Prep Work 2 - DFAs and Logic

CS 234

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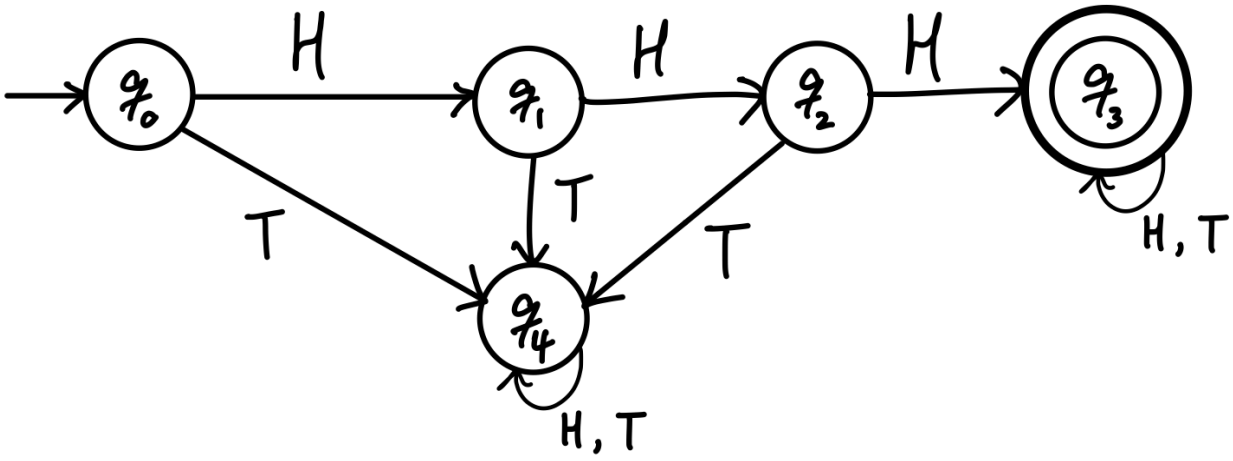
1. DFAs

1. In your own words, what is a DFA, and how does it compute?

DFA is a blueprint of a computation model that is operated for a specific task and it computes by receiving an input from a state one at a time.

2. Draw your own DFA diagram with at least 3 (labelled) states. It should take an input alphabet of at least 2 letters, and 1 or 2 of the states should be final (accepting).

The following DFA diagram is for designing a simple coin flip game in which the condition for winning is to have a flipped coin showing heads three consecutive times for the initial three attempts.



3. What state does ϵ (the empty string) end in when input into your DFA?

It stays in the q0 state because it is neither a head (H) nor a tail (T) so it cannot be computed to move to the next state.

4. Provide a string that your DFA accepts and name the states it goes through, in order, to end at a final state.

A string that is accepted by the DFA: HHH
The states that the accepted string goes through: q0, q1, q2, q3

5. Provide a string that your DFA does not accept and name the states it goes through, in order, to end at a non-final state.

A string that the DFA does not accept: HHT
The states that the string HHT goes through: q0, q1, q2, q4

6. What are the states of your DFA diagram? How does your diagram indicate this?

The states of the DFA diagram: q0, q1, q2, q3, q4

The diagram indicates the states by denoting the symbols (q0, q1, q2, q3, q4) inside each circle.

7. What is the start state of your DFA diagram? How does your diagram indicate this?

Start state: q0

The diagram indicates the start state by denoting it with q0 and an entering arrow to the circle.

8. What is the alphabet of your DFA diagram? How does your diagram indicate this?

The alphabet of the DFA diagram: $\Sigma = \{H, T\}$
The diagram indicates this by labeling the input alongside the arrows that indicate transitions.

9. Give the transition table for your DFA diagram. How does your diagram indicate these transitions?

	H	**T**
q0	**q1**	**q4**
q1	**q2**	**q4**
q2	**q3**	**q4**
q3	**q3**	**q3**
q4	**q4**	**q4**

The transitions are indicated in the diagram by arrows with labels of possible inputs.

10. What are the final states of your DFA diagram? How does your diagram indicate this?

The final state of the DFA diagram is q3 and it is indicated by a double circle.

11. What does it mean for a language to be *regular*?
It means the language can be accepted by a DFA.

2. Logic

1. In your own words, what is a logical statement? (In this class, this term will be used interchangeably with the term proposition.)

A logical statement is a statement that represents either true or false.

2. Give an example of a logical statement.

10 + 10 = 20

3. Is the statement “1+1” a logical statement?

No. It is not a logical statement because we cannot identify that the statement has a value of true or false.

4. Give the symbol and intuitive meaning for conjunction.

$\wedge$
The meaning of conjunction ($\wedge$) can be described as the intersection ($\cap$) of logical statements.

5. Give the symbol and intuitive meaning for disjunction.

$\vee$
The meaning of disjunction ($\vee$) can be described as the union ($\cup$) of logical statements.

6. Give the symbol and intuitive meaning for negation.

$\neg$
The meaning of negation ($\neg$) can be described as converting the value of a logical statement to the opposite value.

7. Give the symbol and intuitive meaning for implication (also called “conditional” in the textbook).

$\Rightarrow$
The meaning of implication ($\Rightarrow$) can be described as an implicative relation of two statements.

8. Give the symbol and intuitive meaning for bi-implication (also called “biconditional” in the textbook).

$\Leftrightarrow$
The meaning of bi-implication ($\Leftrightarrow$) can be described as a conjunction of a statement and a converse of that statement, which has the value of true.

9. Give the symbol and intuitive meaning for universal quantification (also called the "for all quantifier" in the textbook).

$\forall$

The meaning of universal quantification ($\forall$) can be described as an identifier indicating we consider all objects that could exist.

10. Give the symbol and intuitive meaning for existential quantification (also called the "there exists quantifier" in the textbook).

$\exists$

The meaning of existential quantification ($\exists$) can be described as an identifier indicating that there is at least one existing object.

11. Give the plain English meaning of the following statement:
$\forall x, y \in Q. (\neg x = y \rightarrow \exists z \in Q. ((x < z \land z < y) \lor (y < z \land z < x)))$
Break it down a piece at a time if you need to. Logic is a new language for you. With a bit of practice, you'll be able to read this just as easily as your native language!

$(x < z \land z < y)$: x is smaller than y
$(y < z \land z < x)$: y is smaller than x

$(x < z \land z < y) \lor (y < z \land z < x)$: x is smaller than y or y is smaller than x

$\neg x = y \rightarrow \exists z \in Q$: If y is not x, then there exists a rational number z.

$\forall x, y \in Q$: For all x and y, they are rational numbers.

$\forall x, y \in Q. (\neg x = y \rightarrow \exists z \in Q. ((x < z \land z < y) \lor (y < z \land z < x)))$:
For all x and y that are rational numbers, x is smaller than y or y is smaller than x, if y is not x.

12. What is the definition of what it means for f(n) to be O(g(n))?

We say that f(n) is O(g(n)) if there exists n_0, $c > 0$ such that for all $n \geq n_0$ we have that $f(n) \leq cg(n)$.

13. Is $10n^2$ in $O(n^2)$? Why or why not?

$10n^2$ is in $O(n^2)$ because $c = 100$ and $n_0 = 1$, that is, $10n^2 \leq 100n^2$ for all $n \geq 1$. This is true because for all $n \geq 1$, $0 \leq 90n^2$.