Assignment 3 - Quantifiers, Asymptotics, and NFAs

CS 234

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1 Quantifiers and NFAs on Paper

Write the following as English statements with no mathematical notation.

3.47. $\forall n \in \mathbb{N}, 2n \in \mathbb{N}$

For all natural number n, two times n is also a natural number.

3.50. $\forall n \in \mathbb{Z}, \exists m \in Z, m < n$

For all integer n, there exists an integer m such that m is smaller than n.

3.52. $\forall n \in \mathbb{N}, \exists s \in \{0,1\}^*, |s| = n$

For all natural number n, there exists a string s that is in the set of all strings formed by 0s or 1s or both 0s and 1s or the empty string such that the length of string s is n.

3.53. $\forall s \in \{0,1\}^*, \exists q, r \in \{0,1\}^*, s = qr$

For all string s that is in the set of all strings formed by 0s or 1s or both 0s and 1s or the empty string, there exist strings q and r that are in the set of all strings formed by 0s or 1s or both 0s and 1s or the empty string such that string s is equivalent with string qr.

Show the following Big-O relationships by giving constants c and n_0 in each case and showing that they work.

3.55. $7n - 2 \in O(n)$

The function $f(n) = 7n - 2$ is $O(n)$ using $c = 8$ and $n_0 = 4$, that is, $7n - 2 \leq 8n$ for all $n \geq 4$. This is true because $-2 \leq n$ for all $n \geq 4$, so $7n - 2 \leq 7n + n = 8n$ for all $n \geq 4$.

3.56. $2n^2 + 4 \in O\left(n^2\right)$

The function $f(n) = 2n^2 + 4$ is $O\left(n^2\right)$ using $c = 6$ and $n_0 = 1$, that is, $2n^2 + 4 \leq 6n^2$ for all $n \geq 1$. This is true because $4 \leq 4n^2$ for all $n \geq 1$, so $2n^2 + 4 \leq 2n^2 + 4n^2 = 6n^2$ for all $n \geq 1$.

3.57. $3n^2 - 2n + 4 \in O\left(n^2\right)$

The function $f(n) = 3n^2 - 2n + 4$ is $O\left(n^2\right)$ using $c = 9$ and $n_0 = 1$, that is, $3n^2 - 2n + 4 \leq 9n^2$ for all $n \geq 1$. This is true because $-2n \leq 2n^2$ for all $n \geq 1$ and $4 \leq 4n^2$ for all $n \geq 1$, so $3n^2 - 2n + 4 \leq 3n^2 + 2n^2 + 4n^2 = 9n^2$ for all $n \geq 1$.

3.58. $n^3 - n^2 + n - 1 \in O\left(n^3\right)$

The function $f(n) = n^3 - n^2 + n - 1$ is $O\left(n^3\right)$ using $c = 4$ and $n_0 = 1$, that is, $n^3 - n^2 + n - 1 \leq 4n^3$ for all $n \geq 1$. This is true because $-n^2 \leq n^3$ for all $n \geq 1$, $n \leq n^3$ for all $n \geq 1$, and $-1 \leq n^3$ for all $n \geq 1$, so $n^3 - n^2 + n - 1 \leq n^3 + n^3 + n^3 + n^3 = 4n^3$ for all $n \geq 1$.

Negate the following quantified statements.

3.65. $\forall n \in \mathbb{R}, 2n \in \mathbb{R}$

$\exists n \in \mathbb{R}, 2n \notin \mathbb{R}$

3.69. $\forall n \in \mathbb{N}, \exists s \in \{0,1\}^*, |s| = n$

$\exists n \in \mathbb{N}, \forall s \in \{0,1\}^*, |s| \neq n$

3.71.$\exists w \in \Sigma^*, \forall x, y \in \Sigma^*, \exists z \in \Sigma^*, (|wx| \leq |yz|) \vee (|zx| \leq |yw|)$

$\forall w \in \Sigma^*, \exists x, y \in \Sigma^*, \forall z \in \Sigma^*, (|wx| > |yz|) \wedge (|zx| > |yw|)$

Give a sequence of states that show that the following strings are accepted by the following NFAs:

4.5. 101010 for the NFA in Example 4.6

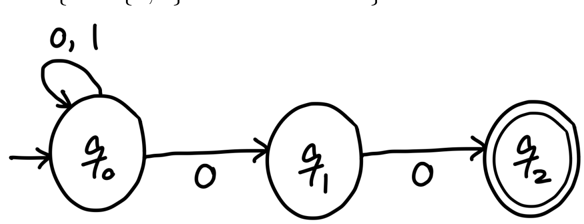
$q_0 \to q_2 \to q_3 \to q_2 \to q_3 \to q_2 \to q_3$

4.7. baaab for the NFA in Example 4.8

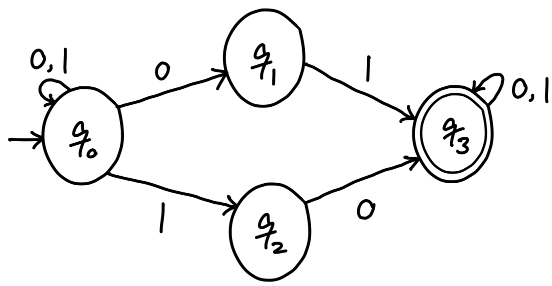
$q_0 \to q_0 \to q_0 \to q_0 \to q_1 \to q_2$

Draw nondeterministic finite automata (that are not also deterministic ones) with no λ transitions for the following languages:

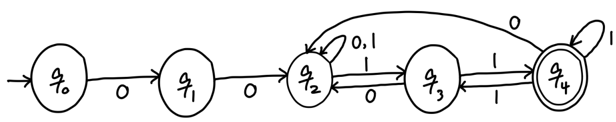
4.8. $\{w \in \{0,1\}^* : w \text{ ends in } 00\}$



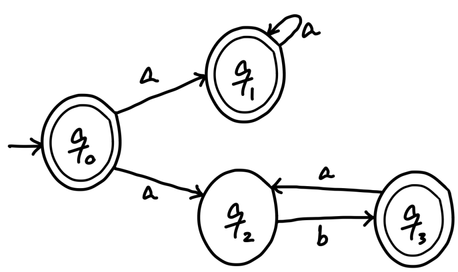
4.10. $\{w \in \{0,1\}^* : w$ has 01 or 10 as a substring $\}$



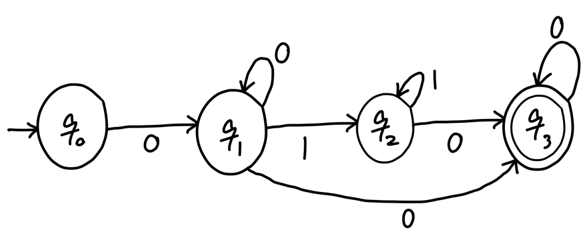
4.13. $\{w \in \{0,1\}^* : w$ starts with 00 and ends with 11$\}$



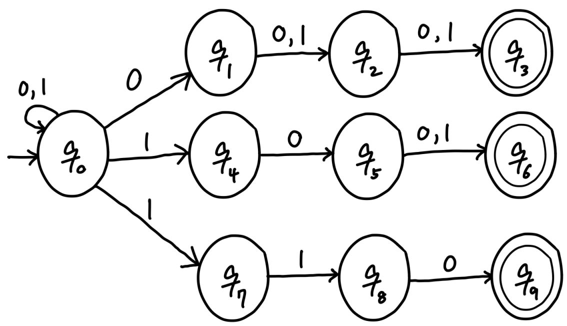
4.14. $\{a\}^* \cup \{(ab)^n : n \geq 1\}$



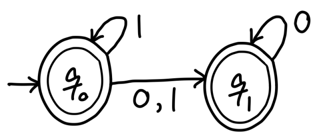
4.16. $\{0^p 1^q 0^r : p \geq 1, q \geq 0, r \geq 1\}$



4.18. $\{w \in \{0,1\}^* :$ at least one of the last three characters from the end is a 0$\}$



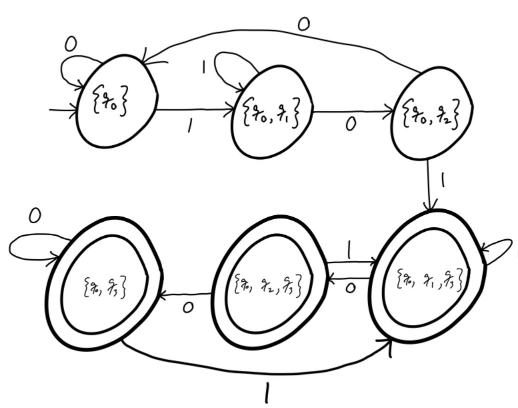
4.19. $\{w \in \{0,1\}^* : w$ does not have 01 as a substring $\}$



Convert the following NFAs to DFAs using the Subset Construction algorithm:

4.20. The NFA in Example 4.4

	0	1
$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
$\{q_0, q_2\}$	$\{q_0\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$



4.22. The NFA in Example 4.8

	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$