Exercise Sheet 1

Due: 1.11.2023, 10:00

Download the files **data1-1.csv**, **data1-2.csv** and **data1-3.csv** from ISIS.

Exercise 1.1 (1P. for each part, a and b)

Denote by $< u, v >:= u^\top v$, with $u, v \in \mathbb{R}^n$, the standard Euclidean inner product. The gradient $\nabla f(x)$ of a differentiable function $f: \mathbb{R}^n \to \mathbb{R}$ at point $x \in \mathbb{R}^n$ is the unique vector satisfying

$$< \nabla f(x), v > = \lim_{t \to 0} \frac{f(x + tv) - f(x)}{t} \quad \forall\, v \in \mathbb{R}^n.$$

a) Compute the gradient of $f(x) = x^\top x$ using the above property.
b) Let $A \in \mathbb{R}^{m,n}$ and $b \in \mathbb{R}^m$. Compute the gradient of $f(x) = (Ax - b)^\top (Ax - b)$ using the above property.

Exercise 1.2

The files data1-1.csv, data1-2.csv and data1-3.csv contain (possibly noisy, incomplete and erroneous) observations (x_i, y_i). For each of the three datasets, find a parametrized model f_θ that may have generated the data, i.e. a model f_θ that reflects the functional relationship $y_i \approx f_\theta(x_i)$ for all i. The model parameters θ should be unspecified variables here. Their values are specified in Exercise 1.3. What could be the distribution for the x-values in each dataset (consider a histogram plot, see `matplotlib.pyplot.hist`).

Exercise 1.3 ***(Demonstrate your hacking skills)***

Find (somehow) suitable model parameters θ for each of the models found in Exercise 1.2. For each model, plot the dataset (using a scatter plot, see `matplotlib.pyplot.scatter`) and the function f_θ with specified model parameters θ you found.

Hint: We will introduce a principled method for estimating model parameters later in this course. Just be creative, your method need not have any theoretical justification, but the result should look meaningful.