CHAPTER-13

Exponents and Powers

Ex 13.1:-

Question 1

Find the value of

(i) 2^6

(ii) 9^3 (iii) 11^2 (iv) 5^4

Solution:

(i) $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(ii) $9^3 = 9 \times 9 \times 9 = 729$

(iii) $11^2 = 11 \times 11 = 121$

(iv) $5^4 = 5 \times 5 \times 5 \times 5 = 625$

Question 2

Exress the following in exponential form:

(i) $6 \times 6 \times 6 \times 6$

(ii) $t \times t$

(iii) $b \times b \times b \times b$

(iv) $5 \times 5 \times 7 \times 7 \times 7$

(v) $2 \times 2 \times a \times a$

(vi) $a \times a \times a \times c \times c \times c \times c \times d$

Solution:

(i) $6 \times 6 \times 6 \times 6 = 6^3$

(ii) $t \times t = t^2$

(iii) $b \times b \times b \times b = b^4$

(iv) $5 \times 5 \times 7 \times 7 \times 7 = 5^2 \times 7^3 = 5^2 \cdot 7^3$

(v) $2 \times 2 \times a \times a = 2^2 \times a^2 = 2^2 \cdot a^2$

(vi) $a \times a \times a \times c \times c \times c \times c \times d = a^3 \times c^4 \times d = a^3 \cdot c^4 \cdot d$

Question 3

Express each of the following numbers using exponential notation:

(i) 512

(ii) 343

(iii) 729

(iv) 3125

Solution:

(i)

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$

(ii)

7	343
7	49
7	7
	1

$7 \times 7 \times 7 = 7^3$

(iii)

3	729
3	243
3	81
3	27
3	9
3	3
	1

$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

(iv)

5	3125
5	625
5	125
5	25
5	5
	1

$5 \times 5 \times 5 \times 5 \times 5 = 5^5$

Question 4

Identify the greater number, wherever possible, in each of the following?

(i) 4^3 or 3^4

(ii) 5^3 or 3^5 (iii) 2^8 or 8^2

(iv) 100^2 or 2^{100} (v) 2^{10} or 10^2

Solution:

(i) 4^3 or 3^4

$4^3 = 4 \times 4 \times 4 = 64$,

$3^4 = 3 \times 3 \times 3 \times 3 = 81$ Since

$81 > 64 \therefore 34$ is greater than

43.

(ii) 5^3 or 3^5

$5^3 = 5 \times 5 \times 5 = 125$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

Since $243 > 125 \therefore 35$ is greater

than 53.

(iii) 2^8 or 8^2

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

$8^2 = 8 \times 8 = 64$ Since

$256 > 64 \therefore 28$ is

greater than 28.

(iv) 100^2 or 2^{100}

$100^2 = 100 \times 100 = 10000$

$2^{100} = 2 \times 2 \times 2 \times \ldots$ 100 times

Here $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 214 =$

16384 Since 16384 > 10,000 $\therefore$ 2100 is greater than 1002.

(v) 2^{10} or 10^2

$2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$

$10^2 = 10 \times 10 = 100$ Since

$1024 > 100 \therefore 210$ is

greater than 102.

Question 5

Express each of the following as the product of powers of their prime

(i) 648

(ii) 405

(iii) 540

(iv) 3600

Solution:

(*i*) 648

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= 2^3 \times 3^4$

(*ii*) 405

3	405
3	135
3	45
3	15
	5

$405 = 3 \times 3 \times 3 \times 3 \times 5$
$= 3^4 \times 5^1$

(*iii*) 540

2	540
2	270
3	135
3	45
3	15
5	5
	1

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 2^2 \times 3^3 \times 5^1$

(*iv*) 3600

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
$= 2^4 \times 3^2 \times 5^2$

Question 6

Simplify: (i) 2×10^3

(ii) $7^2 \times 2^2$

(iii) $2^3 \times 5$

(iv) 3×4^4

(v) 0×10^2

(vi) $5^2 \times 3^3$

(vii) $2^4 \times 3^2$

(viii) $3^2 \times 10^4$

Solution:

(i) $2 \times 10^3 = 2 \times 10 \times 10 \times 10 = = 2000$

(ii) $7^2 \times 2^2 = = 7 \times 7 \times 2 \times 2 = 196$

(iii) $2^3 \times 5 = 2 \times 2 \times 2 \times 5 = 40$

(iv) $3 \times 4^4 = 3 \times 4 \times 4 \times 4 \times 4 = 768$

(v) $0 \times 10^2 = 0 \times 10 \times 10 = = 0$

(vi) $5^2 \times 3^3 = 5 \times 5 \times 3 \times 3 \times 3 = 675$

(vii) $2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$ (viii) $3^2 \times 10^4 = 3 \times 3 \times 10 \times 10 \times 10 \times 10 = 90000$

Question 7

Simplify:

(i) $(-4)^3$

(ii) $(-3) \times (-2)^3$

(iii) $(-3)^2 \times (-5)^2$ (iv) $(-2)^3 \times (-10)^3$

Solution:

(i) $(-4)^2 = (-4) \times (-4) \times (-4) = -64$ [$\because (-a)^{\text{odd number}} = -a^{\text{odd number}}$]

(ii) $(-3) \times (-2)^3 = (-3) \times (-2) \times (-2) \times (-2)$

$= (-3) \times (-8) = 24$

(iii) $(-3)^2 \times (-5)^2 = [(-3) \times (-5)]^2$

$= 15^2 = 225$ [$\because a^m \times b^m = (ab)^m$]

(iv) $(-2)^3 \times (-10)^3 = [(-2) \times (-10)]^3 = 20^2 = 8000$ [$\because a^m \times b^m = (ab)^m$]

Question 8

Compare the following:

(i) 2.7×10^{12}; 1.5×10^8 (ii) 4×10^{14}; 3×10^{14}

Solution:

(i) 2.7×10^{12}; 1.5×10^8

Here, $10^{12} > 10^8$

$\therefore 2.7 \times 10^{12} > 1.5 \times 10^8$

(ii) $4 \times 10^{14}; 3 \times 10^{17}$

Here, $10^{17} > 10^{14}$

$\therefore 4 \times 10^{14} < 3 \times 10^{17}$

Question 1:

Find the value of:

(i) 2^6 (ii) 9^3

(iii) 11^2 (iv)5^4

Answer:

(i) $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(ii) $9^3 = 9 \times 9 \times 9 = 729$

(iii) $11^2 = 11 \times 11 = 121$

(iv)$5^4 = 5 \times 5 \times 5 \times 5 = 625$

Question 2:

Express the following in exponential form:

(i) $6 \times 6 \times 6 \times 6$ (ii) $t \times t$

(iii) $b \times b \times b \times b$ (iv) $5 \times 5 \times 7 \times 7 \times 7$

(v) $2 \times 2 \times a \times a$ (vi) $a \times a \times a \times c \times c \times c \times c \times d$

Answer:

(i) $6 \times 6 \times 6 \times 6 = 6^4$

(ii) $t \times t = t^2$

(iii) $b \times b \times b \times b = b^4$

(iv) $5 \times 5 \times 7 \times 7 \times 7 = 5^2 \times 7^3$

(v) $2 \times 2 \times a \times a = 2^2 \times a^2$

(vi) $a \times a \times a \times c \times c \times c \times c \times d = a^3 c^4 d$

Question 3:

Express the following numbers using exponential notation:

(i) 512 (ii) 343

(iii) 729 (iv) 3125

Answer:

(i) $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$

(ii) $343 = 7 \times 7 \times 7 = 7^3$

(iii) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

(iv) $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

Question 4:

Identify the greater number, wherever possible, in each of the following?

(i) 4^3 or 3^4 (ii) 5^3 or 3^5

(iii) 2^8 or 8^2 (iv) 100^2 or 2^{100}

(v) 2^{10} or 10^2

Answer:

(i) $4^3 = 4 \times 4 \times 4 = 64$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

Therefore, $3^4 > 4^3$

(ii) $5^3 = 5 \times 5 \times 5 = 125$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

Therefore, $3^5 > 5^3$

(iii) $2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

$8^2 = 8 \times 8 = 64$

Therefore, $2^8 > 8^2$

(iv)100^2 or 2^{100}

$2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$

$2^{100} = 1024 \times 1024 \times 1024 \times 1024 \times 1024 \times 1024 \times 1024 \times 1024 \times 1024 \times 1024$

$100^2 = 100 \times 100 = 10000$

Therefore, $2^{100} > 100^2$

(v) 2^{10} and 10^2

$2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$

$10^2 = 10 \times 10 = 100$

Therefore, $2^{10} > 10^2$

Question 5:

Express each of the following as product of powers of their prime factors:

(i) 648 (ii) 405

(iii) 540 (iv) 3,600

Answer:

(i) $648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^3 . 3^4$

(ii) $405 = 3 \times 3 \times 3 \times 3 \times 5 = 3^4 . 5$

(iii) $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 . 3^3 . 5$

(iv) $3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 2^4 . 3^2 . 5^2$

Question 6:

Simplify:

(i) 2×10^3 (ii) $7^2 \times 2^2$

(iii) $2^3 \times 5$ (iv) 3×4^4

(v) 0×10^2 (vi) $5^2 \times 3^3$

(vii) $2^4 \times 3^2$ (viii) $3^2 \times 10^4$

Answer:

(i) $2 \times 10^3 = 2 \times 10 \times 10 \times 10 = 2 \times 1000 = 2000$

(ii) $7^2 \times 2^2 = 7 \times 7 \times 2 \times 2 = 49 \times 4 = 196$

(iii) $2^3 \times 5 = 2 \times 2 \times 2 \times 5 = 8 \times 5 = 40$

(iv) $3 \times 4^4 = 3 \times 4 \times 4 \times 4 \times 4 = 3 \times 256 = 768$

(v) $0 \times 10^2 = 0 \times 10 \times 10 = 0$

(vi) $5^2 \times 3^3 = 5 \times 5 \times 3 \times 3 \times 3 = 25 \times 27 = 675$

(vii) $2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 16 \times 9 = 144$

(viii) $3^2 \times 10^4 = 3 \times 3 \times 10 \times 10 \times 10 \times 10 = 9 \times 10000 = 90000$

Question 7:

Simplify:

(i) $(-4)^3$ (ii) $(-3) \times (-2)^3$

(iii) $(-3)^2 \times (-5)^2$ (iv) $(-2)^3 \times (-10)^3$

Answer:

(i) $(-4)^3 = (-4) \times (-4) \times (-4) = -64$

(ii) $(-3) \times (-2)^3 = (-3) \times (-2) \times (-2) \times (-2) = 24$

(iii) $(-3)^2 \times (-5)^2 = (-3) \times (-3) \times (-5) \times (-5) = 9 \times 25 = 225$

(iv) $(-2)^3 \times (-10)^3 = (-2) \times (-2) \times (-2) \times (-10) \times (-10) \times (-10)$

$= (-8) \times (-1000) = 8000$

Question 8:

Compare the following numbers:

(i) 2.7×10^{12}; 1.5×10^8

(ii) 4×10^{14}; 3×10^{17}

Answer:

(i) 2.7×10^{12}; 1.5×10^8

$2.7 \times 10^{12} > 1.5 \times 10^8$

(ii) 4×10^{14}; 3×10^{17}

$3 \times 10^{17} > 4 \times 10^{14}$

Ex 13.2:-

Question 1

Using laws of exponents, simplify and write the answer in exponential form:

(i) $3^2 \times 3^4 \times 3^8$

(ii) $6^{15} \div 6^{10}$

(iii) $a^3 \times a^2$

(iv) $7^x \times 7^2$

(v) $(5^2)^3 \div 5^3$

(vi) $2^5 \times 5^5$

(vii) $a^4 \times b^4$

(viii) $(3^4)^3$

(ix) $(2^{20} \div 2^{15}) \times 2^3$ (x) $8^t \div 8^2$

Solution:

(i) $3^2 \times 3^4 \times 3^8 = 3^{2+4+8} = 3^{14}$ $[a^m \div a^n = a^{m+n}]$

(ii) $6^{15} \div 6^{10} = 6^{15-10} = 6^5$ $[a^m \div a^n = a^{m-n}]$

(iii) $a^3 \times a^2 = a^{3+2} = a^5$ $[a^m \times a^n = a^{m+n}]$

(iv) $7^x \times 7^2 = 7^{x+2}$ $[a^m \times a^n = a^{m+n}]$

(v) $(5^2)^3 \div 5^3 = 5^{2\times3} \div 5^3 = 5^6 \div 5^3 = 5^{6-3} = 5^3$ $[(a^3)^n = a^{mn}, a^m \div a^n = a^{m-n}]$

(vi) $2^5 \times 5^5 = (2 \times 5)^5 = 10^5$ $[a^m \times b^m = (ab)^m]$

(vii) $a^4 \times b^4 = (ab)^4$ $[a^m \times b^m = (ab)^4]$

(ix) $(2^{20} \div 2^{15}) \times 2^3 = 2^{20-15} \times 2^3$

$=2^5 \times 2^3 = 2^{5+3} = 2^8$

(x) $8^t \div 8^2 = 8^{t-2}$ $[a^m \div a^n = a^{m-n}]$

Question 2

Simplify and express each of the following in exponential form:

(i) $\frac{2^3 \times 3^4 \times 4}{2 \times 32}$

(ii) $[(5^2)^3 \times 5^4] \div 5^7$

(iii) $25^4 \div 5^3$

(iv) $\frac{3 \times 7^2 \times 11^8}{21 \times 11^3}$

(v) $\frac{3^7}{3^4 \times 3^3}$

(vi) $2^0 + 3^0 + 4^0$

(vii) $2^0 \times 3^0 \times 4^0$

(viii) $(3^0 + 2^0) \times 5^0$

(ix) $\frac{2^8 \times a^5}{4^3 \times a^3}$

(x) $\left(\frac{a^5}{a^3}\right) \times a^8$

(xi) $\frac{4^5 \times a^8 b^3}{4^5 \times a^5 b^2}$

(xii) $(2^3 \times 2)^2$

Solution:

$$(i)\ \frac{2^3\times 3^4\times 4}{2\times 32}=\frac{2^3\times 3^4\times 2^2}{2\times 2^5}$$

$$=2^3\times 2^2\times 2^{-1}\times 2^{-5}\times 3^4$$

$$=2^{3+2-1-5}\times 3^4=2^{5-6}\times 3^4$$

$$=2^{-1}\times 3^4=\frac{3^4}{2}$$

$$(ii)\ [(5^2)^3\times 5^4]\div 5^7=[5^{2\times 3}\times 5^4]\div 5^7$$

$$=(5^6\times 5^4)\div 5^7=5^{6+4}\div 5^7$$

$$=5^{10}\div 5^7=5^{10-7}=5^3$$

$$(iii)\ 25^4\div 5^3=(5^2)^4\div 5^3$$

$$=5^{2\times 4}\div 5^3=5^8\div 5^3=5^{8-3}=5^5$$

$$(iv)\ \frac{3\times 7^2\times 11^8}{21\times 11^3}=\frac{3\times 7\times 7\times 11^{8-3}}{21}=7\times 11^5$$

$$(v)\ \frac{3^7}{3^4\times 3^3}=\frac{3^7}{3^{4+3}}=\frac{3^7}{3^7}=3^{7-7}=3^0 \text{ or } 1$$

$$(vi)\ 2^0+3^0+4^0=1+1+1=3 \qquad [\because a^0=1]$$

$$(vii)\ 2^0\times 3^0\times 4^0=1\times 1\times 1=1 \qquad [\because a^0=1]$$

$$(viii)\ (3^0+2^0)\times 5^0=(1+1)\times 1=2\times 1=2$$

$$[\because a^0=1]$$

$$(ix)\ \frac{2^8\times a^5}{4^3\times a^3}=\frac{2^8\times a^{5-3}}{(2^2)^3}=\frac{2^8\times a^2}{2^6}$$

$$=2^{8-6}\times a^2=2^2a^2=(2a)^2$$

$$(x)\ \left(\frac{a^5}{a^3}\right)\times a^8=(a^{5-3})\times a^8=a^2\times a^8=a^{2+8}=a^{10}$$

$$(xi)\ \frac{4^5\times a^8b^3}{4^5\times a^5b^2}=4^{5-5}\times a^{8-5}\,b^{3-2}=4^0\times a^3b$$

$$=1\times a^3b=a^3b \qquad (\because\ a^0=1)$$

$$(xii)\ (2^3\times 2)^2=(2^3)^2\times 2^2=2^6\times 2^2=2^{6+2}=2^8$$

Question 3

Say true or false and justify your answer:

(i) $10\times 10^{11}=100^{11}$

(ii) $2^3>5^2$

(iii) $2^3\times 3^2=6^5$ (iv) $3^{20}=(1000)^0$

Solution:

(i) $10 \times 10^{11} = 10^{1+11} = 10^{12}$

RHS = $100^{11} = (10^2)^{11} = 10^{22}$

$10^{12} \neq 10^{22}$

∴ Statement is false.

(ii) $2^3 > 5^2$

LHS = $2^3 = 8$

RHS = $5^2 2 = 25$

$8 < 25$ ∴

$2^3 < 5^2$

Thus, the statement is false.

(iii) $2^3 \times 3^2 = 6^5$

LHS = $2^3 3 \times 3^2 = 8 \times 9 = 72$

RHS = $6^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7776$

∴ $72 \neq 7776$

∴ The statement is false.

(iv) $3^0 = (1000)^0$

⇒ 1 = 1 True [∵ $a^0 = 1$]

Question 4

Express each of the following as a product of prime factors only in exponential form: (i) 108 × 192

(ii) 270

(iii) 729 × 64 (iv) 768

Solution:

(i) 108 × 192 = 2 × 2 × 3 × 3 × 3 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3

$=2^8 \times 3^4$

2	108
2	54
3	27
3	9
3	3
	1

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

(*ii*) $270 = 2 \times 3 \times 3 \times 3 \times 5$

$= 2 \times 5 \times 3^3$

$= 10 \times 3^3 \times 5$

2	270
3	135
3	45
3	15
5	5
	1

(iii) $729 \times 64 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$=3^6 \times 2^6$

3	729
3	243
3	81
3	27
3	9
3	3
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

(*iv*) $768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$= 2^8 \times 3$

$= 3 \times 2^8$

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Question 5

Simplify:

$(i)\ \dfrac{(2^5)^2 \times 7^3}{8^3 \times 7}$ $\qquad$ $(ii)\ \dfrac{25 \times 5^2 \times t^8}{10^3 \times t^4}$

$(iii)\ \dfrac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

Solution:

$$(i)\ \frac{(2^5)^2 \times 7^3}{8^3 \times 7} = \frac{2^{5\times 2} \times 7^{3-1}}{(2^3)^3}$$

$$= \frac{2^{10} \times 7^2}{2^9} = 2^{10-9} \times 7^2$$

$$= 2 \times 7^2 = 2 \times 49 = 98$$

$$(ii)\ \frac{25 \times 5^2 \times t^8}{10^3 \times t^4} = \frac{5^2 \times 5^2 \times t^{8-4}}{5^3 \times 2^3}$$

$$= \frac{5^4 \times t^4}{5^3 \times 2^3} = \frac{5^{4-3} \times t^4}{2^3}$$

$$= \frac{5 \times t^4}{2^3} = \frac{5t^4}{8}$$

$$(iii)\ \frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5} = \frac{3^5 \times (2\times 5)^5 \times 5 \times 5}{5^7 \times 2^5 \times 3^5}$$

$$= \frac{3^5 \times 2^5 \times 5^5 \times 5^2}{5^7 \times 2^5 \times 3^5} \quad (a \times b)^m = (a^m \times b^m)$$

$$= \frac{3^5 \times 2^5 \times 5^{5+2}}{5^7 \times 2^5 \times 3^5} \quad (a^m \times a^n = a^{m+n})$$

$$= \frac{3^5 \times 2^2 \times 5^7}{5^7 \times 2^5 \times 3^5} = 3^{5-5} \times 2^{5-5} \times 5^{7-7}$$

$$(\because\ a^m \div a^n = a^{m-n})$$

$$= 3^0 \times 2^0 \times 5^0 = 1 \times 1 \times 1 = 1$$

Question 1:

Using laws of exponents, simplify and write the answer in exponential form:

(i) $3^2 \times 3^4 \times 3^8$ (ii) $6^{15} \div 6^{10}$ (iii) $a^3 \times a^2$

(iv) $7^x \times 7^2$ (v) $5^{2^3} \div 5^3$ (vi) $2^5 \times 5^5$

(vii) $a^4 \times b^4$ (viii) $(3^4)^3$

(ix) $\left(2^{20} \div 2^{15}\right) \times 2^3$ (x) $8^t \div 8^2$

Answer:

(i) $3^2 \times 3^4 \times 3^8 = (3)^{2+4+8}$ $(a^m \times a^n = a^{m+n})$

$= 3^{14}$

(ii) $6^{15} \div 6^{10} = (6)^{15-10}$ $(a^m \div a^n = a^{m-n})$

$= 6^5$

(iii) $a^3 \times a^2 = a^{(3+2)}$ $(a^m \times a^n = a^{m+n})$

$= a^5$

(iv) $7^x + 7^2 = 7^{x+2}$ $(a^m \times a^n = a^{m+n})$

(v) $(5^2)^3 \div 5^3$

$= 5^{2 \times 3} \div 5^3$ $(a^m)^n = a^{mn}$

$= 5^6 \div 5^3$

$= 5^{(6-3)}$ $(a^m \div a^n = a^{m-n})$

$= 5^3$

(vi) $2^5 \times 5^5$

$= (2 \times 5)^5$ $[a^m \times b^m = (a \times b)^m]$

$= 10^5$

(vii) $a^4 \times b^4$

$= (ab)^4$ $[a^m \times b^m = (a \times b)^m]$

(viii) $(3^4)^3 = 3^{4 \times 3} = 3^{12}$ $(a^m)^n = a^{mn}$

(ix) $(2^{20} \div 2^{15}) \times 2^3$

$= (2^{20-15}) \times 2^3$ $(a^m \div a^n = a^{m-n})$

$= 2^5 \times 2^3$

$= (2^{5+3})$ $(a^m \times a^n = a^{m+n})$

$= 2^8$

(x) $8^t \div 8^2 = 8^{(t-2)}$ $(a^m \div a^n = a^{m-n})$

Question 2:

Simplify and express each of the following in exponential form:

(i) $\frac{2^3\times3^4\times4}{3\times32}$ (ii) $\left[5^2\times5^4\right]\div5^7$ (iii) $25^4\div5^3$

(iv) $\frac{3\times7^2\times11^8}{21\times11^3}$ (v) $\frac{3^7}{3^4\times3^3}$ (vi) $2^0+3^0+4^0$

(vii) $2^0\times3^0\times4^0$ (viii) $(3^0+2^0)\times5^0$ (ix) $\frac{2^8\times a^5}{4^3\times a^3}$

(x) $\left(\frac{a^5}{a^3}\right)\times a^8$ (xi) $\frac{4^5\times a^8b^3}{4^5\times a^5b^2}$ (xii) $\left(2^3\times2\right)^2$

Answer:

(i)

$$\frac{2^3\times3^4\times4}{3\times32}=\frac{2^3\times3^4\times2\times2}{3\times2\times2\times2\times2\times2}=\frac{2^3\times3^4\times2^2}{3\times2^5}$$

$$=\frac{2^{3+2}\times3^4}{3\times2^5} \qquad (a^m\times a^n=a^{m+n})$$

$$=\frac{2^5\times3^4}{3\times2^5}$$

$$=2^{5-5}\times3^{4-1} \qquad (a^m\div a^n=a^{m-n})$$

$$=2^0\,3^3=1\times3^3=3^3$$

(ii) $[(5^2)^3\times5^4]\div5^7$

$=[5^{2\times3}\times5^4]\div5^7$ $(a^m)^n=a^{mn}$

$=[5^6\times5^4]\div5^7$

$=[5^{6+4}]\div5^7$ $(a^m\times a^n=a^{m+n})$

$=5^{10}\div5^7$

$=5^{10-7}$ $(a^m\div a^n=a^{m-n})$

$=5^3$

(iii) $25^4\div5^3=(5\times5)^4\div5^3$

$=(5^2)^4\div5^3$

$=5^{2\times4}\div5^3$ $(a^m)^n=a^{mn}$

$=5^8\div5^3$

$= 5^{8-3}$ $(a^m \div a^n = a^{m-n})$

$= 5^5$

(iv)

$$\frac{3\times7^2\times11^8}{21\times11^3}=\frac{3\times7^2\times11^8}{3\times7\times11^3}$$

$$=3^{1-1}\times7^{2-1}\times11^{8-3} \qquad (a^m \div a^n = a^{m-n})$$

$$=3^0\times7^1\times11^5$$

$= 1 \times 7 \times 11^5 = 7 \times 11^5$

(v)

$$\frac{3^7}{3^4\times3^3}=\frac{3^7}{3^{4+3}} \qquad (a^m\times a^n = a^{m+n})$$

$$=\frac{3^7}{3^7}=3^{7-7} \qquad (a^m \div a^n = a^{m-n})$$

$$=3^0=1$$

(vi) $2^0 + 3^0 + 4^0 = 1 + 1 + 1 = 3$

(vii) $2^0 \times 3^0 \times 4^0 = 1 \times 1 \times 1 = 1$

(viii) $(3^0 + 2^0) \times 5^0 = (1 + 1) \times 1 = 2$

(ix)

$$\frac{2^8\times a^5}{4^3\times a^3}=\frac{2^8\times a^5}{(2\times2)^3\times a^3}=\frac{2^8\times a^5}{\left(2^2\right)^3\times a^3}$$

$$=\frac{2^8\times a^5}{\left(2^{2\times3}\right)\times a^3} \qquad \left[\left(a^m\right)^n=a^{mn}\right]$$

$$=\frac{2^8\times a^5}{2^6\times a^3}$$

$$= 2^{8-6}\times a^{5-3} \qquad (a^m \div a^n = a^{m-n})$$

$$= 2^2\times a^2=(2\times a)^2 \qquad \left[a^m\times b^m = (a\times b)^m\right]$$

$$=(2a)^2$$

(x)

$$\left(\frac{a^5}{a^3}\right)\times a^8 = a^{5-3} \times a^8 \qquad (a^m \div a^n = a^{m-n})$$

$$= a^2 \times a^8$$

$$= a^{2+8} = a^{10} \qquad (a^m \times a^n = a^{m+n})$$

(xi)

$$\frac{4^5 \times a^8 b^3}{4^5 \times a^5 b^2} = 4^{5-5} \times a^{8-5} \times b^{3-2} \qquad (a^m \div a^n = a^{m-n})$$

$$= 4^0 \times a^3 \times b^1 = 1 \times a^3 \times b = a^3 b$$

(xii) $(2^3 \times 2)^2 = \left(2^{3+1}\right)^2$ $(a^m \times a^n = a^{m+n})$

$= (2^4)^2 = 2^{4 \times 2}$ $(a^m)^n = a^{mn}$

$= 2^8$

Question 3:

Say true or false and justify your answer:

(i) $10 \times 10^{11} = 100^{11}$ (ii) $2^3 > 5^2$

(iii) $2^3 \times 3^2 = 6^5$ (iv) $3^0 = (1000)^0$

Answer:

(i) $10 \times 10^{11} = 100^{11}$

L.H.S. $= 10 \times 10^{11} = 10^{11+1}$ $(a^m \times a^n = a^{m+n})$

$= 10^{12}$

R.H.S. $= 100^{11} = (10 \times 10)^{11} = (10^2)^{11}$

$= 10^{2 \times 11} = 10^{22}$ $(a^m)^n = a^{mn}$

As L.H.S. ≠ R.H.S.,

Therefore, the given statement is false.

(ii) $2^3 > 5^2$

L.H.S. $= 2^3 = 2 \times 2 \times 2 = 8$

R.H.S. $= 5^2 = 5 \times 5 = 25$

As $25 > 8$,

Therefore, the given statement is false.

(iii) $2^3 \times 3^2 = 6^5$

L.H.S. $= 2^3 \times 3^2 = 2 \times 2 \times 2 \times 3 \times 3 = 72$

R.H.S. $= 6^5 = 7776$

As L.H.S. ≠ R.H.S.,

Therefore, the given statement is false.

(iv) $3^0 = (1000)^0$

L.H.S. = $3^0 = 1$

R.H.S. = $(1000)^0 = 1$ = L.H.S.

Therefore, the given statement is true.

Question 4:

Express each of the following as a product of prime factors only in exponential form:

(i) 108 × 192 (ii) 270

(iii) 729 × 64 (iv) 768

Answer:

(i) 108 × 192

= (2 × 2 × 3 × 3 × 3) × (2 × 2 × 2 × 2 × 2 × 2 × 3)

= $(2^2 \times 3^3) \times (2^6 \times 3)$

= $2^{6+2} \times 3^{3+1}$ ($a^m \times a^n = a^{m+n}$)

= $2^8 \times 3^4$

(ii) 270 = 2 × 3 × 3 × 3 × 5 = $2 \times 3^3 \times 5$

(iii) 729 × 64 = (3 × 3 × 3 × 3 × 3 × 3) × (2 × 2 × 2 × 2 × 2 × 2)

= $3^6 \times 2^6$

(iv) 768 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 = $2^8 \times 3$

Question 5:

Simplify:

(i) $\frac{(2^5)^2 \times 7^3}{8^3 \times 7}$ (ii) $\frac{25 \times 5^2 \times t^8}{10^3 \times t^4}$ (iii) $\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

Answer:

(i)

$$\frac{(2^5)^2 \times 7^3}{8^3 \times 7} = \frac{2^{5\times2} \times 7^3}{(2\times2\times2)^3 \times 7} \qquad \left[(a^m)^n = a^{mn}\right]$$

$$= \frac{2^{10} \times 7^3}{(2^3)^3 \times 7} = \frac{2^{10} \times 7^3}{2^{3\times3} \times 7} \qquad \left[(a^m)^n = a^{mn}\right]$$

$$= \frac{2^{10} \times 7^3}{2^9 \times 7} = 2^{10-9} \times 7^{3-1} \qquad (a^m \div a^n = a^{m-n})$$

$$= 2^1 \times 7^2 = 2 \times 7 \times 7 = 98$$

(ii)

$$\frac{25 \times 5^2 \times t^8}{10^3 \times t^4} = \frac{5 \times 5 \times 5^2 \times t^8}{(5\times2)^3 \times t^4} \qquad (a\times b)^m = (a^m \times b^m)$$

$$= \frac{5^{1+1+2} \times t^8}{5^3 \times 2^3 \times t^4} \qquad (a^m \times a^n = a^{m+n})$$

$$= \frac{5^4 \times t^8}{5^3 \times 2^3 \times t^4} = \frac{5^{4-3} \times t^{8-4}}{2^3} \qquad (a^m \div a^n = a^{m-n})$$

$$= \frac{5^1 \times t^4}{2 \times 2 \times 2} = \frac{5t^4}{8}$$

(iii)

$$\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5} = \frac{3^5 \times (2\times5)^5 \times 5 \times 5}{5^7 \times 2^5 \times 3^5}$$

$$= \frac{3^5 \times 2^5 \times 5^5 \times 5^2}{5^7 \times 2^5 \times 3^5} \qquad (a\times b)^m = (a^m \times b^m)$$

$$= \frac{3^5 \times 2^5 \times 5^{5+2}}{5^7 \times 2^5 \times 3^5} \qquad (a^m \times a^n = a^{m+n})$$

$$= \frac{3^5 \times 2^5 \times 5^7}{5^7 \times 2^5 \times 3^5}$$

$$= 3^{5-5} \times 2^{5-5} \times 5^{7-7} \qquad (a^m \div a^n = a^{m-n})$$

$$= 3^0 \times 2^0 \times 5^0 = 1 \times 1 \times 1 = 1$$

Ex 13.3:-

Question 1.

Write the following numbers in the e×panded forms:

279404, 3006194, 2806196, 120719, 20068

Solution:

(i) $279404 = 2 \times 100000 + 7 \times 10000 + 9 \times 1000 + 4 \times 100 + 0 \times 10 + 4$

$= 2 \times 10^5 + 7 \times 10^4 + 9 \times 10^{32} + 4 \times 10^2 + 0 \times 10^1 + 4 \times 10^0$

(ii) $3006194 = 3 \times 1000000 + 0 \times 100000 + 0 \times 10000 + 6 \times 1000 + 1 \times 100 + 9 \times 10 + 4$

$= 3 \times 10^6 + 0 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$

(iii) $2806196 = 2 \times 1000000 + 8 \times 100000 + 0 \times 10000 + 6 \times 1000 + 1 \times 100 + 9 \times 10 + 6$

$= 2 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 6 \times 10^0$

(iv) $120719 = 1 \times 100000 + 2 \times 10000 + 0 \times 1000 + 7 \times 100 + 1 \times 10 + 9$

$= 1 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$

(v) $20068 = 2 \times 10000 + 0 \times 1000 + 0 \times 100 + 6 \times 10 + 8 = 2 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$

Question 2

Find the number from each of the following expanded forms:

(a) $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

(b) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

(c) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$ (d) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

Solution:

(a) $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 101 + 5 \times 10^0$

$= 8 \times 10000 + 6 \times 1000 + 0 \times 100 + 4 \times 10 + 5 \times 1 =$

$80000 + 6000 + 0 + 40 + 5 = 86045$

(b) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

$= 4 \times 100000 + 5 \times 1000 + 3 \times 100 + 2 \times 1 =$

$400000 + 5000 + 300 + 2 = 405302$

(c) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

$= 3 \times 10000 + 7 \times 100 + 5 \times 1 = 30000$

$+ 700 + 5 = 30705$

(d) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

$= 9 \times 100000 + 2 \times 100 + 3 \times 10$

$= 900000 + 200 + 30 = 900230$

Question 3

Express the following numbers in standard form:

(i) 5,00,00,000

(ii) 70,00,000

(iii) 3,18,65,00,000

(iv) 3,90,878

(v) 39087.8 (vi) 3908.78

Solution:

(i) $5,00,00,000 = 5 \times 107^{7}$

(ii) $70,00,000 = 7 \times 10^{6}$

(iii) $3,18,65,00,000 = 3.1865 \times 10^{9}$

(iv) $3,90,878 = 3.90878 \times 10^{5}$

(v) $39087.8 = 3.90878 \times 10^{4}$

(vi) $3908.7\ 8 = 3.90878 \times 10^{3}$

Question 4

Express the number appearing in the following statements in standard form:

(a) The distance between Earth and Moon is 384.0. 000 m.

(b) Speed of light in vacuum is 300,000,000 m/s.

(c) Diameter of the Earth is 1,27,56,000 m.

(d) Diameter of the Sun is 1,400,000,000 m.

(e) In a galaxy there are an average 100,000,000,000 stars.

(f) The universe is estimated to be about 12,000,000,000 years old.

(g) The distance of the Sun from the centre of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000,000 m.

(h) 60,230,000,000,000,000,000,000 molecules are contained in a drop of water weighing 1.8 gm.

(i) The Earth has 1,353,000,000 cubic km of sea water.

(j) The population of India was about 1,027,000,000 in March 2001.

Solution:

(a) $384,000,000 \text{ m} = 3.84 \times 10^{8} \text{ m}$

(b) $300,000,000 \text{ m/s} = 3 \times 10^{8} \text{ m/s}$

(c) $1,27,56,000 \text{ m} = 1.2756 \times 10^{72} \text{ m}$

(d) $1,400,000,000 \text{ m} = 1.4 \times 10^{9} \text{ m}$

(e) $100,000,000,000 \text{ stars} = 1 \times 10^{11} \text{ stars}$

(f) $12,000,000,000 \text{ years old} = 1.2 \times 10^{10} \text{ years old}$

(g) $300,000,000,000,000,000,000 \text{ m} = 3 \times 10^{20} \text{ m}$

(h) $60, 230, 000, 000, 000, 000, 000, 000 \text{ molecules} = 6.023 \times 10^{22} \text{ molecules}$

(i) $1,353,000,000 \text{ cubic km} = 1.353 \times 10^{9} \text{ cubic km}$

(j) $1,0,27,000,000 = 1.027 \times 10^{9}$

Question 1:

Write the following numbers in the expanded forms:

279404, 3006194, 2806196, 120719, 20068

Answer:

$279404 = 2 \times 10^5 + 7 \times 10^4 + 9 \times 10^3 + 4 \times 10^2 + 0 \times 10^1 + 4 \times 10^0$

$3006194 = 3 \times 10^6 + 0 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$

$2806196 = 2 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 6 \times 10^0$

$120719 = 1 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$

$20068 = 2 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$

Question 2:

Find the number from each of the following expanded forms:

(a) $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

(b) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

(c) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

(d) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

Answer:

(a) $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

$= 86045$

(b) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

$= 405302$

(c) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

$= 30705$

(d) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

$= 900230$

Question 3:

Express the following numbers in standard form:

(i) 5, 00, 00, 000 (ii) 70, 00, 000

(iii) 3, 18, 65, 00, 000 (iv) 3, 90, 878

(v) 39087.8 (vi) 3908.78

Answer:

(i) $50000000 = 5 \times 10^7$

(ii) $7000000 = 7 \times 10^6$

(iii) $3186500000 = 3.1865 \times 10^9$

(iv) $390878 = 3.90878 \times 10^5$

(v) $39087.8 = 3.90878 \times 10^4$

(vi) $3908.78 = 3.90878 \times 10^3$

Question 4:

Express the number appearing in the following statements in standard form.

(a) The distance between Earth and Moon is 384, 000, 000 m.

(b) Speed of light in vacuum is 300, 000, 000 m/s.

(c) Diameter of the Earth is 1, 27, 56, 000 m.

(d) Diameter of the Sun is 1, 400, 000, 000 m.

(e) In a galaxy there are on an average 100, 000, 000, 000 stars.

(f) The universe is estimated to be about 12, 000, 000, 000 years old.

(g) The distance of the Sun from the centre of the Milky Way Galaxy is estimated to be 300, 000, 000, 000, 000, 000, 000 m.

(h) 60, 230, 000, 000, 000, 000, 000, 000 molecules are contained in a drop of water weighing 1.8 gm.

(i) The earth has 1, 353, 000, 000 cubic km of sea water.

(j) The population of India was about 1, 027, 000, 000 in March, 2001.

Answer:

(a) 3.84×10^8 m

(b) 3×10^8 m/s

(c) 1.2756×10^7 m

(d) 1.4×10^9 m

(e) 1×10^{11} stars

(f) 1.2×10^{10} years

(g) 3×10^{20} m

(h) 6.023×10^{22}

(i) 1.353×10^9 cubic km

(j) 1.027×10^9