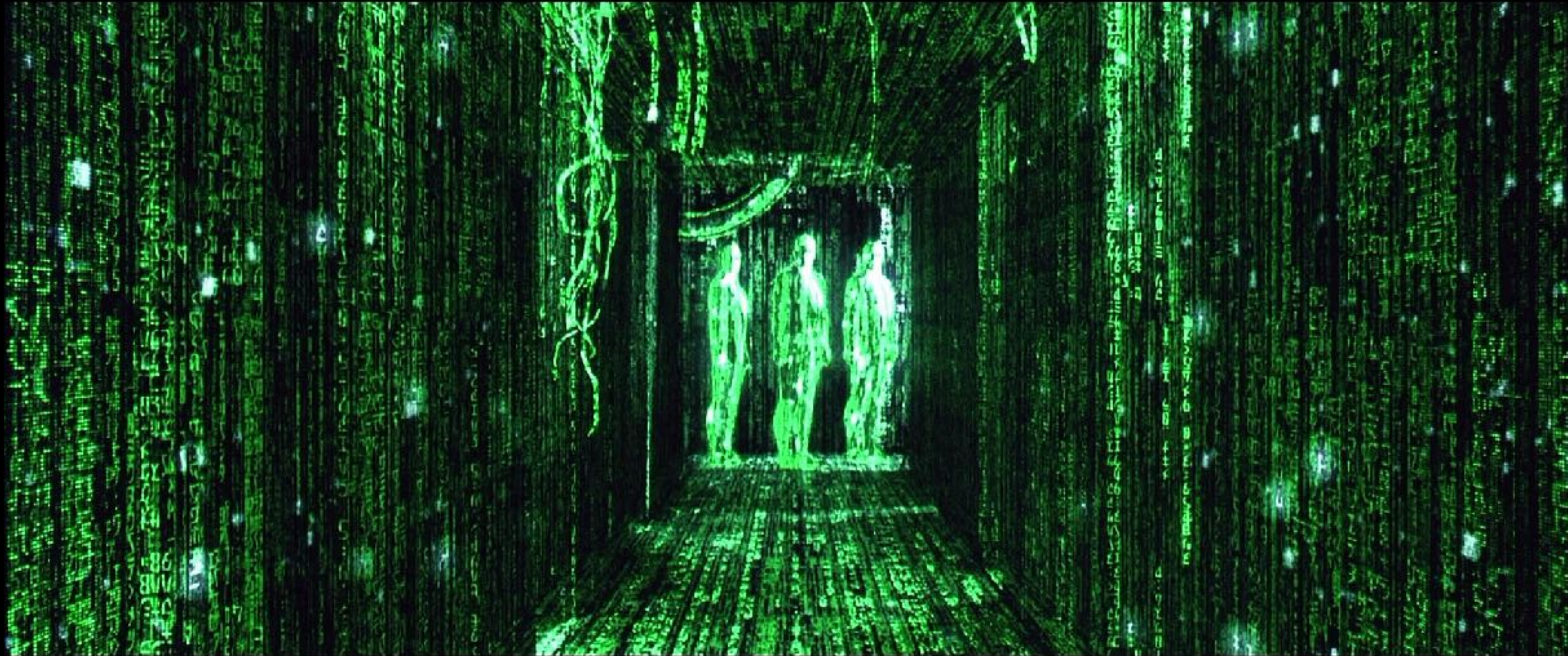
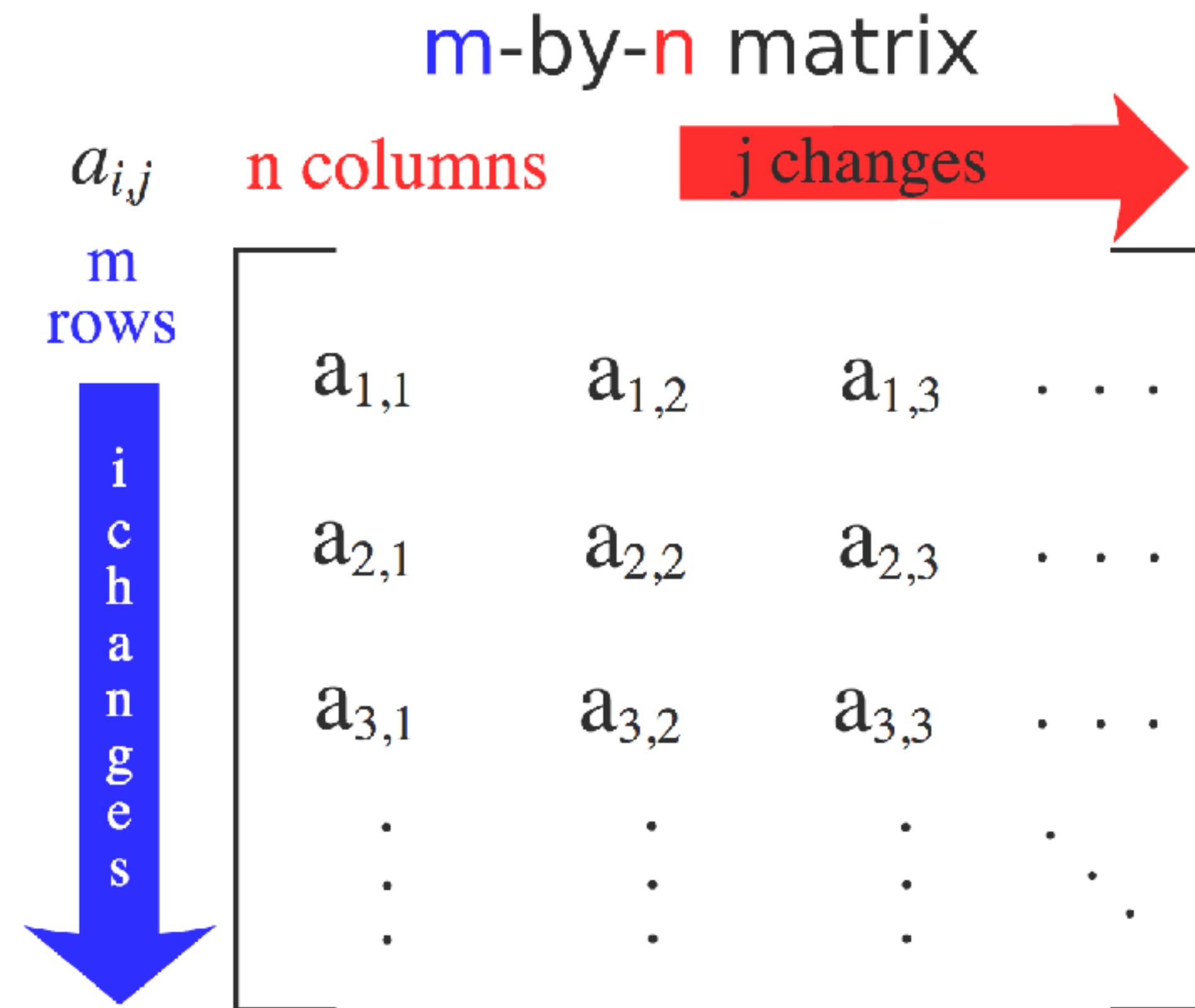


Matrix transformations.



Matrix math.

A matrix.



A 2x3 matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \end{bmatrix}$$

A 3x3 matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \\ 3 & 4 & 2 \end{bmatrix}$$

Matrix operations.

Matrix addition.

To add two matrices, add their corresponding entries.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} + \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} = \begin{bmatrix} A+J & B+K & C+L \\ D+M & E+N & F+O \\ G+P & H+Q & I+R \end{bmatrix}$$

Matrix subtraction.

To subtract two matrices, subtract their corresponding entries.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} - \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} = \begin{bmatrix} A-J & B-K & C-L \\ D-M & E-N & F-O \\ G-P & H-Q & I-R \end{bmatrix}$$

**Matrix addition and subtraction can only
happen
with matrices that are the same size!**

Transpose of a matrix.

Transpose of a matrix is a matrix whose columns are the rows of the original matrix (and its rows are the columns).

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}$$

M

$$\begin{bmatrix} A & D \\ B & E \\ C & F \end{bmatrix}$$

M^T

Matrix/scalar multiplication.

Multiply each entry of the matrix by the scalar.

$$S \times \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = \begin{bmatrix} S \times A & S \times B & S \times C \\ S \times D & S \times E & S \times F \\ S \times G & S \times H & S \times I \end{bmatrix}$$

Matrix/matrix multiplication.

You can only multiply two matrices
if the number of columns of the first matrix
equals the number of rows of the second.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = ?$$

It results in a matrix that is number of rows of first matrix by number of columns of second matrix.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

For each row, find dot product with each column.

The diagram illustrates the dot product of a row and a column. The first matrix has a first row with elements A, B, and C, which are highlighted in a red box with a red arrow pointing to the right. The second matrix has a first column with elements J, M, and P, which are highlighted in a red box with a red arrow pointing downwards. The elements K, N, and Q in the second matrix are shown in blue. An equals sign follows the matrices.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

The diagram shows the calculation of the dot product for the first row and column. The expression $A \times J + B \times M + C \times P$ is shown in red and blue, enclosed in large black brackets.

$$\left[A \times J + B \times M + C \times P \right]$$

For each row, find dot product with each column.

The diagram illustrates the dot product of a row and a column. On the left, a 2x3 matrix is shown with elements A, B, C in the first row and D, E, F in the second row. A red arrow points to the first row, which is highlighted with a red background. On the right, a 3x2 matrix is shown with elements J, M, P in the first column and K, N, Q in the second column. A red arrow points to the second column, which is highlighted with a red background. An equals sign follows the matrices.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

The diagram shows the result of the dot product as a row vector. The first element is the dot product of the first row and first column, calculated as A times J plus B times M plus C times P. The second element is the dot product of the first row and second column, calculated as A times K plus B times N plus C times Q. The entire result is enclosed in large square brackets.

$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \end{bmatrix}$$

For each row, find dot product with each column.

The diagram illustrates the dot product of a row from matrix A with a column from matrix B. Matrix A is represented as a 2x3 grid with elements A, B, C in the first row and D, E, F in the second row. A red arrow points from the first row of A towards the right. Matrix B is represented as a 3x2 grid with elements J, K in the first column and M, N in the second column, and P, Q in the third column. A red arrow points from the first column of B downwards. The two matrices are separated by an equals sign, indicating the result of the dot product operation.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

The diagram shows the resulting dot products for each row of matrix A. The first row of the result is calculated as $A \times J + B \times M + C \times P$ and $A \times K + B \times N + C \times Q$. The second row of the result is calculated as $D \times J + E \times M + F \times P$. The elements are color-coded to match the matrices: A, B, C, D, E, F are red; J, M, P, K, N, Q are blue.

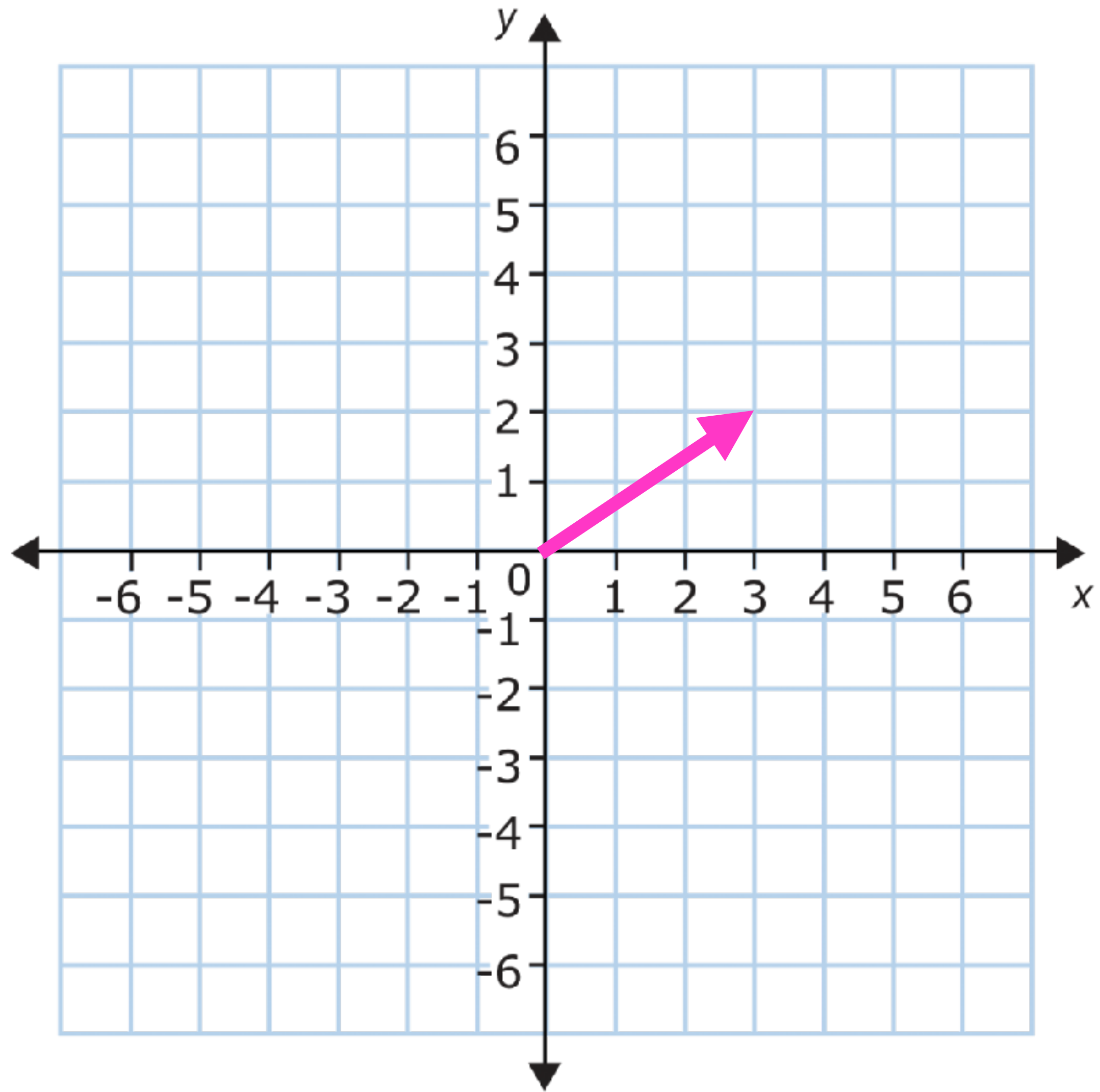
$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \\ D \times J + E \times M + F \times P \end{bmatrix}$$

For each row, find dot product with each column.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \\ D \times J + E \times M + F \times P & D \times K + E \times N + F \times Q \end{bmatrix}$$

Vectors.



A 2 dimensional vector can be represented
as a 2x1 matrix.

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

A 3 dimensional vector can be represented
as a 3x1 matrix.

$$\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

Matrix vector multiplication.

Multiplying a matrix and a vector is just multiplying two matrices.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

Row by row, multiply each column value with the each row of the vector and add them together.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} AX + BY + CZ \end{bmatrix}$$

Row by row, multiply each column value with the each row of the vector and add them together.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} AX + BY + CZ \\ DX + EY + FZ \\ \end{bmatrix}$$

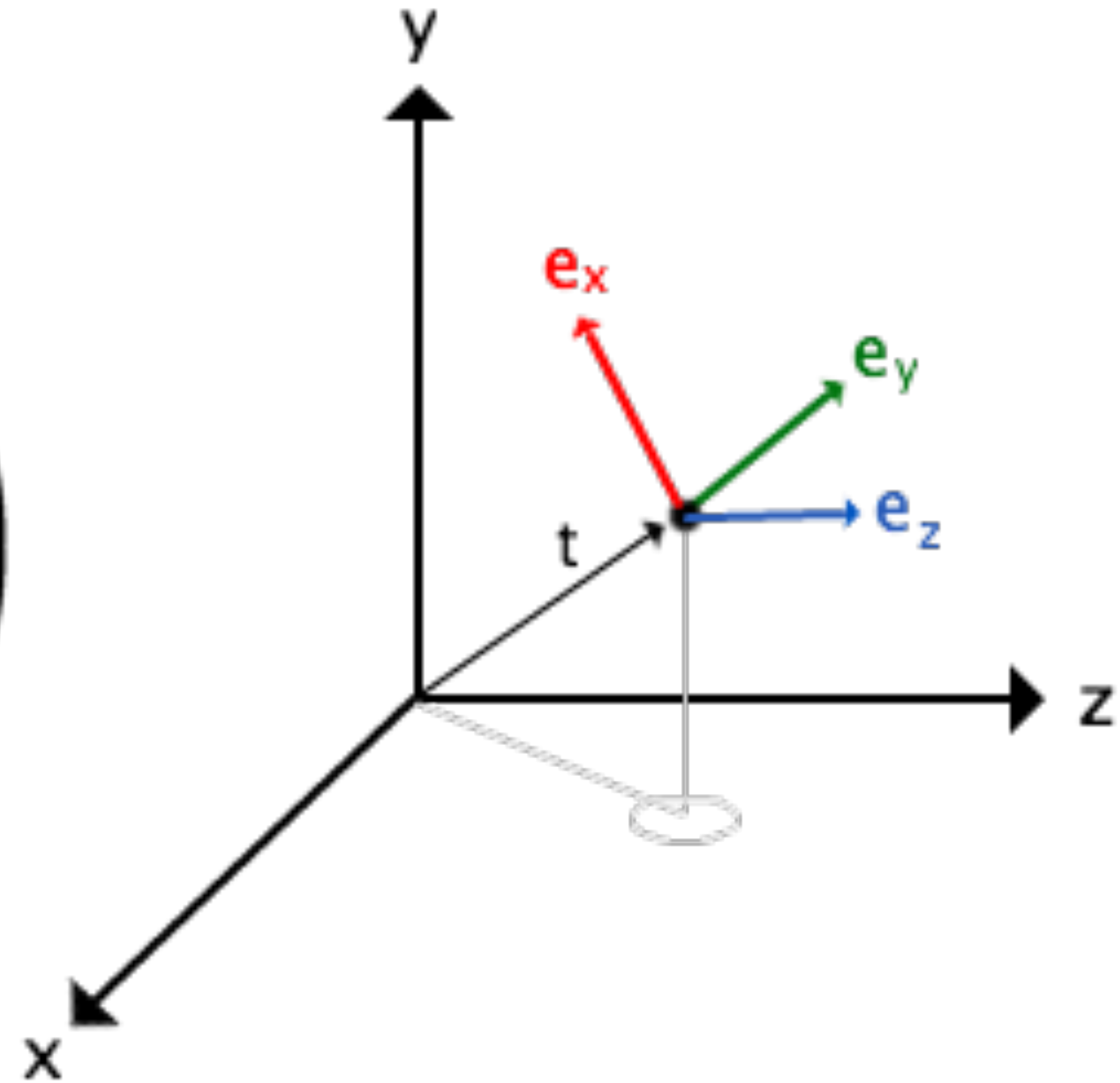
Row by row, multiply each column value with the each row of the vector and add them together.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} AX + BY + CZ \\ DX + EY + FZ \\ GX + HY + IZ \end{bmatrix}$$

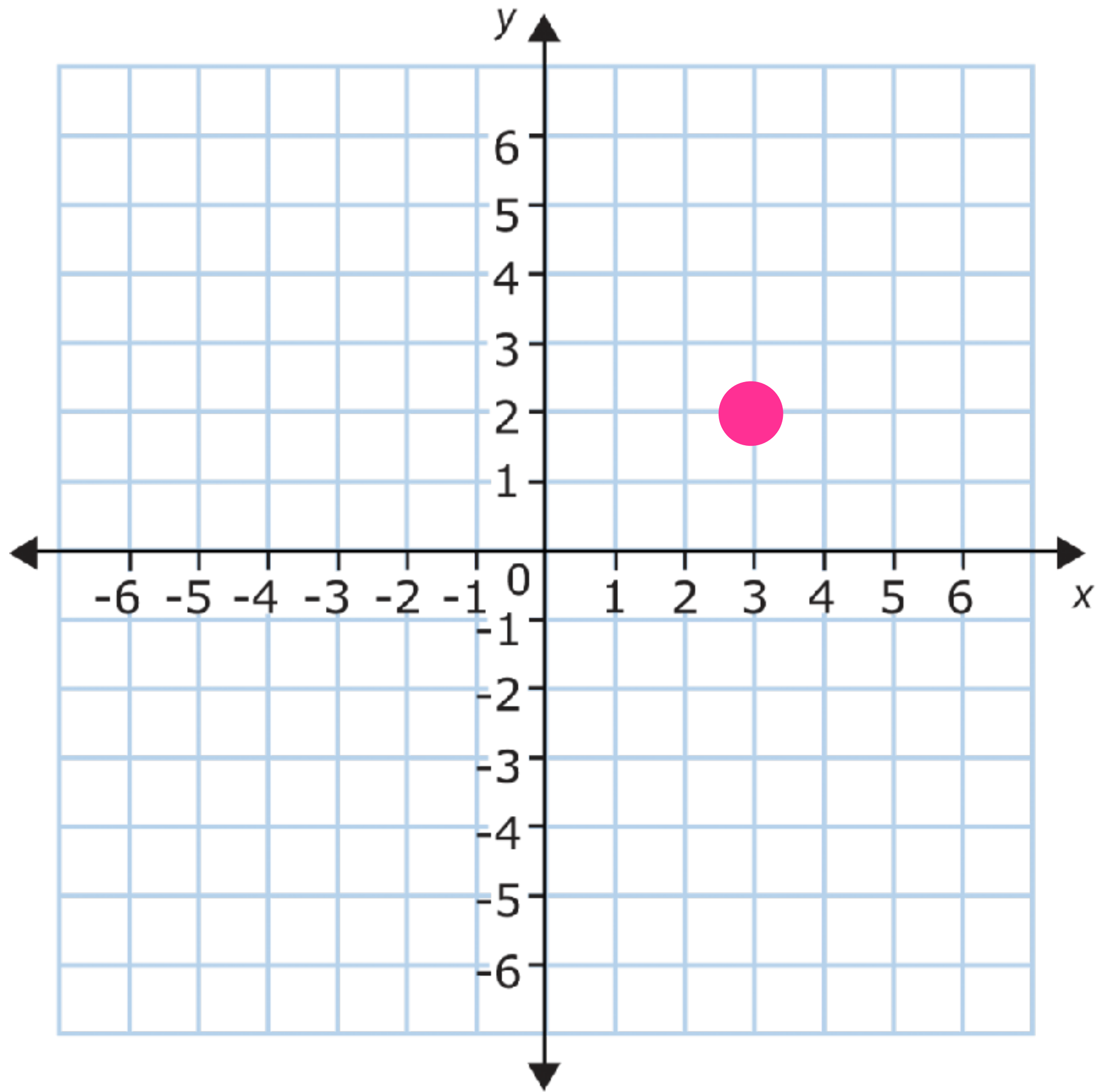
Transformation matrices.

Transformation matrix.

$$\mathcal{T} = \begin{pmatrix} \begin{matrix} \textcolor{red}{e_x} & \textcolor{green}{e_y} & \textcolor{blue}{e_z} & t \\ | & | & | & | \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 1 \end{matrix} \end{pmatrix}$$

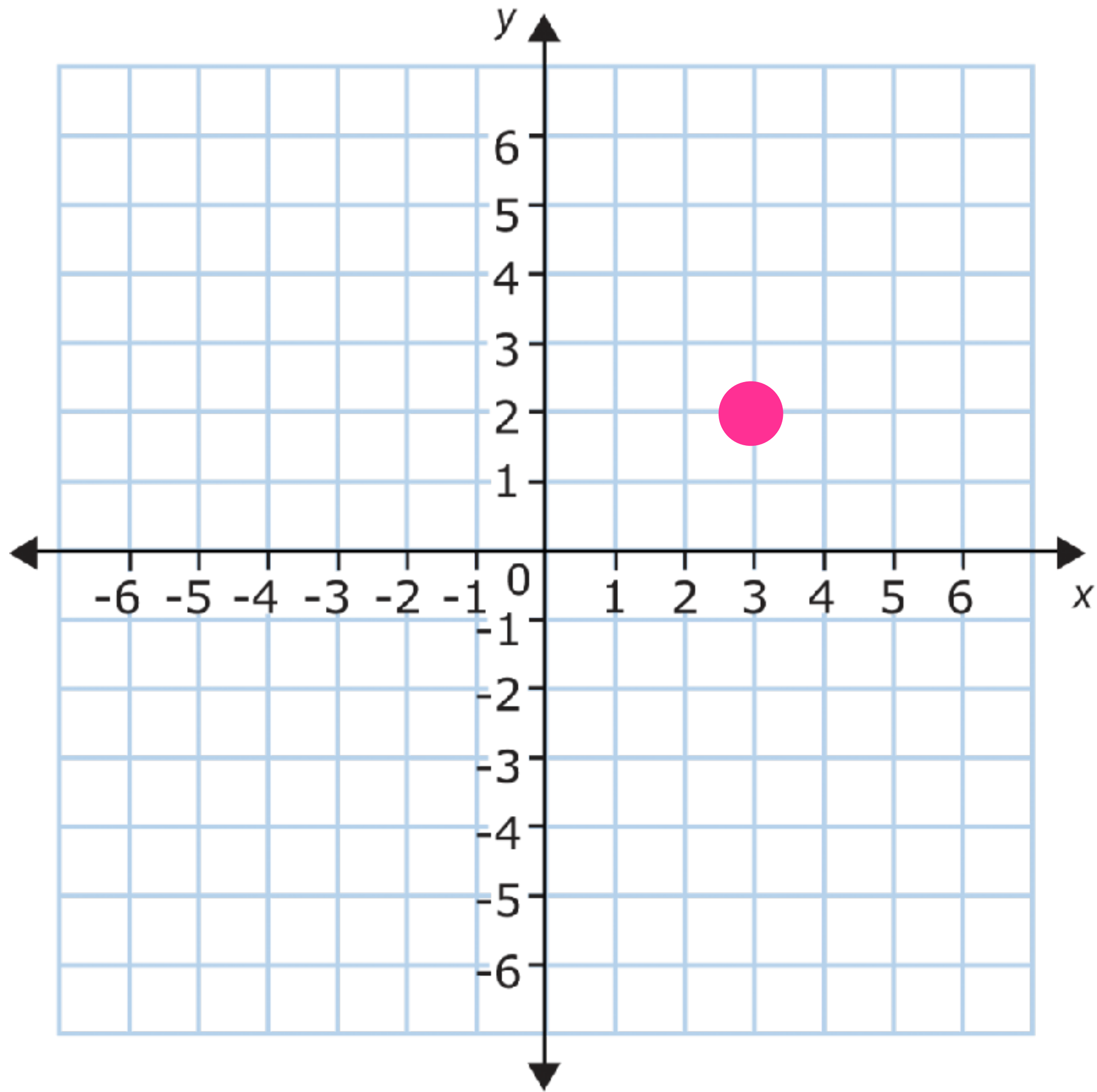


Linear transformations stored
as matrices.



$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Example: scale



$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Scale

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Scale

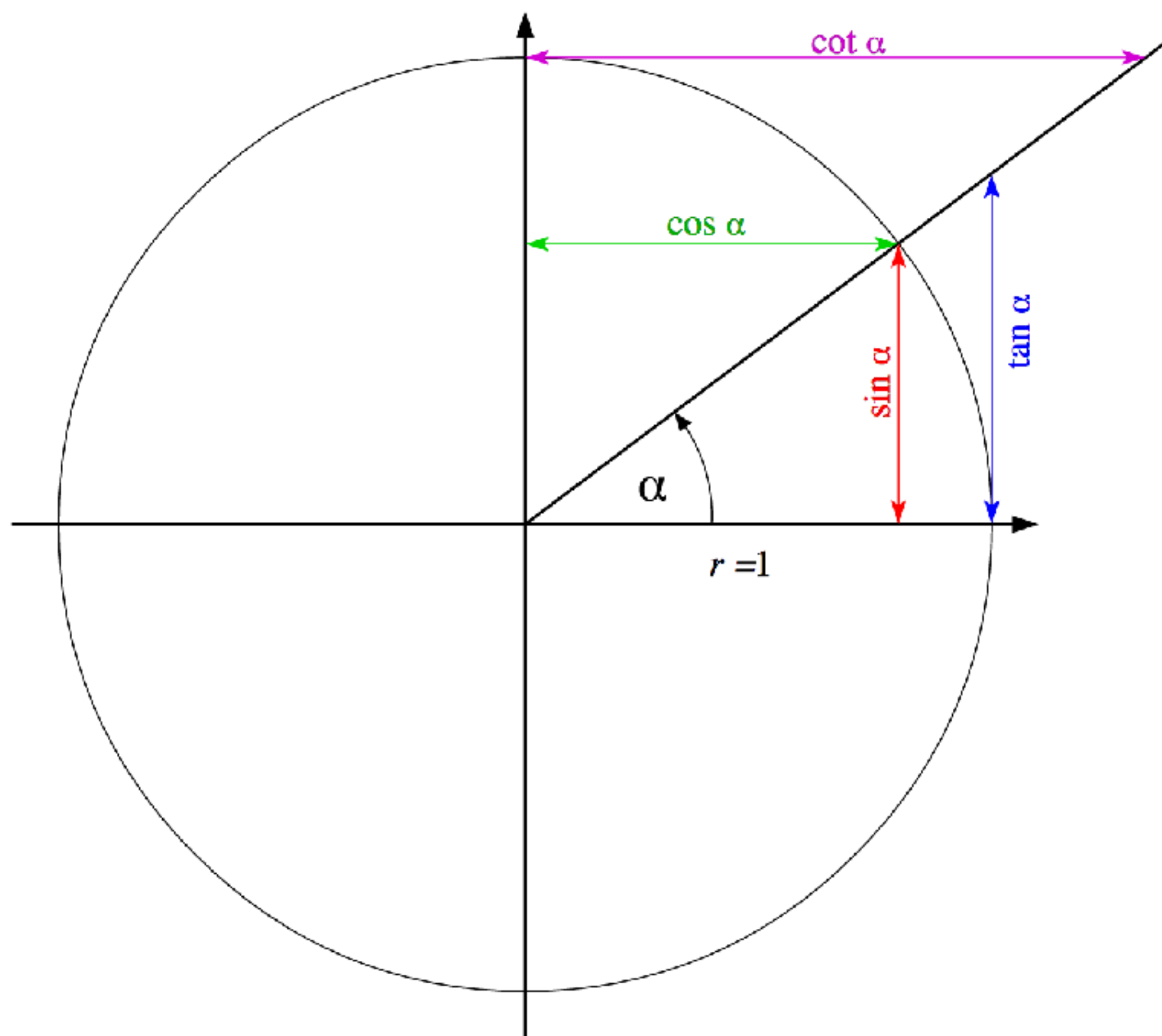
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} AX + BY \\ CX + DY \end{bmatrix}$$

$$\begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} sxX + 0Y \\ 0X + syY \end{bmatrix}$$

Rotation

Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta X + -\sin\theta Y \\ \sin\theta X + \cos\theta Y \end{bmatrix}$$



$$\begin{bmatrix} \cos \theta X + -\sin \theta Y \\ \sin \theta X + \cos \theta Y \end{bmatrix}$$

Affine transformations.

Homogeneous coordinates.

ABCD is the linear part of the affine transformation matrix.

$$\begin{bmatrix} A & B & ? \\ C & D & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Translate

Translate

$$\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1X + 0Y + 1T_x \\ 0X + 1Y + 1T_y \\ 0X + 0Y + 1 \times 1 \end{bmatrix}$$

Identity

Identity

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1X + 0Y + 1 \times 0 \\ 0X + 1Y + 1 \times 0 \\ 0X + 0Y + 1 \times 1 \end{bmatrix}$$

Multiplying affine transformation matrices.

```
matrix.identity();  
matrix.Translate(5.0, 4.0);  
matrix.Scale(2.0, 4.0);
```

```
// draw vertex at 3,2
```

```
matrix.identity();
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

```
matrix.identity(); matrix.Translate(5.0, 4.0);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 1 \end{bmatrix}$$

```
matrix.identity();
```

```
matrix.Translate(5.0, 4.0);
```

```
matrix.Scale(2.0, 4.0);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ 12 \\ 1 \end{bmatrix}$$


```
matrix.identity();  
matrix.Scale(2.0, 4.0);  
matrix.Translate(5.0, 4.0);
```

```
// draw vertex at 3,2
```

```
matrix.identity();
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

```
matrix.identity();    matrix.Scale(2.0, 4.0);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 1 \end{bmatrix}$$

```
matrix.identity();
```

```
matrix.Scale(2.0f, 4.0f);
```

```
matrix.Translate(5.0f, 4.0f);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 10 \\ 0 & 4 & 16 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 24 \\ 1 \end{bmatrix}$$

Moving into 3D

3D identity matrix and 3D position in homogeneous coordinates.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

All transformations in 3D

<p>X-Rotation in 3D</p> $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	<p>Z-Rotation in 3D</p> $\begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	<p>Scale in 3D</p> $\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
<p>Y-Rotation in 3D</p> $\begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	<p>Translation in 3D</p> $\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$	

Projection matrices are the same.

`matrix.setOrthoProjection(l, r, b, t, n, f);`

$$\begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{(r+l)}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{(t+b)}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{(f+n)}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$