Prep Work 6 - Induction

CS 234

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1. Induction

1. What are the 4 parts of an inductive proof? (The textbook calls one of them “state the hypothesis”—in this class we will call that step “stating the inductive predicate” instead. I will also tell you a slightly different way of stating it than the textbook uses, but the textbook is fine for now.)

1. State the hypothesis
2. Base step
3. Inductive step
4. Conclusion

2. What is the inductive predicate? Is it a number? A proposition? A string? Something else?

An inductive predicate is a hypothesis, which is always a logical statement.

3. What do you have to prove for the base case?

We have to prove the hypothesis for the smallest possible input value.

4. What do you get to assume in the inductive step? (We call this assumption the “inductive hypothesis,” which is slightly more specific than how the textbook uses that term.)

We fix an arbitrary value of k and assume the hypothesis for k holds true.

5. Given the inductive hypothesis assumption, what do you then have to prove for the inductive step?

We have to prove the hypothesis for k+1 is also true while making sure to use the inductive hypothesis assumption.

6. In your own words, explain how the base case and inductive step together allow you conclude that the inductive predicate holds for arbitrarily large natural numbers.

Suppose that the smallest possible value of the natural number n is given to the inductive predicate, which is the base case. By verifying this hypothesis, for all $k \geq n$, we show that if the inductive predicate P(k) is true, then P(k+1) is true during the inductive step. Based on this process, we can demonstrate that any value of $k \geq n$ would be valid. This indicates that the inductive predicate holds for arbitrarily large natural numbers since n is a natural number.

7. How is strong induction different from “normal” induction?

Normal induction has one smaller value of n for the induction hypothesis, while strong induction has the induction hypothesis that is true for all smaller values up to n and then prove it for the case of (n+1).

Now let's practice picking out the proof parts at a high level. We want to show that $\forall n \in \mathbb{N}$. $n^2 + n$ is even.

1. What is the appropriate inductive predicate?

For all $n \geq 0$, $P(n)$: $n^2 + n$ is even.

2. What is the base case?

When $n = 0$, which makes the inductive predicate return true.

3. What do we get to assume for the inductive step's inductive hypothesis?

We fix the value of k and assume the inductive hypothesis for k ($P(k)$) is true, which is "$k^2 + k$ is even."

4. What do we have to show for the inductive step, given the assumed inductive hypothesis.

We have to show that $P(k+1)$ is true, which is "$(k+1)^2 + (k+1)$ is even."

5. State what a conclusion to the proof would say.

We know that $P(0)$ is true from the base case. Since we know that $P(0)$ is true and we showed that $P(0)$ implies $P(1)$ in the inductive step (when $k = 0$), this means that $P(1)$ is true. Since we know that $P(1)$ is true and we showed that $P(1)$ implies $P(2)$ in the inductive step (when $k = 1$), this means that $P(2)$ is true. Continuing this manner, we can show that $P(n)$ is true for all $n \geq 0$.