Chapter 3

Trigonometric Functions

"Trigonometric Functions," provides a comprehensive exploration of fundamental trigonometric concepts and their applications. The chapter begins by introducing the basic trigonometric functions—sine, cosine, tangent, cotangent, secant, and cosecant—within the context of right-angled triangles. Emphasis is placed on understanding these functions as ratios of sides in a triangle and their significance in geometry.

Trigonometric ratios are explored in detail, especially for standard angles (0°, 30°, 45°, 60°, and 90°). Reciprocal trigonometric ratios are introduced, enhancing students' familiarity with these fundamental relationships. The chapter then progresses to trigonometric identities, including basic identities, Pythagorean identities, and sum and difference identities, laying the foundation for more advanced concepts.

The application of trigonometry in solving real-life problems, especially those related to properties of triangles, is elucidated. Sine and cosine rules are introduced for solving triangles, and the practical relevance of trigonometry is demonstrated through the analysis of heights and distances in various scenarios.

Inverse trigonometric functions are introduced, providing tools to solve equations involving these functions. The chapter concludes with an exploration of trigonometric identities, emphasizing the ability to prove these identities using algebraic manipulations and simplify complex expressions.

Exercise 3.1

1. Find the radian measures corresponding to the following degree measures:
(i) 25° (ii) – 47°30′ (iii) 240° (iv) 520°

Solution:

(i) 25°
Here 180° = π radian
It can be written as
$$25° = \frac{\pi}{180} \times 25 \text{ radian}$$
So we get
$$= \frac{5\pi}{36} \text{ radian}$$

(ii) – 47° 30′
Here 1° = 60′
It can be written as
$$-47° \; 30' = -47\frac{1}{2} \text{ degree}$$
So we get
$$= \frac{-95}{2} \text{ degree}$$
Here 180° = π radian
$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian}$$
It can be written as
$$= \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$$
We get
$$-47° \; 30' = \frac{-19}{72}\pi \text{ radian}$$

(iii) 240°
Here 180° = π radian
It can be written as
$$240° = \frac{\pi}{180} \times 240 \text{ radian}$$
So we get
$$= \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

Here $180° = \pi$ radian

It can be written as

$$520° = \frac{\pi}{180} \times 520 \text{ radian}$$

So we get

$$= \frac{26\pi}{9} \text{ radian}$$

2. Find the degree measures corresponding to the following radian measures (Use π = 22/7)

(i) 11/16

(ii) -4

(iii) 5π/3

(iv) 7π/6

Solution:

(i) 11/16

Here π radian = 180°

$$\frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ degree}$$

We can write it as

$$= \frac{45 \times 11}{\pi \times 4} \text{degree}$$

So we get

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ degree}$$

$$= \frac{315}{8} \text{ degree}$$

$$= 39\frac{3}{8} \text{ degree}$$

Take $1° = 60'$

$$= 39° + \frac{3 \times 60}{8} \text{ minutes}$$

We get

$$= 39° + 22' + \frac{1}{2} \text{ minutes}$$

Consider $1' = 60''$

$= 39° \, 22' \, 30''$

(ii) -4

Here π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \ \text{degree}$$

We can write it as

$$= \frac{180 \times 7(-4)}{22} \ \text{degree}$$

By further calculation

$$= \frac{-2520}{11} \ \text{degree} = -229\frac{1}{11} \ \text{degree}$$

Take 1° = 60'

$$= -229° + \frac{1 \times 60}{11} \ \text{minutes}$$

So we get

$$= -229° + 5' + \frac{5}{11} \ \text{minutes}$$

Again 1' = 60''

= - 229° 5' 27''

(iii) 5π/3

Here π radian = 180°

$$\frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \ \text{degree}$$

We get

= 300°

(iv) 7π/6

Here π radian = 180°

$$\frac{7\pi}{6} \text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6}$$

We get

= 210°

3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:

It is given that

No. of revolutions made by the wheel in

1 minute = 360

1 second = 360/60 = 6

We know that

The wheel turns an angle of 2π radian in one complete revolution.

In 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian = 12π radian

Therefore, in one second, the wheel turns an angle of 12π radian.

4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use $\pi = 22/7$).

Solution:

Consider a circle of radius r unit with l unit as the arc length which subtends an angle θ radian at the centre

$\theta = l/r$

Here r = 100 cm, l = 22 cm

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ degree}$$

It can be written as

$$= \frac{180 \times 7 \times 22}{22 \times 100} \text{ degree}$$

$$= \frac{126}{10} \text{ degree}$$

So we get

$$= 12\frac{3}{5} \text{ degree}$$

Here $1^\circ = 60'$

$= 12^\circ\, 36'$

Therefore, the required angle is $12^\circ\, 36'$.

5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

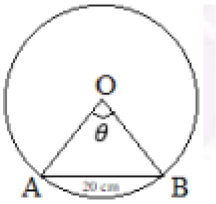
Solution:

The dimensions of the circle are

Diameter = 40 cm

Radius = 40/2 = 20 cm

Consider AB be as the chord of the circle i.e. length = 20 cm



In ΔOAB,

Radius of circle = OA = OB = 20 cm

Similarly AB = 20 cm

Hence, ΔOAB is an equilateral triangle.

$\theta = 60° = \pi/3$ radian

In a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre

We get $\theta = 1/r$

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Therefore, the length of the minor arc of the chord is $20\pi/3$ cm.

6. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Solution:

Consider r_1 and r_2 as the radii of the two circles.

Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1 and an arc of length l subtend an angle of 75° at the centre of the circle of radius r_2.

Here $60^\circ = \pi/3$ radian and $75^\circ = 5\pi/12$ radian

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre

We get

$\theta = l/r$ or $l = r\theta$

We know that

$$l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

By equating both we get

$$\frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

On further calculation

$$r_1 = \frac{r_2 5}{4}$$

So we get

$$\frac{r_1}{r_2} = \frac{5}{4}$$

Therefore, the ratio of the radii is 5: 4.

7. Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution:

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then θ = 1/r

We know that r = 75 cm

(i) l = 10 cm

So we get

θ = 10/75 radian

By further simplification

θ = 2/15 radian

(ii) $l = 15$ cm

So we get

$\theta = 15/75$ radian

By further simplification

$\theta = 1/5$ radian

(iii) $l = 21$ cm

So we get

$\theta = 21/75$ radian

By further simplification

$\theta = 7/25$ radian

Exercise 3.2

Find the values of other five trigonometric functions in Exercises 1 to 5.

1. cos x = -1/2, x lies in third quadrant.

Solution:

It is given that

cos x = - 1/2

sec x = 1/cos x

Substituting the values

$$= \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

Consider

$\sin^2 x + \cos^2 x = 1$

We can write it as

$\sin^2 x = 1 - \cos^2 x$

Substituting the values

$\sin^2 x = 1 - (-1/2)^2$

$\sin^2 x = 1 - 1/4 = 3/4$

$\sin^2 x = \pm \sqrt{3}/2$

Here x lies in the third quadrant so the value of sin x will be negative

$\sin x = - \sqrt{3}/2$

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

2. sin x = 3/5, x lies in second quadrant.

Solution:

It is given that

sin x = 3/5

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

We know that

$\sin^2 x + \cos^2 x = 1$

We can write it as

$\cos^2 x = 1 - \sin^2 x$

Substituting the values

$\cos^2 x = 1 - (3/5)^2$

$\cos^2 x = 1 - 9/25$

$\cos^2 x = 16/25$

$\cos x = \pm 4/5$

Here x lies in the second quadrant so the value of cos x will be negative

$\cos x = - 4/5$

We can write it as

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

Here

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

3. cot x = 3/4, x lies in third quadrant.

Solution:

It is given that

cot x = 3/4

We can write it as

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

We know that

$1 + \tan^2 x = \sec^2 x$

We can write it as

$1 + (4/3)^2 = \sec^2 x$

Substituting the values

$1 + 16/9 = \sec^2 x$

$\cos^2 x = 25/9$

sec x = ± 5/3

Here x lies in the third quadrant so the value of sec x will be negative

sec x = – 5/3

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

By further calculation

$$\sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

4. sec x = 13/5, x lies in fourth quadrant.

Solution:

It is given that

sec x = 13/5

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

We know that

$\sin^2 x + \cos^2 x = 1$

We can write it as

$\sin^2 x = 1 – \cos^2 x$

Substituting the values

$\sin^2 x = 1 – (5/13)^2$

$\sin^2 x = 1 – 25/169 = 144/169$

$\sin^2 x = \pm 12/13$

Here x lies in the fourth quadrant so the value of sin x will be negative

sin x = – 12/13

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

5. tan x = -5/12, x lies in second quadrant.

Solution:

It is given that

tan x = – 5/12

We can write it as

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

We know that

$1 + \tan^2 x = \sec^2 x$

We can write it as

$1 + (-5/12)^2 = \sec^2 x$

Substituting the values

$1 + 25/144 = \sec^2 x$

$\sec^2 x = 169/144$

$\sec x = \pm 13/12$

Here x lies in the second quadrant so the value of sec x will be negative

$\sec x = -13/12$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$-\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

By further calculation

$$\sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Find the values of the trigonometric functions in Exercises 6 to 10.

6. sin 765°

Solution:

We know that values of sin x repeat after an interval of 2π or 360°

So we get

$$\sin 765° = \sin(2 \times 360° + 45°)$$

By further calculation

$= \sin 45°$

$= 1/\sqrt{2}$

7. cosec (–1410°)

Solution:

We know that values of cosec x repeat after an interval of 2π or 360°

So we get

$$\operatorname{cosec}\left(-1410^\circ\right) = \operatorname{cosec}\left(-1410^\circ + 4 \times 360^\circ\right)$$

By further calculation

$$= \operatorname{cosec}\left(-1410^\circ + 1440^\circ\right)$$

= cosec 30° = 2

8. $\tan \frac{19\pi}{3}$

Solution:

We know that values of tan x repeat after an interval of π or 180°

So we get

$$\tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi$$

By further calculation

$$= \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3}$$

We get

= tan 60°

= √3

9. $\sin\left(-\frac{11\pi}{3}\right)$

Solution:

We know that values of sin x repeat after an interval of 2π or 360°

So we get

$$\sin\left(-\frac{11\pi}{3}\right)=\sin\left(-\frac{11\pi}{3}+2\times 2\pi\right)$$

By further calculation

$$=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

10. $\cot\left(-\frac{15\pi}{4}\right)$

Solution:

We know that values of tan x repeat after an interval of π or 180°

So we get

$$\cot\left(-\frac{15\pi}{4}\right)=\cot\left(-\frac{15\pi}{4}+4\pi\right)$$

By further calculation

$$=\cot\frac{\pi}{4}=1$$

Exercise 3.3

Prove that:

1.

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Solution:

Consider

$$\text{L.H.S.} = \sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$$

So we get

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

By further calculation

= 1/4 + 1/4 – 1

= - 1/2

= RHS

2.

$$2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$$

Solution:

Consider

$$\text{L.H.S.} = 2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

By further calculation

$$=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

It can be written as

$$=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

So we get

$$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$$

Here

= 1/2 + 4/4

= 1/2 + 1

= 3/2

= RHS

3.

$$\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$$

Solution:

Consider

$$\text{L.H.S.}=\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$$

So we get

$$=\left(\sqrt{3}\right)^2+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$$

By further calculation

$$=3+\operatorname{cosec}\frac{\pi}{6}+3\times\frac{1}{3}$$

We get

$=3+2+1$

$=6$

$=\text{RHS}$

4.

$$2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$$

Solution:

Consider

$$\text{L.H.S}=2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$$

So we get

$$=2\left\{\sin\left(\pi-\frac{\pi}{4}\right)\right\}^2+2\left(\frac{1}{\sqrt{2}}\right)^2+2(2)^2$$

By further calculation

$$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2\times\frac{1}{2} + 8$$

It can be written as

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$= 1 + 1 + 8$

$= 10$

= RHS

5. Find the value of:

(i) sin 75°

(ii) tan 15°

Solution:

(i) sin 75°

It can be written as

$= \sin(45° + 30°)$

Using the formula $[\sin(x + y) = \sin x \cos y + \cos x \sin y]$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

Substituting the values

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

By further calculation

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15°

It can be written as

$= \tan(45° – 30°)$

Using formula

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

$$=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}$$

Substituting the values

$$=\frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)}=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

By further calculation

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

So we get

$$=\frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{4-2\sqrt{3}}{3-1}=2-\sqrt{3}$$

Prove the following:

6.

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$$

Solution:

Consider LHS =

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

We can write it as

$$=\frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

By further simplification

$$=\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

Using the formula

2 cos A cos B = cos (A + B) + cos (A – B)

-2 sin A sin B = cos (A + B) – cos (A – B)

$$= 2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$$

We get

$$= \cos\left[\frac{\pi}{2}-(x+y)\right]$$

= sin (x + y)

= RHS

7.

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

By using the formula

$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

So we get

$$=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}$$

It can be written as

$$= \frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

= RHS

8.

$$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

It can be written as

$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

So we get

$$= \frac{-\cos^2 x}{-\sin^2 x}$$

= $\cot^2 x$

= RHS

9.

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Solution:

Consider

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

It can be written as

= sin x cos x (tan x + cot x)

So we get

$$= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right]$$

$= 1$

$=$ RHS

10. sin (*n* + 1)*x* sin (*n* + 2)*x* + cos (*n* + 1)*x* cos (*n* + 2)*x* = cos *x*

Solution:

LHS = sin $(n + 1)x$ sin $(n + 2)x$ + cos $(n + 1)x$ cos $(n + 2)x$

By multiplying and dividing by 2

$$= \frac{1}{2}\left[2\sin(n+1)x\sin(n+2)x + 2\cos(n+1)x\cos(n+2)x\right]$$

Using the formula

- 2 sin A sin B = cos (A + B) – cos (A – B)

2 cos A cos B = cos (A + B) + cos (A – B)

$$= \frac{1}{2}\left[\begin{array}{l}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\} \\ +\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{array}\right]$$

By further calculation

$$= \frac{1}{2} \times 2\cos\{(n+1)x-(n+2)x\}$$

= cos (- x)

= cos x

= RHS

11.

$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$$

Solution:

Consider

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$$

Using the formula

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

So we get

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

It can be written as

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

By further calculation

$$=-2\sin\frac{\pi}{4}\sin x$$

Substituting the values

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$= -\sqrt{2}\sin x$

= RHS

12. $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

Consider

L.H.S. = $\sin^2 6x - \sin^2 4x$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

So we get

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

By further calculation

$$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

We get

$= (2\sin 5x \cos x)(2\cos 5x \sin x)$

It can be written as

$= (2\sin 5x \cos 5x)(2\sin x \cos x)$

$= \sin 10x \sin 2x$

= RHS

13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

Consider

L.H.S. = $\cos^2 2x - \cos^2 6x$

Using the formula

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

So we get

= $(\cos 2x + \cos 6x)(\cos 2x - 6x)$

By further calculation

$$= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

We get

= $[2 \cos 4x \cos (-2x)] [-2 \sin 4x \sin (-2x)]$

It can be written as

= $[2 \cos 4x \cos 2x] [-2 \sin 4x (-\sin 2x)]$

So we get

= $(2 \sin 4x \cos 4x) (2 \sin 2x \cos 2x)$

= $\sin 8x \sin 4x$

= RHS

14. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Solution:

Consider

L.H.S. = $\sin 2x + 2 \sin 4x + \sin 6x$

$= [\sin 2x + \sin 6x] + 2 \sin 4x$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$$

By further simplification

$= 2 \sin 4x \cos (- 2x) + 2 \sin 4x$

It can be written as

$= 2 \sin 4x \cos 2x + 2 \sin 4x$

Taking common terms

$= 2 \sin 4x (\cos 2x + 1)$

Using the formula

$= 2 \sin 4x (2 \cos^2 x - 1 + 1)$

We get

$= 2 \sin 4x (2 \cos^2 x)$

$= 4\cos^2 x \sin 4x$

= R.H.S.

15. cot 4*x* (sin 5*x* + sin 3*x*) = cot *x* (sin 5*x* – sin 3*x*)

Solution:

Consider

LHS = cot 4x (sin 5x + sin 3x)

It can be written as

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

Using the formula

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x\cos x\right]$$

So we get

= 2 cos 4x cos x

Similarly

R.H.S. = cot x (sin $5x$ – sin $3x$)

It can be written as

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

Using the formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$= \frac{\cos x}{\sin x}\left[2\cos 4x\sin x\right]$$

So we get

= 2 cos 4x cos x

Hence, LHS = RHS.

16.

$$\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\frac{\sin 2x}{\cos 10x}$$

Solution:

Consider

$$\text{L.H.S}=\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$$

Using the formula

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$=\frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

By further calculation

$$=\frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

So we get

$$=-\frac{\sin 2x}{\cos 10x}$$

= RHS

17.

$$\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$$

Solution:

Consider

$$\text{L.H.S.}=\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

Using the formula

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

By further calculation

$$=\frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

So we get

$$=\frac{\sin 4x}{\cos 4x}$$

= tan 4x

= RHS

18.

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\frac{x-y}{2}$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

Using the formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

By further calculation

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

So we get

$$= \tan\left(\frac{x-y}{2}\right)$$

= RHS

19.

$$\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$$

Solution:

Consider

$$\text{.L.H.S.} = \frac{\sin x+\sin 3x}{\cos x+\cos 3x}$$

Using the formula

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

By further calculation

$$=\frac{\sin 2x}{\cos 2x}$$

So we get

= tan 2x

= RHS

20.

$$\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$$

Using the formula

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\cos^2 A - \sin^2 A = \cos 2A$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

By further calculation

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

So we get

= - 2 (-sin x)

= 2 sin x

= RHS

21.

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

It can be written as

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

Using the formula

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right).$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

By further calculation

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

So we get

$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

= cot 3x

= RHS

22. cot *x* cot 2*x* – cot 2*x* cot 3*x* – cot 3*x* cot *x* = 1

Solution:

Consider

LHS = cot x cot $2x$ – cot $2x$ cot $3x$ – cot $3x$ cot x

It can be written as

= cot x cot $2x$ – cot $3x$ (cot $2x$ + cot x)

= cot x cot $2x$ – cot $(2x + x)$ (cot $2x$ + cot x)

Using the formula

$$\cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$

So we get

= cot x cot 2x – (cot 2x cot x – 1)

= 1

= RHS

23.

$$\tan 4x = \frac{4\tan x(1 - \tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$$

Solution:

Consider

LHS = tan 4x = tan 2(2x)

By using the formula

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

It can be written as

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

By further simplification

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

Taking LCM

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

On further simplification

$$= \frac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$$

We get

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

It can be written as

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

= RHS

24. $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

Solution:

Consider

LHS = cos 4x

We can write it as

$= \cos 2(2x)$

Using the formula $\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\sin^2 2x$

Again by using the formula $\sin 2A = 2\sin A \cos A$

$= 1 - 2(2\sin x \cos x)^2$

So we get

$= 1 - 8\sin^2 x \cos^2 x$

= R.H.S.

25. $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Solution:

Consider

L.H.S. = $\cos 6x$

It can be written as

$= \cos 3(2x)$

Using the formula $\cos 3A = 4\cos^3 A - 3\cos A$

$= 4\cos^3 2x - 3\cos 2x$

Again by using formula $\cos 2x = 2\cos^2 x - 1$

$= 4[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$

By further simplification

$= 4\ [(2\cos^2 x)^3 – (1)^3 – 3\ (2\cos^2 x)^2 + 3\ (2\cos^2 x)] – 6\cos^2 x + 3$

We get

$= 4\ [8\cos^6 x – 1 – 12\cos^4 x + 6\cos^2 x] – 6\cos^2 x + 3$

By multiplication

$= 32\cos^6 x – 4 – 48\cos^4 x + 24\cos^2 x – 6\cos^2 x + 3$

On further calculation

$= 32\cos^6 x – 48\cos^4 x + 18\cos^2 x – 1$

= R.H.S.

2 Marks Questions & Answers

1. Find the radian measure corresponding to $5∘\ 37'\ 30''$

Ans:

Converting the given value to a pure degree form

$$5∘37'30''=5°37'\left(\frac{30}{60}\right)'$$

$$\Rightarrow 5∘37'60''=5°\left(\frac{75}{2}\right)'$$

$$\Rightarrow 5∘37'60''=5°\ \left(\frac{75}{2(60)}\right)°$$

$$\Rightarrow 5∘37'60''=\ \left(\frac{45}{8}\right)°$$

Degree to Radian Conversion

$$\left(\frac{45}{8}\right)\left(\frac{\pi}{180}\right)=\frac{\pi}{32}\text{rad}$$

2. Find the value of $\frac{\mathbf{19\pi}}{\mathbf{3}}$

Ans:

We have $\tan\frac{\mathbf{19\pi}}{\mathbf{3}}$

$\tan\frac{19\pi}{3}=\tan(6\frac{\pi}{3})$

$=\tan(6\pi+\frac{\pi}{3})$

$=\tan(3\times2\pi+\frac{\pi}{3})$

$=\tan(\frac{\pi}{3})$

=3–√=3

3. If tan A= $\frac{a}{a+1}$ **and tan** B= $\frac{1}{2a+1}$**then find the value of A**+B

Ans:

tan (A+B)= $\frac{\tan A+\tan B}{1-\tan A\tan B}$

$$=\frac{\frac{a}{a+1}+\frac{1}{2a+1}}{1-\frac{a}{a+1}\cdot\frac{1}{2a+1}}$$

$$=\frac{\frac{2a2+2a+1}{(a+1)(2a+1)}}{\frac{(a+1)(2a+1)-a}{(a+1)(2a+1)}}$$

= 1

Which can only be possible if A+B=45°

4. If $\cos x=-\frac{1}{3}$ **and** x **lies in quadrant** III **, find the value of** $\sin\frac{x}{2}$

Ans:

We know that $\cos 2x=1-2\sin^2 x$

$\text{Cos }(2(\frac{x}{2})=1-2\sin^2\frac{x}{2}$

$\Rightarrow-\frac{1}{3}=1-2\sin^2\frac{x}{2}$

$\Rightarrow 2\sin^2\frac{x}{2} = 1+\frac{1}{3}$

$\Rightarrow \sin^2\frac{x}{2} = \frac{2}{3}$

$\Rightarrow \sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$

$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{2}{3}}$ [2nd Quadrant]

5. **Find the solution of** Sin $x = -\frac{\sqrt{3}}{2}$

Ans:

We are required to find the general solution for the equation $\sin x = -\frac{\sqrt{3}}{2}$

$\text{Sin } x = -\frac{\sqrt{3}}{2}$

$\Rightarrow \text{Sin } x = \text{Sin }(\pi+\frac{\pi}{3})$

$\Rightarrow \text{Sin } x = \text{Sin}\frac{4\pi}{3}$

When

$\text{Sin } \theta = \text{Sin } \alpha$

$\theta = n\pi + (-1)^n \cdot \alpha$

$x = n\pi + (-1)^n \cdot \frac{4\pi}{3}$

6. Prove that $\tan 56° = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$

Ans:

Starting with the left-hand side and using the trigonometric addition formula for the tangent function, we obtain

L.H.S.=tan56°

=tan(45∘+11∘)

$$= \frac{\tan 45° + \tan 11°}{1 - \tan 45°.\tan 11°}$$

$$= \frac{1 + \tan 11°}{1 - \tan 11°}$$

$$= \frac{\cos 11° + \sin 11°}{\cos 11°.\sin 11°}$$

=R.H.S.

7. **Prove that $\ dfrac {\operator name {Sin} x} {1+\operator name {Cos} x}=\tan \ dfrac {x}{2} $**

Ans:

Starting with the left-hand side and using the trigonometric addition identities for the sine and cosine function, we obtain

L.H.S.= $\frac{sinx}{1+cosx}$

$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos2\frac{x}{2}}$$

$$= \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$

=tan $\frac{x}{2}$

=R.H.S.

8. **Prove that** $\frac{\cos 9° - \sin 9°}{\cos 9° + \sin 9°}$ =tan36°

Ans:

Let us start with the right-hand side and use the trigonometric differences formula for the tangent function.

R.H.S= tan36°

= tan(45° − 9°)

$= \frac{\tan 45° - \tan 9°}{1 + \tan 45° \tan 9°}$

$= \frac{1 - \tan 9°}{1 + \tan 9°}$

$= \frac{\cos 9° - \sin 9°}{\cos 9° + \sin 9°}$

=L.H.S.

9. **Prove** $\text{Cos}4x = 1 - 8\text{Sin}^2 x.\text{Cos}^2 x$

Ans:

Starting with the left-hand side and using the trigonometric addition formula, $\cos 2x = 1 - 2\sin^2 x$

We get,

L.H.S=Cos4xL.H.S

$= 1 - 2\text{Sin}^2 2x$

$= 1 - 2(\sin 2x)^2$

$= 1 - 2(2\sin x.\cos x)^2$

$= 1 - 2(4\text{Sin}^2 x.\text{Cos}^2 x)$

$= 1 - 8\text{Sin}^2 x.\text{Cos}^2 x$

10. Show that tan 3x. tan 2x. tan x = tan 3x tan 2x tan x.

Ans:

Let us start with tan3x and we know 3x=2x+x

tan3x=tan (2x+x)

$\frac{\tan 3x}{1} = \frac{\tan 2x + \tan x}{-\tan 2x.\tan x}$

tan3x (1−tan2x.tanx) = tan2x+tanx

tan3x−tan3x.tan2x.tanx = tan2x+tanx

tan3x.tan2x.tanx = tan3x−tan2x−tanx

Multiple Choice Questions

1. If sin θ and cos θ are the roots of $ax^2 – bx + c = 0$, then the relation between a, b and c will be

(a) $a^2 + b^2 + 2ac = 0$

(b) $a^2 – b^2 + 2ac = 0$

(c) $a^2 + c^2 + 2ab = 0$

(d) $a^2 – b^2 – 2ac = 0$

Correct option: (b) $a^2 – b^2 + 2ac = 0$

Solution:

Given that sin θ and cos θ are the roots of the equation $ax^2 – bx + c = 0$, so sin θ + cos θ = b/a

and sin θ cos θ = c/a

Consider,

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2 \sin\theta \cos\theta$,

$(b/a)^2 = 1 + 2(c/a)$ {using the identity $\sin^2A + \cos^2A = 1$}

$b^2/a^2 = 1 + (2c/a)$

$b^2 = a^2 + 2ac$

$a^2 – b^2 + 2ac = 0$

2. If tan A = 1/2 and tan B = 1/3, then the value of A + B is

(a) π/6

(b) π

(c) 0

(d) $\pi/4$

Correct option: (d) $\pi/4$

Solution:

Given,

$\tan A = 1/2$, $\tan B = 1/3$

We know that,

$\tan(A + B) = (\tan A + \tan B)/(1 - \tan A \tan B)$

$= [(1/2) + (1/3)]/ [1 - (1/2)(1/3)]$

$= [(3 + 2)/6]/ [(6 - 1)/6]$

$= 5/5$

$= 1$

$= \tan \pi/4$

Therefore, $A + B = \pi/4$

3. The value of cos 1° cos 2° cos 3° … cos 179° is

(a) $1/\sqrt{2}$

(b) 0

(c) 1

(d) -1

Correct option: (b) 0

Solution:

$\cos 1° \cos 2° \cos 3° \ldots \cos 179°$

$\cos 1° \cos 2° \cos 3° \ldots \cos 179°$

$= \cos 1° \cos 2° \cos 3° \ldots \cos 89° \cos 90° \cos 91° \ldots \cos 179°$

= cos 1° cos 2° cos 3° … cos 89° (0) cos 91° … cos 179°

= 0 {since the value of cos 90° = 0}

4. The value of sin 50° – sin 70° + sin 10° is equal to

(a) 1

(b) 0

(c) 1/2

(d) 2

Correct option: (b) 0

Solution:

sin 50° – sin 70° + sin 10°

= sin(60° – 10°) – sin(60° + 10°) + sin 10°

Using the formulas

sin(A – B) = sin A cos B – cos A sin B

sin(A + B) = sin A cos B + cos A sin B, we get;

sin 50° – sin 70° + sin 10° = sin 60° cos 10° – cos 60° sin 10° – sin 60° + cos 10° – cos 60° sin 10° + sin 10°

= -2 cos 60° sin 10° + sin 10°

= -2 × (1/2) × sin 10° + sin 10°

= – sin 10° + sin 10°

= 0

5. The value of sin (45° + θ) – cos (45° – θ) is

(a) 2 cosθ

(b) 2 sinθ

(c) 1

(d) 0

Correct option: (d) 0

Solution:

$\sin(45° + \theta) - \cos(45° - \theta)$

$= \sin(45° + \theta) - \sin(90° - (45° - \theta))$ {since $\sin(90° - A) = \cos A$}

$= \sin(45° + \theta) - \sin(45° + \theta)$

$= 0$

Alternative method:

$\sin(45° + \theta) - \cos(45° - \theta)$

Using the formulas

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$, we get;

$\sin(45° + \theta) - \cos(45° - \theta) = \sin 45° \cos\theta + \cos 45° \sin\theta - \cos 45° \cos\theta - \sin 45° \sin\theta$

$= (1/\sqrt{2}) \cos\theta + (1/\sqrt{2}) \sin\theta - (1/\sqrt{2}) \cos\theta - (1/\sqrt{2}) \sin\theta$

$= 0$

6. The value of $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$ is

(a) 0

(b) 1

(c) 1/2

(d) Not defined

Correct option: (b) 1

Solution:

$\tan 1° \tan 2° \tan 3° \ldots \tan 89°$

= [tan 1° tan 2° … tan 44°] tan 45°[tan (90° – 44°) tan (90° – 43°)… tan (90° – 1°)]

= [tan 1° tan 2° … tan 44°] [cot 44° cot 43°……. cot 1°] × [tan 45°]

= [(tan 1° × cot 1°) (tan 2° × cot 2°)…..(tan 44° × cot 44°)] × [tan 45°]

We know that,

tan A × cot A =1 and tan 45° = 1

Hence, the equation becomes as;

= 1 × 1 × 1 × 1 × …× 1

= 1 {As $1^n = 1$}

7. If α + β = π/4, then the value of (1 + tan α) (1 + tan β) is

(a) 1

(b) 2

(c) – 2

(d) Not defined

Correct option: (b) 2

Solution:

Given,

α + β = π/4

Taking "tan" on both sides,

tan(α + β) = tan π/4

We know that,

tan(A + B) = (tan A + tan B)/(1 – tan A tan B)

and tan π/4 = 1.

So, (tan α + tan β)/(1 – tan α tan β) = 1

$\tan \alpha + \tan \beta = 1 - \tan \alpha \tan \beta$

$\tan \alpha + \tan \beta + \tan \alpha \tan \beta = 1$….(i)

$(1 + \tan \alpha)(1 + \tan \beta) = 1 + \tan \alpha + \tan \beta + \tan \alpha \tan \beta$

$= 1 + 1$ [From (i)]

$= 2$

8. If a lies in the second quadrant and 3 tan a + 4 = 0, then the value of (2 cot A – 5 cos A + sin A) is equal to

(a) -53/10

(b) 23/10

(c) 37/10

(d) 7/10

Correct option: (b) 23/10

Solution:

Given that A lies in the second quadrant and 3 tan A + 4 = 0.

$3 \tan A = -4$

$\tan A = -4/3$

$\cot A = 1/\tan A = -3/4$

Using the identity $\sec^2 A = 1 + \tan^2 A$,

$\sec^2 A = 1 + (16/9) = 25/9$

$\sec A = \sqrt{(25/9)}$

$\sec A = -5/3$ (in quadrant II secant is negative)

$\cos A = 1/\sec A = -3/5$

Using the identity $\sin^2 A + \cos^2 A = 1$,

$\sin A = \sqrt{(1 - 9/25)} = \sqrt{(16/25)} = 4/5$ (in quadrant II sine is positive)

Now,

2 cot A – 5 cos A + sin A

= 2(-3/4) – 5(-3/5) + (4/5)

= (-3/2) + 3 + (4/5)

= (-15 + 30 + 8)/10

= 23/10

9. If for real values of x, cos θ = x + (1/x), then

(a) θ is an acute angle

(b) θ is right angle

(c) θ is an obtuse angle

(d) No value of θ is possible

Correct option: (d) No value of θ is possible

Solution:

Given,

cos θ = x + (1/x)

cos θ = $(x^2 + 1)/x$

⇒ $x^2 + 1 = x \cos \theta$

⇒ $x^2 - x \cos \theta + 1 = 0$

We know that for any real root of the equation $ax^2 + bx + c = 0$, $b^2 - 4ac \geq 0$.

⇒ $(-\cos \theta)^2 - 4 \geq 0$

⇒ $\cos^2\theta - 4 \geq 0$

⇒ $\cos^2\theta \geq 4$

⇒ $\cos \theta \geq \pm 2$

We know that $-1 \leq \cos\theta \leq 1$.

Hence, no value of θ is possible.

10. Number of solutions of the equation tan x + sec x = 2 cos x lying in the interval [0, 2π] is

(a) 0

(b) 1

(c) 2

(d) 3

Correct option: (c) 2

Solution:

Given,

$\tan x + \sec x = 2 \cos x$

$(\sin x/\cos x) + (1/\cos x) = 2 \cos x$

$(\sin x + 1)/\cos x = 2 \cos x$

$\sin x + 1 = 2 \cos^2 x$

Using the identity $\sin^2 A + \cos^2 A = 1$,

$\sin x + 1 = 2(1 - \sin^2 x)$

$\sin x + 1 = 2 - 2 \sin^2 x$

$\Rightarrow 2 \sin^2 x + \sin x - 1 = 0$

$\Rightarrow (2 \sin x - 1)(\sin x + 1)=0$

$\Rightarrow \sin x = -1, 1/2$

$\Rightarrow x = \pi/6, 5\pi/6, 3\pi/2 \in [0, 2\pi]$

But for $x = 3\pi/2$, tan x and sec x are not defined.

Therefore, there are only two solutions for the given equation in the interval [0, 2π].

Summary

If in a circle of radius r, an arc of length l subtends an angle of θ radians, then $l = r\,\theta$

- Radian measure = $\frac{\pi}{180}$ × Degree measure
- Degree measure = $\frac{180}{\pi}$× Radian measure
- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \text{cosec}^2 x$
- $\cos(2n\pi + x) = \cos x$
- $\sin(2n\pi + x) = \sin x$
- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
- $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$ $\qquad$ $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$\cos(\pi - x) = -\cos x$ $\qquad$ $\sin(\pi - x) = \sin x$

$\cos(\pi + x) = -\cos x$ $\qquad$ $\sin(\pi + x) = -\sin x$

$\cos(2\pi - x) = \cos x$ $\qquad$ $\sin(2\pi - x) = -\sin x$

- If none of the angles x, y and (x ± y) is an odd multiple of $\frac{\pi}{2}$, then

$$\boldsymbol{Tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}}$$

$$\boldsymbol{Tan(x\text{-}y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}}$$

- If none of the angles x, y and (x ± y) is a multiple of π, then

$$\boldsymbol{\cos(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}}$$

$$\boldsymbol{\cos(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}}$$

- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan 2x}{1 + \tan 2x}$

- $\sin 2x = 2\sin x \cos x = \frac{2 \tan x}{1 + \tan 2x}$

- $\text{Tan } 2x = \frac{2 \tan x}{1 - \tan 2x}$

- $\sin 3x = 3\sin x - 4\sin 3\, x$

- $\cos 3x = 4\cos 3\, x - 3\cos x$

- $\tan 3x = \frac{3 \tan x - \tan 3x}{1 - 3\tan 2x}$

- (i) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

  (ii) $\sin x + \sin y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

  (iii) $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

  (iv) $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

- (i) $2\cos x \cos y = \cos(x + y) + \cos(x - y)$

  (ii) $-2\sin x \sin y = \cos(x + y) - \cos(x - y)$

  (iii) $2\sin x \cos y = \sin(x + y) + \sin(x - y)$

  (iv) $2\cos x \sin y = \sin(x + y) - \sin(x - y)$.