Prep Work 7 - Mutual Induction

CS 234

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1. Mutual Induction

1. In your own words, how does mutual induction differ from "normal" and strong induction?

Unlike "normal" and strong induction, mutual induction has to prove if and only if results for multiple statements from the mutual induction hypothesis.

2. What is the definition of $\hat{\delta}$?

The state that the DFA computation will end at if the string s is processed starting at state q_i.

3. How does the recursive sort of definition given to $\hat{\delta}$ get used in the inductive proof? (Look at the inductive case.)

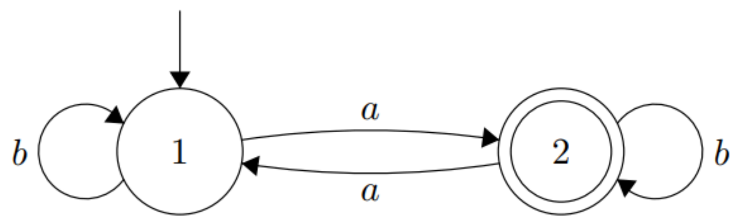
The recursively defined δ^ is being used to identify and use the information that we know based on the induction hypothesis during the process of the inductive step. For example, proof of 1 from example 9.4 in the textbook uses recursively defined δ^ to identify that the string s that is processed starting at state q_0 ends up at q_1 and uses this information to derive the fact that the string w that is processed starting at state q_0 would end up at q_0.

4. When you use mutual induction to prove the language of a DFA, you typically need to prove a predicate for *each* state in the automaton. In our setting, for a state q', we typically use a predicate $P(n)$ that is of the form

$$\forall w \in \Sigma^*.\, |w| = n \to (\hat{\delta}(q_0, s) = q' \leftrightarrow \textit{something})$$

where that "something" is whatever it means for a string of length n to end up in state q'. For example, it might mean the string ends with the symbol a, so that "*something*" is $\exists u \in \Sigma^*.\, w = au$.

Here is a DFA for the language of strings with even numbers of a. Give a predicate P for state 1 and Q for state 2 that each describe what it means for a string of length n to end up in that state.



P(n): ∀w ∈ Σ*. |w| = n → (δ^(1, w) = 1 ↔ w has an even number of as)

Q(n): ∀w ∈ Σ*. |w| = n → (δ^(1, w) = 2 ↔ w has an odd number of as)

5. For an arbitrary n, why would you need to already know $P(n)$ in order to prove $Q(n+1)$? Similarly, why would you need to already know $Q(n)$ in order to prove that $P(n+1)$? (We call it *mutual* induction because each predicate depends on the other in this way.)

We need to know P(n) in order to prove Q(n+1) and Q(n) in order to prove P(n+1) because we might need to refer to another predicate to prove the inductive step. For example, in "proof of 1, if" from example 9.9 in the textbook, it describes that "δ^(q_0, s) must be q_0 or q_2" and in "proof of 1, only if," the textbook describes that "if c= 1, then δ^(q_0, s) is q_0 or q_2." From these parts of the inductive steps, we can notice that we need to be aware of other predicates stated in the inductive hypothesis to prove inductive steps for each inductive predicate.