CHAPTER-12

Factorisation

2MARK Q&A:

Exercise 12.1

1. Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Solution:

(i)$28x^4 = 2\times2\times7\times x\times x\times x\times x$

$56x = 2\times2\times2\times7\times x$

$$28x^4 \div 56x = \frac{2\times2\times7\times x\times x\times x\times x}{2\times2\times2\times7\times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

$$\text{(ii) } -36y^3 \div 9y^2 = \frac{-2\times2\times3\times3\times y\times y\times y}{3\times3\times y\times y} = -4y$$

(iii) $66pq^2r^3 \div 11qr^2 = \dfrac{2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r} = 6pqr$

(iv) $34x^3y^3z^3 \div 51xy^2z^3 = \dfrac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z} = \frac{2}{3}x^2y$

(v) $12a^8b^8 \div (-6a^6b^4) = \dfrac{2 \times 2 \times 3 \times a^8 \times b^8}{-2 \times 3 \times a^6 \times b^4} = -2a^2b^4$

2. Divide the given polynomial by the given monomial.

(i)$(5x^2–6x) \div 3x$

(ii)$(3y^8–4y^6+5y^4) \div y^4$

(iii) $8(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3)\div 4x^2 y^2 z^2$

(iv)$(x^3+2x^2+3x) \div 2x$

(v) $(p^3q^6–p^6q^3) \div p^3q^3$

Solution:

$(i)\ 5x^2 - 6x = x(5x - 6)$

$$(5x^2 - 6x) \div 3x = \frac{x(5x - 6)}{3x} = \frac{1}{3}(5x - 6)$$

$(ii)\ 3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$$(3y^8 - 4y^6 + 5y^4) \div y^4 = \frac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$$

$(iii)\ 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$

$$8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2 = \frac{8x^2y^2z^2(x + y + z)}{4x^2y^2z^2} = 2(x + y + z)$$

$(iv)\ x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$$(x^3 + 2x^2 + 3x) \div 2x = \frac{x(x^3 + 2x^2 + 3)}{2x} = \frac{1}{2}(x^2 + 2x + 3)$$

$(v)\ p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$

$$(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^3(q^3 - p^3)}{p^3q^3} = q^3 - p^3$$

3. Work out the following divisions.

(i) (10x–25) ÷ 5

(ii) (10x–25) ÷ (2x–5)

(iii) 10y(6y+21) ÷ 5(2y+7)

(iv) $9x^2y^2(3z-24) \div 27xy(z-8)$

(v) 96abc(3a–12)(5b–30) ÷ 144(a–4)(b–6)

Solution:

(i) (10x–25) ÷ 5 = 5(2x-5)/5 = 2x-5

(ii) (10x–25) ÷ (2x–5) = 5(2x-5)/(2x-5) = 5

(iii) 10y(6y+21) ÷ 5(2y+7) = 10y×3(2y+7)/5(2y+7) = 6y

(iv) $9x^2y^2(3z-24) \div 27xy(z-8) = 9x^2y^2 \times 3(z-8)/27xy(z-8) = xy$

(v) 96abc(3a –12) (5b–30) ÷ 144(a–4)(b – 6) = $\frac{96\ abc \times 3(a-4) \times 5(b-6)}{144(a-4)\ (b-6)}$ = 10abc

4. Divide as directed.

(i) 5(2x+1)(3x+5)÷ (2x+1)

(ii) 26xy(x+5)(y–4)÷13x(y–4)

(iii) 52pqr(p+q)(q+r)(r+p) ÷ 104pq(q+r)(r+p)

(iv) 20(y+4) (y^2+5y+3) ÷ 5(y+4)

(v) x(x+1) (x+2)(x+3) ÷ x(x+1)

Solution:

$$(i)\ 5(2x+1)(3x+5) \div (2x+1) = \frac{5(2x+1)(3x+5)}{(2x+1)}$$

$$= 5(3x+5)$$

$$(ii)\ 26xy(x+5)(y-4) \div 13x(y-4) = \frac{2 \times 13 \times xy(x+5)(y-4}{13x(y-4)}$$

$$= 2y(x+5)$$

$$(iii)\ 52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$$

$$= \frac{2 \times 2 \times 13 \times p \times q \times r \times (p+q) \times (q+r) \times (r+p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q+r) \times (r+p)}$$

$$= \frac{1}{2} r(p+q)$$

$$(iv)\ 20(y+4)(y^2+5y+3) = 2 \times 2 \times 5 \times (y+4)(y^2+5y+3)$$

$$20(y+4)(y^2+5y+3) \div 5(y+4) = \frac{2 \times 2 \times 5 \times (y+4) \times (y^2+5y+3)}{5 \times (y+4)}$$

$$= 4(y^2+5y+3)$$

$$(v)\ x(x+1)(x+2)(x+3) \div x(x+1) = \frac{x(x+1)(x+2)(x+3)}{x(x+1)}$$

$$= (x+2)(x+3)$$

5. Factorise the expressions and divide them as directed.

(i) $(y^2+7y+10)\div(y+5)$

(ii) $(m^2–14m–32)\div(m+2)$

(iii) $(5p^2–25p+20)\div(p–1)$

(iv) $4yz(z^2+6z–16)\div 2y(z+8)$

(v) $5pq(p^2–q^2)\div 2p(p+q)$

(vi) $12xy(9x^2–16y^2)\div 4xy(3x+4y)$

(vii) $39y^3(50y^2–98) \div 26y^2(5y+7)$

Solution:

(i) $(y^2+7y+10)\div(y+5)$

First, solve the equation $(y^2+7y+10)$

$(y^2+7y+10) = y^2+2y+5y+10 = y(y+2)+5(y+2) = (y+2)(y+5)$

Now, $(y^2+7y+10)\div(y+5) = (y+2)(y+5)/(y+5) = y+2$

(ii) $(m^2–14m–32)\div (m+2)$

Solve for $m^2–14m–32$, we have

$m^2–14m–32 = m^2+2m-16m–32 = m(m+2)–16(m+2) = (m–16)(m+2)$

Now, $(m^2–14m–32)\div(m+2) = (m–16)(m+2)/(m+2) = m-16$

(iii) $(5p^2–25p+20)\div(p–1)$

Step 1: Take 5 common from the equation, $5p^2–25p+20$, we get

$5p^2–25p+20 = 5(p^2–5p+4)$

Step 2: Factorise $p^2–5p+4$

$p^2–5p+4 = p^2–p-4p+4 = (p–1)(p–4)$

Step 3: Solve original equation

$(5p^2–25p+20)÷(p–1) = 5(p–1)(p–4)/(p-1) = 5(p–4)$

(iv) $4yz(z^2 + 6z–16)÷ 2y(z+8)$

Factorising $z^2+6z–16$,

$z^2+6z–16 = z^2-2z+8z–16 = (z–2)(z+8)$

Now, $4yz(z^2+6z–16) ÷ 2y(z+8) = 4yz(z–2)(z+8)/2y(z+8) = 2z(z-2)$

(v) $5pq(p^2–q^2) ÷ 2p(p+q)$

$p^2–q^2$ can be written as $(p–q)(p+q)$ using the identity.

$5pq(p^2–q^2) ÷ 2p(p+q) = 5pq(p–q)(p+q)/2p(p+q) = 5q(p–q)/2$

(vi) $12xy(9x^2–16y^2) ÷ 4xy(3x+4y)$

Factorising $9x^2–16y^2$, we have

$9x^2–16y^2 = (3x)^2–(4y)^2 = (3x+4y)(3x-4y)$ using the identity $p^2–q^2 = (p–q)(p+q)$

Now, $12xy(9x^2–16y^2) ÷ 4xy(3x+4y) = 12xy(3x+4y)(3x-4y) /4xy(3x+4y) = 3(3x-4y)$

(vii) $39y^3(50y^2–98) ÷ 26y^2(5y+7)$
st solve for $50y^2–98$, we have

$50y^2–98 = 2(25y^2–49) = 2((5y)^2–7^2) = 2(5y–7)(5y+7)$

Now, $39y^3(50y^2–98) \div 26y^2(5y+7) =$

$$\frac{3 \times 13 \times y^3 \times 2(5y-7)(5y+7)}{2 \times 13 \times y^2 (5y+7)} = 3y(5y-7)$$

Exercise 12.2

1. 4(x–5) = 4x–5

Solution:

$4(x-5) = 4x - 20 \neq 4x - 5 = RHS$

The correct statement is $4(x-5) = 4x–20$

2. $x(3x+2) = 3x^2+2$

Solution:

$LHS = x(3x+2) = 3x^2+2x \neq 3x^2+2 = RHS$

The correct solution is $x(3x+2) = 3x^2+2x$

3. 2x+3y = 5xy

Solution:

$LHS = 2x+3y \neq$ R. H. S

The correct statement is $2x+3y = 2x+3y$

4. $x+2x+3x = 5x$

Solution:

LHS = $x+2x+3x = 6x \neq$ RHS

The correct statement is $x+2x+3x = 6x$

5. $5y+2y+y-7y = 0$

Solution:

LHS = $5y+2y+y-7y = y \neq$ RHS

The correct statement is $5y+2y+y-7y = y$

6. $3x+2x = 5x^2$

Solution:

LHS = $3x+2x = 5x \neq$ RHS

The correct statement is $3x+2x = 5x$

7. $(2x)^2+4(2x)+7 = 2x^2+8x+7$

Solution:

LHS = $(2x)^2+4(2x)+7 = 4x^2+8x+7 \neq$ RHS

The correct statement is $(2x)^2+4(2x)+7 = 4x^2+8x+7$

8. $(2x)^2+5x = 4x+5x = 9x$

Solution:

LHS = $(2x)^2+5x = 4x^2+5x \neq 9x$ = RHS

The correct statement is $(2x)^2+5x = 4x^2+5x$

9. $(3x + 2)^2 = 3x^2+6x+4$

Solution:

LHS = $(3x+2)^2 = (3x)^2+2^2+2x2x3x = 9x^2+4+12x \neq$ RHS

The correct statement is $(3x + 2)^2 = 9x^2+4+12x$

10. Substituting x = – 3 in

(a) $x^2 + 5x + 4$ gives $(-3)^2+5(-3)+4 = 9+2+4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2-5(-3)+4 = 9-15+4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2+5(-3) = -9-15 = -24$

Solution:

(a) Substituting $x = -3$ in x^2+5x+4, we have

$x^2+5x+4 = (-3)^2+5(-3)+4 = 9-15+4 = -2$. This is the correct answer.

(b) Substituting $x = -3$ in x^2-5x+4

$x^2-5x+4 = (-3)^2-5(-3)+4 = 9+15+4 = 28$. This is the correct answer

(c) Substituting $x = -3$ in x^2+5x

$x^2+5x = (-3)^2+5(-3) = 9-15 = -6$. This is the correct answer

11.$(y–3)^2 = y^2–9$

Solution:

LHS = $(y–3)^2$, which is similar to $(a–b)^2$ identity, where $(a–b)^2 = a^2+b^2-2ab$

$(y – 3)^2 = y^2+(3)^2–2y\times3 = y^2+9 –6y \neq y^2 – 9$ = RHS

The correct statement is $(y–3)^2 = y^2 + 9 – 6y$

12. $(z+5)^2 = z^2+25$

Solution:

LHS = $(z+5)^2$, which is similar to $(a +b)^2$ identity, where $(a+b)^2 = a^2+b^2+2ab$

$(z+5)^2 = z^2+5^2+2\times5\times z = z^2+25+10z \neq z^2+25$ = RHS

The correct statement is $(z+5)^2 = z^2+25+10z$

13. $(2a+3b)(a–b) = 2a^2–3b^2$

Solution:

LHS = $(2a+3b)(a–b) = 2a(a–b)+3b(a–b)$

$= 2a^2–2ab+3ab–3b^2$

$= 2a^2+ab–3b^2$

$\neq 2a^2–3b^2$ = RHS

The correct statement is $(2a +3b)(a –b) = 2a^2+ab–3b^2$

14. $(a+4)(a+2) = a^2+8$

Solution:

LHS = $(a+4)(a+2) = a(a+2)+4(a+2)$

$= a^2+2a+4a+8$

$= a^2+6a+8$

$\neq a^2+8$ = RHS

The correct statement is $(a+4)(a+2) = a^2+6a+8$

15. $(a–4)(a–2) = a^2–8$

Solution:

LHS = $(a–4)(a–2) = a(a–2)–4(a–2)$

$= a^2–2a–4a+8$

$= a^2–6a+8$

$\neq a^2-8$ = RHS

The correct statement is $(a–4)(a–2) = a^2–6a+8$

16. $3x^2/3x^2 = 0$

Solution:

LHS = $3x^2/3x^2 = 1 \neq 0$ = RHS

The correct statement is $3x^2/3x^2 = 1$

17. $(3x^2+1)/3x^2 = 1 + 1 = 2$

Solution:

LHS = $(3x^2+1)/3x^2 = (3x^2/3x^2)+(1/3x^2) = 1+(1/3x^2) \neq 2$ = RHS

The correct statement is $(3x^2+1)/3x^2 = 1+(1/3x^2)$

18. $3x/(3x+2) = ½$

Solution:

LHS = $3x/(3x+2) \neq 1/2$ = RHS

The correct statement is $3x/(3x+2) = 3x/(3x+2)$

19. $3/(4x+3) = 1/4x$

Solution:

LHS = $3/(4x+3) \neq 1/4x$

The correct statement is $3/(4x+3) = 3/(4x+3)$

20. $(4x+5)/4x = 5$

Solution:

LHS = $(4x+5)/4x = 4x/4x + 5/4x = 1 + 5/4x \neq 5$ = RHS

The correct statement is $(4x+5)/4x = 1 + (5/4x)$

21. $\frac{7x+5}{5} = 7x$

Solution:

LHS = (7x+5)/5 = (7x/5)+ 5/5 = (7x/5)+1 ≠ 7x = RHS

The correct statement is (7x+5)/5 = (7x/5) +1

5MARK Q&A:

Exercise 12.3

1. Find the common factors of the given terms.

(i) 12x, 36

(ii) 2y, 22xy

(iii) 14 pq, $28p^2q^2$

(iv) 2x, $3x^2$, 4

(v) 6 abc, $24ab^2$, $12a^2b$

(vi) 16 x^3, – $4x^2$, 32 x

(vii) 10 pq, 20qr, 30 rp

(viii) $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$

Solution:

(i) Factors of 12x and 36

12x = 2×2×3×x

$36 = 2 \times 2 \times 3 \times 3$

Common factors of 12x and 36 are 2, 2, 3

and , $2 \times 2 \times 3 = 12$

(ii) Factors of 2y and 22xy

$2y = 2 \times y$

$22xy = 2 \times 11 \times x \times y$

Common factors of 2y and 22xy are 2, y

and ,$2 \times y = 2y$

(iii) Factors of 14pq and $28p^2q^2$

$14pq = 2x7xpxq$

$28p^2q^2 = 2x2x7xpxpxqxq$

Common factors of 14 pq and 28 p^2q^2 are 2, 7 , p , q

and, $2x7xpxq = 14pq$

(iv) Factors of 2x, $3x^2$and 4

$2x = 2 \times x$

$3x^2 = 3 \times x \times x$

$4 = 2 \times 2$

Common factors of 2x, $3x^2$ and 4 is 1.

(v) Factors of $6abc$, $24ab^2$ and $12a^2b$

$6abc = 2\times3\times a\times b\times c$

$24ab^2 = 2\times2\times2\times3\times a\times b\times b$

$12a^2b = 2\times2\times3\times a\times a\times b$

Common factors of $6abc$, $24ab^2$ and $12a^2b$ are 2, 3, a, b

and, $2\times3\times a\times b = 6ab$

(vi) Factors of $16x^3$, $-4x^2$ and $32x$

$16x^3 = 2\times2\times2\times2\times x\times x\times x$

$-4x^2 = -1\times2\times2\times x\times x$

$32x = 2\times2\times2\times2\times2\times x$

Common factors of $16x^3$, $-4x^2$ and $32x$ are 2,2, x

and, $2\times2\times x = 4x$

(vii) Factors of $10pq$, $20qr$ and $30rp$

$10pq = 2\times5\times p\times q$

$20qr = 2\times2\times5\times q\times r$

$30rp = 2\times3\times5\times r\times p$

Common factors of $10pq$, $20qr$ and $30rp$ are 2, 5

and, $2\times5 = 10$

(viii) Factors of $3x^2y^3$, $10x^3y^2$ and $6x^2y^2z$

$3x^2y^3 = 3\times x\times x\times y\times y\times y$

$10x^3 y^2 = 2\times 5\times x\times x\times x\times y\times y$

$6x^2y^2z = 3\times 2\times x\times x\times y\times y\times z$

Common factors of $3x^2y^3$, $10x^3y^2$ and $6x^2y^2z$ are x^2, y^2

and, $x^2\times y^2 = x^2y^2$

2.Factorise the following expressions.

(i) $7x–42$

(ii) $6p–12q$

(iii) $7a^2+ 14a$

(iv) $-16z+20 z^3$

(v) $20l^2m+30alm$

(vi) $5x^2y-15xy^2$

(vii) $10a^2-15b^2+20c^2$

(viii) $-4a^2+4ab–4 ca$

(ix) $x^2yz+xy^2z +xyz^2$

(x) ax^2y+bxy^2+cxyz

Solution:

$(i)\ \ 7x = 7 \times x$

$42 = 2 \times 3 \times 7$

The common factor is 7

$\therefore\ 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7(x - 6)$

$(ii)\ \ 6p = 2 \times 3 \times p$

$12q = 2 \times 2 \times 3 \times q$

The common factors are 2 and 3

$\therefore\ 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q)$

$= 2 \times 3\ [p - (2 \times q)]$

$= 6(p - 2q)$

$(iii)\ 7a^2 = 7 \times a \times a$

$14a = 2 \times 7 \times a$

The common factors are 7 and a

$\therefore\ 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a)$

$= 7 \times a\ [a + 2] = 7a\ (a + 2)$

$(iv)\ 16z = 2 \times 2 \times 2 \times 2 \times z$

$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$

The common factors are 2, 2, and z.

$\therefore\ -16z + 20z^3 = -(2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$

$= (2 \times 2 \times z)\ \ [-(2 \times 2) + (5 \times z \times z)]$

$= 4z(-4 + 5z^2)$

$(v)\ \ 20l^2m = 2 \times 2 \times 5 \times l \times l \times m$

$30alm = 2 \times 3 \times 5 \times a \times l \times m$

The common factors are 2, 5, l and m

$\therefore\ 20l^2m + 30alm = (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m)$

$= (2 \times 5 \times l \times m)\ [(2 \times l) + (3 \times a)]$

$= 10lm\ (2l + 3a)$

$(vi)\ 5x^2y = 5 \times x \times x \times y$

$15xy^2 = 3 \times 5 \times x \times y \times y$

The common factors are 5, x, and y

$\therefore\ \ 5x^2y - 15xy^2 = (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y)$

$= 5 \times x \times y[x - (3 \times y)]$

$= 5xy\ (x - 3y)$

(vii) $10a^2-15b^2+20c^2$

$10a^2 = 2\times5\times a\times a$

$-15b^2 = -1\times3\times5\times b\times b$

$20c^2 = 2\times2\times5\times c\times c$

Common factor of $10a^2$, $15b^2$ and $20c^2$ is 5

$10a^2-15b^2+20c^2 = 5(2a^2-3b^2+4c^2)$

(viii) $-4a^2+4ab-4ca$

$-4a^2 = -1\times2\times2\times a\times a$

$4ab = 2\times2\times a\times b$

$-4ca = -1\times2\times2\times c\times a$

Common factor of $-4a^2$, $4ab$, $-4ca$ are 2, 2, a i.e. 4a

So,

$-4a^2+4ab-4ca = 4a(-a+b-c)$

(ix) $x^2yz+xy^2z+xyz^2$

$x^2yz = x\times x\times y\times z$

$xy^2z = x\times y\times y\times z$

$xyz^2 = x\times y\times z\times z$

Common factor of x^2yz, xy^2z and xyz^2 are x, y, z i.e. xyz

Now, $x^2yz+xy^2z+xyz^2 = xyz(x+y+z)$

(x) ax^2y+bxy^2+cxyz

$ax^2y = a\times x\times x\times y$

$bxy^2 = b\times x\times y\times y$

$cxyz = c\times x\times y\times z$

Common factors of $a\,x^2y$,bxy^2 and $cxyz$ are xy

Now, $ax^2y+bxy^2+cxyz = xy(ax+by+cz)$

3. Factorise.

(i) $x^2+xy+8x+8y$

(ii) $15xy-6x+5y-2$

(iii) $ax+bx-ay-by$

(iv) $15pq+15+9q+25p$

(v) $z-7+7xy-xyz$

Solution:

$(i)\ x^2 + xy + 8x + 8y = x \times x + x \times y + 8 \times x + 8 \times y$

$= x(x + y) + 8(x + y)$

$= (x + y)\ (x + 8)$

$(ii)\ 15xy - 6x + 5y - 2 = 3 \times 5 \times x \times y - 3 \times 2 \times x + 5xy - 2$

$= 3x(5y - 2) + 1\ (5y - 2)$

$= (5y - 2)\ (3x + 1)$

$(iii)\ ax + bx - ay - by = a \times x + b \times x - a \times y - b \times y$

$= x(a + b) - y(a + b)$

$= (a + b)\ (x - y)$

$(iv)\ 15pq + 15 + 9q + 25p = 15pq + 9q + 25p + 15$

$= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5$

$= 3q\ (5p + 3) + 5\ (5p + 3)$

$= (5p + 3)\ (3q + 5)$

$(v)\ z - 7 + 7xy - xyz = z - x \times y \times z - 7 + 7 \times x \times y$

$= z\ (1 - xy) - 7\ (1 - xy)$

$= (1 - xy)\ (z - 7)$

Exercise 12.4

1. Factorise the following expressions.

(i) $a^2+8a+16$

(ii) $p^2–10p+25$

(iii) $25m^2+30m+9$

(iv) $49y^2+84yz+36z^2$

(v) $4x^2–8x+4$

(vi) $121b^2–88bc+16c^2$

(vii) $(l+m)^2–4lm$ (Hint: Expand $(l+m)^2$ first)

(viii) $a^4+2a^2b^2+b^4$

Solution:

(i) $a^2+8a+16$

$= a^2+2\times4\times a+4^2$

$= (a+4)^2$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(ii) $p^2–10p+25$

$= p^2-2\times5\times p+5^2$

$= (p-5)^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(iii) $25m^2+30m+9$

$= (5m)^2+2\times5m\times3+3^2$

$= (5m+3)^2$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(iv) $49y^2+84yz+36z^2$

$=(7y)^2+2\times7y\times6z+(6z)^2$

$= (7y+6z)^2$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(v) $4x^2–8x+4$

$= (2x)^2-2\times4x+2^2$

$= (2x-2)^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(vi) $121b^2-88bc+16c^2$

$= (11b)^2-2\times11b\times4c+(4c)^2$

$= (11b-4c)^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(vii) $(l+m)^2-4lm$ (Hint: Expand $(l+m)^2$ first)

Expand $(l+m)^2$ using the identity $(x+y)^2 = x^2+2xy+y^2$

$(l+m)^2-4lm = l^2+m^2+2lm-4lm$

$= l^2+m^2-2lm$

$= (l-m)^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(viii) $a^4+2a^2b^2+b^4$

$= (a^2)^2+2\times a^{2\times}b^2+(b^2)^2$

$= (a^2+b^2)^2$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

2. Factorise.

(i) $4p^2–9q^2$

(ii) $63a^2–112b^2$

(iii) $49x^2–36$

(iv) $16x^5–144x^3$ differ

(v) $(l+m)^2-(l-m)^2$

(vi) $9x^2y^2–16$

(vii) $(x^2–2xy+y^2)–z^2$

(viii) $25a^2–4b^2+28bc–49c^2$

Solution:

(i) $4p^2–9q^2$

$= (2p)^2-(3q)^2$

$= (2p-3q)(2p+3q)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(ii) $63a^2–112b^2$

$= 7(9a^2 –16b^2)$

$= 7((3a)^2–(4b)^2)$

$= 7(3a+4b)(3a-4b)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(iii) $49x^2-36$

$= (7x)^2 - 6^2$

$= (7x+6)(7x-6)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(iv) $16x^5-144x^3$

$= 16x^3(x^2-9)$

$= 16x^3(x^2-9)$

$= 16x^3(x-3)(x+3)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(v) $(l+m)^2-(l-m)^2$

$= \{(l+m)-(l-m)\}\{(l+m)+(l-m)\}$

Using the identity $x^2-y^2 = (x+y)(x-y)$

$= (l+m-l+m)(l+m+l-m)$

$= (2m)(2l)$

$= 4\ ml$

(vi) $9x^2y^2-16$

$= (3xy)^2-4^2$

$= (3xy-4)(3xy+4)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(vii) $(x^2–2xy+y^2)–z^2$

$= (x–y)^2–z^2$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

$= \{(x–y)–z\}\{(x–y)+z\}$

$= (x–y–z)(x–y+z)$

Using the identity $x^2-y^2 = (x+y)(x-y)$

(viii) $25a^2–4b^2+28bc–49c^2$

$= 25a^2–(4b^2-28bc+49c^2)$

$= (5a)^2-\{(2b)^2-2(2b)(7c)+(7c)^2\}$

$= (5a)^2-(2b-7c)^2$

Using the identity $x^2-y^2 = (x+y)(x-y)$, we have

$= (5a+2b-7c)(5a-2b+7c)$

3. Factorise the expressions.

(i) ax^2+bx

(ii) $7p^2+21q^2$

(iii) $2x^3+2xy^2+2xz^2$

(iv) $am^2+bm^2+bn^2+an^2$

(v) $(lm+l)+m+1$

(vi) $y(y+z)+9(y+z)$

(vii) $5y^2–20y–8z+2yz$

(viii) $10ab+4a+5b+2$

(ix)$6xy–4y+6–9x$

Solution:

(i) $ax^2+bx = x(ax+b)$

(ii) $7p^2+21q^2 = 7(p^2+3q^2)$

(iii) $2x^3+2xy^2+2xz^2 = 2x(x^2+y^2+z^2)$

(iv) $am^2+bm^2+bn^2+an^2 = m^2(a+b)+n^2(a+b) = (a+b)(m^2+n^2)$

(v) $(lm+l)+m+1 = lm+m+l+1 = m(l+1)+(l+1) = (m+1)(l+1)$

(vi) $y(y+z)+9(y+z) = (y+9)(y+z)$

(vii) $5y^2–20y–8z+2yz = 5y(y–4)+2z(y–4) = (y–4)(5y+2z)$

(viii) $10ab+4a+5b+2 = 5b(2a+1)+2(2a+1) = (2a+1)(5b+2)$

(ix) $6xy–4y+6–9x = 6xy–9x–4y+6 = 3x(2y–3)–2(2y–3) = (2y–3)(3x–2)$

4.Factorise.

(i) $a^4–b^4$

(ii) $p^4–81$

(iii) $x^4–(y+z)^4$

(iv) $x^4–(x–z)^4$

(v) $a^4–2a^2b^2+b^4$

Solution:

(i) $a^4–b^4$

$= (a^2)^2-(b^2)^2$

$= (a^2-b^2)(a^2+b^2)$

$= (a – b)(a + b)(a^2+b^2)$

(ii) $p^4–81$

$= (p^2)^2-(9)^2$

$= (p^2-9)(p^2+9)$

$= (p^2-3^2)(p^2+9)$

$=(p-3)(p+3)(p^2+9)$

(iii) $x^4–(y+z)^4 = (x^2)^2-[(y+z)^2]^2$

$= \{x^2-(y+z)^2\}\{x^2+(y+z)^2\}$

$= \{(x-(y+z)(x+(y+z)\}\{x^2+(y+z)^2\}$

$= (x-y-z)(x+y+z)\ \{x^2+(y+z)^2\}$

(iv) $x^4-(x-z)^4 = (x^2)^2-\{(x-z)^2\}^2$

$= \{x^2-(x-z)^2\}\{x^2+(x-z)^2\}$

$= \{x-(x-z)\}\{x+(x-z)\}\ \{x^2+(x-z)^2\}$

$= z(2x-z)(x^2+x^2-2xz+z^2)$

$= z(2x-z)(2x^2-2xz+z^2)$

(v) $a^4-2a^2b^2+b^4 = (a^2)^2-2a^2b^2+(b^2)^2$

$= (a^2-b^2)^2$

$= ((a-b)(a+b))^2$

$= (a-b)^2\ (a+b)^2$

5. Factorise the following expressions.

(i) p^2+6p+8

(ii) $q^2–10q+21$

(iii) $p^2+6p–16$

Solution:

(i) p^2+6p+8

We observed that $8 = 4×2$ and $4+2 = 6$

p^2+6p+8 can be written as $p^2+2p+4p+8$

Taking Common terms, we get

$p^2+6p+8 = p^2+2p+4p+8 = p(p+2)+4(p+2)$

Again, $p+2$ is common in both the terms.

$= (p+2)(p+4)$

This implies that $p^2+6p+8 = (p+2)(p+4)$

(ii) $q^2–10q+21$

We observed that $21 = -7×-3$ and $-7+(-3) = -10$

$q^2–10q+21 = q^2–3q-7q+21$

$= q(q–3)–7(q–3)= (q–7)(q–3)$

This implies that $q^2–10q+21 = (q–7)(q–3)$

(iii) $p^2+6p–16$

We observed that -16 = -2×8 and 8+(-2) = 6

$p^2+6p–16 = p^2–2p+8p–16$

$= p(p–2)+8(p–2)$

$= (p+8)(p–2)$

So, $p^2+6p–16 = (p+8)(p–2)$

1MARK Q&A

Exercise 12.5

Multiple-choice questions and answers:

Question 1:

Factorize the expression (4a - 12):

a) (4(a - 3))

b) (2(2a - 6))

c) (6(2a - 3))

d) (8(a - 1.5))

Answer 1:

b) (2(2a - 6))

Question 2:

Factorize the expression ($9x^2$ - 16):

a) ((3x + 4)(3x - 4))

b) ((3x - 4)(3x - 4))

c) ((3x + 2)(3x - 2))

d) ((3x - 2)(3x + 2))

Answer 2:

a) ((3x + 4)(3x - 4))

Question 3:

Factorize the expression ($25y^2$ - $9y^2$):

a) ((5y + 3z)(5y - 3z))

b) ((5y + 3z)(5y + 3z))

c) ((5y - 3z)(5y - 3z))

d) ((5y - 3z)(5y + 3z))

Answer 3:

a) ((5y + 3z)(5y - 3z))

Question 4:

Factorize the expression ($2x^2$ + 12x + 18):

a) (2(x + 3)(x + 3))

b) (2(x + 3)(x + 6))

c) (2(x + 6)(x + 3))

d) (2(x + 2)(x + 9))

Answer 4:

b) (2(x + 3)(x + 6))

Question 5:

Factorize the expression ($16a^2$ - 25):

a) ((4a + 5)(4a - 5))

b) $(4a + 5)^2$

c) $(4a - 5)^2$

d) ((4a + 2)(4a - 7))

Answer 5:

a) ((4a + 5)(4a - 5))

Question 6:

Factorize the expression ($3x^2$ - 27):

a) ((3x + 3)(x - 9))

b) ((3x + 9)(x - 3))

c) ((3x + 9)(x + 3))

d) ((3x - 3)(x - 9))

Answer 6:

b) ((3x + 9)(x - 3))

Question 7:

Factorize the expression ($7y^2$ - 14y + 7):

a) $(y - 1)^2$

b) ((y - 1)(y - 6))

c) ((y + 1)(y - 7))

d) $(y - 7)^2$

Answer 7:

a) $(y - 1)^2$

Question 8:

Factorize the expression ($6a^2$ - 15a):

a) (3a(2a - 5))

b) (6a(1a - 2.5))

c) (3a(2a + 5))

d) (2a(3a - 7.5))

Answer 8:

a) (3a(2a - 5))

Question 9:

Factorize the expression (x^2 + 8x + 16):

a) $(x + 4)^2$

b) $(x + 2)^2$

c) ((x + 4)(x + 4))

d) ((x + 2)(x + 6))

Answer 9:

a) $(x + 4)^2$

Question 10:

Factorize the expression (20ab + $15a^2$):

a) (5a(4b + 3a))

b) (5a(4b - 3a))

c) (5a(3b + 4a))

d) (5a(3b - 4a))

Answer:

c) (5a(3b + 4a))

Question 11:

Factorize the quadratic expression (x^2 + 7x + 10):

a) ((x + 5)(x + 2))

b) ((x + 3)(x + 4))

c) ((x + 2)(x + 8))

d) ((x + 1)(x + 10))

Answer:

b) ((x + 3)(x + 4))

Question 12:

Factorize the expression (12x + 18y):

a) (2(6x + 9y))

b) (3(4x + 6y))

c) (6(2x + 3y))

d) (4(3x + 5y))

Answer:

c) (6(2x + 3y))

Exercise 12.6

Fill in the blanks

1. (12x + 18y) can be factorized as 6(-----).

- Answer: 6(2x + 3y)

2. The expression (x^2 - 9) can be factorized(-----)*(-----)

- Answer: ((x + 3) * (x - 3))

3. ($20a^2b + 30ab^2$) can be factorized as 10ab ----

- Answer: 10ab(2a + 3b)

4. The expression ($4x^2 - 25y^2$) can be factorized (----) x (----)

Answer: ((2x + 5y) x (2x - 5y))

5. ($2x^3 + 4x^2$ - 6 x) can be factorized as 2x(-----)

- Answer: (2x(x^2 + 2x - 3))

Summary

Factorisation:

1. Definition: Factorisation is the process of expressing an algebraic expression as a product of its factors or simpler expressions.

2. Common Factors: Factorisation involves identifying common factors shared by the terms of an expression and then factoring them out.

3. Key Concepts:

a. Linear Factors: Expressions can often be factored into linear factors such as (x + a) or (x - a) where 'a' is a constant.

b. Quadratic Factors: Quadratic expressions such as (x^2 - a^2)or (ax^2 + bx + c) can be factored into linear factors.

4. Factorisation Techniques:

a. Common Factor Method: Identify and factor out the common factors shared by all terms of the expression.

b. Difference of Squares: Factorisation of expressions in the form of (a^2 - b^2) as (a + b)(a - b) .

c. Grouping Method: Group terms in pairs, factor out common factors from each pair, and then factorise by grouping.

d. Trial and Error Method: Find the pair of numbers whose product and sum help in factorising the expression.

5. Application and Importance: Factorisation is essential in simplifying complex algebraic expressions, solving equations, finding roots, and solving problems in mathematics and other fields.

6. Role in Mathematics: Understanding factorisation forms the basis for solving higher-level problems in algebra, calculus, and other mathematical areas.

•

****