Chapter-7

Coordinate Geometry

2 MARKS QUESTIONS

1. Find the distance between the following pairs of points:

(i) (2, 3), (4, 1)

(ii) (-5, 7), (-1, 3)

(iii) (a, b), (- a, – b)

Solution:

Distance formula to find the distance between two points (x_1, y_1) and (x_2, y_2) is, say d,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{OR } d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(i) Distance between (2, 3), (4, 1)

$$d = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

(ii) Distance between (-5, 7), (-1, 3)

$$d = \sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$$

(iii) Distance between (a, b), (- a, - b)

$$d = \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$$

2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns, A and B, discussed in Section 7.2?

Solution:

Let us consider town A at point (0, 0). Therefore, town B will be at point (36, 15).

Distance between points (0, 0) and (36, 15)

$$d = \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$$

In section 7.2, A is (4, 0) and B is (6, 0)
$AB^2 = (6 – 4)^2 – (0 – 0)^2 = 4$

The distance between towns A and B will be 39 km. The distance between the two towns, A and B, discussed in Section 7.2, is 4 km.

3. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Solution:

If the sum of the lengths of any two line segments is equal to the length of the third line segment, then all three points are collinear.

Consider, A = (1, 5) B = (2, 3) and C = (-2, -11)

Find the distance between points: say AB, BC and CA

$$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$
$$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$$
$$CA = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$

Since AB + BC ≠ CA

Therefore, the points (1, 5), (2, 3), and (- 2, – 11) are not collinear.

4. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (- 3, 4).

Solution:

Point (x, y) is equidistant from (3, 6) and (- 3, 4).

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$
$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

Squaring both sides, $(x – 3)^2+(y – 6)^2 = (x + 3)^2 +(y – 4)^2$

$x^2 + 9 – 6x + y^2+ 36 – 12y = x^2 + 9 + 6x + y^2 +16 – 8y$

36 – 16 = 6x + 6x + 12y – 8y

20 = 12x + 4y

3x + y = 5

3x + y – 5 = 0

5. Find the coordinates of the point which divides the join of (- 1, 7) and (4, – 3) in the ratio 2:3.

Solution:

Let P(x, y) be the required point. Using the section formula, we get

x = (2×4 + 3×(-1))/(2 + 3) = (8 – 3)/5 = 1

y = (2×-3 + 3×7)/(2 + 3) = (-6 + 21)/5 = 3

Therefore, the point is (1, 3).

6. Find the ratio in which the line segment joining the points (-3, 10) and (6, – 8) is divided by (-1, 6).

Solution:

Consider the ratio in which the line segment joining (-3, 10) and (6, -8) is divided by point (-1, 6) be k :1.

Therefore, $-1 = (6k-3)/(k+1)$

$-k - 1 = 6k - 3$

$7k = 2$

$k = 2/7$

Therefore, the required ratio is 2: 7.

7. Find the coordinates of point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

Solution:

Let the coordinates of point A be (x, y).

Mid-point of AB is (2, – 3), which is the centre of the circle.

Coordinate of B = (1, 4)

$(2, -3) = ((x+1)/2 , (y+4)/2)$

$(x+1)/2 = 2$ and $(y+4)/2 = -3$

$x + 1 = 4$ and $y + 4 = -6$

$x = 3$ and $y = -10$

The coordinates of A(3,-10).

8. If A and B are (-2, -2) and (2, -4), respectively, find the coordinates of P such that AP = 3/7 AB and P lies on the line segment AB.

Solution:

The coordinates of points A and B are (-2,-2) and (2,-4), respectively.

Since AP = 3/7 AB

Therefore, AP:PB = 3:4

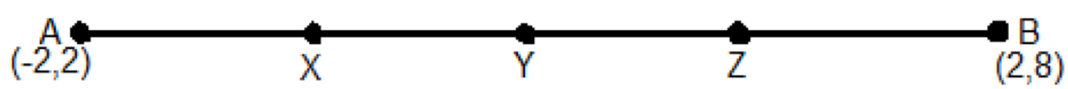
Point P divides the line segment AB in the ratio 3:4.

Coordinate of P = $\left(\frac{3(2)+4(-2)}{3+4}, \frac{3(-4)+4(-2)}{3+4}\right) = \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(-\frac{2}{7}, -\frac{20}{7}\right)$ which is required answer.

9. Find the coordinates of the points which divide the line segment joining A (- 2, 2) and B (2, 8) into four equal parts.

Solution:

Draw a figure, line dividing by 4 points.



From the figure, it can be observed that points X, Y, and Z are dividing the line segment in a ratio 1:3, 1:1, and 3:1, respectively.

Coordinates of X = $\left(\frac{1(2)+3(-2)}{1+3}, \frac{1(8)+3(2)}{1+3}\right)$ = (-1, 7/2)

Coordinates of Y = $\left(\frac{2(1)-2(1)}{1+1}, \frac{2(1)+8(1)}{1+1}\right)$ = (0, 5)

Coordinates of Z = $\left(\frac{3(2)+1(-2)}{1+3}, \frac{3(8)+1(2)}{1+3}\right)$ = (1, 13/2)

4 MARKS QUESTIONS

1. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Solution:

Since two sides of any isosceles triangle are equal, to check whether given points are vertices of an isosceles triangle, we will find the distance between all the points.

Let the points (5, – 2), (6, 4), and (7, – 2) represent the vertices A, B and C, respectively.

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{37}$$
$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{37}$$
$$CA = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{(-2)^2 + (0)^2} = 2$$

Here AB = BC = $\sqrt{37}$

This implies whether given points are vertices of an isosceles triangle.

2. Find the values of y for which the distance between the points P (2, – 3) and Q (10, y) is 10 units.

Solution:

Given: Distance between (2, – 3) and (10, y) is 10.

Using the distance formula,

$$PQ = \sqrt{(10-2)^2 + (y+3)^2} = \sqrt{(8)^2 + (y+3)^2}$$

Since PQ = 10

$$\sqrt{(8)^2 + (y+3)^2} = 10$$

Simplify the above equation and find the value of y.

Squaring both sides,

$64 + (y + 3)^2 = 100$

$(y + 3)^2 = 36$

$y + 3 = \pm 6$

$y + 3 = +6$ or $y + 3 = -6$

$y = 6 - 3 = 3$ or $y = -6 - 3 = -9$

Therefore, $y = 3$ or -9.

3. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (- 3, 4).

Solution:

Point (x, y) is equidistant from (3, 6) and (- 3, 4).

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$
$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

Squaring both sides, $(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$

$36 - 16 = 6x + 6x + 12y - 8y$

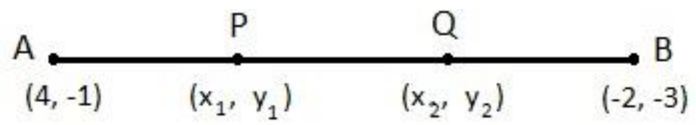
$20 = 12x + 4y$

$3x + y = 5$

$3x + y - 5 = 0$

4. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Solution:



Let P (x_1, y_1) and Q (x_2, y2) be the points of trisection of the line segment joining the given points, i.e. AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2.

x_1 = (1×(-2) + 2×4)/3 = (-2 + 8)/3 = 6/3 = 2

y_1 = (1×(-3) + 2×(-1))/(1 + 2) = (-3 – 2)/3 = -5/3

Therefore: P (x_1, y_1) = P(2, -5/3)

Point Q divides AB internally in the ratio 2:1.

x_2 = (2×(-2) + 1×4)/(2 + 1) = (-4 + 4)/3 = 0

y_2 = (2×(-3) + 1×(-1))/(2 + 1) = (-6 – 1)/3 = -7/3

The coordinates of the point Q are (0, -7/3)

5. Find the ratio in which the line segment joining A (1, – 5) and B (- 4, 5) is divided by the x-axis. Also, find the coordinates of the point of division.

Solution:

Let the ratio in which the line segment joining A (1, – 5) and B (- 4, 5) is divided by the x-axis be k:1. Therefore, the coordinates of the point of division, say P(x, y) is ((-4*k*+1)/(*k*+1), (5*k*-5)/(*k*+1)).

Or P(x, y) = $\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}$

We know that the y-coordinate of any point on the x-axis is 0.

Therefore, (5k – 5)/(k + 1) = 0

5k = 5

or k = 1

So, the *x*-axis divides the line segment in the ratio 1:1.

Now, find the coordinates of the point of division:

P (x, y) = ((-4(1)+1)/(1+1) , (5(1)-5)/(1+1)) = (-3/2 , 0)

6. Find the coordinates of point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

Solution:

Let the coordinates of point A be (x, y).

Mid-point of AB is (2, – 3), which is the centre of the circle.

Coordinate of B = (1, 4)

(2, -3) =((x+1)/2 , (y+4)/2)

(x+1)/2 = 2 and (y+4)/2 = -3

x + 1 = 4 and y + 4 = -6

x = 3 and y = -10

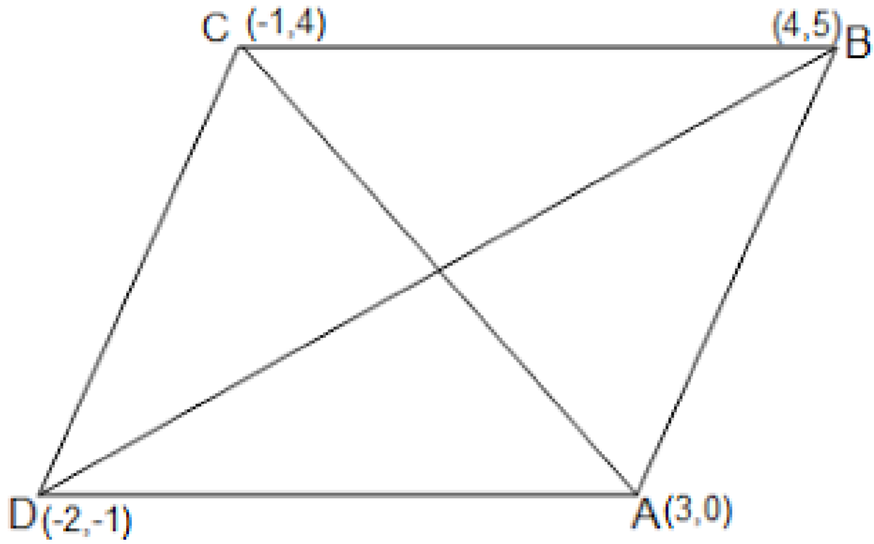
The coordinates of A(3,-10).

7. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4), and (-2,-1) taken in order.

[Hint: Area of a rhombus = 1/2 (product of its diagonals)

Solution:

Let A(3, 0), B (4, 5), C(– 1, 4) and D (– 2, – 1) are the vertices of a rhombus ABCD.



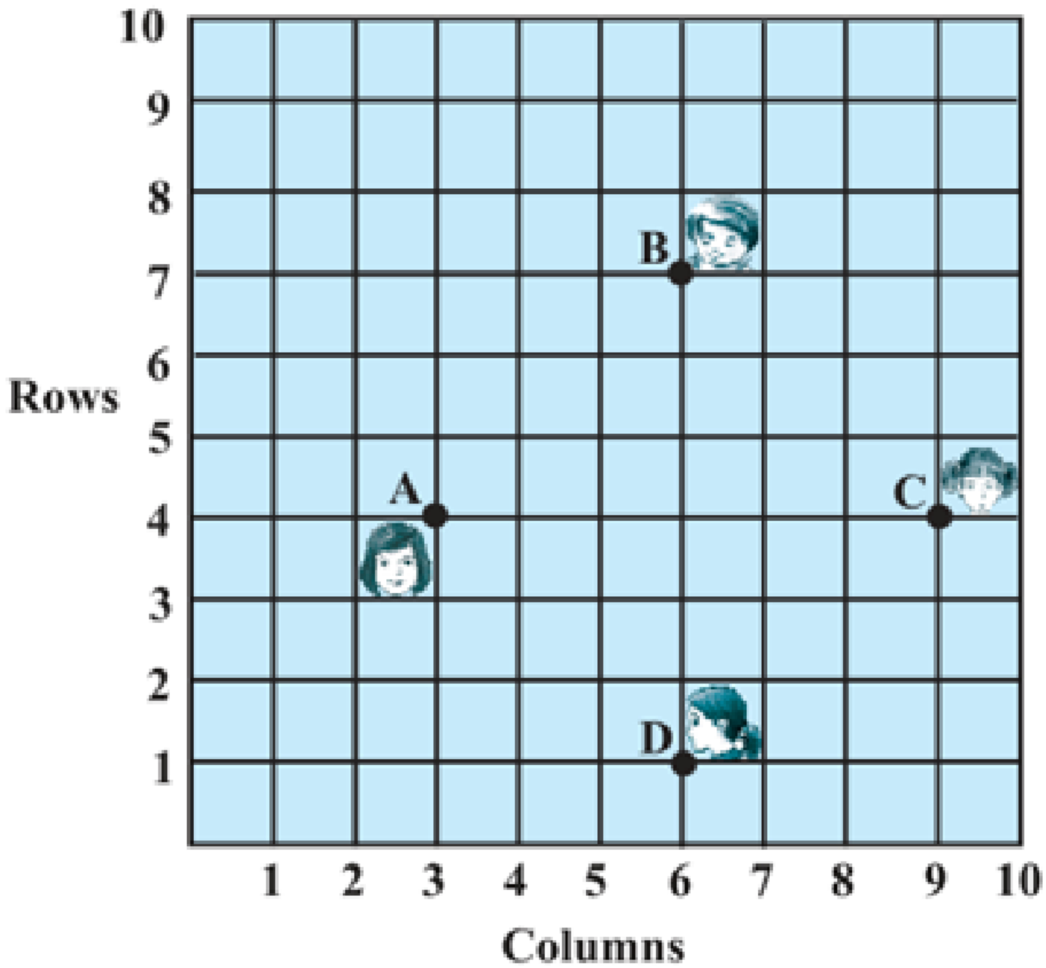
Length of diagonal AC $= \sqrt{(3-(-1)^2+(0-4)^2} = \sqrt{16+16} = 4\sqrt{2}$

Length of diagonal BD $= \sqrt{(4-(-2)^2+(5-(-1))^2} = \sqrt{36+36} = 6\sqrt{2}$

Therefore, area of rhombus ABCD $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ square units

7 MARKS QUESTIONS

1. In a classroom, 4 friends are seated at points A, B, C and D, as shown in Fig. 7.8. Champa and Chameli walk into the class, and after observing for a few minutes, Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using the distance formula, find which of them is correct.



Solution:

From the figure, the coordinates of points A, B, C and D are (3, 4), (6, 7), (9, 4) and (6,1).

Find the distance between points using the distance formula, we get

$$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$DA = \sqrt{(6-3)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$\text{Diagonal } AC = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal } BD = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$

$$AB = BC = CD = DA = 3\sqrt{2}$$

All sides are of equal length. Therefore, ABCD is a square, and hence, Champa was correct.

2. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (- 1, – 2), (1, 0), (- 1, 2), (- 3, 0)

(ii) (- 3, 5), (3, 1), (0, 3), (- 1, – 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Solution:

(i) Let the points (- 1, – 2), (1, 0), (– 1, 2), and (– 3, 0) represent the vertices A, B, C, and D of the given quadrilateral, respectively.

AB = $\sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = 2\sqrt{2}$
BC = $\sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$
CD = $\sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = 2\sqrt{2}$
DA = $\sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = 2\sqrt{2}$
AC = $\sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4$
BD = $\sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4$

Side length = AB = BC = CD = DA = $2\sqrt{2}$

Diagonal Measure = AC = BD = 4

Therefore, the given points are the vertices of a square.

(ii) Let the points (- 3, 5), (3, 1), (0, 3), and (- 1, – 4) represent the vertices A, B, C, and D of the given quadrilateral, respectively.

AB = $\sqrt{(-3-3)^2 + (1-5)^2} = \sqrt{36+16} = 2\sqrt{13}$
BC = $\sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$
CD = $\sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{1+49} = 5\sqrt{2}$
AD = $\sqrt{(-1+3)^2 + (-4-5)^2} = \sqrt{4+81} = \sqrt{85}$

It’s also seen that points A, B and C are collinear.
So, the given points can only form 3 sides, i.e. a triangle and not a quadrilateral which has 4 sides.
Therefore, the given points cannot form a general quadrilateral.

(iii) Let the points (4, 5), (7, 6), (4, 3), and (1, 2) represent the vertices A, B, C, and D of the given quadrilateral, respectively.

$$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10}$$
$$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{9+9} = \sqrt{18}$$
$$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$$
$$AD = \sqrt{(1-4)^2 + (2-5)^2} = \sqrt{9+9} = \sqrt{18}$$
$$AC \text{ (diagonal)} = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0+4} = 2$$
$$BD \text{ (diagonal)} = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = 13\sqrt{2}$$

Opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

3. Find the point on the x-axis which is equidistant from (2, – 5) and (-2, 9).

Solution:

To find a point on the x-axis.

Therefore, its y-coordinate will be 0. Let the point on the x-axis be (x,0).

Consider A = (x, 0); B = (2, – 5) and C = (- 2, 9).

$$AB = \sqrt{(2-x)^2 + (-5-0)^2} = \sqrt{(2-x)^2 + 25}$$
$$AC = \sqrt{(-2-x)^2 + (9-0)^2} = \sqrt{(-2-x)^2 + 81}$$

Since both the distance are equal in measure, so AB = AC

$$\sqrt{(2-x)^2 + 25} = \sqrt{(-2-x)^2 + 81}$$

Simplify the above equation,

Remove the square root by taking square on both sides, we get

$(2 – x)^2 + 25 = [-(2 + x)]^2 + 81$

$(2 – x)^2 + 25 = (2 + x)^2 + 81$

$x^2 + 4 – 4x + 25 = x^2 + 4 + 4x + 81$

$8x = 25 – 81 = -56$

$x = -7$

Therefore, the point is (- 7, 0).

4. Find the values of y for which the distance between the points P (2, – 3) and Q (10, y) is 10 units.

Solution:

Given: Distance between (2, – 3) and (10, y) is 10.

Using the distance formula,

$$PQ = \sqrt{(10-2)^2 + (y+3)^2} = \sqrt{(8)^2 + (y+3)^2}$$

Since PQ = 10

$$\sqrt{(8)^2 + (y+3)^2} = 10$$

Simplify the above equation and find the value of y.

Squaring both sides,

$64 + (y + 3)^2 = 100$

$(y + 3)^2 = 36$

$y + 3 = \pm 6$

$y + 3 = +6$ or $y + 3 = -6$

$y = 6 – 3 = 3$ or $y = – 6 – 3 = -9$

Therefore, y = 3 or -9.

5. If Q (0, 1) is equidistant from P (5, – 3) and R (x, 6), find the values of x. Also, find the distance QR and PR.

Solution:

Given: Q (0, 1) is equidistant from P (5, – 3) and R (x, 6), which means PQ = QR

Step 1: Find the distance between PQ and QR using the distance formula,

PQ = $\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$

QR = $\sqrt{(0-x)^2 + (1-6)^2} = \sqrt{(-x)^2 + (-5)^2} = \sqrt{x^2+25}$

Step 2: Use PQ = QR

$\sqrt{41} = \sqrt{x^2+25}$

Squaring both sides to omit square root

41 = x^2 + 25

x^2 = 16

x = ± 4

x = 4 or x = -4

Coordinates of Point R will be R (4, 6) or R (-4, 6),

If R (4, 6), then QR

QR = $\sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$

PR = $\sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{(1)^2 + (9)^2} = \sqrt{1+81} = \sqrt{82}$

If R (-4, 6), then

QR = $\sqrt{(0+4)^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$

PR = $\sqrt{(5+4)^2 + (-3-6)^2} = \sqrt{(9)^2 + (9)^2} = \sqrt{81+81} = 9\sqrt{2}$

6. To conduct sports day activities in your rectangular-shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs 1/4th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5th the distance AD on the eighth line and posts a red flag. What is the distance between both flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

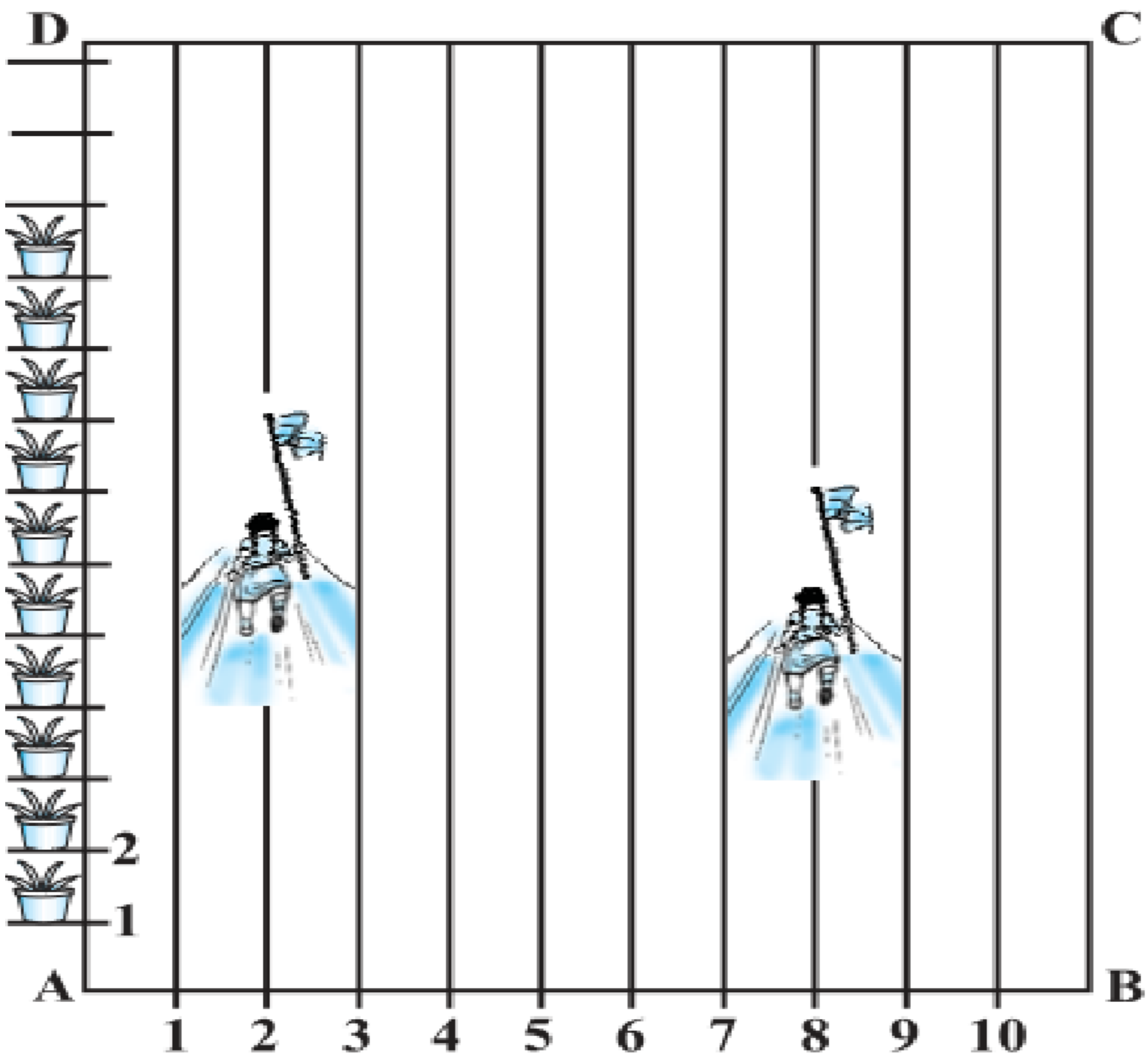


Fig. 7.12

Solution:

From the given instruction, we observed that Niharika posted the green flag at $1/4^{th}$ of the distance AD, i.e. (1/4 ×100) m = 25 m from the starting point of the 2nd line. Therefore, the coordinates of this point are (2, 25).

Similarly, Preet posted a red flag at 1/5 of the distance AD, i.e. (1/5 ×100) m = 20 m from the starting point of the 8th line. Therefore, the coordinates of this point are (8, 20).

Distance between these flags can be calculated by using the distance formula,

Distance between two flags = $\sqrt{(8-2)^2 + (20-25)^2} = \sqrt{36+25} = \sqrt{61}$ m

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let's say this point is P(x, y).

x = (2 + 8)/2 = 10/2 = 5 and y = (20 + 25)/2 = 45/2

Hence, P(*x*, *y*) = (5, 45/2)

Therefore, Rashmi should post her blue flag at 45/2 = 22.5m on the 5th line.

7. Find the ratio in which the line segment joining A (1, – 5) and B (- 4, 5) is divided by the x-axis. Also, find the coordinates of the point of division.

Solution:

Let the ratio in which the line segment joining A (1, – 5) and B (- 4, 5) is divided by the x-axis be k:1. Therefore, the coordinates of the point of division, say P(x, y) is ((-4*k*+1)/(*k*+1), (5*k*-5)/(*k*+1)).

Or P(x, y) = $\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}$

We know that the y-coordinate of any point on the x-axis is 0.

Therefore, (5k – 5)/(k + 1) = 0

5k = 5

or k = 1

So, the *x*-axis divides the line segment in the ratio 1:1.

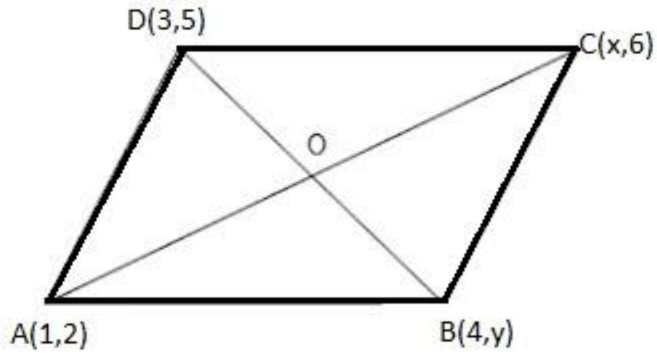
Now, find the coordinates of the point of division:

P (x, y) = ((-4(1)+1)/(1+1) , (5(1)-5)/(1+1)) = (-3/2 , 0)

8. If (1, 2), (4, *y*), (*x*, 6) and (3, 5) are the vertices of a parallelogram taken in order, find *x* and *y*.

Solution:

Let A, B, C and D be the points of a parallelogram: A(1, 2), B(4, *y*), C(*x*, 6) and D(3, 5).



Since the diagonals of a parallelogram bisect each other, the midpoint is the same.

To find the value of x and y, solve for the midpoint first.

Midpoint of AC = ((1+x)/2 , (2+6)/2) = ((1+x)/2 , 4)

Midpoint of BD = ((4+3)/2 , (5+y)/2) = (7/2 , (5+y)/2)

The midpoint of AC and BD are the same, this implies

$(1+x)/2 = 7/2$ and $4 = (5+y)/2$

$x + 1 = 7$ and $5 + y = 8$

$x = 6$ and $y = 3$