Prep Work 9 - Pumping Lemma

CS 234

Daniel Lee

1. Pumping Lemma

1. In your own words, describe the pigeonhole principle.

If the number of entities is strictly greater than the number of containers each entity can fit into, then there must be a container containing more than one entity.

2. How is the pigeonhole principle used in the proof of the pumping lemma?

The pigeonhole principle is used in the proof of the pumping lemma by stating that prefixes for any string w of length at most n in the set of regular language L have n+1 of them by including the prefix of length zero and thus, there must be two of these prefixes that end at the same state of a DFA since the proof initially stated that the DFA has n states.

3. In the statement of the pumping lemma in Theorem 11.2, a string w from a given regular language is divided up into 3 parts xyz. In your own words, what are the 3 properties that the pumping lemma attributes to these parts?

The first property states that y must not be an empty string.
The second property states that the length of xy should be less than or equal to some n that is greater than 0.
The third property states that xy^kz is in the set of a regular language L for all k that is greater than or equal to 0.

4. The pumping lemma is about string membership in a regular language. How can the pumping lemma be applied to determine when a language is not regular?

The pumping lemma can be applied to determine when a language is not regular by demonstrating the proof of contradiction that shows the pumping lemma does not apply to the assumption for a contradiction that the language is regular.

5. In Theorem 11.5, there is an edge case. In your own words, how is the edge case dealt with in the proof?

The edge case is dealt with in the proof by creating multiple cases based on whether (x = λ, y = 01^i, 0 <= i < n, z = 1^(n-i)01^n) or (x = 01^i, y = i^j, 0 <= i < n, 1 <= j < n, z = 1^(n-i-j)01^n).

6. In your own words, why does Theorem 11.6 have to be careful about how many times the string is pumped?

Theorem 11.6 is an example where we should be careful about the number of times the string is pumped because it shows that only $k = 0$ would be valid, as any larger value of k would pump the string into a string in L, which makes an invalid proof by contradiction.