Prep Work 7 - Mutual Induction

CS 234

due March 10, before class

0 Introduction

This assignment has 1 part: mutual induction.

This assignment is to be completed individually, but feel free to collaborate according to the course's external collaboration policy (which can be found in the syllabus).

The deliverables consist of one `.pdf` file. The deliverables should be submitted electronically by the deadline. Put any attribution text in the `.pdf` file.

Every file should be named like `FLast_cs234_pX.ext` where `F` is your first initial, `Last` is your last name, `X` is the assignment number, and `ext` is the appropriate file extension. For example, Éva Tardos's `.pdf` file should be given the name `ETardos_cs234_p7.pdf`. (Éva Tardos is an award-winning researcher in graph and optimization algorithms. I did many asymptotic analyses for her class when I was an undergrad.)

1 Mutual Induction

Read chapter 9 in the textbook. Then complete the following tasks in your `.pdf` submission. Clearly label your responses with the task number.

1. In your own words, how does mutual induction differ from "normal" and strong induction?

2. What is the definition of $\hat{\delta}$?

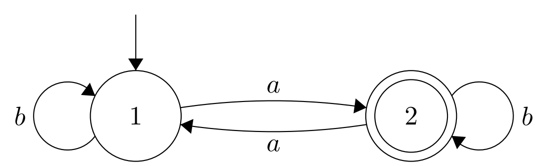
3. How does the recursive sort of definition given to $\hat{\delta}$ get used in the inductive proof? (Look at the inductive case.)

4. When you use mutual induction to prove the language of a DFA, you typically need to prove a predicate for *each* state in the automaton. In our setting, for a state q', we typically use a predicate $P(n)$ that is of the form

$$\forall w \in \Sigma^*.\, |w| = n \rightarrow (\hat{\delta}(q_0, s) = q' \leftrightarrow \textit{something})$$

   where that "something" is whatever it means for a string of length n to end up in state q'. For example, it might mean the string ends with the symbol a, so that "*something*" is $\exists u \in \Sigma^*.\, w = au$.

   Here is a DFA for the language of strings with even numbers of a. Give a predicate P for state 1 and Q for state 2 that each describe what it means for a string of length n to end up in that state.



5. For an arbitrary n, why would you need to already know $P(n)$ in order to prove $Q(n+1)$? Similarly, why would you need to already know $Q(n)$ in order to prove that $P(n+1)$? (We call it *mutual* induction because each predicate depends on the other in this way.)