CHAPTER -7

Functions

2MARK QUESTIONS

1Q: Define a function in mathematics.

Answer:

A function is a relation between two sets, where each element in the domain is related to exactly one element in the codomain, and it assigns a unique output for each input.

2Q: What is the domain and codomain of a function?

Answer:

The domain is the set of all possible input values for a function, while the codomain is the set of all possible output values.

3Q: Explain what is meant by the range of a function.

Answer:

The range of a function is the set of all possible output values it can produce for the given inputs in the domain.

4Q: Differentiate between a one-to-one function and an onto function.

Answer:

A one-to-one (injective) function assigns distinct outputs to distinct inputs, while an onto (surjective) function covers the entire codomain with its range.

5Q: Define the composition of functions.

Answer:

The composition of two functions, f and g, denoted as $(f \circ g)(x)$, is a new function obtained by applying one function after the other.

6Q: What is the inverse of a function?

Answer:

The inverse of a function reverses the roles of inputs and outputs, switching the domain and codomain, denoted as f^(-1)(x).

7Q: When is a function considered even?

Answer:

A function is even if f(x) = f(-x) for all x in its domain.

8Q: State the Horizontal Line Test.

Answer:

The Horizontal Line Test states that if any horizontal line intersects a graph at most once, the function is one-to-one.

9Q: What is the significance of the Vertical Line Test?

Answer:

The Vertical Line Test is used to determine if a graph represents a function. If any vertical line intersects the graph at most once, the relation is a function.

10 Q: Explain the concept of a constant function.

Answer:

A constant function is a type of function where the output is the same constant value for every input in its domain.

4MARK QUESTIONS

1Q: Explain the difference between a function and a relation.

Answer:

A function is a special type of relation where each element in the domain is related to exactly one element in the codomain. In contrast, a relation is a broader concept that simply describes how elements in different sets are related, without the restriction of unique mappings.

2Q: Define the terms "injective," "surjective," and "bijective" with respect to functions.

Answer:

An injective function (one-to-one) assigns distinct outputs to distinct inputs. A surjective function (onto) covers the entire codomain with its range. A bijective function is both injective and surjective, ensuring a one-to-one correspondence between the domain and codomain.

3Q: Illustrate the composition of functions with an example.

Answer:

Let $f(x) = 2x$ and $g(x) = x + 3$. Find $(f \circ g)(x)$. The solution involves substituting $g(x)$ into $f(x)$, yielding $(f \circ g)(x) = 2(x + 3)$.

4Q: Discuss the conditions for the existence of the inverse of a function.

Answer:

For a function to have an inverse, it must be one-to-one (injective) to ensure unique reversibility. In other words, each distinct input in the domain should map to a distinct output in the codomain.

5Q: State and prove the Vertical Line Test for functions.

Answer:

The Vertical Line Test asserts that if any vertical line intersects a graph at more than one point, the graph does not represent a function. The proof involves showing that multiple intersections violate the unique mapping property of a function.

6Q: Explain the concept of a periodic function with an example.

Answer:

A periodic function repeats its values at regular intervals. An example is the sine function (f(x) = sin(x)), which repeats its values every 2π radians.

7Q: Discuss the role of odd and even functions in symmetry.

Answer:

Odd functions satisfy f(-x) = -f(x) for all x in their domain, exhibiting rotational symmetry about the origin. Even functions satisfy f(x) = f(-x) for all x, displaying reflective symmetry about the y-axis.

8Q: Compare and contrast the concepts of the range and the codomain.

Answer:

The range is the set of actual output values a function produces, while the codomain is the set of all possible output values. The range is a subset of the codomain.

9Q: Prove that the composition of two injective functions is injective.

Answer:

Assume f and g are injective functions. To prove $(f \circ g)(x)$ is injective, suppose $(f \circ g)(a) = (f \circ g)(b)$. Then, by the injectivity of f and g, a = b, establishing the injectivity of the composition.

10Q: Discuss the significance of a constant function in real-world applications.

Answer:

A constant function represents a situation where the output remains the same regardless of the input. In real-world applications, it can model scenarios like fixed costs or unchanging quantities over time.

7MARK QUESTIONS

1Q: Elaborate on the concept of a function and its essential components. Provide an example to illustrate.

Answer:

A function is a mathematical relationship between two sets, the domain and codomain, where each element in the domain maps to exactly one element in the codomain. The essential components are the domain, codomain, and the rule that assigns each element in the domain a unique element in the codomain. For example, consider the function $f(x) = 2x$, where x is in the set of real numbers. Here, every real number in the domain has a unique image in the codomain, satisfying the function's definition.

2Q: Define and explain the significance of one-to-one (injective) and onto (surjective) functions. Provide examples for each.

Answer:

A one-to-one (injective) function ensures that distinct elements in the domain map to distinct elements in the codomain. An onto (surjective) function covers the entire codomain with its range, ensuring that every element in the codomain has a pre-image in the domain. For example, the function $f(x) = 2x$ is injective as different inputs result in different outputs, and it is surjective as it covers all real numbers in its range.

3Q: Discuss the composition of functions and its properties. Provide an example to illustrate the composition of two functions.

Answer:

The composition of functions, denoted as $(f \circ g)(x)$, combines two functions by applying one after the other. Properties include associativity and the existence of an identity element. For example, let $f(x) = 2x$ and $g(x) = x + 3$. The composition $(f \circ g)(x)$ is obtained by substituting $g(x)$ into $f(x)$, resulting in $(f \circ g)(x) = 2(x + 3)$.

4Q: Explain the conditions for a function to have an inverse. Derive the inverse function for a specific example.

Answer:

A function must be one-to-one (injective) for it to have an inverse. If $f(x) = 2x$, its inverse, denoted as $f^{-1}(x)$, can be found by swapping x and y and solving for y. In this case, $f^{-1}(x) = x/2$.

5Q: Discuss the graphical significance of odd and even functions. Provide examples and their corresponding graphs.

Answer:

Odd functions exhibit rotational symmetry about the origin, satisfying $f(-x) = -f(x)$. Examples include $f(x) = x^3$. Even functions display reflective symmetry about the y-axis, satisfying $f(x) = f(-x)$. An example is $f(x) = x^2$.

6Q: Prove the Horizontal Line Test and explain its application in determining injectivity.

Answer:

The Horizontal Line Test states that if any horizontal line intersects the graph of a function at most once, the function is injective. Its proof involves assuming the contrary and showing that multiple intersections would violate the definition of injectivity.

7Q: Discuss the concept of a periodic function and provide an example. Explain its significance in modeling real-world phenomena.

Answer:

A periodic function repeats its values at regular intervals. An example is the sine function, $f(x) = \sin(x)$, which repeats every 2π radians. Periodic functions are vital in modeling phenomena with recurring patterns, such as waves and oscillations.

8 Q: Compare and contrast the roles of the range and codomain in defining a function. Provide examples to illustrate.

Answer:

The range is the set of actual output values produced by a function, while the codomain is the set of all possible output values. The range is a subset of the codomain. For example, in the function $f(x) = 2x$, the range is the set of all real numbers, while the codomain could be the set of positive real numbers.

9Q: Discuss the significance of the Vertical Line Test in identifying functions. Provide examples and graphical representations.

Answer:

The Vertical Line Test asserts that if any vertical line intersects a graph at more than one point, the graph does not represent a function. It is a crucial tool in determining the unique mapping property of functions.

10Q: Explore the real-world applications of constant functions. Provide examples and explain their relevance in various scenarios

Answer:

Constant functions, where the output remains the same for all inputs, model scenarios like fixed costs or unchanging quantities. For instance, a function representing the cost of a fixed monthly subscription is a constant function. Constant functions are useful in economics, finance, and other fields where fixed values play a significant role.

Multiple-Choice Questions (MCQs):

1Q: What is the definition of a function in mathematics?

A. A relation with multiple outputs for each input.

B. A relation with no outputs.

C. A relation between two sets where each input is related to exactly one output.

D. A relation where each input has multiple outputs.

Answer: C

2Q: What is the range of a function?

A. The set of all possible input values.

B. The set of actual output values the function produces.

C. The set of all possible output values.

D. The set of integers.

Answer: B

3Q: Which property does an odd function exhibit?

A. Reflective symmetry about the y-axis.

B. Rotational symmetry about the origin.

C. Both A and B.

D. No symmetry.

Answer: B

4Q: What is the composition of functions?

A. The sum of two functions.

B. The multiplication of two functions.

C. The combination of two functions by applying one after the other.

D. The subtraction of two functions.

Answer: C

5Q: Which test is used to determine if a graph represents a function?

A. The Horizontal Line Test.

B. The Vertical Line Test.

C. The Diagonal Line Test.

D. The Curvature Test.

Answer: B

Fill in the Blanks Questions:

Q1: The inverse of a function swaps the roles of ______ and ______.

Answer**:** input; output

Q2: A function is considered even if ______.

Answer: $f(x) = f(-x)$ for all x in its domain

Q3: The ______ function repeats its values at regular intervals.

Answer**:** periodic

Q4: The ______ Line Test is used to determine if a function is injective.

Answer: Horizontal

Q5: A constant function represents a situation where the output remains the same ______.

Answer**:** regardless of the input