Chapter 15

Statistics

Statistics:

It is defined as the process of collection and classification of data, interpretation and presentation of the data, and analysis of data. Statistics also is defined as drawing conclusions from the sample data that is collected using experiments. Statistics is applied in various fields such as sociology, psychology, geology, probability, and so on.

The chapter begins by discussing the concept of data, which can be classified as primary or secondary. Students learn about the presentation of data through various graphical representations, including histograms, bar graphs, and frequency polygons. Measures of central tendency, such as mean, median, and mode, are explored as ways to summarize and describe a dataset.

Statistics Measures of Dispersion:

The observations data and central tendency type measure are used to calculate the dispersion in the data.

The measures of dispersion are represented by several categories. They are the following: standard deviation, quartile deviation, mean deviation, and range.

In statistics, range refers to the difference between a particular data set's maximum and minimum values.

Range is calculated as Maximum Value - Minimum Value.

Statistics Mean Deviation:

The difference between a data point's observed and expected values is known as the "mean deviation."

The two crucial statistics measurements are variance and standard deviation. While the standard deviation measures the distribution of statistical data, variance measures how data values deviate from the mean. Different units are used to measure the variance and the standard deviation.

Analysis of Frequency Distributions:

A frequency distribution is a graphical representation of how frequently an item appears in a data set.

Exercise 15.1

Find the mean deviation about the mean for the data in Exercises 1 and 2.

1. 4, 7, 8, 9, 10, 12, 13, 17

Solution:-

First, we have to find ($\bar{x}$) of the given data.

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = \frac{80}{8} = 10$$

So, the respective values of the deviations from mean,

i.e., $x_i - \bar{x}$ are, 10 – 4 = 6, 10 – 7 = 3, 10 – 8 = 2, 10 – 9 = 1, 10 – 10 = 0,

10 – 12 = – 2, 10 – 13 = – 3, 10 – 17 = – 7

6, 3, 2, 1, 0, -2, -3, -7

Now, absolute values of the deviations,

6, 3, 2, 1, 0, 2, 3, 7

$\therefore \sum_{i=1}^{8} |x_i - \bar{x}| = 24$

MD = sum of deviations/ number of observations

= 24/8

= 3

So, the mean deviation for the given data is 3.

2. 38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution:-

First, we have to find ($\bar{x}$) of the given data.

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = \frac{500}{10} = 50$$

So, the respective values of the deviations from mean,

i.e., $x_i - \bar{x}$ are, 50 – 38 = -12, 50 -70 = -20, 50 – 48 = 2, 50 – 40 = 10, 50 – 42 = 8,

50 – 55 = – 5, 50 – 63 = – 13, 50 – 46 = 4, 50 – 54 = -4, 50 – 44 = 6

-12, 20, -2, -10, -8, 5, 13, -4, 4, -6

Now, absolute values of the deviations,

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

$\therefore \sum_{i=1}^{10} |x_i - \bar{x}| = 84$

MD = sum of deviations/ number of observations

= 84/10

= 8.4

So, the mean deviation for the given data is 8.4.

Find the mean deviation about the median for the data in Exercises 3 and 4.

3. 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solution:-

First, we have to arrange the given observations into ascending order.

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18.

The number of observations is 12.

Then,

Median = ((12/2)th observation + ((12/2)+ 1)th observation)/2

(12/2)th observation = 6th = 13

(12/2)+ 1)th observation = 6 + 1

= 7th = 14

Median = (13 + 14)/2

= 27/2

= 13.5

So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are

3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

$\therefore \sum_{i=1}^{12} |x_i - M| = 28$

Mean Deviation

$$M.D.(M) = \frac{1}{12}\sum_{i=1}^{12} |x_i - M|$$

= (1/12) × 28

= 2.33

So, the mean deviation about the median for the given data is 2.33.

4. 36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Solution:-

First, we have to arrange the given observations into ascending order.

36, 42, 45, 46, 46, 49, 51, 53, 60, 72.

The number of observations is 10.

Then,

Median = ((10/2)th observation + ((10/2)+ 1)th observation)/2

(10/2)th observation = 5th = 46

(10/2)+ 1)th observation = 5 + 1

= 6th = 49

Median = (46 + 49)/2

= 95

= 47.5

So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are

11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

$$\therefore \sum_{i=1}^{10} |x_i - M| = 70$$

Mean Deviation

$$\mathrm{M.D.}(\mathrm{M}) = \frac{1}{10}\sum_{i=1}^{10} |x_i - M|$$

= (1/10) × 70

= 7

So, the mean deviation about the median for the given data is 7.

Find the mean deviation about the mean for the data in Exercises 5 and 6.

5.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

The sum of calculated data,

$$N = \sum_{i=1}^{5} f_i = 25, \sum_{i=1}^{5} f_i x_i = 350$$

Now, we have to find ($\bar{x}$) by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N} \sum_{i=1}^{5} f_i x_i = \frac{1}{25} \times 350 = 14$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

From the table, $\sum_{i=1}^{5} f_i |x_i - \bar{x}| = 158$

Therefore $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i |x_i - \bar{x}|$

= (1/25) × 158

= 6.32

So, the mean deviation about the mean for the given data is 6.32.

6.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

The sum of calculated data,

$$N = \sum_{i=1}^{5} f_i = 80, \sum_{i=1}^{5} f_i x_i = 4000$$

Now, we have to find $(\overline{x})$ by using the formula

$$\Rightarrow \overline{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{80} \times 4000 = 50$$

The absolute values of the deviations from the mean, i.e., $|x_i - \overline{x}|$, as shown in the table.

From the table, $\sum_{i=1}^{5} f_i |x_i - \overline{x}| = 1280$

Therefore $\mathrm{M.D.}(\overline{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i |x_i - \overline{x}|$

$= (1/80) \times 1280$

$= 16$

So, the mean deviation about the mean for the given data is 16.

Find the mean deviation about the median for the data in Exercises 7 and8.

7.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	c.f.	$\|x_i – M\|$	$f_i \|x_i – M\|$
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48

Now, N = 26, which is even.

Median is the mean of the 13th and 14th observations. Both of these observations lie in the cumulative frequency of 14, for which the corresponding observation is 7.

Then,

Median = (13th observation + 14th observation)/2

= (7 + 7)/2

= 14/2

= 7

So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are shown in the table.

Therefore $\sum_{i=1}^{6} f_i = 26$ and $\sum_{i=1}^{6} f_i|x_i - M| = 84$

And $\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - M|$

$= (1/26) \times 84$

$= 3.23$

Hence, the mean deviation about the median for the given data is 3.23.

8.

x_i	**15**	**21**	**27**	**30**	**35**
f_i	**3**	**5**	**6**	**7**	**8**

Solution:-

X_i	**f_i**	**c.f.**	**$\|x_i - M\|$**	**$f_i \|x_i - M\|$**
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40

Let us make the table of the given data and append other columns after calculations.

Now, N = 29, which is odd.

So, 29/2 = 14.5

The cumulative frequency greater than 14.5 is 21, for which the corresponding observation is 30.

Then,

Median = (15th observation + 16th observation)/2

= (30 + 30)/2

= 60/2

= 30

So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are shown in the table.

Therefore $\sum_{i=1}^{5} f_i = 29$ and $\sum_{i=1}^{5} f_i |x_i - M| = 148$

And $\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - M|$

$= (1/29) \times 148$

$= 5.1$

Hence, the mean deviation about the median for the given data is 5.1.

Find the mean deviation about the mean for the data in Exercises 9 and 10.

9.

Income per day in ₹	**0 – 100**	**100 – 200**	**200 – 300**	**300 – 400**	**400 – 500**	**500 – 600**	**600 – 700**	**700 – 800**
Number of persons	**4**	**8**	**9**	**10**	**7**	**5**	**4**	**3**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Income per day in ₹	**Number of persons f_i**	**Midpoints x_i**	**f_ix_i**	**$\lvert x_i - \bar{x}\rvert$**	**$f_i\lvert x_i - \bar{x}\rvert$**
0 – 100	4	50	200	308	1232
100 – 200	8	150	1200	208	1664
200 – 300	9	250	2250	108	972

300 – 400	10	350	3500	8	80
400 – 500	7	450	3150	92	644
500 – 600	5	550	2750	192	960
600 – 700	4	650	2600	292	1160
700 – 800	3	750	2250	392	1176
	50		17900		7896

The sum of calculated data,

$$N = \sum_{i=1}^{8} f_i = 50, \sum_{i=1}^{8} f_i x_i = 17900$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{8} f_i x_i = \frac{1}{50} \times 17900 = 358$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

So, $\sum_{i=1}^{8} f_i |x_i - \bar{x}| = 7896$

And $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{8} f_i |x_i - \bar{x}|$

$= (1/50) \times 7896$

$= 157.92$

Hence, the mean deviation about the mean for the given data is 157.92.

10.

Height in cms	95 – 105	105 – 115	115 – 125	125 – 135	135 – 145	145 – 155
Number of boys	**9**	**13**	**26**	**30**	**12**	**10**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Height in cms	Number of boys f_i	Midpoints x_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
95 – 105	9	100	900	25.3	227.7
105 – 115	13	110	1430	15.3	198.9
115 – 125	26	120	3120	5.3	137.8
125 – 135	30	130	3900	4.7	141
135 – 145	12	140	1680	14.7	176.4
145 – 155	10	150	1500	24.7	247
	100		12530		1128.8

The sum of calculated data,

$$N = \sum_{i=1}^{6} f_i = 100, \sum_{i=1}^{6} f_i x_i = 12530$$

Now, we have to find ($\bar{x}$) by using the formula

$$\Rightarrow \ \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i = \frac{1}{100} \times 12530 = 125.3$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

So $\sum_{i=1}^{6} f_i |x_i - \bar{x}| = 1128.8$

And $\text{M.D.}\,(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \bar{x}|$

= (1/100) ×1128.8

= 11.28

Hence, the mean deviation about the mean for the given data is 11.28.

11. Find the mean deviation about median for the following data.

Marks	0 -10	10 -20	20 – 30	30 – 40	40 – 50	50 – 60
Number of girls	**6**	**8**	**14**	**16**	**4**	**2**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Marks	Number of girls f_i	Cumulative frequency (c.f.)	Mid - points x_i	$\lvert x_i - \text{Med}\rvert$	$f_i\lvert x_i - \text{Med}\rvert$
0 – 10	6	6	5	22.85	137.1
10 – 20	8	14	15	12.85	102.8
20 – 30	14	28	25	2.85	39.9
30 – 40	16	44	35	7.15	114.4
40 – 50	4	48	45	17.15	68.6
50 – 60	2	50	55	27.15	54.3
	50				517.1

The class interval containing $N^{th}/2$ or 25^{th} item is 20-30.

So, 20-30 is the median class.

Then,

Median = l + (((N/2) – c)/f) × h

Where, l = 20, c = 14, f = 14, h = 10 and n = 50

Median = 20 + (((25 – 14))/14) × 10

= 20 + 7.85

= 27.85

The absolute values of the deviations from the median, i.e., $|x_i - \text{Med}|$, as shown in the table.

So $\sum_{i=1}^{6} f_i |x_i - \text{Med.}| = 517.1$

And $\text{M.D.}(\text{M}) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \text{Med.}|$

$= (1/50) \times 517.1$

$= 10.34$

Hence, the mean deviation about the median for the given data is 10.34.

12. Calculate the mean deviation about median age for the age distribution of 100 persons given below.

Age (in years)	**16 – 20**	**21 – 25**	**26 – 30**	**31 – 35**	**36 – 40**	**41 – 45**	**46 – 50**	**51 – 55**
Number	**5**	**6**	**12**	**14**	**26**	**12**	**16**	**9**

[Hint: Convert the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval]

Solution:-

The given data is converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding the 0.5 to the upper limit of each class intervals and append other columns after calculations.

Age	**Number f_i**	**Cumulative frequency (c.f.)**	**Midpoints x_i**	**$\lvert x_i$ – Med$\rvert$**	**$f_i\lvert x_i$ – Med$\rvert$**
15.5 – 20.5	5	5	18	20	100
20.5 – 25.5	6	11	23	15	90
25.5 – 30.5	12	23	28	10	120
30.5 – 35.5	14	37	33	5	70
35.5 – 40.5	26	63	38	0	0
40.5 – 45.5	12	75	43	5	60
45.5 – 50.5	16	91	48	10	160
50.5 – 55.5	9	100	53	15	135
	100				735

The class interval containing $N^{th}/2$ or 50^{th} item is 35.5 – 40.5

So, 35.5 – 40.5 is the median class.

Then,

Median = l + (((N/2) – c)/f) × h

Where, l = 35.5, c = 37, f = 26, h = 5 and N = 100

Median = 35.5 + (((50 – 37))/26) × 5

= 35.5 + 2.5

= 38

The absolute values of the deviations from the median, i.e., $|x_i - \text{Med}|$, as shown in the table.

So $\sum_{i=1}^{8} f_i |x_i - \text{Med.}| = 735$

And $\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \text{Med.}|$

= (1/100) × 735

= 7.35

Hence, the mean deviation about the median for the given data is 7.35.

Exercise 15.2

Find the mean and variance for each of the data in Exercise 1 to 5.

1. 6, 7, 10, 12, 13, 4, 8, 12

Solution:-

We have,

Mean = $\bar{x} = \frac{\sum_{i=1}^{a} x_i}{n}$

Where, n = number of observation

$\sum_{i=1}^{a} x_i$ = sum of total observation

So, $\bar{x}$ = (6 + 7 + 10 + 12 + 13 + 4 + 8 + 12)/8

X_i	Deviations from mean $(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	6 – 9 = -3	9
7	7 – 9 = -2	4
10	10 – 9 = 1	1
12	12 – 9 = 3	9
13	13 – 9 = 4	16
4	4 – 9 = – 5	25
8	8 – 9 = – 1	1
12	12 – 9 = 3	9
		74

= 72/8

= 9

Let us make the table of the given data and append other columns after calculations.

We know that the Variance,

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \bar{x})^2$$

$\sigma^2 = (1/8) \times 74$

= 9.2

∴Mean = 9 and Variance = 9.25

2. First n natural numbers

Solution:-

We know that Mean = Sum of all observations/Number of observations

∴Mean, $\bar{x}$ = ((n(n + 1))2)/n

= (n + 1)/2

and also, WKT Variance,

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \bar{x})^2$

By substituting the value of $\bar{x}$, we get

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \frac{n+1}{2}\right)^2$$

We know that $(a - b)^2 = a^2 - 2ab + b^2$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i)^2 - \frac{1}{n}\sum_{i=1}^{n}2x_i\left(\frac{n+1}{2}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

Substituting the summation values

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \frac{n+1}{n}\left[\frac{n(n+1)}{2}\right] + \frac{(n+1)^2}{4n} \times n$$

Multiplying and Computing

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

By taking LCM and simplifying, we get

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

By taking $(n + 1)$ common from each term, we get

$$= (n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

WKT, $(a + b)(a - b) = a^2 - b^2$

$\sigma^2 = (n^2 - 1)/12$

$\therefore$Mean $= (n + 1)/2$ and Variance $= (n^2 - 1)/12$

3. First 10 multiples of 3

Solution:-

First, we have to write the first 10 multiples of 3.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

We have,

Mean = $\overline{x} = \frac{\sum_{i=1}^{a} x_i}{n}$

Where, n = number of observation

$\sum_{i=1}^{a} x_i$ = sum of total observation

So, $\overline{x}$ = (3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30)/10

= 165/10

= 16.5

Let us make the table of the data and append other columns after calculations.

X_i	**Deviations from mean** $(x_i - \overline{x})$	$(x_i - \overline{x})^2$
3	3 – 16.5 = -13.5	182.25
6	6 – 16.5 = -10.5	110.25
9	9 – 16.5 = -7.5	56.25
12	12 – 16.5 = -4.5	20.25
15	15 – 16.5 = -1.5	2.25
18	18 – 16.5 = 1.5	2.25
21	21 – 16.5 = – 4.5	20.25
24	24 – 16.5 = 7.5	56.25
27	27 – 16.5 = 10.5	110.25
30	30 – 16.5 = 13.5	182.25
		742.5

Then, the Variance

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \overline{x})^2$

= (1/10) × 742.5

= 74.25

∴Mean = 16.5 and Variance = 74.25

4.

x_i	**6**	**10**	**14**	**18**	**24**	**28**	**30**
f_i	**2**	**4**	**7**	**12**	**8**	**4**	**3**

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	$f_i x_i$	Deviations from mean $(x_i - \overline{x})$	$(x_i - \overline{x})^2$	$f_i(x_i - \overline{x})^2$
6	2	12	6 – 19 = 13	169	338
10	4	40	10 – 19 = -9	81	324
14	7	98	14 – 19 = -5	25	175
18	12	216	18 – 19 = -1	1	12
24	8	192	24 – 19 = 5	25	200
28	4	112	28 – 19 = 9	81	324
30	3	90	30 – 19 = 11	121	363
	N = 40	760			1736

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 760/40

= 19

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/40) × 1736

= 43.4

∴Mean = 19 and Variance = 43.4

5.

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	$f_i x_i$	Deviations from mean $(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	92 – 100 = -8	64	192
93	2	186	93 – 100 = -7	49	98
97	3	291	97 – 100 = -3	9	27
98	2	196	98 – 100 = -2	4	8
102	6	612	102 – 100 = 2	4	24
104	3	312	104 – 100 = 4	16	48
109	3	327	109 – 100 = 9	81	243
	N = 22	2200			640

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 2200/22

= 100

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/22) × 640

= 29.09

∴Mean = 100 and Variance = 29.09

6. Find the mean and standard deviation using short-cut method.

x_i	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**
f_i	**2**	**1**	**12**	**29**	**25**	**12**	**10**	**4**	**5**

Solution:-

Let the assumed mean A = 64. Here, h = 1

We obtain the following table from the given data.

X_i	**Frequency f_i**	**$Y_i = (x_i – A)/h$**	**Y_i^2**	**f_iy_i**	**$f_iy_i^2$**
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12

66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
				0	286

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$$

Where A = 64, h = 1

So, $\bar{x}$ = 64 + ((0/100) × 1)

= 64 + 0

= 64

Then, the variance,

$$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$$

$\sigma^2 = (1^2/100^2)\ [100(286) - 0^2]$

= (1/10000) [28600 – 0]

= 28600/10000

= 2.86

Hence, standard deviation = σ = √2.886

= 1.691

∴ Mean = 64 and Standard Deviation = 1.691

Find the mean and variance for the following frequency distributions in Exercises 7 and 8.

7.

Classes	**0 – 30**	**30 – 60**	**60 – 90**	**90 – 120**	**120 – 150**	**150 – 180**	**180 – 210**
Frequencies	**2**	**3**	**5**	**10**	**3**	**5**	**2**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Classes	**Frequency f_i**	**Midpoints x_i**	**f_ix_i**	**$(x_i - \bar{x})$**	**$(x_i - \bar{x})^2$**	**$f_i(x_i - \bar{x})^2$**
0 – 30	2	15	30	-92	8464	16928
30 – 60	3	45	135	-62	3844	11532
60 – 90	5	75	375	-32	1024	5120
90 – 120	10	105	1050	-2	4	40
120 – 150	3	135	405	28	784	2352
150 – 180	5	165	825	58	3364	16820
180 – 210	2	195	390	88	7744	15488
	N = 30		3210			68280

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 3210/30

= 107

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/30) × 68280

= 2276

∴Mean = 107 and Variance = 2276

8.

Classes	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 –50**
Frequencies	**5**	**8**	**15**	**16**	**6**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Classes	Frequency f_i	Midpoints x_i	f_ix_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0 – 10	5	5	25	-22	484	2420
10 – 20	8	15	120	-12	144	1152
20 – 30	15	25	375	-2	4	60
30 – 40	16	35	560	8	64	1024
40 –50	6	45	270	18	324	1944
	N = 50		1350			6600

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 1350/50

= 27

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/50) × 6600

= 132

∴Mean = 27 and Variance = 132

9. Find the mean, variance and standard deviation using the short-cut method.

Height in cms	**70 – 75**	**75 – 80**	**80 – 85**	**85 – 90**	**90 – 95**	**95 – 100**	**100 – 105**	**105 – 110**	**110 – 115**
Frequencies	**3**	**4**	**7**	**7**	**15**	**9**	**6**	**6**	**3**

Solution:-

Let the assumed mean, A = 92.5 and h = 5

Let us make the table of the given data and append other columns after calculations.

Height (class)	**Number of children** **Frequency f_i**	**Midpoint X_i**	**$Y_i = (x_i – A)/h$**	**Y_i^2**	**f_iy_i**	**$f_iy_i^2$**
70 – 75	3	72.5	-4	16	-12	48
75 – 80	4	77.5	-3	9	-12	36
80 – 85	7	82.5	-2	4	-14	28
85 – 90	7	87.5	-1	1	-7	7
90 – 95	15	92.5	0	0	0	0
95 – 100	9	97.5	1	1	9	9
100 – 105	6	102.5	2	4	12	24
105 – 110	6	107.5	3	9	18	54
110 – 115	3	112.5	4	16	12	48
	N = 60				6	254

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$$

Where, A = 92.5, h = 5

So, $\bar{x}$ = 92.5 + ((6/60) × 5)

= 92.5 + ½

= 92.5 + 0.5

= 93

Then, the Variance,

$$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$$

$\sigma^2 = (5^2/60^2)\ [60(254) - 6^2]$

= (1/144) [15240 – 36]

= 15204/144

= 1267/12

= 105.583

Hence, standard deviation = σ = √105.583

= 10.275

∴ Mean = 93, variance = 105.583 and Standard Deviation = 10.275

10. The diameters of circles (in mm) drawn in a design are given below.

Diameters	**33 – 36**	**37 – 40**	**41 – 44**	**45 – 48**	**49 – 52**
No. of circles	**15**	**17**	**21**	**22**	**25**

Calculate the standard deviation and mean diameter of the circles.

[Hint: First, make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 – 48.5, 48.5 – 52.5 and then proceed.]

Solution:-

Let the assumed mean, A = 42.5 and h = 4

Height (class)	Number of children (Frequency f_i)	Midpoint X_i	$Y_i = (x_i - A)/h$	Y_i^2	f_iy_i	$f_iy_i^2$
32.5 – 36.5	15	34.5	-2	4	-30	60
36.5 – 40.5	17	38.5	-1	1	-17	17
40.5 – 44.5	21	42.5	0	0	0	0
44.5 – 48.5	22	46.5	1	1	22	22
48.5 – 52.5	25	50.5	2	4	50	100
	N = 100				25	199

Let us make the table of the given data and append other columns after calculations.

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$$

Where, A = 42.5, h = 4

So, $\bar{x}$ = 42.5 + (25/100) × 4

= 42.5 + 1

= 43.5

Then, the Variance,

$$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$$

$\sigma^2 = (4^2/100^2)[100(199) - 25^2]$

= (1/625) [19900 – 625]

= 19275/625

= 771/25

= 30.84

Hence, standard deviation = $\sigma = \sqrt{30.84}$

= 5.553

∴ Mean = 43.5, variance = 30.84 and Standard Deviation = 5.553.

2Marks Questions & Answers

1.In a test with a maximum marks 2525 , eleven students scored 3,9,5,3,12,10,17,4,7,19,213,9,5,3,12,10,17,4,7,19,21 marks. Respectively. Calculate the range.

Ans: The marks can be arranged in ascending order as

3,3,4,5,7,9,10,12,17,19,21.3,3,4,5,7,9,10,12,17,19,21.

Range = Maximum value – Minimum value

$=21-3$

$=18$

Therefore, the range is 18.

2. Coefficient of variation of two distributions is 70 and 75and their

Standard deviations are 28 and 27 respectively what are their arithmetic mean?

Ans: Given that,

Coefficient of variation of first distribution (C.V) = 70(C.V)= 70

Coefficient of variation of second distribution (C.V) = 75(C.V)= 75

Standard deviation = $\sigma 1=28$

For first distribution,

C.V = $\frac{\sigma 1}{\bar{x}1} \times 100$

Substitute the values,

$70 = \frac{28}{\overline{\overline{x1}}} \times 100$

$\overline{x} = \frac{28}{70} \times 100$

$\overline{x} = 40$

Similarly for second distribution,

C.V = $\frac{\sigma 2}{\text{x2}} \times 100$

Substitute the values,

$75 = \frac{\sigma 2}{\text{x2}} \times 100$

$\overline{x}2 = \frac{27}{75} \times 100$

$\overline{x}2 = 36$

Therefore, the arithmetic mean of the first and second distributions is 40 and 36 respectively.

3. Find the median for the following data.

x_i: 5,7,9,10,12,15

f_i: 8,6,2,2,2,6

Ans:

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6
$c \cdot f$	8	14	16	18	20	26

n=26. Median is the average of 13th and 14th item, both of which lie in the $c \cdot f$ 14

$\therefore x_i = 7$

Thus, Median $= \dfrac{13\text{th observation} + 14\text{th observation}}{2}$

$= \dfrac{7+7}{2}$

$=7$

Therefore, the median is 7.

4. Solve for xx **and** y, $3x + (2x - y)\,i = 6 - 3i$

Ans: $3x = 6$

$x = 2$

$2x - y = -3$

Substitute the values,

$2 \times 2 - y = -3$

$-y = -3 - 4$

$y = 7.$

5. Multiply $3 - 2i$ **by its conjugate.**

Ans: Let $z = 3 - 2i$

The conjugate is,

$\bar{z} = 3 + 2i$

$z\,\bar{z} = (3 - 2i)\,(3 + 2i)$

$= 9 + 6i - 6i - 4i^2$

$= 9 - 4\,(-1)$

$= 13.$

6. Find the multiplicative inverse $4-3i$.

Ans: Let $z=4-3i$

Then, $\bar{z} = 4+3i$

$|z|=\sqrt{16+9} = 5$

$z^{-1} = \frac{\bar{z}}{|z|^2}$

$= \frac{4+3i}{25}$

$= \frac{4}{25} + \frac{3}{25}i$

Therefore, the multiplicative inverse $4-3i$ is $\frac{4}{25} + \frac{3}{25}i$.

7. Write the real and imaginary part $1-2i^2$

Ans: Let $z=1-2i^2$

$=1-2(-1)$

$=1+2$

$=3$

$=3+0.i$

The real part is $\text{Re}(z) = 3$

The imaginary part is $\text{Im}(z) = 0$

8. If $1, w, w^2$ **are three cube root of unity, show that** $(1-w+w2)(1+w-w2)$

Ans: $(1-w+w^2)(1+w-w^2)$

$(1+w^2-w)(1+w-w^2)$

$= (-w-w)(-w^2-w^2)$ $\because [1+w=-w^2 \; 1+w^2=-w]$

$(-2w)(-2w^2)$

$4w^3 \; [w^3=1]$

4×1

$=4$

Hence if 1, w, w^2 are three cube root of unity then
$(1-w+w^2)(1+w-w^2) = 4$.

9. **Find the number of non zero integral solution of the equation** $|1-i|^x = 2^x$

Ans: $|1-i|^x = 2^x$

$\left(\sqrt{(1)^2 + (1)^2}\right)^x = 2^x$

$(\sqrt{2})^x = 2^x$

$(2)^{\frac{1}{2}x} = 2^x$

$\frac{1}{2}x = x$

$\frac{1}{2} = 1$

$=2$ which is false no value of x satisfies.

Multiple Choice Questions

Q.1: Range of a data is equal to:

(a) Range = Max Value – Min Value

(b) Range = Max Value + Min Value

(c) Range = (Max Value – Min Value)/2

(d) Range = (Max Value + Min Value)/2

Answer: (a) Range = Max Value – Min Value

Q.2: Relation between mean, median and mode is given by:

(a) Mode = 2 Median – 3 Mean

(b) Mode = 2 Median + 3 Mean

(c) Mode = 3 Median – 2 Mean

(d) Mode = 3 Median + 2 Mean

Answer: (c) Mode = 3 Median – 2 Mean

Q.3: If the variance of the data is 121, the standard deviation of the data is:

(a) 121

(b) 11

(c) 12

(d) 21

Answer: (b) 11

Explanation:

Given, variance of the data = 121

Now, the standard deviation of the data = $\sqrt{(121)}$

= 11.

Q.4: The geometric mean of series having mean = 25 and harmonic mean = 16 is:

(a) 16

(b) 20

(c) 25

(d) 30

Answer: (b) 20

Explanation:

The relationship between Arithmetic Mean (AM), Geometric Mean (GM) And Harmonic Mean (HM) is

$GM^2 = AM \times HM$

Given AM = 25

HM = 16

So $GM^2 = 25 \times 16$

$\Rightarrow GM = \sqrt{(25 \times 16)}$

$= 5 \times 4$

$= 20$

So, Geometric Mean = 20

Q.5: The coefficient of correlation r satisfies:

(a) $|r| \leq 1$

(b) $0 < r < 1$

(c) $|r| > 1$

(d) $-1 < r < 0$

Answer: (a) I r I ≤ 1

Q.6: The coefficient of variation is computed by:

(i) S.D/.Mean×100

(ii) S.D./Mean

(iii) Mean./S.D×100

(iv) Mean/S.D.

Answer: (ii) S.D./Mean

Q.7: Find the median of 36, 72, 46, 42, 60, 45, 53, 46, 51, 49.

A. 42

B. 45.5

C. 47.5

D. 45

Answer: C. 47.5

Explanation: First we have to arrange the given observations into ascending order,

36, 42, 45, 46, 46, 49, 51, 53, 60, 72.

The number of observations is 10

Then,

Median = ((10/2)th observation + ((10/2)+ 1)th observation)/2

(10/2)th observation = 5th = 46

(10/2)+ 1)th observation = 5 + 1

= 6th = 49

Median = (46 + 49)/2

= 95

= 47.5

Q.8: Find the mean of 6, 7, 10, 12, 13, 4, 8, 12.

A. 9

B. 10

C. 12

D. 13

Answer: A. 9

Explanation: Mean = Sum of observations ÷ Number of observations

= (6 + 7 + 10 + 12 + 13 + 4 + 8 + 12)/8

= 72/8 = 9

Q.9: If the variance is 625, what is the standard deviation?

A. 5

B. 15

C. 25

D. None of the above

Answer: C. 25

Explanation: Given,

Variance, $\sigma^2 = 625$

Standard deviation, $\sigma = \sqrt{625} = 25$

Q.10: If the mean of first n natural numbers is 5n/9, n =

(a) 5

(b) 4

(c) 9

(d) 10

Answer: (c) 9

Explanation:

Given, mean of first n natural number is 5n/9

⇒ (n+1)/2 = 5n/9

⇒ n + 1 = (5n×2)/9

⇒ n + 1 = 10n/9

⇒ 9(n + 1) = 10n

⇒ 9n + 9 = 10n

⇒ 10n – 9n = 9

⇒ n = 9

Summary

- Measures of dispersion Range, Quartile deviation, mean deviation, variance, standard deviation are measures of dispersion. Range = Maximum Value – Minimum Value
- Mean deviation for ungrouped data

$$M.D(\overline{x})=\frac{\sum |xi-\overline{x}|}{n}, \qquad M.D(\overline{x})=\frac{\sum |xi-M|}{n}$$

- Mean deviation for grouped data

$$M.D(\overline{x})=\frac{\sum fi\,|xi-\overline{x}|}{N},\quad M.D(\overline{x})=\frac{\sum fi\,|xi-M|}{N}\quad \text{where } \mathbf{N}=\sum fi$$

- Variance and standard deviation for ungrouped data

$$\sigma^2=\frac{1}{n}\sum(xi-\overline{x}\ )^2 \qquad\qquad \sigma=\sqrt{\frac{1}{n}}\sum(xi-\overline{x}\ )^2$$