Chapter 1

Sets

Introduction of Sets: Understanding the concept of sets and their representation.

Types of sets: Exploring different types of sets, including finite, infinite, equal, and equivalent sets.

Operations of Sets: Performing operations like union, intersection, and complement of sets.

Venn Diagrams: Representing sets using Venn diagrams for better visualization.

Functions:

Introduction to Functions: Defining functions and understanding the relation between inputs and outputs.

Types of Functions: Exploring different types, such as one-to-one (injective), onto (subjective), and one-to-one correspondence.

Composite Functions: understanding the composite functions and their properties.

Inverse Trigonometric Functions: Introducing inverse trigonometric functions and their properties.

Other Concepts:

Binary Operations: Defining binary operations and exploring their properties.

Exercise 1.1

1. Which of the following are sets? Justify your answer.

(i) The collection of all months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) A collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers.

(viii) The collection of questions in this Chapter.

(ix) A collection of most dangerous animals of the world.

Solution:

(i) The collection of all months of a year beginning with the letter J is a well-defined collection of objects as one can identify a month which belongs to this collection.

Therefore, this collection is a set.

(ii) The collection of ten most talented writers of India is not a well-defined collection as the criteria to determine a writer's talent may differ from one person to another.

Therefore, this collection is not a set.

(iii) A team of eleven best-cricket batsmen of the world is not a well-defined collection as the criteria to determine a batsman's talent may vary from one person to another.

Therefore, this collection is not a set.

(iv) The collection of all boys in your class is a well-defined collection as you can identify a boy who belongs to this collection.

Therefore, this collection is a set.

(v) The collection of all natural numbers less than 100 is a well-defined collection as one can find a number which belongs to this collection.

Therefore, this collection is a set.

(vi) A collection of novels written by the writer Munshi Prem Chand is a well-defined collection as one can find any book which belongs to this collection.

Therefore, this collection is a set.

(vii) The collection of all even integers is a well-defined collection as one can find an integer which belongs to this collection.

Therefore, this collection is a set.

(viii) The collection of questions in this chapter is a well-defined collection as one can find a question which belongs to this chapter.

Therefore, this collection is a set.

(ix) A collection of most dangerous animals of the world is not a well-defined collection as the criteria to find the dangerousness of an animal can differ from one animal to another.

Therefore, this collection is not a set.

2. Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol ∈or ∉ in the blank spaces:

(i) 5…A (ii) 8…A (iii) 0…A

(iv) 4…A (v) 2…A (vi) 10…A

Solution:

(i) $5 \in A$

(ii) $8 \notin A$

(iii) $0 \notin A$

(iv) $4 \in A$

(v) $2 \in A$

(vi) $10 \notin A$

3. Write the following sets in roster form:

(i) A = {x: x is an integer and $-3 < x < 7$}.

(ii) B = {x: x is a natural number less than 6}.

(iii) C = {x: x is a two-digit natural number such that the sum of its digits is 8}

(iv) D = {x: x is a prime number which is divisor of 60}.

(v) E = The set of all letters in the word TRIGONOMETRY.

(vi) F = The set of all letters in the word BETTER.

Solution:

(i) A = {x: x is an integer and $-3 < x < 7$}

–2, –1, 0, 1, 2, 3, 4, 5, and 6 only are the elements of this set.

Hence, the given set can be written in roster form as

A = {–2, –1, 0, 1, 2, 3, 4, 5, 6}

(ii) B = {x: x is a natural number less than 6}

1, 2, 3, 4, and 5 only are the elements of this set

Hence, the given set can be written in roster form as

B = {1, 2, 3, 4, 5}

(iii) C = {x: x is a two-digit natural number such that the sum of its digits is 8}

17, 26, 35, 44, 53, 62, 71, and 80 only are the elements of this set

Hence, the given set can be written in roster form as

C = {17, 26, 35, 44, 53, 62, 71, 80}

(iv) D = {x: x is a prime number which is divisor of 60}

2	60
2	30
3	15
	5

Here 60 = 2 × 2 × 3 × 5

2, 3 and 5 only are the elements of this set

Hence, the given set can be written in roster form as

D = {2, 3, 5}

(v) E = The set of all letters in the word TRIGONOMETRY

TRIGONOMETRY is a 12 letters word out of which T, R and O are repeated.

Hence, the given set can be written in roster form as

E = {T, R, I, G, O, N, M, E, Y}

(vi) F = The set of all letters in the word BETTER

BETTER is a 6 letters word out of which E and T are repeated.

Hence, the given set can be written in roster form as

F = {B, E, T, R}

4. Write the following sets in the set-builder form:

(i) (3, 6, 9, 12)

(ii) {2, 4, 8, 16, 32}

(iii) {5, 25, 125, 625}

(iv) {2, 4, 6 …}

(v) {1, 4, 9 … 100}

Solution:

(i) {3, 6, 9, 12}

The given set can be written in the set-builder form as $\{x: x = 3n, n \in N$ and $1 \le n \le 4\}$

(ii) {2, 4, 8, 16, 32}

We know that $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, $16 = 2^4$, and $32 = 2^5$.

Therefore, the given set {2, 4, 8, 16, 32} can be written in the set-builder form as $\{x: x = 2^n, n \in N$ and $1 \le n \le 5\}$.

(iii) {5, 25, 125, 625}

We know that $5 = 5^1$, $25 = 5^2$, $125 = 5^3$, and $625 = 5^4$.

Therefore, the given set {5, 25, 125, 625} can be written in the set-builder form as $\{x: x = 5^n, n \in N$ and $1 \le n \le 4\}$.

(iv) {2, 4, 6 …}

{2, 4, 6 …} is a set of all even natural numbers

Therefore, the given set {2, 4, 6 …} can be written in the set-builder form as {*x*: *x* is an even natural number}.

(v) {1, 4, 9 … 100}

We know that $1 = 1^2$, $4 = 2^2$, $9 = 3^2$ …$100 = 10^2$.

Therefore, the given set {1, 4, 9… 100} can be written in the set-builder form as $\{x: x = n^2, n \in N$ and $1 \le n \le 10\}$.

5. List all the elements of the following sets:

(i) A = {*x*: *x* is an odd natural number}

(ii) B = {*x*: *x* is an integer, $-1/2 < x < 9/2$}

(iii) C = {*x*: *x* is an integer, $x^2 \le 4$}

(iv) D = {*x*: *x* is a letter in the word "LOYAL"}

(v) E = {*x*: *x* is a month of a year not having 31 days}

(vi) F = {*x*: *x* is a consonant in the English alphabet which proceeds *k*}.

Solution:

(i) A = {*x*: *x* is an odd natural number}

So the elements are A = {1, 3, 5, 7, 9 …..}

(ii) B = {*x*: *x* is an integer, $-1/2 < x < 9/2$}

We know that $-1/2 = -0.5$ and $9/2 = 4.5$

So the elements are B = {0, 1, 2, 3, 4}.

(iii) C = {*x*: *x* is an integer, $x^2 \leq 4$}

We know that

$(-1)^2 = 1 \leq 4$; $(-2)^2 = 4 \leq 4$; $(-3)^2 = 9 > 4$

Here

$0^2 = 0 \leq 4$, $1^2 = 1 \leq 4$, $2^2 = 4 \leq 4$, $3^2 = 9 > 4$

So we get

C = {–2, –1, 0, 1, 2}

(iv) D = {*x*: *x* is a letter in the word "LOYAL"}

So the elements are D = {L, O, Y, A}

(v) E = {*x*: *x* is a month of a year not having 31 days}

So the elements are E = {February, April, June, September, November}

(vi) F = {*x*: *x* is a consonant in the English alphabet which proceeds *k*}

So the elements are F = {b, c, d, f, g, h, j}

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(i) {1, 2, 3, 6} (a) {x: x is a prime number and a divisor of 6}

(ii) {2, 3} (b) {x: x is an odd natural number less than 10}

(iii) {M, A, T, H, E, I, C, S} (c) {x: x is a natural number and divisor of 6}

(iv) {1, 3, 5, 7, 9} (d) {x: x is a letter of the word MATHEMATICS}

Solution:

(i) Here the elements of this set are natural number as well as divisors of 6. Hence, (i) matches with (c).

(ii) 2 and 3 are prime numbers which are divisors of 6. Hence, (ii) matches with (a).

(iii) The elements are the letters of the word MATHEMATICS. Hence, (iii) matches with (d).

(iv) The elements are odd natural numbers which are less than 10. Hence, (v) matches with (b).

Exercise 1.2

1. Which of the following are examples of the null set?

(i) Set of odd natural numbers divisible by 2

(ii) Set of even prime numbers

(iii) {x: x is a natural numbers, $x < 5$ and $x > 7$}

(iv) {y: y is a point common to any two parallel lines}

Solution:

(i) Set of odd natural numbers divisible by 2 is a null set as odd numbers are not divisible by 2.

(ii) Set of even prime numbers is not a null set as 2 is an even prime number.

(iii) {x: x is a natural number, $x < 5$ and $x > 7$} is a null set as a number cannot be both less than 5 and greater than 7.

(iv) {y: y is a point common to any two parallel lines} is a null set as the parallel lines do not intersect. Therefore, they have no common point.

2. Which of the following sets are finite or infinite?

(i) The set of months of a year

(ii) {1, 2, 3 …}

(iii) {1, 2, 3 … 99, 100}

(iv) The set of positive integers greater than 100

(v) The set of prime numbers less than 99

Solution:

(i) The set of months of a year is a finite set as it contains 12 elements.

(ii) {1, 2, 3 …} is an infinite set because it has infinite number of natural numbers.

(iii) {1, 2, 3 …99, 100} is a finite set as the numbers from 1 to 100 are finite.

(iv) The set of positive integers greater than 100 is an infinite set as the positive integers which are greater than 100 are infinite.

(v) The set of prime numbers less than 99 is a finite set as the prime numbers which are less than 99 are finite.

3. State whether each of the following set is finite or infinite:

(i) The set of lines which are parallel to the *x*-axis

(ii) The set of letters in the English alphabet

(iii) The set of numbers which are multiple of 5

(iv) The set of animals living on the earth

(v) The set of circles passing through the origin (0, 0)

Solution:

(i) The set of lines which are parallel to the *x*-axis is an infinite set as the lines which are parallel to the *x*-axis are infinite.

(ii) The set of letters in the English alphabet is a finite set as it contains 26 elements.

(iii) The set of numbers which are multiple of 5 is an infinite set as the multiples of 5 are infinite.

(iv) The set of animals living on the earth is a finite set as the number of animals living on the earth is finite.

(v) The set of circles passing through the origin (0, 0) is an infinite set as infinite number of circles can pass through the origin.

4. In the following, state whether A = B or not:

(i) A = {*a*, *b*, *c*, *d*}; B = {*d*, *c*, *b*, *a*}

(ii) A = {4, 8, 12, 16}; B = {8, 4, 16, 18}

(iii) A = {2, 4, 6, 8, 10}; B = {*x*: *x* is positive even integer and *x* ≤ 10}

(iv) A = {*x*: *x* is a multiple of 10}; B = {10, 15, 20, 25, 30 …}

Solution:

(i) A = {a, b, c, d}; B = {d, c, b, a}

Order in which the elements of a set are listed is not significant.

Therefore, A = B.

(ii) A = {4, 8, 12, 16}; B = {8, 4, 16, 18}

We know that 12 ∈ A but 12 ∉ B.

Therefore, A ≠ B

(iii) A = {2, 4, 6, 8, 10};

B = {x: x is a positive even integer and $x \leq 10$} = {2, 4, 6, 8, 10}

Therefore, A = B

(iv) A = {x: x is a multiple of 10}

B = {10, 15, 20, 25, 30 …}

We know that $15 \in B$ but $15 \notin A$.

Therefore, $A \neq B$

5. Are the following pair of sets equal? Give reasons.

(i) A = {2, 3}; B = {x: x is solution of $x^2 + 5x + 6 = 0$}

(ii) A = {x: x is a letter in the word FOLLOW}; B = {y: y is a letter in the word WOLF}

Solution:

(i) A = {2, 3}; B = {x: x is solution of $x^2 + 5x + 6 = 0$}

$x^2 + 5x + 6 = 0$ can be written as

$x(x + 3) + 2(x + 3) = 0$

By further calculation

$(x + 2)(x + 3) = 0$

So we get

$x = -2$ or $x = -3$

Here

$A = \{2, 3\}; B = \{-2, -3\}$

Therefore, $A \neq B$

(ii) A = {x: x is a letter in the word FOLLOW} = {F, O, L, W}

B = {y: y is a letter in the word WOLF} = {W, O, L, F}

Order in which the elements of a set which are listed is not significant.

Therefore, A = B.

6. From the sets given below, select equal sets:

A = {2, 4, 8, 12}, B = {1, 2, 3, 4}, C = {4, 8, 12, 14}, D = {3, 1, 4, 2}

E = {–1, 1}, F = {0, a}, G = {1, –1}, H = {0, 1}

Solution:

A = {2, 4, 8, 12}; B = {1, 2, 3, 4}; C = {4, 8, 12, 14}

D = {3, 1, 4, 2}; E = {–1, 1}; F = {0, *a*}

G = {1, –1}; H = {0, 1}

We know that

$8 \in A, 8 \notin B, 8 \notin D, 8 \notin E, 8 \notin F, 8 \notin G, 8 \notin H$

$A \neq B, A \neq D, A \neq E, A \neq F, A \neq G, A \neq H$

It can be written as

$2 \in A, 2 \notin C$

Therefore, $A \neq C$

$3 \in B, 3 \notin C, 3 \notin E, 3 \notin F, 3 \notin G, 3 \notin H$

$B \neq C, B \neq E, B \neq F, B \neq G, B \neq H$

It can be written as

$12 \in C, 12 \notin D, 12 \notin E, 12 \notin F, 12 \notin G, 12 \notin H$

Therefore, $C \neq D, C \neq E, C \neq F, C \neq G, C \neq H$

$4 \in D, 4 \notin E, 4 \notin F, 4 \notin G, 4 \notin H$

Therefore, $D \neq E, D \neq F, D \neq G, D \neq H$

Here, $E \neq F, E \neq G, E \neq H$

$F \neq G, F \neq H, G \neq H$

Order in which the elements of a set are listed is not significant.

B = D and E = G

Therefore, among the given sets, B = D and E = G.

Exercise 1.3

1. Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:

(i) {2, 3, 4} … {1, 2, 3, 4, 5}

(ii) {*a*, *b*, *c*} … {*b*, *c*, *d*}

(iii) {*x*: *x* is a student of Class XI of your school} … {*x*: *x* student of your school}

(iv) {*x*: *x* is a circle in the plane} … {*x*: *x* is a circle in the same plane with radius 1 unit}

(v) {*x*: *x* is a triangle in a plane}…{*x*: *x* is a rectangle in the plane}

(vi) {*x*: *x* is an equilateral triangle in a plane}… {*x*: *x* is a triangle in the same plane}

(vii) {*x*: *x* is an even natural number} … {*x*: *x* is an integer}

Solution:

(i) $\{2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} \not\subset \{b, c, d\}$

(iii) {*x*: *x* is a student of Class XI of your school} $\subset$ {*x*: *x* student of your school}

(iv) {*x*: *x* is a circle in the plane} $\not\subset$ {*x*: *x* is a circle in the same plane with radius 1 unit}

(v) {*x*: *x* is a triangle in a plane} $\not\subset$ {*x*: *x* is a rectangle in the plane}

(vi) {*x*: *x* is an equilateral triangle in a plane} $\subset$ {*x*: *x* is a triangle in the same plane}

(vii) {*x*: *x* is an even natural number} $\subset$ {*x*: *x* is an integer}

2. Examine whether the following statements are true or false:

(i) $\{a, b\} \not\subset \{b, c, a\}$

(ii) $\{a, e\} \subset \{x$: x is a vowel in the English alphabet$\}$

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$

(iv) $\{a\} \subset \{a. b, c\}$

(v) $\{a\} \in (a, b, c)$

(vi) $\{x$: x is an even natural number less than 6$\} \subset \{x$: x is a natural number which divides 36$\}$

Solution:

(i) False.

Here each element of $\{a, b\}$ is an element of $\{b, c, a\}$.

(ii) True.

We know that a, e are two vowels of the English alphabet.

(iii) False.

$2 \in \{1, 2, 3\}$ where, $2 \notin \{1, 3, 5\}$

(iv) True.

Each element of $\{a\}$ is also an element of $\{a, b, c\}$.

(v) False.

Elements of $\{a, b, c\}$ are a, b, c. Hence, $\{a\} \subset \{a, b, c\}$

(vi) True.

$\{x$: x is an even natural number less than 6$\} = \{2, 4\}$

$\{x$: x is a natural number which divides 36$\} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

3. Let A= {1, 2, {3, 4}, 5}. Which of the following statements are incorrect and why?

(i) {3, 4} ⊂ A

(ii) {3, 4}}∈ A

(iii) {{3, 4}} ⊂ A

(iv) 1 ∈ A

(v) 1⊂ A

(vi) {1, 2, 5} ⊂ A

(vii) {1, 2, 5} ∈ A

(viii) {1, 2, 3} ⊂ A

(ix) Φ ∈ A

(x) Φ ⊂ A

(xi) {Φ} ⊂ A

Solution:

It is given that A= {1, 2, {3, 4}, 5}

(i) {3, 4} ⊂ A is incorrect

Here 3 ∈ {3, 4}; where, 3∉A.

(ii) {3, 4} ∈A is correct

{3, 4} is an element of A.

(iii) {{3, 4}} ⊂ A is correct

{3, 4} ∈ {{3, 4}} and {3, 4} ∈ A.

(iv) 1∈A is correct

1 is an element of A.

(v) 1⊂ A is incorrect

An element of a set can never be a subset of itself.

(vi) $\{1, 2, 5\} \subset A$ is correct

Each element of $\{1, 2, 5\}$ is also an element of A.

(vii) $\{1, 2, 5\} \in A$ is incorrect

$\{1, 2, 5\}$ is not an element of A.

(viii) $\{1, 2, 3\} \subset A$ is incorrect

$3 \in \{1, 2, 3\}$; where, $3 \notin A$.

(ix) $\Phi \in A$ is incorrect

Φ is not an element of A.

(x) $\Phi \subset A$ is correct

Φ is a subset of every set.

(xi) $\{\Phi\} \subset A$ is incorrect

$\Phi \in \{\Phi\}$; where, $\Phi \in A$.

4. Write down all the subsets of the following sets:

(i) $\{a\}$

(ii) $\{a, b\}$

(iii) {1, 2, 3}

(iv) Φ

Solution:

(i) Subsets of $\{a\}$ are

Φ and $\{a\}$.

(ii) Subsets of $\{a, b\}$ are

Φ, $\{a\}$, $\{b\}$, and $\{a, b\}$.

(iii) Subsets of {1, 2, 3} are

Φ, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, and {1, 2, 3}.

(iv) Only subset of Φ is Φ.

5. How many elements has P (A), if A = Φ?

Solution:

If A is a set with *m* elements

n (A) = m then n [P (A)] = 2^m

If A = Φ we get n (A) = 0

n [P(A)] = $2^0 = 1$

Therefore, P (A) has one element.

6. Write the following as intervals:

(i) {x: $x \in$ R, –4 < $x \leq$ 6}

(ii) {x: $x \in$ R, –12 < x < –10}

(iii) {x: $x \in$ R, 0 ≤ x < 7}

(iv) {x: $x \in$ R, 3 ≤ x ≤ 4}

Solution:

(i) {x: $x \in$ R, –4 < $x \leq$ 6} = (–4, 6]

(ii) {x: $x \in$ R, –12 < x < –10} = (–12, –10)

(iii) {x: $x \in$ R, 0 ≤ x < 7} = [0, 7)

(iv) {x: $x \in$ R, 3 ≤ x ≤ 4} = [3, 4]

7. Write the following intervals in set-builder form:

(i) (–3, 0)

(ii) [6, 12]

(iii) (6, 12]

(iv) [–23, 5)

Solution:

(i) (–3, 0) = $\{x: x \in R, -3 < x < 0\}$

(ii) [6, 12] = $\{x: x \in R, 6 \leq x \leq 12\}$

(iii) (6, 12] =$\{x: x \in R, 6 < x \leq 12\}$

(iv) [–23, 5) = $\{x: x \in R, -23 \leq x < 5\}$

8. What universal set (s) would you propose for each of the following?

(i) The set of right triangles

(ii) The set of isosceles triangles

Solution:

(i) Among the set of right triangles, the universal set is the set of triangles or the set of polygons.

(ii) Among the set of isosceles triangles, the universal set is the set of triangles or the set of polygons or the set of two-dimensional figures.

9. Given the sets A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}, which of the following may be considered as universals set (s) for all the three sets A, B and C?

(i) {0, 1, 2, 3, 4, 5, 6}

(ii) Φ

(iii) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(iv) {1, 2, 3, 4, 5, 6, 7, 8}

Solution:

(i) We know that A ⊂ {0, 1, 2, 3, 4, 5, 6}

B ⊂ {0, 1, 2, 3, 4, 5, 6}

So C ⊄ {0, 1, 2, 3, 4, 5, 6}

Hence, the set {0, 1, 2, 3, 4, 5, 6} cannot be the universal set for the sets A, B, and C.

(ii) A ⊄ Φ, B ⊄ Φ, C ⊄ Φ

Hence, Φ cannot be the universal set for the sets A, B, and C.

(iii) A ⊂ {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

B ⊂ {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

C ⊂ {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Hence, the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} is the universal set for the sets A, B, and C.

(iv) A ⊂ {1, 2, 3, 4, 5, 6, 7, 8}

B ⊂ {1, 2, 3, 4, 5, 6, 7, 8}

So C ⊄ {1, 2, 3, 4, 5, 6, 7, 8}

Hence, the set {1, 2, 3, 4, 5, 6, 7, 8} cannot be the universal set for the sets A, B, and C.

Exercise 1.4

1. Find the union of each of the following pairs of sets:

(i) X = {1, 3, 5} Y = {1, 2, 3}

(ii) A = {*a*, *e*, *i*, *o*, *u*} B = {*a*, *b*, *c*}

(iii) A = {*x*: *x* is a natural number and multiple of 3}

B = {*x*: *x* is a natural number less than 6}

(iv) A = {x: x is a natural number and $1 < x \leq 6$}

B = {x: x is a natural number and $6 < x < 10$}

(v) A = {1, 2, 3}, B = Φ

Solution:

(i) X = {1, 3, 5} Y = {1, 2, 3}

So the union of the pairs of set can be written as

X ∪ Y= {1, 2, 3, 5}

(ii) A = {a, e, i, o, u} B = {a, b, c}

So the union of the pairs of set can be written as

A∪ B = {a, b, c, e, i, o, u}

(iii) A = {x: x is a natural number and multiple of 3} = {3, 6, 9 …}

B = {x: x is a natural number less than 6} = {1, 2, 3, 4, 5, 6}

So the union of the pairs of set can be written as

A ∪ B = {1, 2, 4, 5, 3, 6, 9, 12 …}

Hence, A ∪ B = {x: x = 1, 2, 4, 5 or a multiple of 3}

(iv) A = {x: x is a natural number and $1 < x \leq 6$} = {2, 3, 4, 5, 6}

B = {x: x is a natural number and $6 < x < 10$} = {7, 8, 9}

So the union of the pairs of set can be written as

A∪ B = {2, 3, 4, 5, 6, 7, 8, 9}

Hence, A∪ B = {x: $x \in$ N and $1 < x < 10$}

(v) A = {1, 2, 3}, B = Φ

So the union of the pairs of set can be written as

A∪ B = {1, 2, 3}

2. Let A = {*a*, *b*}, B = {*a*, *b*, *c*}. Is A ⊂ B? What is A ∪ B?

Solution:

It is given that

A = {*a*, *b*} and B = {*a*, *b*, *c*}

Yes, A ⊂ B

So the union of the pairs of set can be written as

A∪ B = {*a*, *b*, *c*} = B

3. If a and B are two sets such that a ⊂ B, then what is A ∪ B?

Solution:

If A and B are two sets such that A ⊂ B, then A ∪ B = B.

4. If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}; find

(i) A ∪ B

(ii) A ∪ C

(iii) B ∪ C

(iv) B ∪ D

(v) A ∪ B ∪ C

(vi) A ∪ B ∪ D

(vii) B ∪ C ∪ D

Solution:

It is given that

A = {1, 2, 3, 4], B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}

(i) A ∪ B = {1, 2, 3, 4, 5, 6}

(ii) A ∪ C = {1, 2, 3, 4, 5, 6, 7, 8}

(iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

5. Find the intersection of each pair of sets:

(i) X = {1, 3, 5} Y = {1, 2, 3}

(ii) A = {*a, e, i, o, u*} B = {*a, b, c*}

(iii) A = {*x*: *x* is a natural number and multiple of 3}

B = {*x*: *x* is a natural number less than 6}

(iv) A = {*x*: *x* is a natural number and 1 < *x* ≤ 6}

B = {*x*: *x* is a natural number and 6 < *x* < 10}

(v) A = {1, 2, 3}, B = Φ

Solution:

(i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$

So the intersection of the given set can be written as

$X \cap Y = \{1, 3\}$

(ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$

So the intersection of the given set can be written as

$A \cap B = \{a\}$

(iii) A = {*x*: *x* is a natural number and multiple of 3} = (3, 6, 9 …}

B = {*x*: *x* is a natural number less than 6} = {1, 2, 3, 4, 5}

So the intersection of the given set can be written as

$A \cap B = \{3\}$

(iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$B = \{x: x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$

So the intersection of the given set can be written as

$A \cap B = \Phi$

(v) $A = \{1, 2, 3\}$, $B = \Phi$

So the intersection of the given set can be written as

$A \cap B = \Phi$

6. If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17}; find

(i) A ∩ B

(ii) B ∩ C

(iii) A ∩ C ∩ D

(iv) A ∩ C

(v) B ∩ D

(vi) A ∩ (B ∪ C)

(vii) A ∩ D

(viii) A ∩ (B ∪ D)

(ix) (A ∩ B) ∩ (B ∪ C)

(x) (A ∪ D) ∩ (B ∪ C)

Solution:

(i) $A \cap B = \{7, 9, 11\}$

(ii) $B \cap C = \{11, 13\}$

(iii) $A \cap C \cap D = \{A \cap C\} \cap D$

$= \{11\} \cap \{15, 17\}$

$= \Phi$

(iv) $A \cap C = \{11\}$

(v) $B \cap D = \Phi$

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$= \{7, 9, 11\} \cup \{11\}$

$= \{7, 9, 11\}$

(vii) $A \cap D = \Phi$

(viii) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$

$= \{7, 9, 11\} \cup \Phi$

$= \{7, 9, 11\}$

(ix) $(A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11\}$

(x) $(A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17) \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11, 15\}$

7. If A = {*x*: *x* is a natural number}, B ={*x*: *x* is an even natural number}

C = {*x*: *x* is an odd natural number} and D = {*x*: *x* is a prime number}, find

(i) A ∩ B

(ii) A ∩ C

(iii) A ∩ D

(iv) B ∩ C

(v) B ∩ D

(vi) C ∩ D

Solution:

It can be written as

A = {x: x is a natural number} = {1, 2, 3, 4, 5 …}

B ={x: x is an even natural number} = {2, 4, 6, 8 …}

C = {x: x is an odd natural number} = {1, 3, 5, 7, 9 …}

D = {x: x is a prime number} = {2, 3, 5, 7 …}

(i) A ∩B = {x: x is a even natural number} = B

(ii) A ∩ C = {x: x is an odd natural number} = C

(iii) A ∩ D = {x: x is a prime number} = D

(iv) B ∩ C = Φ

(v) B ∩ D = {2}

(vi) C ∩ D = {x: x is odd prime number}

8. Which of the following pairs of sets are disjoint?

(i) {1, 2, 3, 4} and {x: x is a natural number and $4 \le x \le 6$}

(ii) {a, e, i, o, u}and {c, d, e, f}

(iii) {x: x is an even integer} and {x: x is an odd integer}

Solution:

(i) {1, 2, 3, 4}

{x: x is a natural number and $4 \le x \le 6$} = {4, 5, 6}

So we get

{1, 2, 3, 4} ∩ {4, 5, 6} = {4}

Hence, this pair of sets is not disjoint.

(ii) {a, e, i, o, u} ∩ (c, d, e, f} = {e}

Hence, {a, e, i, o, u} and (c, d, e, f} are not disjoint.

(iii) {x: x is an even integer} ∩ {x: x is an odd integer} = Φ

Hence, this pair of sets is disjoint.

9. If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20},

C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}; find

(i) A – B

(ii) A – C

(iii) A – D

(iv) B – A

(v) C – A

(vi) D – A

(vii) B – C

(viii) B – D

(ix) C – B

(x) D – B

(xi) C – D

(xii) D – C

Solution:

(i) A – B = {3, 6, 9, 15, 18, 21}

(ii) A – C = {3, 9, 15, 18, 21}

(iii) A – D = {3, 6, 9, 12, 18, 21}

(iv) B – A = {4, 8, 16, 20}

(v) C – A = {2, 4, 8, 10, 14, 16}

(vi) D – A = {5, 10, 20}

(vii) B – C = {20}

(viii) B – D = {4, 8, 12, 16}

(ix) C – B = {2, 6, 10, 14}

(x) D – B = {5, 10, 15}

(xi) C – D = {2, 4, 6, 8, 12, 14, 16}

(xii) D – C = {5, 15, 20}

10. If X = {*a*, *b*, *c*, *d*} and Y = {*f*, *b*, *d*, *g*}, find

(i) X – Y

(ii) Y – X

(iii) X ∩ Y

Solution:

(i) X – Y = {*a*, *c*}

(ii) Y – X = {*f*, *g*}

(iii) X ∩ Y = {*b*, *d*}

11. If R is the set of real numbers and Q is the set of rational numbers, then what is R – Q?

Solution:

We know that

R – Set of real numbers

Q – Set of rational numbers

Hence, R – Q is a set of irrational numbers.

12. State whether each of the following statement is true or false. Justify your answer.

(i) {2, 3, 4, 5} and {3, 6} are disjoint sets.

(ii) {*a*, *e*, *i*, *o*, *u* } and {*a*, *b*, *c*, *d*} are disjoint sets.

(iii) {2, 6, 10, 14} and {3, 7, 11, 15} are disjoint sets.

(iv) {2, 6, 10} and {3, 7, 11} are disjoint sets.

Solution:

(i) False

If $3 \in \{2, 3, 4, 5\}$, $3 \in \{3, 6\}$

So we get $\{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$

(ii) False

If $a \in \{a, e, i, o, u\}$, $a \in \{a, b, c, d\}$

So we get $\{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$

(iii) True

Here $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \Phi$

(iv) True

Here $\{2, 6, 10\} \cap \{3, 7, 11\} = \Phi$

Exercise 1.5

1. Let U = {1, 2, 3; 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find

(i) A’

(ii) B’

(iii) (A U C)’

(iv) (A U B)’

(v) (A')'

(vi) (B – C)'

Solution:

It is given that

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {1, 2, 3, 4}

B = {2, 4, 6, 8}

C = {3, 4, 5, 6}

(i) A' = {5, 6, 7, 8, 9}

(ii) B' = {1, 3, 5, 7, 9}

(iii) A U C = {1, 2, 3, 4, 5, 6}

So we get

(A U C)' = {7, 8, 9}

(iv) A U B = {1, 2, 3, 4, 6, 8}

So we get

(A U B)' = {5, 7, 9}

(v) (A')' = A = {1, 2, 3, 4}

(vi) B – C = {2, 8}

So we get

(B – C)' = {1, 3, 4, 5, 6, 7, 9}

2. If U = {*a, b, c, d, e, f, g, h*}, find the complements of the following sets:

(i) A = {*a, b, c*}

(ii) B = {*d, e, f, g*}

(iii) C = {*a, c, e, g*}

(iv) D = {*f, g, h, a*}

Solution:

(i) A = {*a*, *b*, *c*}

So we get

A’ = {d, e, f, g, h}

(ii) B = {*d, e, f, g*}

So we get

B’ = {a, b, c, h}

(iii) C = {*a, c, e, g*}

So we get

C’ = {b, d, f, h}

(iv) D = {*f, g, h, a*}

So we get

D’ = {b, c, d, e}

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) {*x*: *x* is an even natural number}

(ii) {*x*: *x* is an odd natural number}

(iii) {*x*: *x* is a positive multiple of 3}

(iv) {*x*: *x* is a prime number}

(v) {*x*: *x* is a natural number divisible by 3 and 5}

(vi) {*x*: *x* is a perfect square}

(vii) {*x*: *x* is perfect cube}

(viii) {x: x + 5 = 8}

(ix) {x: 2x + 5 = 9}

(x) {x: $x \geq 7$}

(xi) {x: $x \in$ N and 2x + 1 > 10}

Solution:

We know that

U = N: Set of natural numbers

(i) {x: x is an even natural number}´ = {x: x is an odd natural number}

(ii) {x: x is an odd natural number}´ = {x: x is an even natural number}

(iii) {x: x is a positive multiple of 3}´ = {x: $x \in$ N and x is not a multiple of 3}

(iv) {x: x is a prime number}´ ={x: x is a positive composite number and x = 1}

(v) {x: x is a natural number divisible by 3 and 5}´ = {x: x is a natural number that is not divisible by 3 or 5}

(vi) {x: x is a perfect square}´ = {x: $x \in$ N and x is not a perfect square}

(vii) {x: x is a perfect cube}´ = {x: $x \in$ N and x is not a perfect cube}

(viii) {x: x + 5 = 8}´ = {x: $x \in$ N and $x \neq 3$}

(ix) {x: 2x + 5 = 9}´ = {x: $x \in$ N and $x \neq 2$}

(x) {x: $x \geq 7$}´ = {x: $x \in$ N and $x < 7$}

(xi) {x: $x \in$ N and 2x + 1 > 10}´ = {x: $x \in$ N and $x \leq 9/2$}

4. If U = {1, 2, 3, 4, 5,6,7,8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that

(i) (A U B)’ = A’ ∩ B’

(ii) (A ∩ B)’ = A’ U B’

Solution:

It is given that

U = {1, 2, 3, 4, 5,6,7,8, 9}

A = {2, 4, 6, 8}

B = {2, 3, 5, 7}

(i) (A U B)’ = {2, 3, 4, 5, 6, 7, 8}’ = {1, 9}

A’ ∩ B’ = {1, 3, 5, 7, 9} ∩ {1, 4, 6, 8, 9} = {1, 9}

Therefore, (A U B)’ = A’ ∩ B’.

(ii) (A ∩ B)’ = {2}’ = {1, 3, 4, 5, 6, 7, 8, 9}

A’ U B’ = {1, 3, 5, 7, 9} U {1, 4, 6, 8, 9} = {1, 3, 4, 5, 6, 7, 8, 9}

Therefore, (A ∩ B)’ = A’ U B’.

5. Draw appropriate Venn diagram for each of the following:

(i) (A U B)’

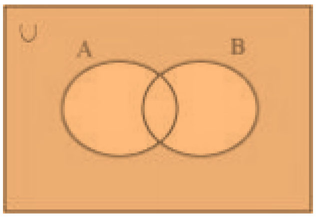
(ii) A’ ∩ B’

(iii) (A ∩ B)’

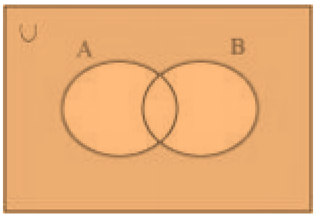
(iv) A’ U B’

Solution:

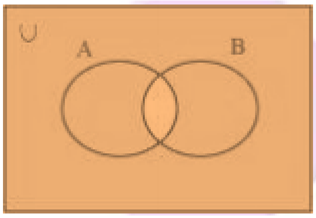
(i) (A U B)’



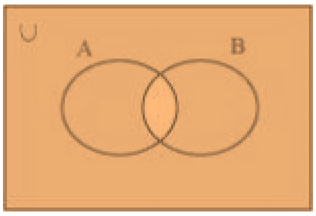
(ii) A’ ∩ B’



(iii) (A ∩ B)’



(iv) A’ U B’



6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A’?

Solution:

A’ is the set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement:

(i) A U A’ = ……..

(ii) Φ′ ∩ A = …….

(iii) A ∩ A’ = …….

(iv) U’ ∩ A = …….

Solution:

(i) A U A’ = U

(ii) Φ′ ∩ A = U ∩ A = A

So we get

Φ′ ∩ A = A

(iii) A ∩ A’ = Φ

(iv) U’ ∩ A = Φ ∩ A = Φ

So we get

U’ ∩ A = Φ

2Marks Questions & Answers

1. Are the following pair of sets equal? Give reasons.

Ans: A={x: x is a letter in the word FOLLOW}

B={x: x is a letter in the word WOLF}

Ans: We can write above mentioned sets as shown

A={F,O,L,W}

n(A)=4

B={W,O,L,F}

n(B)=4

Hence A=B.

2. Are the following pair of sets are equal? Give reasons
A, the set of letters in “ALLOY” and B, the set of letters in “LOYAL”.
Ans: The set of letters in ALLOY is written as

A= {A, L, O, Y}

n (A)=4

Similarly, the set of letters in LOYAL is written as

B= {L, O, Y, A}

n (B)=4

Hence A=B.

3. A= {1, 2, {3, 4}, 5} **which is incorrect and why.**
(i)

{3, 4}⊂A

Ans: Clearly we can see that

$$\{3, 4\} \in A$$

Hence

$$\{3, 4\} \subset A$$

is incorrect

(ii)

$$\{3, 4\} \in A$$

Ans: Clearly we can see that

$$\{3, 4\} \in A$$

Hence $\{3, 4\} \in A$ is correct.

4. Is set $C=\{x:x-5=0\}$ **and**
$E=\{x:$ x is an integral positive root of the equation $x2-2x-15=0\}$ **are equal?**
Ans: From set C we get

$$x=5$$

Hence

$$C= \{5\}$$

Also on solving the equation

$$x2-2x-15=0$$

We get the positive root as shown

$$x=5$$

$E=\{5\}$

So, $C=E$

Hence both the sets are equal.

4. Is pair of sets equal? Give reasons.

$A= (2, 3)$,$B=\{x:x$ is the solution of $x^2+5x+6=0\}$
Ans: Given we have

$$A= \{2,3\}$$

$$B=\{x:x \text{ is the solution of } x^2+5x+6\}$$

Now we can easily find the solution of

$$x^2+5x+6 \text{ to be the set } B=\{-2,-3\}$$

Hence

$$A \neq B$$

So the given pair of sets are not equal

5. If X= {a, b, c, d}
Y= {f, b, d, g}
Find X−Y **and** Y−X
Ans: We are given with the following sets

$$X=\{a, b, c, d\}$$

$$Y=\{f, b, d, g\}$$

Hence X−Y= {a, c}

Similarly, Y−X= {f, g}

6. If A={3,5,7,9,11},B={7,9,11,13},C={11,13,15} **Find** $(A\cap B)\cap(B\cup C)$
Ans: From the data given we have

$$A=\{3, 5, 7, 9, 11\}$$

$$B=\{7, 9, 11, 13\}$$

$$C=\{11, 13, 15\}$$

Now

$$A\cap B=\{7, 9, 11\}$$

$$B\cup C=\{7, 9, 11, 13, 15\}$$

Therefore

$$(A\cap B)\cap (B\cup C) = \{7, 9, 11\}$$

7. If A= [(−3,5),B=(0,6)] **then find**
(i) A−B
Ans: Given we have

$$A=(-3,5)$$

$$B=(0,6)$$

We know that

$$A-B=A\cap B'$$

Hence

$$A-B=[-3,0]$$

(ii) $A\cup B$

Ans: Given we have

$$A=(-3,5)$$

$$B=(0,6)$$

We know that $A\cup B$ means occurrence of at least one

Hence $A\cup B=[-3,6]$

8. A survey shows that 73 **percent of Indians like apples, whereas** 65**percent like oranges. What percent of Indians like both apples and oranges?**

Ans: We will use following notation

A-set of Indians who like apples

O-set of Indians who like oranges

It is given in the question that

$$n(A\cup O)=100$$

$$n(A)=73$$

$$n(O)=65$$

Now we know that

$$n(A\cup O)=n(A)+n(O)-n(A\cap O)$$

Hence on solving the above we get

$$n(A\cap O)=38$$

Therefore 38 percent of Indians like both apples and oranges

9.Let $U=\{1,2,3,4,5,6\},A=\{2,3\},B=\{3,4,5\}$**Find** $A'\cap B'$, $A\cup B$ **and hence show that** $A\cup B=A'\cap B'$**.**

Ans: We know that

$A'=U-A$

$=\{1,4,5,6\}=\{1,4,5,6\}$

$B'=U-B$

$=\{1,2,6\}=\{1,2,6\}$

$A\cup B=\{2,3,4,5\}$

$(A'\cap B')=\{1,6\}$

Hence proved.

10. for any two sets A and B prove by using properties of sets that:

$(A \cap B) \cup (A - B) = A$

Ans: We write LHS and RHS as shown

LHS= $(A \cap B) \cup (A - B)$

$= (A \cap B) \cup (A \cap B')$ (since $(A - B) = (A \cap B')$)

$= A \cap (B \cup B')$

$= A \cap (U)$

$= A$

Multiple Choice Questions

Q.1: How many elements are there in the complement of set A?

A. 0

B. 1

C. All the elements of A

D. None of these

Answer: A. 0

Explanation: The complement of a set A will contain the elements that are not present in set A.

Q.2: Empty set is a _______.

A. Infinite set

B. Finite set

C. Unknown set

D. Universal set

Answer: B. Finite set

Explanation: The cardinality of the empty set is zero, since it has no elements. Hence, the size of the empty set is zero.

Q.3: The number of elements in the Power set P(S) of the set S = {1, 2, 3} is:

A. 4

B. 8

C. 2

D. None of these

Answer: B. 8

Explanation: Number of elements in the set S = 3

Number of elements in the power set of set S = {1,2,3} = 2^3

= 8

Q.4: Order of the power set P(A) of a set A of order n is equal to:

A. n

B. 2n

C. 2^n

D. n^2

Answer: C. 2^n

Explanation: The cardinality of the power set is equal to 2^n, where n is the number of elements in a given set.

Q.5: Which of the following two sets are equal?

A. A = {1, 2} and B = {1}

B. A = {1, 2} and B = {1, 2, 3}

C. A = {1, 2, 3} and B = {2, 1, 3}

D. A = {1, 2, 4} and B = {1, 2, 3}

Answer: C. A = {1, 2, 3} and B = {2, 1, 3}

Explanation: Two sets are said to be equal if they both have the same elements.

Q.6: Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, P = {1, 2, 5}, Q = {6, 7}. Then P ∩ Q' is :

A. P

B. Q

C. Q'

D. None

Answer: A. P

Explanation: Given,

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

P = {1, 2, 5}

Q = {6, 7}

Q' = {1, 2, 3, 4, 5, 8, 9, 10}

Hence,

P ∩ Q' = {1, 2, 5} = P

Q.7: The cardinality of the power set of {x: x∈N, x≤10} is ______.

A. 1024

B. 1023

C. 2048

D. 2043

Answer: A. 1024

Explanation: Given,

Set X = {x: x∈N, x≤10}

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Number of elements of power set of X, P(X) = 2^{10} = 1024

Q.8: Write X = {1, 4, 9, 16, 25,…} in set builder form.

A. X = {x: x is a set of prime numbers}

B. X = {x: x is a set of whole numbers}

C. X = {x: x is a set of natural numbers}

D. X = {x: x is a set of square numbers}

Answer: D. X = {x: x is a set of square numbers}

Explanation: Given,

X = {1, 4, 9, 16, 25,…}

X = $\{1^2, 2^2, 3^2, 4^2, 5^2, \ldots\}$

Therefore,

X = {x: x is a set of square numbers}

Q.9: If A, B and C is any three sets, then A × (B ∪ C) is equal to:

A. (A × B) ∪ (A × C)

B. (A ∪ B) × (A ∪ C)

C. (A × B) ∩ (A × C)

D. None of the above

Answer: A. (A × B) ∪ (A × C)

Explanation: Given,

A, B and C are any three sets.

Now, A × (B ∪ C) = (A × B) ∪ (A × C)

Q.10: The range of the function f(x) = 3x – 2, is:

A. (- ∞, ∞)

B. R – {3}

C. (- ∞, 0)

D. (0, – ∞)

Answer: A. (- ∞, ∞)

Hint:

Let the given function be

$y = 3x - 2$

$\Rightarrow y + 2 = 3x$

$\Rightarrow x = (y + 2)/3$

Since, for all values of y, x has different values. Thus, values of x and y can range from -∞ to ∞.

So, Range {f(x)} = R = (-∞, ∞).

Summary

- This chapter deals with some basic definitions and operations involving sets. These are summarized below:
- A set is a well-defined collection of objects.
- A set which does not contain any element is called empty set.
- A set which consists of a definite number of elements is called finite set, otherwise, the set is called infinite set.
- Two sets A and B are said to be equal if they have exactly the same elements.
- A set A is said to be subset of a set B, if every element of A is also an element of B. Intervals are subsets of R.
- The union of two sets A and B is the set of all those elements which are either in A or in B.
- The intersection of two sets A and B is the set of all elements which are common. The difference of two sets A and B in this order is the set of elements which belong to A but not to B.

- The complement of a subset A of universal set U is the set of all elements of U which are not the elements of A.
- For any two sets A and B, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$