Prep Work 8 - Proof by Contradiction

CS 234

due March 24, before class

0 Introduction

This assignment has 1 part: proof by contradiction.

This assignment is to be completed individually, but feel free to collaborate according to the course's external collaboration policy (which can be found in the syllabus).

The deliverables consist of one `.pdf` file. The deliverables should be submitted electronically by the deadline. Put any attribution text in the `.pdf` file.

Every file should be named like `FLast_cs234_pX.ext` where `F` is your first initial, `Last` is your last name, `X` is the assignment number, and `ext` is the appropriate file extension. For example, Joan Moschovakis's `.pdf` file should be given the name `JMoschovakis_cs234_p8.pdf`. (Joan Moschovakis is researcher in constructive/computable logic and mathematics. She has studied advanced forms of induction like bar induction!)

1 Proof by Contradiction

Read chapter 10 in the textbook. Then complete the following tasks in your `.pdf` submission. Clearly label your responses with the task number.

1. What do you get to assume when using proof by contradiction to prove a proposition P?
2. What do you need to show when using proof by contradiction to prove a proposition P?
3. Let F represent an always-false proposition, like $0 = 1$. Make a truth table that shows $P = (\neg P) \to F$. (You never need a row where F is true because F is always just false.)
4. The previous task shows that proving $(\neg P) \to F$ is sufficient to prove P. What do we get to assume when proving $(\neg P) \to F$ by direct proof?
5. What do we need to show when $(\neg P) \to F$ by direct proof?
6. Given the previous tasks, how does proof by contradiction work?
7. If a number x is *not* irrational, what does this mean about how x can be expressed?
8. In your own words, what is the fundamental theorem of arithmetic?
9. Suppose $w \in L$ and $w \notin L'$. Is it possible that $L = L'$?
10. In your own words, explain how Theorem 10.3 derives its contradiction.