Homework

Brian Chuk

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1 Problem Set 1

1. $p \wedge r \wedge \neg q$
2. True; they are identical
3. It is not a tautology.
4. $w \rightarrow c$
5. It is satisfiable. True when p is false and q is true.
6. false
7. false
8. false
9. false
10. false
11. Not valid
12. Let R(x) be the predicate ”x has read the textbook,” and P(x) be the predicate ”x passed the exam”
    $\forall x(R(x) \rightarrow P(x))$
    $R(Ed) \rightarrow P(Ed)$
    $R(Ed)$
    $P(Ed)$
13. The propositions do not imply the conclusion
14. Existential generalization

15. Let $n = 2a + 1$ and $n^2 = 2b$. So, $4a^2 + 4a + 1 = 2b$. Therefore, $4a^2 + 4a - 2b = -1$. This is a contradiction because there is an even = odd. Thus, n^2 is odd.

16. Let $x = 2a + 1$. Thus $x + 2 = 2a + 3$, which is odd.

17. Let $x + 2 = 2k$. Therefore $x = 2k - 2 = 2(k - 1)$, which is even.

18. Suppose x is odd but $x + 2$ is even. Thus $x = 2a + 1$ and $x + 2 = 2l$. Therefore $x + 2 = 2a + 3 = 2l$. Since $2a - 2l = -3$, there is a contradiction because even = odd.

19. Let $m = 2a$ and $n = 2b$. Therefore $mn = 4ab$, which is a multiple of 4.

20. False when $x = -2$ and $y = 0.5$

21. Let $n = 2a$ and $n^3 + 1 = 2b + 1$. Therefore $n^3 + 1 = 8a^3 + 1 = 2b + 1$. $8a^3 - 2b = 0$. Since it is even = even, if n is even then $n^3 + 1$ is odd. Now, since $n^3 + 1 = 2b + 1$ then $n = \sqrt[3]{2b} = 2a$. The cube root of an even number is always even, therefore this is an even = even equation. Thus, if $n^3 + 1$ is odd then n is even. Thus, the statements n is even and $n^3 + 1$ is odd are equivalent.

    Let $n = 2a$ and $n^3 - 1 = 2b + 1$. Therefore $n^3 + 1 = 8a^3 + 1 = 2b + 1$. As a result, $8a^3 - 2b = 0$. Since this is even = even, if n is even then $n^3 - 1$ is odd. Now, since $n^3 - 1 = 2b + 1$ then $n = \sqrt[3]{2b + 2} = 2a$. $n = \sqrt[3]{2(b + 1)}$ is an even = even equation, therefore if $n^3 - 1$ is odd then n is even.

    Thus the statements n is even and $n^3 - 1$ are equivalent. In conclusion, the three statements are equivalent.

22. If three people were born in each of the months of the year at most, there would be 36 people. Thus, those 4 remaining people must have birthdays in months were at least three others have birthdays. In conclusion, at least four people were born in the same month of the year out of any 40 people.

23. We can reduce the possible values of x and y into a few cases because $2x^2 > 14$ when $|x| \geq 3$ and $y^2 > 14$ when $|y| \geq 4$. This leaves the cases when x equals $-2, -1, 0, 1,$ or 2 and y equals $-3, -2, -1, 0, 1, 2,$ or 3. Since we are only testing for positive integer solutions we can remove the negative integers from the cases we have to test. As a result, none

of the six remaining cases are solutions to the equation. In conclusion, the equation $2x^2 + y^2 = 14$ has no integer solutions.

2 Problem Set 2

1. $\overline{A \bigcap B}$
   $= \{x | x \notin A \bigcap B\}$
   $= \{x | \neg(x \in A \bigcap B)\}$
   $= \{x | \neg(x \in A \bigwedge x \in B)\}$
   $= \{x | \neg(x \notin A \bigvee x \notin B)\}$
   $= \{x | \neg(x \in \overline{A} \bigvee x \in \overline{B})\}$
   $= \{x | \neg(x \in \overline{A} \bigcup \overline{B})\}$
   $\overline{A} \bigcup \overline{B}$

2. true

3. true

4. false

5. 1

6. false

7. true

8. 0

9. uncountable

10. Both (0,1) and (0,2) are uncountably infinite so they have the same cardinality.

11. f is 1-1

12. yes

13. f is not 1-1

14. yes

15. f is not 1-1

16. yes

17. g is 1-1 and onto R

18. {2,3,8}

19. {(8,1),(3,2),(2,3)}

20. yes

21. no

22. $a_n = 3^n + 1 = a_n = 3^n + 1^n$
$(s-3)(s-1) = 0$
$s - 4s + 3 = 0$
$s^2 - As - B = 0$
$A = 4, B = -3$
$a_n = Aa_{n-1} + Ba_{n-2}$
$a_n = 4a_{n-1} - 3a_{n-2}$

23. $a)$ $25000(1 + 0.03)^n$
$b)$ $25000(1 + 0.05)^n$
$c)$ $(25000 + 1000n)(1 + 0.02)^n$