**BLDC Equation for Mathematical Modelling :**

Mathematic Equations The rotor and shaft are assumed to be rigid. Further, the model is assumed to be a viscous friction model. Therefore, the friction torque is proportional to the shaft angular velocity. Based on Newton's 2nd law and Kirchhoff's voltage law, the three phase star connected BLDC motor can be described by the following four governing equations:

𝑇𝑒 = 𝑏𝜃̇𝑚 + 𝐽𝜃̈𝑚 + 𝑇𝐿 (1)

V𝑎𝑏 = R(i𝑎 − i𝑏 ) + L d dt (i𝑎 − i𝑏 ) + 𝑒𝑎 − 𝑒𝑏 (2)

V𝑏𝑐 = R(i𝑏 − i𝑐 ) + L d dt (i𝑏 − i𝑐 ) + 𝑒𝑏 – 𝑒𝑐 (3)

V𝑐𝑎 = R(i𝑐 − i𝑎 ) + L d dt (i𝑐 − i𝑎 ) + 𝑒𝑐 – 𝑒𝑎 (4)

Here 𝑇𝑒 , 𝜃̇𝑚, b, J, 𝑇𝐿 , V, R, I, L and e denote the electrical torque, the mechanical rotational speed, the viscous friction constant, the rotor inertia, the mechanical load torque, the phase-to-phase voltage, the phase resistance, the phase inductance and the phase back emf respectively. Notice that the voltage and current have following relationships:

V𝑎𝑏 + V𝑏𝑐+V𝑐𝑎 = 0 (5)

i𝑎 + i𝑏 + i𝑐 = 0 (6)

Therefore, in order to simply the modelling, only two voltage equations are needed. They can be derived as follows:

2V𝑎𝑏 + V𝑏𝑐 = 3Ri𝑎 + 3L d dt i𝑎 + 2𝑒𝑎 − 𝑒𝑏 – 𝑒𝑐 (7)

−V𝑎𝑏 + V𝑏𝑐 = 3Ri𝑏 + 3L d dt i𝑏 + 2𝑒𝑏 − 𝑒𝑎 − 𝑒𝑐 (8)

In general, the torque generated by a three phase star connected BLDC motor can also be expressed by the following equation:

𝑇𝑒 = (𝑒𝑎i𝑎 + 𝑒𝑏i𝑏 + 𝑒𝑐 i𝑐)/𝜃̇𝑚 (9)

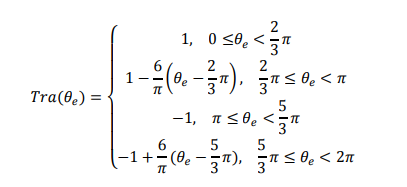
And the trapezoidal back emf can be expressed as:

𝑒𝑎 = 𝑘𝑒 2 𝜃̇𝑚𝑇𝑟(𝜃𝑒) (10)

𝑒𝑏 = 𝑘𝑒 2 𝜃̇𝑚𝑇𝑟(𝜃𝑒 − 2 3 𝜋) (11)

𝑒𝑐 = 𝑘𝑒 2 𝜃̇𝑚𝑇𝑟(𝜃𝑒 − 4 3 𝜋) (12)

Here, 𝑘𝑒 is the back emf constant and 𝜃𝑒 is the electrical angle which is equal to the mechanical angle times the number of pole pairs (𝜃𝑒 = 𝑝𝜃𝑚). Tra(𝜃𝑒 ) is the trapezoidal waveform function and one period of the function can be described as:

(13)

Further, we can simplify Equation 5 using Equation 6, 7, 8, and we can derive the following equation which makes the modelling more convenient:



Source :

1. MODEL-BASED DESIGN OF BRUSHLESS DC MOTOR CONTROL AND MOTION CONTROL , Xiaotian Li
2. BLDC Motor Modelling and Control – A Matlab®/Simulink® Implementation

Master Thesis work by Stefán Baldursson.

1. Three-winding brushless DC motor with trapezoidal flux distribution

https://in.mathworks.com/help/physmod/sps/ref/bldc.html.