

## Chapter-9

### SOME APPLICATIONS OF TRIGONOMETRY

#### 2 MARKS QUESTIONS

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is  $30^\circ$ . (see fig. 9.11)

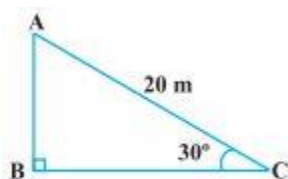


Fig. 9.11

#### Solution:

Length of the rope is 20 m and angle made by the rope with the ground level is  $30^\circ$ .

**Given:**  $AC = 20$  m and angle  $C = 30^\circ$

**To Find:** Height of the pole

Let AB be the vertical pole

In right  $\triangle ABC$ , using sine formula

$$\sin 30^\circ = \frac{AB}{AC}$$

Using value of  $\sin 30$  degrees is  $\frac{1}{2}$ , we have

$$1/2 = AB/20$$

$$AB = 20/2$$

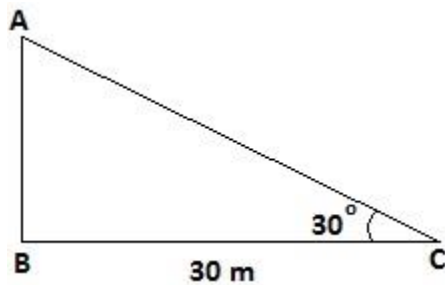
$$AB = 10$$

Therefore, the height of the pole is 10 m.

**2. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is  $30^\circ$ . Find the height of the tower.**

**Solution:**

Let AB be the height of the tower and C is the point elevation which is 30 m away from the foot of the tower.



To Find: AB (height of the tower)

In right ABC

$$\tan 30^\circ = AB/BC$$

$$1/\sqrt{3} = AB/30$$

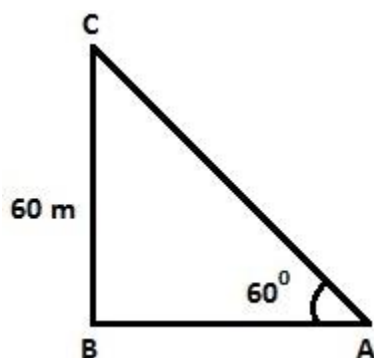
$$\Rightarrow AB = 10\sqrt{3}$$

Thus, the height of the tower is  $10\sqrt{3}$  m.

**3. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is  $60^\circ$ . Find the length of the string, assuming that there is no slack in the string.**

**Solution:**

Draw a figure, based on given instruction,



Let  $BC$  = Height of the kite from the ground,  $BC = 60$  m

$AC$  = Inclined length of the string from the ground and

$A$  is the point where string of the kite is tied.

To Find: Length of the string from the ground i.e. the value of  $AC$

From the above figure,

$$\sin 60^\circ = BC/AC$$

$$\Rightarrow \sqrt{3}/2 = 60/AC$$

$$\Rightarrow AC = 40\sqrt{3} \text{ m}$$

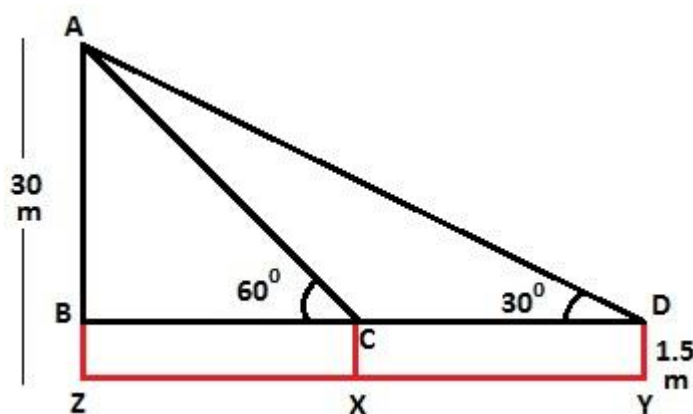
Thus, the length of the string from the ground is  $40\sqrt{3}$  m.

## 4 MARKS QUESTIONS

1. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from  $30^\circ$  to  $60^\circ$  as he walks towards the building. Find the distance he walked towards the building.

### Solution:

Let the boy initially stand at point Y with inclination  $30^\circ$  and then he approaches the building to the point X with inclination  $60^\circ$ .



To Find: The distance boy walked towards the building i.e. XY

From figure,

$$XY = CD.$$

Height of the building =  $AZ = 30$  m.

$$AB = AZ - BZ = 30 - 1.5 = 28.5$$

Measure of AB is 28.5 m

In right  $\triangle ABD$ ,

$$\tan 30^\circ = AB/BD$$

$$1/\sqrt{3} = 28.5/BD$$

$$BD = 28.5\sqrt{3} \text{ m}$$

Again,

In right  $\triangle ABC$ ,

$$\tan 60^\circ = AB/BC$$

$$\sqrt{3} = 28.5/BC$$

$$BC = 28.5/\sqrt{3} = 28.5\sqrt{3}/3$$

Therefore, the length of BC is  $28.5\sqrt{3}/3$  m.

$$XY = CD = BD - BC = (28.5\sqrt{3} - 28.5\sqrt{3}/3) = 28.5\sqrt{3}(1 - 1/3) = 28.5\sqrt{3} \times 2/3 = 57/\sqrt{3} = 19\sqrt{3} \text{ m.}$$

Thus, the distance boy walked towards the building is  $19\sqrt{3}$  m.

**2. From a point on the ground, the angles of elevation of the bottom and the top of a**

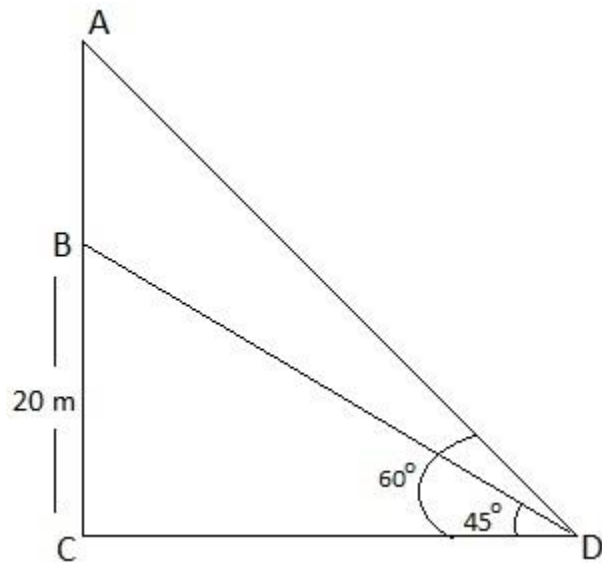
**transmission tower fixed at the top of a 20 m high building are  $45^\circ$  and  $60^\circ$  respectively. Find the height of the tower.**

**Solution:**

Let BC be the 20 m high building.

D is the point on the ground from where the elevation is taken.

$$\text{Height of transmission tower} = AB = AC - BC$$



To Find: AB, Height of the tower

From figure, In right  $\triangle BCD$ ,

$$\tan 45^\circ = BC/CD$$

$$1 = 20/CD$$

$$CD = 20$$

Again,

In right  $\triangle ACD$ ,

$$\tan 60^\circ = AC/CD$$

$$\sqrt{3} = AC/20$$

$$AC = 20\sqrt{3}$$

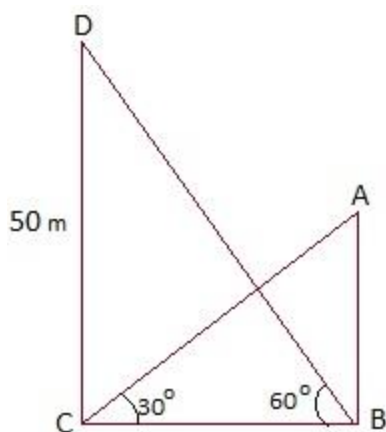
$$\text{Now, } AB = AC - BC = (20\sqrt{3} - 20) = 20(\sqrt{3} - 1)$$

$$\text{Height of transmission tower} = 20(\sqrt{3} - 1) \text{ m.}$$

**3. The angle of elevation of the top of a building from the foot of the tower is  $30^\circ$  and the angle of elevation of the top of the tower from the foot of the building is  $60^\circ$ . If the tower is 50 m high, find the height of the building.**

**Solution:**

Let CD be the height of the tower. AB be the height of the building. BC be the distance between the foot of the building and the tower. Elevation is  $30^\circ$  and  $60^\circ$  from the tower and the building respectively.



In right  $\triangle BCD$ ,

$$\tan 60^\circ = CD/BC$$

$$\sqrt{3} = 50/BC$$

$$BC = 50/\sqrt{3} \dots (1)$$

Again,

In right  $\triangle ABC$ ,

$$\tan 30^\circ = AB/BC$$

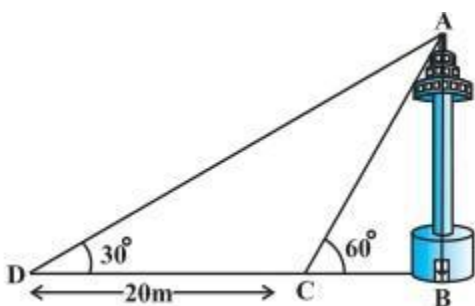
$$\Rightarrow 1/\sqrt{3} = AB/BC$$

Use result obtained in equation (1)

$$AB = 50/3$$

Thus, the height of the building is  $50/3$  m.

**4. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is  $60^\circ$ . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is  $30^\circ$  (see Fig. 9.12). Find the height of the tower and the width of the canal.**



### Solution:

Given, AB is the height of the tower.

$$DC = 20 \text{ m (given)}$$

As per given diagram, In right  $\triangle ABD$ ,

$$\tan 30^\circ = AB/BD$$

$$1/\sqrt{3} = AB/(20+BC)$$

$$AB = (20+BC)/\sqrt{3} \dots (i)$$

Again,

In right  $\triangle ABC$ ,

$$\tan 60^\circ = AB/BC$$



$$\sqrt{3} = AB/BC$$

$$AB = \sqrt{3} BC \dots (ii)$$

From equation (i) and (ii)

$$\sqrt{3} BC = (20+BC)/\sqrt{3}$$

$$3 BC = 20 + BC$$

$$2 BC = 20$$

$$BC = 10$$

Putting the value of BC in equation (ii)

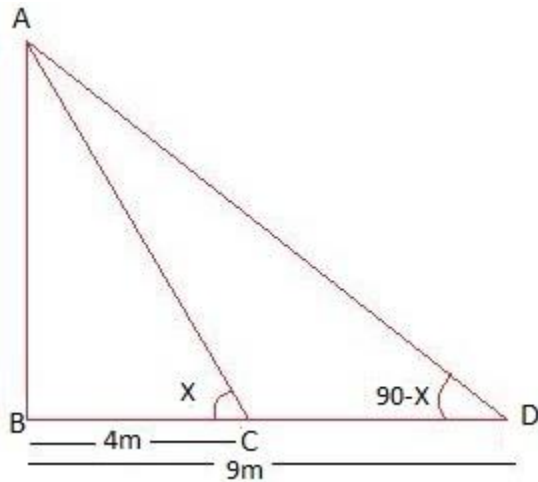
$$AB = 10\sqrt{3}$$

This implies, the height of the tower is  $10\sqrt{3}$  m and the width of the canal is 10 m.

**5. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.**

**Solution:**

Let AB be the tower. C and D be the two points with distance 4 m and 9 m from the base respectively. As per question,



In right  $\triangle ABC$ ,

$$\tan x = AB/BC$$

$$\tan x = AB/4$$

$$AB = 4 \tan x \dots (i)$$

Again, from right  $\triangle ABD$ ,

$$\tan (90^\circ - x) = AB/BD$$

$$\cot x = AB/9$$

$$AB = 9 \cot x \dots (ii)$$

Multiplying equation (i) and (ii)

$$AB^2 = 9 \cot x \times 4 \tan x$$

$$\Rightarrow AB^2 = 36 \text{ (because } \cot x = 1/\tan x \text{)}$$

$$\Rightarrow AB = \pm 6$$

Since height cannot be negative, therefore, the height of the tower is 6 m.

Hence Proved.

## 7 MARKS QUESTIONS

1. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of  $30^\circ$ , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be  $60^\circ$ . Find the time taken by the car to reach the foot of the tower from this point.

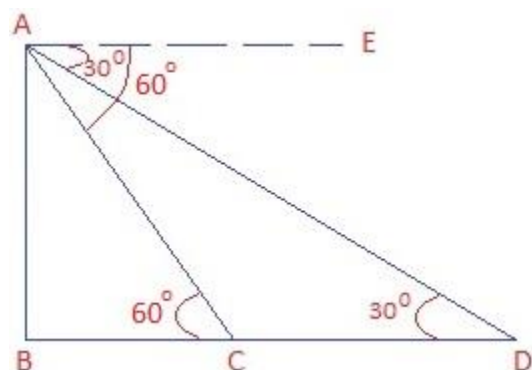
### Solution:

Let AB be the tower.

D is the initial and C is the final position of the car respectively.

Since the man is standing at the top of the tower so, Angles of depression are measured from A.

BC is the distance from the foot of the tower to the car.



Step 1: In right  $\triangle ABC$ ,

$$\tan 60^\circ = AB/BC$$

$$\sqrt{3} = AB/BC$$

$$BC = AB/\sqrt{3}$$

$$AB = \sqrt{3} BC$$

Step 2:

In right  $\triangle ABD$ ,

$$\tan 30^\circ = AB/BD$$

$$1/\sqrt{3} = AB/BD$$

$$AB = BD/\sqrt{3}$$

Step 3: From step 1 and Step 2, we have

$$\sqrt{3} BC = BD/\sqrt{3} \text{ (Since LHS are same, so RHS are also same)}$$

$$3 BC = BD$$

$$3 BC = BC + CD$$

$$2BC = CD$$

$$\text{or } BC = CD/2$$

Here, distance of BC is half of CD. Thus, the time taken is also half.

Time taken by car to travel distance CD = 6 sec. Time taken by car to travel BC =  $6/2 = 3$  sec.

**2. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is  $60^\circ$ . After some time, the angle of elevation reduces to  $30^\circ$  (see Fig. 9.13). Find the distance travelled by the balloon during the interval.**

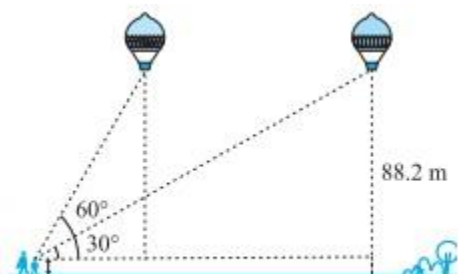


Fig. 9.13

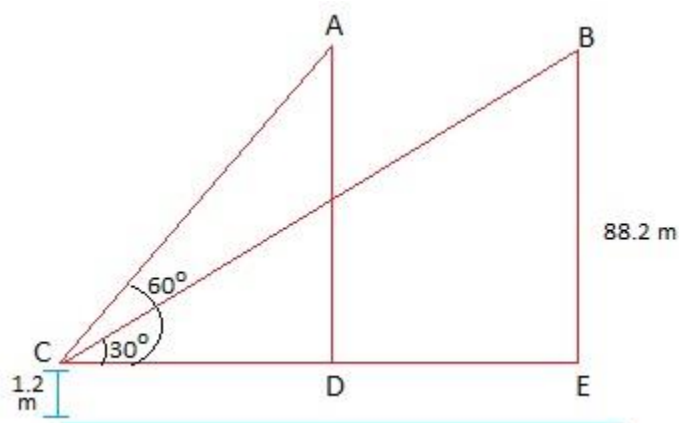
### Solution:

Let the initial position of the balloon be A and final position be B.

Height of balloon above the girl height =  $88.2 \text{ m} - 1.2 \text{ m} = 87 \text{ m}$ .

To Find: Distance travelled by the balloon =  $DE = CE - CD$

Let us redesign the given figure as per our convenient



Step 1: In right  $\triangle BEC$ ,

$$\tan 30^\circ = BE/CE$$

$$1/\sqrt{3} = 87/CE$$

$$CE = 87\sqrt{3}$$

Step 2:

In right  $\triangle ADC$ ,

$$\tan 60^\circ = AD/CD$$

$$\sqrt{3} = 87/CD$$

$$CD = 87/\sqrt{3} = 29\sqrt{3}$$

Step 3:

$$DE = CE - CD = (87\sqrt{3} - 29\sqrt{3}) = 29\sqrt{3}(3 - 1) = 58\sqrt{3}$$

Distance travelled by the balloon =  $58\sqrt{3}$  m.

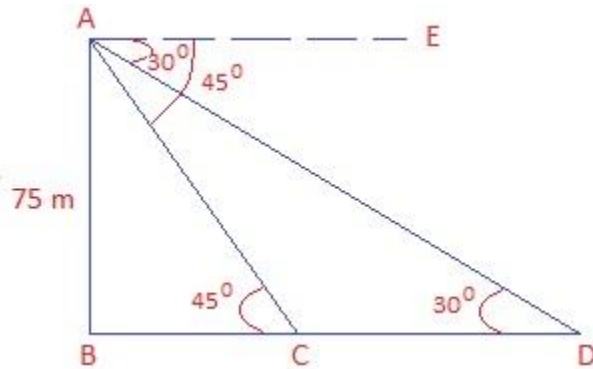
**3. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are  $30^\circ$  and  $45^\circ$ . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.**

**Solution:**

Let AB be the lighthouse of height 75 m. Let C and D be the positions of the ships.

$30^\circ$  and  $45^\circ$  are the angles of depression from the lighthouse.

Draw a figure based on given instructions:



To Find:  $CD$  = distance between two ships

Step 1: From right triangle  $ABC$ ,

$$\tan 45^\circ = AB/BC$$

$$1 = 75/BC$$

$$BC = 75 \text{ m}$$

Step 2: Form right triangle  $ABD$ ,

$$\tan 30^\circ = AB/BD$$

$$1/\sqrt{3} = 75/BD$$

$$BD = 75\sqrt{3}$$

Step 3: To find measure of  $CD$ , use results obtained in step 1 and step 2.

$$CD = BD - BC = (75\sqrt{3} - 75) = 75(\sqrt{3}-1)$$

The distance between the two ships is  $75(\sqrt{3}-1)$  m. Answer!

**4. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is  $60^\circ$  and the angle of depression of its foot is  $45^\circ$ . Determine the height of the tower.**

**Solution:**

Let AB be the building of height 7 m and EC be the height of the tower.

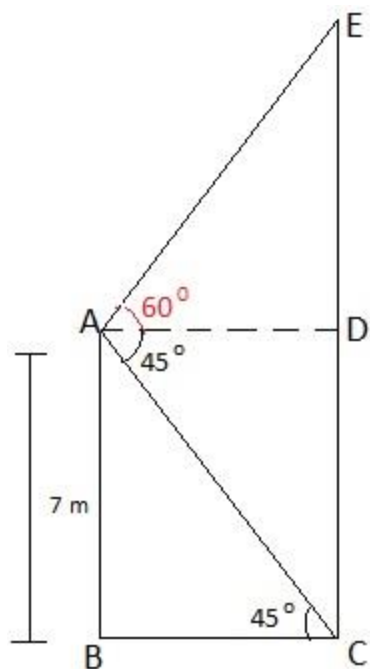
A is the point from where elevation of tower is  $60^\circ$  and the angle of depression of its foot is  $45^\circ$ .

$$EC = DE + CD$$

Also,  $CD = AB = 7$  m. and  $BC = AD$

To Find:  $EC =$  Height of the tower

Design a figure based on given instructions:



In right  $\triangle ABC$ ,

$$\tan 45^\circ = AB/BC$$

$$1 = 7/BC$$



$$BC = 7$$

$$\text{Since } BC = AD$$

$$\text{So } AD = 7$$

Again, from right triangle ADE,

$$\tan 60^\circ = DE/AD$$

$$\sqrt{3} = DE/7$$

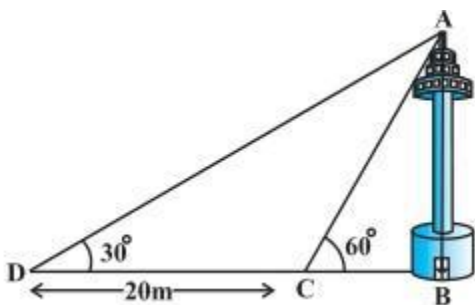
$$\Rightarrow DE = 7\sqrt{3} \text{ m}$$

$$\text{Now: } EC = DE + CD$$

$$= (7\sqrt{3} + 7) = 7(\sqrt{3}+1)$$

Therefore, height of the tower is  $7(\sqrt{3}+1)$  m. Answer!

**5. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is  $60^\circ$ . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is  $30^\circ$  (see Fig. 9.12). Find the height of the tower and the width of the canal.**



**Solution:**

Given, AB is the height of the tower.

$$DC = 20 \text{ m (given)}$$

As per given diagram, In right  $\triangle ABD$ ,

$$\tan 30^\circ = AB/BD$$

$$1/\sqrt{3} = AB/(20+BC)$$

$$AB = (20+BC)/\sqrt{3} \dots (i)$$

Again,

In right  $\triangle ABC$ ,

$$\tan 60^\circ = AB/BC$$

$$\sqrt{3} = AB/BC$$

$$AB = \sqrt{3} BC \dots (ii)$$

From equation (i) and (ii)

$$\sqrt{3} BC = (20+BC)/\sqrt{3}$$

$$3 BC = 20 + BC$$

$$2 BC = 20$$

$$BC = 10$$

Putting the value of BC in equation (ii)

$$AB = 10\sqrt{3}$$

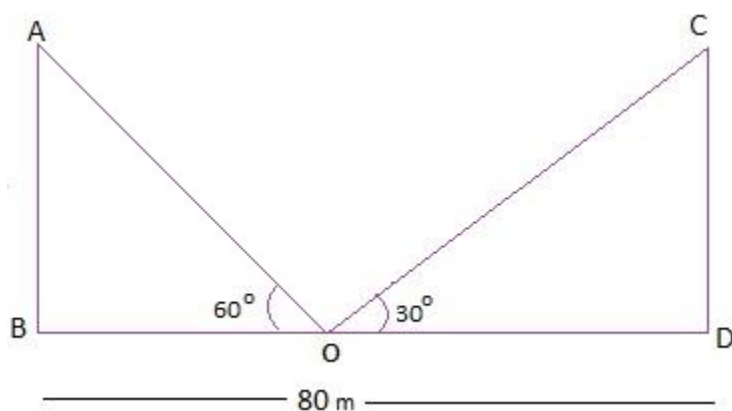
This implies, the height of the tower is  $10\sqrt{3}$  m and the width of the canal is 10 m.

**6. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are  $60^\circ$  and  $30^\circ$ , respectively. Find the height of the poles and the distances of the point from the poles.**

**Solution:**

Let AB and CD be the poles of equal height.

O is the point between them from where the height of elevation taken. BD is the distance between the poles.



As per above figure,  $AB = CD$ ,

$$OB + OD = 80 \text{ m}$$

Now,

In right  $\triangle CDO$ ,

$$\tan 30^\circ = CD/OD$$

$$1/\sqrt{3} = CD/OD$$

$$CD = OD/\sqrt{3} \dots (1)$$

Again,

In right  $\triangle ABO$ ,

$$\tan 60^\circ = AB/OB$$

$$\sqrt{3} = AB/(80-OD)$$

$$AB = \sqrt{3}(80-OD)$$

$$AB = CD \text{ (Given)}$$

$$\sqrt{3}(80-OD) = OD/\sqrt{3} \text{ (Using equation (1))}$$

$$3(80-OD) = OD$$

$$240 - 3 OD = OD$$

$$4 OD = 240$$

$$OD = 60$$

Putting the value of OD in equation (1)

$$CD = OD/\sqrt{3}$$

$$CD = 60/\sqrt{3}$$

$$CD = 20\sqrt{3} \text{ m}$$

Also,

$$OB + OD = 80 \text{ m}$$

$$\Rightarrow OB = (80-60) \text{ m} = 20 \text{ m}$$

Thus, the height of the poles are  $20\sqrt{3}$  m and distance from the point of elevation are 20 m and

60 m respectively.

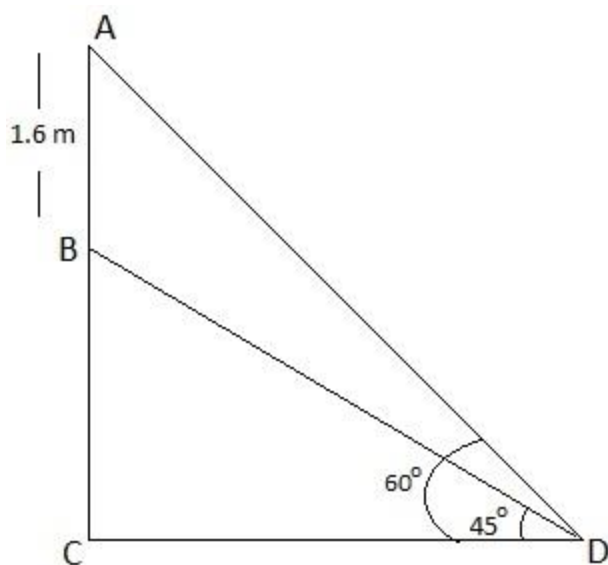
**7. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is  $60^\circ$  and from the same point the angle of elevation of the top of the pedestal is  $45^\circ$ . Find the height of the pedestal.**

**Solution:**

Let AB be the height of statue.

D is the point on the ground from where the elevation is taken.

To Find: Height of pedestal = BC = AC-AB



From figure,

In right triangle BCD,

$$\tan 45^\circ = BC/CD$$

$$1 = BC/CD$$

$$BC = CD \dots\dots(1)$$

Again,

In right  $\triangle ACD$ ,

$$\tan 60^\circ = AC/CD$$

$$\sqrt{3} = (AB+BC)/CD$$

$$\sqrt{3}CD = 1.6 + BC$$

$$\sqrt{3}BC = 1.6 + BC \text{ (using equation (1))}$$

$$\sqrt{3}BC - BC = 1.6$$

$$BC(\sqrt{3}-1) = 1.6$$

$$BC = [(1.6)(\sqrt{3}+1)]/[(\sqrt{3}-1)(\sqrt{3}+1)]$$

$$BC = [1.6(\sqrt{3}+1)]/(2) \text{ m}$$

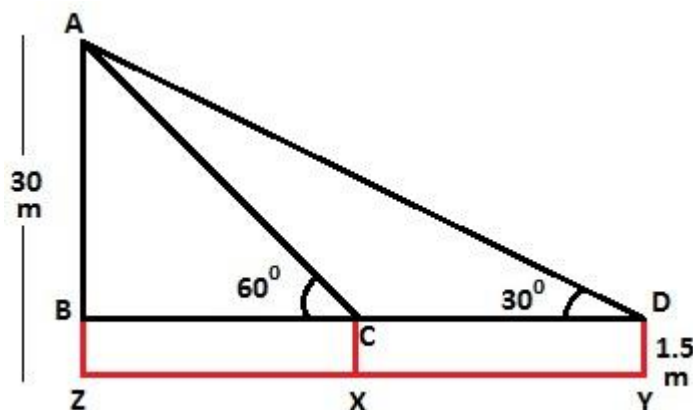
$$BC = 0.8(\sqrt{3}+1)$$

Thus, the height of the pedestal is  $0.8(\sqrt{3}+1)$  m.

**8. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from  $30^\circ$  to  $60^\circ$  as he walks towards the building. Find the distance he walked towards the building.**

**Solution:**

Let the boy initially stand at point Y with inclination  $30^\circ$  and then he approaches the building to the point X with inclination  $60^\circ$ .



To Find: The distance boy walked towards the building i.e. XY

From figure,

$$XY = CD.$$

$$\text{Height of the building} = AZ = 30 \text{ m.}$$

$$AB = AZ - BZ = 30 - 1.5 = 28.5$$

Measure of AB is 28.5 m

In right  $\triangle ABD$ ,

$$\tan 30^\circ = AB/BD$$

$$1/\sqrt{3} = 28.5/BD$$

$$BD = 28.5\sqrt{3} \text{ m}$$

Again,

In right  $\triangle ABC$ ,

$$\tan 60^\circ = AB/BC$$

$$\sqrt{3} = 28.5/BC$$

$$BC = 28.5/\sqrt{3} = 28.5\sqrt{3}/3$$

Therefore, the length of BC is  $28.5\sqrt{3}/3$  m.

$$XY = CD = BD - BC = (28.5\sqrt{3} - 28.5\sqrt{3}/3) = 28.5\sqrt{3}(1 - 1/3) = 28.5\sqrt{3} \times 2/3 = 57/\sqrt{3} = 19\sqrt{3} \text{ m.}$$

Thus, the distance boy walked towards the building is  $19\sqrt{3}$  m.

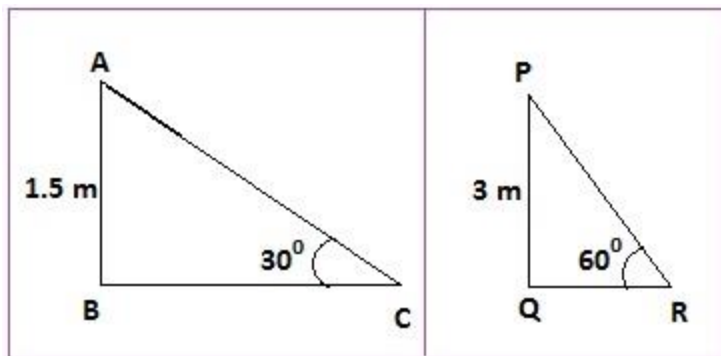
**9. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of  $30^\circ$  to the ground, whereas for elder children, she wants to have a steep slide at a height of 3m, and inclined at an angle of  $60^\circ$  to the ground. What should be the length of the slide in each case?**

**Solution:**

As per contractor's plan,

Ages	Height of Slide	Inclined Angle
Below 5 years	1.5 m	30 degrees
Above 5 years	3 m	60 degrees





Let, ABC is the slide inclined at  $30^\circ$  with length AC and PQR is the slide inclined at

$60^\circ$  with length PR.

To Find: AC and PR

In right  $\triangle ABC$ ,

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{1.5}{AC}$$

$$AC = 3$$

Also,

In right  $\triangle PQR$ ,

$$\sin 60^\circ = \frac{PQ}{PR}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{PR}$$

$$\Rightarrow PR = 2\sqrt{3}$$

Hence, length of the slide for below 5 = 3 m and

Length of the slide for elders children =  $2\sqrt{3}$  m

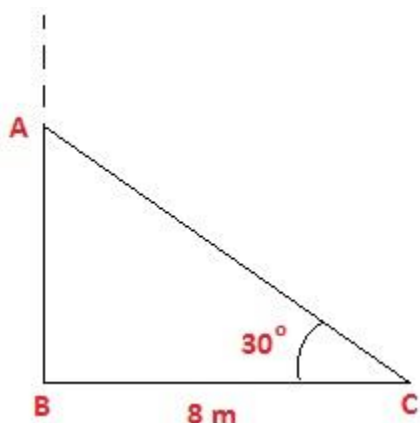
**10. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle  $30^\circ$  with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.**

**Solution:**

Using given instructions, draw a figure. Let AC be the broken part of the tree. Angle C =  $30^\circ$

$$BC = 8 \text{ m}$$

To Find: Height of the tree, which is AB



From figure: Total height of the tree is the sum of AB and AC i.e.  $AB+AC$

In right  $\triangle ABC$ ,

Using Cosine and tangent angles,

$$\cos 30^\circ = BC/AC$$

We know that,  $\cos 30^\circ = \sqrt{3}/2$

$$\sqrt{3}/2 = 8/AC$$

$$AC = 16/\sqrt{3} \dots (1)$$

Also,

$$\tan 30^\circ = AB/BC$$

$$1/\sqrt{3} = AB/8$$

$$AB = 8/\sqrt{3} \dots(2)$$

Therefore, total height of the tree =  $AB + AC = 16/\sqrt{3} + 8/\sqrt{3} = 24/\sqrt{3} = 8\sqrt{3}$  m.