CHAPTER-6

Cubes and Cube Roots

2MARK Q&A:

Exercise 6.1

1. Which of the following numbers are not perfect cubes?

(i) 216

Solution:

By resolving 216 into a prime factor,

2	216
2	108
2	54
3	27
3	9
3	3
	1

$216 = 2\times2\times2\times3\times3\times3$

By grouping the factors in triplets of equal factors,

$216 = (2\times2\times2) \times (3\times3\times3)$

Here, 216 can be grouped into triplets of equal factors,

∴ 216 = (2×3) = 6

Hence, 216 is the cube of 6.

(ii) 128

Solution:

By resolving 128 into a prime factor,

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

128 = 2×2×2×2×2×2×2

By grouping the factors in triplets of equal factors,

128 = (2×2×2) × (2×2×2) ×2

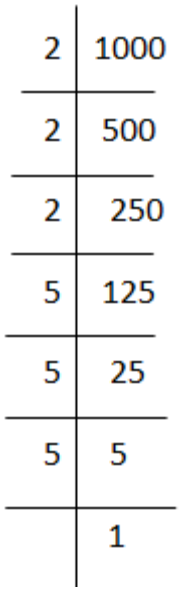
Here, 128 cannot be grouped into triplets of equal factors, and we are left with one factor: 2.

∴ 128 is not a perfect cube.

(iii) 1000

Solution:

By resolving 1000 into prime factor,



$1000 = 2\times2\times2\times5\times5\times5$

By grouping the factors in triplets of equal factors,

$1000 = (2\times2\times2) \times (5\times5\times5)$

Here, 1000 can be grouped into triplets of equal factors.

$\therefore 1000 = (2\times5) = 10$

Hence, 1000 is the cube of 10.

(iv) 100

Solution:

By resolving 100 into a prime factor,

2	100
2	50
5	25
5	5
	1

100 = 2×2×5×5

Here, 100 cannot be grouped into triplets of equal factors.

∴ 100 is not a perfect cube.

(v) 46656

Solution:

By resolving 46656 into prime factor,

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

46656 = 2×2×2×2×2×2×3×3×3×3×3×3

By grouping the factors in triplets of equal factors, 46656 = (2×2×2) × (2×2×2) × (3×3×3) × (3×3×3)

Here, 46656 can be grouped into triplets of equal factors,

∴ 46656 = (2×2×3×3) = 36

Hence, 46656 is the cube of 36.

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2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(i) 243

Solution:

By resolving 243 into a prime factor,

3	243
3	81
3	27
3	9
3	3
	1

243 = 3×3×3×3×3

By grouping the factors in triplets of equal factors, 243 = (3×3×3) ×3×3

Here, 3 cannot be grouped into triplets of equal factors.

∴ We will multiply 243 by 3 to get the perfect cube.

(ii) 256

Solution:

By resolving 256 into a prime factor,

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

256 = 2×2×2×2×2×2×2×2

By grouping the factors in triplets of equal factors,

256 = (2×2×2) × (2×2×2) ×2×2

Here, 2 cannot be grouped into triplets of equal factors.

∴ We will multiply 256 by 2 to get the perfect cube.

(iii) 72

Solution:

By resolving 72 into a prime factor,

2	72
2	36
2	18
3	9
3	3
	1

72 = 2×2×2×3×3

By grouping the factors in triplets of equal factors, 72 = (2×2×2) ×3×3

Here, 3 cannot be grouped into triplets of equal factors.

∴ We will multiply 72 by 3 to get the perfect cube.

(iv) 675

Solution:

By resolving 675 into a prime factor,

3	675
3	225
3	75
5	25
5	5
	1

675 = 3×3×3×5×5

By grouping the factors in triplets of equal factors, 675 = (3×3×3) ×5×5

Here, 5 cannot be grouped into triplets of equal factors.

∴ We will multiply 675 by 5 to get the perfect cube.

(v) 100

Solution:

By resolving 100 into a prime factor,

2	100
2	50
5	25
5	5
	1

100 = 2×2×5×5

Here, 2 and 5 cannot be grouped into triplets of equal factors.

∴ We will multiply 100 by (2×5) 10 to get the perfect cube.

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3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

(i) 81

Solution:

By resolving 81 into a prime factor,

3	81
3	27
3	9
3	3
	1

81 = 3×3×3×3

By grouping the factors in triplets of equal factors, 81 = (3×3×3) ×3

Here, 3 cannot be grouped into triplets of equal factors.

∴ We will divide 81 by 3 to get the perfect cube.

(ii) 128

Solution:

By resolving 128 into a prime factor,

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

128 = 2×2×2×2×2×2×2

By grouping the factors in triplets of equal factors, 128 = (2×2×2) × (2×2×2) ×2

Here, 2 cannot be grouped into triplets of equal factors.

∴ We will divide 128 by 2 to get the perfect cube.

(iii) 135

Solution:

By resolving 135 into prime factor,

3	135
3	45
3	15
5	5
	1

135 = 3×3×3×5

By grouping the factors in triplets of equal factors, 135 = (3×3×3)×5

Here, 5 cannot be grouped into triplets of equal factors.

∴ We will divide 135 by 5 to get the perfect cube.

(iv) 192

Solution:

By resolving 192 into a prime factor,

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

192 = 2×2×2×2×2×2×3

By grouping the factors in triplets of equal factors, 192 = (2×2×2)×(2×2×2)×3

Here, 3 cannot be grouped into triplets of equal factors.

∴ We will divide 192 by 3 to get the perfect cube.

(v) 704

Solution:

By resolving 704 into a prime factor,

2	704
2	352
2	176
2	88
2	44
2	22
11	11
	1

704 = 2×2×2×2×2×2×11

By grouping the factors in triplets of equal factors, 704 = (2×2×2)×(2×2×2)×11

Here, 11 cannot be grouped into triplets of equal factors.

∴ We will divide 704 by 11 to get the perfect cube.

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4. Parikshit makes a cuboid of plasticine with sides 5 cm, 2 cm, and 5 cm. How many such cuboids will he need to form a cube?

Solution:

Given the sides of the cube are 5 cm, 2 cm and 5 cm.

∴ Volume of cube = 5×2×5 = 50

2	50
5	25
5	5
	1

50 = 2×5×5

Here, 2 , 5 and 5 cannot be grouped into triplets of equal factors.

∴ We will multiply 50 by (2×2×5) 20 to get the perfect cube. Hence, 20 cuboids are needed.

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5MARK Q&A:

Exercise 6.2

1. Find the cube root of each of the following numbers by the prime factorization method.

(i) 64

Solution:

$64 = 2\times2\times2\times2\times2\times2$

By grouping the factors in triplets of equal factors, $64 = (2\times2\times2)\times(2\times2\times2)$

Here, 64 can be grouped into triplets of equal factors.

$\therefore 64 = 2\times2 = 4$

Hence, 4 is the cube root of 64.

(ii) 512

Solution:

$512 = 2\times2\times2\times2\times2\times2\times2\times2\times2$

By grouping the factors in triplets of equal factors, $512 = (2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)$

Here, 512 can be grouped into triplets of equal factors.

$\therefore 512 = 2\times2\times2 = 8$

Hence, 8 is the cube root of 512.

(iii) 10648

Solution:

$10648 = 2\times2\times2\times11\times11\times11$

By grouping the factors in triplets of equal factors, $10648 = (2\times2\times2)\times(11\times11\times11)$

Here, 10648 can be grouped into triplets of equal factors.

$\therefore 10648 = 2 \times 11 = 22$

Hence, 22 is the cube root of 10648.

(iv) 27000

Solution:

$27000 = 2\times2\times2\times3\times3\times3\times3\times5\times5\times5$

By grouping the factors in triplets of equal factors, $27000 = (2\times2\times2)\times(3\times3\times3)\times(5\times5\times5)$

Here, 27000 can be grouped into triplets of equal factors.

$\therefore 27000 = (2\times3\times5) = 30$

Hence, 30 is the cube root of 27000.

(v) 15625

Solution:

$15625 = 5\times5\times5\times5\times5\times5$

By grouping the factors in triplets of equal factors, 15625 = (5×5×5)×(5×5×5)

Here, 15625 can be grouped into triplets of equal factors.

∴ 15625 = (5×5) = 25

Hence, 25 is the cube root of 15625.

(vi) 13824

Solution:

13824 = 2×2×2×2×2×2×2×2×2×3×3×3

By grouping the factors in triplets of equal factors,

13824 = (2×2×2)×(2×2×2)×(2×2×2)×(3×3×3)

Here, 13824 can be grouped into triplets of equal factors.

∴ 13824 = (2×2× 2×3) = 24

Hence, 24 is the cube root of 13824.

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(vii) 110592
Solution:

110592 = 2×2×2×2×2×2×2×2×2×2×2×2×3×3×3

By grouping the factors in triplets of equal factors,

110592 = (2×2×2)×(2×2×2)×(2×2×2)×(2×2×2)×(3×3×3)

Here, 110592 can be grouped into triplets of equal factors.

∴ 110592 = (2×2×2×2 × 3) = 48

Hence, 48 is the cube root of 110592.

(viii) 46656

Solution:

46656 = 2×2×2×2×2×2×3×3×3×3×3×3

By grouping the factors in triplets of equal factors,

46656 = (2×2×2)×(2×2×2)×(3×3×3)×(3×3×3)

Here, 46656 can be grouped into triplets of equal factors.

∴ 46656 = (2×2×3×3) = 36

Hence, 36 is the cube root of 46656.

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(ix) 175616

Solution:

175616 = 2×2×2×2×2×2×2×2×2×7×7×7

By grouping the factors in triplets of equal factors,

175616 = (2×2×2)×(2×2×2)×(2×2×2)×(7×7×7)

Here, 175616 can be grouped into triplets of equal factors.

∴ 175616 = (2×2×2×7) = 56

Hence, 56 is the cube root of 175616.

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(x) 91125

Solution:

91125 = 3×3×3×3×3×3×3×5×5×5

By grouping the factors in triplets of equal factors, 91125 = (3×3×3)×(3×3×3)×(5×5×5)

Here, 91125 can be grouped into triplets of equal factors.

∴ 91125 = (3×3×5) = 45

Hence, 45 is the cube root of 91125.

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2. State true or false.

(i) Cube of any odd number is even.

Solution:

False

(ii) A perfect cube does not end with two zeros.

Solution:

True

(iii) If the cube of a number ends with 5, then its cube ends with 25.

Solution:

False

(iv) There is no perfect cube which ends with 8.

Solution:

False

(v) The cube of a two-digit number may be a three-digit number.

Solution:

False

(vi) The cube of a two-digit number may have seven or more digits.

Solution:

False

(vii) The cube of a single-digit number may be a single-digit number.

Solution:

True

3. You are told that 1,331 is a perfect cube. Can you guess without factorization what its cube root is? Similarly, guess the cube roots of 4913, 12167, and 32768.

Solution:

(i) By grouping the digits, we get 1 and 331

We know that since the unit digit of the cube is 1, the unit digit of the cube root is 1.

∴ We get 1 as the unit digit of the cube root of 1331.

The cube of 1 matches the number of the second group.

∴ The ten's digit of our cube root is taken as the unit place of the smallest number.

We know that the unit's digit of the cube of a number having digit as unit's place 1 is 1.

∴ $\sqrt[3]{1331} = 11$

(ii) By grouping the digits, we get 4 and 913

We know that since the unit digit of the cube is 3, the unit digit of the cube root is 7.

∴ we get 7 as the unit digit of the cube root of 4913. We know $1^3 = 1$ and $2^3 = 8$, $1 > 4 > 8$

Thus, 1 is taken as the tens digit of the cube root.

∴ $\sqrt[3]{4913} = 17$

(iii) By grouping the digits, we get 12 and 167.

We know that since the unit digit of the cube is 7, the unit digit of the cube root is 3.

∴ 3 is the unit digit of the cube root of 12167 We know $2^3 = 8$ and $3^3 = 27$, $8 > 12 > 27$

Thus, 2 is taken as the tens digit of the cube root.

∴ $\sqrt[3]{12167} = 23$

(iv) By grouping the digits, we get 32 and 768.

We know that since the unit digit of the cube is 8, the unit digit of the cube root is 2.

∴ 2 is the unit digit of the cube root of 32768. We know $3^3 = 27$ and $4^3 = 64$, $27 > 32 > 64$

Thus, 3 is taken as the tens digit of the cube root.

∴ $\sqrt[3]{32768} = 32$.

1MARK Q&A:

Exercise 6.3

Multiple-choice questions and answers

1. What is the cube of 4?

a) 8

b) 12

c) 16

d) 64

Answer: d) 64

2. Which of the following numbers is a perfect cube?

a) 27

b) 16

c) 12

d) 9

Answer: a) 27

3. What is the cube root of 64?

a) 2

b) 4

c) 6

d) 8

Answer: a) 2

4. If you have a cube with side length 5 cm, what is its volume?

a) 15 cm^3

b) 125 cm^3

c) 250 cm^3

d) 625 cm^3

Answer: b) 125 cm^3

5. What is the cube root of 512?

a) 4

b) 6

c) 8

d) 12

Answer: c) 8

6. If the volume of a cube is 343 cubic centimeters, what is the length of its side?

a) 6 cm

b) 7 cm

c) 8 cm

d) 9 cm

Answer: b) 7 cm

7. Which of the following is not a cube number?

a) 27

b) 64

c) 100

d) 125

Answer: c) 100

8. What is the cube of 2.5?

a) 5.0

b) 6.25

c) 8.0

d) 12.5

Answer: b) 6.25

9. What is the cube root of 1,000?

a) 10

b) 20

c) 30

d) 40

Answer: a) 10

10. If the volume of a cube is 64 cubic inches, what is the length of its side in inches?

a) 2 inches

b) 3 inches

c) 4 inches

d) 5 inches

Answer: c) 4 inches

11. What is the cube of -3?

a) -9

b) -27

c) 3

d) 9

Answer: b) -27

12. If a cube has a volume of 216 cubic units, what is the length of its side?

a) 3 units

b) 4 units

c) 6 units

d) 8 units

Answer: a) 3 units

13. Which of the following numbers is a perfect cube?

a) 20

b) 64

c) 125

d) 216

Answer: d) 216

14. What is the cube root of 729?

a) 9

b) 27

c) 81

d) 243

Answer: a) 9

15. If the volume of a cube is 512 cubic centimeters, what is the length of its side?

a) 4 cm

b) 6 cm

c) 8 cm

d) 12 cm

Answer: a) 4 cm

16. Which of the following numbers is not a cube root?

a) $\sqrt[3]{64}$

b) $\sqrt[3]{125}$

c) $\sqrt[3]{216}$

d) $\sqrt[3]{256}$

Answer: d) $\sqrt[3]{256}$

17. What is the cube of 1/2?

a) 1/8

b) 1/4

c) 1/6

d) 1/16

Answer: b) 1/8

18. What is the cube root of 1?

a) 0

b) 1

c) 2

d) 3

Answer: b) 1

19. If the volume of a cube is 1000 cubic meters, what is the length of its side?

a) 10 meters

b) 20 meters

c) 30 meters

d) 40 meters

Answer: a) 10 meters

20. What is the cube of $\sqrt[3]{8}$?

a) 2

b) 4

c) 8

d) 16

Answer: b) 4

Exercise 6.4:

1. The cube of 4 is _____.

- Answer: 64

2. The cube root of 125 is _____.

- Answer: 5

3. 5^3 is equal to _____.

- Answer: 125

4. The number 8 is a _____ cube.

- Answer: Perfect

5. $\sqrt[3]{64}$ is equal to _____.

- Answer: 4

6. The cube root of 1 is _____.

- Answer: 1

7. $(-3)^3$ is equal to _____.

- Answer: -27

8. The volume of a cube with a side length of 6 cm is _____ cm^3.

- Answer: 216

Summary

1. Understanding Cubes:

- A cube is a three-dimensional shape where all sides are equal in length.

- The cube of a number is obtained by multiplying the number by itself twice.

- For example, the cube of 3 is (3*3*3=27), denoted as $3^3 = 27$.

2. Properties of Cubes:

- Like squares, cubes of even numbers always result in an even number, and cubes of odd numbers always result in an odd number.

- The cube of any number, whether positive or negative, retains the sign of the original number.

3. Perfect Cubes:

- Certain numbers are perfect cubes, meaning they can be expressed as the cube of an integer.

- Examples include 1, 8, 27, 64, 125, etc.

4. Cube Roots:

- The cube root of a number is the value that, when multiplied by itself twice, gives the original number.

- For example, the cube root of 27 is $\sqrt[3]{27}$ =3 because 3*3*3=27

- Cube roots can be positive or negative. For instance, both -3 and 3 are cube roots of 27.

5. Notation and Representation:

- The symbol for cube root is $\sqrt[3]{\ }$ For instance, $\sqrt[3]{64}$=4

- Cube roots of perfect cubes are whole numbers.

- Cube roots of non-perfect cubes are irrational numbers and cannot be represented as simple fractions.

6. Calculating Cube Roots:

- Similar to square roots, cube roots can be calculated using estimation methods or using calculators.

- For manual calculation, methods like prime factorization can be employed.

7. Applications:

- Cubes and cube roots are used in various fields, especially in volume calculations of three-dimensional shapes like cubes, as well as in scientific and engineering calculations.

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