Prep Work 8 - Proof by Contradiction

CS 234

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1. Proof by Contradiction

1. What do you get to assume when using proof by contradiction to prove a proposition P?

We assume for a contradiction that the negation of P is true.

2. What do you need to show when using proof by contradiction to prove a proposition P?

We need to show that we arrive at a contradiction of our initial assumption.

3. Let F represent an always-false proposition, like 0 = 1. Make a truth table that shows P = (¬P) → F. (You never need a row where F is true because F is always just false.)

P	¬P	F	(¬P) → F
T	F	F	T
F	T	F	F

4. The previous task shows that proving (¬P) → F is sufficient to prove P. What do we get to assume when proving (¬P) → F by direct proof?

We assume that ¬P holds.

5. What do we need to show when (¬P) → F by direct proof?

We need to show that F also holds.

6. Given the previous tasks, how does proof by contradiction work?

We first assume that ¬P holds. Based on this assumption, we make an argument that demonstrates that F is a contradiction. Thus, we show that by this contradiction, the proposition P is true.

7. If a number x is not irrational, what does this mean about how x can be expressed?

This means that x can be represented in the form of p/q, where p, q ∈ Z and q = 0.

8. In your own words, what is the fundamental theorem of arithmetic?

The fundamental theorem of arithmetic is that all integers that are greater than 1 are able to be rewritten in the form of prime products without the consideration of the order of those products.

9. Suppose $w \in L$ and $w \notin L'$. Is it possible that $L = L'$?

No, it is not possible. The provided proposition can be translated as the string w is in the language L if and only if the string w is not in the language L’. This means that the two languages, L and L’, cannot be identical languages.

10. In your own words, explain how Theorem 10.3 derives its contradiction.

For the proof of theorem 10.3, the textbook assumes that it is possible for all languages over a certain alphabet to be listed in the form of L_1, L_2, L_3, … for a contradiction. It then shows the argument by creating a new language L’ and states that a string s_j is in the new language L’ if and only if s_j is not in L_j, where j >= 1. This leads to the contradiction of our initial assumption, which was that it is possible for all languages over a certain alphabet to be listed in the form of L_1, L_2, L_3, …, but by the argument, L’ and L_j cannot be the same.