Chapter-2

Polynomials

2 MARKS QUESTIONS

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i) x^2–2x –8

⇒x^2– 4x+2x–8 = x(x–4)+2(x–4) = (x-4)(x+2)

Therefore, zeroes of polynomial equation x^2–2x–8 are (4, -2)

Sum of zeroes = 4–2 = 2 = -(-2)/1 = -(Coefficient of x)/(Coefficient of x^2)

Product of zeroes = 4×(-2) = -8 =-(8)/1 = (Constant term)/(Coefficient of x^2)

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes, respectively.

(i) 4, 1

Solution:

Given,

Sum of zeroes = $\alpha+\beta$ =4

Product of zeroes = $\alpha\beta$ = 1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2–(\alpha+\beta)x+\alpha\beta = 0$

$x^2–4x+1 = 0$

Thus, $x^2–4x+1$ is the quadratic polynomial.

3. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Solution:

Let us consider the cubic polynomial is ax^3+bx^2+cx+d and the values of the zeroes of the polynomials be α, β, γ.

As per the given question,

$\alpha+\beta+\gamma = -b/a = 2/1$

$\alpha\beta +\beta\gamma+\gamma\alpha = c/a = -7/1$

$\alpha\ \beta\gamma = -d/a = -14/1$

Thus, from above three expressions we get the values of coefficient of polynomial.

a = 1, b = -2, c = -7, d = 14

Hence, the cubic polynomial is $x^3-2x^2-7x+14$

4 MARKS QUESTIONS

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Therefore, zeroes of polynomial equation $x^2–2x–8$ are (4, -2)

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Product of zeroes = 4×(-2) = -8 =-(8)/1 = (Constant term)/(Coefficient of x^2)

(ii) $4s^2–4s+1$

$\Rightarrow 4s^2–2s–2s+1 = 2s(2s–1)–1(2s-1) = (2s–1)(2s–1)$

Therefore, zeroes of polynomial equation $4s^2–4s+1$ are (1/2, 1/2)

Sum of zeroes = (½)+(1/2) = 1 = -(-4)/4 = -(Coefficient of s)/(Coefficient of s^2)

Product of zeros = (1/2)×(1/2) = 1/4 = (Constant term)/(Coefficient of s^2)

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) t^2-3, $2t^4+3t^3-2t^2-9t-12$

Solutions:

Given,

First polynomial = t^2-3

Second polynomial = $2t^4+3t^3-2t^2-9t-12$

$$
\begin{array}{rrrrrr}
 & 2t^2 & +3t & +4 & & \\
\cline{2-6}
t^2-3 \;\big) & 2t^4 & +3t^3 & -2t^2 & -9t & -12 \\
 - & & & & & \\
 & 2t^4 & +0t^3 & -6t^2 & & \\
\hline
 & & 3t^3 & +4t^2 & -9t & -12 \\
 & - & & & & \\
 & & 3t^3 & +0t^2 & -9t & \\
\hline
 & & & 4t^2 & +0t & -12 \\
 & & - & & & \\
 & & & 4t^2 & +0t & -12 \\
\hline
 & & & & & 0
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$.

3. On dividing x^3-$3x^2$+x+2 by a polynomial g(x), the quotient and remainder were x–2 and –2x+4, respectively. Find g(x).

Solution:

Given,

Dividend, p(x) = x^3-3x^2+x+2

Quotient = x-2

Remainder = –2x+4

We have to find the value of Divisor, g(x) =?

As we know,

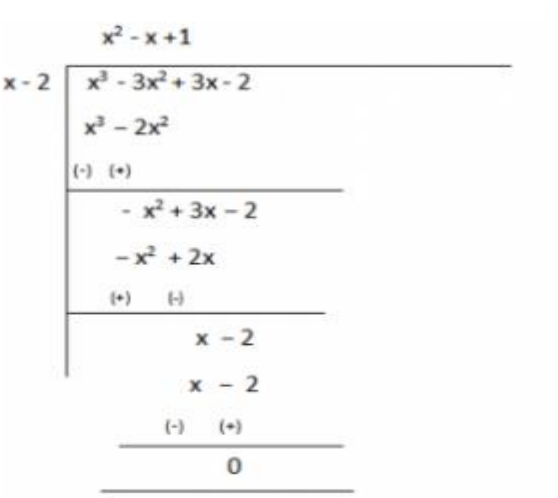
Dividend = Divisor × Quotient + Remainder

∴ $x^3-3x^2+x+2 = g(x)\times(x-2) + (-2x+4)$

$x^3-3x^2+x+2-(-2x+4) = g(x)\times(x-2)$

Therefore, $g(x) \times (x-2) = x^3-3x^2+3x-2$

Now, for finding g(x) we will divide x^3-3x^2+3x-2 with (x-2)



Therefore, **$g(x) = (x^2–x+1)$**

4. If the zeroes of the polynomial x^3-$3x^2$+x+1 are a – b, a, a + b, find a and b.

Solution:

We are given with the polynomial here,

$p(x) = x^3-3x^2+x+1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

$\therefore px^3+qx^2+rx+s = x^3-3x^2+x+1$

p = 1, q = -3, r = 1 and s = 1

Sum of zeroes = a – b + a + a + b

-q/p = 3a

Putting the values q and p.

-(-3)/1 = 3a

a=1

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

$-s/p = 1-b^2$

$-1/1 = 1-b^2$

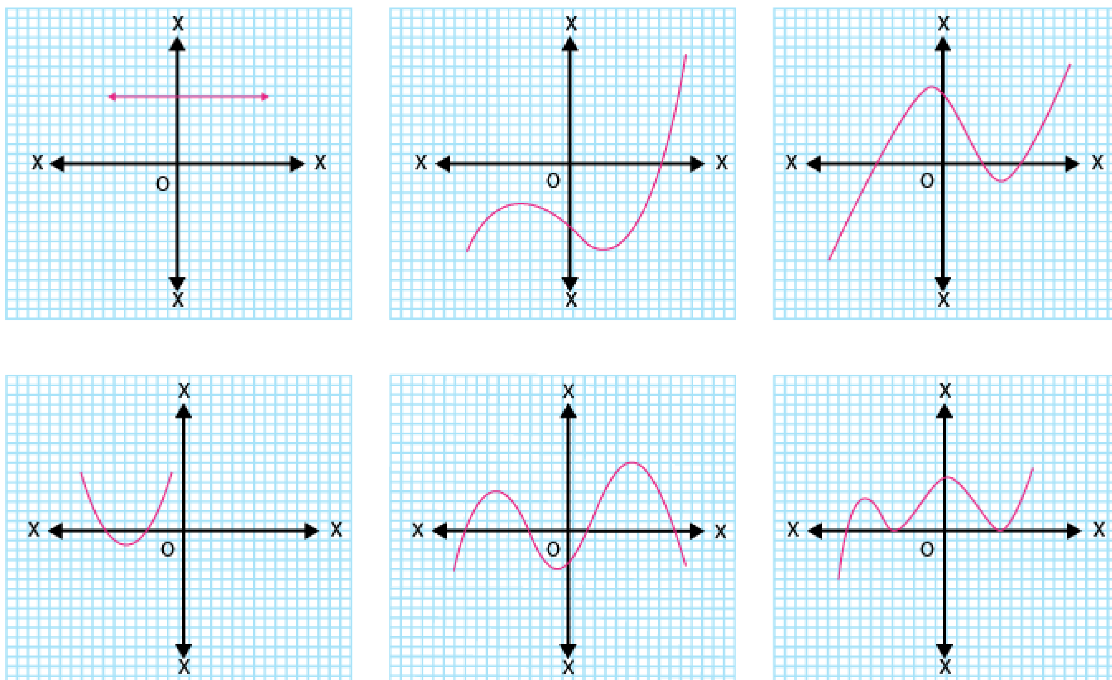
$b^2 = 1+1 = 2$

$b = \pm\sqrt{2}$

Hence,$1-\sqrt{2}$, 1 ,$1+\sqrt{2}$ are the zeroes of x^3-3x^2+x+1.

7 MARKS QUESTIONS

1. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



Solutions:

Graphical method to find zeroes:-

Total number of zeroes in any polynomial equation = total number of times the curve intersects x-axis.

(i) In the given graph, the number of zeroes of p(x) is 0 because the graph is parallel to x-axis does not cut it at any point.

(ii) In the given graph, the number of zeroes of p(x) is 1 because the graph intersects the x-axis at only one point.

(iii) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at any three points.

(iv) In the given graph, the number of zeroes of p(x) is 2 because the graph intersects the x-axis at two points.

(v) In the given graph, the number of zeroes of p(x) is 4 because the graph intersects the x-axis at four points.

(vi) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at three points.

2. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i) $6x^2$–3–7x

$\Rightarrow 6x^2-7x-3 = 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3) = (3x+1)(2x-3)$

Therefore, zeroes of polynomial equation $6x^2$–3–7x are (-1/3, 3/2)

Sum of zeroes = -(1/3)+(3/2) = (7/6) = -(Coefficient of x)/(Coefficient of x^2)

Product of zeroes = -(1/3)×(3/2) = -(3/6) = (Constant term) /(Coefficient of x^2)

(ii) $4u^2$+8u

$\Rightarrow$ 4u(u+2)

Therefore, zeroes of polynomial equation $4u^2$ + 8u are (0, -2).

Sum of zeroes = 0+(-2) = -2 = -(8/4) = = -(Coefficient of u)/(Coefficient of u^2)

Product of zeroes = 0×-2 = 0 = 0/4 = (Constant term)/(Coefficient of u^2)

(iii) $t^2 – 15$

$\Rightarrow t^2 = 15$ or $t = \pm\sqrt{15}$

Therefore, zeroes of polynomial equation $t^2 – 15$ are $(\sqrt{15}, -\sqrt{15})$

Sum of zeroes $=\sqrt{15}+(-\sqrt{15}) = 0= -(0/1)=$ -(Coefficient of t) / (Coefficient of t^2)

Product of zeroes = $\sqrt{15}\times(-\sqrt{15}) = -15 = -15/1 =$ (Constant term) / (Coefficient of t^2)

(iv) $3x^2 – x – 4$

$\Rightarrow 3x^2–4x+3x–4 = x(3x-4)+1(3x-4) = (3x – 4)(x + 1)$

Therefore, zeroes of polynomial equation$3x^2 – x – 4$ are (4/3, -1)

Sum of zeroes = (4/3)+(-1) = (1/3)= -(-1/3) = -(Coefficient of x) / (Coefficient of x^2)

Product of zeroes=(4/3)×(-1) = (-4/3) = (Constant term) /(Coefficient of x^2)

3. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes, respectively.

(i)$\sqrt{2}$, 1/3

Solution:

Sum of zeroes = $\alpha + \beta = \sqrt{2}$

Product of zeroes = $\alpha\beta = 1/3$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2-(\alpha+\beta)x+\alpha\beta = 0$

$x^2-(\sqrt{2})x + (1/3) = 0$

$3x^2-3\sqrt{2}x+1 = 0$

Thus, $3x^2-3\sqrt{2}x+1$ is the quadratic polynomial.

(ii) 0, $\sqrt{5}$

Solution:

Given,

Sum of zeroes = $\alpha+\beta = 0$

Product of zeroes = $\alpha\beta = \sqrt{5}$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly

as:-

$x^2-(\alpha+\beta)x+\alpha\beta = 0$

$x^2-(0)x+\sqrt{5}= 0$

Thus, $x^2+\sqrt{5}$ is the quadratic polynomial.

(iii) 1, 1

Solution:

Given,

Sum of zeroes = $\alpha+\beta = 1$

Product of zeroes = $\alpha\,\beta = 1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2-(\alpha+\beta)x+\alpha\beta = 0$

$x^2-x+1 = 0$

Thus, x^2-x+1 is the quadratic polynomial.

(iv) -1/4, 1/4

Solution:

Given,

Sum of zeroes = $\alpha+\beta = -1/4$

Product of zeroes = $\alpha\,\beta = 1/4$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2-(\alpha+\beta)x+\alpha\beta = 0$

$x^2-(-1/4)x+(1/4) = 0$

$4x^2+x+1 = 0$

Thus, $4x^2+x+1$ is the quadratic polynomial.

4. If the zeroes of the polynomial x^3-$3x^2$+x+1 are a – b, a, a + b, find a and b.

Solution:

We are given with the polynomial here,

$p(x) = x^3-3x^2+x+1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

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