CHAPTER-7

Congruence of Triangles

Ex 7.1:-

Question 1

Complete the following statements:

(a) Two line segments are congruent if _______ .

(b) Among the congruent angles, one has a measure of 70°, the measure of the other angle is _______ .

(c) When we write ∠A = ∠B, we actually mean

Solution:

(a) they have the same length

(b) 70°

(c) m∠A = m ∠B

Question 2

Give any two real life examples for congruent shapes.

Solution:

- Sharing blades of the same brand.
- Biscuits of the same packets.

Question 3

If ΔABC ≅ ΔFED under the correspondence ABC ⟷ FED, write all the corresponding congruent parts of the triangles.

Solution:

Given: ΔABC = ΔFED

and ABC ⟷ FED

∴ $\overline{AB} \leftrightarrow \overline{FE}, \overline{BC} \leftrightarrow \overline{ED}, \overline{AC} \leftrightarrow \overline{FD}\ \angle A \leftrightarrow \angle F, \angle B \leftrightarrow \angle E, \angle C \leftrightarrow \angle D$

Question 4

If ΔDEF ≅ ΔBCA. Write the part of ABCA that correspond to

(i) ∠E

(ii) $\overline{EF}$

(iii) ∠ZF

(iv) $\overline{DF}$

Solution:

Given: ΔDEF ≅ ΔBCA

(i) ∠E ⟷ ∠C

(ii) ∠F ⟷ ∠A

(iii) EF ⟷ ZA

(iv) DF ⟷ BA

Question 1:

Complete the following statements:

(a) Two line segments are congruent if ________.

(b) Among two congruent angles, one has a measure of 70°; the measure of the other angle is ________.

(c) When we write $\angle A = \angle B$, we actually mean ________.

Answer:

(a) They have the same length

(b) 70°

(c) $m\angle A = m\angle B$

Question 2:

Give any two real-life examples for congruent shapes.

Answer:

(i) Sheets of same letter pad

(ii) Biscuits in the same packet

Question 3:

If /ABC ≅ /FED under the correspondence $ABC \leftrightarrow FED$, write all the Corresponding congruent parts of the triangles.

Answer:

If these triangles are congruent, then the corresponding angles and sides will be equal to each other.

$\angle A \leftrightarrow \angle F$

$\angle B \leftrightarrow \angle E$

$\angle C \leftrightarrow \angle D$

$\overline{AB} \leftrightarrow \overline{FE}$

$\overline{CA} \leftrightarrow \overline{DF}$

Question 4:

If /DEF ≅ /BCA, write the part(s) of /BCA that correspond to

(i) ∠E (ii) $\overline{EF}$ (iii) ∠F (iv) $\overline{DF}$

Answer:

(i) ∠C

(ii) $\overline{CA}$

(iii) ∠A

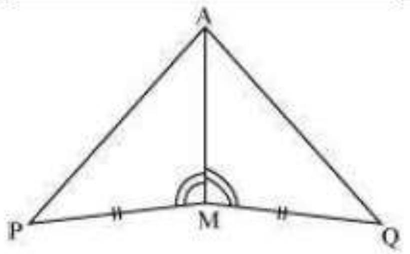
(iv) $\overline{BA}$

Question 5

You have to show that /AMP ≅ AMQ.

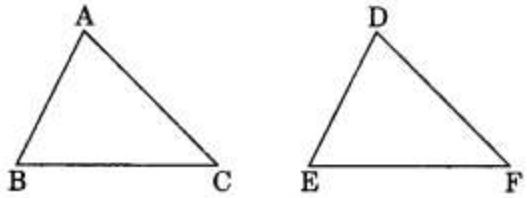
In the following proof, supply the missing reasons.

-	Steps	-	Reasons
(i)	PM = QM	(i)	...
(ii)	∠PMA = ∠QMA	(ii)	...
(iii)	AM = AM	(iii)	...
(iv)	ΔAMP ≅ ΔAMQ	(iv)	...

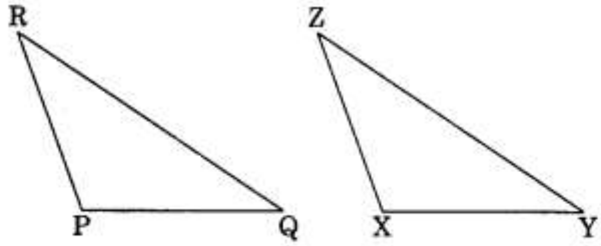


Ex 7.2 :-

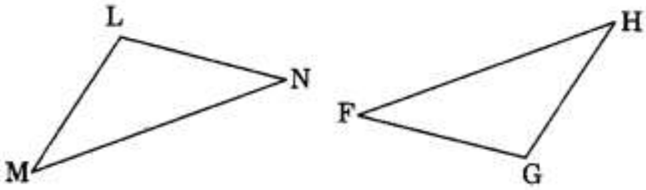
Question 1
Which congruence criterion do you use in the following?
(a) Given:
AC = DF
AB = DE
BC = EF
So, ΔABC = ΔDEF



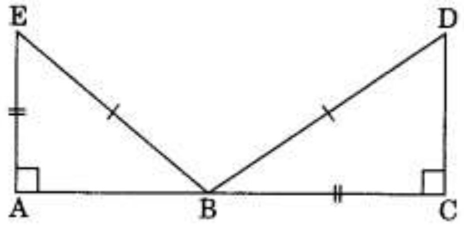
(b) Given:
ZX = RP
RQ = ZY
∠PRQ = ∠XZY
So, ΔPQR ≅ ΔXYZ



(c) Given: ∠MLN = ∠FGH
∠NML = ∠GFH
ML = FG
So, ΔLMN = ΔGFH

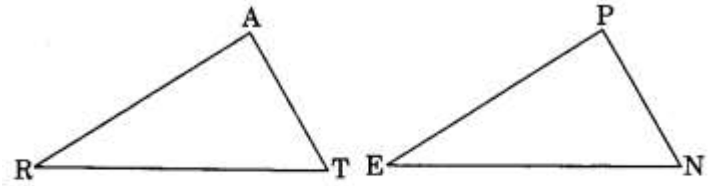


(d) Given:
EB = DB
AE = BC
∠A = ∠C = 90°
ΔABE = ΔCDB



Solution:
(a) ΔABC ≅ ΔDEF (BY SSS rule)
(b) ΔPQR ≅ ΔXYZ (BY SAS rule)
(c) ΔLMN ≅ ΔGFH (BY ASA rule)
(d) ΔABE ≅ ΔCDB (BY RHS rule)

Question 2
You want to show that ΔART = ΔPEN,
(a) If you have to use SSS criterion, then you need to show
(i) AR =
(ii) RT =
(iii) AT =

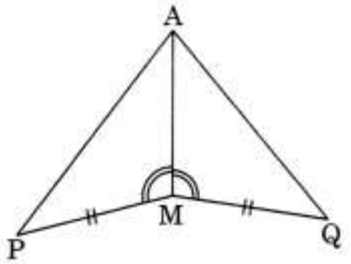


(b) If it is given that ∠T = ∠N and you are to use SAS criterion, you need to have
(i) RT = and
(ii) PN =
(c) If it is given that AT = PN and you are to use ASA criterion, you need to have
(i) ZA
(ii) ZT
Solution:
(a) For SSS criterion, we need
(i) AR = PE
(ii) RT = EN
(iii) AT = PN
(b) For SAS criterion, we need
(i) RT = EN and
(ii) PN = AT
(c) For ASA criterion, we need
(i) ∠A = ∠P
(ii) ∠T = ∠N

Question 3

You have to show that ΔAMP ≅ ΔAMQ. In the fallowing proof, supply the missing reasons.

Steps	Reasons
(i) PM = QM	(i)
(ii) ∠PMA − ∠QMA	(ii)
(iii) AM = AM	(iii)
(iv) ΔAMP = ΔAMQ	(iv)



Solution:

Steps	Reasons
(i) PM = QM	(i) Given
(ii) ∠PMA = ∠QMA	(ii) Given
(iii) AM = AM	(iii) Common
(iv) ΔAMP = ΔAMQ	(iv) SAS rule

Question 4

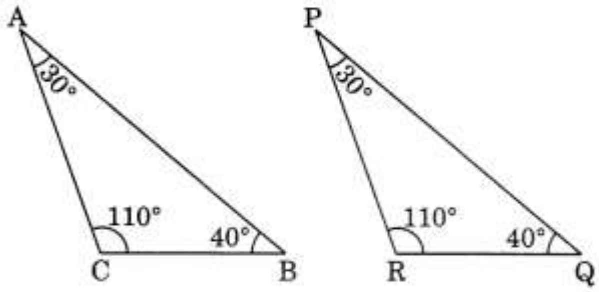
In ΔABC, ∠A = 30°, ∠B = 40° and ∠C = 110°

In ΔPQR, ∠P = 30°, ∠Q = 40° and ∠R = 110°.

A student says that ΔABC = ΔPQR by AAA congruence criterion. Is he justified? Why or why not?

Solution:

The student is not justified because there is not criterion for AAA congruence rule.



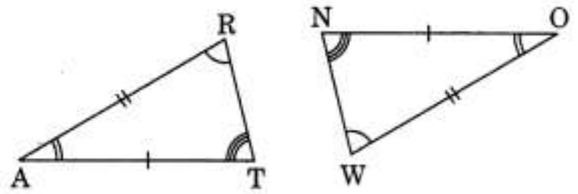
Example: In ΔABC and ΔPQR, we have ∠A = 30°, ∠B = 40°, ∠C = 110°

∠P = 30°, ∠Q = 40°, ∠R = 110°

But ΔABC is not congruent to ΔPQR.

Question 5

In the figure, the two triangles are congruent. The corresponding parts are marked. We can write ΔRAT ≅ ?



Solution:

In ΔRAT and ΔWON

$\overline{AT} = \overline{ON}$ (Given)

$\overline{AR} = \overline{OW}$ (Given)

∠A = ∠O (Given)

∴ ΔRAT ≅ ΔWON (By SAS rule)

Question 6

Complete the congruence statement:

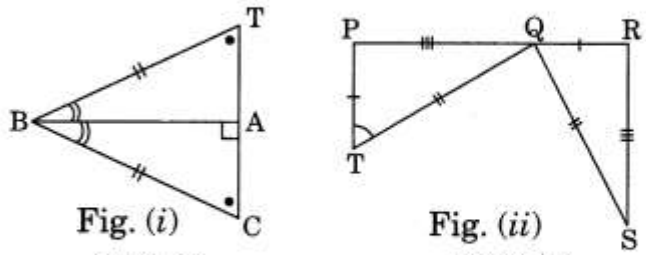


Fig. (i) ΔBCA ≅ ?

Fig. (ii) ΔQRS ≅ ?

Solution:

Refer to Fig. (i)

In ΔBCA and ΔBTA

∠C = ∠T (Given)

$\overline{BC} = \overline{BT}$ (Given)

∠BA = ∠TBA (Given)

∴ ΔBCA = ΔBTA (by ADA rule)

Refer to Fig. (ii)

In ΔQRS and ΔTPQ

$\overline{RS} = \overline{PQ}$ (Given)

$\overline{QS} = \overline{TQ}$ (Given)

∠RSQ = ∠PQT (Given)

∴ ΔQRS = ΔTPQ (by SAS rule)

Question 7

In a squared sheet, draw two triangles of equal areas such that:

(i) the triangles are congruent.

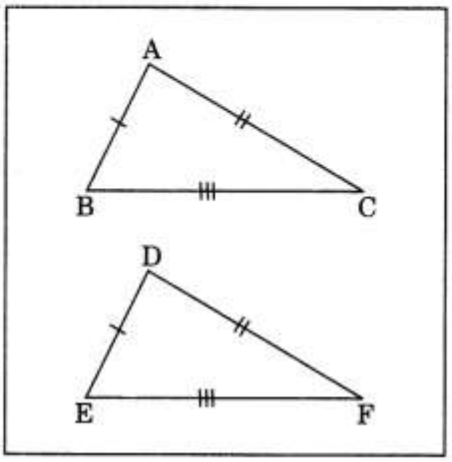
(ii) the triangle are not congruent.

What can you say about their perimeters?

Solution:

(i) On the given square sheet, we have draw two congruent triangles i.e.

ΔABC = ΔDEF



such that

$\overline{AB} = \overline{DE}, \overline{BC} = \overline{EF}$ and $\overline{AC} = \overline{DF}$

On adding, we get

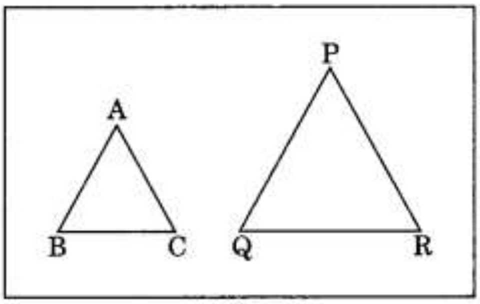
$\overline{AB} + \overline{BC} + \overline{AC} = \overline{DE} + \overline{EF} + \overline{DF}$

i.e. perimeters of ΔABC = Perimeter of ΔDEF

(ii) On the other square sheet, we have drawn two triangles ABC and PQR which are not congruent.

Such that

$$\overline{AB} \neq \overline{PQ}$$
$$\overline{BC} \neq \overline{QR}$$
and $$\overline{AC} \neq \overline{PR}$$



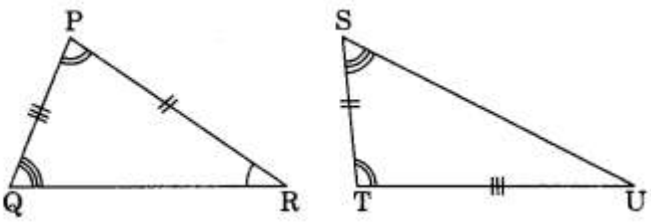
Adding both sides, we get

$\overline{AB} + \overline{BC} + \overline{AC} \neq \overline{PQ} + \overline{QR} + \overline{PR}$

i.e., perimeter of ΔABC ≠ the perimeter of ΔPQR.

Question 8

Draw a rough sketch of two triangles, such that they have five pairs of congruent parts but still
the triangles are not congruent.



Solution:

We have ΔPQR and ΔTSU

$\overline{PQ}=\overline{SU}$ (Given)

$\overline{PR}=\overline{ST}$ (Given)

∠Q = ∠S (Given)

∠P = ∠T (Given)

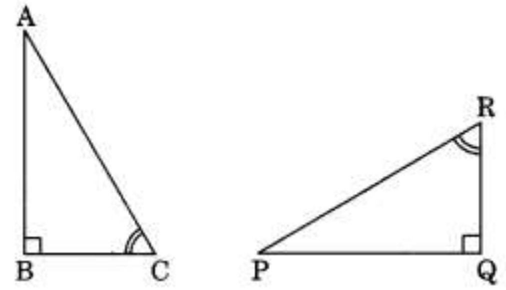
∠R = ∠U (Given)

Since non of the criteria of congruence is relevant here.

∴ ΔPQR and ΔTSU are not congruent.

Question 9

If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



Solution:

In ΔABC and ΔPQR

∠B = ∠Q (Given)

∠C = ∠R (Given)

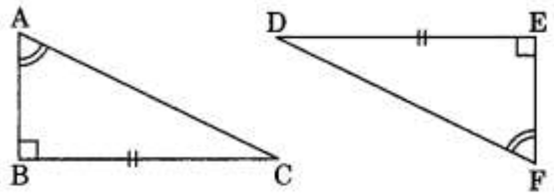
For ΔABC = ΔPQR

BC must equal to $\overline{QR}$ criterion that we used is ASA rule.

Hence, the additional pair of corresponding part is $\overline{BC}=\overline{QR}$.

Question 10

Explain, why ΔABC = ΔFED



Solution:

In ΔABC and ΔFED

∠B = ∠E = 90° (Given)

∠A = ∠F (Given)

∴ ∠A + ∠B = ∠E + ∠F

180° − ∠C = 180° − ∠D

[Angle sum property of triangles]

∴ ∠C =∠D

BC = ED (Given)

∴ ΔABC = ΔFED (By ASA rule)

Question 1:

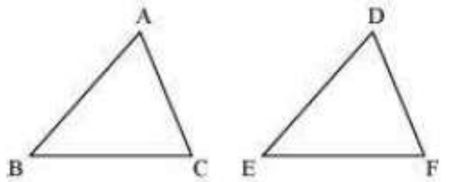
Which congruence criterion do you use in the following?

(a) **Given:** AC = DF

AB = DE

BC = EF

So, /ABC ≅ /DEF

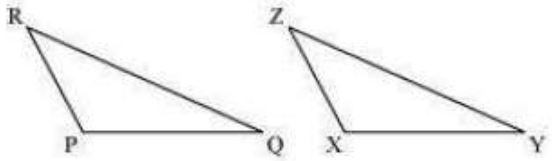


(b) **Given:** ZX = RP

RQ = ZY

∠PRQ = ∠XZY

So, /PQR ≅ /XYZ

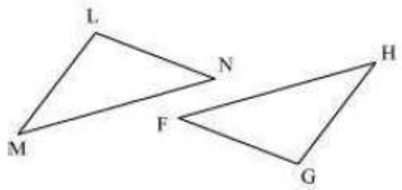


(c) **Given:** ∠MLN = ∠FGH

∠NML = ∠GFH

ML = FG

So, /LMN ≅ /GFH

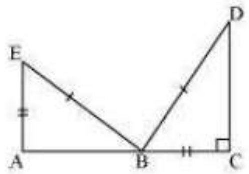


(d) **Given:** EB = DB

AE = BC

∠A = ∠C = 90°

So, /ABE ≅ /CDB



Answer:

(a) SSS, as the sides of /ABC are equal to the sides of /DEF.

(b) SAS, as two sides and the angle included between these sides of /PQR are equal to two sides and the angle included between these sides of /XYZ.

(c) ASA, as two angles and the side included between these angles of /LMN are equal to two angles and the side included between these angles of /GFH.

(d) RHS, as in the given two right-angled triangles, one side and the hypotenuse are respectively equal.

Question 2:

You want to show that /ART ≅ /PEN,

(a) If you have to use SSS criterion, then you need to show

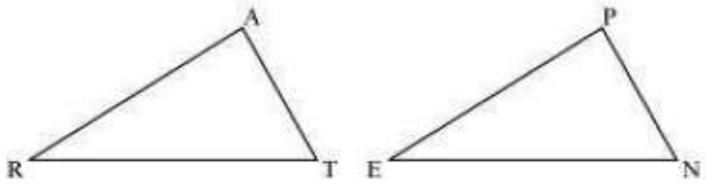
(i) AR = (ii) RT = (iii) AT =

(b) If it is given that ∠T = ∠N and you are to use SAS criterion, you need to have

(i) RT = and (ii) PN =

(c) If it is given that AT = PN and you are to use ASA criterion, you need to have

(i) ? (ii) ?



Answer:

(a)

(i) AR = PE

(ii) RT = EN

(iii) AT = PN

(b)

(i) RT = EN

(ii) PN = AT

(c)

(i) ∠ATR = ∠PNE

(ii) ∠RAT = ∠EPN

Answer:

(i) Given

(ii) Given

(iii) Common

(iv) SAS, as the two sides and the angle included between these sides of /AMP are equal to two sides and the angle included between these sides of /AMQ.

Question 4:

In /ABC, ∠A = 30°, ∠B = 40° and ∠C = 110°

In /PQR, ∠P = 30°, ∠Q = 40° and ∠R = 110°

A student says that /ABC ≅ /PQR by AAA congruence criterion. Is he justified? Why or why not?

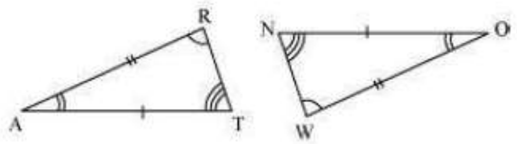
Answer:

No. This property represents that these triangles have their respective angles of equal measure. However, this gives no information about their sides. The sides of these triangles have a ratio somewhat different than 1:1. Therefore, AAA property does not prove the two triangles congruent.

Question 5:

In the figure, the two triangles are congruent.

The corresponding parts are marked. We can write /RAT ≅ ?



Answer:

It can be observed that,

∠RAT = ∠WON

∠ART = ∠OWN

AR = OW

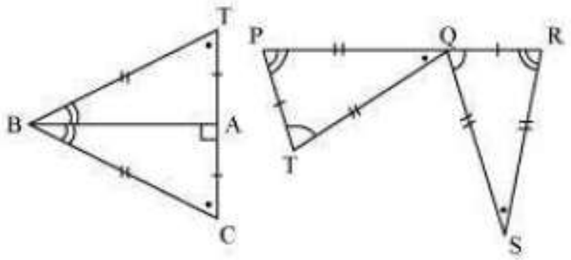
Therefore, /RAT /WON, by ASA criterion.

Question 6:

Complete the congruence statement:

/BCA ≅?

/QRS ≅?



Answer:

Given that, BC = BT

TA = CA

BA is common.

Therefore, /BCA /BTA

Similarly, PQ = RS

TQ = QS

PT = RQ

Therefore, /QRS /TPQ

Question 7:

In a squared sheet, draw two triangles of equal areas such that

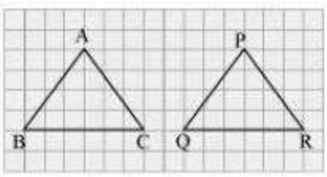
(i) The triangles are congruent.

(ii) The triangles are not congruent.

What can you say about their perimeters?

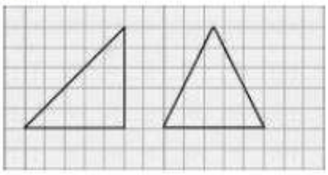
Answer:

(i)



Here, /ABC and /PQR have the same area and are congruent to each other also. Also, the perimeter of both the triangles will be the same.

(ii)



Here, the two triangles have the same height and base. Thus, their areas are equal. However, these triangles are not congruent to each other. Also, the perimeter of both the triangles will not be the same.

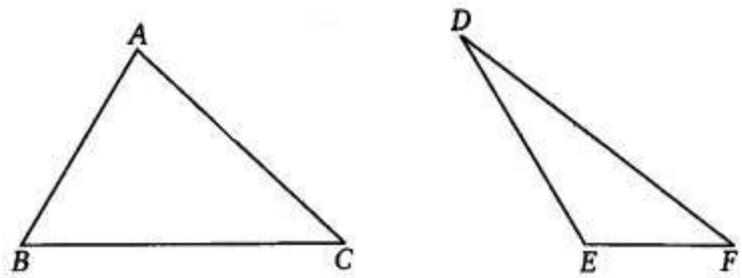
Question 8.

Draw a rough sketch of two triangles such that they have five pairs of congruent parts but still the triangles are not congruent.

Solution:

In some special cases (which depend on the lengths of the sides and the size of the angle involved),

SSA is enough to show congruence. However, it is not always enough. Consider the following triangles :



Here side AB is congruent to side DE (S) side AC is congruent to side DF(S) angle C is congruent to angle F(A)

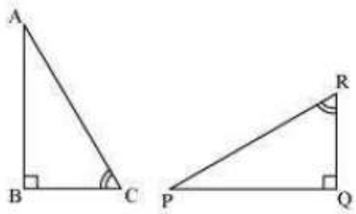
But the triangles are not congruent, as we can see.

What happens is this : If we draw a vertical line through point A in the first triangle, we can sort of “flip” side AB around this line to get the second triangle. If we were to lay one triangle on top of the other and draw the vertical line, this how it would look.

Clearly, side DE is just side AB flipped around the line. So, we have not changed the length of the side, and the other side AC (or DF) is unchanged, as is angle C (or F). So, these two triangles that have the same SSA information, but they are not congruent.

Question 9:

If /ABC and /PQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



Answer:

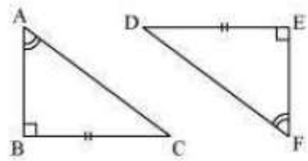
BC = QR

/ABC /PQR (ASA criterion)

Question 10:

Explain, why

/ABC ≅ /FED



Answer:

Given that, ∠ABC = ∠FED (1)

∠BAC = ∠EFD (2)

The two angles of /ABC are equal to the two respective angles of /FED. Also, the sum of all interior angles of a triangle is 180°. Therefore, third angle of both triangles will also be equal in measure.

∠BCA = ∠EDF (3)