CHAPTER-V

KINETIC THEORY

2 MARK QUESTIONS

1. Why is cooling caused by evaporation?

Ans. Evaporation occurs in consequence of faster molecules escaping from the surface of the liquid. The liquid is thus left with molecules having lower speeds. The decrease in the mean speed of molecules results in lowering the temperature and hence cooling is caused.

2.Equal masses of O_2 and He gases are supplied equal amounts of heat. Which gas would undergo a greater rise in temperature and why?

Ans. Helium is a **monatomic** gas while O_2 is a diatomic gas. In the case of helium, the supplied heat has to work in increasing only the translational K.E. of the gas molecules. On the other hand, in the case of oxygen, the supplied heat has to increase the translation, vibrational and rotational K.E. of gas molecules which means all three of them. Thus, helium would undergo a greater rise in temperature.

3. State law of equipartition of energy.

Ans. It states that in equilibrium, the total energy of the system is divided equally in all possible energy modes with each mode i.e. degree of freedom having an average energy equal to 12 k_BT.

4. A glass of water is stirred and then allowed to stand until the water stops moving. What has happened to the K.E. of the moving water?

Ans. The K.E. of moving water is dissipated into internal energy. The temperature of water thus increases.

5. Why does the pressure of a gas increase when it is heated up?

Ans. This is due to the two reasons:

1. The gas molecules move faster than before after getting heating up and so strike the container walls more often.
2. Each impact yields a greater momentum to the walls of the container.

6. R.m.s. velocities of gas molecules are comparable to those of a single bullet, yet a gas takes several seconds to diffuse through a room. Explain why?

Ans. Gas molecules collide with one another at a very high frequency. Hence, a molecule moves along a random and long path to go from one point to another. Therefore, gas takes several seconds to go from one corner of the room to the other.

7. When a gas is heated, its molecules move apart. Does this increase the P.E. or K.E. of the molecules? Explain.

Ans. It increases the K.E. of the molecules. Because of heating, the temperature increases and hence the average velocity of the molecules and their collision with the wall also increases which increases the K.E.

4 MARK QUESTIONS

1. There are n molecules of a gas in a container. If the number of molecules is increased to 2n, what will be:

a. **the pressure of the gas.**
b. **the total energy of the gas.**
c. **r.m.s. speed of the gas molecules.**

Ans.

a. We know that

$P = 13mnC^2$.

where n = no. of molecules per unit volume.

Thus when no. of molecules is increased from n to 2n, no. of molecules per unit volume (n) will increase from n 2n nV to 2nV, hence pressure will become double.

b. The K.E. of a gas molecule is,

$12mC^2 = 32kT$

If the no. of molecules is increased from n to 2n. There is no effect on the average K.E. of a gas molecule, but the total energy is doubled.

c. r.m.s speed of gas is C_{rms} = 3Pρ−−−√=3Pmn−−−√

When n is increased from n to 2n. both n and P become double and the ratio Pn remains unchanged. So there will be no effect of increasing the number of molecules from n to 2n on r.m.s. speed of gas molecules.

2. Two bodies of specific heats S_1 and S_2 having the same heat capacities are combined to form a single composite body. What is the specific heat of the composite body?

Ans. Let m_1 and m_2 be the masses of two bodies having heat capacities S_1 and S_2

respectively.

$\therefore (m_1 + m_2)S = m_1S_1 + m_2S_2 = m_1S_1 + m_1S_1 = 2m_1S_1$

$S = 2m_1S_1m_1+m_2$.

Also, $m_2S_2 = m_1S_1$

or $m_2 = m_1\ S_1\ S_2$

$\therefore S = 2\ m_1\ S_1\ m_1+m_1\ S_1\ S_2=2\ S_1\ S_2\ S_1+S_2$

3.Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3Å.

ANSWER:

Diameter of an oxygen molecule, d = 3Å

Radius, $r = \frac{d}{2} = \frac{3}{2}$ = 1.5 Å = 1.5×10^{-8} cm

Actual volume occupied by 1 mole of oxygen gas at STP = 22400 cm^3

Molecular volume of oxygen gas, $V = \frac{4}{3}\pi r^3 \cdot N$

Where, N is Avogadro's number = 6.023×10^{23} molecules/mole

$$\therefore V = \frac{4}{3} \times 3.14 \times \left(1.5 \times 10^{-8}\right)^3 \times 6.023 \times 10^{23} = 8.51\ \text{cm}^3$$

Ratio of the molecular volume to the actual volume of oxygen $= \frac{8.51}{22400}$

$= 3.8 \times 10^{-4}$

4.Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard

temperature and pressure (STP: 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres.

ANSWER:

The ideal gas equation relating pressure (*P*), volume (*V*), and absolute temperature (*T*) is given as:

$PV = nRT$

Where,

R is the universal gas constant = 8.314 J mol^{-1} K^{-1} *n* =

Number of moles = 1

T = Standard temperature = 273 K

P = Standard pressure = 1 atm = 1.013×10^5 Nm^{-2}

$$\therefore V = \frac{nRT}{P}$$

$$= \frac{1 \times 8.314 \times 273}{1.013 \times 10^5}$$

= 0.0224 m^3

= 22.4 litres

Hence, the molar volume of a gas at STP is 22.4 litres.

5.Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure.

ANSWER:

Volume of the room, *V* = 25.0 m^3

Temperature of the room, T = 27°C = 300 K

Pressure in the room, P = 1 atm = 1 × 1.013 × 10^5 Pa

The ideal gas equation relating pressure (P), Volume (V), and absolute temperature (T) can be written as:

$PV = k_B NT$

Where,

K_B is Boltzmann constant = 1.38 × 10^{-23} m^2 kg s^{-2} K^{-1}

N is the number of air molecules in the room

$\therefore N = \frac{PV}{k_B T}$

$= \frac{1.013 \times 10^5 \times 25}{1.38 \times 10^{-23} \times 300}$ = 6.11 × 10^{26} molecules

Therefore, the total number of air molecules in the given room is

6.11 ×10^{26}.

6.From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 cm^3 s^{-1}. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 cm^3 s^{-1}. Identify the gas.

[Hint: Use Graham's law of diffusion: $R_1/R_2 = (M_2/M_1)^{1/2}$, where R_1, R_2 are diffusion rates of gases 1 and 2, and M_1 and M_2 their respective molecular masses. The law is a simple consequence of kinetic theory.] **ANSWER:**

Rate of diffusion of hydrogen, $R_1 = 28.7 \text{ cm}^3 \text{ s}^{-1}$

Rate of diffusion of another gas, $R_2 = 7.2 \text{ cm}^3 \text{ s}^{-1}$

According to Graham's Law of diffusion, we have:

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

Where,

M_1 is the molecular mass of hydrogen = 2.020 g

M_2 is the molecular mass of the unknown gas

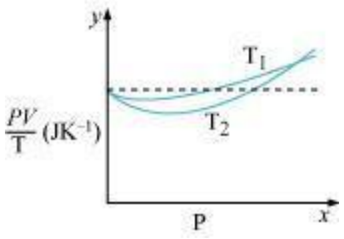
$$\therefore M_2 = M_1\left(\frac{R_1}{R_2}\right)^2$$

$$= 2.02\left(\frac{28.7}{7.2}\right)^2 = 32.09 \text{ g}$$

32 g is the molecular mass of oxygen. Hence, the unknown gas is oxygen.

8 MARK QUESTIONS

1.Figure 13.8 shows plot of *PV/T* versus Pfor 1.00×10^{-3} kg of oxygen gas at two different temperatures.



(a) What does the dotted plot signify?

(b) Which is true: $T_1 > T_2$ or $T_1 < T_2$?

(c) What is the value of PV/T where the curves meet on the y-axis?

(d) If we obtained similar plots for 1.00 $\times 10^{-3}$ kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot)? (Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mo1^{-1} K^{-1}.) ANSWER:

(a) The dotted plot in the graph signifies the ideal behaviour of the gas, i.e., the ratio $\frac{PV}{T}$ is equal. μR (μ is the number of moles and R is the universal gas constant) is a constant quality. It is not dependent on the pressure of the gas.

(b) The dotted plot in the given graph represents an ideal gas. The curve of the gas at temperature T_1 is closer to the dotted plot than the curve of the gas at temperature T_2. A real gas approaches the behaviour of an ideal gas when its temperature increases. Therefore, $T_1 > T_2$ is true for the given plot.

(c) The value of the ratio PV/T, where the two curves meet, is μR. This is because the ideal gas equation is given as:

$PV = \mu RT$

$$\frac{PV}{T} = \mu R$$

Where,

P is the pressure

T is the temperature *V* is the

volume μ is the number of

moles

R is the universal constant

Molecular mass of oxygen = 32.0 g

Mass of oxygen = 1×10^{-3} kg = 1 g

$R = 8.314$ J mole^{-1} K^{-1}

$$\therefore \frac{PV}{T} = \frac{1}{32} \times 8.314$$

= 0.26 J K^{-1}

Therefore, the value of the ratio *PV*/*T*, where the curves meet on the *y*-axis, is

0.26 J K^{-1}.

(d) If we obtain similar plots for 1.00×10^{-3} kg of hydrogen, then we will not get the same value of *PV*/*T* at the point where the curves meet the *y*axis. This is because the molecular mass of hydrogen (2.02 u) is different from that of oxygen (32.0 u).

We have:

$$\frac{PV}{T} = 0.26 \text{ J K}^{-1}$$

$R = 8.314$ J mole^{-1} K^{-1}

Molecular mass (*M*) of H_2 = 2.02 u

$$\frac{PV}{T} = \mu R \text{ at constant temperature}$$

$$\text{Where, } \mu = \frac{m}{M}$$

m = Mass of H_2

$$\therefore m = \frac{PV}{T} \times \frac{M}{R}$$

$$= \frac{0.26 \times 2.02}{8.31}$$

= 6.3×10^{-2} g = 6.3×10^{-5} kg

Hence, 6.3×10^{-5} kg of H_2 will yield the same value of PV/T.

2.An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27 °C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder (R = 8.31 J mol^{-1} K^{-1}, molecular mass of O_2 = 32 u).

ANSWER:

Volume of oxygen, V_1 = 30 litres = 30×10^{-3} m^3

Gauge pressure, P_1 = 15 atm = $15 \times 1.013 \times 10^5$ Pa

Temperature, T_1 = 27°C = 300 K

Universal gas constant, R = 8.314 J mole^{-1} K^{-1}

Let the initial number of moles of oxygen gas in the cylinder be n_1.

The gas equation is given as:

$P_1V_1 = n_1RT_1$

$$\therefore n_1 = \frac{P_1V_1}{RT_1}$$

$$= \frac{15.195 \times 10^5 \times 30 \times 10^{-3}}{(8.314) \times 300}$$ = 18.276

But, $n_1 = \frac{m_1}{M}$

Where, m_1 = Initial mass of

oxygen

M = Molecular mass of oxygen = 32 g $\therefore m_1$ =

$n_1 M$ = 18.276 × 32 = 584.84 g

After some oxygen is withdrawn from the cylinder, the pressure and temperature reduces.

Volume, V_2 = 30 litres = 30 × 10^{-3} m^3

Gauge pressure, P_2 = 11 atm = 11 × 1.013 × 10^5 Pa

Temperature, T_2 = 17°C = 290 K

Let n_2 be the number of moles of oxygen left in the cylinder.

The gas equation is given as:

$P_2 V_2 = n_2 R T_2$

$$\therefore n_2 = \frac{P_2 V_2}{R T_2}$$

$$= \frac{11.143 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290}$$ = 13.86

But, $n_2 = \frac{m_2}{M}$

Where, m_2 is the mass of oxygen remaining in the

cylinder $\therefore m_2 = n_2 M$ = 13.86 × 32 = 453.1 g

The mass of oxygen taken out of the cylinder is given by the relation:

Initial mass of oxygen in the cylinder – Final mass of oxygen in the cylinder

$= m_1 - m_2$

= 584.84 g – 453.1 g

= 131.74 g

= 0.131 kg

Therefore, 0.131 kg of oxygen is taken out of the cylinder.

3.An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12 °C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C?

ANSWER:

Volume of the air bubble, $V_1 = 1.0\ \text{cm}^3 = 1.0 \times 10^{-6}\ \text{m}^3$

Bubble rises to height, $d = 40$ m

Temperature at a depth of 40 m, $T_1 = 12°\text{C} = 285$ K Temperature at the

surface of the lake, $T_2 = 35°\text{C} = 308$ K

The pressure on the surface of the lake:

$P_2 = 1\ \text{atm} = 1 \times 1.013 \times 10^5$ Pa

The pressure at the depth of 40 m:

$P_1 = 1\ \text{atm} + d\rho g$ Where, ρ is the density of water =

10^3 kg/m^3 g is the acceleration due to gravity = 9.8

m/s^2

$\therefore P_1 = 1.013 \times 10^5 + 40 \times 10^3 \times 9.8 = 493300$ Pa

We have: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Where, V_2 is the volume of the air bubble when it reaches the surface

$$V_2 = \frac{P_1V_1T_2}{T_1P_2}$$

$$= \frac{(493300)(1.0 \times 10^{-6})308}{285 \times 1.013 \times 10^5}$$

$= 5.263 \times 10^{-6}$ m^3 or 5.263 cm^3

Therefore, when the air bubble reaches the surface, its volume becomes 5.263 cm^3.

4.Estimate the average thermal energy of a helium atom at (i) room temperature (27 °C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million Kelvin (the typical core temperature in the case of a star).

ANSWER:

(i) At room temperature, T = 27°C = 300 K

Average thermal energy $= \frac{3}{2}kT$

Where k is Boltzmann constant $= 1.38 \times 10^{-23}$ J K^{-1}

∴32kT=32×1.38×10−23×300 J32=32×1.38×10-23×300 J

$= 6.21 \times 10^{-21}$J

Hence, the average thermal energy of a helium atom at room temperature (27°C) is 6.21×10^{-21} J.

(ii) On the surface of the sun, T = 6000 K

Average thermal energy $= \frac{3}{2}kT$

=32kT = 32×1.38×10−23×6000 J32 = 32×1.38×10-23×6000 J

$= 1.241 \times 10^{-19}$ J

Hence, the average thermal energy of a helium atom on the surface of the sun is 1.241×10^{-19} J.

(iii) At temperature, $T = 10^7$ K

Average thermal energy $= \frac{3}{2}kT$

$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^{7}$ J

$= 2.07 \times 10^{-16}$ J

Hence, the average thermal energy of a helium atom at the core of a star is 2.07×10^{-16} J.

5.Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

ANSWER:

Yes. All contain the same number of the respective molecules.

No. The root mean square speed of neon is the largest.

Since the three vessels have the same capacity, they have the same volume.

Hence, each gas has the same pressure, volume, and temperature.

According to Avogadro's law, the three vessels will contain an equal number of the respective molecules. This number is equal to Avogadro's number, $N = 6.023 \times 10^{23}$.

The root mean square speed (v_{rms}) of a gas of mass m, and temperature T, is given by the relation:

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Where, k is Boltzmann constant

For the given gases, k and T are constants.

Hence v_{rms} depends only on the mass of the atoms, i.e.,

$$v_{rms} \propto \sqrt{\frac{1}{m}}$$

Therefore, the root mean square speed of the molecules in the three cases is not the same. Among neon, chlorine, and uranium hexafluoride, the mass of neon is the smallest. Hence, neon has the largest root mean square speed among the given gases.

6.A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

ANSWER:

Length of the narrow bore, L = 1 m = 100 cm

Length of the mercury thread, l = 76 cm

Length of the air column between mercury and the closed end, l_a = 15 cm

Since the bore is held vertically in air with the open end at the bottom, the mercury length that occupies the air space is: 100 – (76 + 15) = 9 cm

Hence, the total length of the air column = 15 + 9 = 24 cm

Let h cm of mercury flow out as a result of atmospheric pressure.

∴Length of the air column in the bore = 24 + h cm

And, length of the mercury column = 76 – h cm

Initial pressure, P_1 = 76 cm of mercury

Initial volume, V_1 = 15 cm^3

Final pressure, P_2 = 76 – (76 – h) = h cm of mercury

Final volume, V_2 = (24 + h) cm^3

Temperature remains constant throughout the process.

$\therefore P_1V_1 = P_2V_2$

76 × 15 = h (24 + h) h^2 +

24h – 1140 = 0

$$\therefore h = \frac{-24 \pm \sqrt{(24)^2 + 4 \times 1 \times 1140}}{2 \times 1}$$

= 23.8 cm or –47.8 cm

Height cannot be negative. Hence, 23.8 cm of mercury will flow out from the bore and 52.2 cm of mercury will remain in it. The length of the air column will be 24 + 23.8 = 47.8 cm.

SUMMARY

The kinetic theory of gases suggests that gases are composed of particles in motion. The continual bombardment of any surface by the gas causes a pressure to be exerted; the greater the density of a gas, the more frequent the number of collisions between molecules and the surface and the greater the pressure exerted.