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B.Sc. (PCM)-18, B.A. (Math)-6
3rd Year Examination, Calendar Batch 2015
Mathematics-VI

Time : 3 Hours]

[Max. Marks : 100

Note. Attempt any **five** questions. Each questions carry equal marks.

Q.1. If the matrix of a linear transformation T on a vector space $V_2(C)$ with respect to the ordered basis $\mathcal{B} = \{(1, 0), (0, -1)\}$ is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. What is the matrix of T with respect to the ordered basis $\mathcal{B}' = \{(1, 1), (1, -1)\}$?

Q.2. If a finite-dimensional vector space $V(F)$ be the direct sum of its two subspaces W_1 and W_2 , then show that

$$\dim. V = \dim. W_1 + \dim. W_2$$

Q.3. Find the dual basis of the basis set $\mathcal{B} = \{(-1, 1, 1), (1, -1, 1), (1, 1, -1)\}$ for $V_3(R)$.

Q.4. Show that the mapping $d: R^2 \times R^2 \rightarrow R^2$ defined by $d(x, y) = \max. (|x_1 - y_1|, |x_2 - y_2|)$, where $x = (x_1, x_2), y = (y_1, y_2) \in R^2$ is metric on R^2 .

Q.5. Let (X, d) be a metric space and let $d_1(x, y) = \min. \{1, d(x, y)\}$, then show that d_1 is a metric on X .

Q.6. Prove that the vectors $\{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$ form a basis for R^3 .

Q.7. If S be an ideal of a ring and T be an ideal of R , containing S , then $R/T \cong \frac{R/S}{T/S}$.

Q.8. (A) The intersection of any two subspaces of a vector space is a subspace.

(B) Is the vector $(2, -5, 3)$ in the subspace of R^3 spanned by the vectors $(1, -3, 2), (2, -4, -1), (1, -5, 7)$?