take a school X.

School X has 10 students and 9 possible ages. So at least 2 students will be of the same age and from the same school.

If we take any other distribution of students, we will always have at least 2 students from the same school who are of the same age. You can take examples to confirm

So both statements together are sufficient.
The correct answer is C.

7. We have a set of 100 consecutive integers. Some of them must be negative and some not (if all of them were negative then the range of 100 consecutive negative integers would be 99, not 80). The greatest negative integer will be -1.

Now, we are told that the range of the negative elements of Set S equals 80: {range} = {largest} - {smallest} --> 80 = -1 - {smallest} --> {smallest} = -81.

So, the set consists of 81 negative elements (from -81 to -1), 0 and 18 positive integers (from 1 to 18), the total of 81+1+18=100 consecutive integers.

The average of integers from 1 to 18 is (first + last)/2 = (1 + 18)/2 = 9.5.
The correct answer is D.

8.
Top 1% expert replies to student queries
"A randomly selected non-negative single digit integer is added to {2, 3, 7, 8}":
We are selecting from non-negative single digit integers, so from {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.
These 10 digits represent the total number of outcomes.

Hence, the total number of outcomes is 10.

We need to find the probability that the median of the set will increase but the range still remains the same.

The median of $\{2, 3, 7, 8\}$ is $(3 + 7)/2 = 5$ --> the number selected must be greater than 5
The range of $\{2, 3, 7, 8\}$ is $8 - 2 = 6$ --> the number selected must be from 2 to 8, inclusive.

To satisfy both conditions the number selected must be 6, 7, or 8.
Hence, the number of favorable outcomes is 3.
P = (favorable)/(total) = 3/10.
The correct answer is B.

9. Both statements say the exact same thing: Set B consists of only prime numbers. But since we don't know what is the largest prime in B, then we cannot get the rage of B.

The correct answer is E.

10. Before analyzing the statements, let‘s consider different scenarios for the range and the median of set A. Since we have an even number of integers in the set, the median of the set will be equal to the average of the two middle numbers. Further, note that integer 2 is the only even prime and it cannot be one of the two middle numbers, since it is the smallest of all primes. Therefore, both of the middle primes will be odd, their sum will be even, and their average (i.e. the median of the set) will be an integer. However, while we know that the median will be an integer, it is unknown whether this integer will be even or odd. For example, the average of 7 and 17 is 12 (even), while the average of 5 and 17 is 11 (odd). Next, let‘s consider the possible scenarios with the range. Remember that the range is the difference between the greatest and the smallest number in the set. Since we are dealing with prime numbers, the greatest prime in the set will always be odd, while the smallest one can be either odd or even (i.e. 2). If the smallest prime in the set is 2, then the range will be odd, otherwise, the range will be even. Now, let‘s consider these scenarios in light of each of the statements. (1) SUFFICIENT: If the smallest prime in the set is 5, the range of the set, i.e. the difference between two odd primes in this case, will be even. Since the median of the set will always be an integer, the product of the median and the range will always be even. (2) INSUFFICIENT: If the largest integer in the set is 101, the range of the set can be odd or even (for example, $101 – 3 = 98$ or $101 – 2 = 99$). The median of the set can also be odd or even, as we discussed. Therefore, the product of the median and the range can be either odd or even.
The correct answer is A.

11. (1) INSUFFICIENT: Statement (1) tells us that the range of S is less than 9. The range of a set is the positive difference between the smallest term and the largest term of the set. In this case, knowing that the range of set S is less than 9, we can answer only MAYBE to the question "Is $(x + y) < 18$". Consider the following two examples: Let $x = 7$ and $y = 7$. The range of S is less than 9 and $x + y < 18$, so we conclude YES. Let $x = 10$ and $y = 10$. The range of S is less than 9 and $x + y > 18$, so we conclude NO. Because this statement does not allow us to answer definitively Yes or No, it is insufficient. (2) SUFFICIENT: Statement (2) tells us that the average of x and y is less than the average of the set S. Writing this as an inequality: $(x + y)/2 < (7 + 8 + 9 + 12 + x + y)/6$; $(x + y)/2 < (36 + x + y)/6$; $3(x + y) < 36 + (x + y)$; $2(x + y) < 36$; $x + y < 18$. Therefore, statement (2) is SUFFICIENT to determine whether $x + y < 18$. **The correct answer is B.**

12. *Question removed*

13. Since the GMAT is scored in 10-point increments, we know from statement (1) that there are a maximum of 19 distinct GMAT scores among the students in the first-year class (600, 610 . . . 770, 780). We also know that are there are 12 months in a year, yielding 12 distinct possibilities for the birth month of a student. Finally, there are 2 possibilities for student gender. Therefore, the number of distinct combinations consisting of a GMAT score, the month of birth, and gender is $19 \times 12 \times 2 = 456$. Because the total number of students is greater than the maximum number of distinct combinations of GMAT score/month of birth/gender, some students must share the same combination. That is, some students must have the same gender, be born in the same month. and have the same GMAT score. Thus, statement (1) is sufficient to answer the question.
Statement (2) provides no information about the range of student GMAT scores in the first-year class. Since there are 61 distinct GMAT scores between 200 and 800, the total number of distinct combinations of GMAT score/month of birth/gender on the basis of statement (2) is $61 \times 12 \times 2$

= 1,464. Since this number is greater than the first-year enrolment, there are potentially enough unique combinations to cover all of the students, implying that there may or may not be some students sharing the same 3 parameters. Since we cannot give a conclusive answer to the question, statement (2) is insufficient.

The correct answer is A.

Alternate Solution from GMATCLUB

Total number of students in class = 478
Possible genders = 2
Total possible scores = 61

Number of months = 12

1. Score range = 600 - 780
Therefore , we have 19 possible scores
Number of combinations =19 *2 * 12 = 456
Since 456< 478 . We will have atleast one repeat of the same gender and birth month.

Sufficient.

2. Number of male = 60 % of 478= 286.8

Not sufficient.
Answer A

14.

Top 1% expert replies to student queries (can skip)

We are given that S is a set of positive integers and the average of the terms in S is equal to the range of the terms in S. We need to determine the sum of all the integers in S.

Statement One Alone:

The range of S is a prime number that is less than 11 and is not a factor of 10.

Using information in statement one, we know that the range of S is 3 or 7. Thus, the average of S is also 3 or 7. However, since we don't know whether it is 3 or 7, nor do we know the number of integers in S, statement one alone is not sufficient to answer the question.

Statement Two Alone:

S is composed of 5 different integers.

Since we don't know any of the 5 integers, statement two alone is not sufficient to answer the question.

Statements One and Two Together:

From statement one, we know that the range and average are either both 3 or both 7. From statement two, we know S is composed of 5 different positive integers. Thus, the sum of these 5 integers is either 15 (if the average is 3) or 35 (if the average is 7). Therefore, we have two cases to consider: range = average = 3 (case 1) and range = average = 7 (case 2).

Case 1: range = average = 3

We can let x = the smallest number, so the largest number = x + 3. We can "squeeze" 2 more integers between x and x + 3, namely x + 1 and x + 2. So, there could be only 4 different total integers. However, remember that there should be 5 different integers in S; thus, case 1 is not possible.

So, it must be case 2: range = average = 7. If that is the case, the sum of the 5 integers is 35.

For example, the 5 integers could be 4, 5, 7, 8, and 11. We see that the range is 11 - 4 = 7 and the sum is 4 + 5 + 7 + 8 + 11 = 35 with an average of 7.

From Statement 1, the range (and thus the average) is either 3 or 7. From Statement 2, we know that the range cannot be 3, since no set with a range of 3 could contain five different integers. So the mean is 7, and we have 5 things in our set, so the sum is 5*7=35 which is what the question asked for.

The correct answer is C.

15. In a set consisting of an odd number of terms, the median is the number in the middle when the terms are arranged in ascending order. In a set consisting of an even number of terms, the median is the average of the two middle numbers. If S has an odd number of terms, we know that the median must be the middle number, and thus the median must be even (because it is a set of even integers). If S has an even number of terms, we know that the median must be the average of the two middle numbers, which are both even, and the average of two consecutive even integers must be odd, and so therefore the median must be odd. The question can be rephrased: ―Are there an even number of terms in the set?

(1) SUFFICIENT: Let X_1 be the first term in the set and let its value equal x. Since S is a set of consecutive even integers, $X_2 = X_1 + 2$, $X_3 = X_1 + 4$, $X_4 = X_1 + 6$, and so on. Recall that the mean of a set of evenly spaced integers is simply the average of the first and last term. Construct a table as follows:

(2)

X_n	Value	Ave *n* Terms	Result	O or E
X_1	x	x	x	Even
X_2	x + 2	(2x+2)/2	x + 1	Odd
X_3	x + 4	(3x+6)/3	x + 2	Even
X_4	x + 6	(4x+12)/4	x + 3	Odd
X_5	x + 8	(5x+20)/5	x + 4	Even

Note that when there is an even number of terms, the mean is odd and when there are an odd number of terms, the mean is even. Hence, since (1) states that the mean is even, it follows that the number of terms must be odd. This is sufficient to answer the question (the answer is ―no). Note of caution: it doesn't matter whether the answer to the question is ―yes or ―no; it is only important to determine whether it is possible to answer the question given the information in the statement.
Alternatively, we can recognize that, in a set of consecutive numbers, the median is equal to the mean, and so the median must be even.

(3) INSUFFICIENT: Let X_1 be the first term in the set and let its value = x. The range of a set is defined as the difference between the largest value and the smallest value. Construct a table as follows:

Term	Value	Range *n* Terms	Div by 6?
X_1	x		
X_2	x + 2	2	No
X_3	x + 4	4	No

X_4	**$x + 6$**	**6**	**Yes**
X_5	$x + 8$	8	No
X_6	$x + 10$	10	No
X_7	**$x + 12$**	**12**	**Yes**

Note that if there are 4 terms in the set, the range of the set is divisible by 6, while if there are 7 terms in the set, the range of the set is still divisible by 6. Hence, it cannot be determined whether the number of terms in the set is even or odd based on whether the range of the set is divisible by 6.

The correct answer is A.

16. The median of a set of numbers is the middle number when the numbers are arranged in increasing order. For a set of 5 scores, the median is the 3rd score. We will call the set of scores A = {A_1, A_2, A_3, A_4, A_5} and B = {B_1, B_2, B_3, B_4, B_5} for Dr. Adam's and Dr. Brown's students, respectively, where the scores are arranged in increasing order within each set.

Rephrasing the question using this notation yields —Is $A_3 > B_3$? (1) INSUFFICIENT: This statement tells us only the highest and lowest score for each set of students, but the only thing we know about the scores in between is that they are somewhere in that range. Since the median is one of the scores in between, this uncertainty means that the statement is insufficient. To illustrate, A_3 could be greater than B_3, making the answer to the question —yes: A = {40, 50, **60**, 70, 80}

B = {50, 55, **55**, 80, 90} However, A_3 could be less than or equal to B_3, making the answer to the question —no:

A = {40, 50, **60**, 70, 80} B = {50, 60, **70**, 80, 90} (2) SUFFICIENT: This statement tells us that for every student pair, the B student scored higher than the A student, or $B_n > A_n$. This statement can be considered qualitatively. Every student in set B scored higher than at least one student in set A. The students in set B not only scored higher individually, but also as a group, so one can reason that the median score for set B is higher than the median score for set A. Therefore, $B_3 > A_3$, and the answer to the question is —no.‖ But let's prove conclusively that the answer cannot be —yes.‖ Constrain A_3 to be greater than B_3, then try to pair the students according to the restriction that $B_n > A_n$. For example, pick any number x between 0 and 100, and let's say that $A_3 > x$, or high (H), and that $B_3 < x$, or low (L). Since the scores are increasing order, the 1st and 2nd scores must be less than or equal to the 3rd, while the 4th and 5th scores must be greater than or equal to the 3rd. Thus we know whether all the other scores are high or low. A = {A_1, A_2, ***H***, A_4, A_5} = {L, L, ***H***, H, H}

B = {B_1, B_2, ***L***, B_4, B_5} = {L, L, ***L***, H, H}

In order to meet the restriction that $B_n > A_n$, each of the 3 high scorers (H) in set A must be paired with a high(er) scorer, but there are only 2 high scorers (H) in set B—not enough to go around! Conversely, the 3 low scorers (L) in set B cannot be paired with a high scorer (H) from set A, leaving only 2 potential study partners for them from set A—not enough to go around! There is no way for A_3 to be greater than B_3 and still meet the restriction that $B_n > A_n$, so $A_3 < B_3$. Thus, the answer can never be —yes, it is always —no, and this statement is sufficient.

The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)

Let the scores of students in Adams' class be A1, A2, A3, A4 and A5, such that A1 < A2 < A3 < A4 < A5
Let the scores of students in Brown's class be B1, B2, B3, B4 and B5, such that B1 < B2 < B3 < B4 < B5

Median score of Adams' class = A3
Median score of Brown's class = B3

Now, we have to find if the median score for Brown's class (B3) > Median score for Adams' class (A3)

Statement 2 :

Statement 2 is saying that if we pair every student from Adams' class with every student from Brown's class, there is a way to pair these 10 students in a way such that the higher scores in each pair is one of Brown's students.

Meaning, for every score in Adams' class, there is a higher score in Brown's class.

So if we arrange the two sets of scores in ascending order, for every score in Adams' class, there is a greater score in Brown's class at the same position.

Meaning,

the least score in Brown's class > least score in Adams' class
the second smallest score in Brown's class > second smallest score in Adams' class
...
...
Since the middle score of Brown's class > middle score of Adams' class, median of Brown's class > median of Adams' class.

So the answer to the question is a definite NO. Sufficient!
The correct answer is B.

17.

In order to determine the median of a set of integers, we need to find the "middle" value.
(1) SUFFICIENT: Statement one tells us that average of the set of integers from 1 to x inclusive is 11. Since this is a set of consecutive integers, the "average" term is always the exact middle of the set. Thus, in order to have an average of 11, the set must be the integers from 1 to 21 inclusive. The middle or median term is also is 11.
(2) SUFFICIENT: Statement two states that the range of the set of integers from 1 to x inclusive is 20. In order for the range of integers to be 20, the set must be the integers from 1 to 21 inclusive. The median term in this set is 11.
The correct answer is D.

Alternate Solution from GMATCLUB

X>7 and the set (let it be S) contains 1 to X inclusive.
I normally read both the statements and start with the easier one. In this case, it is statement 2.
Statement 2: If the range is 20. Then X must be 21. Since X is the last term. In this case, we can easily find the median. So Sufficient.
Statement 1: If we know the average of the set, we can find the sum and number of elements in the set. But this is a tedious process to jot down all the numbers and find the sum.
But we can use statement 2 as a clue to solve this easily without crushing our brain. Since two statements never contradicts each other. If we plugin X as 21. We get the sum as 231(using the sum of n numbers formula) and average as 11.

We can find the median from this statement as well. So Sufficient.
The correct answer is D.

18.
Range before transaction:
112-45=67
Range after transaction:
(94+24)-(56-20)=118-36=82
The difference is: 82-67=15
The correct answer is D.

19.
Range: the difference between the greatest measurement and the smallest measurement.
In the question, combine 1 and 2, we still cannot know the value of q, then, we cannot determine which number of is the greatest measurement.
The correct answer is E.

20.

Prior to median 25, there are 7 numbers.

To make the greatest number as greater as possible, these 7 numbers should cost the range as little as possible. They will be, 24, 23, 22, 21, 20, 19, 18.
So, the greatest value that can fulfil the range is: 18+25=43
The correct answer is D.

GMAT Quant Topic 2: Statistics

Part E: Standard Deviation

1. 1.12 approx

   Find the mean first, i.e. (7+8+9+10)/4 =34/4=8.5

   S.D = √{(7-8.5)^2 + (8-8.5)^2 + (9-8.5)^2 + (10-8.5)^2}/√4

   S.D = √(2.25 + 0.25 + 0.25 + 2.25)/√4

   S.D = √5/√4

   S.D = √5/2 = 1.12 (approx)

2.

Top 1% expert replies to student queries

Set A - {11, 13, 17, 19, 23};
Set B - {5 consecutive even integers}, for example - {12, 14, 16, 18, 20};
Set C - {5 consecutive multiples of 7}, for example - {7, 14, 21, 28, 35}.
Now, the standard deviation of a set shows how much variation there is from the mean, how widespread a given set is. So, a low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.
You can see that set C is most widespread and set B is least widespread, so the correct answer is: B, A, C.
The answer is E, as C has a maximum gap between elements, and A has slightly more than B (has a GAP of 2), because A has a greater gap of 4 in certain cases(from 13 to 17 & from 19 to 23)
The correct answer is E.

3.

Set A - {2, 4, ..., 100};
Set X - {-48, -46, ..., 50};
Set Y - {3, 6, ..., 150};
Set Z - {-2/4, -4/4, ..., -100/4} = {-1/2, -1, -3/2, ..., -25}.

If we add or subtract a constant to each term in a set the SD will not change, so sets A and X will have the same SD.

If we increase or decrease each term in a set by the same percent (multiply by a constant) the SD will increase or decrease by the same percent, so set Y will have 1.5 times greater SD than set A and set Z will have 4 times less SD than set A (note SD cannot be negative so SD of Z will be SD of A divided by 4 not by -4).

So, the ranking of SD's in descending order is: Y, A=X, Z.

The correct answer is D.

4.

SD is always >=0. SD is 0 only when the list contains all identical elements (or which is same only 1 element).

So the answer is clearly E, none.

The correct answer is E.

To elaborate more: Standard deviation shows how much variation there is from the mean, how widespread a given set is. So, a low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values. So, basically we can say that it in a sense measures the distance and the distance cannot be negative. Also if you look at the formula for the standard deviation you'll see that it's square root of some number (variance) and the square root can never be negative.

5.

Top 1% expert replies to student queries

Consider these two sets: Set A {7,9,10,14} and set B {1,8,13,18}
The mean of set A = 10 and the mean of set B = 10
How do the Standard Deviations compare? Well, since the numbers in set B deviate more from the mean than do the numbers in set A, we can see that the standard deviation of set B must be greater than the standard deviation of set A.
Alternatively, let's examine the Average Distance from the Mean for each set.
Set A {7,9,10,14}
Mean = 10
7 is a distance of 3 from the mean of 10
9 is a distance of 1 from the mean of 10
10 is a distance of 0 from the mean of 10
14 is a distance of 4 from the mean of 10
So, the average distance from the mean = (3+1+0+4)/4 = 2
B {1,8,13,18}
Mean = 10
1 is a distance of 9 from the mean of 10
8 is a distance of 2 from the mean of 10
13 is a distance of 3 from the mean of 10
18 is a distance of 8 from the mean of 10
So, the average distance from the mean = (9+2+3+8)/4 = 5.5
IMPORTANT: I'm not saying that the Standard Deviation of set A equals 2, and I'm not saying that the Standard Deviation of set B equals 5.5 (They are reasonably close however).
What I am saying is that the average distance from the mean can help us see that the standard deviation of set B must be greater than the standard deviation of set A.
More importantly, the average distance from the mean is a useful way to think of standard deviation. This model is a convenient way to handle most standard deviation questions on the GMAT.

NOW ONTO THE QUESTION!!
So, for this question, we have:
Mean of set A = 70
Mean of set B = 0
Mean of set C = 40
100 is furthest away from the mean of 0 in set B, so this will cause the GREATEST change in standard deviation.
100 is next furthest away from the mean of 40 in set C, so this will cause the 2nd greatest change in standard deviation.
100 is closest to the mean of 70 in set A, so this will cause the LEAST change in standard deviation.
Answer: E
Use a logical approach:
You don't have to calculate anything. SD measures the distance between each element and mean. If a new element is added which is far away from the mean, it will distort the mean more than if it were added close to the mean.
The means of the 3 sets are 70, 0 and 40.
100 is farthest from 0 so it will change the SD of set B the most (in terms of absolute increase). It is closest to 70 so it will change the SD of set A the least. Hence the answer is B, C, A.
The correct answer is E.

6.
(1) Z - X = 10. No info about y. Not sufficient.
(2) Z - Y = 5. . No info about x. Not sufficient.

(1)+(2) From above x = z - 10 and y = z - 5, so the set in ascending order is {z-10, z-5, z}. Now, if we add or subtract a constant to each term in a set the standard deviation will not change. Adding 20-z to each term in the set we get {10, 15, 20}. So, the standard deviation of {z-10, z-5, z} is equal to that of {10, 15, 20}. Sufficient.

The correct answer is C.

7.
If X – Y > 0, then X > Y and the median of A is greater than the mean of set A. If L – M = 0, then L = M and the median of set B is equal to the mean of set B.

I. NOT NECESSARILY: According to the table, Z > N means that the standard deviation of set A is greater than that of set B. Standard deviation is a measure of how close the terms of a given set are to the mean of the set. If a set has a high standard deviation, its terms are relatively far from the mean. If a set has a low standard deviation, its terms are relatively close to the mean.

Recall that a median separates the set into two as far as the number of terms. There is an equal number of terms both above and below the median. If the median of a set is greater than the mean, however, the terms below the median must collectively be farther from the median than the terms above the median. For example, in the set {1, 89, 90}, the median is 89 and the mean is 60. The median is much greater than the mean because 1 is much farther from 89 than 90 is.

Knowing that the median of set A is greater than the mean of set A just tells us that the terms below set A's median are further from the median than the terms above set A's median. This does not necessarily imply that the terms, overall, are further away from the mean than in set B, where the terms below the median are the same distance from the median as the terms above it. In fact, a set in which the mean and median are equal can have a very high standard deviation if the terms are both far below the mean and far above it.

II. NOT NECESSARILY: According to the table, R > M implies that the mean of set [A + B] is greater than the mean of set B. This is not necessarily true. **When two sets are combined to form a composite set, the mean of the composite set must either be between the means of the individual sets or be equal to the mean of both of the individual sets.** To prove this, consider the simple example of one member sets: A = [3], B = [5], A + B = [3, 5]. In this case the mean of A + B is greater than the mean of A and less than the mean of B. We could easily have reversed this result by reversing the members of sets A and B.

III. NOT NECESSARILY: According to the table, Q > R implies that the median of the set [A + B] is greater than the mean of set [A + B]. We can extend the rule given in statement II to medians as well: **when two sets are combined to form a composite set, the *median* of the composite set must either be between the *medians* of the individual sets or be equal to the median of one or both of the individual sets**. While the median of set A is greater than the mean of set A and the median of set B is equal to the mean of set B, set [A + B] might have a median that is greater or less than the mean of set [A + B]. See the two tables for illustration:

	Set	Median	Mean	Result
A	1, 3, 4	3	2.67	Median > Mean
B	4, 5, 6	5	5	Median = Mean
A + B	1, 3, 4, 4, 5, 6	4	3.83	Median > Mean

	Set	Median	Mean	Result
A	1, 3, 3, 4	3	2.75	Median > Mean
B	10, 11, 12	11	11	Median = Mean
A + B	1, 3, 3, 4, 10, 11, 12	4	6.29	Median < Mean

Therefore none of the statements are necessarily true.
The correct answer is E.

8. 65, 85
Range = Mean + SD
So, Range = 75 + 10
Thus, So, Range = 65 to 85
Answer is 65 to 85.

9.
Top 1% expert replies to student queries
Within 2 stds means that the absolute difference between Elena's score and the class mean is less than or equal to 2 standard deviations.

Let Elena's score be x, Mathematically,

|x - 60| <= 2 * std
|x - 60| < = 30
-30 <= x - 60 <= 30
30 <= x <= 90
Therefore, lowest possible score = 30
Answer is 30.

10. SD of y will be aS.
Taking case if x is 0,1,2,3,4..... SD is S
now for series 1,2,3,4,5....... SD is S still (SD remains same if we add same number in all digits)
but for series 0,5,10,15,20,..... SD is 5S (SD becomes constant*OLD SD when all digits are multiplied by same constant).
Answer is aS.

11. |a/b| × (S)
Top 1% expert replies to student queries (can skip) (additional)
ax + by + c = 0
by = -c - ax
y = -c/b - (a/b)x
x = {x1,x2,........xn} SD = S
-(a/b) x = { -a/b x1, -a/b x2.........-a/b xn }
SD = |a/b|S
Since Standard deviation is always positive or zero , we are taking mod as a ,b can be negative. The mod will make it positive.

Adding -c/b to each term
{(-a/b)x1 -(c/b) , (-a/b)x2 - (c/b).......}
SD = |a/b| S
Standard Deviation remains the same as you are subtracting a constant from each term.

12.
A score of 58 was 2 standard deviations below the mean --> 58 = Mean - 2d
A score of 98 was 3 standard deviations above the mean --> 98 = Mean + 3d

Solving above for Mean = 74.

Answer is: A.

13.

If you add/subtract the same number from each element of a set, the SD does not change.

I. {r - 2, s - 2, t - 2}
Subtracting 2 from each elements will not change the SD.

II. {0, s - t, s - r}

{s - s, s - t, s - r}

Note that whatever the SD of {s, r, t}, the same will be the SD of {-s, -r, -t} because relative distance between them on the number line does not change (all the numbers are just flipped across the Y axis - positive becomes negative, negative becomes positive).
So, when we add s to each term, we still get the same SD.

III. {|r|, |s|, |t|}

This could change the relative distance between them on the number line if r, s and t all do not have the same sign.
e.g.

____________ (-5)____________________0____________(3)____(4)

becomes

______________________________________0____________(3)____(4)____ (5)
Much smaller SD now.

The correct answer is D.

14. Standard deviation is a measure of how far the data points in a set fall from the mean. For example, {5, 5, 6, 7, 7} has a small standard deviation relative to {1, 4, 6, 7, 10}. The values in the second set are much further from the mean than the values in the first set. In general, a value that drastically increases the range of a set will also have a large impact on the standard deviation. In this case, 14 creates the largest spread of the five answer choices, and will therefore be the value that most increases the standard deviation of Set T.
The correct answer is E.

15. The procedure for finding the standard deviation for a set is as follows: 1) Find the difference between each term in the set and the mean of the set. 2) Average the squared "differences." 3) Take the square root of that average. Notice that the standard deviation hinges on step 1: finding **the difference between each term in the set and the mean of the set**. Once this is done, the remaining steps are just calculations based on these "differences." Thus, we can rephrase the question as follows: "What is the difference between each term in the set and the mean of the set?" (1) SUFFICIENT: From the question, we know that Q is a set of consecutive integers. Statement 1 tells us that there are 21 terms in the set. Since, in any consecutive set with an odd number of terms, the middle value is the mean of the set, we can represent the set as 10 terms on either side of the middle term x: [x – 10, x – 9, x – 8, x – 7, x – 6, x – 5, x – 4,
x – 3, x – 2, x – 1, x, x + 1, x + 2, x + 3, x + 4, x + 5, x + 6, x + 7, x + 8, x + 9, x + 10] .
Notice that the difference between the mean (x) and the first term in the set (x – 10) is 10. The difference between the mean (x) and the second term in the set (x – 9) is 9. As you can see, we can actually find the difference between each term in the set and the mean of the set without knowing the specific value of each term in the set! (The only reason we are able to do this is because we know that the set abides by a specified consecutive pattern and because we are told the number of terms in this set.) Since we are able to find the "differences," we can use these to calculate the standard deviation of the set. Although you do not need to do this, here is the actual calculation: Sum of the squared differences:
$10^2 + 9^2 + 8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2 + (-1)^2 + (-2)^2(-3)^2 + (-4)^2 + (-5)^2 + (-6)^2(-7)^2 + (-8)^2 + (-9)^2 + (-10)^2 = 770$.

Average of the sum of the squared differences: 770/21 = 36 2/3

The square root of this average is the standard deviation: $\sqrt{36\frac{2}{3}} \cong 66.66\%$

(2) NOT SUFFICIENT: Since the set is consecutive, we know that the median is equal to the mean. Thus, we know that the mean is 20. However, we do not know how big the set is so we cannot identify the difference between each term and the mean.

The correct answer is A.

Top 1% expert replies to student queries (can skip) (additional) : For Statement 1

Let the 21 numbers be x-10, x-9, x-8, x-7, x-6,.....x, x+1, x+2, x+3, x+4,......x+10

Mean of these numbers = x [For every term x+k, there is a term x-k. Every such pair adds up to 2x. So, total sum = 21x and mean = x]

SD = root{[(x-(x-10))^2 + (x-(x-9))^2 + (x-(x-8))^2 + ... + (x-x)^2 + (x-(x+1))^2 + (x-(x+2))^2 + ...(x-(x+10))^2]/21}

SD = root {[(10)^2 + (9)^2 + (8)^2 + ...0^2 + 1^2 + 2^2 + ...10^2]/21}

SD = root {[2*(1^2 + 2^2 + ...10^2)]/21}

1^2 + 2^2 + 3^2 + ...10^2 = (10 * 11 * 21)/6

SD = root [(2*(10 * 11 * 21)/6 * 21) = root(110/3)

Again, you do not need to calculate the SD. You have to understand that all sets of 21 consecutive integers will have the same standard deviation, irrespective of what those numbers are. So statement 1 is sufficient

16.
(1) SUFFICIENT: The average of data set B = {1, 2, 3} is 2. So, in data set A = {1, 2, x} as x increases above 3, it gets further and further from the average. This necessarily increases its standard deviation, so A necessarily has a greater standard deviation than B.

(2) INSUFFICIENT: Statement (2) is insufficient since there are two different values of x less than 1 that give different answers to the question. For example, let x = 0 so A = {0, 1, 2}. Then A has the same standard deviation as B.

Now let x = -100, so A = {-100, 1, 2}. Clearly A has a larger standard deviation than B since its data is much more spread out. Since we have found two different values of x that give different answers to the question, statement (2) is insufficient.

The correct answer is A.

17.
10-0.3=9.7
10+0.3=10.3
The number within 1 standard deviation should be between 9.7-10.3 so there are six numbers within 1 standard deviation 6/8=75%
The correct answer is D.

18.
Average=100
1 standard deviation below the mean: less than 100-22.4=77.6
Obviously, 70 and 75 can fulfil the requirements.
The correct answer is B.

Top 1% expert replies to student queries (can skip)

Mean of the above distribution = 100
Standard Deviation = 22.4

Mean - 1 standard deviation = 77.6

We need to find the number of children with running times below this figure (Please pay attention to the way the question is framed. How many children have running times "more than 1 sd" below the mean? Meaning, how many running times are lower than the mean by more than 1 sd.)
We see only 2 running times < 77.6. Therefore, our answer is 2.
The correct answer is B.

19.
1 and 2 standard deviations below the mean=>number of the hours at most is 21-6=15, at least is 21-2*6=9.
The correct answer is D

20.
Mean 8.1
Standard deviation 0.3
Within 1.5 standard deviations of the mean=[8.1-0.3*1.5,8.1+0.3*1.5]=[7.65,8.55] All the numbers except 7.51 fall within such interval
Answer is 11
The correct answer is E

Top 1% expert replies to student queries (can skip)

"Within 1.5 standard deviation of the mean" - means in the range {mean-1.5*sd; mean+1.5*sd} = {8.1-1.5*0.3; 8.1+1.5*0.3} = {7.65; 8.55}.
From the 12 listed amounts, only one (7.51) is out of this range and 11 are within this range. Answer: E.

Explanation:
If the Standard Deviation is 0.3 ounces, then 0.3 ounces represents 1 unit of standard deviation.
Similarly, 0.6 ounces represents 2 units of standard deviation,
0.15 ounces represents 0.5 units of standard deviation, and so on.

If the mean is 8.1 ounces, then we say that 8.4 ounces is 1 unit of standard deviation above the mean (since 8.1 + 0.3 = 8.4), and we say that 7.8 ounces is 1 unit of standard deviation below the mean (since 8.1 - 0.3 = 7.8)
We want to know how many measurements in the list are within 1.5 standard deviations of the mean
Well, using the above logic, 0.45 represents 1.5 units of standard deviation.
So, 1.5 units of standard deviation below the mean equals 7.65 (8.1 - 0.45 = 7.65)
Similarly, 1.5 units of standard deviation above the mean equals 8.55 (8.1 + 0.45 = 8.55)
So, any measurement that is between 7.65 ounces and 8.55 will be within 1.5 standard deviations of the mean.
In the given list of measurements, the following meet this requirement:
7.51 8.22 7.86 8.36
8.09 7.83 8.30 8.01
7.73 8.25 7.96 8.53

The correct answer is E

21.
d^2=[(a1-a)^2+(a2-a)^2+ .... +(an-a)^2]/n
When we added 6 and 6, the numerator remained unchanged but the denominator increased, so, the new deviation is less than d.
Answer is E

22.
For statement 1, we know that 68% are within [m-d, m+d], so, the percent greater than m+d will be (1-0.68)/2.
For statement 2, we know that 16% is less than m-d, considering the distribution is symmetric about the mean m, we can get, 16% is greater than m+d.
The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Symmetric about the mean means that the shape of the distribution on the right and left side of the curve are mirror-images of each other.

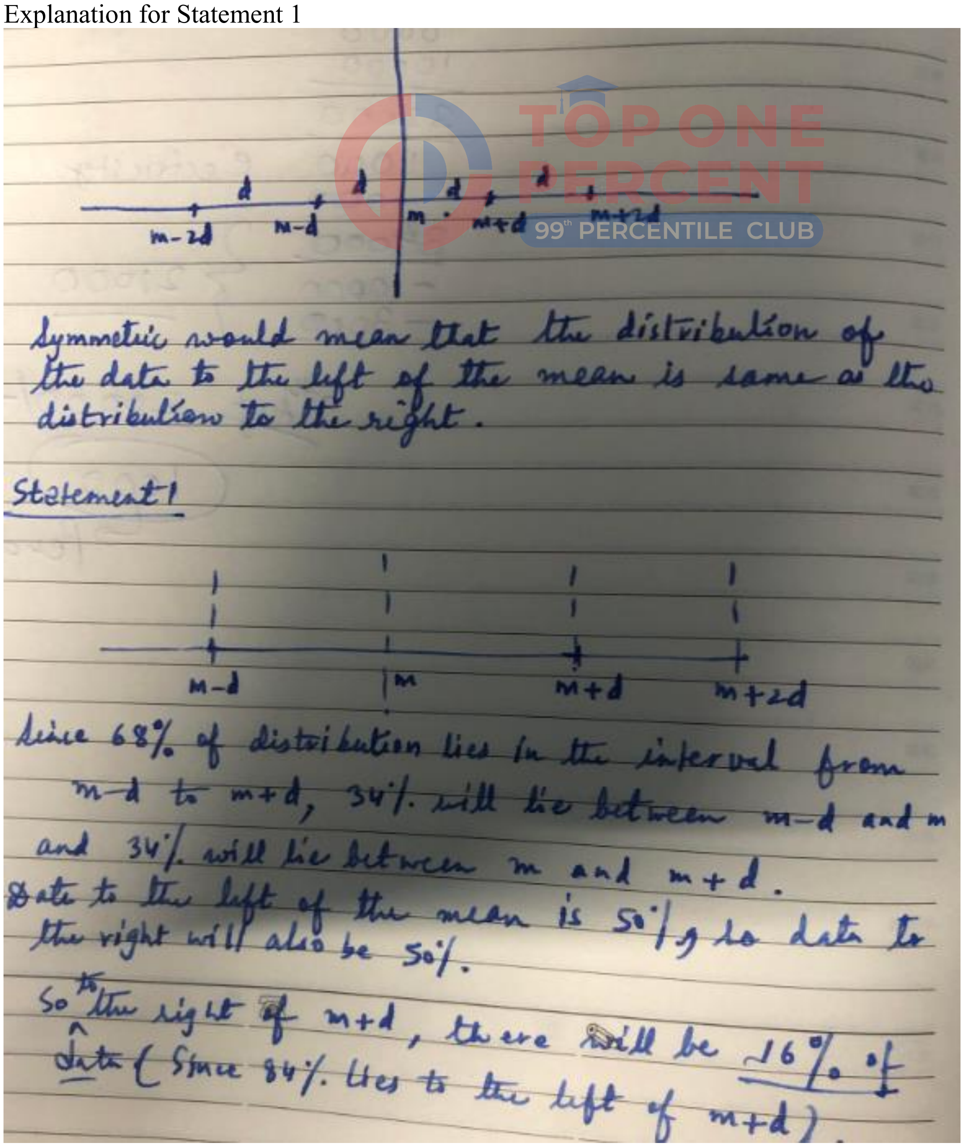
(1) 68% of the distribution lies in the interval from m-d to m+d, inclusive --> 100%-68%=32% is less than m-d and more than m+d. As distribution is symmetric about the mean then exactly half of 32%, or 16%, would be more than m+d. Sufficient.

(2) 16% of the distribution is less than m-d --> again, as distribution is symmetric about the mean then exactly 16%, will be more than m+d. Sufficient.

Answer: D.

Top 1% expert replies to student queries (can skip) (additional) :

Explanation for Statement 1



Symmetric would mean that the distribution of the data to the left of the mean is same as the distribution to the right.

Statement 1

m-d m m+d m+2d

Since 68% of distribution lies in the interval from m-d to m+d, 34% will lie between m-d and m and 34% will lie between m and m+d.
Data to the left of the mean is 50%, so data to the right will also be 50%.
So to the right of m+d, there will be 16% of data (Since 84% lies to the left of m+d).

GMAT Quant Topic 3: Inequalities + Absolute Value (Modulus)

1.
There are two characteristics of x that dictate its exponential behaviour. First of all, it is a decimal with an absolute value of less than 1. Secondly, it is a negative number.

I. True. x^3 will always be negative (negative × negative × negative = negative), and x^2 will always be positive (negative × negative = positive), so x^3 will always be less than x^2.

II. True. x^5 will always be negative, and since x is negative, $1 – x$ will always be positive because the double negative will essentially turn $1 – x$ into $1 + |x|$. Therefore, x^5 will always be less than $1 – x$.

III. True. One useful method for evaluating this inequality is to plug in a number for x. If x = - 0.5,

$x^4 = (-0.5)^4 = 0.0625$

$x^2 = (-0.5)^2 = 0.25$

To understand why this works, it helps to think of the negative aspect of x and the decimal aspect of x separately.

Because x is being taken to an even exponent in both instances, we can essentially ignore the negative aspect because we know the both results will be positive.

The rule with decimals between 0 and 1 is that the number gets smaller and smaller in absolute value as the exponent gets bigger and bigger. Therefore, x^4 must be smaller in absolute value than x^2.

The correct answer is E.

2.
(1) INSUFFICIENT: We can solve this absolute value inequality by considering both the positive and negative scenarios for the absolute value expression $|x + 3|$.
If $x > -3$, making $(x + 3)$ positive, we can rewrite $|x + 3|$ as $x + 3$:
$x + 3 < 4$
$x < 1$
If $x < -3$, making $(x + 3)$ negative, we can rewrite $|x + 3|$ as $-(x + 3)$:
$-(x + 3) < 4$
$x + 3 > -4$
$x > -7$
If we combine these two solutions we get $-7 < x < 1$, which means we can't tell whether x is positive.

(2) INSUFFICIENT: We can solve this absolute value inequality by considering both the positive and negative scenarios for the absolute value expression $|x – 3|$.
If $x > 3$, making $(x – 3)$ positive, we can rewrite $|x – 3|$ as $x – 3$:
$x – 3 < 4$
$x < 7$
If $x < 3$, making $(x – 3)$ negative, we can rewrite $|x – 3|$ as $-(x – 3)$ OR $3 – x$
$3 – x < 4$
$x > -1$
If we combine these two solutions we get $-1 < x < 7$, which means we can't tell whether x is positive.

(1) AND (2) INSUFFICIENT: If we combine the solutions from statements (1) and (2) we get an overlapping range of $-1 < x < 1$. We still can't tell whether x is positive.

The correct answer is E.

3.The question asks: is $x + n < 0$?

(1) INSUFFICIENT: This statement can be rewritten as $x + n < 2n – 4$. This rephrased statement is consistent with $x + n$ being either negative or non-negative. (For example, if $2n – 4 = 1{,}000$, then $x + n$ could be any integer, negative or not, that is less than 1,000.) Statement (1) is insufficient because it answers our question by saying "maybe yes, maybe no".
(2) SUFFICIENT: We can divide both sides of this equation by -2 to get $x < -n$ (remember that the inequality sign flips when we multiply or divide by a negative number). After adding n to both sides of resulting inequality, we are left with $x + n < 0$.

The correct answer is B.

4.
This is a multiple variable inequality problem, so you must solve it by doing algebraic manipulations on the inequalities.

(1) INSUFFICIENT: Statement (1) relates b to d, while giving us no knowledge about a and c. Therefore statement (1) is insufficient.

(2) INSUFFICIENT: Statement (2) does give a relationship between a and c, but it still depends on the values of b and d. One way to see this clearly is by realizing that only the right side of the equation contains the variable d. Perhaps $ab^2 – b$ is greater than $b^2c – d$ simply because of the magnitude of d. Therefore there is no way to draw any conclusions about the relationship between a and c.

(1) AND (2) SUFFICIENT: By adding the two inequalities from statements (1) and (2) together, we can come to the conclusion that $a > c$. Two inequalities can always be added together as long as the direction of the inequality signs is the same:

$ab^2 – b > b^2c – d$
(+) $\quad b > d$
- - - - - - - - - - - - - -
$\quad ab^2 > b^2c$

Now divide both sides by b^2. Since b^2 is always positive, you don't have to worry about reversing the direction of the inequality. The final result: $a > c$.

The correct answer is C.

5.
Since this question is presented in a straightforward way, we can proceed right to the analysis of each statement. On any question that involves inequalities, make sure to simplify each inequality as much as possible before arriving at the final conclusion.
(1) INSUFFICIENT: Let's simplify the inequality to rephrase this statement:
$-5x > -3x + 10$
$5x – 3x < -10$ (don't forget: switch the sign when multiplying or dividing by a negative)
$2x < -10$
$x < -5$

Since this statement provides us only with a range of values for x, it is insufficient.

(2) INSUFFICIENT: Once again, simplify the inequality to rephrase the statement:
$-11x – 10 < 67$
$-11x < 77$
$x > -7$

Since this statement provides us only with a range of values for x, it is insufficient.

(1) AND (2) SUFFICIENT: If we combine the two statements together, it must be that

$-7 < x < -5$. Since x is an integer, $x = -6$.

The correct answer is C.

6. We can start by solving the given inequality for x:

$8x > 4 + 6x$
$2x > 4$
$x > 2$

So, the rephrased question is: "If the integer x is greater than 2, what is the value of x?"

(1) SUFFICIENT: Let's solve this inequality for x as well:

$6 - 5x > -13$
$-5x > -19$
$x < 3.8$

Since we know from the question that $x > 2$, we can conclude that $2 < x < 3.8$. The only integer between 2 and 3.8 is 3. Therefore, $x = 3$.

(2) SUFFICIENT: We can break this inequality into two distinct inequalities. Then, we can solve each inequality for x:

$3 - 2x < -x + 4$
$3 - 4 < x$
$-1 < x$

$-x + 4 < 7.2 - 2x$ x
$< 7.2 - 4$
$x < 3.2$

So, we end up with $-1 < x < 3.2$. Since we know from the information given in the question that $x > 2$, we can conclude that $2 < x < 3.2$. The only integer between 2 and 3.2 is 3. Therefore, $x= 3$.
The correct answer is D.

7.
(1) INSUFFICIENT: The question asks us to compare $a + b$ and $c + d$. No information is provided about b and d.
(2) INSUFFICIENT: The question asks us to compare $a + b$ and $c + d$. No information is provided about a and c.
(1) AND (2) SUFFICIENT: If we rewrite the second statement as $b > d$, we can add the two inequalities:

$$\begin{array}{rl} & a > c \\ + & b > d \\ \hline & a + b > c + d \end{array}$$

This can only be done when the two inequality symbols are facing the same direction.
The correct answer is C.

8.
Let's start by rephrasing the question. If we square both sides of the equation we get:

$$\left(\sqrt{xy}\right)^2 = (xy)^2$$

$$xy = (xy)^2$$

Now subtract xy from both sides and factor:

$(xy)^2 - xy = 0$
$xy(xy - 1) = 0$
So $xy = 0$ or 1

To find the value of $x + y$ here, we need to solve for both x and y. If $xy = 0$, either x or y (or both) must be zero.
If $xy = 1$, x and y are reciprocals of one another.
While we can't come up with a precise rephrasing here, the algebra we have done will help us see the usefulness of the statements.

(1) INSUFFICIENT: Knowing that $x = -1/2$ does not tell us if y is 0 (i.e. $xy = 0$) or if y is - 2 (i.e. $xy = 1$)

(2) INSUFFICIENT: Knowing that y is not equal to zero does not tell us anything about the value of x; x could be zero (to make $xy = 0$) or any other value (to make $xy = 1$).

(1) AND (2) SUFFICIENT: If we know that y is not zero and we have a nonzero value for x, neither x nor y is zero; xy therefore must equal 1. If $xy = 1$, since $x = -1/2$, y must equal -2. We can use this information to find $x + y$, $-1/2 + (-2) = -5/2$.

The correct answer is C.

Top 1% expert replies to student queries (can skip)

root(xy) = xy

Here, xy >=0 because root(xy) can be real only if xy >=0.
So, your case $x = -1/2$ and $y = 2$ is invalid.
Moving on to the solution, Combining statements 1 and 2,
root(xy) = xy
xy - root(xy) = 0
root(xy) [root(xy) - 1] = 0
Now, $x = -1/2$ and y is not equal to 0. So, root(xy) cannot be 0.
Therefore, root(xy) - 1 =0
root(xy) = 1
$xy = 1$
$-y/2 = 1$
$y = -2$
$x + y = -5/2$
Sufficient!
The correct answer is C.

9.
The question asks whether x is greater than y. The question is already in its most basic form, so there is no need to rephrase it; we can go straight to the statements.

(1) INSUFFICIENT: The fact that x^2 is greater than y does not tell us whether x is greater than y. For example, if $x = 3$ and $y = 4$, then $x^2 = 9$, which is greater than y although x itself is less than y. But if $x = 5$ and $y = 4$, then $x^2 = 25$, which is greater than y and x itself is also greater than y.
INSUFFICIENT: We can square both sides to obtain $x < y^2$. As we saw in the examples above, it is possible for this statement to be true whether y is less than or greater than x (just substitute x for y and vice-versa in the examples above).

(1) AND (2) INSUFFICIENT: Taking the statements together, we know that $x < y^2$ and $y < x^2$, but we do not know whether $x > y$. For example, if $x = 3$ and $y = 4$, both of these inequalities hold ($3 < 16$ and $4 < 9$) and $x < y$. But if $x = 4$ and $y = 3$, both of these inequalities still hold ($4< 9$ and $3 < 16$) but now $x > y$.
The correct answer is E.

10.
The equation in question can be rephrased as follows:
$x^2y - 6xy + 9y = 0$
$y(x^2 - 6x + 9) = 0$
$y(x-3)^2 = 0$

Therefore, one or both of the following must be true:
$y = 0$ or $x = 3$

It follows that the product xy must equal either 0 or 3y. This question can therefore be rephrased —What is y?

(1) INSUFFICIENT: This equation cannot be manipulated or combined with the original equation to solve directly for x or y. Instead, plug the two possible scenarios from the original equation into the equation from this statement:

If $x = 3$, then $y = 3 + x = 3 + 3 = 6$, so $xy = (3)(6) = 18$.
If $y = 0$, then $x = y - 3 = 0 - 3 = -3$, so $xy = (-3)(0) = 0$.

Since there are two possible answers, this statement is not sufficient.

(2) SUFFICIENT: If $x^3 < 0$, then $x < 0$. Therefore, x cannot equal 3, and it follows that $y = 0$. Therefore, $xy = 0$.

The correct answer is B.

11.
(1) INSUFFICIENT: We can solve the quadratic equation by factoring: $x^2 - 5x + 6 = 0$
$(x - 2)(x - 3) = 0$
$x = 2$ or $x = 3$

Since there are two possible values for x, this statement on its own is insufficient.

(2) INSUFFICIENT: Simply knowing that $x > 0$ is not enough to determine the value of x.

(1) AND (2) INSUFFICIENT: The two statements taken together still allow for two possible x values: $x = 2$ or 3.

The correct answer is E.

12.
This question is already in simple form and cannot be rephrased.

(1) INSUFFICIENT: This is a second-order or quadratic equation in standard form $ax^2 + bx + c = 0$ where $a = 1$, $b = 3$, and $c = 2$.

The first step in solving a quadratic equation is to reformat or —factor the equation into a product of two factors of the form $(x + y)(x + z)$. The trick to factoring is to find two integers whose sum equals b and whose product equals c. (Informational note: the reason that this works is because multiplying out $(x + y)(x + z)$ results in $x2 + (y + z)x + yz$, hence $y + z = b$ and $yz = c$).

In this case, we have $b = 3$ and $c = 2$. This is relatively easy to factor because c has only two possible combinations of integer multiples: 1 and 2; and -1 and -2. The only combination that also adds up to b is 1 and 2 since $1 + 2 = 3$. Hence, we can rewrite (1) as the product of two factors: $(x + 1)(x + 2) = 0$.

In order for a product to be equal to 0, it is only necessary for one of its factors to be equal to 0. Hence, to solve for x, we must find the x's that would make either of the factors equal to zero.

The first factor is x + 1. We can quickly see that x + 1 = 0 when x = -1. Similarly, the second factor x + 2 is equal to zero when x = -2. Therefore, x can be either -1 or -2 and we do not have enough information to answer the question.

(2) INSUFFICIENT: We are given a range of possible values for x.

(1) and (2) INSUFFICIENT: (1) gives us two possible values for x, both of which are negative. (2) only tells us that x is negative, which does not help us pinpoint the value for x.

The correct answer is E.

13.
When solving an absolute value equation, it helps to first isolate the absolute value expression:

$3|3 - x| = 7$
$|3 - x| = 7/3$

When removing the absolute value bars, we need to keep in mind that the expression inside the absolute value bars (3 – x) could be positive or negative. Let's consider both possibilities:

When (3 – x) is positive:
$(3 - x) = 7/3$
$3 - 7/3 = x$
$9/3 - 7/3 = x$
$x = 2/3$

When (3 – x) is negative:
$-(3 - x) = 7/3$
$x - 3 = 7/3$
$x = 7/3 + 3$
$x = 7/3 + 9/3$
$x = 16/3$

So, the two possible values for x are 2/3 and 16/3. The product of these values is 32/9.

The correct answer is E.

14.
For their quotient to be less than zero, a and b must have opposite signs. In other words, if the answer to the question is "yes," EITHER a is positive and b is negative OR a is negative and b is positive.
The question can be rephrased as the following: "Do a and b have opposite signs?"
(1) INSUFFICIENT: a^2 is always positive so for the quotient of a^2 and b^3 to be positive, b^3 must be positive. That means that b is positive. This does not however tell us anything about the sign of a.
(2) INSUFFICIENT: b^4 is always positive so for the product of a and b^4 to be negative, a must be negative. This does not however tell us anything about the sign of b.
(1) AND (2) SUFFICIENT: Statement 1 tells us that b is positive and statement 2 tells us that a is negative. The yes/no question can be definitively answered with a "yes".

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

Query: Explain how in statement 1, b is positive?
Reply: The question stem asks whether a and b are of the opposite signs.

Statement one in the numerator you have a^2 this means a can be positive or negative (a^2 will be positive) and in the denominator you have b^3, so b should be negative.
So, if a is positive and b is negative → yes answer to the question stem
So, if a is negative and b is negative → no answer to the question stem
So Statement 1 is not sufficient.

15.
The question asks about the sign of d.
(1) INSUFFICIENT: When two numbers sum to a negative value, we have two possibilities:
Possibility A: Both values are negative (e.g., e = -4 and d = -8)
Possibility B: One value is negative and the other is positive. (e.g., e = -15 and d = 3).
(2) INSUFFICIENT: When the difference of two numbers produces a negative value, we have three possibilities:
Possibility A: Both values are negative (e.g., e = -20 and d = -3)
Possibility B: One value is negative and the other is positive (e.g., e = -20 and d = 3).
Possibility C: Both values are positive (e.g., e = 20 and d = 30)
(1) AND (2) SUFFICIENT: When d is ADDED to e, the result (-12) is greater than when d is SUBTRACTED from e. This is only possible if d is a positive value. If d were a negative value than adding d to a number would produce a smaller value than subtracting d from that number (since a double negative produces a positive). You can test numbers to see that d must be positive and so we can definitively answer the question using both statements.
The correct answer is C.

16.
We are given the inequality a – b > a + b. If we subtract a from both sides, we are left with the inequality -b > b. If we add b to both sides, we get 0 > 2b. If we divide both sides by 2, we can rephrase the given information as 0 > b, or b is negative.

I. FALSE: All we know from the given inequality is that 0 > b. The value of a could be either positive or negative.

II. TRUE: We know from the given inequality that 0 > b. Therefore, b must be negative.

III. FALSE: We know from the given inequality that 0 > b. Therefore, b must be negative. However, the value of a could be either positive or negative. Therefore, ab could be positive or negative.

The correct answer is B.

17.
Given that |a| = 1/3, the value of a could be either 1/3 or -1/3. Likewise, b could be either 2/3 or -2/3. Therefore, four possible solutions to a + b exist, as shown in the following table:

a	b	a + b
1/3	2/3	1
1/3	-2/3	-1/3
-1/3	2/3	1/3
-1/3	-2/3	-1

2/3 is the only answer choice that does not represent a possible sum of a + b.

The correct answer is D.

18.
Because we know that |a| = |b|, we know that a and b are equidistant from zero on the number line. But we do not know anything about the signs of a and b (that is, whether they are positive or negative). Because the question asks us which statement(s) MUST be true, we can eliminate any statement that is not always true. To prove that a statement is not always true, we need to find values for a and b for which the statement is false.
I. NOT ALWAYS TRUE: a does not necessarily have to equal b. For example, if a = -3 and b = 3, then |-3| = |3| but -3 ≠ 3.
II. NOT ALWAYS TRUE: |a| does not necessarily have to equal -b. For example, if a = 3 and b = 3, then |3| = |3| but |3| ≠ -3.
III. NOT ALWAYS TRUE: -a does not necessarily have to equal -b. For example, if a = -3 and b = 3, then |-3| = |3| but -(-3) ≠ -3.
The correct answer is E.

19:
A. x^4>=1=>(x^4-1)>=0=>(x^2-1)(x^2+1)>=0=>(x+ 1)(x-1) (x^2+1)>=0 (x^2+1)>0, so (x+1)(x-1)>=0 =>x<-1,x>1
B. x^3<=27=>(x^3-3^3)<=0=>(x-3)(X^2+3x+3^2)<=0, with d=b^2-4ac we can know that (X^2+3x+3^2) has no solution, but we know that (X^2+3x+3^2)=[(x+3/2)^2+27/4]>0, then, (x-3)<=0, x<=3
C. x^2>=16, [x^2-4^2]>=0, (x-4)(x+4)>=0, x>4,x<-4
D. 2<=|x|<=5, 2<=|x|, x>=2, or x<=2
The correct answer is E.

20.
The question asks whether x^n is less than 1. In order to answer this, we need to know not only whether x is less than 1, but also whether n is positive or negative since it is the combination of the two conditions that determines whether x^n is less than 1.

(1) INSUFFICIENT: If x = 2 and n = 2, $x^n = 2^2 = 4$. If x = 2 and n = -2, $x^n = 2^{(-2)} = 1/(2^2) = 1/4$.

(2) INSUFFICIENT: If x = 2 and n = 2, $x^n = 2^2 = 4$. If x = 1/2 and n = 2, $x^n = (1/2)^2 = 1/4$.

(1) AND (2) SUFFICIENT: Taken together, the statements tell us that x is greater than 1 and n is positive. Therefore, for any value of x and for any value of n, x^n will be greater than 1 and we can answer definitively "no" to the question.

The correct answer is C.

21.

Since $3^5 = 243$ and $3^6 = 729$, 3^x will be less than 500 only if the integer x is less than 6. So, we can rephrase the question as follows: "Is x < 6?"

(1) INSUFFICIENT: We can solve the inequality for x.
$4^{x-1} < 4^x - 120$
$4^{x-1} - 4^x < -120$
$4^x(4^{-1}) - 4^x < -120$
$4^x(1/4) - 4^x < -120$
$4^x[(1/4) - 1] < -120$
$4^x(-3/4) < -120$
$4^x > 160$

Since $4^3 = 64$ and $4^4 = 256$, x must be greater than 3. However, this is not enough to determine if x < 6.

(2) INSUFFICIENT: If $x^2 = 36$, then x = 6 or -6. Again, this is not enough to determine if x < 6.

(1) AND (2) SUFFICIENT: Statement (1) tells us that $x > 3$ and statement (2) tells us that $x = 6$ or -6. Therefore, we can conclude that $x = 6$. This is sufficient to answer the question "Is $x < 6$?" (Recall that the answer "no" is sufficient.)

The correct answer is C.

22.
Remember that an odd exponent does not "hide the sign," meaning that x must be positive in order for x^3 to be positive. So, the original question "Is $x^3 > 1$?" can be rephrased "Is $x > 1$?"

(1) INSUFFICIENT: It is not clear whether x is greater than 1. For example, x could be -1, and the answer to the question would be "no," since $(-1)^3 = -1 < 1$ However, x could be 2, and the answer to the question would be "yes," since $2^3 = 8 > 1$.

(2) SUFFICIENT: First, simplify the statement as much as possible.

$2x – (b – c) < c – (b – 2)$
$2x – b + c < c – b + 2$ [Distributing the subtraction sign on both sides]
$2x < 2$ [Cancelling the identical terms (+c and -b) on each side]
$x < 1$ [Dividing both sides by 2]

Thus, the answer to the rephrased question "Is $x > 1$?" is always "no." Remember that for —yes/no data sufficiency questions it doesn't matter whether the answer is —yes or —no; what is important is whether the additional information in sufficient to answer either definitively —yes or definitively —no. In this case, given the information in (2), the answer is always —no therefore, the answer is a definitive —no (1) and (2) is sufficient to answer the question. If the answer were —yes‖ for some values of x and —no 1 for other values of x, it would not be possible to answer the question definitively, and (2) would not be sufficient.

The correct answer is B.

23.
Square both sides of the given equation to eliminate the square root sign:
$(x + 4)^2 = 9$

Remember that even exponents —hide the sign‖ of the base, so there are two solutions to the equation: $(x + 4) = 3$ or $(x + 4) = -3$. On the GMAT, the negative solution is often the correct one, so evaluate that one first.

$(x + 4) = -3$
$x = -3 – 4$
$x = -7$

Watch out! Although -7 is an answer choice, it is not correct. The question does not ask for the value of x, but rather for the value of $x – 4 = -7 – 4 = -11$.

Alternatively, the expression $\sqrt{(x+4)^2}$ can be simplified to $|x + 4|$, and the original equation can be solved accordingly.
If $|x + 4| = 3$, either $x = -1$ or $x = -7$

The correct answer is A.

Top 1% expert replies to student queries (can skip) (additional)

On the GMAT, the square root always gives a non-negative result. And this property is taken into account by the mod function.

So root((x+4)^2) = |x+4| = 3

Solutions are x = -7 and x = -1

x - 4 = -11 or -5

24.
(1) SUFFICIENT: Statement(1) tells us that $x > 2^{34}$, so we want to prove that $2^{34} > 10^{10}$. We'll prove this by manipulating the expression 2^{34}.
$2^{34} = (2^4)(2^{30})$
$2^{34} = 16(2^{10})^3$

Now $2^{10} = 1024$, and 1024 is greater than 10^3. Therefore:

$2^{34} > 16(10^3)^3$
$2^{34} > 16(10^9)$
$2^{34} > 1.6(10^{10})$.

Since $2^{34} > 1.6(10^{10})$ and $1.6(10^{10}) > 10^{10}$, then $2^{34} > 10^{10}$.

(2) SUFFICIENT: Statement (2) tells us that that $x = 2^{35}$, so we need to determine if $2^{35} > 10^{10}$.

Statement (1) showed that $2^{34} > 10^{10}$, therefore $2^{35} > 10^{10}$.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

Stat 1: Compare 2^34 and 10^10. Take square root on both sides: we should compare 2^17 and 10^5=100,000.
Now, 2^17=2^10∗2^7=1,024∗128 > 100,000. Hence, sufficient
You should keep in mind going forward that 2^10 = 1024. It's alright if you were not aware of this before. You can make a note of this approach now

Stat 2: x = 2^35 is given. We have the exact numerical value of x. Hence, we can compare it to 10^10 and get one unique answer to the question. Sufficient
The correct answer is D.

25.

X-2Y < -6 => -X + 2Y > 6
Combined X - Y > -2, we know Y > 4 X
- Y > -2 => -2X + 2Y < 4
Combined X - 2Y < -6, we know -X < -2 => X > 2
Therefore, XY > 0
The correct answer is C.

26. The rules of odds and evens tell us that the product will be odd if all the factors are odd, and the product will be even if at least one of the factors is even. In order to analyze the given statements I, II, and III, we must determine whether x and y are odd or even. First, solve the absolute value equation for x by considering both the positive and negative values of the absolute value expression.

If $x - \frac{9}{2}$ is positive:

$x - \frac{9}{2} = \frac{5}{2}$

$x = \frac{14}{2}$

x = 7

If $x - \frac{9}{2}$ is negative:

$x - \frac{9}{2} = -\frac{5}{2}$

$x = \frac{4}{2}$

x = 2

Therefore, x can be either odd or even.

Next, consider the median (y) of a set of p consecutive integers, where p is odd. Will this median necessarily be odd or even? Let's choose two examples to find out:

Example Set 1: 1, 2, 3 (the median y = 2, so y is even)

Example Set 2: 3, 4, 5, 6, 7 (the median y = 5, so y is odd)

Therefore, y can be either odd or even.

Now, analyze the given statements:

I. UNCERTAIN: Statement I will be true if and only if x, y, and p are all odd. We know p is odd, but since x and y can be either odd or even we cannot definitively say that xyp will be odd. For example, if x = 2 then xyp will be even.

II. TRUE: Statement II will be true if any one of the factors is even. After factoring out a p, the expression can be written as xyp(p + 1). Since p is odd, we know (p + 1) must be even. Therefore, the product of xyp(p + 1) must be even.

III. UNCERTAIN: Statement III will be true if any one of the factors is even. The expression can be written as xxyypp. We know that p is odd, and we also know that both x and y could be odd.

The correct answer is A.

27. The |x| + |y| on the left side of the equation will always add the positive value of x to the positive value of y, yielding a positive value. Therefore, the -x and the -y on the right side of the equation must also each yield a positive value. The only way for -x and -y to each yield positive values is if both x and y are negative.

(A) FALSE: For x + y to be greater than zero, either x or y has to be positive.

(B) TRUE: Since x has to be negative and y has to be negative, the sum of x and y will always be negative.

(C) UNCERTAIN: All that is certain is that x and y have to be negative. Since x can have a larger magnitude than y and vice-versa, x – y could be greater than zero.

(D) UNCERTAIN: All that is certain is that x and y have to be negative. Since x can have a larger magnitude than y and vice versa, x – y could be less than zero.

(E) UNCERTAIN: As with choices (C) and (D), we have no idea about the magnitude of x and y. Therefore, $x^2 - y^2$ could be either positive or negative.

Another option to solve this problem is to systematically test numbers. With values for x and y that satisfy the original equation, observe that both x and y have to be negative. If x = -4 and y = -2, we can eliminate choices (A) and (C). Then, we might choose numbers such that y has a greater magnitude than x, such as x = -2 and y = -4. With these values, we can eliminate choices (D) and (E).

The correct answer is B.

28. The question asks if $xy < 0$. Knowing the rules for positives and negatives (the product of two numbers will be positive if the numbers have the same sign and negative if the numbers have different signs), we can rephrase the question as follows: Do x and y have the same sign?

(1) INSUFFICIENT: We can factor the right side of the equation $y = x^4 – x^3$ as follows:

$y = x^4 – x^3$
$y = x^3(x – 1)$

Let's consider two cases: when x is negative and when x is positive. When x is negative, x^3 will be negative (a negative integer raised to an odd exponent results in a negative), and $(x – 1)$ will be negative. Thus, y will be the product of two negatives, giving a positive value for y.

When x is positive, x^3 will be positive and $(x – 1)$ will be positive (remember that the question includes the constraint that xy is not equal to 0, which means y cannot be 0, which in turn means that x cannot be 1). Thus, y will be the product of two positives, giving a positive value for y.

In both cases, y is positive. However, we don't have enough information to determine the sign of x. Therefore, this statement alone is insufficient.

(2) INSUFFICIENT: Let's factor the left side of the given inequality:

$-12y^2 – y^2x + x^2y^2 > 0$
$y^2(-12 – x + x^2) > 0$
$y^2(x^2 – x – 12) > 0$
$y^2(x + 3)(x – 4) > 0$

The expression y^2 will obviously be positive, but it tells us nothing about the sign of y; it could be positive or negative. Since y does not appear anywhere else in the inequality, we can conclude that statement 2 alone is insufficient (without determining anything about x) because the statement tells us nothing about y.

(1) AND (2) INSUFFICIENT: We know from statement (1) that y is positive; we now need to examine statement 2 further to see what we can determine about x.

We previously determined that $y^2(x + 3)(x – 4) > 0$. Thus, in order for $y^2(x + 3)(x – 4)$ to be greater than 0, $(x + 3)$ and $(x – 4)$ must have the same sign. There are two ways for this to happen: both $(x + 3)$ and $(x – 4)$ are positive, or both $(x + 3)$ and $(x – 4)$ are negative. Let's look at the positive case first.

$x + 3 > 0$
$x > -3$, and
$x – 4 > 0$
$x > 4$

So, for both expressions to be positive, x must be greater than 4. Now let's look at the negative case:

$x + 3 < 0$
$x < -3$, and
$x – 4 < 0$
$x < 4$

For both expressions to be negative, x must be less than -3. In conclusion, statement (2) tells us that $x > 4$ OR $x < -3$. This is obviously not enough to determine the sign of x. Since the sign of x is still unknown, the combination of statements is insufficient to answer the question "Do x and y have the same sign?"

The correct answer is E.

29. This question cannot be rephrased since it is already in a simple form.

(1) INSUFFICIENT: Since x^2 is positive whether x is negative or positive, we can only determine that x is not equal to zero; x could be either positive or negative.

(2) INSUFFICIENT: By telling us that the expression $x \cdot |y|$ is not a positive number, we know that it must either be negative or zero. If the expression is negative, x must be negative (|y| is never negative). However if the expression is zero, x or y could be zero.

(1) AND (2) INSUFFICIENT: We know from statement 1 that x cannot be zero, however, there are still two possibilities for x: x could be positive (y is zero), or x could be negative (y is anything).

The correct answer is E.

30. First, let's try to make some inferences from the fact that $ab^2c^3d^4 > 0$. Since none of the integers is equal to zero (their product does not equal zero), b and d raised to even exponents must be positive, i.e. $b^2 > 0$ and $d^4 > 0$, implying that $b^2d^4 > 0$. If $b^2d^4 > 0$ and $ab^2c^3d^4 > 0$, the product of the remaining variables, a and c^3 must be positive, i.e. $ac^3 > 0$. As a result, while we do not know the specific signs of any variable, we know that $ac > 0$ (because the odd exponent c^3 will always have the same sign as c) and therefore a and c must have the same sign—either both positive or both negative.

Next, let's evaluate each of the statements:

I. UNCERTAIN: While we know that the even exponent a^2 must be positive, we do not know anything about the signs of the two remaining variables, c and d. If c and d have the same signs, then $cd > 0$ and $a^2cd > 0$, but if c and d have different signs, then $cd < 0$ and $a^2cd < 0$.

II. UNCERTAIN: While we know that the even exponent c^4 must be positive, we do not know anything about the signs of the two remaining variables, b and d. If b and d have the same signs, then $bd > 0$ and $bc^4d > 0$, but if b and d have different signs, then $bd < 0$ and $bc^4d < 0$.

III. TRUE: Since $a^3c^3 = (ac)^3$ and a and c have the same signs, it must be true that $ac > 0$ and $(ac)^3 > 0$. Also, the even exponent d^2 will be positive. As a result, it must be true that $a^3c^3d^2 > 0$.

The correct answer is C.

31. It is extremely tempting to divide both sides of this inequality by y or by the |y|, to come up with a rephrased question of —is $x > y$? However, we do not know the sign of y, so this cannot be done.

(1) INSUFFICIENT: On a yes/no data sufficiency question that deals with number properties (positive/negatives), it is often easier to plug numbers. There are two good reasons why we should try both positive and negative values for y: (1) the question contains the expression |y|, (2) statement 2 hints that the sign of y might be significant. If we do that we come up with both a yes and a no to the question.

x	y	$x \cdot \lvert y\rvert > y^2$	?
-2	-4	$-2(4) > (-4)^2$	N
4	2	$4(2) > 2^2$	Y

(2) INSUFFICIENT: Using the logic from above, when trying numbers here we should take care to pick x values that are both greater than y and less than y.

x	y	$x \cdot \lvert y\rvert > y^2$	?
2	4	$2(4) > 4^2$	N

4	2	$4(2) > 2^2$	Y

(1) AND (2) SUFFICIENT: If we combine the two statements, we must choose positive x and y values for which x > y.

x	y	$x \cdot \lvert y \rvert > y^2$	?
3	1	$3(1) > 1^2$	Y
4	2	$4(2) > 2^2$	Y
5	3	$5(3) > 3^2$	Y

Using a more algebraic approach, if we know that y is positive (statement 2), we can divide both sides of the original question by y to come up with "is x > y?" as a new question. Statement 1 tells us that x > y, so both statements together are sufficient to answer the question.

The correct answer is C.

32. (1) INSUFFICIENT: This expression provides only a range of possible values for x.

(2) SUFFICIENT: Absolute value problems often -- ***but not always*** -- have multiple solutions because the expression within the absolute value bars can be either positive or negative even though the absolute value of the expression is always positive. For example, if we consider the equation $|2 + x| = 3$, we have to consider the possibility that 2 + x is already positive and the possibility that 2 + x is negative. If 2 + x is positive, then the equation is the same as 2 + x = 3 and x = 1. But if 2 + x is negative, then it must equal -3 (since $|-3| = 3$) and so 2 + x = -3 and x = -5.

So in the present case, in order to determine the possible solutions for x, it is necessary to solve for x under both possible conditions.

For the case where x > 0:

$x = 3x – 2$
$-2x = -2$
$x = 1$

For the case when x < 0:

$x = -1(3x – 2)$ We multiply by -1 to make x equal a negative quantity.
$x = 2 – 3x$
$4x = 2$
$x = 1/2$

Note however, that the second solution x = 1/2 contradicts the stipulation that x < 0, hence there is no solution for x where x < 0. Therefore, x = 1 is the only valid solution for (2).

The correct answer is B.

33. (1) INSUFFICIENT: If we test values here we find two sets of possible x and y values that yield conflicting answers to the question.

x	$\sqrt{x}$	y	Is x > y?
4	2	1	YES
1/4	1/2	1/3	NO

(2) INSUFFICIENT: If we test values here we find two sets of possible x and y values that yield conflicting answers to the question.

x	x^3	y	Is x > y?
2	8	1	YES
-1/2	-1/8	-1/4	NO

(1) AND (2) SUFFICIENT: Lets start with statement 1 and add the constraints of statement 2. From statement 1, we see that x has to be positive since we are taking the square root of x. There is no point in testing negative values for y since a positive value for x against a negative y will always yield a yes to the question. Lastly, we should consider x values between 0 and 1 and greater than 1 because proper fractions behave different than integers with regard to exponents. When we try to come up with x and y values that fit both conditions, we must adjust the two variables so that x is always greater than y.

x	$\sqrt{x}$	x^3	y	Is x > y?
2	1.4	8	1	YES
1/4	1/2	1/64	1/128	YES

Logically it also makes sense that if the cube and the square root of a number are both greater than another number than the number itself must be greater than that other number.

The correct answer is C.

34. The question "Is |x| less than 1?" can be rephrased in the following way.

Case 1: If x > 0, then |x| = x. For instance, |5| = 5. So, if x > 0, then the question becomes "Is x less than 1?"
Case 2: If x < 0, then |x| = -x. For instance, |-5| = -(-5) = 5. So, if x < 0, then the question becomes "Is -x less than 1?" This can be written as follows:

-x < 1?
or, by multiplying both sides by -1, we get
x > -1?

Putting these two cases together, we get the fully rephrased question:
—Is -1 < x < 1 (and x not equal to 0)"?

Another way to achieve this rephrasing is to interpret absolute value as distance from zero on the number line. Asking "Is |x| less than 1?" can then be reinterpreted as "Is x less than 1 unit away from zero on the number line?" or "Is -1 < x < 1?" (The fact that x does not equal zero is given in the question stem.)
(1) INSUFFICIENT: If x > 0, this statement tells us that x > x/x or x > 1. If x < 0, this statement tells us that x > x/-x or x > -1. This is not enough to tell us if -1 < x < 1.
(2) INSUFFICIENT: When x > 0, x > x which is not true (so x < 0). When x < 0, -x > x or x < 0. Statement (2) simply tells us that x is negative. This is not enough to tell us if -1 < x < 1.
(1) AND (2) SUFFICIENT: If we know x < 0 (statement 2), we know that x > -1 (statement 1). This means that -1 < x < 0. This means that x is definitely between -1 and 1.
The correct answer is C.

35. (1) SUFFICIENT: We can combine the given inequality r + s > 2t with the first statement by adding the two inequalities:

$$\begin{array}{r} r + s > 2t \\ \underline{\quad t > s \quad} \\ r + s + t > 2t + s \end{array}$$

$r > t$

(2) SUFFICIENT: We can combine the given inequality $r + s > 2t$ with the second statement by adding the two inequalities:

$$\begin{array}{r} r + s > 2t \\ \underline{\quad r > s \quad} \\ 2r + s > 2t + s \end{array}$$

$$2r > 2t$$

$$r > t$$

The correct answer is D.

36. The question stem gives us three constraints:

1) a is an integer.
2) b is an integer.
3) a is farther away from zero than b is (from the constraint that $|a| > |b|$).

When you see a problem using absolute values, it is generally necessary to try positive and negative values for each of the variables. Thus, we should take the information from the question, and see what it tells us about the signs of the variables.

For b, we should try negative, zero, and positive values. Nothing in the question stem eliminates any of those possibilities. For a, we only have to try negative and positive values. Why not $a = 0$? We know that b must be closer to zero than a, so a <u>cannot</u> equal zero because there is no potential value for b that is closer to zero than zero itself. So to summarize, the possible scenarios are:

a	b
neg	neg
neg	0
neg	pos
pos	neg
pos	0
pos	pos

(1) INSUFFICIENT: This statement tells us that a is negative, ruling out the positive a scenarios above. Remember that a is farther away from zero than b is.

a	b	$a \cdot \|b\|$	$a - b$	Is $a \cdot \| b \| < a - b$?
neg	neg	neg · |neg| = neg · pos = more negative	neg (far from 0) – neg (close to 0) = neg (far from 0) + pos (close to 0) = less neg	Is more neg < less neg? **Yes.**
neg	0	neg · |0| = neg · 0 = 0	neg – 0 = neg	Is 0 < neg? **No.**

neg	pos	neg · |pos| = neg · pos = at least as negative as a, since b could be 1 or greater	neg – pos = more negative than a	Is at least as negative as a < more negative than a ? **It depends.**

For some cases the answer is —yes, but for others the answer is —no. Therefore, statement (1) is insufficient to solve the problem.

(2) INSUFFICIENT: This statement tells us that a and b must either have the same sign (for ab > 0), or one or both of the variables must be zero (for ab = 0). Thus we can rule out any scenario in the original list that doesn‘t meet the constraints from this statement.

a	b	a · | b |	a – b	Is a · | b | < a - b?
neg	neg	neg · |neg| = neg · pos = more negative	neg (far from 0) - neg (close to 0) = neg (far from 0) + pos (close to 0) = less negative	Is more negative < less negative? **Yes.**
neg	0	neg · |0| = 0	neg - 0 = neg	Is 0 < neg? **No.**
pos	0	pos · |0| = 0	pos - 0 = pos	Is 0 < pos? **Yes.**
pos	pos	pos · |pos| = more positive	pos (far from 0) - pos (close to 0) = less positive	Is more positive < less positive? **No.**

For some cases the answer is —yes, but for others the answer is —no. Therefore, statement (2) is insufficient to solve the problem.

(1) & (2) INSUFFICIENT: For the two statements combined, we must consider only the scenarios with negative a and either negative or zero b. These are the scenarios that are on the list for both statement (1) and statement (2).

a	b	a · | b |	a – b	Is a · | b | < a – b?
neg	neg	neg · |neg| = neg · pos = more negative	neg (far from 0) - neg (close to 0) = neg (far from 0) + pos (close to 0) = less negative	Is more negative < less negative? **Yes**
neg	0	neg · |0| = 0	neg – 0 = neg	Is 0 < neg? **No**

For the first case the answer is —yes, but for the second case the answer is —no. Thus the two statements combined are not sufficient to solve the problem.

The correct answer is E.

37. This is a multiple variable inequality problem, so you must solve it by doing algebraic manipulations on the inequalities.

(1) INSUFFICIENT: Statement (1) relates b to d, while giving us no knowledge about a and c. Therefore statement (1) is insufficient.

(2) INSUFFICIENT: Statement (2) does give a relationship between a and c, but it still depends on the values of b and d. One way to see this clearly is by realizing that only the right side of the equation contains the variable d. Perhaps $ab^2 – b$ is greater than $b^2c – d$ simply because of the magnitude of d. Therefore there is no way to draw any conclusions about the relationship between a and c.

(1) AND (2) SUFFICIENT: By adding the two inequalities from statements (1) and (2) together, we can come to the conclusion that $a > c$. Two inequalities can always be added together as long as the direction of the inequality signs is the same:

$ab^2 – b > b^2c – d$
(+) $b > d$
- - - - - - - - - - - - - -
$ab^2 > b^2c$

Now divide both sides by b^2. Since b^2 is always positive, you don't have to worry about reversing the direction of the inequality. The final result: $a > c$.

The correct answer is C.

38.

The question tells us that $p < q$ and $p < r$ and then asks whether the product pqr is less than p. Statement (1) INSUFFICIENT: We learn from this statement that either p or q is negative, but since we know from the question that $p < q$, p must be negative. To determine whether $pqr < p$, let's test values for p, q, and r. Our test values must meet only 2 conditions: p must be negative and q must be positive.

P	q	r	pqr	Is pqr < p?
-2	5	10	-100	YES
-2	5	-10	100	NO

Statement (2) INSUFFICIENT: We learn from this statement that either p or r is negative, but since we know from the question that $p < r$, p must be negative. To determine whether $pqr < p$, let's test values for p, q, and r. Our test values must meet only 2 conditions: p must be negative and r must be positive.

p	q	r	pqr	Is pqr < p?
-2	-10	5	100	NO
-2	10	5	-100	YES

If we look at both statements together, we know that p is negative and that both q and r are positive. To determine whether $pqr < p$, let's test values for p, q, and r. Our test values must meet 3 conditions: p must be negative, q must be positive, and r must be positive.

p	q	r	pqr	Is pqr < p?
-2	10	5	-100	YES
-2	7	4	-56	YES

At first glance, it may appear that we will always get a "YES" answer. But don't forget to test out fractional (decimal) values as well. The problem never specifies that p, q, and r must be integers.

p	q	r	pqr	Is pqr < p?
-2	.3	.4	-.24	NO

Even with both statements, we cannot answer the question definitively.
The correct answer is E.

39. We are told that $|x|\cdot y + 9 > 0$, which means that $|x|\cdot y > -9$. The question asks whether $x < 6$.
A statement counts as sufficient if it enables us to answer the question with —definitely yes or —definitely no ; a statement that only enables us to say —maybe counts as insufficient.

(1) INSUFFICIENT: We know that $|x|\cdot y > -9$ and that y is a negative integer. Suppose $y = -1$. Then $|x|\cdot(-1) > -9$, which means $|x| < 9$ (since dividing by a negative number reverses the direction of the inequality). Thus x could be less than 6 (for example, x could equal 2), but does not have to be less than 6 (for example, x could equal 7).

(2) INSUFFICIENT: Since the question stem tells us that y is an integer, the statement $|y| \leq 1$ implies that y equals -1, 0, or 1. Substituting these values for y into the expression $|x|\cdot y > -9$, we see that x could be less than 6, greater than 6, or even equal to 6.
If y=0, according to this equation $|x|*y+ 9 > 0$, x can take any value
For example
x=7, $|x|*y+ 9 > 0= |7|*0+ 9 > 0 =16>0$
x=2, $|2|*0+ 9 > 0 =11>0$
x=-2, $|-2|*0+ 9 > 0=11>0$
This is particularly obvious if $y = 0$; in that case, x could be any integer at all. (You can test this by picking actual numbers.)

(1) AND (2) INSUFFICIENT: If y is negative and $|y| \leq 1$, then y must equal -1. We have already determined from our analysis of statement (1) that a value of $y = -1$ is consistent both with x being less than 6 and with x not being less than 6.
The correct answer is E.

40. We can rephrase the question by opening up the absolute value sign. There are two scenarios for the inequality $|n| < 4$.
If $n > 0$, the question becomes —Is $n < 4$? If $n < 0$, the question becomes: —Is $n > -4$?
We can also combine the questions: —Is $-4 < n < 4$? (n is not equal to 0)
(1) SUFFICIENT: The solution to this inequality is $n > 4$ (if $n > 0$) or $n < -4$ (if $n < 0$). This provides us with enough information to guarantee that n is definitely NOT between -4 and 4. Remember that an absolute no is sufficient!
(2) INSUFFICIENT: We can multiply both sides of the inequality by $|n|$ since it is definitely positive. To solve the inequality $|n| \times n < 1$, let's plug values. If we start with negative values, we see that n can be any negative value since $|n| \times n$ will always be negative and therefore less than 1. This is already enough to show that the statement is insufficient because n might not be between -4 and 4.
The correct answer is A.

41. Note that one need not determine the values of both x and y to solve this problem; the value of product xy will suffice.

(1) SUFFICIENT: Statement (1) can be rephrased as follows:

$-4x - 12y = 0$
$-4x = 12y$
$x = -3y$

If x and y are non-zero integers, we can deduce that they must have opposite signs: one positive, and the other negative. Therefore, this last equation could be rephrased as

|x| = 3|y|

We don't know whether x or y is negative, but we do know that they have the opposite signs. Converting both variables to absolute value cancels the negative sign in the expression x = -3y.

We are left with two equations and two unknowns, where the unknowns are |x| and |y|:

|x| + |y| = 32
|x| – 3|y| = 0

Subtracting the second equation from the first yields

4|y| = 32
|y| = 8

Substituting 8 for |y| in the original equation, we can easily determine that |x| = 24. Because we know that one of either x or y is negative and the other positive, xy must be the negative product of |x| and |y|, or -8(24) = -192.

(2) INSUFFICIENT: Statement (2) also provides two equations with two unknowns:

|x| + |y| = 32
|x| - |y| = 16

Solving these equations allows us to determine the values of |x| and |y|: |x| = 24 and |y| = 8. However, this gives no information about the sign of x or y. The product xy could either be -192 or 192.

The correct answer is A.

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When x=-3y, we know that the absolute value of x is 3 times that of y: |x| = 3|y|

Mentioned in solution: "We don't know whether x or y is negative, but we do know that they have the opposite signs...."

|x| = 3|y| This can have 4 possible equations:
a. x= 3y
b. -x= -3y
c. x= -3y
d. -x= 3y
But we cannot use equations a and b, as those make both 'x and y' same sign(mentioned in solution). The only valid solutions are 'c and d'.

Alternate method to solve:
Given:
|X| + |Y| = 32,
so 4 equations:
-X + Y = 32 (I)
X + Y = 32 (II)
-X - Y = 32 (III)
X - Y = 32 (IV)
From (1)
-4X = 12y, so X = -3Y or if X is neg, then Y is pos and vice-versa
This information means equations (II) and (III) above can be eliminated.
Hence, substitute X = -3Y in (I) and (IV) yields Y = -8 or 8.
if Y = -8, then X = +24, so (xy) = -192
if Y = +8, then X = -24, so (xy) = -192 the same as above.

(1) is true

From (II)
$|X| - |Y| = 16$ and given $|X| + |Y| = 32$, subtract both equations yields
$2|X| = 16$, so $X = -8$ or 8. Y is thus -24 or 24... but the combination can be the following (X,Y):
(-8, 24)
(-8, -24)
(8, 24)
(8, -24)
and 'xy' will yield either -192 or +192.

The correct answer is A.

42. (1) INSUFFICIENT: Since this equation contains two variables, we cannot determine the value of y. We can, however, note that the absolute value expression $|x^2 – 4|$ must be greater than or equal to 0. Therefore, $3|x^2 – 4|$ must be greater than or equal to 0, which in turn means that $y – 2$ must be greater than or equal to 0. If $y – 2 > 0$, then $y > 2$.

(2) INSUFFICIENT: To solve this equation for y, we must consider both the positive and negative values of the absolute value expression:

If $3 – y > 0$, then $3 – y = 11$
$y = -8$

If $3 – y < 0$, then $3 – y = -11$
$y = 14$

Since there are two possible values for y, this statement is insufficient.

(1) AND (2) SUFFICIENT: Statement (1) tells us that y is greater than or equal to 2, and statement (2) tells us that y = -8 or 14. Of the two possible values, only 14 is greater than or equal to 2. Therefore, the two statements together tell us that y must equal 14.

The correct answer is C.

43. The question asks whether x is positive. The question is already as basic as it can be made to be, so there is no need to rephrase it; we can go straight to the statements.

(1) SUFFICIENT: Here, we are told that $|x + 3| = 4x – 3$. When dealing with equations containing variables and absolute values, we generally need to consider the possibility that there may be more than one value for the unknown that could make the equation work. In order to solve this particular equation, we need to consider what happens when $x + 3$ is positive and when it is negative (remember, the absolute value is the same in either case). First, consider what happens if $x + 3$ is positive. If $x + 3$ is positive, it is as if there were no absolute value bars, since the absolute value of a positive is still positive:
$x + 3 = 4x – 3$
$6 = 3x$
$2 = x$

So when $x + 3$ is positive, $x = 2$, a positive value. If we plug 2 into the original equation, we see that it is a valid solution:
$|2 + 3| = 4(2) – 3$
$|5| = 8 – 3$
$5 = 5$

Now let's consider what happens when $x + 3$ is negative. To do so, we multiply $x + 3$ by -1:
$-1(x + 3) = 4x – 3$
$-x – 3 = 4x – 3$

$0 = 5x$
$0 = x$

But if we plug 0 into the original equation, it is not a valid solution:
$|0 + 3| = 4(0) – 3$
$|3| = 0 – 3$
$3 = -3$

Therefore, there is no solution when $x + 3$ is negative and we know that 2 is the only solution possible and we can say that x is definitely positive.

Alternatively, we could have noticed that the right-hand side of the original equation must be positive (because it equals the absolute value of $x + 3$). If $4x – 3$ is positive, x must be positive. If x were negative, 4x would be negative and a negative minus a positive is negative.

(2) INSUFFICIENT: Here, again, we must consider the various combinations of positive and negative for both sides. Let's first assume that both sides are positive (which is equivalent to assuming both sides are negative):
$|x – 3| = |2x – 3|$
$x – 3 = 2x – 3$
$0 = x$
So, when both sides are positive, $x = 0$. We can verify that this solution is valid by plugging 0 into the original equation:
$|0 – 3| = |2(0) – 3|$
$|-3| = |-3|$
$3 = 3$

Now let's consider what happens when only one side is negative; in this case, we choose the right-hand side:
$|x – 3| = |2x – 3|$
$x – 3 = -(2x – 3)$
$x – 3 = -2x + 3$
$3x = 6$
$x = 2$

We can verify that this is a valid solution by plugging 2 into the original equation:
$|2 – 3| = |2(2) – 3|$
$|-1| = |1|$
$1 = 1$

Therefore, both 2 and 0 are valid solutions and we cannot determine whether x is positive, since one value of x is zero, which is not positive, and one is positive.
The correct answer is A.

44. Note that the question is asking for the absolute value of x rather than just the value of x. Keep this in mind when you analyze each statement.

(1) SUFFICIENT: Since the value of x^2 must be non-negative, the value of $(x^2 + 16)$ is always positive, therefore $|x^2 + 16|$ can be written $x^2 +16$. Using this information, we can solve for x:
$|x^2 + 16| – 5 = 27$
$x^2 + 16 – 5 = 27$
$x^2 + 11 = 27$
$x^2 = 16$
$x = 4$ or $x = -4$

Since $|-4| = |4| = 4$, we know that $|x| = 4$; this statement is sufficient.

(2) SUFFICIENT:
$x^2 = 8x – 16$

$x^2 – 8x + 16 = 0$
$(x – 4)^2 = 0$
$(x – 4)(x – 4) = 0$
$x = 4$

Therefore, $|x| = 4$; this statement is sufficient.

The correct answer is D.

45. First, let us take the expression, $x^2 – 2xy + y^2 – 9 = 0$. After adding 9 to both sides of the equation, we get $x^2 – 2xy + y^2 = 9$. Since we are interested in the variables x and y, we need to rearrange the expression $x^2 – 2xy + y^2$ into an expression that contains terms for x and y individually. This suggests that factoring the expression into a product of two sums is in order here. Since the coefficients of both the x^2 and the y^2 terms are 1 and the coefficient of the xy term is negative, the most logical first guess for factors is
$(x– y)(x – y)$ or $(x – y)^2$. (We can quickly confirm that these are the correct factors by multiplying out $(x – y)(x – y)$ and verifying that this is equal to $x^2 – 2xy + y^2$.) Hence, we now have $(x – y)^2 = 9$ which means that $x – y = 3$ or $x – y = -3$. Since the question states that $x > y$, $x – y$ must be greater than 0 and the only consistent answer is $x – y = 3$.

We now have two simple equations and two unknowns:

$x – y = 3$
$x + y = 15$

After adding the bottom equation to the top equation we are left with $2x = 18$;
hence $x= 9$.

If we are observant, we can apply an alternative method that uses a —trick to solve this very quickly. Note, of all the answers, $x = 9$ is the only answer that is consistent with both $x > y$ and $x + y = 15$. Hence $x = 9$ must be the answer.

The correct answer is E.

46.

The expression is equal to n if $n \geq 0$, but $–n$ if $n \leq 0$. This means that EITHER $n < 1$ if $n \geq 0$
OR
$–n < 1$ (that is, $n > -1$) if $n \leq 0$.
If we combine these two possibilities, we see that the question is really asking whether $-1 < n < 1$.

(1) INSUFFICIENT: If we add n to both sides of the inequality, we can rewrite it as the following:
$n^x < n$.
Since this is a Yes/No question, one way to handle it is to come up with sample values that satisfy this condition and then see whether these values give us a —yes or a —no to the question.

$n = ½$ and $x = 2$ are legal values since $(1/2)^2 < 1/2$

These values yield a YES to the question, since n is between -1 and 1.

$n = -3$ and $x = 3$ are also legal values since $3^{-3} = 1/27 < 3$

These values yield a NO to the question since n is greater than 1.

With legal values yielding a "yes" and a "no" to the original question, statement (1) is insufficient.

(2) INSUFFICIENT: $x^{–1} = –2$ can be rewritten as $x = -2^{-1} = -½$. However, this statement contains no information about n.

(1) AND (2) SUFFICIENT: If we combine the two statements by plugging the value for x into the first statement, we get $n^{-½} < n$.
The only values for n that satisfy this inequality are greater than 1.

Negative values for n are not possible. Raising a number to the exponent of -½ is equivalent to taking the reciprocal of the square root of the number. However, it is not possible (within the real number system) to take the square root of a negative number.

A fraction less than 1, such as ½, becomes a LARGER number when you square root it ($½^{½} = \sim 0.7$). However, the new number is still less than 1. When you reciprocate that value, you get a number ($½^{-½} = \sim 1.4$) that is LARGER than 1 and therefore LARGER than the original value of n.

Finally, all values of n greater than 1 satisfy the inequality $n^{-½} < n$.
For instance, if n = 4, then $n^{-½} = ½$. Taking the square root of a number larger than 1 makes the number smaller, though still greater than 1 -- and then taking the reciprocal of <u>that</u> number makes the number smaller still.

Since the two statements together tell us that n must be greater than 1, we know the definitive answer to the question "Is n between -1 and 1?" Note that the answer to this question is "No," which is as good an answer as "Yes" to a Yes/No question on Data Sufficiency.

The correct answer is C.

47.

In problems involving variables in the exponent, it is helpful to rewrite an equation or inequality in exponential terms, and it is especially helpful, if possible, to rewrite them with exponential terms that have the same base.

$0.04 = 1/25 = 5^{-2}$

We can rewrite the question in the following way: "Is $5^n < 5^{-2}$?"

The only way 5^n could be less than 5^{-2} would be if n is less than -2. We can rephrase the question:

"Is n < - 2"?

(1) SUFFICIENT: Let's simplify (or rephrase) the inequality given in this statement.

$(1/5)^n > 25$
$(1/5)^n > 5^2$
$5^{-n} > 5^2$
$-n > 2$
$n < -2$ (recall that the inequality sign flips when dividing by a negative number)

This is sufficient to answer our rephrased question.
(2) INSUFFICIENT: n^3 will be smaller than n^2 if n is either a negative number or a fraction between 0 and 1. We cannot tell if n is smaller than -2.

The correct answer is A.

48.

Before we proceed with the analysis of the statements, let's rephrase the question. Note that we can simplify the question by rearranging the terms in the ratio: 2x/3y = (2/3)(x/y). Therefore, to answer the question, we simply need to find the ratio x/y. Thus, we can rephrase the question: "What is x/y?"

(1) INSUFFICIENT: If $x^2/y^2 = 36/25$, you may be tempted to take the positive square root of both sides and conclude that $x/y = 6/5$. However, since even exponents hide the sign of the variable, both 6/5 and -6/5, when squared, will yield the value of 36/25. Thus, the value of x/y could be either 6/5 or -6/5.

(2) INSUFFICIENT: This statement provides only a range of values for x/y and is therefore insufficient.

(1) AND (2) SUFFICIENT: From the first statement, we know that $x/y = 6/5 = 1.2$ or $x/y = -6/5 = -1.2$. From the second statement, we know that $x^5/y^5 = (x/y)^5 > 1$. Note that if $x/y = 1.2$, then $(x/y)^5 = 1.2^5$, which is always greater than 1, since the base of the exponent (i.e. 1.2) is greater than 1. However, if $x/y = -1.2$, then $(x/y)^5 = (-1.2)^5$, which is always negative and does not satisfy the second statement. Thus, since we know from the second statement that $(x/y) > 1$, the value of x/y must be 1.2.

The correct answer is C.

49.

The equation in the question can be rephrased:
$x^y y^{-x} = 1$
$(x^y)(1/y^x) = 1$
Multiply both sides by y^x:
$x^y = y^x$
So the rephrased question is "Does $x^y = y^x$?"

For what values will the answer be "yes"? The answer will be "yes" if x = y. If x does not equal y, then the answer to the rephrased question could still be —yes,‖ but only if x and y have all the same prime factors. If either x or y has a prime factor that the other does not, the two sides of the equation could not possibly be equal. In other words, x and y would have to be different powers of the same base. For example, the pair 2 and 4, the pair 3 and 9, or the pair 4 and 16.

Lets try 2 and 4:
$4^2 = 2^4 = 16$
We see that the pair 2 and 4 would give us a —yes‖ answer to the rephrased question.

If we try 3 and 9, we see that this pair does not:
$3^9 > 9^3$ (because $9^3 = (3^2)^3 = 3^6$)

If we increase beyond powers of 3 (for example, 4 and 16), we will encounter the same pattern. So the only pair of unequal values that will work is 2 and 4. Therefore we can rephrase the question further: "Is x = y, or are x and y equal to 2 and 4?"

(1) INSUFFICIENT: The answer to the question is "yes" if x = y or if x and y are equal to 2 and 4. This is possible given the constraint from this statement that $x^x > y$. For example, x = y =3 meets the constraint that $x^x > y$, because 9 > 3. Also, x = 4 and y = 2 meets the constraint that $x^x > y$, because $4^4 > 2$. In either case, $x^y = y^x$, so the answer is "yes."

However, there are other values for x and y that meet the constraint $x^x > y$, for example x = 10 and y = 1, and these values would yield a "no" answer to the question "Is $x^y = y^x$?"

(2) SUFFICIENT: If x must be greater than y^y, then it is not possible for x and y to be equal. Also, the pair x = 2 and y = 4 is not allowed, because 2 is not greater than 4^4. Similarly, the pair x = 4 and y = 2 is not allowed because 4 is not greater than 2^2. This statement disqualifies all of the scenarios that gave us a "yes" answer to the question. Therefore, it is not possible that $x^y = y^x$, so the answer must be "no."

The correct answer is B.

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Let's re-arrange the question first:

Is $(x)^y * (y)^{-x} = 1$?
Is $(x)^y = (y)^x$?

Check this post for a detailed discussion on this: try-this-one-700-level-number-properties-103461.html#p805817

So, $(x)^y = (y)^x$ when x = y or x and y take values 2,4 or -2,-4

Look at the statements now:

(1) $(x)^x > y$
We know this relation is true for many random values of x and y e.g. x = 4, y = 5 etc. So the answer to the question is NO in this case. $(x)^y$ is not equal to $(y)^x$.
But does it hold for any values which will make $(x)^y = (y)^x$?
Yes it does! If x = y, x^x > y is true for say, x = y = 3. 3^3 is greater than 3. So x and y can take values which will give the answer YES.
Not sufficient.

(2) $x > (y)^y$
Again, it holds for many random values of x and y e.g. x = 10, y = 2 etc. So the answer to the question is NO in this case.
But does it hold for any values which will make $(x)^y = (y)^x$?
Let's see. If x = y, x cannot be greater than y^y. Check for a few values to figure out the pattern.
If x = 4 and y = 2, x is not greater than y^y.
Similarly, it doesn't work for x = -2, y = -4 and x = -4 and y = -2 since x will be negative while y^y will be positive.
Therefore, if $x > (y)^y$, $(x)^y = (y)^x$ cannot hold for any values of x and y. Hence answer to the question stays NO.
Sufficient.

Answer (B).

OR
Is x^y=y^x
In other words:
Is x=y OR Is (x,y) any of the pairs: (2, 4), (4, 2), (-2, -4), (-4, -2)
1. x^x>y
Say (x,y)=(4,2)
x^x=4^4>2; Good. Answer to the question=Yes, x^y is equal to y^x as (x, y) is one of the mentioned pairs.
But say (x,y)=(5,2)
x^x=5^5>2; Good. Answer to the question=No
Not Sufficient.

2. x>y^y
Now,
We can definitely say that x NOT equal to Y.
Let's see whether they can be any of the mentioned pairs.
(x,y)=(2,4); No;
(x,y)=(4,2); No;
(x,y)=(-2,-4); No; as -2 < (-4)^(-4)
(x,y)=(-4,-2); No; as -4 < (-2)^(-2)
So, (x,y) is not one of the pairs that will make the expression true. So, we can definitely conclude that x^y*y^-x NOT equal to 1
A definite NO proves sufficiency.
Sufficient.

The correct answer is B.

50.

(1) INSUFFICIENT: If we multiply this equation out, we get:
$x^2 + 2xy + y^2 = 9a$
If we try to solve this expression for $x^2 + y^2$, we get
$x^2 + y^2 = 9a – 2xy$
Since the value of this expression depends on the value of x and y, we don't have enough information.

(2) INSUFFICIENT: If we multiply this equation out, we get:
$x^2 - 2xy + y^2 = a$
If we try to solve this expression for $x^2 + y^2$, we get
$x^2 + y^2 = a + 2xy$
Since the value of this expression depends on the value of x and y, we don't have enough information.

(1) AND (2) INSUFFICIENT: We can combine the two expanded forms of the equations from the two statements by adding them:

$x^2 + 2xy + y^2 = 9a$
$x^2 - 2xy + y^2 = a$
– – – – – – – – – – – –
$2x^2 + 2y^2 = 10a$
$x^2 + y^2 = 5a$
If we substitute this back into the original question, the question becomes: "Is $5a > 4a$?
Since a is non negative, a can be 0 or positive. When $a = 0$, $5a = 4a$. When $a>0$, $5a$ will always be greater than $4a$.
The correct answer is E.

51.

(1) INSUFFICIENT: Statement (1) is insufficient because y is unbounded when both x and k can vary. Therefore y has no definite maximum.

To show that y is unbounded, let's calculate y for a special sequence of (x, k) pairs. The sequence starts at (-2, 1) and doubles both values to get the next (x, k) pair in the sequence.

$y_1 = | -2 - 1 | - | -2 + 1 | = 3 - 1 = 2$
$y_2 = | -4 - 2 | - | -4 + 2 | = 6 - 2 = 4$
$y_3 = | -8 - 4 | - | -8 + 4 | = -12 + 4 = 8$
etc.

In this sequence y doubles each time so it has no definite maximum, so statement (1) is insufficient.

(2) SUFFICIENT: Statement (2) says that k = 3, so $y = | x - 3 | - | x + 3 |$. Therefore to maximize y we must maximize $| x - 3 |$ while simultaneously trying to minimize $| x + 3 |$. This state holds for very large negative x. Let's try two different large negative values for x and see what happens:

If x = -100 then:
$y = |-100 - 3| - |-100 + 3|$
$y = 103 - 97 = 6$

If x = -101 then:
$y = |-101 - 3| - |-101 + 3|$
$y = 104 - 98 = 6$

We see that the two expressions increase at the same rate, so their difference remains the same. As x decreases from 0, y increases until it reaches 6 when $x = -3$. As x decreases further, y remains at 6 which is its maximum value.

The correct answer is B.

Top 1% expert replies to student queries (can skip)

Think about it this way: The numbers $(x - k)$ and $(x + k)$ are points on the number line separated by a distance of 2k. Now, depending on the way we subtract, the difference might be +2k or -2k, and the absolute values will get tricky when x is close to zero where $(x - k)$ and $(x + k)$ have opposite signs.

Clearly, the value of k will be important in establishing an answer.

What's a little unclear is x. Does x have a single unknown numerical value? In that case, the expression y would have a single value. There would be no question of a "maximum" value. The fact that the question is asking for a "maximum" value implies that x moves over a range.

Statement #1: $x < 0$
We have no information about the value of k, and we would need that to give any sort of answer. This is insufficient.

Statement #2: $k = 3$
We have to assume that x would equal any real number. If x is a large negative number, say x = -20, then
y = (-20 – 3) – (-20 + 3) = -23 – (-17) = +6
When x is closer than 3 to zero on either side, the value of y is less. For example, when x = 0, y = 0.
Now consider a large positive value, say, x = +20.
y = (20 – 3) – (20 + 3) = 17 – 23 = -6
Thus, the maximum value of y is +6. (Notice that this is 2k.) We have a definitive answer. Thus, statement #2, alone and by itself, is sufficient.

The correct answer is B.

52.
When we plug a few values for x, we see that the expression doesn't seem to go below the value of 2. It is important to try both fractions (less than 1) and integers greater than 1. Let's try to mathematically prove that this expression is always greater than or equal to 2. Is $x + \frac{1}{x} \geq 2$?
Since $x > 0$, we can multiply both sides of the inequality by x:

$$x^2 + 1 \geq 2x$$

$$x^2 - 2x + 1 \geq 0$$

$$(x - 1)^2 \geq 0$$

The left side of this inequality is always positive, so in fact the original inequality holds.
The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Query: Explain why option C is incorrect?

Reply: The expression x + 1/x cannot attain a value of 1.5 for real x. It will always be greater than or equal to 2.
Let us take the expression [root(x)-1/root(x)]^2. (x > 0)

We know that [root(x)-1/root(x)]^2 >= 0

x + 1/x - 2 >= 0

x + 1/x >= 2

So the minimum value of (x + 1/x) is 2. [When x = 1]

53.

We can rephrase the question by manipulating it algebraically:

$(|x^{-1} * y^{-1}|)^{-1} > xy$

$(|1/x * 1/y|)^{-1} > xy$

$(|1/xy|)^{-1} > xy$

$1/(|1/(xy)|) > xy$ Is

$|xy| > xy$?

The question can be rephrased as ―Is the absolute value of xy greater than xy?‖ And since |xy| is never negative, this is only true when xy < 0. If xy > 0 or xy = 0, |xy| = xy. Therefore, this question is really asking whether xy < 0, i.e. whether x and y have opposite signs.

(1) SUFFICIENT: If xy > 1, xy is definitely positive. For xy to be positive, x and y must have the same sign, i.e. they are both positive or both negative. Therefore x and y definitely do not have opposite signs and |xy| is equal to xy, not greater. This is an absolute "no" to the question and therefore sufficient.
(2) INSUFFICIENT: $x^2 > y^2$
Algebraically, this inequality reduces to |x| > |y|. This tells us nothing about the sign of x and y. They could have the same signs or opposite signs.
The correct answer is A: Statement (1) alone is sufficient, but statement (2) alone is not.

The correct answer is A

54.

First, rephrase the question stem by subtracting xy from both sides: Is xy < 0? The question is simply asking if xy is negative.

Statement (1) tells us that $\frac{x^2}{y} < 0$.
Since x^2 must be positive, we know that y must be negative. However this does not provide sufficient information to determine whether or not xy is negative.

Statement (2) can be simplified as follows:

$$x^9\left(y^3\right)^3 < \left(x^2\right)^4\left(y^8\right)$$
$$x^9y^9 < x^8y^8$$
$$(xy)^9 < (xy)^8$$

Statement (2) is true for all negative numbers. However, it is also true for positive fractions. Therefore, statement (2) does not provide sufficient information to determine whether or not xy is positive or negative.

There is also no way to use the fact that y is negative (from statement 1) to eliminate either of the two cases for which statement (2) is true. Statement (2) does not provide any information about x, which is what we would need in order to use both statements in conjunction.

Therefore the answer is (E): Statements (1) and (2) TOGETHER are NOT sufficient.
The correct answer is E.

55.

It would require a lot of tricky work to solve this algebraically, but there is, fortunately, a simpler method: picking numbers.

Since $\frac{w}{x} < \frac{y}{z} < 1$, we can pick values for the unknowns such that this inequality holds true. For example, if w=1, x=2, y=3, and z=4, we get $\frac{1}{2} < \frac{3}{4} < 1$, which is true.

Using these values, we see that

$\frac{x}{w} = \frac{2}{1}$; $\frac{z}{y} = \frac{4}{3}$; $\frac{x^2}{w^2} = \frac{4}{1}$; $\frac{xz}{wy} = \frac{8}{3}$; and $\frac{x+z}{w+y} = \frac{6}{4}$.

Placing the numerical values in order, we get

$$1 < \frac{4}{3} < \frac{6}{4} < \frac{2}{1} < \frac{8}{3} < \frac{4}{1}.$$

We can now substitute the unknowns:

$$1 < \frac{z}{y} < \frac{x+z}{w+y} < \frac{x}{w} < \frac{xz}{wy} < \frac{x^2}{w^2}$$

The correct answer is B.

However, for those who prefer algebra.

We know that $\frac{w}{x} < \frac{y}{z} < 1$. If we take the reciprocal of every term, the inequality signs flip, but the relative order remains the same: $\frac{x}{w} > \frac{z}{y} > 1$, which can also be expressed $1 < \frac{z}{y} < \frac{x}{w}$.

Since both $\frac{z}{y}$ and $\frac{x}{w}$ are greater than 1, $\frac{xz}{wy}$ (i.e. their product) must be greater than either of those terms. Also, since $\frac{z}{y} < \frac{x}{w}$, we can multiply both sides by $\frac{x}{w}$ to get $\frac{xz}{wy} < \frac{x^2}{w^2}$. So we now know that $1 < \frac{z}{y} < \frac{x}{w} < \frac{xz}{wy} < \frac{x^2}{w^2}$. All that remains is to place $\frac{x+z}{w+y}$ in its proper position in the order.

Since $\frac{z}{y} < \frac{x}{w}$, we can multiply both sides by wy to get $wz < xy$; adding yz to both sides yields $(wz + yz) < (xy + yz)$, which can be factored into $z(w+y) < y(x+z)$. If we now divide both sides by $y(w + y)$, we get $\frac{z}{y} < \frac{x+z}{w+y}$.

Since $wz < xy$, we can add wx to both sides to get $wx + wz < wx + xy$, which can be factored into $w(x+z) < x(w+y)$. If we divide both sides by $w(w + y)$, we get $\frac{x+z}{w+y} < \frac{x}{w}$. We can now place $\frac{x+z}{w+y}$ in the order:

$$1 < \frac{z}{y} < \frac{x+z}{w+y} < \frac{x}{w} < \frac{xz}{wy} < \frac{x^2}{w^2}.$$

The correct answer is B.

56.

Since Missile 1's rate increases by a factor of $\sqrt{x}$ every 10 minutes, Missile 1 will be traveling at a speed of x^4 miles per hour after 60 minutes:

minutes	0-10	10-20	20-30	30-40	40-50	50-60	60+
speed	x	$x\sqrt{x}$	x^2	$x^2\sqrt{x}$	x^2	$x^3\sqrt{x}$	x^4

And since Missile 2's rate doubles every 10 minutes, Missile 2 will be traveling at a speed of 2^6y after 60 minutes:

minutes	0-10	10-20	20-30	30-40	40-50	50-60	60+
speed	y	$2y$	2^2y	2^3y	2^4y	2^5y	2^6y

The question then becomes: Is $x^4 > 2^6 y$?

Statement (1) tells us that $x = \sqrt{y}$. Squaring both sides yields $x^2 = y$. We can substitute for y: Is $x^4 > 2^6 x^2$? If we divide both sides by x^2, we get: Is $x^2 > 2^6$? We can further simplify by taking the square root of both sides: Is $x > 2^3$? We still cannot answer this, so statement (1) alone is NOT sufficient to answer the question.

Statement (2) tells us that $x > 8$, which tells us nothing about the relationship between x and y. Statement (2) alone is NOT sufficient to answer the question.

Taking the statements together, we know from statement (1) that the question can be rephrased: Is $x > 2^3$? From statement (2) we know certainly that $x > 8$, which is another way of expressing $x > 2^3$. So using the information from both statements, we can answer definitively that after 1 hour, Missile 1 is traveling faster than Missile 2.

The correct answer is C: Statements (1) and (2) taken together are sufficient to answer the question, but neither statement alone is sufficient.

The correct answer is C.

57.

Simplifying the original expression yields:

$$\frac{8xy^3 + 8x^3y}{1} = \frac{2x^2y^2}{2^{-3}}$$

$$2^{-3}(8xy^3 + 8x^3y) = 1(2x^2y^2)$$

$$\frac{1}{8}(8xy^3 + 8x^3y) = 2x^2y^2$$

$$\frac{8xy^3 + 8x^3y}{8} = 2x^2y^2$$

$$\frac{8(xy^3 + x^3y)}{8} = 2x^2y^2$$

$$xy^3 + x^3y = 2x^2y^2$$

$$xy^3 + x^3y - 2x^2y^2 = 0$$

$$xy(y^2 + x^2 - 2xy) = 0$$

$$xy(y - x)^2 = 0$$

Therefore: xy = 0 or y — x = 0. Our two solutions are: xy = 0 or y = x.
Statement (1) says y > x so y cannot be equal to x. Therefore, xy = 0. Statement (1) is sufficient.

Statement (2) says x < 0. We cannot say whether x = y or xy = 0. Statement (2) is not sufficient.
The correct answer is A.

58.

If (a – b)c < 0, the expression (a – b) and the variable c must have opposite signs. Let's check each answer choice:
(A) UNCERTAIN: If a < b, a – b would be negative. It is possible for a – b to be negative according to the question.
(B) UNCERTAIN: It is possible for c to be negative according to the question.
(C) UNCERTAIN: This means that -1 < c < 1, which is possible according to the question.

(D) FALSE: If we rewrite this expression, we get $ac – bc > 0$. Then, if we factor this, we get: $(a – b)c > 0$. This directly contradicts the information given in the question, which states that $(a – b)c < 0$.
(E) UNCERTAIN: If we factor this expression, we get $(a + b)(a – b) < 0$. This tells us that the expressions $a + b$ and $a – b$ have opposite signs, which is possible according to the question.
The correct answer is D.

59.
If $|ab| > ab$, ab must be negative. If ab were positive the absolute value of ab would equal ab. We can rephrase this question: "Is $ab < 0$?"
I. UNCERTAIN: We know nothing about the sign of b.
II. UNCERTAIN: We know nothing about the sign of a.
III. TRUE: This answers the question directly.

The correct answer is C.

60.
Since $c > 0$ and $d > c$, c and d must be positive. b could be negative or positive. Let's look at each answer choice:
(A) UNCERTAIN: bcd could be greater than zero if b is positive.

(B) UNCERTAIN: $b + cd$ could be less than zero if b is negative and its absolute value is greater than that of cd. For example: $b = -12$, $c = 2$, $d = 5$ yields $-12 + (2)(5) = -2$.

(C) FALSE: Contrary to this expression, $b – cd$ must be negative. We could think of this expression as $b + (-cd)$. cd itself will always be positive, so we are adding a negative number to b. If $b < 0$, the result is negative. If $b > 0$, the result is still negative because a positive b must still be less than cd (remember that $b < c < d$ and b, c and d are integers).

(D) UNCERTAIN: This is possible if b is negative.

(E) UNCERTAIN: This is possible if b is negative.
The correct answer is C.

61.
Let's look at the answer choices one by one:
(A) POSSIBLE: c can be greater than b if a is much bigger than d. For example, if $c = 2$, $b = 1$, $a = 10$ and $d = 3$, ab (10) is still greater than cd (6), despite the fact that $c > b$.
(B) POSSIBLE: The same reasoning from (A) applies.
(C) **IMPOSSIBLE**: Since a, b, c and d are all positive we can cross multiply this fraction to yield $ab < cd$, the opposite of the inequality in the question.
(D) DEFINITE: Since a, b, c and d are all positive, we can cross multiply this fraction to yield $ab > cd$, which is the same inequality as that in the question.
(E) DEFINITE: Since a, b, c and d are all positive, we can simply unsquare both sides of the inequality. We will then have $cd < ab$, which is the same inequality as that in the question.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

When you multiply/divide both parts of an inequality by a positive value you should keep the sign. When you multiply/divide both parts of an inequality by a negative value you should flip the sign. Now, if we multiply both parts of the given inequality by $1/(ac)$ (which according to the stem must be positive), then we'll get: $b/c>d/a$, so option C which says that $b/c < d/a$ cannot be true.
The correct answer is C.

OR
This question can be solved by Testing VALUES. Since there are so many variables, and the answers look a "little crazy", the key is to keep your numbers small and simple.
We're told that (A)(B) > (C)(D) and that A, B, C and D are all POSITIVE. We're asked which of the following CANNOT be true. If we can prove that an answer COULD be true, even once, then we can eliminate it.
Let's TEST VALUES:
Starting with Answer A, is there a way for C to be > B?
IF....
A = 3
B = 2
C = 3
D = 1
Here, C is greater than B. Eliminate Answer A.
We can ALSO eliminate a couple of other answers with this example:
Answer D (3/3 > 1/2) and Answer E (3^2 < 6^2) can be eliminated TOO.
With 2 answers remaining, I'm going to deal with the easier-looking option:
With Answer B, is there a way for D to be > A?
IF...
A = 3
B = 2
C = 1
D = 4
Here, D is greater than A. Eliminate Answer B.
The final answer is the only one left.
The correct answer is C.

62.
We can rephrase the question by subtracting y from both sides of the inequality: Is x > -y?

(1) INSUFFICIENT: If we add y to both sides, we see that x is greater than y. We can use numbers here to show that this does not necessarily mean that x > -y. If x = 4 and y = 3, then it is true that x is also greater than -y. However if x = 4 and y = -5, x is greater than y but it is NOT greater than -y.

(2) INSUFFICIENT: If we factor this inequality, we come up (x + y)(x – y) > 0. For the product of (x + y) and (x – y) to be greater than zero, the must have the same sign, i.e. both negative or both positive. This does not help settle the issue of the sign of x + y.

(1) AND (2) SUFFICIENT: From statement 2 we know that (x + y) and (x – y) must have the same sign, and from statement 1 we know that (x – y) is positive, so it follows that (x + y) must be positive as well.

The correct answer is C.

63.
We can rephrase the question by opening up the absolute value sign. In other words, we must solve all possible scenarios for the inequality, remembering that the absolute value is always a positive value. The two scenarios for the inequality are as follows:

If x > 0, the question becomes: Is x < 1?
If x < 0, the question becomes: Is x > -1?
We can also combine the questions: Is -1 < x < 1?

Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.

(1) INSUFFICIENT: There are three possible equations here if we open up the absolute value signs:

1. If x < -1, the values inside the absolute value symbols on both sides of the equation are negative, so we must multiply each through by -1 (to find its opposite, or positive, value):

$|x + 1| = 2|x - 1| \longrightarrow -(x + 1) = 2(1 - x) \longrightarrow x = 3$
(However, this is invalid since in this scenario, $x < -1$.)

2. If $-1 < x < 1$, the value inside the absolute value symbols on the left side of the equation is positive, but the value on the right side of the equation is negative. Thus, only the value on the right side of the equation must be multiplied by -1:

$|x + 1| = 2|x - 1| \longrightarrow x + 1 = 2(1 - x) \longrightarrow x = 1/3$

3. If $x > 1$, the values inside the absolute value symbols on both sides of the equation are positive. Thus, we can simply remove the absolute value symbols:

$|x + 1| = 2|x - 1| \longrightarrow x + 1 = 2(x - 1) \longrightarrow x = 3$

Thus $x = 1/3$ or 3. While 1/3 is between -1 and 1, 3 is not. Thus, we cannot answer the question.

(2) INSUFFICIENT: There are two possible equations here if we open up the absolute value sign:

1. If $x > 3$, the value inside the absolute value symbols is greater than zero. Thus, we can simply remove the absolute value symbols:

$|x - 3| > 0 \longrightarrow x - 3 > 0 \longrightarrow x > 3$

2. If $x < 3$, the value inside the absolute value symbols is negative, so we must multiply through by -1 (to find its opposite, or positive, value).

$|x - 3| > 0 \longrightarrow 3 - x > 0 \longrightarrow x < 3$

If x is either greater than 3 or less than 3, then x is anything but 3. This does not answer the question as to whether x is between -1 and 1.

(1) AND (2) SUFFICIENT: According to statement (1), x can be 3 or 1/3. According to statement (2), x cannot be 3. Thus using both statements, we know that $x = 1/3$ which IS between -1 and 1.

The correct answer is C.

64.
We can rephrase this question as: "Is a farther away from zero than b, on the number-line?" We can solve this question by picking numbers:

Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.
(1) INSUFFICIENT: Picking values that meet the criteria $b < -a$ demonstrates that this is not enough information to answer the question.

a	b	Is $\lvert a\rvert > \lvert b\rvert$?
2	-5	NO
-5	2	YES

(2) INSUFFICIENT: We have no information about b.
(1) AND (2) INSUFFICIENT: Picking values that meet the criteria $b < -a$ and $a < 0$ demonstrates that this is not enough information to answer the question.

a	b	Is $\lvert a\rvert > \lvert b\rvert$?
-2	-5	NO
-5	2	YES

The correct answer is E.

65.

Since |r| is always positive, we can multiply both sides of the inequality by |r| and rephrase the question as: Is $r^2 <$ |r |? The only way for this to be the case is if r is a nonzero fraction between
-1 and 1.

(1) INSUFFICIENT: This does not tell us whether r is between -1 and 1. If r = -1/2, |r| = 1/2 and $r^2 = 1/4$, and the answer to the rephrased question is YES. However, if r = 4, |r| = 4 and $r^2 = 16$, and the answer to the question is NO.
(2) INSUFFICIENT: This does not tell us whether r is between -1 and 1. If r = 1/2, |r| = 1/2 and $r^2 = 1/4$, and the answer to the rephrased question is YES. However, if r = -4, |r| = 4 and r^2=16, and the answer to the question is NO.
(1) AND (2) SUFFICIENT: Together, the statements tell us that r is between -1 and 1. The square of a proper fraction (positive or negative) will always be smaller than the absolute value of that proper fraction.
The correct answer is C.

66.

One way to solve equations with absolute values is to solve for x over a series of intervals. In each interval of x, the sign of the expressions within each pair of absolute value indicators does not change.

In the equation $|x-2|-|x-3|=|x-5|$, there are 4 intervals of interest:
x < 2: In this interval, the value inside each of the three absolute value expressions is negative.
2 < x < 3: In this interval, the value inside the first absolute value expression is positive, while the value inside the other two absolute value expressions is negative.
3 < x < 5: In this interval, the value inside the first two absolute value expressions is positive, while the value inside the last absolute value expression is negative.
5 < x: In this interval, the value inside each of the three absolute value expressions is positive. Use each interval for x to rewrite the equation so that it can be evaluated without absolute value signs.
For the first interval, x < 2, we can solve the equation by rewriting each of the expressions inside the absolute value signs as negative (and thereby remove the absolute value signs):

$$-x+2-(-x+3)=-x+5$$
$$-x+2+x-3=-x+5$$
$$x=6$$

Notice that the solution x = 6 is NOT a valid solution since it lies outside the interval x < 2. (Remember, we are solving the equation for x SUCH THAT x is within the interval of interest). For the second interval 2 < x < 3, we can solve the equation by rewriting the expression inside the first absolute value sign as positive and by rewriting the expressions inside the other absolute values signs as negative:

$$x-2-(-x+3)=-x+5$$
$$x-2+x-3=-x+5$$
$$3x=10$$
$$x=\frac{10}{3}$$

Notice, again, that the solution $x=\frac{10}{3}$ is NOT a valid solution since it lies outside the interval 2 < x < 3.
For the third interval 3 < x < 5, we can solve the equation by rewriting the expressions inside the first two absolute value signs as positive and by rewriting the expression inside the last absolute value sign as negative:

$$x-2-(x-3)=-x+5$$
$$x-2-x+3=-x+5$$
$$x=4$$

The solution x = 4 is a valid solution since it lies within the interval 3 < x < 5.

Finally, for the fourth interval 5 < x, we can solve the equation by rewriting each of the expressions inside the absolute value signs as positive:

$$x - 2 - (x - 3) = x - 5$$
$$x - 2 - x + 3 = x - 5$$
$$x = 6$$

The solution x = 6 is a valid solution since it lies within the interval 5 < x.
We conclude that the only two solutions of the original equation are x = 4 and x = 6. Only answer choice C contains all of the solutions, both 4 and 6, as part of its set.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

1) Get the checkpoints using the absolute value expressions.
|x-2| tells us that x=2 as one checkpoint
|x-3| tells us that x=3 as another checkpoint
|x-5| tells us that x=5 as last checkpoint

2) Let us test for x where x < 2
|x-2| = |x-5| + |x-3|
-(x-2)= -(x-5) - (x-3)
-x +2 = -x +5 -x +3
x=6 Invalid since x=6 is not x<2

3) Let us test for x where 2 < x < 3
(x-2) = -(x-5) -(x-3)
x-2 = -x+5 -x +3
3x = 10
x = 10/3 Invalid since x=3.3 is not within x = (2,3)

4) Let us test for x where 3 < x < 5
x-2 = x-3 - (x-5)
x-2 = 2
x = 4 Valid

5) Let us test for x where x > 5
x-2 = x-3 + x - 5
-x = -6
x=6 Valid

6) Now let's look for 4 and 6 in the sets of answer choices.
(A) {-6, -5, 0, 1, 7, 8}
(B) {-4, -2, 0, 10/3, 4, 5}
(C) {-4, 0, 1, 4, 5, 6}
(D) {-1, 10/3, 3, 5, 6, 8}
(E) {-2, -1, 1, 3, 4, 5}
Only C has both x=4 and x=6.
The correct answer is C.

67.
Statement (1) tells us that a is either 1 or –1, that b is either 2 or –2, and that c is either 3 or –3. Therefore, we cannot find ONE unique value for the expression in the question.

For example, let b = 2, and c = 3. If a = 1, the expression in the question stem evaluates to (1 + 8 + 27) / (1 × 2 × 3) = 36/6 = 6. However, if a = –1, the expression evaluates to (–1 + 8 + 27) / (–1 × 2 × 3) = 34/(–6) = –17/3. Thus, statement (1) is not sufficient to answer the question.

Statement (2) tells us that a + b + c = 0. Therefore, c = – (a + b). By substituting this value of c into the expression in the question, we can simplify the numerator of the expression as follows:

$$\begin{aligned} a^3+b^3+c^3 &= a^3+b^3+(-(a+b))^3 = a^3+b^3-(a+b)^3 \\ &= a^3+b^3-(a^2+2ab+b^2)(a+b) \\ &= a^3+b^3-(a^3+2a^2b+ab^2+a^2b+2ab^2+b^3) \\ &= a^3+b^3-(a^3+b^3+3a^2b+3ab^2) \\ &= -(3a^2b+3ab^2) = -(a+b)(3ab) = c(3ab) \\ &= 3abc \end{aligned}$$

From this, we can rewrite the expression in the question as $\frac{a^3+b^3+c^3}{abc} = \frac{3abc}{abc} = 3$.

Thus, statement (2) alone is sufficient to solve the expression.

The correct answer is B.

68.

First, rewrite the equation for x by breaking down each of the 8's into its prime components (2^3).

Thus, $x = 2^b - [(2^3)^{30} + (2^3)^5] = 2^b - [2^{90} + 2^{15}]$.

The question asks us to minimize the value of w. Given that w is simply the absolute value of x, the question is asking us to find a value for b that makes the expression $2^b - [2^{90} + 2^{15}]$ as close to 0 as possible. In other words, for what value of b, will 2^b approximately equal 2^{90} + 215.

The important thing to keep in mind is that the expression 2^{90} is so much greater than the expression 2^{15} that the expression 2^{15} is basically a negligible part of the equation.

Therefore, in order for 2^b to approximate $2^{90} + 2^{15}$, the best value for b among the answer choices is 90. It is tempting to select an answer such as 91 to somehow "account" for the 2^{15}. However, consider that $2^{91} = 2 \times 2^{90}$. In other words, 2^{91} is twice as large as 2^{90}!

In contrast, 2^{90} is much closer in value to the expression $2^{90} + 2^{15}$, since 2^{15} does not even come close to doubling the size of 2^{90}.

The correct answer is B.

69.

.

(1) |x - |x^2|| = 2. First of all: $|x^2| = x^2$ (as x^2 is a non-negative value). Square both sides: $(x - x^2)^2 = 4$ --> factor out x: $x^2 * (1-x)^2 = 4$ --> as x is an integer then $x = 2$ or $x = -1$ (by trial and error: the product of two perfect square is 4: 1*4=4 or 4*1=4). Not sufficient.

(2) |x^2 - |x|| = 2 --> square both sides: $(x^2 - |x|)^2 = 4$ --> factor out $|x|$: $x^2 * (|x| - 1)^2 = 4$ --> as x is an integer then $x = 2$ or $x = -2$. Not sufficient.

(1)+(2) Intersection of the values from (1) and (2) is $x = 2$. Sufficient.

The correct answer is C.

Top 1% expert replies to student queries (can skip)

Quicker approach:
Question: x=?
Statement 1: |x - |x^2|| = 2
The For 1), it becomes |x^2|=x^2 -> x-x^2=-2, +2. In x-x^2=-2, +2, x^2-x-2=0, (x-2)(x+1)=0 -> x=2, -1, which is not unique and not sufficient. of the given equation are
x=2 and -1
NOT SUFFICIENT
Statement 2: |x^2 - |x|| = 2

For 2), in x^2-|x|=-2, 2, x^2=|x|^2 is derived. |x|^2-|x|=2, |x|^2-|x|-2=0, (|x|-2)(|x|+1)=0 -> x|=2, -1. -1 is impossible. |x|=2 -> x=-2,2, which is not unique and not sufficient.
The solutions of the given equation are
x=2 and -2
NOT SUFFICIENT
Combining the two statement
X has only one value
x = 2
SUFFICIENT

The correct answer is C.

70.

In complex and abstract Data Sufficiency questions such as this one, the best approach is to break the question down into its component parts.

First, we are told that z > y > x > w, where all the unknowns are integers. Then we are asked whether it is true that $|w| > x^2 > |y| > z^2$. Several conditions must be met in order for this inequality to be true in its entirety:

(1) $|y| > z^2$

(2) $x^2 > |y|$

(3) $|w| > x^2$

In order to answer "definitely yes" to the question, we need to establish that all three of these conditions are true. This is a tall order. But in order to answer "definitely no", we need only establish that ONE of these conditions does NOT hold, since all must be true in order for the entire inequality to hold. This is significantly less work. So the better approach in this case is to see whether the statements allow us to disprove any one of the conditions so that we can answer "definitely no".

But in what circumstances would the conditions not be true?

Let's focus first on condition (1): $|y| > z^2$ Since z > y, the only way for $|y| > z^2$ to be true is if y is negative. If y is positive, z must also be positive (since it is greater than y). And taking the absolute value of positive y does not change the size of y, but squaring z will yield a larger value.
So if y is positive, z^2 must be larger than the absolute value of y.

If you try some combinations of actual values where both y and z are positive and z > y, you will see that $z^2 > |y|$ is always true and that $|y| > z^2$ is never true. For example, if z = 3 and y = 2, then $3^2 > |2|$ is true because $z^2 > |y|$. But if z = 3 and y = -10, then $|-10| > 3^2$ is true then $|y| > z^2$. The validity of depends on the specific values (for example, it would not hold true if z = 3 and y = -1), but the only way for $|y| > z^2$ to be true is if y is negative.

And if y must be negative, then x and w must be negative as well, since y > x > w. So if we could establish that any ONE of y, x, or w is positive, we would know that $|y| > z^2$ is NOT true and that the answer to the question must be "no".

Statement (1) tells us that wx > yz. Does this statement allow us to determine whether y is positive or negative? No. Why not? Consider the following:

If z = 1, y = 2, x = -3, and w = -4, then it is true that wx > yz, since (-4)(-3) > (2)(1).

But if z = 1, y = -2, x = -3, and w = -4, then it is also true that wx > yz, since (-4)(-3) > (-2)(1).

In the first case, y is positive and the statement holds true. In the second case, y is negative and the statement still holds true. This is not sufficient to tell us whether y is positive or negative.

Statement (2) tells us that zx > wy. Does this statement allow us to determine whether y is positive or negative? Yes. Why? Consider the following:

If z = 4, y = 3, x = 2, and w = 1, then it is true that zx > wy, since (4)(2) > (1)(3). If z = 3, y =

2, x = 1, and w = -1, then it is true that zx > wy, since (3)(1) > (-1)(2).

If z = 2, y = 1, x = -1, and w = -3, then it is true that zx > wy, since (2)(-1) > (-3)(1).

In all of the cases above, y is positive. But if we try to make y a negative number, zx > wy cannot hold. If y is negative, then x and w must also be negative, but z can be either negative or positive, since z > y > x > w. If y is negative and z is positive, zx > wy cannot hold because zx will be negative (pos times neg) while wy will be positive (neg times neg). If z is negative, then all the unknowns must be negative. But if they are all negative, it is not possible that zx >
wy. Since z > y and x > w, the product zx would be less than wy. Consider the following:

If z = -1, y = -2, x = -3, and w = -4, then zx > wy is NOT true, since (-1)(-2) is NOT greater than (-4)(-3).

Since y is positive in every case where zx > wy is true, y must be positive. If y is positive, then $|y| > z^2$ cannot be true. If $|y| > z^2$ cannot be true, then $|w| > x^2 > |y| > z^2$ cannot be true and we can answer "definitely no" to the question.

Statement (2) is sufficient.

The correct answer is B: Statement (2) alone is sufficient but statement (1) alone is not.

The correct answer is B.

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Plot the number line thus,
<----w-------x-------y--------z------> and read the question. Basically the question asks where is the 0 on this number line with respect of y?
Because |y| > z^2 only if y<0.
After that we can get options that state definitely where the zero is. We can choose it without knowing the traditional solution. So we can look at the options do a little imagination of the following sort :
<----- (-6) --- (-3) ---- (-2) ---- 0 ----(2)----(3) -----(6)------>

1) wx>yz : This statement tells me (i) <------(w)---(x)--(0)---(y)--(z)--> (kind of to scale)
or (ii) <-----(w)---(x)-----(y)---0----(z)--------->
Therefore, INSUFFICIENT.

2) zx>wy: (i)<---------(w)------(x)-------(0)-------(y)------(z)
and we know that B is sufficient to tell me where the zero is.

The correct answer is B.

Alternate Solution from Gmatclub

w, x, y, and z are integers. If z > y > x > w, is |w| > x^2 > |y| > z^2?

Given: w, x, y, and z are integers and $z > y > x > w$: --W--X--y--Z--
Question: is $|w| > x^2 > |y| > z^2$?

Now, since $z > y$, then in order $|y| > z^2$ to hold true y must be negative and since $y > x > w$, then x and w must be negative. So in order the answer to be YES, y, x and w must be negative (notice that the reverse is not always true: they might be negative but the answer could be NO, **but if answer is YES then they must be negative**). So if we get that either of them is not negative then we could conclude that the answer to the question is NO.

(1) wx > yz --> the product of two smaller integers (w and x) is more than the product of to larger integers (y and z). It's possible for example that all but z are negative but even in this case we could have two different answers:
If $(z=1) > (y=-2) > (x=-3) > (w=-4)$ --> in this case answer to the question "is $|w| > x^2 > |y| > z^2$?" will be NO;
If $(z=1) > (y=-2) > (x=-3) > (w=-10)$ --> in this case answer to the question "is $|w| > x^2 > |y| > z^2$?" will be YES.
Not sufficient.

(2) zx > wy --> y is positive, because if it's negative then x and w are also negative (since $y > x > w$) and in this case no matter whether z is positive or negative: $zx < wy$ (if z>0 then zx<0<wy and if z<0 then the product of two "more negative" numbers w and y will be more then the product of two "less negative" numbers z and x, so again we would have: zx<wy). Thus we have that y must be positive, which makes $|w| > x^2 > |y| > z^2$ impossible (as discussed), hence the answer to the question is NO. Sufficient,

Answer: B.

The correct answer is B.

71.

The first step we need to take is to simplify the left side of the inequality:

$$\left(\frac{a-b}{a^{-1}+b^{-1}}\right)^{-1} \rightarrow$$
$$\frac{a^{-1}+b^{-1}}{a-b} \rightarrow$$
$$\frac{\left(\frac{1}{a}+\frac{1}{b}\right)}{(a-b)} \rightarrow$$
$$\frac{\left(\frac{a+b}{ab}\right)}{(a-b)} \rightarrow$$
$$\frac{(a+b)}{(ab)(a-b)}$$

We can now rephrase the question as "Is $\frac{(a+b)}{(ab)(a-b)} > (a+b)$?"

Statement (1) tells us that the absolute value of a is greater than the absolute value of b. Immediately we need to consider whether different sets of values for a and b would yield different answers.

Since the question deals with absolute value and inequalities, it is wise to select values to cover multiple bases. That is, choose sets of values to take into account different combinations of positive and negative, fraction and integer, for example.

Let's first assume that a and b are positive integers. Let a equal 4 and b equal 2, since the absolute value of a must be greater than that of b. If we plug these values into the inequality, we get 3/8 on the left and 6 on the right, yielding an answer of "no" to the question.

Now let's assume that a and b are negative integers. Let a equal -4 and b equal -2, since the absolute value of a must be greater than that of b. If we plug these values into the inequality, we get 3/8 on the left and -6 on the right, yielding an answer of "yes" to the question.

Since statement (1) yields both "yes" and "no" depending on the values chosen for a and b, it is insufficient.

Statement (2) tells us that a is less than b. Again, we should consider whether different sets of values for a and b would yield different answers.

Let's assume that a and b are negative integers. Let a equal -4 and b equal -2, since a must be less than b. If we plug these values into the inequality, we get 3/8 on the left and -6 on the right, yielding an answer of "yes" to the question.

Now let's assume that a is a negative fraction and that b is a positive fraction. Let a equal -1/2 and b equal 1/5. If we plug these values into the inequality, we get 30/7 on the left and on the right we get -3/10, yielding an answer of "no" to the question.

Do not forget that if a question does not specify that an unknown is an integer you CANNOT assume that it is. In fact, you must ask yourself whether the distinction between integer and fraction makes any difference in the question.

Since statement (2) yields both "yes" and "no" depending on the values chosen for a and b, it is insufficient.

Now we must consider the information from the statements taken together. From both statements, we know that the absolute value of a is greater than that of b and that a is less than b. If a equals -4 and b equals -2, both statements are satisfied and we can answer "yes" to the question. However, if a equals -1/2 and b equals 1/5, both statements are also satisfied but we can answer "no" to the question.

Even pooling the information from both statements, the question can be answered either "yes" or "no" depending on the values chosen for a and b. The statements in combination are therefore insufficient.

The correct answer is E: Statements (1) and (2) together are not sufficient.

The correct answer is E.

72.

For $|a| + |b| > |a + b|$ to be true, a and b must have opposite signs. If a and b have the same signs (i.e. both positive or both negative), the expressions on either side of the inequality will be the same. The question is really asking if a and b have opposite signs.

(1) INSUFFICIENT: This tells us that $|a| > |b|$. This implies nothing about the signs of a and b.

(2) INSUFFICIENT: Since the absolute value of a is always positive, this tells us that $b < 0$. Since we don't know the sign of a, we can't answer the question.

(1) AND (2) INSUFFICIENT: We know the sign of b from statement 2 but statement 1 does not tell us the sign of a. For example, if $b = -4$, a could be 5 or -5.

The correct answer is E

73.

In order to answer the question, Is $\sqrt{x}$ a prime number? we must first solve for x.

The key to solving this week's problem is understanding that certain types of equations have more than one solution.

One such equation type, is an equation that involves absolute value, like the equation in the first statement. Let's solve for x in statement one.

The first solution to an absolute value equation assumes that the expression inside the brackets yields a positive result (and therefore the absolute value brackets do not actually change the sign of this expression).

$$|3x-7| = 2x+2$$
$$3x-7 = 2x+2$$
$$x-7 = 2$$
$$x = 9$$

Notice that in the "positive" version of the absolute value equation, we simply remove the absolute value brackets with no change to the expression inside.

The second solution to an absolute value equation assumes that the expression inside the brackets yields a negative result (and therefore the absolute value brackets DO change the sign of this expression from negative to positive).

$$|3x-7| = 2x+2$$
$$-3x+7 = 2x+2$$
$$7 = 5x+2$$
$$5 = 5x$$
$$1 = x$$

Notice that in the "negative" version of the absolute value equation, when we remove the absolute value brackets, we must reverse the sign of all the terms inside.

Thus, according to statement 1, x may be 1 or 9. From this, we know that $\sqrt{x}$ must be 1 or 3. Since 1 is not a prime number, but 3 is a prime number, it is NOT possible to answer the original question using statement one, alone.

Now let's analyze statement two ($x^2 = 9x$) alone. Although it may be tempting to simply divide both sides of this equation by x, and find that x = 9, this neglects an important second solution.

We must first realize that statement two is a quadratic equation. Generally, quadratic equations have two solutions. To see this, let's rewrite this equation in quadratic form and then factor as follows:

$$x^2 = 9x$$
$$x^2 - 9x = 0$$
$$x(x-9) = 0$$

Factoring produces two solutions:
Either x = 0 or x – 9 = 0. Thus, x can be 0 or 9.

Thus, according to statement two, x may be 0 or 9. From this, we know that $\sqrt{x}$ must be 0 or 3. Since 0 is not a prime number, but 3 is a prime number, it is NOT possible to answer the original question using statement two, alone.

Now let's analyze both statements together. From statement one, we know that x must be 1 or 9. From statement two, we know that x must be 0 or 9. Thus, since both statements must be true, we can deduce that x must be 9.

Therefore $\sqrt{x} = 3$, which is a prime number. Using both statements, we can answer the question in the affirmative: Yes, $\sqrt{x}$ is a prime number.

Since BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

The correct answer is C.

74.

The question asks for the average of x and |y|. Taking the absolute value of a number has no effect if that number is positive; on the other hand, taking the absolute value of a negative number changes the sign to positive. The most straightforward way to approach this question is to test positive and negative values for y.

(1) INSUFFICIENT: We know that the sum of x and y is 20. Here are two possible scenarios, yielding different answers to the question:

x	y	Sum	Average of x and |y|
10	10	20	10
25	–5	20	15

(2) INSUFFICIENT: We know that |x + y| = 20. The same scenarios listed for statement (1) still apply here. There is more than one possible value for the average of x and |y|,
(1) AND (2) INSUFFICIENT: We still have the same scenarios listed above. Since there is more than one possible value for the average of x and |y|, both statements taken together are NOT sufficient.
The correct answer is E.

75.
First, let's simplify the question:

$$\left(x^{-1}+y^{-1}\right)^{-1} > \left(\left(x^{-1}\right)\left(y^{-1}\right)\right)^{-1}$$

$$\left(\frac{1}{x}+\frac{1}{y}\right)^{-1} > \left(\left(\frac{1}{x}\right)\left(\frac{1}{y}\right)\right)^{-1}$$

$$\left(\frac{x+y}{xy}\right)^{-1} > \left(\frac{1}{xy}\right)^{-1}$$

$$\text{Is } \frac{xy}{x+y} > xy\ ?$$

(1) SUFFICIENT: If we plug x = 2y into our simplified question we get the following:
Is $2y^2/3y > 2y^2$? Since $2y^2$ must be positive we can divide both sides of the inequality by $2y^2$ which leaves us with the following: Is 1/3y > 1? If we investigate this carefully, we find that if y is a nonzero integer, 1/3y is never greater than 1. Try y = 2 and y = -2, In both cases 1/3y is less than 1.
(2) INSUFFICIENT: Lets plug in values to investigate this statement. According to this statement, the x and y values we choose must have a positive sum. Lets choose a set of values that will yield a positive xy and a set of values that will yield a negative xy.

x	y	
3	1	xy/(x + y) < xy
3	-1	xy/(x + y) > xy

This not does yield a definitive yes or no answer so statement (2) is not sufficient.
The correct answer is A.

76.
The question can first be rewritten as Is p(pq) > q(pq)?

If pq is positive, we can divide both sides of the inequality by pq and the question then becomes:
Is p > q?
If pq is negative, we can divide both sides of the inequality by pq and change the direction of the inequality sign and the question becomes: Is p < q?

Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.

(1) INSUFFICIENT: Knowing that pq < 0 means that the question becomes Is p < q? We know that p and q have opposite signs, but we don't know which one is positive and which one is negative so we can't answer the question Is p < q?

(2) INSUFFICIENT: We know nothing about q or its sign
(1) AND (2) SUFFICIENT: From statement (1), we know we are dealing with the question —
Is p < q?, and that p and q have opposite signs. Statement (2) tells us that p is negative, which means that q is positive. Therefore p is in fact less than q.
The correct answer is C.

77.
We can rephrase the question: "Is m – n > 0?"
(1) INSUFFICIENT: If we solve this inequality for m – n, we get m – n < 2. This does not answer the question "Is m – n > 0?".

(2) SUFFICIENT: If we solve this inequality for m – n, we get $m - n < -2$. This answers the question "Is $m - n > 0$?" with an absolute NO.
The correct answer is B.

78.
Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.

(1) INSUFFICIENT: We can substitute 2p for q in the inequality in the question: $3^p > 2^{2p}$. This can be simplified to $3^p > (2^2)^p$ or $3^p > 4^p$.

If $p > 0$, $3^p < 4^p$ (for example $3^2 < 4^2$ and $3^{0.5} < 4^{0.5}$) If $p < 0$, $3^p > 4^p$ (for example $3^{-1} > 4^{-1}$)
Since we don't know whether p is positive or negative, we cannot tell whether 3^p is greater than 4^p.
(2) INSUFFICIENT: This tells us nothing about p.
(1) AND (2) SUFFICIENT: If $q > 0$, then p is also greater than zero since $p = 2q$. If $p > 0$, then $3^p < 4^p$. The answer to the question is a definite NO.
The correct answer is C.

79.
To begin, list all of the scenarios in which mp would be greater than m. There are only 2 scenarios in which this would occur.
Scenario 1: m is positive and p is greater than 1 (since a fractional or negative p will shrink m). Scenario 2: m is negative and p is less than 1 -- in other words, p can be a positive fraction, 0 or any negative number. A negative value for p will make the product positive, 0 will make it 0 and a positive fraction will make a negative m greater).

NOTE: These scenarios could have been derived algebraically by solving the inequality $mp > m$: $mp - m > 0$
$m(p - 1) > 0$
Which means either $m > 0$ and $p > 1$ OR $m < 0$ and $p < 1$.
(1) INSUFFICIENT: This eliminates the second scenario, but doesn't guarantee the first scenario. If $m = 100$ and $p = .5$, then $mp = 50$, which is NOT greater than m. On the other hand, if $m = 100$ and $p = 2$, then $mp = 200$, which IS greater than m.

(2) INSUFFICIENT: This eliminates the first scenario since p is less than 1, but it doesn't guarantee the second scenario. m has to be negative for this to always be true. If $m = –100$ and $p = –2$, then $mp = 200$, which IS greater than m. But if $m = 100$ and $p = .5$, then $mp = 50$, which is NOT greater than m.

(1) AND (2) SUFFICIENT: Looking at statements (1) and (2) together, we know that m is positive and that p is less than 1. This contradicts the first and second scenarios, therebyensuring that mp will NEVER be greater than m. Thus, both statements together are sufficient to answer the question. Note that the answer to the question is "No" -- which is a definite, and therefore sufficient, answer to a "Yes/No" question in Data Sufficiency.
The correct answer is C.

80.
In order to answer the question, we must compare w and y.
(1) INSUFFICIENT: This provides no information about y.
(2) INSUFFICIENT: This provides no information about w.
(1) AND (2) INSUFFICIENT: Looking at both statements together, it is possible that w could be less than y. For example w could be 1.305 and y could be 100. It is also possible that w could be greater than y. For example, w could be 1.310 and y could be 1.305. Thus, it is not possible to determine definitively whether w is less than y.
The correct answer is E.

81.

Let us start be examining the conditions necessary for $|a|b > 0$. Since $|a|$ cannot be negative, both $|a|$ and b must be positive. However, since $|a|$ is positive whether a is negative or positive, the only condition for a is that it must be non-zero.

Hence, the question can be restated in terms of the necessary conditions for it to be answered "yes":

"Do both of the following conditions exist: *a* is non-zero AND *b* is positive?"
(1) INSUFFICIENT: In order for $a = 0$, $|a^b|$ would have to equal 0 since 0 raised to any power is always 0. Therefore (1) implies that a is non-zero. However, given that a is non-zero, b can be anything for $|a^b| > 0$ so we cannot determine the sign of b.

(2) INSUFFICIENT: If $a = 0$, $|a| = 0$, and $|a|^b = 0$ for any b. Hence, a must be non-zero and the first condition (a is not equal to 0) of the restated question is met. We now need to test whether the second condition is met. (Note: If a had been zero, we would have been able to conclude right away that (2) is sufficient because we would answer "no" to the question is $|a|b > 0$?) Given that a is non-zero, $|a|$ must be positive integer. At first glance, it seems that b must be positive because a positive integer raised to a negative integer is typically fractional (e.g., $a^{-2} = 1/a^2$). Hence, it appears that b cannot be negative. However, there is a special case where this is not so:

If $|a| = 1$, then b could be anything (positive, negative, or zero) since $|1|^b$ is always equal to 1, which is a non-zero integer . In addition, there is also the possibility that $b = 0$. If $|b| = 0$, then $|a|^0$ is always 1, which is a non-zero integer.

Hence, based on (2) alone, we cannot determine whether b is positive and we cannot answer the question.

An alternative way to analyze this (or to confirm the above) is to create a chart using simple numbers as follows:

a	*b*	Is $|a|^b$ a non-zero integer?	Is $|a|b > 0$?
1	2	Yes	Yes
1	-2	Yes	No
2	1	Yes	Yes
2	0	Yes	No

(Continued on next page)

We can quickly confirm that (2) alone does not provide enough information to answer the question.

(1) AND (2) INSUFFICIENT: The analysis for (2) shows that (2) is insufficient because, while we can conclude that a is non-zero, we cannot determine whether b is positive. (1) also implies that a is non-zero, but does not provide any information about b other than that it could be anything. Consequently, (1) does not add any information to (2) regarding b to help answer the question and (1) and (2) together are still insufficient. (Note: As a quick check, the above chart can also be used to analyze (1) and (2) together since all of the values in column 1 are also consistent with (1)).
The correct answer is E.

82.

450<x<550, 350<y<450
Combined 450<X<500 and 350<y<400, we know that 800<x+y<900, but we still don't know which multiple of 100 is closest to x+y.
The correct answer is E.

83:
From 1, 30=1*2*3*5, the three digits could be 1/6/5 or 2/3/5. So, the number could be 651, which is greater than 550. Insufficient.
From 2, sum is 10, three digits only could be 2, 3, and 5.
Combined 1 and 2, we can know that the number must less than 550.
The correct answer is C.

84:
The <u>GREATEST possible value</u> of x would be 3. x can be 2,1,0,-1,-2 etc. but the **greatest value** of x would still be **3**
X^4+Y^4=100 ==> x^4<100 => x^2<10 => 3<X<6
The correct answer is B.

85.

(1) $2x-3y=-2$ --> question becomes: is $-2<x^2$? as square of a number is always non-negative ($x^2\geq 0$) then $x^2\geq 0>-2$. Sufficient.

(2) $x>2$ and $y>0$ --> is $2x-3y<x^2$ --> is $x(x-2)+3y>0$ --> as $x>2$ then $x(x-2)$ is a positive number and as $y>0$ then $3y$ is also a positive number --> sum of two positive numbers is more than zero, hence $x(x-2)+3y>0$ is true. Sufficient.

Answer: D.

The correct answer is D.

86:
(1)+(2), we can know that z>0, then, m>3z>0
Together, m+z>0
The correct answer is C.

87.
Please notice that it says "could be true", not "must be true"
I. X=1,Y=1/2,Z=1/3, can fulfill X > Y > Z and X > Y^2 > Z^4
II. Z=1/2,Y=1/3,X=1/4, can fulfill Z > Y > X and X > Y^2 > Z^4
III.X=1,Z=1/2,Y=1/3, can fulfill X > Z > Y and X > Y^2 > Z^4
The correct answer is E

.

88.
Since if
1) $x < 8/9$
2) $Y < 1/8$
$x+y$ could be >1, $=1$ or <1.
The correct answer is E.

89.
For 1, $x<0$, $x+|x|=0$
For 2, $y<1$, noticed that y is an integer, y only can be 0
The correct answer is D.

90.
$x-y+1-(x+y-1)=2-2y$, we just need to know the situation of y.
From statement 2, we know that $y<0$, so, $2-2y>0$
The correct answer is B.

91.
1. $w>-2$, insufficient.
2. $w>1$ or $w<-1$, insufficient.

1+2, $w>1$ or $-2<w<-1$, still insufficient.
The correct answer is E.

92.
From 1, $n+1>0$, $n>-1$. n is an integer, so, $n>=0$
From 2, $np>0$.
Combined 1 and 2, $p>0$
The correct answer is C.

93.
1). $3.5<x+y<4.5$
2). $0.5<x-y<1.5$
Combined 1 and 2, $4<2x<6 \Rightarrow 2<x<3$. We know that x is not an integer, then, we cannot determine the specify value of x.
The correct answer is E.

GMAT Quant Topic 4: Numbers
Part A: Types of Numbers

1.
We cannot rephrase the given question so we will proceed directly to the statements.
(1) INSUFFICIENT: n could be divisible by any square of a prime number, e.g. 4 (2^2), 9 (3^2), 25 (5^2), etc.
(2) INSUFFICIENT: This gives us no information about n. It is not established that y is an integer, so n could be many different values.
(1) AND (2) SUFFICIENT: We know that y is a prime number. We also know that y^4 is a two-digit odd number. The only prime number that yields a two-digit *odd* integer when raised to the fourth power is 3: $3^4 = 81$. Thus $y = 3$.
We also know that n is divisible by the square of y or 9. So n is divisible by 9 and is less than 99, so n could be 18, 27, 36, 45, 54, 63, 72, 81, or 90. We do not know which number n is but we do know that all of these two-digit numbers have digits that sum to 9.
The correct answer is C.

2.
There is no obvious way to rephrase this question. Note that $x!$ is divisible by all integers up to and including x; likewise, $x! + x$ is definitely divisible by x. However, it's impossible to know anything about $x! + x + 1$. Therefore, the best approach will be to test numbers. Note that since the question is Yes/No, all you need to do to prove insufficiency is to find one Yes and one No.
(1) INSUFFICIENT: Statement (1) says that $x < 10$, so first we'll consider $x = 2$.
$2! + (2 + 1) = 5$, which is prime.

Now consider $x = 3$.
$3! + (3 + 1) = 6 + (3 + 1) = 10$, which is not prime.

Since we found one value that says it's prime, and one that says it's not prime, statement (1) is NOT sufficient.

(2) INSUFFICIENT: Statement (2) says that x is even, so let's again consider $x = 2$:
$2! + (2 + 1) = 5$, which is prime.

Now consider $x = 8$:
$8! + (8 + 1) = (8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 9$.
This expression must be divisible by 3, since both of its terms are divisible by 3.
Therefore, it is not a prime number.

Since we found one case that gives a prime and one case that gives a non-prime, statement (2) is NOT sufficient.

(1) and (2) INSUFFICIENT: since the number 2 gives a prime, and the number 8 gives a non-prime, both statements taken together are still insufficient. **The correct answer is E.**

3.
When we take the square root of any number, the result will be an integer only if the original number is a perfect square. Therefore, in order for $\sqrt{x+y}$ to be an integer, the quantity $x + y$ must be a perfect square. We can rephrase the question as "Is $x + y$ a perfect square?"
(1) INSUFFICIENT: If $x^3 = 64$, then we take the cube root of 64 to determine that x must equal 4. This tells us nothing about y, so we cannot determine whether $x + y$ is a perfect square.
(2) INSUFFICIENT: If $x^2 = y - 3$, then we can rearrange to $x^2 - y = -3$. There is no way to rearrange this equation to get $x + y$ on one side, nor is there a way to find x and y separately, since we have just one equation with two variables.
(1) AND (2) SUFFICIENT: Statement (1) tells us that $x = 4$. We can substitute this into the equation given in statement two: $4^2 = y - 3$. Now, we can solve for y. $16 = y - 3$, therefore $y = 19$. $x + y = 4 + 19 = 23$. The quantity $x + y$ is not a perfect square. Recall that "no" is a definitive answer; it is sufficient to answer the question.
The correct answer is C.

4.
(1) INSUFFICIENT: Start by listing the cubes of some positive integers: 1, 8, 27, 64, 125. If we set each of these equal to $2x + 2$, we see that we can find more than one value for x which is prime. For example, $x = 3$ yields $2x + 2 = 8$ and $x = 31$ yields $2x + 2 = 64$. With at least two possible values for x, the statement is insufficient.

(2) INSUFFICIENT: In a set of consecutive integers, the mean is always equal to the median. When there are an odd number of members in a consecutive set, the mean/median will be a member of the set and thus an integer (e.g. 5,6,7,8,9; mean/median = 7). In contrast when there are an even number of members in the set, the mean/median will NOT be a member of the set and thus NOT an integer (e.g. 5,6,7,8; mean/median = 6.5). Statement (2) tells us that we are dealing with an integer mean; therefore x, the number of members in the set, must be odd. This is not sufficient to give us a specific value for the prime number x.

(1) AND (2) INSUFFICIENT: The two x values that we came up with for statement (1) also satisfy the conditions of statement (2).

The correct answer is E.

5.
The least number in the list is -4, so, the list contains -4,-3,-2,-1, 0, 1, 2, 3, 4, 5, 6, 7.
So, the range of the positive integers is 7-1=6.
Answer is 6

6.
1). m/y=x/r, the information is insufficient to determine whether m/r=x/y or not.
2). (m+x)/(r+y)=x/y=>(m+x)*y=(r+y)*x=>my=rx=> m/r=x/y, sufficient.
The correct answer is B.

7.
8!=1*2*3*4*5*6*7*8=2^7*3^2*5*7
From 1, a^n=64, where 64 could be 8^2, 4^3, 2^6, a could be 8, 4, and 2, insufficient.
From 2, n=6, only 2^6 could be a factor of 8!, sufficient.
The correct answer is B.

8. Since x is the sum of six consecutive integers, it can be written as:
x = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5)
x = 6n + 15

Note that x must be odd since it is the sum of the even term 6n and the odd term 15, and an even plus an odd gives an odd.

I. TRUE: Since 6n and 15 are both divisible by 3, x is divisible by 3.

II. FALSE: Since x is odd, it CANNOT be divisible by 4.

III. FALSE: Since x is odd, it CANNOT be divisible by 6.

The correct answer is A.

9.
If the least number was 3, then 3*4=12<15, does not fulfil the requirement. So, the least number is 4.
If the greatest number is 14, then 14*15=210>200, does not fulfil the requirement. So, answer is 4 and 13
The correct answer is C.

10.
From statement 1, p/4=n, n is prime number, could be 2, 3, 5, 7,11, insufficient.
From statement 2, p/3=n, n is an integer, could be 1, 2, 3, 4, 5, … insufficient.
Combined 1 and 2, only when p=12 can fulfil the requirement
The correct answer is C.

11.
1^2+5^2+7^2=75, so the sum of there 3 integers is 13.

The correct answer is E.

Top 1% expert replies to student queries (can skip)

List down all perfect squares less than 75: 1, 4, 9, 16, 25, 36, 49, 64

75 will be the sum of 3 of these 8 numbers. Also, we can notice that as 75 is an odd number then either all three numbers must be odd (odd+odd+odd=odd) OR two must be even and one odd (even+even+odd=odd).

Then, using trial & error, 75 equals to 1+25+49=1^2+5^2+7^2=75 --> 1+5+7=13
The correct answer is E.

12.
Let tens digit of n be x, units digit could be kx
Then, n=10x+kx=x(10+k)
n>20, then x>2, n contains at least two nonzero factors x and 10+k. Statement 2 alone is insufficient.
The correct answer is A.

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This is a fairly simple problem to attack.

n is > 20 (given)

If we take St 1:

The tens digit can be 2, and then the units digit can be 2, 4, 6, 8 -> Numbers can be 22, 24, 26, 28
The tens digit can be 3, and then the units digit can be 3, 6, 9 -> Numbers can be 33, 36, 39
The tens digit can be 4, and then the units digit can be 4, 8 -> Numbers can be 44, 48
The tens digit can be 5, 6, 7 8 or 9, and then the units digit can only be 5, 6, 7, 8 or 9 respectively (as the second multiple of these numbers themselves are two-digit numbers and cannot be in the units digit) -> Numbers can be 55, 66, 77, 88, 99 respectively

You can see in this exhaustive list above that all numbers are composite -> This condition is sufficient

If we take St 2:

Tens digit is 2, so units digit can be 4 as an example (number is 24) or 3 as an example (number is 23). 24 is not prime, but 23 is -> This condition is not sufficient

The correct answer is A.

13.
p^2q is a multiple of 5, only can ensure that pq is a multiple of 5.
So, only (pq)^2 can surely be a multiple of 25.
The correct answer is D.

14.

From 2, |t-r|=|t-(-s)|=|t+s|.
From 1, we know that s>0, so t+s>0; t is to the right of r, so t-r>0. Combine 1 and 2, t+s=t-r=>s=-r=>s+r=0. Zero is halfway between r and s
The correct answer is C.

15.
The first 10 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
An easy way to add these numbers is as follows:
(29 + 11) + (23 + 7) + (17 + 13) + (2 + 5 + 3) + 19 = 40 + 30 + 30 + 10 + 19 = 129.
The correct answer is D.

16.
1/5, 2/5, 3/5, 4/5=>7/35, 14/35, 21/35, 28/35
1/7, 2/7, 3/7, 4/7, 5/7, 6/7=>5/35, 10/35, 15/ 35, 20/35, 25/35, 30/35
It is easy to find that the least distance between any two of the marks is 1/35
The correct answer is B.

17.

In order for ab/cd to be positive, ab and cd must share the same sign; that is, both either positive or negative.There are two sets of possibilities for achieving this sufficiency. First, if all four integers share the same sign- positive or negative- both ab and cd would be positive. Second, if any two of the four integers are positive while the other two are negative, ab and cd must share the same sign. The following table verifies this claim:

Positive Pair	Negative Pair	ab Sign	cd Sign
a, b	c, d	+	+
a, c	b, d	-	-
a, d	b, c	-	-
b, c	a, d	-	-
b, d	a, c	-	-
c, d	a, b	+	+

For the first and last cases, ab/cd will be positive. On the other hand, it can be shown that if only one of the four integers is positive and the other three negative, or vice versa, ab/cd must be negative. This question can most tidily be rephrased as —Among the integers a, b, c and d, are an even number (zero, two, or all four) of the integers positive?

(1) SUFFICIENT: This statement can be rephrased as ad = -bc. For the signs of ad and bc to be opposite one another, either precisely one or three of the four integers must be negative. The answer to our rephrased question is "no," and, therefore, we have achieved sufficiency.

(2) SUFFICIENT: For the product abcd to be negative, either precisely one or three of the four integers must be negative. The answer to our rephrased question is "no," and, therefore, we have achieved sufficiency.

The correct answer is D.

18.

1) J = 2 * 3 * 5 => J has 3 different prime factors, insufficient.
2) K = 2^3 * 5^3 => K has 2 different prime factors, insufficient. 1 + 2
=> J has more different prime factors

The correct answer is C.

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Question basically asks: is the # of distinct prime factors of j more than the # of distinct prime factors of k?
(1) j is divisible by 30 --> j=30*n=(2*3*5)*n --> j has at least three distinct prime factors 2, 3, and 5. Not sufficient as no info about k.
(2) k = 1000 --> k=1,000=2^3*5^3 --> k has exactly two distinct prime factors 2 and 5. Not sufficient as no info about j.
(1)+(2) The # of distinct prime factors of j, which is at least 3, is more than the # of distinct prime factors of k, which is exactly 2. Sufficient.
The correct answer is C.

19.

990=11*9*5*2, where 11 is a prime number. So, to guarantee that the produce will be a multiple of 990, the least possible value of n is 11
The correct answer is B.

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19. n → +ve int

⇒ 1 . 2 . 3 . . . . . n = 990 k

⇒ n! = 990 k

⇒ n! = 9 × 11 × 10 k ⇒ $2^1 \times 3^2 \times 5^1 \times 11^1 \times k$

⇒ (You have to make Right hand side equal to some factorial of a Number 'n').

∵ you see an '11' on the right, you are sure that n should be atleast 11 (How can you get 11 in a factorial ? when n is atleast 11).

So least possible value of n = 11.

20.

I have found any shortcut to solve such question. So, we must try it one by one.

E:F(x)=-3x, then F(a)=-3a,f(b)=-3b,f(a+b)=-3(a+b)=-3a-3b=F(a)+f(b)

The correct answer is E.

21.

A............9............C............8............D--1--B

A--1--D...8...............C...............9...............B

Both the two situations can fulfill the requirements.

The correct answer is E.

22.

Statement 1: the greatest number*the least number is positive, means all the numbers should be positive or negative.

All are positive integers, the product of all integers is positive.

All are negative integers, we need to know even or odd the number of the integers is. From 2, we know the number of the integers is even. Thus, the product is positive.

The correct answer is C.

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As per statement 1, we can't have 'zero' in the set, as the product of greatest and smallest of the integers is positive: Either all are positive or all are negative.

(1) The product of the greatest and smallest of the integers in the list is positive.

Two cases:

A. all integers in the list are positive: in this case product of all integers would be positive;

OR

B. all integers in the list are negative: now, if there is even number of integers, then product of all integers would be positive BUT if there is odd number of integers, then product of all integers would be negative. Not sufficient.

(2) There is an even number of integers in the list.

Clearly insufficient. {-2, 2} - answer NO; {2,4} - answer YES.

(1)+(2) Now if we have scenario A (from 1) then the answer is YES. If we have scenario B, then as there are even numbers of integers (from 2) the product of all integers still would be positive, so the answer is still YES. Sufficient.

The correct answer is C.

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Combine stat 1 & 2: Please refer to below image

(I) product of smallest (S) & largest (L) integers is +ve.
case ① S & L both positive.
∴ each int. in list positive → Yes answer
case ② S & L negative → Each Int is negative
if even # then positive → Yes
if odd # then negative → No
NS

(II) No info on sign of Int.
Eg. (−2,2) → No
(2,4) → Yes
NS

Combine:
case ① If S & L positive → Yes
case ② If S & L negative,
# of Int is even
∴ product → positive → Yes

The correct answer is C.

23.
For statement 1, the two numbers can only be 38, 39
For statement 2, the tens digit of x and y must be 3, then, only 9+8 can get the value 17. Two numbers must be 38, 39 as well.
The correct answer is D.

Top 1% expert replies to student queries (can skip)
x and y cannot be more than 2 digits long, and they both have the same tens digit - so they are each 2 digits long The units digit is 7, so the sum of the two units digits is 7. This can be (0,7), (7,0), (1,6), (6,1), (2,5), (5,2), (3,4), (4,3), (8,9), (9,8)

Just a little bit of visualization of the two additions will tell you the units digits have to be 8 and 9 and then the tens digit has to be 3. So x and y and (38, 39) or (39, 38). Irrespective of the order, the product xy will always be the same.
Both statements, by themselves, are sufficient.
The correct answer is D.

24. For statement 1, for example, 0, 0, 0, 2 can fulfil the requirement. Insufficient.
The correct answer is B.

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Question stem states more than 2 numbers, i.e at least 3 numbers:

is each of the numbers in the list equal to 0?
one positive, one negative, and what about the rest?
Statement 2: The sum of any two numbers in the list equal to 0.
Since we're dealing with MORE than 2 numbers, this Fact provides a specific 'restriction' - we CAN'T have ANY non-0 numbers because then we could end up with a sum that is NOT 0.
IF...
{0, 0, 0} then the answer to the question is YES
IF....
{-1, 1, 1} then we could end up with (1) + (1) = 2, which does NOT fit the given Fact. Thus, this example is NOT possible and neither is any other example that could lead to a Non-0 sum. By extension, that means that EVERY number in the group MUST be 0.
Fact 2 is SUFFICIENT.

The correct answer is B.

25.

2-$\sqrt{5}$ is less than zero, so $\sqrt{2-\sqrt{5}}$ is not the real number

Real Numbers are: Integers, Fractions and Irrational Numbers. Non-real numbers are even roots (such as square roots) of negative numbers.

We have $\sqrt{1-\sqrt{2-\sqrt{x}}}$. For $x=5$ expression becomes: $\sqrt{1-\sqrt{2-\sqrt{5}}}$ and $2-\sqrt{5}<0$, thus square root from this expression is not a real number.

Answer: E.

The correct answer is E.

26.

My understanding is that, the question asks you how many pairs of consecutive terms in the sequence have a negative product. In the sequence shown, there are three pairs.
Namely, 1&(-3), (-3)&2, 5&(-4)
Answer is 3.

The correct answer is C.

27.
xy+z=x(y+z)
xy+z=xy+xz
z=xz
z(x-1)=0
x=1 or z=0
The correct answer is E.

28.
For 1, 3*2>3, * can be multiply or add, while (6*2)*4=6*(2*4).
For 2, 3*1=3, * can be multiply or divide. The information cannot determine whether (6*2)*4=6*(2*4).
The correct answer is A.

Top 1% expert replies to student queries (can skip)

(6∗2)∗4=6∗(2∗4) holds true only if * denotes add or multiply
1) 3∗2>3
* can be Add or multiply
* can NOT be subtract or divide
we have only one answers for question "Is (6∗2)∗4=6∗(2∗4)(6∗2)∗4=6∗(2∗4)? "
Yes if * denotes multiply
Yes if * denotes add
SUFFICIENT

2) 3∗1=3
* can be multiply or divide
we have two different answers for question "Is (6∗2)∗4=6∗(2∗4)(6∗2)∗4=6∗(2∗4)? "
Yes if * denotes multiply
No if * denotes divide
NOT SUFFICIENT
The correct answer is A.

29.
As the following shows, the value of r cannot be determined.
................0....m(6)......12.......r(18)................
--m(-12)....0..................12..................r(36)--
On the other way, r also could be -18 and -36
The correct answer is E.

30.
x+4=1, y+4=-5=> x=-3, y=-9, z+4=m
x+e=7, y+e=n => e=10, n=1, z=0, z+e=10 z=0
and z+4=m, m=4 =>m+n=4+1=5
Answer is 5

31.
A perfect square is an integer whose square root is an integer. For example: 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 are all perfect squares.
(1) INSUFFICIENT: There are many possible values for y and z. For example:
y = 7 and z = 2. The sum of y and z (9) is a perfect square but the difference of y and z (5) is NOT a perfect square.
y = 10 and z = 6. The sum of y and z (16) is a perfect square and the difference of y and z (4) is ALSO a perfect square.
Thus, statement (1) alone is not sufficient.
(2) INSUFFICIENT: The fact that z is even has no bearing on whether y – z is a perfect square. The two examples we evaluated above used an even number for z, but we still were not able to answer the question.
(1) AND (2) INSUFFICIENT: Using the same two examples as tested for Statement (1) alone, we can see that knowing that y + z is a perfect square and that z is even still does not allow us to determine whether y – z is a perfect square.
The correct answer is E.

GMAT Quant Topic 4: Numbers
Part B: Odds and Evens

1.
(1) SUFFICIENT: If z/2 = even, then z = 2 × even. Thus, z must be even, because it is the product of 2 even numbers.

Alternatively, we could list numbers according to the criteria that z/2 is even.
z/2: 2, 4, 6, 8, 10, etc.
Multiply the entire list by the denominator 2 to isolate the possible values of z: z: 4, 8, 12, 16, 20, etc. All of those values are even.

(2) INSUFFICIENT: If 3z = even, then z = even/3. There are no odd and even rules for division, mainly because there is no guarantee that the result will be an integer. For example, if 3z = 6, then z is the even integer 2. However, if 3z = 2, then z = 2/3, which is not an integer at all.

The danger in evaluating this statement is forgetting about the fractional possibilities. A way to avoid that mistake is to create a full list of numbers for z that meet the criteria that 3z is even. 3z: 2, 4, 6, 8, 10, 12, etc.

Divide the entire list by the coefficient 3 to isolate the possible values of z:
z: 2/3, 4/3, 2, 8/3, 10/3, 4, etc. Some of those values are even, but others are not.
The correct answer is A.

2. (1) INSUFFICIENT: Given that $m = p^2 + 4p + 4$,

If p is even:
$m = (\text{even})^2 + 4(\text{even}) + 4$
m = even + even + even
m = even

If p is odd:
$m = (\text{odd})^2 + 4(\text{odd}) + 4$
m = odd + even + even
m = odd

Thus, we don't know whether m is even or odd. Additionally, we know nothing about n.

(2) INSUFFICIENT: Given that $n = p^2 + 2m + 1$

If p is even:
$n = (\text{even})^2 + 2(\text{even or odd}) + 1$
n = even + even + odd
n = odd

If p is odd:
$n = (\text{odd})^2 + 2(\text{even or odd}) + 1$
n = odd + even + odd
n = even

Thus we don't know whether n is even or odd. Additionally, we know nothing about m.

(1) AND (2) SUFFICIENT: If p is even, then m will be even and n will be odd. If p is odd, then m will be odd and n will be even. In either scenario, m + n will be odd.

The correct answer is C.

3.
We can first simplify the exponential expression in the question:
$b^{a+1} – ba^b$
$b(b^a) - b(a^b)$
$b(b^a - a^b)$
So we can rewrite this question then as is $b(b^a - a^b)$ odd? Notice that if either b or $b^a - a^b$ is even, the answer to this question will be no.
(1) SUFFICIENT: If we simplify this expression we get 5a - 8, which we are told is odd. For the difference of two numbers to be odd, one must be odd and one must be even. Therefore, 5a must be odd, which means that a itself must be odd. To determine whether or not this is enough to dictate the even/oddness of the expression $b(b^a - a^b)$, we must consider two scenarios, one with an odd b and one with an even b:

a	b	$b(b^a - a^b)$	odd/even
3	1	$1(1^3 - 3^1) = -2$	even
3	2	$2(2^3 - 3^2) = -2$	even

(2) SUFFICIENT: It is probably easiest to test numbers in this expression to determine whether it implies that b is odd or even.

b	$b^3 + 3b^2 + 5b + 7$	odd/even
2	$2^3 + 3(2^2) + 5(2)+ 7 = 37$	odd
1	$1^3 + 3(1^2) + 5(1) + 7 = 16$	even

We can see from the two values that we plugged that only even values for b will produce odd values for the expression $b^3 + 3b^2 + 5b + 7$, therefore b must be even. Knowing that b is even tells us that the product in the question, $b(b^a - a^b)$, is even so we have a definitive answer to the question.
The correct answer is D, EACH statement ALONE is sufficient to answer the question.
The correct answer is D.

4.
The question asks simply whether x is odd. Since we cannot rephrase the question, we must go straight to the statements.
(1) INSUFFICIENT: If y is even, then $y^2 + 4y + 6$ will be even, since every term will be even.
For example, if y = 2, then $y^2 + 4y + 6 = 4 + 8 + 6 = 18$. But if y is odd, then $y^2 + 4y + 6$ will be odd.
For example, if y = 3, then $y^2 + 4y + 6 = 9 + 12 + 6 = 27$.
(2) SUFFICIENT: If z is even, then $9z^2 + 7z – 10$ will be even.
For example, if z = 2, then $9z^2 + 7z – 10 = 36 + 14 – 10 = 40$. If z is odd, then $9z^2 + 7z – 10$ will still be even.
For example, if z = 3, then $9z^2 + 7z – 10 = 81 + 21 – 10 = 92$. So, no matter what the value of z, x will be even and we can answer "no" to the original question.
The correct answer is B.

5.
A perfect square is an integer whose square root is an integer. For example: 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 are all perfect squares.
(1) INSUFFICIENT: There are many possible values for y and z. For example:
y = 7 and z = 2. The sum of y and z (9) is a perfect square but the difference of y and z (5) is NOT a perfect square.
y = 10 and z = 6. The sum of y and z (16) is a perfect square and the difference of y and z (4) is ALSO a perfect square.
Thus, statement (1) alone is not sufficient.
(2) INSUFFICIENT: The fact that z is even has no bearing on whether y – z is a perfect square. The two examples we evaluated above used an even number for z, but we still were not able to answer the question.
(1) AND (2) INSUFFICIENT: Using the same two examples as tested for Statement (1) alone, we can see that knowing that y + z is a perfect square and that z is even still does not allow us to determine whether y – z is a perfect square.
The correct answer is E.

6.
First, let's simplify the inequality in the original question:
$3x + 5 < x + 11$
$2x < 6$
$x < 3$

Since x is less than 3 and must be a positive integer, the only way that x can be a prime number is when x = 2. Therefore, we can rephrase the question: "Does x equal 2?"

(1) INSUFFICIENT: We can infer from this statement that x and y are either both even or both odd. Since we do not have any information about the value of y, we cannot determine the value of x.

(2) SUFFICIENT: Since the product of x and y is odd, we know that x and y are both odd. Therefore, x cannot be a prime number, since the only prime number less than 3 is 2, i.e. an even number. Thus, since x is odd, we know that it is not a prime, and this statement is sufficient to yield a definitive answer "no" to the main question.

The correct answer is B.

7.
This question asks simply whether the positive integer p is even. This question cannot be rephrased.

(1) INSUFFICIENT: $p^2 + p$ can be factored, resulting in $p(p + 1)$. This expression equals the product of two consecutive integers and we are told that this product is even. In order to make the product even, either p or p + 1 must be even, so $p(p + 1)$ will be even regardless of whether p is odd or even. Alternatively, we can try numbers. For p = 2, 2(2 + 1) = 6. For p = 3, 3(3 + 1) = 12. So, when p(p + 1) is even, p can be even or odd.

(2) INSUFFICIENT: Multiplying any positive integer p by 4 (an even number) will always result in an even number. Adding an even number to an even number will always result in an even number. Therefore, 4p + 2 will always be even, regardless of whether p is odd or even. Alternatively, we can try numbers. For p = 2, 4(2) + 2 = 10. For p = 3, 4(3) + 2 = 14. So, when 4p + 2 is even, p can be even or odd.

(1) AND (2) INSUFFICIENT: Because both statements (1) and (2) are true for all positive integers, combining the two statements is insufficient to determine whether p is even. Alternatively, notice that for each statement, we tried p = 2 and p = 3. We can also use these two numbers when we combine the two statements and we are left with the same result: p can be even or odd.

The correct answer is E

8.
First, let us simplify the original expression: $p + q + p = 2p + q$
Since the product of an even number and any other integer will always be even, the value of 2p must be even. If q were even, 2p + q would be the sum of two even integers and would thus have to be even. But the problem stem tells us that 2p + q is odd. Therefore, q cannot be even, and must be odd.
Alternatively, we can reach this same conclusion by testing numbers. We simply test even and odd values of p and q to see whether they meet our condition that p + q + p must be odd.
1) even + even + even = even (for example, 4 + 2 + 4 = 10). The combination (p even, q even) does **not** meet our condition.
2) odd + odd + odd = odd (for example, 5 + 3 + 5 = 13). The combination (p odd, q odd) **does** meet our condition.
3) even + odd + even = odd (for example, 4 + 3 + 4 = 11). The combination (p even, q odd) **does** meet our condition.
4) odd + even + odd = even (for example, 3 + 4 + 3 = 10). The combination (p odd, q even) does **not** meet our condition.

If we examine our results, we see that q has to be odd, while p can be either odd or even. Our question asks us which answer must be odd; since q is an answer choice, we don't have to test the more complicated answer choices.

The correct answer is B.

9.

If ab^2 were odd, the quotient would never be divisible by 2, regardless of what c is. To prove this try to divide an odd number by any integer to come up with an even number; you can't. If ab^2 is even, either a is even or b is even.

(I) TRUE: Since a or b is even, the product ab must be even

(II) NOT NECESSARILY: For the quotient to be positive, a and c must have the same sign since b^2 is definitely positive. We know nothing about the sign of b. The product of ab could be negative or positive.

(III) NOT NECESSARILY: For the quotient to be even, ab^2 must be even but c could be even or odd. An even number divided by an odd number could be even (ex: 18/3), as could an even number divided by an even number (ex: 16/4).

The correct answer is A.

10.

(A) UNCERTAIN: k could be odd or even.

(B) UNCERTAIN: k could be odd or even.

(C) TRUE: If the sum of two integers is odd, one of them must be even and one of them must be odd. Whether k is odd or even, 10k is going to be even; therefore, y must be odd.

(D) FALSE: If the sum of two integers is odd, one of them must be even and one of them must be odd. Whether k is odd or even, 10k is going to be even; therefore, y must be odd.

(E) UNCERTAIN: k could be odd or even.

The correct answer is C.

11. Since the digits of G are halved to derive those of H, the digits of G must both be even. Therefore, there are only 16 possible values for G and H and we can quickly calculate the possible sums of G and H:

G	H	G + H
88	44	132
86	43	129
84	42	126
82	41	123
68	34	102
66	33	99
64	32	96
62	31	93
48	24	72
46	23	69
44	22	66
42	21	63
28	14	42
26	13	39
24	12	36
22	11	33

Alternately, we can approach this problem algebraically. Let's call x and y the tens digit and the units digit of G. Thus H can be expressed as $5x + .5y$. And the sum of G and H can be written as $15x + 1.5y$.

Since we know that x and y must be even, we can substitute 2a for x and 2b for y and can rewrite the expression for the sum of G and H as: $15(2a) + 1.5(2b) = 30a + 3b$. This means that the sum of G and H must be divisible by 3, so we can eliminate C and E.

Additionally, since we know that the maximum value of G is less than 100, then the maximum value of H must be less than 50. Therefore the maximum value of G + H must be less than 150. This eliminates answer choices A and B. This leaves answer choice D, 129. This can be written as 86 + 43.

The correct answer is D.

Alternate Solution from GMATCLUB

$G + G/2 = 3G/2$ --> the sum is a multiple of 3.
G is a two-digit number --> $G < 100$ --> $3G/2 < 150$.
Among the answer choices the only multiple of 3 which is less than 150 is 129.
The correct answer is D.

12.
Let's look at each answer choice:
(A) EVEN: Since a is even, the product ab will always be even. Ex: $2 \times 7 = 14$.

(B) UNCERTAIN: An even number divided by an odd number might be even if the the prime factors that make up the odd number are also in the prime box of the even number. Ex: $6/3 = 2$.

(C) NOT EVEN: An odd number is never divisible by an even number. By definition, an odd number is not divisible by 2 and an even number is. The quotient of an odd number divided by an even number will not be an integer, let alone an even integer. Ex: $15/4 = 3.75$

(D) EVEN: An even number raised to any integer power will always be even. Ex: $2^1 = 2$

(E) EVEN: An even number raised to any integer power will always be even. Ex: $2^3 = 8$

The correct answer is C.

13. Let's look at each answer choice:
(A) UNCERTAIN: x could be the prime number 2.

(B) UNCERTAIN: x could be the prime number 2, which when added to another prime number (odd) would yield an odd result. Ex: $2 + 3 = 5$

(C) UNCERTAIN: Since x could be the prime number 2, the product xy could be even.

(D) UNCERTAIN: $y > x$ and they are both prime so y must be odd. If x is another odd prime number, the expression will be: (odd) + (odd)(odd), which equals an even (O + O = E).

(E) FALSE: 2x must be even and y must be odd (since it cannot be the smallest prime number 2, which is also the only even prime). The result is even + odd, which must be odd.
The correct answer is E.

14.
If q, r, and s are consecutive even integers and $q < r < s$, then $r = s – 2$ and $q = s – 4$. The expression $s^2 – r^2 – q^2$ can be written as $s^2 – (s – 2)^2 – (s – 4)^2$. If we multiply this out, we get: $s^2 – (s – 2)^2 – (s – 4)^2 = s^2 – (s^2 – 4s + 4) – (s^2 – 8s + 16) = s^2 – s^2 + 4s – 4 – s^2 + 8s – 16 =$ $\boldsymbol{-s^2 + 12s - 20}$

The question asks which of the choices CANNOT be the value of the expression $\boldsymbol{-s^2 + 12s - 20}$ so we can test each answer choice to see which one violates what we know to be true about s, namely that s is an even integer.
Testing (E) we get:
$-s^2 + 12s – 20 = 16$

$-s^2 + 12s – 36 = 0$
$s^2 + 12s – 36 = 0$
$(s – 6)(s – 6) = 0$
s = 6. This is an even integer so this works.
Testing (D) we get:
$-s^2 + 12s – 20 = 12$
$-s^2 + 12s – 32 = 0$
$s^2 + 12s – 32 = 0$
$(s – 4)(s – 8) = 0$
s = 4 or 8. These are even integers so this works.
Testing (C) we get:
$-s^2 + 12s – 20 = 8$
$-s^2 + 12s – 28 = 0$
$s^2 + 12s – 28 = 0$
Since there are no integer solutions to this quadratic (meaning there are no solutions where s is an integer), 8 is not a possible value for the expression.

Alternately, we could choose values for q, r, and s and then look for a pattern with our results. Since the answer choices are all within twenty units of zero, choosing integer values close to zero is logical. For example, if q = 0, r = 2, and s = 4, we get $4^2 – 2^2 – 0^2$ which equals
16 – 4 – 0 = 12. Eliminate answer choice D.

Since there is only one value greater than 12 in our answer choices, it makes sense to next test q = 2, r = 4, s = 6. With these values, we get $6^2 – 4^2 – 2^2$ which equals 36 – 16 – 4 = 16.
Eliminate answer choice E.

We have now eliminated the two greatest answer choices, so we must test smaller values for q, r, and s. If q = -2, r = 0, and s = 2, we get $2^2 – 0^2 – 2^2$ which equals 4 – 0 – 4 = 0. Eliminate answer choice B.

At this point, you might notice that as you choose smaller (more negative) values for q, r, and s, the value of $s^2 < r^2 < q^2$. Thus, any additional answers will yield a negative value. If not, simply choose the next logical values for q, r, and s: q = -4, r = -2, and s = 0. With these values we get $0^2 – (-2)^2 – (-4)^2 = 0 – 4 – 16 = -20$. Eliminate answer choice A.

The correct answer is C.

15.
Sequence problems are often best approached by charting out the first several terms of the given sequence. In this case, we need to keep track of n, t_n, and whether t_n is even or odd.

n	**t_n**	**Is t_n even or odd?**
0	$t_n = 3$	Odd
1	$t_1 = 3 + 1 =$ **4**	Even
2	$t_2 = 4 + 2 =$ **6**	Even
3	$t_3 = 6 + 3 =$ **9**	Odd
4	$t_4 = 9 + 4 =$ **13**	Odd
5	$t_5 = 13 + 5 =$ **18**	Even
6	$t_6 = 18 + 6 =$ **24**	Even
7	$t_7 = 24 + 7 =$ **31**	Odd
8	$t_8 = 31 + 8 =$ **39**	Odd

Notice that beginning with n = 1, a pattern of even-even-odd-odd emerges for t_n.

Thus t_n is even when n = 1, 2 . . . 5, 6 . . . 9, 10 . . . 13, 14 . . . etc. Another way of conceptualizing this pattern is that t_n is even when n is either
(a) 1 plus a multiple of 4 (n = 1, 5, 9, 13, etc.) or
(b) 2 plus a multiple of 4 (n = 2, 6, 10, 14, etc).

From this we see that only Statement (2) is sufficient information to answer the question. If n – 1 is a multiple of 4, then n is 1 plus a multiple of 4. This means that t_n is always even.

Statement (1) does not allow us to relate n to a multiple of 4, since it simply tells us that n + 1 is a multiple of 3. This means that n could be 2, 5, 8, 11, etc. Notice that for n = 2 and n = 5, t_n is in fact even. However, for n = 8 and n = 11, t_n is odd.

Thus, Statement (2) alone is sufficient to answer the question but Statement (1) alone is not.
The correct answer is B.

16.
In order for the square of (y + z) to be even, y + z must be even. In order for y + z to be even, either both y and z must be odd or both y and z must be even.
(1) SUFFICIENT: If y – z is odd, then one of the integers must be even and the other must be odd. Thus, the square of y + z will definitely NOT be even. (Recall that "no" is a sufficient answer to a yes/no data sufficiency question; only "maybe" is insufficient.)
(2) INSUFFICIENT: If yz is even, then it's possible that both integers are even or that one of the integers is even and the other integer is odd. Thus, we cannot tell, whether the square of y + z will be even.
The correct answer is A.

17.
From 1, 4y is even, then, 5x is even, and x is even.
From 2, 6x is even, then, 7y is even, and y is even.
The correct answer is D.

18.
From 1, [x,y] = [2,2] & [3,2] though fulfill requirement but results contradict each other
From 2, x, y are not specified to be odd or even
Together, prime>7 is always odd thus make y+1 always even, therefore x(y+1) is made always even.
The correct answer is C.

19.

(9)=27, (6)=3, so, (9)*(6)=81Only (27) equals to 81.
So, [m]=3m when m is odd, [m]=(1/2)*m when m is even.
As 9 is odd then [9] equals to 3*9=27;
As 6 is even then [6] equals to 1/2*6=3;
So [9]*[6]=27*3=81. Note that numbers in the answer choices are also in boxes, so we have: [m]=81. m could be 27 (in this case as 27 is odd [27]=3*27=81) OR 162 (in this case as 162 is even [162]=162/2=81) --> only [27] is in the answer choices.
The correct answer is D.

20.
Statement 1, m and n could be both odd or one odd, one even. Insufficient.
Statement 2, when n is odd, n^2+5 is even, then m+n is even, m is odd; when n is even, n^2+5=odd, m+n is odd, then m is odd. Sufficient.
The correct answer is B.

21.
Even=even*even or Even=even*odd
We know that d+1 and d+4 cannot be even together, and both c(d+1), (c+2)(d+4) are even. Therefore, c or c+2 must be even to fulfill the requirement. That is, c must be even.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

Start with a second statement:
$(c+2)(d+4)=cd+4c+2d+8=even$ => "cd" is even.
Now statement one says :
$c(d+1)=cd+c$ is even.
Now combine:
From statement 2, we know that "cd" is even => for statement 1 (i.e. cd+c=even) to be true, "c" has to be even. Sufficient.
The correct answer is C.

22.
Statement 1 alone is sufficient.
Statement 2 means that the units digit of x^2 cannot be 2, 4, 5, 6, 8, and 0, only can be 1, 3, 7, 9.
Then, the units digit of x must be odd, and $(x^2+1)(x+5)$ must be even.
The correct answer is D.

23.
Let's look at each answer choice:
(A) EVEN: Since a is even, the product ab will always be even. Ex: $2 \times 7 = 14$.

(B) UNCERTAIN: An even number divided by an odd number might be even if the prime factors that make up the odd number are also in the prime box of the even number. Ex: $6/3 = 2$.

(C) NOT EVEN: An odd number is never divisible by an even number. By definition, an odd number is not divisible by 2 and an even number is. The quotient of an odd number divided by an even number will not be an integer, let alone an even integer. Ex: $15/4 = 3.75$

(D) EVEN: An even number raised to any integer power will always be even. Ex: $2^1 = 2$

(E) EVEN: An even number raised to any integer power will always be even. Ex: $2^3 = 8$

The correct answer is C.

24.
In order for the square of $(y + z)$ to be even, $y + z$ must be even. In order for $y + z$ to be even, either both y and z must be odd or both y and z must be even.
(1) SUFFICIENT: If $y – z$ is odd, then one of the integers must be even and the other must be odd. Thus, the square of $y + z$ will definitely NOT be even. (Recall that "no" is a sufficient answer to a yes/no data sufficiency question; only "maybe" is insufficient.)
(2) INSUFFICIENT: If yz is even, then it's possible that both integers are even or that one of the integers is even and the other integer is odd. Thus, we cannot tell, whether the the square of $y + z$ will be even.
The correct answer is A.

25.
Let's look at each answer choice:

(A) UNCERTAIN: x could be the prime number 2.
(B) UNCERTAIN: x could be the prime number 2, which when added to another prime number (odd) would yield an odd result. Ex: $2 + 3 = 5$
(C) UNCERTAIN: Since x could be the prime number 2, the product xy could be even.
(D) UNCERTAIN: $y > x$ and they are both prime so y must be odd. If x is another odd prime number, the expression will be: (odd) + (odd)(odd), which equals an even (O + O = E).
(E) FALSE: 2x must be even and y must be odd (since it cannot be the smallest prime number 2, which is also the only even prime). The result is even + odd, which must be odd.
The correct answer is E

GMAT Quant Topic 4: Numbers
Part C: Unit's digits, factorial powers

1.
When raising a number to a power, the units digit is influenced only by the units digit of that number. For example 16^2 ends in a 6 because 6^2 ends in a 6.

17^{27} will end in the same units digit as 7^{27}.
The units digit of consecutive powers of 7 follows a distinct pattern:

Power of 7	Ends in a ...
7^1	7
7^2	9
7^3	3
7^4	1
7^5	7

The pattern repeats itself every four numbers so a power of 27 represents 6 full iterations of the pattern ($6 \times 4 = 24$) with three left over. The "leftover three" leaves us back on a "3," the third member of the pattern 7, 9, 3, 1.
The correct answer is C.

2.
When a number is divided by 10, the remainder is simply the units digit of that number. For example, 256 divided by 10 has a remainder of 6. This question asks for the remainder when an integer power of 2 is divided by 10. If we examine the powers of 2 (2, 4, 8, 16, 32, 64, 128, and 256…), we see that the units digit alternates in a consecutive pattern of 2, 4, 8, 6. To answer this question, we need to know which of the four possible units digits we have with 2^p.

(1) INSUFFICIENT: If s is even, we know that the product rst is even and so is p. Knowing that p is even tells us that 2^p will have a units digit of either 4 or 6 ($2^2 = 4$, $2^4 = 16$, and the pattern continues).

(2) SUFFICIENT: If $p = 4t$ and t is an integer, p must be a multiple of 4. Since every fourth integer power of 2 ends in a 6 ($2^4 = 16$, $2^8 = 256$, etc.), we know that the remainder when 2^p is divided by 10 is 6.

The correct answer is B.

3.
When a whole number is divided by 5, the remainder depends on the units digit of that number. Thus, we need to determine the units digit of the number $1^1+2^2+3^3+...+10^{10}$. To do so, we need to first determine the units digit of each of the individual terms in the expression as follows:

Term	Last (Units) Digit
1^1	**1**
2^2	**4**
3^3	**7**
4^4	**6**
5^5	**5**
6^6	**6**
7^7	**3**
8^8	**6**
9^9	**9**
10^{10}	**0**

To determine the units digit of the expression itself, we must find the sum of all the units digits of each of the individual terms:

$1 + 4 + 7 + 6 + 5 + 6 + 3 + 6 + 9 = 47$

Thus, **7** is the units digit of the number $1^1+2^2+3^3+...+10^{10}$. When an integer that ends in 7 is divided by 5, the remainder is 2. (Test this out on any integer ending in 7.)

Thus, the correct answer is C.

4.
The easiest way to approach this problem is to chart the possible units digits of the integer p. Since we know p is even, and that the units digit of p is positive, the only options are 2, 4, 6, or 8.

UNITS DIGIT OF p	Units Digit of p^3	Units Digit of p^2	Units Digits of $p^3 - p^2$
2	8	4	4
4	4	6	8
6	6	6	**0**
8	2	4	8

Only when the units digit of p is 6, is the units digit of $p^3 - p^2$ equal to 0. The question asks for the units digit of p + 3. This is equal to 6 + 3, or 9.
The correct answer is D.

5.

For problems that ask for the units digit of an expression, yet seem to require too much computation, remember the Last Digit Shortcut. Solve the problem step-by-step, but recognize that you only need to pay attention to the last digit of every intermediate product. Drop any other digits.

So, we can drop any other digits in the original expression, leaving us to find the units digit of:
$(4)^{(2x+1)}(3)^{(x+1)}(7)^{(x+2)}(9)^{(2x)}$

This problem is still complicated by the fact that we don't know the value of x. In such situations, it is often a good idea to look for patterns. Let's see what happens when we raise the bases 4, 3, 7, and 9 to various powers. For example: $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$, and so on. The units digit of the powers of three follow a pattern that repeats every fourth power: 3, 9, 7, 1, 3, 9, 7, 1, and so on. The patterns for the other bases are shown in the table below:

	Exponent						
Base	**1**	**2**	**3**	**4**	**5**	**6**	**Pattern**
4	4	6	4	6	4	6	4, 6, repeat
3	3	9	7	1	3	9	3, 9, 7, 1, repeat
7	7	9	3	1	7	9	7, 9, 3, 1, repeat
9	9	1	9	1	9	1	9, 1, repeat

The patterns repeat at least every fourth term, so let's find the units digit of
$(4)^{(2x+1)}(3)^{(x+1)}(7)^{(x+2)}(9)^{(2x)}$ for at least four consecutive values of x:
x = 1: units digit of $(4^3)(3^2)(7^3)(9^2)$ = units digit of (4)(9)(3)(1) = units digit of 108 = 8 x = 2: units digit of $(4^5)(3^3)(7^4)(9^4)$ = units digit of (4)(7)(1)(1) = units digit of 28 = 8 x = 3: units digit of $(4^7)(3^4)(7^5)(9^6)$ = units digit of (4)(1)(7)(1) = units digit of 28 = 8 x = 4: units digit of $(4^9)(3^5)(7^6)(9^8)$ = units digit of (4)(3)(9)(1) = units digit of 108 = 8

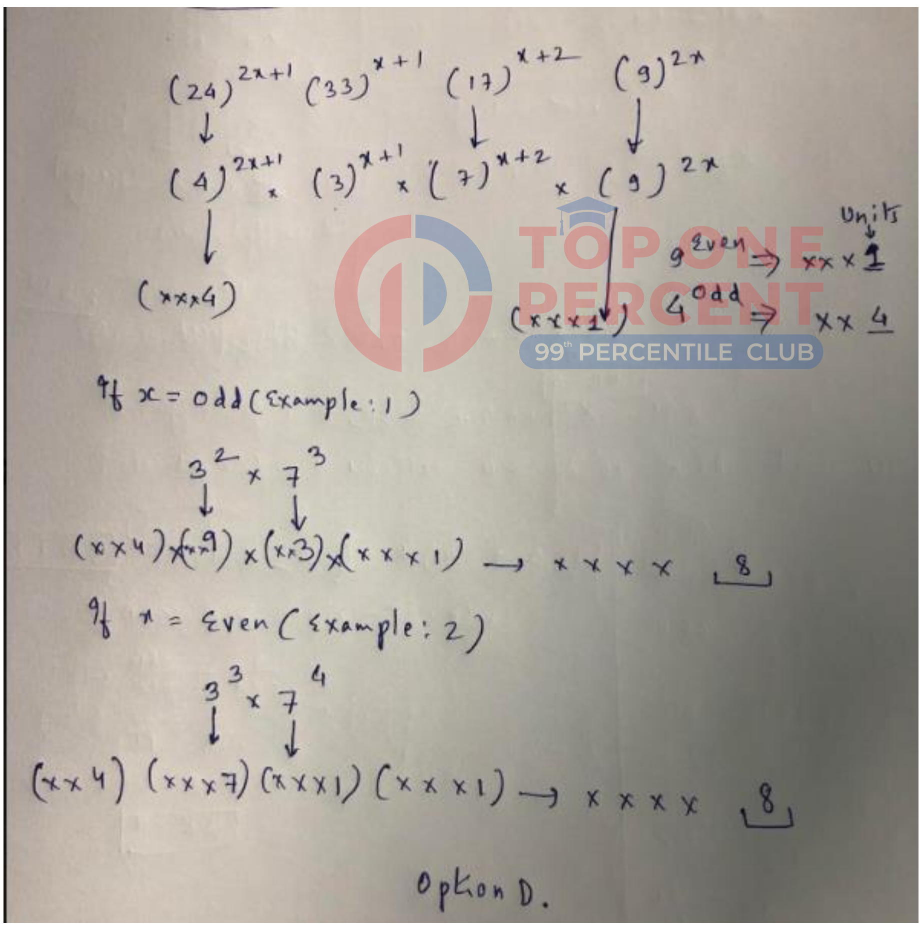
The units digit of the expression in the question must be 8.

Alternatively, note that x is a positive integer, so 2x is always even, while 2x + 1 is always odd. Thus, $(4)^{(2x + 1)} = (4)^{(odd)}$, which always has a units digit of 4 $(9)^{(2x)} =$
$(9)^{(even)}$, which always has a units digit of 1

That leaves us to find the units digit of $(3)^{(x + 1)}(7)^{(x + 2)}$. Rewriting, and dropping all but the units digit at each intermediate step,

$(3)_{(x + 1)}(7)_{(x + 2)}$
$= (3)_{(x + 1)}(7)_{(x + 1)}(7)$
$= (3 \times 7)^{(x + 1)}(7)$
$= (21)^{(x + 1)}(7)$
$= (1)^{(x + 1)}(7) = 7$, for any value of x.
So, the units digit of $(4)^{(2x + 1)}(3)^{(x + 1)}(7)^{(x + 2)}(9)^{(2x)}$ is (4)(7)(1) = 28, then once again drop all but the units digit to get 8.
The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)



6.
If a is a positive integer, then 4^a will always have a units digit of 4 or 6. We can show this by listing the first few powers of 4:

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$

The units digit of the powers of 4 alternates between 4 and 6. Since $x = 4^a$, x will always have a units digit of 4 or 6.

Similarly, if b is a positive integer, then 9^b will always have a units digit of 1 or 9. We can show this by listing the first few powers of 9:

$9^1 = 9$
$9^2 = 81$
$9^3 = 729$
$9^4 = 6561$

The units digit of the powers of 9 alternates between 1 and 9. Since $y = 9^b$, y will always have a units digit of 1 or 9.

To determine the units digit of a product of numbers, we can simply multiply the units digits of the factors. The resulting units digit is the units digit of the product. For example, to find the units digit of (23)(39) we can take (3)(9) = 27. Thus, 7 is the units digit of (23)(39). So, the units digit of xy will simply be the units digit that results from multiplying the units digit of x by the units digit of y. Let's consider all the possible units digits of x and y in combination:

$4 \times 1 = 4$, units digit = 4
$4 \times 9 = 36$, units digit = 6
$6 \times 1 = 6$, units digit = 6
$6 \times 9 = 54$, units digit = 4

The units digit of xy will be 4 or 6.

The correct answer is B.

7.
Since every multiple of 10 must end in zero, the remainder from dividing xy by 10 will be equal to the units‘ digit of xy. In other words, the units‘ digit will reflect by how much this number is greater than the nearest multiple of 10 and, thus, will be equal to the remainder from dividing by 10. Therefore, we can rephrase the question: ―What is the units‘ digit of xy?

Next, let‘s look for a pattern in the units‘ digit of 3^{21}. Remember that the GMAT will not expect you to do sophisticated computations; therefore, if the exponent seems too large to compute, look for a shortcut by recognizing a pattern in the units' digits of the exponent:
$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$

As you can see, the pattern repeats every 4 terms, yielding the units digits of 3, 9, 7, and 1. Therefore, the exponents 3^1, 3^5, 3^9, 3^{13}, 3^{17}, and 3^{21} will end in 3, and the units‘ digit of 3^{21} is 3.

Next, let‘s determine the units‘ digit of 6^{55} by recognizing the pattern:
$6^1 = 6$
$6^2 = 36$
$6^3 = 256$
$6^4 = 1,296$

As shown above, all positive integer exponents of 6 have a units‘ digit of 6. Therefore, the units' digit of 6^{55} will also be 6.

Finally, since the units digit of 3^{21} is 3 and the units‘ digit of 6^{55} is 6, the units' digit of $3^{21} \times 6^{55}$ will be equal to 8, since $3 \times 6 = 18$. Therefore, when this product is divided by 10, the remainder will be 8.

The correct answer is E.

8.
To find the remainder when a number is divided by 5, all we need to know is the units digit, since every number that ends in a zero or a five is divisible by 5.

For example, 23457 has a remainder of 2 when divided by 5 since 23455 would be a multiple of 5, and 23457=23455+ 2.

Since we know that x is an integer, we can determine the units digit of the number $7^{12x+3} + 3$. The first thing to realize is that this expression is based on a power of 7. The units digit of any integer exponent of seven can be predicted since the units digit of base 7 values follows a patterned sequence:

Units Digit = **7**	Units Digit = **9**	Units Digit = **3**	Units Digit = **1**
7^1	7^2	7^3	7^4
7^5	7^6	7^7	7^8
			7^{12x}
7^{12x+1}	7^{12x+2}	7^{12x+3}	

We can see that the pattern repeats itself every 4 integer exponents.

The question is asking us about the 12x+3 power of 7. We can use our understanding of multiples of four (since the pattern repeats every four) to analyze the 12x+3 power.

12x is a multiple of 4 since x is an integer, so 7^{12x} would end in a 1, just like 7^4 or 7^8.
7^{12x+3} would then correspond to 7^3 or 7^7 (multiple of 4 plus 3), and would therefore end in a 3.

However, the question asks about $7^{12x+3} + 3$.
If 7^{12x+3} ends in a three, $7^{12x+3} + 3$ would end in a 3 + 3 = 6.
If a number ends in a 6, there is a remainder of 1 when that number is divided by 5.

The correct answer is B

9.

Units digit questions often times involve recognition of a pattern.

The units digit of n is determined solely by the units digit of the expressions 5^x and $7^{y + 15}$, because when two numbers are added together, the units digit of the sum is determined solely by the units digits of the two numbers.

Since x is a positive integer, 5^x always ends in a 5 ($5^2 = 2\mathbf{5}$, $5^3 = 12\mathbf{5}$, $5^4 = 62\mathbf{5}$). This property is also shared by the integer 6.
The units digit of a power of 7 is not consistent. The value of x becomes a non-factor here.

The question can be rephrased as "what is the units digit of $7^{y + 15}$?" or potentially just "what is y?"

(1) INSUFFICIENT: This statement cannot be used to find the value of y or the units digit of 7^{y+15}.

(2) SUFFICIENT: This statement can be used to solve for two potential values for y. The quadratic can be factored:
$(y – 5)(y – 1) = 0$, so y = 1 or 5. This does NOT sufficiently answer the question "what is y?" but it DOES provide a single answer to the question "what is the units digit of $7^{y + 15}$?"

Powers of 7 have units digits that follow a specific pattern:

7^0	**1**
7^1	**7**
7^2	4**9**
7^3	34**3**
7^4	240**1**

The pattern is 1, 7, 9 and 3, repeating in iterations of four. The two possible values for y according to statement (2) are 1 and 5, which means that 7^{y+15} is either 7^{16} or 7^{20}. Both 7^{16} and 7^{20} have a units digit of 1 (according to the pattern). Ultimately this means that n will have a units digit of 5 + 1 = 6.

The correct answer is (B), statement (2) ALONE is sufficient to answer the question, but statement (1) alone is not.
The correct answer is B.

10.
Since the question only asks about the units digit of the final solution, focus only on computing the units digit for each term. Thus, the question can be rewritten as follows:

$(1)^5(6)^3(3)^4 + (7)(8)^3$.
The units digit of 1^5 is 1.
The units digit of 6^3 is 6.
The units digit of 3^4 is 1.
The units digit of $(1 \times 6 \times 1)$ is **6**.
The units digit of 7 is 7.
The units digit of 8^3 is 2.
The units digit of (7×2) is **4**.
The solution is equal to the units digit of (6 + 4), which is 0.
The correct answer is A.

11.

In order to answer this, we need to recognize a common GMAT pattern: the difference of two squares. In its simplest form, the difference of two squares can be factored as follows:
$x^2 - y^2 = (x+y)(x-y)$. Where, though, is the difference of two squares in the question above? It pays to recall that all even exponents are squares. For example,
$x^4 = (x^2)(x^2)$
$x^{56} = (x^{28})(x^{28})$.

Because the numerator in the expression in the question is the difference of two even exponents, we can factor it as the difference of two squares and simplify:

$$\frac{(13!)^{16} - (13!)^8}{(13!)^8 + (13!)^4} = a$$

$$\frac{\left[\cancel{(13!)^8 + (13!)^4}\right]\left[(13!)^8 - (13!)^4\right]}{\cancel{(13!)^8 + (13!)^4}} = a$$

$$(13!)^8 - (13!)^4 = a$$

$$(13!)^4\left[(13!)^4 - 1\right] = a$$

$$(13!)^4 - 1 = \frac{a}{(13!)^4}$$

The units digit of the left side of the equation is equal to the units digit of the right side of the equation (which is what the question asks about). Thus, if we can determine the units digit of the expression on the left side of the equation, we can answer the question.

Since $(13!) = 13 \times 12 \times 11 \times 10 \ldots \times 1$, we know that 13! contains a factor of 10, so its units digit must be 0. Similarly, the units digit of $(13!)^4$ will also have a units digit of 0. If we subtract 1 from this, we will be left with a number ending in 9.

Therefore, the units digit of $\frac{a}{(13!)^4}$ is 9.
The correct answer is E.

12.

Since the question asks only about the units digit, we can look for patterns in each of the numbers.

Lets begin with 177^{28} :

Expression	177^1	177^2	177^3	177^4	177^5
Units Digit	1	9	3	1 again	9 again

Since this pattern will continue, the units digit of 177^{28} will be **1**.

Next, lets follow the same procedure with 133^{23}

Expression	133^1	133^2	133^3	133^4	133^5
Units Digit	3	9	7	1	3 again

Since this pattern will continue, the units digit of 133^{23} will be **7**.
Therefore in calculating the expression 177^{28}- 133^{23}, we can determine that the units digit of the solution will equal 1-7
Since, **7** is greater than **1**, the subtraction here requires that we carry over from the tens place.
Thus, we have 7, yielding the units digit **4**

The correct answer is C.

13.
A quotient of two integers will be an integer if the numerator is divisible by the denominator, so we need 50! to be divisible by 10^m. To check divisibility, we must compare the prime boxes of these two numbers (The prime box of a number is the collection of prime numbers that make up that number. The product of all the elements of a number's prime box is the number itself. For example, the prime box of 12 contains the numbers 2,2,3).

Since $10 = 2 \times 5$, the prime box of 10^m is comprised of only 2's and 5's, namely m 2's and m 5's. That is because $10^m = (2 \times 5)^m = (2^m) \times (5^m)$. Now, some x is divisible by some y if x's prime box contains all the numbers in y's prime box. So in order for 50! to be divisible by 10^m, it has to have at least m 5's and m 2's in its prime box.

Let's count how many 5's 50! has in its prime box.
$50! = 1 \times 2 \times 3 \times ... 50$, so all we have to do is add the number of 5's in the prime boxes of 1, 2, 3, ..., 50.
The only numbers that contribute 5's are the multiples of 5, namely 5, 10, 15, 20,
25, 30, 35, 40, 45, 50. But don't forget to notice that 25 and 50 are both divisible by 25, so they each contribute two 5's.

That makes a total of 10 + 2 = twelve 5's in the prime box of 50!.

As for 2's, we have at least 25 (2, 4, 6, ..., 50), so we shouldn't waste time counting the exact number. The limiting factor for m is the number of 5's, i.e. 12. Therefore, the greatest integer m that would work here is 12.

The correct answer is E.

14.
To determine how many terminating zeroes a number has, we need to determine how many times the number can be divided evenly by 10. (For example, the number 404000 can be divided evenly by 10 three times, as follows:

$$404000 \div 10 = 40400 \quad \rightarrow \quad 40400 \div 10 = 4040 \quad \rightarrow \quad 4040 \div 10 = 404$$

We can see that the number has three terminating zeroes because it is divisible by 10 three times.) Thus, to arrive at an answer, we need to count the factors of 10 in 200!
Recall that $200! = 200 \times 199 \times 198 \times 197 \ldots \times 4 \times 3 \times 2 \times 1$.

Each factor of 10 consists of one prime factor of 2 and one prime factor of 5. Let's start by counting the factors of 5 in 200!. Starting from 1, we get factors of 5 at 5, 10, 15, . . . , 190, 195, and 200, or every 5th number from 1 to 200. Thus, there are 200/5 or 40 numbers divisible by five from 1 to 200. Therefore, there are at least 40 factors of 5 in 200!
We cannot stop counting here, because some of those multiples of 5 contribute more than just one factor of 5. Specifically, any multiple of 5^2(or 25) and any multiple of 5^3(or 125) contribute additional factors of 5. There are 8 multiples of 25 (namely, 25, 50, 75, 100, 125, 150, 175, 200). Each of these 8 numbers contributes one additional factor of 5 (in addition to the one we already counted) so we now have counted 40 + 8, or 48 factors of 5. Finally, 125 contributes a third additional factor of 5, so we now have 48 + 1 or 49 total factors of 5 in 200!

Let us now examine the factors of 2 in 200!. Since every even number contributes at least one factor of 2, there are at least 100 factors of 2 in 200! (2, 4, 6, 8 . . .etc). Since we are only interested in the factors of 10 — a factor of 2 paired with a factor of 5 — and there are more factors of 2 than there are of 5, the number of factors of 10 is constrained by the number of factors of 5. Since there are only 49 factors of 5, each with an available factor of 2 to pair with, there are exactly 49 factors of 10 in 200!.

It follows that 200! has 49 terminating zeroes.
The correct answer is C.

15.
We know from the question that x and y are integers and also that they are both greater than 0. Because we are only concerned with the units digit of n and because both bases end in 3 (243 and 463), we simply need to know x + y to figure out the units digit for n. Why? Because, to get the units digit, we are simply going to complete the operation $3^x \times 3^y$ which, using our exponent rules, simplifies to $3^{(x+y)}$.

So we can rephrase the question as "What is x + y?"

(1) SUFFICIENT: This tells us that x + y = 7. Therefore, the units digit of the expression in the question will be the same as the units digit of 37.

(2) INSUFFICIENT: This gives us no information about y.

The correct answer is A.

16.
First, let's identify the value of the square of the only even prime number. The only even prime is 2, so the square of that is $2^2 = 4$. Thus, $x = 4$ and y is divisible by 4. With this information, we know we will be raising 4 to some power divisible by 4. The next step is to see if we can establish a pattern.

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1024$
We will quickly notice that 4 raised to any odd power has a units digit of 4. And 4 raised to any even number has a units digit of 6. Therefore, because we are raising 4 to a number divisible by 4, which will be an even number, we know that the units digit of x^y is 6
The correct answer is D.

17.
(1) INSUFFICIENT: This statement does not provide enough information to determine the units digit of x^2. For example, x^4 could be 1 in which case $x = 1$ and the units digit of x^2 is 1, or x^4 could be 81 in which case $x = 3$ and the units digit of x^2 is 9.

(2) SUFFICIENT: Given that the units digit of x is 3, we know that the units digit of x^2 is 9.

The correct answer is B.

GMAT Quant Topic 4: Numbers
Part D: Decimals

1.
To determine the value of 10 – x, we must determine the exact value of x. To determine the value of x, we must find out what digits a and b represent. Thus, the question can be rephrased: What is a and what is b?
(1) INSUFFICIENT: This tells us that x rounded to the nearest hundredth must be 1.44. This means that a, the hundredths digit, might be either 3 (if the hundredths digit was rounded up to 4) or 4 (if the hundredths digit was rounded down to 4). This statement alone is NOT sufficient since it does not give us a definitive value for a and tells us nothing about b.
(2) SUFFICIENT: This tells us that x rounded to the nearest thousandth must be 1.436. This means, that a, the hundredths digit, is equal to 3. As for b, the thousandths digit, we know that it is followed by a 5 (the ten-thousandths digit); therefore, if x is rounded to the nearest thousandth, b must rounded UP. Since b is rounded UP to 6, then we know that b must be equal to 5. Statement (2) alone is sufficient because it provides us with definitive values for both a and b.

The correct answer is B.

2.
For fraction p/q to be a terminating decimal, the numerator must be an integer and the denominator must be an integer that can be expressed in the form of 2^x5^y where x and y are nonnegative integers. (Any integer divided by a power of 2 or 5 will result in a terminating decimal.)
The numerator p, 2^a3^b, is definitely an integer since a and b are defined as integers in the question.

The denominator q, $2^c3^d5^e$, could be rewritten in the form of 2^x5^y if we could somehow eliminate the expression 3^d. This could happen if the power of 3 in the numerator (b) is greater than the power of 3 in the denominator (d), thereby cancelling out the expression 3^d. Thus, we could rephrase this question as, is b > d?

(1) INSUFFICIENT. This does not answer the rephrased question "is b > d"? The denominator q is not in the form of 2^x5^y so we cannot determine whether or not p/q will be a terminating decimal.

(2) SUFFICIENT. This answers the question "is b > d?"

The correct answer is B.

3.

(1) SUFFICIENT: If the denominator of d is exactly 8 times the numerator, then d can be simplified to 1/8. Rewritten as a decimal, this is 0.125. Thus, there are not more than 3 non zero digits to the right of the decimal.

(2) INSUFFICIENT: Knowing that d is equal to a non-repeating decimal does not provide any information about how many nonzero digits are to the right of the decimal point in the decimal representation of d.
The correct answer is A.

4.
The question asks us to determine whether the number (5/28)(3.02)(90%)(x) can be represented in a finite number of non-zero decimal digits. A number can be represented in a finite number of non-zero decimal digits when the denominator of its reduced fraction contains only integer powers of 2 and 5 (in other words, 2 raised to an integer and 5 raised to an integer). For example, 3/20 CAN be represented by a finite number of decimal digits, since the denominator equals 4 times 5 which are both integer powers of 2 and 5 (that is, 2 to the 2nd power and 5 to the 1st power).
We can manipulate the original expression as follows:

(5/28) (3.02) (90%) x

(5/28) (302/100) (90/100) x

The 100's in the denominator consist of powers of 2 and 5, so the only problematic number in the denominator is the 28 -- specifically, the factor of 7 in the 28. So any value of x that removes the 7 from the denominator will allow the entire fraction to be represented in a finite number of non-zero decimal digits. We have to make sure that this 7 doesn't cancel with anything already present in the combined numerator, but none of the numbers in the numerator (that is, 5, 302, and 90) contain a factor of 7.

(1) INSUFFICIENT: Statement (1) says that x is greater than 100. If x has a factor of 7, say 112, then the expression can be reduced to a finite number of non-zero decimal digits. Otherwise the number will be represented with an infinite number of (repeating) decimal digits.

(2) SUFFICIENT: Statement (2) tells us that x is divisible by 21. Multiplying the expression by any multiple of 21 will remove the factor of 7 from the denominator, so the resultant number can be represented by a finite number of digits. For example, when x = 21, the expression can be manipulated as follows:

(5/28) (302/100) (90/100) (21)
= (5/28) (21) (302/100) (90/100)
= (5) (3/4) (302/100) (90/100)

All the factors in the combined denominator are powers of 2 and 5, so it can be represented in a finite number of digits.

The correct answer is B.

5.

A fraction will always yield a terminating decimal as long as the denominator has only 2 and 5 as its prime factors. In this case, since we know that a, b, c, and d are integers greater than or equal to 0, the denominator potentially has 2, 3, and 5 as its prime factors. The only "problematic" factor is 3. Therefore, this complex looking question can actually be rephrased as follows:

Is b = 0?

If b = 0, then the decimal terminates, since $3^b = 3^0 = 1$, which would leave 2 and 5 as the only prime factors in the denominator. If, b not equal to zero, then the denominator has 3 as a prime factor which means that the fraction may or may not terminate (depending on the value of the numerator).
Statement (1) does not provide any information about b so it is not sufficient to answer the question.

Statement (2) provides an equation that can be factored and simplified as follows:

$$b = (1+a)(a^2 - 2a + 1) - (a-1)(a^2 - 1)$$
$$b = (1+a)(a-1)(a-1) - (a-1)(a+1)(a-1)$$
$$b = (a+1)(a-1)^2 - (a+1)(a-1)^2$$
$$b = 0$$

Since b = 0, the denominator of the fraction contains only 2's and 5's in its prime factorization and therefore it IS a terminating decimal.

The correct answer is B: Statement (2) alone is sufficient, but statement (1) alone is not sufficient.
The correct answer is B.

6.
From statement (1), we know that d – e must equal a positive perfect square. This means that d is greater than e. In addition, since any single digit minus any other single digit can yield a maximum of 9, d – e could only result in the perfect squares 9, 4, or 1.
However, this leaves numerous possibilities for the values of d and e respectively. For example, two possibilities are as follows:
d = 7, e = 3 (d – e = the perfect square 4)

d = 3, e = 2 (d – e = the perfect square 1)
In the first case, the decimal .4de would be .473, which, when rounded to the nearest tenth, is equal to .5. In the second case, the decimal would be .432, which, when rounded to the nearest tenth, is .4. Thus, statement (1) is not sufficient on its own to answer the question.

Statement (2) tells us that $\sqrt{d} > e^2$. Since d is a single digit, the maximum value for d is 9, which means the maximum square root of d is 3. This means that e^2 must be less than 3. Thus the digit e can only be 0 or 1.
However, this leaves numerous possibilities for the values of d and e respectively. For example, two possibilities are as follows:
d = 9, e = 1
d = 2, e = 0
In the first case, the decimal .4de would be .491, which, when rounded to the nearest tenth, is equal to .5. In the second case, the decimal would be .420, which, when rounded to the nearest tenth, is .4. Thus, statement (2) is not sufficient on its own to answer the question.

Taking both statements together, we know that e must be 0 or 1 and that d – e is equal to 9, 4 or 1. This leaves the following 4 possibilities:

d = 9, e = 0
d = 5, e = 1
d = 4, e = 0
d = 1, e = 0
These possibilities yield the following four decimals: .490, .451, .440, and .410 respectively. The first two of these decimals yield .5 when rounded to the nearest tenth, while the second two decimals yield .4 when rounded to the nearest tenth.
Thus, both statements taken together are not sufficient to answer the question. The correct answer is E: Statements (1) and (2) TOGETHER are NOT sufficient.
The correct answer is E.

7.

Since neither of the terms are divisible by 5, we take each term separately.

$3^4 = 81$ and 81 divided by 5 gives a remainder of 1. So, $3^{21} = (3^4)^5 * 3$

81^5, when divided by 5 gives a remainder of 1 and we are left with 3 for this term.

Since 6 divided by 5 gives a remainder of 1, therefore 6^{55} divided by 5 gives a remainder of 1.

So, $R[\frac{3^{21} * 6^{55}}{5}] = R[\frac{3*1}{5}] = 3$

The correct answer is D.

8.
Let's first calculate x by summing a, b, and c and rounding the result to the tenths place.

a + b + c = 5.45 + 2.98 + 3.76 = 12.17
12.17 rounded to the tenths place = 12.2
x = 12.2

Next, let's find y by first rounding a, b, and c to the tenths place and then summing the resulting values.

5.45 rounded to the tenths place = 5.5
2.98 rounded to the tenths place = 3.0
3.76 rounded to the tenths place = 3.8

5.5 + 3.0 + 3.8 = 12.3
y = 12.3

y – x = 12.3 – 12.2 = .1

The correct answer is D.

9.
To answer the question, let's recall that the tenths digit is the first digit to the right of the decimal point. Let's evaluate each statement individually:

(1) INSUFFICIENT: This statement provides no information about the tenths digit.

(2) INSUFFICIENT: Since the value of the rounded number is 54.5, we know that the original tenths digit prior to rounding was either 4 (if it was rounded up) or 5 (if it stayed the same); however, we cannot answer the question with certainty.

(1) AND (2) SUFFICIENT: Since the hundredths digit of number x is 5, we know that when the number is rounded to the nearest tenth, the original tenths digit increases by 1. Therefore, the tenths digit of number x is one less than that of the rounded number: 5 – 1 = 4.

The correct answer is C.

10.
The question asks whether 8.3xy equals 8.3 when it's rounded to the nearest tenth. This is a Yes/No question, so all we need is a definite "Yes" or a definite "No" for the statement to be sufficient.

(1) SUFFICIENT: When x = 5, then 8.35y rounded to the nearest tenth equals 8.4. Therefore, we have answered the question with a definite "No," so statement (1) is sufficient.

(2) INSUFFICIENT: When y = 9, then 8.3x9 can round to either 8.3 or to 8.4 depending on the value of x. For example, if x = 0, then 8.309 rounds to 8.3. If x = 9, then 8.99 rounds to 8.4. Therefore statement (2) is insufficient.

The correct answer is A.

11.
To determine the value of y rounded to the nearest tenth, we only need to know the value of j.
This is due to the fact that 3 is the hundredths digit (the digit that immediately follows j), which means that j will not be rounded up. Thus, y rounded to the nearest tenth is simply 2.j. We are looking for a statement that leads us to the value of j.
(1) INSUFFICIENT: This does not provide information that allows us to determine the value of j.
(2) SUFFICIENT: Since rounding y to the nearest hundredth has no effect on the tenths digit j, this statement is essentially telling us that j = 7. Thus, y rounded to the nearest tenth equals 2.7. This statement alone answers the question.
The correct answer is B.

12.
(1) SUFFICIENT: If the denominator of d is exactly 8 times the numerator, then d can be simplified to 1/8. Rewritten as a decimal, this is 0.125. Thus, there are not more than 3 nonzero digits to the right of the decimal.

(2) INSUFFICIENT: Knowing that d is equal to a non-repeating decimal does not provide any information about how many nonzero digits are to the right of the decimal point in the decimal representation of d.
The correct answer is A

13.

One way to think about this problem is to consider whether the information provided gives us any definitive information about the digit y.

If the sum of the digits of a number is a multiple of 3, then that number itself must be divisible by 3. The converse holds as well: If the sum of the digits of a number is NOT a multiple of 3, then that number itself must NOT be divisible by 3. Thus, from Statement (1), we know that the numerator of decimal d is NOT a multiple of 3. This alone does not provide us with sufficient information to determine anything about the length of the decimal d. It also does not provide us any information about the digit y.
Statement (2) tells us that 33 is a factor of the denominator of decimal d. Since 33 is composed of the prime factors 3 and 11, we know that the denominator of decimal d must be divisible by both 3 and 11. The denominator 441,682,36y will only be divisible by 11 for ONE unique value of y. We know this because multiples of 11 logically occur once in every 11 numbers. Since there are only 10 possible values for y (the digits 0 through 9), only one of those values will yield a denominator that is a multiple of 11. (It so happens that the value of y must be 2, in order to make the denominator a multiple of 11. It is not essential to determine this - we need only understand that only one value for y will work.)

Given that statement (2) alone allows us to determine one unique value for y, we can use this information to determine the exact value for d and thereby answer the question.

The correct answer is B: Statement (2) alone is sufficient but statement (1) alone is not sufficient to answer the question.
The correct answer is B.

GMAT Quant Topic 4: Numbers
Part E: Sequences and Series

1.
First, let us simplify the problem by rephrasing the question. Since any even number must be divisible by 2, any even multiple of 15 must be divisible by 2 and by 15, or in other words, must be divisible by 30. As a result, finding the sum of even multiples of 15 is equivalent to finding the sum of multiples of 30. By observation, the first multiple of 30 greater than 295 will be equal to 300 and the last multiple of 30 smaller than 615 will be equal to 600.

Thus, since there are no multiples of 30 between 295 and 299 and between 601 and 615, finding the sum of all multiples of 30 between 295 and 615, inclusive, is equivalent to finding the sum of all multiples of 30 between 300 and 600, inclusive. Therefore, we can rephrase the question: ―What is the greatest prime factor of the sum of all multiples of 30 between 300 and 600, inclusive?

The sum of a set = (the mean of the set) × (the number of terms in the set)

Since 300 is the 10th multiple of 30, and 600 is the 20th multiple of 30, we need to count all multiples of 30 between the 10th and the 20th multiples of 30, inclusive.
There are 11 terms in the set: 20th – 10th + 1 = 10 + 1 = 11
The mean of the set = (the first term + the last term) divided by 2: (300 + 600) / 2 = 450
k = the sum of this set = 450 × 11

Note, that since we need to find the greatest prime factor of k, we do not need to compute the actual value of k, but can simply break the product of 450 and 11 into its prime factors: k = 450 × 11 = 2 × 3 × 3 × 5 × 5 × 11

Therefore, the largest prime factor of k is 11.

The correct answer is C.

2. For sequence S, any value S_n equals 6n. Therefore, the problem can be restated as determining the sum of all multiples of 6 between 78 (S_{13}) and 168 (S_{28}), inclusive. The direct but time-consuming approach would be to manually add the terms: 78 + 84 = 162; 162 + 90 = 252; and so forth.

The solution can be found more efficiently by identifying the median of the set and multiplying by the number of terms. Because this set includes an even number of terms, the median equals the average of the two _middle terms, S_{20} and S_{21}, or (120 + 126)/2 = 123.
Given that there are 16 terms in the set, the answer is 16 (123) = 1,968.

The correct answer is D.

3. Let the five consecutive even integers be represented by x, x + 2, x + 4, x + 6, and x +8. Thus, the second, third, and fourth integers are x + 2, x + 4, and x + 6. Since the sum of these three integers is 132, it follows that

3x + 12 = 132, so
3x = 120, and x = 40.

The first integer in the sequence is 40 and the last integer in the sequence is x + 8, or 48.
The sum of 40 and 48 is 88

The correct answer is C.

4. 84 is the 12th multiple of 7. (12 x 7 = 84) 140 is the 20th multiple of 7.
The question is asking us to sum the 12th through the 20th multiples of 7.

The sum of a set = (the mean of the set) x (the number of terms in the set)
There are 9 terms in the set: 20th - 12th + 1 = 8 + 1 = 9
The mean of the set = (the first term + the last term) divided by 2: (84 + 140)/2 = 112
The sum of this set = 112 x 9 = 1008

Alternatively, one could list all nine terms in this set (84, 91, 98 ... 140) and add them.
When adding a number of terms, try to combine terms in a way that makes the addition Easier (i.e. 98 + 112 = 210, 119 + 91 = 210, etc).

The correct answer is C.

5. We can write a formula of this sequence: $S_n = 3S_{n-1}$
(1) SUFFICIENT: If we know the first term $S_1 = 3$, the second term $S_2 = (3)(3) = 9$.
The third term $S_3 = (3)(9) = 27$
The fourth term $S_4 = (3)(27) = 81$
(2) INSUFFICIENT: We can use this information to find the last term and previous terms, however, we don't know how many terms there are between the second to last term and the fourth term.
The correct answer is A.

Top 1% expert replies to student queries (can skip):

Statement 2) The second to last term is 3^10.
This means last term is 3*(3^10) = 3^11
If there are 4 terms in total, then the last term or forth term is 3^11.
If there are 5 terms in total, then the last term or fifth term is 3^11. In this case, the forth term(or second to last term) is 3^10.
Basically, if there are 4 terms in the set, the fourth term is 3^11. However, if there are 5 terms in the set, the fourth term is 3^10.
Since we get 2 different values for the 4th term, this statement is not sufficient.

6.

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Since the units digit of 350 is zero then the number of terms must be multiple of 10. Only answer choice which is multiple of 10 is C (40).

To illustrate consider adding:

*7
*7
...
77
77

=350

So, several 7's and several 77's, note that the # of rows equals to the # of terms. Now, to get 0 for the units digit of the sum the # of rows (# of terms) must be multiple of 10. Only answer choice which is multiple of 10 is C (40).
The correct answer is C.

OR
Let x = the number of 7's and y = the number of 77's.

Total number of terms:
Since the OA represents the total number of terms, we get:
x + y = OA.

Sum of the terms:
Since the sum of the terms is 350, we get:
7x + 77y = 350
7(x + 11y) = 350
x + 11y = 50.

Subtracting the red equation from the blue equation, we get:
(x + 11y) - (x + y) = 350 - OA
10y = 350 - OA
OA = 350 - 10y = (multiple of 10) - (multiple of 10) = multiple of 10.
The correct answer is C.

7.
2+2^2+2^3+2^4+2^5+2^6+2^7+2^8 is a geometric progression
S=(A1+An*q)/(1-q)=-2+2^9
2+S=2^9
The correct answer is A.

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See for pattern:
2 + 2 =2^2
2^2+2^2=2^3
2^3 + 2^3=2^4
....
2^8 + 2^8=2^9
Solving it would not take more than 30 seconds.

Basically,
Just observe the series
The addition of first 2 terms = 3rd term = 2^2
Addition of first 3 terms = 4th term = 2^3
Addition of first 4 terms = 5th term and so on...................
Addition of first 8 terms = 9th term = 2^8
2^8 + 2^8 = 2^9
The correct answer is A.

8.

T= 1/2-1/2^2+1/2^3-...-1/2^10

= 1/4+1/4^2+1/4^3+1/4^4+1/4^5

Notice that 1/4^2+1/4^3+1/4^4+1/4^5 < 1/4,

we can say that 1/4<T<1/2.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

For every integer k from 1 to 10, inclusive the "k"th term of a certain sequence is given by $(-1)^{(k+1)} * (\frac{1}{2^k})$ if T is the sum of the first 10 terms in the sequence, then T is

A. Greater than 2

B. Between 1 and 2

C. Between 1/2 and 1

D. Between 1/4 and 1/2

E. Less than 1/4

First of all we see that there is set of 10 numbers and every even term is negative.

Second it's not hard to get this numbers: $\frac{1}{2}$, $-\frac{1}{4}$, $\frac{1}{8}$, $-\frac{1}{16}$, $\frac{1}{32}$ ... enough for calculations, we see pattern now.

And now the main part: adding them up is quite a job, after calculations you'll get $\frac{341}{1024}$. You can add them up by pairs but it's also time consuming. Once we've done it we can conclude that it's more than $\frac{1}{4}$ and less than $\frac{1}{2}$, so answer is D.

The correct answer is D.

BUT there is shortcut:

Sequence $\frac{1}{2}$, $-\frac{1}{4}$, $\frac{1}{8}$, $-\frac{1}{16}$, $\frac{1}{32}$ ...

represents geometric progression with first term $\frac{1}{2}$ and the common ratio of $-\frac{1}{2}$.

Now, **the sum of infinite geometric progression with common ratio $|r|<1$, is**

$sum=\frac{b}{1-r}$**, where b is the first term.**

So, if the sequence were infinite then the sum would be: $\frac{\frac{1}{2}}{1-(-\frac{1}{2})}=\frac{1}{3}$

This means that no matter how many number (terms) we have their sum will never be more then $\frac{1}{3}$ (A, B and C are out). Also this means that the sum of our sequence is very close to $\frac{1}{3}$ and for sure more than $\frac{1}{4}$ (E out). So the answer is D.

The correct answer is D.

9. Sequence A defines the infinite set: 10, 13, 16 ... 3n + 7.
Set B is a finite set that contains the first x members of sequence A.

Set B is based on an evenly spaced sequence, so its members are also evenly spaced. All evenly spaced sets share the following property: the mean of an evenly spaced set is equal to the median. The most common application of this is in consecutive sets, a type of evenly spaced set. Since the median and mean are the same, we can rephrase this question as: "What is either the median or the mean of set B?"

(1) SUFFICIENT: Only one set of numbers with the pattern 10, 13, 16 ... will add to 275, which means only one value for n (the number of terms) will produce a sum of 275.
For example,
If n = 2, then 10 + 13 = 23
If n = 3, then 10 + 13 + 16 = 39
We can continue these calculations until we reach a sum of 275, at which point we know the value of n. If we know the value for n, then we can write out all of the terms, allowing us to find the median (or the mean). In this case, when n = 11, 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31 + 34 + 37 + 40 = 275. The median is 25. (Though we don't have to do these calculations to see that this statement is sufficient.)
Alternatively, the sum of a set = (the number of terms in the set) × (the mean of the set). The number of the terms of set B is n. The first term is 10 and the last (or nth) term in the set will have a value of 3n + 7, so the mean of the set = (10 + 3n + 7)/2.

Therefore, we can set up the following equation: 275 = n(10 + 3n +7)/2 Simplifying, we get the quadratic: $3n^2 + 17n – 550 = 0$.

This quadratic factors to (3n + 50)(n – 11) = 0, which only has one positive integer root, 11.

Note that we can see that this is sufficient without actually solving this quadratic, however. The - 550 implies that if there are two solutions (not all quadratics have two solutions) there must be a positive and a negative solution. Only the positive solution makes sense for the number of terms in a set, so we know we will have only one positive solution.

Once we have the number of terms in the set, we can use this to calculate the mean (though, again, because this is data sufficiency, we can stop our calculations prior to this point):
= (10 + 3n + 7)/2 = (10 + 3(11) + 7)/2 = 50/2 = 25.

(2) SUFFICIENT: The first term of set B is 10. If the range is 30, the last term must be 10 + 30 = 40. The mean of the set then must be (10 + 40)/2 = 25. This is sufficient.

The correct answer is D.

10.
The first few terms of the sequence are 2, 22, and 222 and each subsequent term has an additional 2 added on. The 30th term then is a string of 30 2's. If we line up the first 30 terms of the sequence to add them up, we will get rows in the following pattern:

2
22
222
2222
22222
:

:

(30) 2's

To find p, the sum of the first 30 terms of S, we would simply be adding columns of 2's. The key here is to see a pattern in the addition process. Starting with the units digit column, all 30 of the terms have a 2 in that position so the sum of the units column would be 30 x 2 = 60. A zero would be written as the units digit of the sum and a six would be carried over to the tens column.

In the tens column, 29 of the 30 terms would have a 2 because the first term has no tens digit. The sum of the tens digits would be 29 x 2 = 58, to which we must add the 6 for a total of 64. The 4 gets written down as the second digit of p and the 6 is carried over to the hundreds column.

In the hundreds column, 28 of the 30 terms would have a 2, the sum of the hundreds digits would be 28 x 2 = 56, to which we must add the 6 again for a total of 62. The 2 gets written down as the third digit of p and the 6 is carried over to the thousands column.

There are two ways to finish this problem. We can do out the remaining 8 columns and find that the 11th digit (i.e. the 10 billions column) will have a sum of 2(20) + 4 = 44 (where the 4 was carried over from the 10th column). 4 then will be the 11th digit (from the right) of p (and a 4 will be carried over into the 12th column).

We could also have seen that each column has one less 2 than the previous, so if we started out with 30 2's in the first column, the 11th column must have 11 – 1 = 10 less 2's, for a total of 20 2's. The amount that is carried over from the previous column could be calculated by realizing that the 10th column had 21 2's for a total of 42. Since there is no way that the 10th column inherited more than 8 from the 9th column, the total must be forty-something and the amount that is carried over to the 11th column MUST BE 4. This makes the total for the 11th column 40 + 4 = 44 and the 11th digit of p 4.

The correct answer is C.

Top 1% expert replies to student queries (can skip)

Focus on last digits of the last 11 numbers: You can just add the numbers (since the numbers consist of only digits of '2', counting becomes very easy:
Formula Sk = S(k–1) + 2(10^(k–1)) gives numbers of the sequence: 2, 22, 222, 2222, ...
2
22
222
2222
22222
222222
2222222
22222222
222222222
2222222222
22222222222
41975308642
(2*11)+ 2(reminder)= 24.
Hence, the eleventh digit of p, counting right to left from the unit digit is 4. C, it is.

OR
This method will also not take a lot of time:
---2
--22
--222
---2,222
---------------------------------------22,222
...
222,222,222,222,222,222,222,222,222,222

Total 30 numbers.

For the first digit (units place) we should add 30 2's --> 30*2=60, so 0 will be units digit and 6 will be carried over;

For the second digit (tens place) we should add 29 2's --> 29*2=58+6=64, so 4 will be written for this digit and 6 will be carried over;

.
.
.
.
.

For the 10th digit we should add 21 2's --> 21*2=42, so the minimum value for the number carried over is 4. Max value is also 4, because even if the carry remained 6, as we had at the beginning, still --> 42+6=48, so still 4 will be carried over;

For the 11th digit we should add 20 2's --> 20*2+4=44, so the 11th digit will be 4.

The correct answer is C.

11.

We can use the formula to calculate the first 10 values of S:

$S_1 = 3$	$S_2 = 2(3) – 2 = 4$	$S_3 = 2(4) – 2 = 6$
$S_4 = 2(6) – 2 = 10$	$S_5 = 2(10) – 2 = 18$	$S_6 = 2(18) – 2 = 34$
$S_7 = 2(34) – 2 = 66$	$S_8 = 2(66) – 2 = 130$	$S_9 = 2(130) – 2 = 258$
$S_{10} = 2(258) – 2 = 514$		

$S_{10} – S_9 = 514 – 258 = 256$.

Alternatively, we could solve this problem by noticing the following pattern in the sequence:

$S_2 – S_1 = 1$ or (2^0)
$S_3 – S_2 = 2$ or (2^1)
$S_4 – S_3 = 4$ or (2^2)
$S_5 – S_4 = 8$ or (2^3)

We could extrapolate this pattern to see that $S_{10} – S_9 = 2^8 = 256$.

The correct answer is E.

12.

If $S_7 = 316$, then $316 = 2S_6 + 4$, which means that $S_6=156$. We can then solve for S_5:

$156 = 2S_5 + 4$, so $S_5 = 76$

Since $S_5 = Q_4$, we know that $Q_4 = 76$ and we can now solve for previous Q_n's to find the first n value for which Q_n is an integer.

To find Q_3: $76 = 4Q_3 + 8$, so $Q_3 = 17$

To find Q_2: $17 = 4Q_2 + 8$, so $Q_2 = 9/2$

It is clear that Q_1 will also not be an integer so there is no need to continue. Q_3 ($n = 3$) is the first integer value.

The correct answer is C.

13.

Noting that a1 = 1, each subsequent term can be calculated as follows:
a1 = 1
a2 = a1 + 1
a3 = a1 + 1 + 2
a4 = a1 + 1 + 2 + 3
ai = a1 + 1 + 2 + 3 + ... + i-1

In other words, ai = a1 plus the sum of the first i - 1 positive integers. In order to determine the sum of the first i - 1 positive integers, find the sum of the first and last terms, which would be 1 and i - 1 respectively, plus the sum of the second and penultimate terms, and so on, while working towards the median of the set. Note that the sum of each pair is always equal to i:

1 + (i - 1) = i
2 + (i – 2) = i
3 + (i – 3) = i
…

Because there are (i - 1)/2 such pairs in a set of i - 1 consecutive integers, this operation can be summarized by the formula i(i - 1)/2. For this problem, substituting a1 = 1 and using this formula for the sum of the first (i-1) integers yields:
ai = 1 + (i)(i - 1)/2

The sixtieth term can be calculated as:
a60 = 1 + (59)(60)/2
a60 = 1,771

The correct answer is D.

14.

To find each successive term in *S*, we add 1 to the previous term and add this to the reciprocal of the previous term plus 1.

$S_1 = 201$

$$S_2 = (201+1) + \frac{1}{(201+1)} = 202 + \frac{1}{202}$$

$$S_3 = \left(202 + \frac{1}{202} + 1\right) + \frac{1}{\left(202 + \frac{1}{202} + 1\right)} = 203 + \frac{1}{202} + \frac{1}{203 + \frac{1}{202}}$$

The question asks to estimate (*Q*), the sum of the first 50 terms of *S.* If we look at the endpoints of the intervals in the answer choices, we see have quite a bit of leeway as far as our estimation is concerned. In fact, we can simply ignore the fractional portion of each term. Let's use $S_2 \approx 202$, $S_3 \approx 203$. In this way, the sum of the first 50 terms of *S* will be approximately equal to the sum of the fifty consecutive integers 201, 202 … 250.

To find the sum of the 50 consecutive integers, we can multiply the mean of the integers by the number of integers since average = sum / (number of terms).

The mean of these 50 integers = (201 + 250) / 2 = 225.5

Therefore, the sum of these 50 integers = 50 x 225.5 = 11,275, which falls between 11,000 and 12,000.

The correct answer is C.

15.

The equation of the sequence can be written as follows: $a_n = (a_{n-1})(x)$, where *x* is the integer constant. So for every term after the first, multiply the previous term by *x*. Essentially, then, all we are doing is multiplying the first term by *x* over and over again. For example, $a_2 = (a_1)(x)$ and $a_3 = (a_2)(x)$ or $a_3 = ((a_1)(x))(x)$, which is the same as $a_3 = (a_1)(x^2)$.

If we keep going, we'll see that $a_3 = (a_1)(x^2)$ and so on for the rest of the sequence. We can thus rewrite the equation of the sequence as $a_n = (a_1)(x^{n-1})$, for all *n* >1.

We also know from the question that $a_5 < 1000$, which means that $(a_1)(x^4) < 1000$.

We are asked for the maximum number of possible nonnegative integer values for a_1; we can get this by minimizing the value of the integer constant, *x*. Since *x* is an integer constant greater than 1, the smallest possible value for *x* is 2. When *x* = 2, then $x^4 = 16$.

We can solve for a_1 as follows:

$(a_1)(x^4) < 1000$
$(a_1)(16) < 1000$
$(a_1) < 62.5$

Thus all the integers from 1 to 62, inclusive, are permissible for a_1. So far we have 62 permissible values.

If a_1 = 0, then it doesn't matter what *x* is, since every term in the sequence will always be 0. So 0 is one more permissible value for a_1.

There is a maximum of 62 +1 (or 63) nonnegative integer values for a_1 in which $a_5 < 1000$.
The correct answer is D.

Alternate sol from gmatclub (additional)

Given sequence:
x;
$x * r$;
$x * r^2$;
$x * r^3$;
$x * r^4 < 1,000$ (where x is the first term and r is the constant greater than 1).

To maximize the # of ***non-negative integer*** values possible for x, we should minimize the value of r and since $r = integer > 1$ then $r = 2$. (**General rule for such kind of problems: to maximize one quantity, minimize the others and to minimize one quantity, maximize the others.**)

Thus, $x * 2^4 < 1,000$ --> $x < \frac{1,000}{16} = 62,5$ --> as the first term must be ***a non-negative integer*** then: $x_{max} = 62$ and $x_{min} = 0$ --> total of 63 values possible for the first term x: {0, 1, 2, ..., 62}.

Answer: D.

16.
The key to solving this problem is to represent the sum of the squares of the second 15 integers as follows: $(15 + 1)^2 + (15 + 2)^2 + (15 + 3)^2 + . . . + (15 + 15)^2$.

Recall the popular quadratic form, $(a + b)^2 = a^2 + 2ab + b^2$. Construct a table that uses this expansion to calculate each component of each term in the series as follows:

$(a + b)^2$	a^2	2ab	b^2
$(15 + 1)^2$	225	2(15)1 =30	1^2
$(15 + 2)^2$	225	2(15)2 =60	2^2
$(15 + 3)^2$	225	2(15)3 =90	3^2
. .	. .	. .	. .
.	.	.	.
$(15 + 15)^2$	225	2(15)15 =450	15^2
TOTALS =	15(225)=3375	(30+450)/2 × 15 = 3600	1240

In order to calculate the desired sum, we can find the sum of each of the last 3 columns and then add these three subtotals together. Note that since each column follows a simple pattern, we do not have to fill in the whole table, but instead only need to calculate a few terms in order to determine the sums.

The column labelled a^2 simply repeats 225 fifteen times; therefore, its sum is 15(225) = 3375.

The column labelled 2ab is an equally spaced series of positive numbers. Recall that the average of such a series is equal to the average of its highest and lowest values; thus, the average term in this series is (30 + 450) / 2 = 240. Since the sum of n numbers in an equally spaced series is simply n times the average of the series, the sum of this series is 15(240) = 3600.

The last column labelled b^2 is the sum of the squares of the first 15 integers. This was given to us in the problem as 1240.

Finally, we sum the 3 column totals together to find the sum of the squares of the second 15 integers: 3375 + 3600 + 1240 = 8215.

The correct answer is D.

17.

In order for the average of a consecutive series of n numbers to be an integer, n must be odd. (If n is even, the average of the series will be the average of the two middle numbers in the series, which will always be an odd multiple of 1/2.)
Statement (1) tells us that n is odd so we know that the average value of the series is an integer. However, we have no way of knowing whether this average is divisible by 3.

Statement (2) tells us that the first number of the series plus $\frac{n-1}{2}$ is an integer divisible by 3.

Since some integer plus $\frac{n-1}{2}$ yields another integer, we know that $\frac{n-1}{2}$ must itself be an integer.

In order for $\frac{n-1}{2}$ to be an integer, n must be odd. (Test this with real numbers for n to see why.) Given that n is odd, let's examine some sample series:
If k is the first number in a series where n = 5, the series is { k, k + 1, k + 2, k + 3, k + 4 }. Note that $\frac{n-1}{2}=\frac{5-1}{2}=2$. Thus, the first term in the series + $\frac{n-1}{2}$ = k + 2. Notice that k + 2 is the middle term of the series.
Now let's try n = 7. The series is now {k, k + 1, k + 2, k + 3, k + 4, k + 5, k + 6}. Note again that $\frac{n-1}{2}=\frac{7-1}{2}=3$. Thus, the first term in the series + $\frac{n-1}{2}$ = k + 3. Notice (again) that k + 3 is the middle term of the series.
This can be generalized for any odd number n. That is, if there are an odd number n terms in a consecutive series of positive integers with first term k then $\frac{n-1}{2}$ = the middle term of the series.
Recall that the middle term of a consecutive series of integers with an odd number of terms is also the average of that series (there are an equal number of terms equidistant from the middle term from both above and below in such a series, thereby canceling each other out). Hence, statement (2) is equivalent to saying that the middle term is an integer divisible by 3. Since the middle term in such a series IS the average value of the series, the average of the series is an integer divisible by 3.
Thus statement (2) alone is sufficient to answer the question and B is the correct answer choice.

The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)

Let the first term of the series be 'a'.
Common difference = 1.
Number of terms = n

Sum of AP = n/2 [2a + (n-1)d] = n/2 [2a + (n-1)]

Average of the series = Sum/n = {n/2 [2a + (n-1)] }/n = a + (n-1)/2

We want to check if the average is an integer divisible by 3.

Statement 2 :

the sum of the first number of the series and (n-1)/2 is an integer divisible by 3.

a + (n-1)/2 is an integer divisible by 3.

But this is exactly the expression [a + (n-1)/2] whose divisibility by 3 we need to evaluate.

a + (n-1)/2 is the average of the series and from statement 2, we know that this expression is an integer divisible by 3. Therefore, statement 2 is sufficient
The correct answer is B.

18.
According to the rule, to form each new term of the series, we multiply the previous term by k, an unknown constant. Thus, since the first term in the series is 64, the second term will be $64k$, the third term will be $64k^2$, the fourth term will be $64k^3$, and so forth.

According to the pattern above, the 25th term in the series will be $64k^{24}$.

Since we are told that the 25th term in the series is 192, we can set up an equation to solve for k as follows:

$$64k^{24} = 192$$
$$k^{24} = 3$$
$$k = 3^{1/24}$$

Now that we have a value for the constant, k, we can use the rule to solve for any term in the series. The 9th term in the series equals $64k^8$.

Plugging in the value for k, yields the following:

$$S_9 = 64k^8 = 64(3^{1/24})^8 = 64(3^{8/24}) = 64(3^{1/3}) = 64\sqrt[3]{3}$$

Answer is $64\sqrt[3]{3}$

19.
In complex sequence questions, the best strategy usually is to look for a pattern in the sequence of terms that will allow you to avoid having to compute every term in the sequence.

In this case, we know that the first term of S_k is 1 and the first term of A_n is 9. So, when $n = 1$ and $k = 1$, $q = 9 + 1 = 10$ and the sum of the digits of q is $1 + 0 = 1$.

Since $S_1 = 1$, $S_2 = (10)(1) + (2) = 10 + 2 = 12$. Since $A_1 = 9$, $A_2 = (10)(9) + (9 - (2 - 1)) = 90 + (9 - 1) = 90 + 8 = 98$. So when $k = 2$ and $n = 2$, $q = 12 + 98 = 110$ and the sum of the digits of q is $1 + 1 + 0 = 2$.

Since $S_2 = 12$, $S_3 = (10)(12) + 3 = 120 + 3 = 123$. Since $A_2 = 98$, it is true that $A_3 = (10)(98) + (9 - (3 - 1)) = 980 + (9 - 2) = 980 + 7 = 987$. So when $n = 3$ and $k = 3$, $q = 123 + 987 = 1110$ and the sum of the digits of q is $1 + 1 + 1 + 0 = 3$.

At this point, we can see a pattern: S_k proceeds as 1, 12, 123, 1234..., and A_n proceeds as 9, 98, 987, 9876.... The sum q therefore proceeds as 10, 110, 1110, 11110... The sum of the digits of q, therefore, will equal 9 when q consists of nine ones and one zero. Since the number of ones in q is equal to the value of n and k (when n and k are equal to each other), the sum of the digits of q will equal 9 when $n = 9$ and $k = 9$: $S_9 = 123456789$ and $A_9 = 987654321$. By way of illustration:

$$\begin{array}{r} 987654321 \\ +\ 123456789 \\ \hline 1111111110 \end{array}$$

When $n > 9$ and $k > 9$, the sum of the digits of q is not equal to 9 because the pattern of 10, 110, 1110..., does not hold past this point and the additional digits in q will cause the sum of the digits of q to exceed 9.

Therefore, the maximum value of $k + n$ (such that the sum of the digits of q is equal to 9) is $9 + 9 = 18$.

The correct answer is E.

20.

At the end of the first week, there are 5 members. During the second week, $5x$ new members are brought in (x new members for every existing member). During the third week, the previous week's new members ($5x$) each bring in x new members: $(5x)*x = 5x^2$ new members. If we continue this pattern to

the twelfth week, we will see that $5x^{11}$ new members join the club that week. Since y is the number of new members joining during week 12, $y = 5x^{11}$.

If $y = 5x^{11}$, we can set each of the answer choices equal to $5x^{11}$ and see which one yields an integer value (since y is a specific number of people, it must be an integer value). The only choice to yield an integer value is (D):

$$5x^{11} = 3^{11}5^{12}$$

$$\frac{5x^{11}}{5} = \frac{3^{11}5^{12}}{5}$$

$$x^{11} = 3^{11}5^{11}$$

$$\sqrt[11]{x^{11}} = \sqrt[11]{3^{11}5^{11}}$$

$$x = (3)(5)$$

Therefore $x = 15$.

Since choice (D) is the only one to yield an integer value, it is the correct answer.

The correct answer is D.

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The questions asks which of the options could be y. That means we will have to somehow eliminate the options apart from the correct one.
At the end of the first week, there are 5 new members;
At the end of the second week, there are 5x new members (since each 5 new members from the previous week brings x new members);
At the end of the third week, there are 5x^2 new members (since each 5x new members from the previous week brings x new members);
...
At the end of the twelfth week, there are 5x^11 new members (since each 5x^10 new members from the previous week brings x new members).
We are given that 5x^11=y. Out of the answers only D yields integer value for x (x=members=integers):
5x^11=3^11 * 5^12 --> x=3*5=15.
Option 1 : 5^1/12 = 5 X^11 => x^11 = 5^(1/12 -1)= 5^(-11/12) (x is not an integer)
Option 2 : 3^11 * 5^11 = 5 X^11 => 3^11 *5^10 = x^11 (x is not an integer)
Option 3 : 3^12 * 5^12 = 5 X^11 => 3^12 * 5^11 = x^11 (x is not an integer)
Option 4 : 3^11 * 5^12 =5* 3^11 * 5^11 = 5 x^11 => 3^11 * 5^11 = x^11 => x = 15 (x is an integer)
Option 5 : 60^12 = 5 X^11 => 12 * 60^12 = x^11 (x is not an integer)
The correct answer is D.

21.

To solve this problem within the time constraints, we can use algebraic expressions to simplify before doing arithmetic. The integers being squared are 9 consecutive integers.
As such we can notate them as x, x+1, x+2, ... x+8, where $x = 36$.
We can then simplify the expression $x^2 + (x+1)^2 + (x+2)^2 + ...(x+8)^2$.
However, there's an even easier way to notate the numbers here. Let's make $x = 40$.
The 9 consecutive integers would then be: x-4, x-3, x-2, x-1, x, x+1, x+2, x+3, x+4.
This way when we square things out, we will have more terms that will cancel. In addition $x = 40$ is an easier value to work with.
The expression can now be simplified as $(x-4)^2 + (x-3)^2 + (x-2)^2 + (x-1)^2 + x^2 + (x+1)^2 + (x+2)^2 + (x+3)^2 + (x+4)^2$:

Combine related terms:
x^2
$(x-1)^2 + (x+1)^2 = 2x^2 + 2$ (notice that the $-2x$ and $2x$ terms cancel out)
$(x-2)^2 + (x+2)^2 = 2x^2 + 8$ (again the $-4x$ and the $4x$ terms cancel out)
$(x-3)^2 + (x+3)^2 = 2x^2 + 18$ (the $-6x$ and $6x$ terms cancel out)
$(x-4)^2 + (x+4)^2 = 2x^2 + 32$ (the $-8x$ and $8x$ terms cancel out)
If we total these groups together, we get $9x^2 + 60$.
If $x = 40$, $x^2 = 1600$.
$9x^2 + 60 = 14400 + 60 = 14460$
The correct answer is C.

22.
Since the membership of the new group increases by 700% every 10 months, after the first 10-month period the new group will have (8)(4) members (remember that to increase by 700% is to increase eightfold). After the second 10-month period, it will have (8)(8)(4) members. After the third 10-month period, it will have (8)(8)(8)(4) members. We can now see a pattern: the number of members in the new

group can be expressed as $(4)(8^x)$, where x is the number of 10-month periods that have elapsed.

Since the membership of the established group doubles every 5 months (remember, to increase by 100% is to double), it will have (2)(4096) after the first 5-month period. After another 5 months, it will have (2)((2)(4096)) members. After another 5 months, it will have (2)((2)(2)(4096)). We can now see a pattern: the number of members in the established group will be equal to $(4096)(2^y)$, where y represents the number of 5-month periods that have elapsed.

The question asks us after how many months the two groups will have the same number of members. Essentially, then, we need to know when $(4096)(2^y) = (4)(8^x)$. Since y represents the number of 5- month periods and x represents the number of 10-month periods, we know that $y = 2x$. We can rewrite the equation as $(4096)(2^y) = (4)(8^x)$. We now need to solve for x, which represents the number of 10-month periods that elapse before the two groups have the same number of members. The next step we need to take is to break all numbers down into their prime factors:

$4 = 2^2$
$8 = 2^3$
$4096 = 2^{12}$

We can now rewrite the equation:

$(2^{12})(2^{2x}) = (2^2)(2^3)^x \rightarrow$
$2^{2x+12} = (2^2)(2^{3x}) \rightarrow$
$2^{2x+12} = 2^{3x+2}$

Since the bases are equal on both sides of the equation, the exponents must be equal as well. Therefore, it must be true that $2x + 12 = 3x + 2$. We can solve for x:

$2x + 12 = 3x + 2 \rightarrow$
$10 = x$

If $x = 10$, then 10 ten-month periods will elapse before the two groups have equal membership rolls, for a total of 100 months.

The correct answer is E.

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Couple of things should tingle in your mathematical sense on seeing this question:

1) I see 4096, 4 and things becoming 2 times and 8 times (increasing by 100% and 700% respectively). This is all about powers of 2

2) The number of months after which the two orgs will have equal number of members will be divisible by both 5 and 10

Let the number of months be x

At each multiplication of the established org, things become times 2. In x months, there are (x/5) multiplications that will happen. So the final number will be 4096*2^(x/5)

At each multiplication of the new org, things become times 8 (= 2^3). In x months, there are (x/10) multiplications that will happen. So the final number will be 4*2^(3x/10)

So 12 + x/5 = 2 + 3x/10

Or, x/10 = 10

Or, x = 100 months

The correct answer is E.

23.

The ratio of A_n to $x(1 + x(1 + x(1 + x(1 + x))))$ will look like this:

$$\frac{x^{n-1} + x^n + x^{n+1} + x^{n+2} + x^{n+3}}{x(1 + x(1 + x(1 + x(1 + x))))}$$

So the question is: For what value of *n* is

$$\frac{x^{n-1} + x^n + x^{n+1} + x^{n+2} + x^{n+3}}{x(1 + x(1 + x(1 + x(1 + x))))} = x^5$$

First, let's distribute the expression $x(1 + x(1 + x(1 + x(1 + x))))$, starting with the inner most parentheses:

$$x(1 + x(1 + x(1 + x(1 + x)))) \rightarrow$$
$$x(1 + x(1 + x(1 + x + x^2))) \rightarrow$$
$$x(1 + x(1 + x + x^2 + x^3)) \rightarrow$$
$$x(1 + x + x^2 + x^3 + x^4) \rightarrow$$
$$x^1 + x^2 + x^3 + x^4 + x^5$$

Now we can rephrase the question: For what value of *n* is $\frac{x^{n-1} + x^n + x^{n+1} + x^{n+2} + x^{n+3}}{x^1 + x^2 + x^3 + x^4 + x^5} = x^5$?

We can cross-multiply:

$$x^{n-1} + x^n + x^{n+1} + x^{n+2} + x^{n+3} = x^5(x^1 + x^2 + x^3 + x^4 + x^5) \rightarrow$$
$$x^{n-1} + x^n + x^{n+1} + x^{n+2} + x^{n+3} = x^6 + x^7 + x^8 + x^9 + x^{10}$$

Therefore. *n* must equal 7.

The correct answer is B.

24.

We are given $x = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}}$.

This can be rewritten as: $x = \sqrt{2+\left(\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}\right)}$.

Since the entire right-hand-side of the equation repeats itself an infinite number of times, we can say that the expression inside the parentheses is actually equal to *x*.

Consequently, we can replace the expression within the parentheses by x as follows:

$$x = \sqrt{2+x}$$

Now we have an equation for which we can solve for x as follows:

$$x = \sqrt{2+x}$$
$$x^2 = 2+x$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = \{2, -1\}$$

Since *x* was specified to be a positive number, $x = 2$.

The correct answer is B.

25.

The sum of a set of integers = (mean of the set) × (number of terms in the set) The mean of the set of consecutive even integers from 200 to 400 is (200 + 400)/2 = 300 (i.e. the same as the mean of the first and last term in the consecutive set). The number of terms in the set is 101. Between 200 and 400, inclusive, there are 201 terms. 100 of them are odd, 101 of them are even, since the set begins and ends on an even term. Sum of the set = 300 × 101 = 30,300.
The correct answer is C.

26.
The best approach to this problem is to attempt to find a pattern among the numbers. If we scan the table, we see that there are five sets of consecutive integers represented in the five columns:

98, 99, 100, 101, 102

-196, -198, -200, -202, -204

290, 295, 300, 305, 310

-396, -398, -400, -402, -404

498, 499, 500, 501, 502

To find the sum of a set of consecutive integers we can use the formula:
Sum of consecutive set = (number of terms in the set) × (mean of the set). Each group contains 5 consecutive integers and the mean of a consecutive set is always equal to the median (or the middle term if there is an odd number of terms). In this way we can find the sum of the five sets:
5(100) = 500
5(-200) = -1,000
5(300) = 1,500
5(-400) = -2,000

5(500) = 2,500

Therefore, the sum of all the integers is:
500 + (-1,000) + 1,500 + (-2,000) + 2,500 = 1,500.

The correct answer is D.

27.
The key to solving this problem quickly is organization. Build a table listing n and f(n). Go ahead and list them all, but save time where possible. For instance, notice that you can drop the odd factors – since the question deals only with factors of 2.

n	f(n)
1	$2! \div 1! = 2 \times 1$
2	$4! \div 2! = 4 \times 3$
3	$6! \div 3! = 6 \times 5 \times 4$
4	$8! \div 4! = 8 \times 7 \times 6 \times 5$
5	$10! \div 5! = 10 \times 8 \times 6$ (realize you can drop the odds)
6	$12! \div 6! = 12 \times 10 \times 8$
7	$14! \div 7! = 14 \times 12 \times 10 \times 8$
8	$16! \div 8! = 16 \times 14 \times 12 \times 10$
9	$18! \div 9! = 18 \times 16 \times 14 \times 12 \times 10$
10	$20! \div 10! = 20 \times 18 \times 16 \times 14 \times 12$

Since x is defined as the product of the first ten terms of the sequence, we must sum all of the factors of 2 for each term, using the following factor principles:

*Multiples of 4 have at least 2 factors of 2. Therefore, evens that are not multiples of 4 (for example, 2 or 6) have only 1 factor of 2.
*Multiples of 8 have at least 3 factors of 2. Therefore, if a multiple of 4 is not also a multiple of 8 (for example, 4 or 12), then that multiple of 4 has exactly 2 factors of 2.
*Multiples of 16 have at least 4 factors of 2. Multiples of 8 that are not also multiples of 16 have exactly 3 factors of 2.

A third column can now be added to the chart as follows:

n	f(n)	Powers of 2
1	2	1
2	4	2
3	6×4	1+2 = 3
4	8×6	3+1 = 4
5	$10 \times 8 \times 6$	1+3+1 = 5
6	$12 \times 10 \times 8$	2+1+3 = 6
7	$14 \times 12 \times 10 \times 8$	1+2+1+3 = 7
8	$16 \times 14 \times 12 \times 10$	4+1+2+1 = 8
9	$18 \times 16 \times 14 \times 12 \times 10$	1+4+1+2+1 = 9
10	$20 \times 18 \times 16 \times 14 \times 12$	2+1+4+1+2 = 10

You may notice a pattern before having to complete the chart. The sum of all the factors of 2 in the product of the first 10 terms in the sequence is 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 +10 = **55**.

Therefore, 2^{55} is the greatest factor of 2.
The correct answer is E.

Alternate Solution from Gmatclub

Given: $f(n) = \frac{(2n)!}{n!}$, so $f(1) = \frac{(2)!}{1!}$, $f(2) = \frac{(4)!}{2!}$, $f(3) = \frac{(6)!}{3!}$, ...

$x = \frac{(2)!}{1!} * \frac{(4)!}{2!} * \frac{(6)!}{3!} * ... * \frac{(20)!}{10!}$.

Now, try to figure out the pattern in powers of 2 instead of brute force (reducing, factoring out 2's,...).

$\frac{(2)!}{1!} = 2^1;$

$\frac{(4)!}{2!} = 2^2 * 3;$

$\frac{(6)!}{3!} = 2^2 * 5 * 6 = 2^3 * something;$

$\frac{(8)!}{4!} = 2^4 * 3 * 5 * 7 = 2^4 * something;$

...

$\frac{(20)!}{10!} = 2^{10} * something$

Basically the power of 2 goes up by 1 with each step.

So the greatest factor of x (greatest power of 2 which is a factor of x) will be $2^1 * 2^2 * 2^1 * ... * 2^{10} = 2^{1+2+3+4+5+6+7+8+9+10} = 2^{55}$ (mean*# of terms=5.5*10).

Answer: E.

The correct answer is E.

GMAT Quant Topic 4: Numbers
Part F: Remainders, Divisibility

1.
If there is a remainder of 5 when x is divided by 9, it must be true that x is five more than a multiple of 9. We can express this algebraically as $x = 9a + 5$, where a is a positive integer.

The question asks for the remainder when 3x is divided by 9. If $x = 9a + 5$, then 3x can be expressed as $3x = 27a + 15$ (we just multiply the equation by 3). If we divide the right side of the equation by 9, we get $3a + 15/9$. 9 will go once into 15, leaving a remainder of 6.

Alternatively, we can pick numbers. If we add the divisor (in this case 9) to the remainder (in this case 5) we get the smallest possibility for x. $9 + 5 = 14$ (and note that 14/9 leaves a remainder of 5). 3x then gives us $3(14) = 42$. 42/9 gives us 4 remainder 6 (since $4 \times 9 = 36$ and $36 + 6 = 42$).

The correct answer is E.

2. The definition given tells us that when x is divided by y a remainder of (x # y) results. Consequently, when 16 is divided by y a remainder of (16 # y) results. Since (16 # y) = 1, we can conclude that when 16 is divided by y a remainder of 1 results.

   Therefore, in determining the possible values of y, we must find all the integers that will divide into 16 and leave a remainder of 1. These integers are 3 , 5, and 15. The sum of these integers is 23.

   The correct answer is D.

3.
The value $\sqrt{288kx}$ can be simplified to $12\sqrt{2kx}$. Given that x is divisible by 6, for the purpose of solving this problem x might be restated as 6y, where y may be any positive integer.
The expression $\sqrt{288kx}$ could then be further simplified to

$12\sqrt{12ky}$

or

$24\sqrt{3ky}$

Therefore, each answer choice CAN be a solution if and only if there is an integer y such that $24\sqrt{3ky}$ equals that answer choice.
The following table shows such an integer value of y for four of the possible answer choices, which therefore CAN be a solution.

y	Solution
1	$24\sqrt{3k}$
2	$24\sqrt{6k}$
3	$72\sqrt{k}$
k	$24k\sqrt{3}$

The answer choice that cannot be the value of $\sqrt{288kx}$ is $24\sqrt{k}$. For this expression to be a possible solution, y would have to equal 1/3, which is not a positive integer. Put another way, this solution would require that $x = 2$, which cannot be true because x is divisible by 6.

The correct answer is B.

Alternate sol from gmatclub (additional)

x is divisible by 6 --> $x = 6n$ for some positive integer n.

$$\sqrt{288kx} = \sqrt{2^5 * 3^2 * k * 6n} = \sqrt{2^6 * 3^3 * kn} = 2^3 * 3\sqrt{3kn} = 24\sqrt{3kn}.$$

Now, for any values of positive integers k and n $24\sqrt{3kn}$ is always more than $24\sqrt{k}$ (B): $24\sqrt{3kn} > 24\sqrt{k}$ --> $\sqrt{3n} > 1$.

Answer: B.

4.
First consider an easier expression such as $10^5 - 560$. Doing the computation yields 99,440, which has 2 9's followed by 440.
From this, we can extrapolate that $10^{25} - 560$ will have a string of 22 9's followed by 440. Now simply apply your divisibility rules:

You might want to skip 11 first because there is no straightforward rule for divisibility by 11. You can always return to this if necessary. [One complex way to test divisibility by 11 is to assign opposite signs to adjacent digits and then to add them to see if they add up to 0. For example, we know that 121 is divisible by 11 because -1 +2 -1 equals zero. In our case, the twenty-two 9s, when assigned opposite signs, will add up to zero, and so will the digits of 440, since +4 -4 +0 equals zero.]

If the last three digits of the number are divisible by 8, the number is divisible by 8. Since 440 is divisible by 8, the entire expression is divisible by 8.

If the last two digits of the number are divisible by 4, the number is divisible by 4. Since 40 is divisible by 4, the enter expression is divisible by 4.

If a number ends in 0 or 5, it is divisible by 5. Since the expression ends in 0, it is divisible by 5. For a number to be divisible by three, the sum of the digits must be divisible by three. The sum of the 22 9's will be divisible by three but when you add the sum of the last three digits, 8 (4 + 4 + 0), the result will not be divisible by 3. Thus, the expression will NOT be divisible by 3.

The correct answer is E.

5.
The problem states that when x is divided by y the remainder is 6. In general, the divisor (y in this case) will always be greater than the remainder. To illustrate this concept, let's look at a few examples:

15/4 gives 3 remainder 3 (the divisor 4 is greater than the remainder 3)

25/3 gives 8 remainder 1 (the divisor 3 is greater than the remainder 1)

46/7 gives 6 remainder 4 (the divisor 7 is greater than the remainder 4)

In the case at hand, we can therefore conclude that y must be greater than 6.

The problem also states that when a is divided by b the remainder is 9. Therefore, we can conclude that b must be greater than 9.

If y > 6 and b > 9, then y + b > 6 + 9 > 15. Thus, 15 cannot be the sum of y and b.

The correct answer is E.

6.
After considering the restrictions in the original problem, we have four possibilities: 3 teams of 8 players each, 4 teams of 6 players each, 6 teams of 4 players each, or 8 teams of 3 players each.

(1) INSUFFICIENT: If one person sits out, then 12 new players are evenly distributed among the teams. This can be achieved if there are 3, 4 or 6 teams, since 12 is a multiple of 3, 4, and 6

(2) INSUFFICIENT: If one person sits out, then 6 new players are evenly distributed among the teams. This can be achieved if there are 3 or 6 teams, since 6 is a multiple of 3 and 6.

(1) AND (2) INSUFFICIENT: If we combine the information in both statements, we can determine that the number of teams must be either 3 or 6 (since either number of teams would agree with the information contained in either statement). We cannot, however, determine whether we have 3 teams or 6 teams. Therefore, we cannot answer the question.

The correct answer is E.

7.
The remainder is what is left over after 4 has gone wholly into x as many times as possible. For example, suppose that x is 10. 4 goes into 10 two whole times ($2 \times 4 = 8 < 10$), but not quite three times ($3 \times 4 = 12 > 10$). The remainder is what is left over: $10 - 8 = 2$.

(1) INSUFFICIENT: This statement tells us that x/3 must be an odd integer, because that is the only way we would have a remainder of 1 after dividing by 2. Thus, x is (3 × an odd integer), and (odd × odd = odd), so x must be an odd multiple of 3. The question stem tells us that x is positive. So, x could be any positive, odd integer that is a multiple of 3: 3, 9, 15, 21, 27, 33, 39, 45, etc. Now we need to answer the question ―when x is divided by 4, is the remainder equal to 3? for every possible value of x on the list. For x = 15, the answer is ―yes, since 15/4 results in a remainder of 3. For x = 9, the answer is ―no, since 9/4 results in a remainder of 1. The answer to the question might be ―yes or ―no, depending on the value of x, so we are not able to give a definite answer based on the information given.

(2) INSUFFICIENT: This statement tells us that x is a multiple of 5. The question stem tells us that x is a positive integer. So, x could be any positive integer that is a multiple of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, etc. Now we need to answer the question ―when x is divided by 4, is the remainder equal to 3?‖ for every possible value of x on the list. For x = 15, the answer is ―yes, since 15/4 results in a remainder of 3. For x = 5, the answer is ―no, since 5/4 results in a remainder of 1. The answer to the question might be ―yes or ―no, depending on the value of x, so we are not able to give a definite answer based on the information given.

(1) AND (2) INSUFFICIENT: From the two statements, we know that x is an odd multiple of 3 and that x is a multiple of 5. In order for x to be both a multiple of 3 and 5, it must be a multiple of 15 ($15 = 3 \times 5$). The question stem tells us that x is a positive integer. So, x could be any odd, positive integer that is a multiple of 15: 15, 45, 75, 105, etc. Now we need to answer the question ―when x is divided by 4, is the remainder equal to 3? for every possible value of x on the list. For x = 15, the answer is ―yes, since 15/4 results in a remainder of 3. For x = 45, the answer is ―no, since 45/4 results in a remainder of 1. The answer to the question might be ―yes or ―no, depending on the value of x, so we are not able to give a definite answer based on the information given.

The correct answer is E.

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Statement 1) When x/3 is divided by 2, the remainder is 1
i.e. x/3 is Odd number
i.e. x/3 could be 1, 3, 5, 7 etc
i.e. x = 3, 9, 15, 21 etc

i.e. on Dividing x by 4 we get remainder = 3, 1, 3, 1 etc respectively
i.e. remainders are not consistent therefore NOT SUFFICIENT
Statement 2) x is divisible by 5
i.e. x could be 5, 10, 15, 20 etc
i.e. on Dividing x by 4 we get remainder = 1, 2, 3, 0 etc respectively
i.e. remainders are not consistent therefore NOT SUFFICIENT
Combining the two statements:
x can be an odd multiple of 3 and 5
i.e. x can be only an odd multiple of 15
i.e. x can be 15, 45, 75 etc.
On dividing possible values of x by 4 the remainder are 3, 1, 3 etc.
i.e. remainders are not consistent therefore NOT SUFFICIENT.
The correct answer is E.

8.
A remainder, by definition, is always smaller than the divisor and always an integer. In this problem, the divisor is 7, so the remainders all must be integers smaller than 7. The possibilities, then, are 0, 1, 2, 3, 4, 5, and 6. In order to calculate the sum, we need to know which remainders are created.

(1) INSUFFICIENT: The range is defined as the difference between the largest number and the smallest number in a given set of integers. In this particular question, a range of 6 indicates that the difference between the largest remainder and the smallest remainder is 6. However, this does not tell us any information about the rest of the remainders; though we know the smallest term is 0, and the largest is 6, the other remainders could be any values between 0 and 6, which would result in varying sums.

(2) SUFFICIENT: By definition, when we divide a consecutive set of seven integers by seven, we will get one each of the seven possibilities for remainder. For example, let's pick 11, 12, 13, 14, 15, 16, and 17 for our set of seven integers (x_1 through x_7). The remainders are as follows:
$x_1 = 11$. $11/7 = 1$ remainder 4
$x_2 = 12$. $12/7 = 1$ remainder 5
$x_3 = 13$. $13/7 = 1$ remainder 6
$x_4 = 14$. $14/7 = 2$ remainder 0
$x_5 = 15$. $15/7 = 2$ remainder 1
$x_6 = 16$. $16/7 = 2$ remainder 2
$x_7 = 17$. $17/7 = 2$ remainder 3

Alternatively, you can solve the problem algebraically. When you pick 7 consecutive integers on a number line, one and only one of the integers will be a multiple of 7. This number can be expressed as 7n, where n is an integer. Each of the other six consecutive integers will cover one of the other possible remainders: 1, 2, 3, 4, 5, and 6. It makes no difference whether the multiple of 7 is the first integer in the set, the middle one or the last. To prove this consider the set in which the multiple of 7 is the first integer in the set. The seven consecutive integers will be: $7n$, $7n + 1$, $7n + 2$, $7n + 3$, $7n + 4$, $7n + 5$, $7n + 6$. The sum of the remainders here would be $0 + 1 + 2 + 3 + 4 + 5 + 6 = 21$.

The correct answer is B.

9.
If the integer x divided by y has a remainder of 60, then x can be expressed as: $x = ky + 60$, where k is an integer (i.e. y goes into x k times with a remainder of 60)

We could also write an expression for the quotient x/y: $x/y = k+60/y$

Notice that k is still the number of times that y goes into x evenly. 60/y is the decimal portion of the quotient, i.e. the remainder over the divisor y.
The first step to solving this problem is realizing that k, the number of times that y goes into x evenly, can be anything for this question since we are only given a value for the remainder. The integer values before the decimal point in answers I, II and III are irrelevant.

The decimal portion of the possible quotients in I, II and III are another story. From the equation we have above, for a decimal to be possible, it must be something that can be expressed as 60/y, since that is the portion of the quotient that corresponds to the decimal. But couldn't any decimal be expressed as 60 over some y? The answer is NO because we are told in the question that y is an integer.

Let's look at answer choice I first. Is 60 / y = 0.15, where y is an integer? This question is tantamount to asking if 60 is divisible by 0.15 or if 6000 is divisible by 15? 6000 IS divisible by 15 because it is divisible by 5 (ends in a 0) and by 3 (sum of digits, 6, is divisible by 3) **Therefore, answer choice I is CORRECT.** Using the same logic for answer choice II, we must check to see if 6000 is divisible by 16. 6000 IS divisible by 16 because it is can be divided by 2 four times: 3000, 1500, 750, 375. **Therefore, answer choice II is CORRECT.** 6000 IS NOT divisible by 17 because 17 is prime and not part of the prime make-up of 6000. **Therefore, answer choice III is NOT CORRECT.**

The correct answer is D.

10.

(1) INSUFFICIENT: At first glance, this may seem sufficient since if 5 is the remainder when k is divided by j, then there will always exist a positive integer m such that k = jm + 5. In this case, m is equal to the integer quotient and 5 is the remainder. For example, if k = 13 and j = 8, and 13 divided by 8 has remainder 5, it must follow that there exists an m such that k = jm + 5: m = 1 and 13 = (8)(1) + 5.

However, the logic does not go the other way: 5 is not necessarily the remainder when k is divided by j. For example, if k = 13 and j = 2, there exists an m (m = 4) such that k = jm + 5: 13 = (2)(4) + 5, consistent with statement (1), yet 13 divided by 2 has remainder 1 rather than 5.

When j < 5 (e.g., 2 < 5); this means that j can go into 5 (e.g., 2 can go into 5) at least one more time, and consequently m is not the true quotient of k divided by j and 5 is not the true remainder. Similarly, if we let k = 14 and j = 3, there exists an m (e.g., m = 3) such that statement (1) is also satisfied [i.e., 14 = (3)(3) + 5], yet the remainder when 14 is divided by 3 is 2, a different result than the first example.

Statement (1) tells us that k = jm + 5, where m is a positive integer. That means that k/j = m + 5/j = integer + 5/j. Thus, the remainder when k is divided by j is either 5 (when j > 5), or equal to the remainder of 5/j (when j is 5 or less). Since we do not know whether j is greater than or less than 5, we cannot determine the remainder when k is divided by j.

(Continued on next page)

(2) INSUFFICIENT: This only gives the range of possible values of j and by itself does not give any insight as to the value of the remainder when k is divided by j.

(1) AND (2) SUFFICIENT: Statement (1) was not sufficient because we were not given whether 5 > j, so we could not be sure whether j could go into 5 (or k) any additional times. However, (2) tells us that j > 5, so we now know that j cannot go into 5 any more times. This means that m is the exact number of times that k can be divided by j and that 5 is the true remainder.

Another way of putting this is: From statement (1) we know that k/j = m + 5/j = integer + 5/j. From statement (2) we know that j > 5. Therefore, the remainder when k is divided by j must always be 5.
The correct answer is C.

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Que

$j, k \rightarrow$ +ve int   Remainder when 'k' is divided by 'j'.

$k > j$

St1 $k = j\underset{\text{int}}{m} + 5$.

when k is divided by 'j'.

$\frac{k}{j} = \frac{jm}{j} + \frac{5}{j}$

This term is exactly divisible by j, so remainder is '0'.

We don't know what 'j' here.

Example: $k = 6, j = 1$.

So $6 = 1 \times 1 + 5$

when you divide 6 by 1, the remainder is '0'.

$k = 7, j = 2$

$7 = 2 \times 1 + 5$

when you divided 7 by 2, the remainder is '1'.

Hence, Not Sufficient.

St2 $j > 5$

$j = 6, k = 7$, Remainder 1
$j = 6, k = 8$, Remainder 2   Not Sufficient.

St1 + 2 → $\frac{k}{j} = \underbrace{\frac{jm}{j}}_{\text{1st term}} + \underbrace{\frac{5}{j}}_{\text{2nd term}}$, $j > 5$ ⇒ 1st term is divisible by 'j'.
⇒ 2nd term will leave a remainder '5' as j is greater than 5.

Hence, Remainder ⇒ 5, (Option C)

The correct answer is C.

11.
In a set of consecutive integers with an odd number of terms, the average of the set is the middle term. Since the question tells us that we have 5 consecutive integers, we know that the average of the set is the middle term. For example, in the set {1, 2, 3, 4, 5}, the average is 15/5
= 3, which is the middle term. We are also told that the average is odd, which means the 5 integers of the set must go as follows: {odd, even, odd, even, odd}.

We are then asked for the remainder when the largest of the five integers is divided by 4. Since the largest integer must be odd, we know that it cannot be a multiple of 4 itself. So the remainder depends on how far this largest integer is from the closest multiple of 4 smaller than it. Since there are five numbers in the set, at least one of them must be a multiple of 4 (remember that counting from 1, every fourth integer is a multiple of 4).

Statement 1 tells us that the third of the five integers is a prime number. The third integer is the middle integer. Knowing that it is a prime number does not tell us which of the five integers is a multiple of 4. If the middle number is 17, then the second number is 16 (a multiple of 4) and the largest number is 19, yielding a remainder of 3 when divided by 4. But if the middle number is 7, then the fourth number is 8 (a multiple of 4) and the largest number is 9, yielding a remainder of 1 when divided by 4. Insufficient.

Statement 2 tells us that the second of the integers is the square of an integer. Since the middle integer is odd, we know the second integer is even. If the second integer is even **and** the square of an integer, it must be a multiple of 4.

To see this clearly, let's think about the square root of the second integer. Since the second integer is even, its square root must be even. We can call this root 2x (since all even numbers are multiples of 2). Now, to find the second integer, we must square 2x to get $4x^2$. So the second integer must be a multiple of 4. Therefore, the largest integer can be expressed as $4x^2 + 3$. So the remainder when the largest integer is divided by 4 will be 3. Sufficient.

The correct answer is B: Statement 2 alone is sufficient to answer the question but statement 1 alone is not.
The correct answer is B.

12.
For the cookies to be split evenly between Laurel and Jean without leftovers, the number of cookies, c, must be even. We can rephrase the question: "Is c even?"
(1) INSUFFICIENT: If there is one cookie left over when c is divided among three people, then c = 3x + 1, where x is an integer. This does not tell us if c is odd or even. The expression 3x could be odd or even (depending on x) so adding 1 to it could result in an odd or even answer. For example, if x = 1, then c = 4, which is even. If x = 2, then c = 7, which is odd.

(2) SUFFICIENT: If the cookies will divide evenly by two if three cookies are first eaten, then c - 3 is even. c itself must be odd: an odd minus an odd is even. This answers our rephrased question with a definite NO. (Recall that "no" is a sufficient answer to a yes/no data sufficiency question. Only "maybe" is insufficient.)
The correct answer is B.

13.
(1) INSUFFICIENT: We are told that 5n/18 is an integer. This does not allow us to determine whether n/18 is an integer. We can come up with one example where 5n/18 is an integer and where n/18 is **NOT** an integer. We can come up with another example where 5n/18 is an integer and where n/18 **IS** an integer.

Let's first look at an example where 5n/18 is equal to the integer 1.

If 5n/18= 1, then n/18 = 1/5. In this case n/18 is NOT an integer.
Let's next look at an example where 5n/18 is equal to the integer 15.
If 5n/18= 15, then n/18 = 3 In this case n/18 is an integer.
Thus, Statement (1) is NOT sufficient.

(2) INSUFFICIENT: We can use the same reasoning for Statement (2) that we did for statement (1). If 3n/18 is equal to the integer 1, then n/18 is NOT an integer. If 3n/18 is equal to the integer 9, then n/18 IS an integer.
(1) AND (2) SUFFICIENT: If 5n/18 and 3n/18 are both integers, n/18 must itself be an integer. Let's test some examples to see why this is the case.

The first possible value of n is 18, since this is the first value of n that ensures that **both** 5n/18 and 3n/18 are integers. If n = 18, then n/18 is an integer. Another possible value of n is 36. (This value also ensures that both 5n/18 and 3n/18 are integers). If n = 36, then n/18 is an integer.

A pattern begins to emerge: the fact that 5n/18 AND 3n/18 are both integers limits the possible values of n to multiples of 18. Since n must be a multiple of 18, we know that n/18 must be an integer. The correct answer is C.

Another way to understand this solution is to note that according to (1), n = (18/5)*integer, and according to (2), n = 6*integer. In other words, n is a multiple of both 18/5 and 6. The least common multiple of these two numbers is 18. In order to see this, write 6 = 30/5. The LCM of the numerators 18 and 30 is 90. Therefore, the LCM of the fractions is 90/5 = 18.

Again, **the correct answer is C.**

14.
There is no obvious way to rephrase this question. There are too many possibilities for a and b that would yield an "a + b" which is a multiple of 3.

(1) SUFFICIENT: The two-digit number "ab" can be represented by the expression 10a + b.

Since 10a + b is a multiple of 3, 10a + b = 3k, where k is some integer.
This can be rewritten as 9a + (a + b) = 3k (we are being asked about a + b). If we solve for the expression a + b, we get: a + b = 3k - 9a.

3k – 9a can be factored to 3(k – 3a).
Since both k and a are integers, 3(k – 3a) must be a multiple of 3. Therefore a + b is also a multiple of 3.

(2) SUFFICIENT: Since a – 2b is a multiple of 3, a – 2b = 3k, where k is some integer. This can be rewritten as a + b – 3b = 3k (we are being asked about a + b).
If we solve for the expression a + b, we get: a + b = 3k + 3b .

3k + 3b can be factored to 3(k + b).
Since both k and b are integers, 3(k + b) must be a multiple of 3. Therefore a + b is also a multiple of 3.

The correct answer is D.

15.
If the ratio of cupcakes to children is 104 to 7, we can first express the number of cupcakes and children as 104n and 7n, where n is some positive integer. If n = 1, for example, there are 104 cupcakes and 7 children; if n = 2, there are 208 cupcakes and 14 children, etc.
We are told in the problem that each of the children eats exactly x cupcakes and that there are some number of cupcakes leftover (i.e. a remainder) that is less than the number of children. Let's call the remainder R. This means that means that the number of children, 7n, goes into the number of cupcakes, 104n, x times with a remainder of R. We can use this to write out the following equation:
104n = 7nx + R.
We are asked here to find out information about the divisibility R. Often times with remainder questions the easiest thing to do is to try numbers:
If n = 1, the problem becomes what is true of the remainder when you divide 104 by 7.
n = 1 104/7 = 14 remainder 6
n = 2 208/14 = 14 remainder 12
n = 3 312/21 = 14 remainder 18

Notice the pattern here. With 104 and 7, we started out with a remainder of 6. When we doubled both the numerator (104) and the denominator (7), the quotient remained the same (14), and the remainder (6) simply doubled. In this particular problem, the remainder when n = 1 was 6, which as we know is divisible by 2 and 3. Since all subsequent multiples of 104 and 7 (i.e. n = 2, 3, 4…) will yield remainders that are multiples of this original 6, the remainder will always be divisible by 2 and 3 and the answer here is D.
There is a more algebraic reason why the remainder always remains a multiple of the original remainder, 6. Let's take for example a number x that when divided by y, qives a quotient of q with a a remainder of r. An equation can be written: x = qy + r. If we multiply p by some constant, c, we must multiply both sides of the equation above, i.e. xc = cqy + cr. Notice that it is not just the x and y that get multiplied by a factor of c, but also r, the remainder! We can generalize to say that if x divided by y has a quotient of q and a remainder of r, a multiple of x divided by that same multiple of y will have the original quotient and the same multiple of the original remainder.

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Let the number of children be 7n
Let the number of cupcakes be 104n
Since each child eats x cupcakes, total cupcakes that are eaten are 7nx

Number of uneaten cupcakes = Total cupcakes - Uneaten cupcakes = 104n - 7nx = n (104 - 7x)
We are given that the number of uneaten cupcakes is less than the number of children
Therefore,

$104n - 7nx < 7n$
$104 - 7x < 7$
$7x > 97$
$x > 97/7$

We know that x is a positive integer. So, x has to be more than or equal to 14
Uneaten Cupcakes = 104n - 7nx
Now, if x = 14, Uneaten cupcakes = 104n - 98n = 6n.
Now, if x = 15, Uneaten cupcakes = 104n - 105n = -n (Not possible)
So, x has to be 14.
This would make the number of uneaten cupcakes = 6n.
6n must be a multiple of 2 and 3.

The correct answer is D.

16.

If x divided by 11 has a quotient of y and a remainder of 3, x can be expressed as x = 11y + 3, where y is an integer (by definition, a quotient is an integer). If x divided by 19 also has a remainder of 3, we can also express x as x = 19z + 3, where z is an integer.

We can set the two equations equal to each other:
$11y + 3 = 19z + 3$
$11y = 19z$

The question asks us what the remainder is when y is divided by 19. From the equation we see that 11y is a multiple of 19 because z is an integer. y itself must be a multiple of 19 since 11, the coefficient of y, is not a multiple of 19.

If y is a multiple of 19, the remainder must be zero.

The correct answer is A.

17. The key to this problem is to recognize that in order for any integer to be divisible by 5, it must end in 0 or 5. Since we are adding a string of powers of 9, the question becomes "Does the sum of these powers of 9 end in 0 or 5?" If we knew the units digits of each power of nine, we'd be able to figure out the units digit of their sum.

9 raised to an even exponent will result in a number whose units digit is 1 (e.g., $9^2 = 81$, $9^4 = 6561$, etc.). If 9 raised to an even exponent always gives 1 as the units digit, then 9 raised to an odd exponent will therefore result in a number whose units digit is 9 (think about this: $9^2 = 81$, so 9^3 will be 81 x 9 and the units digit will be 1 x 9).

Since our exponents in this case are x, x+1, x+2, x+3, x+4, and x+5, we need to know whether x is an integer in order to be sure the pattern holds. (NEVER assume that an unknown is an integer unless expressly informed). If x is in fact an integer, we will have 6 consecutive integers, of which 3 will necessarily be even and 3 odd. The 3 even exponents will result in 1's and the 3 odd exponents will result in 9's. Since the three 1's can be paired with the three 9's (for a sum of 30), the units digit of y will be 0 and y
will thus be divisible by 5. But we don't know whether x is an integer. For that, we need to check the statements.

Statement (1) tells us that 5 is a factor of x, which means that x must be an integer. Sufficient.

Statement (2) tells us that x is an integer. Sufficient.
The correct answer is D.

18.
When n is divided by 4 it has a remainder of 1, so n = 4x + 1, where x is an integer. Likewise when n is divided by 5 it has a remainder of 3, so n = 5y + 3, where y is an integer. To find the two smallest values for n, we can list possible values for n based on integer values for x and y. To be a possible value for n, the value must show up on both lists:

n = 4x + 1	n = 5y + 3
5	8
9	**13**
13	18
17	23
21	28
25	**33**
29	38
33	43

The first two values for n that work with both the x and y expressions are 13 and 33. Their sum is 46.
The correct answer is B.

19.
If x is divided by 4 and has a quotient of y and a remainder of 1, then x = 4y + 1. And if x divided by 7 and has a quotient of z and a remainder of 6, then x = 7z + 6. If we combine these two equations, we get:
4y + 1 = 7z + 6
4y = 7z + 5, so we have y = (7z + 5) / 4.
You could also solve this problem by picking a value for x. The trick is to pick a value that works with the constraints given in the problem.
One such value is x = 13. This means that y is equal to the quotient of x ÷ 4, which is 3. The remainder would be 1, which meets the constraint given in the problem.
Given that x = 13, z is equal to the quotient of x ÷ 7, which is 1. The remainder would be 6, which meets the constraint given in the problem.
Thus x = 13, y = 3, and z = 1 meet the constraints given in the problem. Plug the value z = 1 into each answer choice to see which choice yields the correct value for y, which is 3. Only

answer choice D works.

The correct answer is D.

20.

The prime factors of n^4 are really four sets of the prime factors of the integer n.

Since n^4 is divisible by 32 (or 2^5), n^4 must be divisible by 2 at least 5 times. What does this tell us about the integer n?

If n is divisible by only one 2, then n^4 would be divisible by exactly four 2's (since the prime factors of n^4 have no source other than the integer n).

But we know that n^4 is divisible by at least five 2's! This means that n must be divisible by at least two 2's (which means that n^4 must be divisible by eight 2's). Thus, we know that the integer n must be divisible by 4.

Now that we know that n is divisible by 4, we can consider what happens when we divide n by 32. If we divide n by 32 we can represent this mathematically as follows:

$n = 32b + c$ (where b is the number of times 32 goes into n and c is the integer remainder) We know that n is divisible by 4 so we can rewrite this as:

$4x = 32b + c$ (where x is an integer)

This equation can be simplified, by dividing both sides by 4 as follows:

$x = 8b + c/4$

Since we know that x is an integer, the sum of 8b and c/4 must yield an integer. We know that 8b is an integer so c/4 must be also be an integer. Therefore, c, the remainder, must be divisible by 4. Only answer choice B qualifies. The remainder when n is divided by 32 could be 4. It could not be any of the another answer choices.

The correct answer is B.

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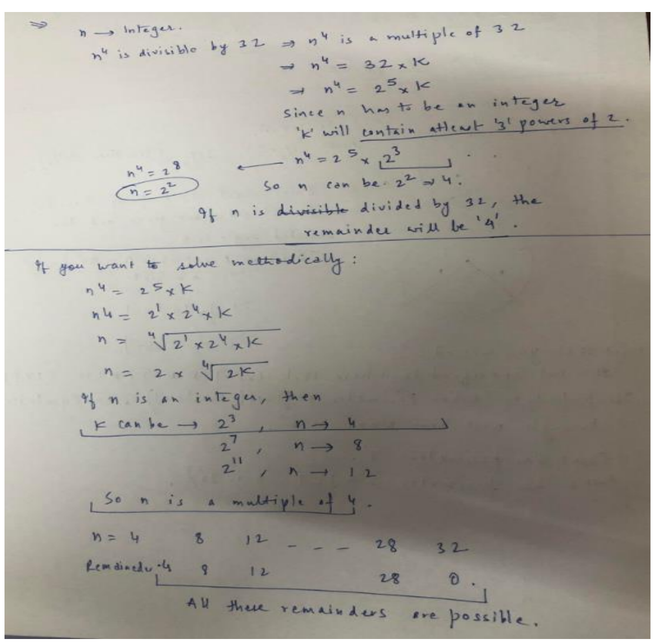
$\Rightarrow$ $n \rightarrow$ Integer.

n^4 is divisible by 32 $\Rightarrow$ n^4 is a multiple of 32

$\Rightarrow n^4 = 32 \times K$

$\Rightarrow n^4 = 2^5 \times K$

Since n has to be an integer 'K' will contain atleast '3' powers of 2.

$n^4 = 2^5 \times \underbrace{2^3}$ $\leftarrow$ $n^4 = 2^8$, $n = 2^2$

So n can be $2^2 \Rightarrow 4$.

If n is ~~divisible~~ divided by 32, the remainder will be '4'.

If you want to solve methodically:

$n^4 = 2^5 \times K$

$n^4 = 2^1 \times 2^4 \times K$

$n = \sqrt[4]{2^1 \times 2^4 \times K}$

$n = 2 \times \sqrt[4]{2K}$

If n is an integer, then

K can be $\rightarrow 2^3$, $n \rightarrow 4$

2^7, $n \rightarrow 8$

2^{11}, $n \rightarrow 12$

So n is a multiple of 4.

$n = 4$ 8 12 - - - 28 32

Remainder - 4 8 12 28 0.

All these remainders are possible.

21.

The statement —when x_1 is divided by 3, the remainder is 1 can be translated mathematically as follows:

There exists an integer n such that $x_1 = 3n+1$.

Similarly, the statement "when x_2 is divided by 12, the remainder is 4" can be translated

mathematically as follows: There exists an integer m such that x_2=12m+4. .

$$y = 2x_1 + x_2$$
$$= 2(3n+1) + (12m+4)$$
$$= 6n + 2 + 12m + 4$$

Therefore: $$= 6(n + 2m + 1)$$

The expression (n + 2m + 1) must be an integer (since n and m are both integers). Therefore, y is equal to some integer multiplied by 6. This means that y is divisible by 6. Any number that is divisible by 6 must also be divisible by both 2 and 3. Hence, y must be both even and divisible by 3 Consequently, I and III must be true.

The correct answer is D.

22.

The question asks whether x is the square of an integer (otherwise known as a perfect square). This is a yes/no question. If the statements allow us to say —definitely yes‖ or —definitely no‖ to the question, we have sufficiency.

Statement (1) tells us that x=12k+6, where k is a positive integer. The GMAT does not expect you to try out all perfect squares to see whether any fit the equation. Instead, look for a shorter way to analyze the statement.

Since x is the sum of two even numbers, we know that x is even. In order for x to be both even AND the square of an integer, x would have to be a multiple of 4. This is so because all even

numbers can be expressed as 2y, where y is an integer. Squaring 2y yields $4y^2$; therefore all squares of even numbers must be multiples of 4. You can verify this by testing out numbers: pick any even perfect square and you will see that it is a multiple of 4. (Note that all even perfect squares are multiples of 4, but not all multiples of 4 are perfect squares).

However, since statement (1) tells us that x = 12k+6, we know that x CANNOT be a multiple of Why? 12k is a multiple of 4, but adding 6 to 12k will bypass the next multiple of 4, while falling 2 short of the one beyond that.

Since x is even but not a multiple of 4, we know that x is definitely NOT the square of an integer. Statement (1) is therefore sufficient to answer the question.

Statement (2) tells us that x =3q+9where q is a positive integer. The equation itself does not allow us to deduce much about x. The easiest thing to do in this case is try to eliminate the statement by showing that it can go either way. So, for example, if q = 9, then x = 36 and x definitely IS a perfect square. But if q = 1, then x = 12 and x is definitely NOT a perfect square. Thus, statement (2) is not sufficient to answer the question.

The correct answer is A: Statement (1) alone is sufficient, but statement (2) alone is not sufficient.

The correct answer is A.

23.

This question can be rephrased: Is r – s divisible by 3? Or, are r and s each divisible by 3?

Statement (1) tells us that r is divisible by 735. If r is divisible by 735, it is also divisible by all the factors of 735. 3 is a factor of 735. (To test whether 3 is a factor of a number, sum its digits; if the sum is divisible by 3, then 3 is a factor of the number.) However, statement (1) does not tell us anything about whether or not s is divisible by 3. Therefore it is insufficient.

Statement (2) tells us that r + s is divisible by 3. This information alone is insufficient. Consider each of the following two cases:

CASE ONE: If r = 9, and s = 6, r + s = 15 which is divisible by 3, and r – s = 3, which is also divisible by 3.

CASE TWO: If r = 7 and s = 5, r + s = 12, which is divisible by 3, but r – s = 2, which is NOT divisible by 3.

Let's try both statements together. There is a mathematical rule that states that if two integers are each divisible by the integer x, then the sum or difference of those two integers is also divisible by x.
We know from statement (1) that r is divisible by 3. We know from statement (2) that r + s is divisible by 3. Using the converse of the aforementioned rule, we can deduce that s is divisible by
3. Then, using the rule itself, we know that the difference r – s is also divisible by 3.
The correct answer is C: BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
The correct answer is C.

24.
Statement (1) tells us that n is a prime number.

If n = 2, the lowest prime number, then $n^2 - 1 = 4 - 1 = 3$, which is not divisible by 24. If n = 3, the next prime number, then $n^2 - 1 = 9 - 1 = 8$ which is not divisible by 24.

However, if n = 5, then $n^2 - 1 = 25 - 1 = 24$, which is divisible by 24.

Thus, statement (1) alone is not sufficient.
Now let us examine statement (2). By design, this number is large enough so that it would not be easy to check numbers directly. Thus, we need to go straight to number properties.

For an expression to be divisible by 24, it must be divisible by 2, 2, 2, and 3 (since this is the prime factorization of 24). In other words, the expression must be divisible by 2 at least three times and by 3 at least once.

The expression $n^2 - 1 = (n - 1)(n + 1)$.

If we think about 3 consecutive integers, with n as the middle number, the expression $n^2 - 1$ is the product of the smallest number (n – 1) and the largest number (n + 1) in the consecutive set.

Given 3 consecutive positive integers, the product of the smallest number and the largest number will be divisible by 2 three times if the middle number is odd. Thus, if n is odd, the product (n – 1)(n + 1) must be divisible by 2 three times.

(Consider why: If the middle number of 3 consecutive integers is odd, then the smallest and largest numbers of the set will be consecutive even integers - their product must therefore be divisible by 2 at least twice. Further, since the smallest and the largest number are consecutive even integers, one of them must be divisible by 4. Thus the product of the smallest and largest number must actually be divisible by 2 at least three times!)

Additionally, given 3 consecutive positive integers, exactly ONE of those three numbers must be divisible by 3. To ensure that the product of the smallest number and the largest number will be divisible by 3, the middle number must NOT be divisible by 3. Thus, for the expression $n^2 - 1$ to be divisible by 24, n must be odd and must NOT be divisible by 3.
Statement (2) alone tells us that n > 191. Since, this does not tell us whether n is even, odd, or divisible by 3, it is not sufficient to answer the question.

Taking statements (1) and (2) together, we know that n is a prime number greater than 191. Every prime number greater than 3 must, by definition, be ODD (since the only even prime number is 2), and must, by definition, NOT be divisible by 3 (otherwise it would not be prime!)

Thus, so long as n is a prime number greater than 3, the expression $n^2 - 1$ will always be divisible by 24.
The correct answer is C: Statements (1) and (2) TAKEN TOGETHER are sufficient to answer the question, but NEITHER statement ALONE is sufficient.
The correct answer is C.

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(1) n is a prime number --> if n=2, then the answer is NO but if n=5, then the answer is YES. Not sufficient.

(2) n is greater than 191. Clearly insufficient (consider n=24^2 for a NO answer and n=17^2 for an YES answer).

(1)+(2) Given that n is a prime number greater than 191 so n is odd and not a multiple of 3. n^2-1=(n-1)(n+1) --> out of three consecutive integers (n-1), n and n+1 one must be divisible by 3, since it's not n then it must be either (n-1) or (n+1), so (n-1)(n+1) is divisible by 3. Next, since n is odd then (n-1) and (n+1) are consecutive even numbers, which means that one of them must be a multiple of 4, so (n-1)(n+1) is divisible by 2*4=8. We have that (n-1)(n+1) is divisible by both 3 and 8 so (n-1)(n+1) is divisible by 3*8=24. Sufficient.
The correct answer is C.

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We need to determine whether (n^2 - 1)/24 = integer. Notice that 24 is 2^3 x 3, or 8 x 3.
Since n^2 - 1 = (n + 1)(n - 1), when n is odd and not a multiple of 3, we will have the product of two consecutive even integers, one of which is a multiple of 3, and thus n^2 - 1 is divisible by 24. For instance, when n is 5, we have 4 x 6, which is divisible by 24.
Statement One Alone:
n is a prime number.
When n is 5, we see that 24/24 = integer; however, when n = 2, 3/24 does not equal an integer. Statement one is not sufficient to answer the question.
Statement Two Alone:
n is greater than 191.
If n = 192, then n^2 - 1 will be odd and will not be divisible by 24. If n = 199, then n^2 - 1 = (199 + 1)(199 - 1) = 200 x 198 is divisible by 24, since 200 is divisible by 8 and 198 is divisible by 3. Statement two is not sufficient to answer the question.
Statements One and Two together:
From both statements, we see that n is a prime that is greater than 191, and thus it satisfies the case that n is odd and not a multiple of 3. So, n^2 - 1 will be divisible by 24.
The correct answer is C.

25.

We are first told that the sum of all the digits of the number q is equal to the three-digit number

x13. Then we are told that the number q itself is equal to 10^n-49. Finally, we are asked for the value of n.

The first step is to recognize that 10^n-49 will have to equal a series of 9's ending with a 5 and

a 1 (99951, for example, is10^5-49). So q is a series of 9's ending with a 5 and a 1. Since the sum of all the digits of q is equal to x13, we know that x13 is the sum of all those 9's plus 5 plus
1. So if we subtract 5 and 1 from x13, we are left with x07.

This three-digit number x07 is the sum of all the 9's alone. So x07 must be a multiple of 9. For any multiple of 9, the sum of all the digits of that multiple must itself be a multiple of 9 (for example, 585 = (9)(65) and 5 + 8 + 5 = 18, which is a multiple of 9). So it must be true that x + 0 + 7 is a multiple of 9. The only single-digit value for x that will yield a multiple of 9 when added to 0 and 7 is 2. Therefore, x = 2 and the sum of all the 9's in q is 207.

Since 207 is a multiple of 9, we can set up the equation 9y = 207, where y is a positive integer. Solving for y, we get y = 23. So we know q consists of a series of twenty-three 9's followed by a 5 and a 1: 9999999999999999999999951. If we add 49 to this number, we get 10,000,000,000,000,000,000,000,000.

Since the exponent in every power of 10 represents the number of zeroes (e.g. 10^2=100 , which has two zeroes; 10^3=100, which has three zeroes, etc.), we must be dealing with 10^{25}. Thus n = 25.

The correct answer is B.

26.
Question Stem Analysis:

We choose three consecutive integers between 1 and 50, inclusive, and we need to determine the sum of the remainders when each integer is divided by x. No other information is provided in the question stem.

Statement One Alone:

⇒⇒ When the greatest of the consecutive integers is divided by x, the remainder is 0.

If $x = 1$, then the remainder when each of the integer is divided by x is 0.
Thus, the answer is $0 + 0 + 0 = 0$ in this case.

If $x = 3$, then the remainder when the middle integer is divided by x is 2, and the remainder when the smallest integer is divided by x is 1. Thus, the answer is $1 + 2 + 0 = 3$ in this case.

Since there are more than one possible answers, statement one alone is not sufficient.

Eliminate answer choices A and D.

Statement Two Alone:

⇒⇒ When the least of the consecutive integers is divided by x, the remainder is 1.

If $x = 3$, then the remainder when the middle integer is divided by x is 2, and the remainder when the largest integer is divided by x is 0. In this case, the answer is $1 + 2 + 0 = 3$.

If $x > 3$, then the remainder when the middle integer is divided by x is 2, and the remainder when the largest integer is divided by x is 3. In this case, the answer is $1 + 2 + 3 = 6$.

Since there are more than one possible answers, statement two alone is not sufficient.

Eliminate answer choice B.

Statements One and Two Together:

If the remainder when the smallest integer is divided by x is 1 and the remainder when the largest integer is divided by x is 0, then the remainder when the middle integer is divided by x can only equal 2. Thus, the sum of the three remainders is $1 + 2 + 0 = 3$.

Statements one and two together are sufficient.

The correct answer is C.

27.

In order for any number to be divisible by 6, it must be divisible by both 2 and 3. Thus, in order for y to be divisible by 6, y must be even (which is the same as being divisible by 2) and y must be a multiple of 3.

We can analyze each statement more effectively by breaking y into its 2 components: [$3^{(x-1)}$] and [x].

Statement (1) tells us that x is a multiple of 3.

Since x is a positive integer, $3^{(x-1)}$ is simply 3 raised to some integer power. This component of y will always be a multiple of 3.

From statement (1) we also know that the second component of y, which is simply x, is also a multiple of 3.

Subtracting one multiple of 3 from another multiple of 3 will yield a multiple of 3. Therefore, statement (1) tells us that y must be divisible by 3. However, this does not tell us whether y is even or not. Therefore, this is not enough information to tell us whether y is divisible by 6.

Statement (2) tells us that x is a multiple of 4. This means that x must be even.

Since x is a positive integer, $3^{(x-1)}$ is simply 3 raised to some integer power. This component of y will always be odd.

From statement (2) we also know that the second component of y, which is simply x, is even. Subtracting

an even number from an odd number yields an odd number. Therefore, statement (2) tells us that y must be odd. Since y is odd, it cannot be divisible by 2, which means y is NOT divisible by 6. This is sufficient information to answer the question.

The correct answer is B: Statement (2) alone is sufficient, but statement (1) alone is not sufficient.

The correct answer is B.

28.
m/n will be an integer if m is divisible by n. For m to be divisible by n, the elements of n's prime box (i.e. the prime factors that make up n) must also appear in m's prime box.
(1) INSUFFICIENT: If 2m is divisible by n, the elements of n's prime box are in 2m's prime box. However, since 2m contains a 2 in its prime box because of the coefficient 2, m alone may not have all of the elements of n's prime box. For example, if 2m = 6 and n = 2, 2m is divisible by n but m is not.
(2) SUFFICIENT: If m is divisible by 2n, m's prime box contains a 2 and the elements of n's prime box. Therefore m must be divisible by n.
The correct answer is B.

29.
If n is divisible by both 4 and 21, its prime factors include 2, 2, 3, and 7. Therefore, any integer that can be constructed as the product of these prime factors is also a factor of n. In this case, 12 is the only integer that can definitively be constructed from the prime factors of n, since 12 = 2 x 2 x 3.

The correct answer is B.

30. Since the supermarket sells apples in bundles of 4, we can represent the number of apples that Susie buys from the supermarket as 4x, where x can be any integer ≥ 0. If the number of apples that Susie buys from the convenience store is simply y, the total

number of apples she buys is (4x + y). We are asked to find the smallest possible value of y such that (4x + y) can be a multiple of 5.

We can solve this problem by testing numbers. Since the question asks us what is the minimum value for y such that (4x + y) can be a multiple of 5, it makes sense to begin by testing the smallest of the given answer choices. If y=0, can (4x + y) be a multiple of 5? Yes, because x could equal 5. (The value of (4(5) + 0) is 20, which is a multiple of 5.)

The correct answer is A.

31.
(A) 2 ÷ 11 has a quotient of 0 and a remainder of 2.
(B) 13 ÷ 11 has a quotient of 1 and a remainder of 2.
(C) 24 ÷ 11 has a quotient of 2 and a remainder of 2.
(D) 57 ÷ 11 has a quotient of 5 and a remainder of 2.
(E) 185 ÷ 11 has a quotient of 16 and a remainder of 9.

The correct answer is E.

32.
According to the information given, n=3k+2.
Combined statement 1, n-2=5m, that is n=5m+2, we can obtain n=15p+2.
According to the information given, t=5s+3.
Combined statement 2, t is divisible by 3, we can obtain t=15q+3.
nt=(15p+2)(15q+3)=(15^2)pq+45p+30q+6, when divided by 15, the remainder is 6.
The correct answer is C.

(*Continued on next page)*

Alternate sol from gmatclub (additional)

From the stem: $n=3p+2$ and $t=5q+3$.
$nt=15pq+9p+10q+6$, we should find the remainder when this expression is divided by 15.

(1) $n-2=5m$--> $n=5m+2=3p+2$ -> $5m=3p$, $15m=9p$ --> \(nt=15pq+**9p**+10q+6=15pq+**15m**+10q+6\). Clearly $15pq$ and $15m$ are divisible by 15, so remainder by dividing these components will be 0. But we still know nothing about $10q+6$. Not sufficient.

(2) t is divisible by 3 means that $5q+3$ is divisible by 3 --> 5q is divisible by 3 or q is divisible by 3 --> $5q=5*3z=15z$ --> $10q=30z$ --> \(nt=15pq+9p+**10q**+6=15pq+9p+**30z**+6\). $15pq$ and $30z$ are divisible by 15. Know nothing about $9p+6$. Not sufficient.

(1)+(2) $9p=15m$ and $10q=30z$ -> \(nt=15pq+**9p**+**10q**+6=15pq+15m+30z+6\). Remainder when this expression is divided by 15 is 6. Sufficient.

Answer: C.

OR:

From the stem: $n=3p+2$ and $t=5q+3$.

(1) n-2 is divisible by 5 --> $n-2=5m$ --> $n=5m+2$ and $n=3p+2$ --> general formula for n would be $n=15k+2$ (about deriving general formula for such problems at: good-problem-90442.html#p723049 and manhattan-remainder-problem-93752.html#p721341) --> $nt=(15k+2)(5q+3)=15*5kq+15*3k+10q+6$ --> first two terms are divisible by 15 ($15*5kq+15*3k$) but we don't know about the last two terms ($10q+6$). Not sufficient.

(2) t is divisible by 3 --> $t=3r$ and $t=5q+3$ -> general formula for t would be $t=15x+3$ --> $nt=(3p+2)(15x+3)=15*3px+9p+15*2x+6$. Not sufficient.

(1)+(2) $nt=(15k+2)(15x+3)=15*15kx+15*3k+15*2x+6$ this expression divided by 15 yields remainder of 6. Sufficient.

Answer: C.

Top 1% expert replies to student queries (can skip) (additional)

Given: $n = 3p + 2$ $\quad t = 5q + 3$

So $n \times t = 15pq + \underbrace{9p} + 10q + 6$

St 1: $(n-2)$ is divisible of 5

$(n-2)$ is a multiple of 5

$(n-2) = 5m$

$\left.\begin{array}{l} n = 5m + 2 \\ n = 3p + 2 \end{array}\right\} \rightarrow 5m + 2 = 3p + 2$

$5m = 3p$

So $(5m) \times 3 = (3p) \times 3$

$15m = 9p$.

So $n \times t = \underbrace{15pq + \underbrace{15m}}_{\downarrow} + 10q + 6$

These 2 ~~expressi~~ terms are divisible by 15, but no information on $\underline{10q}$, hence not sufficient.

St 2: t is divisible by 3, So 5q+3 is divisible by 3

So 5q+3 is a multiple of 3.

** $\boxed{\text{This means 'q' should be a multiple of 3.}}$

→ So $5(q) = 5(3z) = 15z$

So $5q = 15z$

$\times 2$: $\boxed{10q = 30z}$

$n \times t = 15pq + 9p + \underbrace{30z}_{10q} + 6$

(1st and 3rd term divisible by 15, but no info. on 9p). So, Insufficient.

St 1 + 2: $nt = 15pq + \underbrace{15m} + \underbrace{30z} + 6$

→ 1st, 2nd, 3rd term is divisible by 15, Hence '6' will remain. (Sufficient)

33.

To resolve such questions, at first we must find a general term for the number. Usually, general term is in the following form:

S=Am+B, where A and B are constant numbers. How to get A and B?

A is the least multiple of A1 and A2; B is the least possible value of S that let S1=S2.

For example:

When divided by 7, a number has remainder 3, when divided by 4, has remainder 2. S1=7A1+3
S2=4A2+2

The least multiple of A1 and A2 is 28; when A1=1, A2=2, S1=S2 and have the least value of 10.

Therefore, the general term is: S=28m+10

Back to our question:

1). n=2k+1

2). n=3s+1 or 3s+2

Combine 1 and 2, n=6m+1 or n=6m+5 (n=6m-1)

So, (n-1)(n+1)=6m(6m+2)=12m(3m+1). Because m*(3m+1) must be even, then 12m(3m+1) must be divisible by 24, the remainder r is 0

Or, (n-1)(n+1)=(6m-2)6m=12m(3m-1). Result is the same.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

We want to find the remainder 'r' when (n-1)(n+1) is divided by 24.

Statement 1 : n is not divisible by 2.

n = 2k+1

n-1 = 2k
n+1 = 2k+2

So, (n-1)(n+1) = 4k(k+1).

If k = 1, then 4k(k+1) = 8. Remainder = 8
If k = 2, then 4k(k+1) = 24. Remainder = 0
If k = 4, then 4k(k+1) = 80. Remainder = 8

So we can clearly see that the remainder is not unique. Insufficient!

Statement 2: n is not divisible by 3.

Case 1 : n = (3k+2)

n-1 = 3k+1

n+1 = 3k+3

So, (n-1)(n+1) = 3(3k+1)(k+1).

If k = 1, then 3(3k+1)(k+1) = 24. Remainder = 0
If k = 2, then 3(3k+1)(k+1) = 63. Remainder = 15
If k = 3, then 3(3k+1)(k+1) = 120. Remainder = 0

Case 1 : n = (3k+1)

n-1 = 3k
n+1 = 3k+2

So, (n-1)(n+1) = (3k)(3k+2)

If k = 1, then (3k)(3k+2) = 15. Remainder = 15
If k = 2, then (3k)(3k+2) = 48. Remainder = 0
If k = 3, then (3k)(3k+2) = 99. Remainder = 3

As can be seen from the 2 cases, we again do not have a unique value for the remainder. Insufficient!

Combining 1 and 2,

n is not a multiple of 2 and 3.

n can only take the forms (6k+1) and (6k+5) [All other forms will be multiples of 2 or 3]

If n = 6k+1,

n - 1 = 6k
n+1 = 6k+2

So (n-1)(n+2) = 12k(k+3) [multiple of 12]

Now, if k is odd, then k+3 will be even and k(k+3) will be even.
If k is even, then k(k+3) will be even.

So k(k+3) will always be even.

Therefore, the expression 12k(k+3) will be a multiple of 12*2 = 24

If n = 6k+5,

n - 1 = 6k + 4
n+1 = 6k+6

So (n-1)(n+2) = 12(k+1)(3k+2) [multiple of 12]

Now, if k is odd, then k+1 will be even and (k+1)(3k+2) will be even.
If k is even, then 3k+2 will be even and (k+1)(3k+2) will be even.

So (k+1)(3k+2) will always be even.

Therefore, the expression 12(k+1)(3k+2) will be a multiple of 12*2 = 24

So we see that in both cases, the expression is a multiple of 24 and therefore remainder = 0. Sufficient!

The correct answer is C.

34.

1). N is odd, then $n=2k+1$, $n^2 - 1=(2k+1)^2-1=4k^2+4k=4k(k+1)$. One of k and k+1 must be even, therefore, 4k(k+1) is divisible by 8.
The correct answer is A.

35.
4+7n=3+3n+3n+1+n=3(1+n)+3n+(1+n)
From 1), n+1 is divisible by 3, then 4+7n is divisible by 3. Thus, r=0
The correct answer is A.

36.

1). n could be 22,24, 26, ...insufficient
2). n=28K+3, then (28K+3)/7, the remainder is 3
The correct answer is B.

37.
1). The general term is x=6k+3. So, the remainder is 3. [look for the "general term" in this page, you can find the explanation about it.]
2). The general term is x=12k+3. So, remainder is 3 as well.
The correct answer is D.

38.
If x divided by 11 has a quotient of y and a remainder of 3, x can be expressed as x = 11y + 3, where y is an integer (by definition, a quotient is an integer). If x divided by 19 also has a remainder of 3, we can also express x as x = 19z + 3, where z is an integer.

We can set the two equations equal to each other:
11y + 3 = 19z + 3
11y = 19z

The question asks us what the remainder is when y is divided by 19. From the equation we see that 11y is a multiple of 19 because z is an integer. y itself must be a multiple of 19 since 11, the coefficient of y, is not a multiple of 19.

If y is a multiple of 19, the remainder must be zero.

The correct answer is A

39.
If x is divided by 4 and has a quotient of y and a remainder of 1, then x = 4y + 1. And if x divided by 7 and has a quotient of z and a remainder of 6, then x = 7z + 6. If we combine these two equations, we get:
4y + 1 = 7z + 6
4y = 7z + 5
y=(7z+5)/4

The correct answer is D.

GMAT Quant Topic 4: Numbers
Part G: Factors, Divisors, Multiples, LCM, HCF

1.
Since n must be a non-negative integer, n must be either a positive integer or zero. Also, note that the base of the exponent 12^n is even and that raising 12 to the n^{th} exponent is equivalent to multiplying 12 by itself n number of times. Since the product of even integers is always even, the value of 12^n will always be even as long as n is a positive integer. For example, if n = 1, then $12^1 = 12$; if n = 2, then $12^2 = 144$, etc.

Since integer 3,176,793 is odd, it cannot be divisible by an even number. As a result, if n is a positive integer, then 12^n (an even number) will never be a divisor of 3,176,793. However, if n is equal to zero, then $12^n = 12^0 = 1$. Since 1 is the only possible divisor of 3,176,793 that will result from raising 12 to a non-negative integer exponent (recall that all other outcomes will be even and thus will not be divisors of an odd integer), the value of n must be 0.

$0^{12} – 12^0 = 0 – 1 = -1$

The correct answer is B.

2. If the square root of p^2 is an integer, p^2 is a perfect square. Let's take a look at 36, an example of a perfect square to extrapolate some general rules about the properties of perfect squares.

Statement I: 36's factors can be listed by considering pairs of factors (1, 36) (2, 18) (3,12) (4, 9) (6, 6). We can see that they are 9 in number. In fact, for any perfect square, the number of factors will always be odd. This stems from the fact that factors can always be listed in pairs, as we have done above. For perfect squares, however, one of the pairs of factors will have an identical pair, such as the (6,6) for 36. The existence of this ―identical pair‖ will always make the number of factors odd for any perfect square. Any number that is not a perfect square will automatically have an even number of factors. Statement I must be true.

Statement II: 36 can be expressed as 2 x 2 x 3 x 3, the product of 4 prime numbers.
A perfect square will always be able to be expressed as the product of an even number of prime factors because a perfect square is formed by taking some integer, in this case 6, and squaring it. 6 is comprised of one two and one three. What happens when we square this number? $(2 \times 3)^2 = 2^2 \times 3^2$. Notice that each prime element of 6 will show up twice in 6^2. In this way, the prime factors of a perfect square will always appear in pairs, so there must be an even number of them. Statement II must be true.

Statement III: p, the square root of the perfect square p^2 will have an odd number of factors if p itself is a perfect square as well and an even number of factors if p is not a perfect square. Statement III is not necessarily true.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

This Roman Numeral question is based on a series of Number Property rules, but you don't need to know the rules to get the correct answer - you can TEST VALUES and do some brute-force math.

From the prompt, we know that P is a positive INTEGER greater than 1. We're asked which of the 3 Roman Numerals MUST be true. In most Roman Numeral questions, the 'key' is to DISPROVE the Roman Numerals so that we can quickly eliminate answer choices. Here though, we're going to prove that patterns exist.

Since P is a positive integer, we know that P^2 is a perfect square.

I. P^2 has an odd number of positive factors

IF...
P = 2, $P^2 = 4$ and the factors are 1, 2 and 4... so there IS an odd number of factors
P = 3, $P^2 = 9$ and the factors are 1, 3 and 9... so there IS an odd number of factors
P = 4, $P^2 = 16$ and the factors are 1, 2, 4, 8 and 16... so there IS an odd number of factors

Notice the pattern here. Since P^2 is a perfect square, there will ALWAYS be an odd number of factors, so Roman Numeral 1 IS true.
Eliminate Answers B, C and E.

From the answers that remain, we only have to deal with Roman Numeral II.

II. P^2 can be expressed as the product of an even number of positive prime factors

Using the same examples from Roman Numeral I, you can prove this pattern too:
P = 2, $P^2 = 4$ and we can get to 4 by multiplying (2)(2)... an even number of positive prime factors
P = 3, $P^2 = 9$ and we can get to 9 by multiplying (3)(3)... an even number of positive prime factors
P = 4, $P^2 = 16$ and we can get to 16 by multiplying (2)(2(2)(2))... an even number of positive prime factors
Since P^2 is a perfect square there will ALWAYS be a product of positive prime factors that will end in P^2, so Roman Numeral 2 IS true.
Eliminate Answer A.
The correct answer is D.

3.
The greatest common factor (GCF) of two integers is the largest integer that divides both of them evenly (i.e. leaving no remainder).

One way to approach this problem is to test each answer choice:

Answer choice	n	GCF of n and 16	GCF of n and 45
(A)	6	2	3
(B)	8	8	1
(C)	9	1	9
(D)	**12**	**4**	**3**
(E)	15	1	15

Alternatively, we can consider what the GCFs stated in the question stem tell us about n:

The greatest common factor of n and 16 is 4. In other words, n and 16 both are evenly divisible by 4 (i.e., they have the prime factors 2 × 2), but have absolutely no other factors in common. Since 16 = 2 × 2 × 2 × 2, n must have <u>exactly</u> two prime factors of 2--no more, no less.

The greatest common factor of n and 45 is 3. In other words, n and 45 both are evenly divisible by 3, but have absolutely no other factors in common. Since 45 = 3 × 3 × 5, n must have <u>exactly</u> one prime factor of 3--no more, no less. Also, n cannot have 5 as a prime factor.

So, n must include the prime factors 2, 2, and 3. Additional prime factors are OK, as long as they do not include more 2s, more 3s, or any 5s.

(A) 6 = 2 × 3 missing a factor of 2
(B) 8 = 2 × 2 × 2 missing a factor of 3, too many factors of 2
(C) 9 = 3 × 3 missing factors of 2, too many factors of 3
(D) 12 = 2 × 2 × 3 OK
(E) 15 = 3 × 5 missing factors of 2, cannot have a factor of 5

The correct answer is D.

4.

The factors of 104 are {1, 2, 4, 8, 13, 26, 52, 104}. If x – 1 is definitely one of these factors, OR if x – 1 is definitely NOT one of these factors, then the statement is sufficient.

(1) INSUFFICIENT: x could be any one of the infinite multiples of 3. If x = 3, x – 1 would equal 2, which is a factor of 104. If x = 6, x – 1 would equal 5, which is not a factor of 104.

(2) INSUFFICIENT: The factors of 27 are {1, 3, 9, 27}. Subtracting 1 from each of these values yields the set {0, 2, 8, 26}. The non-zero values are all factors of 104, but 0 is not.

(1) AND (2) SUFFICIENT: Given that x is a factor of 27 and also divisible by 3, x must equal one of 3, 9 or 27. x – 1 must therefore equal one of 2, 8, or 26 - all factors of 104.

The correct answer is C.

5.
36^2 can be expressed as the product of its prime factors, raised to the appropriate exponents:
$36^2 = (2^2 \times 3^2)^2 = 2^4 \times 3^4$
So, the prime box of 36^2 contains four 2's and four 3's, as shown:
2 2 2 2 3 3 3 3
Now, if you pick any combination of these primes and multiply them all together, the product will be a factor of 36^2. As you take primes from this prime box to construct a factor of 36^2, note that you can choose up to four 2's and up to four 3's. In fact, you have FIVE choices for the number

of 2's you put into the factor: zero, one, two, three, or four 2's. Likewise, you have the same FIVE choices for the number of 3's you put into the factor: zero, one, two, three, or four. (It doesn't matter what order you pick the factors, since order doesn't matter in multiplication.) Note that you are allowed to pick zero 2's and zero 3's at the same time. By doing so, you are constructing the factor $2^0 \times 3^0 = 1$, which is a separate, valid factor of 36^2.
Since you have five independent choices for the number of 2's you pick AND you have five independent choices for the number of 3's you pick, you MULTIPLY the number of choices together to get the number of options you have overall. Thus you have 5 × 5 = 25 different ways to construct a factor. This means that there are 25 different factors of 36^2.
The correct answer is D.

6.
88,000 is the product of an unknown number of 1's, 5's, 11's and x's. To figure out how many x's are multiplied to achieve this product, we have to figure out what the value of x is. Remember that a number's prime box shows all of the prime factors that when multiplied together produce that number: 88,000's prime box contains one 11, three 5's, and six 2's, since $88,000 = 11 \times 5^3 \times 2^6$.

The 11 in the prime box must come from a red chip, since we are told that $5 < x < 11$ and therefore x could not have 11 as a factor. In other words, the factor of 11 definitely did not come from the selection of a purple chip, so we can ignore that factor for the rest of our solution.

So, turning to the remaining prime factors of 88,000: the three 5's and six 2's. The 2's must come from the purple chips, since the other colored chips have odd values and thus no factor of two. Thus, we now know something new about x: it must be even.
We already knew that $5 < x < 11$, so now we know that x is 6, 8, or 10.

However, x cannot be 6: 6 = 2 × 3, and our prime box has no 3's.

x seemingly might be 10, because 10 = 2 × 5, and our prime box does have 2's and 5's. However, our prime box for 88,000 only has three 5's, so a maximum of three chips worth 10 points are possible. But that leaves three of the six factors of 2 unaccounted for, and we know those factors of two must have come from the purple chips.

So x must be 8, because $8 = 2^3$ and we have six 2's, or two full sets of three 2's, in the prime box. Since x is 8, the chips selected must have been 1 red (one factor of 11), 3 green (three factors of 5), 2 purple (two factors of 8, equivalent to six factors of 2), and an indeterminate number of blue chips.

The correct answer is B.

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Blue = 1 point;
Green = 5 points;
Purple = x points ($5<x<11$);
Red = 11 points.

Since the product is $88000=2^6 * 5^3 *11$, then there were exactly 3 green chips and 1 red chip selected. $2^6=64$. The only factor of 64 between 5 and 11 is '8'. Thus 64 is a product of 2 purple chips: 8*8=64(the product of the point values of the selected chips) , so two purple chips were drawn.

The correct answer is B.

7.
The problem asks us to find the greatest possible value of (length of x + length of y), such that x and y are integers and $x + 3y < 1,000$ (note that x and y are the numbers themselves, not the lengths of the numbers - lengths are always indicated as "length of x" or "length of y," respectively).

Consider the extreme scenarios to determine our possible values for integers x and y based upon our constraint $x + 3y < 1,000$ and the fact that both x and y have to be greater than 1. If $y = 2$, then $x \leq 993$. If $x = 2$, then $y \leq 332$. Of course, x and y could also be somewhere between these extremes.

Since we want the maximum possible sum of the lengths, we want to maximize the length of our x value, since this variable can have the largest possible value (up to 993).
The greatest number of factors is calculated by using the smallest prime number, 2, as a factor as many times as possible. $2^9 = 512$ and $2^{10} = 1,024$, so our largest possible length for x is 9.
If x itself is equal to 512, that leaves 487 as the highest possible value for 3y (since $x + 3y < 1,000$). The largest possible value for integer y, therefore, is 162 (since 487 / 3 = 162 remainder 1). If $y < 162$, then we again use the smallest prime number, 2, as a factor as many times as possible for a number less than 162. Since $2^7 = 128$ and $2^8 = 256$, our largest possible length for y is 7.

If our largest possible length for x is 9 and our largest possible length for y is 7, our largest sum of the two lengths is 9 + 7 = 16.
What if we try to maximize the length of the y value rather than that of the x value? Our maximum y value is 332, and the greatest number of prime factors of a number smaller than 332 is $2^8 = 256$, giving us a length of 8 for y.
That leaves us a maximum possible value of 231 for x (since $x + 3y < 1,000$). The greatest number of prime factors of a number smaller than 231 is $2^7 = 128$, giving us a length of 7 for x. The sum of these lengths is 7 + 8 = 15, which is smaller than the sum of 16 that we obtained when we maximized the x value. Thus 16, not 15, is the maximum value of (length of x + length of y).
The correct answer is D.

Top 1% expert replies to student queries (can skip)

Basically the length of an integer is the sum of the powers of its prime factors. For example the length of 24 is 4 because $24=2^3*3$ --> 3+1=4.
Given: $x+3y<1,000$. Now, to maximize the length of x or y (to maximize the sum of the powers of their primes) we should minimize their prime bases. Minimum prime base is 2: so if x=2^9=512 then its length is 9 --> 512+3y<1,000 --> y<162.7 --> maximum length of y can be 7 as 2^7=128 --> 9+7=16.
The correct answer is D.

OR

we know that : $x > 1$, $y > 1$, and $x + 3y < 1000$,
and it is given that length means no of factors.
for any value of x and y, the max no of factors can be obtained only if the factor is smallest no & all factors are equal.
Hence, let's start with smallest no 2.
$2^{1} = 2$
$2^{2} = 4$
$2^{3}=8$
$2^{4}=16$
$2^{5}=32$
$2^{6}=64$
$2^{7}=128$
$2^{8}=256$
$2^{9}=512$
$2^{10} = 1024$ (opps//it exceeds 1000, so, x can't be 2^{10})
so, the max value that X can take is 2^9 , for which the "length of integer" is 9.
now, since x =512 , & x+3y<1000
so, 3y<488
==> y<162
so, y can take any value which is less than 162. and to get the maximum number of factors of the smallest integer, we can say y=2^7
for 2^7 the "length of integer" is 7.
SO, combined together: 9+7 = 16.
The correct answer is D.

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We need to choose x and y such that length(x) + length(y) = maximum possible value, given the constraints $x + 3y < 1000$ and $x,y>1$.

Let us first look at x. We need to choose a value of x that is less than 1000 and has the maximum possible length. That value is 2^9. Any other less than 1000 will have a smaller length. That's because 2 is the smallest prime number and 2^9 is the largest power of 2 that is less than 1000.

Now that we have x, we need y. x = 2^9 = 512

$512 + 3y < 1000$
$3y < 488$
$y < 162$

What is the largest power of 2 that is less than 162? It is 2^7 = 128. Therefore, y = 2^7.

Length of x = 9
Length of y = 7

Therefore, max (length x + length y) = 16

8. *Question removed*

9.

This is a very tricky problem. We're told that a and b are both positive integers and that 6 is a divisor of both numbers. We could certainly determine whether 6 is the greatest common divisor, or greatest common factor (GCF), if we know the individual values for a and b. We do not have to know the individual values, however; we only have to be able to prove either that there cannot be a GCF greater than 6 or that there is a GCF greater than 6.

(1) SUFFICIENT: We are already told in the question stem that 6 is a divisor of both a and b. This statement tells us that a is exactly 6 more than 2b. If one number is x units away from another number, and x is also a factor of both of those numbers, than x is also the GCF of those two numbers. This always holds true because x is the greatest number separating the two; in order to have a larger GCF, the two numbers would have to be further apart.

This statement, then, tells us that the GCF of a and 2b is 6. The GCF of a and b can't be larger than the GCF of a and 2b, because b is smaller than 2b; since we were already told that 6 is a factor of b, the GCF of a and b must be also be 6.

This can also be tested with real numbers. If b = 6, then a would be 18 and the GCF would be 6. If b = 12, then a would be 30 and the GCF would be 6. If b = 18, then a would be 42 and the GCF would still be 6 (and so on).

(2) INSUFFICIENT: There are no mathematical rules demonstrated in this statement to help us determine whether 6 is the GCF of a and b. This can also be tested with real numbers. If b = 6, then a would be 18 and the GCF would be 6. If, however, b = 12, then a would be 36 and the GCF would be 12.

Note that solving with the combined statements (1) and (2) would allow us to determine the individual values for a and b, which also allows us to determine the GCF. C cannot be the correct answer, however, because it specifically states that "NEITHER one ALONE is sufficient" and, in this case, statement (1) alone is sufficient. In fact, it is so easy to see here that both statements together would provide an answer that one should naturally be suspicious of C.

The correct answer is A.

Top 1% expert replies to student queries (can skip) (additional)

a, b → divisible by 6.

$a = 6x$ where $x = 1, 2, 3 \ldots$

$b = 6y$ where $y = 1, 2, 3 \ldots$

Question: Is 6 the HCF of a and b?

(i) $a = 2b + 6$

$a = 2(6y) + 6$

$a = 6(2y + 1)$

Create YES scenario (we will always get a YES here)

i.e 6 is the HCF

when $y = 1$, $b = 6$, $a = 6(3) = 18$

$y = 2$, $b = 12$, $a = 6(5) = 30$

$y = 3$, $b = 18$, $a = 6(7) = 42$

(6 will always be the HCF)

Sufficient

(ii) $a = 3b$

$a = 3(6y)$

$a = 18y$

HCF = 6

$y = 1$, $b = 6$, $a = 18$ (YES)

$y = 2$, $b = 12$, $a = 36$ (NO)

HCF = 12

Not Sufficient.

The correct answer is A.

10.
This question looks daunting, but we can tackle it by thinking about the place values of the unknowns. If we had a three-digit number abc, we could express it as $100a + 10b + c$ (think of an example, say 375: $100(3) + 10(7) + 5$). Thus, each additional digit increases the place value tenfold.

If we have abcabc, we can express it as follows:

$100000a + 10000b + 1000c + 100a + 10b + c$

If we combine like terms, we get the following:

$100100a + 10010b + 1001c$

At this point, we can spot a pattern in the terms: each term is a multiple of 1001. On the GMAT, such patterns are not accidental. If we factor 1001 from each term, the expression can be simplified as follows:

$1001(100a + 10b + c)$ or $1001(abc)$.

Thus, abcabc is the product of 1001 and abc, and will have all the factors of both. Since we don't know the value of abc, we cannot know what its factors are. But we can see whether one of the answer choices is a factor of 1001, which would make it a factor of abcabc.

1001 is not even, so 16 is not a factor. 1001 doesn't end in 0 or 5, so 5 is not a factor. The sum of the digits in 1001 is not a multiple of 3, so 3 is not a factor. It's difficult to know whether 13 is a factor without performing the division: $1001/13 = 77$. Since 13 divides into 1001 without a remainder, it is a factor of 1001 and thus a factor of abcabc.

The correct answer is B.

11.

The greatest common divisor is the largest integer that evenly divides both $35x$ and $20y$.
(A) CAN be the greatest common divisor.
First, $35x / 5 = 7x$, which is an integer for every possible value of x.
Second, $20y / 5 = 4y$, which is an integer for every possible value of y.
Therefore, 5 is a common divisor. It will be the greatest common divisor when $7x$ and $4y$ share no other factors. To illustrate, if $x = 1$ and $y = 1$, then $35x = 35$ and $20y = 20$, and their greatest common divisor is 5.

(B) CAN be the greatest common divisor.
First, $35x/[5(x-y)]= 7x(x-y)$, which can be an integer in certain cases: when x is a multiple of $(x – y)$. Second, $20y/[5(x-y)] = 4y/(x-y)$, which can be an integer in certain cases: when y is a multiple of $(x – y)$. So, this answer choice will be a divisor if both x and y are multiples of $(x – y)$. Since x and y are integers, the easiest way to meet the requirement is to select $(x – y) = 1$, for example $x = 3$ and $y = 2$. To illustrate, if $x = 3$ and $y = 2$, then $35x = 105$ and $20y = 40$, and their greatest common divisor is $5 = 5(x – y)$.

(C) CANNOT be the greatest common divisor.
Regardless of the values of x and y, $35x / 20x = 35/20 = 7/4$, which is not an integer. Therefore, $20x$ does not evenly divide one of the numbers in question. It is not a divisor, and certainly not the greatest common divisor.

(D) CAN be the greatest common divisor.
First, $35x / 20y = 7x / 4y$, which can be an integer in certain cases: one such case is when $x = 4$ and $y = 1$. Second, $20y / 20y = 1$, which is an integer. To illustrate, if $x = 4$ and $y = 1$, then $35x = 140$ and $20y = 20$, and their greatest common divisor is $20 = 20y$.

(E) CAN be the greatest common divisor.
First, $35x / 35x = 1$, which is an integer.

Second, $20y / 35x = 4y / 7x$, which can be an integer in certain cases: one such case is when $x = 1$ and $y = 7$. To illustrate, if $x = 1$ and $y = 7$, then $35x = 35$ and $20y = 140$, and their greatest common divisor is $35 = 35x$.

The correct answer is C.

12.
Let's begin by analyzing the information given to us in the question:

If P, Q, R, and S are positive integers, and $\frac{P}{Q} = \frac{R}{S}$, is R divisible by 5 ?
It is often helpful on the GMAT to rephrase equations so that there are no denominators. We can do this by cross-multiplying as follows:

$$\frac{P}{Q} = \frac{R}{S} \to PS = RQ$$

Now let's analyze Statement (1) alone: P is divisible by 140.
Most GMAT divisibility problems can be solved by breaking numbers down to their prime factors (this is called a "prime factorization").
The prime factorization of 140 is: $140 = 2 \times 2 \times 5 \times 7$.
Thus, if P is divisible by 140, it is also divisible by all the prime factors of 140. We know that P is divisible by 2 twice, by 5, and by 7. However, this gives us no information about R so Statement (1) is not sufficient to answer the question.

Next, let's analyze Statement (2) alone: $Q = 7^x$, where x is a positive integer.
From this, we can see that the prime factorization of Q looks something like this: $Q = 7 \times 7 \times 7 \times \ldots$ Therefore, we know that 7 is the only prime factor of Q. However, this gives us no information about R so Statement (2) is not sufficient to answer the question.
Finally, let's analyze both statements taken together:
From Statement (1), we know that P has 5 as one of its prime factors. Since 5 is a factor of P and since P is a factor of PS, then by definition, 5 is a factor of PS.

Recall that the question told us that PS = QR . From this, we can deduce that PS must have the same factors as QR. Since 5 is a factor of PS, 5 must also be a factor of QR.
From Statement (2), we know that 7 is the only prime factor of Q. Therefore, we know that 5 is NOT a factor of Q. However, we know that 5 must be a factor of QR. The only way this can be the case is if 5 is a factor of R.
Thus, by combining both statements we can answer the question: Is R divisible by 5? Yes, it must be divisible by 5. Since BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
The correct answer is C.

13.
According to the question, the ―star function is only applicable to four digit numbers. The function takes the thousands, hundreds, tens and units digits of a four-digit number and applies them as exponents for the bases 3, 5, 7 and 11, respectively, yielding a value which is the product of these exponential expressions.

Let's illustrate with a few examples:
$*2234* = (3^2)(5^2)(7^3)(11^4)$
$*3487* = (3^3)(5^4)(7^8)(11^7)$

According to the question, the four-digit number m must have the digits of rstu, since $*m* = (3^r)(5^s)(7^t)(11^u)$.

If $*n* = (25)(*m*)$
$*n* = (5^2)(3^r)(5^s)(7^t)(11^u)$
$*n* = (3^r)(5^{s+2})(7^t)(11^u)$

n is also a four digit number, so we can use the *n* value to identify the digits of n: thousands = r, hundreds = s + 2, tens = t, units = u.

All of the digits of n and m are identical except for the hundreds digits. The hundreds digits of n is two more than that of m, so n – m = 200.

The correct answer is B.

14.
We are asked whether y is a common divisor of x and w. In other words, is y a factor of both x and w?

Statement (1) tells us that $\frac{w}{x} = \frac{1}{z} + \frac{1}{x}$. Since w is greater than x, the quotient w/x must be greater than 1. For example, $\frac{17}{6} = 2\frac{5}{6}$.

Since $\frac{W}{x}$ is greater than 1, the other side of the equation must be greater than 1 as well. In this case, since x and z are integers, $\frac{1}{z} + \frac{1}{x}$ cannot be greater than 1 unless either x or z is equal to 1. x cannot be equal to 1 because at least two integers (y and z) stand between it and 0 on the number line. So z must equal 1. We can now rewrite the equation as $\frac{W}{x} = 1 + \frac{1}{x}$.
At this point, we can finish up using algebra:

$$\frac{w}{x} = 1 + \frac{1}{x} \rightarrow$$

$$w = x + \frac{x}{x} \rightarrow$$

$$w = x + 1$$

If w = x + 1, then w and x must be consecutive integers. Since the distance between two consecutive integers is always 1, no number other than 1 can be a factor of both. Note, for example, that all multiples of 2 are at least 2 spaces apart, all multiples of 3 are at least 3 spaces apart, and so on. So in order for y to be a common factor of w and x, it would have to equal 1.
But because y is greater than 1 (at least 1 integer stands between it and 0 on the number line), it cannot be a common divisor of w and x. Thus, statement (1) alone is sufficient to answer the question.

Statement (2) gives us an equation that we can factor into w(w – y – 2) = 0 . This implies that either w or (w - y - 2) is equal to 0. We know that w is greater than 0, so it must be true that (w - y - 2) = 0. We can add y and 2 to both sides to get w = y + 2. So w is 2 greater than y on the number line. Since x falls between w and y, we know that w, x, and y are consecutive integers. No integer greater than 1 can be a factor of the two integers that follow it on the number line.
Since y is greater than 1, it cannot be a factor of both w and x. Thus, statement (2) alone is sufficient to answer the question

The correct answer is D.

Top 1% expert replies to student queries (can skip)

1 -
$w/x = 1/x + 1/z$
$(w-1)/x = 1/z$
$x = z(w-1)$

If $z > 1$. then $x > w$ but this contradicts the inequality given $w > x$.

Therefore, $z = 1$.
$x = w - 1$

Therefore, x and w are consecutive integers.

Therefore, x and w have no common factor other than 1. But $y > 1$ [Since the minimum value of z is 1 and $y > z$]. As a result, y cannot be a common factor of x and w. Sufficient!

2 -

$w^2 - wy - 2w = 0$
$w(w - y - 2) = 0$

w cannot be 0.
Therefore,

$w - y - 2 = 0$
$w = y + 2$
But $w > x > y$.

Since the minimum difference between w and z is 2, which would be when w, x and y are consecutive integers.

Therefore, w and x are consecutive integers. And $y > 1$.

So, again, y cannot be a common factor of w and x. Sufficient!

The correct answer is D.

15.

Let's start by finding the cost of the lobster, per bowl, in terms of the variables given (d, v, and b).

(d dollars/6 pounds) x (1 pound/v vats) x (1 vat/b bowls) = (d/6vb)

The problem states that this value, the cost of the lobster per bowl, or (d/6vb), is an integer. In other words, d is divisible by 6vb. To make d as small as possible, we need to make 6vb as small as possible. Since v and b are different prime integers, the smallest value of 6vb is 36 (using the two smallest prime integers, $v = 2$ and $b = 3$, or $v = 3$ and $b = 2$).

In order to make the cost of the lobster per bowl an integer, d must be divisible by 36. In other words, d must be a multiple of 36. What's the smallest possible multiple of 36? The smallest multiple of 36 is 36.

The correct answer is C.

16.

The question tells us that (a + b) (c – d) = r, where r is an integer, and then asks whether $\sqrt{c+d}$ is an integer. In order for $\sqrt{c+d}$ to be an integer, (c + d) must be the square of an integer.

Statement (1) tells us that a + b) (c + d) = r^2. Therefore, $(a+b)(c+d)=(a+b)^2(c-d)^2$. If we divide both sides of the equation by (a + b), we get $(c+d)=(a+b)(c-d)^2$. Is this sufficient to determine whether (c + d) is the square of an integer? No. Since $(c-d)^2$ is a square, (a + b) must also be a square in order for (c + d) to be a square. We do not know whether (a + b) is a square, so this statement is insufficient.

Statement (2) tell us that the prime factorization of (a + b) is $x^4y^6z^2$ This does not tell us anything about (c + d), so it must be insufficient.

If we combine the information from both statements, we know that $(c+d)=(a+b)(c-d)^2$

(Continued on next page)

and that (a + b) = $x^4y^6z^2$. In order for the information from statement (1) to be sufficient, we would need to know that (a + b) is itself a square. Knowing that the prime factorization of (a + b) is $x^4y^6z^2$ tells us that it must be true that (a + b) is a square, because all of its prime factors come in pairs.

Any integer that has an even number of each of its prime factors must be a square.

We can see this by expressing (a + b) as $(a + b) = (x^2y^3z)(x^2y^3z)$.

Therefore, (c + d) must be a square and $\sqrt{(c + d)}$ must be an integer.

The correct answer is C: Taken together, the statements are sufficient, but neither statement alone is sufficient.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

a,b,c,d are positive integers.

(a+b)(c-d) = r, where "r" is an integer.

We want to know whether the root(c+d) is an integer or not.

Statement 1:

(a+b)(c+d) = r^2.

Using the original equation and the equation in statement 1,

(a+b)(c+d) = (a+b)^2 * (c-d)^2

(c+d) = (a+b) * (c-d)^2

root(c+d) = root(a+b) * (c-d)

We know that (c-d) is an integer. But we don't know if root(a+b) is an integer, or not. If it is, then root(c+d) will be an integer. If it is not, then root(c+d) will not be an integer. So statement 1 is insufficient.

Statement 2 :

(a+b) = x^4 * y^6 * z^2.

This will give us the value of (c-d). But this would not tell us if root(c+d) is an integer or not. So statement 2 is insufficient!

Combining statements 1 and 2,

root(c+d) = root(a+b) * (c-d) AND (a+b) = x^4 * y^6 * z^2.

Here, root(a+b) is an integer (root(a+b) = x^2 * y^3 * z))

So root(c+d) is an integer. Sufficient! So the answer is C

17.

Any product pq must have the following factors: {1, p, q, and pq}. If the product pq has no additional factors, the p and q must be prime.
Statement (1) tells us that the sum of p and q is an odd integer. Therefore either p or q must be even, while the other is odd. Since we know p and q are prime, either p or q must be equal to 2 (the only even prime number). However, statement (1) does not provide enough information for us to know which of the variables, p and q, is equal to 2.
Statement (2) tells us that $q < p$, which does not give us any information about the value of p. From both statements taken together, we know that, since 2 is the smallest prime number, q must equal 2. However, we cannot determine the value of p.
The correct answer is E.

18.

(1) SUFFICIENT: Since x and y are distinct prime numbers (we know that $x < y$), their sum must be greater than or equal to 5 (i.e. greater than or equal to the sum of the two smallest primes, 2 and 3). Further, the factors of 57 are 1, 3, 19, and 57. Note that the sum of x and y cannot be equal to 57, since if the sum of two distinct primes is odd, one of the two primes must be even (i.e. equal to 2). Since 55 (i.e.57 – 2) is not a prime number, 57 cannot be equal to the sum of two primes.

Since we already figured that the sum of two distinct primes must exceed 5, the only factor of 57 that can be equal to the sum of two primes is 19. Again, since the sum of the two primes is odd, one of the primes must be even. Thus, $x = 2$ and $y = 19 - 2 = 17$. Finally, since we know that x is even, it cannot be a factor of the odd integer z, since any even integer is never a factor of any odd integer. Therefore, we have sufficient information to conclude that x will not be a factor of z.

(2) INSUFFICIENT: First, note that z has to be greater than or equal to 3 (recall that $x < y < z$, where x, y, and z are positive integers). Thus, since the factors of 57 are 1, 3, 19, and 57, it follows that z can be equal to 3, 19, or 57. Since we have no further information about x, we cannot conclude whether x is a factor of z.

The correct answer is A.

19.
There is no conceptual or formulaic approach for solving this question. One must simply try out various integers.

(2) INSUFFICIENT: We can start with the second statement first because it is clear that it is insufficient to solve the question what is value of the positive integer n?

(1) INSUFFICIENT: We must first understand what this statement is saying. If all of n's factors (other than n itself) are added up, they equal n.

We can begin our search by considering prime factors. By definition prime factors have only two factors, themselves and 1. It is impossible that the factors "other-than-the number" add up
to the number for any prime number. Thus we can begin our search for such n's with the number 4.

4 does not equal 1 + 2
6 DOES EQUAL 1 + 2 + 3
9 does not equal 1 + 3
10 does not equal 1 + 2 + 5
12 does not equal 1 + 2 + 3 + 4 + 6
14 does not equal 1 + 2 + 7
15 does not equal 1 + 3 + 5

At this point we might be tempted to think that this is a property that is unique to 6 and is unlikely to come around again (i.e. that the answer is A). It would behoove us to keep searching though and to at least cover

the range defined by the second statement (i.e. $n < 30$) . If we do that we see that this property repeats itself one other time in the remaining integers that are less than 30.
16 does not equal 1 + 2 + 4 + 8
18 does not equal 1 + 2 + 9
20 does not equal 1 + 2 + 4 + 5 + 10
21 does not equal 1 + 3 + 7
22 does not equal 1 + 2 + 11
24 does not equal 1 + 2 + 3 + 4 + 6 + 8 + 12
25 does not equal 1 + 5
26 does not equal 1 + 2 + 13
27 does not equal 1 + 3 + 9
28 DOES EQUAL 1 + 2 + 4 + 7 + 14

The correct answer is E.

20.

Let's start by breaking 80 down into its prime factorization: $80 = 2 \times 2 \times 2 \times 2 \times 5$. If p^3 is divisible by 80, p^3 must have 2, 2, 2, 2, and 5 in its prime factorization. Since p^3 is actually $p \times p \times p$, we can conclude that the prime factorization of $p \times p \times p$ must include 2, 2, 2, 2, and 5.

Let's assign the prime factors to our p's. Since we have a 5 on our list of prime factors, we can give the 5 to one of our p's:

p: 5
p:
p:

Since we have four 2's on our list, we can give each p a 2:

p: 5×2
p: 2
p: 2

But notice that we still have one 2 leftover. This 2 must be assigned to one of the p's:

p: $5 \times 2 \times 2$
p: 2
p: 2

We must keep in mind that each p is equal in value to any other p. Therefore, all the p's must have exactly the same prime factorization (i.e. if one p has 5 as a prime factor, all p's must have 5 as a prime factor). We must add a 5 and a 2 to the 2nd and 3rd p's:

p: $5 \times 2 \times 2 = 20$
p: $5 \times 2 \times 2 = 20$
p: $5 \times 2 \times 2 = 20$

We conclude that p must be at least 20 for p^3 to be divisible by 80. So, let's count how many factors 20, or p, has:

1×20
2×10
4×5

20 has 6 factors. If p must be at least 20, p has at least 6 distinct factors.
The correct answer is C.

21.
Any integer can be broken down into prime factors, and these prime factors can be used to find the total number of factors. The factors of 12, for example, can be found by listing all of the different combinations of 12's prime factors, 2, 2 and 3, viz., 1, 2, 3, 4, 6, 12. Another way to list the factors of 12, however, is to simply consider, by pairs, all of the numbers that divide 12 evenly. Start the list with 1 and the number itself and then search for other pairs by increasing by increments of 1. For 12, you easily come across two other pairs of distinct factors: (2,6) and (3,4). In this way, the factors of any reasonably sized integer can be listed and counted. This question is asking if p an odd number of distinct factors. This begs the question: if factors of an integer can always be listed in factor pairs, how could an integer ever have anything other than an even number of factors? Let's look at the two statements to shed some light on the issue.

Statement (1) tells us that p is a perfect square, i.e. a number that when you take its square root, the result is an integer. Let's take 36 to investigate. Any perfect square will have an even number of prime factors when you break it down. What effect does this have on the number of factors however? Well if we resort to our listing of pairs we have: (1,36), (2,18), (3,12), (4,9) and (6,6). As always, the factors can be listed in pairs. However, in this example one of the pairs has a single factor that is repeated, i.e. (6,6). When we count the total number of distinct factors of 36, the answer is nine - an odd number. In fact, this will be true for any perfect square because one (and only one) of the factor pairs will have a single factor repeated twice. The converse is also true - non-perfect square integers will always have an even number of factors, since none of the factor pairs will have a repeated integer. Statement (1) is sufficient so the answer is either (A) or (D).
If we look at statement (2), it tells us that p is an odd integer. Knowing that p is odd, however, doesn't tell us if the number of factors is odd or even. Odd numbers that are perfect squares would have an odd number of factors (9 = 1, 3 and 9) and odd numbers that are not perfect squares would have an even number of factors (15 = 1, 3, 5 and 15). Statement (2) is not sufficient and the answer is (A). This problem could also be solved by plugging legal values based on the two statements to see if they are sufficient to answer the question. When evaluating the sufficiency of statement (1), it would be best to try perfect squares that are both odd and even to see if all perfect squares have an odd number of factors. When evaluating the sufficiency of statement (2), it would be best to try odd numbers that are both perfect squares and not to see if all odd numbers have an odd number of factors.

The correct answer is A.

22.
(1) INSUFFICIENT: For a number to be divisible by 16, it must be divisible by 2 four times. The expression $m(m + 1)(m + 2) \dots (m + n)$ represents the product of $n + 1$ consecutive integers. For example if $n = 5$, the expression represents the product of 6 consecutive integers.

To find values of n for which the product will be divisible by 16 let's first consider $n = 3$. This implies a product of four consecutive integers. In any set of four consecutive integers, two of the integers will be even and two will be odd. In addition one of the two even integers will be a multiple of 4, since every other even integer on the number line is a multiple of 4 (4 yes, 6 no, 8 yes, etc...). This accounts for a total of three 2's that can definitely divide into the product of 4 consecutive integers. This however is not enough.

With five integers (i.e. $n = 4$), there may or may not be an additional 2: there could be two or three even integers. Six integers (i.e. $n = 5$) will guarantee three even integers and a product that is divisible by four 2's. Anything above six integers will naturally be divisible by 16 as well, so we can conclude that n is greater than or equal to 5.

(2) INSUFFICIENT: This expression factors to $(n - 4)(n - 5) = 0$. There are two solutions for n, 4 or 5.

(1) AND (2) TOGETHER SUFFICIENT: If n must be greater than or equal to 5 and either 4 or 5, then n must be equal to 5.

The correct answer is C.

23.
(1) INSUFFICIENT: a and b could be 12 and 8, with a greatest common factor of 4; or they could be 11 and 7, with a greatest common factor of 1.
(2) INSUFFICIENT: This statement tells us that b is a multiple of 4 but we have no information about a.

(1) AND (2) SUFFICIENT: Together, we know that b is a multiple of 4 and that a is the next consecutive multiple of 4. For any two positive consecutive multiples of an integer n, n is the greatest common factor of those multiples, so the greatest common multiple of a and b is 4.

The correct answer is C.
.

24.

The first 7 integer multiples of 5 are 5, 10, 15, 20, 25, 30, and 35. The question is asking for the least common multiple (LCM) of these 7 numbers. Let's construct the prime box of the LCM.

In order for the LCM to be divisible by 5, one **5** must be in the prime box.
In order for the LCM to be divisible by 10, a 5 (already in) and a **2** must be in the prime box. In order for the

LCM to be divisible by 15, a 5 (already in) and a **3** must be in the prime box.

In order for the LCM to be divisible by 20, a 5 (already in), a 2 (already in), and a second **2** must be in the prime box.

In order for the LCM to be divisible by 25, a 5 (already in) and a second **5** must be in the prime box.

In order for the LCM to be divisible by 30, a 5 (already in), a 2 (already in) and a 3 (already in) must be in the prime box.

In order for the LCM to be divisible by 35, a 5 (already in) and a **7** must be in the prime box. Thus, the prime box of the LCM contains a 5, 2, 3, 2, 5, and 7. The value of the LCM is the product of these prime factors, 2100.

The correct answer is D

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First 7 multiples of 5 are:

5 = 5*1
10 = 5*2
15 = 5*3
20 = 5*2*2
25 = 5*5
30 = 5*2*3
35 = 5*7

What prime factors will be needed to create a number that is divisible by the first 7 multiples of 5?

5,5,2,2,3,7.[We have taken 2 2s because the number should be divisible by 20. We have taken 2 5s because the number should be divisible by 25.]

Therefore, the number = 2 * 2 * 3 * 5 * 5 * 7 = 2100
The correct answer is D.

25.
Factorial notation is a shorthand notation. Write out statement (1) in its expanded form:

$$n \times (n-1) \times (n-2) \times (n-3) \ldots = n \times [(n-1) \times (n-2) \times (n-3) \ldots]$$

This does not provide any useful information about the value of n. Statement (2) can be rearranged and factored as follows:

$n^3 + 3n^2 + 2n = n(n^2 + 3n + 2) = n(n + 1)(n + 2)$

This is a set of three consecutive integers. Any set of three consecutive integers must contain one multiple of three. Therefore, it must be divisible by three. This does not provide any useful information about the value of n either.
Both statements are true for all integers; therefore, they do not provide sufficient information to figure out the value of n.
The correct answer is E.

26. When a perfect square is broken down into its prime factors, those prime factors always come in "pairs." For example, the perfect square 225 (which is 15 squared) can be broken down into the prime factors 5 × 5 × 3 × 3. Notice that 225 is composed of a pair of 5's and a pair of 3's.

The problem states that x is a perfect square. The prime factors that build x are p, q, r, and s. In order for x to be a perfect square, these prime factors must come in pairs. This is possible if either of the following two cases hold:

Case One: The exponents a, b, c, and d are even. In the example $3^2 5^4 7^2 11^6$, all the exponents are even so all the prime factors come in pairs.

Case Two: Any odd exponents are complemented by other odd exponents of the same prime. In the example $3^1 5^4 3^3 11^6$, notice that 3^1 and 3^3 have odd exponents but they complement each other to create an even exponent (3^4), or "pairs" of 3's. Notice that this second case can only occur when p, q, r, and s are NOT distinct. (In this
example, both p and r equal 3.)

Statement (1) tells us that 18 is a factor of both ab and cd. This does not give us any information about whether the exponents a, b, c, and d are even or not.

Statement (2) tells us that 4 is not a factor of ab and cd. This means that neither ab nor cd has two 2's as prime factors. From this, we can conclude that at least two of the exponents (a, b, c, and d) must be odd. As we know from Case 2 above, if $p^a q^b r^c s^d$ is a perfect square but the exponents are not all even, then the primes p, q, r and s must NOT be distinct.

The correct answer is B.

27.

Using the rules for dividing exponential expressions with common bases, we can rewrite the function W as follows: $5^{a-p}2^{b-q}7^{c-r}3^{d-s}$. Clearly, this function represents a product of powers of the prime numbers 5, 2, 7, and 3.

The question stems states that W = 16, which is simply 2^4. Since 2 is the only number that is a factor of W, then it must be true that the powers of the other prime bases that compose
W (namely, 5, 7, and 3) are each zero. Otherwise, the value of the function W would be divisible by these primes.
Thus, we can conclude that $W = 5^{a-p}2^{b-q}7^{c-r}3^{d-s} = 5^0 2^{b-q} 7^0 3^0 = 16$. This means that b – q = 4. Since b and q each represent the hundreds digit of the integers K and L respectively, we know that the hundreds digit of K is 4 greater than the hundreds digit of L. Also, since the exponents of 5, 7, and 3 are equal to zero, the differences between the thousands, tens, and units digits of K and L are zero, implying that K and L differ only in their hundreds digit.

Since the hundreds digit of K is 4 greater than that of L, the difference between K and L is 4 × 100 = 400. Therefore K – L = 400. Since Z is defined as (K – L) ÷ 10, we can determine that Z = 400 ÷ 10 = 40.
The correct answer is D.

28.

264,600 can be broken into its prime factors as follows: 2^3 x 3^3 x 5^2 x7^2.

To determine the total number of factors, we need to calculate the number of ways that the various powers of 2, 3, 5, and 7 can combine.

There are 4 powers of 2 (including the zero power): 2^0, 2^1, 2^2, and 2^3.

There are 4 powers of 3 (including the zero power): 3^0, 3^1, 3^2, and 3^3.

There are 3 powers of 5 (including the zero power): 5^0, 5^1, and 5^2.

There are 3 powers of 7 (including the zero power): 7^0, 7^1, and 7^2.

Consequently, there are 4 x 4 x 3 x 3 = 144 different ways to combine the prime factors of 264,600. These 144 combinations form all the factors of 264,600, from the first factor (2^0 x 3^0 x 5^0 x 7^0 = 1) to the last factor (2^3 x 3^3 x 5^2 x 7^2= 264,600).

Now we need to eliminate the factors of 6. Recall that a factor of six must have at least one 2 and one 3. So it must have either 2^1, 2^2, or 2^3 AND 3^1, 3^2, or 3^3 as its factors. There are 3 x 3 = 9 different ways the powers of 2 and 3 can combine to generate distinct numbers divisible by 6. There are 3 x 3 = 9 different numbers that can be formed from the powers 5 and 7 (using 5^0, 5^1, and 5^2 and 7^0, 7^1, and 7^2). Any of these 9 numbers can combine with any of the 9 multiples of 6 to form 9 x 9 = 81 distinct multiples of 6.

Hence, the number of factors of 264,600 that are not divisible by 6 is 144 – 81 = 63.
The correct answer is D.

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The total number of factors of 264600 are 144.

264600 = (2^3) * (3^3) * (5^2) * (7^2).

We have to subtract from 144 all the factors are multiples of 6.

Total factors divisible by 6 = (2+1)(2+1)(2+1)(2+1) = 81

Note we have not calculated this as (3+1)(3+1)(2+1)(2+1). This is because this expression includes the (2^0) * (3^0) case, which we don't want since we're looking for factors that are multiples of 6.

Therefore, factors not divisible by 6 = 144 - 81 = 63.
The correct answer is D.

29.

The question stem states that n^2/n yields an integer greater than 0, which can be simplified to state that n is an integer. The question can be rephrased as follows: Does the integer n have 2, 3, and 5 as prime factors? Statement (1) tells us that n^2 divisible by 20. Thus, n^2 has 2, 2, and 5 as its prime factors. What does this tell us about n? Since we know that n is an integer, n^2 must be a perfect square. The prime factors of any perfect square come in pairs.

For example, the perfect square 36 can be broken down into 2 x 2 x 3 x 3, or a pair of 2's and a pair of 3's. Taking one prime from each pair, yields 2 x 3, or 6, the integer root of the perfect square 36.

Since we are told in Statement (1) that n^2 has 2, 2, and 5 as its prime factors we can assume that n^2 actually has a pair of 2's and **a pair** of 5's as well (remember, all perfect squares can be broken into pairs of primes).

Thus, taking one prime from each pair, we know that n must be divisible by 2 and 5. However, this is not sufficient to answer the question, since we do not know whether or not n is divisible by 3.
Statement (2) tells us that n^3 is divisible by 12. Thus n^3 must have 2, 2, and 3 as its prime factors. We also know that, since n is an integer, n^3 is a perfect cube. Using the same logic as in the previous statement, n^3 must be divisible by a triplet of 2's and a triplet of 3's. Taking one prime from each triplet, we know that n must be divisible by 2 and 3. However, this is not sufficient to answer the question since we do not know whether or not n is divisible by 5.
Taking both statements together, we know that n is divisible by 2, 3, and 5. Therefore n is divisible by 30.
The correct answer is C.

30. First, let's rephrase the complex wording of this question into something easier to handle. The question asks whether x is a non-integer which is the reverse of an easier question: Is x an integer?

Surely, if we can answer this question, we can answer the original question.

We can isolate x by rewriting the given equation as follows: $x = \frac{ab}{30}$.

In order for x to be an integer, ab must be divisible by 30. In order for a number to be divisible by 30, it must have 2, 3, and 5 as prime factors (since 30 = 2 x 3 x 5).

Thus, the question becomes: Does ab have 2, 3, and 5 as prime factors?

We know from the question that a and b are consecutive positive integers. Thus, either a or b is an even number, which means that the product ab must be divisible by 2.
Since we know that 2 is a prime factor of ab, the question can be further simplified: Does ab have 3 and 5 as prime factors?

Statement (1) tells us that 21 is a factor of a^2 which means that 3 and 7 are prime factors of a^2.

We can deduce from this that 3 and 7 must also be factors of a itself. (How? We know that a is a positive integer, which means that a^2 is a perfect square. All prime factors of perfect squares come in pairs. Thus, if a^2 is divisible by 3, then a^2 must be divisible by a pair of 3's, which means that a itself must be divisible by at least one 3. You can test this using a real value for a^2.)

Knowing that 3 is a prime factor of a tells us that 3 is a factor of ab but is not sufficient to answer our rephrased question, since we know nothing about whether 5 is a factor of ab.

Statement (2) tells us that 35 is a factor of b^2 which means that 5 and 7 are prime factors of b^2.

Using the same logic as for the previous statement, we can deduce that 5 and 7 must be factors of b itself. Knowing that 5 is a prime factor of b tells us that 5 is a factor of ab but it is not sufficient to answer our rephrased question, since we know nothing about whether 3 is a factor of ab.

If we combine both statements, we know that ab must be divisible by both 3 and 5, which is sufficient information to answer the original question.
The correct answer is C.

31.
Divisibility problems can be solved using prime factorization.

The prime factorization of $504 = 2^3 3^2 7$.

Therefore, using the given equation, we can see that:

$$\sqrt{ABC} = 504 = 2^3 3^2 7$$
$$\left(\sqrt{ABC}\right)^2 = \left(2^3 3^2 7\right)^2$$
$$ABC = 2^6 3^4 7^2$$

To answer the question, we must determine whether ***B*** contains ***one*** of the six 2's in the prime factorization.

Statement (1) alone tells us that $C = 168 = 2^3(3)(7)$.

This tells us that ***C*** has ***three*** of the 2's in the prime factorization. However, since we have no information about ***A*** or ***B***, this is not sufficient information to answer the question.

Statement (2) alone tells us that **A** is a perfect square.

This tells us that if ***A*** contains any 2's as prime factors, it must have an even number of 2's. (The only way a number can be a perfect square is if its prime factors come in pairs). This, again is not sufficient information to answer the question.

Using both statements together, we know that ***C*** has ***three*** of the 2's. We also know that ***A*** can have either ***zero*** of the 2's or ***two*** of the 2's, but, since ***A*** is a perfect square, it cannot have all ***three*** of the remaining 2's.

Thus, ***B*** must have at least one 2 as a prime factor.

The correct answer is C.

32.

Any factor of a nonprime integer is the product of prime factors of that integer. For example, 90 has the prime factors 2, 3, 3, and 5, and all other factors of 90 are the products of some combination of these factors (e.g., 6 = (2)(3); 9 = (3)(3); 10 = (2)(5); 15 = (3)(5); 18 = (2)(3)(3); 30 = (2)(3)(5); 45 = (3)(3)(5); 90 = (2)(3)(3)(5)).

So to determine the number of factors that a nonprime integer has, we need to determine how many different combinations of factors that integer's prime factorization will allow. Let's look at

90 again. Its prime factorization is $2^1 3^2 5^1$. This means that we have one 2, two 3's, and one 5. If we had one hat, two shirts, and one pair of pants to combine to make outfits, we could

make 1*2*1=2 outfits. By analogy, 90 should have 1*2*1=2 factors. But 90 has 12 factors (including 1 and 90), so where do the other 10 factors come from?

Think of each prime factor as a category: 2, 3, and 5. In the 2 category, we have two options: 2^0 and 2^1.In the 3 category, we have 3 options: 3^0.3^1 and 3^2 .In the 5 category, we have 2 options: 5^0 and 5^1. Note that a nonzero number raised to the zero power always equals 1, so when we choose a prime factor raised to the zero power, we are simply introducing a 1 into our

multiplication. For example, $2^0 \times 3^1 \times 5^1 = 1 \times 3 \times 5 = 15$. When we choose the zero power from each category of prime factor, we get 1 as the product, yielding 1 as a factor. For example, $2^0 \times 3^0 \times 5^0 = 1 \times 1 \times 1 = 1$.

So instead of 1*2*1=2, which leaves out the zero power in each category, we need to add 1 to the exponent of each prime factor in the prime factorization to account for the zero power.

For example, the prime factorization of 90 is $2^1 \cdot 3^2 \cdot 5^1$, but since there are really two powers in the 2 category, three powers in the 3 category, and two powers in the 5 category (to account for

the zero powers), the number of possible combinations of prime factors is actually 2*3*2=12

A chart may make this clear:

Combination of Prime Factors	Factor Yielded
$2^0 \times 3^0 \times 5^0$	1
$2^1 \times 3^0 \times 5^0$	2
$2^0 \times 3^1 \times 5^0$	3
$2^0 \times 3^0 \times 5^1$	5
$2^1 \times 3^1 \times 5^0$	6
$2^0 \times 3^2 \times 5^0$	9
$2^1 \times 3^0 \times 5^1$	10
$2^0 \times 3^1 \times 5^1$	15
$2^1 \times 3^2 \times 5^0$	18
$2^1 \times 3^1 \times 5^1$	30
$2^0 \times 3^2 \times 5^1$	45
$2^1 \times 3^2 \times 5^1$	90

The question asks which choice could be the number of factors of the integer q if the prime factorization of q can be expressed as $a^{2x} \cdot b^{x} \cdot c^{3x-1}$ The number of factors will not be equal to $(2x)(x)(3x-1)$ but rather to $(2x+1)(x+1)(3x-1+1)$, to take into account the zero power in each category of prime factor (i.e., a^0, b^0, and c^0). The product of these terms will be the number of factors of q:

$$(2x+1)(x+1)(3x-1+1) \rightarrow$$
$$(2x+1)(x+1)(3x) \rightarrow$$
$$(2x^2+3x+1)(3x) \rightarrow$$
$$6x^3+9x^2+3x$$

Note that all three terms are multiples of 3 and can be factored: $3(2x^3+3x^2+x)$
So the number of factors of q must be a multiple of 3. Which choice could potentially be a multiple of 3?

3j + 4 cannot be a multiple of 3 because 3j is a multiple of 3 and adding 4 to it will bypass the next multiple of 3. Eliminate A.

5k + 5 could be a multiple of 3 if k = 17: 5(17) + 5 = 90. Keep B.

6l + 2 cannot be a multiple of 3 because 6l is a multiple of 3 and adding 2 to it will fall 1 short of the next multiple of 3. Eliminate C.

9m + 7 cannot be a multiple of 3 because 9m is a multiple of 3 and adding 7 to it will bypass the next two multiples of 3. Eliminate D.

10n + 1 can be a multiple of 3 if n = 8: 10(8) + 1 = 81. Keep E. Which is the correct answer, B or E?

Let's reconsider the expression $3(2x^3+3x^2+x)$. If x is even, the expression will be even (the sum of three evens is even and the product of even and odd is even). If x is odd, the expression will be even (the sum of two odds and an even is even and the product of an even and odd is even). So regardless of the value of x, the number of factors of q must be even.

10n + 1 can never be even because 10n is even and adding 1 to it will result in an odd number. Eliminate E. Therefore, **the correct answer is B.**

33. An integer is ―divisible‖ by a number if the integer can be divided evenly by that number (meaning that there is no remainder). For example, 15 is divisible by 3 because it can be divided evenly by 3 (15/3 = 5), but 15 is not divisible by 4 (15/4 = 3 r. 3).

One way to answer this question is to test each answer choice. The smallest integer that is divisible by 12, 11, 10, 9, and 8 will be the correct answer.

(A) 7,920 is divisible by every value on the list.

(B) 5,940 IS NOT divisible by 8.

(C) 3,960 IS divisible by every value on the list.

(D) 2,970 IS NOT divisible by 12 or 8.

(E) 890 IS NOT divisible by 12, 11, 9, or 8.

Alternatively, the least common multiple can be calculated directly. Consider the prime factors of the numbers on our list:
$12 = 2^2 \times 3$
$11 = 11^1$
$10 = 2^1 \times 5^1$
$9 = 3^2$
$8 = 2^3$

The smallest integer that is divisible by 12, 11, 10, 9, and 8 will have the prime factors that appear in the list above, but no more of each than is necessary. In order to be divisible by 8, we need three 2's. In order to be divisible by 9, we need two 3's. In order to be divisible by 10, we need one 2 and one 5. But since we already have three 2's in the 8, we need only the 5. In order to be divisible by 11, we need the entire 11 (because it is prime). In order to be divisible by 12, we need two 2's and one 3, but since we already have three 2's in the 8 and two 3's in the 9, we do not need to take any more.
Thus, the integer will have the following prime factors: three 2s, two 3s, one 5, and one 11. $2^3 \times 3^2 \times 5 \times 11 = 3{,}960$
The correct answer is C.

34.
The question stem tells us that x is a positive integer. Then we are asked whether x is prime; it is helpful to remember that all prime numbers have exactly two factors. Since we cannot rephrase the question, we must go straight to the statements.

(1) SUFFICIENT: If x has the same number of factors as y^2, then x cannot be prime. A prime number is a number that has only itself and 1 as factors. But a square has at least 3 prime factors. For example, if y is prime, y = 2, then $y^2 = 4$, which has 1, 2, and 4 as factors. If the root (in this case y) is not prime, then the square will have more than 3 factors. For example, if y

= 4, then $y^2 = 16$, which has 1, 2, 4, 8, and 16 as factors. In either case, x will have at least 3 factors, establishing it as nonprime.

(2) INSUFFICIENT: If z is prime, then x will have only two factors, making it prime. But if z is nonprime, it will have either one (if z = 1) or more than two factors, which means x will have either one or more than two factors, making x nonprime. Since we do not know which case we have, we cannot tell whether x is prime.

The correct answer is A.

35.
h(100)=2*4*6*...*100, h(100) is the multiple of 2, 3, 5, 7, 11, ?3.
If a integer m is multiple of integer n(none 1), m+1 is not the multiple of n definitely. So, p is greater than 47.
Answer is p>40
We use the property that if m is a multiple of n then m+1 **cannot** be a multiple of n.
Now h(100) = 2*4*6 .... 96*98*100.
The maximum possible prime factor for h(100) can be calculated by trying 98/2 = 49 (not prime), 94/2 = 47 (prime).
So maximum possible prime factor of h(100) =47.
So h(100) is a multiple of 47(and all primes less than 47), hence h(100)+1 cannot be a multiple of 47 (or primes less than 47).
Since p is a prime factor of h(100)+1, so p should atleast be greater than 47 to be a prime factor of h(100)+1. hence p>47.
The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

$h(100) + 1 = 2*4*6*...*100 + 1 = 2^{50} * (1*2*3*..*50) + 1 = 2^{50} * 50! + 1$

Now, two numbers $h(100) = 2^{50} * 50!$ and $h(100) + 1 = 2^{50} * 50! + 1$ are consecutive integers. Two consecutive integers are co-prime, which means that they don't share ANY common factor but 1. For example 20 and 21 are consecutive integers, thus only common factor they share is 1.

As $h(100) = 2^{50} * 50!$ has ALL prime numbers from 1 to 50 as its factors, then, according to the above, $h(100) + 1 = 2^{50} * 50! + 1$ won't have ANY prime factor from 1 to 50. Hence p (> 1), the smallest prime factor of $h(100) + 1$ must be more than 50.

The correct answer is C.

36.
We have 5,10,15,20,25,30 and some 2s to see how many zeros there are in the tail of the result 5x2, 15x2 contributes 1 zero each
10, 20, 30 => 3 zeros
25x4 => be careful, it contributes 2 zeros
So totally there are 1x2+3+2=7 zeros in the tails of 30! For 1, d
can be 1,2,3,4,5,6,7
For 2, d can be any integer >6
Together, d can only be 7 to fulfil both requirements.
The correct answer is C

37.
For 1, k >4!=24, it maybe 29, 31, ? insufficient
For 2, k=n*(1+13!/n),where 2<=n<=13, we are sure to find p=n to fulfil the requirement.
The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)

We have to check if for an integer k, there exists a factor p such that 1 < p < k. Meaning we have to check if k is prime or not.

Statement 1 : k > 4!

This is clearly insufficient.

For example, k can be 29, which is a prime number
Or k can be 28, which is a composite number.

So, statement 1 is insufficient!

Statement 2 :

13! + 2 <= k <= 13! + 13

If k = 13! + 2,

k = 13*12*11*10*9*8*7*6*5*4*3*2 + 2

k = 2 * [13*12*11*10*9*8*7*6*5*4*3 + 1] = 2k, where k = 13*12*11*10*9*8*7*6*5*4*3 + 1   [We've taken 2 common from the 2 expressions]

Similarly,

if k = 13! + 3,

k = 13*12*11*10*9*8*7*6*5*4*3*2 + 3

k = 3 * [13*12*11*10*9*8*7*6*5*4*2 + 1] = 3k, where k = 13*12*11*10*9*8*7*6*5*4*2 + 1

We can perform this exercise for numbers till 13! + 13, and we will find that all these numbers are composite. Therefore, statement 2 is sufficient!
The correct answer is B.

38.
Alternate Solution from Gmatclub

(1) $3y^2+7y=x$ --> $y(3y+7)=x$ --> as $3y+7=integer$, then $y*integer=x$ --> x is a multiple of y. Sufficient.

(2) x^2-x is a multiple of y --> $x(x-1)$ is a multiple of y --> x can be multiple of y ($x=2$ and $y=2$) OR $x-1$ can be multiple of y ($x=3$ and $y=2$) or their product can be multiple of y ($x=3$ and $y=6$). Not sufficient.

Answer: A.

The correct answer is A.

39.
Because P is a prime number, so, all the positive numbers less than p can fulfill the requirements.
The number of these numbers is p-1.
The correct answer is A.

40.
x/y=1/2 or 1/6 both can fulfill the requirement.
The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

The least common denominator is the least common multiple of the denominators of a set of fractions. For example, the least common denominator of 1/2 and 1/3 is 6.
(1) The least common denominator of x/y and 1/3 is 6 --> LCM of y and 3 is 6 --> y=2 or y=6. Not sufficient.

(2) x = 1 --> no info about y. Not sufficient.

(1)+(2) y can still be 2 or 6. Not sufficient.

The correct answer is E.

Intuitive Approach - Solution:
Let me remind you first that LCM is the lowest common multiple. It is that smallest number which is a multiple of both the given numbers.

Say, we have two fractions: 1/4 and 1/2. What is their LCM? It's 1/2 because 1/2 is the smallest fraction which is a multiple of both 1/2 and 1/4. It will be easier to understand in this way:

1/2 = 2/4. (Fractions with the same denominator are comparable.)

LCM of 2/4 and 1/4 will obviously be 2/4.

If this is still tricky to see, think about their equivalents in decimal form:

1/2 = 0.50 and 1/4 = 0.25. You can see that 0.50 is the lowest common multiple they have.

Let's look at GCF now.

What is GCF of two fractions? It is that greatest factor which is common between the two fractions.
Again, let's take 1/2 and 1/4. What is the greatest common factor between them?

Think of the numbers as 2/4 and 1/4. The greatest common factor between them is 1/4.

(Note that 1/2 and 1/4 are both divisible by other factors too e.g. 1/8, 1/24 etc but 1/4 is the greatest such common factor)

Now think, what will be the LCM of 2/3 and 1/8?

We know that 2/3 = 16/24 and 1/8 = 3/24.
LCM = 16*3/24 = 48/24 = 2

LCM is a fraction greater than both the fractions or equal to one or both of them (when both fractions are equal). When you take the LCM of the numerator and GCF of the denominator, you are making a fraction greater than (or equal to) the numbers.

Also, what will be the GCF of 2/3 and 1/8?

We know that 2/3 = 16/24 and 1/8 = 3/24.

GCF = 1/24

GCF is a fraction smaller than both the fractions or equal to one or both of them (when both fractions are equal). When you take the GCF of the numerator and LCM of the denominator, you are making a fraction smaller than (or equal to) the numbers.

The correct answer is E.

41:
The least prime number is 2. We notice that 2^6=64 and 2^5*3=96. Any 2 combined, the number will be greater than 100. So, only 64 and 96 fulfill the requirement.
So, answer is 2

The correct answer is C.

42.
1). n=k1*5;t=k2*5(k1,k2 are natural numbers), n*t=k1*k2*5^2. k1 and k2 are unknown, thus, we cannot obtain the greatest prime factor of nt.
2). 105=1*3*5*7.
The least common multiple of n and t is 105=>the greatest value of n and t is 105, and the other one must be less than 105 and be composed with numbers of 1, 3, 5, 7.
So, the greatest prime factor is 7
The correct answer is B.

43.
14n/60 is an integer, than n=30*k, k=1,2,...6 30
has 3 prime factors, 2, 3, and 5.
k is from 1 to 6, at most contains 2,3 and 5.
Above all, 30k has three different positive prime factors: 2, 3, and 5
The correct answer is B.

44.

Given:
x is a factor of y --> $y = mx$, for some non-zero integer m;
y is a factor of z --> $z = ny$, for some non-zero integer n;
So, $z = mnx$.

Question: is z even? Note that z will be even if either x or y is even

(1) xz even --> either z even, so the answer is directly YES or x is even (or both). But if x is even and as $z = mnx$ then z must be even too (one of the multiples of z is even, so z is even too). Sufficient.

(2) y even --> as $z = ny$ then as one of the multiples of z even --> z even. Sufficient.

Answer: D.

The correct answer is D.

45.
1). K is prime, 20K=2^2*5*K, k could be 2, 5, or other prime, so, number of the different prime factors of 20k cannot be determined.
2). K=7, 20k=2^2*5*7, has 3 different prime factors.
The correct answer is B.

46:
From 1: 12u = 8y + 12 => 3u = 2y + 3 (inconclusive)
From 2: x = 12(8z + 1) => x = 12 * an odd number
Given y = 12 z => the greatest common divisor between z and (8z + 1) is 1 => the greatest common divisor between x and y is 12
The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)

x=8y+12 = 4(2y+3)
i.e. x is a Multiple of 4
Statement 1: X=12u
i.e. x is a multiple of 12
i.e. y must be a multiple of 3
but since y may be an even multiple of 3 or an odd multiple of 3 so GCD will have different values. Hence,
NOT SUFFICIENT
Statement 2: Y=12z
i.e. y must be a multiple of 3 as well 4
for such value of y, x must be a multiple of 12
e.g. @y=12, x = 4*27, GCD = 12
@y=24, x = 4*51, GCD = 12
but since y is an even multiple of 3 so GCD will have constant value. Hence,
SUFFICIENT
The correct answer is B.

OR

x = 8y + 12
(1) x = 12u, where u is an integer.
If x is a multiple of 12, it means 8y is a multiple of 12. Since 8 already has three 2s, we NEED y to have a 3 but it COULD have 2s and/or other factors too.
So we cannot say what the GCD of x and y is.
Not sufficient.
(2) y = 12z, where z is an integer.
If y is a multiple of 12, x is a multiple of 12 too. So they certainly have 12 in common. Let's see

what else they could have in common.
x is 12 more than a multiple of y so the only common factors they could have are the factors of 12.
We already know that they both have 12 in them. So GCD must be 12.
Sufficient.
The correct answer is B.

47.
4^17 = (2^2)^17 = 2^34
2^34 - 2^28 = 2^28*(2^6 - 1) = 2^28*63 = 2^28 * 7 * 3^2
Answer is 7

48.

(1) 4 different prime numbers are factors of 2n --> if n itself has 2 as a factor (eg $n = 2 * 3 * 5 * 7$) than its total # of primes is 4 **but** if n doesn't have 2 as a factor (eg $n = 3 * 5 * 7$) than its total # of primes is 3. Not sufficient.

(2) 4 different prime numbers are factors of n^2 --> n^x (where x is an integer ≥ 1) will have as many different prime factors as integer n, exponentiation doesn't "produce" primes. So, 4 different prime numbers are factors of n. Sufficient.

Answer: B.

The correct answer is B.

49.
(1) INSUFFICIENT: a and b could be 12 and 8, with a greatest common factor of 4; or they could be 11 and 7, with a greatest common factor of 1.

(2) INSUFFICIENT: This statement tells us that b is a multiple of 4 but we have no information about a.

(1) AND (2) SUFFICIENT: Together, we know that b is a multiple of 4 and that a is the next consecutive multiple of 4. For any two positive consecutive multiples of an integer n, n is the greatest common factor of those multiples, so the greatest common factor of a and b is 4.

The correct answer is C

50.
The first 7 integer multiples of 5 are 5, 10, 15, 20, 25, 30, and 35. The question is asking for the least common multiple (LCM) of these 7 numbers. Let's construct the prime box of the LCM.

In order for the LCM to be divisible by 5, one **5** must be in the prime box.
In order for the LCM to be divisible by 10, a 5 (already in) and a **2** must be in the prime box. In order for

the LCM to be divisible by 15, a 5 (already in) and a **3** must be in the prime box.

In order for the LCM to be divisible by 20, a 5 (already in), a 2 (already in), and a second **2** must be in the prime box.

In order for the LCM to be divisible by 25, a 5 (already in) and a second **5** must be in the prime box.

In order for the LCM to be divisible by 30, a 5 (already in), a 2 (already in) and a 3 (already in) must be in the prime box.

In order for the LCM to be divisible by 35, a 5 (already in) and a **7** must be in the prime box. Thus, the prime box of the LCM contains a 5, 2, 3, 2, 5, and 7. The value of the LCM is the product of these prime factors, 2100.

The correct answer is D

GMAT Quant Topic 4: Numbers

Part H: Consecutive Integers

1.
For any set of consecutive integers with an **odd number of terms**, the sum of the integers is **always** a multiple of the number of terms. For example, the sum of 1, 2, and 3 (three consecutives -- an odd number) is 6, which is a multiple of 3. For any set of consecutive integers with an **even number of terms**, the sum of the integers is **never** a multiple of the number of terms. For example, the sum of 1, 2, 3, and 4 (four consecutives -- an even number) is 10, which is not a multiple of 4.
The question tells us that y = 2z, which allows us to deduce that y is even. Since y is even, then the sum of y integers, x, cannot be a multiple of y. Therefore, x/y cannot be an integer; choice C is the correct answer.
We can verify this by showing that the other choices could indeed be true:
(A) The sum x can equal the sum w: 4 + 5 + 6 + 7 + 8 + 9 = 12 + 13 + 14 = 39, for example.
(B) The sum x can be greater than the sum w: 1 + 2 + 3 + 4 > 1 + 2, for example.
(D) z could be odd (the question does not restrict this), making the sum w a multiple of z. Thus, w/z could be an integer. For example, if z = 3, then we are dealing with three consecutive integers. We can choose any three: 2, 3, and 4, for example. 2 + 3 + 4 = 9 and 9/3 = 3, which is an integer.
(E) x/z could be an integer. If z = 2 and if x is an even sum, then x/z would be an integer. For example, if z = 2, then y = 4. We can choose any four consecutive integers: 1 + 2 + 3 + 4, for example. So the sum x of these four integers is 10. 10/2 = 5, which is an integer.
The correct answer is C.

2.
The quadratic expression $k^2 + 4k + 3$ can be factored to yield (k + 1)(k + 3). Thus, the expression in the question stem can be restated as (k + 1)(k + 2)(k + 3), or the product of three consecutive integers. This product will be divisible by 4 if one of two conditions are met:

If k is odd, both k + 1 and k + 3 must be even, and the product (k + 1)(k + 2)(k + 3) would be divisible by 2 twice. Therefore, if k is odd, our product must be divisible by 4.

If k is even, both k + 1 and k + 3 must be odd, and the product (k + 1)(k + 2)(k + 3) would be divisible by 4 only if k + 2, the only even integer among the three, were itself divisible by 4.

The question might therefore be rephrased —Is k odd, OR is k + 2 divisible by 4? Note that a ‘yes’ to either of the conditions would suffice, but to answer 'no' to the question would require a ‘no’ to both conditions.

(1) SUFFICIENT: If k is divisible by 8, it must be both even and divisible by 4. If k is divisible by 4, k + 2 cannot be divisible by 4. Therefore, statement (1) yields a definitive ‘no’ to both conditions in our rephrased question; k is not odd, and k + 2 is not divisible by 4.

(2) INSUFFICIENT: If k + 1 is divisible by 3, k + 1 must be an odd integer, and k an even integer. However, we do not have sufficient information to determine whether k or k + 2 is divisible by 4.

The correct answer is A.

3.
One way to approach this problem is to test the values given in the answer choices.

	Product	Product when x =						
k	x(x – 1)(x – k)	0	1	2	3	4	5	Comments
-4	x(x – 1)(x + 4)	0	0	(2)(1)(6)	(3)(2)(7)	(4)(3)(8)	(5)(4)(9)	always divisible by 3

-2	x(x – 1)(x + 2)	0	0	**(2)(1)(4)**	(3)(2)(5)	(4)(3)(6)	**(5)(4)(7)**	NOT always divisible by 3
-1	x(x – 1)(x + 1)	0	0	(2)(1)(3)	(3)(2)(4)	(4)(3)(5)	(5)(4)(6)	always divisible by 3
2	x(x – 1)(x – 2)	0	0	(2)(1)(0)	(3)(2)(1)	(4)(3)(2)	(5)(4)(3)	always divisible by 3
5	x(x – 1)(x – 5)	0	0	(2)(1)(-3)	(3)(2)(-2)	(4)(3)(-1)	(5)(4)(0)	always divisible by 3

Alternatively, use the rule that the product of three consecutive integers will always be a multiple of three. The rule applies because any three consecutive integers will always include one multiple of three. So, if (x), (x – k) and (x – 1) are consecutive integers, then their product must be divisible by three. Note that (x) and (x – 1) are consecutive, so the three terms would be consecutive if (x – k) is either the lowest of the three, or the greatest of the three:

(x – k), (x – 1), and (x) are consecutive when (x – k) = (x – 2), or k = 2
(x – 1), (x), and (x – k) are consecutive when (x – k) = (x + 1), or k = -1

Note that the difference between k = -1 and k = 2 is 3. Every third consecutive integer would serve the same purpose in the product x(x – 1)(x – k): periodically serving as the multiple of three in the list of consecutive integers. Thus, k = -4 and k = 5 would also give us a product that is always divisible by three.

The correct answer is B.

4.
The possible values of n should be computed right away, to rephrase and simplify the question.
Note that n consecutive positive integers that sum to 45 have a mean of 45/n, which is also the median of the set; therefore, the set must be arranged around 45/n. Also, any set of consecutive integers must have either an integer mean (if the number of integers is odd) or a mean that is an integer + 1/2 (if the number of integers is even). So, if we compute 45/n and see that it is neither an integer nor an integer +1/2, then we can eliminate this possibility right away.
Setting up a table that tracks not only the value of n but also the value of 45/n is useful.

n	45/n	n positive consecutive integers summing to 45
1	45	45
2	22.5	22, 23
3	15	14, 15, 16
4	11.25	none
5	9	7, 8, 9, 10, 11
6	7.5	5, 6, 7, 8, 9, 10
7	6 3/7	none
8	5 5/8	none
9	5	1, 2, 3, 4, 5, 6, 7, 8, 9
10	4.5	0, 1, 2, 3, 4, 5, 6, 7, 8, 9 -- but this doesn't work, because not all are positive integers
...	...	impossible (the set will include negative integers, if an integer set can be found at all)

(1) INSUFFICIENT: If n is even, n could be either 2 or 6. Statement (1) is NOT sufficient.

Alternatively, to find these values algebraically, you can use the following procedure. The sum of two consecutive integers can be represented as $n + (n + 1) = 2n + 1$ The sum of three consecutive integers $= n + (n + 1) + (n + 2) = 3n + 3$
The sum of four consecutive integers $= 4n + 6$ The sum of five consecutive integers $= 5n + 10$ The sum of six consecutive integers $= 6n + 15$

Since the expressions $2n + 1$ and $6n + 15$ can both yield 45 for integer values of n, 45 can be the sum of two or six consecutive integers.

(2) INSUFFICIENT: If $n < 9$, n could again take on either of the values 2 or 6 (or 3 or 5 according to the table or the expressions above)

(1) and (2) INSUFFICIENT: if we combine the two statements, n must be even and less than 9, so n could still be either of the values: 2 or 6.

The correct answer is E.

5.
The question is in very simple form already; rephrasing the question isn't useful. The statements, however, can be rephrased.

Statement (1) gives the formula $n^3 - n$. We can first factor out an n to get $n(n^2 – 1)$. Next, we can factor $(n^2 – 1)$ to get $(n + 1)(n – 1)$. So, $n^3 - n$ factors to $n(n + 1)(n – 1)$. Notice that the three factored terms represent consecutive integers: $n – 1$, n, and $n + 1$.

Now, let's rephrase statement (2). We first factor out an n to get $n(n^2 + 2n + 1)$. Next, we can factor $(n^2 + 2n + 1)$ to get $(n + 1)(n + 1)$. So, $n^3 + 2n^2 + n$ factors to $n(n + 1)(n + 1)$. Notice that the factored terms represent two consecutive integers, with the larger of the two represented twice.
(1) INSUFFICIENT: $(n – 1)(n)(n + 1)$ is a multiple of 3. Any three consecutive positive integers include exactly one multiple of 3 (if you don't remember this rule, try some real numbers and prove it to yourself). Of the three terms $n – 1$, n, and $n + 1$, one is a multiple of 3 but we have no way to determine which one.
(2) SUFFICIENT: $n(n + 1)(n + 1)$ is a multiple of 3. This tells us that either n or $n + 1$ is a multiple of 3. The question asks whether the term $n – 1$ is a multiple of 3. Recall that $n – 1$, n, and $n + 1$ represent three consecutive integers and also recall that any three consecutive integers include exactly one multiple of 3. Note that we do not need to get this information from statement (1). If the multiple of 3 is either the n or the $n + 1$ term, then the $n – 1$ term cannot be a multiple of 3.

The correct answer is B.

6.
Since the product of b, c, and d is equal to twice that of a, b, and c, we can set up an equation and discover something about the relationship between d and a:

$bcd = 2abc$
$d(bc) = 2a(bc)$

Note that bc appears on both sides of the equation. It is multiplied by d on the left side, and by 2a on the right side. Since the left side of the equation must equal the right side of the equation:

$d = 2a$

Since a, b, c, and d are consecutive integers, d must be 3 more than a, or:

$d = a + 3$

We can combine both equations and solve for a:

$2a = a + 3$
$a = 3$

If a = 3, we know that b = 4, c = 5, and d = 6. Therefore, bc = (4)(5) = 20.

The correct answer is D.

7.

Question Removed

8.
The number of integers between 51 and 107, inclusive, is (107 - 51) + 1 = 57.
When a list is **inclusive** of the extremes, don‘t forget to "add one before you‘re done."

The correct answer is D.

9.
First, note that a product of three consecutive positive integers will always be divisible by 8 if the set of these integers contains 2 even terms. These two even terms will represent consecutive multiples of 2 (note that z = x + 2), and since every other multiple of 2 is also a multiple of 4, one of these two terms will always be divisible by 4. Thus, if one of the two even terms is divisible by 4 and the other even term is divisible by 2 (since it is even), the product of 3 consecutive positive terms containing 2 even numbers will always be divisible by 8. Therefore, to address the question, we need to determine whether the set contains 2 even terms. In other words, the remainder from dividing xyz by 8 will depend on whether x is even or odd.

(1) SUFFICIENT: This statement tells us that the product xz is even. Note that since z = x + 2, x and z can be only both even or both odd. Since their product is even, it must be that both x and z are even. Thus, the product xyz will be a multiple of 8 and will leave a remainder of zero when divided by 8.

(2) SUFFICIENT: If $5y^3$ is odd, then y must be odd. Since y = x +1, it must be that x = y – 1. Therefore, if y is odd, x is even, and the product xyz will be a multiple of 8, leaving a remainder of zero when divided by 8.

The correct answer is D.

10.
If we factor the equation in the question, we get n = x(x – 1)(x + 1) or n = (x – 1)x(x +1). n is the product of three consecutive integers. What would it take for n to be divisible by 8? To be divisible by 8, is to be divisible by 2 three times, or to have three 2's in the prime box.
The easiest way for this to happen is if x is odd. If x is odd, both x – 1 and x + 1 will be even or divisible by 2. Furthermore, if x is odd, x – 1 and x + 1 will also be consecutive even integers. Among consecutive even integers, every other even integer is divisible not only by 2 but also by 4. Thus, either x – 1 or x + 1 must be divisible by 4. With one number divisible by 2 and the other by 4, the product represented by n will be divisible by 8 if x is odd.
(1) SUFFICIENT: This tells us that x is odd. If 3x divided by 2 has a remainder, 3x is odd. If 3x is odd, x must be odd as well.
(2) SUFFICIENT: This statement tells us that x divided by 4 has a remainder of 1. This also tells us that x is odd because an even number would have an even remainder when divided by 4. Alternative method: if we rewrite this statement as x – 1 = 4y, we see that x – 1 is divisible by 4, which means that x + 1 is also even and the product n is divisible by 8.
The correct answer is D.

11.
We can express the sum of consecutive integers algebraically. For example, the sum of three consecutive integers can be expressed as n + (n + 1) + (n + 2) = 3n + 3. So we need to see which values of x and y cannot be equated algebraically.

(A) Can the sum of two consecutive integers be equal to the sum of 6 consecutive integers? We can express this as $r + (r + 1) = s + (s + 1) + (s + 2) + (s + 3) + (s + 4) + (s + 5)$ --> $2r + 1 = 6s + 15$.
Subtract 1 from both sides: $2r = 6s + 14$.
Divide both sides by 2: $r = 3s + 7$. So if $s = 2$, for example, then $r = 13$.
Is it true that $13 + 14 = 2 + 3 + 4 + 5 + 6 + 7$? Yes, $27 = 27$. So this pair of values can work.
We can express this as $3r + 3 = 6s + 15$.
Divide through by 3: $r + 1 = 2s + 5$, subtract 1 from both sides: $r = 2s + 4$. Whatever the value of s, we will find an integer value of r. This can work.

(B) We can express this as $7r + 21 = 9s + 36$.
Subtract 21 from both sides: $7r = 9s + 15$.
If $s = 3$, then $7r = 27 + 15 = 42$, and $r = 6$. This can work.

(C) We can express this as $10r + 45 = 4s + 6$.
We can see here that the left side will be odd (10r is even and 45 is odd --> even + odd = odd). But the right side will be even (4s is even and 6 is even --> even + even = even).
Since an odd sum can never equal an even sum, these cannot be equal. This cannot work and is therefore the correct answer.

(D) We can express this as $10r + 45 = 7s + 21$.
Subtract 21 from both sides: $10r + 24 = 7s$.
If $r = 6$, then $10(6) + 24 = 7s$ --> $60 + 24 = 7s$ --> $84 = 7s$ and s therefore is equal to 12. This can work.

The correct answer is D.

12.
Since $30 = 2 \times 3 \times 5$, the question stem can be rephrased as follows: Does x have at least one factor each of 2, 3, and 5?
Since we are trying to determine ―divisibility‖ for an expression, we need to transform the given expressions from ―sums and differences‖ (polynomial form) into ―products‖ (factored form) as completely as possible. The most logical thing to do is to keep factoring as long as you can.

We can factor the expression in statement (1) as follows:

$x = k(m^3 - m) = k(m(m^2 - 1)) = k(m - 1)m(m + 1)$

Notice that x can be expressed as the product of k and 3 consecutive integers $[(m - 1)m(m + 1)]$.

Since three consecutive integers must include at least one even number and one factor of 3, the product of three consecutive integers MUST have be divisible by both 2 and 3.

However, there is no way to determine whether the product of 3 consecutive integers is divisible by 5. We also don't know whether the integer k is divisible by 5. Therefore, we don't know whether x is divisible by 5 and so statement (1) is not sufficient.

We can now try factoring the expression in statement (2) as follows:

$$\begin{aligned} n^5 - n &= n(n^4 - 1) \\ &= n\left[(n^2)^2 - 1\right] \\ &= n(n^2 - 1)(n^2 + 1) \\ &= n(n - 1)(n + 1)(n^2 + 1) \end{aligned}$$

We see that x is the product of 3 consecutive integers x $(n^2 + 1)$.

We already know that the product of 3 consecutive integers must be divisible by both 2 and 3. Hence, we need to determine whether the expression must also be divisible by 5 for all possible n. We can use logic to do so.

For any integer n, n must either be divisible by 5, or have a remainder of 1, 2, 3, or 4 when divided by 5. We must prove that there is a factor of 5 in the expression for ALL five cases. If n is divisible by 5, then the expression surely must be divisible by 5.

If n divided by 5 has a remainder of 1, then the (n - 1) term must be divisible by 5 and so, again, the expression must be divisible by 5.

If n divided by 5 has a remainder of 4, then the (n + 1) term must be divisible by 5 and so, again, the expression must be divisible by 5.

Now we are left only with the cases where n divided by 5 has a remainder of 2 or a remainder of 3. For these cases, we need to check the $(n^2 + 1)$ term of the expression.
If n divided by 5 has a remainder of 2, then we can express n as $n = 5j + 2$ (where j is a positive integer). Hence, $n^2 + 1 = (5j + 2)^2 + 1 = (25j^2 + 20j + 4) + 1 = 25j^2 + 20j + 5 = 5(5j^2 + 4j + 1)$. Thus, $n^2 + 1$ is divisible by 5.

If n divided by 5 has remainder 3, we can express n as $n = 5j + 3$ (where j is a positive integer). Hence, $n^2 + 1 = (5j + 3)^2 + 1 = 25j^2 + 30j + 9 + 1 = 25j^2 + 30j + 10 = 5(5j^2 + 6j + 2)$.
Thus, $n^2 + 1$ is divisible by 5.

Thus, x is divisible by 5 for all possible n‘s so statement (2) is sufficient to answer the question.

The correct answer is (B).

13.

The question stem tells us that $z = x^{1/2}$, which is simply another way of stating that $z = \sqrt{x}$. We are also told that x, y, and z are positive integers and that $x > y$. Then we are asked whether x and y are consecutive perfect squares. For example, if $y = 9$ and $x = 16$, then y and x are consecutive perfect squares. In order for x and y to be consecutive perfect squares, given that x is greater than y, it would have to be true that $\sqrt{x} = \sqrt{y} + 1$.
For example, $\sqrt{16} = \sqrt{9} + 1$.
[Another way of thinking about this: If y is the square of 3, then x must be the square of 4, or the square of (3 + 1).]
Statement (1) says that $x + y = 8z + 1$. Using the fact that $z = \sqrt{x}$, we get $z^2 = x$. We can substitute for x into the given equation as follows: $z^2 + y = 8z + 1$. We can rearrange this into $z^2 - 8z - 1 = -y$ and other similar equations. Unfortunately, these equations are not useful as no factoring is possible. So instead, let‘s try to prove insufficiency by picking values that demonstrate that statement (1) can go either way.
Let‘s begin by picking a value for x. We know that x must be a perfect square (since the square root of x is the integer z) so it makes sense to simply start picking small perfect squares for x. If x is 4, then $z = 2$. Substituting these values into the equation in statement (1) yields the following: $y = 8z + 1 - x = 8(2) + 1 - 4 = 13$. This does not meet the constraint given in the question that $x > y$, so we cannot use this value for x.
If x is 9, then $z = 3$ and y is 16. Again, this does not meet the constraint given in the question that $x > y$ so we cannot use this value for x.
If x is 25 then $z = 5$ and y is 16. In this case the answer to the question is YES: y and x (16 and 25) are consecutive perfect squares.

If x is 36 then $z = 6$, which means that y is 13. In this case the answer to the question is NO: y and x (13 and 36) are not consecutive perfect squares.

Therefore Statement (1) alone is not sufficient to answer the question.

Statement (2) says that $x - y = 2z - 1$. Again, using the fact that $z = \sqrt{x}$, we get $z^2 = x$. We can substitute for x into the given equation as follows: $z^2 - y = 2z - 1$. We can rearrange this to get $z^2 - 2z + 1 = y$, which we can factor into $(z - 1)(z - 1) = y$. Therefore, $z - 1 = \sqrt{y}$.

We can replace z with $\sqrt{x}$ to get $\sqrt{x} - 1 = \sqrt{y}$, which yields $\sqrt{x} = \sqrt{y} + 1$. Thus, x and y are always consecutive perfect squares. Statement (2) alone is sufficient to answer the question.

The correct answer is B.

Alternate sol from gmatclub (additional)

If x and y are consecutive perfect squares, then $\sqrt{y}$ and $\sqrt{x}$ must be consecutive integers. So, the question becomes: is $\sqrt{y} = \sqrt{x} - 1$? --> square both sides: is $y = x - 2\sqrt{x} + 1$?

(1) x + y = 8z +1 --> $x + y = 8\sqrt{x} + 1$ --> $y = 8\sqrt{x} + 1 - x$. Now, question becomes is $8\sqrt{x} + 1 - x = x - 2\sqrt{x} + 1$? --> is $10\sqrt{x} = 2x$? --> is $5\sqrt{x} = x$? --> is $\sqrt{x} = 5$? --> is $x = 25$? But we don't know that, hence insufficient.

(2) x - y = 2z - 1 --> $y = x - 2\sqrt{x} + 1$, which is exactly what we needed to know. Sufficient.

Answer: B.

GMAT Quant Topic 4: Numbers
Part I: Digits

1.
In digit problems, it is usually best to find some characteristic that must be true of the correct solution. In looking at the given addition problem, the only promising feature is that the digit b is in the hundredths place in both numbers that are being added.

What does this mean? Adding together two of the same numbers is the same as multiplying the number by 2. In other words, $b + b = 2b$. This implies that the hundredths place in the correct solution should be an even number (since all multiples of 2 are even).

However, this implication is ONLY true if their is no "carry over" into the hundredths column. If the addition of the units and tens digits requires us to "carry over" a 1 into the hundredths column, then this will throw off our logic. Instead of just adding $b + b$ to form the hundredths digit of the solution, we will be adding $1 + b + b$ (which would sum to an odd digit in the hundredths place of the solution).

The question then becomes, will there be a "carry over" into the hundredths column? If not, then the hundredths digit of the solution MUST be even. If there is a carry over, then the hundredths digit of the solution MUST be odd.

The only way that there would be a "carry over" into the hundredths column is if the sum of the units and tens places is equal to 100 or greater.

The sum of the units place can be written as $c + a$.
The sum of the tens place can be written as $10d + 10c$.
Thus, the sum of the units and tens places can be written as $c + a + 10d + 10c$ which simplifies to $10d + 11c + a$.

The problem states that $10d + 11c < 100 – a$. This can be rewritten as $10d + 11c + a < 100$. In other words, the sum of the units and ten places totals to less than 100. Therefore, there is no "carry over" into the hundredths column and so the hundredths digit of the solution MUST be even.

The problem asks us which of the answer choices could NOT be a solution to the given addition problem, so we simply need to find an answer choice that does NOT have an even number in the hundredths place.

The only answer choice that qualifies is 8581.
The correct answer is C.

Alternate Solution from Gmatclub

$10d + 11c < 100 - a$;
$10d + 11c + a < 100$;
$(10d + c) + (10c + a) < 100$.

(10d+c) is the way of writing two-digit integer **dc** and (10c+a) is the way of writing two-digit integer **ca.** Look at the sum:

ab**dc**
+db**ca**

Now, as two-digit integer **dc** + two-digit integer **ca** is less than 100, then there won't be carried over 1 to the hundreds place and as b+b=2b=even then the hundreds digit of the given sum must be even too. Thus 8581 **could not be the sum** of abdc+dbca (for any valid digits of a, b, c, and d).

Answer: C.

The correct answer is C.

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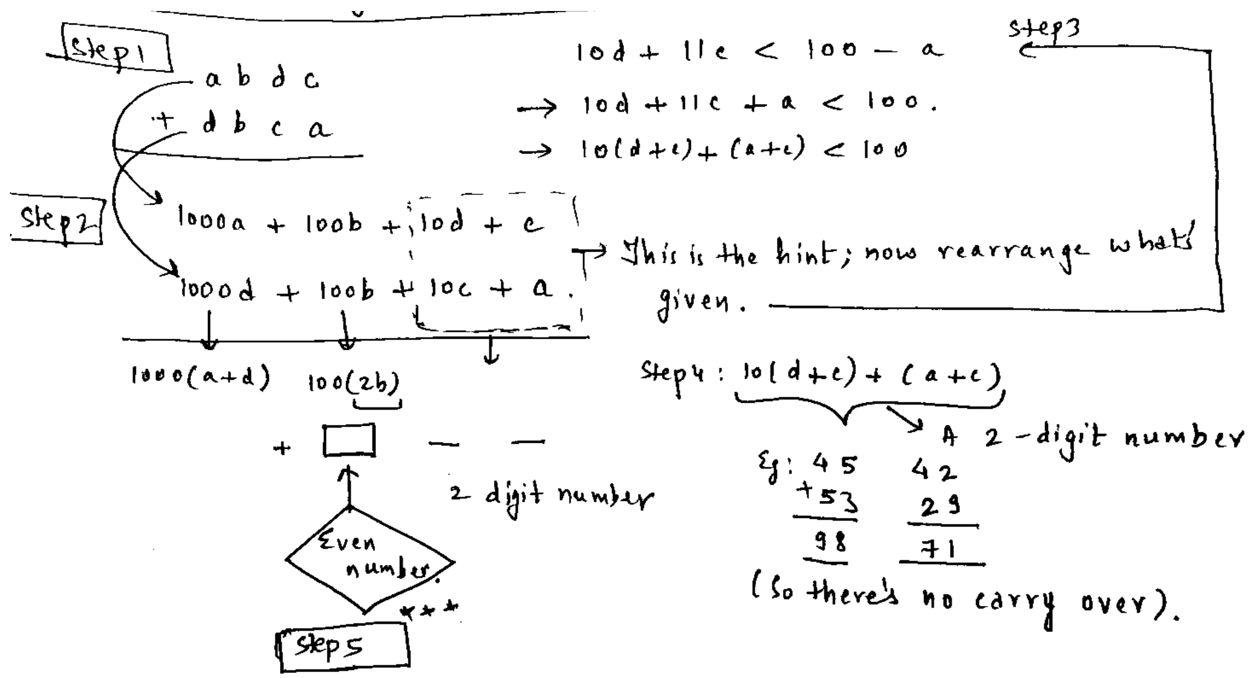


2.

Solving this problem requires a bit of logic. A quick look at the ones column tells us that a value of 1 must be 'carried' to the tens column. As a result, p must equal the ones digit from the sum of k + 8 + 1, or k + 9 (note that it would be incorrect to say that p = k + 9).

Now, given that k is a non-zero digit, k + 9 must be greater than or equal to 10. Furthermore, since k is a single digit and must be less than 10, we can also conclude that k + 9 < 20.
Therefore, we know that a value of 1 will be 'carried' to the hundreds column as well.

We are now left with some basic algebra. In the hundreds column, 8 + k + 1 = 16, so k = 7. Recall that p equals the ones digit of k + 9. k + 9 = 7 + 9 = 16, so p = 6.

The correct answer is A.

3.
There are 3!, or 6, different three-digit numbers that can be constructed using the digits a, b, and c:

abc	bac	cab
acb	bca	cba

The value of any one of these numbers can be represented using place values. For example, the value of abc is 100a + 10b + c.
Therefore, you can represent the sum of the 6 numbers as:

$$\begin{array}{rcrcr} 100a & + & 10b & + & c \\ 100a & + & b & + & 10\,c \\ 10a & + & 100b & + & c \\ a & + & 100b & + & 10c. \\ 10a & + & b & + & 100c. \\ a & + & 10b & + & 100c. \\ \hline 222a & + & 222b & + & 222c \end{array} \;=\; 222(a+b+c)$$

x is equal to 222(a + b + c). Therefore, x must be divisible by 222.

The correct answer is E.

4.

If the sum of the digits of the positive two-digit number x is 4, then x must be 13, 31, 22, or 40. We can rephrase this question as —Is the value of x 13, 31, 22 or 40?

(1) INSUFFICIENT: If x is odd, x can be 13 or 31.

(2) SUFFICIENT: From the statement, 2x < 44, so x < 22. This means that x must be 13.

The correct answer is B.

5. The sum of the units digit is 2 + 3 + b = 10. We know that 2 + 3 + b can't equal 0, because b is a positive single digit. Likewise 2 + 3 + b can't equal 20 (or any higher value) because b would need to be 15 or greater—not a single digit. Therefore, b must equal 5.

We know that 1 is carried from the sum of the units digits and added to the 2, a, and 4 in the tens digit of the computation, and that those digits sum to 9. Therefore 1 + 2 + a + 4 = 9, or a = 2.

Thus, the value of the two digit integer ba is 52.

The correct answer is E

6.

The problem states that all 9 single digits in the problem are different; in other words, there are no repeated digits.

(1) SUFFICIENT: Given 3a = f = 6y, the only possible value for y = 1. Any greater value for y, such as y = 2, would make f greater than 9. Since y = 1, we know that f = 6 and a = 2. We can now rewrite the problem as follows:

```
  2 b c
 +d e 6
 ______
  x 1 z
```

In order to determine the possible values for z in this scenario, we need to rewrite the problem using place values as follows:

(Continued on next page)

$200 + 10b + c + 100d + 10e + 6 = 100x + 10 + z$
This can be simplified as follows:
$196 = 100(x – d) – 10(b + e) + 1(z – c)$
Since our focus is on the units digit, notice that the units digit on the left side of the equation is 6 and the units digit on the right side of the equation is $(z – c)$. Thus, we know that $6 = z – c$.
Since z and c are single positive digits, let's list the possible solutions to this equation.
$z = 9$ and $c = 3$
$z = 8$ and $c = 2$
$z = 7$ and $c = 1$
However, the second and third solutions are NOT possible because the problem states that each digit in the problem is different. The second solution can be eliminated because c cannot be 2 (since a is already 2). The third solution can be eliminated because c cannot be 1 (since y is already 1). Thus, the only possible solution is the first one, and so z must equal 9.

(2) INSUFFICIENT: The statement $f – c = 3$ yields possible values of z. For example f might be 7 and c might be 4. This would mean that $z = 1$. Alternatively, f might be 6 and c might be 3. This would mean that $z = 9$.

The correct answer is A.

7.
According to the question, the ―star function‖ is only applicable to four digit numbers. The function takes the thousands, hundreds, tens and units digits of a four-digit number and applies them as exponents for the bases 3, 5, 7 and 11, respectively, yielding a value which is the product of these exponential expressions.

Let‘s illustrate with a few examples:
$*2234* = (3^2)(5^2)(7^3)(11^4)$
$*3487* = (3^3)(5^4)(7^8)(11^7)$
According to the question, the four-digit number m must have the digits of rstu, since
$*m* = (3^r)(5^s)(7^t)(11^u)$.

If $*n* = (25)(*m*)$
$*n* = (5^2)(3^r)(5^s)(7^t)(11^u)$
$*n* = (3^r)(5^{s+2})(7^t)(11^u)$

n is also a four digit number, so we can use the *n* value to identify the digits of n: thousands =

r, hundreds = s + 2, tens = t, units = u.

All of the digits of n and m are identical except for the hundreds digits. The hundreds digits of n is two more than that of m, so $n – m = 200$.

The correct answer is B.

8.
The question states that a, b, and c are each positive single digits. Statement (1) says that $a = 1.5b$ and $b = 1.5c$. This means that $a = 1.5(1.5c) = 2.25c$. Nine is the only positive single digit that is a multiple of 2.25. Therefore $a = 9$, $b = 6$, and $c = 4$. Statement (1) is sufficient to determine that abc is 964.
Statement (2) says that $a = 1.5x + b$ and $b = x + c$, where x represents a positive single digit. There are several three digit numbers for which these equations would hold true:
631: If $x = 2$ and $c = 1$, then $b = 2 + 1 = 3$ and $a = 1.5(2) + 3 = 6$. Thus abc could be 631.

742 : If $x = 2$ and $c = 2$, then $b = 2 + 2 = 4$ and $a = 1.5(2) + 4 = 7$. Thus abc could be 742.

853 : If $x = 2$ and $c = 3$, then $b = 2 + 3 = 5$ and $a = 1.5(2) + 5 = 8$. Thus abc could be 853.

964: If $x = 2$ and $c = 4$, then $b = 2 + 4 = 6$ and $a = 1.5(2) + 6 = 9$. Thus abc could be 964.

Therefore, Statement (2) is not sufficient to answer the question.
The correct answer is A.

9.
The question asks for the value of the three-digit number *SSS* and tells us that *SSS* is the sum of the three-digit numbers *ABC* and *XYZ*. We can represent this relationship as:

$$\begin{array}{r} ABC \\ +XYZ \\ \hline SSS \end{array}$$

Statement (1) tells us that $S = 1.75X$. Since *X* is a digit between 0 and 9, inclusive, *S* must equal 7. This is because *X* must be a multiple of 4 in order for 1.75*X* to yield an integer (remember that 1.75 is the decimal equivalent of 7/4). Therefore, *X* must be either 4 or 8. However, *X* cannot be 8 because 1.75(8) = 14, which is not a digit and thus cannot be the value of *S*. So *X* must be 4 and 1.75(4) = 7. Therefore, the value of *SSS* is 777. Statement (1) is sufficient.

Statement (2) tells us that $S^2 = \frac{49}{8}ZX$. If we take the square root of both sides and simplify, we get:

$$\sqrt{S^2} = \sqrt{\frac{49}{8}ZX} \rightarrow$$
$$S = 7\sqrt{\frac{ZX}{8}}$$

Since *S* is an integer, $7\sqrt{\frac{ZX}{8}}$ must be an integer as well. And since *S* must be less than 10, $7\sqrt{\frac{ZX}{8}}$ must also be less than 10. The only way in the present circumstances for this to happen is if $\sqrt{\frac{ZX}{8}} = 1$. Therefore, *S* = (7)(1) = 7 and the value of *SSS* is 777. Statement (2) is sufficient.

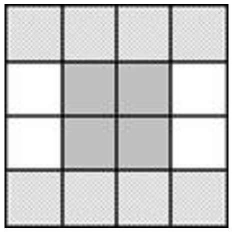
The correct answer is D.

10. The key to this problem is to consider the implications of the fact that every column, row, and major diagonal must sum to the same amount.

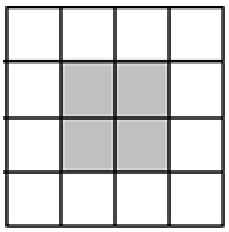
If the cells contain the consecutive integers from 37 to 52, inclusive, then the sum of all the cells must be 37 + 38 + 39 +. 52. You can find this sum quickly by adding the largest and smallest values (37 + 52) and multiplying that sum by the number of high/low pairs in the set (e.g., 38 + 51, 39 + 50, etc.) Note that this works only when you have an even number of evenly spaced terms. If you have an odd number of evenly spaced terms, you can find all the high/low pairs, but then you must add in the unpaired, middle value. For example, in the set {2, 4, 6, 8, 10}, note that 2 + 10 and 4 + 8 both sum to 12, but 6 has no mate. So the sum of this set would be 2 x 12 + 6 = 30.

So in the case at hand, we have 8 pairs with a value of 89 each, for a total sum of 8 x 89 = 712. Since there are 4 rows, each with the same sum, each row must have a sum of 712/4 or 178. The same holds true for each column. And since each diagonal has the same sum as each row and column, each diagonal must also have a sum of 178. We can now use this insight to solve the problem.

If we add the two diagonals and the two center columns, we end up with a grid that looks like this:



The four center squares (darker shading) have been counted twice, however, once in each diagonal and once in each center column. Overall, this pattern has a value of 4 x 178 (two diagonals and two columns). If we subtract the top and bottom rows (each with a value of 178), we are left with a grid that looks like this:



Since the pattern before had a value of 4 x 178 and we subtracted 2 x 178, this pattern must have a value of 2 x 178. But since each of these center cells has been counted twice, the value of the 4 center cells without overcounting must be 1 x 178 or 178.

The correct answer is C.

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Note:-The question asks for a POSSIBLE value:
Easier approach:
The sum of all numbers in the grid = 712
The average value of a cell is 712/16 = 44.5

44.5	44.5	44.5	44.5	x
44.5	44.5	44.5	44.5	x
44.5	44.5	44.5	44.5	x
44.5	44.5	44.5	44.5	x
x	x	x	x	712

(Also given : sum of values in column 2 = sum of values in column 3 = sum of values in row 2 = sum of values in row 3.)
To satisfy this, if we assign the average value of a cell to each cell, the 4 cells will get a total of 44.5*4 = 178.
Hence C is the answer.

OR
Conventional approach:-
Sum of the numbers from 37 to 52 = (n/2)[2a + (n-1)d] = (16/2) [74+15] = 8 * 89 = 712

Therefore the sum of all numbers in the grid = 712
If x is the sum of all numbers in a row/column/major diagonal, then

4x = 712

=> x = 178 = sum of all numbers in any row = sum of all numbers in any diagonal = sum of all numbers in either major diagonal

Now, consider the grid as follows:

1' 2' 3' 4'

5' 6' 7' 8'

9' 10' 11' 12'

13' 14' 15' 16'

We know that 1' + 6' + 11' + 16' = 178

4' + 7' + 10' + 13' = 178

=> 1' + 6' + 11' + 16' + 4' + 7' + 10' + 13' = 356

=> 6' + 7' + 10' + 11' + 1' + 13' + 4' + 16' = 356

Also 5' + 6' + 7' + 8' + 9' + 10' + 11' + 12' = 356

and 1' + 5' + 9' + 13' + 4' + 8' + 12' + 16' = 356

=> 6' + 7' + 10' + 11' - 1' - 13' - 4' - 16' = 0

Therefore 6' + 7' + 10' + 11' = 356/2 = 178 = sum of the middle four numbers

The correct answer is C.

GMAT Quant Topic 5: Geometry
Part A: Lines and Angles

1.

The angles labeled (2a)° and (5a + 5)° are supplementary (add up to 180°) because together they form a line. We can solve for a as follows:

$2a + (5a + 5) = 180$
$a = 25$

The angles labeled (4b +10)° and (2b – 10)° are supplementary (add up to 180°) as well. We can solve for b as follows:

$(4b + 10) + (2b - 10) = 180$
$b = 30$

Now that we know both a and b, we can find a + b:

$a + b = 25 + 30 = 55$.

Alternatively, you could solve this problem by using the fact that opposite angles are equal, which implies that

$5a + 5 = 4b + 10$, **and** $2a = 2b - 10$

It is possible to solve this system of two equations for a and b, though the algebra required is slightly more difficult than what we used earlier to find that a + b = 55.

The correct answer is C.

2.

Because l_1 is parallel to l_2, the transversal that intersects these lines forms eight angles with related measurements. All of the acute angles are equal to one another, and all of the obtuse angles are equal to one another. Furthermore, each acute angle is the supplement of each obtuse angle (i.e., they add up to 180°).

Therefore, $2x + 4y = 180$.
Dividing both sides of the equation by 2 yields: $x + 2y = 90$.

The correct answer is A.

3.

(1) INSUFFICIENT: We don't know any of the angle measurements.

(2) INSUFFICIENT: We don't know the relationship of x to y.

(1) AND (2) INSUFFICIENT: Because l1 is parallel to l2, we know the relationship of the four angles at the intersection of l2 and l3 (l3 is a transversal cutting two parallel lines) and the same four angles at the intersection of l1 and l3. We do not, however, know the relationship of y to those angles because we do not know if l3 is parallel to l4.

The correct answer is E.

4. The figure is one triangle superimposed on a second triangle. Since the sum of the 3 angles inside each triangle is 180°, the sum of the 6 angles in the two triangles is 180° + 180° = 360°.

The correct answer is D.

5.

We are given two triangles and asked to determine the degree measure of z, an angle in one of them.

The first step in this problem is to analyze the information provided in the question stem. We are told that x - q = s - y. We can rearrange this equation to yield x + y = s + q. Since x + y + z = 180 and since q + s + r = 180, it must be true that z = r. We can now look at the statements.

Statement (1) tells us that xq + sy + sx + yq = zr. In order to analyze this equation, we need to rearrange it to facilitate factorization by grouping like terms: xq + yq + sx + sy = zr. Now we can factor:

$$xq + yq + sx + sy = zr \rightarrow$$
$$q(x + y) + s(x + y) = zr \rightarrow$$
$$(x + y)(q + s) = zr$$

Since x + y = q + s and z = r, we can substitute and simplify:

$$(x + y)(q + s) = zr \rightarrow$$
$$(x + y)(x + y) = (z)(z) \rightarrow$$
$$\sqrt{(x + y)^2} = \sqrt{z^2} \rightarrow$$
$$x + y = z$$

Is this sufficient to tell us the value of z? Yes. Why? Consider what happens when we substitute z for x + y:

$$x + y + z = 180 \rightarrow$$
$$z + z = 180 \rightarrow$$
$$2z = 180 \rightarrow$$
$$z = 90$$

It is useful to remember that when the sum of two angles of a triangle is equal to the third angle, the triangle must be a right triangle. Statement (1) is sufficient.

Statement (2) tells us that zq - ry = rx - zs. In order to analyze this equation, we need to rearrange it:

$$zq - ry = rx - zs \rightarrow$$
$$zq + zs = rx + ry \rightarrow$$
$$z(q + s) = r(x + y) \rightarrow$$
$$z = \frac{r(x + y)}{(q + s)} \rightarrow$$
$$\frac{z}{r} = \frac{x + y}{q + s}$$

Is this sufficient to tell us the value of z? No. Why not? Even though we know the following:

z = r
x + y = q + s

$x + y + z = 180$
$q + r + s = 180$

we can find different values that will satisfy the equation we derived from statement (2):

$$\frac{90}{90} = \frac{30 + 60}{40 + 50}$$

or

$$\frac{100}{100} = \frac{40 + 40}{10 + 70}$$

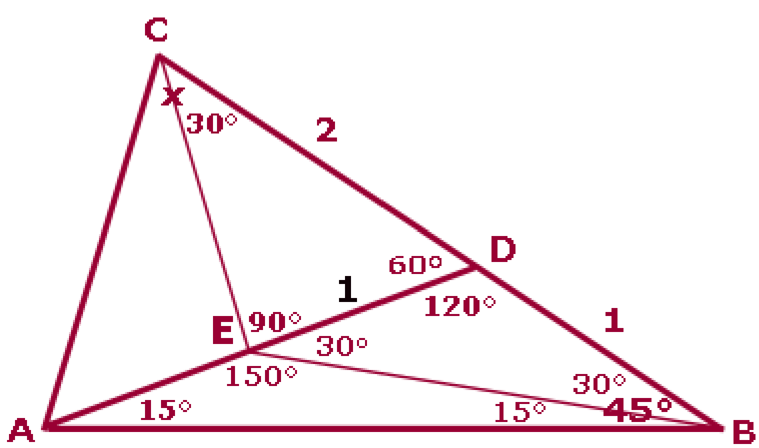
These are just two examples. We could find many more. Since we cannot determine the value of z, statement (2) is insufficient.

The correct answer is A.

6.

As first, it appears that there is not enough information to compute the rest of the angles in the diagram. When faced with situations such as this, look for ways to draw in new lines to exploit any special properties of the given diagram.

For example, note than the figure contains a 60° angle, and two lines with lengths in the ratio of 2 to 1. Recall that a 30-60-90 triangle also has a ratio of 2 to 1 for the ratio of its hypotenuse to its short leg. This suggests that drawing in a line from C to line AD and forming a right triangle may add to what we know about the figure. Let's draw in a line from C to point E to form a right triangle, and then connect points E and B as follows:



Triangle CED is a 30-60-90 triangle. Using the side ratios of this special triangle, we know that the hypotenuse is two times the smallest leg. Therefore, segment ED is equal to 1.

From this we see that triangle EDB is an isosceles triangle, since it has two equal sides (of length 1). We know that EDB = 120°; therefore angles DEB and DBE are both 30°.

Now notice two other isosceles triangles:

(1) Triangle CEB is an isosceles triangle, since it has two equal angles (each 30 degrees). Therefore segment CE = segment EB.

(2) Triangle AEB is an isosceles triangle, since CEA is 90 degrees, angles ACE and EAC must be equal to 45 degrees each. Therefore angle x = 45 + 30 = 75 degrees.
The correct answer is D.

7. The question asks us to find the degree measure of angle a. Note that a and e are equal since they are vertical angles, so it's also sufficient to find e.

Likewise, you should notice that e + f + g = 180 degrees. Thus, to find e, it is sufficient to find f + g. The question can be rephrased to the following: "What is the value of f + g?"

(1) SUFFICIENT: Statement (1) tells us that b + c = 287 degrees. This information allows us to calculate f + g. More specifically:

b + c = 287
(b + f) + (c + g) = 180 + 180 Two pairs of supplementary angles.
b + c + f + g = 360
287 + f + g = 360
f + g = 73

(2) INSUFFICIENT: Statement (2) tells us that d + e = 269 degrees. Since e = a, this is equivalent to d + a = 269. There are many combinations of d and a that satisfy this constraint, so we cannot determine a unique value for a.

The correct answer is A.

GMAT Quant Topic 5: Geometry
Part B: Triangles

1.
Because angles BAD and ACD are right angles, the figure above is composed of three similar right triangles: BAD, ACD and BCA. [Any time a height is dropped from the right angle vertex of a right triangle to the opposite side of that right triangle, the three triangles that result have the same 3 angle measures. This means that they are similar triangles.] To solve for the length of side CD, we can set up a proportion, based on the relationship between the similar triangles ACD and BCA: BC/AC = CA/CD or ¾ = 4/CD or CD = 16/3.
The correct answer is D.

2.
If the triangle marked T has sides of 5, 12, and 13, it must be a right triangle. That's because 5, 12, and 13 can be recognized as a special triple that satisfies the Pythagorean theorem: $a^2 + b^2 = c^2$ ($5^2 + 12^2 = 13^2$). Any triangle that satisfies the Pythagorean Theorem must be a right triangle.

The area of triangle T = 1/2 × base × height
= 1/2(5)(12)
= 30

The correct answer is B.

3.
(1) INSUFFICIENT: This tells us that AC is the height of triangle BAD to base BD. This does not help us find the length of BD.
(2) INSUFFICIENT: This tells us that C is the midpoint of segment BD. This does not help us find the length of BD.
(1) AND (2) SUFFICIENT: Using statements 1 and 2, we know that AC is the perpendicular bisector of BD. This means that triangle BAD is an isosceles triangle so side AB must have a length of 5 (the same length as side AD). We also know that angle BAD is a right angle, so side BD is the hypotenuse of right isosceles triangle BAD. If each leg of the triangle is 5, the hypotenuse (using the Pythagorean theorem) must be $5\sqrt{2}$.

The correct answer is C.

4.
(1) SUFFICIENT: If we know that ABC is a right angle, then triangle ABC is a right triangle and we can find the length of BC using the Pythagorean theorem. In this case, we can recognize the common triple 5, 12, 13 - so BC must have a length of 12.

(2) INSUFFICIENT: If the area of triangle ABC is 30, the height from point C to line AB must be 12 (We know that the base is 5 and area of a triangle = 0.5 × base × height). There are only two possibilities for such such a triangle. Either angle CBA is a right triangle, and CB is 12, or angle BAC is an obtuse angle and the height from point C to length AB would lie outside of the triangle. In this latter possibility, the length of segment BC would be greater than 12.
The correct answer is A.

5.
If the hypotenuse of isosceles right triangle ABC has the same length as the height of equilateral triangle DEF, what is the ratio of a leg of triangle ABC to a side of triangle DEF?

One approach is to use real values for the unspecified values in the problem. Let's say the hypotenuse of isosceles right triangle ABC is 5. The ratio of the sides on an isosceles right triangle (a 45-45-90 triangle) is $1:1:\sqrt{2}$. Therefore, each leg of triangle ABC has a length of $5/\sqrt{2}$.

We are told that the hypotenuse of triangle ABC (which we chose as 5) is equal to the height of equilateral triangle DEF. Thus, the height of DEF is 5. Drawing in the height of an equilateral triangle effectively cuts that triangle into two 30-60-90 triangles.

The ratio of the sides of a 30-60-90 triangle is $1:\sqrt{3}:2$ (short leg: long leg: hypotenuse).

The long leg of the 30-60-90 is equal to the height of DEF. In this case we chose this as 5.
They hypotenuse of the 30-60-90 is equal to a side of DEF. Using the side ratios, we can calculate this as $10/\sqrt{3}$.

Thus, the ratio of a leg of ABC to a side of DEF is:

$$\frac{\frac{5}{\sqrt{2}}}{\frac{10}{\sqrt{3}}} = \frac{5}{\sqrt{2}} * \frac{\sqrt{3}}{10} = \frac{\sqrt{3}}{2\sqrt{2}}$$

The perimeter of a triangle is equal to the sum of the three sides.

(1) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

(2) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

Together, the two statements are SUFFICIENT. Triangle ABC is an isosceles triangle which means that there are theoretically 2 possible scenarios for the lengths of the three sides of the triangle:
(1) AB = 9, BC = 4 and the third side, AC = 9 **OR** (1) AB = 9, BC = 4 and the third side AC = 4.

These two scenarios lead to two different perimeters for triangle ABC, HOWEVER, upon careful observation we see that the second scenario is an IMPOSSIBILITY. A triangle with three sides of 4, 4, and 9 is not a triangle. Recall that any two sides of a triangle must sum up to be greater than the third side. $4 + 4 < 9$ so these are not valid lengths for the side of a triangle.

Therefore the actual sides of the triangle must be AB = 9, BC = 4, and AC = 9. The perimeter is 22.

The correct answer is C.

6.
Let‘s begin by looking at the largest triangle (the border of the figure) and the first inscribed triangle, a mostly white triangle. We are told that all of the triangles in the figure are equilateral. To inscribe an equilateral triangle in another equilateral triangle, the inscribed triangle must touch the midpoints of each of the sides of the larger triangle.

Using similar triangles, we could show that each side of the inscribed equilateral triangle must be 1/2 that of the larger triangle. It also follows that the area of the inscribed triangle must be equal to 1/4 that of the larger triangle. This is true because area is comprised of two linear components, base and height, which for the inscribed triangle would each have a value of 1/2 the base and height of the larger triangle.

To see how this works, think of the big triangle‘s area as 1/2(bh); the inscribed triangle‘s area would then be 1/2(1/2b)(1/2h) = (1/8)bh, which is 1/4 of the area of the big triangle.

The mathematical proof notwithstanding, you could probably have guessed that the inscribed triangle‘s area is 1/4 that of the larger triangle by ―eyeing it. On the GMAT, unless a figure is explicitly marked as ―not drawn to scale,‖ estimation can be a very valuable tool.

Thus, if we consider only the first equilateral triangle (the entire figure) and the white inscribed triangle, we can see that the figure is 3/4 shaded. This, however, is not the end of the story. We are told that this inscribed triangle and shading pattern continues until the smallest triangle has a side that is 1/128 or $1/2^7$ that of the largest triangle.

We already established that the white second triangle (the first inscribed triangle) has a side 1/2 that of the largest triangle (the entire figure). The third triangle would have a side 1/2 that of the second triangle or 1/4 that of the largest. The triangle with a side $1/2^7$ that of the largest would be the 8th triangle.

Now that we know that there are 8 triangles, how do we deal with the shading pattern? Perhaps the easiest way to deal with the pattern is to look at the triangles in pairs, a shaded triangle with its inscribed white triangle. Let‘s also assign a variable to the area of the whole figure, n. Looking at the first "pair" of triangles, we see (3/4)n of the total area is shaded.

The next area that we will analyze is the second pair of triangles, comprised of the 3rd (shaded) and 4th (white) triangles. Of course, this area is also 3/4 shaded. The total area of the third triangle is n/16 or $n/2^4$ so the area of the second —pair‖ is $(3/4)(n/2^4)$. In this way the area of the third "pair" would be $(3/4)(n/2^8)$, and the area of the fourth pair would be $(3/4)(n/2^{12})$. The sum of the area of the 4 pairs or 8 triangles is then:

$$\frac{3}{4}n+\frac{3}{4}\cdot\frac{n}{2^4}+\frac{3}{4}\cdot\frac{n}{2^8}+\frac{3}{4}\cdot\frac{n}{2^{12}}$$

which can be factored to

$$\frac{3}{4}n\left(1+2^{-4}+2^{-8}+2^{-12}\right)$$

But remember that t

The question asks to find the fraction of the total figure that is shaded. We assigned the total figure an area of n; if we put the above expression of the shaded area over the total area n, the

n‘s cancel out and we get $\frac{3}{4}\left(2^0+2^{-4}+2^{-8}+2^{-12}\right)$, or answer choice C.

Notice that the 1 from the factored expression above was rewritten as 2^0 in the answer choice to emphasize the pattern of the sequence.

Note that one could have used estimation in this problem to easily eliminate three of the five answer choices. After determining that the figure is more than 3/4 shaded, answer choices A, B and E are no longer viable. Answer choices A and B are slightly larger than 1/4. Answer choice E is completely illogical because it ludicrously suggests that more than 100% of the figure is shaded.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

Let's try to solve the easy way..
1/128 = 1/2^7 . So the center triangle will go upto 8th triangle in the center and will be unshaded.
Let A be the area of a bigger triangle. Lets number the biggest triangle as 1 and then smaller center triangles as 2 and next smaller center triangle as 3 and so on.

So, First focusing on the 1st bigger triangle no. 1..
Shaded region = 3/4 A
Now , In the 2nd smaller triangle no. 2 , we have only a shaded region from 1/4th of the part i.e from triangle no.3. So we will consider it in the next calculation in triangle no.3

In triangle 3,
Shaded region = 3/4 * triangle 3 = 3/4 *(1/4)^2 A

In triangle no. 4 , we have only a shaded region from 1/4th of the part i.e from triangle no.5. So we will consider it in the next calculation in triangle no.5

In triangle 5,

Shaded region = 3/4 * triangle 5 = 3/4* 1/4 *1/4 * triangle 3 = 3/4 *(1/4)^2 * (1/4)^2 A

In triangle 6, we have only a shaded region from 1/4th of the part i.e from triangle no.7. So we will consider it in the next calculation in triangle no.7
Shaded region = 3/4 * (1/4)^2 * (1/4)^2 * (1/4)^2 A

Sum of shaded regions = 3/4 A + 3/4 *(1/4)^2 A + 3/4 *(1/4)^2 * (1/4)^2 A + 3/4 * (1/4)^2 * (1/4)^2 * (1/4)^2 A
= 3/4 (1/2^0 + 1/2^4 + 1/2^8 + 1/2^12) A

The correct answer is C.

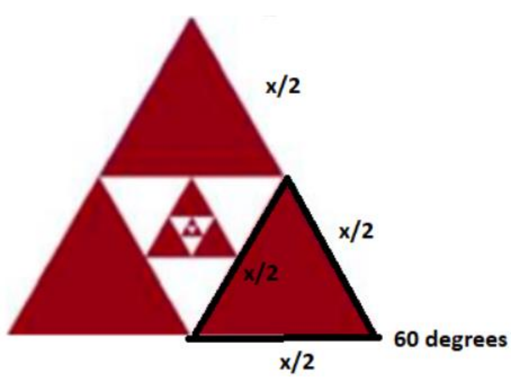
Top 1% expert replies to student queries (can skip)

Area of the next triangle is 1/4 of preceding triangle not 1/2.
Consider it this way:
1/128 = 1/2^7 . So center triangle will go upto 8th triangle in the center and will be unshaded.
Let A be the area of bigger triangle. Lets number biggest triangle as 1 and then smaller center triangles as 2 and next smaller center triangle as 3 and so on.
So, First focusing on 1st bigger triangle no. 1..
Shaded region = 3/4 A
Now , In 2nd smaller triangle no. 2 , we have only shaded region from 1/4th of the part i.e from triangle no.3. So we will consider it in the next calculation in triangle no.3
In triangle 3,
Shaded region = 3/4 * triangle 3 = 3/4 *(1/4)^2 A
In triangle no. 4 , we have only shaded region from 1/4th of the part i.e from triangle no.5. So we will consider it in the next calculation in triangle no.5
In triangle 5,
Shaded region = 3/4 * triangle 5 = 3/4* 1/4 *1/4 * triangle 3 = 3/4 *(1/4)^2 * (1/4)^2 A
In triangle 6, we have only shaded region from 1/4th of the part i.e from triangle no.7. So we will consider it in the next calculation in triangle no.7
Shaded region = 3/4 * (1/4)^2 * (1/4)^2 * (1/4)^2 A
Sum of shaded regions = 3/4 A + 3/4 *(1/4)^2 A + 3/4 *(1/4)^2 * (1/4)^2 A + 3/4 * (1/4)^2 * (1/4)^2 * (1/4)^2 A
= 3/4 (1/2^0 + 1/2^4 + 1/2^8 + 1/2^12) A

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

Look at the figure below



Let the side length of the outer triangle be x

Pay attention to the triangle highlighted in black. Its two outer sides will be x/2. Since the outer triangle is equilateral, all its angles will be 60 degrees. In the highlighted triangle, one of the angles is 60 degrees, as shown. The other two angles have to be equal, because the sides opposite to those angles are equal. So, the other two angles will also be 60 degrees. So, the highlighted triangle is an equilateral triangle with side length x./2.

Now, we can perform the same exercise for the inner triangles. By symmetry, every inner triangle will have half the side length of its outer triangle.

So, in this case,

Side length of 1st inner triangle = x/2
Side length of 2nd inner triangle = (x/2)/2 = x/4 and so on.

The pattern continues till the innermost triangle has a side length of 1/128 times the side length of the outermost triangle, that is x/128

Let the number of inner triangles = n

Side length of nth inner triangle = x/(2)^n

Therefore, x/(2)^n = x/128

Therefore, n = 7. That is, we have 7 inner triangles, and 8 triangles in total.

if we count the outermost triangle is counted as triangle 1, the shaded triangles are triangle 1, 3, 5 and 7

Area of triangle 1 = root(3)/4 * x^2

Area of triangle 3 = root(3)/4 * (x/4)^2   [Side length of triangle 3 = x/4]

Area of triangle 5 = root(3)/4 * (x/16)^2   [Side length of triangle 3 = x/16]

Area of triangle 7 = root(3)/4 * (x/64)^2   [Side length of triangle 3 = x/16]

Area of shaded portion = Area of triangle 1 + Area of triangle 3 + Area of triangle 5 + Area of triangle 7

Area of shaded portion = root(3)/4 * [x^2 + (x/4)^2 + (x/16)^2 + (x/64)^2] = root(3)/4 * x^2 * [1 + (1/4)^2 + (1/16)^2 + (1/64)^2]

We can use the sum of a GP formula to calculate the sum.

The first term is 1 and the common difference is (1/4)^2 = 1/16. Number of terms is 4.

1 + (1/4)^2 + (1/16)^2 + (1/64)^2 = [1 - (1/16)^4]/(1-1/16)]

Area of shaded portion = root(3)/4 * x^2 * [1 - (1/16)^4]/(1-1/16)]

In calculating the total area of all the 8 triangles,

First term = 1
Common difference = (1/2)^2 =1/4
Number of terms = 8

Area of all 8 triangles = root(3)/4 * x^2 * [1 - (1/4)^8]/(1-1/4)]

Fraction of the area that is shaded = Area of shaded portion / Area of all 8 triangles

= {root(3)/4 * x^2 * [1 - (1/16)^4]/(1-1/16)]} / {root(3)/4 * x^2 * [1 - (1/4)^8]/(1-1/4)]}

The correct answer is C.

7.

The question stem tells us that ABCD is a rectangle, which means that triangle ABE is a right triangle.

The formula for the area of any triangle is: 1/2 (Base X Height).

In right triangle ABE, let's call the base AB and the height BE. Thus, we can rephrase the questions as follows: **Is 1/2 (AB X BE) greater than 25?**

Let's begin by analyzing the first statement, taken by itself. Statement (1) tells us that the length of AB = 6. While this is helpful, it provides no information about the length of BE. Therefore there is no way to determine whether the area of the triangle is greater than 25 or not.

Now let's analyze the second statement, taken by itself. Statement (2) tells us that length of diagonal AE = 10. We may be tempted to conclude that, like the first statement, this does not give us the two pieces of information we need to know (that is, the lengths of AB and BE respectively). However, knowing the length of the diagonal of the right triangle actually does provide us with some very relevant information about the lengths of the base (AB) and the height (BE).

Consider this fact: Given the length of the diagonal of a right triangle, it **IS** possible to determine the maximum area of that triangle.

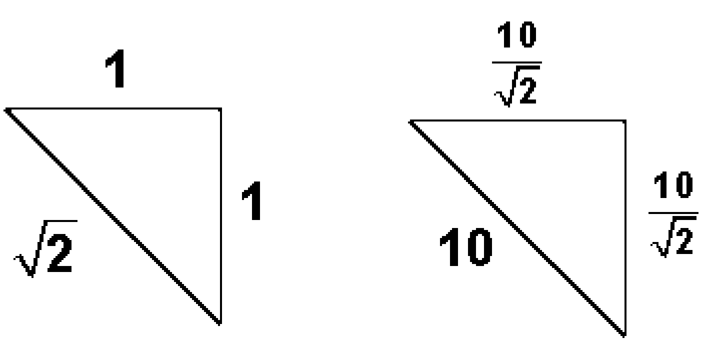
How? The right triangle with the largest area will be an isosceles right triangle (where both the base and height are of equal length).

If you don't quite believe this, see the end of this answer for a visual proof of this fact. (See "visual proof" below).

Therefore, given the length of diagonal AE = 10, we can determine the largest possible area of triangle ABE by making it an isosceles right triangle.

If you plan on scoring 700+ on the GMAT, you should know the side ratio for all isosceles right triangles (also known as 45-45-90 triangles because of their degree measurements).

That important side ratio is $1 : 1 : \sqrt{2}$ where the two **1**'s represent the two legs (the base and the height) and $\sqrt{2}$ represents the diagonal. Thus if we are to construct an isosceles right triangle with a diagonal of 10, then, using the side ratios, we can determine that each leg will have a length of $\frac{10}{\sqrt{2}}$.



Now, we can calculate the area of this isosceles right triangle:

$$\frac{1}{2}(AB \times BE) = \frac{1}{2}\left(\frac{10}{\sqrt{2}} \times \frac{10}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{100}{2}\right) = \frac{1}{2}(50) = 25$$

Since an isosceles right triangle will yield the maximum possible area, we know that 25 is the maximum possible area of a right triangle with a diagonal of length 10.

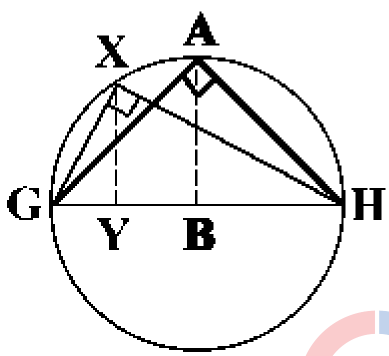
Of course, we don't really know if 25 is, in fact. the area of triangle ABE, but we do know that 25 is the maximum possible area of triangle ABE. Therefore we are able to answer our original question: Is the area of triangle ABE greater than 25? NO it is not greater than 25, because the maximum area is 25.

Since we can answer the question using Statement (2) alone.
The correct answer is B.

Visual Proof:

Given a right triangle with a fixed diagonal, why will an ISOSCELES triangle yield the triangle with the greatest area?

Study the diagram below to understand why this is always true:



In the circle above, GH is the diameter and AG = AH. Triangles GAH and GXH are clearly both right triangles (any triangle inscribed in a semicircle is, by definition, a right triangle).

Let's begin by comparing triangles GAH and GXH, and thinking about the area of each triangle. To determine this area, we must know the base and height of each triangle.

Notice that both these right triangles share the same diagonal (GH). In determining the area of both triangles, let's use this diagonal (GH) as the base. Thus, the bases of both triangles are equal.

Now let's analyze the height of each triangle by looking at the lines that are perpendicular to our base GH. In triangle GAH, the height is line AB. In triangle GXH, the height is line XY.

Notice that the point A is HIGHER on the circle's perimeter than point X. This is because point A is directly above the center of the circle, it the highest point on the circle.

Thus, the perpendicular line extending from point A down to the base is LONGER than the perpendicular line extending from point X down to the base. Therefore, the height of triangle GAH (line AB) is greater than the height of triangle GXH (line XY).

Since both triangles share the same base, but triangle GAH has a greater height, then the area of triangle GAH must be greater than the area of triangle GXH.

We can see that no right triangle inscribed in the circle with diameter GH will have a greater area than the isosceles right triangle GAH.

(Note: Another way to think about this is by considering a right triangle as half of a rectangle. Given a rectangle with a fixed perimeter, which dimensions will yield the greatest area? The rectangle where all sides are equal, otherwise known as a square! Test it out for yourself.
Given a rectangle with a perimeter of 40, which dimensions will yield the greatest area? The one where all four sides have a length of 10.)
The correct answer is B.

8.

Since BC is parallel to DE, we know that Angle ABC = Angle BDE, and Angle ACB = Angle CED. Therefore, since Triangle ABC and Trianlge ADE have two pairs of equal angles, they must be ***similar triangles***. Similar triangles are those in which all corresponding angles are equal and the lengths of corresponding sides are in proportion.

For Triangle ABC, let the base = **b**, and let the height = **h**.

Since Triangle ADE is similar to triangle ABC, apply multiplier "**m**" to **b** and **h**. Thus, for Triangle ADE, the base = **mb** and the height = **mh**.

Since the Area of a Triangle is defined by the equation $\frac{\text{base} \times \text{height}}{2}$, and since the problem tells us that the area of triangle ABC is 1/12 the area of Triangle ADE, we can write an equation comparing the areas of the two triangles:

$$\frac{b \times h}{2} = \frac{1}{12}\left(\frac{mb \times mh}{2}\right)$$

Simplifying this equation yields:

$$bh = \frac{1}{12} mbmh$$
$$12bh = m^2 bh$$
$$12 = m^2$$
$$\sqrt{12} = m$$
$$2\sqrt{3} = m$$

Thus, we have determined that the multipier (**m**) is $2\sqrt{3}$. Therefore the length of $AE = 2\sqrt{3} \times AC$

We are told in the problem that $AC = 3$, so $AE = 3\left(2\sqrt{3}\right) = 6\sqrt{3}$.

The problem asks us to solve for x, which is the difference between the length of **AE** and the length of **AC**.

Therefore, $x = AE - AC = 6\sqrt{3} - 3$

The correct answer is D.

9.

By simplifying the equation given in the question stem, we can solve for x as follows:

$$\sqrt{x^8} = 81$$
$$x^4 = 81$$
$$x = 3$$

Thus, we know that one side of Triangle A has a length of 3.

Statement (1) tells us that Triangle A has sides whose lengths are consecutive integers. Given that one of the sides of Triangle A has a length of 3, this gives us the following possibilities: (1, 2, 3) OR (2, 3, 4) OR (3, 4, 5). However, the first possibility is NOT a real triangle, since it does not meet the following condition, which is true for all triangles: The sum of the lengths of any two sides of a triangle must always be greater than the length of the third side. Since 1 + 2 is not greater than 3, it is impossible for a triangle to have side lengths of 1, 2 and 3.

Thus, Statement (1) leaves us with two possibilities. Either Triangle A has side lengths 2, 3, 4 and a perimeter of 9 OR Triangle A has side lengths 3, 4, 5 and a perimeter of 12. Since there are two possible answers, Statement (1) is not sufficient to answer the question.

Statement (2) tells us that Triangle A is NOT a right triangle. On its own, this is clearly not sufficient to answer the question, since there are many non-right triangles that can be constructed with a side of length 3.

Taking both statements together, we can determine the perimeter of Triangle A. From Statement (1) we know that Triangle A must have side lengths of 2, 3, and 4 OR side lengths of 3, 4, and 5. Statement (2) tells us that Triangle A is not a right triangle; this eliminates the possibility that Triangle A has side lengths of 3, 4, and 5 since any triangle with these side lengths is a right triangle (this is one of the common Pythagorean triples). Thus, the only remaining possibility is that Triangle A has side lengths of 2, 3, and 4, which yields a perimeter of 9.

The correct answer is C.

10.
The formula for the area of a triangle is 1/2(bh). We know the height of ΔABC. In order to solve for area, we need to find the length of the base. We can rephrase the question:

What is BC?

(1) INSUFFICIENT: If angle ABD = 60°, ΔABD must be a 30-60-90 triangle. Since the proportions of a 30-60-90 triangle are x: $x\sqrt{3}$: $2x$ (shorter leg: longer leg: hypotenuse), and AD = $6\sqrt{3}$, BD must be 6. We know nothing about DC.

(2) INSUFFICIENT: Knowing that AD = $6\sqrt{3}$, and AC = 12, we can solve for CD by recognizing that ΔACD must be a 30-60-90 triangle (since it is a right triangle and two of its sides fit the 30- 60-90 ratio), or by using the Pythagorean theorem. In either case, CD = 6, but we know nothing about BD.

(1) AND (2) SUFFICIENT: If BD = 6, and DC = 6, then BC = 12, and the

area of ΔABC = 1/2(bh) = 1/2(12)($6\sqrt{3}$) = $36\sqrt{3}$.

The correct answer is C

11.

Since BE || CD, triangle ABE is similar to triangle ACD (parallel lines imply two sets of equal angles). We can use this relationship to set up a ratio of the respective sides of the two triangles:

AB/AC = AE/AD

3/6 = 4/AD

AD = 8

We can find the area of the trapezoid by finding the area of triangle CAD and subtracting the area of triangle ABE.

Triangle CAD is a right triangle since it has side lengths of 6, 8 and 10, which means that triangle BAE is also a right triangle (they share the same right angle).
Area of trapezoid = area of triangle CAD – area of triangle BAE
= (1/2)bh – (1/2)bh
= 0.5(6)(8) – 0.5(3)(4)
= 24 – 6
= 18
The correct answer is B.

12.
According to the Pythagorean Theorem, in a right triangle $a^2 + b^2 = c^2$.

(1) INSUFFICIENT: With only two sides of the triangle, it is impossible to determine whether $a^2 + b^2 = c^2$.

(2) INSUFFICIENT: With only two sides of the triangle, it is impossible to determine whether $a^2 + b^2 = c^2$.

(1) AND (2) SUFFICIENT: With all three side lengths, we can determine if $a^2 + b^2 = c^2$. It turns out that $17^2 + 144^2 = 145^2$, so this is a right triangle. However, even if it were not a right triangle, this formula would still be sufficient, so it is unnecessary to finish the calculation.

The correct answer is C.

Top 1% expert replies to student queries (can skip)

This is a Data Sufficiency question; All you have to see is whether you have enough details to get to the solution. In this question, if you get to know the value of all sides, it is sufficient to find out whether it is a right angle triangle; you don't actually have to find out whether it is a right angled triangle in the exam.

(1) The length of side BC is 144 --> there are infinitely many triangles possible with the sides of 17 and 144, two of them will be right triangles (17^2+144^2=x^2, where x is a hypotenuse and 17^2+y^2=144^2, where y is an another leg) and others will not. Not sufficient.
(2) The length of side AC is 145. Not sufficient for the same reason.
(1)+(2) We know all the sides hence, even without actual calculation(in fact we don't have to do the calculation), we can find out whether the triangle is right or not. Sufficient. (We don't need to find out this: just additional info- As a matter of fact it is a right triangle: 17^2+144^2=145^2)

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)

If you're given two sides of a triangle, the angle between those 2 sides OR the length of the third side will decide whether the triangle is right angled.

Here, we're given that AB = 17.

Statement 1 : BC = 144.

This is insufficient, since there can be infinitely many triangles with 2 sides 17 and 144. Keep changing angle B and you'll see for yourself.

Statement 1 : AC = 145.

This again is insufficient, since there can be infinitely many triangles with 2 sides 17 and 145. Keep changing angle A and you'll see for yourself.

Combining, we now have all 3 sides lengths and this allows us to check whether the triangle is right angled or not. Simply use the Pythagoras theorem and check. Sufficient!

The correct answer is C.

13.

For GMAT triangle problems, one useful tool is the similar triangle strategy. Triangles are defined as similar if all their corresponding angles are equal or if the lengths of their corresponding sides have the same ratios.

(1) INSUFFICIENT: Just knowing that x = 60° tells us nothing about triangle EDB. To illustrate, note that the exact location of point E is still unknown. Point E could be very close to the circle, making DE relatively short in length. However, point E could be quite far away from the circle, making DE relatively long in length. We cannot determine the length of DE with certainty.

(2) SUFFICIENT: If DE is parallel to CA, then (angle EDB) = (angle ACB) = x. Triangles EBD and ABC also share the angle ABC, which of course has the same measurement in each triangle. Thus, triangles EBD and ABC have two angles with identical measurements. Once you find that triangles have 2 equal angles, you know that the third angle in the two triangles must also be equal, since the sum of the angles in a triangle is 180°.

So, triangles EBD and ABC are similar. This means that their corresponding sides must be in proportion:

CB/DB = AC/DE
radius/diameter = radius/DE
3.5/7 = 3.5/DE

Therefore, DE = diameter = 7.

The correct answer is B.

14.

First, recall that in a right triangle, the two shorter sides intersect at the right angle. Therefore, one of these sides can be viewed as the base, and the other as the height. Consequently, the area of a right triangle can be expressed as one half of the product of the two shorter sides (i.e., the same as one half of the product of the height times the base).
Also, since AB is the hypotenuse of triangle ABC, we know that the two shorter sides are BC and AC and the area of triangle ABC = (BC × AC)/2. Following the same logic, the area of triangle KLM = (LM × KM)/2.

Also, the area of ABC is 4 times greater than the area of KLM: (BC × AC)/2 = 4(LM × KM)/2
BC × AC = 4(LM × KM)

(1) SUFFICIENT: Since angle ABC is equal to angle KLM, and since both triangles have a right angle, we can conclude that the angles of triangle ABC are equal to the angles of triangle KLM, respectively (note that the third angle in each triangle will be equal to 35 degrees, i.e., 180 – 90 – 55 = 35). Therefore, we can conclude that triangles ABC and KLM are similar. Consequently, the respective sides of these triangles will be proportional, i.e. AB/KL = BC/LM = AC/KM = x, where x is the coefficient of proportionality (e.g., if AB is twice as long as KL, then AB/KL = 2 and for every side in triangle KLM, you could multiply that side by 2 to get the corresponding side in triangle ABC).

We also know from the problem stem that the area of ABC is 4 times greater than the area of KLM, yielding BC × AC = 4(LM × KM), as discussed above.

Knowing that BC/LM = AC/KM = x, we can solve the above expression for the coefficient of proportionality, x, by plugging in BC= x(LM) and AC = x(KM):
BC × AC = 4(LM × KM) x(LM) × x(KM) = 4(LM × KM) $x^2 = 4$
x = 2 (since the coefficient of proportionality cannot be negative)

Thus, we know that AB/KL = BC/LM = AC/KM = 2. Therefore, AB = 2KL = 2(10) = 20

(2) INSUFFICIENT: This statement tells us the length of one of the shorter sides of the triangle KLM. We can compute all the sides of this triangle (note that this is a 6-8-10 triangle) and find its

area (i.e., (0.5)(6)(8) = 24); finally, we can also calculate that the area of the triangle ABC is equal to 96 (four times the area of KLM). We determined in the first paragraph of the explanation, above, that the area of ABC = (BC × AC)/2. Therefore: 96 = (BC × AC)/2 and 192 = BC × AC. We also know the Pythagorean theorem: $(BC)^2 + (AC)^2 = (AB)^2$. But there is no way to convert BC × AC into $(BC)^2 + (AC)^2$ so we cannot determine the hypotenuse of triangle ABC.

The correct answer is A.

15.

We are given a right triangle PQR with perimeter 60 and a height to the hypotenuse QS of length 12. We're asked to find the ratio of the area of the larger internal triangle PQS to the area of the smaller internal triangle RQS.

First let's find the side lengths of the original triangle. Let c equal the length of the hypotenuse PR, and let a and b equal the lengths of the sides PQ and QR respectively. First of all we know that:

(1) $a^2 + b^2 = c^2$ Pythagorean Theorem for right triangle PQR
(2) $ab/2 = 12c/2$ Triangle PQR's area computed using the standard formula (1/2*b*h) but using a different base-height combination:
- We can use base = leg a and height = leg b to get Area of PQR = ab/2
- We can also use base = hypotenuse c and height = 12 (given) to get Area of PQR = 12c/2
- The area of PQR is the same in both cases, so I can set the two equal to each other: ab/2 = 12c/2.

(3) $a + b + c = 60$ The problem states that triangle PQR's perimeter is 60

(4) $a > b$ PQ > QR is given

(5) $(a + b)^2 = (a^2 + b^2) + 2ab$ Expansion of $(a + b)^2$
(6) $(a + b)^2 = c^2 + 24c$ Substitute (1) and (2) into right side of (5)
(7) $(60 – c)^2 = c^2 + 24c$ Substitute (a + b) = 60 – c from (3) (8)
$3600 – 120c + c^2 = c^2 + 24c$
(9) $3600 = 144c$
(10) $25 = c$

Substituting c = 25 into equations (2) and (3) gives us:

(11) $ab = 300$
(12) $a + b = 35$

which can be combined into a quadratic equation and solved to yield a = 20 and b = 15. The other possible solution of the quadratic is a = 15 and b = 20, which does not fit the requirement that a > b.

Remembering that a height to the hypotenuse always divides a right triangle into two smaller triangles that are similar to the original one (since they all have a right angle and they share another of the included angles), therefore all three triangles are similar to each other. Therefore their areas will be in the ratio of the square of their respective side lengths. The larger internal triangle has a hypotenuse of 20 (= a) and the smaller has a hypotenuse of 15 (= b), so the side lengths are in the ratio of 20/15 = 4/3. You must square this to get the ratio of their areas, which is $(4/3)^2 = 16/9$.

The correct answer is D.

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We know whatever the answer is going to be it is going to satisfy this equation. A1/A2=S1^2/S2^2=(S1/S2)^2. Now even after ratio S1/S2 is reduced to the smallest possible fraction. The answer choices have to satisfy the condition that the numbers are squares. The only answer choice (E) in this problem where the answer choices are squares is 16/9=4^2/3^2.

We can also approach this way:

We are given:

QS = 12
PQ>QR
Perimeter = 60

Fist off, we are dealing with a right triangle, it is glaring, and the first thing that pops up in my mind is: Pythagorean Triples.
Let's just recall the basic one because any other is just a multiple of the basic one.
3+4+5 = 12 not really close to 60.
15+20+25= 60 there we go!

Now we know that PQ= 20, QR= 15 and PR=25.

Since QS is perpendicular to PR the two smaller triangles are also right triangles. Let's figure out the length of the sides of PSQ.
Once again the smaller side (QS) turns out to be our fundamental Pythagorean triplet multiplied by 4.
Once realized this we can quickly gauge the length of the remaining side and hypothenuse. 12:16:20

Now we have all the elements that we need to find out our answer.

Area PQS- Area PQS = 54 (=Area QSR) and our ratio is going to be 16/9.

The correct answer is D.

16.

Triangle DBC is inscribed in a semicircle (that is, the hypotenuse CD is a diameter of the circle). Therefore, angle DBC must be a right angle and triangle DBC must be a right triangle.

(1) SUFFICIENT: If the length of CD is twice that of BD, then the ratio of the length of BD to the length of the hypotenuse CD is 1 : 2. Knowing that the side ratios of a 30-60- 90 triangle are 1 : $\sqrt{3}$: 2, where 1 represents the short leg, $\sqrt{3}$ represents the long leg, and 2 represents the hypotenuse, we can conclude that triangle DBC is a 30-60-90 triangle. Since side BD is the short leg, angle x, the angle opposite the short leg, must be the smallest angle (30 degrees).

(2) SUFFICIENT: If triangle DBC is inscribed in a semicircle, it must be a right triangle. So, angle DBC is 90 degrees. If y = 60, x = 180 – 90 – 60 = 30.

The correct answer is D.

17.

We are given a right triangle that is cut into four smaller right triangles. Each smaller triangle was formed by drawing a perpendicular from the right angle of a larger triangle to that larger triangle's hypotenuse. When a right triangle is divided in this way, two similar triangles are created. And each one of these smaller similar triangles is also similar to the larger triangle from which it was formed.

Thus, for example, triangle ABD is similar to triangle BDC, and both of these are similar to triangle ABC. Moreover, triangle BDE is similar to triangle DEC, and each of these is similar to triangle BDC, from which they were formed. If BDE is similar to BDC and BDC is similar to ABD, then BDE must be similar to ABD as well.

Remember that similar triangles have the same interior angles and the ratio of their side lengths are the same. So the ratio of the side lengths of BDE must be the same as the ratio of the side lengths of ABD. We are given the hypotenuse of BDE, which is also a leg of triangle ABD. If we had even one more side of BDE, we would be able to find the side lengths of BDE and thus know the ratios, which we could use to determine the sides of ABD.

(1) SUFFICIENT: If BE = 3, then BDE is a 3-4-5 right triangle. BDE and ABD are similar triangles, as discussed above, so their side measurements have the same proportion. Knowing the three side measurements of BDE and one of the side measurements of ABD is enough to allow us to calculate AB.

To illustrate:
BD = 5 is the hypotenuse of BDE, while AB is the hypotenuse of ABD.
The longer leg of right triangle BDE is DE = 4, and the corresponding leg in ABD is BD = 5.

Since they are similar triangles, the ratio of the longer leg to the hypotenuse should be the same in both BDE and ABD.
For BDE, the ratio of the longer leg to the hypotenuse = 4/5. For ABD,
the ratio of the longer leg to the hypotenuse = 5/AB.

Thus, 4/5 = 5/AB, or AB = 25/4 = 6.25

(2) SUFFICIENT: If DE = 4, then BDE is a 3-4-5 right triangle. This statement provides identical information to that given in statement (1) and is sufficient for the reasons given above.

The correct answer is D.

18.
The third side of a triangle must be less than the sum of the other two sides and greater than their difference (i.e. $|y - z| < x < y + z$).

In this question:

$|BC - AC| < AB < BC + AC$
$9 - 6 < AB < 9 + 6$
$3 < AB < 15$

Only 13.5 is in this range. $9\sqrt{3}$ is approximately equal to 9(1.7) or 15.3.

The correct answer is C.

19. In order to find the area of the triangle, we need to find the lengths of a base and its associated height. Our strategy will be to prove that ABC is a right triangle, so that CB will be the base and AC will be its associated height.

(1) INSUFFICIENT: We now know one of the angles of triangle ABC, but this does not provide sufficient information to solve for the missing side lengths.

(2) INSUFFICIENT: Statement (2) says that the circumference of the circle is 18π. Since the circumference of a circle equals π times the diameter, the diameter of the circle is 18. Therefore AB is a diameter. However, point C is still free to "slide" around the circumference of the circle giving different areas for the triangle, so this is still insufficient to solve for the area of the triangle.

(1) AND (2) SUFFICIENT: Note that inscribed triangles with one side on the diameter of the circle must be right triangles. Because the length of the diameter indicated by Statement (2) indicates that segment AB equals the diameter, triangle ABC must be a right triangle. Now, given Statement (1) we recognize that this is a 30-60-90 degree triangle. Such triangles always have side length ratios of $1:\sqrt{3}:2$

Given a hypotenuse of 18, the other two segments AC and CB must equal 9 and $9\sqrt{3}$ respectively. This gives us the base and height lengths needed to calculate the area of the triangle, so this is sufficient to solve the problem.

The correct answer is C.

20.
Let the hypotenuse be x, then the length of the leg is x/root2.
x+2x/root2=16+16*root2
x+root*x=16+16*root2
So, x=16
The correct answer is B.

Alternate sol from gmatclub (additional)

An IMPORTANT point to remember is that, in any isosceles right triangle, the sides have length x, x, and x√2 for some positive value of x.

Note: x√2 is the length of the hypotenuse, so our goal is to find the value of x√2

From here, we can see that the perimeter will be x + x + x√2

In the question, the perimeter is 16 + 16√2, so we can create the following equation:
x + x + x√2 = 16 + 16√2,
Simplify: 2x + x√2 = 16 + 16√2
IMPORTANT: Factor x√2 from the left side to get : x√2(√2 + 1) = 16 + 16√2
Now factor 16 from the right side to get: x√2(√2 + 1) = 16(1 + √2)
Divide both sides by (1 + √2) to get: x√2 = 16

Answer = B

GMAT Quant Topic 5: Geometry
Part C: Quadrilaterals

1.
(1) INSUFFICIENT: The diagonals of a parallelogram bisect one another. Knowing that the diagonals of quadrilateral ABCD (i.e. AC and BD) bisect one another establishes that ABCD is a parallelogram, but not necessarily a rectangle.
(2) INSUFFICIENT: Having one right angle is not enough to establish a quadrilateral as a rectangle.

(1) AND (2) SUFFICIENT: According to statement (1), quadrilateral ABCD is a parallelogram. If a parallelogram has one right angle, all of its angles are right angles (in a parallelogram opposite angles are equal and adjacent angles add up to 180), therefore the parallelogram is a rectangle.
The correct answer is C.

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All squares are rectangles. So, the answer(C) would still be YES. A rectangle is a square when both pairs of opposite sides are the same length. This means that a square is a specialized case of the rectangle and is indeed a rectangle.
The correct answer is C.

2.
(1) SUFFICIENT: The diagonals of a rhombus are perpendicular bisectors of one another. This is in fact enough information to prove that a quadrilateral is a rhombus.

(2) SUFFICIENT: A quadrilateral with four equal sides is by definition a rhombus.
The correct answer is D.

3.
(1) INSUFFICIENT: Not all rectangles are squares.
(2) INSUFFICIENT: Not every quadrilateral with two adjacent sides that are equal is a square. (For example, you can easily draw a quadrilateral with two adjacent sides of length 5, but with the third and fourth sides not being of length 5.)

(1) AND (2) SUFFICIENT: ABCD is a rectangle with two adjacent sides that are equal. This implies that all four sides of ABCD are equal, since opposite sides of a rectangle are always equal. Saying that ABCD is a rectangle with four equal sides is the same as saying that ABCD is a square.
The correct answer is C.

4.
Consider one of the diagonals of ABCD. It doesn't matter which one you pick, because the diagonals of a rectangle are equal to each other. So, lets focus on BD.

BD is part of triangle ABD. Since ABCD is a rectangle, we know that angle A is a right angle, so BD is the hypotenuse of right triangle ABD. Whenever a right triangle is inscribed in a circle, its hypotenuse is a diameter of that circle. Therefore, BD is a diameter of the circle P.

Knowing the length of a circle's diameter is enough to find the area of the circle. Thus, we can rephrase this question as "How long is BD?"

(1) INSUFFICIENT: With an area of 100, rectangle ABCD could have an infinite number of diagonal lengths. The rectangle could be a square with sides 10 and 10, so that the diagonal is $10\sqrt{2}$. Alternatively, if the sides of the rectangle were 5 and 20, the diagonal would have a length of $5\sqrt{17}$

(2) INSUFFICIENT: This does not tell us the actual length of any line in the diagram, so we don't have enough information to say how long BD is.
(1) AND (2) SUFFICIENT: If we know that ABCD is a square and we know the area of the square,

we can find the diagonal of the square - in this case $10\sqrt{2}$.

The correct answer is C.

5. We can let the height of the triangle (which is also the height of the trapezoid) = h.

   Let's first determine the area of triangle ABD in terms of h:

   area = (1/2)(base)(height) = (1/2)(6)(h) = 3h

   Next we can determine the area of trapezoid BACE in terms of h:

   area = (1/2)(base 1 + base 2)(height) = (1/2)(18 + 6)(h) = 12h

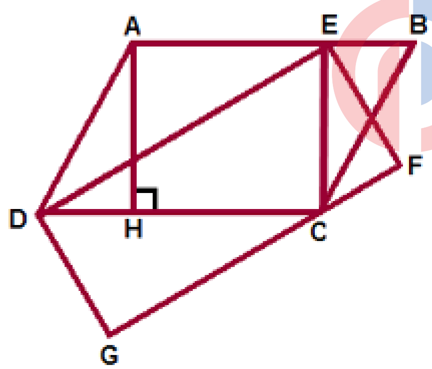
   Thus, the fraction of the trapezoid that is shaded is 3h/12h = 1/4.

   The correct answer is B.

6.

At first, it looks as if there is not enough information to solve this problem. Whenever you have a geometry problem that does not look solvable, one strategy is to look for a construction line that will add more information.

Lets draw a line from point E to point C as shown in the picture below:



Now look at triangle DEC. Note that triangle DEC and parallelogram ABCD share the same base (line DC). They also necessarily share the same height (the line perpendicular to base DC that passes through the point E). Thus, the area of triangle DEC is exactly one-half that of parallelogram ABCD.

We can also look at triangle DEC another way, by thinking of line ED as its base. Notice that ED is also a side of rectangle DEFG. This means that triangle DEC is exactly one-half the area of rectangle DEFG.

We can conclude that parallelogram ABCD and DEFG have the same area!

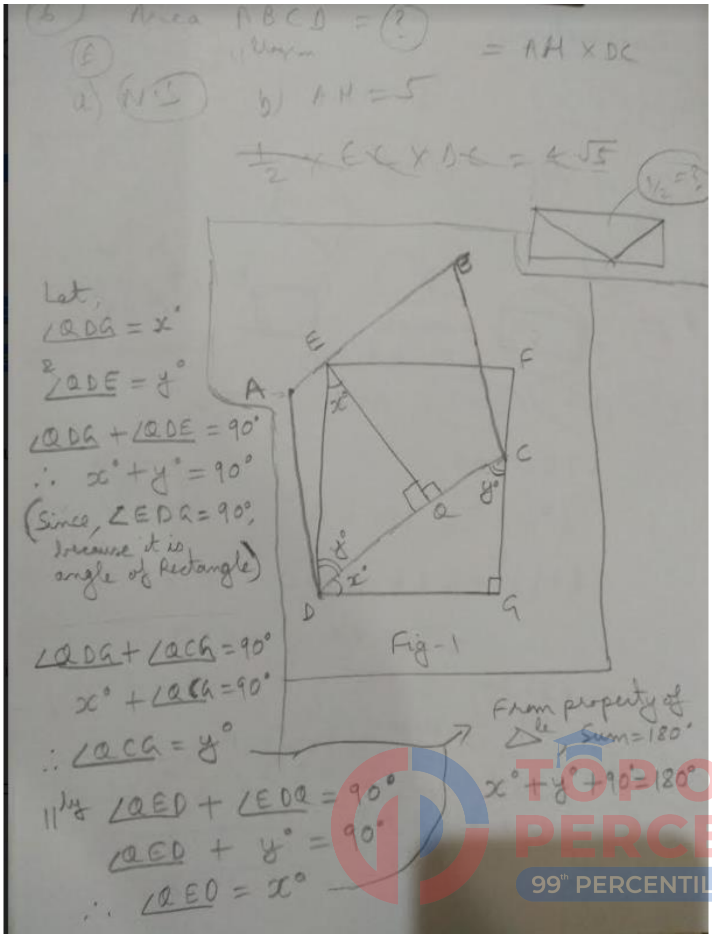
Thus, since statement (1) gives us the area of the rectangle, it is clearly sufficient, on its own, to determine the area of the parallelogram.

Statement (2) gives us the length of line AH, the height of parallelogram ABCD. However, since we do not know the length of either of the bases, AB or DC, we cannot determine the area of ABCD. Note also that if the length of AH is all we know, we can rescale the above figure horizontally, which would change the area of ABCD while keeping AH constant. (Think about stretching the right side of parallelogram ABCD.) Hence, statement (2) is not sufficient on its own.

The correct answer is A.

Top 1% expert replies to student queries (can skip)

Please go through the diagram that I have drawn



In the diagram below, notice I have constructed segment EQ, which is perpendicular to CD. This segment is the height of the parallelogram, so that times the length of CD would be the area of the parallelogram.

Look at ΔDGC and ΔEQD. Those two triangles are similar. Why?

Well, first of all, ∠QDG (let's say x°) and ∠EDQ (let's say y°) are complementary: they both add up to the 90° angle of ∠EDG. Also, ∠QDG (x°) and ∠QCG (becomes y°) are complementary, because they are the acute angles of a right triangle. Since ∠EDQ and ∠QCG are both complementary to the same angle (∠QDG), they are congruent: ∠EDQ≅ ∠QCG.

Since we know ∠EDQ≅ ∠QCG and we know ∠EQD≅ ∠G (both right angles), we know two angles in ΔDGC are congruent to two angles in ΔEQD. By the AA Similarity Theorem, they must be similar triangles.

ΔDGC ~ ΔEQD.

Similar triangles have proportional sides. In particular, we can set up a proportion:

ED /EQ = DC/ DG

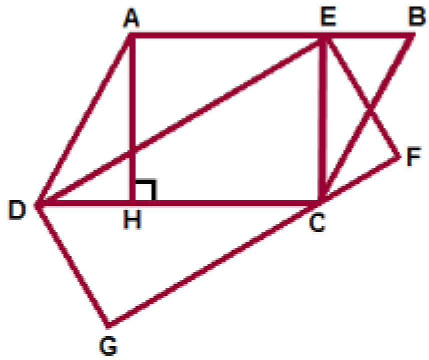
(ED)*(DG) = (EQ)*(DC)

After cross-multiplying, we get two equal products.

(ED)*(DG) = the area of the rectangle. (EQ)*(DC) = the area of the parallelogram.

The correct answer is A.

Top 1% expert replies to student queries (can skip) (additional)



In the figure, let us construct the line connecting points E and C, thereby completing triangle CDE.

If you look closely at the rectangle DEFG, you'll find the triangle contained within, sharing the base DE.

Area of rectangle DEFG = (DE) * (EF) ----[Equation 1] [Area of rectangle = Base * Height]

Area of triangle CDE = 1/2 * (DE) * (Perpendicular from C to the base DE) [But perpendicular drawn will be equal to EF, since the triangle and the rectangle share a base]

So, Area of triangle CDE = 1/2 * (DE) * (EF) -----[Equation 2]

From Equations 1 and 2. we get -

Area of triangle CDE = 1/2 * Area of rectangle DEFG

If you look closely at the parallelogram ABCD, you'll find the triangle CDE contained within, sharing the base CD.

Area of parallelogram ABCD = (DC) * (AH) [Equation 3] [Area of parallelogram = Base * Height]
Area of triangle CDE = 1/2 * (DC) * (Perpendicular from E to the base DC) [But perpendicular drawn will be equal to AH, since the triangle and the parallelogram share a base]
So, Area of triangle CDE = 1/2 * (DC) * (AH) [Equation 4]

From Equations 3 and 4. we get -

Area of triangle CDE = 1/2 * Area of parallelogram ABCD

From the two bolded equations above, we get that :

Area of parallelogram ABCD = Area of rectangle DEFG

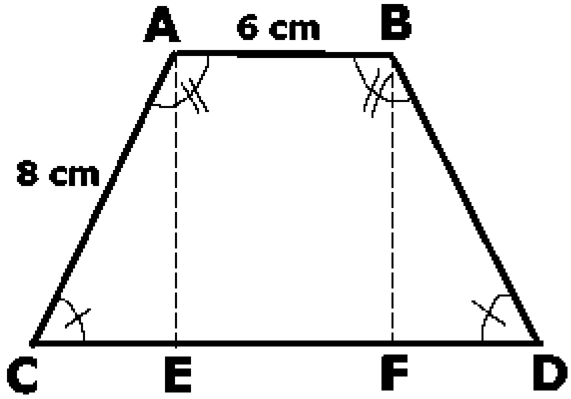
Statement 1 : Area of rectangle = $8\sqrt{5}$. Therefore, area of parallelogram = $8\sqrt{5}$. Sufficient!

Statement 2 : AH = 5. Doesn't tell us anything about the area of either figure. Insufficient!

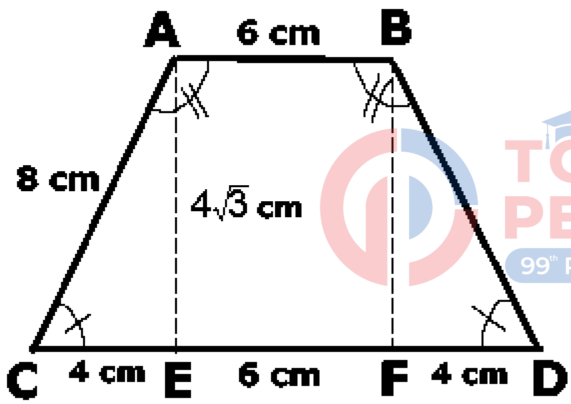
The answer therefore is A.

7.
The area of a trapezoid is equal to the average of the bases multiplied by the height. In this problem, you are given the top base (AB = 6), but not the bottom base (CD) or the height. (Note: 8 is NOT the height!) In order to find the area, you will need a way to figure out this missing data.

Drop 2 perpendicular lines from points A and B to the horizontal base CD, and label the points at which the lines meet the base E and F, as shown.



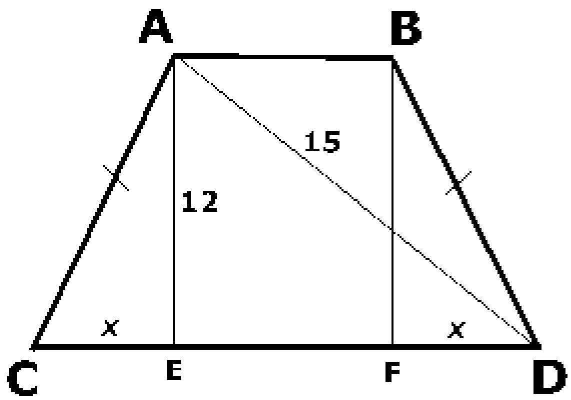
EF = AB = 6 cm. The congruent symbols in the drawing tell you that Angle A and Angle B are congruent, and that Angle C and Angle D are congruent. This tells you that AC = BD and CE = FD.
Statement (1) tells us that Angle A = 120. Therefore, since the sum of all 4 angles must yield 360 (which is the total number of degrees in any four-sided polygon), we know that Angle B = 120, Angle C = 60,



and Angle D = 60. This means that triangle ACE and triangle BDF are both 30-60-90 triangles. The relationship among the sides of a 30-60-90 triangle is in the ratio of $x : x\sqrt{3} : 2x$, where x is the shortest side. For triangle ACE, since the longest side AC = 8 , CE = 4 and AE = $4\sqrt{3}$. The same measurements hold for triangle BFD. Thus we have the length of the bottom base (4 + 6 + 4) and the height and we can calculate the area of the trapezoid.

Statement (2) tells us that the perimeter of trapezoid ABCD is 36. We already know that the lengths of sides AB (6), AC (8), and BD (8) sum to 22. We can deduce that CD = 14. Further, since EF = 6, we can determine that CE = FD = 4. From this information, we can work with either Triangle ACE or Triangle BDF, and use the Pythagorean theorem to figure out the height of the trapezoid. Now, knowing the lengths of both bases, and the height, we can calculate the area of the trapezoid.
The correct answer is D.

8.



By sketching a drawing of trapezoid ABDC with the height and diagonal drawn in, we can use the Pythagorean theorem to see the ED = 9. We also know that ABDC is an isosceles trapezoid, meaning that AC = BD; from this we can deduce that CE = FD, a value we will call x. The area of a trapezoid is equal to the average of the two bases multiplied by the height.
The bottom base, CD, is the same as CE + ED, or x + 9. The top base, AB, is the same as ED – FD, or 9 – x.

Thus, the average of the two bases is $\frac{(x+9)+(9-x)}{2} = \frac{18}{2} = 9$.

Multiplying this average by the height yields the area of the trapezoid: $9 \times 12 = 108$.

The correct answer is D.

9.

Assume the larger red square has a side of length x + 4 units and the smaller red square has a side of length x - 4 units. This satisfies the condition that the side length of the larger square is 8 more than that of the smaller square.

Therefore, the area of the larger square is $(x + 4)^2$ or $x^2 + 8x + 16$. Likewise, the area of the smaller square is $(x - 4)^2$ or $x^2 - 8x + 16$. Set up the following equation to represent the combined area:

$(x^2 + 8x + 16) + (x^2 - 8x + 16) = 1000$
$2x^2 + 32 = 1000$
$2x^2 = 968$

It is possible, but not necessary, to solve for the variable x here.

The two white rectangles, which are congruent to each other, are each x + 4 units long and x - 4 units high. Therefore, the area of either rectangle is $(x + 4)(x - 4)$, or $x^2 - 16$. Their combined area is $2(x^2 - 16)$, or $2x^2 - 32$.

Since we know that $2x^2 = 968$, the combined area of the two white rectangles is 968 - 32, or 936 square units.

The correct answer is B.

10. This question is simply asking if the two areas--the area of the circle and the area of quadrilateral ABCD--are equal.

We know that the area of a circle is equal to πr^2, which in this case is equal to $\pi(XY)^2$
If ABCD is a square or a rectangle, then its area is equal to the length times the width. Thus, in order to answer this question, we will need to be given (1) the exact shape of quadrilateral ABCD (just because it appears visually to be a square or a
rectangle does not mean that it is) and (2) some relationship between the radius of the circle and the side(s) of the quadrilateral that allows us to relate their respective areas.

Statement 1 appears to give us exactly what we need. Using the information given, one might deduce that since all of its sides are equal, quadrilateral ABCD is a square.

Therefore, its area is equal to one of its sides squared or $(AB)^2$. Substituting for the value of AB given in this statement, we can calculate that the area of ABCD equals $(\sqrt{\pi}XY)^2 = \pi XY^2$. This suggests that the area of quadrilateral ABCD is in fact equal to the area of the circle. However, this reasoning is INCORRECT.

A common trap on difficult GMAT problems is to seduce the test-taker into making assumptions that are not verifiable; this is particularly true when unspecified figures are involved. Despite the appearance of the drawing and the fact that all sides of ABCD are equal, ABCD does not HAVE to be a square. It could, for example, also be a rhombus, which is a quadrilateral with equal sides, but one that is not necessarily composed of four right angles. The area of a rhombus is not determined by squaring a side, but rather by taking half the product of the diagonals, which do not have to be of equal length. Thus, the information in Statement 1 is NOT sufficient to determine the shape of ABCD. Therefore, it does not allow us to solve for its area and relate this area to the area of the circle.

Statement 2 tells us that the diagonals are equal--thus telling us that ABCD has right angle corners (The only way for a quadrilateral to have equal diagonals is if its corners are 90 degrees.) Statement 2 also gives us a numerical relationship between the diagonal of ABCD and the radius of the circle. If we assume that ABCD is a square, this relationship would allow us to determine that the area of the square and the area of the circle are equal. However, once again, we cannot assume that ABCD is a square.

Statement 2 tells us that ABCD has 90 degree angle corners but it does not tell us that all of its sides are equal; thus, ABCD could also be a rectangle. If ABCD is a rectangle then its length is not necessarily equal to its width which means we are unable to determine its exact area (and thereby relate its area to that of the circle). Statement 2 alone is insufficient.

Given BOTH statements 1 and 2, we are assured that ABCD is a square since only squares have both equal sides AND equal length diagonals. Knowing that ABCD must be a square, we can use either numerical relationship given in the statements to confirm that the area of the quadrilateral is equal to the area of the circle.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

Basically, the question is asking whether the area of quadrilateral ABCD and the area of the circle are equal.

Statement 1:
AB = BC = CD = DA and AB = XY√π (AB = BC = CD = DA means all the 4 sides are equal, so the figure/quadrilateral can be square or rhombus)
ABCD could be square or a rhombus:
CASE I : let ,ABCD is a square, area of ABCD =(AB)^2 , area of Circle = π(XY)^2 =(AB)^2 ; -->Area of ABCD(square) = area of Circle (EQUAL)
CASE II : let ,ABCD is a rhombus ,
since area of rhombus < area of an square of same side
Area of ABCD(Rhombus) ≠ area of Circle (NOT EQUAL)
So, Statement (1) insufficient .
INSUFFICIENT

Statement 2:
AC = BD and AC = XY√2π (AC = BD means both the diagonals are equal, so the figure/quadrilateral can be square or rectangle)
CASE I : let ,ABCD is a square, area of ABCD =(AB)^2 =(AC/√2)^2=(AC)^2/2, area of Circle = π(XY)^2 =(AC)^2/2 ; --> Area of ABCD (square) = area of Circle (EQUAL)
CASE II : let ,ABCD is a rectangle ,
since area of rectangle < area of an square of same diagonal [suppose , d= 5 , area of square = (5/√2)^2= 12.5 whereas area of rectangle = 3*4 = 12 (5^2 = 3^2+4^2)]
Area of ABCD (rectangle) ≠ area of Circle (NOT EQUAL)

Combine (1) & (2):
ABCD has to be a square with the same area as that of the circle --> Area of ABCD (square) = area of Circle (EQUAL)

The correct answer is C.

11.
A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. The opposite sides of a parallelogram also have equal length.

(1) SUFFICIENT: We know from the question stem that opposite sides PS and QR are parallel, while this statement tells us that they also have equal lengths. The opposite sides PQ and RS must also be parallel and equal in length. This is the definition of a parallelogram, so the answer to the question is —Yes.

(2) INSUFFICIENT: We know from the question stem that opposite sides PS and QR are parallel, but have no information about their respective lengths. This statement tells us that the opposite sides PQ and RS are equal in length, but we don't know their respective angles; they might be parallel, or they might not be. According to the information given, PQRS could be a trapezoid with PS not equal to QR. On the other hand, PQRS could be a parallelogram with PS = QR. The answer to the question is uncertain.

The correct answer is A.

12.
To prove that a quadrilateral is a square, you must prove that it is both a rhombus (all sides are equal) and a rectangle (all angles are equal).

(1) INSUFFICIENT: Not all parallelograms are squares (however all squares are parallelograms).

(2) INSUFFICIENT: If a quadrilateral has diagonals that are perpendicular bisectors of one another, that quadrilateral is a rhombus. Not all rhombuses are squares (however all squares are rhombuses).

If we look at the two statements together, they are still insufficient. Statement (2) tells us that ABCD is a rhombus, so statement one adds no more information (all rhombuses are parallelograms). To prove that a rhombus is a square, you need to know that one of its angles is a right angle or that its diagonals are equal (i.e. that it is also a rectangle).

The correct answer is E

13. Because we do not know the type of quadrilateral, this question cannot be rephrased in a useful manner.

(1) INSUFFICIENT: We do not have enough information about the shape of the quadrilateral to solve the problem using Statement (1). For example, ABCD could be a rectangle with side lengths 3 and 5, resulting in an area of 15, or it could be a square with side length 4, resulting in an area of 16.

(2) INSUFFICIENT: This statement gives no information about the size of the quadrilateral.

(1) AND (2) SUFFICIENT: The four sides of a square are equal, so the length of one side of a square could be determined by dividing the perimeter by 4. Therefore, each side has a length of 16/4 = 4 and the area equals 4(4) = 16.

The correct answer is C

14.
The rectangular yard has a perimeter of 40 meters (since the fence surrounds the perimeter of the yard). Let's use l for the length of the fence and w for the width.

Perimeter = $2l + 2w$
$40 = 2l + 2w$
$20 = l + w$

The area of the yard = lw
$64 = lw$

If we solve the perimeter equation for w, we get $w = 20 – l$.
Plug this into the area equation:
$64 = l(20 – l)$
$64 = 20l – l^2$
$l^2 – 20l + 64 = 0$
$(l – 16)(l – 4) = 0$
$l = 4$ or 16

This means the width is either 16 or 4 ($w = 20 – l$).
By convention, the length is the longer of the two sides so the length is 16.

We could also solve this question by back solving the answer choices.

Let's start with C, the middle value. If the length of the yard is 12 and the perimeter is 40, the width would be 8 (perimeter – $2l = 2w$).
With a length of 12 and a width of 8, the area would be 96. This is too big of an area.

It may not be intuitive whether we need the length to be longer or shorter, based on the above outcome. Consider the following geometric principle: **for a fixed perimeter, the maximum area will be achieved when the values for the length and width are closest to one another.** A 10 × 10 rectangle has a much bigger area than an 18 × 2 rectangle. Put differently, when dealing with a fixed perimeter, the greater the disparity between the length and the width,

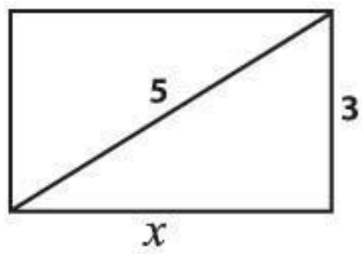
the smaller the area.

Since we need the area to be smaller than 96, it makes sense to choose a longer length so that the disparity between the length and width will be greater.

When we get to answer choice E, we see that a length of 16 gives us a width of 4 (perimeter – $2l = 2w$). Now the area is in fact 16 × 4 = 64.

The correct answer is E.

15.
If the square has an area of 9 square inches, it must have sides of 3 inches each. Therefore, sides AD and BC have lengths of 3 inches each. These sides are lengthened to x inches, while the other two remain at 3 inches. This gives us a rectangle with two opposite sides of length x and two opposite sides of length 3. Then we are asked by how much the two lengthened sides were extended. In other words, what is the value of $x – 3$? In order to answer this, we need to find the value of x itself.
(1) SUFFICIENT: If the resulting rectangle has a diagonal of 5 inches, we end up with the following:



We can now see that we have a 3-4-5 right triangle, since we have a leg of 3 and a hypotenuse (the diagonal) of 5. The missing leg (in this case, x) must equal 4. Therefore, the two sides were each extended by 4 – 3 = 1 inch.
(2) INSUFFICIENT: It will be possible, no matter what the value of x, to divide the resulting rectangle into three smaller rectangles of equal size. For example, if x = 4, then the area of the rectangle is 12 and we can have three rectangles with an area of 4 each. If x = 5, then the area of the rectangle is 15 and we can have three rectangles with an area of 5 each. So it is not possible to know the value of x from this statement.
The correct answer is A.

16. A rhombus is a parallelogram with four sides of equal length. Thus, AB = BC = CD = DA.

The diagonals of a parallelogram bisect each other, meaning that AC and BD intersect at their midpoints, which we will call E. Thus, AE = EC = 4 and BE = ED = 3. Since ABCD is a rhombus, diagonals AC and BD are also perpendicular to each other.

Labelling the figure with the lengths above, we can see that the rhombus is divided by the diagonals into four right triangles, each of which has one side of length 3 and another side of length 4.

Remembering the common right triangle with side ratio = 3: 4: 5, we can infer that the unlabelled hypotenuse of each of the four triangles has length 5.

Thus, AB = BC = CD = DA = 5, and the perimeter of ABCD is 5 × 4 = 20.
The correct answer is C.

17.

Sum of inner angels of quadrilateral is 360 degrees. (Sum of inner angles of polygon=180*(n-2), where n is # of sides)

(1) Two of the interior angles of ABCD are right angles --> angles can be 90+90 + any combination of two angels totaling 180. Not sufficient.

(2) The degree measure of angle ABC is twice the degree measure of angle BCD --> <ABC=2<BCD. Not sufficient

(1)+(2) Angles can be 90+**90+45**+135 Or 90+90+**60+120**. Not sufficient.

The correct answer is E.

GMAT Quant Topic 5: Geometry
Part D: Circles

1. Let us say that line segment RT has a length of 1. RT is the radius of circle R, so circle R has a radius of 1. Line segment QT is the diameter of circle R, so it has a length of 2 (twice the radius of circle R). Segment QT also happens to be the radius of circle Q, which therefore has a radius of 2. Line segment PT, being the diameter of circle Q, has a length of 4. Segment PT also happens to be the radius of circle P, which therefore has a radius of 4. The question is asking us what fraction of circle P is shaded. The answer will be (shaded area) ÷ (area of circle P). The area of circle P is $\pi(4)^2$, which equals 16π. The shaded area is just the area of circle Q (i.e. $\pi\ (2)^2$, which equals 4π) minus the area of circle R (i.e. $\pi\ (1)^2$, which equals π). Therefore, the answer to our question is $(4\pi - \pi) / 16\pi = 3/16$

The correct answer is A.

2. Lets first consider the relationship between (1/a + a) and (1/a^2 + a^2). If we square (1/a + a), we get:

$$\left(\frac{1}{a} + a\right)^2 \rightarrow \left(\frac{1}{a} + a\right)\left(\frac{1}{a} + a\right) \rightarrow \frac{1}{a^2} + 1 + 1 + a^2 \rightarrow \frac{1}{a^2} + a^2 + 2$$

We can use this information to manipulate the equation (1/a +a = 3) provided in the question.

$$\left(\frac{1}{a} + a\right) = 3$$

$$\left(\frac{1}{a} + a\right)^2 = 3^2$$

$$\frac{1}{a^2} + a^2 + 2 = 9$$

$$\frac{1}{a^2} + a^2 = 7$$

If we square both sides of the given equation, we arrive at the following:

$$\frac{1}{a^2} + a^2$$

The question asks for the approximate circumference of a circle with a diameter of ,
which we have determined is equal to 7.

Circumference = diameter $\times \pi = 7\pi \approx 22$.

The correct answer is B.

3.
Consider the RT = D formula (rate × time = distance).

D, the total distance travelled by the car, will be equal to C, the circumference of each tire, times N, the number of 360° rotations made by each tire. Thus, we can rewrite our formula as RT = CN.

The question is asking us how many 360° rotations each tire makes in 10 minutes. In other words, it is asking us for the value of N when T = 10. We can rewrite our equation thus:

R(10) = CN

N = R(10)/C

Clearly, in order to answer the question —what is N we need to know both R and C.

(1) INSUFFICIENT: This only gives us R, the speed of the car.

(2) INSUFFICIENT: This gives us the radius of the tire, which enables us to find C, the circumference of the tire. Knowing C is not, however, sufficient to figure out N.

(1) AND (2) SUFFICIENT: Now that we know both R and C, we can figure out what N is from the equation N = R(10)/C.

The correct answer is C.

4.
If we know the ratio of the radii of two circles, we know the ratio of their areas. Area is based on the square of the radius [$A = \pi (\text{radius})^2$]. If the ratio of the radii of two circles is 2:1, the ratio of their areas is $2^2 : 1^2$ or 4:1.

In this question, the radius of the larger circular sign is twice that of the smaller circular sign therefore, the larger sign's area is four times that of the smaller sign and would require 4 times as much paint.
The correct answer is D.

5.
It would be exceedingly difficult to find the area of each shaded band directly, so we will use alternate methods to find their combined area indirectly.

The simplest approach is to use actual values in place of the unknown x. Let's assume that the radius x of the outermost quarter-circle has a value of 10. The radii of the other quarter-circles would then be 9, 8, 7, and 6, respectively. If we were dealing with whole circles, we would have five circles with respective areas of 100π, 81π, 64π, 49π and 36π.

To find the area of the innermost quarter-circle, we just need 36π. .

To find the area of the middle band, we need to subtract the area of the second-smallest quarter-circle from that of the middle circle: $64\pi - 49\pi = 15\pi$.

To find the area of the outermost band, we need to subtract the area of the second-largest quarter-circle from that of the largest: $100\pi - 81\pi = 19\pi$.

So the combined area of the shaded bands (as whole circles) is $36\pi + 15\pi + 19\pi = 70\pi$. Since

we are dealing with quarter-circles, we need to divide this by 4: $70\pi/4$. Now we need to substitute 10 for x in the answer choices. The choice that yields $70\pi/4$ is the correct answer. The only one to do so is choice A:

$$\frac{\left(x^2 - 4x + 10\right)\pi}{4} \rightarrow$$
$$\frac{\left(10^2 - 4(10) + 10\right)\pi}{4} \rightarrow$$
$$\frac{\left(100 - 40 + 10\right)\pi}{4} \rightarrow$$
$$\frac{70\pi}{4}$$

We can also solve the problem algebraically:

First, we need the area of the smallest quarter-circle. The smallest quarter-circle will have a radius of (x - 4) and thus an area of $\frac{(x-4)^2\pi}{4}$ (we must divide by 4 because it is a quarter- circle).

Now, we need the area of the middle band. We can find this by subtracting the area of the second

quarter-circle from that of the third: $\frac{(x-2)^2\pi}{4} - \frac{(x-3)^2\pi}{4}$. Now, we need the area of the outer band. We

$$\left(\frac{(x)^2\pi}{4} - \frac{(x-1)^2\pi}{4}\right) + \left(\frac{(x-2)^2\pi}{4} - \frac{(x-3)^2\pi}{4}\right) + \frac{(x-4)^2\pi}{4} \rightarrow$$

$$\frac{\pi\left(\left(x^2 - \left(x^2 - 2x + 1\right)\right) + \left(\left(x^2 - 4x + 4\right) - \left(x^2 - 6x + 9\right)\right) + x^2 - 8x + 16\right)}{4} \rightarrow$$

$$\frac{\pi\left(x^2 - x^2 + 2x - 1 + x^2 - 4x + 4 - x^2 + 6x - 9 + x^2 - 8x + 16\right)}{4} \rightarrow$$

$$\frac{\left(x^2 - 4x + 10\right)\pi}{4}$$

can find this by subtracting the area of the second-largest quarter-circle from that of the largest:

$\frac{(x)^2\pi}{4} - \frac{(x-1)^2\pi}{4}$. Finally, we need to add all these areas:

Note that we factored pi out of every term and that you must remember to distribute the minus signs as if they were -1.

The correct answer is C.

6.

Since sector PQ is a quarter-circle, line segments QB and PB must be radii of the circle. Let r be the radius of the circle. So, the question is, what is the value of r? Since QB and PB are radii of the circle, QB = r and PB = r.

If AC = 100, and triangle ABC is a right triangle, it must be true that $AB^2 + CB^2 = AC^2$. We can use this to solve for r.

CQ + QB = CB. Since CQ = 2/7QB and QB = r, we can construct the following equation:

$r + \frac{2}{7}r = \frac{9}{7}r$. Therefore, $CB = \frac{9}{7}r$.

AP + PB = AB. Since AP = 1/2PB and PB = r, we can construct the following equation:

$r + \frac{1}{2}r = \frac{3}{2}r$. Therefore, $AB = \frac{3}{2}r$.

Since $AB^2 + CB^2 = AC^2$, it must be true that $\left(\frac{9}{7}r\right)^2 + \left(\frac{3}{2}r\right)^2 = 100^2$.

We can find a common denominator:

$$\left(\frac{2 \times 9}{2 \times 7}r\right)^2 + \left(\frac{7 \times 3}{7 \times 2}r\right)^2 = 100^2$$

Therefore,

$$\left(\frac{18}{14}r\right)^2+\left(\frac{21}{14}r\right)^2=100^2$$

Now we can solve for r:

$$\frac{324}{196}r^2+\frac{441}{196}r^2=100^2 \rightarrow$$
$$\frac{765}{196}r^2=100^2 \rightarrow$$
$$\sqrt{\frac{765}{196}r^2}=\sqrt{100^2} \rightarrow$$
$$\frac{\sqrt{765}}{14}r=100 \rightarrow$$
$$\sqrt{765}(r)=1400 \rightarrow$$
$$r=\frac{1400}{\sqrt{765}} \rightarrow$$
$$r=\frac{1400}{\sqrt{765}}\left(\frac{\sqrt{765}}{\sqrt{765}}\right) \rightarrow$$
$$r=\frac{1400\sqrt{765}}{765} \rightarrow$$
$$r=\frac{\cancel{(5)}(280)\sqrt{9\times 85}}{\cancel{(5)}(153)} \rightarrow$$
$$r=\frac{280\times \cancel{3}\sqrt{85}}{\cancel{3}\times 51} \rightarrow$$
$$r=\frac{280\sqrt{85}}{51}$$

The correct answer is A.

7.
Since the notches start in the same position and move in opposite directions towards each other, they will trace a circle together when they pass for the first time, having covered a joint total of 360^0. When the notches meet for the second time, they will have traced two full circles together for a total of 720^0.

Since the circumference of the large gear is 4 times greater than that of the small gear, the large notch will cover only 1/4 of the number of degrees that the small notch does. We can represent this as an equation (where x is the number of degrees covered by the large notch):

$$4x+x=720 \rightarrow$$
$$5x=720 \rightarrow$$
$$x=144$$

So the large notch will have covered 144^0 when the notches pass for the second time. Since the circumference of the large gear is 96π, we can set up the following proportion to solve for the linear distance (call it d) covered by the large notch:

$$\frac{144}{360} = \frac{d}{96\pi} \rightarrow$$
$$\frac{2}{5} = \frac{d}{96\pi} \rightarrow$$
$$5d = 192\pi \rightarrow$$
$$d = 38.4\pi$$

The correct answer is C.

Top 1% expert replies to student queries (can skip)

According to the Q:
Speed of rotation is same (linear speed)
Diameters of Large : Small are in ratio: 4:1 (96:24)
So, 4 rotations on small gear = 1 rotation on large gear.
Or, 1 rotation on small gear = 0.25 rotation on large gear.
Direction of rotation: opposite (small anti clockwise, large clockwise)

To calculate: distance traveled by a notch on large gear when notch of large gear meets notch on small gear for the second time.

Instance #1:
Small gear 1 rot = notch back at 12 o clock
Large gear= 0.25 rot = notch at 3 o o clock i.e. 0.25 on the circumstance

Instance #2:
Small gear Another 0.5 rot= notch at 6 o clock
Large gear = 0.25/2 rot = notch at 4.30 on clock i.e. 0.25 + 0.125= 0.375 on circumference

Meeting Point:
Using relative distance and speed concept

Relative Distance between two notches is 1/8 of circumference on large gear= 0.125

Angular speed is in ratio- small:large= 4:1
Relative speed: 5x

Time to meet: 0.125/5x = 0.025x

In this time, distance traveled by large gear speed x: 0.025

Position of large gear is= 96pi/(0.375+0.025) = 96pi/0.4= 38.4pi

Ans C

Top 1% expert replies to student queries (can skip) :

Query : How will the large notch cover only 1/4 of the number of degrees that the small notch covers ?
Reply: There are two circles sitting on top of each other, the smaller one moving anticlockwise, the larger one clockwise (the respective directions don't matter, just that they are rotating in opposite directions). Each has a notch at 12 o'clock position.

Now picturize what is happening here carefully. At the same time the two circles start moving at the same speed, so the two notches are separating from each other in a circular arc. Since the motion is circular, the distance travelled is basically in degrees, which also translates to the length of the arc travelled (when any point on the circle travels 360 degrees, it travels the circumference of the circle in terms of distance, or a distance of 2pi x r, from which we can find the corresponding distance travelled for any other degree amount travelled).

Each notch is essentially a point in the corresponding circle that is travelling in the way described above. So when they pass each other for the first time, the notch on the clockwise circle has travelled some degrees, the notch on the counter-clockwise circle has travelled some degrees, but together they have travelled 360 degrees. Then they again separate and travel in opposite circular directions and again meet a second time. Exactly similarly, when they meet for the second time, they have together travelled 720 degrees.

Now, we saw that a point travels 2pi r distance when it travels 360 degrees (r is the radius of the circle). Here we have two radii - r and 4r

Let's take one full rotation of the small circle as an example. This is a distance of 2pi r for the notch on it, and 360 travelled. Since the speeds of rotation are the same, the notch on the bigger circle has also travelled 2pi r distance, but simple unitary method will tell you, because its radius is 4r, the degrees the point has travelled is 90, or 1/4th of the degrees travelled by the point on the smaller circle. So degrees travelled by each point will be in the inverse ratio of the radii of the circles

Then when they meet for the second time, if the larger notch has travelled a total of x degrees, the smaller notch would have travelled a total of 4x degrees and 5x = 720 as we obtained earlier, then x = 144. So the notch on the larger circle has travelled 144 degrees. Use unitary method to find its distance in cm (hint: if it travels 360 degrees, it travels 2 pi x 96 cm, so if it travels 1 degree it travels (2 pi x 96 / 360) cm and so on)

Finally, a distinction you need to be aware of that, the solution says the circumference of the larger circle is 4 times greater than that of the smaller circle. This is incorrect as it would make the circumference five times that of the of the smaller. The correct verbiage (and distinction) is that it is 4 times that of the smaller circle

8.
Since the question asks for a ratio, it will be simplest to define the radius of the circle as 1.

Inscribed angle AXY (given as 105^0 intercepts are ACY). By definition, the measure of an inscribed angle is equal to half the measure of its intercepted arc. Thus, arc ACY = 2 x 105 = 210^0.

We are given that points A, B and C are all on the diameter of the circle. Using this information, we can think of arc ACY in two chunks: arc AC plus arc CY. Since arc AC defines a semicircle, it is equal to 180^0

Therefore, arc CY = $210^0 - 180^0 = 30^0$

We can now use a proportion to determine the area of sector CBY. Since arc CY represents 30/360 or 1/12 the measure of the entire circle, the area of sector CBY = 1/12 the area of the entire circle. Given that we have defined the circle as having a radius of 1, the area of sector

CBY = $(1/12)\pi r^2$

= $(1/12)\pi(1)^2$

= $\pi/12$

Next, we must determine the area of the small grey semi-circle with diameter BC. Since BC = 1, the radius of this semi-circle is .5. The area of this semi-circle is $(1/2)\pi r^2 = (1/2)\ \pi\ (.5)^2 = \pi/8$

Thus, the area of the shaded region below the red line is $\pi/12 + \pi/8 = \pi/5$

Now we must determine the area of the shaded region above the red line. Using the same logic as above, angle YBA is $180^0 - 30^0 = 150^0$. We can now use a proportion to determine the area of sector YBA.

Since arc YA represents 150/360 or 5/12 the measure of the entire circle, the area of sector

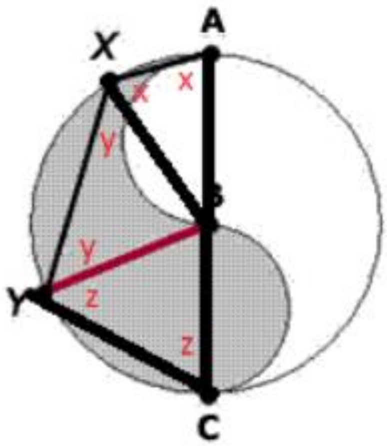
YBA = 5/12 the area of the entire circle. Given that we have defined the circle as having a radius of 1, the area of sector YBA = (5/12) is $(5/12)\ \pi r^2 = (1/2)\ \pi\ (1)^2 = 5\pi/12$

We must now determine the area of the small white semi-circle with diameter AB. Since AB = 1, the radius of this semi-circle is .5. The area of this semi-circle is = $(1/2)\pi r^2 = (1/2)\ \pi\ (.5)^2 = \pi/8$

Thus, the area of the shaded region above the red line is $5\pi/12 - \pi/8 = 7\pi/24$.
Finally, then, the ratio of the two areas is $(7\pi/24) / (5\pi/24) = 7/5$.
The correct answer is D.

Top 1% expert replies to student queries (can skip)

Three new triangles can be formed which will be isosceles triangles (please refer the attachment)



Hence,
x+y = 105 (given)
2(x+y+z) = 360 since ACYX is a quadrilateral
x+y+z = 180
-> z = 75

in Triangle, BYC, <YBC will be 180 - 2*z = 180 - 2*75 = 30 degrees.

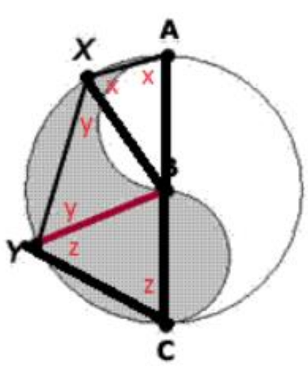
(1) Area of the total shaded portion is half the area of the circle pi*(r^2)/2

(2) Area below the red line = area of segment BYC of circle + area of shaded semicircle BC
= (30/360)*(pi*r^2) + pi ((r/2)^2)/2
=pi*(r^2)/12+pi*(r^2)/8
=5 * pi * (r^2) / 24

(3) Area above the red line is (1)-(2) above
= [pi * (r^2)/2] - [5 * pi* (r^2) / 24]
= 7 * pi * (r^2) / 24

Answer is (3) / (2) which is 7/5

Top 1% expert replies to student queries (can skip) (additional)



In the figure above, let the radius of the circle be R.

The radius of the 2 inner semicircles = R/2

Now, observe the figure drawn below carefully

We have labelled the 2 inner semicircles as semi-circle 1 and 2

Area of the shaded region = Area of semicircle AXYC - Area of semicircle 1 + Area of semicircle 2
[Please let me know if this equation is not clear. We're simply following the shaded region.]

Now, Area of semicircle 1 = Area of semicircle 2 = 1/2 * pi * (R/2)^2

Therefore,

Area of the shaded region = Area of semicircle AXYC = pi * (R^2/2)

Now, we have been given that angle YXA = 105 degrees.

If we join points Y and C, we'll get the cyclic quadrilateral AXYC. We know that in a cyclic quadrilateral, the opposite angles are supplementary (That is, they add up to 180 degrees).

Therefore, angle YXA + angle YCB = 180

105 + angle YCB = 180

Therefore, angle YCA = 75 degrees

In the triangle YCB, BY = BC = R.

Therefore angles opposite to sides BY and BC will also be equal (Angles opposite to equal sides are equal in a triangle).

So, angle YCB = angle BYC = 75 degrees

In triangle BYC, sum of all the angles should be 180 degrees,

75 + 75 + angle YBC = 180 OR angle YBC = 30 degrees

Now, area of shaded region below line YB = Area of arc BYC + Area of semicircle 2

Area of arc BYC = (angle YBC/360 degrees) * pi * R^2 = (30/360) * pi * R^2 = pi * R^2/12

Area of semicircle 2 = 1/2 * pi * (R/2)^2

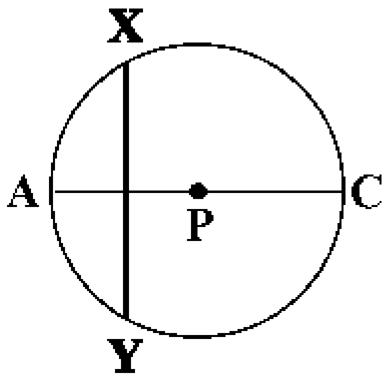
Therefore, shaded region area below the line YB = pi * R^2/12 + pi * R^2/8 = pi * (5R^2/24)

Total area of shaded region = pi * (R^2/2)

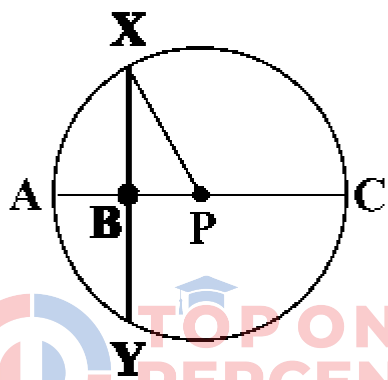
Therefore, shaded region area above the line YB = Total area of shaded region - shaded region area above the line YB = pi * (R^2/2) - pi * (5R^2/24) = pi * (7R^2/24)

Therefore, area of upper region/ area of lower region = 7/5

9.
Since no picture is given in the problem, draw it. Below, find the given circle with center P and cord XY bisecting radius AP.



Although, the picture above is helpful, drawing in an additional radius is often an important step towards seeing the solution. Thus, we will add to the picture by drawing in radius XP as shown below.



Since XY bisects radius AP at point B, segment BP is half the length of any radius.

Since BP is half the length of radius XP, right triangle XBP must be a 30-60-90 triangle with sides in the ratio of $1 : \sqrt{3}:2$.

Therefore, finding the length of any side of the triangle, will give us the lengths of the other two sides.

Finding the length of radius XP will give us the length of XB, which is half the length of cord XY. Thus, in order to answer the question--**what is the length of cord *XY*?**--we need to know only one piece of information:

The length of the radius of the circle.

Statement (1) alone tells us that the circumference of the circle is twice the area of the circle. Using this information, we can set up an equation and solve for the radius as follows:

$$\text{Circum} = 2 \times \text{Area}$$
$$2\pi r = 2\left(\pi r^2\right)$$
$$2r = 2r^2$$
$$r = r^2$$
$$r = 1$$

Therefore Statement (1) alone is sufficient to answer the question.

Statement (2) alone tells us that the length of Arc XAY $= \frac{2\pi}{3}$.

Arc XAY is made up of arc XA + arc AY.

Given that triangle XBP is a 30:60: 90 triangle, we know that <XPA = 60 degrees and can deduce that <APY = 60 degrees as well. Therefore Arc XAY =120 degrees or 1/3 of the circumference of the circle. Using this information, we can solve for the radius of the circle by setting up an equation as follows: Therefore, Statement (2) alone is also sufficient to answer the question.

$$\text{Arc } XAY = \frac{1}{3}\text{Circum}$$

$$\frac{2\pi}{3} = \frac{1}{3}(2\pi r)$$

$$2\pi = 2\pi r$$

$$1 = r$$

The correct answer is D.

10.

The probability of the dart landing outside the square can be expressed as follows:

$$P = \frac{\text{area of circle} - \text{area of square}}{\text{area of circle}}$$

To find the area of the circle, we need to first determine the radius. Since arc ADC has a length of $\pi\sqrt{x}$,we know that the circumference of the circle must be double this length or $2\pi\sqrt{x}$. Using the formula for circumference, we can solve for the radius:

$$2\pi r = 2\pi\left(\sqrt{x}\right)$$

$$r = \sqrt{x}$$

Therefore, the area of the circle $= \pi r^2 = \pi\left(\sqrt{x}\right)^2 = \pi(x)$

To find the area of the square, we can use the fact that the diagonal of the square is equal to twice the radius of the circle. Thus, the diagonal of the square is $2\sqrt{x}$. Since the diagonal of the square represents the hypotenuse of a 45-45-90 triangle, we use the side ratios of this special triangle $(1:1:\sqrt{2})$ to determine that each side of the square measures $\frac{2\sqrt{x}}{\sqrt{2}} = \sqrt{2x}$.

Thus, the area of the square $= (\sqrt{2x})^2 = 2x$.

Now we can calculate the total probability that the dart will land inside the circle but outside the square:

$$P = \frac{\text{area of circle} - \text{area of square}}{\text{area of circle}} = \frac{\pi(x) - 2x}{\pi(x)} = 1 - \frac{2}{\pi}$$

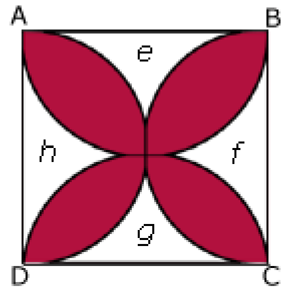
Interestingly, the probability does not depend on x at all, meaning that this probability holds true for any square inscribed in a circle (regardless of size).

The correct answer is D.

11.

Instead of trying to find the area of the red shaded region directly by somehow measuring the area of the clover leaf, it is easier to find the area of the un-shaded portions and subtract that from the area of the square as a whole.

Let's call the four uncolored portions e, f, g, and h, respectively.



To find the combined area of h and f, we need to take the area of the square and subtract the combined area of the semicircles formed by arcs AB and DC.

To make the problem less abstract, it helps to pick a number for the variable x. Let's say each side of the square has a length of 8. Therefore, the area of the square is 64.

Since the square has a side of length 8, the diameter of each semicircle is 8, which means that the radius is 4. So, the area of each semicircle is $\frac{\pi r^2}{2} = \frac{16\pi}{2} = 8\pi$.

If we take the area of the square and subtract the combined area of the two semicircles formed by arcs AB and DC, we get $64 - 2(8\pi) = 64 - 16\pi$. This represents the combined area of h and f. The combined area of e and g is the same as that of h and f, so altogether the four uncolored portions have an area of $2(64 - 16\pi) = 128 - 32\pi$.

If we take the area of the square and subtract out the area of the uncolored regions, we are left with the area of the colored red region:

$64 - (128 - 32\pi) \rightarrow 64 - 128 + 32\pi \rightarrow -64 + 32\pi \rightarrow 32\pi - 64$

Now we need to plug in 8 for x in each answer choice. The one that produces the expression is the $32\pi - 64$ correct answer. The only one to do so is choice (B).

(Note: One other way to think about the problem is to recognize that the combined area of the 4 semicircles (AB, AD, BC, BD) minus the area of the square is equal to the area of the red shaded region. Then one can logically proceed from this by picking a value for x in a manner similar to the procedure explained above.)

12.

One way to begin this problem is to assign the overlapping section an area of x. Therefore, the area of the non-overlapping section of circle R is equal to (R^2 – x) and the area of the non- overlapping section of circle r is equal to (r^2 – x).

The difference in the non-overlapping areas is $(\cdot R^2 - x) - (\cdot r^2 -$.

Hence, the question stem restated is: What is $R^2 - r^2$?

Statement (1) states that $R = r + 3k$. This is equivalent to saying that R – r = 3k, but it is not sufficient to answer the question stem.

Statement (2) states that $\frac{kR}{kr-6} = -1$. This can be rewritten as follows: $kR = 6 - kr$
$kR + kr = 6$
$R + r = 6/k$

This is not sufficient to answer the question.
By combining statements (1) and (2) we have, R – r = 3k and R + r = 6/k.

Therefore, $(R – r)(R + r) = R^2 – r^2 = 3k(6/k) = 18$. This answers the restated question stem. The correct answer is C: BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
The correct answer is C

Top 1% expert replies to student queries (can skip) (additional)

Query: In this question we do not know how much overlap exists between the 2 circles, Shouldn't the answer be E in that case?
Reply: The difference in the areas of the non-overlapping parts will be independent of the overlap.

Let the area of the overlapping region be x.

Non overlapping region of the larger = pi*R^2 - x [Radius of larger circle = R]
Non overlapping region of the smaller circle = pi*r^2 - x [Radius of smaller circle = r]

Therefore, difference of the non-overlapping areas = (pi*R^2 - x) - (pi*r^2 - x) = (pi*R^2 - pi*r^2)

As you can see, the difference is independent of the overlap area.

13.
Triangle DBC is inscribed in a semicircle (that is, the hypotenuse CD is a diameter of the circle). Therefore, angle DBC must be a right angle and triangle DBC must be a right triangle.

(1) SUFFICIENT: If the length of CD is twice that of BD, then the ratio of the length of BD to the length of the hypotenuse CD is 1 : 2. Knowing that the side ratios of a 30-60-90 triangle are 1 : $\sqrt{3}$: 2, where 1 represents the short leg, $\sqrt{3}$ represents the long leg, and 2 represents the hypotenuse, we can conclude that triangle DBC is a 30-60-90 triangle. Since side BD is the short leg, angle x, the angle opposite the short leg, must be the smallest angle (30 degrees).

(2) SUFFICIENT: If triangle DBC is inscribed in a semicircle, it must be a right triangle. So, angle DBC is 90 degrees. If y = 60, x = 180 – 90 – 60 = 30.

The correct answer is D.

14.
The radius of the small circle is 2. Calculating the area gives:

Area = πr^2
Area small circle = $\pi (2)^2 = 4\pi$.

The radius of the large circle is 5. Calculating the area gives:

Area = πr^2
Area large circle = $\pi (5)^2 = 25\pi$

So, the ratio of the area of the large circle to the area of the small circle is: $25\pi/4\pi = 25/4$

The correct answer is A

15.According to the question, the ratio of the lengths of arc AC to arc AB to arc BC is 6: 2: 1. An inscribed angle cuts off an arc that is twice its measure in degrees. For example, if angle ACB is 30°, minor arc AB is 60° (or 60/360 = 1/6 of the circumference of the circle).

The angles of triangle ABC are all inscribed angles of the circle, so we can deduce the ratio of the angles of triangle ABC from the ratio of the lengths of minor arcs AC, AB and BC. The ratio of the angles will be the same as the ratios of the arcs.

The ratio of angle ABC to angle BCA to angle BAC is also 6: 2: 1. Using the unknown multiplier strategy, whereby a variable can be multiplied by each segment of a ratio to define the actual values described by that ratio, we might say that the interior angles equal 6x: 2x: x. The sum of the angles in a triangle is 180, so x + 2x + 6x = 180.
9x = 180
x = 20
Angle BCA = 2x or 40

The correct answer is B

Top 1% expert replies to student queries (can skip) (additional)

We should solve questions as follows:

6x + 2x + x = 2*pi*R, (since the sum of the lengths of the 3 arcs is the circumference of the circle)

Now, angle BCA is the angle subtended by arc AB at the circumference of the circle.

2(Angle subtended by arc AB at the circumference of the circle) = (Angle subtended by arc AB at the center of the circle) [We know this property]

Now, the length of AB = 2x.

Fraction of circumference occupied by the arc AB = 2x/9x = 2/9

Angle subtended by arc AB at the center of the circle = 2/9 * 360 = 80 degrees

So, angle subtended by arc AB at the circumference of the circle = 80/2 = 40 degrees
The correct answer is B

16.
We are given a diagram of a circle with point O in the interior and points P and Q on the circle, but are not given any additional information. We are asked to find the value of the radius.
(1) INSUFFICIENT: This statement tells us the length of arc PQ but we are not told what portion of the overall circumference this represents. Although angle POQ looks like it is 90 degrees, we are not given this information and we cannot assume anything on data sufficiency; the angle could just as easily be 89 degrees. (And, in fact, we're not even told that O is in the center of the circle; if it is not, then we cannot use the degree measure to calculate anything.)
(2) INSUFFICIENT: Although we now know that O is the center of the circle, we have no information about any actual values for the circle.
(1) AND (2) INSUFFICIENT: Statement 2 corrected one of the problems we discovered while examining statement 1: we know that O is the center of the circle. However, we still do not know the measure of angle POQ. Without it, we cannot determine what portion of the overall circumference is represented by arc PQ.
The correct answer is E

17.

The circumference of the circle is $4\sqrt{\Pi\sqrt{3}}$.
We can use this information to find the area of the circular base.

$$Circumference = 2\pi r$$

$$4\sqrt{\pi\sqrt{3}} = 2\pi r$$

$$\left(4\sqrt{\pi\sqrt{3}}\right)^2 = (2\pi r)^2$$

$$16\pi\sqrt{3} = 4\pi^2 r^2$$

$$r^2 = \frac{4\sqrt{3}}{\pi}$$

$$Area = \pi r^2$$

$$A = \pi\left(\frac{4\sqrt{3}}{\pi}\right)$$

$$A = 4\sqrt{3}$$

Because the probability of the grain of sand landing outside the triangle is 3/4 , the triangle must comprise 1/4 of the area of the circular base.

The height of an equilateral triangle splits the triangle into two 30-60-90 triangles (Each 30-60-90 triangle has sides in the ratio of 1: $\sqrt{3}$: 2). Because of this, the area for an equilateral triangle can be expressed in terms of one side. If we call the side of the equilateral triangle, s, the height must be (s $\sqrt{3}$) / 2 (using the 30-60-90 relationships).

The area of a triangle = 1/2 × base × height, so the area of an equilateral triangle can be expressed as: 1/2 × s × (s $\sqrt{3}$) / 2.

Here the triangle has an area of $\sqrt{3}$, so: $\sqrt{3}$ = 1/2 × s × (s$\sqrt{3}$) / 2
s = 2

The correct answer is E.

18.
Given that line CD is parallel to the diameter, we know that angle DCB and angle CBA are equal.
Thus x = 30°.
First, let's calculate the length of arc CAE. Since arc CAE corresponds to an inscribed angle of 60° (2x = 2×30° = 60°), it must correspond to a central angle of 120° which is 1/3 of the 360° of the circle.
Thus we can take 1/3 of the circumference to give us the arc length CAE.
The circumference is given as 2π*r, where r is the radius. Thus the circumference equals 10π and arc length CAE equals (10/3)π.
Now we need to calculate the length of CB and BE. Since they have the same angle measure, these lengths are equal so we can just find one length and double it. Let us find the length of CB. If we draw a line from A to C we have a right triangle because any inscribed triangle that includes the diameter is a right triangle.
Also, we know that x = 30° so we have a 30-60-90

triangle. The proportions of the length of the sides of a 30-60-90 triangle are 1- $\sqrt{3}$ -2 for the side opposite each respective angle. We know the hypotenuse is the diameter which is 2*r = 10. So length AC must equal 5 and length CB must equal 5$\sqrt{3}$.

Putting this all together gives us (10/3) π + 2 × 5$\sqrt{3}$= (10/3)π + 10$\sqrt{3}$.

The correct answer is D.

19.

The area of the large circle is πr^2
The area of the large circle is $\pi(r-s)^2$
Then the area of the shade region: $\pi r^2-(\pi r^2-2\pi rs+\pi s^2) = 2\pi rs-\pi s^2$

The correct answer is E.

20.
The area of the circle is πr^2
The wire for the square is $40-2\pi r$, so, the side length of the square is $(40-2\pi r)/4 = (10- 1/2\ \pi r)$
area of the square is $(10- 1/2\ \pi r)^2$
Total area is $\pi r^2+(10- 1/2\ \pi r)^2$

The correct answer is E.

GMAT Quant Topic 5: Geometry
Part E: Polygons

1.
The formula for the sum of the interior angles of a non-convex polygon is (n – 2)(180), where n represents the number of sides. To find the sum of the interior angles of the polygon then, we need to know the number of sides. We can therefore rephrase the question:

How many sides does the game board have?

(1) INSUFFICIENT: It tells us nothing about the number of sides. The sum of the exterior angles for any non-convex polygon is 360.

(2) SUFFICIENT: The sum of the exterior angles = 5 × length of each spoke × number of spokes.

$360 = 5(8)(x)$
$360 = 40x$
$9 = x$

The game board has nine sides. The sum of its interior angles is (9 – 2)(180) = 1260.

The correct answer is B.

2.
We are told that the smallest angle measures 136 degrees--this is the first term in the consecutive set. If the polygon has S sides, then the largest angle--the last term in the consecutive set--will be (S - 1) more than 136 degrees.

The sum of consecutive integers =

$$(\text{Average Term}) \times (\#\text{ of Terms}) = \frac{\text{First} + \text{Last}}{2} \times (\#\text{ of Terms})$$.

Given that there are S terms in the set, we can plug in for the first and last term as follows:

$\frac{(136) + (136 + (S - 1))}{2} \times (S)$ = sum of the angles in the polygon.

We also know that the sum of the angles in a polygon = $180(S - 2)$ where S represents the number of sides.

Therefore: $180(S-2) = \frac{(136 + 136 + (S-1))}{2} \times (S)$. We can solve for S by cross-multiplying and simplifying as follows:

$$2(180)(S-2) = (272 + (S-1)) \times S$$
$$360S - 720 = (271 + S) \times S$$
$$360S - 720 = 271S + S^2$$
$$S^2 - 89S + 720 = 0$$

A look at the answer choices tells you to try (S - 8), (S - 9), or (S - 10) in factoring.
As it turns out $(S-9)(S-80) = 0$, which means S can be 9 or 80. However S cannot be 80 because this creates a polygon with angles greater than 180.

Therefore S equals 9; there are 9 sides in the polygon.

The correct answer is B.

Top 1% expert replies to student queries (can skip)

Sum of interior angles for n sides: (n-2)*180
And each interior angle is increasing by 1.
This can be written as : 136+137+138+139+... = 136+(136+1)+(136+2)+(136+3)+... (136+(n-1))
smallest: a=136; largest: a+(n-1)
sum consec integers: avg (smallest,largest) * n
sum interior angles polygon: 180(n-2)
For n sides
{[a+a+n-1]/2}*n=180(n-2)
{[136+136+n-1]/2}*n=180(n-2)
n^2-89n+720=0
factors of (720)=720*1 or 360*2 or 180*4 or 90*8 or **80*9**
(n-80)(n-9)=0
n=9
ans (B)

3.
Recall that the sum of the interior angles of a polygon is computed according to the following formula: 180(n – 2), where n represents the number of sides in the polygon. Let's use this formula to find the values of x and y:

x = the sum of interior angles of a regular hexagon = 180(6 – 2) = 720 y = the sum of interior angles of a regular pentagon = 180(5 – 2) = 540 x – y = 720 – 540 = 180

Thus, the value of (x – y), i.e.180, is equal to the sum of interior angles of a triangle.

The correct answer is A.

4. The relationship between the number of sides in a polygon and the sum of the interior angles is given by 180(n – 2) = (sum of interior angles), where n is the number of sides. Thus, if we know the sum of the interior angles, we can determine the number of sides. We can rephrase the question as follows: "What is the sum of the interior angles of Polygon X?"

(1) SUFFICIENT: Using the relationship 180(n – 2) = (sum of interior angles), we could calculate the sum of the interior angles for all the polygons that have fewer than 9 sides. Just the first two are shown below; it would take too long to calculate all of the possibilities:

polygon with 3 sides: 180(3 – 2) or 180 × 1 = 180
polygon with 4 sides: 180(4 – 2) or 180 × 2 = 360

Notice that each interior angle sum is a multiple of 180. Statement 1 tells us that the sum of the interior angles is divisible by 16. We can see from the above that each possible sum will consist of 180 multiplied by some integer.

The prime factorization of 180 is (2 × 2 × 3 × 3 × 5). The prime factorization of 16 is (2 × 2 × 2 × 2). Therefore, two of the 2's that make up 16 can come from the 180, but the other two 2's will have to come from the integer that is multiplied by 180. Therefore, the difference (n – 2) must be a multiple of 2 × 2, or 4. Our possibilities for (n – 2) are:

3 sides: 180(3 – 2) or 180 × 1
4 sides: 180(4 – 2) or 180 × 2

5 sides: 180(5 – 2) or 180 × 3
6 sides: 180(6 – 2) or 180 × 4
7 sides: 180(7 – 2) or 180 × 5
8 sides: 180(8 – 2) or 180 × 6

Only the polygon with 6 sides has a difference (n – 2) that is a multiple of 4.

(2) INSUFFICIENT: Statement (2) tells us that the sum of the interior angles of Polygon X is divisible by 15. Therefore, the prime factorization of the sum of the interior angles will include 3× 5. Following the same procedure as above, we realize that both 3 and 5 are included in the prime factorization of 180. As a result, every one of the possibilities can be divided by 15 regardless of the number of sides.

The correct answer is A.

Top 1% expert replies to student queries (can skip)

Quicker approach:
Sum of inner angles of polygon=180*(x-2), where x is # of sides. Given x<9. Question x=?
(1) The sum of the interior angles of Polygon X is divisible by 16 --> 180*(x-2)=16k --> 45(x-2)=4k --> x-2 must be a multiple of 4 (as 45 is not) --> since x<9 then the only acceptable value of x is 6. Sufficient.
(2) The sum of the interior angles of Polygon X is divisible by 15 --> 180*(x-2)=15m --> 12(x-2)=m --> x can be any integer from 3 to 8, inclusive. Not sufficient. (We could even not consider this statement at all: as the sum of inner angles of a polygon is 180*(x-2) and 180 is a multiple of 15, then all polygons will have the sum of the interior angles divisible by 15.)
Answer: A.

5. Shaded area = Area of the hexagon – (area of circle O) – (portion of circles A, B, C, D, E, F that is in the hexagon)

With a perimeter of 36, the hexagon has a side that measures 6. The regular hexagon is comprised of six identical equilateral triangles, each with a side measuring 6. We can find the area of the hexagon by finding the area of the equilateral triangles.

The height of an equilateral triangle splits the triangle into two 30-60-90 triangles (Each 30-60-90 triangle has sides in the ratio of 1:$\sqrt{3}$: 2). Because of this, the area for an equilateral triangle can be expressed in terms of one side. If we call the side of the equilateral triangle, s, the height must be (s $\sqrt{3}$) / 2 (using the 30-60-90 relationships).

The area of a triangle = 1/2 × base × height, so the area of an equilateral triangle can be expressed as: 1/2 × s × (s $\sqrt{3}$) / 2 = 1/2 × 6 × (3$\sqrt{3}$) = 9.

Area of hexagon ABCDEF = 6 × 9 $\sqrt{3}$ = 54 $\sqrt{3}$.
For circles A, B, C, D, E, and F to have centers on the vertices of the hexagon and to be tangent to one another, the circles must be the same size. Their radii must be equal to half of the side of the hexagon, 3. For circle O to be tangent to the other six circles, it too must have a radius of 3.

Area of circle O =πr^2 = 9π. To find the portion of circles A, B, C, D, E, and F that is inside the hexagon, we must consider the angles of the regular hexagon. A regular hexagon has external angles of 360/6 = 60°, so it has internal angles of 180 – 60 = 120°. This means that each circle has 120/360 or 1/3 of its area inside the hexagon.

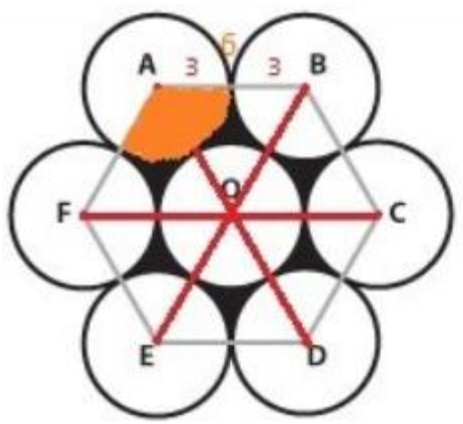
The area of circles A, B, C, D, E, and F inside the hexagon = 1/3(9π) × 6 circles = 18π.
Thus, the shaded area = 54 $\sqrt{3}$ – 9π– 18π = 54 $\sqrt{3}$ - 27π.

The correct answer is E.

Top 1% expert replies to student queries (can skip)

Area of hexagon = 6 * area of a triangle (As all triangles are equilateral with the same side(as all the angles of regular hexagon = 120 degrees, internal angles of the triangle is 60 degrees)).
= 6∗√(3/4) ∗62
= 54√3

Area of orange part in the diagram:
pie∗ r^2 ∗120/360= pie ∗9*(1/3)
3∗pie
Total of 6 areas like this + the central circle = [6*(3∗pie)] + [9∗pie]=27∗pie



Now area of black shaded region = area point 1 - area point 2.
= (54√3)- (27∗pie) -->Answer E.

GMAT Quant Topic 5: Geometry
Part F: General Solids (Cube, Box, Sphere)

1.
The question stem tells us that the four spheres and three cubes are arranged in order of increasing volume, with no two solids of the same type adjacent in the line-up. This allows only one possible arrangement: sphere, cube, sphere, cube, sphere, cube, sphere.

Then we are told that the ratio of one solid to the next in line is constant. This means that to find the volume of any solid after the first, one must multiply the volume of the previous solid by a constant value. If, for example, the volume of the smallest sphere were 2, the volume of the first cube (the next solid in line) would be 2x, where x is the constant. The volume of the second sphere (the third solid in line) would be 2(x)(x) and the volume of the second cube (the fourth solid in line) would then be 2(x)(x)(x), and so on. By the time we got to the largest sphere, the volume would be $2x^6$.

We are not given the value of the constant, but are told that the radius of the smallest sphere is 1/4 that of the largest. We can use this information to determine the value of the constant. First, if the radius of the smallest sphere is r, then the radius of the largest sphere must be 4r. So if the volume of the smallest sphere is $\frac{4}{3}\pi r^3$, then the volume of the largest sphere must be $\frac{4}{3}\pi(4r)^3$ or $\frac{4}{3}\pi 64r^3$. So the volume of the largest sphere is 64 times larger than that of the smallest.

Using the information about the constant from the question stem, we can set up and simplify the following equation:

$$\frac{4}{3}\pi 64r^3 = \frac{4}{3}\pi r^3 x^6 \rightarrow$$
$$64 = x^6 \rightarrow$$
$$2 = x$$

Therefore, the value of the constant is 2. This means that the volume of each successive solid is twice that of the preceding solid. We are ready to look at the statements.

Statement (1) tells us that the volume of the smallest cube is 72π. This means that the volume of the smallest sphere (the immediately preceding solid) must be half, or 36π. If we have the volume of the smallest sphere, we can find the radius of the smallest sphere:

$$36\pi = \frac{4}{3}\pi r^3 \rightarrow$$
$$\left(\frac{3}{4}\right)36\pi = \left(\frac{3}{4}\right)\frac{4}{3}\pi r^3 \rightarrow$$
$$27\pi = \pi r^3 \rightarrow$$
$$27 = r^3 \rightarrow$$
$$3 = r$$

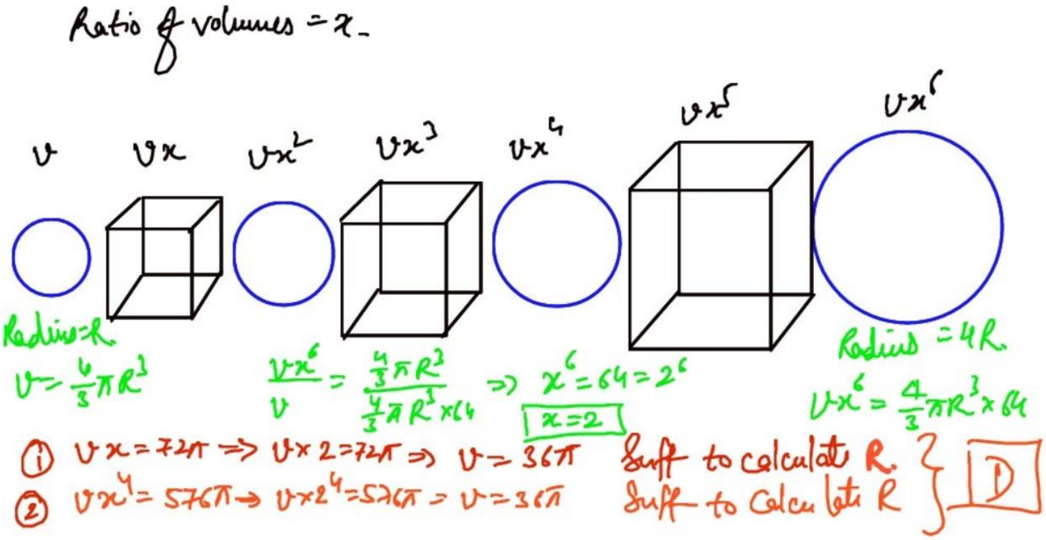
Therefore, the radius of the smallest sphere is 3. Statement (1) is sufficient.

Statement (2) tells us that the volume of the second largest sphere is 576π. Applying the same logic that we used to evaluate statement (1), we can find the radius of the smallest sphere by Dividing 576π by 2 four times (because there are four solids smaller than the second largest sphere) until we get to the volume of the smallest sphere: 36π. At this point we can solve for the radius of the smallest sphere as we did in our analysis of statement (1) above. Statement (2) is sufficient.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

Look at the diagram attached below for more clarity:



Top 1% expert replies to student queries (can skip) (additional)

Query: Can we directly interpret in such kind of stem: radius of smallest sphere is given + volume is mentioned so without calculating D is correct answer?

Reply: In this case, this method would work. Just a couple of clarifications.

We have 4 spheres and 3 cubes. No two solids can be adjacent. So, the only possible arrangement is:

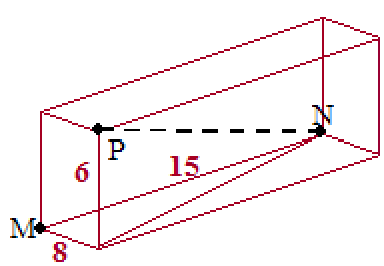
S1 C1 S2 C2 S3 C3 S4

Since we know this order, any information about the volume of any one of the seven figures is sufficient for us to calculate the radius of the smallest sphere.

So, the answer is D.

2.

The diagram represents the situation described in the problem.



The plane (P) is at an altitude of 6 miles and is flying due north towards The Airport (M). Notice that if the plane flies in a straight line toward The Airport it would be flying along the diagonal of a right triangle with sides of length 6 and 8. Thus, taking a direct approach, the plane would fly exactly 10 miles towards The Airport (a 6-8-10 right triangle).

However, the control tower instructs the plane to fly toward a new airport (N), which lies 15 miles due east of The Airport. If the plane flies in a straight line towards the new airport, it will be flying along the diagonal (the dotted black line) of the rectangular solid. To determine this length, we first need to determine the diagonal of the bottom face of the solid. Using the Pythagorean theorem, we can determine that the bottom face right triangle with sides of lengths 8 and 15 must have a diagonal of 17 miles (an 8-15-17 right triangle).

Now the dotted line diagonal of the solid can be calculated as follows:

$$x^2 = 6^2 + 17^2$$
$$x^2 = 325$$
$$x = 5\sqrt{13}$$

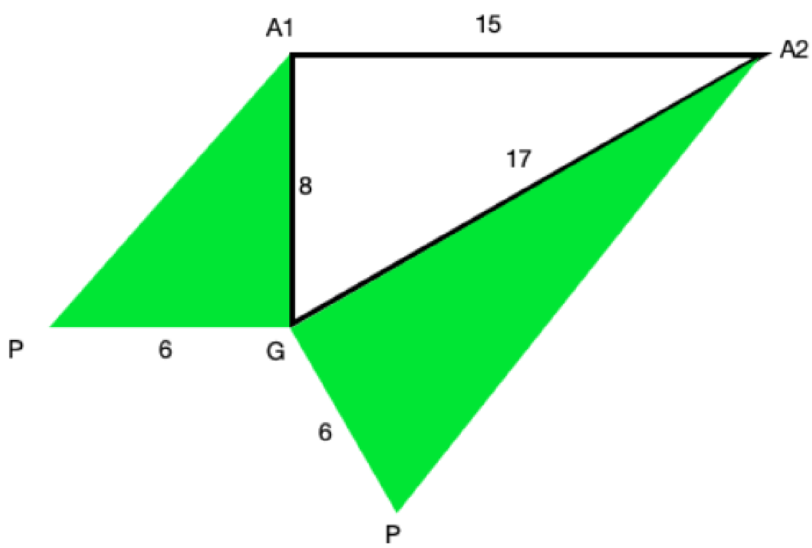
In order to arrive at the new airport at 8:00 am, the pilots flying time must remain 3 minutes, or 1/20 of an hour. The flight distance, however, has increased from 10 miles to $5\sqrt{3}$ miles. The planes rate must increase accordingly. Using the formula that rate = distance/ time, we can calculate the rate increase as follows:

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}} = \frac{(5\sqrt{13} - 10)\text{ miles}}{1/20\text{ hr}} = 100\sqrt{13} - 200 \text{ miles/hr}$$

The correct answer is C.

Top 1% expert replies to student queries (can skip)

There are 3 right triangles here in different planes. On the ground, we have the two airports 15 miles apart (due east) and the plane is 8 miles to the south of A1 (point G). So this is the 8-15-17 triangle.



The points P represent the current location of the plane at height 6. The green triangles are in the air with base on the ground. The plane needs to travel from P to A1 or A2 in the two situations.
P to A1 - Distance is 10
Time taken = 3 mins
Speed= 10/ (3/60) miles/hour =200 miles/hour
P to A2 - Distance is √[(17^2) + (6^2)] = 5√13
Time taken = 3 mins
Speed=5√13 / (3/60) = 100√13 miles/hour
Increase = (100√13 - 200) miles/hour

Answer (C)

3.
The question asks for the ratio between a cube and a rectangular solid identical to the cube except that its length has been doubled. Since the original cube is smaller the ratio of their surface areas will be less than 1.

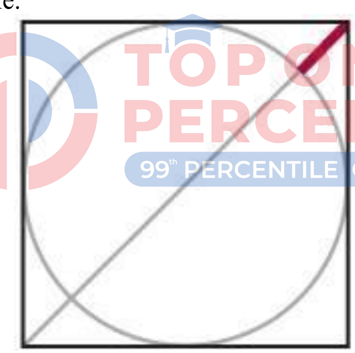
First let's calculate the surface area of the original cube. Since lengths were not specified, it's easiest to assign the cube a side length of 1. Each side then has a surface area of 1 square unit, and there are 6 sides giving a total surface area of 6 square units.

Now picture a rectangular solid with the length stretched out to 2 units. The two end caps will still be 1×1 squares having a combined surface area of 2 square units. In addition, there will be 4 side rectangles each having dimensions of 1×2, having a combined surface area of $4 \times 2 = 8$. Therefore the total surface area of the rectangular solid is $2 + 8 = 10$ square units.

The ratio will therefore be 6/10 which reduces to 3/5.

The correct answer is D.

4. The shortest distance from a vertex of the cube to the sphere would be ½ the length of the diagonal of the cube minus the radius of the sphere. To understand why, think of the parallel situation in two dimensions. In the diagram of the circle inscribed in the square to the right, the shortest possible distance from one of the vertices of the square to the circle would be ½ the diagonal of the square minus the radius of the circle.



The diagonal of a cube of side x is $x\sqrt{3}$. This can be found by applying the Pythagorean Theorem twice (first to find the diagonal of a face of the cube, $x\sqrt{2}$, and then to find the diagonal through the center, $x\sqrt{3}$). Like the sides of the circle in the diagram above, the sides of a sphere inscribed in a cube will touch the sides of the cube. Therefore, a sphere inscribed in a cube will have a radius equal to half the length of the side of that cube.

Diagonal of the cube = $x\sqrt{3} = 10\sqrt{3}$
Radius of the sphere = 5
½ diagonal of the cube – radius of the sphere = $5\sqrt{3} - 5 = 5(\sqrt{3} - 1)$

The correct answer is D.

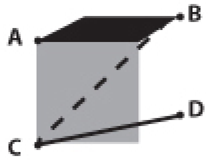
Top 1% expert replies to student queries (can skip)

Let the edge length for the cube be 'a'.
Surface area of the cube = 6a^2

Dimensions of the rectangular solid = 2a * a * a [The solid is identical to the cube, except that its length has been doubled]
Surface area of the solid = 2[(2a)(a) + (a)(a) + (a)(2a)] = 10a^2

Ratio = 6a^2/10^2 = 3/5
The correct answer is D.

5.



Let x be the length of an edge of the cube. We can find the length of BC by first finding the length of CD. CD must be $x\sqrt{2}$ since it is the hypotenuse of a 45-45-90 triangle with legs of length x.

Using the Pythagorean theorem, BC can be calculated: $BC = \sqrt{x^2 + (x\sqrt{2})^2} = x\sqrt{3}$

$AB = CD = x\sqrt{2}$, so $BC - AB = x\sqrt{3} - x\sqrt{2}$.

If we factor this expression and simplify, $x(\sqrt{3} - \sqrt{2}) \approx x(1.7 - 1.4) \approx 0.3x$.

Since $BC - AB \approx 0.3x$ and $AC = x$, the difference between BC and AB is equal to approximately 30% of AC.

The correct answer is C.

GMAT Quant Topic 5: Geometry
Part G: Cylinders

1.
The old volume is πR^2H. Let's look at each answer choice to see which one is farthest away from twice this volume:

(A) a 100% increase to R and a 50% decrease to H:
The new volume = $\pi (2R)^2(.5H) = 2\pi R^2H$ = exactly twice the original volume.

(B) a 30% decrease to R and a 300% increase to H:
The new volume = $\pi (.7R)^2(4H) = (.49)(4)\ \pi R^2H \approx 2\pi R^2H$ = approximately twice the original volume.

(C) a 10% decrease to R and a 150% increase to H:
The new volume = $\pi(.9R)^2(2.5H) = \pi(.81)(2.5)\ R^2H \approx 2\pi R^2H$ = approximately twice the original volume.

(D) a 40% increase to R and no change to H:
The new volume = $\pi(1.4R)^2(H) = (1.96)\pi R^2H \approx 2\pi R^2H$ = approximately twice the original volume.

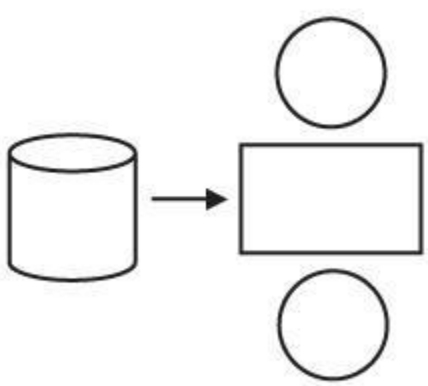
(E) a 50% increase to R and a 20% decrease to H:
The new volume = $\pi(1.5R)^2(.8H) = (2.25)(.8)\pi R^2H = 1.8\pi R^2H$. This is the farthest away from twice the original volume.

The correct answer is E.

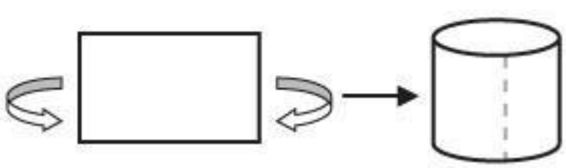
2.
The surface area of a cylinder = $2\pi r^2 + 2\pi rh$, where r = the radius and h = the height.
Surface area of Cylinder A = $2\pi x^2 + 2\pi xy$
Surface area of Cylinder B = $2\pi y^2 + 2\pi xy$
Surface area of Cylinder A – surface area of Cylinder B = $2\pi x^2 + 2\pi xy - (2\pi y^2 + 2\pi xy) = 2\pi(x^2 - y^2)$

The correct answer is D.

3.
The surface of a right circular cylinder can be broken into three pieces as follows:



To understand why the curved side of the cylinder can be represented as a rectangle, imagine taking a sheet of 8.5x11 paper and rolling the sides forward until the edges meet in the front:



The resulting shape is the curved side of a cylinder without the top or bottom. Unrolling this side gives the rectangular surface that becomes one piece of the cylinders surface area.

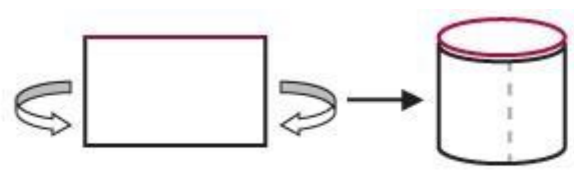
So, the total surface area can be determined by:

SA = Area of circular top + Area of rectangular side + Area of circular bottom Inserting

mathematical expressions for these areas yields

$SA = \pi r^2 + lw + \pi r^2$

where r is the radius of the top and bottom circles, l is the length of the rectangle and w is the width. Notice that l (in red) can be expressed as the circumference of the circle and w (dotted line) can be expressed as the height of the cylinder:



Now the equation becomes:

$SA = \pi r^2 + 2\pi rh + \pi r^2 = 2\pi r^2 + 2\pi rh$

Now that we have an expression for the surface area, we can begin to analyze the question. It might be tempting at this point to decide that, in order to calculate the SA, we would need to know the values of r and h explicitly. However, after some manipulation of the equation (factoring out 2r).

$SA = 2\pi r^2 + 2\pi rh$
$SA = 2\pi r(r + h)$

We can see that knowing the value of r(r + h) will be enough to determine the SA. In this case, we DON'T need to know the values of r and h explicitly. Rather, determining the value of a combination of r and h will be sufficient. So, the rephrased question becomes:

What is the value of r(r + h)?

Statement (1), $r = 2h - 2/h$, cannot be manipulated to isolate r(r +h). We can also see that different values of h we yield different surface areas.
For example:
If h = 2, then r = 3 and the surface area is 30π.
But if h = 4, then r = 7.5 and the surface area is $(172.5)\pi$.

So, statement (1) does not give one and only one value for the surface area. Statement (1) is therefore NOT sufficient.

Statement (2), however can be manipulated as follows:

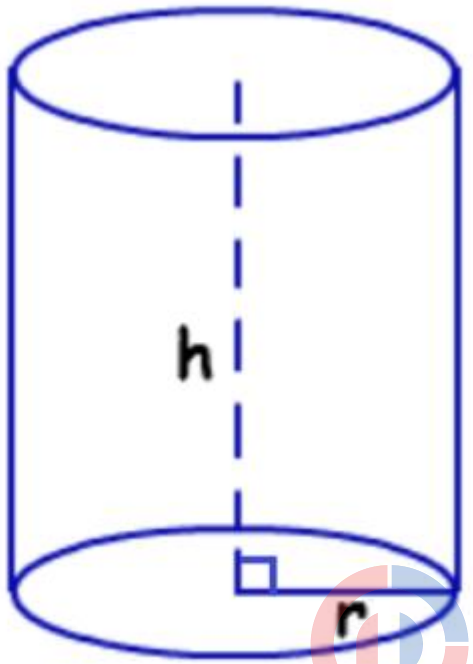
$h = 15/r - r$
(Multiply through by r)
$hr = 15 - r^2$
(Add r^2 to both sides)
$r^2 + hr = 15$
(factor out an r)
$r(r + h) = 15$

Statement (2) is sufficient.

The correct answer is B.

Top 1% expert replies to student queries (can skip)

Surface Area of a Cylinder is given by:



TWO of the Circular Base Areas + Area of Rectangle that wraps around the Cylinder =
2 * (pi * (r)^2) + (2pi * r)(h) =
2pi * [(r)^2 + rh] = ?
or
2pi*r * (r + h) = ?
if we can find either:
(r)^2 + rh
OR
r * (r + h)
the Statements would be Sufficient
S1: r = 2h - (2/h)
r = (2(h)^2 - 2) / (h)
rh = 2(h)^2 - 2
From this point we could try re-arranging and isolating 1 Variable and Substituting, but we will never be able to get the Variables to cancel out such that we will get a Unique Value for the Surface Area
S1 NOT Sufficient
S2: h = (15/r) - r
h + r = 15/r
Plug this equation above into the Surface Area Equation of:
2pi* r * (r + h) = ?
2pi * r * (15/r) = ?
----the Variable of r Cancels Out ----
2pi * 15 = 30pi = Surface Area of Cylinder
S2 is Sufficient Alone
B is correct.

4.
Lets start by calculating the volume of the original cylinder.

$$V_{cylinder} = \pi r^2 h$$
$$= \pi(10)^2 10 = 1000\pi$$

The new conical tank must have a volume of twice that of the cylinder, i.e. 2000π.
How does this relate to the radius and the height of the new cone?

We are given the formula for the volume of a cone $V_{cone} = \frac{1}{3}\pi r^2 h$.
Lets set this equal to the known volume of the new conical tank to find an equation relating the height and radius of the new cone.

$$2000\pi = \frac{1}{3}\pi r^2 h$$
$$6000\pi = \pi r^2 h$$
$$r^2 h = 6000$$

In scenario 1, the radius of the cone must remain 10, with only the height changing:

$(10)^2 h = 6000$
$h = 60$

In scenario 2, the height of the cone must remain 10, with only the radius changing:

$$r^2(10) = 6000$$
$$r^2 = 600$$
$$r \approx 25 \quad (25^2 = 625)$$

The difference between the height of the cone in scenario 1 and the radius of the cone in scenario 2 is 60 – 25 = 35.

The correct answer is D.

5.

To solve this problem, we need to (a) find the volume of the tire and then (b) solve a rate problem to determine how long it will take to inflate the tire.

To find the volume of just the tire, we can find the volume of the entire object and then subtract out the volume of the hub. In order to do this, we will first need to determine the radius of the hub.

If the radius of the hub is r, then its area equals πr^2

The area of the entire object is then $\pi(r+6)^2$.

This means that the area of just the tire equals $\pi(r+6)^2 - \pi r^2$.

The problem also tells us that the ratio of the area of the tire to the area of the entire object is 1/3. We can use this information to set up the following equation:

$$\frac{\text{area of hub}}{\text{area of tire}} = \frac{1}{3} = \frac{\pi r^2}{\pi(r+6)^2 - \pi r^2}$$

Now, we can solve for r as follows:

$$\frac{1}{3} = \frac{\pi r^2}{\pi(r+6)^2 - \pi r^2}$$

$$\frac{1}{3} = \frac{r^2}{(r+6)^2 - r^2}$$

$$3r^2 = (r^2 + 12r + 36) - r^2$$

$$3r^2 = 12r + 36$$

$$r^2 = 4r + 36$$

$$r^2 - 4r - 12 = 0$$

$$(r-6)(r+2) = 0$$

Thus, r is either 6 or –2. Since the radius must be positive, we know that r = 6.

To calculate volume, we simply multiply the area by a third dimension. This third dimension is the thickness of the tire (we'll call this h), which is defined in the problem as 3 inches when the tire is fully inflated. Now, we can determine the volume of the tire using the following:

Volume of Entire Object – Volume of Hub = Volume of Tire

$$h\pi(r+6)^2 - h\pi r^2 = \text{Volume of Tire}$$

This can be simplified as followed:

$$= h\pi(6+6)^2 - h\pi 6^2$$

$$= 3\pi 12^2 - 3\pi 36$$

$$= 432\pi - 108\pi$$

$$= 324\pi$$

Thus, the volume of the tire is 324π cubic inches. Using the formula work = rate × time, we can set the volume of the tire as the total work to be done and use the rate given in the problem to determine the total time required to inflate the tire.

$$\text{Work} = \text{Rate} \times \text{Time}$$

$$324\pi in^3 = \frac{4\pi in^3}{\text{second}} \times t$$

$$t = 81 \text{ seconds}$$

The correct answer is D.

Top 1% expert replies to student queries (can skip)

If the radius of the hub is r, then its area equals πr^2

The area of the entire object is then $\pi(r+6)^2$

This means that the area of just the tyre equals $\pi(r+6)^2 - \pi r^2$

The problem also tells us that the ratio of the area of the tyre to the area of the entire object is $\frac{1}{3}$. We can use this information to set up the following equation:

$$\frac{(Area of Hub)}{(Area of Tyre)} = \frac{1}{3} = \frac{(\pi r^2)}{(\pi(r+6)^2 - \pi r^2)}$$

Which boils down to $r^2 - 4r - 12 = 0$.

So $(r-6)(r+2) = 0$

So $r = 6$ or $r = -2$

This will give $r = 6$ as radius can never be negative.

So Total Radius $= 6 + 6 = 12$ inches
So Total Area $= \pi r^2 = \pi 12 * 12 = 144\pi$
So Area of Hub $= \pi r^2 = \pi 6 * 6 = 36\pi$
So Area of Just tyre $= 144\pi - 36\pi = 108\pi$
Thickness of Tyre When inflated $= 3$ inches
So Total Volume of Trye in $inches^3 = 3 * 108\pi = 324\pi$

Air is filled at a rate of 4π
So total time $= \frac{324\pi}{4\pi} = 81$ seconds

The correct answer is D.

6.
The volume for a cylinder can be calculated by multiplying the area of the base times the height. The base is a circle with an area of r^2, where r is the radius of the circle. Thus, the volume of a cylinder is $r^2 \times h$, where h represents the height of the figure.

(1) SUFFICIENT: If we call the radius of the smaller cylinder r, and the height of the smaller cylinder h, the volume of the smaller cylinder would be r^2h. If the radius of the larger cylinder is twice that of the smaller one, as is the height, the volume of the larger cylinder would be $(2r)^2(2h) = 8r^2h$. The volume of the larger cylinder is eight times larger than that of the smaller one. If the contents of the smaller silo, which is full, are poured into the larger one, the larger one will be 1/8 full.

(2) INSUFFICENT: This question is about proportions, and this statement tells us nothing about volume of the smaller silo relative to the larger one.

The correct answer is A

7.
Since water is filling the tank at a rate of 22 cubic meters per hour, after one hour there will be a "cylinder of water" in the tank (smaller than the tank itself) with a volume of $22m^3$. Since the water level rises at 0.7 meters/hour, this "cylinder of water" will have a height of 0.7 meters. The radius of this "cylinder of water" will be the same as the radius of the cylindrical tank.

$Volume_{cylinder} = \pi r^2 h$

$22 = \pi r^2(0.7)$ (use $\pi \approx 22/7$)

$22 \approx 22/7\ (7/10)\ r^2$

$r^2 \approx 10$
$r \approx \sqrt{10}$
The correct answer is B.

8. One of the cylinders has a height of 6 and a base circumference of 10; the other has a height of 10 and a base circumference of 6.

The cylinder with a height of 6 and a base circumference of 10 has a radius of $(5/\pi)$. Its volume is equal to $\pi r^2 h$, or $\pi(5/\pi)^2(6)$ or $150/\pi$.

The cylinder with a height of 10 and a base circumference of 6, however, has a radius of $(3/\pi)$. Its volume is equal to $\pi r^2 h$, or $(3/\pi)^2(10)$ or $90/\pi$.

We can see that the volume of the cylinder with a height of 6 is $60/\pi$ inches greater than that of the cylinder with a height of 10. It makes sense in this case that the cylinder with the greater radius will have the greater volume since the radius is squared in the volume formula.

The correct answer is B.

GMAT Quant Topic 6: Co-ordinate Geometry

1. Firstly, we assume that a*b>0. Let a=1, b=2, then (-a,b)=(-1,2), (-b, a)=(-2,1), the two point are in the second quadrant. From 2), ax>0, x and a are both positive or both negative, as well as the -x and –a. From 1), xy>0, x and y are both positive or both negative, while -x and y are different. Above all, point (-a,b) and (-x, y) are in the same quadrant. Then we assume that a*b<0. Let a-1, b2, then (-a,b)=1,2), (-b,a)=(-2,-1), in different quadrant. This is conflict to the question, need no discussion.
   Answer is C

Top 1% expert replies to student queries (can skip)

ab != 0, then any of a and b are just not individually 0, they can be positive or negative.

a is positive, b is positive: -a is negative, -b is negative; (-a, b) is in quadrant II, (-b, a) is in quadrant II - Possible
a is positive, b is negative -a is negative, -b is positive; (-a, b) is in quadrant III, (-b, a) is in quadrant I - Not possible
a is negative, b is positive: -a is positive, -b is negative; (-a, b) is in quadrant I, (-b, a) is in quadrant III - Not possible
a is negative, b is negative: -a is positive, -b is positive; (-a, b) is in quadrant IV, (-b, a) is in quadrant IV - Possible

So a and b are either both positive (and the quadrant we are considering is quadrant II) or they are both negative (and the quadrant we are considering is quadrant IV)

Statement 1 - xy > 0 => x and y are individually both positive or both negative. If they are both positive, (-x, y) is in quadrant II, if they are both negative, (-x, y) is in quadrant IV. But let's say (a, b) are both positive, then the first bold line above is true (i.e. (-a, b) and (-b, a) both lie in quadrant II) and let's say at the same time (x, y) are both negative (i.e. (-x, y) lies in quadrant IV). Or let's say (a, b) are bth negative, then the second bold line above is true (i.e. (-a, b) and (-b, a) both lie in quadrant IV) and let's say at the same time (x, y) are both positive (i.e. (-x, y) lies in quadrant II) In these two combinations, nothing in the stem or statement 1 is violated, yet we cannot say for sure (-x, y) lies in the same quadrant as (-a, b) and (-b, a). This statement by itself is not sufficient

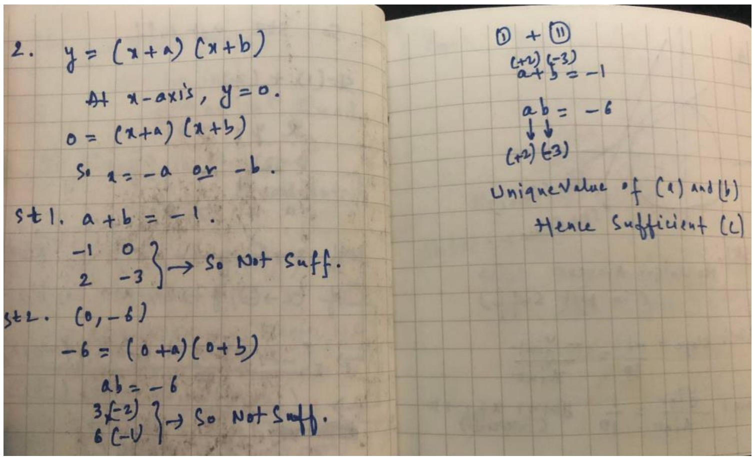
Statement 2 - ax > 0. However, we know nothing about y. This statement by itself is not sufficient

Combining the two statements - ax > 0, implies a and x are both positive or are both negative. Let's say they are both positive. From bold line 1 above, we see if a is positive, the quadrant we are considering is quadrant II. Also, x is positive, then y is also positive (-x, y) is in quadrant II both from Statement 1 above). Similarly, if both a and x are negative, you can trace the exact same logic path to come to the fact that all points are not in quadrant IV. The two statements together are sufficient

2. From 1, a+b=-1. From 2, x=0, so ab=6. (x+a)*(x+b)=0x^2+(a+b)x+ab=0

So, x=-3, x=2 **The answer is C.**

Top 1% expert replies to student queries (can skip)



2. $y=(x+a)(x+b)$

At x-axis, $y=0$.

$0=(x+a)(x+b)$

So $x=-a$ or $-b$.

St1. $a+b=-1$.

$\left.\begin{matrix}-1 & 0\\ 2 & -3\end{matrix}\right\} \rightarrow$ So Not Suff.

St2. $(0,-6)$

$-6=(0+a)(0+b)$

$ab=-6$

$\left.\begin{matrix}3(-2)\\ 6(-1)\end{matrix}\right\} \rightarrow$ So Not Suff.

(I) + (II)

$(+2)(-3)$

$a+b=-1$

$ab=-6$

$\downarrow\downarrow$

$(+2)(-3)$

Unique Value of (a) and (b)

Hence Sufficient (C)

Top 1% expert replies to student queries (can skip) (additional)

y=(x+a)(x+b)=x^2+x(a+b)+ab

For the graph to intersect x-axis we have to find the value of x at y=0.
So we need to know a+b and ab

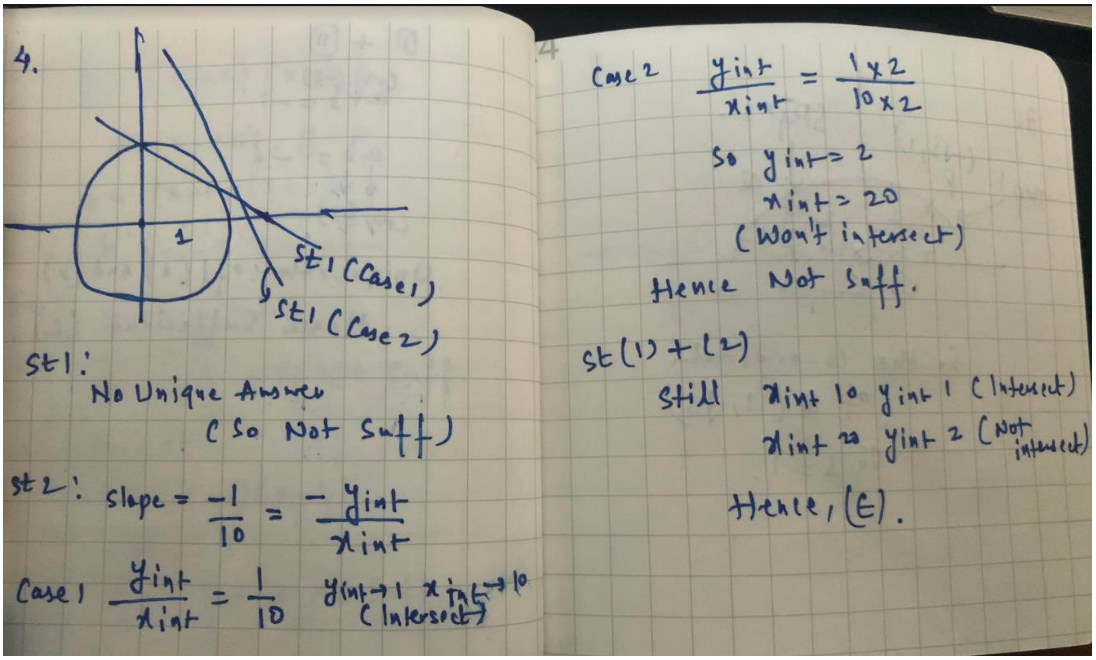
1)Gives a+b
2)We can substitute for x,y and get
-6=(a)(b)
ab=-6

So combining these two we can now solve the quadratic .
So **The answer is C.**

3. (0+6+x)/3=3,x=3 (0+0+y)/3=2,y=6 **Answer is B**

4. Just image that, when we let the x-intercept great enough, the line k would not intersect circle c, even the absolute value of its slope is very little.
Answer is E

Top 1% expert replies to student queries (can skip)


4.

$St1$: No Unique Answer (So Not Suff)

$St\,2$: Slope $= \frac{-1}{10} = -\frac{y_{int}}{x_{int}}$

Case 1 $\frac{y_{int}}{x_{int}} = \frac{1}{10}$ $y_{int} \to 1$ $x_{int} \to 10$ (Intersect)

Case 2 $\frac{y_{int}}{x_{int}} = \frac{1\times 2}{10\times 2}$

So $y_{int} = 2$

$x_{int} = 20$

(Won't intersect)

Hence Not Suff.

$St(1)+(2)$

Still x_{int} 10 y_{int} 1 (Intersect)

x_{int} 20 y_{int} 2 (Not intersect)

Hence, (E).

5. We need to know whether $r^2+s^2=u^2+v^2$ or not. From statement 2,

$u^2+v^2=(1-r)^2+(1-s)^2=r^2+s^2+2-2(r+s)$

Combined statement 1, $r+s=1$, we can obtain that $r^2+s^2=u^2+v^2$.

Answer is C.

6. $y = kx + b$ 1). $k=3b$ 2). $-b/k=-1/3 \Rightarrow k=3b$

So, answer is E

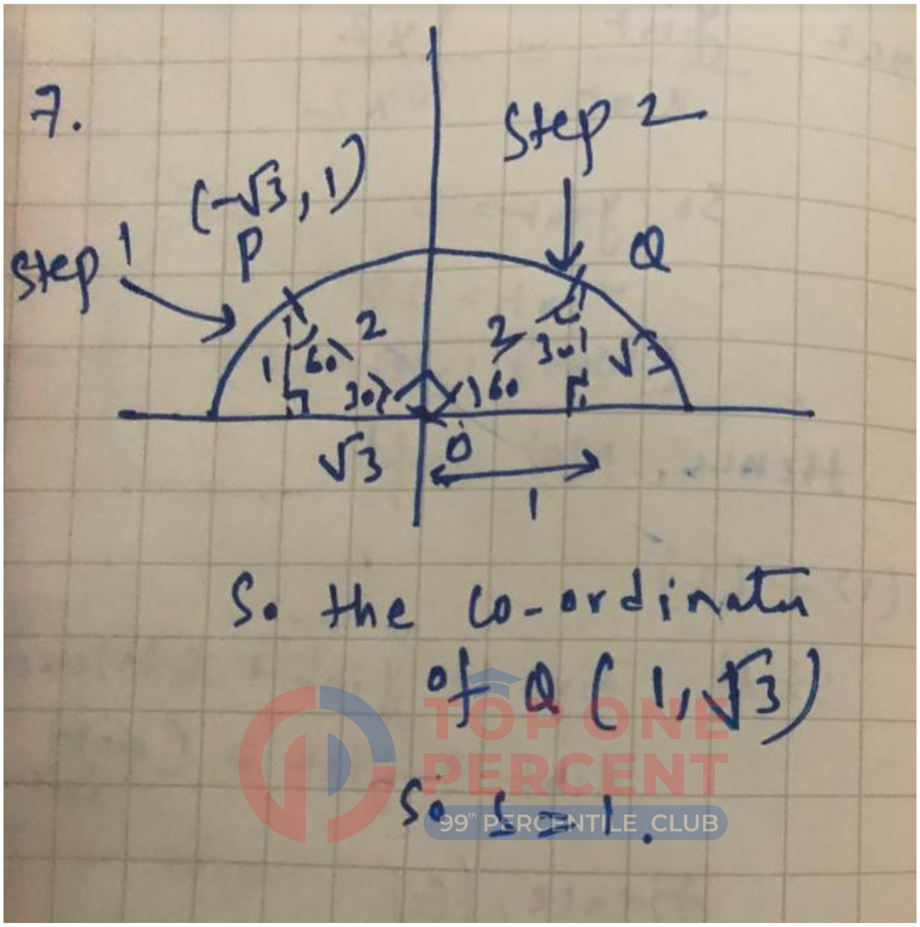
7. Slope of line OP is $-1/\sqrt{3}$ and slope of OQ is t/s, so$(t/s) * (-1/\sqrt{3})=-1$, $t=\sqrt{3}*s$

$OQ=OP=2$, $t^2+s^2=4$.

Combined above, $s=+/-1 \Rightarrow s=1$

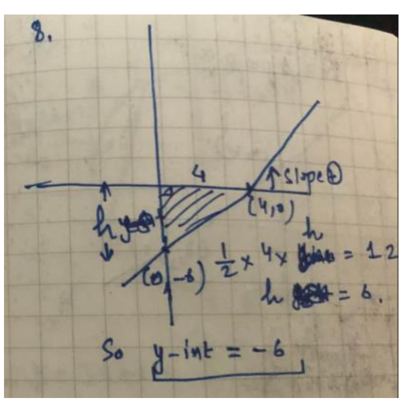
Answer is B

Top 1% expert replies to student queries (can skip)



8. The two intersections: (0,4) and (y, 0)
So, 4 * y/ 2 = 12 => y= 6
Slope is positive => y is below the x-axis => y = -6
Answer is A

Top 1% expert replies to student queries (can skip)



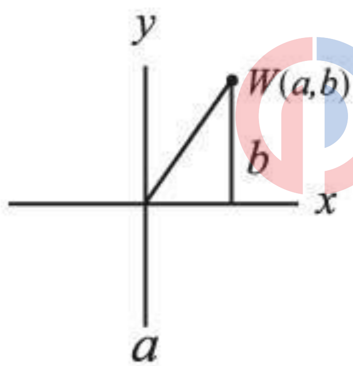
9. First, lets rewrite both equations in the standard form of the equation of a line:
Equation of line *l*: $y = 5x + 4$
Equation of line *w*: $y = -(1/5)x - 2$

Note that the slope of line *w*, -1/5, is the negative reciprocal of the slope of line *l*. Therefore, we can conclude that line *w* is perpendicular to line *l*.

Next, since line *k* does not intersect line *l*, lines *k* and *l* must be parallel. Since line *w* is perpendicular to line *l*, it must also be perpendicular to line *k*. Therefore, lines *k* and *w* must form a right angle, and its degree measure is equal to 90 degrees.

The correct answer is D.

10. To find the distance from the origin to any point in the coordinate plane, we take the square root of the sum of the squares of the point's *x*- and *y*-coordinates. So, for example, the distance from the origin to point *W* is the square root of $(a^2 + b^2)$. This is because the distance from the origin to any point can be thought of as the hypotenuse of a right triangle with legs whose lengths have the same values as the *x*- and *y*-coordinates of the point itself:



We can use the Pythagorean Theorem to determine that $a^2 + b^2 = p^2$, where p is the length of the hypotenuse from the origin to point *W*.

We are also told in the question that $a^2 + b^2 = c^2 + d^2$, therefore point *X* and point *W* are equidistant from the origin. And since $e^2 + f^2 = g^2 + h^2$, we know that point *Y* and point *Z* are also equidistant from the origin.

If the distance from the origin is the same for points *W* and *X*, and for points *Z* and *Y*, then the length of *WY* must be the same as the length of *XZ*. Therefore, the value of length *XZ* – length *WY* must be 0.

The correct answer is C.

11. The question asks us to find the slope of the line that goes through the origin and is equidistant from the two points *P*=(1, 11) and *Q*=(7, 7). It's given that the origin is one point on the requested line, so if we can find another point known to be on the line we can calculate its slope. Incredibly the midpoint of the line segment between *P* and *Q* is also on the requested line, so all we have to do is calculate the midpoint between *P* and *Q*! (This proof is given below).

Let's call *R* the midpoint of the line segment between *P* and *Q*. *R*'s coordinates will just be the respective average of *P*'s and *Q*'s coordinates. Therefore *R*'s *x*-coordinate equals 4, the average of 1 and 7. Its *y*-coordinate equals 9, the average of 11 and 7. So *R*=(4, 9).

Finally, the slope from the (0, 0) to (4, 9) equals 9/4, which equals 2.25 in decimal form.

Proof

To show that the midpoint *R* is on the line through the origin that's equidistant from two points *P* and *Q*, draw a line segment from *P* to *Q* and mark *R* at its midpoint. Since *R* is the midpoint then *PR* = *RQ*.

Now draw a line *L* that goes through the origin and *R*. Finally draw a perpendicular from each of *P* and *Q* to the line *L*. The two triangles so formed are congruent, since they have three equal angles and *PR* equals *RQ*. Since the triangles are congruent their perpendicular distances to the line are equal, so line *L* is equidistant from *P* and *Q*.

The correct answer is B.

Top 1% expert replies to student queries (can skip)

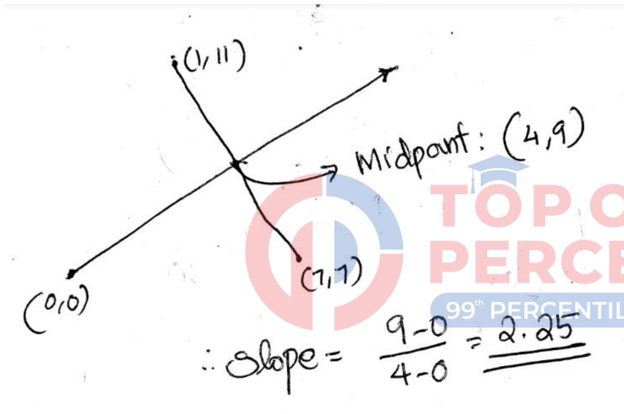
There are two ways 2 points can be equidistant from a line:
Case 1) Either a line that is equidistant from the two points must be passing from the gap between the two points. In this case every point on the line will be equidistant from each of the two given points. (The line passes through the mid point of the two given points)
Since the line is equidistant from P = (1, 11) and Q = (7, 7), it must pass through the midpoint between P = (1, 11) and Q = (7, 7). We can use the midpoint formula:
Midpoint = ((x1 + x2)/2, (y1 + y2)/2)
Midpoint = ((1 + 7)/2, (11 + 7)/2)
Midpoint = (4, 9)
Since the line also passes through the origin, (0, 0), the slope is:
Slope = change in y/change in x
(9 - 0)/(4 - 0) = 9/4 = 2.25
Answer: B
OR
Case 2) The line that is equidistant must be parallel to the line joining the two points (7, 7) and (1, 11). But this case is applicable only when the perpendicular distance of the Line from point is discussed. (The line between the given point P & Q is parallel to a line passing through the origin.)
But in case 1) the line will have a positive slope and in case 2) the line will have a negative slope [Slope = (11-7) / (1-7) = - (2/3)] (Slope of parallel lines are the same, hence the slope of the line parallel to PQ will also be 2/3) and here we have not been given any option of Negative slope. Also the question doesn't mention anything about the perpendicular distance of line from point specifically.

Therefore, We will have to consider case 1 only and find the slope of the line that passes through the gap between two lines and is equidistant from two points.

The correct answer is B.

Top 1% expert replies to student queries (can skip)

Since the line is equidistant from P = (1, 11) and Q = (7, 7), it must pass through the midpoint between P = (1, 11) and Q = (7, 7). We can use the midpoint formula:
Midpoint = ((x1 + x2)/2, (y1 + y2)/2)
Midpoint = ((1 + 7)/2, (11 + 7)/2)
Midpoint = (4, 9)
Since the line also passes through the origin, (0, 0), the slope is:
Slope = change in y/change in x
(9 - 0)/(4 - 0) = 9/4 = 2.25
Answer: B



12.

To find the slope of a line, it is helpful to manipulate the equation into slope-intercept form:
$y = mx + b$, where m equals the slope of the line (incidentally, b represents the y-intercept). After isolating y on the left side of the equation, the x coefficient will tell us the slope of the line.

$x + 2y = 1$
$2y = -x + 1$
$y = -x/2 + 1/2$

The coefficient of x is -1/2, so the slope of the line is -1/2.

The correct answer is C

13. Each side of the square must have a length of 10. If each side were to be 6, 7, 8, or most other numbers, there could only be four possible squares drawn, because each side, in order to have integer coordinates, would have to be drawn on the x- or y-axis. What makes a length of 10 different is that it could be the hyptoneuse of a pythagorean triple, meaning the vertices could have integer coordinates without lying on the x- or y-axis.

For example, a square could be drawn with the coordinates (0,0), (6,8), (-2, 14) and (-8, 6). (It is tedious and unnecessary to figure out all four coordinates for each square).

If we label the square *abcd*, with *a* at the origin and the letters representing points in a clockwise direction, we can get the number of possible squares by figuring out the number of unique ways *ab* can be drawn.

a has coordinates (0,0) and *b* could have coordinates:

(-10,0)
(-8,6)
(6,8)
(0,10)
(6,8)
(8,6)
(10,0)
(8, -6)
(6, -8)
(0, 10)
(-6, -8)
(-8, -6)

There are 12 different ways to draw *ab*, and so there are 12 ways to draw *abcd*.

The correct answer is E.

Top 1% expert replies to student queries (can skip) (additional)

We have been given that the side length of the square is 10 and that all the vertices have integer coordinates.

Let one of the vertices have the coordinates (x,y).

The distance between (0,0) and (x,y) should be 10.

Therefore, x^2 + y^2 = 100

We have to find integer solutions of this equation.

if x = (+-)10, y = 0 [2 solutions]
if x = 0, y = (+-)10 [2 solutions]
if x = 6, y = (+-)8 [2 solutions]
if x = -6, y = (+-)8 [2 solutions]
if x = 8, y = (+-)6 [2 solutions]
if x = -8, y = (+-)6 [2 solutions]

Therefore, number of solutions = 12
The correct answer is E.

14. At the point where a curve intercepts the x-axis (i.e. the x intercept), the y value is equal to 0. If we plug $y = 0$ in the equation of the curve, we get $0 = (x - p)(x - q)$. This product would only be zero when x is equal to p or q. The question is asking us if (2, 0) is an x-intercept, so it is really asking us if either p or q is equal to 2.

(1) INSUFFICIENT: We can't find the value of p or q from this equation.

(2) INSUFFICIENT: We can't find the value of p or q from this equation.

(1) AND (2) SUFFICIENT: Together we have enough information to see if either p or q is equal to 2. To solve the two simultaneous equations, we can plug the p-value from the first equation, $p = -8/q$, into the second equation, to come up with $-2 + 8/q = q$.
This simplifies to $q^2 + 2q - 8 = 0$, which can be factored $(q + 4)(q - 2) = 0$, so $q = 2, -4$.
If $q = 2$, $p = -4$ and if $q = -4$, $p = 2$. Either way either p or q is equal to 2.

The correct answer is C.

15. Lines are said to intersect if they share one or more points. In the graph, line segment QR connects points (1, 3) and (2, 2). The slope of a line is the change in y divided by the change in x, or rise/run. The slope of line segment QR is $(3 - 2)/(1 - 2) = 1/-1 = -1$.

(1) SUFFICIENT: The equation of line S is given in $y = mx + b$ format, where m is the slope and b is the y-intercept. The slope of line S is therefore -1, the same as the slope of line segment QR. Line S and line segment QR are parallel, so they will not intersect <u>unless</u> line S passes through both Q and R, and thus the entire segment. To determine whether line S passes through QR, plug the coordinates of Q and R into the equation of line S. If they satisfy the equation, then QR lies on line S.

Point Q is (1, 3):
$y = -x + 4 = -1 + 4 = 3$
Point Q is on line S.

Point R is (2, 2):
$y = -x + 4 = -2 + 4 = 2$
Point R is on line S.

Line segment QR lies on line S, so they share many points. Therefore, the answer is "yes," Line S intersects line segment QR.

(2) INSUFFICIENT: Line S has the same slope as line segment QR, so they are parallel. They might intersect; for example, if Line S passes through points Q and R. But they might never intersect; for example, if Line S passes above or below line segment QR.

The correct answer is A.

Top 1% expert replies to student queries (can skip)

Q = (1,3) and R is (2,2).

Let us first calculate the slope of line QR. Slope = (y2 - y1)/(x2 - x1) = (2-3)/(2-1) = -1

We know that 2 different lines with the same slope do not intersect. So, if the slope of Line S is -1, it won't be intersecting Line QR. (Unless they overlap)

Also, we can find the equation of line QR using the slope point formula. y = y1 + m(x - x1) = 3 + (-1)*(x-1)

Therefore, the Line QR is y = -x + 4

Statement 1 says that the equation of line S is y = -x + 4. We know that Line QR also has the same equation. So, the two lines overlap, and consequently, they intersect. Sufficient!

Statement 2 says that the slope of S = -1. We have to be careful here. While it is easy to assume this statement is sufficient, there is a case here which will result in the two lines overlapping. (Since slope of S = -1, the line S is of the form y = -x + c, where c is the intercept. If c = 4, the two lines will overlap and intercept. However, if c is not equal to 4, then the two lines will be parallel and won't intersect.) Therefore, this statement is insufficient.

The correct answer is A.

16. First, we determine the slope of line L as follows:

$$L = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{q-3}{p-2} = \frac{2-3}{p-2} = \frac{-1}{p-2} = \frac{1}{2-p}$$

If line *m* is perpendicular to line *L*, then its slope is the negative reciprocal of line *L*'s slope. (This is true for all perpendicular lines.) Thus:

If the slope of $L = \frac{a}{b}$, then the slope of $m = -\frac{b}{a}$.

Therefore, the slope of line *m* can be calculated using the slope of line *L* as follows:

$$m = -\left(\frac{1}{\frac{1}{2-p}}\right) = p - 2$$

This slope can be plugged into the slope-intercept equation of a line to form the equation of line *m* as follows:

$y = (p - 2)x + b$ [where $(p - 2)$ is the slope and b is the y-intercept]

This can be rewritten as $y = px - 2x + b$ or $2x + y = px + b$ as in answer choice A.

An alternative method: Plug in a value for p. For example, let's say that $p = 4$.

Thus, the slope of line $L = \frac{\Delta y}{\Delta x} = \frac{q-3}{p-2} = \frac{2-3}{4-2} = \frac{-1}{2} = -\frac{1}{2}$

The slope of line m is the negative inverse of the slope of line L. Thus, the slope of line m is 2.

Therefore, the correct equation for line m is the answer choice that yields a slope of 2 when the value 4 is plugged in for the variable p.

(A) $2x + y = px + 7$ **yields** $y = 2x + 7$
(B) $2x + y = -px$ **yields** $y = -6x$

(C) $x + 2y = px + 7$ **yields** $y = (3/2)x + 7/2$
(D) $y - 7 = x \div (p - 2)$ **yields** $y = (1/2)x + 7$
(E) $2x + y = 7 - px$ **yields** $y = -6x + 7$

Only answer choice A yields a slope of 2.

The correct answer is A.

17. The distance between any two points (x, y) and (x_2, y_2) in the co-ordinate plane is defined by the distance formula. D.

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2A + 4 - A)^2 + (\sqrt{2A+9} - 0)^2}$$

$$= \sqrt{(A + 4)^2 + (\sqrt{2A+9} - 0)^2}$$
$$= \sqrt{A^2 + 8A + 16 + 2A + 9}$$
$$= \sqrt{A^2 + 10A + 25}$$
$$= \sqrt{(A+5)^2}$$
$$= A + 5$$

Thus, the distance between point K and point G is A + 5.

Statement (1) tells us that:

$$A - 5A - 6 = 0$$
$$(A-6)(A+1) = 0$$

Thus A = 6 or A = –1.

Using this information, the distance between point K and point G is either 11 or 4. This is not sufficient to answer the question.

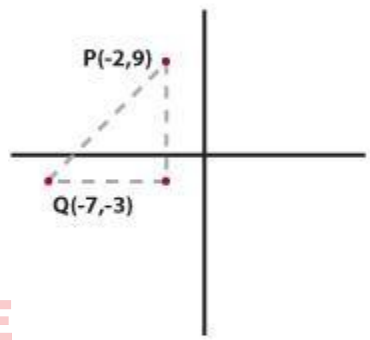
Statement (2) alone tells us that A > 2, which is not sufficient to answer the question.

When we combine both statements, we see that A must be 6, which means the distance between point K and point G is 11. This is a prime number and we are able to answer the question.

The correct answer is C.

18. The formula for the distance between two points (x_1, y_1) and (x_2, y_2) is:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



One way to understand this formula is to understand that the distance between any two points on the coordinate plane is equal to the hypotenuse of a right triangle whose legs are the difference of the x-values and the difference of the y-values (see figure). The difference of the x-values of P and Q is 5 and the difference of the y-values is 12. The hypotenuse must be 13 because these leg values are part of the known right triangle triple: 5, 12, 13.

We are told that this length (13) is equal to the height of the equilateral triangle XYZ. An equilateral triangle can be cut into two 30-60-90 triangles, where the height of the equilateral triangle is equal to the long leg of each 30-60-90 triangle. We know that the height of XYZ is 13 so the long leg of each 30-60-90 triangle is equal to 13. Using the ratio of the sides of a 30-60-90 triangle ($1:\sqrt{3}:2$), we can determine that the length of the short leg of each 30-60-90 triangle is equal to $13/\sqrt{3}$. The short leg of each 30-60-90 triangle is equal to half of the base of equilateral triangle XYZ. Thus the base of $XYZ = 2(13/\sqrt{3}) = 26/\sqrt{3}$.

The question asks for the area of XYZ, which is equal to 1/2 × base × height:

$$\frac{1}{2} \times \frac{26}{\sqrt{3}} \times 13 = \frac{169}{\sqrt{3}} = \frac{169\left(\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}\right)} = \frac{169\sqrt{3}}{3}$$

The correct answer is A.

19. To find the area of equilateral triangle ABC, we need to find the length of one side. The area of an equilateral triangle can be found with just one side since there is a known ratio between the side and the height (using the 30: 60: 90 relationship). Alternatively, we can find the area of an equilateral triangle just knowing the length of its height.

(1) INSUFFICIENT: This does not give us the length of a side or the height of the equilateral triangle since we don't have the coordinates of point A.

(2) SUFFICIENT: Since C has an x-coordinate of 6, the height of the equilateral triangle must be 6.

The correct answer is B.

20. If we put the equation $3x + 4y = 8$ in the slope-intercept form ($y = mx + b$), we get:

$y = (-3/4)x + 2$

This means that m (the slope) = -3/4 and b (the y-intercept) = 2.

We can graph this line by going to the point (0, 2) and going to the right 4 and down 3 to the point (0 + 4, 2 - 3) or (4, -1).

If we connect these two points, (0, 2) and (4, -1), we see that the line passes through quadrants I, II and IV.

The correct answer is C.

21. To determine in which quadrant the point $(p, p – q)$ lies, we need to know the sign of p and the sign of $p – q$.

(1) SUFFICIENT: If (p, q) lies in quadrant IV, p is positive and q is negative. $p – q$ must be positive because a positive number minus a negative number is always positive [e.g. 2 – (-3) = 5].

(2) SUFFICIENT: If $(q, -p)$ lies in quadrant III, q is negative and p is positive. (This is the same information that was provided in statement 1).

The correct answer is D.

22. Point B is on line AC, two-thirds of the way between Point A and Point C. To find the coordinates of point B, it is helpful to imagine that you are a point traveling along line AC.

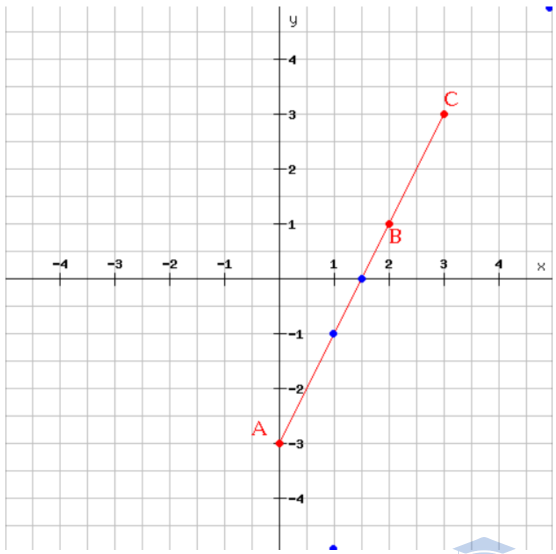
When you travel all the way from point A to point C, your x-coordinate changes 3 units (from $x = 0$ to $x = 3$). Two-thirds of the way there, at point B, your x-coordinate will have changed 2/3 of this amount, i.e. 2 units. The x-coordinate of B is therefore $x = 0 + 2 = 2$.

When you travel all the way from point A to point C, your y-coordinate changes 6 units (from $y = -3$ to $y = 3$). Two-thirds of the way there, at point B, your y-coordinate will have changed 2/3 of this amount, i.e. 4 units. The y-coordinate of B is therefore $y = -3 + 4 = 1$.

Thus, the coordinates of point B are (2,1).

The correct answer is C.

The easiest way to find the solution is to draw the line segment AC and you will literally see the answer:

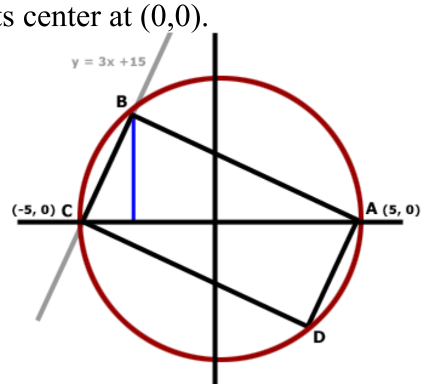


Since AB is twice the length of BC then the only acceptable choice is B (2, 1). The two points (1, -√5) and (√ 5, √ 5) do not lie on AC at all, (1.-1) is closer to A than to C and (1.5, 0) divides AC in half. After eliminating 4 wrong options, we are left with only C.

Answer: C.

23. The equation of a circle given in the form $x^2 + y^2 = r^2$ indicates that the circle has a radius of r and that its center is at the origin (0,0) of the xy-coordinate

system. Therefore, we know that the circle with the equation $x^2 + y^2 = 25$ will have a radius of 5 and its center at (0,0).



If a rectangle is inscribed in a circle, the diameter of the circle must be a diagonal of the rectangle (if you try inscribing a rectangle in a circle, you will see that it is impossible to do so unless the diagonal of the rectangle is the diameter of the circle). So diagonal AC of rectangle $ABCD$ is the diameter of the circle and must have length 10 (remember, the radius of the circle is 5). It also cuts the rectangle into two right triangles of equal area. If we find the area of one of these triangles and multiply it by 2, we can find the area of the whole rectangle.

We could calculate the area of right triangle *ABC* if we had the base and height. We already know that the base of the triangle, *AC*, has length 10. So we need to find the height.

The height will be the distance from the *x*-axis to vertex *B*. We need to find the coordinate of point B in order to find the height. Since the circle intersects triangle *ABCD* at point *B*, the coordinates of point *B* will satisfy the equation of the circle $x^2 + y^2 = 25$. Point *B* also lies on the line $y = 3x + 15$, so the coordinates of point *B* will satisfy that equation as well.
Since the values of *x* and *y* are the same in both equations and since $y = 3x + 15$, we can substitute (3*x* + 15) for *y* in the equation $x^2 + y^2 = 25$ and solve for *x*:

$x^2 + y^2 = 25 \rightarrow$

$x^2 + (3x + 15)^2 = 25 \rightarrow$

$x^2 + (3x + 15)(3x + 15) = 25 \rightarrow$

$x^2 + 9x^2 + 90x + 225 = 25 \rightarrow$

$10x^2 + 90x + 200 = 0 \rightarrow$

$x^2 + 9x + 20 = 0 \rightarrow$

$(x + 4)(x + 5) = 0$

So the two possible values of *x* are -4 and -5. Therefore, the two points where the circle and line intersect (points *B* and *C*) have *x*-coordinates -4 and -5, respectively. Since the *x*-coordinate of point *C* is -5 (it has coordinates (-5, 0)), the x-coordinate of point *B* must be -4. We can plug this into the equation $y = 3x + 15$ and solve for the *y*-coordinate of point *B*:

$y = 3(-4) + 15 \rightarrow$

$y = -12 + 15 \rightarrow$

$y = 3$

So the coordinates of point *B* are (-4, 3) and the distance from the *x*-axis to point *B* is 3, making the height of triangle *ABC* equal to 3. We can now find the area of triangle *ABC*:

$$\text{area of } \triangle ABC = \frac{1}{2}(10)(3) \rightarrow$$

$$\text{area of } \triangle ABC = \frac{1}{2}(30) \rightarrow$$

$$\text{area of } \triangle ABC = 15$$

The area of rectangle *ABCD* will be twice the area of triangle *ABC*. So if the area of triangle *ABC* is 15, the area of rectangle *ABCD* is (2)(15) = 30.

The correct answer is B.

Alternate Solution from GMATCLUB

Circle's equation: $x^2 + y^2 = 5^2$
Circle has a radius of 5 and is centered at (0,0)

AC is the diagonal of the rectangle and lies on x-axis; means AC=10

B lies in 2nd quadrant and D lies in 4th quadrant. See the image.

BC lies on line "y=3x+15". B and C are two vertices of the rectangle. We can find B and C if we find the solutions for x and y for both line and circle. Line y=3x+15 must intersect the circle on two points giving us the vertices B and C. These two points can be found by solving the simultaneous equations for the circle and the line.

Line: $y = 3x + 15$ ------ 1
Circle: $y^2 + x^2 = 25$ ------ 2

Substituting 1 in 2:
$(3x + 15)^2 + x^2 = 25$
$9x^2 + 225 + 90x + x^2 = 25$
$10x^2 + 200 + 90x = 0$
$x^2 + 9x + 20 = 0$
$x^2 + 9x + 20 = 0$
$(x + 5)(x + 4) = 0$
x=-5 and x=-4

if x=-5; y= 3x+15 = 3*(-5)+15=0
if x=-4; y= 3x+15 = 3*(-4)+15=3

We found the vertices B and C now; B(-4,3) and C(-5,0)

Length of BC;
Distance between two points $(x_1, y_1) and (x_2, y_2)$ is found using following formula:
$BC = sqrt(y_2 - y_1)^2 + (x_2 - x_1)^2$

Distance between B(-4,3) and C(-5,0)
$BC = sqrt(0 - 3)^2 + (-5 - (-4))^2 = sqrt9 + 1 = sqrt10$

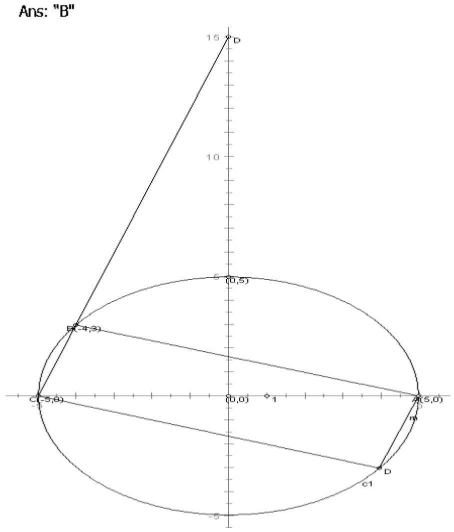
We now know $BC = \sqrt{10} \& AC = 10$

We can find AB; $\triangle ABC$ is a right angled triangle with hypotenuse as AC. We can use Pythagoras theorem to find AB

$(AC)^2 = (AB)^2 + (BC)^2$
$10^2 = (AB)^2 + (\sqrt{10})^2$
$100 = (AB)^2 + 10$
$(AB)^2 = 100 - 10 = 90$
$AB = \sqrt{90}$

Area of the rectangle ABCD
$BC * AB = \sqrt{10} * \sqrt{90} = \sqrt{900} = 30$

Ans: "B"



24. First, rewrite the line y = 4-2x as y = -2x+4 . The equation is now in the form y = mx + b where m represents the slope and b represents the y-intercept. Thus, the slope of this line is -2 .

By definition, if line F is the perpendicular bisector of line G, the slope of line F is the negative inverse of the slope of line G. Since we are told that the line y = -2x + 4 is the perpendicular bisector of line segment RP, line segment RP must have a slope of 1/2(which is the negative inverse of).

Now we know that the slope of the line containing segment RP is (1/2) but we do not know its y-intercept. We can write the equation of this line as y = (1/2)x+ b, where b represents the unknown y-intercept.

To solve for b, we can use the given information that the coordinates of point R are (4, 1). Since point R is on the line y = (1/2)x + b , we can plug 4 in for x and 1 in for y as follows:

$$y = \frac{1}{2}x + b$$

$$1 = \frac{1}{2}(4) + b$$

$$-1 = b$$

Now we have a complete equation for the line containing segment RP: y = (1/2) x - 1
We also have the equation of the perpendicular bisector of this line: y = -2x + 4.
To determine the point M at which these two lines intersect, we can set these two equations to equal each other as follows:

$$\frac{1}{2}x - 1 = -2x + 4$$
$$\frac{5}{2}x = 5$$

$$x = 2$$

Thus, the intersection point M has x-coordinate 2. Using this value, we can find the y coordinate of point M:

$$y = -2x + 4$$
$$y = -2(2) + 4$$
$$y = 0$$

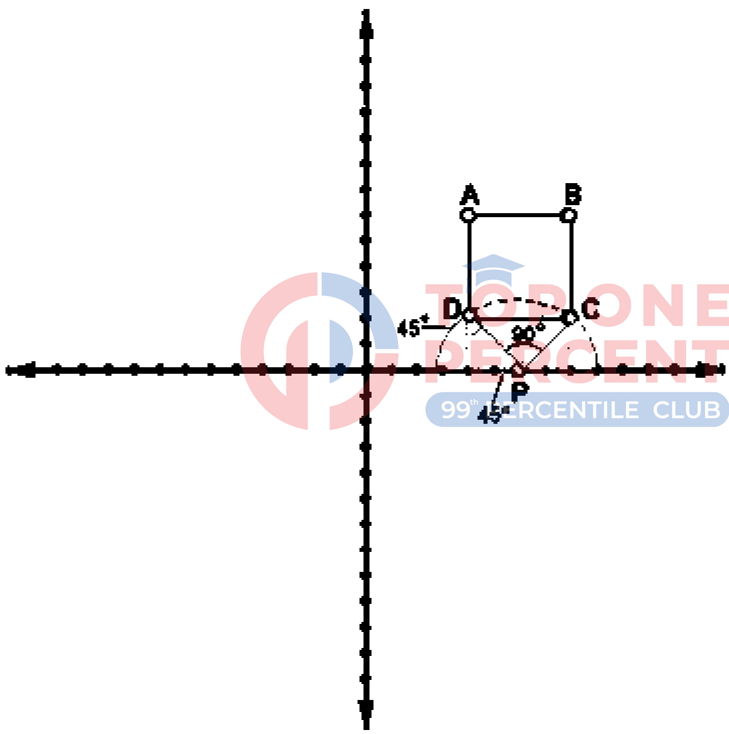
Thus the perpendicular bisector intersects line segment RP at point M, which has the coordinates (2, 0). Since point M is on the bisector of RP, point M represents the midpoint on line segment RP; this means that it is equidistant from point R and point P.

We know point R has an x-coordinate of 4. This is two units away from the x-coordinate of midpoint M, 2. Therefore the x-coordinate of point P must also be two units away from 2, which is 0.

We know point R has a y-coordinate of 1. This is one unit away from the y-coordinate of midpoint M, 0. Therefore, the y-coordinate of point P must also be one unit away from 0, which is –1.

The coordinates of point P are (0, -1).
The correct answer is D.

25.



The most difficult part of this question is conceptualizing what you're asked to find. The best way to handle tricky problems like this is to break them down into their component parts, resolve each part, then reconstruct them as a whole. You start off with a coordinate plane and four points (A, B, C, and D) that form a square when joined. Beneath the square, on the x-axis, lies another point, P. Then you are asked to determine the probability that a line randomly drawn through P will not also pass through square ABCD. In any probability question, the first thing you need to determine is the number of total possibilities. In this case, the total number of possibilities will be the total number of lines that can be drawn through P. The problem, though, is that there are literally infinitely many lines that satisfy this criterion. We cannot use infinity as the denominator of our probability fraction. So what to do?

This is where some creative thinking is necessary. First, you need to recognize that there must be a limited range of possibilities for lines that pass through both P and ABCD. If that were not the case, the probability would necessarily be 1 or

0. What is this range? Well, drawing out a diagram of the problem will help enormously here. Any line that passes through both P and ABCD has to pass through a triangle formed by ABP. This triangle is an isosceles right triangle. We know this because if we drop perpendiculars from points A and B, we end up with two isosceles right triangles, with angles of 45 degrees on either side of P. Since angles along a straight line must equal 180, we know that angle APB must measure 90 degrees. So if angle APB measures 90 degrees, we can use that to figure out the proportion of lines that pass through both P and ABCD. If we draw a semicircle with P as its center, we can see that the proportion of lines passing through P and ABCD will be the same as the proportion of the semicircle occupied by sector APB. Since a semicircle contains 180 degrees, sector APB occupies 1/2 of the semicircle. So 1/2 of all possible lines through P will also pass though ABCD, which means the 1/2 will NOT pass through ABCD and we have our answer.

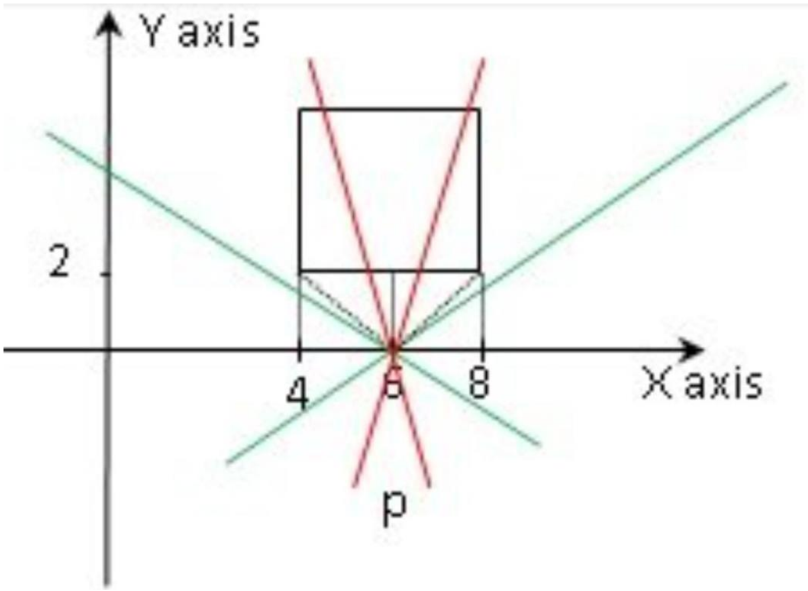
The correct answer is C.

Top 1% expert replies to student queries (can skip)

If one can notice in the original picture posted, coordinates of point C ,D and P are such that angle DPC is 90. (This can be verified using coordinates or slopes)
Thus, for any line of the form y=mx+b, passing through point P. There is a 90' region from which it can not pass, else it will go inside the square ABCD.
Outside of this 90 it can pass wherever it wants. it will be ok.
There is an overall 180' (180 degrees) region, starting from the x-axis and going anticlockwise, where this line can be drawn. (considering only above x-axis - as once you rotate line enough to go below x-axis, the other end of line would be above x-axis)
Thus probability = Total favourable area/Total area = 1/2
Now, coming to how it is 90' and 180'. We can consider this solution only in the first quadrant or in the first & fourth quadrant both. However when we consider both quadrants, it will provide a mirror image across the x-axis.
total area, which is not desired, formed by line = 90*2 and total area possible for line = 180*2 = 360
Still we get the same answer 1/2.

The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)



The simplest way to solve this:
PFA the diagram.
Any line passing through the point P which is at 6 on the x axis will pass through one of the 4 equal dotted regions (except x axis). Out of these, lines passing through 2 of the regions are not acceptable (red lines) while those passing through other 2 are acceptable (green lines).
Hence, 1/2 of the region is fine.
Probability that the line does not pass through ABCD is 1/2.
The correct answer is C.

26.
Because we are given two points, we can determine the equation of the line. First, we'll calculate the slope by using the formula $(y_2 - y_1) / (x_2 - x_1)$:

$$\frac{[0-(5)]}{(7-0)} = \frac{-5}{7}$$

Because we know the line passes through (0,5) we have our y-intercept which is 5. Putting these two pieces of information into the slope-intercept equation gives us $y = (-5/7)x + 5$. Now all we have to do is plug in the x-coordinate of each of the answer choices and see which one gives us the y-coordinate.

(A) (-14, 10)
$y = -5/7(-14) + 5 = 15$; this does not match the given y-coordinate.
(B) (-7, 5)
$y = -5/7(-7) + 5 = 10$; this does not match the given y-coordinate.
(C) (12, -4)
$y = -5/7(12) + 5 = -60/7 + 5$, which will not equal an integer; this does not match the given y-coordinate.
(D) (-14, -5)

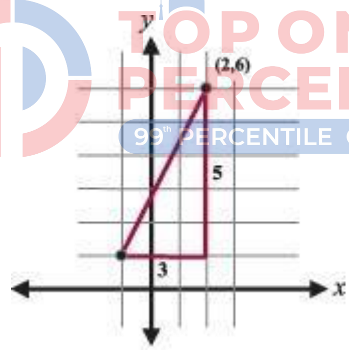
y = -5/7(-14) + 5 = -5; this matches the given y-coordinate so we have found our answer.
(E) (21, -9)
y = -5/7(21) + 5 = -15/7 + 5, which will not equal an integer; this does not match the given y-coordinate. (Note that you do not have to test this answer choice if you've already discovered that D works.)

The correct answer is D

27.
If we put the equation $3x + 4y = 8$ in the slope-intercept form ($y = mx + b$), we get:
Y = -(3/4)x+2, which means that m (the slope) = -¾
Among the answer The slope of a line perpendicular to this line must be 4/3, the negative reciprocal of -3/4

choices, only E gives an equation with a slope of 4/3.
The correct answer is E

28.
We are essentially asked to find the distance between two points. The simplest method is to sketch a coordinate plane and draw a right triangle using the two given points:



We can now see that one leg of the triangle is 3 and the other leg is 5. Because it is a right triangle, we can use the Pythagorean Theorem to calculate the hypotenuse, which is the line segment whose length we are asked to calculate.
$3^2 + 5^2 = c^2$
$34 = c^2$
$c = \sqrt{34}$
Note: always be careful! Some people will notice the lengths 3 and 5 and automatically assume this is a 3-4-5 right triangle. This one is *not*, however, because the hypotenuse must always be the longest side and, in this problem, the *leg* is 5 units long, not the hypotenuse.
The correct answer is C

29.
For this question it is helpful to remember that lines are perpendicular when their slopes are the negative reciprocals of each other.
(1) SUFFICIENT: Because we know the lines pass through the origin, we can figure out if the slopes are negative reciprocals of each other. The slope of *m* is -1 so if the slope of *n* is 1 (-1/-1) then we know the lines are perpendicular and the angle between them is 90°. Because we know two points for line n, (0, 0) and (-*a*, -*a*), we can calculate the slope:

$$\frac{0-(-a)}{0-(-a)} = \frac{a}{a} = 1$$

Thus the lines are perpendicular and the angle between them is 90°.
(2) SUFFICIENT: Reciprocals, when multiplied together, equal 1. Solving for one slope in terms of the other, we get $x = -1/y$. Thus the slopes are the negative reciprocals of each other and therefore the lines are perpendicular. Thus the angle between them must be 90°.
The correct answer is D

30.
Two lines are perpendicular if their slopes are opposite reciprocals. For example, the lines given by the equations $y = 3x + 4$ and $y = -1/3x + 7$ must be perpendicular because the slopes (3 and -1/3) are opposite reciprocals.

The slope of a line can be found using the following equation:
slope = $(y_2 - y_1) \div (x_2 - x_1)$
We are given the coordinate pairs (3, 2) and (-1, -2). The slope of the line on which these points lie is therefore (2 – (-2)) ÷ (3 – (-1)) = 4/4 = 1. So any line that is perpendicular to this line must have a slope of -1. Now we can check the choices for the pair that does NOT have a slope of -1.
(A) (8 – 9) ÷ (5 – 4) = -1/1 = -1.
(B) (-1 – (-2)) ÷ (3 – 4) = 1/(-1) = -1.
(C) (6 – 9) ÷ (-1 – (-4)) = -3/3 = -1.

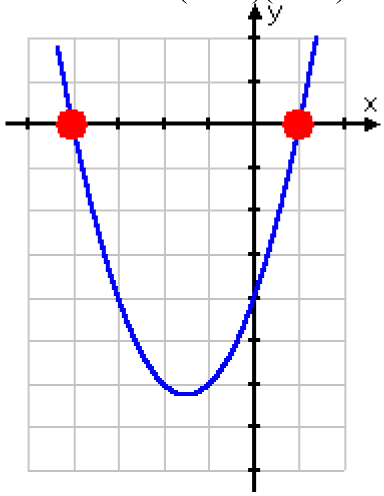
(D) (5 – 2) ÷ (2 – (-3)) = 3/5.
(E) (1 – 2) ÷ (7 – 6) = -1/1 = -1.
The only pair that does not have a slope of -1 is (2, 5) and (-3, 2).
The correct answer is D.

31. **Top 1% expert replies to student queries (can skip) (additional)**

Query: While drawing the parabola for the equation, how do we determine what the vertex should be?
Reply: For an upward facing parabola, the vertex will lie at the minima of the curve. This will be at x = -b/2a
For a downward facing parabola, the vertex will lie at the maxima of the curve. This will be at x = -b/2a

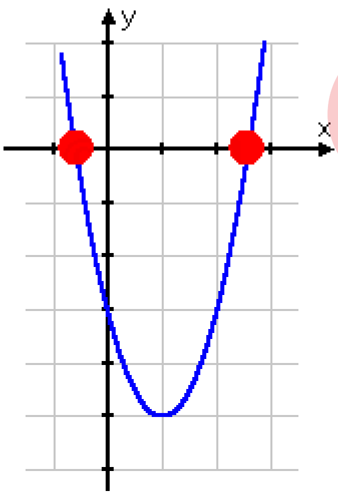
a. Note first that this quadratic happens to factor:

$$x^2 + 3x - 4 = (x + 4)(x - 1) = 0$$



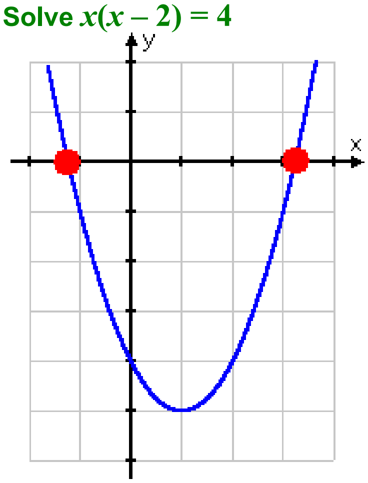
b.

Solve $2x^2 - 4x - 3 = 0$.



c.

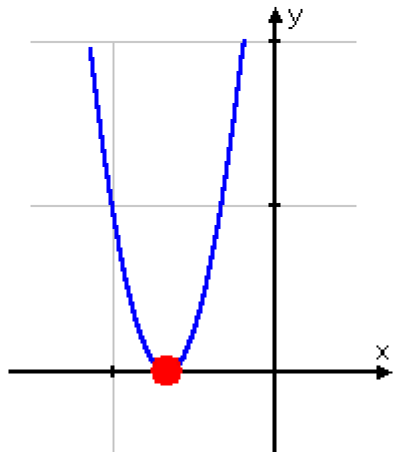
Solve $x(x - 2) = 4$



d.

Solve $9x^2 + 12x + 4 = 0$.

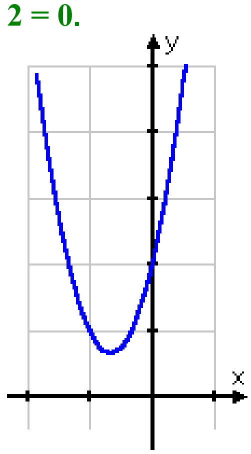
Then the answer is $x = {}^{-2}/_{3}$.



e.

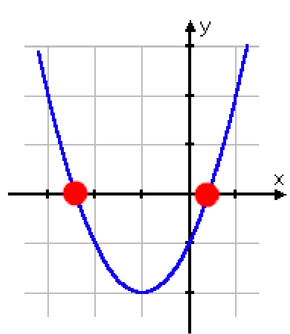
Solve $3x^2 + 4x + 2 = 0$.

Here's the graph:



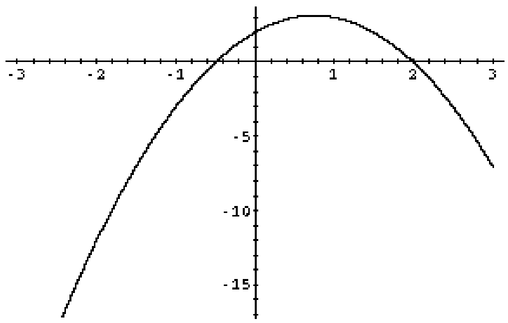
f.
Solve $x^2 + 2x = 1$.

Here's the graph:

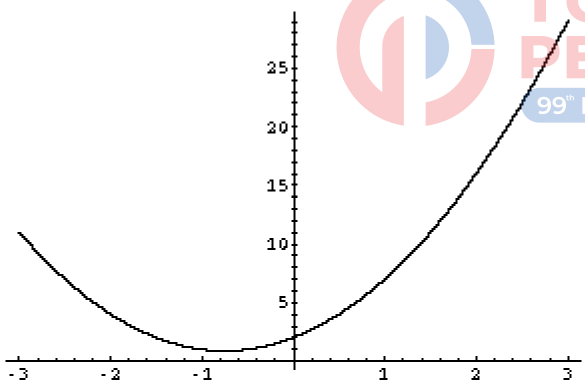


g.

y(x)=-2x^2+3x+2.



h. y(x)=2x^2+3x+2.



GMAT Quant Topic 7: Permutations and Combinations

1. 4!, 11!/ (2!*2!*2!).

Top 1% expert replies to student queries

As ‘mathematics’ contains 11 letters so we can arrange them in 11! Ways but m, a and t are repeated or say are 2 times so we have to subtract repeated words to get exact count of words. Hence, we will divide it by 2! 3 times to get the actual number of words. Why do we have to divide? As for every word if we interchange both m's position we get the exact word again. As we can see we have a copy of every word. So, we have to divide the whole number of words into half to get rid of copies. Similarly, we have to again divide into half for two t's and a's.
So total no. of words = 11!/ (2!*2!*2!).

2. 7P4
We know that the total number of ways of arranging r items from a total n items given is nPr.
So, here we have to arrange 4 people of the 7 people given.
So, answer will be 7P4.
Solving we get,
7P4 = 840.

3. 4P2 + 4P3 + 4P4

The # of ways to visit 2 cities when order matters = $P^2_4=\frac{4!}{(4-2)!}=12$: AB, BA, AC, CA, AD, DA, BC, CB, BD, DB. CD, DC.

The # of ways to visit 3 cities when order matters = $P^3_4=\frac{4!}{(4-3)!}=24$: ABC, ACB, BAC, BCA, CAB, CBA, ...

The # of ways to visit 4 cities when order matters = $P^4_4=\frac{4!}{(4-4)!}=24$: ABCD, ABDC, ADCD, ...

12 + 24 + 24 = 60.

Answer: C.

4. 4!/2!
Answer: The number of different linear arrangements can be generated by arranging these balls is 12.

Step-by-step explanation:

The number of ways to arrange n things in a line where a things are like and b things are like is $\frac{n!}{a!\ b!\ ....}$

Given : There are 2 black balls, one red ball and one green ball, identical in shape and size.

Total balls = 2+1+1=4
Here 2 black balls are alike.

So , the number of different linear arrangements can be generated by arranging these balls would be $\frac{4!}{2!}=\frac{4\times3\times2!}{2!}=12$

Hence, the number of different linear arrangements can be generated by arranging these balls is 12.

5. 10P2 + 10P3

We can have either two-song lists or three-song lists. We will count them separately and add.
Note that having the same songs in different orders count as different playlists, according to this setup.

For two-song lists, the DJ has 10 choices for the first song, and then 9 remaining choices for the second song. This means
# of two-song playlists = $^{10}P_2 = 10*9 = 90$

For three-song lists, the DJ has 10 choices for the first song, then 9 remaining choices for the second song, then 8 remaining choices for the third song. This means
# of three-song playlists = $^{10}P_3 = 10*9*8 = 10*72 = 720$

Add those
total number of playlists = $^{10}P_2 + {}^{10}P_3 = 720 + 90 = 810$

6. $({}^{10}P_8 + {}^{10}P_9 + {}^{10}P_{10}) \times 12$ seconds

Answer:

6048 hours is required to guarantee access to the database

Explanation:

If a password contains 8 distinct digits, then there are **P(10,8) permutations** possible

P(10,8)=10! / (10-8)! = 10!/2! = 1814400

Since it takes 12 seconds to try one combination,

1814400×12 = **21772800** seconds is required to guarantee access to the database.

Ans E

7. 6! – 5! × 2!

The best way to deal with the questions like this is to find the total # of arrangements and then subtract # of arrangements for which opposite of restriction occur:

Total # of arrangements of 6 people (let's say A, B, C, D, E, F) is $6!$.
# of arrangement for which 2 particular persons (let's say A and B) **are adjacent** can be calculated as follows: consider these two persons as one unit like {AB}. We would have total 5 units: {AB}{C}{D}{E}{F} - # of arrangement of them 5!, # of arrangements of A and B within their unit is 2!, hence total # of arrangement when A and B are adjacent is $5! * 2!$.

# of arrangement when A and B are not adjacent is $6! - 5! * 2! = 480$.

8.

7C4

Hint: According to given in the question we have to determine the number of ways can a committee of 4 people be selected from a group of 7 people Step by step solution. So, first of all we have to use the permutation and combination to obtain the required number of ways for which we have to choose the required number of people from the total number of people.

Now, as mentioned in the question that there are total 7 people in which we have to make the committee of 4 people so for this selection we have to user the formula which is as mentioned below:

Formula used:

$$\Rightarrow c_r^n = \frac{n!}{r!\,(n-r)!}\ldots\ldots\ldots\ldots\ldots\ldots(A)$$

Where, n is the total number of ways or we can say that n is the total number of people in the given group and r is the required number of ways.

Now, to solve the factor of a given number n we have to use the formula to find the factor which is as mentioned below:

Formula used:

$$\Rightarrow n! = n(n-1)\times(n-2)\times\ldots\ldots\times 2\times 1\ldots\ldots\ldots\ldots(B)$$

Hence, with the help of the formula (B) we can determine the value factor for the given number.

Complete step-by-step answer:

Step 1: First of all we have to use the permutation and combination to obtain the required number of ways for which we have to choose the required number of people which are 4 from the total number of people which are 7.

Step 2: Now, as mentioned in the question, there are a total 7 people in which we have to make a committee of 4 people so for this selection we have to use the formula (A) which is as mentioned in the solution hint. Hence,

$$\Rightarrow c_4^7 = \frac{7!}{4!\,(7-4)!}$$

$$\Rightarrow c_4^7 = \frac{7!}{4!\times 3!}\ldots\ldots\ldots\ldots(1)$$

Step 3: Now, to solve the expression (1) as obtained in the solution step 2 we have to use the formula (B) which is used to determine the factorial of a given number. Hence,

$$\Rightarrow c_4^7 = \frac{7\times 6\times 5\times 4!}{4!\times 3!}$$

$$\Rightarrow c_4^7 = \frac{7\times 6\times 5}{3\times 2\times 1}$$

On solving the expression as obtained just above,

$$\Rightarrow c_4^7 = 35$$

Hence, with the help of the formula (A) and (B) we have determined the required number of ways a committee of 4 people can be selected from a group of 7 people is 35.

9. 12C9
6 students play at both levels
Thus, # of students playing only at state level = 8 - 6 = 2
Thus, # of students playing only at national level = 10 - 6 = 4
Total # of students = Both + Only state + Only national = 6 + 2 + 4 = 12
Selecting 9 out of 12 students = 12C9 = 220

10. ${}^{2}C_{1} \times {}^{3}C_{1} \times ({}^{7}C_{4} - {}^{5}C_{2})$
Answer= Total number combinations - Total number of combinations with constraints

Total number of combinations = 2C1*3C1*7C4= 210
Total number of combinations with constraints = 2C1*3C1*5C2=60

Answer=210-60=150

11. ${}^{5}C_{3} \times 3! - {}^{3}C_{1}*2! *2!$

Alternate sol from gmatclub

Total no. of permutations = 5*4 * 3 = 60
Considering the cases when A & B are in the final three, we have the following 18 combinations

AB* ABC ABD ABE

BA* BAC BAD BAE

*AB CAB DAB EAB

*BA CBA DBA EBA

A*B ACB ADB AEB

B*A BCA BDA BEA

This is also obtained by (1*1*3)*3! = 18 ways

But the question says only that A & B should not be together in a straight line. So the last 6 - A*B and B*A should be also ok with the results.

If that is true, then it should be only 12 ways that are not allowed (and not 18).

This can also be arrived at by considering in the following way:

A and B can be arranged within in 2 ways

(AB) can be arranged with 3 other letters DEF in 3 ways

And position of AB and D/E/F can be interchanged in 2 ways.

Hence, 2*3*2 = 12 ways.

Final answer should be 60-12 = 48. (5 C3 × 3! – 3 C1*2! *2!)

12. (4! / 2!) × 2! = 24 ways

Top 1% expert replies to student queries

The nickel can face either heads up or tails up, thus we multiply the total # of ways in which we can arrange NDQQ by 2. The quarters and the dime have to face heads up, so only 1 choice for both of them.
Basically, it says that quarters and dime have to be faced heads up. ONLY Nickel can be either head or tail. So, Nickel can be placed in two ways.

we have four coins in total, let's name them as per requirement

Nickel = can have 2 face N_H or N_T = Nickel$_{(Head)}$ or Nickel$_{(Tail)}$

Dime = can have 1 face D_H = Dime$_{(Head)}$

quarters = 2 quarters, each quarter must have 1-face = Q_H . Q_H

So we have N_{Head} or N_{Tail} + Dime$_{head}$ + Quarter$_{(head)}$ + Quarter$_{(head)}$

arrangements can be → $N_H D_H Q_H Q_H = \frac{4!}{2!} = 12$ ways

(i) with N = head

(ii) Nickel = Tail → $N_T D_H Q_H Q_H = \frac{4!}{2!} = 12$ ways

24 ways

13. 7C3

Top 1% expert replies to student queries

The chemicals are identified by an unordered combination' means that we don't care about the order in which the colours appear. All we care about is the colours that appear. So, BAC is the same as ABC, which is the same as CBA.

Therefore, number of chemicals that can be named = number of ways of choosing 3 colors from 7 colors = 7C3 = 35

14. 7P3

The Q is asking us to find 3 out of 7 and then arrange them.... so it is a permutation Q..

$$7P3 = \frac{7!}{(7-3)!} = 7 * 6 * 5 = 210$$

15. 5! / 2!

By the rule of permutations/Arrangements for an n letter word containing x similar letters of 1 type, y similar letters of another type and so on, the number of words that can be formed = n! / (x! * y!)

TWIST has 5 letters, with 2 T's. therefore, the number of words that can be formed = 5! / 2! = 120 / 2 = 60

16. 4P3

We're told that an ID code must consist of 3 non-repeating digits and each digit is in the code must be a PRIME number. We're asked for the number of ID codes that can be generated.

Since we're limited to just PRIME numbers/digits, we can only use the numbers 2, 3, 5 and 7. With a 3-digit code that uses NON-REPEATING digits, we can calculate the total number of possibilities by working through one digit at a time:

For the 1st digit, there are 4 options. Once we choose one....
For the 2nd digit, there are 3 options. Once we choose one...
For the 3rd digit, there are 2 options...

(4)(3)(2) = 24 possible ID codes.

17. 4 × 4 = 16

There are 4 flavors of pizza and each can be:
1. Without cheese and mushrooms;
2. With cheese;
3. With mushrooms;
4. With cheese and mushrooms.

So, the total number of pizza varieties is 4*4 = 16.

Answer: D.

18. Another version: $2 \times {}^{5}C_{4} \times 2 = 20$

Mario's Pizza:
2 choices of crust.
5 choices of toppings.
2 choices of cheese (extra-cheese or regular).

Linda decides to order a pizza with four toppings.

How many 4-topping pizzas are possible from 5? $C_5^4 = 5$. But the pizza can have 2 different crusts and two different cheese, therefore the final answer is 5*2*2 = 20.

19. (3 ! × 2 ! × 3 !) × 3 !
The store has 3/8 x 8 = 3 novels, 1/4 x 8 = 2 study guides, and 3 textbooks.

We can let n = a novel, s = study guide, and t = textbook.

Since the each category of book has to be next to each other, we have:

[n(1)-n(2)-n(3)] [s(1)-s(2)] [t(1)-t(2)-t(3)]

We have 3 categories of books (novels, study guides, and textbooks), and so those categories of books can be arranged in 3! = 6 ways.

Now, let's arrange the books within each category. The novels can be arranged in 3! = 6 ways, the study guides in 2! = 2 ways, and the textbooks in 3! = 6 ways.

Thus, the total number of ways to arrange the books is 6 x 6 x 2 x 6 = 432 ways.

Answer: E

20. $3! \times 2$

Top 1% expert replies to student queries

The question basically asks about the probability that Bob and Lisa sit at the ends.

The total # of sitting arrangements is 5!.

Desired arrangement is either BXYZL or LXYZB. Now, XYZ can be arranged in 3! ways, therefore total # of favorable arrangements is 2*3!.

Top 1% expert replies to student queries (can skip)

The question is saying that Bob and Lisa want to sit next to one other student. This means the two will sit at the extreme ends of the line.

Case 1 : The arrangement is as follows.

Bob X Y Z Lisa

Number of ways of arranging the students in this way, keeping Bob and Lisa fixed = 3! (Since we can only arrange X, Y and Z here)

Case 2 : The arrangement is as follows.

Lisa X Y Z Bob

Number of ways of arranging the students in this way, keeping Bob and Lisa fixed = 3! (Since we can only arrange X, Y and Z here)

Therefore, total ways = 3! + 3! = 12

21. $1 \times 10 \times 10 \times 5$

Total no of ways of selecting first digit : 10 (0-10)
Total no of ways of selecting second digit : 10 (0-10)
Total no of ways of selecting third digit : 5 (0,2,4,6,8)
Total no of ways : 10 x 10 x 5 = 500 ways

22. ${}^5C_3 \times {}^4C_3$

Top 1% expert replies to student queries

He eats only on weekdays
5 days (so 5 lunch and 5 dinner slots)
Also Friday dinner is fixed. So he has got 4 dinner slots
And he can have 3 lunch and 3 dinners. In how many ways can he do it?
${}^5C_3 * {}^4C_3 = 10*4=40$

Top 1% expert replies to student queries (can skip) (additional)

We have to choose only from weekdays. There are 5 weekdays.

For lunch, we have 5 possible options, from which we have to choose 3. Number of ways = ${}^{5}C_{3}$
For dinner, we have 4 possible options (We can't include Friday), from which we have to choose 3. Number of ways = ${}^{4}C_{3}$

Therefore, total number of ways = ${}^{5}C_{3}$ * ${}^{4}C_{3}$

23. $5! \times 2!$

Since president and vice president must sit next to each other, they will be considered as single unit.
So total number of units to be arranged = 5
Number of ways of arranging president and vice president will be 2.
Thus total number of arrangements=
2(5!) = 240
Answer:- B

24. ${}^{5}C_{3} + {}^{5}C_{4}$
Since the order in which we select the pictures does not matter, we can use combinations.

We must consider two possible cases:
1) Veronica includes 3 of her pictures
We can select 3 pictures from 5 pictures in ${}^{5}C_{3}$ways
${}^{5}C_{3}$= 10

2) Veronica includes 4 of her pictures
We can select 4 pictures from 5 pictures in ${}^{5}C_{4}$ways
${}^{5}C_{4}$= 5

ASIDE: Below, there's a video that shows you how to quickly calculate combinations (like ${}^{5}C_{3}$) in your head

Total number of possibilities = 10 + 5
= 15
= D

25. ${}^{3}C_{2} \times 1 \times {}^{3}C_{2}$

2 distributors in east coast and 2 in west coast. And 1 in Midwest.

So,

East Coast= 3C2: $\frac{3!}{2!1!}$ = 3.

West Coast= 3C2: $\frac{3!}{2!1!}$ = 3.

Midwest= 1C1= 1

So The total number of ways will be,

$3 * 1 * 3 = 9.$

Answer is B.

26. $3! \times 2! \times 2! \times 2!$

Six spots to fill. Restriction = No couple should be separated

First spot has 6 choices
Second spot has only 1
Third spot = 4 choices
Fourth spot = 1
Fifth spot = 2
Sixth spot = 1

Thus total arrangements = 6 * 4 * 2 = 48

27. TTTHHH, HTTTHH, HHTTTH, HHHTTT;

total 4

28. 26
The only combination of odd is $5 \times 5 \times 5$.
So total required = $3 \times 3 \times 3 - 1 = 26$.

29. ${}^{6}P_{4} \times 2 = 720$

[X][YYYYYY]

X: Inna or Jake: P_1^2
Y: Lena , Fred, John and (Jake or Inna): P_4^6

$$N = P_1^2 * P_4^6 = 2 * \frac{6!}{2!} = 6 * 5 * 4 * 3 * 2 = 720$$

30. $2 \times 2 \times 2 \times 4 \times 4 - 2 \times 1 \times 1 \times 4 \times 4 = 96$

Top 1% expert replies to student queries

2 multiples-choice questions can be answered in = 4 x 4 = 16 ways
3 true-false questions can be answered in = 2x2x2 = 8 ways
But out of the 8 ways, 2 ways [(True-True-True) (False-False-False)]will contain same answers.
Thus 3 true-false questions can be answered in = (2x2x2) -2 = 6 ways
Total ways to answer the quiz = 16 x 6 = 96
Answer C.

Top 1% expert replies to student queries(can skip)

Number of ways = (3 true false questions and 2 multiple choice questions)
= [(true or false)*(true or false)*(true or false) - (when all the answers are same TTT /FFF)] and (4Choices* 4Choices)
= (2*2*2 - 2) * (4*4)

= 6*4*4
= 96 ways **Answer C.**

Top 1% expert replies to student queries (can skip) (additional)

The line basically means that the answers to the 3 true-false questions cannot be all TRUE or all FALSE.

In other words, let the 3 true-false questions be called Q1, Q2 and Q3.

The answers to these questions cannot be all TRUE or all FALSE.

The 3 true-false questions can be 6 possible answer combinations. (All combinations except {True True True} and {False False False})

The 2 multiple choice questions can be answered in 4^2 = 16 ways. [The first question can be answered in 4 ways and the second question can also be answered in 4 ways]

Therefore, total numbers of ways = 6 * 16 = 96
Answer C.

31. 10
Let t = TOTAL number of candidates
So,
# of ways to select a president = x
# of ways to select a vice-president = x - 1
Total number of ways to select both = (x)(x - 1)
We're told that 90 ways are possible
So, (x)(x - 1) = 90
Expand: $x^2 - x = 90$
Rearrange: $x^2 - x - 90 = 0$
Factor: (x - 10)(x + 9) = 0
So, x = 10 OR x = -9
Since x cannot be negative, it must be the case that x = 10
Answer: D

Top 1% expert replies to student queries (can skip) (additional)

If there are n total students, I can select the president in n ways, and then I can select the vice president in (n-1) ways

Then n(n-1) = 90

or, n^2 - n - 90 = 0

or, n^2 - 10n + 9n - 90 = 0

or, n (n-10) + 9 (n-10) = 0

or, (n+9)(n-10) = 0

n cannot be -9 (n has to be a positive integer, i.e. > 0). So n = 10

There are a total of 10 students in the committee

32. $3\,! \times 3\,! = 36$

We are given that a pod of 6 dolphins always swims single file, with 3 females at the front and 3 males in the rear. Thus:

The number of ways to arrange the 3 female dolphins in the front is 3! = 3 x 2 x 1 = 6.

The number of ways to arrange the 3 male dolphins is in the rear is 3! = 3 x 2 x 1 = 6.

Thus, the number of ways to arrange all the dolphins is 6 x 6 = 36 ways.

Answer: B

33. **Top 1% expert replies to student queries**
Since the question is asking for longest possible time, we need to consider that all the attempts take the maximum of 3 seconds. This means that even the last attempt takes the full 3 seconds (focus on longest possible time).
Basically the answer should be = no. of attempts* maximum time per attempt.
$^6C_2 \times 9 \times 3 = 405$ seconds

Top 1% expert replies to student queries (can skip) (additional)

Query: Explain how we choose pair for this question?
Reply: Let the six buttons be B1, B2, B3, B4, B5 and B6.

In this question, we're pressing the 2 buttons simultaneously. The order does not matter. Meaning pressing B1 first and B2 seconds later is the same as pressing B2 first and B1 seconds later.

In other words, we need to choose 2 buttons from 6. This can be done in 6C2 ways.

34. $9! / (5! \times 4!) = 126$

In order to travel exactly nine blocks Casey should go 5 block down and 4 block left - DDDDDLLLL.

# of permutations of 9 letters DDDDDLLLL out of which there are 5 identical D's and 4 identical L's is $\frac{9!}{5!4!}$.

35. $6 \times 2 \times 1 \times 3 \times 2 \times 1 = 72$
Top 1% expert replies to student queries
Let the 3 dwarfs be D1, D2 and D3
Let the 3 elves be E1, E2 and E3

From the conditions, it is clear that the dwarfs and the elves will be seated alternately

Case1 :

D1 E1 D2 E2 D3 E3

Now,

The dwarfs here can be arranged in 3! ways
The elves here can be arranged in 3! ways

Total number of ways of arranging = 3! * 3! = 36

Case1 :

E1 D1 E2 D2 E3 D3

Now,

The dwarfs here can be arranged in 3! ways
The elves here can be arranged in 3! ways

Total number of ways of arranging = 3! * 3! = 36

Therefore, total number of ways = 36 + 36 = 72

36. (5! / 2! – 4! / 2!) = 48.

Top 1% expert replies to student queries

Five dolls are D1, D1, D2, D3, D4 ; five nieces are N1, N2, N3, N4, N5

Let's say N5 doesn't want D4

N5 — 4 ways
N4 — 4 ways
N3 — 3 ways
N2 — 2 ways
N1 — 1 way

Total ways — 96. This you have got

BUT, and you already know this, but haven't applied - in all of these arrangements, two items are identical. So unique number of arrangements will be obtained by dividing the total number of ways by 2! in this case (as an example - N1 - D2, N2 - D1, N3 - D4, N4 - D1, N5 - D3 this arrangement will appear twice, and 47 other such arrangements, all of which have to be discarded to obtain the total number of unique arrangements)

Then unique number of arrangements is 48

37. In order to answer this question, we need to be able to determine the value of x. Thus, this question can be rephrased: What is x?

(1) SUFFICIENT: In analyzing statement (1), consider how many individuals would have to be available to create 126 different 5 person teams. We don't actually have to figure this out as long as we know that we *could* figure this out. Certainly by testing some values, we *could* figure this out. It turns out that if there are 9 available individuals, then we could create exactly 126 different 5-person teams (since 9! ÷ [(5!)(4!)] = 126). This value (9) represents $x + 2$. Thus x would equal 7.

(2) SUFFICIENT: The same logic applies to statement (2). Consider how many individuals would have to be available to create 56 different 3-person teams. Again, we don't actually have to figure this out as long as we know that we *could* figure this out. It turns out that if there are 8 available individuals, then we could create exactly 56 different 3-person teams (since 8! ÷ [(5!)(3!)] = 56). This value
(8) represents $x + 1$. Thus x would equal 7. Statement (2) alone IS sufficient.

The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

Query: In this question, how can we determine whether or not we'll find a unique value for X without solving it?
Reply: Statement 1 :

(x+2)C5 = 126

This will always have a unique solution. Think of it this way. The expression basically means that the number of ways of choosing 5 people from (x+2) people is 126.

Can two different values of the total number of people ever give the same number of 5 member teams that can be formed?

For example, we know that 10C2 = 45. Can there be any number other than 10 that will give us the same result of 45? No, right? If n < 10, then the number of ways will be less than 45. If n > 10, then the number of ways will be more than 45. But for n = 10, the number of ways = 45.

Similarly, for the equation above, there has to be a unique value of x.
Use the same logic for statement 2.
If we say xCy = z, in this case, y can have 2 values (y and x-y).
For example, 10C2 = 10C8 = 45.

Meaning, the number of ways of choosing 2 people out of 10 is the same as the number of ways of choosing 8 people out of 10. But 10 here will remain constant.

38. This question is simply asking us to come up with the number of permutations that can be formed when *x* people are seated in *y* chairs. It would seem that all we require is the values of *x* and *y*. Let's keep in mind that the question stem adds that *x* and *y* must be prime integers.

(1) SUFFICIENT: If *x* and *y* are prime numbers and add up to 12, *x* and *y* must be either 7 and 5 or 5 and 7. Would the number of permutations be the same for both sets of values? Let's start with *x* = 7, *y* = 5. The number of ways to seat 7 people in 5 positions (chairs) is 7!/2!. We divide by 2! because 2 of the people are not selected in each seating arrangement and the order among those two people is therefore not significant. An anagram grid for this permutation would look like this:

A	B	C	D	E	F	G
1	2	3	4	5	N	N

But what if *x* = 5 and *y* = 7? How many ways are there to position five people in 7 chairs? It turns out the number of permutations is the same. One way to think of this is to consider that in addition to the five people (A,B,C,D,E), you are seating two ghosts (X,X). The number of ways to seat A,B,C,D,E,X,X would be 7!/2!. We divide by 2! to eliminate order from the identical X's.
Another way to look at this is by focusing on the chairs as the pool from which you are choosing. It's as if we are fixing the people in place and counting the number of ways that different chair positions can be assigned to those people. The same anagram grid as above would apply, but now the letters would correspond to the 7 chairs being assigned to each of the five fixed people. Two of the chairs would be unassigned, and thus we still divide by 2! to eliminate order between those two chairs.

(2) INSUFFICIENT: This statement does not tell us anything about the values of *x* and *y*, other than *y* > *x*. The temptation in this problem is to think that you need statement 2 in conjunction with statement 1 to distinguish between the *x* = 5, *y* = 7 and the *x* = 7, *y* = 5 scenarios.

The correct answer is A.

Alternate sol from gmatclub (additional)

(1) x + y = 12. Since x and y are primes, then x=5 and y=7 OR x=7 and y=5.

If x=5 and y=7, the number of arrangements would be $C_7^5 * 5! = \frac{7!}{2!}$, where C_7^5 is the number of way to choose 5 chairs from 7, and 5! is the number of arrangements of 5 people on those chairs.

If x=7 and y=5, the number of arrangements would be $C_7^5 * 5! = \frac{7!}{2!}$, where C_7^5 is the number of way to choose 5 people who will get the chairs, and 5! is the number of arrangements of 5 people on those chairs.

Since both possible cases give the same exact answer, then the statement sufficient.

(2) There are more chairs than people. Clearly insufficient.

Answer: A.

39. Let W be the number of wins and L be the number of losses. Since the total number of hands equals 12 and the net winnings equal \$210, we can construct and solve the following simultaneous equations: w + l = 12, 100 w – 10 l = 210. so l = 9, w = 3. So we know that the gambler won 3 hands and lost 9. We do not know where in the sequence of 12 hands the 3 wins appear. So when counting the possible outcomes for the first 5 hands, we must consider these possible scenarios:
1) Three wins and two losses 2) Two wins and three losses 3) One win and four losses 4) No wins and five losses
In the first scenario, we have WWWLL. We need to know in how many different ways we can arrange these five letters:
5!/2!3! = 10. So there are 10 possible arrangements of 3 wins and 2 losses. The second scenario -- WWLLL -- will yield the same result: 10. The third scenario -- WLLLL -- will yield 5 possible arrangements, since the one win has only 5 possible positions in the sequence. The fourth scenario -- LLLLL -- will yield only 1 possible arrangement, since rearranging these letters always yields the same sequence. Altogether, then, there are 10 + 10 + 5 + 1 = 26 possible outcomes for the gambler's first five hands.
The correct answer is C.

40.
First, let's consider the different medal combinations that can be awarded to the 3 winners: (1) If there are NO TIES then the three medals awarded are: GOLD, SILVER, BRONZE. (2) What if there is a 2-WAY tie? --If there is a 2-WAY tie for FIRST, then the medals awarded are: GOLD, GOLD, SILVER. --If there is a 2-WAY tie for SECOND, then the medals awarded are: GOLD, SILVER, SILVER. --There cannot be a 2-WAY tie for THIRD (because exactly three medals are awarded in total). (3) What if there is a 3-WAY tie? --If there is a 3-WAY tie for FIRST, then the medals awarded are: GOLD, GOLD, GOLD. --There are no other possible 3-WAY ties. Thus, there are 4 possible medal combinations: (1) G, S, B (2) G, G, S (3) G, S, S (4) G, G, G. Now let's determine how many different ways each combination can be distributed. We'll do this by considering four runners: Albert, Bob, Cami, and Dora.

COMBINATION 1: **Gold, Silver, Bronze**

Gold Medal	**Silver Medal**	**Bronze Medal**
Any of the 4 runners can receive the gold medal.	There are only 3 runners who can receive the silver medal. Why? One of the runners has already been awarded the Gold Medal.	There are only 2 runners who can receive the bronze medal. Why? Two of the runners have already been awarded the Gold and Silver medals.
4 possibilities	**3 possibilities**	**2 possibilities**

Therefore, there are $4 \times 3 \times 2 = 24$ different ***victory circles*** that will contain 1 GOLD, 1 SILVER, and 1 BRONZE medallist.

COMBINATION 2: **Gold, Gold, Silver.**

Using the same reasoning as for Combination 1, we see that there are 24 different ***victory circles*** that will contain 2 GOLD medalists and 1 SILVER medalist. However, it is important to realize that these 24 ***victory circles*** must be reduced due to "overcounting." To illustrate this, consider one of the 24 possible Gold-Gold-Silver ***victory circles***: Albert is awarded a GOLD. Bob is awarded a GOLD. Cami is awarded a SILVER. Notice that this is the exact same ***victory circle*** as the following: Bob is awarded a GOLD. Albert is awarded a GOLD. Cami is awarded a SILVER. Each ***victory circle*** has been "overcounted" because we have been counting each different ordering of the two gold medals as a unique ***victory circle***, when, in reality, the two different orderings consist of the exact same ***victory circle***. Thus, the 24 ***victory circles*** must be cut in half; there are actually only **12** unique ***victory circles*** that will contain 2 GOLD medalists and 1 SILVER medalist. (Note that we did not have to worry about "overcounting" in Combination 1, because each of those 24 possibilities ***was*** unique.)

<u>COMBINATION 3</u>: Gold, Silver, Silver.

Using the same reasoning as for Combination 2, we see that there are 24 possible ***victory circles,*** but only **12 *unique victory circles*** that contain 1 GOLD medalist and 2 SILVER medalists.

<u>COMBINATION 4</u>: Gold, Gold, Gold.

Here, once again, there are 24 possible ***victory circles***. However, because all three winners are gold medalists, there has been a lot of "overcounting!" How much overcounting? Let's consider one of the 24 possible Gold-Gold-Gold ***victory circles***: Albert is awarded a GOLD. Bob is awarded a GOLD. Cami is awarded a GOLD. Notice that this ***victory circle*** is exactly the same as the following ***victory circles***: Albert-GOLD, Cami-GOLD, Bob-GOLD. Bob-GOLD, Albert-GOLD, Cami-GOLD. Bob-GOLD, Cami-GOLD, Albert-GOLD. Cami-GOLD, Albert-GOLD, Bob-GOLD. Cami-GOLD, Bob-GOLD, Albert-GOLD. Each unique ***victory circle*** has actually been counted 6 times! Thus we must divide 24 by 6 to find the number of unique ***victory circles***. There are actually only $24 \div 6 = 4$ unique ***victory circles*** that contain 3 GOLD medalists. **FINALLY,** then, we have the following:

(Combination 1) 24 unique GOLD-SILVER-BRONZE ***victory circles***. (Combination 2) 12 unique GOLD-GOLD-SILVER ***victory circles***. (Combination 3) 12 unique GOLD-SILVER-SILVER ***victory circles***. (Combination 4) 4 unique GOLD-GOLD-GOLD ***victory circles***. Thus, there are $24 + 12 + 12 + 4 = 52$ unique ***victory circles***.

The correct answer is B

Top 1% expert replies to student queries (can skip) (additional)

Case 1: Gold - Silver - Bronze. Number of ways = 4C3 * 3! = 24 (Number of ways of choosing 3 winners from 4 = 4C3. Arranging them in three positions = 3!)

Case 2: Gold - Gold - Bronze. Number of ways = 4C3 * 3! /2! = 12

Case 3: Gold - Silver - Silver. Number of ways = 4C3 * 3! /2! = 12

Case 4: Gold - Gold - Gold. Number of ways = 4C3 * 1 = 4

Total number of ways of forming the victory circle = 24 + 12 + 12 + 4 = 52

The correct answer is B

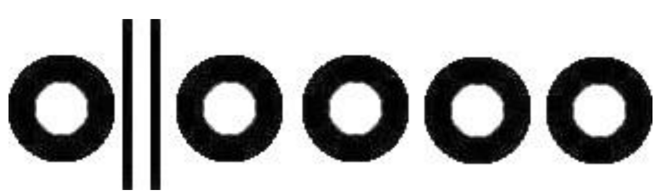
41.

This question is not as complicated as it may initially seem. The trick is to recognize a recurring pattern in the assignment of the guards. First, we have five guards (let's call them a, b, c, d, and e) and we have to break them down into pairs. So how many pairs are possible in a group of five distinct entities? We could use the combinations formula: nCr. where n is the number of items you are selecting from (the pool) and k is the number of items you are selecting (the subgroup). Here we would get 5C2 = 10. So there are 10 different pairs in a group of 5 individuals. However, in this particular case, it is actually more helpful to write them out (since there are only 5 guards and 10 pairs, it is not so onerous): ab, ac, ad, ae, bc, bd, be, cd, ce, de. Now, on the first night (Monday), any one of the ten pairs may be assigned, since no one has worked yet. Let's say that pair ab is assigned to work the first night. That means no pair containing either a or b may be assigned on Tuesday night. That rules out 7 of the 10 pairs, leaving only cd, ce, and de available for assignment. If, say, cd were assigned on Tuesday, then on Wednesday no pair containing either c or d could be assigned. This leaves only 3 pairs available for Wednesday: ab, ae, and be. At this point the savvy test taker will realize that on any given night after the first, including Saturday, only 3 pairs will be available for assignment. Those test takers who are really on the ball may have realized right away that the assignment of any two guards on any night necessarily rules out 7 of the 10 pairs for the next night, leaving only 3 pairs available on all nights after Monday.

The correct answer is D.

42. The key to this problem is to avoid listing all the possibilities. Instead, think of an arrangement of five donuts and two dividers. The placement of the dividers determines which man is allotted which donuts, as pictured below:

In this example, the first man receives one donut, the second man receives three donuts, and the third man receives one donut. Remember that it is possible for either one or two of the men to be allotted no donuts at all. This situation would be modelled with the arrangement below:



Here, the second man receives no donuts. Now all that remains is to calculate the number of ways in which the donuts and dividers can be arranged: There are 7 objects. The number of ways in which 7 objects can be arranged can be computed by taking 7!: $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$ However, the two dividers are identical, and the five donuts are identical. Therefore, we must divide 7! by 2! and by 5!:

$$\frac{5040}{5!2!} = \frac{5040}{(5\times4\times3\times2\times1)(2\times1)} = \frac{5040}{240} = 21$$

The correct answer is A.

Alternate Solution from Gmatclub

Consider this: we have 5 donuts d and 2 separators , like: $ddddd$. How many permutations (arrangements) of these symbols are possible? Total of 7 symbols (5−2=7), out of which 5 d's and 2 's are identical, so $\frac{7!}{5!2!} = 21$.

We'll get combinations like: $dd|d|dd$ this would mean that Larry got 2 donuts, Michael got 1 donut and Doug got 2 donuts, so to the left of the first separator are Larry's donuts, between the separators are Michael's donuts and to the right of the second separator are Doug's donuts

Answer: A.

This can be done with direct formula as well:

The total number of ways of dividing n identical items (5 donuts in our case) among r persons or objects (3 persons in our case), each one of whom, can receive **0, 1, 2** or more items (from zero to 5 in our case) is ${}^{n+r-1}C_{r-1}$.

In our case we'll get: ${}^{n+r-1}C_{r-1} = {}^{5+3-1}C_{3-1} = 7C2 = \frac{7!}{5!2!} = 21$.

Top 1% expert replies to student queries (can skip)

Visualise it: The other way to look at it is to simply have 5 donuts, with two dividers. You have 7 slots:

_ _ _ _ _ _ _

In those slots, you can either put a donut, O, or a divider, |. The number preceding the first divider goes to Person 1, the number between the two dividers goes to Person 2, and the number after the 2nd divider goes to Person 3.

For example:

O O | O O | O
| | O O O O O
| O | O O O O

etc.

You choose 2 spots out of the 7 to put the dividers, hence the answer is 7C2. 21 is the answer.

OR
If you get confused about combinations, there's a simple way to count these combinations as well, by counting the number of ways 5 can be summed with 3 numbers.

{5,0,0} = 3 possibilities.
{4,1,0} = 6 possibilities.
{3,2,0} = 6 possibilities.
{3,1,1} = 3 possibilities.
{2,2,1} = 3 possibilities.

Total = 21 possibilities.

Tip: For each set, we only have to consider numbers less than the first; for instance, we wouldn't consider {2,3,0} because that's already accounted for in a permutation of {3,2,0}

43. There are two possibilities in this problem. Either Kim and Deborah will both get chocolate chip cookies or Kim and Deborah will both get oatmeal cookies. If Kim and Deborah both get chocolate chip cookies, then there are 3 oatmeal cookies and 2 chocolate chip cookies left for the remaining four children. There are 5!/3!2! = 10 ways for these 5 remaining cookies to be distributed--four of the cookies will go to the children, one to the dog. (There are 5! ways to arrange 5 objects but the three oatmeal cookies are identical so we divide by 3!, and the two chocolate chip cookies are identical so we divide by 2!.) If Kim and Deborah both get oatmeal cookies, there are 4 chocolate chip cookies and 1 oatmeal cookie left for the remaining four children. There are 5!/4! = 5 ways for these 5 remaining cookies to be distributed--four of the cookies will go to the children, one to the dog. (There are 5! ways to arrange 5 objects but the four chocolate chip cookies are identical so we divide by 4!.) Accounting for both possibilities, there are 10 + 5 = 15 ways for the cookies to be distributed.
The correct answer is D.

44. In order to determine how many 10-flavor combinations Sammy can create, we simply need to know how many different flavors Sammy now has. If Sammy had x flavors to start with and then threw out y flavors, he now has $x - y$ flavors. Therefore, we can rephrase this questions as: What is $x - y$? According to statement (1), if Sammy had $x - y - 2$ flavors, he could have made exactly 3,003 different 10-flavor bags. We could use the combination formula below to determine the value of $x - y - 2$, which is equal to n in the equation below:

$$\frac{n!}{10!(n-10)!} = 3{,}003$$

Solving this equation would require some time and more familiarity with factorials than is really necessary for the GMAT. However, keep in mind that you do not need to solve this equation; you merely need to be certain that the equation is solvable. (Note, if you begin testing values for n, you will soon find that n = 15.) Once we know the value of n, we can easily determine the value of $x - y$, which is simply 2 more than n. Thus, we know how many different flavors Sammy has, and could determine how many different 10-flavor combinations he could make. Statement (2) is tells us that $x = y + 17$. Subtracting y from both sides of the equation yields the equation $x - y = 17$. Thus, Sammy has 17 different flavors. This information is sufficient to determine the number of different 10-flavor combinations he could make.
The correct answer is D.

Alternate sol from gmatclub (additional)

In order to calculate how many 10-flavor bags can Sammy make from the remaining (x-y) flavors, we should know the value of x-y. The answer would simply be C^{10}_{x-y}. For example if he has 11 flavors (if x-y=11), then he can make $C^{10}_{11} = 11$ different 10-flavor bags.

(1) If Sammy had thrown away 2 additional flavors of candy, he could have made exactly 3,003 different 10-flavor bags. We are told that $C^{10}_{n} = 3,003$, where $n = (x-y) - 2$: he can make 3,003 10-flavor bags out of n flavors. Now, n can take only one particular value, so we can find n (it really doesn't matter what is the value n, important is that we can find it), hence we can find the value of x-y (x-y=n+2). Sufficient.

(2) x = y + 17 --> x-y=17. Directly gives us the value of x-y. Sufficient.

Answer: D.

45. The three-dice combinations fall into 3 categories of outcomes:
1) All three dice have the same number 2) Two of the dice have the same number with the third having a different number than the pair 3) All three dice have different numbers By calculating the number of combinations in each category, we can determine the total number of different possible outcomes by summing the number of possible outcomes in each category.
First, let‟s calculate how many combinations can be made if all 3 dice have the same number. Since there are only 6 numbers, there are only 6 ways for all three dice to have the same number (i.e., all 1‟s, or all 2‟s, etc.). Second, we can determine how many combinations can occur if only 2 of the dice have the same number. There are 6 different ways that 2 dice can be paired (i.e., two 1‟s, or two 2‟s or two 3‟s, etc.). For each given pair of 2 dice, the third die can be one of the five other numbers. (For example if two of the dice are 1‟s, then the third die must have one of the 5 other numbers: 2, 3, 4, 5, or 6.) Therefore there are 6 x 5 = 30 combinations of outcomes that involve 2 dice with the same number. Third, determine how many combinations can occur if all three dice have different numbers. Think of choosing three of the 6 items (each of the numbers) to fill three "slots." For the first slot, we can choose from 6 possible items. For the second slot, we can choose from the 5 remaining items. For the third slot, we can choose from the 4 remaining items. Hence, there are 6 x 5 x 4 = 120 ways to fill the three slots. However, we do not care about the order of the items, so permutations like {1,2,5}, {5, 2, 1}, {2, 5, 1}, {2, 1, 5}, {5, 1, 2}, and {1, 5, 2} are all considered to be the same result. There are 3! = 6 ways that each group of three numbers can be ordered, so we must divide 120 by 6 in order to obtain the number of combinations where order does not matter (every 1 combination has 6 equivalent permutations). Thus there are 120/6 = 20 combinations where all three dice have different numbers. The total number of combinations is the sum of those in each category or 6 + 30 + 20 = 56.
The correct answer is C.

Alternate sol from gmatclub (additional)

If the order of the dice does not matter then we can have 3 cases:
1. XXX - all dice show alike numbers: 6 outcomes (111, 222, ..., 666);
2. XXY - two dice show alike numbers and third is different: $6*5=30$, 6 choices for X and 5 choices for Y;
3. XYZ - all three dice show distinct numbers: $C^3_6=20$, selecting three different numbers from 6;

Total: 6+30+20=56.

Answer: C.

46. There are two different approaches to solving this problem. The first employs a purely algebraic approach, as follows: Let us assume there are n teams in a double-elimination tournament. In order to crown a champion, n – 1 teams must be eliminated, each losing exactly two games. Thus, the *minimum* number of games played in order to eliminate all but one of the teams is 2(n – 1). At the time when the (n – 1)th team is eliminated, the surviving team (the division champion) either has one loss or no losses, adding at most one more game to the total played. Thus, the *maximum* number of games that can be played in an n-team double-elimination tournament is 2(n – 1) + 1. There were four divisions with 9, 10, 11, and 12 teams each. The maximum number of games that could have been played in order to determine the four division champions was (2(8) + 1) + (2(9) + 1) + (2(10) + 1) + (2(11) + 1) = 17 + 19 + 21 + 23 = 80. The four division champions then played in a single-elimination tournament. Since each team that was eliminated lost exactly one game, the elimination of three teams required exactly three more games. Thus, the maximum number of games that could have been played in order to crown a league champion was 80 + 3 = 83. The correct answer choice is (B). Another way to approach this problem is to use one division as a concrete starting point. Let's think first about the 9-team division. After 9 games, there are 9 losses. Assuming that no team loses twice (thereby *maximizing* the number of games played), all 9 teams remain in the tournament. After 8 additional games, only 1 team remains and is declared the division winner. Therefore, 9 + 8 = 17 games is the maximum # of games than can be played in this tournament. We can generalize this information and apply it to the other divisions. To maximize the # of games in the 10-team division, 10 + 9 = 19 games are played. To maximize the # of games in the 11-team division, 11 + 10 = 21 games are played. To maximize the # of games in the 12-team division, 12 + 11 = 23 games are played. Thus, the maximum number of games that could have been played in order to determine the four division champions was 17 + 19 + 21 + 23 = 80. After 3 games in the single elimination tournament, there will be 3 losses, thereby eliminating all but the one championship team. Thus, the maximum number of games that could have been played in order to crown a league champion was 80 + 3 = 83.

The correct answer is B.

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We need to keep track of losses. Let's focus on those and forget about the wins. Every time a game is played, someone loses. You can give at most 2 losses to a team since after that it is out of the tournament.
Consider the division which has 9 teams. What happens when 18 games are played? There are 18 losses and each team gets 2 losses (you can't give more than 2 to a team since it gets kicked out after 2 losses) so all are out of the tournament. But we need a winner so we play only 17 games so that the winning team get only 1 loss.

Similarly, the division with 10 teams can have at most 2*10 - 1 = 19 games.
The division with 11 teams can have at most 2*11 - 1 = 21 games.
The division with 12 teams can have at most 2*12 - 1 = 23 games.
This totals up to 80 games (note that the average of 17, 19, 21 and 23 will be 20 so the sum will be 4*20 = 80).

Now you have 4 teams. 1 loss gets a team kicked out. If you have 3 games, there are 3 losses and 3 teams are kicked out. You have a final winner!
Hence the total number of games = 80 + 3 = 83

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Let's name the teams in group 1 as 1,2,3,4,5,6,7,8,9.

Case 1; team1 played with every other team and won all of its matches.
so total number of matches =8
case 2: team2 , played with team 3,4,5,6,7,8,9 and won all of its matches.
total number of matches =7
after case 1 and case 2 we have only two teams remaining in the group1 which are team 1 and team 2. Now since question asks us for the maximum no. of matches. Therefore we must include the extra case in which team 2 defeated team 1. Now both team 2 and team 1 have 1 loss each. Now in the final match, we will found out about the eventual winner in group 1.
maximum no. of matches in group 1 are 8+7+1(in which team2 defeated team1) + 1 (final) =17

Similarly in group 2 we have 9 + 8 +1 +1 =19
group 3 = 10+9+1+1 =21
group 4 = 11+10+1+1=23

After this we will have 4 winner from each group. lets name them as w1,w2,w3,w4
Let's assume w1 won all of its matches from the remaining three teams and eventually emerged as a winner. Therefore total matches among four winners=3

Therefore maximum total no. of matches played are 17+19+21+23+3=83

47. The simplest way to solve this problem is to analyze one row at a time, and one square at a time in each row if necessary. Let"s begin with the top row. First, let"s place a letter in left box; we have a choice of 3 different letters for this box: X, Y, or Z. Next, we place a letter in the top center box. Now we have only 2 options so as not to match the letter we placed in the left box. Finally, we only have 1 letter to choose for the right box so as not to match either of the letters in the first two boxes. Thus, we have 3 x 2 x 1 or 6 ways to fill in the top row without duplicating a letter across it. Now let"s analyze the middle row by assuming that we already have a particular arrangement of the top row, say the one given in the example above (XYZ).

X	Y	Z	Given Arrangement of Top Row
Middle Row LEFT Options	Middle Row CENTER Options	Middle Row RIGHT Options	
Y	X	Z	Not Allowed: Z is in Right column twice

	Z	X	Permissible
Z	X	Y	Permissible
	Y	X	Not Allowed: Y is in Center column twice

Now lets analyze the bottom row by assuming that we already have a particular arrangement of the top and middle rows. Again, lets use top and middle row arrangements given in the example above.

X	Y	Z	Given Arrangement of Top Row
Y	Z	X	Give Arrangment of Middle Row
Bottom Row LEFT OPtions	Bottom Row CENTER Options	Bottom Row RIGHT Options	
Z	X	Y	Only this option is permissible.

We can see that given fixed top and middle rows, there is only 1 possible bottom row that will work. (In other words, the 3rd row is completely determined by the arrangement of the 1st and 2nd rows). By combining the information about each row, we calculate the solution as follows: 6 possible top rows × 2 possible middle rows × 1 possible bottom row = 12 possible grids.

The correct answer is D.

48. This is a counting problem that is best solved using logic. First, let"s represent the line of women as follows:

0000
0000

where the heights go from 1 to 8 in increasing order and the unknowns are designated 0s. Since the women are arranged by their heights in increasing order from left to right and front to back, we know that at a minimum, the line-up must conform to this:

0008
1000

Lets further designate the arrangement by labelling the other individuals in the top row as X, Y and Z, and the individuals in the bottom row as A, B, and C.

XYZ8
1ABC

Note that Z must be greater than at least 5 numbers (X, Y, B, A, and 1) and less than at least 1 number (8). This means that Z can only be a 6 or a 7. Note that Y must be greater than at least 3 numbers (X, A and 1) and less than at least 2 numbers (8 and Z). This means that Y can only be 4, 5, or 6. Note that X must be greater than at least 1 number (1) and less than at least 3 numbers (8, Z and Y), This means that X must be 2, 3, 4, or 5. This is enough information to start counting the total

number of possibilities for the top row. It will be easiest to use the middle unknown value Y as our starting point. As we determined above, Y can only be 4, 5, or 6. Let‟s check each case, making our conclusions logically: *If Y is 4*, Z has 2 options (6 or 7) and X has 2 options (2 or 3). This yields 2 x 2 = 4 possibilities. *If Y is 5*, Z has 2 options (6 or 7), and X has 3 options (2, 3, or 4). This yields 2 x 3 = 6 possibilities. *If Y is 6*, Z has 1 option (7), and X has 4 options (2, 3, 4, or 5). This yields 1 x 4 = 4 possibilities. For each of the possibilities above, the bottom row is completely determined because we have 3 numbers left, all of which must be in placed increasing order. Hence, there are 4 + 6 + 4 = 14 ways for the women to pose.
The correct answer is B.

49. One way to approach this problem is to pick an actual number to represent the variable *n.* This helps to make the problem less abstract. Let's assume that *n* = 6. Since each of the regional offices must be represented by exactly one candidate on the committee, the committee must consist of 6 members. Further, because the committee must have an equal number of male and female employees, it must include 3 men and 3 women. First, let's form the female group of the committee. There are 3 women to be selected from 6 female candidates (one per region). One possible team selection can be represented as follows, where A, B, C, D, E, & F represent the 6 female candidates:

A	B	C	D	E	F
Yes	Yes	Yes	No	No	No

In the representation above, women A, B, and C are on the committee, while women D, E, and F are not. There are many other possible 3 women teams. Using the combination formula, the number of different combinations of three female committee members is 6! / (3! × 3!) = 720/36 = 20. To ensure that each region is represented by exactly one candidate, the group of men must be selected from the remaining three regions that are not represented by female employees. In other words, three of the regions have been “used up” in our selection of the female candidates. Since we have only 3 male candidates remaining (one for each of the three remaining regions), there is only one possible combination of 3 male employees for the committee. Thus, we have 20 possible groups of three females and 1 possible group of three males for a total of 20 × 1 = 20 possible groups of six committee members. Now, we can plug 6 in for the variable *n* in each of the five answer choices. The answer choice that yields the solution 20 is the correct expression. Therefore, **D is the correct answer** since plugging 6 in for *n,* yields the following: $\frac{n!}{(.5n)!^2} = \frac{6!}{(3!)^2} = \frac{6\times5\times4\times3\times2\times1}{3\times2\times1\times3\times2\times1} = 20$

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There are 6 offices. You can list out the 6 offices as A through F.

And each office must recommend 1 male and 1 female.
(the recommendation is of 12 , but ultimately there are only 6 members in the audit committee)

From each office exactly 1 person will go on the committee.

This means the committee will have 6 people.

Further, we are told that there must be an equal number of males and females on the committee. Therefore, we know that there must be 3 males and 3 females chosen on the 6 person committee.

Since each office will send one person ——-> and the order in which the people are selected does not matter (only the makeup of the group selected is important)

all we care about is which 3 offices will send their male representative and which 3 offices will send their female representatives.

3 Males and 3 Females from 6 offices.
(MMMFFF)
3 males from the first three offices
Or
(MMFFFM)
2 males from the first two and 1 male from the last office.
and so on.

Hence, 6!/3!3! = 20.

50. This is a relatively simple problem that can be fiendishly difficult unless you have a good approach to solving it and a solid understanding of how to count. We will present two different strategies here. Strategy 1: This problem seems difficult, because you need to figure out how many distinct orientations the cube has relative to its other sides. Given that you can rotate the cube in an unlimited number of ways, it is *very* difficult to keep track of what is going on – unless you have a system. Big hint: In order to analyze how multiple things behave or compare or are arranged *relative to each other*, the first thing one should do is *pick a reference point and fix it*. Here is a simple example. Let"s say you have a round table with four seat positions and you want to know how many distinct ways you can orient 4 people around it *relative to each other* (i.e., any two orientations where all 4 people have the same person to their left and to their right are considered equivalent). Let"s pick person A as our reference point and anchor her to the North position. Think about this next statement and convince yourself that it is true: By choosing A as a fixed reference, *all distinct arrangements of the other 3 people relative to A* will constitute the *complete set of distinct arrangements of all 4 people relative to each other*. Hence, fixing the location of one person makes it significantly easier to keep track of what is going on. Given A is fixed at the North, the 3 other people can be arranged in the 3 remaining seats in 3! = 6 ways, so there are 6 distinct orientations of 4 people sitting around a circular table. Using the same principle, we can conclude that, in general, if there are *N* people in a circular arrangement, after fixing one person at a reference point, we have (*N*-1)! distinct arrangements *relative to each other*. Now let"s solve the problem. Assume the six sides are: Top (or T), Bottom (or B), N, S, E, and W, and the six colors are designated 1, 2, 3, 4, 5, and 6. Following the first strategy, let"s pick color #1 and

fix it on the Top side of the cube. If #1 is at the Top position, then one of the other 5 colors must be at the Bottom position and each of those colors would represent a distinct set of arrangements. Hence, since there are exactly 5 possible choices for the color of the Bottom side, the number of unique arrangements relative to #1 in the Top position is a multiple of 5. For each of the 5 colors paired with #1, we need to arrange the other 4 colors in the N, S, E, and W positions in distinct arrangements. Well, this is exactly like arranging 4 people around a circular table, and we have already determined that there are (n- 1)! or 3! ways to do that. Hence, the number of distinct patterns of painting the cube is simply 5 x 3! = 30.

The correct answer is B.

Strategy 2: There is another way to solve this kind of problem. Given one distinct arrangement or pattern, you can try to determine how many equivalent ways there are to represent that one particular arrangement or pattern within the set of total permutations, then divide the total number of permutations by that number to get the number of distinct arrangements. This is best illustrated by example so let"s go back to the 4 people arranged around a circular table. Assume A is in the North position, then going clockwise we get B, then C, then D. Rotate the table 1/4 turn clockwise. Now we have a different arrangement where D is at the North position, followed clockwise by A, then B, then C. BUT, this is merely a rotation of the distinct relative position of the 4 people (i.e., everyone still has the same person to his right and to his left) so they are actually the same arrangement. We can quickly conclude that there are 4 *equivalent* or non-distinct arrangements for every distinct relative positioning of the 4 people. We can arrange 4 people in a total of 4! = 24 ways. However, each DISTINCT arrangement has 4 equivalents, so in order to find the number of distinct arrangements, we need to divide 4! by 4, which yields 3! or 6 distinct ways to arrange 4 people around a circular table, the same result we got using the "fixed reference" method in Strategy 1. Generalizing, if there are N! ways to arrange N people around a table, each distinct relative rotation can be represented in N ways (each 1/Nth rotation around the table) so the number of distinct arrangements is $N!/N = (N-1)!$ Now let"s use Strategy #2. Consider a cube that is already painted in a particular way. Imagine putting the cube on the table, with color #1 on the top side. Note, that by rotating the cube, we have 4 different orientations of this particular cube given color #1 is on top. Using symmetry, we can repeat this analysis when #1 is facing any of the other 5 directions. Hence, for each of the six directions that the side painted with #1 can face, there are 4 ways to orient the cube. Consequently, there are 6 x 4 = 24 total orientations of any one cube painted in a particular manner. Since there are 6 sides and 6 colors, there are 6! or 720 ways to color the six sides each with one color. However, we have just calculated that each DISTINCT pattern has 24 equivalent orientations, so 720 must be divided by 24 to get the number of distinct patterns. This yields 720/24 = 30, confirming the answer found using Strategy #1. Again, **the correct answer is B**.

Top 1% expert replies to student queries (can skip)

Consider the cube lying on one of its faces (say F2) on a table and F1 is the face parallel to the ceiling (exactly opposite F2). If you paint F1 and F2 say two colours C1 and C2 respectively and then, irrespective of what you painted the rest of the four faces, flipped the cube over 180 degrees (so that it is now sitting on F1 and F2 is parallel to the ceiling). Again, irrespective of what you painted the other four faces of the cube (which have a circular symmetry btw), this new cube is exactly similar to another cube which was originally lying on F2 and that you had painted F2-C2 and F1-C1. Per the question these two are not distinct arrangements / squares. So you are looking at fixing any one face and then painting the others to obtain distinct arrangements.

That way, with no loss of generality, you can paint the top face parallel to the ceiling any colour, then 5 options for the bottom face, and (4-1)! = 3! = 6 arrangements for the other faces because of circular symmetry. Total ways are then 5x6 = 30

Top 1% expert replies to student queries (can skip)

If you have six distinct colors, say ROYGBP, you could place them in the slots below in 6! ways. Why? Any of the 6 colors could go in the first slot, any of the five remaining could go in the second, and so on. So the total is 6*5*4*3*2*1=6!

_ _ _ _ _ _

Now the cube is a combination of slots and a circular arrangement. Here is my sophisticated diagram for that:

_
_ _ _ _
_

The four slots between the top and bottom 'faces' actually wrap around the whole cube. So, let's say for the four 'slots' you choose ROYG.

ROYG =GROY = YGRO = OYGR = ROYG

All of these arrangements are the same because essentially each one is just a rotation of the cube by 90 degrees, not a different paint job.

So we treat the four 'slots' in the middle the same way we would a circular table. Hence the above solutions: you can choose any color for the top face, you can choose one of the five remaining colors for the bottom face (5 ways), and since the four middle faces are 'in a circle' they can be arranged (4-1)!=3! ways. So the total is 5*3!

The tricky part is that we don't count the ways in which we can choose the color of the first face, since every color is going to be chosen anyway. Essentially, you are finding the ways you can paint the other sides relative to one of painted sides. Otherwise you are including in your total the number of different ways you can look at the cube (which don't constitute a new paint job).

The correct answer is B.

51. The first thing to recognize here is that this is a *permutation with restrictions* question. In such questions it is always easiest to tackle the restricted scenario(s) first. The restricted case here is when all of the men actually sit together in three adjacent seats. Restrictions can often be dealt with by considering the limited individuals as one unit. In this case we have four women (w_1, w_2, w_3, and w_4) and three men (m_1, m_2, and m_3). We can consider the men as one unit, since we can think of the 3 adjacent seats as simply 1 seat. If the men are one unit (m), we are really looking at seating 5 individuals (w_1, w_2, w_3, w_4, and m) in 5 seats. There are 5! ways of arranging 5 individuals in a row. This means that our group of three men is sitting in any of the "five" seats. Now, imagine that the one seat that holds the three men magically splits into three seats. How many different ways can the men arrange themselves in those three seats? 3!. To calculate the total number of ways that the men and women can be arranged in 7 seats such that the men **ARE** sitting together, we must multiply these two values: 5!3!. However this problem asks for the number of ways the theatre-goers can be seated such that the men are **NOT** seated three in a row. Logically, this must be equivalent to the following: (Total number of all seat arrangements) – (Number of arrangements with 3 men in a row). The total number of all seat arrangements is simply 7! so the final calculation is 7! – 5!3!.

The correct answer is C.

52. It is important to first note that our point of reference in this question is all the possible subcommittees that include Michael. We are asked to find what percent of these subcommittees also include Anthony. Let's first find out how many possible subcommittees there are that must include Michael. If Michael must be on each of the three-person committees that we are considering, we are essentially choosing people to fill the two remaining spots of the committee. Therefore, the number of possible committees can be found by considering the number of different two-people groups that can be formed from a pool of 5 candidates (not 6 since Michael was already chosen). Using the anagram method to solve this combinations question, we assign 5 letters to the various board members in the first row. In the second row, two of the board members get assigned a Y to signify that they were chosen and the remaining 3 get an N, to signify that they were not chosen:

A	B	C	D	E
Y	Y	N	N	N

The number of different combinations of two-person committees from a group of 5 board members would be the number of possible anagrams that could be formed from the word YYNNN = 5! / (3!2!) = 10. Therefore there are 10 possible committees that include Michael. Out of these 10 possible committees, of how many will Anthony also be a member? If we assume that Anthony and Michael must be a member of the three-person committee, there is only one remaining place to fill. Since there are four other board members, there are four possible three-person committees with both Anthony and Michael. Of the 10 committees that include Michael, 4/10 or 40% also include Anthony. **The correct answer is C.** As an alternate method, imagine splitting the original six-person board into two equal groups of three. Michael is automatically in one of those groups of three.

Now, Anthony could occupy any one of the other 5 positions -- the 2 on Michael's committee and the 3 on the other committee. Since Anthony has an equal chance of winding up in any of those positions, his chance of landing on Michael's committee is 2 out of 5, or 2/5 = 40%. Since that probability must correspond to the ratio of committees asked for in the problem, the answer is achieved. **Answer choice C is correct**.

53. The easiest way to solve this question is to consider the restrictions separately. Lets start by considering the restriction that one of the parents must drive, temporarily ignoring the restriction that the two sisters won't sit next to each other. This means that…

2 people (mother or father) could sit in the drivers seat
4 people (remaining parent or one of the children) could sit in the front passenger seat 3 people 0 could sit in the first back seat
2 people could sit in the second back seat
1 person could sit in the remaining back seat

The total number of possible seating arrangements would be the product of these various possibilities: 2 × 4 × 3 × 2 × 1 = 48. We must subtract from these 48 possible seating arrangements the number of seating arrangements in which the daughters are sitting together. The only way for the daughters to sit next to each other is if they are both sitting in the back. This means that…

2 people (mother or father) could sit in the drivers seat
2 people (remaining parent or son) could sit in the front passenger seat

Now for the back three seats we will do something a little different. The back three seats must contain the two daughters and the remaining person (son or parent). To find out the number of arrangements in which the daughters are sitting adjacent, let‟s consider the two daughters as one unit. The remaining person (son or parent) is the other unit. Now, instead of three seats to fill, we only have two "seats," or units, to fill.
There are 2 × 1 = 2 ways to seat these two units. However,
the daughter-daughter unit could be d_1d_2 or d_2d_1
We must consider both of these possibilities so we multiply the 2 by 2! for a total of 4 seating possibilities in the back. We could also have manually counted these possibilities: d_1d_2X, d_2d_1X, Xd_1d_2, Xd_2d_1
Now we must multiply these 4 back seat scenarios by the front seat scenarios we calculated earlier:
(2 × 2) × 4 = 16
front back
If we subtract these 16 "daughters-sitting-adjacent" scenarios from the total number of "parent-driving" scenarios, we get: 48 – 16 = 32.
The correct answer is B.

Top 1% expert replies to student queries (can skip)

Overall, think of the arrangement of the car like this

F1 is the front left seat - the driver's seat

F2 - front right
B1 B2 B3 - the three back seats

F1 F2
B1 B2 B3

Now, one of the two parents has to drive AND the two sisters cannot sit together. Then we can find the answer by this approach. Also note here that I have used sister / daughter and brother / son interchangeably in the following solution

(Total number of arrangements possible - Bracket I) - (Those in which ONLY the parents are not driving, i.e. the sisters are not together - Bracket II) - (Those in which ONLY the sisters are together, i.e. one of the parents is driving - Bracket III) - (Those in which the parents are not driving AND the sisters are together - Bracket IV)

I did this in my mind first, so I am going to talk about the simple brackets first, and then go to the difficult ones

Bracket I - Total number of arrangements. This is simple = 5! = 120

Bracket IV - If sisters are together, they have to sit in (B1 B2) or (B2 B3). The parents are also not driving. Then the son has to drive. If the son is driving and the daughters are sitting in (B1 B2), then we have 2 arrangements (F2 B3) where the parents can sit, and between the daughters, they can also move in 2 ways between (B1 B2). Then a total of 4 arrangements. Exactly in a similar manner, another 4 arrangements when the daughters are sitting together in (B2 B3). So a total of 8 arrangements for Bracket IV

Bracket III - One of the parents is driving, so F1 can be filled in 2 ways. After this, the two daughters are together. So exactly as above, there are 8 arrangements in which the two daughters sit together. So total arrangements for Bracket III is 2 x 8 = 16.

Bracket II - This is the toughest bracket - the parents are not driving and the sisters are not together. If the parents are not driving, the two sisters and the brother can drive. Say any one sister is driving. Then the second condition of the two sisters not being together is already taken care of. So we simply have to check for the total number of arrangements possible. This is 4! = 24. Same when the other sister is driving. Another 24 arrangements. Now when the son is driving, the total number of arrangements is 4! = 24. Out of these, by the logic we used above, the sisters are together in 8 arrangements. Then when the son is driving, the total number of arrangements in which the two sisters are not together = 24 - 8 = 16. So total number of arrangements for Bracket II = 24 + 24 + 16 = 64

Then going back to our initial bracket formula that we set up, the answer is 120 - 64 - 16 - 8 = 32 arrangements

54.
Ignoring Frankie's requirement for a moment, observe that the six mobsters can be arranged 6! or 6 x 5 x 4 x 3 x 2 x 1 = 720 different ways in the concession stand line. In each of those 720 arrangements, Frankie must be either ahead of or behind Joey. Logically, since the combinations favor neither Frankie nor Joey, each would be behind the other in

precisely half of the arrangements. Therefore, in order to satisfy Frankie's requirement, the six mobsters could be arranged in 720/2 = 360 different ways.
The correct answer is D.

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The probability of Frankie standing behind Joey = The probability of Joey standing behind Frankie. (The number of ways in which Joey stands in front of Frankie is equal to the number of ways he stands behind him).
Therefore, The probability of Frankie standing behind Joey = 50% and The probability of Joey standing behind Frankie = 50%.

Frankie and Joey are absolutely identical elements of this arrangement.
Consider this simple example:
Say, I have 3 elements: A, B and C.
I can arrange them in 3! ways:
ABC
ACB
BAC
BCA
CAB
CBA
Look at them carefully.
In 3 of them A is before B and in other 3, B is before A (The number of ways in which "A" stands in front of "B" is equal to the number of ways "A" stands behind "B").
The probability of A standing behind B = 50% and The probability of B standing behind A = 50%.
It will be this way because A and B are equal elements. There is no reason why A should be before B in more cases than B before A. Similarly, you can compare B and C or A and C.
Hence, when we arrange all 6 people in 6! ways, in half of them Frankie will be before Joey and in the other half, Joey will be before Frankie.
The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

We want to arrange 6 people, such that Frankie is always behind Joe.

Number of ways of arranging 6 people = 6! = 720

Now think about this. What are the possible combinations that Frankie and Joe can be arranged in?

1 - Frankie is behind Joe
2 - Joe is behind Frankie

All 720 arrangements will have one of the above arrangements. Meaning,

Number of arrangements in which Frankie is behind Joe + Number of arrangements in which Joe is behind Frankie = 720

But Number of arrangements in which Frankie is behind Joe = Number of arrangements in which Frankie is behind Joe (due to symmetry. We can very well make the argument that since for every arrangement in which Frankie is behind Joe, we can switch Frankie's and Joe's position to get an arrangement where Joe is behind Frankie. So for every arrangement in which Frankie is behind Joe, there'll be an arrangement in which Joe is behind Frankie).

So, Number of arrangements in which Frankie is behind Joe = 720/2 = 360
The correct answer is D.

55.
We can imagine that the admissions committee will choose a “team” of students to receive scholarships. However, because there are three different levels of scholarships, it will not suffice to simply count the number of possible “scholarship teams.” In other words, the committee must also place the “scholarship team” members according to scholarship level. So, order matters. If we knew the number of scholarships to be granted at each of the three scholarship levels, we could use the anagram method to count the number of ways the scholarships could be doled out among the 10 applicants. To show that this is true, lets invent a hypothetical case for which there are 3 scholarships to be granted at each of the three levels. If we assign letters to the ten applicants from A to J, and if T represents a \$10,000 scholarship, F represents a \$5,000 scholarship, O represents a \$1,000 scholarship, and N represents no scholarship, then the anagram grid would look like this:

A	B	C	D	E	F	G	H	I	J
T	T	T	F	F	F	O	O	O	N

Our “word” is TTTFFFOOON. To calculate the number of different “spellings” of this “word,” we use the following shortcut:

$$\frac{10!}{(3!)(3!)(3!)(1!)}$$

The 10 represents the 10 total applicants, the 3‟s represent the 3 T‟s, 3 F‟s, and 3 O‟s, and the 1 represents the 1 N. Simplifying this expression would yield the number of ways to distribute the scholarships among the 10 applicants. So, knowing the number of scholarships to be granted at each of the three scholarship levels allows us to calculate the answer to the question. Therefore, the rephrased question is: "How many scholarships are to be granted at each of the three scholarship levels?" (1) INSUFFICIENT: While this tells us the total number of scholarships to be granted, we still dont know how many from each level will be granted. (2) INSUFFICIENT: While this tells us that the number of scholarships from each level will be equal, we still dont know how many from each level will be granted. (1) AND (2) SUFFICIENT: If there are 6 total scholarships to be granted and the same number from each level will be granted, there will be two \$10,000 scholarships, two \$5,000 scholarships, and two \$1,000 scholarships granted.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

Option A is clearly insufficient:
A certain number of $10,000, $5,000, and $1,000 scholarships" means that the numbers of each type of scholarship are FIXED. We just don't know these numbers.
If we were asked to calculate # of ways to distribute all possible numbers of scholarships (<=10), with all possible breakdowns among them (10-$10,000, 0-$5,000, 0-$1,000; 9-$10,000, 1-$5,000, 0-$1,000; ... 3-$10,000, 1-$5,000, 5-$1,000 ... huge # of combinations), then we could calculate it. But this is not what the question is asking.

Easier way to solve this problem:
T= number of 10,000 scholarships
F= number of 5,000 scholarships
O= number of 1,000 scholarships
The question stem tells us there will be three types of scholarships awarded to 10 students; however, we don't know how many of each type of scholarship will be awarded, nor do we know how many students will get scholarships. However we know that between 3 and 10 scholarships will be awarded.
Statement 1. We know six scholarships will be awarded, so T+F+O = 6.
However it could be many different combinations:
4T+1F+1O = 6 and the combinations would be 10C4*6C1*5C1
3T+2F+1O = 6 and the combinations would be 10C3*7C2*5C1
Insufficient
Statement 2. Now we know that we have either 1T+1F+1O, or 2T+2F+2O, or 3T+3F+3O, but we don't know which.
Insufficient
Together (1+2):
It must be 2T+2F+2O
So the combinations are 10C2*8C2*6C2
Answer C

Top 1% expert replies to student queries (can skip)

There are 3 types of scholarships - 10k, 5k and 1k
How many and of what type, we don't know.
We have 10 total students.
(1) In total, six scholarships will be granted.
Ok, so we know that we need to select 6 of the 10 students. But how do we split the scholarships among them? (1-1-4) or (1-2-3) or (2-2-2) etc. We don't know. We need to know how many of each type there are.
Not sufficient.
(2) An equal number of scholarships will be granted at each scholarship level.
But how many total will be granted? Is it (1-1-1) or (2-2-2) or (3-3-3)? We don't know.
Not sufficient

(1) & (2) Both together, we know that we have 2 scholarships of each type.
So for two 10k scholarships, select 2 students in 10C2 ways.
For two 5k scholarships, select 2 students in 8C2 ways.
Now for two 1k scholarships, select 2 students in 6C2 ways.
Total ways = 10C2 * 8C2 * 6C2
Answer (C)

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For statement 1, we know that the number of scholarships granted are 6. Meaning, we need to first choose 6 students out of 10, and then these 6 students will be given 6 scholarships.

Number of ways of choosing 6 students from 10 = 10C6

Now, we need to arrange 6 scholarships among 6 students, when we know some scholarships are worth 10000, some are worth 5000 and some are worth 1000? The answer is 'we don't know'.

This is because we don't know the number of scholarships worth 1000, 5000 and 10000.

For example, if we have 3 1000 dollar scholarships, 2 5000 dollar scholarships and 1 10000 scholarships.

Then, the number of ways of distributing these scholarships to 6 students = 6! /(3! * 2! * 1!) [This is just one possible case]

On the other hand, if we have 2 1000 dollar scholarships, 2 5000 dollar scholarships and 2 10000 scholarships.

Then, the number of ways of distributing these scholarships to 6 students = 6! /(2! * 2! * 2!)

As can be seen, the answer is different in the above 2 cases. Since we don't know the individual number of scholarships, we cannot answer this question. Insufficient!

We don't need to make different cases and add them all up. That is not what the question is asking. It is asking for the number of ways of awarding 6 scholarships to 6 students.
Answer (C)

56.
In order to know how many panels we can form when choosing three women and two men, we need to know how many women and men we have to choose from. In this case, we need to know the value of x (the number of women to choose from) and the value of y (the number of men to choose from). The number of panels will be equal to the number of groups of three that could be chosen from x women multiplied by the number of groups of two that could be chosen from y men. (1)

INSUFFICIENT: This statement tells us that choosing 3 from $x + 2$ would yield 56 groups. One concept that you need to know for the exam is that when dealing with combinations and permutations, each result corresponds to a unique set of circumstances. For example, if you have z people and know that choosing two of them would result in 15 different possible groups of two, it must be true that $z = 6$. No other value of z would yield exactly 15 different groups of two. So if you know how many subgroups of a certain size you can choose from an unknown original larger group, you can deduce the size of the larger group. In the present case, we know that choosing three women from $x + 2$ women would

yield 56 groups of 3. These numbers must correspond to a specific value of x. Do not worry if you do not know what value of x would yield these results (in this case, x must equal 6, because the only way to obtain 56 groups if choosing 3 is to choose from a group of 8. Since the statement tells us that 8 is 2 more than the value of x, x must be 6). The GMAT does not expect you to memorize all possible results. It is enough to understand the underlying concept: if you know the number of groups yielded (in this case 56), then you know that there is only one possible value of x.
(2) INSUFFICIENT: Knowing only that $x = y + 1$ tells us nothing specific about the values of x and y. Infinitely many values of x and y satisfy this equation, thus yielding infinitely many answers to the question. (1) AND (2) SUFFICIENT: Taking the statements together, we know that (1) gives us the value of x and that (2) allows us to use that value of x to determine the value of y. Remember that, with data sufficiency, we do not actually need to calculate the values for x and y; it is enough to know that we *can* calculate them. **The correct answer is C.**

57.
To find the total number of possible committees, we need to determine the number of different five-person groups that can be formed from a pool of 8 candidates. We will use the anagram method to solve this combinations question. First, let's create an anagram grid and assign 8 letters in the first row, with each letter representing one of the candidates. In the second row, 5 of the candidates get assigned a Y to signify that they were chosen for a committee; the remaining 3 candidates get an N, to signify that they were not chosen:

A	B	C	D	E	F	G	H
Y	Y	Y	Y	Y	N	N	N

The total number of possible five-person committees that can be created from a group of 8 candidates will be equal to the number of possible anagrams that can be formed from the word YYYYYNNN = 8! / (5!3!) = 56. Therefore, there are a total of 56 possible committees.
The correct answer is D.

Top 1% expert replies to student queries (can skip)

The answer is 8C5.
Since we are selecting a committee, the order of selection does not matter and we have a combination.
We need to select 5 members from 8; thus 8C5:

8C5 = 8!/[5!(8-5)!] = 8!/(5!3!) = (8 x 7 x 6)/3! = (8 x 7 x 6)/(3 x 2) = 8 x 7 = 56

Answer: D

58. Using the anagram method to solve this combinations question, we assign 10 letters to the 10 teams in the first row. In the second row, three of the teams are assigned numbers (1,2,3) representing gold, silver and bronze medals. The remaining seven teams get an N, to signify that they do NOT receive a medal.

A	B	C	D	E	F	G	H	I	J
1	2	3	N	N	N	N	N	N	N

The above anagram represents ONE possible way to assign the medals. The number of different possible ways to assign the three medals to three of the 10 competing teams is equal to the number of possible anagrams (arrangements of letters) that can be formed from the word 123NNNNNNN. Since there are 10 letters and 7 repeats, this equals 10! /7!

Ans A.

59.

This problem cannot be solved through formula. Given that the drawer contains at least three socks of each color, we know that at least one matched pair of each color can be removed. From the first nine socks, we can therefore make three pairs, leaving three 'orphans'. To think through the problem, it is useful to conceptualize removing those nine socks from the drawer. We will need additional information about any socks left in the drawer to solve the problem.

(1) INSUFFICIENT: Once those first nine socks have been removed, only two socks remain, but we do not have sufficient information about the color of the two socks to solve the problem. If the two remaining socks are a matched pair, we can add this final pair to the first three. This scenario results in four pairs and three orphans. However, if the final two socks are mismatched, each will make a new pair with one of the original three orphans, resulting in five pairs and one orphan.

(2) INSUFFICIENT: This statement gives no information about how many socks are in the drawer.

(1) AND (2) INSUFFICIENT: Given that the drawer contains 11 socks and that there are an equal number of black and gray socks, there are two possible scenarios. Three black, three gray, and five blue socks would yield four pairs total. Four black, four gray, and three blue socks would yield five pairs total.

The correct answer is E.

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Statement 1 tells us that the sum of the 3 different colors = 11.
With the given constraint of at least 3 of each color, we can have 5 black socks, 3 blue and 3 gray. This would give us 4 matching pairs (2*2 black, 2*1 blue, 2*1 gray).
We could also have 4 black, 4 gray, and 3 blue. This would give us 5 matching pairs. (2*2 black, 2*2 gray, 2*1 blue).

We get 2 different answers. We can eliminate A and D.

Statement 2 tells us that there is an equal number of black and gray. No need to explain this. We can have 100 black, 100 gray, hence 100 different matching pairs. (50*2 black, 50*2 gray). We could also have 4 gray and 4 black.
Unlimited different answers from this. We can eliminate B.

Statement 1+2 together:
Black and gray are equal, total number of socks = 11.
Ex: 5 blue, 3 gray and 3 black. = 4 different pairs.
4 black, 4 gray, 3 blue = 5 different pairs.
Eliminate C.
E is the correct answer.

60.

There are 3 × 2 × 4 = 24 possible different shirt-sweater-hat combinations that Kramer can wear. He wears the first one on a Wednesday. The following Wednesday he will wear the 8th combination. The next Wednesday after that he will wear the 15th combination. The next Wednesday after that he will wear the 22nd combination. On Thursday, he will wear the 23rd combination and on Friday he will wear the 24th combination.

Thus, the first day on which it will no longer be possible to wear a new combination is Saturday.

The correct answer is E.

61.
With one letter, 26 stocks can be designated.
With two letters, 26*26 stocks can be designated.
With three letters, 26*26*26 stocks can be designated. So, 26+262+263, the units digit is 8.
Answer is E

62.

Total # of different groups of 3 out of 10 people: $C_{10}^{3} = 120$;
# of groups with only junior partners (so with zero senior memeber): $C_{6}^{3} = 20$;

So the # of groups with at least one senior partner is **{all}** - **{none}** = **{at least one}** = **120** - **20** = **100**.

Answer: B.

63.

Notice that we are told that "Each number is to consist of four different digits from 0 to 9, inclusive, except that the first digit cannot be 0".

The first digit can take 9 values from 1 to 9 inclusive;
The second digit can also take 9 values (9 digits minus the one we used for the first digit plus 0);
The third digit can take 8 values;
The fourth digit can take 7 values.

Total = 9*9*8*7 = something with the units digit of 6.

Answer: B.

64.

In order the length to be minimum Pat should only go UP and RIGHT: namely thrice UP and twice RIGHT.

So combination of UUURR: # of permutations of 5 letters out of which there are 3 identical U's and 2 identical R's is 5!/3!2!=10.

Answer: C.

If there were 5 streets and 4 avenues then the answer would be combination of UUUURRR: # of permutations of 7 letters out of which there are 4 identical U's and 3 identical R's is 7!/4!3!=35.

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To reach from X to Y in the shortest possible way, we will have to take 4 steps in the x-direction and 3 steps in the y-direction. But these 7 steps can be arranged in different ways.

So, number of ways of arranging 7 steps, such that 3 steps are of one kind and the other 4 of another = 7!/(4! * 3!) = 35

65.

A derangement is a permutation in which none of the objects appear in their "natural" (i.e., ordered) place. For example, the only derangements of (1, 2, 3) are (2, 3, 1) and (3, 1, 2), so !3 = 2. The function giving the number of distinct derangements on n elements is called the sub- factorial !n and is equal to

$$!n = n!\sum_{k=0}^{n} \frac{(-1)^k}{k!}$$

If some, but not necessarily all, of the items are not in their original ordered positions, the configuration can be referred to as a partial derangement. Among the n! possible permutations of n distinct items, examine the number R(n, k) of these permutations in which exactly k items are in their original ordered positions. Then

R(n, n) = 1

R (n, n-1) = 0

$R(n,k) = \binom{n}{k} \times !(n-k)$, where $\binom{n}{k}$ denotes ${}^nC_k$ and $!(n-k)$ is the sub-factorial.

In the given question, n = 4, k = 1, R (4, 1) = ${}^4C_1 \times !(4-1) = 4 \times 2 = 8$. Total number of arrangements = ${}^4P_4 = 4! = 24$. **Answer = 8/24 = 1/3.**

Top 1% expert replies to student queries (can skip)

Fundamental law of counting first:
What is the sample space, i.e., what is the total number of ways in which 4 letters can be put into 4 envelopes and swirled around? 4! = 24 ways.

Let's say the four letters are L1, L2, L3 and L4; the four addresses are A1, A2, A3, A4.

Now let's say only L1 will go into A1 - that can happen in 1 way. Now L2 cannot go into A2, so one of L3 or L4 will go into A2 (2 ways). Let's say L3 does. That leaves us L2, L4 and A3, A4. Now L4 cannot go into A4, so it has to go into A3 (1 way). Then L2 has to go into A4 (1 way). Going back to the bolded part (that is where our logic branched out), If L4 goes into A2, you can again check that L2 and L3 can go into A3 and A4 in 1 way each. So in total, when L1 goes into A1, here are 1x2x1x1 ways = 2 ways to arrange the letters. Also, at this point it should be amply clear to you that this is perfectly symmetrical for all letter and address pairs. Then if L2, A2 was the correct pair, we have another 2 ways and so on. Then total number of ways in which only one letter-address pair will be correct is 2+2+2+2 = 8 ways.

Probability = 8/24 = 1/3.

Let's look at an nCr-based explanation now:
Again the sample space is 4! = 24 ways

You can select a correct letter-address pair in 4C1 ways, and total number of ways in which only one letter-address pair is correct is 4C1 x (all other pairs are incorrect)
= 4C1 x (Total number of ways the other three letters can be arranged in envelopes (I) - Ways in which all three are in their correct envelopes (II) - Ways in which two letters are in the correct envelopes (III) - Ways in which one letter in its correct envelope (IV))

I - This is 3! = 6 ways
II - This is only 1 way
III - Note that when two letters out of three are in their correct envelopes, then all three are in their correct envelopes by default. Then this can also be done 1 way and has already been taken into account in II, so doesn't need to be double-counted
IV - Number of ways of selecting 1 letter out of 3 is 3C1 and then there is only 1 way in which the other two can be put in respectively incorrect envelopes. Then this can be done in 3 ways

Then total number of ways = 4C1 x (6 - 1 - 3) = 4 x 2 = 8 ways

Probability = 8/24 = 1/3

66.

We have a case of circular arrangement.

The number of arrangements of n distinct objects in a row is given by $n!$.
The number of arrangements of n distinct objects in a circle is given by $(n-1)!$.

From Gmat Club Math Book (combinatorics chapter):
"The difference between placement in a row and that in a circle is following: if we shift all object by one position, we will get different arrangement in a row but the same relative arrangement in a circle. So, for the number of circular arrangements of n objects we have:

$R=\frac{n!}{n}=(n-1)!$"

$(n-1)!=(5-1)!=24$

Answer: C.

GMAT Quant Topic 8: Probability

1.
There are 2 possible outcomes on each flip: heads or tails. Since the coin is flipped three times, there are $2 \times 2 \times 2 = 8$ total possibilities: HHH, HHT, HTH, HTT, TTT, TTH, THT, THH.
Of these 8 possibilities, how many involve exactly two heads? We can simply count these up: HHT, HTH, THH. We see that there are 3 outcomes that involve exactly two heads. Thus, the correct answer is 3/8.
Alternatively, we can draw an anagram table to calculate the number of outcomes that involve exactly 2 heads.

A	B	C
H	H	T

The top row of the anagram table represents the 3 coin flips: A, B, and C. The bottom row of the anagram table represents one possible way to achieve the desired outcome of exactly two heads. The top row of the anagram yields 3!, which must be divided by 2! since the bottom row of the anagram table contains 2 repetitions of the letter H. There are $3!/2! = 3$ different outcomes that contain exactly 2 heads.
The probability of the coin landing on heads exactly twice is (# of two-head results) ÷ (total # of outcomes) = 3/8.
The correct answer is B.

2.
Let us say that there are n questions on the exam. Let us also say that p_1 is the probability that Patty will get the first problem right, and p_2 is the probability that Patty will get the second problem right, and so on until p_n , which is the probability of getting the last problem right. Then the probability that Patty will get all the questions right is just $p_1 \times p_2 \times \ldots \times p_n$. We are being asked whether $p_1 \times p_2 \times \ldots \times p_n$ is greater than 50%.
(1) INSUFFICIENT: This tells us that for each question, Patty has a 90% probability of answering correctly. However, without knowing the number of questions, we cannot determine the probability that Patty will get all the questions correct.
(2) INSUFFICIENT: This gives us some information about the number of questions on the exam but no information about the probability that Patty will answer any one question correctly.
(1) AND (2) INSUFFICIENT: Taken together, the statements still do not provide a definitive "yes" or "no" answer to the question. For example, if there are only 2 questions on the exam, Patty's probability of answering all the questions correctly is equal to $.90 \times .90 = .81 = 81\%$. On the other hand if there are 7 questions on the exam, Patty's probability of answering all the questions correctly is equal to $.90 \times .90 \times .90 \times .90 \times .90 \times .90 \times .90 \approx 48\%$. We cannot determine whether Patty's chance of getting a perfect score on the exam is greater than 50%.
The correct answer is E.

3.
In order to solve this problem, we have to consider two different scenarios. In the first scenario, a woman is picked from room A and a woman is picked from room B. In the second scenario, a man is picked from room A and a woman is picked from room B.
The probability that a woman is picked from room A is 10/13. If that woman is then added to room B, this means that there are 4 women and 5 men in room B (Originally there were 3 women and 5 men). So, the probability that a woman is picked from room B is 4/9.
Because we are calculating the probability of picking a woman from room A AND then from room B, we need to multiply these two probabilities:
$10/13 \times 4/9 = 40/117$
The probability that a man is picked from room A is 3/13. If that man is then added to room B, this means that there are 3 women and 6 men in room B. So, the probability that a woman is picked from room B is 3/9.
Again, we multiply these two probabilities:

3/13 x 3/9 = 9/117
To find the total probability that a woman will be picked from room B, we need to take both scenarios into account. In other words, we need to consider the probability of picking a woman and a woman OR a man and a woman. In probabilities, OR means addition. If we add the two probabilities, we get:
40/117 + 9/117 = 49/117
The correct answer is B.

4.
The period from July 4 to July 8, inclusive, contains 8 – 4 + 1 = 5 days, so we can rephrase the question as ―What is the probability of having exactly 3 rainy days out of 5?‖
Since there are 2 possible outcomes for each day (R = rain or S = shine) and 5 days total, there are 2 x 2 x 2 x 2 x 2 = 32 possible scenarios for the 5 day period (RRRSS, RSRSS, SSRRR, etc…) To find the probability of having exactly three rainy days out of five, we must find the total number of scenarios containing exactly 3 R‘s and 2 S‘s, that is the number of possible RRRSS anagrams:
= 5! / 2!3! = (5 x 4)/2 x 1 = 10
The probability then of having exactly 3 rainy days out of five is 10/32 or 5/16.
Note that we were able to calculate the probability this way because the probability that any given scenario would occur was the same. This stemmed from the fact that the probability of rain = shine = 50%. Another way to solve this question would be to find the probability that one of the favourable scenarios would occur and to multiply that by the number of favorable scenarios. In this case, the probability that RRRSS (1st three days rain, last two shine) would occur is (1/2)(1/2)(1/2)(1/2)(1/2) = 1/32. There are 10 such scenarios (different anagrams of RRRSS) so the overall probability of exactly 3 rainy days out of 5 is again 10/32. This latter method works even when the likelihood of rain does not equal the likelihood of shine.
The correct answer is C.

5.
There are four possible ways to pick exactly one defective car when picking four cars: DFFF, FDFF, FFDF, FFFD (D = defective, F = functional).
To find the total probability we must find the probability of each one of these scenarios and add them together (we add because the total probability is the first scenario OR the second OR…). The probability of the first scenario is the probability of picking a defective car first (3/20) AND then a functional car (17/19) AND then another functional car (16/18) AND then another functional car (15/17).
The probability of this first scenario is the product of these four probabilities: 3/20 x 17/19 x 16/18 x 15/17 = 2/19
The probability of each of the other three scenarios would also be 2/19 since the chance of getting the D first is the same as getting it second, third or fourth.
The total probability of getting exactly one defective car out of four = 2/19 + 2/19 + 2/19 + 2/19 = 8/19.

6.
The simplest way to solve the problem is to recognize that the total number of gems in the bag must be a multiple of 3, since we have 2/3 diamonds and 1/3 rubies. If we had a total number that was not divisible by 3, we would not be able to divide the stones into thirds. Given this fact, we can test some multiples of 3 to see whether any fit the description in the question.
The smallest number of gems we could have is 6: 4 diamonds and 2 rubies (since we need at least 2 rubies). Is the probability of selecting two of these diamonds equal to 5/12?
4/6 × 3/5 = 12/30 = 2/5. Since this does not equal 5/12, this cannot be the total number of gems.
The next multple of 3 is 9, which yields 6 diamonds and 3 rubies:

6/9 × 5/8 = 30/72 = 5/12. Since this matches the probability in the question, we know we have 6 diamonds and 3 rubies. Now we can figure out the probability of selecting two rubies:
3/9 × 2/8 = 6/72 = 1/12
The correct answer is C.

Top 1% expert replies to student queries (can skip) (additional)
Let the total number of gemstones be x

Number of diamonds = 2x/3
Number of rubies = x/3

Probability of choosing 2 diamonds from the bag = ${}^{(2x/3)}C_2$ / ${}^{x}C_2$ = (2x/3)(2x/3 - 1)/x(x-1) = 5/12

[2x/3 * (2x/3 - 1)] / x * (x-1) = 5/12

12 * 2x/3 * (2x/3 - 1) = 5x(x-1)

8x(2x/3 - 1) = 5x^2 - 5x

16x^2/3 - 8x = 5x^2 - 5x

3x = x^2/3

x = 9 (Total number of gemstones)

Number of diamonds = 2x/3 = 6
Number of rubies = x/3 = 3

So, probability of choosing 2 rubies without replacement = 3C2 / 9C2 = 3/36 = 1/12

The correct answer is C

7.
For probability, we always want to find the number of ways the requested event could happen and divide it by the total number of ways that any event could happen.
For this complicated problem, it is easiest to use combinatorics to find our two values. First, we find the total number of outcomes for the triathlon. There are 9 competitors; three will win medals and six will not. We can use the Combinatorics Grid, a counting method that allows us to determine the number of combinations without writing out every possible combination.

A	B	C	D	E	F	G	H	I
Y	Y	Y	N	N	N	N	N	N

Out of our 9 total places, the first three, A, B, and C, win medals, so we label these with a "Y." The final six places (D, E, F, G, H, and I) do not win medals, so we label these with an "N." We translate this into math: 9! / 3!6! = 84. So our total possible number of combinations is 84. (Remember that ! means factorial; for example, 6! = 6 × 5 × 4 × 3 × 2 × 1.)
Note that although the problem seemed to make a point of differentiating the first, second, and third places, our question asks only whether the brothers will medal, not which place they will win. This is why we don't need to worry about labeling first, second, and third place distinctly. Now, we need to determine the number of instances when at least two brothers win a medal. Practically speaking, this means we want to add the number of instances two brothers win to the number of instances three brothers win.
Let's start with all three brothers winning medals, where B represents a brother.

A	B	C	D	E	F	G	H	I
B	B	B	N	N	N	N	N	N

Since all the brothers win medals, we can ignore the part of the counting grid that includes those who don't win medals. We have 3! / 3! = 1. That is, there is only one instance when all three brothers win medals.
Next, let's calculate the instances when exactly two brothers win medals.

A	B	C	D	E	F	G	H	I
B	B	Y	B'	N	N	N	N	N

Since brothers both win and don't win medals in this scenario, we need to consider both sides of the grid (i.e. the ABC side and the DEFGHI side). First, for the three who win medals, we have 3! / 2! = 3. For the six who don't win medals, we have 6! / 5! = 6. We multiply these two numbers to get our total number: 3 × 6 = 18.

Another way to consider the instances of at least two brothers medalling would be to think of simple combinations with restrictions.

If you are choosing 3 people out of 9 to be winners, how many different ways are there to chose a specific set of 3 from the 9 (i.e. all the brothers)? Just one. Therefore, there is only one scenario of all three brothers medalling.

If you are choosing 3 people out of 9 to be winners, if 2 specific people of the 9 have to be a member of the winning group, how many possible groups are there? It is best to think of this as a problem of choosing 1 out of 7 (2 must be chosen). Choosing 1 out of 7 can be represented as 7! / 1!6! = 7. However, if 1 of the remaining 7 cannot be a member of this group (in this case the 3rd brother) there are actually only 6 such scenarios. Since there are 3 different sets of exactly two brothers (B_1B_2, B_1B_3, B_2B_3), we would have to multiply this 6 by 3 to get 18 scenarios of only two brothers medalling.

The brothers win at least two medals in 18 + 1 = 19 circumstances. Our total number of circumstances is 84, so our probability is 19 / 84.

The correct answer is B.

Top 1% expert replies to student queries (can skip)

B1 B2 B3 are the 3 brothers, P1 P2 P3 P4 P5 P6 ae the other 6 people, for a total of 9 contestants

How many ways can 3 people win medals? We are not concerned about who wins what medal, just the number of ways in which people can win medals. That is the number of ways of selecting 3 people out of 9 = 9C3 = 9!/3!6! = 84

Now if all three brothers have won medals, then it becomes the number of ways of selecting 3 people among B1 B2 B3 and 0 people out of P1 P2 P3 P4 P5 P6

This is achieved in 3C3 x 6C0 = 1 way

If two brothers have won medals, it becomes a case of selecting any 2 brothers out of B1 B2 B3 and selecting one other person to medal from among P1 P2 P3 P4 P5 P6

Then this is 3C2 x 6C1 = 3 x 6 = 18 ways

Total number of ways in which at least 2 brothers have won medals = 1 + 18 = 19

Probability = 19/84

Top 1% expert replies to student queries (can skip) (additional)

Let the 3 brothers be A, B and C

Case 1 : 2 out of A,B and C get 2 medals and the remaining medal goes to someone from the remaining 6 people.

Number of ways of distributing the medals = 3C2 * 6C1 * 3! = 3*6*6 = 108

Case 2 : All 3 medals go to the triplets

Number of ways of distributing the medals = 3C3 * 6C0 * 3! = 6

Therefore, number of ways at least 2 triplets win medals = 114

Total number of ways of distributing 3 medals among 9 people? 9P3 = 504

Probability = 114/504 = 57/252 = 19/84

8.
If set S is the set of all prime integers between 0 and 20 then:
S = {2, 3, 5, 7, 11, 13, 17, 19}
Let's start by finding the probability that the product of the three numbers chosen is a number less than 31. To keep the product less than 31, the three numbers must be 2, 3 and 5. So, what is the probability that the three numbers chosen will be some combination of 2, 3, and 5?
Here's the list all possible combinations of 2, 3, and 5:
case A: 2, 3, 5
case B: 2, 5, 3
case C: 3, 2, 5
case D: 3, 5, 2
case E: 5, 2, 3
case F: 5, 3, 2
This makes it easy to see that when 2 is chosen first, there are two possible combinations. The same is true when 3 and 5 are chosen first. The probability of drawing a 2, AND a 3, AND a 5 in case A is calculated as follows (remember, when calculating probabilities, AND means multiply): case A: (1/8) x (1/7) x (1/6) = 1/336
The same holds for the rest of the cases. case B: (1/8) x (1/7) x (1/6) = 1/336 case C: (1/8) x (1/7) x (1/6) = 1/336 case D: (1/8) x (1/7) x (1/6) = 1/336 case E: (1/8) x (1/7) x (1/6) = 1/336 case F: (1/8) x (1/7) x (1/6) = 1/336
So, a 2, 3, and 5 could be chosen according to case A, OR case B, OR, case C, etc. The total probability of getting a 2, 3, and 5, in any order, can be calculated as follows (remember, when calculating probabilities, OR means add):
(1/336) + (1/336) + (1/336) + (1/336) + (1/336) + (1/336) = 6/336
Now, let's calculate the probability that the sum of the three numbers is odd. In order to get an odd sum in this case, 2 must NOT be one of the numbers chosen. Using the rules of odds and evens, we can see that having a 2 would give the following scenario:
even + odd + odd = even
So, what is the probability that the three numbers chosen are all odd? We would need an odd AND another odd, AND another odd:
(7/8) x (6/7) x (5/6) = 210/336
The positive difference between the two probabilities is:
(210/336) – (6/336) = (204/336) = 17/28
The **correct answer is C.**

Top 1% expert replies to student queries (can skip)

Simpler and straightforward way to solve this:
(1) probability product less than 31
only combo works with 2, 3, 5 --> 3C3
Total combinations= 8C3
Probability = 3C3/8C3 = 1/56
(2) probability odd
Any combo that doesn't have 2 works
7C3 ways to pick without choosing 2
Total combinations= 8C3
Probability = 7C3/8C3 = 35/56 = 5/8
35/56-1/56 =34/56= 17/28
The **correct answer is C.**

Top 1% expert replies to student queries (can skip) (additional)

1st 8 prime numbers. Then 8C3 because I have to take any 3 numbers which would go in denominator of both the probabilities.
8C3=56

1. 3 numbers less than 31 which when multiplied should give 30 are 2, 3, and 5. 3c3 is 1.

1/56

2. Sum of these 3 numbers odd: 7C3. Not considering 2 bcos O +O + E would make it even. So, 7C3/8C3= 5/8

1/56- 5/8 = 1-35/56 = 34/56 = 17/28

9.
To find the probability of forming a code with two adjacent I's, we must find the total number of such codes and divide by the total number of possible 10-letter codes.
The total number of possible 10-letter codes is equal to the total number of anagrams that can be formed using the letters ABCDEFGHII, that is 10!/2! (we divide by 2! to account for repetition of the I's).
To find the total number of 10 letter codes with two adjacent I's, we can consider the two I's as ONE LETTER. The reason for this is that for any given code with adjacent I's, wherever one I is positioned, the other one must be positioned immediately next to it. For all intents and purposes, we can think of the 10 letter codes as having 9 letters (I-I is one). There are 9! ways to position 9 letters.
Probability = (# of adjacent I codes) / (# of total possible codes)
= 9! ÷ (10! / 2!) = (9!2! / 10!) = (9!2! / 10(9!)) = 1/5
The correct answer is C.

10.
If we factor the right side of the equation, we can come up with a more meaningful relationship between p and q: $p^2 - 13p + 40 = q$ so $(p - 8)(p - 5) = q$. We know that p is an integer between 1 and 10, inclusive, so there are ten possible values for p. We see from the factored equation that the sign of q will depend on the value of p. One way to solve this problem would be to check each possible value of p to see whether it yields a positive or negative q.
However, we can also use some logic here. For q to be negative, the expressions $(p - 8)$ and $(p - 5)$ must have opposite signs. Which integers on the number line will yield opposite signs for the expressions $(p - 8)$ and $(p - 5)$? Those integers in the range $5 < p < 8$ (notice 5 and 8 are not included because they would both yield a value of zero and zero is a nonnegative integer). That means that there are only two integer values for p, 6 and 7, that would yield a negative q. With a total of 10 possible p values, only 2 yield a negative q, so the probability is 2/10 or 1/5.
The correct answer is B.

11.
The simplest way to approach a complex probability problem is not always the direct way. In order to solve this problem directly, we would have to calculate the probabilities of all the different ways we could get two opposite-handed, same-colored gloves in three picks. A considerably less taxing approach is to calculate the probability of NOT getting two such gloves and subtracting that number from 1 (remember that the probability of an event occurring plus the probability of it NOT occurring must equal 1).
Let's start with an assumption that the first glove we pick is blue. The hand of the first glove is not important; it could be either right or left. So our first pick is any blue. Since there are 3 pairs of blue gloves and 10 gloves total, the probability of selecting a blue glove first is 6/10.
Let's say our second pick is the same hand in blue. Since there are now 2 blue gloves of the same hand out of the 9 remaining gloves, the probability of selecting such a glove is 2/9.
Our third pick could either be the same hand in blue again or any green. Since there is now 1 blue glove of the same hand and 4 green gloves among the 8 remaining gloves, the probability of such a pick is (1 + 4)/8 or 5/8.
The total probability for this scenario is the product of these three individual probabilities: 6/10 x 2/9 x 5/8 = 60/720.
We can summarize this in a chart:

Pick	**Color/Hand**	**Probability**
1st	blue/any	6/10
2nd	blue/same	2/9
3rd	blue/same or any green	5/8
total		6/10 x 2/9 x 5/8 = **60/720**

We can apply the same principles to our second scenario, in which we choose blue first, then any green, then either the same-handed green or the same-handed blue:

Pick	Color/Hand	Probability
1st	blue/any	6/10
2nd	green/any	4/9
3rd	green/same or blue/same	(2+1)/8
total		6/10 x 4/9 x 3/8 = **72/720**

But it is also possible to pick green first. We could pick any green, then the same-handed green, then any blue:

Pick	Color/Hand	Probability
1st	green/any	4/10
2nd	green/same	1/9
3rd	blue/any	6/8
total		4/10 x 1/9 x 6/8 = **24/720**

Or we could pick any green, then any blue, then the same-handed green or same-handed blue:

Pick	Color/Hand	Probability
1st	green/any	4/10
2nd	blue/any	6/9
3rd	green/same or blue/same	(1 + 2)/8
total		4/10 x 6/9 x 3/8 = **72/720**

The overall probability of NOT getting two gloves of the same color and same hand is the SUM of the probabilities of these four scenarios: **60/720 + 72/720 + 24/720 + 72/720 = 228/720 = 19/60.**

Therefore, the probability of getting two gloves of the same color and same hand is **1 - 19/60 = 41/60**.

The correct answer is D.

12.
Every player has an equal chance of leaving at any particular time. Thus, the probability that four particular players leave the field first is equal to the probability that any other four players leave the field first. In other words, the answer to this problem is completely independent of which four players leave first.

Given the four players that leave first, there are 4! or 24 orders in which these players can leave the field - only one of which is in increasing order of uniform numbers. (For example, assume the players have the numbers 1, 2, 3, and 4. There are 24 ways to arrange these 4 numbers: 1234, 1243, 1324, 1342, 1423, 1432, . . . , etc. Only one of these arrangements is in increasing order.)

Thus, the probability that the first four players leave the field in increasing order of their uniform numbers is 1/24.
The correct answer is D.

13.

In order for one number to be the reciprocal of another number, their product must equal 1. Thus, this question can be rephrased as follows:

What is the probability that $\frac{\frac{u}{v}}{w} \times \frac{\frac{x}{y}}{z} = 1$?

This can be simplified as follows:

What is the probability that $\frac{\frac{ux}{vy}}{wz} = 1$?

What is the probability that $\frac{ux}{vy} = wz$?

Finally: What is the probability that ux = vywz ?

Statement (1) tells us that vywz is an integer, since it is the product of integers. However, this gives no information about u and x and is therefore not sufficient to answer the question.

Statement (2) tells us that ux is NOT an integer. This is because the median of an even number of consecutive integers is NOT an integer. (For example, the median of 4 consecutive integers - 9, 10, 11, 12 - equals 10.5.) However, this gives us no information about vywz and is therefore not sufficient to answer the question.

Taking both statements together, we know that vywz IS an integer and that ux is NOT an integer. Therefore vywz CANNOT be equal to ux. The probability that the fractions are reciprocals is zero.

The correct answer is C.

14.

Since each die has 6 possible outcomes, there are 6 × 6 = 36 different ways that Bill can roll two dice. Similarly there are 6 × 6 = 36 different ways for Jane to roll the dice. Hence, there are a total of 36 × 36 = 1296 different possible ways the game can be played.

One way to approach this problem (the hard way) is to consider, in turn, the number of ways that Bill can get each possible score, compute the number of ways that Jane can beat him for each score, and then divide by 1296.

The number of ways to make each score is: 1 way to make a 2 (1 and 1), 2 ways to make a 3 (1 and 2, or 2 and 1), 3 ways to make a 4 (1 and 3, 2 and 2, 3 and 1), 4 ways to make a 5 (use similar reasoning…), 5 ways to make a 6, 6 ways to make a 7, 5 ways to make an 8, 4 ways to make a 9, 3 ways to make a 10, 2 ways to make an 11, and 1 way to make a 12.

We can see that there is only 1 way for Bill to score a 2 (1 and 1). Since there are 36 total ways to roll two dice, there are 35 ways for Jane to beat Bob's 2.

Next, there are 2 ways that Bob can score a 3 (1 and 2, 2 and 1). There are only three ways in which Jane would not beat Bob: if she scores a 2 (1 and 1), she would lose to Bob or if she scores a 3 (1 and 2, 2 and 1), she would tie Bob. Since there are 36 total ways to roll the dice, Jane has 33 ways to beat Bob.

Using similar logic, we can quickly create the following table:

Score	**Ways Bill Can Make It**	**Ways Jane Can Beat Bill**	**Total Combinations**
2	1	35	1 × 35 = 35
3	2	33	2 × 33 = 66

4	3	30	3 × 30 = 90
5	4	26	4 × 26 = 104
6	5	21	5 × 21 = 105
7	6	15	6 × 15 = 90
8	5	10	5 × 10 = 50
9	4	6	4 × 6 = 24
10	3	3	3 × 3 = 9
11	2	1	2 × 1 = 2
12	1	0	1 × 0 = 0
			Total = 575

Out of the 1296 possible ways the game can be played, 575 of them result in Jane winning the game. Hence, the probability the Jane will win is 575/1296 and the correct answer is C.
There is a much easier way to compute this probability. Observe that this is a —symmetric‖ game in that neither Bill nor Jane has an advantage over the other. That is, each has an equal change of winning. Hence, we can determine the number of ways each can win by subtracting out the ways they can tie and then dividing the remaining possibilities by 2.

Note that for each score, the number of ways that Jane will tie Bill is equal to the number of ways that Bill can make that score (i.e., both have an equal number of ways to make a particular score).

Thus, referring again to the table above, the total number of ways to tie are: $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 146$. Therefore, there are 1296 – 146 = 1150 non-ties. Since this is a symmetric problem, Jane will win 1150/2 or 575 times out of the 1296 possible games. Hence, the probability that she will win is 575/1296.

Top 1% expert replies to student queries (can skip)

There are three possible outcomes in the game:
1. B Wins
2. J Wins
3. Draw

So, 1=P(J)+P(B)+P(Draw)

The probability of B winning or J winning the game is same.
so, 1=2*P(J)+P(Draw)

So, we need to calculate the probability of a draw.
A draw can happen when both will have the same sum.
The possible sums are - 2,3,4,5,6,7,8,9,10,11,12
Prob. of each sum is -
2 - 1,1 - 1/36
3 - 1 2, 2 1 - 2/36
4 - 1 3, 2 2, 3 1 - 3/36
5 - 1 4, 2 3, 3 2, 41 - 4/36
6 - 1 5, 2 4, 3 3, 4 2, 5 1 - 5/36
7 - 1 6, 2 5, 3 4, 4 3, 5 2, 61 - 6/36
8 - 2 6, 3 5, 4 4, 5 3, 6 2, - 5/36
9 - 3 6, 4 5, 5 4, 63 - 4/36
10 - 4 6, 5 5, 6 4 - 3/36
11 - 5 6, 6 5 - 2/36
12 - 6 6 - 1/36

Now, since both have to get the same sum, the probability will be same for both and the combined probability will be the square of the probabilities
i.e. 2 - (1/36)^2
3 - (2/36)^2
etc.

Squaring the probabilities and adding them will give 146/1296

So, P(D) = 146/1296
1 = 2*P(J) + P(D)
2*P(J) = 1150/1296
P(J) = 575 / 1296

Top 1% expert replies to student queries (can skip)

Consider the case of Bill
He can have 6C1*6C1= 36 possibilities for the throw.

Same for Jane, She can have 6C1*6C1= 36 possibilities for the throw.

Thus total possible cases= 36*36= 1296

Number of ways both Bill and Jane get a 2:
Bill: (1,1) and Jane: (1,1) = 1
This is same for the case when the both get 12:
Bill: (6,6) and Jane: (6,6) = 1

Number of ways both Bill and Jane get a 3:
Bill: (1,2|2,1) and Jane: (1,2|2,1) = 2C1*2C1= 4
This is same for the case when the both get 11:
Bill: (5,6|6,5) and Jane: (5,6|6,5) = 4

Number of ways both Bill and Jane get a 4:
Bill: (1,3|3,1|2,2) and Jane: (1,3|3,1|2,2) = 3C1*3C1= 9
This is same for the case when the both get 10:
Bill: (4,6|6,4|5,5) and Jane: (4,6|6,4|5,5) = 9

Number of ways both Bill and Jane get a 5:
Bill: (1,4|4,1|2,3|3,2) and Jane: (1,4|4,1|2,3|3,2) = 4C1*4C1= 16
This is same for the case when the both get 9:
Bill: (3,6|6,3|5,4|4,5) and Jane: (3,6|6,3|5,4|4,5) = 16

Number of ways both Bill and Jane get a 6:
Bill: (2,4|4,2|3,3|1,5|5,1) and Jane: (2,4|4,2|3,3|1,5|5,1) = 5C1*5C1= 25
This is same for the case when the both get 8:
Bill: (2,6|6,2|5,3|3,5|4,4) and Jane: (2,6|6,2|5,3|3,5|4,4) = 25

Number of ways both Bill and Jane get a 7:
Bill: (3,4|4,3|2,5|5,2|6,1|1,6) and Jane: (3,4|4,3|2,5|5,2|6,1|1,6) = 6C1*6C1= 36

Total number of ways for draw:
2*1 + 2*4 + 2*9 + 2*16 + 2*25 + 36= 146

Thus total ways in which either Bill or Jane wins:
1296-146= 1150

Jane will win in half the cases whereas Bill will win in half the cases
Hence Jane wins in 1150/2= 575 cases

Thus total probability
575/1296

Top 1% expert replies to student queries (can skip) (additional)

Query: Why are the number of possibilities squared?
Reply: Case 1:

Bill and Jane both get the sum as 2 [Sum = 1 cannot be obtained]

Number of ways of getting 2 as the total = (1,1)

Number of ways Bill can get 2 as the sum = 1C1 [We have only 1 possibility of getting 2 as the sum. Number of ways of choosing 1 way from these possibilities = 1C1]
Number of ways Jane can get 2 as the sum = 1C1 [We have only 1 possibility of getting 2 as the sum. Number of ways of choosing 1 way from these possibilities = 1C1]

Number of ways of both Jane and Bill would both get 2 as the sum = 1C1 * 1C1 = (1)^2

Case 2 :

Bill and Jane both get the sum as 3

Number of ways of getting 2 as the total = [(2,1), (1,2)]

Number of ways Bill can get 3 as the sum = 2C1 [We have 2 possibilities of getting 3 as the sum. Number of ways of choosing 1 way from these possibilities = 2C1]
Number of ways Jane can get 3 as the sum = 2C1 [We have 2 possibilities of getting 3 as the sum. Number of ways of choosing 1 way from these possibilities = 2C1]

Number of ways of both Jane and Bill would both get 3 as the sum = 2C1 * 2C1 = (2)^2

Case 2 :

Bill and Jane both get the sum as 4

Number of ways of getting 2 as the total = [(2,2), (1,3), (3,1)]

Number of ways Bill can get 4 as the sum = 3C1 [We have 3 possibilities of getting 4 as the sum. Number of ways of choosing 1 way from these possibilities = 3C1]
Number of ways Jane can get 4 as the sum = 3C1 [We have 3 possibilities of getting 4 as the sum. Number of ways of choosing 1 way from these possibilities = 3C1]

Number of ways of both Jane and Bill would both get 4 as the sum = 3C1 * 3C1 = (3)^2

Continue this analysis for all possible sums till 12.

The total number of ways Jane and Bill get the same score = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 146

15.
Let's consider the different scenarios:

If Kate wins all five flips, she ends up with $15.
If Kate wins four flips, and Danny wins one flip, Kate is left with $13. If Kate wins three flips, and Danny wins two flips, Kate is left with $11. If Kate wins two flips, and Danny wins three flips, Kate is left with $9. If Kate wins one flip, and Danny wins four flips, Kate is left with $7. If Kate loses all five flips, she ends up with $5.

The question asks for the probability that Kate will end up with more than $10 but less than $15. In other words, we need to determine the probability that Kate is left with $11 or $13 (since there is no way Kate can end up with $12 or $14).

The probability that Kate ends up with $11 after the five flips:
Since there are 2 possible outcomes on each flip, and there are 5 flips, the total number of possible outcomes is 2 x 2 x 2 x 2 x 2 = 32. Thus, the five flips of the coin yield 32 different outcomes.

To determine the probability that Kate will end up with $11, we need to determine how many of these 32 outcomes include a combination of exactly three winning flips for Kate.

We can create a systematic list of combinations that include three wins for Kate and two wins for Danny:

DKKKD, DKKDK, DKDKK, DDKKK, KDKKD, KDKDK, KDDKK, KKDKD, KKDDK, KKKDD = 10 ways

Alternatively, we can consider each of the five flips as five spots. There are 5 potential spots for Kate's first win. There are 4 potential spots for Kate's second win (because one spot has already been taken by Kate's first win). There are 3 potential spots for Kate's third win. Thus, there Are 5 x 4 x 3 = 60 ways for Kate's three victories to be ordered.

However, since we are interested only in unique winning combinations, this number must be reduced due to overcounting. Consider the winning combination KKKDD: This one winning combination has actually been counted 6 times (this is 3! or three factorial) because there are 6 different orderings of this one combination:

$K_1K_2K_3DD$, $K_1K_3K_2DD$, $K_2K_3K_1DD$, $K_2K_1K_3DD$, $K_3K_2K_1DD$, $K_3K_1K_2DD$

This overcounting by 6 is true for all of Kate's three-victory combinations. Therefore, there are Only 60 / 6 = 10 ways for Kate to have three wins and end up with $11 (as we had discovered earlier from our systematic list).

The probability that Kate ends up with $13 after the five flips:

To determine the probability that Kate will end up with $13, we need to determine how many of the 32 total possible outcomes include a combination of exactly four winning flips for Kate.

Again, we can create a systematic list of combinations that include four wins for Kate and one win for Danny: KKKKD, KKKDK, KKDKK, KDKKK, DKKKK = 5 ways.

Alternatively, using the same reasoning as above, we can determine that there Are 5 x 4 x 3 x 2 = 120 ways for Kate's four victories to be ordered. Then, reduce this by 4! (four factorial) or 24 due to overcounting. Thus, there are 120/24 = 5 ways for Kate to have four wins and end up with $13 (the same answer we found using the systematic list).

The total probability that Kate ends up with either $11 or $13 after the five flips:

There are 10 ways that Kate is left with $11. There are 5 ways that Kate is left with $13. Therefore, there are 15 ways that Kate is left with more than $10 but less than $15. Since there are 32 possible outcomes, the correct answer is 15/32, answer choice **D**.

16.

There is a strong temptation to solve this problem by simply finding the probability that it will snow (90%) and the probability that schools will be closed (80%) and multiplying these two probabilities. This approach would yield the incorrect answer (72%), choice D.

However, it is only possible to multiply probabilities of separate events if you know that they are independent from each other. This fact is not provided in the problem. In fact, we would assume that school being closed and snow are, at least to some extent, dependent on each other.

However, they are not entirely dependent on each other; it is possible for either one to happen without the other. Therefore, there is an unknown degree of dependence; hence there is a range of possible probabilities, depending on to what extent the events are dependent on each other.

Set up a matrix as shown below. Fill in the probability that schools will not be closed and the probability that there will be no snow.

	Schools closed	Schools not closed	TOTAL
Snow			
No snow			**10**
TOTAL		**20**	**100**

Then use subtraction to fill in the probability that schools will be closed and the probability that there will be snow.

	Schools closed	Schools not closed	TOTAL
Snow			**90**
No snow			**10**
TOTAL	**80**	**20**	**100**

To find the greatest possible probability that schools will be closed and it will snow, fill in the remaining cells with the largest possible number in the upper left cell.

	Schools closed	Schools not closed	TOTAL
Snow	**80**	10	**90**
No snow	0	10	**10**
TOTAL	**80**	**20**	**100**

The greatest possible probability that schools will be closed and it will snow is 80%.
The correct answer is E.

Top 1% expert replies to student queries (can skip)

Snowing and schools closing are two non-mutually exclusive, non-independent events. That is to say, one happening doesn't preclude the other from happening in any way, but one happening distinctly has an effect on the other (you should see Canadian winters, schools are sometimes closed for days simply because it is impossible to move around with that much ice and snow!)

Now, when two events are non-mutually exclusive but non-independent, what is a general formula we have?

P(A) = Probability of event that it snows = 1 - Probability of event that it does not snow = 1 - 0.1 = 0.9
P(B) = Probability of event that schools are closed = 1 - Probability of event that schools are not closed = 1 - 0.2 = 0.8

P(A union B) = P(A) + P(B) - P(A intersection B) ...(i)

P(A union B) means the probability of at least one of the two events happening
P(A intersection B) = Probability that both the events happen

We are interested in finding the maximum value of P(A intersection B)

Rearranging equation (i) above, we have

P(A intersection B) = P(A) + P(B) - P(A union B)

P(A) and P(B) are as is and cannot be changed, then P(A intersection B) will be maximised when P(A union B) is the lowest it can be

What is the minimum value of P(A union B) i.e. when is the probability that A happens or B happens or both happen, minimized? As these are non-independent (i.e. dependent) events, this probability cannot possibly be lower than the higher of the two probabilities; so it is 0.9

Then max P(A intersection B) = 0.9 + 0.8 - 0.9 = 0.8
The correct answer is E.

17.

The easiest way to attack this problem is to pick some real, easy numbers as values for y and n. Let's assume there are 3 travellers (A, B, C) and 2 different destinations (1, 2). We can chart out the possibilities as follows:

Destination 1	Destination 2
ABC	
AB	C
AC	B
BC	A
	ABC
C	AB
B	AC
A	BC

Thus there are 8 possibilities and in 2 of them all travelers end up at the same destination. Thus the probability is 2/8 or 1/4. By plugging in y = 3 and n = 2 into each answer choice, we see that only answer choice D yields a probability of 1/4.
Alternatively, consider that each traveler can end up at any one of n destinations. Thus, for each traveler there are n possibilities. Therefore, for y travelers, there are n^y possible outcomes.
Additionally, the "winning" outcomes are those where all travelers end up at the same destination.
Since there are n destinations there are n "winning" outcomes.

Thus, the probability = $\frac{\text{winning outcomes}}{\text{total outcomes}} = \frac{n}{n^y} = \frac{1}{n^{y-1}}$.

The answer is D.

Top 1% expert replies to student queries (can skip)

Query : Why exactly is the number of winning outcomes equal to n?
Reply : We have n destinations here. Let the destinations be labelled 1,2 and so till N

Let the travellers be Y1, Y2 and so on

We want all the travellers to end up at the same destination.

Let's assume Y1 chooses 1 as the destination. How many choices did Y1 have? NC1, right?
Y1 had to choose 1 destination from N destinations.

Now, how many choices does Y2 have? Only 1, since all the travellers have to end up at the same destination. Similar is the case for all the other travellers.

So, total number of ways in which the travellers end up at the same destination = NC1 = N

18.

There are four scenarios in which the plane will crash. Determine the probability of each of these scenarios individually:

CASE ONE: Engine 1 fails, Engine 2 fails, Engine 3 works = $\frac{1}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{24}$

CASE TWO: Engine 1 fails, Engine 2 works, Engine 3 fails = $\frac{1}{3} \times \frac{3}{4} \times \frac{1}{2} = \frac{3}{24}$

CASE THREE: Engine 1 works, Engine 2 fails, Engine 3 fails = $\frac{2}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{2}{24}$

CASE FOUR: Engine 1 fails, Engine 2 fails, Engine 3 fails = $\frac{1}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{24}$

To determine the probability that any one of these scenarios will occur, sum the four probabilities:

$$\frac{1}{24} + \frac{3}{24} + \frac{2}{24} + \frac{1}{24} = \frac{7}{24}$$

The correct answer is D. There is a 7/24 chance that the plane will crash in any given flight.

19.

Two pieces of fruit are selected out of a group of 8 pieces of fruit consisting only of apples and bananas. What is the probability of selecting exactly 2 bananas?

Say there are x bananas and y ($y = 8 - x$) apples. The question is $P(bb) = \frac{x}{8} * \frac{x-1}{7} =?$. Basically we need to find how many bananas are there.

(1) The probability of selecting exactly 2 apples is greater than 1/2 --> $\frac{y}{8} * \frac{y-1}{7} > \frac{1}{2}$ --> $y(y-1) > 28$ --> y can be 6, 7, or 8, thus **x can be 2, 1, or 0**. not sufficient.

(2) The probability of selecting 1 apple and 1 banana in either order is greater than 1/3. $2 * \frac{x}{8} * \frac{8-x}{7} > \frac{1}{3}$ --> $x(8-x) > \frac{28}{3} = 9\frac{1}{3}$, thus **$x$ can be 2, 3, 4, 5, or 6**. Not sufficient.

(1)+(2) From above x can only be 2. Sufficient.

Answer: C.

Therefore, the correct answer to this problem is C.

20.

Since each of the 4 children can be either a boy or a girl, there are 2 x 2 x 2 x 2 = 16possible ways that the children might be born, as listed below:
BBBB (all boys)
BBBG, BBGB, BGBB, GBBB, (3 boys, 1 girl)
BBGG, BGGB, BGBG, GGBB, GBBG, GBGB (2 boys, 2 girls)
GGGB, GGBG, GBGG, BGGG (3 girls, 1 boy)
GGGG (all girls)
Since we are told that there are at least 2 girls, we can eliminate 5 possibilities--the one possibility in which all of the children are boys (the first row) and the four possibilities in which only one of the children is a girl (the second row).
That leaves 11 possibilities (the third, fourth, and fifth row) of which only 6 are comprised of two boys and two girls (the third row). Thus, the probability that Ms. Barton also has 2 boys is 6/11 and **the correct answer is E.**

Top 1% expert replies to student queries (can skip)

What is the number of ways there can be 2 girls and 2 boys out of 4 children? Note here that while the children are all distinct, girls are one type here and boys are one type here.

Out of 4 people, I can select 2 people in 4C2 ways = 6 ways. These two will be girls / boys and the other two will be boys / girls and we will have 6 ways in which 2 children out of 4 will be girls

Similarly, I can say 3 children out of 4 will be girls in 4C3 ways (I can select any 3 in 4C3 = 4 ways)

All 4 children can be girls in 1 way

Then total number of ways she has 2 girls or 3 girls or 4 girls = 6 + 4 + 1 = 11

Out of this, she has exactly 2 girls in 6 cases, so required probability = 6/11

Note that because this is a case of conditional probability (we are already given she has at least 2 girls), we are not dividing by the total number of ways of having any combination of boy and girl children
the correct answer is E.

21.

The question require us to determine whether Mike's odds of winning are better if he attempts 3 shots instead of 1. For that to be true, his odds of making 2 out of 3 must be better than his odds of making 1 out of 1.
There are two ways for Mike to at least 2 shots: Either he hits 2 and misses 1, or he hits all 3:

Odds of hitting 2 and missing 1 $p \times p \times (1-p)$	**# of ways to hit 2 and miss 1** 3 (HHM, HMH, MHH)	**Total Probability** $3p^2(1-p)$
Odds of hitting all 3 $p \times p \times p$	**# of ways to hit all 3** 1 (HHH)	p^3
Mike's probability of hitting at least 2 out of 3 free throws =		$3p^2(1-p)+p^3$

Now, we can rephrase the question as the following inequality:

Is $3p^2(1-p)+p^3 > p$? (Are Mike's odds of hitting at least 2 of 3 greater than his odds of hitting 1 of 1?)
This can be simplified as follows:

$3p^2(1-p)+p^3 > p$

$3p^2-3p^3+p^3 > p$

$3p^2-2p^3 > p$ (we can divide by p since $p>0$)

$3p-2p^2 > 1$

$-2p^2+3p-1>0$ (divide by -2, flipping the inequality)

$p^2-1.5p+.5<0$

$(p-.5)(p-1)<0$

In order for this inequality to be true, p must be greater than .5 but less than 1 (since this is the only way to ensure that the left side of the equation is negative). But we already know that p is less than 1 (since

Mike occasionally misses some shots). Therefore, we need to know whether p is greater than .5. If it is, then the inequality will be true, which means that Mike will have a better chance of winning if he takes 3 shots.
Statement 1 tells us that $p < .7$. This does not help us to determine whether $p > .5$, so statement 1 is not sufficient.
Statement 2 tells us that $p > .6$. This means that p must be greater than .5. This is sufficient to answer the question.
The correct answer is B: Statement (2) alone is sufficient, but statement (1) alone is not sufficient.

22.

Although this may be counter-intuitive at first, the probability that any card in the deck will be a heart before any cards are seen is 13/52 or 1/4.

One way to understand this is to solve the problem analytically for any card by building a probability "tree" and summing the probability of all of its "branches."

For example, let's find the probability that the 2nd card dealt from the deck is a heart. There are two mutually exclusive ways this can happen: (1) both the first and second cards are hearts or (2) only the second card is a heart.

CASE 1: Using the multiplication rule, the probability that the first card is a heart AND the second card is a heart is equal to the probability of picking a heart on the first card (or 13/52, which is the number of hearts in a full deck divided by the number of cards) times the probability of picking a heart on the second card (or 12/51, which is the number of hearts remaining in the deck divided by the number of cards remaining in the deck).

13/52 x 12/51 = 12/204

CASE 2: Similarly, the probability that the first card is a non-heart AND the second card is a heart is equal to the probability that the first card is NOT a heart (or 39/52) times the probability of subsequently picking a heart on the 2nd card (or 13/51).

39/52 x 13/51 = 39/204

Since these two cases are mutually exclusive, we can add them together to get the total probability of getting a heart as the second card: 12/204 + 39/204 = 51/204 = 1/4.

We can do a similar analysis for any card in the deck, and, although the probability tree gets more complicated as the card number gets higher, the total probability that the nth card dealt will be a heart will always end up simplifying to 1/4.

The correct answer is A.

Top 1% expert replies to student queries (can skip)

First thing - in probability, mutually exclusive and independent are completely different things, and you must not use them interchangeably. Two events are mutually exclusive means one cannot happen if the other happens. A car turning left and right at the same time are mutually exclusive events. Independent events are those where the occurrence of one does not preclude in any way the occurrence of the other. In other words, the occurrence of one has no effect whatsoever on the occurrence of the other. If I shuffle a deck of normal playing cards nicely and pull one, replace it, then pull another, these are independent events - what the second card will be has absolutely no dependence on what the first card was because the sample space got restored to its original sanctity

In this question, I am taking out one card and then another and so on, without replacing the cards. These are not independent events - if I take out any suite (say heart), the probability the first card was heart was

13/52, but the probability the second card is heart now depends on what the first card was. If it was heart, the probability that the second card is heart is 12/51, if the first card was not heart, the probability the second card is heart is 13/51.

Now understand this - two events are mutually exclusive means they cannot happen together, correct? What that means is, the probability of one is dependent on the other (probability of one event goes to 0 if the other event has occurred). Then mutually exclusive events are always dependent as well.

Then let's break down the event universe as follows and understand intersection and union in each case.

i. Events are mutually exclusive
Probability of two such events (A and B) occurring together is 0. We call this the intersection of the two events. Then P(A intersection B) = 0. Probability of A or B occurring (at least one event occurring) is known as the union of the two events. P(A union B) = P(A) + P(B)

ii. Events are not mutually exclusive but independent
P(A intersection B) = P(A) x P(B)
P(A union B) = P(A) + P(B) - P(A intersection B)

iii. Events are not mutually exclusive and dependent
Think of the two events of drawing one card one after the other without replacement. As we saw above, probability of the second card being heart depends on what the first card was. So the two events are not independent. Now, the probability the second card was heart given that the first card was heart is called conditional probability. This conditional probability is given by Bayes Theorem, and is applicable to dependent events (such as drawing cards one by one without replacement). This is expressed as P(A given B) or P(A|B) and P(A|B) = P(A intersection B) / P(B). That means the probability of the first card being heart AND the probability of the second card being heart (P(A intersection B)) is P(A|B) x P(B). Also as P(A intersection B) is not zero, these events are not mutually exclusive. These distinctions are vitally important in advanced math

Now this problem we can approach in this way:
Let the event B be drawing a heart on the 10th draw
Now the previous nine draws can be any permutation - let's say heart is H, non-heart card is N; then HNNNNNNNN, NHNNNNNNN, NNHNNNNNN...etc, also HHNNNNNNN etc etc etc. Each position can be H or N, so 2^9 possibilities - it is a mind boggling number by the way.
Then the event A can be each of these individual permutations in the first 9 draws

Except for the first, none of the draws are independent. Then Baye's Theorem applies. Our job becomes to find P(B|A), where A is each of these permutations. Then we have to add all of these up to obtain the overall probability. This is because the outcome of any permutation in A (say HNNNNNNNN) and any other permutation in A and B cannot happen together, so these two events are mutually exclusive, and we can find the union probability of these by adding them up individually (so adding up all the 2^9 conditional probabilities). This is an incredibly difficult task for even a computer, so then we take a limit of the probability to find its converging value. That is a level of calculus I don't even want to so much as start explaining here.

If the OA is 1/4, take it as such and move on with life, but know and understand that there is a lot more to this problem than meets the eye

23.

Since the first two digits of the license plate are known and there are 10 possibilities for each of the remaining two digits (each can be any digit from 0 to 9), the total number of combinations for digits on the license plate will equal 10 ×10 = 100.
Because there are only 3 letters that can be used for government license plates (A, B, or C), there are a total of nine two-letter combinations that could be on the license plate (3 possibilities for first letter × 3 possibilities for the second letter).

Given that we have 100 possible digit combinations and 9 possible letter combinations, the total number of vehicles to be inspected will equal 100 × 9 = 900.

Since it takes 10 minutes to inspect one vehicle, the police will have time to inspect 18 vehicles in three hours (3 hours = 180 minutes). Thus, the probability of locating the transmitter within the allotted time is 18/900 = 1/50.
The correct answer is D.

24.

Trying to figure this problem out directly is time-consuming and risky. The safest and most efficient way to handle this is to assign a value to x, figure out the probability, and then plug that value into the answer choices until you find one choice that yields the correct probability.
Since x must be greater than 2, let's assign x a value of 3. This produces a 3-by-3 grid as follows (where each letter represents a bulb:

ABC
DEF
GHI

In order to determine the probability, we need to first figure out how many different groups of 4 bulbs could be illuminated. Since we have 9 bulbs, we can represent one way that exactly four bulbs could be illuminated as follows (each letter represents a bulb):

A	B	C	D	E	F	G	H	I
Yes	Yes	Yes	Yes	No	No	No	No	No

There are many other ways this could happen. Using the permutation formula, there are 9!/(4!)(5!) = 126 different combinations of exactly four illuminated bulbs.

How many of these 126 groups of 4 form a 2-by-2 square? If you analyze the 3-by-3 grid above you'll see there are only 4 groups that form a 2-by-2 square (ABDE, BCEF, DEGH, & EFHI).

Thus the correct probability is 4/126 or 2/63. If we plug in 3 for x in the answer choices, only choice (B) reduces to the same answer.

The correct answer is B.

For those interested in the direct solution:

The total number of possible combinations of 4 light bulbs chosen from an x-by-x grid can be expressed as follows:

$$\frac{(x^2)!}{4!(x^2-4)!}$$

This expression can be simplified as follows:

$$\frac{(x^2)!}{4!(x^2-4)!} = \frac{x^2(x^2-1)(x^2-2)(x^2-3)(x^2-4)!}{24(x^2-4)!}$$

$$= \frac{x^2(x^2-1)(x^2-2)(x^2-3)}{24} = \frac{x^2(x+1)(x-1)(x^2-2)(x^2-3)}{24}$$

The above expression represents the total # of possible combinations of 4 light bulbs, which is the denominator of our probability fraction.

The numerator of our probability fraction can be represented by the total # of 2-by-2 grids available in any x-by-x grid. Testing this out with several different values for x should enable you to see that there are $(x-1)^2$ possible 2-by-2 grids available in any x-by-x grid. Thus putting the numerator over the denominator yields the following probability:

$$\frac{(x-1)^2}{\frac{x^2(x+1)(x-1)(x^2-2)(x^2-3)}{24}} = \frac{24(x-1)^2}{x^2(x+1)(x-1)(x^2-2)(x^2-3)}$$

$$= \frac{24(x-1)}{x^2(x+1)(x^2-2)(x^2-3)}$$

25.
In order to determine the probability that the World Series will last fewer than 7 games, we can first determine the probability that the World Series WILL last exactly 7 games and then subtract this value from 1.

In order for the World Series to last exactly 7 games, the first 6 games of the series must results in 3 wins and 3 losses for each team.

Let's analyze one way this could happen:

Game 1	Game 2	Game 3	Game 4	Game 5	Game 6
T1 Wins	T1 Wins	T1 Wins	T1 Loses	T1 Loses	T1 Loses

There are many other ways this could happen. Using the permutation formula, there are 6!/(3!)(3!) = 20 ways for the two teams to split the first 6 games (3 wins for each).

There are then 2 possible outcomes to break the tie in Game 7. Thus, there are a total of 20 × 2 = 40 ways for the World Series to last the full 7 games.

The probability that any one of these 40 ways occurs can be calculated from the fact that the probability of a team winning a game equals the probability of a team losing a game = 1/2.

Given that 7 distinct events must happen in any 7 game series, and that each of these events has a probability of 1/2, the probability that any one particular 7 game series occurs is $\left(\frac{1}{2}\right)^7 = \frac{1}{128}$.

Since there are 40 possible different 7 game series, the probability that the World Series will last exactly 7 games is:

$$40 \times \frac{1}{128} = \frac{40}{128} = .3125 = 31.25\%$$

Thus the probability that the World Series will last fewer than 7 games is 100% - 31.25% = 68.75%.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

Let's say either team won in the first 4 games - then we are selecting the first 4 games out of 4 in 1 way, any one team in 2C1 way and then the probability of that team winning all four consecutively is (1/2)^4. I don't know your mathematical rigour, but notice that this is a Bernoulli Distribution - we have k successes out of n (here n = k = 4), probability of success = p = 1/2, probability of failure = 1-p = 1/2 and success has happened in 4 out of 4 trials. You don't have to know Bernoulli Distribution for the GMAT, this is just for your knowledge / opening up your mind to this possibility. So total probability in this case is (2)(1/2)^4

Now say any of the two teams won in 5 games, i.e. they won 4 out of the first 5 games. This is where I made a mistake in my original solution. If you simply select 4 out of 5 games in 5C4 ways, you are also selecting this case - WWWWL. This is not a 5 game case at all - this is a 4 game scenario, and so needs to be discounted from the counting. So the problem here becomes how do I arrange 4 W's and 1 L among themselves, such that the case with 4 Ws right at the start isn't counted? Total cases are 5!/4!1! and we have to subtract 1 case from this, so (5!/4!1!) - 1 = 5 - 1 = 4. Then total probability is (2)(4)(1/2)^5. The first term 2 is present because any one team can be selected in 2C1 ways, and the last term (1/2)^5 is because of the Bernoulli Distribution structure of this - the team winning won 4 games with probability (1/2)^4 AND lost 1 game with probability (1/2)

Finally, if 6 games were played, we again cannot simply select 4 games out of 6 in 6C4 ways, as these cases WWWWLL, WWWLWL, WWLWWL, WLWWWL, LWWWWL will be counted in those, and we need to discount these. Total cases = (6!/4!2!) - 5 = 10. Then total probability is (2)(10)(1/2)^6

Now add up these individually and you will get 68.xx%

26.

Let's consider the different scenarios:

If Harriet wins all five flips, she ends up with $15.
If Harriet wins four flips, and Tran wins one flip, Harriet is left with $13. If Harriet wins three flips, and Tran wins two flips, Harriet is left with $11.
If Harriet wins two flips, and Tran wins three flips, Harriet is left with $9. If Harriet wins one flip, and Tran wins four flips, Harriet is left with $7. If Harriet loses all five flips, she ends up with $5.

The question asks for the probability that Harriet will end up with more than $10 but less than $15. In other words, we need to determine the probability that Harriet is left with $11 or $13 (since there is no way Harriet can end up with $12 or $14).

The probability that Harriet ends up with $11 after the five flips:

Since there are 2 possible outcomes on each flip, and there are 5 flips, the total number of possible outcomes is 2 x 2 x 2 x 2 x 2 = 32. Thus, the five flips of the coin yield 32 different outcomes.

To determine the probability that Harriet will end up with $11, we need to determine how many of these 32 outcomes include a combination of exactly three winning flips for Harriet and exactly two winning flips for Tran.
This is equivalent to figuring out the possible rearrangements of THREE H's and TWO T's in a FIVE letter word.
We can create a systematic list of combinations that include three wins for Harriet and two wins for Tran:
THHHT, THHTH, THTHH, TTHHH, HTHHT, HTHTH, HTTHH, HHTHT, HHTTH, HHHTT = 10 ways.

Alternatively, we can count the combinations by applying the anagram method:

A	B	C	D	E
H	H	H	T	T

We take the factorial of the top and divide by the factorial of each repeated letter on the bottom. Since there are two repeated letters, we get 5! / (3! * 2!) = 10 combinations.

Thus the probability that Harriet ends up with exactly $11 after 5 flips is 10/32.

The probability that Harriet ends up with $13 after the five flips:

To determine the probability that Harriet will end up with $13, we need to determine how many of the 32 total possible outcomes include a combination of exactly four winning flips for Harriet.

Again, we can create a systematic list of combinations that include four wins for Harriet and one win for Tran: HHHHT, HHHTH, HHTHH, HTHHH, THHHH = 5 ways.

Alternatively, using the same reasoning as above, we can write

A	B	C	D	E
H	H	H	H	T

The formula yields 5! / 4! = 5 combinations.

Thus the probability that Harriet ends up with exactly $13 after 5 flips is 5/32.

The total probability that Harriet ends up with either $11 or $13 after the five flips: There are 10 ways that Harriet is left with $11. There are 5 ways that Harriet is left with $13. Therefore, there are 15 ways that Harriet is left with more than $10 but less than $15.
Since there are 32 possible outcomes, the correct answer is 15/32.
Alternatively, we can observe that the two possible ways to ―succeed‖ according to the terms of the problem are connected by a logical OR: Harriet can end up with $11 OR $13. When we have two avenues to success that are connected by a logical OR, we add the probabilities:
10/32 + 5/32 = 15/32.
The correct answer is D.

27.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. The values here are percents, and no actual number of students is given or requested. Therefore, we can assign a value of 100 to the total number of students at College X. From the given information in the question we have:

	Blue Eyes	Not Blue Eyes	Total
Brown Hair			40
Not Brown Hair			60
Total	70	30	100

The question asks for the difference between maximum value and the minimum value of the central square, that is, the percent of students who have neither brown hair nor blue eyes. The maximum value is 30, as shown below:

	Blue Eyes	Not Blue Eyes	Total
Brown Hair	40	0	40
Not Brown Hair	30	30	60
Total	70	30	100

Therefore the maximum probability of picking such a person is 0.3. Likewise, the minimum value of the central square is zero, as shown below:

	Blue Eyes	Not Blue Eyes	Total
Brown Hair	10	30	40
Not Brown Hair	60	0	60
Total	70	30	100

Therefore the minimum probability of picking such a person is 0, and the difference between the maximum and the minimum probability is 0.3.

28.
Begin by counting the number of relationships that exist among the 7 individuals whom we will call A, B, C, D, E, F, and G.
First consider the relationships of individual A: AB, AC, AD, AE, AF, AG = 6 total. Then consider the relationships of individual B without counting the relationship AB that was already counted before: BC, BD, BE, BF, BG = 5 total. Continuing this pattern, we can see that C will add an additional 4 relationships, D will add an additional 3 relationships, E will add an additional 2 relationships, and F will add 1 additional relationship. Thus, there are a total of 6 + 5 + 4 + 3 + 2 + 1 = 21 total relationships between the 7 individuals.

Alternatively, this can be computed formulaically as choosing a group of 2 from 7: 7C2 = 21.

We are told that 4 people have exactly 1 friend. This would account for 2 "friendship" relationships (e.g. AB and CD). We are also told that 3 people have exactly 2 friends. This would account for another 3 "friendship" relationships (e.g. EF, EG, and FG). Thus, there are 5 total "friendship" relationships in the group.
The probability that any 2 individuals in the group are friends is 5/21. The probability that any 2 individuals in the group are NOT friends = 1 – 5/21 = 16/21.
The correct answer is E.

29.
The chance of getting AT LEAST one pair of cards with the same value out of 4 dealt cards should be computed using the 1-x technique. That is, you should figure out the probability of getting NO PAIRS in those 4 cards (an easier probability to compute), and then subtract that probability from 1.
First card: The probability of getting NO pairs so far is 1 (since only one card has been dealt). Second card: There is 1 card left in the deck with the same value as the first card. Thus, there are 10 cards that will NOT form a pair with the first card. We have 11 cards left in the deck.
Probability of NO pairs so far = 10/11.
Third card: Since we have gotten no pairs so far, we have two cards dealt with different values. There are 2 cards in the deck with the same values as those two cards. Thus, there are 8 cards that will not form a pair with either of those two cards. We have 10 cards left in the deck.
Probability of turning over a third card that does NOT form a pair in any way, GIVEN that we have NO pairs so far = 8/10.
Cumulative probability of avoiding a pair BOTH on the second card AND on the third card = product of the two probabilities above = (10/11) (8/10) = 8/11.
Fourth card: Now we have three cards dealt with different values. There are 3 cards in the deck with the same values; thus, there are 6 cards in the deck that will not form a pair with any of the three dealt cards. We have 9 cards left in the deck.
Probability of turning over a fourth card that does NOT form a pair in any way, GIVEN that we have NO pairs so far = 6/9.
Cumulative probability of avoiding a pair on the second card AND on the third card AND on the fourth card = cumulative product = (10/11) (8/10) (6/9) = 16/33.
Thus, the probability of getting AT LEAST ONE pair in the four cards is 1 - 16/33 = 17/33.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

What re-arrangements!!! The probability of picking the first 3 and the second 3 is the same as picking the second 3 and the first 3. Picking the same numbers (for e.g. 3,3) happens in only one way. There are no re-arrangements in such cases. But when the numbers are different, the ways/arrangements matter For e.g (3,4 is different from 4,3), So, 2 ways exist.
We can choose any card for the first one - 12/12
Next card can be any card but 1 of the value we've already chosen -10/11(if we've picked 3, then there are one more 3 left and we can choose any but this one card out of 11 cards left);
Next card can be any card but 2 of the values we've already chosen - 8/10(if we've picked 3 and 5, then there are one 3 and one 5 left and we can choose any but these 2 cards out of 10 cards left);
Last card can be any card but 3 of the value we've already chosen - 6/9 (if we've picked 3, 5 and 6, then there are one 3, one 5 and one 6 left and we can choose any but these 3 cards out of 9 cards left);
P(none)=12/12 * 10/11 * 8/10 * 6/9 = 16/33
P= 1- P(none)= 1 - (16/33) = 17/33.

30.
12 people will be selected from a pool of 15 people: 10 men (2/3 of 15) and 5 women (1/3 of 15). The question asks for the probability that the jury will comprise at least 2/3 men, or at least 8 men (2/3 of 12 jurors = 8 men).
The easiest way to calculate this probability is to use the —1-x shortcut. The only way the jury will have fewer than 8 men is if a jury of 7 men and 5 women (the maximum number of women available) is selected. There cannot be fewer than 7 men on the jury, since the jury must have 12 members and only 5 women are available to serve on the jury.

The total number of juries that could be randomly selected from this jury pool is:

$$\frac{15!}{12!3!} = \frac{(15)(14)(13)}{(3)(2)} = 455$$

The number of ways we could select 7 men from a pool of 10 men is:

$$\frac{10!}{7!3!} = \frac{(10)(9)(8)}{(3)(2)} = 120$$

The number of ways we could select 5 women from a pool of 5 women is:

5!/5! = 1

This makes practical sense, in addition to mathematical sense. All of the women would have to be on the jury, and there is only one way that can happen.
Putting these selections together, the number of ways a jury of 7 men and 5 women could be selected is:
120 × 1 = 120

The probability that the jury will be comprised of fewer than 8 men is thus 120/455 = 24/91.
Therefore, the probability that the jury will be comprised of at least 8 men is 1 – (24/91) = 67/91.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

Taking, 8 , 9 and 10 men (for at least 8) and dividing by total
Here, in the jury of 12 members, you are considering only men and not the women.
We should include women also in the jury, for.eg. if you are considering 8 men in the jury, we should include 4 women to make a total of 12-member jury. Similarly- 9 men and 3 women OR 10 men and 2 women.
So the total becomes: (10c8*5c4) + (10c9*5c3) + (10c10*5c2) / {15c12} = 67/91

The correct answer is D.

31.
First we must find the total number of 5 member teams, with or without John and Peter. We can solve this using an anagram model in which each of the 9 players (A – I) is assigned either a Y (for being chosen) or an N (for not being chosen):

Player	A	B	C	D	E	F	G	H	I
Chosen ?	Y	Y	Y	Y	Y	N	N	N	N

It is the various arrangements of Y's and N's above that would yield all of the different combinations, so we can find the number of possible teams here by considering how many anagrams of YYYYYNNNN exist:

$$\frac{9!}{5!\,4!} = \frac{9 \times 8 \times 7 \times 6 \times 5}{5 \times 4 \times 3 \times 2 \times 1} = (3 \times 7 \times 6) = 126$$

(because there are 9! ways to order 9 objects) (because the 5Y's and 4N's are identical)

So there are 126 possible teams of 5. Since the question asks for the probability of choosing a team that includes John and Peter, we need to determine how many of the 126 include John and Peter. If we reserve two of the 5 spots on a team for John and Peter, there will be 3 spots left, which must be filled by 3 of the remaining 7 players (remember that John and Peter were already selected). Therefore the number of teams including John and Peter will be equal to the number of 3-player teams that can be formed from a 7-player pool. We can approach the problem as we did above:

Player	A	B	C	D	E	F	G
Chosen	Y	Y	Y	N	N	N	N

The number of possible YYYNNNN anagrams is:

$$\frac{7!}{3!\,4!} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35$$

Since 35 of the total possible 126 teams include John and Peter, the probability of selecting a team with both John and Peter is 35/126 or 5/18.

The correct answer is D.

Top 1% expert replies to student queries (can skip) (additional)

If you subtract the number of cases in which you do not choose John and Peter from the total number of cases, that does not give you the number of cases in which you choose BOTH JOHN AND PETER.

Some of the cases will be where you've chosen just one of the two and not the other.

So, if you really want to use the subtraction method (which is tedious), you will also have to subtract the cases where you've chosen just one of the two.

Number of cases in which you choose either John or Peter = (7C4)(2) = 70 [7C4 because we have already chosen one and rejected the other. So we have to choose 4 people from the remaining 7. The factor of 2 because this is done for both John and Peter]

Total number of cases to be subtracted = 21 + 70 = 91

Remaining cases (Favourable cases) = 126 - 91 = 35

Probability = 35/126 = 5/18

32.
First, we must calculate the total number of possible teams (let's call this t). Then, we must calculate how many of these possible teams have exactly 2 women (let's call this w). The probability that a randomly selected team will have exactly 2 women can be expressed as w/t.

To calculate the number of possible teams, we can use the Anagram Grid method. Since there are 8 employees, 4 of whom will be on the team (represented with a Y) and 4 of whom will not (represented with an N), we can arrange the following anagram grid:

A	B	C	D	E	F	G	H
Y	Y	Y	Y	N	N	N	N

To make the calculation easier, we can use the following shortcut: t = (8!)/(4!)(4!). The (8!) in the numerator comes from the fact that there are 8 total employees to choose from. The first (4!) in the denominator comes from the fact that 4 employees will be on the team, and the other (4!) comes from the fact that 4 employees will not be on the team. Simplifying yields:

$$t = \frac{8!}{4!4!}$$

$$t = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(4 \times 3 \times 2 \times 1) \times (4 \times 3 \times 2 \times 1)}$$

$$t = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}$$

t = 70

So, there are 70 possible teams of 4 employees.

Next, we can use a similar method to determine w, the number of possible teams with exactly 2 women. We note that in order to have exactly 2 women on the team, there must also be 2 men on the team of 4. If we calculate the number of ways that 2 out of 5 women can be selected, and the number of ways that 2 out of 3 men can be selected, we can then multiply the two to get the total number of teams consisting of 2 men and 2 women. Let's start with the women:

A	B	C	D	E
Y	Y	Y	N	N

$$\frac{5!}{3!2!} = 10$$

So, the number of ways that 2 women can be selected is 10. Now the men:

A	B	C
Y	Y	N

$$\frac{3!}{2!1!} = 3$$

Thus, the number of ways that 2 men can be selected is 3. Now we can multiply to get the total number of 2 women teams: w = (10)(3) = 30. Since there are 30 possible teams with exactly 2 women, and 70 possible teams overall, w/t = 30/70 = 3/7.

The correct answer is D.

Alternate sol from gmatclub (additional)

The problem here is that you are arranging the people. When you select a woman out of 8 as 5/8, you are saying that you are picking a woman first. You are arranging them in this way:
WWMM
Now, if you want to un-arrange them, multiply it by the total number of arrangements i.e. 4!/(2!*2!) (because there are 2 men and 2 women so you divide by 2!s to get the total number of arrangements)

When you do that, you get 1/14 * 4!/(2!*2!) = 3/7

The correct answer is D.

33.
To determine the probability that jelly donuts will be chosen on the first and second selections, we must find the probability of both events and multiply them together.

The probability of picking a jelly donut on the first pick is 4/12. However, the probability of picking a jelly donut on the second pick is NOT 4/12. If a jelly donut is selected on the first pick, the number of donuts in the box has decreased from 12 to 11, and the number of jelly donuts has decreased from 4 to 3. Thus, the probability of picking a jelly donut on the second pick is 3/11.

Since overall probability is calculated by multiplying the probabilities of both events, the probability of picking two jelly donuts is 4/12 × 3/11 = 12/132 = 1/11.

The correct answer is A.

34. The probability that Memphis does NOT win the competition is equal to 1 – p, where p is the probability that Memphis DOES win the competition. Statement (1) states that the probability that Memphis (or any of the other cities) does not win the competition is 7/8. This explicitly answers the question so this statement alone is sufficient. Statement (2) gives us 1/8 as the value for p, the probability that Memphis DOES win

the competition. We can use this to calculate the probability that Memphis does NOT win the competition: 1 – 1/8 = 7/8. This statement alone is sufficient to answer the question.
The correct answer is D

35.

To find the probability that two independent events will occur, one after the other, multiply the probability of the first event by the probability of the second event.

Probability of a non-nickel on **first** pick = (5 pennies + 4 dimes) / 15 coins = 3/5 Probability of a

non-nickel on **second** pick = (8 non-nickel coins) / 14 coins = 4/7

Notice that for the second pick both the non-nickel pool and the total coin pool diminished by one coin after a non-nickel was selected on the first pick.

Total probability = 3/5 x 4/7 = 12/35.
The correct answer is B.

36.
No matter what sign Jim throws, there is one sign Renee could throw that would beat it, one that would tie, and one that would lose. Renee is equally likely to throw any one of the three signs.
Therefore, the probability that Jim will win is 1/3.

For example, Jim could throw a Rock sign. He will win only if Renee throws a Scissors sign. There is a one in three chance that Renee will do so.

In fact:
Probability that Renee will win = 1/3
Probability of a tie = 1/3
Probability that Jim will win = 1/3

The correct answer is E.

37.
If we know the number of red balls and the number of green balls in the box, we can determine the probability of selecting one red ball at random. We can also determine this probability if we know the ratio of red balls to green balls in the box, even if we do not know the exact number of either color.
(1) SUFFICIENT: This statement tells us that the red balls make up two-thirds of all the balls in the box. Thus, two out of every three balls in the box are red. Therefore, the probability of selecting a red ball at random is 2/3.
(2) SUFFICIENT: This statement tells us that the probability of selecting a green ball from the box is 1/3. Thus, the probability of selecting a red ball must be 1 – 1/3 or 2/3, because the probability of selecting red plus the probability of selecting green must equal 1.
The correct answer is D.

38. 41/50

We are given that the 40 vehicles equipped with air conditioning represent 80% of all cars available for sale. We can let n = the total number of cars and create the following equation:

40 = 0.8n

n = 40/0.8 = 50

Thus, there are a total of 50 cars at the dealership. Furthermore, we are given that there are 15 convertibles, and 14 of them are equipped with air conditioning.

So, the the probability that a randomly selected vehicle will be a convertible or a car with air conditioning is:

[(15 + 40) - 14]/50 = 41/50

Note that we double-counted the 14 cars that are both convertibles and are cars with air conditioning, and so we had to subtract the 14 from the total.

39. 1/221

The best possible combination is the case when he wins in his first two draws

There are 52 cards in a deck of cards with 4 aces in it.

Probability of first card to be ace = 4/52
Probability of Second card to be ace = 3/51

Probability of Best possible case = (4/52)*(3/51) = 1/221

P.S. The language of this question ("Best possible combination") is not the Language that GMAT uses in the questions.
Answer: Option A

40. 1/1770

Top 1% expert replies to student queries

The first PDA goes to Derek and the second to Lena - 1/60*1/59;
OR
The first PDA goes to Lena and the second to Derek - 1/60*1/59;
Thus, the overall probability that both PDA will go to the couple is 2*1/60*1/59=1/1770.

Top 1% expert replies to student queries (can skip) (additional)

The question talks about lucky draw, there are two PDA's available as prizes, PDA stands for Personal Digital Assistants, kind of pocket PC.

Hence question is asking what is the probability that the couple gets both the prizes.

The first PDA goes to Derek and the second to Lena - 1/60*1/59;
The first PDA goes to Lena and the second to Derek - 1/60*1/59

Ans - 1/60*59 + 1/60*59

41. 2/9

To have equal number of black and white balls, the yellow urn must have 5 black and 5 white balls. So the balls we select out of the green urn must be both white.
Probability (first ball is white) = 5/10
Probability(second ball is white) = 4/9
Probability (both balls are white) = (5/10)*(4/9) = 2/9

42. 1/216
Top 1% expert replies to student queries
Given: the player's score is determined as a sum of three throws of a single die.

To guarantee that Jim will get some monetary payoff he must score the maximum score of 6+6+6=18, because if he gets even one less than that so 17, someone can get 18 and Jim will get nothing. P(18)=1/6*1/6*1/6 (Getting 6 in each of the 3 throws)= 1/6^3=1/216.

43. 3/25 = 12%

Top 1% expert replies to student queries

Shelf can accomodate 25 books but 20% of it is empty so there are in all 20 books.
If no of Chem books = x, number of Physics book = x+4, number of Math books = 2(x+4)

x + x+4 + 2(x+4) = 4x + 12 = 20
x = 2

Chem books = 2, Phy books = 6, Math books = 12

Probability of picking a Math book = 12/20
Probability of picking a Chem book = 2/20

Total Probability = Pick Math first and then Chem + Pick Chem first and then Math
= (12/20) * (2/20) + (2/20) * (12/20) = 12/100 = 12%

Top 1% expert replies to student queries (can skip) (additional)
Number of empty shelf spots = 20% of 25 = 5
Number of filled spots = 20.

Let the number of physics books be x.
Number of mathematics books will be 2x
Number of chemistry books will be (x - 4)

Total number of books in the shelf = x + 2x + x - 4 = 20

4x = 24
x = 6

So, the number of physics books is 6.
The Number of mathematics books is 12.
The Number of chemistry books is 2.

We have to find the probability that Ricardo reads one Mathematics and one Chemistry book. {Please note that the picked book is returned, so the total number of books during any pick is the same, that is 20}

Case 1 : The first book that Ricardo picks is Mathematics and the second book is Chemistry.

First pick : Probability of picking a mathematics book from the shelf = Number of mathematics books/total number of books = 12/20 = 3/5

Second pick : Probability of picking a chemistry book from the shelf = Number of chemistry books/total number of books = 2/20 = 1/10

Case 2 : The first book that Ricardo picks is Chemistry and the second book is Mathematics.

First pick : Probability of picking a chemistry book from the shelf = Number of chemistry books/total number of books = 2/20 = 1/10

Second pick : Probability of picking a mathematics book from the shelf = Number of mathematics books/total number of books = 12/20 = 3/5

Therefore, total probability = Probability of case 1 + Probability of case 2 = (3/5)(1/10) + (1/10)(3/5) = 3/25

Now, coming to your solution, the denominator you have used is incorrect. 20C2 would have been correct if we were choosing 2 books simultaneously. In that case, we would have had 19 options for the second book. However, since we're replacing the book we picked initially, we will have 20 options for the second book.

So the probability = 2 * (12C1 * 2C1) / (20 * 20) = 3/25

44. $3.6

If the total pencils are t, the probability of picking first as black = 4/t.
As it is again replaced, the probability in next will also be the same = 4/t

P to pick black pencil in both picks = $\frac{4}{t} * \frac{4}{t} = \frac{1}{36}$
$t^2 = 16 * 36 \ldots\ldots . t = 4 * 6 = 24$

Price of these 24 pencils = 15*24 = $3.6

D

45. 7/8

A coin is tossed 3 times
Prob of getting heads atleast once = Prob(All cases) - Prob(Getting no heads)
Also, prob of all cases = 1
And, prob of getting no heads -> Means getting 3 tails = 1/2*1/2*1/2 = 1/8
Thus, prob(Getting heads atleast once) = 1 - 1/8 = 7/8

46. 9/10

Solution from Gmatclub

$S = \{2, 3, 6, 48, 164\}$ and set of first 10 non-negative integers, say $T = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

$K = s * t$, where s and t are random numbers from respective sets.

678,463 is an odd number.

The only case when 6^k IS a factor of 678,463 is when k equals to 0 (in this case $6^k = 6^0 = 1$ and 1 is a factor of every integer). Because if $k > 0$, then $6^k = even$ and even number cannot be a factor of odd number 678,463.

Hence 6^k NOT to be a factor of 678,463 we should pick any number from S and pick any number but 0 from T:
$P = 1 * \frac{9}{10} = \frac{9}{10}$.

Top 1% expert replies to student queries (can skip) (additional)

How many values can 'k' assume? 60 [every number in set S can be multiplied with any of the first 10 non-negative integers. 10*6 = 60]

Now, z = 6^(k).

z will be 1 for k = 0. [k will be 0 when any number in set S is multiplied by 0. So, we have 6 cases when k = 0 and z = 1]
We need to find the probability that 678463 is not a multiple of z.
We know that 6^(z) will end with 6 if z is not equal to 0. If 6^z ends with 6, it will not be a factor of 678463
For z = 0, 6^z will end with 1. If 6^z ends with 1, it will be a factor of 678463
Therefore, in 6 out of 60 cases, 6^z will be a factor of 678463. And so in the remaining 54 cases, it won't be.
Probability = 54/60 = 0.9

47. 143/152

Top 1% expert replies to student queries

There is only one way that she will not be able to afford a 25 cent chocolate bar: she must draw Green balls FOUR Times: two green from each urn. G-G-G-G ———> 5 + 5 + 5 + 5 = 20 cents
Any other combination of balls will allow her to buy the candy bar. The next lowest amount she could possibly get is: G-G-G-R ——-> 5 + 5 + 5 + 10 = 25 cents, which is enough.
To find the probability that she will be able to afford the candy bar, we can take:
1 - (Prob. of Unfavorable Outcome) = Prob. of Favorable Outcome
Probability of Pulling 2 Green balls from the Black urn, when the 1st ball is replaced is:
(10/20) * (10/20) = (1/2) * (1/2) = 1/4
AND
Probability of pulling 2 Green balls from the White urn when the balls are NOT replaced after taking one:
(10/20) * (9/19) = (1/2) * (9/19) = 9/38
Probability of the Unfavorable Outcome of pulling 2 green from the Black urn and pulling 2 green from the White urn therefore is = (1/4) * (9/38) = 9/152
1 - (9/152) = 143/152

48. 13/14

There are several ways to solve this question.

First, 15 members are interested in investment banking (IB) and 6 are NOT interested in IB

We want P(have at least 1 interested in IB)
When it comes to probability questions involving "at least," it's best to try using the complement.
That is, P(Event A happening) = 1 - P(Event A not happening)
So, here we get: P(have at least 1 interested in IB) = 1 - P(not have at least 1 interested in IB)
What does it mean to **not** have at least 1 interested in IB? It means getting ZERO members interested in IB.
So, we can write: P(have at least 1 interested in IB) = 1 - P(have ZERO interested in IB)

Okay, let's go...
P(have ZERO interested in IB) = P(1st person is NOT interested in IB **AND** 2nd person is NOT interested in IB)
= P(1st person is NOT interested in IB) **x** P(2nd person is NOT interested in IB)
= 6/21 **x** 5/20
= 1/14

So, P(at least 1 interested in IB) = 1 - P(not at least 1 interested in IB)
= 1 - **1/14**
= 13/14

Answer: E

49.

Top 1% expert replies to student queries

In these types of probability questions, it's often easier to calculate what we DON'T WANT to have happen. We can then subtract that fraction from 1 to get the probability of what we DO want to have happen.

Here, we're going to throw 4 (6-sided) dice. We're asked for the probability of throwing AT LEAST one pair of matching numbers.

Since it's actually easier to calculate how to throw 0 pairs of matching numbers (and then subtract that fraction from 1), I'll approach the question that way...

1st roll = any number = 6/6
2nd roll = not a match to the first = 5/6
3rd roll = not a match to the 1st or 2nd = 4/6
4th roll = not a match to the 1st or 2nd or 3rd = 3/6

(6/6)(5/6)(4/6)(3/6) =
(1)(5/6)(2/3)(1/2) =
10/36

10/36 is the probability of rolling 0 matching numbers, so...

1 - 10/36 = 26/36 = the probability of rolling at least one matching pair of numbers

26/36 = 13/18

Answer: 13/18

50. 0

Top 1% expert replies to student queries

Any multiple of 6 is even.
Any two-digit prime number is odd.
(even+odd)/2 is not an integer. Thus # does not yield an integer at all.
Therefore P=0.
Answer: A.
OR
All two-digit multiples of 6 are even, and all two-digit prime numbers are odd. Thus, we are adding even + odd, which always yields an odd number. When that odd number is divided by 2, the result is a non-integer. If operation "#" is repeated, we will again obtain a non-integer, and, in fact, operation "#" will never yield an integer. Thus, the probability of yielding at least 2 integers is zero.

51. 1/5
In such Qs, always look for peculiarities in the given number.
And here 89 is a PRIME number.

All prime numbers have only TWO factors- 1 and itself.
So look for all prime numbers between 50 and 69 inclusive.
They are 53, 59, 61 and 67. so 4 numbers.
Total numbers= (69-50+1) = 20.

Probability=4/20=1/5
Answer is B

52. 271/1000
If the current month is January, then in the next 16 months there will be 3 launches - 2 in June and 1 in October. 16 months brings us to April of the following year.

To find the probability of at least one launch being delayed during that time, we should look for the opposite case - the probability of no launches being delayed in the next 16 months - and subtract that from 1.

Probability that any given launch is not delayed = 0.9
Probability that 3 launches are not delayed = 0.9*0.9*0.9 = 0.729
Probability that at least one launch will be delayed = 1-0.729 = 0.271 = 27.1%

53. 7/216

The total number of ways to throw 3 dice is 6 x 6 x 6 = 216
The only way to get a product of the numbers appearing on the top of 3 dice to be an odd number that is divisible by 25 is if two of the numbers are both 5 and the third number is an odd number. Thus, we have:
{1, 5, 5}, {5, 1, 5}, {5, 5, 1}, {3, 5, 5}, {5, 3, 5}, {5, 5, 3} and {5, 5, 5}
Thus the probability is 7/216.

54. 2/25

Probability of 1st correct # is 1/5
probability of 2nd correct # is 1/5
So the probability of choosing the correct combination is 1/5* 1/5 = 1/25.
Choosing the wrong combination : 1 - 1/25 = 24/25
Now, we hv 2 cases.
Case 1 : 1st attempt is wrong and second attempt is right.
24/25*1/25 = 24/625
Case 2 : hit the eye in 1st attempt :
1/5
Total probability :
24/625 + 1/25 = 49/625 ~ 50/625 ~ 2/25
OR
[Probability (getting both digits right)] is NOT the same as [1 - Probability (getting both digits wrong)]
Since if either digit is wrong, the guessed number is wrong.
[Probability (getting both digits right)] = 1 - [Probability (getting both digits wrong) + Probability (getting either digit wrong)]
= 1 - [4/5 + (4/5 * 1/5) + (1/5 * 4/5)]
= 1/25
Now you have two tries to guess the correct number.
Probability (getting it right the first time) = 1/25
Probability (getting it wrong the first time, and right the second time) = (1 - 1/25) * 1/25 = 24/625
Add the two together, you get 49/625, closest to 50/625 ~2/25

55. ½

Set of all non-negative single digit integers - 0 to 9 i.e. 10 numbers

For $5N^3/8$ to be an integer, $5N^3$ should be completely divisible by 8. This will happen only if N^3 is divisible by 8 i.e. if N has 2 as a factor. So for $5N^3/8$ to be an integer, N should be even.

Out of the 10 numbers 0 - 9, 5 are even and 5 are odd.
Probability that $5N^3/8$ is an integer is 1/2.

56. 8/25
Let's suppose that he gets into the school in his 1st attempt. The probability is 15/100. We don't need to consider the 2nd case because he has already gotten into the school.

So the 1st case is 15/100 -------1)

In the second case he doesn't qualify in the 1st round which means probability is 85/100. We'll multiply it with 20/100 because he is supposed to clear it in his 2nd attempt.

The second case is 85/100*20/100 = 17/100-------2)

On adding 1) and 2) we get 32/100= 32%

57. 1/24
There are 3 single digit multiple of 3, that is, 3,6,9.
There are 8 prime nos less than 20 - 2,3,5,7,11,13,17,19
Total outcome - 8*3 = 24
Favourable outcome = 1 (9*5)

Hence required probability = 1/24.
Answer C.

58. B
(1) There are 6 black and 4 orange pencils among the pencils in the drawer. This one is clearly insufficient because we know nothing about red pencils.

(2) There are three times as many red pencils in the drawer as pencils of all the other colors combined. So, if there are x other color pencils, then there are 3x red pencils, so the probability of picking red is (red)/(total) = 3x/(x+3x) = 3/4. Sufficient.

Answer: B.

59. A
(1) There are twice as many white balls as balls of all other colors combined. - Suff
White balls : Other Balls = 2:1
White Balls : All Balls = 2:3 = 66.6% probability
(2) There are 30 more white balls than balls of all other colors combined. - Insuff
Let white balls be 40
Let other balls be 10
Probability of white = 4/5

Let white balls be 50
Let other balls be 20
Probability of white = 5/7

Two different answers hence not sufficient

60. B
The probability that the gym will have both a swimming pool and a squash court = (Number of gyms that have both a swimming pool and a squash court)/12
Find number of gyms that have both a swimming pool and a squash court.

S1: The statement implies that 10 gyms have squash court while 2 do not. We have no information about the number of gyms with pools. Insufficient.

S2: The statement implies that 'Number of gyms that have both a swimming pool and a squash court' = 9.
Thus, the probability that the gym will have both a swimming pool and a squash court = 9/12 = 3/4. Sufficient.

Answer: B

61. C
1) Could have 3 black and 3 white or 8 black and 8 white. Probability differs for both cases. Insufficient
2) Need to know how many white are there in the jar. This doesn't provide that information.

1) & 2) combined There are 8 black and 8 white in the jar. Now it should be easy! Sufficient

62. D

(1) The probability of no rain throughout the first two days is 36%.

$P(B) * P(B) = 0,36 \text{ --> } P(B) = 0,6$

$P(A) = 1 - P(B) = 1 - 0.6 = 0.4$

$P(A)^3 = 0,4^3 = 0,064$

SUFFICIENT

(2) The probability of rain on the third day is 40%.

$P(A) = 0,4$
$P(A)^3 = 0,4^3 = 0,064$

SUFFICIENT

Correct answer is D

63.

Top 1% expert replies to student queries
The trap is (1) doesn't define "refined number" and (2) does.
For example, you can say "an even number must be an integer", this doesn't define an even number, because not every integer is an even number. Similarly with (1), every refined number must be divided by 22. But perhaps they also must be divided by 5 (or something else): in this case refined numbers are only 110, 220 etc.
OR
1. Statement 1 tells us that a refined number must be a multiple of 22. This is a necessary, but not sufficient condition for a refined number. While every refined number is a multiple of 22, every multiple of 22 is not necessarily a refined number. Since we have no other info about the definition of a refined number, we cannot determine how many integers from 1-1000 fit that definition. Insufficient.

2. Statement 2 provides us with a definition of a refined number- an even multiple of 11. We can find the number of even multiples of 11 in the set, so this is sufficient.

Hence Ans is B.

Top 1% expert replies to student queries (can skip) (additional)
Statement 1 says any refined number is a multiple of 22. But it does not say any multiple of 22 is a refined number. There are many multiples of 22 from 1 to 1,000. Any refined number between 1 to 1,000 will be in this group of multiples of 22, but not necessarily this entire group of multiples of 22 will be refined numbers.

22, 44, 66, 88...are all multiples of 22. What Statement 1 says is that if there is a refined number, it has to be within this group. What it does not say is that every number in this group is a refined number. 76 cannot be a refined number, because a refined number has to be a multiple of 22 (and 76 is not); but it is not necessary that each of 22, 44, 66, 88... is a refined number.

Statement 2 says ANY even multiple of 11 (i.e. the number has at least one 11 and one 2 in its prime factorization) is a refined number. So according to this meaning, all multiples 11 such as 22, 44, 66...are refined numbers. We don't have to find the total number of such multiples from 1 to 1,000, but suffice to know that we can. So this statement is sufficient by itself.

64. 4/7

Top 1% expert replies to student queries

The probability of an event happening is equal to

# of Desired Outcomes_______
Total # of Outcomes

Here, "Desired Outcomes" are those in which
(N + K) = odd number
1) Elements of the probability calculation
N = 11, 13, 17 ,19 ,23, 29, 31, 37 (8 terms)
K = 10, 15, 20, 25, 30, 35, 40 (7 terms)
All N are odd.
For (N + K) to be odd, K must be even:
(odd + EVEN) = odd
2) Number of POSSIBLE outcomes for N + K?
8 * 7 = 56
3) Number of DESIRED outcomes, where N + K is odd?
All N are odd.
4 of K's terms are even.
So for (N + K) to be odd, the 4 even multiples of five (10, 20, 30, 40), in K, could be paired with all 8 odd primes in N. That means:
4 * 8 = 32 favorable outcomes
4) Probability that N + K is odd? Favorable/Possible:
32/56 = **4/7**

65. A
set = {3,6,9,12,...96,99}

1) We know only prime #'s have just two factors - 1 and the number itself.
We also know from the set that 3 is the only prime in it and of course 3 is > 2
SUFF => AD

2) every other number in the set is odd.
NOT SUFF => A

66.D
The probability that A is playing a song he likes is 0.3;
The probability that B is playing a song he likes is 0.7*0.3=0.21;
The probability that C is playing a song he likes is 0.7*0.7*0.3=0.147; So, the total probability is 0.3+0.21+0.147=0.657

Top 1% expert replies to student queries (can skip)

Station B or C is not 0.3*0.3. Try to understand and apply the concept, mentioned below.
The probability that Leo will hear a song he likes on the way to work is the probability he will not turn off his radio. That is, either station A will be on for the entire trip, or station B or C will be on by the end of the trip.
The probability that station A will be on for the entire trip is 0.3.
Station B will be on by the end of the trip if station A did not play a song he likes AND station B did play a song he likes. The probability is 0.7 x 0.3 = 0.21.
Station C will be on by the end of the trip if station A did not play a song he likes AND station B did not play a song he likes AND station C did play a song he likes. The probability is 0.7 x 0.7 x 0.3 = 0.147.
Since these events are mutually exclusive, we add their probabilities, so the probability that a station will be on by the end of the trip is 0.3 + 0.21 + 0.147 = 0.657.
Answer: D
OR
We can also consider the probability of something not happening and subtracting that value from 1.
1 - {Probability Leo doesn't like A} * {Probability Leo doesn't like B} * {Probability Leo doesn't like C}
= 1 -(0.7 * 0.7 * 0.7)= 0.657
Answer: D

67.A
Top 1% expert replies to student queries (can skip) (additional)
Let us assume a pair of brothers J and S, where J is from the junior class and S is from the senior class.
The first expression (60/1000 * 1/800) includes this case. But so does the second expression (60/800 * 1/1000). So we should only include one of these cases.
The answer should only be (60/800 * 1000) = 3/400000
The correct answer is A

68.
P(white+even)=Pw+Pe-P(w&e)
From 1, we know that P(w&e) From
2, we know that Pw-Pe=0.2.
But we still don't know what is Pw+Pe, so **answer is E**

69.
We need to know whether R/(B+W+R)>W/(B+W+R). B, W, R are positive, so, we just need to know is R>W.
For 1, R/(B+W)> W/(B+R)=>R/(B+W)-W/(B+R) >0
[R(B+R)-W(B+W)]/(B+W)(B+R) >0
(R-W)(R+W+B)/(B+W)(B+R) >0
As (R+W+B)>0, (B+W)(B+R)>0, so, R-W>0.
Statement 1 is sufficient.
For 2, B-W > R is insufficient to determine R>W.
Answer is A.

70.
The probability that none fashion book will be selected is:
4/8*3/7*2/6=1/14
Then the probability asked is 1-1/14=13/14
Answer is E

71.
When x=-10 or 2.5, the function is equal to 0. So,
the probability is 1/6
Answer is B

MISCELLANEOUS QUESTIONS

Part A: Word Problems

1. To determine the greatest possible number of contributors we must assume that each of these individuals contributed the minimum amount, or \$50. We can then set up an inequality in which n equals the number of contributors:

50n is less than or equal to \$1,749

Divide both sides of the equation by 50 to isolate n, and get

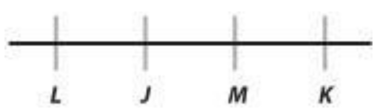
n is less than or equal to 34.98

Since n represents individual people, it must be the greatest whole number less than 34.98. Thus, the greatest possible value of n is 34.

Alternately, we could have assumed that the fundraiser collected \$1,750 rather than \$1,749. If it had, and we assumed each individual contributed the minimum amount, there would have been exactly 35 contributors (\$50 x 35 = \$1,750). Since the fundraiser actually raised one dollar less than \$1,750, there must have been one fewer contributor, or 34.

The correct answer is B.

2.
It may be easiest to represent the ages of Joan, Kylie, Lillian and Miriam (*J*, *K*, *L* and *M*) on a number line. If we do so, we will see that the ages represent consecutive integers as shown in the diagram.



Since the ages are consecutive integers, they can all be expressed in terms of L: L, $L + 1$,
$L + 2$, $L + 3$. The sum of the four ages then would be $4L + 6$. Since L must be an integer (its Lillians age), the expression $4L + 6$ describes a number that is two more than a multiple of 4:

$4L + 6 = (4L + 4) + 2$
[$4L + 4$ describes a multiple of 4, since it can be factored into $4(L + 1)$ or 4 * an integer.]

54 is the only number in the answer choices that is two more than a multiple of 4 (namely, 52).

The correct answer is D.

3. This is an algebraic translation problem dealing with ages. For this type of problem, an age chart can help us keep track of the variables:

	NOW	IN 6 YEARS
JANET	J	$J + 6$
CAROL	C	$C + 6$

Using the chart in combination with the statements given in the question, we can derive equations to relate the variables. The first statement tells us that Janet is now 25 years younger than her mother Carol. Since we have used J to represent Janets current age, and C to represent Carols current age, we can translate the statement as follows: $J = C – 25$.

The second statement tells us that Janet will be half Carols age in 6 years. Since we have used $(J + 6)$ to represent Janets age in 6 years, and $(C + 6)$ to represent Carols age in 6 years, we can translate the statement as follows: $J + 6 = (1/2)(C + 6)$.

Now, we can substitute the expression for C ($C = J + 25$) derived from the first equation into the second equation (note: we choose to substitute for C and solve for J because the question asks us for Janet's age 5 years ago):

$J + 6 = (1/2)(J + 25 + 6)$
$J + 6 = (1/2)(J + 31)$
$2J + 12 = J + 31$
$J = 19$
If Janet is now 19 years old, she was 14 years old 5 years ago.

The correct answer is B.

4.
The \$1,440 is divided into 12 equal monthly allocations.
1440/12 = \$120
The company has \$120 allocated per month for entertainment, so the allocation for three months is 120 × 3 = 360
Since the company has spend a total of \$300 thus far, it is \$360 - \$300 = \$60 under budget.

The correct answer is A.

5.
Since this problem includes variables in both the question and the answer choices, we can try solving by plugging in smart numbers. For x, we want to choose a multiple of 2 because we will have to take x/2 later. Let's say that ACME produces 4 brooms per month from January to April, so x = 4. The total number of brooms produced was (4 brooms x 4 months), or 16 brooms.

ACME sold x/2 brooms per month, or 2 brooms per month (because we chose x = 4). Now we need to start figuring out the storage costs from May 2nd to December 31st. Since ACME sold 2 brooms on May 1st, it needed to store 14 brooms that month, at a cost of $14. Following the same logic, we see that ACME sold another two brooms June 1st and stored 12 brooms, which cost the company $12. We now see that the July storage costs were $10, August were $8, September $6, October $4, November $2, and for December there were no storage costs since the last 2 brooms were sold on December 1st.

So ACME's total storage costs were 14 + 12 + 10 + 8 + 6 + 4 + 2 = $56. Now we just need to find the answer choice that gives us $56 when we plug in the same value, x = 4, that we used in the question. Since 14 x 4 = 56, $14x must be the correct value.

The correct answer is E.

While plugging in smart numbers is the *preferred* method for VIC problems such as this one, it is not the only method. Below is an alternative, algebraic method for solving this problem:

ACME accumulated an inventory of $4x$ brooms during its four-month production period. If it sold $0.5x$ brooms on May 1st, then it paid storage for $3.5x$ brooms in May, or $\$3.5x$. Again, if ACME sold $0.5x$ brooms on June 1st, it paid storage for $3x$ brooms in June, or $\$3x$. The first row of the table below shows the amount of money spent per month on storage. Notice that since ACME liquidated its stock on December 1st, it paid zero dollars for storage in December.

MAY	JUN	JUL	AUG	SEP	OCT	N
$\$3.5x$	$\$3x$	$\$2.5x$	$\$2x$	$\$1.5x$	$\$1x$	$

If we add up these costs, we see that ACME paid $\$14x$ for storage.

6.
The bus will carry its greatest passenger load when P is at its maximum value. If $P = -2(S-4)^2 + 32$, the maximum value of P is 32 because $(S-4)^2$ will never be negative, so the expression $-2(S-4)^2$ will never be positive. The maximum value for P will occur when $-2(S-4)^2 = 0$, i.e. when $S = 4$.

The question asks for the number of passengers two stops after the bus reaches its greatest passenger load, i.e. after 6 stops ($S = 6$).

$P = -2(6-4)^2 + 32$
$P = -2(2)^2 + 32$
$P = -8 + 32$
$P = 24$

The correct answer is C.

Alternatively, the maximum value for *P* can be found by building a table, as follows:

S	*P*
0	0
1	14
2	24
3	30
4	32
5	30
6	24

The maximum value for *P* occurs when $S = 4$. Thus, two stops later at $S = 6$, $P = 24$.
Answer choice C is correct.

7. John was 27 when he married Betty, and since they just celebrated their fifth wedding anniversary, he is now 32.
   Since Betty's age now is 7/8 of John's, her current age is (7/8) × 32, which equals 28.
   The correct answer is C.

8. Joe uses 1/4 of 360, or 90 gallons, during the first week. He has 270 gallons remaining (360 –90 = 270).
During the second week, Joe uses 1/5 of the remaining 270 gallons, which is 54 gallons.
Therefore, Joe has used 144 gallons of paint by the end of the second week (90 + 54 = 144).
The correct answer is B.

9. One way to do this problem is to recognize that the star earned \$8M more (\$32M - \$24M = \$8M) when her film grossed \$40M more (\$100M - \$60M = \$40M). She wants to earn \$40M on her next film, or \$8M more than she earned on the more lucrative of her other two films. Thus, her next film would need to gross \$40M more than \$100M, or \$140M.

   Alternatively, we can solve this problem using algebra. The star's salary consists of a fixed amount and a variable amount, which is dependent on the gross revenue of the film. We know what she earned for two films, so we can set up two equations, where *f* is her fixed salary and *p* is her portion of the gross, expressed as a decimal:

   She earned \$32 million on a film that grossed \$100 million: \$32M = *f* + *p*(\$100M)
   She earned \$24 million on a film that grossed \$60 million: \$24M = *f* + *p*(\$60M)

We can solve for p by subtracting the second equation from the first:

$$\begin{array}{r} \$32\text{M} = f + p(\$100\text{M}) \\ \underline{-[\$24\text{M} = f + p(\$60\text{M})]} \\ \$8\text{M} = p(\$40\text{M}) \\ 0.2 = p \end{array}$$

We can now plug in 0.2 for p in either of the original equations to solve for f:

$\$32\text{M} = f + 0.2(\$100\text{M})$

$\$32\text{M} = f + \$20\text{M}$

$\$12\text{M} = f$

Now that we know her fixed salary and the percentage of the gross earnings she receives, we can rewrite the formula for her total earnings as:

Total earnings = $12M + 0.2(*gross*)

Finally, we just need to figure out how much gross revenue her next film needs to generate in order for her earnings to be $40 million:

$\$40\text{M} = \$12\text{M} + 0.2(\textit{gross})$

$\$28\text{M} = 0.2(\textit{gross})$

$\$28\text{M}/0.2 = \$140\text{M} = \textit{gross}$

The correct answer is D.

10. I. UNCERTAIN: It depends on how many bicycles Norman sold.

For example, if $x = 4$, then Norman earned $44 [= $20 + (4 × $6)] last week. In order to double his earnings, he would have to sell a minimum of 9 bicycles this week (y = 9), making $92 [= $20 + (6 × $6) + (3 × $12)]. In that case, $y > 2x$.

However, if $x = 6$ and $y = 11$, then Norman would have earned $56 [= $20 + (6 × $6)] last week and $116 [= $20 + (6 × $6) + (5 × $12)] this week. In that case, $116 > 2 × $56, yet $y < 2x$.

So, it is possible for Norman to more than double his earnings without selling twice as many bicycles.

II. TRUE: In order to earn more money this week, Norman must sell more bicycles.

III. TRUE: If Norman did not sell any bicycles at all last week ($x = 0$), then he would have earned the minimum fixed salary of $20. So he must have earned at least $40 this week. If $y = 3$, then Norman earned $38 [= $20 + (3 × $6)] this week. If $y = 4$, then Norman earned $44 [= $20 + (4 × $6)] this week. Therefore, Norman must have sold at least 4 bicycles this week, which can be expressed $y > 3$.

The correct answer is D.

11. In order to determine the greatest number of points that an individual player might have scored, assume that 11 of the 12 players scored 7 points, the minimum possible. The 11 low scorers would therefore account for $7(11) = 77$ points out of 100. The number of points scored by the twelfth player in this scenario would be $100 - 77 = 23$.

The correct answer is E.

12. Since we are not given any actual spending limits, we can pick numbers. In problems involving fractions, it is best to pick numbers that are multiples of the denominators.

We can set the spending limit for the gold account at \$15, and for the platinum card at \$30. In this case, Bruce is carrying a balance of \$5 (which is 1/3 of \$15) on her gold card, and a balance of \$6 (1/5 of \$30) on her platinum card. If she transfers the balance from her gold card to her platinum card, the platinum card will have a balance of \$11. That means that \$19 out of her \$30 spending limit would remain unspent.

Alternatively, we can solve this algebraically by using the variable x to represent the spending limit on her platinum card:

$(1/5)x + (1/3)(1/2)x =$

$(1/5)x + (1/6)x =$

$(6/30)x + (5/30)x =$

$(11/30)x$

This leaves 19/30 of her spending limit untouched.

The correct answer is D.

Top 1% expert replies to student queries (can skip)

G-card spending limit: x

G-card balance: (1/3)x

P-card spending limit: 2x

P-card balance: (2/5)x

Portion used on P-card: ((1/3)x + (2/5)x)/2x = 11/30

Portion unused on P-card: 1-(11/30) = 19/30

Answer is D

13.

The problem talks about Martina and Pam's incomes but never provides an actual dollar value, either in the question or in the answer choices. We can, therefore, use smart numbers to solve the problem. Because the dollar value is unspecified, we pick a dollar value with which to solve the problem. To answer the question, we need to calculate dollar values for the portion of income each earns during the ten months not including June and August, and we also need to calculate dollar values for each player's annual income.

Let's start with Martina, who earns 1/6 of her income in June and 1/8 in August. The common denominator of the two fractions is 24, so we set Martina's annual income at \$24. This means that she earns \$4 (1/6 × 24) in June and \$3 (1/8 × 24) in August, for a total of \$7 for the two months. If Martina earns \$7 of \$24 in June and August, then she earns \$17 during the other ten months of the year.

The problem tells us that Pam earns the same dollar amount during the two months as Martina does, so Pam also earns \$7 for June and August. The \$7 Pam earns in June and August represents 1/3 + 1/4 of her annual income. To calculate her annual income, we solve the equation: $7 = (1/3 + 1/4)x$, with x representing Pam's annual income. This simplifies to $7 = (7/12)x$ or $12 = x$. If Pam earns \$7 of \$12 in June and August, then she earns \$5 during the other ten months of the year. [NOTE: we cannot simply pick a number for Pam in the same way we did for Martina because we are given a relationship between Martina's income and Pam's income. It is a coincidence that Pam's income of \$12 matches the common denominator of the two fractions assigned to Pam, 1/3 and 1/4 - if we had picked \$48 for Martina's income, Pam's income would then have to be \$24, not \$12.]

Combined, the two players earn \$17 + \$5 = \$22 during the other ten months, out of a combined annual income of \$24 + \$12 = \$36. The portion of the combined income earned during the other ten months, therefore, is 22/36 which simplifies to 11/18.

Note first that you can also calculate the portion of income earned during June and August and then subtract this fraction from 1. The portion of income earned during June and August, 7/18, appears as an answer choice, so be careful if you decide to solve it this way.

Note also that simply adding the four fractions given in the problem produces the number 7/8, an answer choice. 1/8 (or 1 – 7/8) is also an answer choice. These two answers are "too good to be true" - that is, it is too easy to arrive at these numbers.

The correct answer is D.

14. This fraction problem contains an "unspecified" total (the x liters of water in the lake). Pick an easy "smart" number to make this problem easier. Usually, the smart number is the lowest common denominator of all the fractions in the problem. However, if you pick 28, you will quickly see that this yields some unwieldy computation.

The easiest number to work with in this problem is the number 4. Let's say there are 4 liters of water originally in the lake. The question then becomes: During which year is the lake reduced to *less than 1 liter* of water?

At the end of 2076, there are 4 × (5/7) or 20/7 liters of water in the lake. This is not less than 1.

At the end of 2077, there are (20/7) × (5/7) or 100/49 liters of water in the lake. This is not less than 1.

At the end of 2078, there are (100/49) × (5/7) or 500/343 liters of water in the lake. This is not less than 1.

At the end of 2079, there are (500/343) × (5/7) or 2500/2401 liters of water in the lake. This is not less than 1.

At the end of 2080, there are (2500/2401) × (5/7) or 12500/16807 liters of water in the lake. **This is less than 1.**

Notice that picking the number 4 is essential to minimizing the computation involved, since it is very easy to see when a fraction falls below 1 (when the numerator becomes less than the denominator.) The only moderately difficult computation involved is multiplying the denominator by 7 for each new year.

The correct answer is D.

15. This fraction problem contains an unspecified total (the number of married couples) and is most easily solved by a picking a "smart" number for that total. The

smart number is the least common denominator of all the fractions in the problem. In this case, the smart number is 20.
Let's say there are 20 married couples.
15 couples (3/4 of the total) have more than one child.
8 couples (2/5 of the total) have more than three children.
This means that 15 – 8 = 7 couples have either 2 or 3 children. Thus 7/20 of the married couples have either 2 or 3 children.

The correct answer is C.

16. We can back solve this question by using the answer choices. Let's first check to make sure that each of the 5 possible prices for one candy can be paid using exactly 4 coins:

8 = 5+1+1+1
13 = 10+1+1+1
40 = 10+10+10+10
53 = 50+1+1+1
66 = 50+10+5+1

So far we can't make any eliminations. Now let's check two pieces of candy:

16 = 5 + 5 + 5 + 1
26 = 10 + 10 + 5 + 1
80 = 25 + 25 + 25 + 5
106 = 50 + 50 + 5 + 1
132 = 50 + 50 + 25 + 5 + 1 + 1

We can eliminate answer choice E here. Now three pieces of candy:

24 = 10 + 10 + 1 + 1 + 1 + 1
39 = 25 + 10 + 1 + 1 + 1 + 1
120 = 50 + 50 + 10 + 10
159 = 50 + 50 + 50 + 5 + 1 + 1 + 1 + 1.

We can eliminate answer choices A, B and D.
Notice that at a price of 40¢, Billy can buy four and five candies with exactly 4 coins as well:
160 = 50 + 50 + 50 + 10
200 = 50 + 50 + 50 + 50

This problem could also have been solved using divisibility and remainders. Notice that all of the coins are multiples of 5 except pennies. In order to be able to pay for a certain number of candies with exactly four coins, the total price of the candies cannot be a value that can be expressed as $5x + 4$, where x is a positive integer. In other words, the total price cannot be a number that has a remainder of 4 when divided by 5. Why? The remainder of 4 would alone require 4 pennies.
We can look at the answer choices now just focusing on the remainder when each price and its multiples are divided by 5:

Price per candy	Remainder when price for 1 candy is	Remainder when price for 2 candies is	Remainder when price for 3 candies is	Remainder when price for 4 candies is

	divided by 5	divided by 5	divided by 5	divided by 5
8	3	1	4	1
13	3	1	4	2
40	0	0	0	0
53	3	1	4	2
66	1	2	3	4

The only price for which none of its multiples have a remainder of 4 when divided by 5 is 40¢.
Notice that not having a remainder of 4 does *not guarantee* that exactly four coins can be used; however, having a remainder of 4 does guarantee that exactly for coins cannot be used!

The correct answer is C.

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Billy bought 5 candies with 4 coins on Friday . So maximum amount paid for 5 candies on Friday = 200¢
From this we can eliminate 53 ¢ and 66¢.
Since we have coins of denomination 1, 5, 10, 25 and 50 and we are supposed to use only 4 coins, my first thought is that I cannot make a sum which ends in a 4 or a 9 (except 4 itself). To make a sum ending in 4 or 9, I would need four coins of 1¢ and some more coins to make whatever is left e.g. to make 9, we need 5+1+1+1+1
So we cannot make 8*3 = 24
13*3 = 39
53 and 66 are anyway out as discussed above. The only option left is 40.
Answer C.

17.
From the question we know that 40 percent of the violet/green mix is blue pigment. We also know that 30 percent of the violet paint and 50 percent of the green paint is blue pigment. Since the blue pigment in the violet/green mix is the same blue pigment in the original violet and green paints, we can construct the following equation:

$.3v + .5g = .4(v + g)$
$.3v + .5g = .4v + .4g$
$.1g = .1v$
$g = v$

Therefore, the amount of violet paint is equal to the amount of green paint in the brown mixture, each contributing 50 percent of the total. Since the red pigment represents 70 percent of the weight of the violet paint, it must account for 70 percent of 50 percent of the weight of the brown mix. This represents (.7)(.5) = .35, or 35% of the total weight of the brown mix. Since we have 10 grams of the brown paint, the red pigment must account for (.35)(10) = 3.5 grams of the brown paint.

There is an alternative way to come up with the conclusion that there must be equal amounts of green and violet paints in the mix. Since there is blue paint in both the violet and green paints, when we combine the two paints, the percentage of blue paint in the mix will be a *weighted average* of the percentages of blue in the violet paint and the percentage of blue in the green paint. For example, if there is twice as much violet as green in the brown mix, the percentage of blue in the violet will get double weighted. From looking at the numbers, however, 40% is exactly the simple average of the 30% blue in violet and the 50% blue in green. This means that there must be an equal amount of both paints in the mix.

Since there are equal amounts of violet and green paint in the 10 grams of brown mixture, there must be 5 grams of each. The violet paint is 70% red, so there must be (.7)(5) = 3.5 grams of red paint in the mix.

The correct answer is B.

18.
This question requires us to untangle a series of ratios among the numbers of workers in the various years in order to find the number of workers after the first year. We can solve this problem by setting up a grid to keep track of the information:

Before	After Year 1	After Year 2	After Year 3	After Year 4

We are told initially that after the four-year period, the company has 10,500 employees:

Before	After Year 1	After Year 2	After Year 3	After Year 4
				10,500

We are then told that the ratio of the number of workers after the fourth year to the number of workers after the second year is 6 to 1. This implies that the number of workers after the fourth year is six times greater than that after the second year. Thus the number of workers after the second year must be 10,500/6 = 1,750:

Before	After Year 1	After Year 2	After Year 3	After Year 4
		1,750		10,500

We are then told that the ratio of the number of workers after the third year to the number after the first year is 14 to 1. We can incorporate this into the chart:

Before	After Year 1	After Year 2	After Year 3	After Year 4
	x	1,750	$14x$	10,500

Now we are told that the ratio of the number of workers after the third year to that before the period began is 70 to 1. We can incorporate this into the chart as well:

Before	After Year 1	After Year 2	After Year 3	After Year 4
Y	x	1,750	$14x$ $70y$	10,500

From the chart we can see that $14x = 70y$. Thus $x = 5y$:

Before	After Year 1	After Year 2	After Year 3	After Year 4
Y	5y	1,750	$70y$	10,500

Since the ratio between consecutive years is always an integer and since after three years the number of workers is 70 times greater, we know that the series of ratios for the first three years must include a 2, a 5, and a 7 (because 2 x 5 x 7 = 70). But this fact by itself does not tell us the order of the ratios. In other words, is it 2 - 5 - 7 or 7 - 2 - 5 or 5 - 2 - 7, etc? We do know, however, that the factor of 5 is accounted for in the first year. So we need to know whether the number of workers in the second year is twice as many or seven times as many as in the first year.

Recall that the number of workers after the fourth year is six times greater than that after the second year. This implies that the ratios for the third and fourth years must be 2 and 3 or 3 and 2. This in turn implies that the ratio of 7 to 1 must be between the first and second years. So 1,750 is 7 times greater than the number of workers after the first year. Thus, 1,750/7 = 250.

Alternatively, since the question states that the ratio between any two years is always an integer, we know that 1,750 must be a multiple of the number of workers after the first year. Since only 70 and 250 are factors of 1750, we know the answer must be either choice B or choice C. If we assume that the number of workers after the first year is 70, however, we can see that this must cannot work. The number of workers always increases from year to year, but if 70 is the number of workers after the first year and if the number of workers after the third year is 14 times greater than that, the number of workers after the third year would be 14 x 70 = 980, which is less than the number of workers after the second year. So choice B is eliminated and the answer must be choice C.

The correct answer is choice C: 250.

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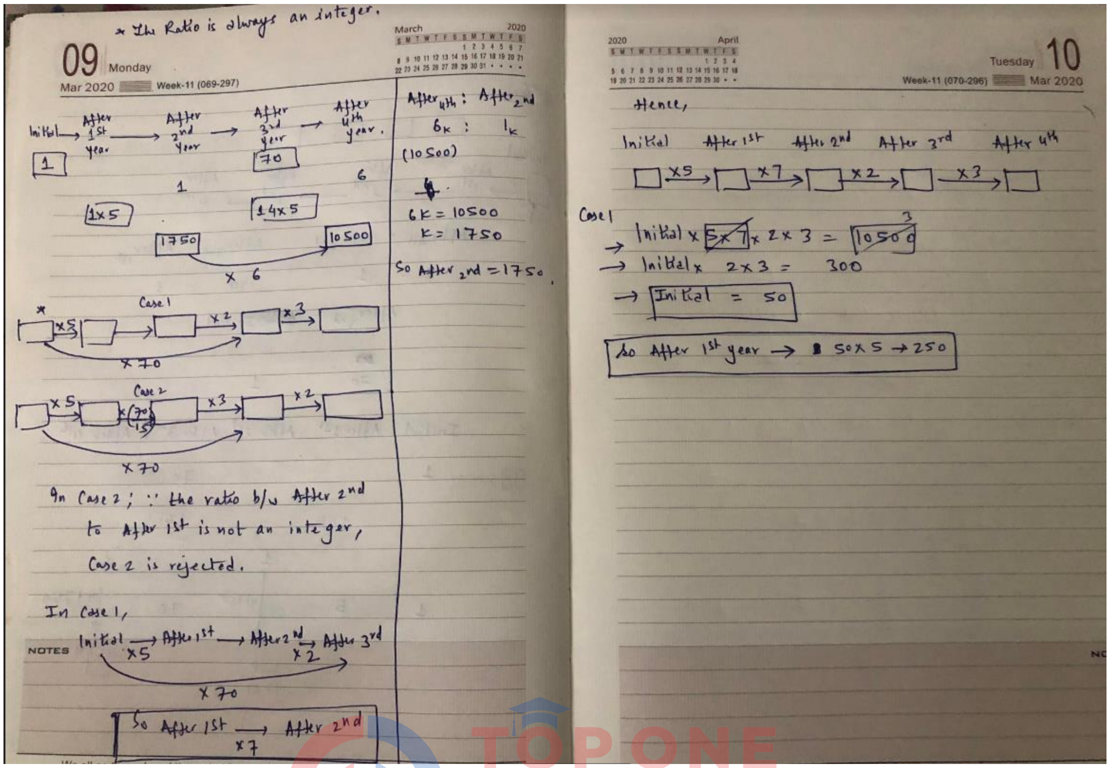

Top 1% expert replies to student queries (can skip) (additional)

Let the number of workers before the 4 year period began be x0. Let the number of workers after 1 year be x1. So on and so forth.

We're given that:

x4/x2 = 6

x3/x1 = 14/1

x3/x0 = 70

We also know that x4 = 10500

x2 = x4/6 = 1750

x3 = 14x1

x3/x0 = 70

14x1/x0 = 70

x1 = 5x0
x3 = 70x0

We know that x1 has to be a multiple of x2 = 1750 and x2 = 1750 has to be a multiple of x3.

So,
x3/x2 = 70 * x0/1750 = k, where k is an integer.
x0 = 25k.
x1 = 5x0 = 125k. So x1 has to be a multiple of 125.

From the options, if x1 = 250, then x0 = 50, x2 = 1750, x3 = 3500 and x4 = 10500
From the options, if x1 = 750, then x0 = 150, x2 = 1750, x3 = 10500 and x4 = 10500
(But this is not possible, since the number of workers has to increase every year).

So x1 = 250

The correct answer is choice C

19.
It is important to remember that if the ratio of one group to another is x:y, the total number of objects in the two groups together must be a multiple of $x + y$. So since the ratio of rams to ewes on the farm is 4 to 5, the total number of sheep must be a multiple of 9 (4 parts plus 5 parts). And since the ratio of rams to ewes in the first pen is 4 to 11, the total number of sheep in the first pen must be a multiple of 15 (4 parts plus 11 parts). Since the number of sheep in each pen is the same, the total number of sheep must be a multiple of both 9 and 15.

If we assume that the total number of sheep is 45 (the lowest common multiple of 9 and 15), the number of rams is 20 and the number of ewes is 25 (ratio 4:5).

45/3 = 15, so there are 15 sheep in each pen. Therefore, there are 4 rams and 11 ewes in the first pen (ratio 4:11). This leaves 20 - 4 = 16 rams and 25 - 11 = 14 ewes in the other two pens. Since the second and third pens have the same ratio of rams to ewes, they must have 16/2 = 8 rams and 14/2 = 7 ewes each, for a ratio of 8:7 or 8/7.

Alternatively, we can answer the question algebraically.

Since the ratio of rams to ewes in the first pen is 4:11, let the number of rams in the first pen be $4x$ and the number of ewes be $11x$. Let r be the number of rams in the second pen and let e be the number of ewes in the second pen. Since the number of sheep in each pen is the same, we can construct the following equation: $4x + 11x = r + e$, or $15x = r + e$.

Since the number of sheep in each pen is the same, we know that the number of rams in the second and third pens together is $2r$ and the number of ewes in the second and third pens together is $2e$. Therefore, the total number of rams is $4x + 2r$. The total number of ewes is $11x + 2e$. Since the overall ratio of rams to ewes on the farm is 4:5, we can construct and simplify the following equation:

$$\frac{4x + 2r}{11x + 2e} = \frac{4}{5} \rightarrow$$

$$20x + 10r = 44x + 8e \rightarrow$$

$$10r - 8e = 24x$$

We can find the ratio of r to e by setting the equations we have equal to each other. First, though, we must multiply each one by coefficients to make them equal the same value:

$$5(10r - 8e = 24x) \rightarrow$$
$$50r - 40e = 120x$$

$$8(r + e = 15x) \rightarrow$$
$$8r + 8e = 120x$$

Since both equations now equal $120x$, we can set them equal to each other and simplify:

$$50r - 40e = 8r + 8e \rightarrow$$
$$42r = 48e \rightarrow$$
$$\frac{42r}{e} = 48 \rightarrow$$
$$\frac{r}{e} = \frac{48}{42} \rightarrow$$
$$\frac{r}{e} = \frac{8}{7}$$

The correct answer is A.

20.
We can find a ratio between the rates of increase and decrease for the corn and wheat:

$$\frac{\text{rate of corn price increase}}{\text{rate of wheat price decrease}} = \frac{5x}{x\left(\sqrt{2} - 1\right)}$$

To get rid of the radical sign in the denominator, we can multiply top and bottom by $\sqrt{2} + 1$ and simplify:

$$\frac{5x}{x(\sqrt{2} - 1)} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} \rightarrow$$
$$\frac{5\sqrt{2}(x) + 5x}{x(2 - 1)} \rightarrow$$
$$\frac{\cancel{x}(5\sqrt{2} + 5)}{\cancel{x}(2 - 1)} \rightarrow$$
$$\frac{5\sqrt{2} + 5}{1}$$

This ratio indicates that for every cent that the price of wheat decreases, the price of corn increases by $5\sqrt{2} + 5$ cents. So if the price of wheat decreases by x cents, the price of corn will increase by $\left(5\sqrt{2} + 5\right)x$ cents.

Since the difference in price between a peck of wheat and a bushel of corn is currently \$2.60 or 260 cents, the amount by which the price of corn increases plus the amount by which the price of wheat decreases must equal 260 cents. We can express this as an equation:

Amount Corn Increases + *Amount Wheat Decreases* = 260

We can then rewrite this word equation using variables. Let *c* be the decrease in the price of wheat in cents:

$$\left(5\sqrt{2} + 5\right)c + c = 260 \rightarrow$$
$$\left(5\sqrt{2}\right)c + 5c + c = 260 \rightarrow$$
$$\left(5\sqrt{2}\right)c + 6c = 260 \rightarrow$$
$$\left(5\left(1.4\right)\right)c + 6c = 260 \rightarrow$$
$$7c + 6c = 260 \rightarrow$$
$$13c = 260 \rightarrow$$
$$c = 20$$

Notice that the radical 2 was replaced with its approximate numerical value of 1.4 because the question asks for the approximate price. We need not be exact in this particular instance.

If $c = 20$, we know that the price of a peck of wheat had decreased by 20 cents when it reached the same level as the increased price of a bushel of corn. Since the original price of a peck of wheat was \$5.80, its decreased price is \$5.80 - \$0.20 = \$5.60.

(By the same token, since $c = 20$, the price of a bushel of corn had increased by $20(5\sqrt{2} + 5)$ cents when it reached the same level as the decreased price of a peck of wheat. This is equivalent to an increase of approximately 240 cents. Thus the increased price of a bushel of corn = \$3.20 + \$2.40 = \$5.60.)

The correct answer is E.

21.
Let *s* represent the number of science majors, *m* represent the number of math majors, *h* represent the number of history majors, and *l* represent the number of linguistics majors.

We can set up the following equations:

$s = (1/3)h$

$m = (2/3)h$

$s + m + h + l = 2000$

We can substitute and isolate the number of linguistics majors.

$(1/3)h + (2/3)h + h + l = 2000$

$2h + l = 2000$
$l = 2000 – 2h$

We can rephrase the question: "How many students major in history?"

(1) SUFFICIENT: If $l = m$, and $m = (2/3)h$, we can solve for h:

$(1/3)h + (2/3)h + h + l = 2000$
$(1/3)h + (2/3)h + h + (2/3)h = 2000$
$(8/3)h = 2000$
$h = 2000(3/8)$
$h = 750$

If $h = 750$, $l = (2/3)h = 500$.

(2) SUFFICIENT: If $m = s + 250$, and $m = (2/3)h$ and $s = (1/3)h$, we can substitute and solve for h:

$(2/3)h = (1/3)h + 250$
$(1/3)h = 250$
$h = 750$

If $h = 750$, $l = 2000 – 2(750) = 500$.
The correct answer is D.

22.
We can think of the liquids in the red bucket as liquids A, B, C and E, where E represents *the totality of every other kind of liquid that is not A, B, or C.* In order to determine the percentage of E contained in the red bucket, we will need to determine the total amount of A + B + C and the total amount of E.
It is TEMPTING (but incorrect) to use the following logic with the information given in Statement (1).
Statement (1) tells us that the total amount of liquids A, B, and C now in the red bucket is 1.25 times the total amount of liquids A and B initially contained in the green bucket.

Let's begin by assuming that, initially, there are 10 ml of liquid A in the green bucket. Using the percentages given in the problem we can now determine that the composition of the green bucket was as follows:

10% A = 10 ml
10% B = 10 ml
80% E = 80 ml

Since there were 20 total ml of A and B in the green bucket, we know from statement (1) that there must be 25 ml of A + B + C now in the red bucket (since 25 is 1.25 times 20).
From this we can deduce that, there must have been 5 ml of C in the blue bucket. We can use the percentages given in the problem to determine the exact initial composition of the blue bucket:

10% C = 5 ml
90% E = 45 ml
Since the liquid in the red bucket is simply the totality of all the liquids in the green bucket plus all the liquids in the blue bucket, we can use this information to determine the total amount of A + B + C (25 ml) and the total amount of E (80 + 45 = 125 ml) in the red bucket. Thus, the percentage of liquid now in the red bucket that is NOT A, B, or C is equal to 125/150 = 83 1/3 percent.
This ratio (or percentage) will always remain the same no matter what initial amount we choose for liquid A in the green bucket. This is because the relative percentages are fixed. We can generalize that given an initial amount x for liquid A in the green bucket, we know that the amount of liquid B in the green bucket must also be x and that the amount of E in the green bucket must be 8x. We also know that the amount of liquid C in the blue bucket must be .5x, which means that the amount of E in the blue bucket must be 4.5x.

Thus the total amount of A + B + C in the red bucket is x + x + .5x = 2.5x and the total amount of liquid E in the red bucket is 8x + 4.5x = 12.5x. Thus the percentage of liquid now in the red bucket that is NOT A, B, or C is equal to 12.5x/15x or 83 1/3 percent.

However, the above logic is FLAWED because it assumes that the green bucket does not contain liquid C and that the blue bucket does not contain liquids A or B.
In other words, the above logic assumes that knowing that there are x ml of A in the green bucket implies that there are 8x ml of E in the green bucket. Remember, however, that E is defined as *the totality of every liquid that is NOT A, B,* ***or C****!* While the problem gives us information about the percentages of A and B contained in the green bucket, it does not tell us anything about the percentage of C contained in the green bucket and we cannot just assume that this is 0. If the percentage of C in the green bucket is not 0, then this will change the percentage of E in the green bucket as well as changing the relative amount of liquid C in the blue bucket.

For example, let's say that the green bucket contains 10 ml of liquids A and B but also contains 3 ml of liquid C. Take a look at how this changes the logic:
Green bucket:
10% A = 10 ml
10% B = 10 ml
3% C = 3 ml
77% E = 77 ml
Since there are 20 total ml of A and B in the green bucket, we know from statement (1) that there must be 25 ml of A + B + C in the red bucket (since 25 is 1.25 times 20).
Since the green bucket already contributes 23 ml of this total, we know that there must be 2 total ml of liquids A, B and C in the blue bucket. If the blue bucket does not contain liquids A or B (which we cannot necessarily assume), then the composition of the blue bucket would be the following:
10% C = 2 ml
90% E = 18 ml
Note, however, that if the blue bucket does contain some of liquids A or B, then the composition of the blue bucket might also be the following:
10% C = 1 ml
10% A = 1 ml
80% E = 8 ml

Notice that it is impossible to ascertain the exact amount of E in the red bucket - since this amount will change depending on whether the green bucket contains liquid C and/or the blue bucket contains liquids A or B.
Thus statement (1) by itself is NOT sufficient to answer this question.
Statement (2) tells us that the green and blue buckets did not contain any of the same liquids. As such, we know that the green bucket did not contain liquid C and that the blue bucket did not contain liquids A or B. On its own, this does not help us to answer the question. However, taking Statement (2) together with Statement (1), we can definitively answer the question.

The correct answer is C: BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

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ASK = % of non (Liq A, B and C) in red bucket

But we do not know by the description in the question itself that
we have any liquid A or liquid B in green bucket ? or we have any liquid C in bucket green ?

Answer to this is given in statement B : alone B can't suffice

Question asks us to find = non liquid (A , B and C) in red bucket / total liquid in red bucket
$= [(x+ y) - \frac{x}{10} - \frac{x}{10} - \frac{y}{10}] / (x+ y)$

$= 1 - \frac{(2x+y)}{10(x+y)}$

statement 1 : $\frac{x}{10} + \frac{x}{10} + \frac{y}{10} = 1.25(\frac{x}{10} + \frac{x}{10})$

$= 2x + y = 2.5x$
$= y = 0.5x$ or $x = 2y$

answer : 1 - 5y/30y = 5/6 or 83.33% = Ans C

	GREEN BUCKET	BLUE BUCKET
LIQ A	X/10	0(frm ST-2)
LIG B	X/10	0(frm ST-2)
LIQ C	0 (frm ST- 2)	Y/10
OTHERS	8x/10	9y/10
TOTAL	X	Y

	RED BUCKET
GREEN BUCKET	X
BLUE BUCKET	Y
TOTAL	X + Y

Top 1% expert replies to student queries (can skip)

First, the important parts of the premise:
1) It asks you to identify the percentage of non-ABC liquids in the bucket. Therefore, you don't need to be able to have an exact quantity.
2) Since we are adding two buckets and need to know how the two buckets mix, we DO need to know the relative size of the two buckets. (e.g., how much bigger is the blue bucket than the green bucket)

With that, let's start with statement 1:
The trap here is to start calculating a bunch of hypothetical numbers or assigning tons of variables to get the answer. But, if you remember that you need to have some info on the relative size of the two buckets, you can quickly eliminate this statement as insufficient since it doesn't tell you anything about the size of the blue bucket. For example, the green bucket could be 1 liter, and the blue bucket is 1000 liters (or vise versa). Since we don't know how much C is in the green bucket or how much A or B is in the blue bucket, sizes could be off the charts differently.

Eliminate A and D.

On to statement 2:
Remember here that you have to forget about statement 1. Again, this statement doesn't tell you anything about the relative size of the buckets and now you also don't have info about how the proportions of liquids end up in the red bucket. Therefore this is insufficient.

Eliminate B.

Both statements combined:
Statement 1 tells you the ratio of A+B in Green to A+B+C in Red. Statement 2 tells you that there is no C in Green and no A or B in Blue. Therefore, you know that:
since A+B+C in red = 1.25 (A+B) in green

and C in green = 0
And A+B in blue = 0
then,
C in blue = .25(A+B) in green

Since A and B are equal portions in green,
then A = B
then C in blue = .25 (A+B) = .25 (A+A) = .25 (2A) = 0.5A
If C is half of A, but still constitutes the same percentage of the blue bucket as A of the green bucket, you know that the blue bucket must be half the size of the green bucket. Now you have your relative size.

You also know from the second statement that blue doesn't have any A or B and green doesn't have any C. So now you can find the solution.

Assume the green bucket is 1000 liters and blue bucket is 500 liters (any ratio of 2:1 would work here).
A = 100 liters
B = 100 liters
C = 50 liters
D = (1000 liters +500 liters) - (100 liters + 100 liters + 50 liters) = 1500 - 250 = 1250.

D as a percentage = 1250/1500 = 83.33%

Therefore both statements together are sufficient

23.
When solving such kind of questions, we just need to know the ratio one price to another price. It is time waste to calculate one by one.
Both two statements do not give the information, as well as their combination.
Answer is E

Top 1% expert replies to student queries (can skip)
We know that $5x + 3y \leq 10$. We have to answer whether $4x+4y \leq 10$

Stat 1: x<1 given. If x is a little less than 1 but almost 1, then 5x is little less than 5. Thus, we get that y is less than 5/3
4x will be a little less than 4 and 4y will be less than 4x5/3 i.e. 6.66. Together, 4x + 4y will be less than 10.66. It may be less than 10 or a little more than 10, hence this is not sufficient

Stat 2: x<10/11 given. If x is a little less than 10/11 but almost 10/11, then 5x is little less than 50/11. Thus, we get that y is less than 20/11
4x will be a little less than 40/11 and 4y will be less than 4x20/11 i.e. 80/11. Together, 4x + 4y will be less than 120/11 i.e. less than 10.9. It may be less than 10 or more, we do not know, hence this is not sufficient

Together also they won't be sufficient. **Hence, E is the correct choice**

24. This question can be restated in several ways. Let *Work* = amount earned (i.e., amount needed to purchase the jacket). Recall, *Work* = *Rate* x *Time*. Since the number of hours that either Jim or Tom need to work in order to purchase the jacket is given, we need only know either person's rate of pay to determine the cost of the jacket; hence, the question can be restated as either: "What is x?" or "What is y?".

Also, since the amount of time needed for either Jim or Tom to purchase the jacket is given, it can be shown that the amount of time needed for them working together to purchase the jacket can also be calculated. The formula *Work* = *Rate* x *Time* also applies when Jim and Tom work together; hence, only the combined rate of Jim and Tom working together is required. Since the combined rate of two people working together is equal to the sum of their individual rates, the question can also be restated as: "What is $X + Y$?"

(1) INSUFFICIENT: This statement gives only the relative earning power of Jim and Tom. Since the original question states the amount of time needed for either Jim or Tom to earn enough money to purchase the jacket, it also gives us the relative earning power of Jim and Tom. Hence, statement (1) does not add any information to the original question.

(2) SUFFICIENT: Let $Z = 1$ jacket. Since Tom and Jim must 4 and 5 hours, respectively, to earn enough to buy 1 jacket, in units of "jacket per hour," Jim works at the rate of 1/4 jackets per hour and Tom works at the rate of 1/5 jackets per hour. Their combined rate is 1/4 + 1/5 = 5/20 + 4/20 = 9/20 jackets per hour. Since *Time* = *Work/Rate*, *Time* = 1 jacket/(9/20 jackets per hour) = 20/9 hours.

Since the combined pay rate of the Jim and Tom is equal to the sum of the individual pay rates of the two; hence, the combined pay rate in dollars per hour is $X + Y$. When the two work together, *AmountEarned* = *CombinedPayRate* x *Time* = $(X + Y)$ x 9/20. Since statement (2) states that $X + Y$ = \$43.75, this statement is sufficient to compute the cost of the jacket (it is not necessary to make the final calculation).

The correct answer is B.

Note: It is also not necessary to explicitly compute the time needed for Jim and Tom working together to earn the jacket (20/9 hours). It is only necessary to recognize that this number *can be calculated* in order to determine that (2) is sufficient.

25. The question stem tells us that Bill has a stack of \$1, \$5, and \$10 bills in the ratio of 10 : 5 : 1 respectively. We're trying to find the number of \$10 bills.

(1) INSUFFICIENT: Since the ratio of the number of \$1 bills to \$10 bills is 10 : 1, the dollar value of the \$1 and \$10 bills must be equal. Therefore statement (1) gives us no new information, and we cannot find the number of \$10 bills.

(2) SUFFICIENT: The problem states that the number of \$1, \$5, and \$10 bills is in the ratio of

10 : 5 : 1, so let's use an unknown multiplier x to solve the problem.

	\$1 bills	\$5 bills	\$10 bills	Total
Number	10 x	5 x	1 x	16 x
Value	\$10 x	\$25 x	\$10 x	\$45 x

Using x, we can see that there are $10x$ \$1 bills with a value of \$$10x$. Furthermore, there are $5x$ \$5 bills with a value of \$$25x$. Finally, there are $1x$ \$10 bills with a value of \$$10x$. Statement (2) says that the total amount he has is \$225, so we can set up an equation as follows:

$\$10x + \$25x + \$10x = \$225$
$\$45x = \$225$
$x = 5$

Since there are $1x$ \$10 bills this means that there are 5 \$10 bills.

The correct answer is B.

26. It is tempting to view the information in the question as establishing a pattern as follows:
Green, Yellow, Red, Green, Yellow, Red, . . .
However, consider that the following non-pattern is also possible: Green, Yellow, Red, Green, Green, Green, Green .
.
INSUFFICIENT: This tells us that the 18th tile is Green or Red but this tells us nothing about the 24th tile. Statement (1) alone is NOT sufficient.
INSUFFICIENT: This tells us that the 19th tile is Yellow or Red but this tells us nothing about the 24th tile. Statement (2) alone is NOT sufficient.
AND (2) INSUFFICIENT: Together, the statements yield the following possibilities for the 18th and 19th tiles:
GY, GR, RY, or RR
However, only GY adheres to the rules given in the question. Thus, we know that tile 18 is green and tile 19 is yellow. However, this does not help us to determine the color of the next tile, much less tile 24 (the one asked in the question). For example, the *next* tile (tile 20) could be green or red. Thus, the statements taken together are still not sufficient.
The correct answer is E.

27. Each basket must contain at least one of each type of fruit. We also must ensure that every basket contains less than twice as many apples as oranges. Therefore, the minimum number of apples that we need is equal to the number of baskets, since we can simply place one apple per basket (even if we had only 1 apple and 1 orange per basket, we would not be violating any conditions). If we are to divide the 20 oranges evenly, we know we will have 1, 2, 4, 5, 10, or 20 baskets (the factors of 20). But because we don't know the exact number of baskets, we do not know how

many apples we need. Thus, the question can be rephrased as: "How many baskets are there?"

INSUFFICIENT: This tells us only that the number of baskets is even (halving an odd number of baskets would result in half of a basket). Since we have 20 oranges that must be distributed evenly among an even number of baskets, we know we have 2, 4, 10, or 20 baskets. But because we still do not know exactly how many baskets we have, we cannot know how many apples we will need.

SUFFICIENT: This tells us that 10 oranges (half of the original 20) would not be enough to place an orange in every basket. So we must have more than 10 baskets. Since we know the number of baskets is 1, 2, 4, 5, 10, or 20, we know that we must have 20 baskets. Therefore, we know how many apples we will need.

The correct answer is B.

28.

Let the cost of each coat be x, the sales price be y. We just want to know what is 20(y-x).

For 1, we knew that 20(2y-x)=2400, insufficient to find y-x

For 2, we knew that 20(y+2-x)=440, we can get 20(y-x)=400. It's sufficient.

Answer is B

29.

For 1, country A can send 9 representatives, total number will

9+8+7+6+5+41=76>75.

Answer is E

30.

Let number of rows is a, number of the chairs in a row is b.

So, b-a=1

From 1, ab72, a8, b9, sufficient alone.

From 2, 2b-1=17, b=9, sufficient alone.

Answer is D.

31. Statement 1 is obviously insufficient

Statement 2, let Friday be x. To obtain the least value of x, the other five days should be, x-1, x-2, x-3, x-4, x-5

So, 38+x+x-1+x-2+x-3+x-4+x-5=90

6x=67

x=67/6>11

Sufficient.

Answer is B.

32.

Let attend fee be x, number of person be y:

Form 1, (x-0.75)(y+100)=xy --- 100x-0.75y-75=0

From 2, (x+1.5)(y-100)=xy ----100x+1.5y-150=0

Combine 1 and 2, we can get specific value of x and y.

Answer is C

33.
Combined 1 and 2, three situations need to be studied:
----Last week + this week<36, then x=(510-480)/2=15, the number of the items is 480/15=32
----Last week=35, then x=480/35=160/7. Or x can be resolved in the way: x=(510-480)/(1+3/2)=12, two result are conflict.
----Last week>=36, then x=30/(2*3/2)=10. The number of the items more than 36 =(480-36*10)/20=6, so, total number is 36+6=42
Above all, **answer is E**

Alternate Solution from GMATCLUB

First let's set the equation for Bob's income:

When $n \leq 36$, then $I = nx$;

When $n > 36$, then $I = 36x + (n-36)1.5x = 1.5nx - 18x$.

The question asks to find the value of n.

(1) Last week Bob was paid total of $480 for the items that he produced that week.

So, $I = 480$.

Either $I = 480 = nx$ OR $I = 480 = 1.5nx - 18x$.

Not sufficient.

(2) This week Bob produced 2 items more than last week and was paid a total of $510 for the items that he produced this week.

So, $I' = 510$ and $n' = n + 2$.

Either: $I' = 510 = (n+2)x$ OR $I' = 510 = 1.5(n+2)x - 18x$.

Not sufficient.

(1)+(2) We can have three different systems of equations:

$480 = nx$ and $510 = (n+2)x$, meaning that $n+2 \leq 36$. In this case $x = 15$ and $n = 32$.

OR:

$480 = nx$ and $510 = 1.5(n+2)x - 18x$, meaning that $n+1 \leq 36$ and $n+2 > 36$ ($n = 35$). In this case n has no integer value, so this system doesn't work;

OR:

$480 = 1.5nx - 18x$ and $510 = 1.5(n+2)x - 18x$, meaning that $n > 36$. In this case $x = 10$ and $n = 44$.

So we can have two values for n: 32 and 44. Not sufficient.

Answer: E.

The last step can be done in another way:

We know that 2 more items resulted 30$ more.

If these two items were paid by 1.5x rate (n>=36) --> 1.5x+1.5x=30 --> x=10 and as n>=36, we should substitute this value in the second equation from (1), which gives n=44

If these two items were paid by x rate (n<=34) --> x+x=30 --> x=15 and as n<=34, we should substitute this value in the first equation from (1), which gives --> n=32

Already two different answers for n (no need to check for the third case when one item is paid by regular rate and another with overtime rate), hence insufficient.

Answer: E.

34.
1) is sufficient.
2). No two members sold same number of tickets; the least numbers of the tickets they sold would be 0, 1, 2.
Answer is D

35.
"one kilogram of a certain coffee blend consists of X kilogram of type I and Y kilogram of type II" means that X+Y=1
Combined C=6.5X+8.5Y, we get:
X=(8.5-C)/2, Y=(C-6.5)/2
Combined C>=7.3, X=(8.5-C)/2<1.2/2=0.6
Answer is B

36.
It is somewhat tricky.
Usually, we need two equations to solve two variables.
For example, in this question, from 1, x=y=6, from 2, 21x+23y=130, **the answer should be C.**

Actually, the variables in such questions should be integers. Thus, hopefully, we can solve them with only one equation.
21x+23y=130, we try x=1, 2, 3, 4,5..and find that only x=4, y=2 can fulfill the requirements. **Answer is B.**
To sum up, please be careful when you met such questions.

37.
More than 10 Paperback books, at least is 11, and cost at least
$88 From 1, 150/25=6, at least 6 hardcover books.

From 2, 260-150-88=22, is not enough to buy a hardcover book.
Combined 1 and 2, we know that Juan bought 6 hardcover books.
Answer is C

38.
c=kx+t
In last month, cost=1000k+t; profit=1000(k+60) –(1000k+t) =60000-t, so, we need to solve t.
From 1, 150000=1000*(k+60), there is no information about t.
From 2, (1000k+t)-(500k+t)=45000, still cannot solve out t.
Answer is E

39. Answer is B

Set up equation: $\frac{x}{60,000}=\frac{2+5+6+4}{5,000+12,000+18,000+16,000}$ --> $x=20$;

Or: $2+5+6+4=17$ defective chips in $5,000+12,000+18,000+16,000=51,000$ chips, so $\frac{17}{51,000}=\frac{1}{3,000}$: 1 in 3,000. So, expected number of defective chips in a shipment of 60,000 chips is $\frac{60,000}{3,000}=20$.

Answer: B.

40.
The fine for one day: $0.1
The fine two days: $0.2, as it is less than $0.1+$0.3
The fine for three days: $0.4, as it is less than $0.2+$0.3
The fine for four days: $0.4+$0.3=$0.7, as it is less than $0.4*2
Answer is B

41.
Let x be the height of the tree increase each year, then:
[4+6x-(4+4x)]/(4+4x) = 1/5
10x = 4+ 4x
x= 2/3

42.
In the origin plan, each one should pay X/T.
Actually, each of the remaining coworkers paid X/(T-S).
Then, X/(T-S) - X/T = S*X / T(T-S)

43. The business produced a total of 4x rakes from November through February. The storage situations were shown in the following table:
So, the total cost is 14X*0.1=1.4X

Month	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Total
Storage	7x/2	3x	5x/2	2x	3x/2	x	x/2	0	14x

44. In order to realize a profit, the company's revenue must be higher than the company's costs. We can express this as an inequality using the information from the question:

$$12 - p < p(6 - p)$$

If we distribute and move all terms to one side, we get:

$$12 - p < p(6 - p) \rightarrow$$
$$12 - p < 6p - p^2 \rightarrow$$
$$p^2 - 7p + 12 < 0$$

We can factor this result:

$$p^2 - 7p + 12 < 0 \rightarrow$$
$$(p - 3)(p - 4) < 0$$

When the value of p makes this inequality true, we know we will have a profit. When the value of p does NOT make the inequality true, we will not have a profit. When p equals 3 or 4, the product is zero. So the values of p that will make the inequality true (i.e., will yield a negative product) must be either greater than 4, less than 3, or between 3 and 4. To determine which is the case, we can test a sample value from each interval.

If we try $p = 5$, we get:

$$(5 - 3)(5 - 4) \rightarrow$$
$$(2)(1) = 2$$

Since 2 is positive, we know that values of p greater than 4 will not make the inequality true and thus will not yield a profit.

If we try $p = 2$, we get:

$$(2 - 3)(2 - 4) \rightarrow$$
$$(-1)(-2) = 2$$

Since 2 is positive, we know that the values of p less than 3 will not make the inequality true and thus will not yield a profit.

If we try $p = 3.5$, we get:

$$(3.5 - 3)(3.5 - 4) \rightarrow$$
$$(.5)(-.5) = -.25$$

Since -.25 is negative, we know that values between 3 and 4 will make the inequality true and will thus yield a profit. Since p can be any positive value less than 100 (we cannot have a negative price or a price of zero dollars), there are 100 possible intervals between consecutive integer values of p. The interval $3 < p < 4$ is just one. Therefore, the probability that the company will realize a profit is 1/100 and the probability that it will NOT realize a profit is 1 - 1/100 or 99/100.

The correct answer is D.

45. To calculate the average daily deposit, we need to divide the sum of all the deposits up to and including the given date by the number of days that have elapsed so far in the month. For example, if on June 13 the sum of all deposits to that date is $20,230, then the average daily deposit to that date would be $\frac{20,230}{13}$.

We are told that on a randomly chosen day in June the sum of all deposits to that day is a prime integer greater than 100. We are then asked to find the probability that the average daily deposit up to that day contains fewer than 5 decimal places.

In order to answer this question, we need to consider how the **numerator** (the sum of all deposits, which is defined as a prime integer greater than 100) interacts with the **denominator** (a randomly selected date in June, which must therefore be some number between 1 and 30).

First, are there certain denominators that – no matter the numerator – will always yield a quotient that contains fewer than 5 decimals?

Yes. A fraction composed of any integer numerator and a denominator of 1 will always yield a quotient that contains fewer than 5 decimal places. This takes care of June **1**.

In addition, a fraction composed of any integer numerator and a denominator whose prime factorization contains only 2s and/or 5s will always yield a quotient that contains fewer than 5 decimal places. This takes cares of June **2**, **4**, **5**, **8**, **10**, **16**, **20**, and **25**.

Why does this work? Consider the chart below:

Denominator	Any Integer divided by this denominator will yield either an integer quotient or a quotient ending in:	# of Decimal Places
2	.5	1
4	multiples of .25	maximum of 2
5	multiples of .2	1
8	multiples of .125	maximum of 3
10	multiples of .1	1
16	multiples of .0625	maximum of 4
20	multiples of .05	maximum of 2
25	multiples of .04	maximum of 2

What about the other dates in June?

If the chosen day is any other date, the denominator (of the fraction that makes up the average daily deposit) will contain prime factors other than 2 and/or 5 (such as 3 or 7). Recall that the numerator (of the fraction that makes up the average daily deposit) is defined as a prime integer greater than 100 (such as 101).

Thus, the denominator will be composed of at least one prime factor (other than 2 and/or 5) that is not a factor of the numerator. Therefore, when the division takes place, it will result in an infinite decimal. (To understand this principle in greater detail read the explanatory note that follows this solution.)

Therefore, of the 30 days in June, only 9 (June 1, 2, 4, 5, 8, 10, 16, 20, and 25) will produce an average daily deposit that contains fewer than 5 decimal places: $\frac{9}{30} = \frac{3}{10}$.

The correct answer is D.

Explanatory Note: Why will an infinite decimal result whenever a numerator is divided by a denominator composed of prime factors (other than 2 and/or 5) that are not factors of the numerator?

Consider division as a process that ends when a remainder of 0 is reached.

Let's look at 1 (the numerator) divided by 7 (the denominator), for example. If you divide 1 by 7 on your calculator, you will see that it equals .1428... This decimal will go on infinitely because 7 will never divide evenly into the remainder. That is, a remainder of 0 will never be reached.

$\begin{array}{r} .1 \\ 7\overline{)1.0} \end{array}$ r. 3 $\rightarrow$ $\begin{array}{r} 4 \\ 7\overline{)3.0} \end{array}$ r. 2 $\rightarrow$ $\begin{array}{r} 2 \\ 7\overline{)2.0} \end{array}$ r. 6 $\rightarrow$ $\begin{array}{r} 8 \\ 7\overline{)6.0} \end{array}$ r. 4 , and so on...

For contrast, let's look at 23 divided by 5:

$\begin{array}{r} 4 \\ 5\overline{)23} \end{array}$ r. 3 $\rightarrow$ $\begin{array}{r} .6 \\ 5\overline{)3.0} \end{array}$ r. 0

So 23/5 = 4.6. When the first remainder is divided by 5, the division will end because the first remainder (3) is treated as if it were a multiple of 10 to facilitate the division and 5 divides evenly into multiples of 10.

By the same token, the remainder when an odd number is divided by 2 is always 1, which is treated as if it were 10 to facilitate the division. 10 divided by 2 is 5 (hence the .5) with no remainder.

When dividing by primes that are not factors of 10 (e.g., 3, 7, 11, etc.), however, the process continues infinitely because the remainders will always be treated as if they were multiples of 10 but the primes cannot divide cleanly into 10, thus creating an endless series of remainders to be divided.

If the divisor contains 2's and/or 5's in addition to other prime factors, the infinite decimal created by the other prime factors will be divided by the 2's and/or 5's but will still be infinite.

46. It might be tempting to think that either statement is sufficient to answer this question. After all, pouring water from the larger container to the smaller container will leave exactly 2 gallons of water in the larger container. Repeating this operation twice will yield 4 gallons of water.

    The problem is - where would these 4 gallons of water accumulate? We will need to use one of the containers. However, neither statement alone tells us whether one of the containers will hold 4 gallons of water.

    On the other hand, statements (1) and (2) taken together ensure that the first container can hold at least 4 gallons of water. We know this because (from statement 1) the first container holds 2 gallons more than the second container, which (from statement 2) holds 2 gallons more than the third container, which must have a capacity greater than 0.

    Since we know that the first container has a capacity of at least 4 gallons, there are several ways of measuring out this exact amount.

    One method is as follows: Completely fill the first container with water. Then pour out just enough water from the first container to fill the third container to the brim. Now, 4 gallons of water remain in the first container.

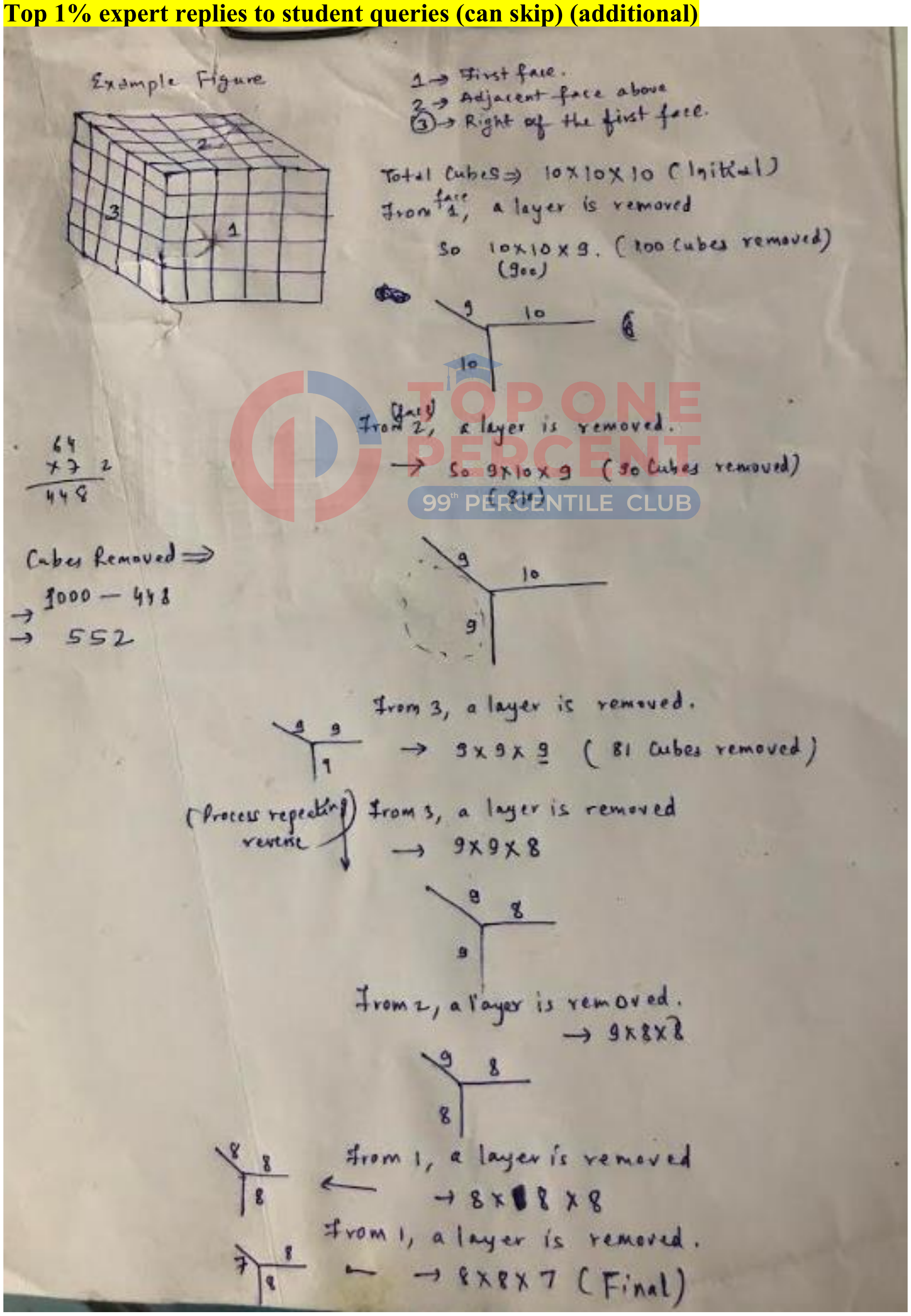
    Alternatively: Fill the first container to the brim. Pour out just enough water from the first container to fill the second container to the brim. There are now 2 gallons of water in the first container. Now pour water from the second container to fill the third container to the brim.
    There are now 2 gallons of water in the second container. Finally, pour all the water from the second container into the first container. There are now 4 gallons of water in the first container.
    The correct answer is C.

47. We can answer this by keeping track of how many cubes are lopped off of each side as the cube is trimmed (10 x 10 + 10 x 9 + 9 x 9 + ...), but this approach is tedious and error prone. A more efficient method is to determine the final dimensions of the trimmed cube, then find the difference between the dimensions of the trimmed and original cubes.

Let's call the first face A, second face B, and third face C. By the end of the operation, we will have removed 2 layers each from faces B and C, and 3 layers from face A. So B now is 8 cubes long, C is 8 cubes long, and A is 7 cubes long. The resulting solid has dimensions 8 x 8 x 7 cubes or 448 cubes. We began with 1000 cubes, so 1000 - 448 = 552. Thus, 552 cubes have been removed.

The correct answer is B.

Top 1% expert replies to student queries (can skip) (additional)



48. In order to determine the length of the line, we need to know how many people are standing in it. Thus, rephrase the question as follows: How many people are standing in the line? Statement (1) says that there are three people in front of Chandra and three people behind Ken. Consider the following different scenarios:

The line might look like this: (Back) X X X Ken X Chandra X X X (Front)

OR

The line might looks like this: (Back) Chandra X X Ken (Front)

The number of people in the line depends on several factors, including whether Chandra is in front of Ken and how many people are standing between Chandra and Ken. Since there are many different scenarios, statement (1) is not sufficient to answer the question.

Statement (2) says that two people are standing between Chandra and Ken. Here, we don't know how many people are ahead or behind Ken and Chandra. Since there are many different scenarios, statement (2) is not sufficient to answer the question.

Taking both statements together, we still don't know whether Chandra is in front of Ken or vice versa, and therefore we still have two different possibilities:

The line might look like this: (Back) X X X Ken X X Chandra X X X (Front)

OR

The line might look like this: (Back) Chandra X X Ken (Front)

Therefore, the correct answer is (E): Statements (1) and (2) TOGETHER are NOT sufficient.

49. Begin by rephrasing, or simplifying, the original question. Since the rules of the game involve the *negative* of the sum of two dice, one way of restating this problem is that whoever gets the higher sum LOSES the game. Thinking about the sum of the two dice is easier than thinking about the *negative* of the sum of the two dice. Thus, let's rephrase the question as: Who *lost* the game? (Knowing this will obviously allow us to answer the original question, who *won* the game.)

Statement (1) gives us information about the first of Nina's dice, but it does not tell us anything about the second. Consider the following two possibilities:

	Nina's First Roll	Nina's Second Roll	Teri's Sum	Higher Sum = Loser
CASE ONE	**3**	5	**–1**	Nina
CASE TWO	**3**	–5	**–1**	Teri

Notice that in both cases, Nina's first roll is greater than Teri's Sum. However, in Case One Nina loses, but in Case Two Teri loses. Thus, this information is not sufficient to answer the question.

Statement (2) gives us information about the second of Nina's dice, but it does not tell us anything about the first. Using the same logic as for the previous statement, this is not sufficient on its own to answer the question.

Combining the information contained in both statements, one may be tempted to conclude that Nina's sum must be higher than Teri's sum. However, one must test scenarios involving both positive and negative rolls. Consider the following two possibilities.

	Nina's First Roll	Nina's Second Roll	Teri's Sum	Higher Sum = Loser
CASE ONE	**3**	**4**	**–5**	Nina
CASE TWO	**–3**	**–4**	**–5**	Teri

Notice that in both cases, Nina's first roll is greater than Teri's Sum and Nina's second roll is greater than Teri's sum. However, in Case One Nina loses, but in Case Two Teri loses. Thus, this information is not sufficient to answer the question.

The correct answer is E: Statements (1) and (2) TOGETHER are NOT sufficient.

Top 1% expert replies to student queries (can skip)

So Teri cannot get a sum greater than 4, as Nina won't be able to get more than 5 on a single throw of the die.

Statement 1: Say Teri got a sum of 4, then her score was -4. Nina got 5 in the first throw. Now she can get anything on the second try. Say she got 0, her score is -5 and Teri won. Say Nina got -2, her sum is 3, score is -3, and she wins. This statement by itself is insufficient.

Statement 2: By the same logic as above, because we don't know the first throw, this statement by itself is insufficient

Combining the two: Say Teri got a sum of 4, so score of -4. Nina got 5 and 5 on her two throws, sum of 10, and score of -10. Teri won. Now say Teri got a sum of -5 (-4 and -1), and hence score of 5. Say Nina scored -4 and -4 on her two throws (each throw is individually greater than the sum of Teri's throws. Nina scored 8 and Nina won. Both together are not sufficient

The correct answer is E: Statements (1) and (2) TOGETHER are NOT sufficient.

50. Every third Alb gives a click. This means no click is awarded until the third Alb is captured. The second click is not awarded until the sixth Alb is captured. Similarly, a tick is not awarded until the fourth Berk is captured.

We are told that the product clicks x ticks = 77. Thus, there are four possibilities: 1 × 77, 7 × 11, 11 × 7, 77 × 1.

Clicks Awarded	Albs Captured	Ticks Awarded	Berks Captured
1	3, 4 or 5	77	308, 309, 310, or 311
7	21, 22, or 23	11	44, 45, 46 or 47
11	33, 34, or 35	7	28, 29, 30 or 31
77	231, 232 or 233	1	4, 5, 6, or 7

Statement (1) tells us that the difference between Albs captured and Berks captured is 7. Looking at the chart, the only way to get a difference of 7 between Albs captured and Berks captured is with 35 Albs and 28 Berks. Therefore, statement (1) is sufficient to answer the question--there must have been 35 Albs captured.

Statement (2) says the number of Albs captured is divisible by 4. Again, looking at the chart, we see that the number of Albs captured must be 4 or 232. Therefore, statement (2) is not sufficient to answer the question--we do not know how many Albs were captured.

The correct answer is A: Statement (1) alone is sufficient, but statement (2) alone is not sufficient.

Top 1% expert replies to student queries (can skip)

Let's say tick=T and berk=B. To solve this case, you need to understand that one T can be granted only if the 3rd Alb is captured, that means 1t can be = 3, 4, or 5. Similarly, for B = 4, 5, 6, 7.

Now, we are given BxT=77
Only four possibilities satisfy this: 1 × 77, 77 × 1, 7 × 11, 11 × 7.

Clicks Scored----Albs can be-----------Ticks Scored-------Berks can be
1---------------3, 4 or 5--------------77----------------308, 309, 310, or 311
77--------------231, 232 or 233--------1------------------4, 5, 6, or 7
7---------------21, 22, or 23-----------11-----------------44, 45, 46 or 47

11--------------33, 34, or 35-----------7------------------28, 29, 30 or 31

S(1): The difference between Albs captured and Berks captured is 7.
On the above list, only 35 Albs and 28 Berks meets the S1. Sufficient.
S(2) The numbers of Albs captured is divisible by 4.
Values can be 4 or 232. So, one value cannot be definitely identified. **Insuffiicient**.
Hence, OA is A

51.
The question gives a function with two unknown constants and two data points. In order to solve for the position of the object after 4 seconds, we need to first solve for the constants r and b. We can do this by creating two equations from the two data points given:

$p(2) = 41 = r(2) - 5(2)^2 + b$
$41 = 2r - 20 + b$
$61 = 2r + b$

$p(5) = 26 = r(5) - 5(5)^2 + b$
$26 = 5r - 125 + b$
$151 = 5r + b$

We can now solve these equations for r and b using substitution:

$61 = 2r + b$
$(61 - 2r) = b$

$151 = 5r + b$
$151 = 5r + (61 - 2r)$
$151 = 3r + 61$
$90 = 3r$
$r = 30$

Substituting back in, we can find b:

$61 = 2r + b$
$61 = 2(30) + b$
$b = 1$

So, we can rewrite the original function and plug in t = 4 to find our answer:

$p(t) = 30t - 5t^2 + 1$
$p(4) = 30(4) - 5(4)^2 + 1$
$p(4) = 120 - 80 + 1$
$p(4) = 41$

The correct answer is D.

52.

Let us call the Trussian's current age a. Therefore the Trussian's current weight is $\sqrt{a}$. Seventeen years after he is twice as old as he is now, the Trussian's age will be 2a + 17 and his weight will therefore be $\sqrt{2a+17}$.We are told that the Trussian's current weight, $\sqrt{a}$, is three keils less than his future weight, $\sqrt{2a+17}$. Therefore, $\sqrt{a} + 3 = \sqrt{2a+17}$.

We can solve the equation as follows:

$$\sqrt{a} + 3 = \sqrt{2a + 17}$$
$$(\sqrt{a} + 3)^2 = (\sqrt{2a + 17})^2$$
$$a + 6\sqrt{a} + 9 = 2a + 17$$
$$6\sqrt{a} = a + 8$$
$$(6\sqrt{a})^2 = (a + 8)^2$$
$$36a = a^2 + 16a + 64$$
$$a^2 - 20a + 64 = 0$$
$$(a - 16)(a - 4) = 0$$

a = 16 or 4. However, we are told that the Trussian is a teenager so he must be 16 years old.

The correct answer is C.

53.

This problem is easier to think about with real values.

Let's assume that there are 2 high level officials. This means that each of these 2 high level officials supervises 4 (or x^2) mid-level officials, and that each of these 4 mid-level officials supervises 8 (or x^3) low-level officials.

It is possible that the supervisors do not share any subordinates. If this is the case, then, given 2 high level officials, there must be 2(4) = 8 mid-level officials, and 8(8) = 64 low-level officials.

Alternatively, it is possible that the supervisors share all or some subordinates. In other words, given 2 high level officials, it is possible that there are as few as 4 mid-level officials (as each of the 2 high-level officials supervise the same 4 mid-level officials) and as few as 8 low-level officials (as each of the 4 mid-level officials supervise the same 8 low-level officials).

Statement (1) tells us that there are fewer than 60 low-level officials. This alone does not allow us to determine how many high-level officials there are. For example, there might be 2 high level officials, who each supervise the same 4 mid-level officials, who, in turn, each supervise the same 8 low-level officials. Alternatively, there might be 3 high-level officials, who each supervise the same 9 mid-level officials, who, in turn, each supervise the same 27 low-level officials.

Statement (2) tells us that no official is supervised by more than one person, which means that supervisors do not share any subordinates. Alone, this does not tell us anything about the number of high-level officials.

Combining statements 1 and 2, we can test out different possibilities.

If x = 1, there is 1 high-level official, who supervises 1 mid-level official ($1^2 = 1$), who, in turn, supervises 1 low-level official ($1^3 = 1$).

If x = 2, there are 2 high-level officials, who each supervise a unique group of 4 mid-level officials, yielding 8 mid-level officials in total. Each of these 8 mid-level officials supervise a unique group of 8 low-level officials, yielding 64 low-level officials in total. However, this cannot be the case since we are told that there are fewer than 60 low-level officials.

Therefore, based on both statements taken together, there must be only 1 high-level official.
The correct answer is C.

Top 1% expert replies to student queries (can skip)

Second statement says that 'No' official is supervised by more than one person.' This means that there is no overlap. 1 mid-level official is not managed by 2 or more high-level officials. 1 low-level official is not managed by 2 or more mid-level officials

There can be two cases as below. Stat 2 eliminates case 2

Case 1 : NO OVERLAP (no official is supervised by more than one person)
High-level X
Mid-level X(X2)=X3
Low-level (X3)(X3)=X6

Case 2 : MAXIMUM OVERLAP (ALL lower-level official are supervised by the SAME higher-level official)
High-level X
Mid-level X2
Low-level X3

54.
Use algebra to solve this problem as follows:

Let the x = the number of donuts Jim originally ordered. Since he paid $15 for these donuts, the price per donut for his original order is $15/x.

When he leaves, Jim receives 3 free donuts changing the price per donut to $15/(x + 3). In addition, we know that the price per dozen donuts was $2 per dozen cheaper when he leaves, equivalent to a per donut savings of $2/12 = 1/6 dollars.

Using this information, we can set up an equation that states that the original price per donut less 1/6 of a dollar is equal to the price per donut after the addition of 3 donuts:

$$\frac{15}{x} - \frac{1}{6} = \frac{15}{x+3}$$

We can now solve for x as follows:

$$\frac{15}{x} - \frac{1}{6} = \frac{15}{x+3}$$
$$\frac{90-x}{6x} = \frac{15}{x+3}$$
$$(90-x)(x+3) = 90x$$
$$-x^2 - 3x + 90x + 270 = 90x$$
$$0 = x^2 + 3x - 270$$
$$0 = (x-15)(x+18)$$
$$x = \{15, -18\}$$

The only positive solution of x is 15. Hence, Jim left the donut shop with x + 3 = 18 donuts.

The correct answer is A.

55.
In questions like this, it helps to record the given information in a table. Upon initial reading, the second sentence is probably very confusing but what is clear is that it discusses the ages of the two boys at two different points in time: let's refer to them as ―now‖, and ―then‖. So, let's construct a table such as the one below. Let x and y denote the boys' ages ―now‖:

	Johnny's age	Bobby's age
Now	x	y
then		

Now, re-read the first few words of the second sentence: ―Johnny's age now is the same as Bobby's age . . . _then'‖. We can fill in one more entry of the table as shown:

	Johnny's age	Bobby's age
Now	x	y
then		x

Finally, the rest of the second sentence tells us that ―then‖ was the time when Johnny's age was half Bobby's current age; i.e., Johnny's age ―then‖ was (1/2)y. We can complete the table as follows:

	Johnny's age	Bobby's age
Now	x	y
then	(1/2)y	x

One way to solve this problem is to realize that, as two people age, the ratio of their ages changes but the difference in their ages remains constant. In particular, the difference in the boys ages ―now'‖ must be the same as the difference in their ages ―then‖. This leads to the equation: $y - x = x - (1/2)y$, which reduces to $x = (3/4)y$; Johnny is currently three-fourths as old as Bobby.

Without another equation, however, we can't solve for the values of either x or y. (Alternatively, we could compute the elapsed time between ―then‖ and ―now‖ for each boy and set the two equal; this leads to the same equation as above.)
(1) INSUFFICIENT: Bobby's age at the time of Johnny's birth is the same as the difference between their ages, $y - x$. So statement (1) tells us that $y = 4(y - x)$, which reduces to $x = (3/4)y$. This adds no more information to what we already knew! Statement (1) is insufficient.
(2) SUFFICIENT: This tells us that Bobby is 6 years older than Johnny; i.e., $y = x + 6$. This gives us a second equations in the two unknowns so, except in some rare cases, we should be able to solve for both x and y -- statement (2) is sufficient. Just to verify, substitute $x = (3/4)y$ into the second equation to obtain $y = (3/4)y + 6$, which implies $y = 24$. Bobby is currently 24 and Johnny is currently 18.

The correct answer is B.

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Let Johny's current age be x
Let Bobby's current age be y.

The past age of Johny we're talking about here is y/2 [Since Johnny was half as old as Bobby is now]
Number of years in the past = Johny's current age - Johnny's age in the past = (x - y/2)
Therefore, Bobby's age in the past = Bobby's current age - Number of years in the past = y - (x - y/2) = 3y/2 - x

We have been given that Johnny's current age = Bobby's age in the past.

Therefore,

x = 3y/2 - x
2x = 3y/2
x = 3y/4

Statement 1 :

Bobby is currently four times as old as he was when Johnny was born.

Let Bobby's age when Johnny was born be k

Johnny's age when he was born = 0
Johnny's current age = x
Years passed = x

Therefore, Bobby's current age = k + x = y = 4k [Bobby is currently 4 times as old as he was when Johnny was born]

This does not help us figure out Bobby's current age. Insufficient!

Statement 2 :

Bobby was six years old when Johnny was born. Here, k = 6

Let Bobby's age when Johnny was born be k

Johnny's age when he was born = 0
Johnny's current age = x
Years passed = x

Therefore, Bobby's current age = x + 6 = y

We know that x = 3y/4

Therefore,

3y/4 + 6 = y

y/4 = 6

y = 24

Therefore, Bobby's current age is 24. Sufficient!

Answer is B

56.

First, let c be the number of cashmere blazers produced in any given week and let m be the number of mohair blazers produced in any given week. Let p be the total profit on blazers for any given week. Since the profit on cashmere blazers is $40 per blazer and the profit on mohair blazers is $35 per blazer, we can construct the equation p = 40c + 35m. In order to know the maximum potential value of p, we need to know the maximum values of c and m.

Statement (1) tells us that the maximum number of cutting hours per week is 200 and that the maximum number of sewing hours per week is 200.

Since it takes 4 hours of cutting to produce a cashmere blazer and 4 hours of cutting to produce a mohair blazer, we can construct the following inequality: $4c + 4m \leq 200$.

Since it takes 6 hours of sewing to produce a cashmere blazer and 2 hours of sewing to produce a mohair blazer, we can construct the following inequality: $6c + 2m \leq 200$.

In order to maximize the number of blazers produced, the company should use all available cutting and sewing time. So we can construct the following equations:

$4c + 4m = 200$
$6c + 2m = 200$

Since both equations equal 200, we can set them equal to each other and solve:

$4c + 4m = 6c + 2m \rightarrow$

$2m = 2c \rightarrow$

$m = c \rightarrow$

$4m + 4(m) = 200 \rightarrow$

$8m = 200 \rightarrow$

$m = 25 \rightarrow$

$m = c \rightarrow$

$c = 25$

So when m = 25 and c = 25, all available cutting and sewing time will be used. If p = 40c + 35m, the profit in this scenario will be 40(25) + 35(25) or $1,875. Is this the maximum potential profit?

Since the profit margin on cashmere is higher, might it be possible that producing only cashmere blazers would be more profitable than producing both types? If no mohair blazers are made, then the largest number of cashmere blazers that could be made will be the value of c that satisfies 6c = 200 (remember, it takes 6 hours of sewing to make a cashmere blazer). So c could have a maximum value of 33 (the company cannot sell 1/3 of a blazer). So producing only cashmere blazers would net a potential profit of 40(33) or $1,320. This is less than $1,875, so it would not maximize profit.

Since mohair blazers take less time to produce, perhaps producing only mohair blazers would yield a higher profit. If no cashmere blazers are produced, then the largest number of mohair blazers that could be made will be the value of m that satisfies 4m = 200 (remember, it takes 4 hours of cutting to produce a mohair blazer). So m would have a maximum value of 50 in this scenario and the profit would be 35(50) or $1,750. This is less than $1,875, so it would not maximize profit.

So producing only one type of blazer will not maximize potential profit, and producing both types of blazer maximizes potential profit when m and c both equal 25.

Statement (1) is sufficient.

Statement (2) tells us that the wholesale cost of cashmere cloth is twice that of mohair cloth. This information is irrelevant because the cost of the materials is already taken into account by the profit margins of $40 and $35 given in the question stem.

Statement (2) is insufficient.

The answer is A: Statement (1) alone is sufficient, but statement (2) alone is not.

57.
Each year, the age of the boy increases by 1. Each year, the sum of the ages of the two girls increases by 2 (as each girl gets older by one year, and there are two of them).
Let's say that the age of the boy today is equal to x, while the combined ages of the girls today is equal to y. Then, next year the figures will be x + 1 and y + 2, respectively. The problem states that these two figures will be equal, which yields the following equation:

x + 1 = y + 2 which can be simplified to ***x* = *y* + 1**

(This is consistent with the fact that the sum of the ages of the two girls today is smaller than the age of the boy today.)
Three years from now, the combined age of the girls will be y + 3(2) = y + 6. Three years from now, the boy's age will be x + 3. Using the fact (from above) that x = y + 1, the boy's age three years from now can be written as x + 3 = (y + 1) + 3 = y + 4.
The problem asks for the difference between the age of the boy three years from today and the combined ages of the girls three years from today. This difference equals y + 4 – (y + 6) = –2**. The correct answer is D.**

Plug in real numbers to see if this makes sense.
Let the girls be 4 and 6 in age. The sum of their ages today is 10. The boy's age today is then (10 + 1) = 11. Three years from today, the girls will be 7 and 9 respectively, so their combined age will be 16. Three years from today, the boy will be 14.

Be careful: The question asks for the difference between the boy's age and the sum of the girls ages three years from today. Which one will be younger? The boy. So the difference between the boy's age and the combined age of the girls will be a negative value: 14 – 16 = – 2.

58.
Use a chart to keep track of the ages in this problem:

	x years ago	NOW	in x years
Corey	C - x	C	C + x
Tania	T - x	T	T + x

Then write algebraic expressions to represent the information given in the problem:

x years ago, Cory was one fifth as old as Tania	In x years, Tania will be twice as old as Cory
$5(C - x) = T - x$ $5C - 5x = T - x$ $5C - T = 4x$	$2(C + x) = T + x$ $2C + 2x = T + x$ $T - 2C = x$

Substitute T - 2C in for x in the first equation and solve:

$5C - T = 4(T - 2C)$
$5C - T = 4T - 8C$
$13C = 5T$
$\frac{C}{T} = \frac{5}{13}$

The correct answer is C.

59.
The question does not ask for the actual number of years ago that animal z became extinct. Instead it asks for t, the number of years scientists predicted it would take for animal z to become extinct.

(1) INSUFFICIENT: This tells us that animal z became extinct 4 years ago but it does not provide information about t.

(2) INSUFFICIENT: This provides a relationship between the predicted time of extinction time and the actual time of extinction but does not provide any actual values for either.

(1) AND (2) INSUFFICIENT: The easiest way to approach this problem is to imagine a time line from 0 to 10. The scientists made their prediction 10 years ago, or at 0 years.

From statement (1) we know that animal z became extinct 4 years ago, or at 6 years.

From statement (2) we know that if the scientists had extended their prediction by 3 years they would have been incorrect by 2 years. The key to this question is to realize that "incorrect by 2 years" could mean 2 years in either direction: 6 + 2 = 8 years or 6 – 2 = 4 years.

From here, we can write two simple equations:

$t + 3 = 8$ OR $t + 3 = 4$
$t = 5$ $t = 1$

This gives us two different values for t, which means that (1) and (2) together are not sufficient to come up with one definitive value for t.
The correct answer is E.

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(1) Animal z became extinct 4 years ago. The only thing we can get from this statement is when animal z actually extincted: 4 years ago or 6 years after the prediction. The point here is that it is not necessary that the prediction is accurate. Not sufficient.

(2) If the scientists had extended their extinction prediction for animal z by 3 years, their prediction would have been incorrect by 2 years. Also not sufficient: t+3=actual extinction +/- 2.

(1)+(2) Animals extincted 6 years after the prediction: t+3=6-2 --> t=1 OR t+3=6+2 --> t=5. Two answers, not sufficient.
Answer: E.

Note: Predictions are not 100% precise. But even if you are confused by the first statement, the second one should help to realise that the predicted extinction date and the actual extinction date are not the same.

60.
To answer this question, we need to minimize the value of $l = (7.5 - x)^4 + 8.97^{1.05}$. Since we do not need to determine the actual minimum longevity, we do not need to find the value of the second component in our formula, $8.97^{1.05}$, which will remain constant for any level of x. Therefore, to minimize longevity, we need to minimize the value of the first component in our formula,

i.e. $(7.5 - x)^4$. Since we are raising the expression $(7.5 - x)$ to an even exponent, 4, the value of $(7.5 - x)^4$ will always be non-negative, i.e. positive or zero. Thus, to minimize this outcome, we need to find the value of x for which $(7.5 - x)^4 = 0$.
$(7.5 - x)^4 = 0$
$7.5 - x = 0$
$x = 7.5$

Therefore, the metal construction will have minimal longevity for the value of $x = 7.5$, i.e. when the density of the underlying material will be equal to 7.5 g/cm^3.

The correct answer is C.

MISCELLANEOUS QUESTIONS
Part B: Calculations, Exponents, Basic Algebra

1.
The first culprit in this expression is the radical in the denominator. Radicals in the denominator are dealt with by multiplying the fraction with an expression that is equal to 1 but contains a ―cancelling radical‖ in both the numerator and denominator.

For example, $\frac{6}{\sqrt{3}}$ is simplified by multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$.

In the case of a complex radical, such as $5+2\sqrt{6}$, we multiply by the conjugate, $5-2\sqrt{6}$ as follows:

$$\sqrt{\sqrt{96}+\frac{2}{5+2\sqrt{6}}\left(\frac{5-2\sqrt{6}}{5-2\sqrt{6}}\right)}=\sqrt{\sqrt{96}+\frac{10-4\sqrt{6}}{25-4(6)}}$$

$$=\sqrt{\sqrt{96}+\frac{10-4\sqrt{6}}{1}}=\sqrt{\sqrt{96}+10-4\sqrt{6}}$$

$$=\sqrt{\sqrt{16\times 6}+10-4\sqrt{6}}=\sqrt{4\sqrt{6}+10-4\sqrt{6}}=\sqrt{10}$$

Simplifying radicals in the denominator with conjugate radical expressions is very useful on challenging GMAT radical questions.

The correct answer is C.

2.
The GMAT does not require you to know how to evaluate an integral ***root*** of any general integer, but you are expected to understand how to evaluate an integer raised to an integer ***power***.
Hence, you should immediately realize that there must be a way we can transform each of the expressions into an expression that we can evaluate.
The most obvious way to transform a root into a power is to raise it to a higher power. Since we are trying to compare the expressions, a reasonable transformation is to raise each of the expressions to the same power. What power should we use?

Since $\left(\sqrt[m]{x}\right)^n=\left(x^{\frac{1}{m}}\right)^n=x^{\frac{n}{m}}$, in order to get integral powers of x (i.e., n/m is an integer) we should raise all of the roots to the Least Common Multiple of the m‘s. We have roots of 3, 6, 10, and 15, so the LCM is equal to 30. Therefore, we should raise each of the expressions to the 30th power as follows:

$$\left(\sqrt[3]{2}\right)^{30}=2^{10}=1024$$
$$\left(\sqrt[6]{5}\right)^{30}=5^{5}=3125$$
$$\left(\sqrt[10]{10}\right)^{30}=10^{3}=1000$$
$$\left(\sqrt[15]{30}\right)^{30}=30^{2}=900$$

Thus, the original expressions in increasing order are as follows:

$\sqrt[15]{30}\quad \sqrt[10]{10}\quad \sqrt[3]{2}\quad \sqrt[6]{5}$.

3.
In order to rid the expression of square roots, let's first square the entire expression. We are allowed to do this as long as we remember to "unsquare" whatever solution we get at that end.

$$\sqrt{24+5\sqrt{23}}+\sqrt{24-5\sqrt{23}} \quad\rightarrow\quad \left(\sqrt{24+5\sqrt{23}}+\sqrt{24-5\sqrt{23}}\right)^2$$

Notice that the new expression is of the form $(x+y)^2$ where
$x=\sqrt{24+5\sqrt{23}}$ and $y=\sqrt{24-5\sqrt{23}}$.

Recall that $(x+y)^2 = x^2+y^2+2xy$. This is one of the GMAT's favorite expressions.
Returning to our expression:

$x^2 = 24+5\sqrt{23}$, while $y^2 = 24-5\sqrt{23}$ and $2xy = 2\left(\sqrt{24+5\sqrt{23}}\right)\left(\sqrt{24-5\sqrt{23}}\right)$.

Notice that x^2+y^2 neatly simplifies to 48. This leaves only the $2xy$ expression left to simplify.

In order to simplify $2\left(\sqrt{24+5\sqrt{23}}\right)\left(\sqrt{24-5\sqrt{23}}\right)$, recall that $\left(\sqrt{a}\right)\left(\sqrt{b}\right)=\sqrt{ab}$.

Thus, $2\left(\sqrt{24+5\sqrt{23}}\right)\left(\sqrt{24-5\sqrt{23}}\right)=2\sqrt{\left(24+5\sqrt{23}\right)\left(24-5\sqrt{23}\right)}$.

Notice that the expression under the square root sign is of the form $(x+y)(x-y)$. And recall that $(x+y)(x-y)=x^2-y^2$. This is another one of the GMAT's favorite expressions.
Returning to our expression:

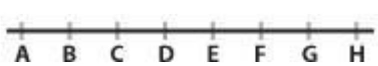
$$2\sqrt{\left(24+5\sqrt{23}\right)\left(24-5\sqrt{23}\right)}=2\sqrt{24^2-\left(5\sqrt{23}\right)^2}=2\sqrt{24^2-(25)(23)}=2\sqrt{576-575}=2\sqrt{1}=2$$
.

Finally then: $x^2+y^2+2xy = 48+2 = 50$.
But now we must remember to "unsquare" (or take the square root of) our answer:
$\sqrt{50}=\sqrt{25\times 2}=5\sqrt{2}$.

Therefore, the correct answer is D.

4. We can simplify the question as follows:
$8^a(1/4)^b = ?$ [Break all non-primes down to primes.]
$(2^3)^a(2^{-2})^b = ?$ [Multiply exponents taken on the same base.]
$(2^{3a})(2^{-2b}) = ?$ [Add exponents since the two bases are equal.]
$2^{3a-2b} = ?$
We can rephrase the question as "what is 3a -2b?"
(1) SUFFICIENT: b = 1.5a, so 2b = 3a. This means that 3a - 2b = 0.
(2) INSUFFICIENT: This statement gives us no information about b.
The correct answer is A

5.



The distance from G to H is $5^{13} - 5^{12}$.

The distance between and two consecutive points is constant, so the distance from A to G will be 6 times the distance from G to H or $6(5^{13} - 5^{12})$.

The value of A, therefore, will be equal to the value of G minus the distance from A to G: 5^{12} –

$6(5^{13} - 5^{12}) \longrightarrow 5^{12} - 6[5^{12}(5 - 1)] \longrightarrow 5^{12} - 6(5^{12})(4) \longrightarrow$

$5^{12}(1 - 24) \longrightarrow (-23)5^{12}$.

The correct answer is B.

6. $(3^{5x} + 3^{5x} + 3^{5x})(4^{5x} + 4^{5x} + 4^{5x} + 4^{5x}) = 3^{5x}(1 + 1 + 1) \times 4^{5x}(1 + 1 + 1 + 1) =$
$3(3^{5x}) \times 4(4^{5x}) = 3^{5x+1} \times 4^{5x+1} = (3 \times 4)^{5x+1} = 12^{5x+1}$

Remember that when you multiply different bases raised to the SAME exponent, the product is simply the product of the bases raised to their common exponent.

The correct answer is A.

7.

First, let us simplify the exponential equation:

$(2^a)(3^b)(5^c) = 12(2^k)(3^l)(5^m)$

$(2^a)(3^b)(5^c) = (3)(4)(2^k)(3^l)(5^m)$

$(2^a)(3^b)(5^c) = (3^1)(2^2)(2^k)(3^l)(5^m)$

$(2^a)(3^b)(5^c) = (2^{k+2})(3^{l+1})(5^m)$

When the bases on both sides of an equation are equal and the bases are prime numbers, the exponents of the respective bases must also be equal: a = k + 2; b = l + 1; and c = m. Now recall that a, b, and c represent the hundreds, tens, and units digits of the three-digit integer x; similarly, k, l, and m represent the hundreds, tens, and units digits of the three-digit integer y.

Therefore, the hundreds digit of x is 2 greater than the hundreds digit of y; the tens digit of x is 1 greater than the tens digit of y; finally, the units digit of x is equal to the units digit of y.

i.e., a−k=2, b−l=1, and c−m=0

Using this information, we can set up our subtraction problem and find the value of (x – y):

abc – klm = 210.

The correct answer is C.

8. (1) SUFFICIENT: Statement(1) tells us that $x > 2^{34}$, so we want to prove that $2^{34} > 10^{10}$. We'll prove this by manipulating the expression 2^{34}.

$2^{34} = (2^4)(2^{30})$

$2^{34} = 16(2^{10})^3$

Now $2^{10} = 1024$, and 1024 is greater than 10^3. Therefore:

$2^{34} > 16(10^3)^3$

$2^{34} > 16(10^9)$

$2^{34} > 1.6(10^{10})$.

Since $2^{34} > 1.6(10^{10})$ and $1.6(10^{10}) > 10^{10}$, then $2^{34} > 10^{10}$.

(2) SUFFICIENT: Statement (2) tells us that that $x = 2^{35}$, so we need to determine if $2^{35} > 10^{10}$. Statement (1) showed that $2^{34} > 10^{10}$, therefore $2^{35} > 10^{10}$.

The correct answer is D.

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Once you do x > (2^10)(2^24) you are basically comparing that to (2^10)(5^10) to be conclusively able to answer if x is also > (2^10)(5^10).

The only elements different in the two are (2^24) and (5^10). Three possibilities here

(2^24) > (5^10) —> We can definitely say x > (2^10)(5^10)

(2^24) = (5^10) —> We can definitely say x > (2^10)(5^10)

BUT if (2^24) < (5^10) —> We CANNOT definitely say x > (2^10)(5^10). x may be > (2^10)(2^24) but that does not give us any indication where it lies w.r.t. (2^10)(5^10)

You won't be able to answer conclusively like this, hence in this approach this statement by itself is not sufficient.

I will tell you two more ways in which Statement 1 will not be sufficient.

Say x = (2^10)(2^24); y = (2^10)(5^10)

x/y = (2^24)/(5^10) [since 2^10 not equal to 0, it can be cancelled out]

Now in the above equation, whether the right side is >, < or = 1 will determine whether x is >, <, or = y. So this statement is not sufficient. Yes there is a subtlety here wherein you could say hey, I know exactly what the value of numerator and denominator are, so I know what the fraction is. But the gMAT is not expecting that these values are known (or indeed can be calculated - just try doing 2^24 and see how that turns out)

Now, coming to what you were asking.

The situation is x > (2^10)(2^24) and (2^24) < (5^10). The question is, can we conclusively say x > (2^10)(5^10)

Let's take an easier to visualize example.

x > (20)(2) and 2 < 100. Can we conclusively say x > (20)(100)?

x can be (20)(5), in which case x < (20)(100) or x can be (20)(1000), in which case x > (20)(100).

For even more clarity, let's say (2^24) < (5^10). Picturize a vertical column of numbers (higher numbers are greater than lower numbers), and somewhere on that vertical column (2^10)(2^24) lies on the bottom, there is a vertical gap, and (2^10)(5^24) lies on the top. x > (2^10)(2^24), so x can be somewhere in the gap between the two numbers, or x can be in the gap above the greater number - we can't say for sure

The correct answer is D.

9. A radical expression in a denominator is considered non-standard. To eliminate a radical in the denominator, we can multiply both the numerator and the denominator by the conjugate of that denominator.

$$= \sqrt{3\sqrt{80} + \frac{3}{9+4\sqrt{5}}\left(\frac{9-4\sqrt{5}}{9-4\sqrt{5}}\right)}$$

$$= \sqrt{3\sqrt{16 \cdot 5} + \frac{27-12\sqrt{5}}{81-(16)(5)}}$$

$$= \sqrt{12\sqrt{5} + 27 - 12\sqrt{5}}$$

$$= \sqrt{27}$$

$$= 3\sqrt{3}$$

The correct answer is C

10. First rewrite the expression in the question using only prime bases (4 is not prime), as follows: $2^a 2^{2b}$.

(1) SUFFICIENT: We can substitute -2b for a into the expression in the question. What is the value of $(2^{-2b})(2^{2b})$?

This can be simplified to $2^{-2b+2b} = 2^0 = 1$.

(2) INSUFFICIENT: We have no information about the value of a.

The correct answer is A.

11. An effective strategy for problems involving exponents is to break the bases of all the exponents into prime factors. This technique will allow us to combine like terms:

$27^{4x+2} \times 162^{-2x} \times 36^x \times 9^{6-2x} = 1$
$(3^3)^{4x+2} \times (2 \times 3^4)^{-2x} \times (2^2 \times 3^2)^x \times (3^2)^{6-2x} = 1$
$3^{12x+6} \times 2^{-2x} \times 3^{-8x} \times 2^{2x} \times 3^{2x} \times 3^{12-4x} = 1$
$2^{-2x+2x} \times 3^{12x+6-8x+2x+12-4x} = 1$
$2^0 \times 3^{2x+18} = 1$
$3^{2x+18} = 1$
$3^{2x+18} = 3^0$
$2x + 18 = 0$
$2x = -18$
$x = -9$

The correct answer is A.

12.
Let's rewrite the right side of the equation in base 2 and base 3: $(2^{2x+1})(3^{2y-1}) = (2^3)^x(3^3)^y$. This can be rewritten as: $(2^{2x+1})(3^{2y-1}) = 2^{3x}3^{3y}$
Since both bases on either side of the equation are prime, we can set the exponents of each respective base equal to one another:
$2x + 1 = 3x$, so $x = 1$
$2y - 1 = 3y$, so $y = -1$

Therefore, $x + y = 1 + (-1) = 0$.

The correct answer is C

13.

Before dealing with this expression, it is helpful to remember several general exponent rules: When

multiplying expressions with the same base, ADD the exponents first:
$(3^2)(3^3) = (3)(3)(3)(3)(3) = 3^5 = 3^{2+3}$

When dividing expressions with the same base, SUBTRACT the exponents first:
$(3^5)/(3^2) = (3)(3)(3)(3)(3) / (3)(3) = (3)(3)(3) = 3^3 = 3^{5-2}$

When raising a power to a power, combine exponents by MULTIPLYING them:
$(3^2)^4 = (3^2)(3^2)(3^2)(3^2) = (3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3) = (3)(3)(3)(3)(3)(3)(3)(3) = 3^8 = 3^{2(4)}$

In this question, we are asked to solve an expression with many exponents, but none of them have common bases, at least not as currently written. However, some of the bases have prime factors in common. Look at each part of the expression and break each into its factored form.

If we look at the terms in the numerator on the left side:

$6^2 = (2 \times 3)^2 = 2^2 \times 3^2$
$44 = 2 \times 2 \times 11 = 2^2 \times 11^1$
5^x cannot be factored
$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$
Note that several of the terms in the numerator have bases in common, so the numerator simplifies by ADDING the exponents of those terms:
$(2^2 \times 3^2)(2^2 \times 11^1)(5^x)(2^2 \times 5^1) = (2^6)(3^2)(5^{(x+1)})(11^1)$

If we look at the terms in the denominator on the left side:

$8^2 = (2 \times 2 \times 2)^2 = (2^3)^2 = 2^6$
$9 = 3 \times 3 = 3^2$

On the right side of the equation:

$1375 = 5 \times 5 \times 5 \times 11 = 5^3 \times 11^1$

Now that each exponent and large number is expressed in terms of its prime factors, we can put the equation back together. Then we'll see what can cancel to simplify the entire equation: $(2^6)(3^2)(5^{(x+1)})(11^1) \div (2^6)(3^2) = 5^3 \times 11^1$
Cancelling the 2^6 and 3^2 that appear in the numerator and denominator of the left side, and cancelling the 11 that appears on each side of the equal sign:

$5^{(x+1)} = 5^3$
$x + 1 = 3$
$x = 2$

The correct answer is D.

14.
If we know y, we can solve for x. Thus, we can rephrase the question, "What is y?"

(1) SUFFICIENT: If $y^2 = 625$, we know that y = 25 or -25. However, since 5^x will be positive no matter what x is, and $5^x = y$, then y must be positive. Thus, y must be 25. If y is 25, we know that x = 2.

(2) SUFFICIENT: If $y^3 = 15{,}625$, y must equal 25. (Tip: In order to understand a number like 15,625 exponentially, try breaking it down into its roots. $15{,}625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 25 \times 25 \times 25 = 25^3$.) If y is 25, we know that x = 2.

The correct answer is D.

15.
This question might be rephrased ―How many golf balls do Wendy and Pedro have combined?‖ Otherwise, we must simply find the number of balls possessed by Jim.

(1) INSUFFICIENT: Observe that Jim could have 2 balls and Wendy 6, or Jim could have 3 balls while Wendy has 9.

(2) INSUFFICIENT: If Pedro has 1/2 of the golf balls, Pedro has 12 balls. However, this statement gives no information about the number of balls possessed by Jim or Wendy.

(1) AND (2) SUFFICIENT: Statement (2) tells us that Pedro has 12 balls. Therefore, Wendy and Jim collectively have the remaining 12 balls. Statement (1) tells us that Jim has 1/3 of the number of Wendy‘s golf balls. Let j = the number of Jim‘s golf balls and w = the number of Wendy‘s golf balls.

$j = w/3$

Multiplying both sides by 3 yields

$3j = w$

Given that $j + w = 12$, we can substitute that $j + 3j = 12$. If $4j = 12$, $j = 3$.

The correct answer is C.

16.
The best way to answer this question is to use the exponential rules to simplify the question stem, then analyze each statement based on the simplified equation.

$(3^{27})(35^{10})(z) = (5^{8})(7^{10})(9^{14})(x^{y})$ Break up the 35^{10} and simplify the 9^{14} $(3^{27})(5^{10})(7^{10})(z) = (5^{8})(7^{10})(3^{28})(x^{y})$ Divide both sides by common terms 5^{8}, 7^{10}, 3^{27} $(5^{2})(z) = 3x^{y}$

(1) SUFFICIENT: Analyzing the simplified equation above, we can conclude that z must have a factor of 3 to balance the 3 on the right side of the equation. Statement (1) says that z is prime, so z cannot have another factor besides the 3. Therefore z = 3.

Since z = 3, the left side of the equation is 75, so $x^{y} = 25$. The only integers greater than 1 that satisfy this equation are x = 5 and y = 2, so statement (1) is sufficient. Put differently, the expression x^{y} must provide the two fives that we have on the left side of the equation. The only way to get two fives if x and y are integers greater than 1 is if x = 5 and y = 2.

(2) SUFFICIENT: Analyzing the simplified equation above, we can conclude that x must have a factor of 5 to balance out the 5^{2} on the left side. Since statement (2) says that x is prime, x cannot have any other factors, so x = 5. Therefore statement (2) is sufficient.

The correct answer is D.

Alternate Solution from Gmatclub

Split everything into prime factors:

$$(3^{27})(35^{10})(z) = (5^8)(7^{10})(9^{14})(x^y)$$

$$(3^{27})(5^{10})(7^{10}) * (z) = (3^{28})(5^8)(7^{10})(x^y)$$

Now powers of prime factors on both sides of the equation should match since all variables are integers. If you have only 3^{27} on left hand side, it cannot be equal to the right hand side which has 3^{28}. Prime factors cannot be created by multiplying other numbers together and hence you must have the same prime factors with the same powers on both sides of the equation.

Stmnt 1: z is prime
Note that you have 3^{28} on Right hand side but only 3^{27} on left hand side. This means z must have at least one 3. Since z is prime, z MUST be 3 only. You get

$$(3^{28})(5^{10})(7^{10}) = (3^{28})(5^8)(7^{10})(x^y)$$

Now 5^2 is missing on the right hand side since we have 5^{10} on left hand side but only 5^8 on right hand side. So x^y must be 5^2. x MUST be 5.
Sufficient.

Stmnt2: x is prime
If x is prime, it must be 5 since 5^2 is missing on the right hand side. This would give us $x^y = 5^2$. Sufficient.

Answer (D)

17. It is tempting to express both sides of the equation $4^{4x} = 1600$ as powers of 4 and to try and solve for x. However, if we do that, we get a power of five on the right side as well:
$4^{4x} = 16 \times 100$
$4^{4x} = 4^2 \times 4 \times 25$
$4^{4x} = 4^3 \times 5^2$
It becomes clear that x is not an integer and that we can't solve the question this way.

Let's try manipulating the expression about which we are being asked. $(4^{x-1})^2 = 4^{2x-2}$
If we further simplify we get the expression $4^{2x}/4^2$
To solve this expression, all we need is to find the value of 4^{2x}

Now let's look back at our original equation. If $4^{4x} = 1600$, we can find the value of 4^{2x} by taking the square root of both sides of the equation. Taking the square root of an exponential expression is tantamount to halving its exponent.

$$4^{4x} = 1600$$
$$\sqrt{4^{4x}} = \sqrt{1600}$$
$$4^{2x} = 40$$

Since the question asks for $4^{2x}/4^2$, the answer is $40/4^2$, which simplifies to 40/16 or 5/2.

The correct answer is D

18.
$(15^x + 15^{x+1}) = 15^y4^y$ $[15^x + 15^x(15^1)] = 15^y4^y$
$(15^x)(1 + 15) = 15^y4^y$
$(15^x)(16) = 15^y4^y$
$(3^x)(5^x)(2^4) = (3^y)(5^y)(2^{2y})$

Since both sides of the equation are broken down to the product of prime bases, the respective exponents of like bases must be equal.
$2y = 4$ so $y = 2$.
$x = y$ so $x = 2$.

The correct answer is A

19.
In problems that involve exponential expressions on both sides of the equation, it is imperative to rewrite the bases so that either the same base or the same exponent appears on both sides of the equation. Here, we can get a common base by replacing the 9 with 3^2.

$3^m 3^m 3^m = 9^n$
$3^m 3^m 3^m = (3^2)^n$
$3^{3m} = 3^{2n}$

Since the bases are the same, the exponents must be equal. $3m = 2n$
$m/n = 2/3$

The correct answer is B

20.
(1) SUFFICIENT: We can rewrite this equation in a base of 3: $3^{b+2} = 3^5$, which means that $b + 2 = 5$ and therefore $b = 3$.
We can plug this value into the equation $a = 3^{b-1}$ to solve for a.
(2) SUFFICIENT: We can set the right side of this equation equal to the right side of the equation in the quesiton (both sides equal a).
$3^{b-1} = 3^{2b-4}$, which means that $b - 1 = 2b - 4$ and therefore $b = 3$. We can plug this value into the equation $a = 3^{b-1}$ to solve for a.
The correct answer is D.

21.
Recognize here the basic form $(x - y)^2$, which equals $x^2 - 2xy + y^2$.
$\sqrt{7+\sqrt{29}}$ corresponds here to x, and $\sqrt{7-\sqrt{29}}$ corresponds to y.

So the expression can be simplified to:
$$\left(\sqrt{7+\sqrt{29}}\right)^2 - 2\left(\sqrt{7+\sqrt{29}}\right)\left(\sqrt{7-\sqrt{29}}\right) + \left(\sqrt{7-\sqrt{29}}\right)^2 \rightarrow$$
$$7+\sqrt{29} - 2\sqrt{\left(7+\sqrt{29}\right)\left(7-\sqrt{29}\right)} + 7 - \sqrt{29} \rightarrow$$
$$14 - 2\sqrt{\left(7+\sqrt{29}\right)\left(7-\sqrt{29}\right)}$$

Under the radical, recognize the basic form $(a + b)(a - b)$, which equals $a^2 - b^2$. The

expression can be further simplified to:

$$14 - 2\sqrt{49-29} \rightarrow$$
$$14 - 2\sqrt{20} \rightarrow$$
$$14 - 4\sqrt{5}$$
The correct answer is C.

22.
The key to this question is to recognize the common algebraic identity:
$a^2 - b^2 = (a + b)(a - b)$

In this question, the a term is x/3 and the b term is 2/y, which makes the identity from the question equal to:
$x^2/9 - 4/y^2 = (x/3 - 2/y)(x/3 + 2/y) = 12$

(1) SUFFICIENT:
Substituting the information from this statement into the equation from the question: $(x/3 - 2/y)(x/3 + 2/y) = 12$
$(x/3 - 2/y)(6) = 12$
$x/3 - 2/y = 2$

We now have two equations with x and y:
$x/3 + 2/y = 6$ (from this statement)
$x/3 - 2/y = 2$ (from the substitution above)

Combine the two equations (by adding), then simplify:
$(2)(x/3) = 8$ (the y terms cancelled)
$x/3 = 4$
$x = 12$

(2) SUFFICIENT:
Substituting the information from this statement into the equation from the question: $(x/3 - 2/y)(x/3 + 2/y) = 12$
$(2)(x/3 + 2/y) = 12$
$x/3 + 2/y = 6$

We now have two equations with x and y:
$x/3 - 2/y = 2$ (from this statement)
$x/3 + 2/y = 6$ (from the substitution above)

Combine the two equations (by adding), then simplify:
$(2)(x/3) = 8$ (the y terms cancelled)
$x/3 = 4$
$x = 12$

The correct answer is D.

23.
We can determine the value of $(a + b)^2$ in one of three ways: by figuring out the sum of a and b, by determining what a and b are separately, or by determining the value of $a^2 + 2ab + b^2$ (which is the quadratic form of our product of factors).

(1) INSUFFICIENT: After multiplying both sides by b we can determine that $ab = 15$, but we know nothing else.

(2) INSUFFICIENT: If we FOIL $(a - b)^2$ we can learn that $a^2 - 2ab + b^2 = 4$. Alternatively, this statement also indicates that $(a - b) = 2$ or -2. However, neither of these manipulations allow us to determine the sum of a and b, the respective values of a and b individually, or the value of $a^2 + 2ab + b^2$.

(1) AND (2) SUFFICIENT: Statement 1 tells us $ab = 15$. If we substitute this into the quadratic equation from the second statement, we can determine the value of $a^2 + b^2$ in the following manner:

$a^2 - 2(15) + b^2 = 4$
$a^2 - 30 + b^2 = 4$
$a^2 + b^2 = 34$

If we know the value of $a^2 + b^2$, and the value of ab, we can determine the value of $a^2 + 2ab + b^2$.

$a^2 + 2ab + b^2 =$

$a^2 + b^2 + 2ab =$
$34 + 2(15) = 64$

The correct answer is C.

24.
The equation $x^2y^2 = 18 – 3xy$ is really a quadratic, with the xy as the variable.

$x^2y^2 + 3xy – 18 = 0$
$(xy + 6)(xy – 3) = 0$
$xy = 3$ or -6

However, we are told that x and y are positive so xy must equal 3.
Therefore, $x = 3/y$ and $x^2 = 9/y^2$.
Alternatively, this is a VIC (variable in choice) and can be solved by plugging numbers. If we plug a value for y and find the corresponding value of x, we can check the answers to see which one matches the value of x.

Looking at the values 3 and 18 in the equation, a y value of 3 makes sense.
$x^2(3)^2 = 18 – 3(x)(3)$
$9x^2 = 18 – 9x$
$9x^2 – 9x + 18 = 0$
$x^2 – x + 2 = 0$
$(x + 2)(x – 1) = 0$
$x = 1, -2$
But since x cannot be negative, $x = 1$

If we plug $y = 3$ into each of the answer choices, (C) and (D) both give an x value of 1.
(A) 1/3
(B) 2
(C) 1
(D) 1
(E) 4

We must now plug another value of y to decide between (C) and (D). Ultimately, only (D) represents the correct value each time.
The correct answer is D.

25.
This equation can be manipulated into a quadratic equation by squaring both sides:

$y = \sqrt{3y + 4}$
$y^2 = 3y + 4$
$y^2 – 3y – 4 = 0$

This quadratic equation can be factored to $(y + 1)(y – 4) = 0$

There are two possible solutions for y: -1 and 4.
But y cannot be negative as it equals root of some expression ($\sqrt{expression} \geq 0$), so only one solution is valid. i.e $y = 4$.
Therefore, Sum of roots = 4.

The correct answer is D.

26.

We know that the sum of the cubes of a and b is 8: $a^3 + b^3 = 8$. We also know that $a^6 – b^6 = 14$. Using our knowledge of the quadratic template for the difference of two squares, $x^2 – y^2 = (x + y)(x – y)$, we can rewrite $a^6 – b^6 = 14$ as follows:

$(a^3)^2 – (b^3)^2 = 14$
$(a^3 – b^3)(a^3 + b^3) = 14$

Substituting for $a^3 + b^3$ gives:

$(a^3 – b^3)(8) = 14$
$(a^3 – b^3) = 14/8 = 7/4$
The correct answer is D.

27.

One way to answer this question is to substitute 1/x for x and 1/y for y in the expression, then simplify the resulting expression.

$$\frac{1/x + 1/y}{1/x - 1/y}$$

Multiply the numerator and the denominator by xy to eliminate the fractions.

$$\frac{xy(1/x + 1/y)}{xy(1/x - 1/y)} = \frac{(y + x)}{(y - x)}$$

Since this is not one of the answer choices, it is necessary to simplify further. With the knowledge that $y + x = x + y$ and $y – x = -(x – y)$, it can be stated that

$$\frac{(y + x)}{(y - x)} = -\frac{(x + y)}{(x - y)}$$

The correct answer is A.

28.
When a problem involves variables raised to the fourth power, it is often useful to represent them as a square of another square, since this approach will allow us to apply manipulations of squares. Also note that since we are dealing with high exponents, the approach of plugging numbers would prove time consuming and prone to error in this case.
Therefore, let's use algebra to solve this problem. Note that $9x^4 – 4y^4 = (3x^2)^2 – (2y^2)^2$. We can represent this expression using the formula for the difference of two squares: $9x^4 – 4y^4 = (3x^2)^2 – (2y^2)^2 = (3x^2 + 2y^2)(3x^2 – 2y^2)$.
Let's use this shortcut to simplify the equation:
$9x^4 – 4y^4 = 3x^2 + 2y^2$
$(3x^2 + 2y^2)(3x^2 – 2y^2) = (3x^2 + 2y^2)$

At this step, our first instinct may be to divide both sides of the equation by $(3x^2 + 2y^2)$. Remember that in order to divide both sizes of the equation by an algebraic expression, we need to know that the value of this expression is not equal to zero, since dividing by zero results in an undefined outcome. By looking again at the question, we see that x and y are both non-zero integers, so $(3x^2 + 2y^2)$ cannot be equal to zero. Therefore, we can indeed simplify the equation further by dividing out the $(3x^2 + 2y^2)$ from each side:
$(3x^2 – 2y^2) = 1$
$3x^2 = 2y^2 + 1$
$x^2 = (2y^2+1)/3$
Note that we could also have found the correct answer by plugging in numbers at this stage (since we have eliminated the high exponents in the equation). For example, if we plug in $y = 2$ to the equation $(3x^2 – 2y^2) = 1$, we see that $x^2 = 3$. Now we can plug $y = 2$ into each of the answer choices to find that only C also gives us $x^2 = 3$.
Finally, if we are careful, we might also see that answer choices A and B cannot be correct because they are negative, and no non-zero integer squared can equal a negative number.

The correct answer is C

29. To find the ratio of r to s, we need to be able to solve EITHER for r/s OR for r and s independently.

(1) INSUFFICIENT: The equation provided in statement 1 cannot be rewritten in the form r/s = some value.
(2) INSUFFICIENT: The equation provided in statement 2 can be simplified as follows:

$r2 – s2 = 7$
$(r + s)(r – s) = 7$.
However, this cannot be rewritten in the form r/s = some value.
(1) AND (2) SUFFICIENT: We can substitute the information from statement (1) in the equation from statement 2 as follows:
$(r + s)(r – s) = 7$.
$(7)(r – s) = 7$.
$r – s = 1$.
Adding this equation to the equation from the first statement allows us to solve for r.

$$\begin{array}{rl} & (r - s = 1) \\ + & (r + s = 7) \\ \hline & 2r \quad = 8 \end{array}$$

Thus, r = 4. If r is 4, then s must be 3. The ratio of r to s is 4 to 3.
The correct answer is C.

30.
z is the difference between the number of men and the number of women in the choir; hence z = x – y. In order to answer the question —what is z?‖, we need to be able to determine either the value of the quantity x – y, or the values of both x and y from which quantity x – y can be computed.

(1) SUFFICIENT: After adding 9 to both sides of the equation, we get $x^2 - 2xy + y^2 = 9$. Since we are interested in the variables x and y, it would be helpful to rearrange the expression $x^2 - 2xy + y^2$ into an expression that contains terms for x and y individually. This suggests that factoring the expression into a product of two sums is in order here. Since the coefficients of both the x^2 and the y^2 terms are 1 and the coefficient of the xy term is negative, the most logical first guess for factors is $(x - y)(x - y)$ or $(x - y)^2$. (We can quickly confirm that these are the correct factors by multiplying out $(x - y)(x - y)$ and verifying that this is equal to $x^2 - 2xy + y^2$.) Hence, we now have $(x - y)^2 = 9$ or $x - y = 3$ or -3. Since the stimulus states that z or x – y is a physical quantity (—there are z more men than women…‖), the only answer that makes logical sense is $x - y = 3$.

(2) INSUFFICIENT: After adding 225 to both sides of the equation, we get $x^2 + 2xy + y^2 = 225$. Since we are interested in the variables x and y, it would be helpful to rearrange the expression $x^2 + 2xy + y^2$ into an expression that contains terms for x and y individually. This suggests that factoring the expression into a product of two sums is in order here. Since the coefficients of both the x^2 and the y^2 terms are 1 and the coefficient of the xy term is positive, the most logical first guess for factors is $(x - y)(x - y)$ or $(x - y)^2$. (We can quickly confirm that these are the correct factors by multiplying out $(x - y)(x - y)$ and verifying that this is equal to $x^2 - 2xy + y^2$.) Hence, we now have $(x + y)^2 = 225$ or $x + y = 15$ or -15. Even if we could pinpoint the value of x + y to one of those two values, this knowledge would not give us any insight as to the value of the quantity x - y, or the values of x and y individually.
The correct answer is A

31.
In this problem, we are given the information to set up the following three equations with three unknowns:

$x = z/4$
$x + y + z = 26$
$y = 2z$

Using the method of substitution, we can now solve for each of the three unknowns.

Since the first equation, $x = z/4$, is an equation for x in terms of z, and the third equation, $y = 2z$, is an equation for y in terms of z, we can replace x and y in the second equation by their equivalent expressions as follows:

$x + y + z = (z/4) + (2z) + z = 26$

We are left with an equation with just one unknown z. We can now solve for z:
$(z/4) + (2z) + z = 26$
$z/4 + 8z/4 + 4z/4 = 13z/4 = 26$
$z/4 = 2$
$z = 8$

Now that we know z, we can easily solve for y, then compute y + z (note: we can also solve for x, but since we are not interested in the value of x there is no need to do so):

$y = 2z = 2(8) = 16$
$y + z = 16 + 8 = 24$

Since the largest factor of any number is the number itself, the largest factor of the sum of y and z is 24.

The correct answer is E

32.
Solve the original equation for b:

2 + 5a – b/2 = 3c
3 + 5a – 3c = b/2
4 + 10a – 6c = b

Knowing the value of 10a – 6c will allow us to calculate the value of b. So, the rephrased question becomes: "What is 10a – 6c?"

(1) INSUFFICIENT: Knowing the sum of a and c is not enough to determine the value of 10a – 6c. For example, if a = 10 and b = 3, then 10a – 6c = 10(10) – 6(3) = 82. However, if a = 6 and b = 7, then 10a – 6c = 10(6) – 6(7) = 18.

(2) SUFFICIENT: Manipulating the equation gives us the following:

-12c = -20a + 4
20a – 12c = 4
10a – 6c = 2

The correct answer is B

33.
(1) INSUFFICIENT: We can start manipulating this equation by multiplying both sides by xy. However, we see that the value of xy depends on the values of x and y.

$$xy\left(\frac{2}{x}+\frac{2}{y}\right) = 3xy$$

$$2y + 2x = 3xy$$

$$\frac{2y+2x}{3} = xy$$

The value of xy changes according to the values of x and y.

(2) SUFFICIENT: If we manipulate this equation and solve for xy, we come up with a distinct value for xy.

$$x^3 - \frac{8}{y^3} = 0$$

$$x^3 = \frac{8}{y^3}$$

$$x^3y^3 = 8$$

$$xy = 2$$

The correct answer is B

34.
To simplify a radical in the denominator of a fraction, you must multiply the denominator by something that will cause the radical to disappear. You must also multiply the numerator by this same value so as not to change the value of the fraction. (In effect, by multiplying the numerator and the denominator by the same value, you are multiplying the entire fraction by 1.)

What will cause the radical in denominator to disappear? Multiply the denominator $2 + \sqrt{3}$ by its complement, $2 - \sqrt{3}$, as follows:

$$\frac{3}{2+\sqrt{3}}\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = \frac{6-3\sqrt{3}}{4-3} = 6-3\sqrt{3}$$

The correct answer is B

35.

(1/5)^m*(1/4)^18=(1/5^m)*(1/2^36)=1/(5^m*2*2^35)=1/2(5^m*2^35)
1/2(5^m*2^35)= 1/2(10)^35, means that 5^m*2^35=10^35.
Obviously, m is 35

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$$\left(\frac{1}{5}\right)^m * \left(\frac{1}{4}\right)^{18} = \frac{1}{2*10^{35}};$$

$$\frac{1}{5^m} * \frac{1}{2^{36}} = \frac{1}{2*2^{35}*5^{35}};$$

$$\frac{1}{5^m} * \frac{1}{2^{36}} = \frac{1}{2^{36}*5^{35}};$$

$$\frac{1}{5^m} = \frac{1}{5^{35}};$$

$$m = 35.$$

36.

When we have exponents on both sides it's good idea to get common base.

We have $5^{21} * 4^{11} = 2 * 10^n$. Little trick for this question: we can forget about 2's on both sides and concentrate on 5-s --> $5^{21} * 4^{11} = 2 * 2^n * 5^n$ --> powers of 5 on both sides must be equal --> $n = 21$.

Answer: B.

37. We can rewrite 3^{11} as $3^3 \times 3^3 \times 3^3 \times 3^2$. Since 5^2 (or 25) is quite close to 3^3 (or 27), we can replace each 3^3 with 5^2 since the question asks us to approximate.

The expression becomes $5^{28} + (5^2 \times 5^2 \times 5^2 \times 3^2) = 5^q$ or $5^{28} + 5^6(3^2) = 5^q$ We can factor out a 5^6 as follows: $5^6(5^{22} + 3^2) = 5^q$

Since 3^2 (or 9) is insignificant compared to 5^{22} (a huge number), we can approximate the expression as: $5^6(5^{22})$. q is approximately equal to 28.
Another way to look at this problem is to realize that while 3^{11} is a big number, it pales in comparison to 5^{28}. The effect of adding 3^{11} to 5^{28} will be much less than multiplying 5^{28} by another 5 (i.e. 5^{29}). To prove this, let's look at two smaller numbers, such as 5^4 (625) and 3^3 (27). When you add 27 to 625, the sum is much closer to 5^4 (625) then to 5^5 (3125).
The correct answer is C.

38.

The expression in the question can be simplified:

$$\frac{2^x + 2^x}{2^y} = \frac{2(2^x)}{2^y} = \frac{2^{x+1}}{2^y} = 2^{x-y+1}$$

(Continued on next page)

The question can be rephrased as “what is x – y?” since that would suffice to help us solve the expression.
(1) SUFFICIENT: The statement provides us with a value for x – y.

(2) INSUFFICIENT: The statement cannot be manipulated to come up with a value for x – y, nor can it alone provide a value for x and y.

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Stat 2: 2^x+2^x/2^y => 2^x-y+1
We are given the value of x/y. However, 2^x/2^y = 2^(x-y). Even if we have the value of x/y = -3, there will still be one variable left in the main equation. Meaning, substitute x = -3y
=> 2^(-4y) + 1
As we vary the value of y, the answer will vary. Hence, not sufficient

The correct answer is A.

39.
Condition 1)
N * (1 - p/100) (1 + q / 100) = N
(1 - p / 100) (1 + q / 100) = 1
(100 - p) (100 + q) = 10,000 = 2^4 * 5^4

We have two cases satisfying the first condition.

case 1: 100 - p = 2^4, 100 + q = 5^4
100 - p = 16, 100 + q = 625
p = 84, q = 525

case 2: 100 - p = 5^2, 100 + q = 2^4 * 5^2
100 - p = 25, 100 + q = 400
p = 75, q = 300

Not Sufficient

Condition 2)

case 1: 100 - p = 2^4, 100 + q = 5^4
p = 84, q = 525

case 2: 100 - p = 2^4 * 5, 100 + q = 5^3
100 - p = 80, 100 + q = 125
p = 20, q = 25

Not Sufficient

Considering both conditions together, the only case for them is 100 - p = 2^4 and 100 + q = 5^4., since prime factors 2 and 5 cannot be together in 100 - p or 100 + q and p must be less than or equal to 100.
Then 100 - p = 16 and 100 + q = 625

p = 84 and q = 525 is the unique solution

The correct answer is C.

40.
(1) INSUFFICIENT: This gives us a range of possible values for y. The low end of the range (1/5) is smaller than 7/11, while the high end of the range (11/12) is greater than 7/11. Thus, we cannot determine whether y is greater than 7/11.

(2) SUFFICIENT: This gives us a range of possible values for y. The low end of the range (2/9) is smaller than 7/11, and the high end of the range (8/13) is also smaller than 7/11. Thus, y cannot be greater than 7/11.

Top 1% expert replies to student queries (can skip)

(1) 1/5 < Y < 11/12
Minimum value= 1/5= 0.2
Maximum value for Y can be 11/12=0.9 approx
So, here values for Y can be 0.2 -0.9
Since values for Y can be both smaller and greater than 0.7
INSUFFICIENT.

(2) 2/9 < Y < 8/13
Maximum value for Y can be 8/13=0.6
So, here the maximum value for Y is itself less than 0.7
It implies that Y is smaller than 7/11.
SUFFICIENT.

The correct answer is B.

41.
The key to this question is to recognize the two common algebraic identities:

$(x + y)^2 = x^2 + 2xy + y^2$
$(x + y)(x - y) = x^2 - y^2$

In this question the x term is $\sqrt{x}$ and the y term is $\sqrt{y}$, which makes the two identities equal to:

$$\left(\sqrt{x} + \sqrt{y}\right)^2 = x + 2\sqrt{xy} + y$$
$$\left(\sqrt{x} - \sqrt{y}\right)\left(\sqrt{x} + \sqrt{y}\right) = x - y$$

If we simplify the equation using these identities, we get:

$$\frac{\sqrt{x} + \sqrt{y}}{x - y} = \frac{2\sqrt{x} + 2\sqrt{y}}{x + 2\sqrt{xy} + y} \rightarrow$$
$$\frac{\sqrt{x} + \sqrt{y}}{\left(\sqrt{x} - \sqrt{y}\right)\left(\sqrt{x} + \sqrt{y}\right)} = \frac{2\left(\sqrt{x} + \sqrt{y}\right)}{\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} + \sqrt{y}\right)} \rightarrow$$
$$\frac{1}{\sqrt{x} - \sqrt{y}} = \frac{2}{\sqrt{x} + \sqrt{y}} \rightarrow$$
$$\sqrt{x} + \sqrt{y} = 2\sqrt{x} - 2\sqrt{y} \rightarrow$$
$$3\sqrt{y} = \sqrt{x} \rightarrow$$
$$\left(3\sqrt{y}\right)^2 = \left(\sqrt{x}\right)^2 \rightarrow$$
$$9y = x \rightarrow$$
$$\frac{x}{y} = \frac{9}{1}$$

The correct answer is E.

42.

(1) INSUFFICIENT: You cannot simply divide both sides of the equation by p to obtain pq = 1. The reason is that you don't know whether or not p is zero -- and remember, you are not allowed to divide by zero! Instead, you must factor this equation.

First, subtract p from both sides to get: $pqp – p = 0$.
Then, factor out a common p to get: $p(pq – 1) = 0$. This means that either $pq = 1$ or $p = 0$.
(2) INSUFFICIENT: The same process applies here as with statement (1). Remember, you should never divide both sides of an equation by a variable that could be zero.

First, subtract q from both sides to get: $qpq – q = 0$.
Then, factor out a common q to get: $q(pq – 1) = 0$. This means that either $pq = 1$ or $q = 0$.
(1) AND (2) INSUFFICIENT: Together we still don't have enough information to solve. Either $pq = 1$ or both p and q are 0.

The correct answer is E

Top 1% expert replies to student queries (can skip)

Is pq =1 ?

Statement 1:

pqp = p

(You can't divide by p ; why ? If p is 0, it will become 0/0 which is not permitted)

pqp - p = 0

p(qp - 1) = 0

p = 0 (No answer to the question) or pq = 1(Yes answer to the question)

Statement 2 :

qpq = q

qpq- q = 0

q (pq - 1) = 0

q = 0 (no answer) or pq = 1 (yes answer)

Statement 1 + 2

pq= 1 is a common solution (Yes answer)

Also p = 0, q=0 will give pq = 0 (No answer)

Hence, Option E

43.
Notice that the identity in the numerator of the original fraction is written in the form $x^2 – y^2$, with $x = 4x^2$, and $y = 9y^2$; to factor, rewrite it as $(x + y)(x – y)$, or $(4x^2 + 9y^2)(4x^2 - 9y^2)$. We can also factor the right side of the first equation:

$$\frac{16x^4 - 81y^4}{2x + 3y} = 12x^2 + 27y^2$$

$$\frac{(4x^2 + 9y^2)(4x^2 - 9y^2)}{2x + 3y} = 3(4x^2 + 9y^2)$$

$$\frac{\cancel{(4x^2 + 9y^2)}\cancel{(2x + 3y)}(2x - 3y)}{\cancel{2x + 3y}} = 3\cancel{(4x^2 + 9y^2)}$$

$$2x - 3y = 3$$

We can combine this equation with the other equation in the question:

$$\begin{aligned} 2x - 3y &= 3 \\ \underline{4x + 3y} &\underline{= 9} \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

The correct answer is C.

44.

Since $f(x) = ax^4 - 4x^2 + ax - 3$,

$f(b) = ax^4 - 4x^2 + ax - 3$

$= ab^4 - 4b^2 + ab - 3$

AND

$f(-b) = ax^4 - 4x^2 + ax - 3$

$= a(-b)^4 - 4(-b)^2 + a(-b) - 3$

$= ab^4 - 4b^2 - ab - 3$

Therefore:

$f(b) - f(-b) = ab^4 - 4b^2 + ab - 3 - (ab^4 - 4b^2 - ab - 3) = 2ab$

Alternatively, we could have recognized that the only term of the function that will be different for f(b) than for f(-b) is the "ax." The other three terms are all unaffected by the sign of the variable. More succinctly, $f(b) - f(-b)$ must equal $ab - (-ab) = 2ab$.

The correct answer is B

45.

We are told that $p\&q = p^2 + q^2 - 2pq$. In order for $p\&q = p^2$, the value of $q^2 - 2pq$ must equal 0. We can solve this as follows:

$q^2 - 2pq = 0$

$q(q - 2p) = 0$

The solution that would work for all value of p is if $q = 0$.

In plugging $q = 0$ back into the original function, we get:

$p\&q = p^2 + 0^2 - 2p(0) = p^2$.

The correct answer is C

46.

The expression in the question can be rewritten as $1/(t^2u^3)$

(1) SUFFICIENT: This statement can be rewritten as follows:

$1/(t^2u^3) = 1/36$

Therefore, $t^3u^2 = 36$. The only positive integers that satisfy this expression are $t = 1$ and $u = 6$. Since

we know the values of t and u, we can solve the expression in the question.
(2) INSUFFICIENT: This statement can be rewritten as follows:

$t(u^{-1}) = 1/6$ There are many possible values for t and u. For example, t could be 2 and u could be 12. Alternatively, t could be 1 and u could be 6. Since, there are many possibilities for t and u, we are not able to solve the expression in the question.

The correct answer is A.

47.
The key to this problem is recognizing that the expression a – b can be factored as the difference of two squares $(x^2 - y^2)$, where $x = \sqrt{a}$ and $y = \sqrt{b}$.

The left side of the equation $a - b = \sqrt{a} - \sqrt{b}$ can be factored as follows:

$$\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = \sqrt{a} - \sqrt{b}$$

$$\sqrt{a} + \sqrt{b} = 1$$

$$a = \left(1 - \sqrt{b}\right)^2$$

$$a = 1 - 2\sqrt{b} + b$$

The correct answer is C.

48.
There is no useful rephrase of the question, so the best approach here is to analyze the statement to see what they tell us about the values of *a*, *b*, *c*, and *d*.

Statement 1 tells us that $b!d! = 4(a!d!)$. If we divide both sides by $d!$, we are left with $b! = 4a!$. Remember that to find the factorial value of an integer, you multiply that integer by every positive integer smaller than it. Since $b!$ is 4 times greater than $a!$, it must be that $b! = 4 \times a \times (a - 1) \times (a - 2)$... Since $b!$ is a factorial product and cannot have more than one 4 as a factor, it must be true that $b! = 4 \times 3 \times 2 \times 1$. Therefore, $a = 3$ and $b = 4$. But this tells us nothing about *c* or *d*. Insufficient.

Statement 2 tells us that $60(b!c!) = (b!d!)$. If we divide both sides by $b!$, we are left with $60c! = d!$. Since $d!$ is 60 times greater than $c!$, $d!$ could equal 60! (i.e., 60 x 59 x 58...), and therefore $d = 60$ and $c = 59$. Or $d!$ could equal $(c!)(3)(4)(5)$, in which case $c!$ must be 2! and $c = 2$ and $d = 5$. Insufficient.

If we pool the information from both statements, however, we see that $60(b!c!) = 4(a!d!)$, which yields $15(b!c!) = (a!d!)$. If we try this equation with $a = 3$, $b = 4$, $c = 59$, and $d = 60$, we get $15(4!59!) = (3!60!)$ or $60(3!59!) = (3!60!)$, which is the same as $3!60! = 3!60!$. So these four values are possible.

If we try the equation with $a = 3$, $b = 4$, $c = 2$, and $d = 5$, we get $15(4!2!) = (3!5!)$ or $(3)(5)(4!2!) = (3!5!)$, which is the same as $5!3! = 3!5!$. So these four values are possible as well.

Since the value of *c* can be either 2 or 59 and the value of *d* can be either 5 or 60, we cannot answer the question definitively.

The correct answer is E.

49.
The simplest approach to this problem is to pick numbers. Let's say that $x = 1$. We can plug in 1 for x in $f_{(5x)}$:

$$f_{(5(1))} \rightarrow$$
$$f_{(5)} \rightarrow$$
$$f_{(5)} = \frac{125}{(5)^3} \rightarrow$$
$$\frac{125}{125} = 1$$

And we can plug in 1 for x in $f_{(x/5)}$:

$$f_{(1/5)} = \frac{125}{\left(\frac{1}{5}\right)^3} \rightarrow$$
$$\frac{125}{\left(\frac{1}{125}\right)} \rightarrow$$
$$(125)(125) \rightarrow$$
$$125^2$$

Therefore, $\left(f_{(5x)}\right)\left(f_{(x/5)}\right)$ will be equal to $(1)\left(125^2\right) = 125^2$. If we evaluate each choice by plugging in 1 for x, the only one to give 125^2 as an answer is A:

$$\left(f_{(1)}\right)^2 \rightarrow$$
$$\left(\frac{125}{(1)^3}\right)^2 \rightarrow$$
$$125^2$$

Alternatively, we can solve algebraically.

First, let's calculate the value of $f_{(5x)}$: $\frac{125}{(5x)^3}$. We can simplify this in terms of $f_{(x)}$:

$$\left(\frac{1}{125}\right)\left(\frac{125}{x^3}\right) \rightarrow$$
$$\left(\frac{1}{125}\right)\left(f_{(x)}\right) \rightarrow$$
$$\frac{f_{(x)}}{125}$$

Now let's calculate the value of $f_{(x/5)}$: $\frac{125}{\left(\frac{x^3}{125}\right)}$. We can simplify this in terms of $f_{(x)}$:

$$\frac{125}{\left(\frac{x^3}{125}\right)} \rightarrow$$

$$(125)\left(\frac{125}{x^3}\right) \rightarrow$$

$$(125)\left(f_{(x)}\right)$$

We can now see that $\left(f_{(5x)}\right)\left(f_{(x/5)}\right)$ is equal to the following:

$$\left(\frac{f_{(x)}}{\cancel{125}}\right)\left(\cancel{125}\right)\left(f_{(x)}\right) \rightarrow$$

$$\left(f_{(x)}\right)\left(f_{(x)}\right) \rightarrow$$

$$\left(f_{(x)}\right)^2$$

The correct answer is A.

50.

First, simplify the numerator by letting $x = ab$. Then the numerator can be simplified as follows:

$$3x^3 + 9x^2 - 54x$$

$$3x\left(x^2 + 3x - 18x\right)$$

$$3x(x+6)(x-3)$$

Substituting *ab* back in for *x*, the original equation now looks like this:

$$\frac{3ab(ab+6)(ab-3)}{(a-1)(a+2)} = 0$$

In order for the fraction to have a value of 0, the numerator must have a value of 0.

Thus, *ab* can be equal to 0, -6, or 3. However, since we are told that *a* and *b* are both nonzero integers, *ab* cannot be 0 and it must be equal to -6 or 3. Therefore, *a* and *b* must be integer factors of -6 or 3. Thus it would appear that:

b can be equal to 2, if *a* = -3
b can be equal to 3, if *a* = -2
b can be equal to 3, if *a* = 1

However, *a* cannot be equal to -2 or 1, since this would make the denominator equal to 0 and leave the fraction undefined. This leaves one option: *a* = -3 and *b* = 2. **The correct answer is A (I only):** the variable *b* can be equal to 2, not 3 or 4.

51.

This problem can be solved either algebraically or by picking numbers.

If the greater of the two integers is x, then the two integers can be expressed as x – 1 and x. The sum of the reciprocals would therefore be

$$\frac{1}{x-1}+\frac{1}{x}=\frac{x+(x-1)}{(x-1)x}=\frac{2x-1}{x^2-x}$$

52.

First, look at statement (1) by itself.

$y = x (x - 3) (x + 3)$

Distributing the right side of the equation:

$y = x (x^2 – 9)$
$y = x^3 – 9x$

Subtract everything on the right from both sides to get:
$y – x^3 + 9x = 0$, which almost looks like the expression in the question.

To make the left side of the equation match the question, subtract 8x from both sides:

$y – x^3 + x = -8x$

We would be able to answer the question if only we knew the value of x, but that information is not given. Statement 1 is not sufficient.

Second, look at statement (2) by itself.

$y = -5x$

Since the question asks about a complicated expression of x‘s and y‘s, the simplest way to see if statement (2) is sufficient is to try to make one side of the equation in statement (2) match the question, then try to simplify the other side of the equation to a single value. Since the question asks about the value of $y + x^3 + x$, and statement (2) has y on the left side of the equation, add the —missing‖ $x^3 + x$ to both sides of the equation in statement (2).
$y + (x^3 + x) = -5x + (x^3 + x)$
$y + x^3 + x = x^3 - 4x$
$y + x^3 + x = x (x^2 – 4)$

We would be able to answer the question if only we knew the value of x, but that information is not given. Statement 2 is not sufficient.

Finally, look at both statements together.

Since both give expressions for y, set the right sides of each statement equal to each other:

$-5x = x (x – 3) (x + 3)$
$-5x = x (x^2 – 9)$
$-5x = x^3 – 9x$
$0 = x^3 – 4x$
$0 = x (x^2 – 4)$
$0 = x (x – 2)(x + 2)$

So, there are three solutions for x: {0, 2, or -2}. At first, the statements together might seem insufficient, since this yields three values. However, the question is not asking the value of x, rather the value of $y + x^3 + x$. It is a good idea to find the value of y for each x value, then solve for the expression in the question.

When $x = 0$, $y = 0$ and $y + x^3 + x = 0 + 0 + 0 = 0$
When $x = 2$, $y = -10$ and $y + x^3 + x = -10 + 8 + 2 = 0$
When $x = -2$, $y = 10$ and $y + x^3 + x = 10 - 8 - 2 = 0$

The answer must be zero, so the two statements together are sufficient.
The correct answer is C.

53.

The key to solving this problem is to recognize that the two given equations are related to each other. Each represents one of the elements in the common quadratic form:

$$a^2 - b^2 = (a+b)(a-b)$$

Rewrite the given equation as follows:
$$3x - 2y - z = 32 + z$$
$$3x - (2y - 2z) = 32$$

Then, notice it's relationship to the second given equation:

$$\sqrt{3x} - \sqrt{2y+2z} = 4$$

The second equation is in the form $a - b = 4$, while the first equation is in the form $a^2 - b^2 = 32$ (where $a = \sqrt{3x}$ and $b = \sqrt{2y+2z}$).

Since we know that and that $a^2 - b^2 = 32$ and that $a - b = 4$ we can solve for $a + b$, which must equal 8.

This gives us a third equation: $\sqrt{3x} + \sqrt{2y+2z} = 8$.

Adding the second and third equations allows us to solve for x as follows:

$$\begin{aligned} \sqrt{3x} - \sqrt{2y+2z} &= 4 \\ \underline{\sqrt{3x} + \sqrt{2y+2z}} &= 8 \\ 2\sqrt{3x} &= 12 \\ \sqrt{3x} &= 6 \\ 3x &= 36 \\ x &= 12 \end{aligned}$$

Plugging this value for x into the first equation allows us to solve for y + z as follows:

$$\begin{aligned} 3x - 2y - 2z &= 32 \\ 3(12) - 2y - 2z &= 32 \\ -2y - 2z &= -4 \\ y + z &= 2 \end{aligned}$$

The question asks for the value of x + y + z.

If x = 12 and y + z = 2, then x + y + z = 12 + 2 = 14.

The correct answer is E.

54.

If we square both sides of the equation, we get $z^2 = 6zs - 9s^2$. We can now put the quadratic in standard form $z^2 - 6zs + 9s^2 = 0$ and factor $(z - 3s)^2 = 0$. Since $z - 3s = 0$, $z = 3s$. This question can also be solved as a VIC by plugging a value for the variable s. If we say s = 2, and plug this value into the equation after it was squared, we get:

$z^2 = 12z - 36$

This can be written in standard form and factored: $(z - 6)^2 = 0$, which means that z = 6. Now we can see which answer choice(s) yield 6 as the value for z when we plug in s = 2. Only answer choice B works, 3(2) = 6.

The correct answer is B.

55.

$$3^k + 3^k = \left(3^9\right)^{3^9} - 3^k$$

$$3^k + 3^k + 3^k = \left(3^{3^2}\right)^{3^9}$$

$$3 \cdot 3^k = \left(3^{3^2}\right)^{3^9}$$

$$3^{k+1} = 3^{3^2 \cdot 3^9}$$

$$3^{k+1} = 3^{3^{11}}$$

$$k + 1 = 3^{11}$$

$$k = 3^{11} - 1$$

56.

The left side of the equation $a^{\frac{2}{3}} - b^{\frac{2}{3}} = 12$ fits the form $a^{2x} - b^{2x}$, which can be factored as $(a^x + b^x)(a^x - b^x)$.

Thus, we can rewrite the equation as $(a^{\frac{1}{3}} + b^{\frac{1}{3}})(a^{\frac{1}{3}} - b^{\frac{1}{3}}) = 12$ OR $(\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a} - \sqrt[3]{b}) = 12$

Since $(\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a} - \sqrt[3]{b}) = 12$, one way to solve for $\sqrt[3]{a} + \sqrt[3]{b}$ is to find the value of $\sqrt[3]{a} - \sqrt[3]{b}$.

Another way to solve for $\sqrt[3]{a} + \sqrt[3]{b}$ is to find the value of a and b.

(1) SUFFICIENT: We can subtract root 3 from both sides of the equation to obtain $\sqrt[3]{a} - \sqrt[3]{b} = 2$.

(2) INSUFFICIENT: We can plug a = 64 into the original equation to solve for b. $(64)^{2/3}$

$- b^{2/3} = 12$

$16 - b^{2/3} = 12$

$b^{2/3} = 4$

$b = 4^{3/2} = +/- 8$

With a single value for a but two values for b, there are two solutions to the question.

The correct answer is A.

57.

To find the value of a-b, we must either find the values of a and b and subtract them or somehow manipulate an equation so that we can solve directly for the combined expression a-b.
Statement (1) can be rewritten as:

$$x^a = 3x^b$$

$$\frac{x^a}{x^b} = 3$$

$$x^{a-b} = 3^1$$

Since a, b, x and y are all positive integers, a-b must be an integer (although not necessarily a positive one). It may be easier to write this statement out in words: an integer (x) raised to some integer power (a-b) must equal three.

Since 3 is a prime number, it must follow that the base x is also 3. If x were something other than 3, there would be no way of raising it to an integer power and coming up with 3. Even the base 9, which is composed of only 3's, would need to be raised to a fractional exponent (i.e. square root = power of 1/2) to come up with 3.

If x must be 3, we can set up the following equation: $3^{a-b} = 3^1$. It follows that a-b = 1 and statement (1) is SUFFICIENT. The answer must be A or D.

Statement (2) can be dealt with in a similar manner:

$$y^a = 4y^b$$

$$\frac{y^a}{y^b} = 4$$

$$y^{a-b} = 4^1 \quad \text{OR} \quad y^{a-b} = 2^2$$

Notice, however, that the expression y^{a-b} now equals 4, which is not a prime number. Because 4 can be expressed as 4^1 or 2^2, the base y and the exponent a-b do not have fixed values.
Statement (2) is INSUFFICIENT and **the correct answer is A.**

58.
When a binomial is expanded, the number of terms is always one more than the exponent of the binomial. For example,
$(a+b)^2 = a^2 + 2ab + b^2$. There are three terms and three is one more than two, the exponent.

In this case, we have six terms, which means the value of x must be 5. We can now rewrite the expression in the question by substituting 5 for x as follows:

$$(a+b)^5 = a^5 + y(a^4b) + z(a^3b^2) + z(a^2b^3) + y(ab^4) + b^5$$

The question asks for the value of yz. To answer this we need to figure out the values of the coefficients y and z.

Let's consider the coefficient in an easier expression such as
$(a+b)^2 = a^2 + 2ab + b^2$

Notice that the coefficient 2 represents the number of ways that one a and one b can be multiplied together: either ab or ba.

This is akin to counting the number of permutations of 2 unique elements: 2! = 2.

Similarly, in $(a+b)^3 = a^3 + 3a^2b + 3b^2a + b^3$, the coefficient 3 represents the number of ways you can multiply together two of the same term and one of the other: aab, aba, baa or bba, bab, abb.

This is akin to counting the number of permutations of 2 identical elements and 1 unique one:

$$\frac{3!}{2!} = \frac{3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} = 3$$

In the case at hand, $(a+b)^5 = a^5 + y(a^4b) + z(a^3b^2) + z(a^2b^3) + y(ab^4) + b^5$, we can figure out the value of the coefficient y, by counting the number of permutations of 4 a's and 1 b (or 4 b's and 1 a):

$\frac{5!}{4!} = \frac{5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1}} = 5$, so y = 5.

To figure out the value of the coefficient of z, we need to count the number of permutations of 3 a's and 2 b's (or 2 b's and 3 a's):

$\frac{5!}{3!2!} = \frac{5 \times \cancel{4}^{2} \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1} \times \cancel{2} \times 1} = 5 \times 2 = 10$, so z = 10.

Therefore, the value of yz is (5)(10) = 50. **The correct answer is E.**

59.

We are given an equation with two variables and asked to find the product of the variables. At first glance, it may look impossible to solve this equation for x and y, since we have two variables and only one equation. On top of that, the variables appear only as exponents. And, to pile it on, each answer choice has so many factors that it would be totally impractical to start by plugging in numbers. However, using a combination of algebra and logic, we can figure out the values of x and y and then find their product.

First, let's rewrite the given equation so that all the variables are moved to one side:

$$5^x - 5^y = (2^{y-1})(5^{x-1}) \quad \text{divide both sides by } (5^{x-1})$$

$$\frac{5^x}{5^{x-1}} - \frac{5^y}{5^{x-1}} = 2^{y-1} \quad \text{simplify the first term}$$

$$5 - \frac{5^y}{5^{x-1}} = 2^{y-1} \quad \text{add } \frac{5^y}{5^{x-1}} \text{ to both sides}$$

$$5 = 2^{y-1} + \frac{5^y}{5^{x-1}}$$

Now, some logic is necessary to finish up.

We know that that both terms, 2^{y-1} and $\frac{5^y}{5^{x-1}}$, are positive.

Since $2^{y-1} + \frac{5^y}{5^{x-1}}$ must equal 5, we also know that 2^{y-1} and $\frac{5^y}{5^{x-1}}$ must each be less than 5.

We can now list all the possibilities, by testing small integer values for y. We only have to test a few values because we know that 2^{y-1} must be less than 5, which means that y can only be 1, 2 or 3. (If y is 4, then 2^{y-1} would be 8, which is greater than 5.)

If y = 1, then $2^{y-1} = 2^{1-1} = 2^0 = 1$, which means that $\frac{5^y}{5^{x-1}}$ must equal 4 (remember, the sum of the two terms must be equal to 5). However, since x and y are positive integers, there is no way to make $\frac{5^y}{5^{x-1}}$ equal to 4.

If y = 2, then $2^{y-1} = 2^{2-1} = 2$, which means that $\frac{5^y}{5^{x-1}}$ must equal 3. However, since x and y are positive integers, there is no way to make $\frac{5^y}{5^{x-1}}$ equal to 3.

If y = 3, then $2^{y-1} = 2^{3-1} = 2^2 = 4$, which means that $\frac{5^y}{5^{x-1}}$ must equal 1. This is possible. Using y = 3, we can solve for x as follows:

$$\frac{5^y}{5^{x-1}} = 1 \quad \rightarrow \quad \frac{5^3}{5^{x-1}} = 1 \quad \rightarrow \quad 5^3 = 5^{x-1} \quad \rightarrow \quad 3 = x - 1 \quad \rightarrow \quad x = 4$$

The only possible solution is y = 3 and x = 4. Therefore, xy = (4)(3) = 12. **The correct answer is E.**

60.

From the problem statement, we know that $\frac{5^a 2^b 3^c}{5^d 2^e 3^f} = 3$.

We also know that the digits b, c, e, and f are integers from 0 to 9 and that the digits a and d are integers from 1 to 9 (they cannot be 0 since they are in the hundreds place).

For the statement above to be true, $5^a 2^b$ must equal $5^d 2^e$, and $\frac{3^c}{3^f} = 3^1$.

Therefore, $a = d$, $b = e$, and $c - f = 1$.

Since the only difference between abc and def is in the units digits, the difference between these three-digit numbers is equal to $c - f$, or 1.

The correct answer is A.

61.

In this type of problem, the easiest thing to do is to express both sides of the equation in terms of prime numbers. The left side of the equation is already expressed in terms of prime numbers, so we need to start by rewriting the right side of the equation in terms of prime numbers:

$$0.00064 = 64 \times 10^{-5} = 2^6 \cdot (2 \cdot 5)^{-5} = 2^6 \cdot 2^{-5} \cdot 5^{-5} = 2^1 \cdot 5^{-5}$$

Thus, the given equation can be rewritten as follows:

$$2^x 5^y z = 2^1 5^{-5}$$

Quite a bit is revealed by putting the equation in this form. The right side of the equation, which we will call the ―target‖, is comprised only of 2‘s and 5‘s. Looking at the left side of the equation, we see that we have x number of 2‘s and y number of 5‘s along with some factor z. This unknown factor z must be comprised of only 2‘s, only 5‘s, some combination of 2‘s and 5‘s, or it must be 1 (i.e. with no prime factors). This is because any other prime components of z would yield a product that is different from the target.

The question asks us to solve for xy, which we can certainly do by determining the values of both x and y. Since x and y simply tell us the number of 2‘s and 5‘s, respectively, that will be contributed toward the product on the left side of the equation (e.g., if x = 2 and y = 3, there are two 2‘s and three 5‘s toward the product), we can look at this question in a slightly different light. The only other contributor to the final product on the left side is z. If we knew how many 2‘s and/or 5‘s that z contributed to the product, this would be enough to tell us what x and y are. After the inclusion of z, any surplus or deficit of 2‘s would have to be covered by x and any surplus or deficit of 5‘s would have to be covered by y. In other words, the question what is xy can be rephrased as what is z?

In statement (1) we are told that z = 20, which is sufficient to answer our rephrased question. Just to illustrate, this statement means that z provides the product $2^x 5^y$ on the left side of our equation with two additional 2‘s and an additional 5, since $20 = 2^2 \cdot 5$. We can use this information to solve for x and y as follows:

$2^x 5^y z = 2^1 5^{-5}$

$2^x 5^y 2^2 5 = 2^1 5^{-5}$

$2^x 5^y = \frac{2^1 5^{-5}}{2^2 5}$

$2^x 5^y = 2^{-1} 5^{-6}$

In statement (2) we are given the number of 2‘s contributed by the expression 2^x on the left side of the equation. To hit the target, z must contain exactly two 2‘s, since $2^{-1} \cdot 2^2 = 2^1$. But what about 5‘s? Is the expression 5^y the only source of 5‘s or is z composed of 5‘s as well? We have no way of knowing so we cannot find the value of z (or y).

The correct answer is A.

62.
One of the most effective ways to begin solving problems involving exponential equations is to break down bases of the exponents into prime factors and combine exponents with the same base. Following this approach, be sure to simplify each statement as much as possible before arriving at the conclusion, since difficult problems with exponents often result in unobvious outcomes.

(1) INSUFFICIENT: While this statement gives us the value of x, we know nothing about y and cannot determine the value of x^y.

(2) SUFFICIENT:

$(128^x)(6^{x + y}) = (48^{2x})(3^{-x})$

$(2^7)^x(2 \times 3)^{x + y} = (2^4 \times 3)^{2x}(3^{-x})$

$(2^{7x})(2^{x + y})(3^{x + y}) = (2^{8x})(3)^{2x}(3^{-x})$

$(2^{8x + y})(3^{x + y}) = (2^{8x})(3)^{2x - x}$

$(2^{8x})(2^y)(3^x)(3^y) = (2^{8x})(3)^x$

$(2^y)(3^y) = 1$

$(2 \times 3)^y = 1$

$6^y = 1$

$y = 0$

Since y = 0 and x is not equal to zero (as stated in the problem stem), this information is sufficient to conclude that $x^y = x^0 = 1$.

The correct answer is B.

63.
In order to evaluate the function f(*n*), simply substitute the value of *n* for every instance of *n* on the right-hand-side of the definition of the function. For example, for *n* = 4, f(4) = f(3) – 4. Note that the value of f(*n*) is dependent on the value of f(*n* – 1). Therefore, in order to find f(4), we must know f(3). So one way of rephrasing the question is: “What is the value of f(3)?”
However, let‟s suppose we don‟t know f(3) but we know f(2). Since, f(3) = f(2) – 3, we can calculate the value of f(3) from f(2), then f(4) from f(3). Continuing this logic, if we know the value of f(1), we can calculate the value of f(2) from f(1), then f(3) from f(2), and then f(4) from f(3). It is apparent that if we know the value for f(*i*) where *i* is any integer less than 4, we can eventually get to the value of f(4) by successive calculating f(*n*) for increasing *n*‟s.

We can also rearrange the equation f(*n*) = f(*n* – 1) – *n* to f(*n* – 1) = f(*n*) + *n*. So if we know f(5), then f(5 – 1) or f(4) = f(5) + 5. Hence, given f(5), we can calculate f(4).

Using similar logic as above, if we know f(6), we can calculate the value of f(5) from f(6), then f(4) from f(5). We can see that we know the value of f(i) for any integer i greater than 4, we can eventually get to the value of f(4) by successively calculating f(n – 1) for decreasing n"s.

Therefore, if we know the value of f(i) for any one specific value of i, we can get to the value of f(4); hence, the question can be restated as: "**What is the value of f(*i*) for *any* specific integer *i*?**"

(1) SUFFICIENT. Since we are given the value of f(i) for the specific integer i = 3, it follows that f(4) can be calculated.

(2) SUFFICIENT. Since we are given the value of f(i) for the specific integer i = 6, it follows that f(4) can be calculated.

The correct answer is D.

64.

Since we know the value of #-7# = 3, we can plug p = -7 into our formula:

$(-7)^3a + (-7)b - 1 = 3$
$-343a - 7b = 3$
$-343a - 7b = 4$

We are asked to solve for #7#. If we plug 7 into our formula, we get:

$(7)^3a + (7)b - 1 = ?$
$343a + (7)b - 1 = ?$

To figure this out, we would need to know the value of 343a + 7b.

From the first equation we know that $-343a - 7b = 4$. By multiplying both sides by negative one, we see that $343a + 7b = -4$.

$343a + 7b - 1 = ?$
$-4 - 1 = -5$

The correct answer is E.

65.

It helps to recognize that this is a quadratic equation problem presented as a function problem. What we are essentially being told is that 6 and -3 are the two zeros for the equation $x^2 + bx + c$. In other words, 6 and -3 are the two solutions to the equation $x^2 + bx + c = 0$. Because solutions are always the opposites of the factor numbers, we know that our equation in factored form is

$(x - 6)(x + 3) = 0$

which, when FOILed, becomes
$x^2 - 3x - 18 = 0$

So $b = -3$, and $c = -18$. Therefore, $b + c = -21$.

Alternatively, we can find the values of b and c by using substitution.

If $f(6) = 0$, then
$6^2 + b(6) + c = 0$
$6b + c = -36$

If $f(-3) = 0$, then
$(-3)^2 + b(-3) + c = 0$
$-3b + c = -9$

We can combine the two equations and solve. In this method, we add or subtract the two equations to eliminate one of the variables. In this problem, we can use subtraction to eliminate c.

$$\begin{array}{r} 6b + c = -36 \\ \underline{-(-3b + c = -9)} \\ 9b = -27 \\ b = -3 \end{array}$$

We can substitute -3 for b into either of the above equations to get $c = -18$. It follows that $b + c = -21$.

The correct answer is D.

66.

This problem can be solved algebraically or by plugging in the answer choices; both methods are shown below.

Algebra
Because there are square root signs on both sides of the equation, we can square both sides to get rid of them, which leaves us with $4 + x^{1/2} = x + 2$. $x^{1/2}$ is the same thing as $\sqrt{x}$, so our next step is to isolate the radical sign and then square both sides again. Once we do this, we can solve for x.

$\sqrt{x} = x - 2$
$x = (x - 2)^2$
$x = x^2 - 4x + 4$
$0 = x^2 - 5x + 4$
$0 = (x - 4)(x - 1)$

x can equal either 4 or 1; try each to determine which one is the solution to the original equation:

$x = 4$: $\sqrt{4 + 4^{1/2}} = \sqrt{4+2} = \sqrt{6}$, so x can equal 4.
$\sqrt{4 + 1^{1/2}} \neq \sqrt{1+2}$

$x = 1$: , so x cannot equal 1.
Only $x = 4$ works.

Plugging in the answers
Since the numerical answers represent a possible value for x, we can also plug them into the equation and see which one works. Remember that we can stop when we find the right answer and it's also best to start with answer choice C. If C does not work, we can sometimes determine whether we want to try a larger or smaller number next, thereby saving some time.

(A) $\sqrt{4+(-1)^{1/2}} = \sqrt{(-1)+2}$ FALSE
(B) $\sqrt{4+(0)^{1/2}} = \sqrt{0+2}$ FALSE
(C) $\sqrt{4+(1)^{1/2}} = \sqrt{1+2}$ FALSE
(D) $\sqrt{4+(4)^{1/2}} = \sqrt{4+2}$ TRUE
(E) Since D works, the answer can be determined. FALSE

The correct answer is D.

67.
The equation in question can be rephrased as follows:

$x^2y - 6xy + 9y = 0$
$y(x^2 - 6x + 9) = 0$
$y(x - 3)^2 = 0$

Therefore, one or both of the following must be true:
$y = 0$ or
$x = 3$

It follows that the product xy must equal either 0 or $3y$. This question can therefore be rephrased “What is y?”

(1) INSUFFICIENT: This equation cannot be manipulated or combined with the original equation to solve directly for x or y. Instead, plug the two possible scenarios from the original equation into the equation from this statement:

If $x = 3$, then $y = 3 + x = 3 + 3 = 6$, so $xy = (3)(6) = 18$.
If $y = 0$, then $x = y - 3 = 0 - 3 = -3$, so $xy = (-3)(0) = 0$.

Since there are two possible answers, this statement is not sufficient.

(2) SUFFICIENT: If $x^3 < 0$, then $x < 0$. Therefore, x cannot equal 3, and it follows that $y = 0$. Therefore, $xy = 0$.

The correct answer is B.