Gibasová, Martinka

$u_0(x) = x^3 + x^2 = u(x;0)$ $\qquad u(0;0) = a(0)$ ✓

$a(t) = t$ $\qquad u(1;0) = b(0)$ ✓

$b(t) = 2+t$

$u_t - u_{xx} = 0 \; ; \; x \in (0;1) \; ; \; t > 0$

$u(x;0) = x^3 + x^2$

$u(0;t) = t \; ; \; u(1;t) = 2+t$

$u(x;t) = v(x;t) + w(x;t) \longrightarrow$ $\qquad w(x;t) \to w(0;t) = t$ $\qquad [0;t]$ $\qquad [1;2+t]$

$v(0;t) = v(1;t) = 0$ $\qquad w(1;t) = 2+t$ $\qquad \cancel{1+t+x}$

$w(0;t) + x\,(w(1;t) - w(0;t))$

$t + x[2+\cancel{t}-\cancel{t}] = t+2x \;\to$ spĺňa

nulové okr. pod.

$u(x;t) = v(x;t) + t + 2x$ $\qquad u(x;t) = \sum_{i=1}^{\infty} d_i(t) \cdot \sin(i\pi x)$

$u(x;0) = v(x;0) + 2x$ $\qquad \Rightarrow \quad u(x;0) = \sum_k d_i(0) \sin(i\pi x)$

$v(x;0) = (x^3+x^2) - 2x$ $\qquad d_i(0) = 2\int_0^1 v(x;0)\sin(i\pi x)\,dx$

$(k=i)$

a ideme riešiť: $d_k(0)$ $\qquad$ 3x per par $\quad$ 2krát per $\quad$ 1 per

$d_k(0) = 2\int_0^1 (x^3+x^2-2x)\sin(k\pi x)\,dx = 2\int_0^1 x^3\sin(k\pi x)dx + 2\int_0^1 x^2\sin(k\pi x)dx - 2\int_0^1 x\sin(k\pi x)dx$

$=$ (radšej wolfram lebo sa ešte pomýlim)

$2\left[\dfrac{3(\pi^2k^2-2)\underbrace{\sin(k\pi)}_{0} + k\pi(6-\pi^2k^2)\cos(\pi k)}{\pi^4k^4}\right] + 2\left[\dfrac{(2-\pi^2k^2)\cos(\pi k) + 2\pi k\underbrace{\sin(\pi k)}_{0} - 2}{\pi^3k^3}\right]$

$-4\left[\dfrac{\underbrace{\sin(k\pi)}_{0} - \pi k\cos(\pi k)}{\pi^2k^2}\right] \cdots = \dfrac{\cdots 8\pi k\cos(\pi k) - 2\pi k}{\pi^4k^4} = \dfrac{k8\pi(-1)^{k} - 2\pi k}{\pi^4k^4} \cdot 2$

$u_t = v_t + 1 \; ; \; u_x = v_x + 2$

$u_{xx} = v_{xx}$

$\Downarrow$

$v_t + 1 - v_{xx} = 0$

$v_t - v_{xx} = -1$

Pelikan

rozvoj pravej strany

$$f_k(t) = 2\int_0^1 (-1)\sin(k\pi x)\,dx = +2\left[\frac{\cos(k\pi x)}{k\pi}\right]_0^1 = \frac{+2}{k\pi}\left[\underset{(-1)^{k}}{\cos(k\pi)} - 1\right] =$$

$$= \frac{(-1)^k - 1}{k\pi}\cdot 2$$

$$\alpha_k'(t) + k^2\pi^2\alpha_k(t) = \frac{(-1)^k - 1}{k\pi}\cdot 2$$

$$\alpha_k^h(t) = c\,e^{-k^2\pi^2 t} \quad ; \quad \alpha_k^p(t) = c_1$$

$$0 + k^2\pi^2\alpha_k^p = 2\cdot\frac{(-1)^k - 1}{k\pi}$$

$$\alpha_k^p = \frac{2[(-1)^k - 1]}{(k\pi)^3} + c \leftarrow c = \alpha_k(0) - 2\left(\frac{(-1)^k - 1}{(k\pi)^3}\right)$$

$$c = \frac{8k\pi(-1)^{k-1} - 2k\pi - 2k\pi[(-1)^k - 1]}{(\pi k)^4} = \frac{8k\pi(-1)^{k-1} - 2k\pi(-1)^k}{(\pi k)^4}$$

$$= \frac{2\cancel{k\pi}[4\cdot(-1)^{k-1} + (-1)(-1)^k]}{(\pi k)^{\cancel{4}\,3}} = \frac{2[4(-1)^{k-1} + (-1)^{k-1}]}{(k\pi)^3} = \frac{10(-1)^{k-1}}{(k\pi)^3}$$

$$v(x,t) = \sum_{k=1}^{\infty}\left[\left(\frac{10(-1)^{k-1}}{(k\pi)^3}\cdot e^{-k^2\pi^2 t} + 2\,\frac{(-1)^k - 1}{(k\pi)^3}\right)\sin(k\pi x)\right]$$

$$\boxed{u(x,t) = v(x,t) + 2x + t}$$ oprava nižšie

oprava cecka : $\cancel{8k\pi(-1)^{k-1} - 2k\pi - 2k\pi}$

$$c = \frac{16k\pi(-1)^k - 4k\pi - 2k\pi[(-1)^k - 1]}{(k\pi)^4} = \frac{16k\pi(-1)^k - 2k\pi - 2k\pi(-1)^k}{(k\pi)^4}$$

$$= \frac{14\cancel{k\pi}(-1)^k - 2\cancel{k\pi}}{(k\pi)^{\cancel{4}\,3}} = \frac{14(-1)^k - 2}{(k\pi)^3} = \gamma$$

$$v(x,t) = \sum_{k=1}^{\infty}\left(\left[\frac{(-1)^k - 2}{(k\pi)^3}\cdot e^{-k^2\pi^2 t} + 2\cdot\frac{(-1)^k - 1}{(k\pi)^2}\right]\sin(k\pi x)\right)$$

$$u(x,t) = v(x,t) + 2x + t$$