Automne 2021

IFT6512 – Programmation stochastique
Devoir 1

Date de remise : 20 octobre 2021.

1. Consider the second-stage problem defined by

$$\begin{aligned} \min_y \ & 2y_1 + y_2 \\ \text{s.t. } & y_1 + y_2 \geq 1 - x_1, \\ & y_1 \geq \boldsymbol{\xi} - x_1 - x_2, \\ & y_1, y_2 \geq 0. \end{aligned}$$

   Show that this program has a finite recourse if $\mathbb{E}[\boldsymbol{\xi}]$ is finite.

2. Consider the second-stage problem defined by

$$\begin{aligned} \min_y \ & 2y_1 + y_2 \\ \text{t.q. } & y_1 - y_2 \leq 2 - \boldsymbol{\xi} x_1, \\ & y_2 \leq x_2, \\ & y_1, y_2 \geq 0. \end{aligned}$$

   Determine $K_2(\boldsymbol{\xi})$ and K_2 for

   (a) $\boldsymbol{\xi} \sim U[0, 1]$;

   (b) $\boldsymbol{\xi} \sim \text{Poisson}(\lambda)$, $\lambda > 0$. ($P[X = k] = \lambda^k e^{-\lambda}/k!$)

   What properties can we reasonably expect for K_2 ?

3. We consider a baker who decides the morning how many breads he will cook, and can sell his/her production during the day with profit. In the evening, the unsold items can be sold at a reduced price as the breads are less fresh. Two types of bread can be produced : white bread and whole wheat bread. A unit of white bread cost 1.5\$ to produce, while the whole wheat bread costs 1.8\$. The are sold at 3\$ and 4\$ per unit respectively. The unsold breads are sold the evening at 1\$ et 1.2\$. 200g of flour are required to produce one unit of whole wheat bread, while for one unit of white bread, 150g are sufficient. The baker has a total of 12kg of flour. We assume that the demand for white bread follows a normal distribution with mean 50 and standard devision 5, and the demand for whole wheat bread follows a normal distribution with mean 30 and standard deviation 2. The two demands are correlated, with a covariance between of 0.4.

- Describe the problem as a two-stage stochastic program. Explain the choice of first- and second-stage decision variables, and form the mathematical program.
- Give the expression of K_1, K_2, and K_2^P. Is the recourse complete, relatively complete, simple ?
- Draw 100 demand scenarios and implement the problem using Julia and JuMP.
- Interpret the solution.
- Adapt the Julia code in order to obtain integer solutions.