Assignment 2 - DFAs and Propositional Logic

CS 234

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1 DFAs and Propositional Logic on Paper

Give the full 5-tuple for DFAs for the following languages:

2.2. $\{w \in \{0,1\}^* : w$ ends with a $1\}$

$(\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_1\})$

δ	0	1
q_0	q_0	q_1
q_1	q_0	q_1

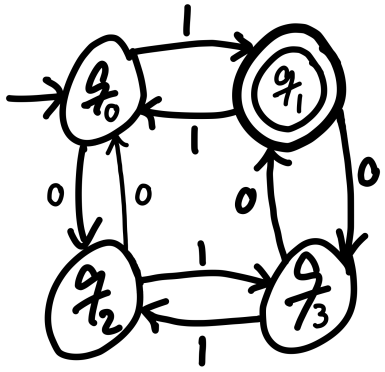
2.4. $\{w \in \{0,1\}^* : w$ has 01 as a substring $\}$

$(\{q_0, q_1, q_2\}, \{0,1\}, \delta, q_0, \{q_2\})$

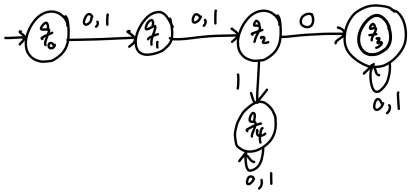
δ	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_2	q_2

Draw DFAs for the following languages:

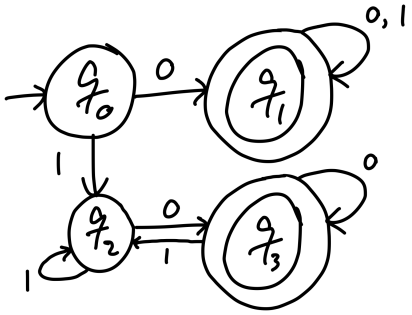
2.8. $\{w \in \{0,1\}^* : w$ has an even number of 0s and an odd number of 1s$\}$



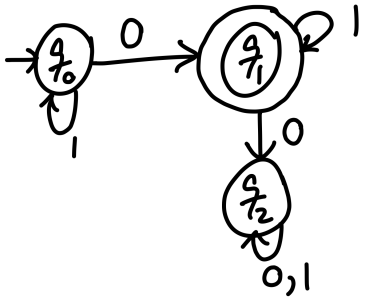
2.12. $\{w \in \{0,1\}^* :$ the third character in w is $0\}$



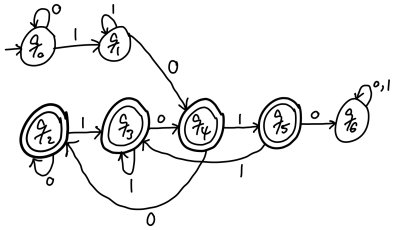
2.14. $\{w \in \{0,1\}^* : w$ starts or ends with a $0\}$



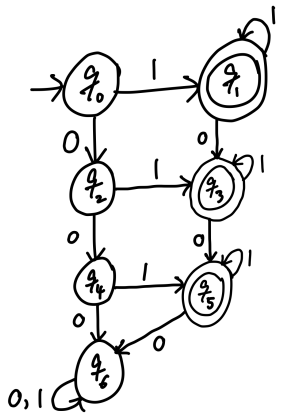
2.15. $\{w \in \{0,1\}^* : w$ has exactly one $0\}$



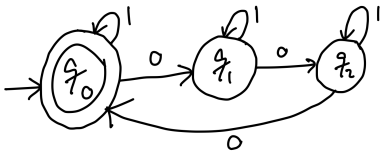
2.16. $\{w \in \{0,1\}^* : w$ contains 10 but not $1010\}$



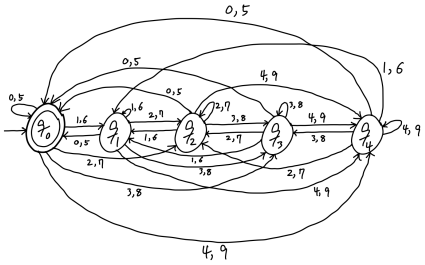
2.18. $\{w \in \{0,1\}^* : w$ contains at most two 0s and at least one $1\}$



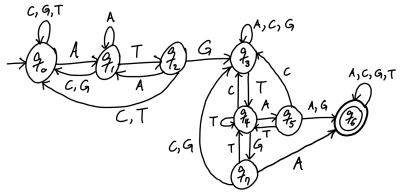
2.19. $\{w \in \{0,1\}^* :$ the number of 0s in w is divisble by $3\}$



2.24. $\{w \in \{0,\ldots,9\}^* : w$ is divisible by 5 in decimal $\}$ (Let λ be in the language and ignore leading 0s.)



2.27. $\{w \in \{A,C,G,T\}^* : w$ has the substring ATG followed later by the substring TAA or TAG or TGA $\}$



Determine whether each of the following statements is true or false. Give a one line explanation for each.

3.4. (Assume that $x = 3$) $(x \leq 3) \vee (x \leq 2)$

True. The above statement's value is true if either $(x \leq 3)$ or $(x \leq 2)$ is true and the given assumption makes the statement $(x \leq 3)$ to true.

3.5. (Assume that $x = 3$ and $y = 4$) $(x < y) \wedge (x + 1 \leq y)$

True. Based on the given assumption, the statement $(x < y)$ is true and also $(x + 1 \leq y)$ is true, which the conjunction's value that is based on these two statement results true.

Write each of the following statements in symbolic form in terms of $\neg$, $\wedge$, and $\vee$ operations

3.13. The value x is smaller than zero or at least eleven.

$(x < 0) \vee (x \geq 11)$

3.19. Exactly two of the values x, y, z are equal to ten.

$(x = 10 \wedge y = 10 \wedge \neg(z = 10)) \vee (x = 10 \wedge \neg(y = 10) \wedge z = 10) \vee (\neg(x = 10) \wedge y = 10 \wedge z = 10)$

Show whether the following pairs of expressions are equivalent by creating truth tables for them. Be sure to show each of the intermediate steps for each expression.

3.27. $A \wedge (A \vee B)$ and A

A	B	$A \vee B$	$A \wedge (A \vee B)$
F	F	F	F
F	T	T	F
T	F	T	T
T	T	T	T

3.33. $A \vee (B \wedge C)$ and $(A \vee B) \wedge (A \vee C)$

A	B	C	$A \vee B$	$A \vee C$	$B \wedge C$	$A \vee (B \wedge C)$	$(A \vee B) \wedge (A \vee C)$
F	F	F	F	F	F	F	F
F	F	T	F	T	F	F	F
F	T	F	T	F	F	F	F
F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T
T	F	T	T	T	F	T	T
T	T	F	T	T	F	T	T
T	T	T	T	T	T	T	T

Write each of the following statements in the form $P \Rightarrow Q$ or $P \Leftrightarrow Q$, stating the values of P and Q for each.

3.36. The sum of two numbers is an integer if both numbers are integers.

P: Both numbers are integers.

Q: The sum of two numbers is an integer.

$P \Rightarrow Q$

3.38. It is safe to divide by x only if $x \neq 0$.

P: It is safe to divide by x.

Q: $x \neq 0$

$P \Rightarrow Q$

Show whether the following pairs of expressions are equivalent by creating truth tables for them. Be sure to show each of the intermediate steps for each expression.

3.40. $A \vee B$ and $\neg A \Rightarrow B$

A	B	$\neg A$	$A \vee B$	$\neg A \Rightarrow B$
F	F	T	F	F
F	T	T	T	T
T	F	F	T	T
T	T	F	T	T

3.41. $A \wedge B$ and $A \Rightarrow \neg B$

A	B	$\neg B$	$A \wedge B$	$A \Rightarrow \neg B$
F	F	T	F	T
F	T	F	F	T
T	F	T	F	T
T	T	F	T	F

3.42. $A \Leftrightarrow B$ and $(\neg A \vee B) \wedge (A \vee \neg B)$

A	B	$\neg A$	$\neg B$	$\neg A \vee B$	$A \vee \neg B$	$(\neg A \vee B) \wedge (A \vee \neg B)$	$A \Leftrightarrow B$
F	F	T	T	T	T	T	T
F	T	T	F	T	F	F	F
T	F	F	T	F	T	F	F
T	T	F	F	T	T	T	T

3.44. $(A \Rightarrow B) \vee (A \Rightarrow C)$ and $A \Rightarrow (B \vee C)$

A	B	C	$A \Rightarrow B$	$A \Rightarrow C$	$B \vee C$	$(A \Rightarrow B) \vee (A \Rightarrow C)$	$A \Rightarrow (B \vee C)$
F	F	F	T	T	F	T	T
F	F	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	T	F	T	F	T	T	T
T	T	T	T	T	T	T	T

3.46. $(A \Rightarrow C) \wedge (B \Rightarrow C)$ and $(A \vee B) \Rightarrow C$

A	B	C	$A \Rightarrow C$	$B \Rightarrow C$	$A \vee B$	$(A \Rightarrow C) \wedge (B \Rightarrow C)$	$(A \vee B) \Rightarrow C$
F	F	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	T	F	T	F	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	T	T	F	F
T	F	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	T	T	T	T	T	T	T