Chapter-12

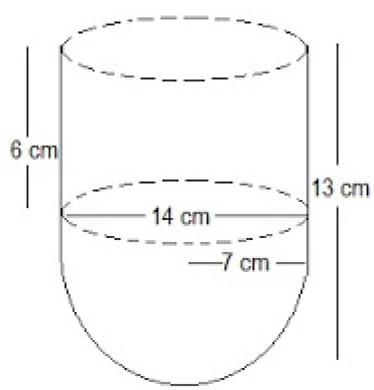
SURFACE AREAS AND VOLUMES

2 MARKS QUESTIONS

1. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm, and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Solution:

The diagram is as follows:



Now, the given parameters are:

The diameter of the hemisphere = D = 14 cm

The radius of the hemisphere = r = 7 cm

Also, the height of the cylinder = h = (13-7) = 6 cm

And the radius of the hollow hemisphere = 7 cm

Now, the inner surface area of the vessel = CSA of the cylindrical part + CSA of the hemispherical part

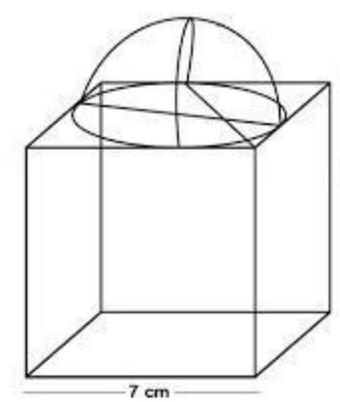
$(2\pi rh+2\pi r^2)$ cm^2 = $2\pi r(h+r)$ cm^2

$2\times(22/7)\times7(6+7)$ cm^2 = 572 cm^2

2. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Solution:

It is given that each side of the cube is 7 cm. So, the radius will be 7/2 cm.



We know,

The total surface area of solid (TSA) = surface area of the cubical block + CSA of the hemisphere – Area of the base of the hemisphere

∴ TSA of solid = $6\times(\text{side})^2+2\pi r^2-\pi r^2$

= $6\times(\text{side})^2+\pi r^2$

= $6\times(7)^2+(22/7)\times(7/2)\times(7/2)$

= $(6\times49)+(77/2)$

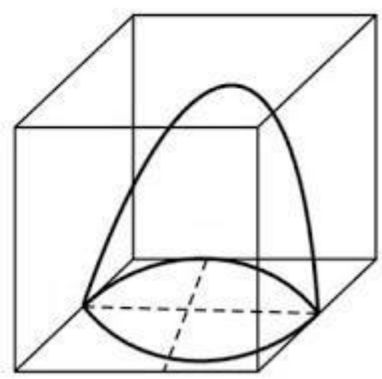
= 294+38.5 = 332.5 cm^2

So, the surface area of the solid is 332.5 cm^2

3. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution:

The diagram is as follows:



Now, the diameter of the hemisphere = Edge of the cube = l

So, the radius of the hemisphere = l/2

∴ The total surface area of solid = surface area of cube + CSA of the hemisphere – Area of the base of the hemisphere

The surface area of the remaining solid = $6\,(\text{edge})^2+2\pi r^2-\pi r^2$

$= 6l^2 + \pi r^2$

$= 6l^2+\pi(l/2)^2$

$= 6l^2+\pi l^2/4$

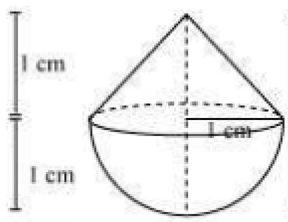
$= l^2/4(24+\pi)$ sq. units

4. A solid is in the shape of a cone standing on a hemisphere, with both their radii being equal to 1 cm and the height of the cone being equal to its radius. Find the volume of the solid in terms of π.

Solution:

Here r = 1 cm and h = 1 cm.

The diagram is as follows.



Now, Volume of solid = Volume of conical part + Volume of hemispherical part

We know the volume of cone = $\frac{1}{3}\pi r^2 h$

And,

The volume of the hemisphere = $\frac{2}{3}\pi r^3$

So, the volume of the solid will be

$$= \frac{1}{3}\pi(1)^2[1+2(1)]\text{cm}^3 = \frac{1}{3}\pi \times 1 \times [3]\text{cm}^3$$

= π cm^3

5. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution:

It is given that radius of the sphere (R) = 4.2 cm

Also, the radius of the cylinder (r) = 6 cm

Now, let the height of the cylinder = h

It is given that the sphere is melted into a cylinder.

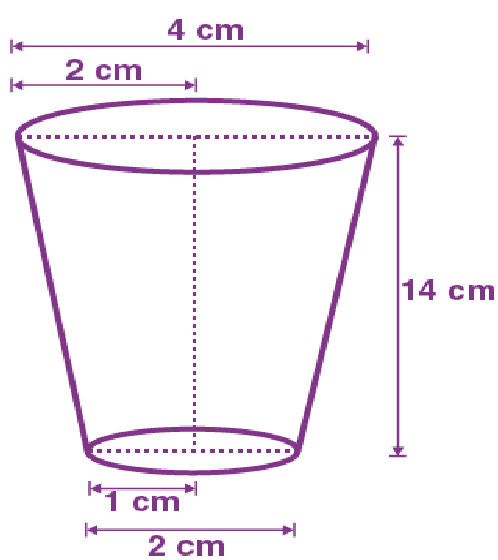
So, the volume of the sphere = Volume of the cylinder

$\therefore (4/3)\times\pi\times R^3 = \pi\times r^2\times h$.

h = 2.74 cm

6. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Solution:



Radius (r_1) of the upper base = 4/2 = 2 cm

Radius (r_2) of lower the base = 2/2 = 1 cm

Height = 14 cm

Now, the capacity of glass = Volume of the frustum of the cone

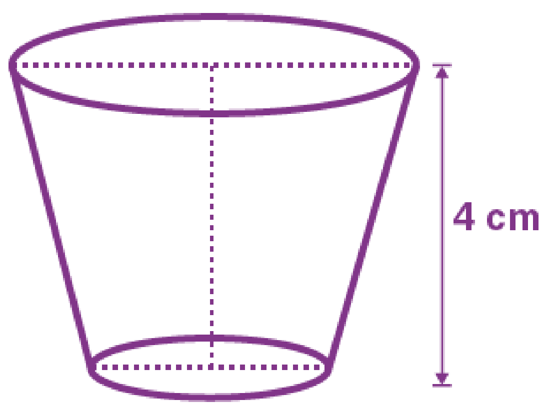
So, Capacity of glass = $(⅓)\times\pi\times h(r_1^2+r_2^2+r_1r_2)$

$= (⅓)\times\pi\times(14)(2^2+1^2+ (2)(1))$

$\therefore$ The capacity of the glass = $102\times(⅔)$ cm^3

7. The slant height of a frustum of a cone is 4 cm, and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the surface area of the frustum.

Solution:



Given,

Slant height (l) = 4 cm

Circumference of upper circular end of the frustum = 18 cm

$\therefore 2\pi r_1 = 18$

Or, $r_1 = 9/\pi$

Similarly, the circumference of the lower end of the frustum = 6 cm

$\therefore 2\pi r_2 = 6$

Or, $r_2 = 3/\pi$

Now, the surface area of the frustum = $\pi(r_1+r_2) \times l$

$= \pi(9/\pi+3/\pi) \times 4$

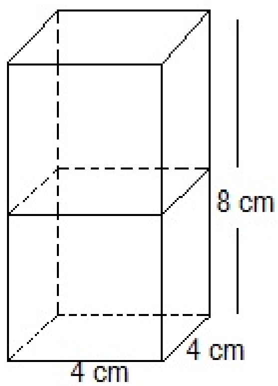
$= 12\times4 = 48\ cm^2$

4 MARKS QUESTIONS

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Solution:

The diagram is given as:



Given,

The Volume (V) of each cube is = 64 cm^3

This implies that $a^3 = 64$ cm^3

∴ a = 4 cm

Now, the side of the cube = a = 4 cm

Also, the length and breadth of the resulting cuboid will be 4 cm each, while its height will be 8 cm.

So, the surface area of the cuboid = 2(lb+bh+lh)

= 2(8×4+4×4+4×8) cm^2

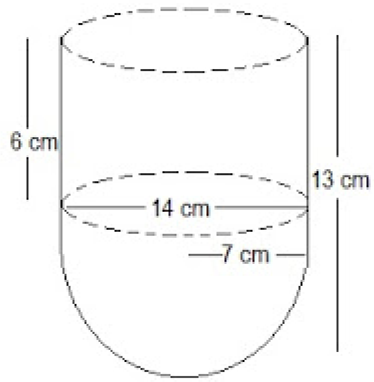
= 2(32+16+32) cm^2

$= (2\times80)\ cm^2 = 160\ cm^2$

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm, and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Solution

The diagram is as follows:



Now, the given parameters are:

The diameter of the hemisphere = D = 14 cm

The radius of the hemisphere = r = 7 cm

Also, the height of the cylinder = h = (13-7) = 6 cm

And the radius of the hollow hemisphere = 7 cm

Now, the inner surface area of the vessel = CSA of the cylindrical part + CSA of the hemispherical part

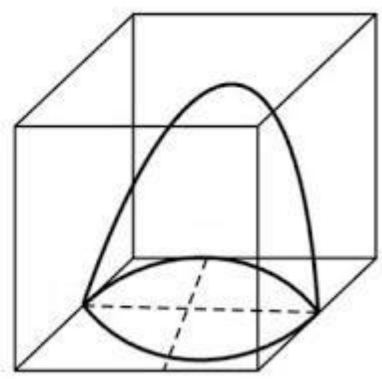
$(2\pi rh+2\pi r^2)\ cm^2 = 2\pi r(h+r)\ cm^2$

$2\times(22/7)\times7(6+7)\ cm^2 = 572\ cm^2$

3. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution:

The diagram is as follows:



Now, the diameter of the hemisphere = Edge of the cube = l

So, the radius of the hemisphere = l/2

∴ The total surface area of solid = surface area of cube + CSA of the hemisphere – Area of the base of the hemisphere

The surface area of the remaining solid = $6\ (\text{edge})^2 + 2\pi r^2 - \pi r^2$

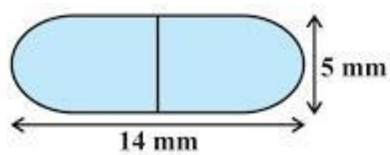
$= 6l^2 + \pi r^2$

$= 6l^2 + \pi(l/2)^2$

$= 6l^2 + \pi l^2/4$

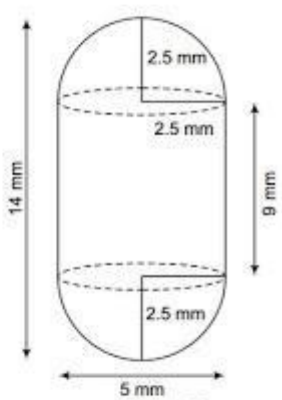
$= l^2/4(24+\pi)$ sq. units

4. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm, and the diameter of the capsule is 5 mm. Find its surface area.



Solution:

Two hemispheres and one cylinder are shown in the figure given below.



Here, the diameter of the capsule = 5 mm

∴ Radius = 5/2 = 2.5 mm

Now, the length of the capsule = 14 mm

So, the length of the cylinder = 14-(2.5+2.5) = 9 mm

∴ The surface area of a hemisphere = $2\pi r^2$ = 2×(22/7)×2.5×2.5

= 275/7 mm^2

Now, the surface area of the cylinder = $2\pi rh$

= 2×(22/7)×2.5×9

(22/7)×45 = 990/7 mm^2

Thus, the required surface area of the medicine capsule will be

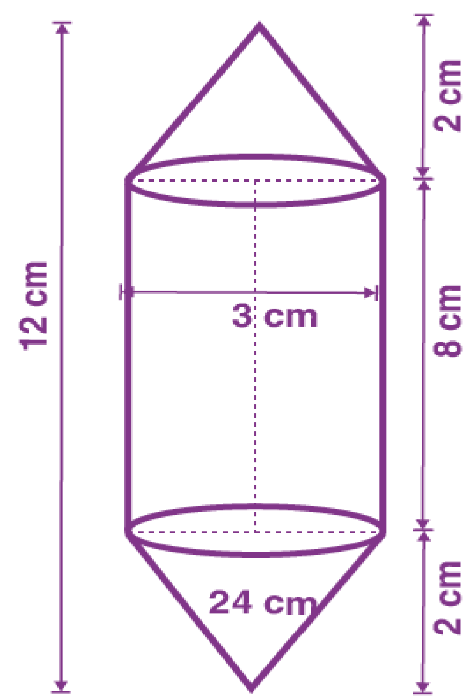
= 2×surface area of hemisphere + surface area of the cylinder

= (2×275/7) × 990/7

= (550/7) + (990/7) = 1540/7 = 220 mm^2

5. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm, and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model are nearly the same.)

Solution:



Given,

Height of cylinder = 12–4 = 8 cm

Radius = 1.5 cm

Height of cone = 2 cm

Now, the total volume of the air contained will be = Volume of cylinder+2×(Volume of the cone)

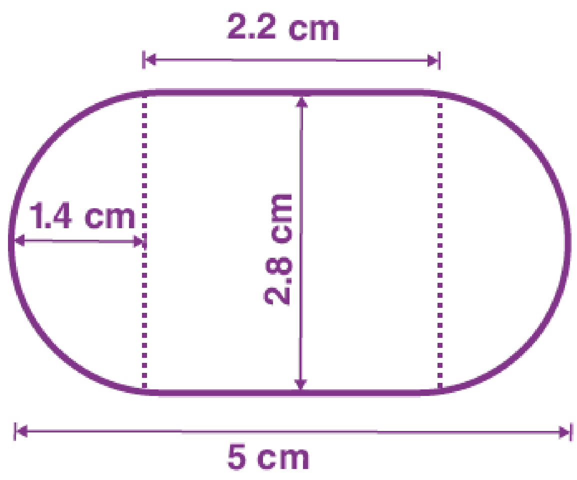
∴ Total volume = $\pi r^2 h+[2\times(\frac{1}{3}\pi r^2 h)]$

= 18 π+2(1.5 π)

= 66 cm^3.

6. A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with a length of 5 cm and a diameter of 2.8 cm (see figure).

Solution:



It is known that the gulab jamuns are similar to a cylinder with two hemispherical ends.

So, the total height of a gulab jamun = 5 cm.

Diameter = 2.8 cm

So, radius = 1.4 cm

∴ The height of the cylindrical part = 5 cm–(1.4+1.4) cm

=2.2 cm

Now, the total volume of one gulab jamun = Volume of cylinder + Volume of two hemispheres

$= \pi r^2 h+(4/3)\pi r^3$

$= 4.312\pi+(10.976/3)\ \pi$

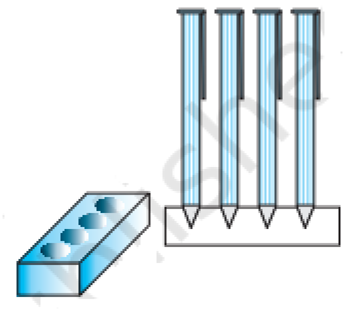
$= 25.05\ cm^3$

We know that the volume of sugar syrup = 30% of the total volume

So, the volume of sugar syrup in 45 gulab jamuns = $45\times30\%(25.05\ cm^3)$

$= 45\times7.515 = 338.184\ cm^3$

7. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm, and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig.).



Solution:

The volume of the cuboid = length x width x height

We know the cuboid's dimensions as 15 cmx10 cmx3.5 cm

So, the volume of the cuboid = 15x10x3.5 = 525 cm^3

Here, depressions are like cones, and we know,

Volume of cone = $(\frac{1}{3})\pi r^2 h$

Given, radius (r) = 0.5 cm and depth (h) = 1.4 cm

∴ Volume of 4 cones = $4 \times (\frac{1}{3})\pi r^2 h$

= 1.46 cm^2

Now, the volume of wood = Volume of the cuboid – 4 x volume of the cone

= 525-1.46 = 523.54 cm^2

8. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution:

For the cone,

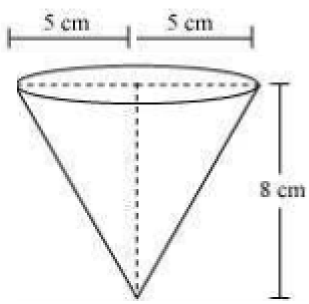
Radius = 5 cm,

Height = 8 cm

Also,

Radius of sphere = 0.5 cm

The diagram will be like



It is known that,

The volume of cone = volume of water in the cone

= $\frac{1}{3}\pi r^2 h$ = (200/3)π cm^3

Now,

Total volume of water overflown= (¼)×(200/3) π =(50/3)π

The volume of lead shot

$= (4/3)\pi r^3$

$= (1/6)\ \pi$

Now,

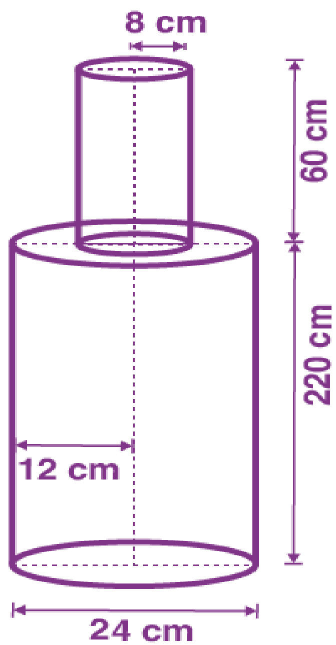
The number of lead shots = Total volume of water overflown/Volume of lead shot

$= (50/3)\pi/(1/6)\pi$

$= (50/3)\times 6 = 100$

9. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass.

Solution:



Given the height of the big cylinder (H) = 220 cm

The radius of the base (R) = 24/2 = 12 cm

So, the volume of the big cylinder = $\pi R^2 H$

$= \pi(12)^2 \times 220\ cm^3$

$= 99565.8\ cm^3$

Now, the height of the smaller cylinder (h) = 60 cm

The radius of the base (r) = 8 cm

So, the volume of the smaller cylinder = $\pi r^2 h$

$= \pi(8)^2 \times 60\ cm^3$

$= 12068.5\ cm^3$

∴ The volume of iron = Volume of the big cylinder+ Volume of the small cylinder

= 99565.8 + 12068.5

$=111634.5\ cm^3$

We know,

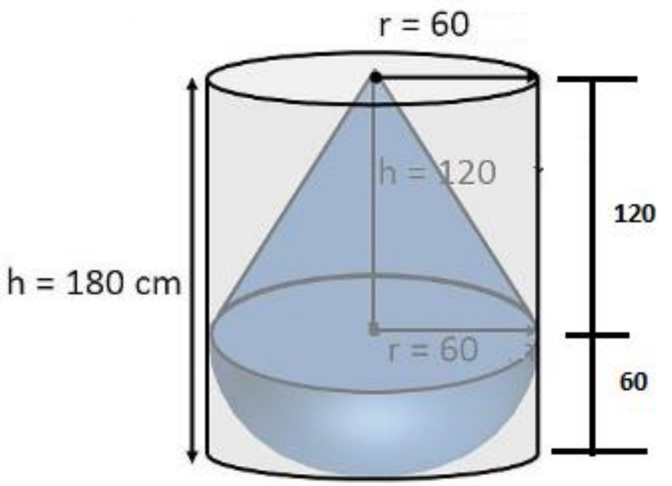
Mass = Density x volume

So, the mass of the pole = 8×111634.5

= 893 Kg (approx.)

10. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder if the radius of the cylinder is 60 cm and its height is 180 cm.

Solution:



Here, the volume of water left will be = Volume of the cylinder – Volume of solid

Given,

Radius of cone = 60 cm,

Height of cone = 120 cm

Radius of cylinder = 60 cm

Height of cylinder = 180 cm

Radius of hemisphere = 60 cm

Now,

The total volume of solid = Volume of Cone + Volume of the hemisphere

Volume of cone = $1/3\pi r^2h = 1/3 \times \pi\times60^2\times120cm^3 = 144\times10^3\pi\ cm^3$

Volume of hemisphere = $(⅔)\times\pi\times60^3\ cm^3 = 144\times10^3\pi\ cm^3$

So, total volume of solid = $144\times10^3\pi\ cm^3 + 144\times10^3\pi\ cm^3 = 288\times10^3\pi\ cm^3$

Volume of cylinder = $\pi\times60^2\times180 = 648000 = 648\times10^3\ \pi\ cm^3$

Now, the volume of water left will be = Volume of the cylinder – Volume of solid

$= (648-288)\times10^3\times\pi = 1.131\ m^3$

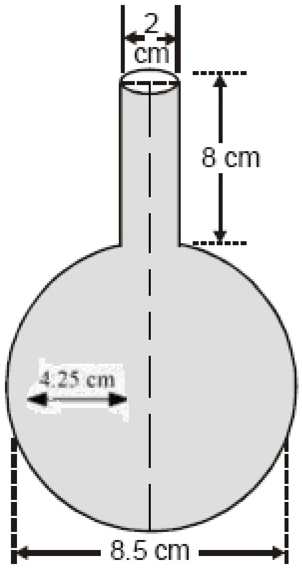
11. A spherical glass vessel has a cylindrical neck 8 cm long and 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements and π = 3.14.

Solution:

Given,

For the cylinder part, Height (h) = 8 cm and Radius (R) = (2/2) cm = 1 cm

For the spherical part, Radius (r) = (8.5/2) = 4.25 cm



Now, volume of this vessel = Volume of cylinder + Volume of sphere

$= \pi\times(1)^2\times8+(4/3)\pi(4.25)^3$

$= 346.51\ cm^3$

Hence, the child's calculation is not correct.

12. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution:

For Sphere 1:

Radius (r_1) = 6 cm

$\therefore$ Volume (V_1) = $(4/3)\times\pi\times r_1^3$

For Sphere 2:

Radius (r_2) = 8 cm

∴ Volume (V_2) = $(4/3)\times\pi\times r_2^3$

For Sphere 3:

Radius (r_3) = 10 cm

∴ Volume (V_3) = $(4/3)\times \pi\times r_3^3$

Also, let the radius of the resulting sphere be "r"

Now,

The volume of the resulting sphere = $V_1+V_2+V_3$

$(4/3)\times\pi\times r^3 = (4/3)\times\pi\times r_1^3+(4/3)\times\pi\times r_2^3 +(4/3)\times\pi\times r_3^3$

$r^3 = 6^3+8^3+10^3$

$r^3 = 1728$

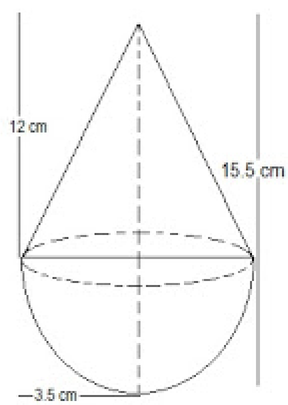
r = 12 cm

7 MARKS QUESTIONS

1. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Solution:

The diagram is as follows:



Given that the radius of the cone and the hemisphere (r) = 3.5 cm or 7/2 cm

The total height of the toy is given as 15.5 cm.

So, the height of the cone (h) = 15.5-3.5 = 12 cm

Slant height of the cone(l) = $\sqrt{h^2 + r^2}$

$\Rightarrow l = \sqrt{12^2 + (3.5)^2}$

$\Rightarrow l = \sqrt{12^2 + (7/2)^2}$

$\Rightarrow l = \sqrt{144 + 49/4} = \sqrt{(576 + 49)/4} = \sqrt{625/4}$

$\Rightarrow l = 25/2$

∴ The curved surface area of the cone = πrl

$(22/7)\times(7/2)\times(25/2) = 275/2\ cm^2$

Also, the curved surface area of the hemisphere = $2\pi r^2$

$2\times(22/7)\times(7/2)^2$

$= 77\ cm^2$

Now, the Total surface area of the toy = CSA of the cone + CSA of the hemisphere

$= (275/2)+77\ cm^2$

$= (275+154)/2\ cm^2$

$= 429/2\ cm^2 = 214.5cm^2$

So, the total surface area (TSA) of the toy is $214.5cm^2$

2. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution:

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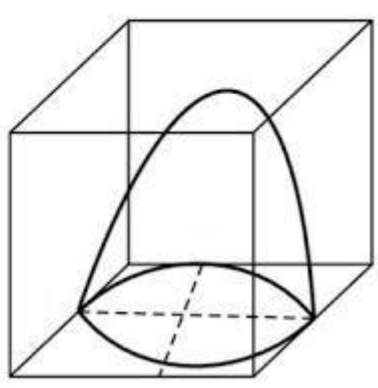


Now, the diameter of the hemisphere = Edge of the cube = l

So, the radius of the hemisphere = l/2

∴ The total surface area of solid = surface area of cube + CSA of the hemisphere – Area of the base of the hemisphere

The surface area of the remaining solid = 6 $(edge)^2+2\pi r^2-\pi r^2$

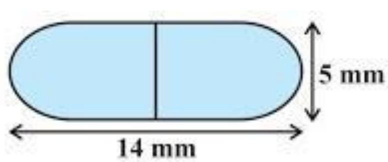
$= 6l^2 + \pi r^2$

$= 6l^2+\pi(l/2)^2$

$= 6l^2+\pi l^2/4$

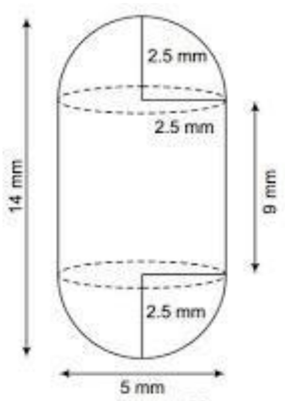
$= l^2/4(24+\pi)$ sq. units

3. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm, and the diameter of the capsule is 5 mm. Find its surface area.



Solution:

Two hemispheres and one cylinder are shown in the figure given below.



Here, the diameter of the capsule = 5 mm

∴ Radius = 5/2 = 2.5 mm

Now, the length of the capsule = 14 mm

So, the length of the cylinder = 14-(2.5+2.5) = 9 mm

∴ The surface area of a hemisphere = $2\pi r^2$ = 2×(22/7)×2.5×2.5

= 275/7 mm^2

Now, the surface area of the cylinder = $2\pi rh$

= 2×(22/7)×2.5×9

(22/7)×45 = 990/7 mm^2

Thus, the required surface area of the medicine capsule will be

= 2×surface area of hemisphere + surface area of the cylinder

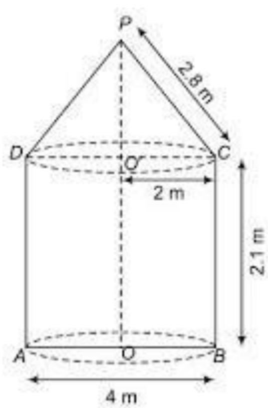
= (2×275/7) × 990/7

= (550/7) + (990/7) = 1540/7 = 220 m

4. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m, respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Solution:

It is known that a tent is a combination of a cylinder and a cone.



From the question, we know that

Diameter = 4 m

The slant height of the cone (l) = 2.8 m

Radius of the cone (r) = Radius of cylinder = 4/2 = 2 m

Height of the cylinder (h) = 2.1 m

So, the required surface area of the tent = surface area of the cone + surface area of the cylinder

$= \pi rl+2\pi rh$

$= \pi r(l+2h)$

$= (22/7)\times 2(2.8+2\times 2.1)$

= (44/7)(2.8+4.2)

= (44/7)×7 = 44 m^2

∴ The cost of the canvas of the tent at the rate of ₹500 per m^2 will be

= Surface area × cost per m^2

44×500 = ₹22000

So, Rs. 22000 will be the total cost of the canvas.

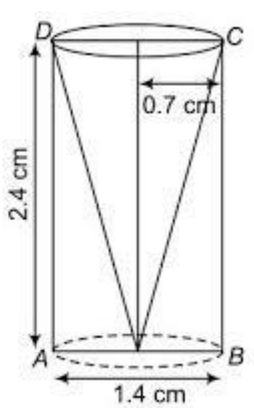
5. From a solid cylinder whose height is 2.4 cm and diameter is 1.4 cm, a conical cavity of the

same height and same diameter is hollowed out. Find the total surface area of the

remaining solid to the nearest cm^2.

Solution:

The diagram for the question is as follows:



From the question, we know the following:

The diameter of the cylinder = diameter of conical cavity = 1.4 cm

So, the radius of the cylinder = radius of the conical cavity = 1.4/2 = 0.7

Also, the height of the cylinder = height of the conical cavity = 2.4 cm

$\therefore$ Slant height of the conical cavity $(l) = \sqrt{h^2 + r^2}$

$$= \sqrt{(2.4)^2 + (0.7)^2}$$

$$= \sqrt{5.76 + 0.49} = \sqrt{6.25}$$

$$= 2.5 \text{ cm}$$

Now, the TSA of the remaining solid = surface area of conical cavity + TSA of the cylinder

$= \pi rl+(2\pi rh+\pi r^2)$

$= \pi r(l+2h+r)$

$= (22/7)\times 0.7(2.5+4.8+0.7)$

$= 2.2\times 8 = 17.6 \text{ cm}^2$

So, the total surface area of the remaining solid is 17.6 cm^2

6. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution:

For the cone,

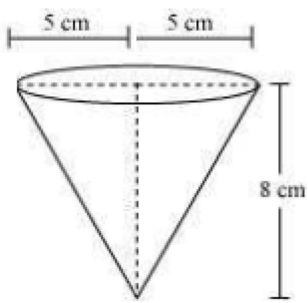
Radius = 5 cm,

Height = 8 cm

Also,

Radius of sphere = 0.5 cm

The diagram will be like



It is known that,

The volume of cone = volume of water in the cone

= $\frac{1}{3}\pi r^2 h$ = (200/3)π cm^3

Now,

Total volume of water overflown= (¼)×(200/3) π =(50/3)π

The volume of lead shot

$= (4/3)\pi r^3$

$= (1/6)\ \pi$

Now,

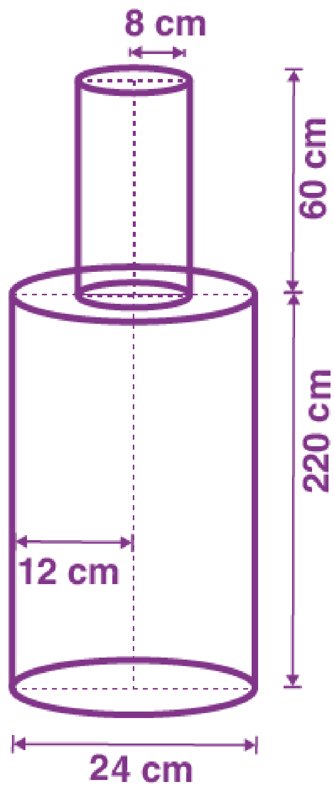
The number of lead shots = Total volume of water overflown/Volume of lead shot

$= (50/3)\pi/(1/6)\pi$

$= (50/3)\times 6 = 100$

7. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass.

Solution:



Given the height of the big cylinder (H) = 220 cm

The radius of the base (R) = 24/2 = 12 cm

So, the volume of the big cylinder = $\pi R^2 H$

$= \pi(12)^2 \times 220\ cm^3$

$= 99565.8\ cm^3$

Now, the height of the smaller cylinder (h) = 60 cm

The radius of the base (r) = 8 cm

So, the volume of the smaller cylinder = $\pi r^2 h$

$= \pi(8)^2 \times 60\ cm^3$

$= 12068.5\ cm^3$

∴ The volume of iron = Volume of the big cylinder+ Volume of the small cylinder

= 99565.8 + 12068.5

=111634.5 cm^3

We know,

Mass = Density x volume

So, the mass of the pole = 8×111634.5

= 893 Kg (approx.)

8. Water in a canal, 6 m wide and 1.5 m deep, flows at a speed of 10 km/h. How much area will it irrigate in 30 minutes if 8 cm of standing water is needed?

Solution:

It is given that the canal is the shape of a cuboid with dimensions as:

Breadth (b) = 6 m and Height (h) = 1.5 m

It is also given that

The speed of canal = 10 km/hr

Length of canal covered in 1 hour = 10 km

Length of canal covered in 60 minutes = 10 km

Length of canal covered in 1 min = (1/60)x10 km

Length of canal covered in 30 min (l) = (30/60)x10 = 5km = 5000 m

We know that the canal is cuboidal in shape. So,

The volume of the canal = lxbxh

$= 5000 \times 6 \times 1.5\ m^3$

$= 45000\ m^3$

Now,

The volume of water in the canal = Volume of area irrigated

= Area irrigated x Height

So, Area irrigated = 56.25 hectares

∴ The volume of the canal = lxbxh

45000 = Area irrigatedx8 cm

45000 = Area irrigated x (8/100)m

Or, Area irrigated = 562500 m^2 = 56.25 hectares.