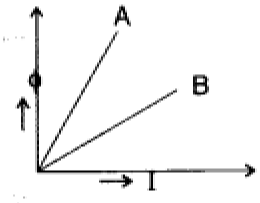
PHYSICS

Class XII

Chapter 6 - Electromagnetic Induction

1 Mark Questions

Question 1.
A plot of magnetic flux (ϕ) versus current (I) is shown in the figure for two inductors A and B. Which of the two has larger value of self inductance?

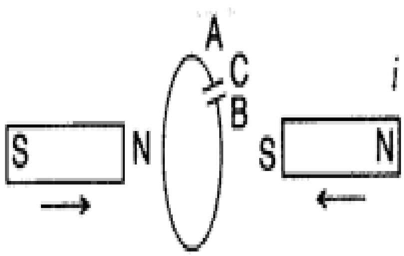


Answer:
Since $\phi = LI$
$\therefore L = \phi I$ = slope
Slope of A is greater than slope of B
∴ Inductor A has larger value of self inductance than inductor B.

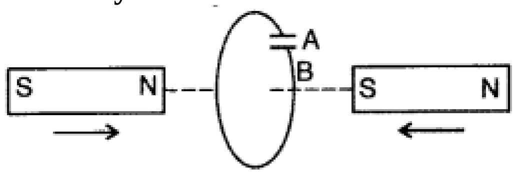
Question 2.
Define self-inductance of a coil. Write its S.I. unit.
Answer:
Self induction is the property of a coil by virtue of which it opposes the growth or decay of the current flowing through it.
S.I. unit of self-inductance is henry (H).

Question 3.
Two bar magnets are quickly moved towards a metallic loop connected across a capacitor 'C' as shown in the figure. Predict the polarity of the capacitor.

Answer:
When both magnets move towards loop, the A side plate of cL capacitor will be positive while the lower plate B is negative, making the induced current in a clockwise direction.

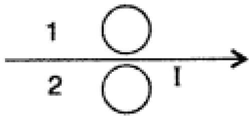


Question 4.
Predict the polarity of the capacitor when the two magnets are quickly moved in the directions marked by arrows.



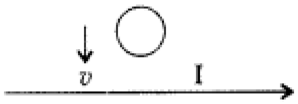
Answer:
Curerent in the coil will be anti-clockwise, when seen from the left, therefore plate A will become + ve (positive) and plate B will be negative.

Question 5.
Predict the directions of induced currents in metal rings 1 and 2 lying in the same plane where current I in the wire is increasing steadily.



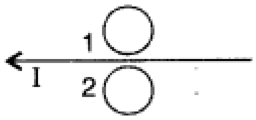
Answer:
In metal ring 1, the induced current flows in the clockwise direction.
In metal ring 2, the induced current flows in the anticlockwise direction.

Question 6.
Predict the direction of induced current in a metal ring when the ring is moved towards a conductor is carrying current I in the direction shown in the figure.



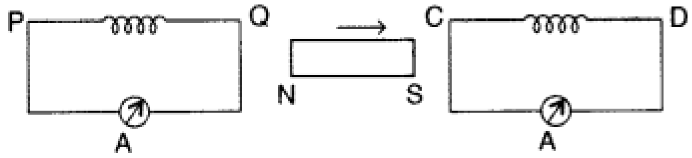
Answer:
Clockwise direction.

Question 7.
Predict the directions of induced current in metal rings 1 and 2 when current I in the wire is steadily decreasing?



Answer:
In metal ring 1, the induced current flows in Anticlockwise direction.
In metal ring 2, the induced current flows in the Clockwise direction.

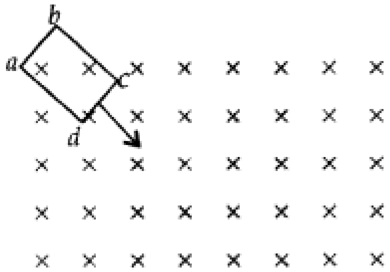
Question 8.
A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil.



Answer:
By Lenz's law, the ends of both the coils closer to the magnet behave as south pole. Hence the current induced in both the coils will flow clockwise when seen from the magnet side.

Question 9.
State Lenz's law.
Answer:
Lenz's law states that "the polarity of induced emf is such that it tends to produce a current, which opposes the change in magnetic flux that induced it".

Question 10.
Predict the direction of the induced current in the rectangular loop abed as it is moved into the region of a uniform magnetic field $\vec{B}$ directed normal to the plane of the loop.



Answer:
The direction of the induced current in the given rectangular loop is anti-clockwise, i.e., cbadc.

Question 11.
State Faraday's law of electromagnetic induction.
Answer:
Faraday's law states that "The magnitude of emf induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit". Mathematically, we can write,
e=−dϕdt …where [dϕ is the small change in magnetic flux in small time dt

Question 12.
How does the mutual inductance of a pair of coils change when
(i) distance between the coils is increased and
(ii) number of turns in the coils is increased
Answer:
(i) Mutual inductance decreases', because flux linked with the secondary coil decreases.
(ii) M=μ0n1n2Al, so when n_1 and n_2 increase, mutual inductance (M) increases.

Question 13.
A light metal disc on the top of an electromagnet is thrown up as the current is switched on. Why? Give reason.
Answer:
Because of Eddy Current

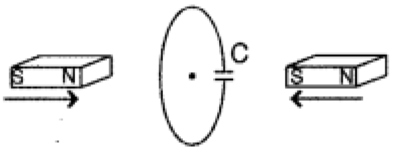
If the upper force of the core of the electromagnet acquires north polarity, then according to Lenz's Law, the lower face of the disc will also acquire north polarity. Due to the force of repulsion between the lower face (N-pole) of the core of the electromagnet, the disc jumps upto a certain height.

Question 14.
The motion of copper plate is damped when it is allowed to oscillate between the two poles of a magnet. What do the cause of this damping?
Answer:
The cause of this damping is eddy current.

Question 15.
The motion of copper plates is damped when it is allowed to oscillate between the two poles of a magnet. If slots are cut in the plate, how will the damping be affected?
Answer:
Eddy current will decrease due to which damping reduces.

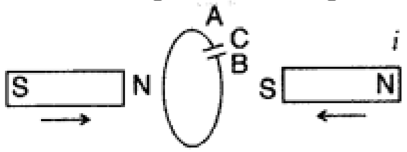
Question 16.
How does the mutual inductance of a pair of coils change when
(i) distance between the coils is decreased and
(ii) number of turns in the coils is decreased?
Answer:
(i) increases.
(ii) decreases, because
M=m0n1n2Al
where [N_1 and N_2 are number of turns

Question 17.
Predict the polarity of the capacitor in the situation described in the figure.



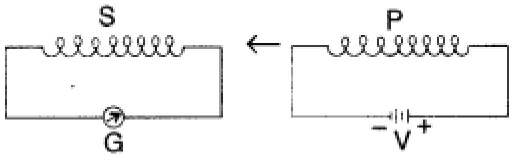
Answer:
When both magnets move towards loop, the A side plate of cL capacitor will be positive while the lower

plate B is negative, making the induced current in a clockwise direction.



2 Mark Questions

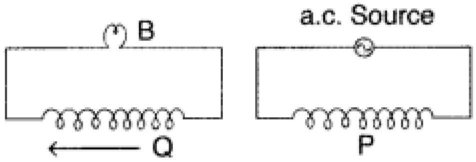
Question 1.
(i) When primary coil P is moved towards secondary coil S (as shown in the figure) the galvanometer shows momentary deflection. What can be done to have larger deflection in the galvanometer with the same battery?



(ii) State the related law.
Answer:
(i) To have larger deflection in the galvanometer with the same battery, coil P has to be moved faster towards S so that rate of change of magnetic flux is more.

(ii) The related law governing this phenomenon is Faraday’s second law of electromagnetic induction which states that induced emf is set up in a circuit when magnetic flux linked with it changes. The magnitude of induced emf is proportional to the rate of change of magnetic flux.

Question 2.
A coil Q is connected to low voltage bulb B and placed near another coil P as shown in the figure. Give reasons to explain the following observations :

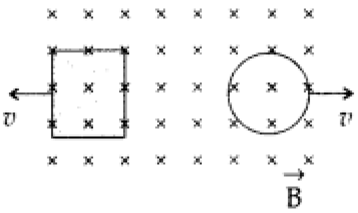


(a) The bulb ‘B’ lights.
(b) Bulb gets dimmer if the coil Q is moved towards left.
Answer:

1. The bulb B lights on account of emf induced in the coil Q due to mutual induction between P and Q.
2. When coil Q is moved towards left, magnetic flux linked with Q decreases and may even reduce to zero at some distance. The emf induced may decrease and the bulb B gets dimmer.

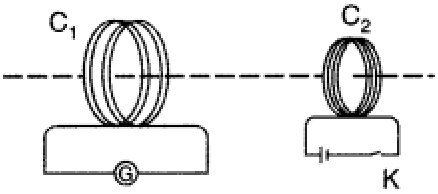
Question 3.
Two identical loops, one of copper and the other of aluminium, are rotated with the same angular speed in the same magnetic field. Compare
(i) the induced emf and
(ii) the current produced in the two coils. Justify your answer.
Answer:
(i) Induced emf in a coil is $\varepsilon = NBA\omega \sin \omega t$
As the angular speed is same, induced emf will also be same in both the loops.
(ii) Current induced in a loop is $I=\varepsilon R=\varepsilon A\rho l$
As the resistivity of copper is lesser, more amount of current is induced in it.

Question 4.
A rectangular loop and a circular loop are moving out of a uniform magnetic field to a field-free region with a constant velocity 'v' as shown in the figure. Explain in which loop do you expect the induced emf to be constant during the passage out of the field region. The magnetic field is normal to the loops.



Answer:
In case of rectangular loop, induced emf will be constant. It is because rate of change of the area of rectangular loop is uniform whereas that of a circular loop is not constant.

Question 5.
A current is induced in coil C_1 due to the motion of current carrying coil C_2.

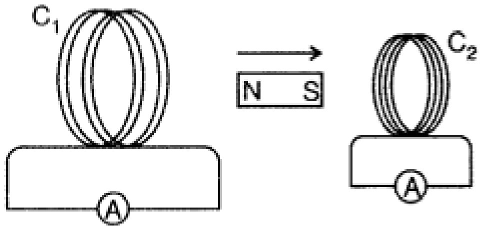


(a) Write any two ways by which a large deflection can be obtained in the galvanometer G.
(b) Suggest an alternative device to demonstrate the induced current in place of a galvanometer .
Answer:
(a) To obtain a large deflection, one or more of the following steps can be taken :

1. Use a rod made of soft iron inside the coil C_2.
2. Connect the coil to a powerful battery.
3. Move the arrangement rapidly towards the test coil C_1

(b) Replace the galvanometer by a small torch bulb.

Question 6.
A magnet is quickly moved in the direction indicated by an arrow between two coils C_1 and C_2 as shown in the figure. What will be the direction of induced current in each coil as seen from the magnet? Justify your answer.



Answer:
When magnet moves in given direction, induced current will be clockwise in both the coils as magnet is going away from C_1 and moving towards C_2 making C_1 as S pole and C_2 also as S pole according to Lenz rule. So C_1 will try to attract and C_2 will try to repel the motion of magnet.

Question 7.
What are eddy currents? Write any two applications of eddy currents.
Answer:
(a) Eddy current : Due to change in magnetic flux, if there is induced current in the volume (bulk) of the material, it is called as eddy currents. It is a necessary evil in an arrangement as it can be used in applications like electric brakes, induction furnaces and dead-beat galvanometers and brings loss of energy with heat production etc.
(b) (i) As the arm RS of length l is moved with a uniform speed, there is a change in area. It is given by dA = Idx = Ivdt

The emf induced, $e = -\dfrac{d\phi}{dt}$

...where [ϕ is the magnetic flux,
$e = \dfrac{BdA}{dt} = -Blv$

(ii) The current flow due to this,

$$I = \frac{e}{R} = -\frac{Blv}{R}$$

...where [R is the net resistance in the network with arm RS

The force experienced is therefore

$$f = IlB = \frac{B^2l^2v}{R}.$$

(iii) Power dissipated or required for the movement, $P = \vec{F}.\vec{v} = Fv = \left(\dfrac{B^2l^2v^2}{R^2}\right)$

Question 8.
Define self-inductance of a coil. Show that magnetic energy required to build up the current I in a coil of self inductance L is given by – 12I^2.
Answer:
The self-inductance of a coil may be defined as the induced emf set up in the coil due to a unit rate of change of current through it.
Let I be the current through the inductor L at any instant t
The current rises at the rate dlldt, so the induced

emf is $e = -L\frac{dI}{dt}$

Work done against the induced emf in small time dt is

$$dW = |e|Idt = LI\frac{dI}{dt}dt = LIdI$$

Total work done in building up the current from 0 to I in

$$W = \int dW = \int_0^1 LIdI = L\int_0^1 IdI = L\left[\frac{I^2}{2}\right]_0^1 = \frac{1}{2}LI^2$$

This work done is stored as the magnetic field energy U in the inductor

$$\therefore \quad U = \frac{1}{2}LI^2$$

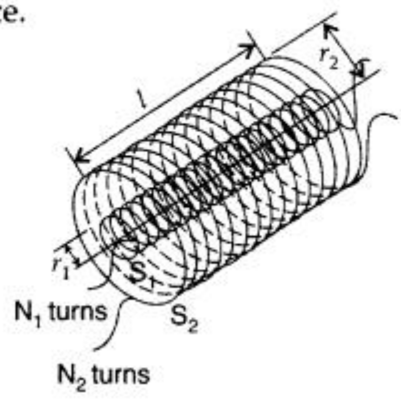
Question 9.
Define mutual inductance between two long coaxial solenoids. Find out the expression for the mutual inductance of inner solenoid of length l having the radius r_1 and the number of turns n_1 per unit length due to the second outer solenoid of same length and n_2 number of turns per unit length.
Answer:
Mutual induction is the phenomenon of inducing emf in a coil due to the rate of change of current in a nearby coil.

When a current I_2 is set up through S_2 (outer solenoid), it in turn sets up a magnetic flux through S_1. Let us denote it by ϕ_1, The corresponding flux linkage with solenoid S_1 is,

$$N_1\phi_1 = M_{12}I_2 \quad \ldots(1)$$

(Here N_1 is total no. of turns with S_1)

M_{12} is referred to as coeficient of mutual inductance.



The flux linkage with coil S_1 is,

$$N_1\phi_1 = (n_1 l)(\pi r_1^2)(\mu_0 n_2 I_2)$$

$$= \mu_0 n_1 n_2 \pi r_1^2 l I_2 \quad \ldots(2)$$

from equation (1)

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l$$

Now consider reverse case

$$N_2\phi_2 = M_{21} I_1 \quad \ldots(3)$$

(Here N_2 is total no. of turns with S_2)

M_{21} is referred to as coefficient of mutual inductance of solenoid S_2 with respect to solenoid S_1.

Thus, flux linkage with solenoid S_2 is

$$N_2\phi_2 = (n_2 l)(\pi r_1^2)(\mu_0 n_1 I_1) \quad \ldots(4)$$

$\therefore$ from equation (3), we get

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$

Using equations (2) and (3), we get

$M_{12} = M_{21} = M$

If a medium of relative permeability (it had been present the mutual inductance would be

$$\boxed{M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l}$$

Question 10.

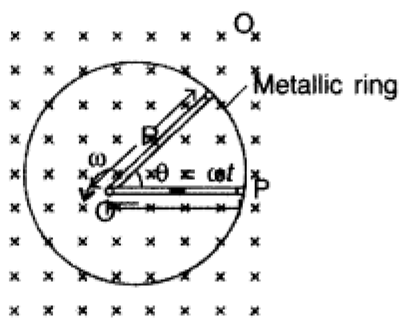
A metallic rod of 'L' length is rotated with angular frequency of 'ω' with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius L, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere. Deduce the expression for the emf between the centre and the metallic ring.

Answer:

The magnitude of the emf, generated across a length dr of the rod, as it moves at right angles to the magnetic field, is given by

$d\varepsilon = Bvdr$

$$\therefore \quad \varepsilon = \int d\varepsilon = \int_0^R Bv\,dr = \int_0^R B\omega r\,dr = \frac{B\omega R^2}{2}$$



Alternatively, The potential difference across the resistor is equal to the induced emf and equal B x .(rate of change of area of loop), If θ is the angle between the rod and the radius of the circle at P at time t, the area of the sector OPQ (as shown in the figure) is given by

$$\pi r^2 \times \frac{\theta}{2\pi} = \frac{1}{2}R^2\theta$$

where [R is the radius of the circle]

$$\varepsilon = B \times \frac{d}{dt}\left[\frac{1}{2}R^2\theta\right] = \frac{1}{2}BR^2\frac{d\theta}{dt} = \frac{B\omega R^2}{2}$$

Question 11.
Derive the expression for the self inductance of a long solenoid of cross sectional area A and. length l, having n turns per unit length.
Answer:
Self-induction of a long solenoid : Consider a long solenoid of length l and radius r with r << 1 and having n turns per unit length. If a current I flows through the coil, then the magnetic field inside the coil is almost constant and is given by

$$B = \mu_0 nI$$

Magnetic flux linked with each turn,

$$BA = \mu_0 nIA$$

where [n = number of turns per unit length, I = current flowing]

When $A = \pi r^2$

= the cross-sectional area of the solenoid

∴ Magnetic flux linked with the entire solenoid is,

$$\phi = \mu_0 nIA \times nl = \mu_0 n^2 IAl$$

But $\phi = LI$

∴ **Self inductance of the long solenoid** is,

$$L = \mu_0 n^2 Al$$

Question 12.
State Lenz's Law.
A metallic rod held horizontally along east-west direction, is allowed to fall under gravity. Will there be an emf induced at its ends? Justify your answer.
Answer:
Lenz's law states that "the polarity of induced emf is such that it tends to produce a current, which oppose the change in magnetic flux that induced it".

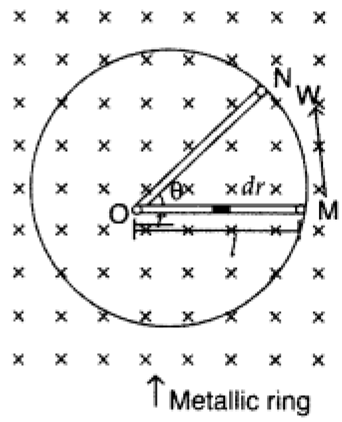
Yes there will be an emf induced as the horizontal component of field of earth, velocity of the motion of the rod and the length of the rod are all perpendicular to each other.

The magnetic flux due to vertical component of Earth's magnetic field keeps on changing as the metallic rod falls down.

4 Mark Questions

Question 1.
A metallic rod of length l is rotated at a constant angular speed to, normal to a uniform magnetic field B. Derive an expression for the current induced in the rod, if the resistance of the rod is R.
Answer:
emf produced/induced across a small-section (dr) on the rod

$$d\varepsilon = Bvdr$$
$$d\varepsilon = Br\omega dr \qquad [\because v = r\omega]$$



r = radius
l = length

Net emf induced across the length 'l' of the rod

$$\varepsilon = \int_0^l B\omega r dr \quad \text{or} \quad \varepsilon = \frac{B\omega l^2}{2}$$

If resistence of the rod is 'R', current induced is,

$$I = \frac{\varepsilon}{R} = \frac{B\omega l^2}{2R}.$$

Question 2.
An inductor 200 mH, capacitor 500 µF, resistor 10 Ω are connected in series with a 100 V variable frequency a.c. source. Calculate the
(i) frequency at which the power factor of the circuit is unity
(ii) current amplitude at this frequency
(iii) Q-factor
Answer:
(i) Power factor will be unity at resonance,

because then $Z = R$ and $\cos\phi = \frac{R}{Z} = 1$

$$\therefore \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

...where $[f_r$ = frequency at resonance

$$= \frac{1}{2\pi\sqrt{200 \times 10^{-3} \times 500 \times 10^{-6}}} \text{ Hz}$$

$$= \frac{1}{2\pi\sqrt{10^5 \times 10^{-9}}} \text{ Hz}$$

$$= \frac{1}{2\pi \times 10^{-2}} = \frac{50}{\pi} \text{ Hz}$$

$$(ii)\ I_0 = \frac{\varepsilon_0}{R} = \frac{\sqrt{2}\varepsilon_{rms}}{R} = \frac{1.414 \times 100}{10} = \mathbf{14.14\ A}$$

(iii) Q-factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{200 \times 10^{-3}}{500 \times 10^{-6}}}$$

$$= \frac{1}{10}\sqrt{\frac{2 \times 10^{-1}}{5 \times 10^{-4}}} = \frac{20}{10} = \mathbf{2}$$

Question 3.
A coil of number of turns N, area A, is rotated at a constant angular speed ω, in a uniform magnetic field B, and connected to a resistor R. Deduce expressions for :
(i) Maximum emf induced in the coil.
(ii) Power dissipation in the coil.
Answer:
We know that induced emf

$$(i)\ e = -\frac{d\phi}{dt} \quad \text{or } e = \frac{-d}{dt}(NAB\cos\omega t)$$

$$\therefore \quad e = NAB\omega \sin\omega t \quad \text{...where } [e_{max} = NAB\omega]$$

(ii) Power dissipation,

$$P = \frac{\varepsilon^2{}_{rms}}{R} = \frac{N^2A^2B^2\omega^2\sin^2 wt}{R}$$

Power dissipated over a complete cycle is,

$$P = \frac{N^2B^2A^2\omega^2}{2R}$$

Question 4.
(a) Define self inductance. Write its S.I. units.
(b) Derive an expression for self inductance of a long solenoid of length l, cross-sectional area A having N number of turns.
Answer:
(a) Self-inductance of a coil
Since flux $\phi = LI$

emf induced $\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

where [L is coefficient of self-induction or self inductance]
Self inductance is numerically equal to the magnetic flux linked with the coil when unit current passes through it.
Its S.I. unit is henry.
(b)
Consider a long solenoid of length l and radius r with r << 1 and having n turns per unit length. If a current I flows through the coil, then the magnetic field inside the coil is almost constant and is given by
$B = \mu_0 nI$
Magnetic flux linked with each turn = BA = $\mu_0 nIA$
...where [$A = \pi r^2$ = cross-sectional area of the solenoid

$\therefore$ Magnetic flux linked with the entire solenoid is

ϕ = Flux linked with each turn × Total number of turns

$= \mu_0 nIA \times nl = \mu_0 n^2 IAl$

But $\phi = LI$

$\therefore$ Self-inductance of the long solenoid is

$L = \mu_0 n^2 lA$

If N is the total number of turns in the solenoid

then $n = \frac{N}{l}$

$\therefore \quad L = \frac{\mu_0 N^2 A}{l}$

Question 5.
(i) State Faraday's law of electromagnetic induction.
(ii) A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°?
Answer:
(i) First law : Whenever the magnetic flux linked with a closed circuit changes, an emf (and hence a current) is induced in it which lasts only so long as the change in flux is taking place. This phenomenon is called electromagnetic induction.
Second law : The magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the closed circuit.

Mathematically $|\varepsilon| = \frac{d\phi}{dt}$

(*ii*) v = 1800 km/h in the west, l = 25 m, B_R = 5×10^{-4} tesla, $\delta = 30°$

Since v, l and B are to be perpendicular to get induce emf, the vertical component alone will contribute to the induction

$$B_V = B_R \sin\delta = 5\times10^{-4}\times\sin 30°$$
$$= 2.5\times10^{-4}\text{ tesla}$$

The induced emf $= -B_V vl$

$$= -2.5\times10^{-4}\times\frac{1800\times10^3}{3600}\times 25$$

$$= -\frac{62.5}{2}\times10^{-1} = -3.125\text{ volt}$$

– ve sign signifies that the emf will oppose the change in magnetic flux causing it.

7 Marks Questions

Question 1.

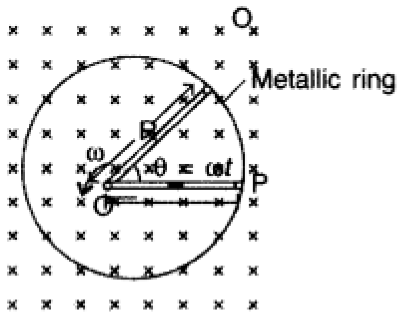
A metallic rod of length 'l' is rotated with a frequency v with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius r, about an axis passing through the centre and perpendicular to the plane of the ring. A constant uniform magnetic field B parallel to the axis is present every where. Using Lorentz force, explain how emf is induced between the centre and the metallic ring and hence obtain the expression for it.

Answer:

The magnitude of the emf, generated across a length dr of the rod, as it moves at right angles to the magnetic field, is given by

$d\varepsilon = Bvdr$

$$\therefore \quad \varepsilon = \int d\varepsilon = \int_0^R Bvdr = \int_0^R B\omega r dr = \frac{B\omega R^2}{2}$$



Alternatively, The potential difference across the resistor is equal to the induced emf and equal B x .(rate of change of area of loop), If θ is the angle between the rod and the radius of the circle at P at time t, the area of the sector OPQ (as shown in the figure) is given by

$$\pi r^2\times\frac{\theta}{2\pi} = \frac{1}{2}R^2\theta$$

where [R is the radius of the circle]

$$\varepsilon = B\times\frac{d}{dt}\left[\frac{1}{2}R^2\theta\right] = \frac{1}{2}BR^2\frac{d\theta}{dt} = \frac{B\omega R^2}{2}$$

Question 2.
Starting from the expression for the energy w = 12LI2, stored in a solenoid of self-inductance L to build up the current I, obtain the expression for the magnetic energy in terms of the magnetic field B, area A and length l of the solenoid having n number of turns per unit length. Hence show that the energy density is given by $B^2/2\mu_0$.
Answer:

Given : $W = \frac{1}{2}LI^2$

We know that $e = -N\frac{d\phi}{dt}$ Also $e = -L\frac{dI}{dt}$

From these two equations, we get

$$L = \frac{N d\phi/dt}{dI/dt} = \frac{N\phi}{I} \qquad (N = n \times l)$$

$$\Rightarrow L = \frac{N(\vec{B} \cdot A)}{I} = \frac{N(\mu_0 n I)A}{I}$$

$$\Rightarrow L = \frac{nl(\mu_0 n I)A}{I} = \mu_0 n^2 A l$$

Since magnetic energy $= \frac{1}{2}LI^2 = \frac{1}{2}(\mu_0 n^2 A l)I^2$

$\Rightarrow$ Expression for magnetic energy

$$= \frac{1}{2}(\mu_0 n^2 I^2 A l)$$

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$$= \frac{1}{2}\frac{\mu_0^2 n^2 I^2 (Al)}{\mu_0 (Al)} \qquad (V = Al)$$

$$= \frac{1}{2}\frac{(\mu_0 n I)^2}{\mu_0}$$

$$\therefore \text{Energy density} = \frac{B^2}{2\mu_0}$$

Question 3.
A metallic rod of length 'l' is rotated with a uniform angular speed ω, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius R = l, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present eveywhere. Deduce the expression for the emf induced in the rod.
If r is the resistance of the rod and the metallic ring has negligible resistance, obtain the expression for the power generated.
Answer:
Are swept per rotation = $\pi R^2 = \pi l^2$ [$\because$ R = l]

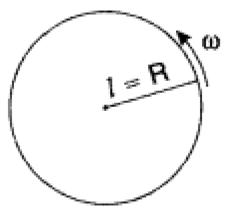
Area swept in v rotation per sec = $v\pi l^2 = \frac{dA}{dt}$

$$\frac{dA}{dt} = v\pi l^2$$

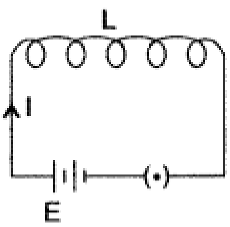
$$\varepsilon = -\frac{d\phi}{dt} \quad \Rightarrow \varepsilon = \frac{d}{dt}(BA)$$

$$\Rightarrow \varepsilon = B\frac{dA}{dt} \quad \Rightarrow \varepsilon = Bv\pi l^2$$

$$\Rightarrow \varepsilon = B\frac{\omega}{2\pi} \times \pi l^2 \qquad \therefore \varepsilon = \frac{1}{2}Bl^2\omega$$



Question 4.
Write its SI unit for self-inductance of a coil. Derive the expression for self-inductance of a long solenoid of cross-sectional area 'A', length 'l' having 'n' turns per unit length.
Answer:
Coefficient of self induction.
Consider a coil L as shown in Figure. Suppose a current I flows through the coil at any instant then magnetic flux ϕ linked with the coil is directly proportional to the current passing through it at that instant.



$\therefore \quad \phi \propto I$
$\Rightarrow \phi$ = LI where [L is called coefficient of self induction]
If I = 1, then ϕ = L
Thus, self inductance of a coil is numerically equal to the magnetic flux linked with the coil, when a unit current flows through it.

The SI unit of self inductance is henry (H).

Expression for self-inductance : Consider a long solenoid of length Z and cross-sectional area A having n turns per unit length.

The magnetic field due to a current flowing in the solenoid is B = $\mu_0$nl
Total flux linked with the solenoid is,

$$\phi = (nl)(\mu_0 nI)\,A = \mu_0 n^2\,Al\,I$$

where [nl is the total number of turns.]
Thus, the self inductance is, ϕI

$$= \frac{\mu_0 n^2 AlI}{I} = \mu_0 n^2\,Al$$

If we fill the inside of the solenoid with a material of relative permeability μ_r then
L = $\mu_r\mu_0 n^2 Al$

Question 5.
A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the place is 0.4 G and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased?
Answer:
If a rod length 'l' rotates with angular speed ω in the uniform magnetic field B,

$$\varepsilon = \frac{1}{2}Bl^2\omega$$

In case of earth's magnetic field $B_H = |B_e|\cos\delta$
and $B_v = |B_e|\sin\delta$

$$\varepsilon = \frac{1}{2}|B_e|\cos\delta l^2\omega$$

$$= \frac{1}{2}\times 0.4\times 10^{-4}\cos 60°\times(0.5)^2\times 2\pi\nu$$

$$= \frac{1}{2}\times 0.4\times 10^{-4}\times\frac{1}{2}\times(0.5)^2\times 2\pi\times\left(\frac{120\text{ rev}}{60\text{ sec}}\right)$$

$$= 10^{-5}\times 0.25\times 2\times 3.14\times 2 = 3.14\times 10^{-5}\text{ volt}$$

The induced emf will not change with the increase in the number of spokes.

Question 6.
Define the term 'mutual inductance' between the two coils.
Obtain the expression for mutual inductance of a pair of long coaxial solenoids each of length l and radii r_1 and r_2 ($r_2 >> r_1$). Total number of turns in the two solenoids are N_1 and N_2 respectively.
Answer:
(i) Mutual inductance. The mutual inductance of two coils is numerically equal to the induced emf produced in one coil, when the rate of change of current is unit in the other coil.

$$\therefore\ \varepsilon_1 = -M\frac{dI}{dt}$$

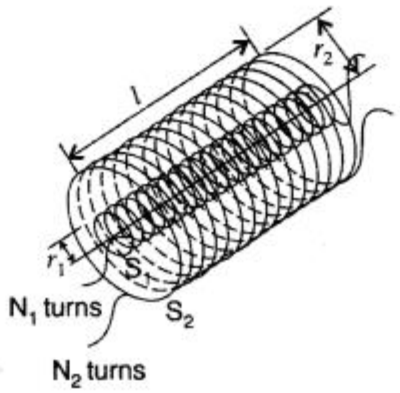
(ii)

(a) Lenz's law (Direction) : "The direction of induced current in a circuit is such as to oppose the change that causes it. So emf=−dϕdt

When north pole of a magnet, for example, is brought near a coil, then the nearer end of the coil will acquire north polarity and mechanical work will be needed to cause the relative motion between the coil and the magnet. This work, in fact, converts itself in the form of induced emf. Hence, the phenomenon of electromagnetic induction is in accordance with the law of conservation of energy.

(b) When a current I_2 is set up through S_2, it in turn sets up a magnetic flux through S_1. Let us denote it by ϕ_1. The corresponding flux linkage with solenoid S_1 is

$$N_1\phi_1 = M_{12}I_2 \qquad \ldots(i)$$



M_{12} is called the mutual inductance of solenoid S_1 with respect to solenoid S_2.

The magnetic field due to the current I_2 in S_2 is $\mu_0 n_2 I_2$. The resulting flux linkage with coil S_1 is,

$$N_1\phi_1 = (n_1 l)(\pi r_1^2)(\mu_0 n_2 I_2)$$
$$= \mu_0 n_1 . n_2 \pi r_1^2 l I_2 \qquad \ldots(ii)$$

where $[n_1 l$ is the total number of turns in solenoid S_1

Thus $M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l \qquad \ldots(iii)$

We now consider the reverse case :

A current I_1, is passed through the solenoid S_1 and the flux linkage with coil S_2 is,

$$N_2\phi_2 = M_{21}I_1 \qquad \ldots(iv)$$

M_{21} is called the mutual inductance of the solenoid S_2 with respect to solenoid S_1The flux due to the current I_1 in S_1 can be assumed to be confined solely inside S_1 since the solenoid are very long. Thus, flux linkage with solenoid S_2 is

$$N_2\phi_2 = (n_2 l)(\pi r_1^2)(\mu_0 n_1 I_1) \qquad \ldots(v)$$

where $[n_2 l$ is the total number of turns of S_2

From equation (iv)

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$

From equations (ii) and (iii), we get

$$M_{12} = M_{21} = M \text{ (say)}$$

$$\therefore \boxed{M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l}$$

Question 7.
Define the term self-inductance of a solenoid. Obtain the expression for the magnetic energy stored in an inductor of self-inductance L to build up a current I through it.
Answer:
Self-inductance: emf is induced in a single isolated coil due to change of flux through the coil by means of varying the current through the same coil. This phenomenon is called self-induction. In this case, flux linkage through a coil of N turns is proportional to the current through the coil and is expressed as

$$N\phi_B \propto I, \qquad N\phi_B = LI$$

where constant of proportionality L is called self-inductance of the coil. It is also called the coefficient of self-induction of the coil. When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil. Using the above equation the induced emf is given by

$$\varepsilon = \frac{d(N\phi_B)}{dt} \quad \text{or } \varepsilon = -L\frac{dI}{dt}$$

Thus, the self-induced emf always opposes any change (increase or decrease) of current in the coil. It is possible to calculate the self-inductance for circuits with simple geometries. Let us calculate the self-inductance of a long solenoid of cross-sectional area A and length Z, having n turns per unit length. The magnetic field due to a current I flowing in the solenoid is $B = \mu_0 nI$ (neglecting edge effects, as before). The total flux linked with the solenoid is

$$N\phi_B = (nl)(\mu_0 nI)(A) = \mu_0 n^2 AlI$$

where [*nl* is the total number of turns.

$$\text{Thus, the self-inductance is, } L = \frac{N\phi_B}{I} = \mu_0 n^2 Al$$

If we fill the inside of the solenoid with a material of relative permeability (e.g. soft iron, which has a high value of relative permiability), then $L = \mu_r \mu_0 n^2 Al$

The self-inductance of the coil depends on its geometry and on the permeability of the medium. The self-induced emf is also called the back emf as it opposes any change in the current in a circuit. Physically, the self-inductance plays the role of inertia. It is the electromagnetic analogue of mass in mechanics. So, work needs to be done against the back emf (ε) in establishing the current. This work done is stored as magnetic potential energy. For the current I at an instant in a circuit, the rate of work done is

$$\frac{dW}{dt} = |\varepsilon| I$$

If we ignore the resistive losses and consider only inductive effect, then using

$$\varepsilon = -L\frac{dI}{dt}, \quad \frac{dW}{dt} = LI\frac{dI}{dt}$$

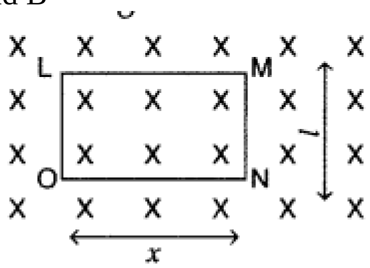
Total amount of work done in establishing the current I is,

$$W = \int dW = \int_0^I LI\, dI = \frac{1}{2}LI^2$$

Thus, the magnetic energy required to build up the current I is,

$$\text{the current I is, } W = \frac{1}{2}LI^2$$

Question 8.
(a) A rod of length l is moved horizontally with a uniform velocity 'v' in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod.
(b) How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor? Explain.
Answer:
(a) Induced emf in rotating metallic rod: Suppose a rectangular loop LMNO is placed in a uniform magnetic field B



Suppose at any instant, length ON = x
Area of the loop LMNO = lx
Flux through the loop, ϕ = Blx

...[$\because$ Max flux, ϕ = BA]

$$\text{Induced emf, } E = -\frac{d\phi}{dt} = -\frac{d}{dt}Blx$$
$$= -Bl\frac{dx}{dt} = Blv$$

where $\left[\frac{dx}{dt} = -v\right.$, that is the velocity of conductor MN.

(b) The induced emf produced in a rod (l) moved with velocity (v) kept in a magnetic field (perpendicular to the plane of length of the rod) is given by e = BIv …(i)

This induced emf is called motional emf, by moving a conductor instead of varying the magnetic field; that is by changing the magnetic flux enclosed by the circuit.

We can explain motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor.
Lorentz force acting on charge q is

$$|F| = |qvB| \qquad \ldots(ii)$$

The work done in moving, the charge through a distance is : W = qvBl
Since the emf is the work done per unit

$$\text{charge, } e = \frac{W}{q} = \frac{qvBl}{q} = Blv \qquad \ldots(iii)$$

This expression is the same as given in (i)

Question 9.
Derive the expression for the magnetic energy stored in a solenoid in terms of magnetic field B, area A and length l of the solenoid carrying a steady current I. How does this magnetic energy per unit volume compare with the electrostatic energy density stored in a parallel plate capacitor?
Answer:
Magnetic Energy in a Solenoid,

Rate of work done, $\frac{dW}{dt} = |\varepsilon| \quad I = (LI)\frac{dI}{dt}$

$\because \quad dW = LIdI$

Total amount of work done,

$$\int dW = \int LI\,dI, \qquad W = \frac{1}{2}LI^2$$

For the solenoid, we know

Inductance, $L = \mu_0 n^2 Al$; also, $B = \mu_0 nI$

$$\therefore \quad W = U_B = \frac{1}{2}LI^2 = \frac{1}{2}(\mu_0 n^2 Al)\left(\frac{B}{\mu_0 n}\right)^2$$

$$= \frac{B^2 Al}{2\mu_0} = \frac{\mathbf{B^2 V}}{\mathbf{2\mu_0}}$$

$\because$ Magnetic energy per unit volume (E_M)

$$= \frac{B^2}{2\mu_0} = \frac{\mathbf{1}}{\mathbf{2\mu_0}}\mathbf{B^2} \qquad \ldots(i)$$

The electrostatic energy stored per unit volume for a parallel plate capacitor,

$$E_S = \frac{1}{2}\epsilon_0 E^2 \qquad \ldots(ii)$$

These two expressions are similiar in nature.

Fill in the Blanks

1. Which of the following factors is the induced charge in an electromagnetic induction independent of----------------------**(Time)**

2. Which of the following states that an emf is induced whenever there is a change in the magnetic field linked with electric circuits--------------**(None)**

3. Which of the following gives the polarity of the induced emf-----------------

(Lenz's Law)

4. Electrical Inertia is the measure of----------------**(Self Inductance)**

5. Which of the following laws is the consequence of the Law of conservation of energy------------ **(Lenz's Law)**

6. Which of the following apparatus construction uses electromagnetic induction is---------- **(Generator)**

Multiple choice questions

Q.1. Whenever the magnetic flux linked with an electric circuit changes, an emf is induced in the circuit. This is called
(a) electromagnetic induction
(b) lenz's law
(c) hysteresis loss
(d) kirchhoff's laws

Answer:(a)

Q.2. In electromagnetic induction, the induced charge is independent of
(a) change of flux
(b) time.
(c) resistance of the coil
(d) None of these

Answer:(b)

Q.3. An induced e.m.f. is produced when a magnet is plunged into a coil. The strength of the induced e.m.f. is independent of
(a) the strength of the magnet
(b) number of turns of coil
(c) the resistivity of the wire of the coil
(d) speed with which the magnet is moved

Answer:(c)

Q.4. According to Faraday's law of electromagnetic induction
(a) electric field is produced by time varying magnetic flux.
(b) magnetic field is produced by time varying electric flux.
(c) magnetic field is associated with a moving charge.
(d) None of these

Answer:(a)

Q.5. A moving conductor coil produces an induced e.m.f. This is
in accordance with
(a) Lenz's law
(b) Faraday's law
(c) Coulomb's law
(d) Ampere's law

Answer:(b)

Q.6. A coil of insulated wire is connected to a battery. If it is taken to galvanometer, its pointer is deflected, because
(a) the induced current is produced
(b) the coil acts like a magnet
(c) the number of turns in the coil of the galvanometer are changed
(d) None of these

Answer:(a)

Q.7. The polarity of induced emf is given by
(a) Ampere's circuital law
(b) Biot-Savart law
(c) Lenz's law
(d) Fleming's right hand rule

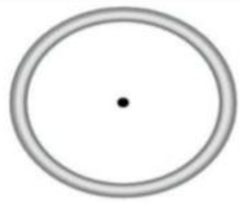
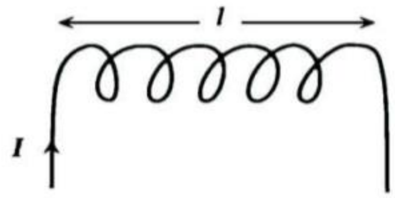
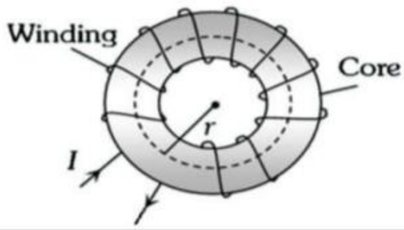
Answer:(c)

Q.8. The self inductance of a coil is a measure of
(a) electrical inertia
(b) electrical friction
(c) induced e.m.f.
(d) induced current

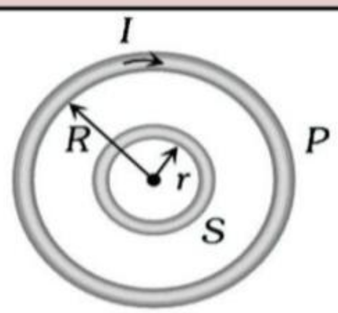
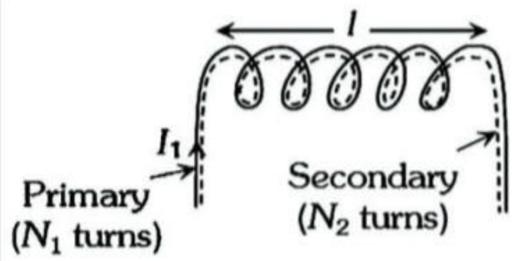
Answer:(a)

Diagrams

The various formulae for L

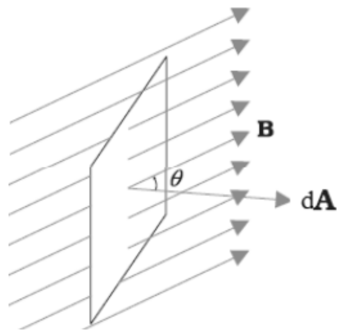
Condition	Figure
Circular coil $L = \dfrac{\mu_0 \pi N^2 r}{2}$	
Solenoid $L = \dfrac{\mu_0 \mu_r N^2 A}{l} = \dfrac{\mu N^2 A}{l}$ $(\mu = \mu_0 \mu_r)$	
Toroid $L = \dfrac{\mu_0 N^2 r}{2}$	

The various formulae for M

Condition	Figure
Two concentric coplanar circular coils $M = \dfrac{\pi \mu_0 N_1 N_2 r^2}{2R}$	
Two Solenoids $M = \dfrac{\mu_0 N_1 N_2 A}{l}$	

SUMMARY

- **Magnetic Flux:**
  Magnetic flux through a plane of area d*A* placed in a uniform magnetic field B

  $$\phi = \int \vec{B}.d\vec{A}$$
  If the surface is closed, then
  $$\phi = \int \vec{B}.d\vec{A}$$
  This is because magnetic lines of force are closed lines and free magnetic poles do not exist.

- **Faraday's Law:**
  a) First Law: whenever there is a change in the magnetic flux linked with a circuit with time, an induced emf is produced in the circuit which lasts as long as the change in magnetic flux continues.
  b) Second Law: According to this law,
  Induced emf, $E \propto \left(\frac{d\phi}{dt}\right)$

- **Lenz's Law:**
  The direction of the induced emf or current in the circuit is such that it opposes the cause due to which it is produced, so that,
  $$E = -N\left(\frac{d\phi}{dt}\right)$$
  Where N is the number of turns in coil
  Lenz's law is based on energy conservation.

- **Induced EMF and Induced Current:**
  a) Induced EMF,
  $$E = -N\frac{d\phi}{dt}$$
  $$= -\frac{N(\phi_2 - \phi_1)}{t}$$
  b) Induced current,
  $$I = \frac{E}{R} = -\frac{N}{R}\left(\frac{d\phi}{dt}\right)$$
  $$= -\frac{N}{R}\frac{(\phi_2 - \phi_1)}{t}$$
  Charge depends only on net change in flux does not depends on time.

- **Induced Emf due to Linear Motion of a Conducting Rod in a Uniform Magnetic Field**
  The induced emf,

$$E = -\vec{l}.(\vec{v} x \vec{B})$$

If $\vec{e}, \vec{v}$ and $\vec{B}$ are perpendicular to each other, then

$$E = Bvl$$

- **Induced EMF due to Rotation of a Conducting Rod in a Uniform Magnetic Field:**
  The induced emf,

  $$E = \frac{1}{2} B\omega l^2 = B\pi n l^2 = BAn$$

  Where n is the frequency of rotation of the conducting rod.
- **Induced EMF due to Rotation of a Metallic Disc in a Uniform Magnetic Field:**

  $$E_{OA} = \frac{1}{2} B\omega R^2 = B\pi R^2 n = BAn$$

- **Induced EMF, Current and Energy Conservation in a Rectangular Loop Moving in a Non – Uniform Magnetic Field with a Constant Velocity:**

a) The net increase in flux crossing through the coil in time Δt is,

$$\Delta\phi = (B_2 - B_1) lv\Delta t$$

b) Induced emf in the coil is,

$$E = (B_1 - B_2) lv$$

c) If the resistance of the coil is R, then the induced current in the coil is,

$$I = \frac{E}{R} = \frac{(B_1 - B_2)}{R} lv$$

d) Resultant force acting on the coil is

$$F = Il(B_1 - B_2)(\text{ towards left})$$

e) The work done against the resultant force

$$W = (B_1 - B_2)^2 \frac{l^2 v^2}{R} \Delta t \text{ joule}$$

Energy supplied in this process appears in the form of heat energy in the circuit.

f) Energy supplied due to flow of current I in time Δt is,

$$H = I^2 R\Delta t$$

$$Or\ H = (B_1 - B_2)^2 \frac{l^2 v^2}{R} \Delta t \text{ joule}$$

Or H = W

- **Rotation of Rectangular Coil in a Uniform Magnetic Field:**

a) Magnetic flux linked with coil

$$\phi = BAN \cos\theta$$

$$= BAN \cos\omega t$$

b) Induced emf in the coil

$$E = \frac{d\phi}{dt} = BAN\omega \sin\omega t = E_0 \sin\omega t$$

c) Induced current in the coil.

$$I = \frac{E}{R} = \frac{BAN\omega}{R}\sin\omega t$$

$$= \frac{E_0}{R}\sin\omega t$$

d) Both Emf and current induced in the coil are alternating

- **Self-Induction and Self Inductance:**

a) The phenomenon in which an induced emf is produced by changing the current in a coil is called self in induction.

$$\phi \propto I \text{ or } \phi = LI$$

$$or\ \mathrm{L} = \frac{\phi}{I}$$

$$E = -L\frac{dI}{dt}$$

$$L = \frac{E}{-(dI/dt)}$$

where L is a constant, called self inductance or coefficient of self – induction.

b) Self inductance of a circular coil

$$L = \frac{\mu_0 N^2 \pi R}{2} = \frac{\mu_0 N^2 A}{2R}$$

c) Self inductance of a solenoid

$$L = \frac{\mu_0 N^2 A}{l}$$

d) Two coils of self – inductances L_1 and L_2, placed far away (i.e., without coupling) from each other.

i) For series combination:

$$L = L_1 + L_2 \ldots\ldots\ldots L_n$$

ii) For parallel combination:

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$$

- **Mutual Induction and Mutual Inductance:**

a) On changing the current in one coil, if the magnetic flux linked with a second coil changes and induced emf is produced in that coil, then this phenomenon is called mutual induction.

$$\phi_2 \propto I_1 \text{ or } \phi_2 = MI_1$$

$$\text{Or } M = \frac{\phi_2}{I_1}$$

$$E_2 = -\frac{d\phi_2}{dt} = -M\frac{dI_1}{dt}$$

$$M = \frac{E_2}{-(dI_1/dt)}$$

Therefore, $M_{12} = M_{21} = M$

b) Mutual inductance two coaxial solenoids

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

c) If two coils of self- inductance L_1 and L_2 are wound over each other, the mutual inductance is,

$$M = K\sqrt{L_1 L_2}$$

Where K is called coupling constant.

d) Mutual inductance for two coils wound in same direction and connected in series

$$L = L_1 + L_2 + 2M$$

e) Mutual inductance for two coils wound in opposite direction and connected in series

$$L = L_1 + L_2 - 2M$$

f) Mutual inductance for two coils in parallel

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}$$

- **Energy Stored in an Inductor:**

$$U_B = \frac{1}{2} L I^2_{max}$$

- **Magnetic Energy Density:**

$$U_B = \frac{B^2}{2\mu_0}$$

- **Eddy Current:**
When a conductor is moved in a magnetic field, induced currents are generated in the whole volume of the conductor. These currents are called eddy currents.

- **Transformer:**

a) It is a device which changes the magnitude of alternating voltage or current.

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = K$$

b) For ideal transformer:

$$\frac{I_p}{I_s} = \frac{n_s}{n_p}$$

c) In an ideal transformer:

$$E_p I_p = E_s I_s$$

d) In step – up transformer:

$n_s > n_p$ or K > 1

$E_s > E_p$ and $I_s < I_p$

e) In step – down transformer:

$n_s < n_p$ or K < 1

$E_s < E_p$ and $I_s > I_p$

f) Efficiency

$$\eta = \frac{E_s I_s}{E_p I_p} x100\%$$

- **Generator or Dynamo:**

  It is a device by which mechanical energy is converted into electrical energy. It is based on the principle of electromagnetic induction.

- **Different Types of Generator:**

a) AC Generator

It consists of field magnet, armature, slip rings and brushes.

b) DC Generator

It consists of field magnet, armature, commutator and brushes.

- **Motor:**

  It is a device which converts electrical energy into mechanical energy.

  Back emf $e \propto \omega$

  Current flowing in the coil,

$$i_a = \frac{E - e_b}{R}$$

$$E = e_b + i_a R$$

Where R is the resistance of the coil.

Out put Power = $i_a e_b$

Efficiency,

$$\eta = \frac{e_b}{E} \times 100\%$$