Chapter-6

Triangles

2 MARKS QUESTIONS

1. Fill in the blanks using correct word given in the brackets:-

(i) All circles are __________. (congruent, similar)

Answer: Similar

(ii) All squares are __________. (similar, congruent)

Answer: Similar

(iii) All __________ triangles are similar. (isosceles, equilateral)

Answer: Equilateral

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are __________ and (b) their corresponding sides are __________. (equal, proportional)

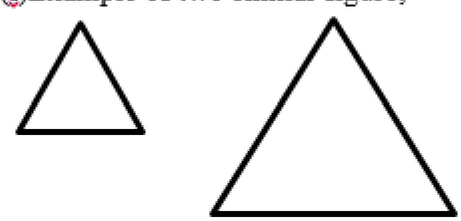
Answer: (a) Equal

(b) Proportional

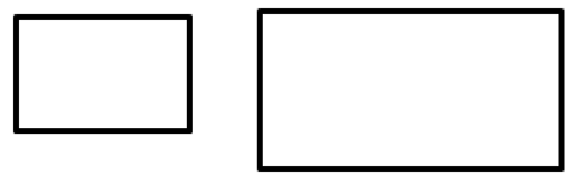
2. Give two different examples of pair of
(i) Similar figures
(ii) Non-similar figures

Solution:

(i)Example of two similar figure;

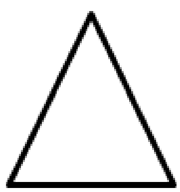


Two Equilateral Triangle

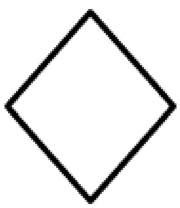


Two Rectangle

(ii) Example of two Non-similar figure;

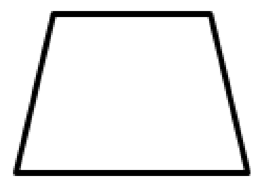


Triangle



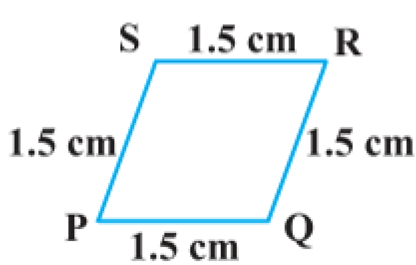
Rhombus

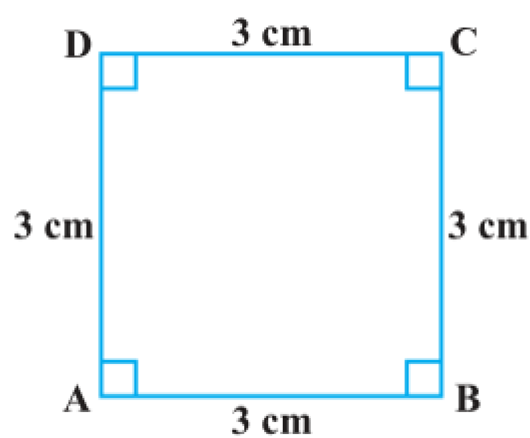
Rectangle



Trapezium

3. State whether the following quadrilaterals are similar or not:

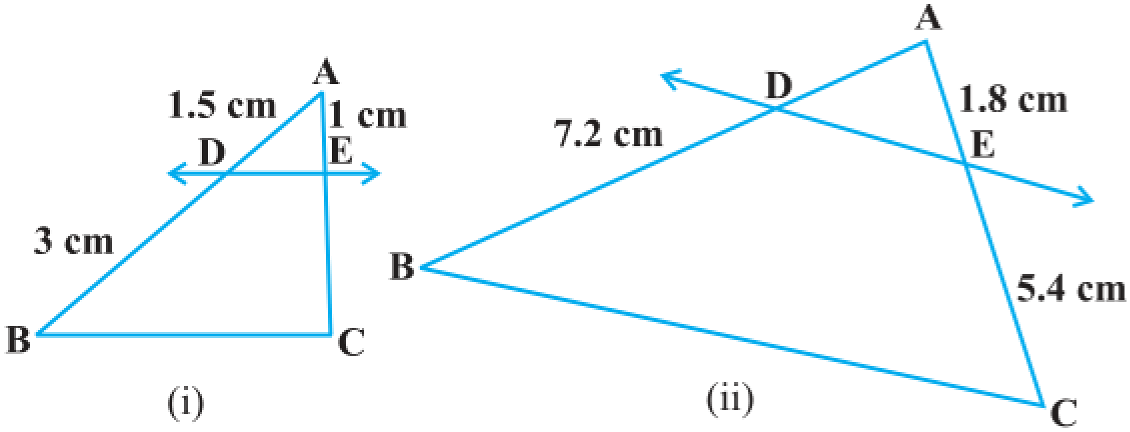




Solution:

From the given two figures, we can see their corresponding angles are different or unequal. Therefore, they are not similar.

4. In figure. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



Solution:

(i) Given, in △ ABC, DE∥BC

∴ AD/DB = AE/EC [Using Basic proportionality theorem]

⇒1.5/3 = 1/EC

⇒EC = 3/1.5

EC = 3×10/15 = 2 cm

Hence, EC = 2 cm.

(ii) Given, in △ ABC, DE∥BC

∴ AD/DB = AE/EC [Using Basic proportionality theorem]

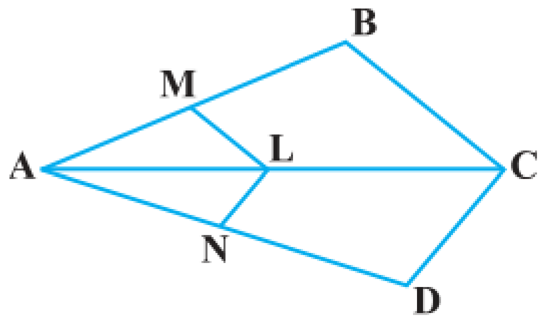
⇒ AD/7.2 = 1.8 / 5.4

⇒ AD = 1.8 ×7.2/5.4 = (18/10)×(72/10)×(10/54) = 24/10

⇒ AD = 2.4

Hence, AD = 2.4 cm.

5. In the figure, if LM || CB and LN || CD, prove that AM/AB = AN/AD



Solution:

In the given figure, we can see, LM || CB,

By using basic proportionality theorem, we get,

AM/AB = AL/AC……………………..**(i)**

Similarly, given, LN || CD and using basic proportionality theorem,

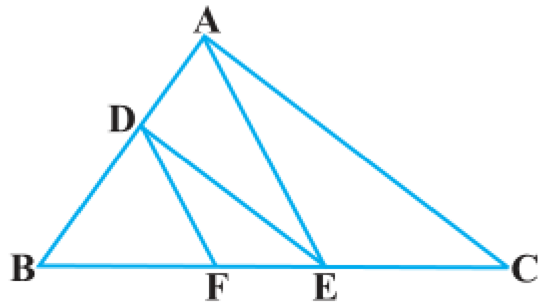
∴AN/AD = AL/AC……………………………**(ii)**

From equation **(i)** and **(ii)**, we get,

AM/AB = AN/AD

Hence, proved.

6. In the figure, DE||AC and DF||AE. Prove that BF/FE = BE/EC



Solution:

In ΔABC, given as, DE || AC

Thus, by using Basic Proportionality Theorem, we get,

∴BD/DA = BE/EC ………………………………………………**(i)**

In ΔBAE, given as, DF || AE

Thus, by using Basic Proportionality Theorem, we get,

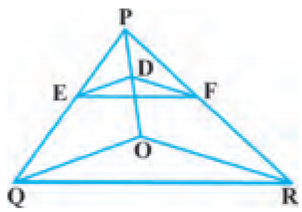
∴BD/DA = BF/FE ………………………………………………**(ii)**

From equation **(i)** and **(ii)**, we get

BE/EC = BF/FE

Hence, proved.

7. In the figure, DE||OQ and DF||OR, show that EF||QR.



Solution:

Given,

In ΔPQO, DE || OQ

So by using Basic Proportionality Theorem,

PD/DO = PE/EQ……………… **..(i)**

Again given, in ΔPOR, DF || OR,

So by using Basic Proportionality Theorem,

PD/DO = PF/FR………………… **(ii)**

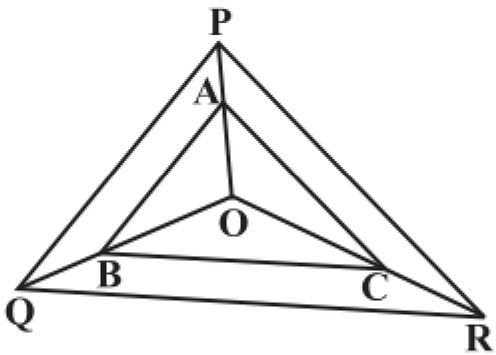
From equation **(i)** and **(ii)**, we get,

PE/EQ = PF/FR

Therefore, by converse of Basic Proportionality Theorem,

EF || QR, in ΔPQR.

8. In the figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Solution:

Given here,

In ΔOPQ, AB || PQ

By using Basic Proportionality Theorem,

OA/AP = OB/BQ…………….**(i)**

Also given,

In ΔOPR, AC || PR

By using Basic Proportionality Theorem

∴ OA/AP = OC/CR……………(ii)

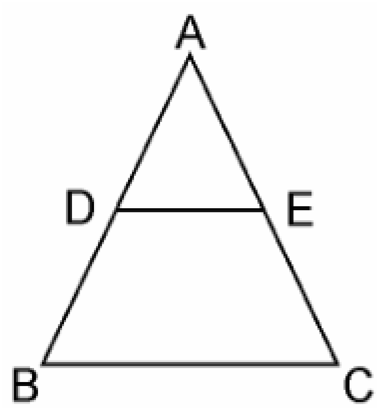
From equation **(i)** and **(ii)**, we get,

OB/BQ = OC/CR

Therefore, by converse of Basic Proportionality Theorem,

In ΔOQR, BC || QR.

9. Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



Solution:

Given, in ΔABC, D is the midpoint of AB such that AD=DB.

A line parallel to BC intersects AC at E as shown in above figure such that DE || BC.

We have to prove that E is the mid point of AC.

Since, D is the mid-point of AB.

∴ AD=DB

⇒AD/DB = 1 …………………………. **(i)**

In ΔABC, DE || BC,

By using Basic Proportionality Theorem,

Therefore, AD/DB = AE/EC

From equation (i), we can write,

⇒ 1 = AE/EC

∴ AE = EC

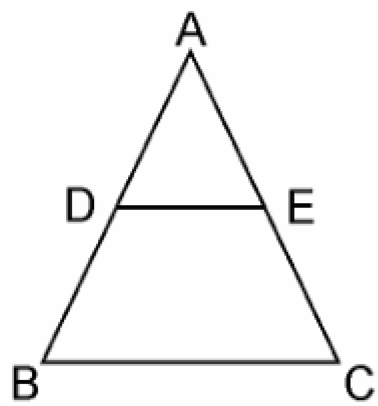
Hence, proved, E is the midpoint of AC.

10. Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Solution:

Given, in ΔABC, D and E are the mid points of AB and AC, respectively, such that,

AD=BD and AE=EC.



We have to prove that: DE || BC.

Since, D is the midpoint of AB

∴ AD=DB

⇒AD/BD = 1………………………………. **(i)**

Also given, E is the mid-point of AC.

∴ AE=EC

⇒ AE/EC = 1

From equation **(i)** and **(ii)**, we get,

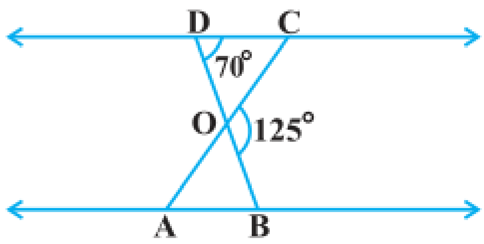
AD/BD = AE/EC

By converse of Basic Proportionality Theorem,

DE || BC

Hence, proved.

11. In figure 6.35, ΔODC ~ ΔOBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.



Solution:

As we can see from the figure, DOB is a straight line.

Therefore, ∠DOC + ∠ COB = 180°

⇒ ∠DOC = 180° – 125° (Given, ∠ BOC = 125°)

= 55°

In ΔDOC, sum of the measures of the angles of a triangle is 180º

Therefore, ∠DCO + ∠ CDO + ∠ DOC = 180°

⇒ ∠DCO + 70º + 55º = 180°(Given, ∠ CDO = 70°)

⇒ ∠DCO = 55°

It is given that, ΔODC ~ ΔOBA,

Therefore, ΔODC ~ ΔOBA.

Hence, corresponding angles are equal in similar triangles

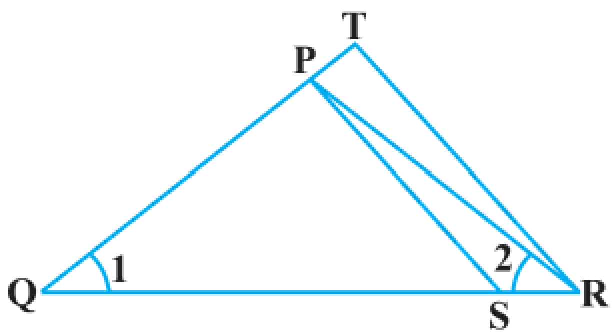
∠OAB = ∠OCD

⇒ ∠ OAB = 55°

∠OAB = ∠OCD

⇒ ∠OAB = 55°

12. In the fig.6.36, QR/QS = QT/PR and ∠1 = ∠2. Show that ΔPQS ~ ΔTQR.



Solution:

In ΔPQR,

∠PQR = ∠PRQ

∴ PQ = PR ……………………….**(i)**

Given,

QR/QS = QT/PRUsing equation **(i)**, we get

QR/QS = QT/QP**……………….(ii)**

In ΔPQS and ΔTQR, by equation (ii),

QR/QS = QT/QP

∠Q = ∠Q

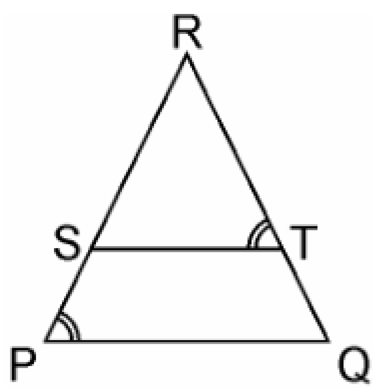
∴ ΔPQS ~ ΔTQR [By SAS similarity criterion]

13. S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Solution:

Given, S and T are point on sides PR and QR of ΔPQR

And ∠P = ∠RTS.



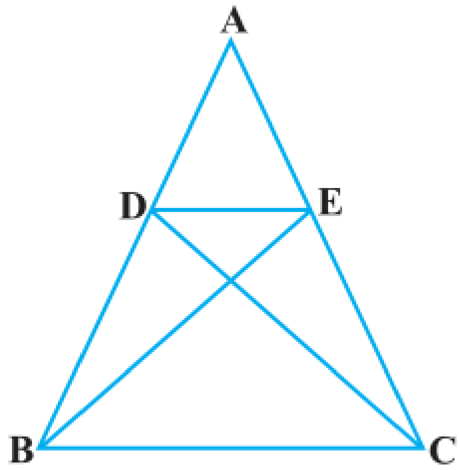
In ΔRPQ and ΔRTS,

∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (AA similarity criterion)

14. In the figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Solution:

Given, ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] ………………………………**(i)**

And, AD = AE [By CPCT] ……………………………**(ii)**

In ΔADE and ΔABC, dividing eq.(ii) by eq(i),

AD/AB = AE/AC

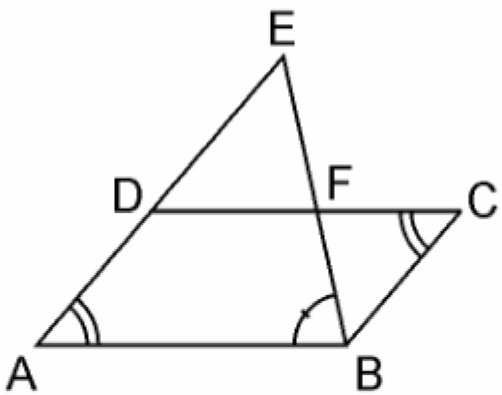
∠A = ∠A [Common angle]

∴ ΔADE ~ ΔABC [SAS similarity criterion]

15. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.

Solution:

Given, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Consider the figure below,



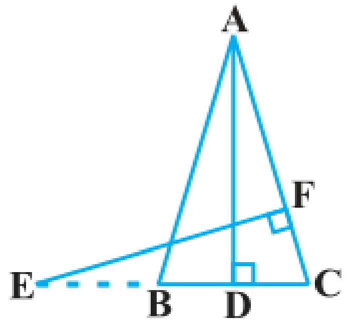
In ΔABE and ΔCFB,

∠A = ∠C (Opposite angles of a parallelogram)

∠AEB = ∠CBF (Alternate interior angles as AE || BC)

∴ ΔABE ~ ΔCFB (AA similarity criterion)

16. In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF.



Solution:

Given, ABC is an isosceles triangle.

∴ AB = AC

⇒ ∠ABD = ∠ECF

In ΔABD and ΔECF,

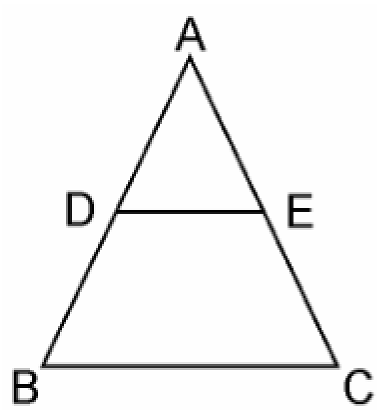
∠ADB = ∠EFC (Each 90°)

∠BAD = ∠CEF (Already proved)

∴ ΔABD ~ ΔECF (using AA similarity criterion)

4 MARKS QUESTIONS

1. Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



Solution:

Given, in ΔABC, D is the midpoint of AB such that AD=DB.

A line parallel to BC intersects AC at E as shown in above figure such that DE || BC.

We have to prove that E is the mid point of AC.

Since, D is the mid-point of AB.

∴ AD=DB

⇒AD/DB = 1 ………………………….. **(i)**

In ΔABC, DE || BC,

By using Basic Proportionality Theorem,

Therefore, AD/DB = AE/EC

From equation (i), we can write,

⇒ 1 = AE/EC

∴ AE = EC

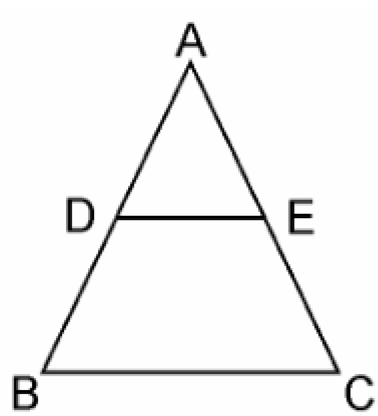
Hence, proved, E is the midpoint of AC.

2. Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Solution:

Given, in ΔABC, D and E are the mid points of AB and AC, respectively, such that,

AD=BD and AE=EC.



We have to prove that: DE || BC.

Since, D is the midpoint of AB

∴ AD=DB

⇒AD/BD = 1………………………………. **(i)**

Also given, E is the mid-point of AC.

∴ AE=EC

⇒ AE/EC = 1

From equation **(i)** and **(ii)**, we get,

AD/BD = AE/EC

By converse of Basic Proportionality Theorem,

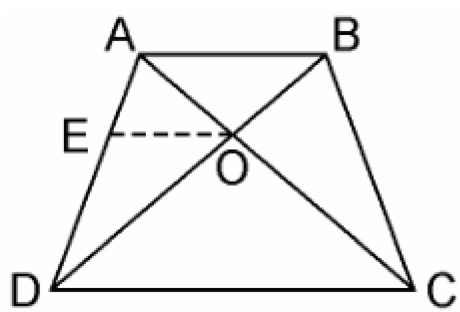
DE || BC

Hence, proved.

3. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/BO = CO/DO.

Solution:

Given, ABCD is a trapezium where AB || DC and diagonals AC and BD intersect each other at O.



We have to prove, AO/BO = CO/DO

From the point O, draw a line EO touching AD at E, in such a way that,

EO || DC || AB

In ΔADC, we have OE || DC

Therefore, by using Basic Proportionality Theorem

AE/ED = AO/CO **……………..(i)**

Now, In ΔABD, OE || AB

Therefore, by using Basic Proportionality Theorem

DE/EA = DO/BO**…………….(ii)**

From equation **(i)** and **(ii)**, we get,

AO/CO = BO/DO

⇒AO/BO = CO/DO

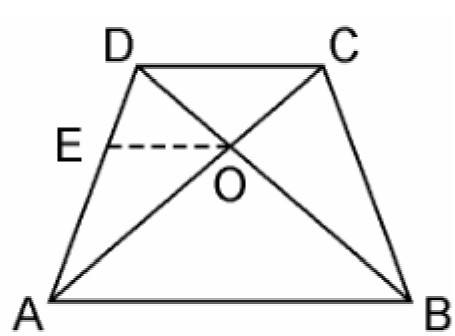
Hence, proved.

4. The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/BO = CO/DO. Show that ABCD is a trapezium.

Solution:

Given, Quadrilateral ABCD where AC and BD intersect each other at O such that,

AO/BO = CO/DO.



We have to prove here, ABCD is a trapezium

From the point O, draw a line EO touching AD at E, in such a way that,

EO || DC || AB

In ΔDAB, EO || AB

Therefore, by using Basic Proportionality Theorem

DE/EA = DO/OB ……………………(i)

Also, given,

AO/BO = CO/DO

⇒ AO/CO = BO/DO

⇒ CO/AO = DO/BO

⇒DO/OB = CO/AO **…………………………..(ii)**

From equation **(i)** and **(ii)**, we get

DE/EA = CO/AO

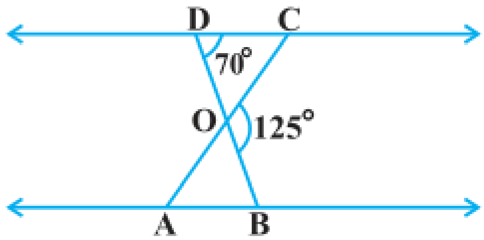
Therefore, by using converse of Basic Proportionality Theorem,

EO || DC also EO || AB

⇒ AB || DC.

Hence, quadrilateral ABCD is a trapezium with AB || CD.

5. In figure 6.35, ΔODC ~ ΔOBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.



Solution:

As we can see from the figure, DOB is a straight line.

Therefore, ∠DOC + ∠ COB = 180°

⇒ ∠DOC = 180° – 125° (Given, ∠ BOC = 125°)

= 55°

In ΔDOC, sum of the measures of the angles of a triangle is 180º

Therefore, ∠DCO + ∠ CDO + ∠ DOC = 180°

⇒ ∠DCO + 70º + 55º = 180°(Given, ∠ CDO = 70°)

⇒ ∠DCO = 55°

It is given that, ΔODC ~ ΔOBA,

Therefore, ΔODC ~ ΔOBA.

Hence, corresponding angles are equal in similar triangles

∠OAB = ∠OCD

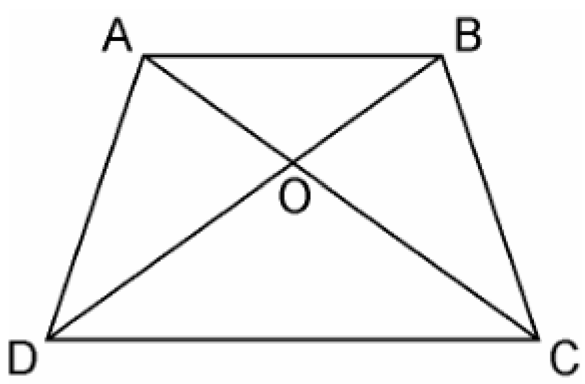
⇒ ∠ OAB = 55°

∠OAB = ∠OCD

⇒ ∠OAB = 55°

6. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that AO/OC = OB/OD

Solution:



In ΔDOC and ΔBOA,

AB || CD, thus alternate interior angles will be equal,

∴∠CDO = ∠ABO

Similarly,

∠DCO = ∠BAO

Also, for the two triangles ΔDOC and ΔBOA, vertically opposite angles will be equal;

∴∠DOC = ∠BOA

Hence, by AAA similarity criterion,

ΔDOC ~ ΔBOA

Thus, the corresponding sides are proportional.

DO/BO = OC/OA

⇒OA/OC = OB/OD

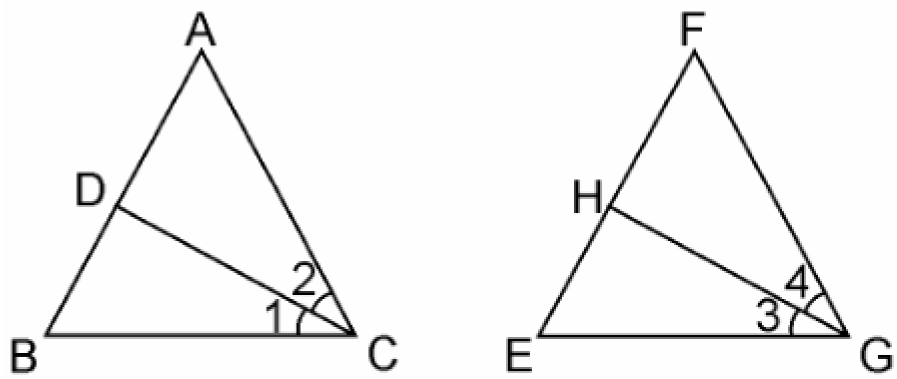
Hence, proved.

7. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) CD/GH = AC/FG
(ii) ΔDCB ~ ΔHGE
(iii) ΔDCA ~ ΔHGF

Solution:

Given, CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG, respectively.



(i) From the given condition,

ΔABC ~ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

Since, ∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,

$\angle A = \angle F$

$\angle ACD = \angle FGH$

$\therefore \Delta ACD \sim \Delta FGH$ (AA similarity criterion)

$\Rightarrow CD/GH = AC/FG$

(ii) In ΔDCB and ΔHGE,

$\angle DCB = \angle HGE$ (Already proved)

$\angle B = \angle E$ (Already proved)

$\therefore \Delta DCB \sim \Delta HGE$ (AA similarity criterion)

(iii) In ΔDCA and ΔHGF,

$\angle ACD = \angle FGH$ (Already proved)

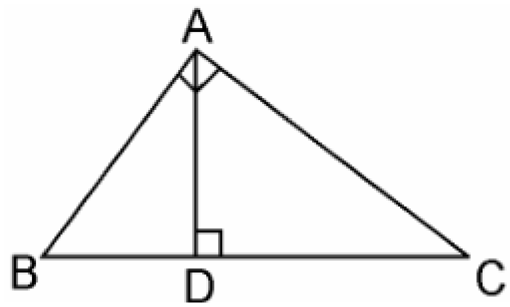
$\angle A = \angle F$ (Already proved)

$\therefore \Delta DCA \sim \Delta HGF$ (AA similarity criterion)

8. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$

Solution:

Given, D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$.



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Already given)

∠ACD = ∠BCA (Common angles)

∴ ΔADC ~ ΔBAC (AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

∴ CA/CB = CD/CA

⇒ $CA^2 = CB.CD$.

Hence, proved.

9. A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

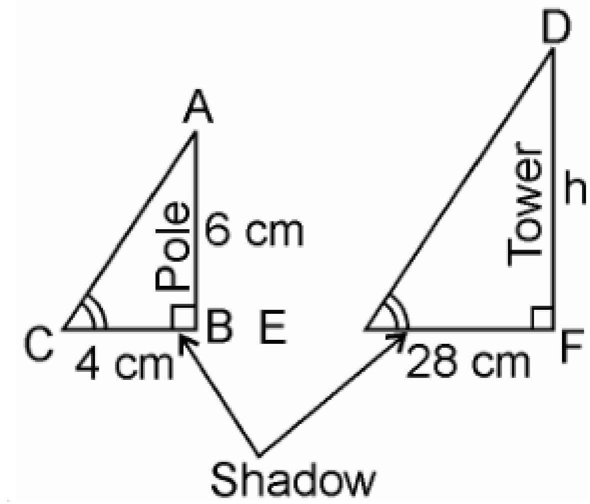
Solution:

Given, Length of the vertical pole = 6m

Shadow of the pole = 4 m

Let Height of tower = *h* m

Length of shadow of the tower = 28 m



In ΔABC and ΔDEF,

$\angle C = \angle E$ (angular elevation of sum)

$\angle B = \angle F = 90°$

$\therefore \Delta ABC \sim \Delta DEF$ (AA similarity criterion)

$\therefore AB/DF = BC/EF$ (If two triangles are similar corresponding sides are proportional)

$\therefore 6/h = 4/28$

$\Rightarrow h = (6 \times 28)/4$

$\Rightarrow h = 6 \times 7$

$\Rightarrow h = 42$ m

Hence, the height of the tower is 42 m.

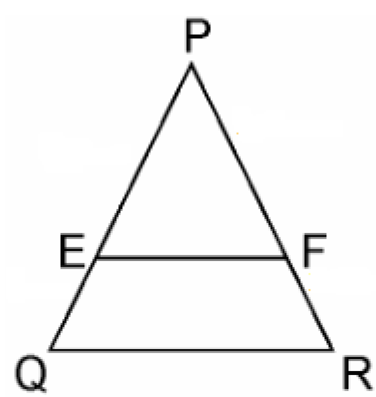
7 MARKS QUESTIONS

1. E and F are points on the sides PQ and PR, respectively of a ΔPQR. For each of the following cases, state whether EF || QR.
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm
(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Solution:

Given, in ΔPQR, E and F are two points on side PQ and PR, respectively. See the figure below;



(i) Given, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2,4 cm

Therefore, by using Basic proportionality theorem, we get,

PE/EQ = 3.9/3 = 39/30 = 13/10 = 1.3

And PF/FR = 3.6/2.4 = 36/24 = 3/2 = 1.5

So, we get, PE/EQ ≠ PF/FR

Hence, EF is not parallel to QR.

(ii) Given, PE = 4 cm, QE = 4.5 cm, PF = 8cm and RF = 9cm

Therefore, by using Basic proportionality theorem, we get,

PE/QE = 4/4.5 = 40/45 = 8/9

And, PF/RF = 8/9

So, we get here,

PE/QE = PF/RF

Hence, EF is parallel to QR.

(iii) Given, PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

From the figure,

EQ = PQ – PE = 1.28 – 0.18 = 1.10 cm

And, FR = PR – PF = 2.56 – 0.36 = 2.20 cm

So, PE/EQ = 0.18/1.10 = 18/110 = 9/55**…………. (i)**

And, PE/FR = 0.36/2.20 = 36/220 = 9/55**………… (ii)**

So, we get here,

PE/EQ = PF/FR

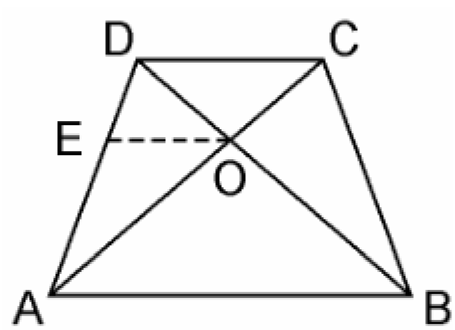
Hence, EF is parallel to QR.

2. The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/BO = CO/DO. Show that ABCD is a trapezium.

Solution:

Given, Quadrilateral ABCD where AC and BD intersect each other at O such that,

AO/BO = CO/DO.



We have to prove here, ABCD is a trapezium

From the point O, draw a line EO touching AD at E, in such a way that,

EO || DC || AB

In ΔDAB, EO || AB

Therefore, by using Basic Proportionality Theorem

DE/EA = DO/OB ……………………(i)

Also, given,

AO/BO = CO/DO

⇒ AO/CO = BO/DO

⇒ CO/AO = DO/BO

⇒DO/OB = CO/AO **…………………………..(ii)**

From equation **(i)** and **(ii)**, we get

DE/EA = CO/AO

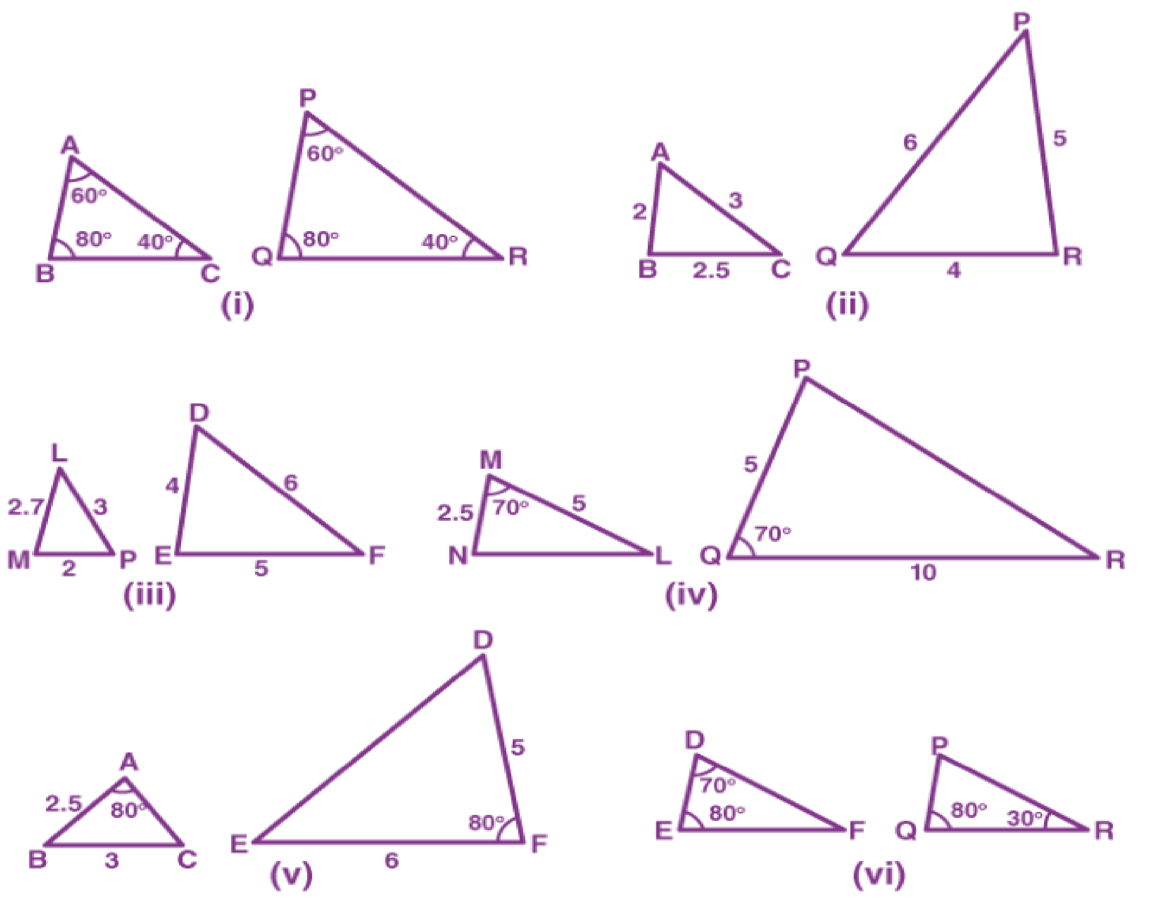
Therefore, by using converse of Basic Proportionality Theorem,

EO || DC also EO || AB

⇒ AB || DC.

Hence, quadrilateral ABCD is a trapezium with AB || CD.

3. State which pairs of triangles in the figure are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



Solution:

(i) Given, in ΔABC and ΔPQR,

$\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Therefore, by AAA similarity criterion,

∴ ΔABC ~ ΔPQR

(ii) Given, in ΔABC and ΔPQR,

AB/QR = 2/4 = 1/2,

BC/RP = 2.5/5 = 1/2,

CA/PA = 3/6 = 1/2

By SSS similarity criterion,

ΔABC ~ ΔQRP

(iii) Given, in ΔLMP and ΔDEF,

LM = 2.7, MP = 2, LP = 3, EF = 5, DE = 4, DF = 6

MP/DE = 2/4 = 1/2

PL/DF = 3/6 = 1/2

LM/EF = 2.7/5 = 27/50

Here , MP/DE = PL/DF ≠ LM/EF

Therefore, ΔLMP and ΔDEF are not similar.

(iv) In ΔMNL and ΔQPR, it is given,

MN/QP = ML/QR = 1/2

∠M = ∠Q = 70°

Therefore, by SAS similarity criterion

∴ ΔMNL ~ ΔQPR

(v) In ΔABC and ΔDEF, given that,

AB = 2.5, BC = 3, ∠A = 80°, EF = 6, DF = 5, ∠F = 80°

Here , AB/DF = 2.5/5 = 1/2

And, BC/EF = 3/6 = 1/2

⇒ ∠B ≠ ∠F

Hence, ΔABC and ΔDEF are not similar.

(vi) In ΔDEF, by sum of angles of triangles, we know that,

∠D + ∠E + ∠F = 180°

⇒ 70° + 80° + ∠F = 180°

⇒ ∠F = 180° – 70° – 80°

⇒ ∠F = 30°

Similarly, In ΔPQR,

∠P + ∠Q + ∠R = 180 (Sum of angles of Δ)

⇒ ∠P + 80° + 30° = 180°

⇒ ∠P = 180° – 80° -30°

⇒ ∠P = 70°

Now, comparing both the triangles, ΔDEF and ΔPQR, we have

$\angle D = \angle P = 70°$

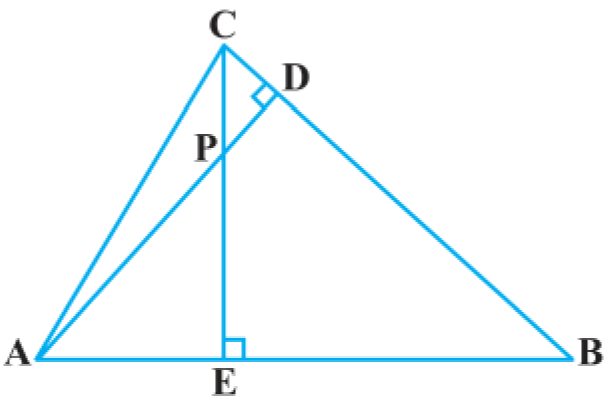
$\angle F = \angle Q = 80°$

$\angle F = \angle R = 30°$

Therefore, by AAA similarity criterion,

Hence, ΔDEF ~ ΔPQR

4. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) ΔAEP ~ ΔCDP
(ii) ΔABD ~ ΔCBE
(iii) ΔAEP ~ ΔADB
(iv) ΔPDC ~ ΔBEC

Solution:

Given, altitudes AD and CE of ΔABC intersect each other at the point P.

(i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP$ (90° each)

$\angle APE = \angle CPD$ (Vertically opposite angles)

Hence, by AA similarity criterion,

$\Delta AEP \sim \Delta CDP$

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ (90° each)

$\angle ABD = \angle CBE$ (Common Angles)

Hence, by AA similarity criterion,

$\Delta ABD \sim \Delta CBE$

(iii) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ (90° each)

$\angle PAE = \angle DAB$ (Common Angles)

Hence, by AA similarity criterion,

$\Delta AEP \sim \Delta ADB$

(iv) In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ (90° each)

$\angle PCD = \angle BCE$ (Common angles)

Hence, by AA similarity criterion,

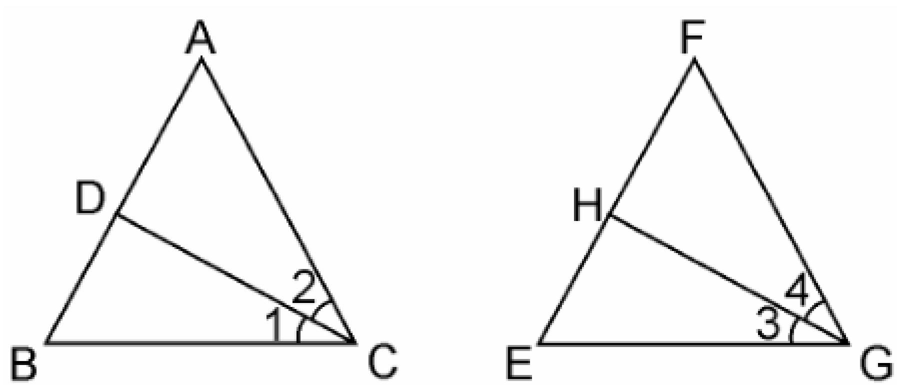
$\Delta PDC \sim \Delta BEC$

5. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) CD/GH = AC/FG
(ii) ΔDCB ~ ΔHGE
(iii) ΔDCA ~ ΔHGF

Solution:

Given, CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG, respectively.



(i) From the given condition,

ΔABC ~ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

Since, ∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,

∠A = ∠F

∠ACD = ∠FGH

∴ ΔACD ~ ΔFGH (AA similarity criterion)

⇒CD/GH = AC/FG

(ii) In ΔDCB and ΔHGE,

∠DCB = ∠HGE (Already proved)

∠B = ∠E (Already proved)

∴ ΔDCB ~ ΔHGE (AA similarity criterion)

(iii) In ΔDCA and ΔHGF,

∠ACD = ∠FGH (Already proved)

∠A = ∠F (Already proved)

∴ ΔDCA ~ ΔHGF (AA similarity criterion)

6. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

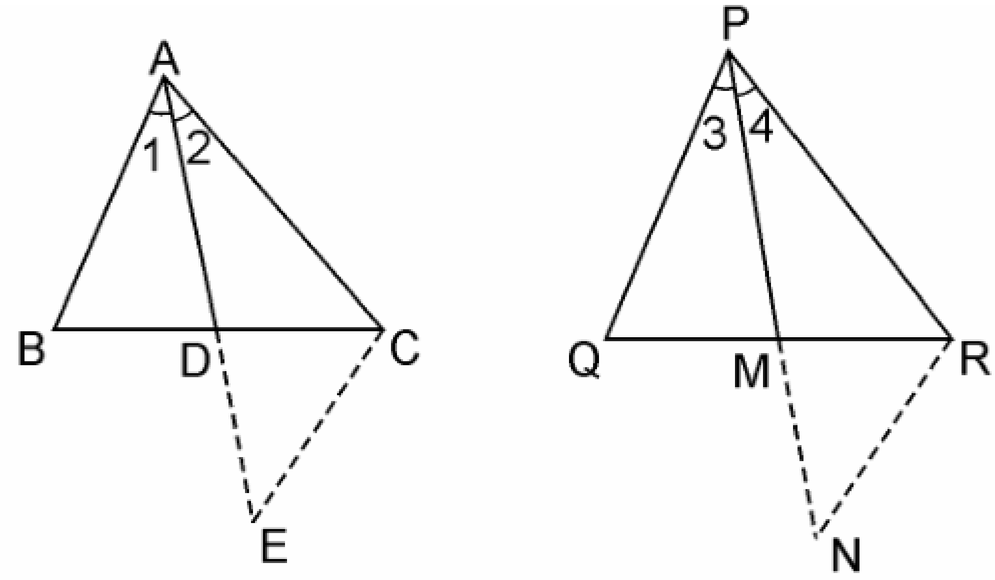
Solution:

Given: Two triangles ΔABC and ΔPQR in which AD and PM are medians such that;

AB/PQ = AC/PR = AD/PM

We have to prove, ΔABC ~ ΔPQR

Let us construct first: Produce AD to E so that AD = DE. Join CE, Similarly produce PM to N such that PM = MN, also Join RN.



In ΔABD and ΔCDE, we have

AD = DE [By Construction.]

BD = DC [Since, AP is the median]

and, ∠ADB = ∠CDE [Vertically opposite angles]

∴ ΔABD ≅ ΔCDE [SAS criterion of congruence]

⇒ AB = CE [By CPCT] …………………………..**(i)**

Also, in ΔPQM and ΔMNR,

PM = MN [By Construction.]

QM = MR [Since, PM is the median]

and, ∠PMQ = ∠NMR [Vertically opposite angles]

∴ ΔPQM = ΔMNR [SAS criterion of congruence]

⇒ PQ = RN [CPCT] ………………………………**(ii)**

Now, AB/PQ = AC/PR = AD/PM

From equation **(i)** and **(ii)**,

⇒CE/RN = AC/PR = AD/PM

⇒ CE/RN = AC/PR = 2AD/2PM

⇒ CE/RN = AC/PR = AE/PN [Since 2AD = AE and 2PM = PN]

∴ ΔACE ~ ΔPRN [SSS similarity criterion]

Therefore, ∠2 = ∠4

Similarly, ∠1 = ∠3

∴ ∠1 + ∠2 = ∠3 + ∠4

⇒ ∠A = ∠P ……………………………………………**(iii)**

Now, in ΔABC and ΔPQR, we have

AB/PQ = AC/PR (Already given)

From equation (iii),

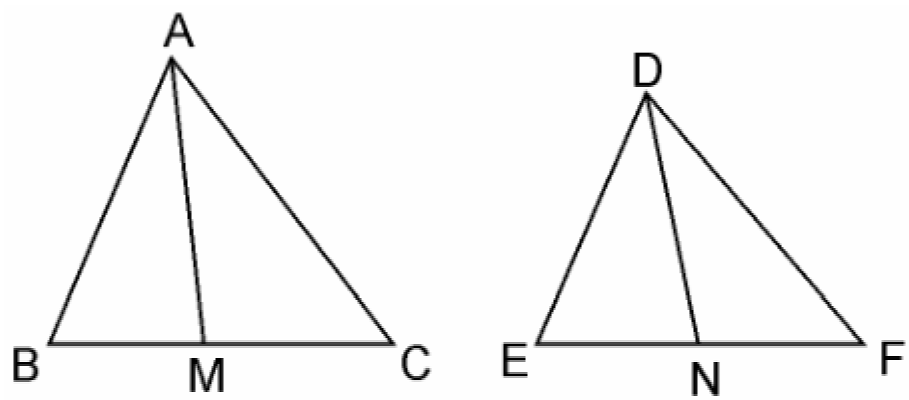
∠A = ∠P

∴ ΔABC ~ ΔPQR [SAS similarity criterion]

7. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution:

Given: AM and DN are the medians of triangles ABC and DEF respectively and ΔABC ~ ΔDEF.



We have to prove: Area(ΔABC)/Area(ΔDEF) = AM^2/DN^2

Since, ΔABC ~ ΔDEF (Given)

∴ Area(ΔABC)/Area(ΔDEF) = (AB^2/DE^2) ……………………………**(i)**

and, AB/DE = BC/EF = CA/FD ……………………………………**(ii)**

$$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{CD}{FD}$$

In ΔABM and ΔDEN,

Since ΔABC ~ ΔDEF

∴ ∠B = ∠E

AB/DE = BM/EN [Already Proved in equation **(i)**]

∴ ΔABC ~ ΔDEF [SAS similarity criterion]

⇒ AB/DE = AM/DN ……………………………………………………..**(iii)**

∴ ΔABM ~ ΔDEN

As the areas of two similar triangles are proportional to the squares of the corresponding sides.

∴ area(ΔABC)/area(ΔDEF) = $AB^2/DE^2 = AM^2/DN^2$

Hence, proved.

8. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm

Solution:

(i) Given, sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of the sides of the, we will get 49, 576, and 625.

49 + 576 = 625

$(7)^2 + (24)^2 = (25)^2$

Therefore, the above equation satisfies, Pythagoras theorem. Hence, it is right angled triangle.

Length of Hypotenuse = 25 cm

(ii) Given, sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will get 9, 64, and 36.

Clearly, $9 + 36 \neq 64$

Or, $3^2 + 6^2 \neq 8^2$

Therefore, the sum of the squares of the lengths of two sides is not equal to the square of the length of the hypotenuse.

Hence, the given triangle does not satisfies Pythagoras theorem.

(iii) Given, sides of triangle's are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will get 2500, 6400, and 10000.

However, $2500 + 6400 \neq 10000$

Or, $50^2 + 80^2 \neq 100^2$

As you can see, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle does not satisfies Pythagoras theorem.

Hence, it is not a right triangle.

(iv) Given, sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will get 169, 144, and 25.

Thus, $144 +25 = 169$

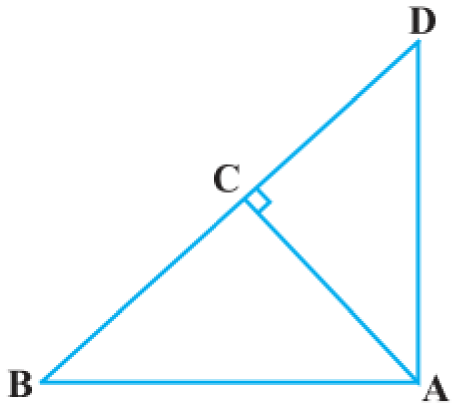
Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

Hence, length of the hypotenuse of this triangle is 13 cm.

9. In Figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that
(i) $AB^2 = BC \times BD$
(ii) $AC^2 = BC \times DC$
(iii) $AD^2 = BD \times CD$



Solution:

(i) In ΔADB and ΔCAB,

∠DAB = ∠ACB (Each 90°)

∠ABD = ∠CBA (Common angles)

∴ ΔADB ~ ΔCAB [AA similarity criterion]

⇒ AB/CB = BD/AB

⇒ $AB^2 = CB \times BD$

(ii) Let ∠CAB = x

In ΔCBA,

∠CBA = 180° – 90° – x

∠CBA = 90° – x

Similarly, in ΔCAD

$\angle CAD = 90° – \angle CBA$

$= 90° – x$

$\angle CDA = 180° – 90° – (90° – x)$

$\angle CDA = x$

In ΔCBA and ΔCAD, we have

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA$ (Each 90°)

$\therefore$ $\Delta CBA \sim \Delta CAD$ [AAA similarity criterion]

$\Rightarrow AC/DC = BC/AC$

$\Rightarrow AC^2 = DC \times BC$

(iii) In ΔDCA and ΔDAB,

$\angle DCA = \angle DAB$ (Each 90°)

$\angle CDA = \angle ADB$ (common angles)

$\therefore$ $\Delta DCA \sim \Delta DAB$ [AA similarity criterion]

$\Rightarrow DC/DA = DA/DA$

$\Rightarrow AD^2 = BD \times CD$