Prep Work 5 - Definitions and Direct Proofs

CS 234

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1. Direct Proofs

1. Restate the 6 tips for writing proofs in your own words.

1. Proofs should be based on full, grammatical sentences with necessary punctuation, including periods after mathematical expressions.
2. Proofs should be in paragraph form.
3. Proofs should clearly show where the proof starts and ends.
4. Proofs should be written considering the audience for the degree of formality.
5. Proofs should define variables before using them.
6. Proofs should not start a sentence with mathematical expressions.

2. Look at definition 7.2. This definition tells you exactly what condition must be met for the definition of "odd length" to apply to a string s: the existential proposition $\exists n \in \mathbb{N}.|s| = 2n + 1$. We show such an existential is true by providing the concrete n witnessing that $|s| = 2n+1$. With this in mind, use definition 7.2 to argue that the string "hello" is odd length. (Remember the tips for proof writing too!)

Let x be the odd length of the string "hello".

By the definition of odd length, we know that there exists some integer $m \geq 0$ such that $|x|=2m+1$.

Since the value of x is 5, we can identify that the value of m is 2.

This means that the string "hello" is odd length.

3. When trying to prove a statement of the form $P \rightarrow Q$ by direct proof, what do we start by assuming, and what do we want to show? (Look back at the start of section 7.3 if you can't remember.)

We start by assuming that P is true, and we show a sequence of logical arguments to show that Q then also must be true.

4. We can also try to prove a statement of the form $P \rightarrow Q$ by using direct proof with their contrapositive. What do we start by assuming then, and what do we want to show?

We start by assuming that $\neg Q$ is true, and we show a sequence of logical arguments to show that $\neg P$ then also must be true, ultimately leading to the conclusion that the statement of the form $P \rightarrow Q$ itself must be true.

5. We call a set “closed” under some operation when that operation’s domain and codomain are both contained within the set. For example, natural numbers are closed under addition. Name two more operations that natural numbers are closed under.

Multiplication, Squares