**Question 1 –**

a. B ∨ C

The truth table for this sentence would be as follows:

| **A** | **B** | **C** | **D** | **B ∨ C** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | T | T | F | T |
| T | T | F | T | T |
| T | T | F | F | T |
| T | F | T | T | T |
| T | F | T | F | T |
| T | F | F | T | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | T | T | F | T |
| F | T | F | T | F |
| F | T | F | F | F |
| F | F | T | T | T |
| F | F | T | F | T |
| F | F | F | T | F |
| F | F | F | F | F |

There are 9 models in which the sentence B ∨ C is true.

b. ¬A ∨ ¬B ∨ ¬C ∨ ¬D

The truth table for this sentence would be as follows:

| **A** | **B** | **C** | **D** | **¬A ∨ ¬B ∨ ¬C ∨ ¬D** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | T | T | F | T |
| T | T | F | T | T |
| T | T | F | F | T |
| T | F | T | T | T |
| T | F | T | F | T |
| T | F | F | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | T | T | F | T |
| F | T | F | T | T |
| F | T | F | F | T |
| F | F | T | T | T |
| F | F | T | F | T |
| F | F | F | T | T |
| F | F | F | F | T |

There is only 1 model in which the sentence ¬A ∨ ¬B ∨ ¬C ∨ ¬D is true.

c. (A ⇒ B) ∧ A ∧ ¬B ∧ C ∧ D

The truth table for this sentence would be as follows:

| **A** | **B** | **C** | **D** | **(A ⇒ B) ∧ A ∧ ¬B ∧ C ∧ D** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | T | T | F | T |
| T | T | F | T | T |
| T | T | F | F | T |
| T | F | T | T | F |
| T | F | T | F | F |
| T | F | F | T | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | T | T | F | T |
| F | T | F | T | T |
| F | T | F | F | T |
| F | F | T | T | T |
| F | F | T | F | T |
| F | F | F | T | T |
| F | F | F | F | T |

There are 9 models in which the sentence (A ⇒ B) ∧ A ∧ ¬B ∧ C ∧ D is true.

d. (A ∧ B) ∨ (C ∧ D)

The truth table for this sentence would be as follows:

| **A** | **B** | **C** | **D** | **(A ∧ B) ∨ (C ∧ D)** |
| --- | --- | --- | --- | --- |
| T | T | T | T | T |
| T | T | T | F | T |
| T | T | F | T | T |
| T | T | F | F | F |
| T | F | T | T | T |
| T | F | T | F | F |
| T | F | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | T | T | F | F |
| F | T | F | T | F |
| F | T | F | F | F |
| F | F | T | T | F |
| F | F | T | F | F |
| F | F | F | T | F |
| F | F | F | F | F |

There are 8 models in which the sentence (A ∧ B) ∨ (C ∧ D) is true.

e. B ⇒ (A ∧ B)

The truth table for this sentence would be as follows:

| **A** | **B** | **B ⇒ (A ∧ B)** |
| --- | --- | --- |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | T |

There are 4 models in which the sentence B ⇒ (A ∧ B) is true.

In summary

a. B ∨ C: 9 models

b. ¬A ∨ ¬B ∨ ¬C ∨ ¬D: 1 model

c. (A ⇒ B) ∧ A ∧ ¬B ∧ C ∧ D: 9 models

d. (A ∧ B) ∨ (C ∧ D): 8 models

e. B ⇒ (A ∧ B): 4 models

**Question 2 –**

(a) Smoke ⇒ Smoke

The truth table for this sentence would be as follows:

| **Smoke** | **Smoke ⇒ Smoke** |
| --- | --- |
| T | T |
| F | T |

The sentence "Smoke ⇒ Smoke" is valid because it is true for all possible truth values of Smoke.

(b) Smoke ⇒ Fire

The truth table for this sentence would be as follows:

| **Smoke** | **Fire** | **Smoke ⇒ Fire** |
| --- | --- | --- |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

The sentence "Smoke ⇒ Fire" is neither valid nor unsatisfiable because it is true in some cases (when Smoke is false) and false in other cases.

(c) (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

We can use logical equivalences to simplify this sentence:

(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire) (¬Smoke ∨ Fire) ⇒ (Smoke ∨ ¬Fire) (¬Smoke ∨ Fire) ⇒ ¬(¬Fire) (Double negation elimination) (¬Smoke ∨ Fire) ⇒ Fire (Negation elimination)

Now let's construct the truth table:

| **Smoke** | **Fire** | **(¬Smoke ∨ Fire) ⇒ Fire** |
| --- | --- | --- |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

The sentence "(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)" is neither valid nor unsatisfiable because it is true in some cases (when Smoke is false, and Fire is true) and false in other cases.

(d) Smoke ∨ Fire ∨ ¬Fire

The truth table for this sentence would be as follows:

| **Smoke** | **Fire** | **¬Fire** | **Smoke ∨ Fire ∨ ¬Fire** |
| --- | --- | --- | --- |
| T | T | F | T |
| T | F | T | T |
| F | T | F | T |
| F | F | T | T |

The sentence "Smoke ∨ Fire ∨ ¬Fire" is valid because it is true for all possible truth values of Smoke and Fire.

(e) ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

We can use logical equivalences to simplify this sentence:

((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)) (¬(Smoke ∧ Heat) ∨ Fire) ⇔ ((¬Smoke ∨ Fire) ∨ (¬Heat ∨ Fire)) ((¬Smoke ∨ ¬Heat) ∨ Fire) ⇔ ((¬Smoke ∨ Fire) ∨ (¬Heat ∨ Fire)) (¬Smoke ∨ ¬Heat ∨ Fire) ⇔ (¬Smoke ∨ Fire ∨ ¬Heat)

Now let's construct the truth table:

| **Smoke** | **Heat** | **Fire** | **(¬Smoke ∨ ¬Heat ∨ Fire) ⇔ (¬Smoke ∨ Fire ∨ ¬Heat)** |
| --- | --- | --- | --- |
| T | T | T | T |
| T | T | F | T |
| T | F | T | T |
| T | F | F | T |
| F | T | T | T |
| F | T | F | T |
| F | F | T | T |
| F | F | F | T |

The sentence "((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))" is valid because it is true for all possible truth values of Smoke, Heat, and Fire.

(f) (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)

We can use logical equivalences to simplify this sentence:

(Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire) (¬Smoke ∨ Fire) ⇒ (¬(Smoke ∧ Heat) ∨ Fire) (¬Smoke ∨ Fire) ⇒ (¬Smoke ∨ ¬Heat ∨ Fire)

Now let's construct the truth table:

| **Smoke** | **Heat** | **Fire** | **(¬Smoke ∨ Fire) ⇒ (¬Smoke ∨ ¬Heat ∨ Fire)** |
| --- | --- | --- | --- |
| T | T | T | T |
| T | T | F | F |
| T | F | T | T |
| T | F | F | F |
| F | T | T | T |
| F | T | F | T |
| F | F | T | T |
| F | F | F | T |

The sentence "(Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)" is valid because it is true for all possible truth values of Smoke, Heat, and Fire.

(g) Big ∨ Dumb ∨ (Big ⇒ Dumb)

We can use logical equivalences to simplify this sentence:

Big ∨ Dumb ∨ (Big ⇒ Dumb) Big ∨ Dumb ∨ (¬Big ∨ Dumb) Big ∨ Dumb

Now let's construct the truth table:

| **Big** | **Dumb** | **Big ∨ Dumb** |
| --- | --- | --- |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

The sentence "Big ∨ Dumb ∨ (Big ⇒ Dumb)" is valid because it is true for all possible truth values of Big and Dumb.

In summary:

(a) Smoke ⇒ Smoke: Valid (b) Smoke ⇒ Fire: Neither valid nor unsatisfiable (c) (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire): Neither valid nor unsatisfiable (d) Smoke ∨ Fire ∨ ¬Fire: Valid (e) ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)): Valid (f) (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire): Valid (g) Big ∨ Dumb ∨ (Big ⇒ Dumb): Valid

**Question 3 –**

**Question 4 –**

a. Any apartment in London has lower rent than some apartments in Paris.

∀x [Apt(x) ∧ In(x, London)] ⇒ ∃y ([Apt(y) ∧ In(y, Paris)] ⇒ (Rent(x) < Rent(y)))

The first-order logic sentence is a good translation of the English sentence. It correctly captures the idea that for any apartment in London, there exists at least one apartment in Paris with higher rent.

b. There is exactly one apartment in Paris with rent below $1000.

∃x Apt(x) ∧ In(x, Paris) ∧ ∀y [Apt(y) ∧ In(y, Paris) ∧ (Rent(y) < Dollars(1000))] ⇒ (y=x)

The first-order logic sentence is not a good translation of the English sentence. The sentence seems to be attempting to state that there is exactly one apartment in Paris with rent below $1000, but the usage of the variable y and the implication (⇒) is incorrect. The correct translation would be:

∃x [Apt(x) ∧ In(x, Paris) ∧ (Rent(x) < Dollars(1000))] ∧ ∀y [(Apt(y) ∧ In(y, Paris) ∧ (Rent(y) < Dollars(1000))] ⇒ (x = y)

This translation states that there exists an apartment in Paris with rent below $1000, and for any other apartment in Paris with rent below $1000, it must be the same apartment (ensuring uniqueness).

c. If an apartment is more expensive than all apartments in London, it must be in Moscow.

∀x Apt(x) ∧ [ ∀y Apt(y) ∧ In(y, London) ∧ (Rent(x) > Rent(y))] ⇒ In(x, Moscow)

The first-order logic sentence is a good translation of the English sentence. It correctly expresses the condition that if an apartment is more expensive than all apartments in London, then it must be in Moscow.