This question paper contains 7 printed pages.

Your Roll No. 71

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(Write your Roll No. on the top immediately on receipt of this question paper.)

Section A is compulsory. Attempt any ***four*** *questions from Section B. Parts of a question must be answered together. Use of Simple Calculator is allowed. The symbols have their usual meanings.*

SECTION A

1. (i) What do you mean by convex set? Define an extreme point of a convex region. Show that the set $S = \{(x_1, x_2) \mid x_1 + x_2 \leq 1, x_1, x_2 \geq 0\}$ is convex. 5

(ii) Consider the following LP with two variables :

Maximize $z = 2x_1 + 4x_2$

Subject to

$$x_1 + 3x_2 \leq 9$$

P.T.O.

$3x_1 + 2x_2 \le 18$

$x_1, x_2 \ge 0.$

Determine all the basic solutions of the problem, and classify them as feasible and infeasible. Verify optimal solution graphically. Show how infeasible basic solutions are represented on graphical solution space. 5

(iii) Write the dual of the following LPP :

Maximize $z = 5x_1 + 6x_2$

Subject to $x_1 + 2x_2 = 5$

$-x_1 + 5x_2 \ge 3$

$4x_1 + 7x_2 \le 8$

x_1 unrestricted, $x_2 \ge 0.$ 5

(iv) A client needs 4 different servers to do 4 different tasks. Given table summarizes the time to process j^{th} job on i^{th} processor. Determine an assignment to minimize the total processing time using Hungarian Method. 5

Tasks

		1	2	3	4
Servers	1	\$1	\$4	\$6	\$3
	2	\$9	\$7	\$10	\$9
	3	\$4	\$5	\$11	\$7
	4	\$8	\$7	\$8	\$5

(v) Let $\{X_n, n \ge 0\}$ be a Markov chain having state space $S = \{1, 2, 3, 4\}$ and transition probability matrix as given below. Prove that the states 1 and 2 are ergodic. 6

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix} \end{array}$$

(vi) Define basis and dimension of a vector space. Show that vectors (1, 1) and (–3, 2) form a basis of R^2. 4

(vii) For what values of k and l will the given system of linear equations have NO solution?

$x + y + z = 6$

$x + 4y + 6z = 20$

$x + 4y + kz = l.$ 5

SECTION B

2. Solve the following LPP using two-phase method :

Maximize $z = 4x_1 + x_2$

Subject to :

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \ge 6$

$x_1 + 2x_2 \le 4$

$x_1, x_2 \ge 0.$ 10

3. (a) John must work at least 20 hours a week to supplement his income while attending school. He has the opportunity to work in two retail stores. In store 1, he can work between 4.5 and 12 hours a week, and in store 2, he is allowed between 5.5 and 10 hours. Both stores pay the same hourly

wage. In deciding how many hours to work in each store, John wants to base his decision on work stress. Based on interviews with present employees, John estimates that, on an ascending scale of 1 to 10, the stress factors are 8 and 6 at stores 1 and 2 respectively. Because stress mounts by the hour, he assumes that the total stress for each store at the end of the week is proportional to the number of hours he works in the store. Formulate the above as a Linear Programming Problem. 5

(b) Consider the following LPP:

maximize $z = x_1$

subject to,

$$5x_1 + x_2 = 4$$
$$6x_1 + x_3 = 4$$
$$3x_1 + x_4 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Solve the problem by inspection (do not use the Gauss-Jordon row operations). 5

4. (a) An engineering professor acquires a new computer once every two years. The professor can choose from three models: MI, M2, and M3. If the present model is M1, the next computer can be M2 with the probability .2, or M3 with the probability .15. If the present model is M2, the probabilities of switching to MI and M3 are .6 and .25, respectively. And, if the present model is M3, then the

probabilities of purchasing M1 and M2 are .5 and .1, respectively. Represent the situation as a Markov chain. And determine the probability that the professor will purchase the current model in 2 years. 5

(b) Classify the states of the following Markov chain. If a state is periodic, determine its period.

$$\text{States} \quad \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.1 & 0 & 0.9 \\ 0.7 & 0.3 & 0 \\ 0.2 & 0.7 & 0.1 \end{pmatrix} \end{array}$$ 5

5. (a) Solve the following Linear Programming Problem using simplex algorithm and comment on the nature of the solution :

Maximize $z = 2x_1 + 4x_2$

Subject to

$$x_1 + 2x_2 \leq 5$$
$$x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$ 5

(b) B & K Groceries operates with three checkout counters. The manager uses the following schedule to determine the number of counters in operation, depending on the number of customers in line :

Number of customers in store	*Number of counters in operation*
1 to 3	1
4 to 6	2
More than 6	3

Customers arrive in the counters area according to a Poisson distribution with a mean rate of 10 customers per hour. The average checkout time per customer is exponential with a mean 12 minutes. Determine the steady-state probability p_n of n customers in the checkout area. 5

6. Consider the following LP model :

Minimize $z = 2x_1 + x_2$

Subject to

$$3x_1 + 5x_2 - x_3 = 3$$
$$4x_1 + 3x_2 - x_4 = 6$$
$$x_1 + 2x_2 + x_5 = 3$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Compute the entire simplex tableau associated with the following basic solution, and check it for optimality and feasibility :

Basic variables = (x_1, x_2, x_5),

Inverse $\begin{pmatrix} 3/5 & -1/5 & 0 \\ -4/5 & 3/5 & 0 \\ 1 & -1 & 1 \end{pmatrix}$ 10

7. (a) In the unbalanced transportation problem given below, if a unit of source is not shipped out (to any of the destinations), a storage cost is incurred at the rate of \$5, \$4, and \$3 per unit for sources 1, 2 and 3, respectively. Additionally, all the supply at source 2 must be shipped out completely to make room for a new product. Use Vogel's starting solution,

and determine all the iterations leading to the optimum shipping schedule. 5

	D1	D2	D3	Supply
S1	\$1	\$2	\$1	20
S2	\$3	\$4	\$5	40
S3	\$2	\$3	\$3	30
Demand	30	20	20	

90

10

(b) A barber shop serves one customer at a time and provides three seats for waiting customers. If the place is full, customers go elsewhere. Arrivals occur according to a Poisson distribution with mean 4 per hour. The time to get a haircut is exponential with mean 15 minutes. Determine the steady state probabilities and expected number of customers in the shop. 5