Mathematics
Class-x

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Chapter-1

Real Numbers

2 MARKS QUESTIONS

1. Use Euclid's division algorithm to find the HCF of:

i. 867 and 255

Solution:

As we know, 867 is greater than 255. Let us apply now Euclid's division algorithm on 867, to get,

$867 = 255 \times 3 + 102$

Remainder $102 \neq 0$, therefore taking 255 as divisor and applying the division lemma method, we get,

$255 = 102 \times 2 + 51$

Again, $51 \neq 0$. Now 102 is the new divisor, so repeating the same step we get,

$102 = 51 \times 2 + 0$

The remainder is now zero, so our procedure stops here. Since, in the last step, the divisor is 51, therefore, HCF (867,255) = HCF(255,102) = HCF(102,51) = 51.

Hence, the HCF of 867 and 255 is 51.

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

Solutions:

Expressing 26 and 91 as product of its prime factors, we get,

$26 = 2 \times 13 \times 1$

$91 = 7 \times 13 \times 1$

Therefore, LCM (26, 91) = $2 \times 7 \times 13 \times 1 = 182$

And HCF (26, 91) = 13

Verification

Now, product of 26 and 91 = $26 \times 91 = 2366$

And product of LCM and HCF = $182 \times 13 = 2366$

Hence, LCM × HCF = product of the 26 and 91.

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Solutions:

Writing the product of prime factors for all the three numbers, we get,

$12=2\times2\times3$

$15=5\times3$

$21=7\times3$

Therefore,

HCF(12,15,21) = 3

LCM(12,15,21) = 2 × 2 × 3 × 5 × 7 = 420

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution:

As we know that,

HCF×LCM=Product of the two given numbers

Therefore,

9 × LCM = 306 × 657

LCM = (306×657)/9 = 22338

Hence, LCM(306,657) = 22338

5. Check whether 6^n can end with the digit 0 for any natural number n.

Solution:

If the number 6^n ends with the digit zero (0), then it should be divisible by 5, as we know any number with unit place as 0 or 5 is divisible by 5.

Prime factorization of $6^n = (2\times3)^n$

Therefore, the prime factorization of 6^n doesn’t contain prime number 5.

Hence, it is clear that for any natural number n, 6^n is not divisible by 5, and thus it proves that 6^n cannot end with the digit 0 for any natural number n.

6.There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution:

Since, Both Sonia and Ravi move in the same direction and at the same time, the method to find the time when they will be meeting again at the starting point is LCM of 18 and 12.

Therefore, LCM(18,12) = $2\times3\times3\times2\times1=36$

Hence, Sonia and Ravi will meet again at the starting point after 36 minutes.

7. Prove that the following are irrationals:

(i) $6 +\sqrt{2}$

Solution:

Let us assume $6 +\sqrt{2}$ is a rational number.

Then we can find co-primes x and y ($y \neq 0$) such that $6 +\sqrt{2} = x/y$·

Rearranging, we get,

$\sqrt{2} = (x/y) – 6$

Since, x and y are integers, thus $(x/y) – 6$ is a rational number and therefore, $\sqrt{2}$ is rational. This contradicts the fact that $\sqrt{2}$ is an irrational number.

Hence, we can conclude that $6 +\sqrt{2}$ is irrational.

4 MARKS QUESTIONS

1. Use Euclid's division algorithm to find the HCF of:

i. 135 and 225

ii. 196 and 38220

Solutions:

i. 135 and 225

As you can see from the question, 225 is greater than 135. Therefore, by Euclid's division algorithm, we have,

$225 = 135 \times 1 + 90$

Now, remainder $90 \neq 0$, thus again using division lemma for 90, we get,

$135 = 90 \times 1 + 45$

Again, $45 \neq 0$, repeating the above step for 45, we get,

$90 = 45 \times 2 + 0$

The remainder is now zero, so our method stops here. Since, in the last step, the divisor is 45, therefore, HCF (225,135) = HCF (135, 90) = HCF (90, 45) = 45.

Hence, the HCF of 225 and 135 is 45.

ii. 196 and 38220

In this given question, 38220>196, therefore the by applying Euclid's division algorithm and taking 38220 as divisor, we get,

$38220 = 196 \times 195 + 0$

We have already got the remainder as 0 here. Therefore, HCF(196, 38220) = 196.

Hence, the HCF of 196 and 38220 is 196.

2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Solution:

Let a be any positive integer and b = 6. Then, by Euclid's algorithm, a = 6q + r, for some integer q ≥ 0, and r = 0, 1, 2, 3, 4, 5, because $0 \le r < 6$.

Now substituting the value of r, we get,

If r = 0, then a = 6q

Similarly, for r= 1, 2, 3, 4 and 5, the value of a is 6q+1, 6q+2, 6q+3, 6q+4 and 6q+5, respectively.

If a = 6q, 6q+2, 6q+4, then a is an even number and divisible by 2. A positive integer can be either even or odd Therefore, any positive odd integer is of the form of 6q+1, 6q+3 and 6q+5, where q is some integer.

3. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Solutions:

Let x be any positive integer and y = 3.

By Euclid's division algorithm, then,

$x = 3q + r$ for some integer $q\geq0$ and $r = 0, 1, 2$, as $r \geq 0$ and $r < 3$.

Therefore, $x = 3q$, $3q+1$ and $3q+2$

Now as per the question given, by squaring both the sides, we get,

$x^2 = (3q)^2 = 9q^2 = 3 \times 3q^2$

Let $3q^2 = m$

Therefore, $x^2= 3m$ ……………………..(1)

$x^2 = (3q + 1)^2 = (3q)^2+1^2+2\times3q\times1 = 9q^2 + 1 +6q = 3(3q^2+2q) +1$

Substitute, $3q^2+2q = m$, to get,

$x^2= 3m + 1$ ……………………………. (2)

$x^2= (3q + 2)^2 = (3q)^2+2^2+2\times3q\times2 = 9q^2 + 4 + 12q = 3 (3q^2 + 4q + 1)+1$

Again, substitute, $3q^2+4q+1 = m$, to get,

$x^2= 3m + 1$…………………………… (3)

Hence, from equation 1, 2 and 3, we can say that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

4. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 510 and 92

(ii) 336 and 54

Solutions:

(i) 510 and 92

Expressing 510 and 92 as product of its prime factors, we get,

$510 = 2 \times 3 \times 17 \times 5 \times 1$

$92 = 2 \times 2 \times 23 \times 1$

Therefore, $LCM(510, 92) = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

And $HCF (510, 92) = 2$

Verification

Now, product of 510 and 92 = $510 \times 92 = 46920$

And Product of LCM and HCF = $23460 \times 2 = 46920$

Hence, LCM × HCF = product of the 510 and 92.

(ii) 336 and 54

Expressing 336 and 54 as product of its prime factors, we get,

$336 = 2 \times 2 \times 2 \times 2 \times 7 \times 3 \times 1$

$54 = 2 \times 3 \times 3 \times 3 \times 1$

Therefore, $LCM(336, 54) = = 3024$

And $HCF(336, 54) = 2 \times 3 = 6$

Verification

Now, product of 336 and 54 = 336 × 54 = 18,144

And product of LCM and HCF = 3024 × 6 = 18,144

Hence, LCM × HCF = product of the 336 and 54.

5. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 17, 23 and 29

(ii) 8, 9 and 25

Solutions:

(i) 17, 23 and 29

Writing the product of prime factors for all the three numbers, we get,

$17=17\times1$

$23=23\times1$

$29=29\times1$

Therefore,

HCF(17,23,29) = 1

LCM(17,23,29) = 17 × 23 × 29 = 11339

(ii) 8, 9 and 25

Writing the product of prime factors for all the three numbers, we get,

$8=2\times2\times2\times1$

$9=3\times3\times1$

$25=5\times5\times1$

Therefore,

HCF(8,9,25)=1

LCM(8,9,25) = $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution:

By the definition of composite number, we know, if a number is composite, then it means it has factors other than 1 and itself. Therefore, for the given expression;

$7 \times 11 \times 13 + 13$

Taking 13 as common factor, we get,

$=13(7 \times 11 \times 1+1) = 13(77+1) = 13 \times 78 = 13 \times 3 \times 2 \times 13$

Hence, $7 \times 11 \times 13 + 13$ is a composite number.

Now let's take the other number,

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

Taking 5 as a common factor, we get,

$=5(7 \times 6 \times 4 \times 3 \times 2 \times 1+1) = 5(1008+1) = 5 \times 1009$

Hence, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number.

7. Prove that 3 + 2√5 + is irrational.

Solutions:

Let us assume 3 + 2√5 is rational.

Then we can find co-prime x and y (y ≠ 0) such that 3 + 2√5 = x/y

Rearranging, we get,

$$2\sqrt{5} = \frac{x}{y} - 3$$
$$\sqrt{5} = \frac{1}{2}(\frac{x}{y} - 3)$$

Since, x and y are integers, thus,

$\frac{1}{2}(\frac{x}{y} - 3)$ is a rational number.

Therefore, √5 is also a rational number. But this contradicts the fact that √5 is irrational.

So, we conclude that 3 + 2√5 is irrational.

7 MARKS QUESTIONS

1. Prove that the following are irrationals:

(i) $1/\sqrt{2}$

(ii) $7\sqrt{5}$

(iii) $6 + \sqrt{2}$

Solutions:

(i) $1/\sqrt{2}$

Let us assume $1/\sqrt{2}$ is rational.

Then we can find co-prime x and y ($y \neq 0$) such that $1/\sqrt{2} = x/y$

Rearranging, we get,

$\sqrt{2} = y/x$

Since, x and y are integers, thus, $\sqrt{2}$ is a rational number, which contradicts the fact that $\sqrt{2}$ is irrational.

Hence, we can conclude that $1/\sqrt{2}$ is irrational.

(ii) $7\sqrt{5}$

Let us assume $7\sqrt{5}$ is a rational number.

Then we can find co-prime a and b ($b \neq 0$) such that $7\sqrt{5} = x/y$

Rearranging, we get,

$\sqrt{5} = x/7y$

Since, x and y are integers, thus, $\sqrt{5}$ is a rational number, which contradicts the fact that $\sqrt{5}$ is irrational.

Hence, we can conclude that 7√5 is irrational.

(iii) 6 +√2

Let us assume 6 +√2 is a rational number.

Then we can find co-primes x and y ($y \neq 0$) such that 6 +√2 = x/y·

Rearranging, we get,

√2 = (x/y) – 6

Since, x and y are integers, thus (x/y) – 6 is a rational number and therefore, √2 is rational. This contradicts the fact that √2 is an irrational number.

Hence, we can conclude that 6 +√2 is irrational.

2.Prove that √5 is irrational.

Solution:

Let us assume, that √5 is rational number.

i.e. √5 = x/y (where, x and y are co-primes)

y√5= x

Squaring both the sides, we get,

$(y\sqrt{5})^2 = x^2$

$\Rightarrow 5y^2 = x^2$……………………………….. (1)

Thus, x^2 is divisible by 5, so x is also divisible by 5.

Let us say, x = 5k, for some value of k and substituting the value of x in equation (1), we get,

$5y^2 = (5k)^2$

$\Rightarrow y^2 = 5k^2$

is divisible by 5 it means y is divisible by 5.

Clearly, x and y are not co-primes. Thus, our assumption about √5 is rational is incorrect.

Hence, √5 is an irrational number.

3. Express each number as a product of its prime factors:

(i) 140

(ii) 156

(iii) 3825

(iv) 5005

(v) 7429

Solutions:

(i) 140

By taking the LCM of 140, we will get the product of its prime factor.

Therefore, $140 = 2 \times 2 \times 5 \times 7 \times 1 = 2^2 \times 5 \times 7$

(ii) 156

By Taking the LCM of 156, we will get the product of its prime factor.

Hence, $156 = 2 \times 2 \times 13 \times 3 \times 1 = 2^2 \times 13 \times 3$

(iii) 3825

By taking the LCM of 3825, we will get the product of its prime factor.

Hence, $3825 = 3 \times 3 \times 5 \times 5 \times 17 \times 1 = 3^2 \times 5^2 \times 17$

(iv) 5005

By Taking the LCM of 5005, we will get the product of its prime factor.

Hence, 5005 = 5 × 7 × 11 × 13 × 1 = 5 × 7 × 11 × 13

(v) 7429

By taking the LCM of 7429, we will get the product of its prime factor.

Hence, 7429 = 17 × 19 × 23 × 1 = 17 × 19 × 23

4. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Solution:

Let x be any positive integer and y = 3.

By Euclid's division algorithm, then,

x = 3q+r, where q≥0 and r = 0, 1, 2, as r ≥ 0 and r < 3.

Therefore, putting the value of r, we get,

x = 3q

or

x = 3q + 1

or

x = 3q + 2

Now, by taking the cube of all the three above expressions, we get,

Case (i): When r = 0, then,

$x^2= (3q)^3 = 27q^3= 9(3q^3)= 9m$; where $m = 3q^3$

Case (ii): When r = 1, then,

$x^3 = (3q+1)^3 = (3q)^3 + 1^3 + 3\times 3q\times 1(3q+1) = 27q^3+1+27q^2+9q$

Taking 9 as common factor, we get,

$x^3 = 9(3q^3+3q^2+q)+1$

Putting = m, we get,

Putting $(3q^3+3q^{2+}q)$ = m, we get ,

$x^3 = 9m+1$

Case (iii): When r = 2, then,

$x^3 = (3q+2)^3 = (3q)^3+2^3+3\times 3q\times 2(3q+2) = 27q^3+54q^2+36q+8$

Taking 9 as common factor, we get,

$x^3=9(3q^3+6q^2+4q)+8$

Putting $(3q^3+6q^2+4q)$ = m, we get ,

$x^3 = 9m+8$

Therefore, from all the three cases explained above, it is proved that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.