Work Energy and Power

Short Answer Questions (2 Marks Questions)

Q1. A body is only allowed to move on the z-axis of a coordinate system. The body is subjected to a constant force F = -i + 2j + 3k N. Calculate the work done to move the body a distance of 4m along the z-axis.

Ans. Given the force F = -i + 2j + 3k N

Displacement = 4k m (where k is unit vector along z-axis)

Work done = Force * displacement

W = F.s

= (-i + 2j + 3k) .(4k)

= 12 J

The work done to move the body is 12 J

Q2. A ball in a container hits a horizontal wall with some speed making an angle of 60° at normal, and rebounds with the same speed. Can we say that momentum is conserved in the collision? Is it an elastic or inelastic collision?

Ans. Yes, the collision is elastic. The **momentum** of the ball is conserved independent of the collision being elastic or inelastic. The ball has some velocity v and strikes the stationary wall of the container and rebounds with the same speed v. Therefore, the total kinetic energy of the ball is conserved during the collision.

Q3. The bob X of a pendulum released at an angle of 60° to the vertical hits another bob Y of the same mass at rest on a table. How high will the bob X rise after the collision? The radius of the bob is negligible and the collision is elastic.

Ans. Bob X will not rise after the collision. In an elastic collision between two objects with equal masses in which one is at rest, while the other is moving with some **velocity**, the stationary mass acquires the same velocity, while the moving mass will immediately come to rest after the collision. In this case, a complete transfer of momentum takes place from the moving object to the stationary object. Thus, bob X of mass m will come to rest after colliding with bob Y, while bob Y will move with the velocity of bob X after the collision.

Q4. A trolley with a mass of 300 kg contains a sandbag of 25 kg. It is moving uniformly with a speed of 27 km/h on a frictionless track. After some interval of time, the sand begins leaking out of a hole on the floor of the trolley, at the rate of 0.05 kg/s. What is the speed of the trolley after the sandbag is empty?

Ans. The sandbag is placed on a trolley which is moving with a uniform speed of 27 km/h. The external forces acting on the whole system, which is (sandbag + trolley) is zero. As the leaking of the sand does not produce any external force on the system, the sand starts leaking from the bag with no change in the velocity of the trolley. Thus, the speed of the trolley will remain as it was before.

Q5. Two bodies are at different temperatures Ta and Tb. They are brought in thermal contact but they do not necessarily settle down to the mean temperature. Why?

Ans. When two bodies at different temperatures Ta and Tb come in contact with each other, they do not always settle at their mean temperature because they might have different thermal capacities.

Q6. Calculate the work done by the centripetal force?

Ans. The work done by the centripetal force is always zero. This is because the angle, centripetal force makes with the displacement vector is always 90 degrees. Therefore the dot product is zero and hence work done is 0.

4 Marks Answers and Questions

Q1. A ball by mistake fell from a table on the floor. The height of the table is 2cm. It rises up to a height of 1.5m after its collision with the ground. Assume that 40% of mechanical energy is lost and it goes to thermal energy into the ball. Calculate the increase in temperature of the ball during the collision. Specific heat capacity of the ball = 800J/k. g = 10m/s2

Ans. Given in the question (initial height) h1 = 2m

(Final height) h2 = 1.5m

It is known that potential energy(PE) = mechanical energy for a body at rest (as kinetic energy is 0)

ME lost = |mg(h1-h2)|

= |1 x 10 (105 -2)|

= 5 J

Now,

ME x 40% = Cm▵▵T

40/100 x 5 = 800 x 1 x T

Therefore,

▵▵T = 100 x 800 /(40 x 5) = 2.5 x 10-3 oC .

This is the increase in the temperature.

Q2. A thermometer is found to have the wrong calibration. The melting point of ice is read by it as 100oC. It reads 600oC in place of 500oC. What temperature will it show for the boiling point of water?

Ans. Given to us,

The lower fixed point on the wrong scale is 100**o**C

Let n be athe number of divisions between upper and lower fixed points on the scale

Let Q = reading on this scale,

(C−0)/100=Q−(−10)/n

Now, C = wrong reading = 600**o**C

Q = correct reading = 500**o**C

So,

(50−0)/100 = (60−(−10))/n

50/100=70/n

n = 140

Now,

C−0100=Q−(−10)n

Boiling point of water on the Celsius scale is 100**o**C

So, (100−0)/100 = (Q+10)/140

Q = 130**o**C,

This is the temperature of the boiling point of water on this scale.

Q3. A body is moving in a single direction under the influence of the source of constant power. Its displacement in time t is proportional to

(i) t1/2

(ii) t

(iii) t3/2

(iv) t2

Ans. The correct option is option (iii) t3/2

Power can be written as,

P = Fv

= mav = mv dv/dt = k (constant)

⇒vdv=k/m dt

Integrating both the sides

v2/2=k/m x t

v=√(2kt/m)(2��/�)

Displacement of the body,

$$v = \frac{dx}{dt} = \sqrt{\frac{2k}{m}}t^{\frac{1}{2}}$$

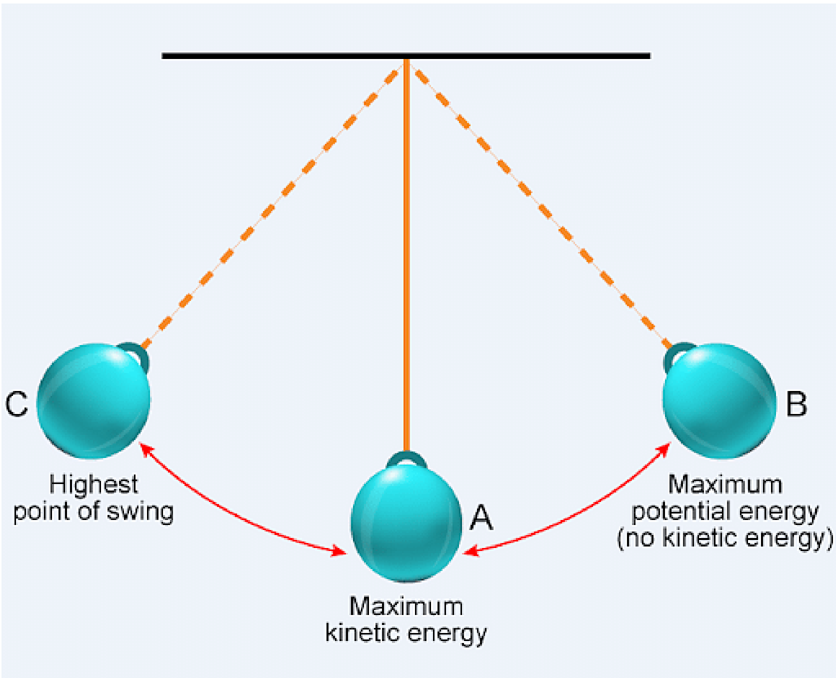
Dx = k′ t1/2dt

Where k′ =√(2k/3)(2�/3)

Integrating both sides,

X = 2/3k′ t3/2

⇒x∝t3/2



Conservation of Energy

Q4. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms-1. It hits the floor of the elevator (length of elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?
Ans:

Mass of the bolt, m = 0.3 kg

Potential energy of the bolt = mgh = 0.3 x 9.8 x 3 = 8.82 J

The bolt does not rebound. So, the whole of the potential energy gets converted to heat energy. The heat produced will remain the same even if the lift is stationary since the value of acceleration due to gravity is the same in all inertial systems.

Q5. A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW?
(b) Compare this area to that of the roof of a typical house.

Ans:

(a) Power used by the family, p = 8 KW = 8000 W

Solar energy received per square metre = 200 W/m2

Percentage of energy converted to useful electrical energy = 20%

As solar energy is incident at a rate of 200 Wm-2

The area required to generate the desired energy is A.

Useful electrical energy produced per second

= (20/100) A x 200 = 8000

A= 4000 W/200 Wm-2 =200 m2

(b) The area needed is comparable to the roof of a large house of dimension 14m × 14m.

Q6. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?

Ans:

Length of the pendulum, l= 1.5 m

Potential of the bob at the horizontal position = mgh = mgl

The initial energy dissipated against air resistance when the bob moves from the horizontal position to the lowermost point= 5%

The total kinetic energy of the bob at the lowermost position = 95% of the total potential energy at the horizontal position

(1/2)mv2 = (95/100) mgl

v2 =2 [(95/100) x 9.8 x 1.5]

$v2 = 2\ (13.965) = 27.93$

$v = \sqrt{27.93} = 5.28$ m/s

Q7. On a frictionless track, a trolley weighing 200 kg moves at a constant speed of 36 km/h. A child of 20 kg runs on the trolley from one end to the other (10 m away) at a speed of 4 relative to the trolley in the opposite direction of its motion and jumps out. What is the trolley's top speed? How far has the trolley moved since the child began to run?

Ans. In the above question, it is given that,

Mass of the trolley, M = 200 kg

Speed of the trolley, v = 36 km/h = 10 m/s

Mass of the boy, m = 20 kg

The initial momentum of the system of the boy and the trolley

$= (M+ m)v$

$= (200+20)\times10$

= 2200 kg m/s

Let 'v' be the final velocity of the trolley with respect to the ground.

The final velocity of the boy with respect to the ground = $v'-4$

Final momentum = $Mv'+m(v'-4)$

$= 200v'+20v'-80$

$= 220v'-80$

As per the law of conservation of momentum,

Initial momentum = Final momentum

$2200 = 220v'-80$

$\Rightarrow v' = 2280220 = 10.36$m/s

Length of the trolley, l = 10 m

Speed of the boy, $v'' = 4$ m/s

Time taken by the boy to run, t = 104 = 2.5s.

Thus, the distance moved by the trolley = $v''\times t = 10.36\times2.5 = 25.9$m.

7 Marks Answers and Questions

Q1. A bullet that has a mass of 0.012 kg and Vx = 70 ms-1 hits a block of wood of mass 0.4 kg and instantly comes to rest w.r.t the block. The block is suspended from the ceiling by a thin string. Calculate the height upto which the block will rise. Calculate the amount of heat produced.

Ans. Given to us that $m1 = 0.012$ kg ,$u1 = 70$m/s

$m2 = 0.4$ kg = 400g and $u2 = 0$m/s

The bullet and wood behave as one body because the bullet comes to rest with respect to the wood. Let v be the final velocity gained by the system.

As per the principle of conservation of **linear momentum**,

So, $v = m1u1/(m1 + m2) = 0.012 \times 70 /(0.012 + 0.4) = 0.84 / 0.412 = 2.04$ m/s.

Let us assume that the block will rise up to a height h.

Total P.E = total K.E

$(m1 + m2)gh = ½ (m1 + m2)v2$

$H = v2/2g$

$= 2.04 \times 2.04 /(2 \times 9.8) = 0.212$m

The energy lost in the process is actually the heat lost.

Therefore Work done = initial energy - final energy

$W = ½ m1 u12 - ½ (m1 + m2) v2$

$= ½ X 0.012 (70)2 - ½ X 0.412 (2.04)2$

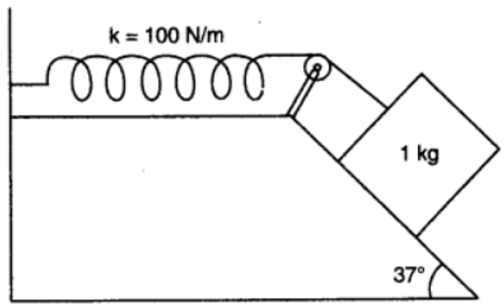
$= 29.4 - 0.86$

$= 28.54$ J

Heat lost = $W/J = 28.54 / 4.2 = 6.8$ cal.

The heat produced will be 6.8 cal.

Q2. A 1 kg block is placed on a rough inclined plane. It is connected to a spring having a spring constant of 100 N/m as shown in the diagram. The block is released from rest with the spring in a relaxed position. The block moves about 10 cm down the inclined plane before it comes to rest. Find (coefficient of friction) between the block and the inclined plane. Assume the spring has negligible mass and the pulleys are frictionless.



Ans. From the above figure,

Applying NLM of 1kg block

R = mg cos 37°

f= μR = mg sin 37°

Net force on the block down the incline = mg sin 37° – f

= mgsin 37° – μmgcos 37°

= mg(sin 37° – μcos 37°)

The distance moved by the object = 10cm = 0.1m

Let the distance moved to be x

In the equilibrium position, Work done = P.E of spring

i.e., $mg(\sin 37° - \mu \cos 37°)x = \left(\frac{1}{2}\right) kx^2$

$1 \times 9.8\ (\sin 37^0 - \mu \cos 37°) = \left(\frac{1}{2}\right) \times 100 \times (0.1)$

$$0.602 - \mu \times 0.799 = 0.510$$

$\therefore \qquad \mu = \frac{0.092}{0.799} = 0.115$

Q3. A raindrop having a radius of 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration due to air drag, until it reaches half its original height. It then attains its maximum speed and moves with uniform speed after that. Calculate what is the work done by the gravitational force on the drop in the 1st and 2nd half of its journey. What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10m/s?

Ans. Given to us

Radius of the drop, r = 2 mm = 2×10^{-3}m

Density of water is known to us, ρ=103 kg/m3

Volume of the raindrop, V = 4/3 π�r3

= 4/3 × 3.14 × (2×10−3)3 × 103 kg

Mass of the drop, m=ρ x V

=4/3×3.14×(2×10−3)3×103 kg

F = mg (**gravitational force**)

=4/3×3.14×(2×10−3)3×103 kg × 9.8N

The work done by the gravitational force on the drop in the first half,

W1= F. s

=4/3×3.14×(2×10−3)3×103 kg×9.8×250

= 0.082 J

This work done is mathematically equal to the work done by the gravitational force on the drop in the second half of its journey

W2 = 0.082 J

The **law of conservation of energy** states that if no external force is present, the total energy of the system remains the same.

Total energy at the top,

ET = mgh + 0

=4/3×3.14×(2×10−3)3×103 kg×9.8×500×10−5

=0.164J

Due to the presence of air drag (resistive force), the drop hits the ground with a velocity of 10 m/s.

Total energy at the ground will be:

EG = 12m = 4/3×3.14×(2×10−3)3×103 kg×9.8×102

= 1.675×10−3 Joules

Wr (Work done by the resistive force) = EG − ET = −0.162J

Q4. A bullet of mass 0.012 kg and horizontal speed 70 m s–1 strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Ans:

Mass of the bullet, $m_1 = 0.012$ kg

Initial speed of the bullet, $u_1 = 70$ m/s
Mass of the wooden block, $m_2 = 0.4$ kg
Initial speed of the wooden block, $u_2 = 0$
Final speed of the system of the bullet and the block = v m/s
Applying the law of conservation of momentum,
$m_1u_1 + m_2u_2 = (m_1+m_2)v$
$(0.012\times70)+(0.4\times0)=(0.012+0.4)v$
$v=0.84/0.412$
$=2.04$ m/s

Let h be the height to which the block rises.

Applying the law of conservation of energy to this system,
Potential energy of the combination = Kinetic energy of the combination

$(m_1 + m_2)gh = (1/2)(m_1 + m_2)v^2$

$h= v^2/2g$
$= (2.04)^2/2\times9.8$

$=0.212$m
The wooden block will rise to a height of 0.212m.

The heat produced = Initial kinetic energy of the bullet – final kinetic energy of the combination
$=(1/2)m_1u_1^2 -(1/2)(m_1 + m_2)v^2$
$=(1/2)\times0.012\times(70)^2 -(1/2)\times(0.012 + 0.4)\times(2.04)^2$
$=29.4-0.857=28.54$J

Multiple Choice Questions

1.An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another.
This is because,

(a) the two magnetic forces are equal and opposite, so they produce no net effect.

(b) the magnetic forces do no work on each particle.

(c) the magnetic forces do equal and opposite (but non-zero) work on each particle.

(d) the magenetic forces are necessarily negligible.

Answer:(b) the magnetic forces do no work on each particle.

2. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time t, the work done on the two moving charged particles is

(a) same as the same force law is involved in the two experiments.

(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.

(c) more for the case of a positron, as the positron moves away a larger distance.

(d) same as the work done by charged particle on the stationary proton.

Answer:(c) more for the case of a positron, as the positron moves away a larger distance.

3.A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is

(a) constant and equal to mg in magnitude.

(b) constant and greater than mg in magnitude.

(c) variable but always greater than mg.

(d) at first greater than mg, and later becomes equal to mg.

Answer:(d) at first greater than mg, and later becomes equal to mg.

4.A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is

(a) + 2000J

(b) – 200J

(c) zero

(d) – 20,000J

Answer:(c) zero

5.A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall?

(a) Kinetic energy.

(b) Potential energy.

(c) Total mechanical energy.

(d) Total linear momentum.

Answer:(c) Total mechanical energy.

6.During inelastic collision between two bodies, which of the following quantities always remain conserved?

(a) Total kinetic energy.

(b) Total mechanical energy.

(c) Total linear momentum.

(d) Speed of each body.

Answer:(c) Total linear momentum.

7.A man, of mass m, standing at the bottom of the staircase, of height L climbs it and stands at its top.

(a) Work done by all forces on man is equal to the rise in potential energy mgL.

(b) Work done by all forces on man is zero.

(c) Work done by the gravitational force on man is mgL.

(d) The reaction force from a step does not do work because the point of application of the force does not move while the force exists.

Answer:(b),(d)

8.Two blocks M_1 and M_2 having equal mass are free to move on a horizontal frictionless surface. M_2 is attached to a massless spring as shown in Fig. 6.10. Iniially M_2 is at rest and M_1 is moving toward M_2 with speed v and collides head-on with M_2.

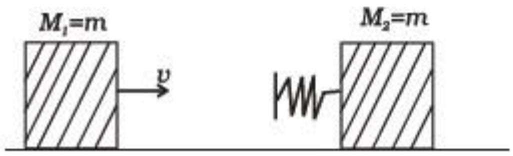


Fig. 6.10

(a) While spring is fully compressed all the KE of M_1 is stored as PE of spring.

(b) While spring is fully compressed the system momentum is not conserved, though final momentum is equal to initial momentum.

(c) If spring is massless, the final state of the M_1 is state of rest.

(d) If the surface on which blocks are moving has friction, then collision cannot be elastic.

Answer:(c) If spring is massless, the final state of the M_1 is state of rest.

9.In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1m s^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m s^{-2}, the kinetic energy of the shotput when it just reaches the ground will be

(a) 2.5 J

(b) 5.0 J

(c) 52.5 J

(d) 155.0 J

Answer:(d)155.0 J

10.A cricket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat.
Assuming that collision between ball and bat is completely elastic and the two remain

in contact for 0.001s, the force that the batsman had to apply to hold the bat firmly at its place would be

(a) 10.5 N

(b) 21 N

(c) 1.05 $\times 10^4$ N

(d) 2.1 $\times$ 10^4 N

Answer:(c) 1.05 $\times 10^4$ N

Fill in the blanks

1.In a perfectly elastic collision in two dimensions between two particles of ………………….mass,the two particles move at …………………..to each other.

Answer: In a perfectly elastic collision in two dimensions between two particles of same mass,the two particles move at 90° to each other.

2.Work is said to be done by a force when ……………in the …………………..of the force

Answer: Work is said to be done by a force when a body moves actually in the direction of the force

3.Absolute unit energy on SI is …………………..and on CGS system,it is …………..where…………=…………

Answer: Absolute unit energy on SI is joule and on CGS system,it is erg where 1 joule=10 power 7 erg

4.A weight lifter does……………in lifting the weight………………..the ground but ……………in………………..the weight up.

Answer: A weight lifter does same work in lifting the weight above the ground but does no work in holding the weight up.

5.Gravitational force is ……………and frictional force is a…………….. .

Answer: Gravitational force is conservative force and frictional force is a non conservative force .

6.Power of a person / machine is defined as the …………………..at which ……………………….. .

Answer: Power of a person / machine is defined as the time rate at which work is done .

7.According to work energy principle, work done by …………………..in displacing body …………………to…………………… in………………. .

Answer: According to work energy principle, work done by net force in displacing body is equal to change in kinetic energy of the body.

8.A collision is an isolated event in which……………..exert……………….on one another for……………… .

A collision is an isolated event in which two or more colliding bodies exert relatively strong force on one another for relatively short time .