CHAPTER-6
The Triangle and its Properties

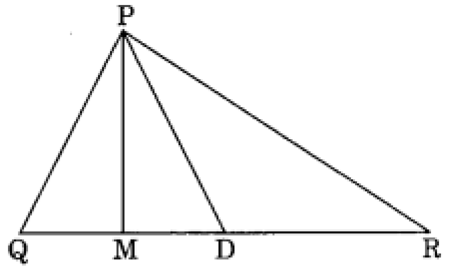
Ex 6.1:-

Question 1

In ΔPQR, D is the mid-point of $\overline{QR}$,

$\overline{PM}$ is

$\overline{PD}$ is



If QM = MR?

Solution:

$\overline{PM}$ is altitude.

$\overline{PD}$ is median.

No, QM ≠ MR.

Question 2

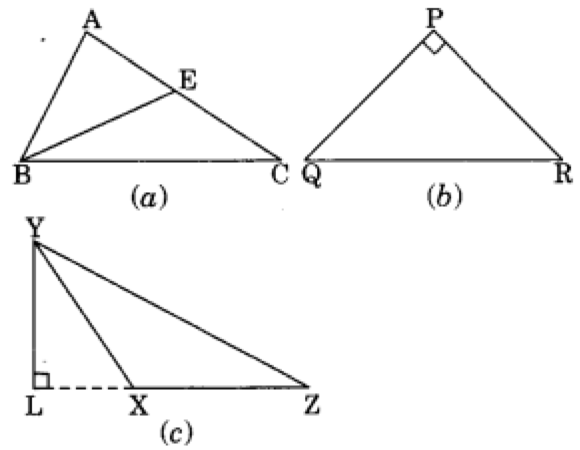
Draw rough sketches for the following:

(a) In ΔABC, BE is a median.

(ib) In ΔPQR, PQ and PR are altitudes of the triangle.

(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

Solution:

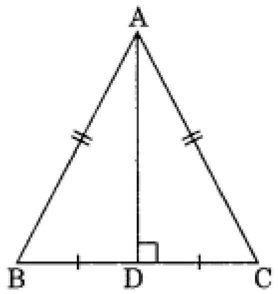


Question 3

Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

Solution:

ΔABC is an isosceles triangle in which AB = AC



Draw AD as the median of the triangle.
Measure the angle ADC with the help of protractor we find, $\angle ADC = 90°$
Thus, AD is the median as well as the altitude of the ΔABC. Hence Verified.

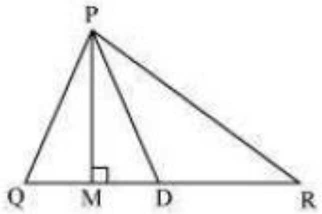
Question 1:

In ΔPQR, D is the mid-point of $\overline{QR}$.

$\overline{PM}$ is __________.

PD is __________.

Is QM = MR?



Answer:

(i) Altitude

(ii) Median

(iii) No

Question 2:

Draw rough sketches for the following:

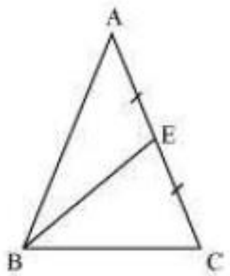
(a) In ΔABC, BE is a median.

(b) In ΔPQR, PQ and PR are altitudes of the triangle.

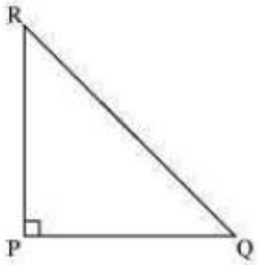
(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

Answer:

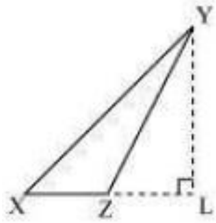
(a)



(b)



(c)

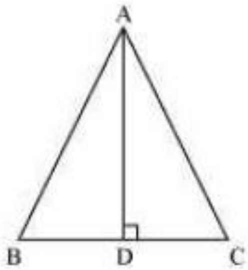


Here, it can be observed that for ΔXYZ, YL is an altitude drawn exterior to side XZ which is extended up to point L.

Question 3:

Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

Answer:

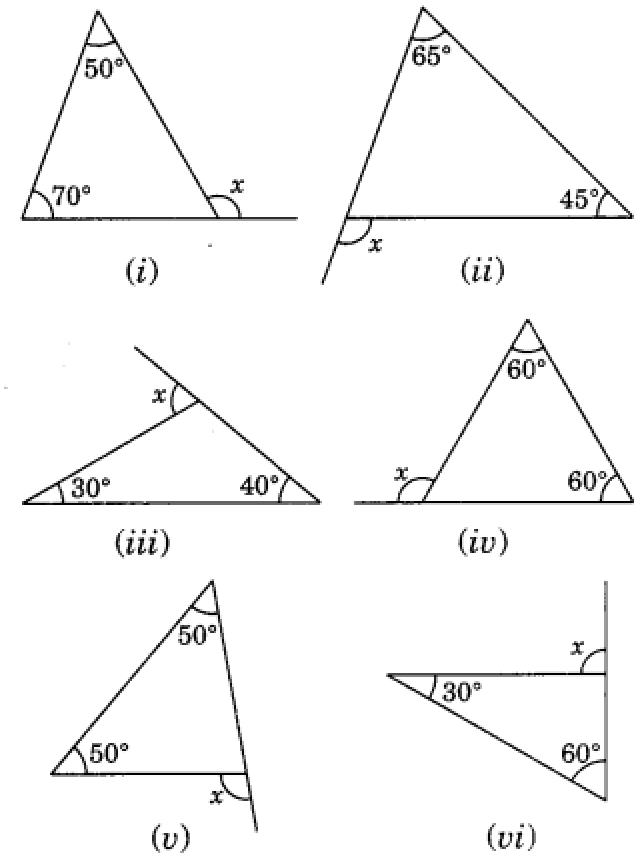


Draw a line segment AD perpendicular to BC. It is an altitude for this triangle. It can be observed that the length of BD and DC is also same. Therefore, AD is also a median of this triangle.

Ex 6.2:-

Question 1

Find the value of the unknown exterior angle x in the following diagrams:



Solution:

(i) ∠x = 50° + 70° = 120° (Exterior angle is equal to sum of its interior opposite angles)

(ii) ∠x = 65°+ 45° = 110° (Exterior angle is equal to sum of its interior opposite angles)

(iii) ∠x = 30° + 40° = 70° (Exterior angle is equal to sum of its interior opposite angles)

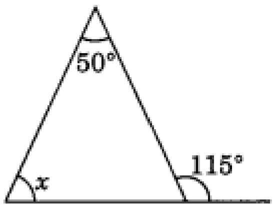
(iv) ∠x = 60° + 60° = 120° (Exterior angle is equal to sum of its interior opposite angles)

(v) ∠x = 50° + 50° =100° (Exterior angle is equal to sum of its interior opposite angles)

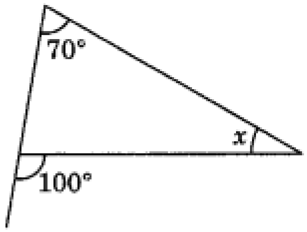
(vi) ∠x = 30° + 60° = 90° (Exterior angle is equal to sum of its interior opposite angle)

Question 2

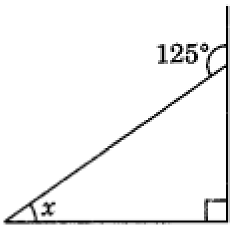
Find the value of the unknown interior angle x in the following figures:



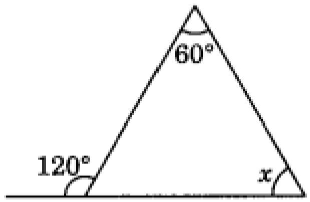
(i)



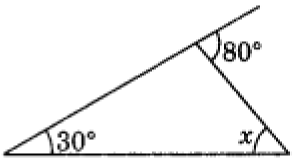
(ii)



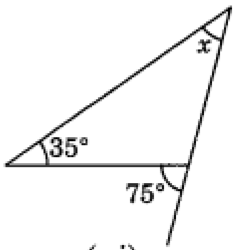
(iii)



(iv)



(v)



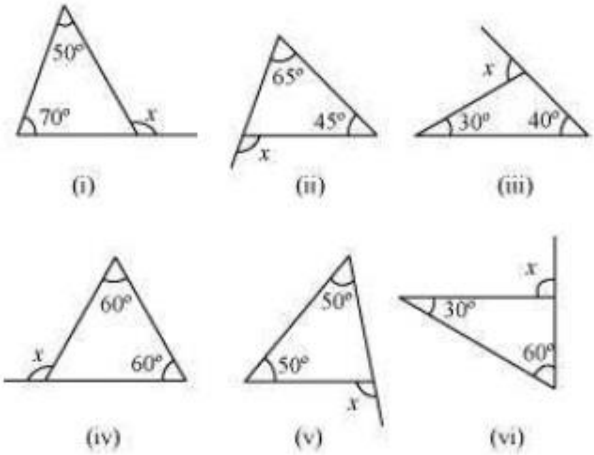
(vi)

Solution:

(i) ∠x + 50° = 115° (Exterior angle of a triangle)
∴ ∠x = 115°- 50° = 65°
(ii) ∠x + 70° = 110° (Exterior angle of a triangle)
∴ ∠x = 110° − 70° = 40°
(iii) ∠ x + 90° = 125° (Exterior angle of a right triangle)
∴ ∠x = 125° − 90° = 35°
(iv) ∠x + 60° = 120° (Exterior angle of a triangle)
∴ ∠x = 120° − 60° = 60°
(v)∠ X + 30° = 80° (Exterior angle of a triangle)
∴ ∠x = 80° − 30° = 50°
(vi) ∠ x + 35° = 75° (Exterior angle of a triangle)
∴ ∠ x = 75° − 35° = 40°

Question 1:

Find the value of the unknown exterior angle x in the following diagrams:



Answer:

(i) $x = 50° + 70°$ (Exterior angle theorem)

$x = 120°$

(ii) $x = 65° + 45°$ (Exterior angle theorem)

$= 110°$

(iii) $x = 40° + 30°$ (Exterior angle theorem)

$= 70°$

(iv) $x = 60° + 60°$ (Exterior angle theorem)

$= 120°$

(v) $x = 50° + 50°$ (Exterior angle theorem)

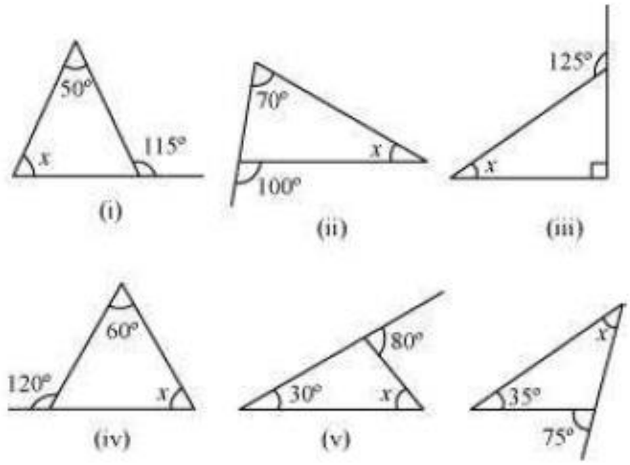
$= 100°$

(vi) $x = 30° + 60°$ (Exterior angle theorem)

$= 90°$

Question 2:

Find the value of the unknown interior angle x in the following figures:



Answer:

(i) $x + 50° = 115°$ (Exterior angle theorem)

$x = 115° - 50° = 65°$

(ii) $70° + x = 100°$ (Exterior angle theorem)

$x = 100° - 70° = 30°$

(iii) $x + 90° = 125°$ (Exterior angle theorem)

$x = 125° - 90° = 35°$

(iv) $x + 60° = 120°$ (Exterior angle theorem)

$x = 120° - 60° = 60°$

(v) $x + 30° = 80°$ (Exterior angle theorem)

$x = 80° - 30° = 50°$

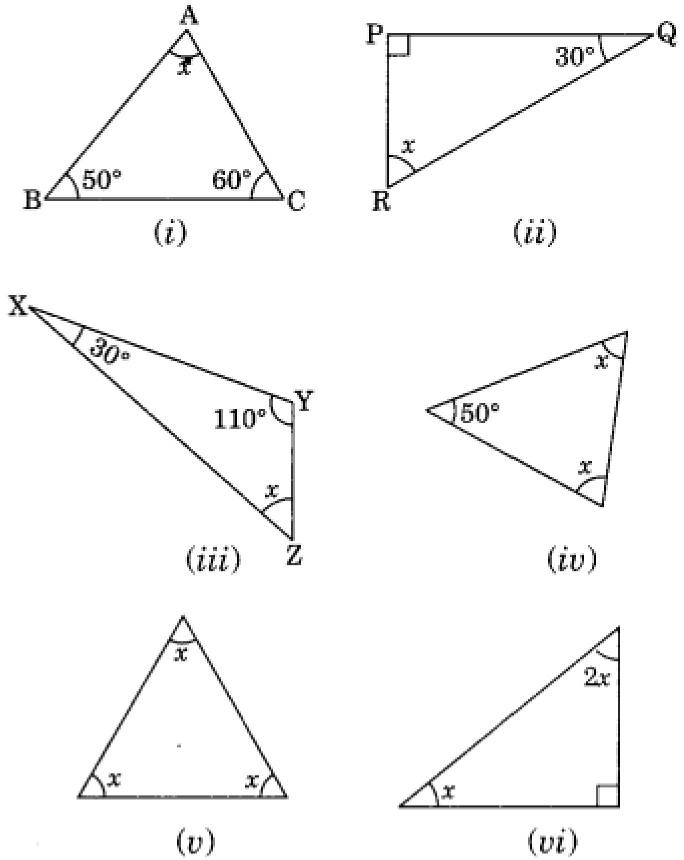
(vi) $x + 35° = 75°$ (Exterior angle theorem)

$x = 75° - 35° = 40°$

Ex 6.3:-

Question 1

Find the value of the unknown x in the following diagrams:



Solution:

(i) By angle sum property of a triangle, we have

$\angle x + 50^\circ + 60^\circ = 180^\circ$

$\Rightarrow \angle x + 110^\circ = 180^\circ$

$\therefore \angle x = 180^\circ - 110^\circ = 70^\circ$

(ii) By angle sum property of a triangle, we have

$\angle x + 90^\circ + 30 = 180^\circ$ [Δ is right angled triangle]

$\Rightarrow \angle x + 120^\circ = 180^\circ$

$\therefore \angle x - 180^\circ - 120^\circ = 60^\circ$

(iii) By angle sum property of a triangle, we have

$\angle x + 30^\circ + 110^\circ - 180^\circ$

$\Rightarrow \angle x + 140^\circ = 180^\circ$

$\therefore \angle x = 180^\circ - 140^\circ = 40^\circ$

(iv) By angle sum property of a triangle, we have

$\angle x + \angle x + 50^\circ = 180^\circ$

$\Rightarrow 2x + 50^\circ = 180^\circ$

$\Rightarrow 2x = 180^\circ - 50^\circ$

$\Rightarrow 2x = 130^\circ$

$\therefore x = \frac{130^\circ}{2} = 65^\circ$

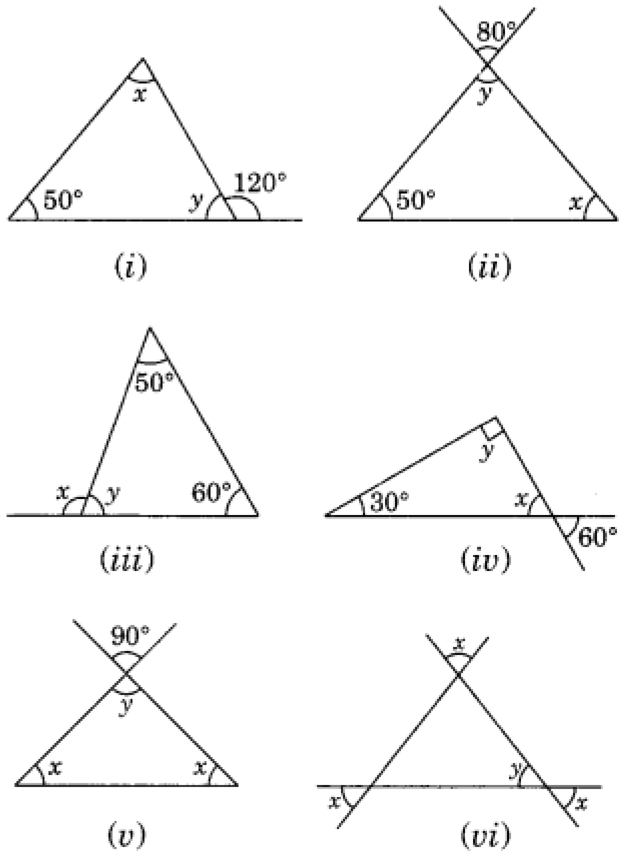
(v) By angle sum property of a triangle, we have

$\angle x + \angle x + \angle x = 180^\circ$

⇒ 3 ∠x = 180°
∴ $\angle x = \frac{180^\circ}{3} = 60^\circ$
(vi) By angle sum property of a triangle, we have
x + 2 x + 90° = 180° (Δ is right angled triangle)
⇒ 3x + 90° = 180°
⇒ 3x = 180° – 90°
⇒ 3x = 90°
∴ $x = \frac{90^\circ}{3} = 30^\circ$

Question 2

Find the values of the unknowns x and y in the following diagrams:



Solution:

(i) ∠x + 50° = 120° (Exterior angle of a triangle)
∴ ∠x = 120°- 50° = 70°
∠x + ∠y + 50° = 180° (Angle sum property of a triangle)
70° + ∠y + 50° = 180°
∠y + 120° = 180°
∠y = 180° – 120°
∴ ∠y = 60°
Thus ∠x = 70 and ∠y – 60°

(ii) ∠y = 80° (Vertically opposite angles are same)
∠x + ∠y + 50° = 180° (Angle sum property of a triangle)
⇒ ∠x + 80° + 50° = 180°
⇒ ∠x + 130° = 180°

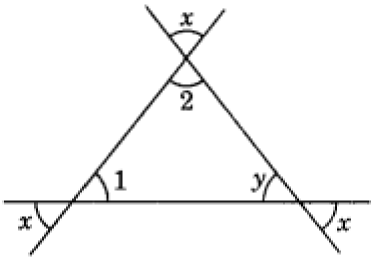
∴ ∠x = 180° – 130° = 50°
Thus, ∠x = 50° and ∠y = 80°

(iii) ∠y + 50° + 60° = 180° (Angle sum property of a triangle)
∠y + 110° = 180°
∴ ∠y = 180°- 110° = 70°
∠x + ∠y = 180° (Linear pairs)
⇒ ∠x + 70° = 180°
∴ ∠x = 180° – 70° = 110°
Thus, ∠x = 110° and y = 70°

(iv) ∠x = 60° (Vertically opposite angles)
∠x + ∠y + 30° = 180° (Angle sum property of a triangle)
⇒ 60° + ∠y + 30° = 180°
⇒ ∠y + 90° = 180°
⇒ ∠y = 180° – 90° = 90°
Thus, ∠x = 60° and ∠y = 90°

(v) ∠y = 90° (Vertically opposite angles)
∠x + ∠x + ∠y = 180° (Angle sum property of a triangle)
⇒ 2 ∠x + 90° = 180°
⇒ 2∠x = 180° – 90°
⇒ 2∠x = 90°
∴ ∠x=90∘2=45∘
Thus, ∠x = 45° and ∠y = 90°
(vi) From the given figure, we have

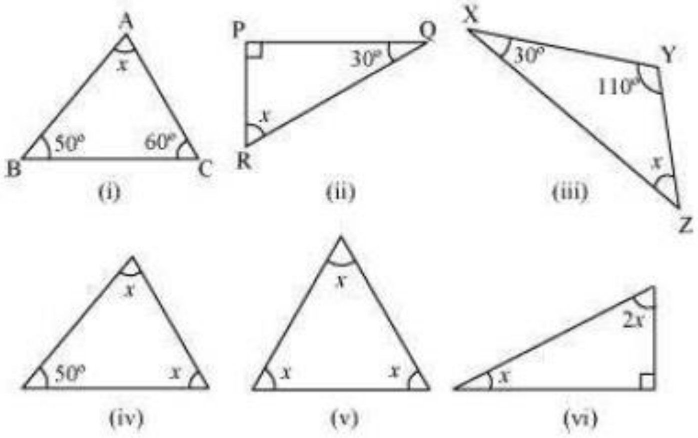
$$\left.\begin{aligned}\angle y &= \angle x\\ \angle 1 &= \angle x\\ \angle 2 &= \angle x\end{aligned}\right\}$$



Adding both sides, we have
∠y + ∠1 + ∠2 = 3∠x
⇒ 180° = 3∠x (Angle sum property of a triangle)
∴ ∠x=180∘3=60∘
∠x = 60°, ∠y = 60°

Question 1:

Find the value of the unknown x in the following diagrams:



Answer:

The sum of all interior angles of a triangle is 180°. By using this property, these problems can be solved as follows.

(i) $x + 50° + 60° = 180°$

$x + 110° = 180°$

$x = 180° - 110° = 70°$

(ii) $x + 90° + 30° = 180°$

$x + 120° = 180°$

$x = 180° - 120° = 60°$

(iii) $x + 30° + 110° = 180°$

$x + 140° = 180°$

$x = 180° - 140° = 40°$

(iv) $50° + x + x = 180°$

$50° + 2x = 180°$

$2x = 180° - 50° = 130°$

$$x = \frac{130°}{2} = 65°$$

(v) $x + x + x = 180°$

$3x = 180°$

$$x = \frac{180}{3} = 60°$$

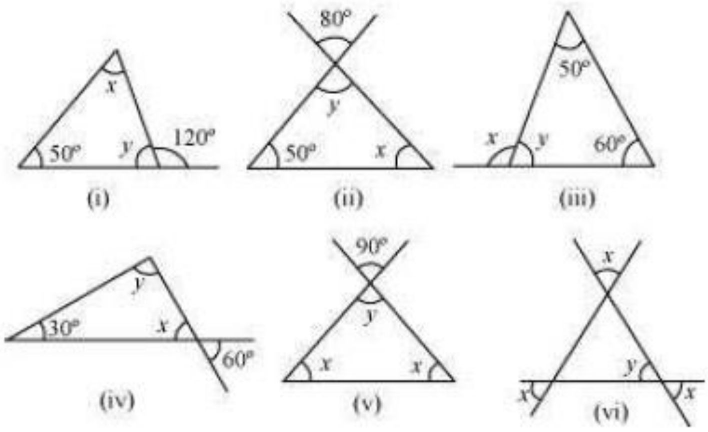
(vi) $x + 2x + 90° = 180°$

$3x = 180° - 90° = 90°$

$$x = \frac{90°}{3} = 30°$$

Question 2:

Find the value of the unknowns x and y in the following diagrams:



Answer:

(i) $y + 120° = 180°$ (Linear pair)

$y = 180° - 120° = 60°$

$x + y + 50° = 180°$ (Angle sum property)

$x + 60° + 50° = 180°$

$x + 110° = 180°$

$x = 180° - 110° = 70°$

(ii) $y = 80°$ (Vertically opposite angles)

$y + x + 50° = 180°$ (Angle sum property)

$80° + x + 50° = 180°$

$x + 130° = 180°$

$x = 180° - 130° = 50°$

(iii) $y + 50° + 60° = 180°$ (Angle sum property)

$y = 180° - 60° - 50° = 70°$

$x + y = 180°$ (Linear pair)

$x = 180° - y = 180° - 70° = 110°$

(iv) $x = 60°$ (Vertically opposite angles)

$30° + x + y = 180°$

$30° + 60° + y = 180°$

$y = 180° - 30° - 60° = 90°$

(v) $y = 90°$ (Vertically opposite angles)

$x + x + y = 180°$ (Angle sum property)

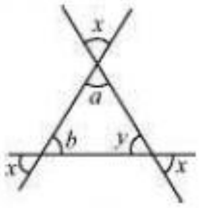
$2x + y = 180°$

$2x + 90° = 180°$

$2x = 180° - 90° = 90°$

$$x = \frac{90°}{2} = 45°$$

(vi)



$y = x$ (Vertically opposite angles)

$a = x$ (Vertically opposite angles)

$b = x$ (Vertically opposite angles)

$a + b + y = 180°$ (Angle sum property)

$x + x + x = 180°$

$3x = 180°$

$$x = \frac{180°}{3} = 60°$$

$y = x = 60°$

Ex 6.4:-

Question 1

Is it possible to have a triangle with the following sides?
(i) 2 cm, 3 cm, 5 cm
(ii) 3 cm, 6 cm, 7 cm
(iii) 6 cm, 3 cm, 2 cm

Solution:

We know that for a triangle, the sum of any two sides must be greater than the third side.

(i) Given sides are 2 cm, 3 cm, 5 cm

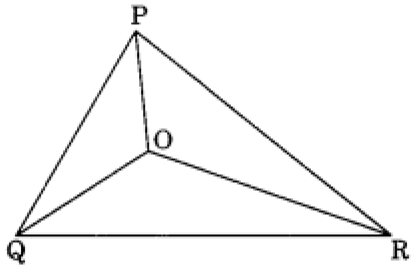
Sum of the two sides = 2 cm + 3 cm = 5 cm Third side = 5 cm

We have, Sum of any two sides = the third side i.e. 5 cm = 5 cm
Hence, the triangle is not possible.

(ii) Given sides are 3 cm, 6 cm, 7 cm
Sum of the two sides = 3 cm + 6 cm = 9 cm Third side = 7 cm
We have sum of any two sides > the third
side. i.e. 9 cm > 7 cm
Hence, the triangle is possible.

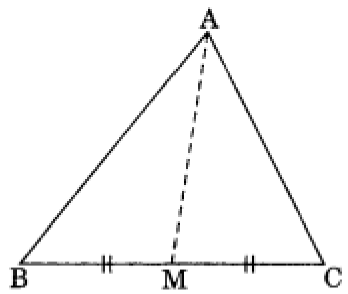
(iii) Given sides are 6 cm, 3 cm, 2 cm
Sum of the two sides = 3 cm + 2 cm – 5 cm Third side = 6 cm
We have, sum of any two sides < the third sid6 i.e. 5 cm < 6 cm Hence, the triangle is not possible.

Question 2
Take any point O in the interior of a triangle PQR . Is



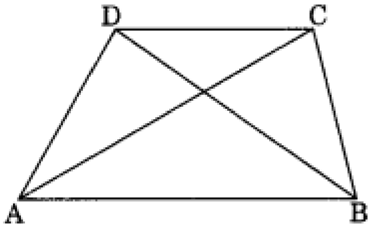
(i) OP + OQ > PQ?
(ii) OQ + OR > QR?
(iii) OR + OP > RP?
Solution:
(i) Yes, In ΔOPQ, we have
OP + OQ > PQ
[Sum of any two sides of a triangle is greater than the third side]
(ii) Yes, In ΔOQR, we have OQ + OR > QR
[Sum of any two sides of a triangle is greater than the third side]
(iii) Yes, In ΔOPR, we have OR + OP > RP
[Sum of any two sides of a triangle is greater than the third side]

Question 3
AM is a median of a triangle ABC.
Is AB + BC + CA > 2AM ?
(Consider the sides of triangles ΔABM and ΔAMC)
Solution:
Yes, In AABM, we have

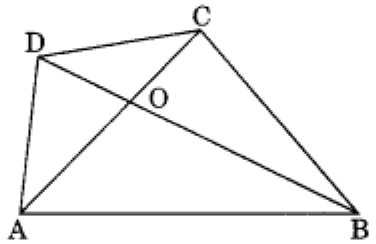


AB + BM > AM ...(i)
[Sum of any two sides of a triangle is greater than the third side]
In ΔAMC, we have
AC + CM > AM ...(ii)
[Sum of any two sides of a triangle is greater than the third side]
Adding eq (i) and (ii), we have
AB + AC + BM + CM > 2AM
AB + AC + BC + > 2AM
AB + BC + CA > 2AM
Hence, proved.

Question 4
ABCD is a quadrilateral.
Is AB + BC + CD + DA > AC + BD?
Solution:
Yes, Given Undefined control sequence \squareABCD in which AC and BD are its diagonals.



In ΔABC, we have
AB + BC > AC ...(i)
[Sum of any two sides is greater than the third side]
In ΔBDC, we have
BC + CD > BD ...(ii)
[Sum of any two sides is greater than the third side]
In ΔADC, we have
CD + DA > AC ...(iii)
[Sum of any two sides is greater than the third side]
In ΔDAB, we have
DA + AB > BD ...(iv)
[Sum of any two sides is greater than the third side]
Adding eq. (i), (ii), (iii) and (iv), we get
2AB + 2BC + 2CD + 2DA > 2AC + 2BD or AB + BC + CD + DA > AC + BD [Dividing both sides by 2]
Hence, proved.

Question 5
ABCD is a quadrilateral.
Is AB + BC + CD + DA < 2(AC + BD)?
Solution:
Yes, we have a quadrilateral ABCD.


In ΔAOB, we have AB < AO + BO …(i)
[Any side of a triangle is less than the sum of other two sides]
In ΔBOC, we have
BC < BO + CO …(ii)
[Any side of a quadrilateral is less than the sum of other two sides]
In ΔCOD, we have
CD < CO + DO …(iii)
[Any side of a triangle is less than the sum of other two sides]
In ΔAOD, we have
DA < DO + AO …(iv)
[Any side of a triangle is less than the sum of other two sides]
Adding eq. (i), (ii), (iii) and (iv), we have
AB + BC + CD + DA
∠2AO + ∠BO + ∠CO + ∠DO
∠2(AO + BO + CO + DO)
∠2 [(AO + CO) + (BO + DO)]
∠2(AC + BD)
Thus, AB + BC + CD + DA < 2(AC + BD)
Hence, proved.

Question 6
The length of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?
Solution:
Sum of two sides
= 12 cm + 15 cm = 27 cm
Difference of the two sides
= 15 cm – 12 cm = 3 cm
∴ The measure of third side should fall between 3 cm and 27 cm.

Question 1:

Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm (ii) 3 cm, 6 cm, 7 cm

(iii) 6 cm, 3 cm, 2 cm

Answer:

In a triangle, the sum of the lengths of either two sides is always greater than the third side.

(i) Given that, the sides of the triangle are 2 cm, 3 cm, 5 cm.

It can be observed that,

2 + 3 = 5 cm

However, 5 cm = 5 cm

Hence, this triangle is not possible.

(ii) Given that, the sides of the triangle are 3 cm, 6 cm, 7 cm.

Here, 3 + 6 = 9 cm > 7 cm

6 + 7 = 13 cm > 3 cm

3 + 7 = 10 cm > 6 cm

Hence, this triangle is possible.

(iii) Given that, the sides of the triangle are 6 cm, 3 cm, 2 cm.

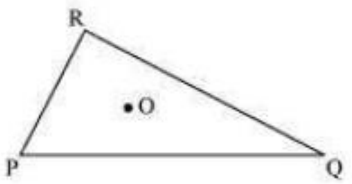
Here, 6 + 3 = 9 cm > 2 cm

However, 3 + 2 = 5 cm < 6 cm

Hence, this triangle is not possible.

Question 2:

Take any point O in the interior of a triangle PQR. Is



(i) OP + OQ > PQ?

(ii) OQ + OR > QR?

(iii) OR + OP > RP?

Answer:

If O is a point in the interior of a given triangle, then three triangles ΔOPQ, ΔOQR, and ΔORP can be constructed. In a triangle, the sum of the lengths of either two sides is always greater than the third side.

(i) Yes, as ΔOPQ is a triangle with sides OP, OQ, and PQ.

OP + OQ > PQ

(ii) Yes, as ΔOQR is a triangle with sides OR, OQ, and QR.

OQ + OR > QR

(iii) Yes, as ΔORP is a triangle with sides OR, OP, and PR.

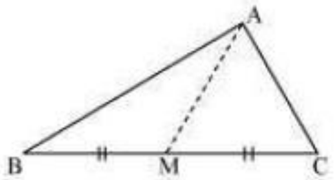
OR + OP > PR

Question 3:

AM is a median of a triangle ABC.

Is AB + BC + CA > 2 AM?

(Consider the sides of triangles ΔABM and ΔAMC.)



Answer:

In a triangle, the sum of the lengths of either two sides is always greater than the third side.

In ΔABM,

AB + BM > AM (*i*)

Similarly, in ΔACM,

AC + CM > AM (*ii*)

Adding equation (*i*) and (*ii*),

AB + BM + MC + AC > AM + AM

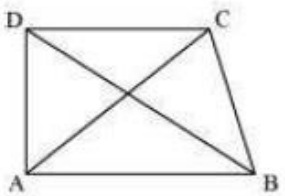
AB + BC + AC > 2AM

Yes, the given expression is true.

Question 4:

ABCD is quadrilateral.

Is AB + BC + CD + DA >AC + BD?



Answer:

In a triangle, the sum of the lengths of either two sides is always greater than the third side.

Considering ΔABC,

AB + BC > CA (*i*)

In ΔBCD,

BC + CD > DB (*ii*)

In ΔCDA,

CD + DA > AC (*iii*)

In ΔDAB,

DA + AB > DB (*iv*)

Adding equations (*i*), (*ii*), (*iii*), and (*iv*), we obtain

AB + BC + BC + CD + CD + DA + DA + AB > AC + BD + AC + BD

2AB + 2BC + 2CD +2DA > 2AC + 2BD

2(AB + BC + CD + DA) > 2(AC + BD)

(AB + BC + CD + DA) > (AC + BD)

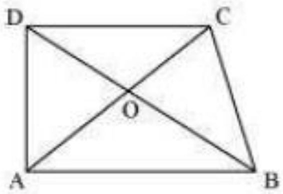
Yes, the given expression is true.

Question 5:

ABCD is quadrilateral.

Is AB + BC + CD + DA < 2 (AC + BD)?

Answer:



In a triangle, the sum of the lengths of either two sides is always greater than the third side.

Considering ΔOAB,

OA + OB > AB (*i*)

In ΔOBC,

OB + OC > BC (*ii*)

In ΔOCD,

OC + OD > CD (*iii*)

In ΔODA,

OD + OA > DA (*iv*)

Adding equations (*i*), (*ii*), (*iii*), and (*iv*), we obtain

$OA + OB + OB + OC + OC + OD + OD + OA > AB + BC + CD + DA$

$2OA + 2OB + 2OC + 2OD > AB + BC + CD + DA$

$2OA + 2OC + 2OB + 2OD > AB + BC + CD + DA$

$2(OA + OC) + 2(OB + OD) > AB + BC + CD + DA$

$2(AC) + 2(BD) > AB + BC + CD + DA$

$2(AC + BD) > AB + BC + CD + DA$

Yes, the given expression is true.

Question 6:

The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

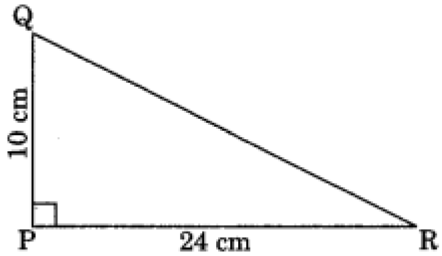
Answer:

In a triangle, the sum of the lengths of either two sides is always greater than the third side and also, the difference of the lengths of either two sides is always lesser than the third side. Here, the third side will be lesser than the sum of these two (i.e., 12 + 15 = 27) and also, it will be greater than the difference of these two (i.e., 15 − 12 = 3). Therefore, those two measures are 27cm and 3 cm.

Ex 6.5 :-

Question 1

PQR is a triangle, right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.



Solution:

In right angled triangle PQR, we have

$QR^2 = PQ^2 + PR^2$ From Pythagoras property)

$= (10)^2 + (24)^2$

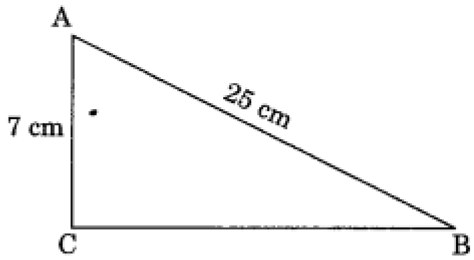
$= 100 + 576 = 676$

$\therefore QR = \sqrt{676} = 26$ cm

The, the required length of QR = 26 cm.

Question 2

ABC is a triangle, right angled at C. If AB = 25 cm and AC = 7 cm, find BC.



Solution:

In right angledΔABC, we have

$BC^2 + (7)^2 = (25)^2$ (By Pythagoras property)

$\Rightarrow BC^2 + 49 = 625$

$\Rightarrow BC^2 = 625 - 49$

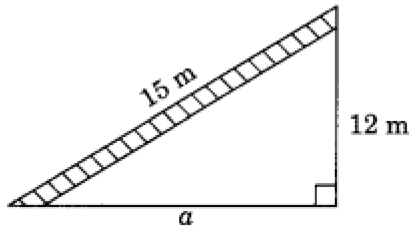
$\Rightarrow BC^2 = 576$

$\therefore BC = \sqrt{576} = 24$ cm

Thus, the required length of BC = 24 cm.

Question 3

A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



Solution:

Here, the ladder forms a right angled triangle.

$\therefore a^2 + (12)^2 = (15)^2$ (By Pythagoras property)

$\Rightarrow a^2 + 144 = 225$

$\Rightarrow a2 = 225 - 144$

$\Rightarrow a^2 = 81$

$\therefore a = \sqrt{81} = 9$ m

Thus, the distance of the foot from the ladder = 9m

Question 4

Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm.

(ii) 2 cm, 2 cm, 5 cm.

(iii) 1.5 cm, 2 cm, 2.5 cm

Solution:

(i) Given sides are 2.5 cm, 6.5 cm, 6 cm.

Square of the longer side = $(6.5)^2 = 42.25$ cm.

Sum of the square of other two sides

$= (2.5)^2 + (6)^2 = 6.25 + 36$

$= 42.25$ cm.

Since, the square of the longer side in a triangle is equal to the sum of the squares of other two sides.

$\therefore$ The given sides form a right triangle.

(ii) Given sides are 2 cm, 2 cm, 5 cm .
Square of the longer side = $(5)^2$ = 25 cm Sum of the square of other two sides
= $(2)^2 + (2)^2$ =4 + 4 = 8 cm
Since 25 cm ≠ 8 cm
∴ The given sides do not form a right triangle.

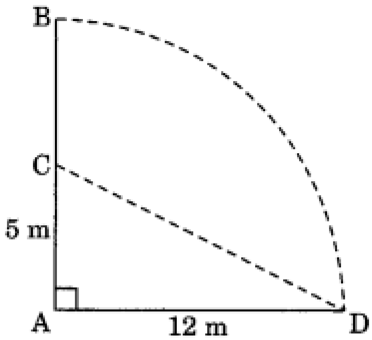
(iii) Given sides are 1.5 cm, 2 cm, 2.5 cm
Square of the longer side = $(2.5)^2$ = 6.25 cm Sum of the square of other two sides
= $(1.5)^2 + (2)^2$ = 2.25 + 4
Since 6.25 cm = 6.25 cm = 6.25 cm
Since the square of longer side in a triangle is equal to the sum of square of other two sides.
∴ The given sides form a right triangle.

Question 5

A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree . Find the original height of the tree.

Solution:

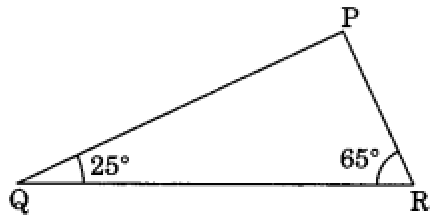
Let AB be the original height of the tree and broken at C touching the ground at D such that



AC = 5 m
and AD = 12 m
In right triangle ΔCAD,
$AD^2 + AC^2 = CD^2$ (By Pythagoras property)
⇒ $(12)^2 + (5)^2 = CD^2$
⇒ $144 + 25 = CD^2$
⇒ $169 = CD^2$
∴ CD = $\sqrt{169}$ = 13 m
But CD = BC
AC + CB = AB
5 m + 13 m = AB
∴ AB = 18 m .
Thus, the original height of the tree = 18 m.

Question 6

Angles Q and R of a APQR are 25° and 65°. Write which of the following is true.
(i) $PQ^2 + QR^2 = RP^2$
(ii) $PQ^2 + RP^2 = QR^2$
(iii) $RP^2 + QR^2 = PQ^2$



Solution:

We know that

$\angle P + \angle Q + \angle R = 180°$ (Angle sum property)

$\angle P + 25° + 65° = 180°$

$\angle P + 90° = 180°$

$\angle P = 180° - 90° - 90°$

ΔPQR is a right triangle, right angled at P

(i) Not True

$\therefore PQ^2 + QR^2 \neq RP^2$ (By Pythagoras property)

(ii) True

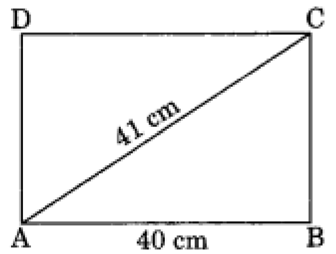
$\therefore PQ^2 + RP^2 = QP^2$ (By Pythagoras property)

(iii) Not True

$\therefore RP^2 + QR^2 \neq PQ^2$ (By Pythagoras property)

Question 7

Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.



Solution:

Given: Length AB = 40 cm

Diagonal AC = 41 cm

In right triangle ABC, we have

$AB^2 + BC^2 = AC^2$ (By Pythagoras property)

$\Rightarrow (40)^2 + BC^2 = (41)^2$

$\Rightarrow 1600 + BC^2 = 1681$

$\Rightarrow BC^2 = 1681 - 1600$

$\Rightarrow BC^2 = 81$

$\therefore BC = \sqrt{81} = 9$ cm

$\therefore AB = DC = 40$ cm and $BC = AD = 9$ cm (Property of rectangle)

$\therefore$ The required perimeter

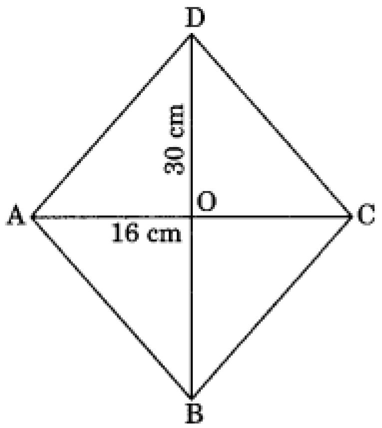
= AB + BC + CD + DA

= (40 + 9 + 40 + 9) cm

= 98 cm

Question 8

The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.



Solution:

Let ABCD be a rhombus whose diagonals intersect each other at O such that AC = 16 cm and BD = 30 cm

Since, the diagonals of a rhombus bisect each other at 90°.

∴ OA = OC = 8 cm and OB = OD = 15 cm

In right ΔOAB,

$AB^2 = OA^2 + OB^2$ (By Pythagoras property)

$= (8)^2 + (15)^2 = 64 + 225$

$= 289$

∴ AB = $\sqrt{289}$ = 17 cm

Since AB = BC = CD = DA (Property of rhombus)

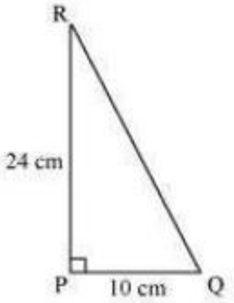
∴ Required perimeter of rhombus

= 4 × side = 4 × 17 = 68 cm.

Question 1:

PQR is a triangle right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Answer:



By applying Pythagoras theorem in ΔPQR,

$(PQ)^2 + (PR)^2 = (RQ)^2$

$(10)^2 + (24)^2 = RQ^2$

$100 + 576 = (QR)^2$

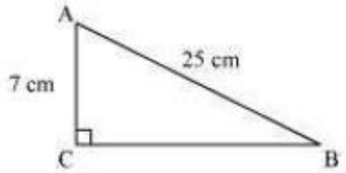
$676 = (QR)^2$

QR = 26 cm

Question 2:

ABC is a triangle right angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Answer:



By applying Pythagoras theorem in ΔABC,

$(AC)^2 + (BC)^2 = (AB)^2$

$(BC)^2 = (AB)^2 - (AC)^2$

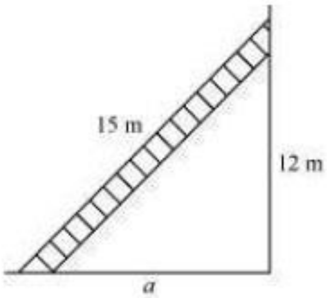
$(BC)^2 = (25)^2 - (7)^2$

$(BC)^2 = 625 - 49 = 576$

BC = 24 cm

Question 3:

A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance *a*. Find the distance of the foot of the ladder from the wall.



Answer:

By applying Pythagoras theorem,

$(15)^2 = (12)^2 + a^2$

$225 = 144 + a^2$

$a^2 = 225 - 144 = 81$

$a = 9$ m

Therefore, the distance of the foot of the ladder from the wall is 9 m.

Question 4:

Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm

(ii) 2 cm, 2 cm, 5 cm

(iii) 1.5 cm, 2 cm, 2.5 cm

In the case of right-angled triangles, identify the right angles.

Answer:

(i) 2.5 cm, 6.5 cm, 6 cm

$(2.5)^2 = 6.25$

$(6.5)^2 = 42.25$

$(6)^2 = 36$

It can be observed that,

$36 + 6.25 = 42.25$

$(6)^2 + (2.5)^2 = (6.5)^2$

The square of the length of one side is the sum of the squares of the lengths of the remaining two sides. Hence, these are the sides of a right-angled triangle. Right angle will be in front of the side of 6.5 cm measure.

(ii) 2 cm, 2 cm, 5 cm

$(2)^2 = 4$

$(2)^2 = 4$

$(5)^2 = 25$

Here, $(2)^2 + (2)^2 \neq (5)^2$

The square of the length of one side is not equal to the sum of the squares of the lengths of the remaining two sides. Hence, these sides are not of a right-angled triangle.

(iii) 1.5 cm, 2 cm, 2.5 cm

$(1.5)^2 = 2.25$

$(2)^2 = 4$

$(2.5)^2 = 6.25$

Here,

$2.25 + 4 = 6.25$

$(1.5)^2 + (2)^2 = (2.5)^2$

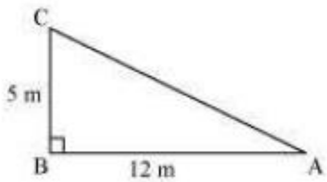
The square of the length of one side is the sum of the squares of the lengths of the remaining two sides. Hence, these are the sides of a right-angled triangle.

Right angle will be in front of the side of 2.5 cm measure.

Question 5:

A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Answer:



In the given figure, BC represents the unbroken part of the tree. Point C represents the point where the tree broke and CA represents the broken part of the tree. Triangle ABC, thus formed, is right-angled at B.

Applying Pythagoras theorem in ΔABC,

$AC^2 = BC^2 + AB^2$

$AC^2 = (5\text{ m})^2 + (12\text{ m})^2$

$AC^2 = 25\text{ m}^2 + 144\text{ m}^2 = 169\text{ m}^2$

$AC = 13\text{ m}$

Thus, original height of the tree = AC + CB = 13 m + 5 m = 18 m

Question 6:

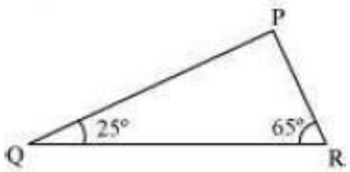
Angles Q and R of a ΔPQR are 25° and 65°.

Write which of the following is true:

(i) $PQ^2 + QR^2 = RP^2$

(ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$



Answer:

The sum of the measures of all interior angles of a triangle is 180°.

$\angle PQR + \angle PRQ + \angle QPR = 180°$

$25° + 65° + \angle QPR = 180°$

$90° + \angle QPR = 180°$

$\angle QPR = 180° - 90° = 90°$

Therefore, Δ PQR is right-angled at point P.

Hence, $(PR)^2 + (PQ)^2 = (QR)^2$

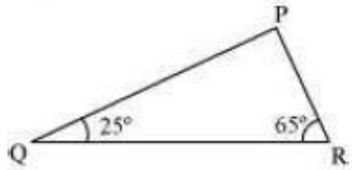
Angles Q and R of a ΔPQR are 25° and 65°.

Write which of the following is true:

(i) $PQ^2 + QR^2 = RP^2$

(ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$



Answer:

The sum of the measures of all interior angles of a triangle is 180°.

$\angle PQR + \angle PRQ + \angle QPR = 180^\circ$

$25^\circ + 65^\circ + \angle QPR = 180^\circ$

$90^\circ + \angle QPR = 180^\circ$

$\angle QPR = 180^\circ - 90^\circ = 90^\circ$

Therefore, Δ PQR is right-angled at point P.

Hence, $(PR)^2 + (PQ)^2 = (QR)^2$

Let ABCD be a rhombus (all sides are of equal length) and its diagonals, AC and BD, are intersecting each other at point O. Diagonals in a rhombus bisect each other at 90°. It can be observed that

$$AO=\frac{AC}{2}=\frac{16}{2}=8 \text{ cm}$$

$$BO=\frac{BD}{2}=\frac{30}{2}=15 \text{ cm}$$

By applying Pythagoras theorem in ΔAOB,

$OA^2 + OB^2 = AB^2$

$8^2 + 15^2 = AB^2$

$64 + 225 = AB^2$

$289 = AB^2$

$AB = 17$

Therefore, the length of the side of rhombus is 17 cm.

Perimeter of rhombus = 4 × Side of the rhombus = 4 × 17 = 68 cm