Chapter 16

Probability

Probability is the branch of mathematics concerning numerical descriptions of how likely an event is to occur, or how likely it is that a proposition is true. The probability of an event is a number between 0 and 1, where, roughly speaking, 0 indicates impossibility of the event and 1 indicates certainty. Probability is a measure of the likelihood of an event to occur. Many events cannot be predicted with total certainty. We can predict only the chance of an event to occur i.e., how likely they are going to happen, using it.

In daily life, probability is quite important. In the analysis of political strategies, the determination of blood types, sports and gaming strategies, purchasing or selling insurance, online shopping, and online games.

Exercise 16.1

1. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

So, S = (1, 2, 3, 4, 5, 6)

As per the conditions given in the question,

E be the event "die shows 4"

E = (4)

F be the event "die shows even number".

F = (2, 4, 6)

$E \cap F = (4) \cap (2, 4, 6)$

= 4

$4 \neq \varphi$ … [because there is a common element in E and F]

Therefore, E and F are not mutually exclusive events.

2. A die is thrown. Describe the following events:

(i) A: a number less than 7 (ii) B: a number greater than 7

(iii) C: a multiple of 3 (iv) D: a number less than 4

(v) E: an even number greater than 4 (vi) F: a number not less than 3

Also, find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F^I$, F^I

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

So, S = (1, 2, 3, 4, 5, 6)

As per the conditions given in the question,

(i) A: a number less than 7

All the numbers in the die are less than 7,

A = (1, 2, 3, 4, 5, 6)

(ii) B: a number greater than 7

There is no number greater than 7 on the die.

Then,

B= (φ)

(iii) C: a multiple of 3

There are only two numbers which are multiple of 3.

Then,

C= (3, 6)

(iv) D: a number less than 4

D= (1, 2, 3)

(v) E: an even number greater than 4

E = (6)

(vi) F: a number not less than 3

F= (3, 4, 5, 6)

Also, we have to find, A U B, A ∩ B, B U C, E ∩ F, D ∩ E, D – E, A – C, E ∩ F', F'

So,

A ∩ B = (1, 2, 3, 4, 5, 6) ∩ (φ)

= (φ)

B U C = (φ) U (3, 6)

= (3, 6)

E ∩ F = (6) ∩ (3, 4, 5, 6)

= (6)

D ∩ E = (1, 2, 3) ∩ (6)

= (φ)

D – E = (1, 2, 3) – (6)

= (1, 2, 3)

A – C = (1, 2, 3, 4, 5, 6) – (3, 6)

= (1, 2, 4, 5)

F’ = S – F

= (1, 2, 3, 4, 5, 6) – (3, 4, 5, 6)

= (1, 2)

E ∩ F’ = (6) ∩ (1, 2)

= (φ)

3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events: A: the sum is greater than 8, B: 2 occurs on either die C: the sum is at least 7 and a multiple of 3. Which pairs of these events are mutually exclusive?

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

In the question, it is given that pair of die is thrown, so the sample space will be

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5). (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

A: the sum is greater than 8

$\therefore$ A= $\begin{Bmatrix} (3,6), (4,5), (5,4), (6,3), (4,6), \\ (5,5), (6,4), (5,6), (6,5), (6,6) \end{Bmatrix}$

Possible sum greater than 8 are 9, 10, 11 & 12

B: 2 occurs on either die

B= $\begin{Bmatrix} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (1,2), (3,2), (4,2), (5,2), (6,2) \end{Bmatrix}$

In this conditions possibilities are there that the number 2 will come on either first die or second die or both the die simultaneously

C: The sum is at least 7 and multiple of 3

C= $\{(3,6), (4,5), (5,4), (6,3), (6,6)\}$

So the sum can be only 9 or 12

Now, we shall find whether pairs of these events are mutually exclusive or not.

(i) A∩ B = φ

There is no common element in A and B.

Therefore, A & B are mutually exclusive.

(ii) B ∩ C = φ

There is no common element between.

Therefore, B and C are mutually exclusive.

(iii) A ∩ C {(3,6), (4,5), (5,4), (6,3), (6,6)}

⇒ {(3,6), (4,5), (5,4), (6,3), (6,6)} ≠ φ

A and C have common elements.

Therefore, A and C are mutually exclusive.

4. Three coins are tossed once. Let A denotes the event 'three heads show", B denotes the event "two heads and one tail show", C denotes the event" three tails show and D denote the event 'a head shows on the first coin". Which events are

(i) Mutually exclusive? (ii) Simple? (iii) Compound?

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

But, now three coins are tossed once, so the possible sample space contains

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTH}

Now,

A: 'three heads'

A= (HHH)

B: "two heads and one tail"

B= (HHT, THH, HTH)

C: 'three tails'

C= (TTT)

D: a head shows on the first coin

D= (HHH, HHT, HTH, HTT)

(i) Mutually exclusive

$A \cap B = (HHH) \cap (HHT, THH, HTH)$

$= \varphi$

Therefore, A and C are mutually exclusive.

$A \cap C = (HHH) \cap (TTT)$

$= \varphi$

There, A and C are mutually exclusive.

$A \cap D = (HHH) \cap (HHH, HHT, HTH, HTT)$

$= (HHH)$

$A \cap D \neq \varphi$

So they are not mutually exclusive

$B \cap C = (HHT, HTH, THH) \cap (TTT)$

$= \varphi$

Since there is no common element in B & C, so they are mutually exclusive.

$B \cap D = (HHT, THH, HTH) \cap (HHH, HHT, HTH, HTT)$

$= (HHT, HTH)$

$B \cap D \neq \varphi$

There are common elements in B & D.

So, they are not mutually exclusive.

$C \cap D = (TTT) \cap (HHH, HHT, HTH, HTT)$

$= \varphi$

There is no common element in C & D.

So they are not mutually exclusive.

(ii) Simple event

If an event has only one sample point of a sample space, it is called a simple (or elementary) event.

$A = (HHH)$

$C = (TTT)$

Both A & C have only one element.

So they are simple events.

(iii) Compound events

If an event has more than one sample point, it is called a Compound event.

B= (HHT, HTH, THH)

D= (HHH, HHT, HTH, HTT)

Both B & D have more than one element.

So, they are compound events.

5. Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

(iii) Two events which are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

(v) Three events which are mutually exclusive but not exhaustive.

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

But, now three coins are tossed once, so the possible sample space contains

S= (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)

(i) Two events which are mutually exclusive.

Let us assume A be the event of getting only head,

A = (HHH)

And also, let us assume B be the event of getting only Tail,

B = (TTT)

So, $A \cap B = \varphi$

Since there is no common element in A& B, these two are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

Now,

Let us assume P be the event of getting exactly two tails,

P = (HTT, TTH, THT)

Let us assume Q be the event of getting at least two heads,

Q = (HHT, HTH, THH, HHH)

Let us assume R be the event of getting only one tail,

C= (TTT)

P ∩ Q = (HTT, TTH, THT) ∩ (HHT, HTH, THH, HHH)

= φ

There is no common element in P and Q.

Therefore, they are mutually exclusive.

Q ∩ R = (HHT, HTH, THH, HHH) ∩ (TTT)

= φ

There is no common element in Q and R.

Hence, they are mutually exclusive.

P ∩ R = (HTT, TTH, THT) ∩ (TTT)

= φ

There is no common element in P and R.

So they are mutually exclusive.

Now, P and Q, Q and R, and P and R are mutually exclusive

∴ P, Q, and R are mutually exclusive.

And also,

$P \cup Q \cup R$ = (HTT, TTH, THT, HHT, HTH, THH, HHH, TTT) = S

Hence P, Q and R are exhaustive events.

(iii) Two events which are not mutually exclusive.

Let us assume 'A' be the event of getting at least two heads,

A = (HHH, HHT, THH, HTH)

Let us assume 'B' be the event of getting only head,

B= (HHH)

Now, $A \cap B$ = (HHH, HHT, THH, HTH) $\cap$ (HHH)

= (HHH)

$A \cap B \neq \varphi$

There is a common element in A and B.

So they are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

Let us assume 'P' be the event of getting only Head,

P = (HHH)

Let us assume 'Q' be the event of getting only tail,

Q = (TTT)

$P \cap Q$ = (HHH) $\cap$ (TTT)

$= \varphi$

Since there is no common element in P and Q,

These are mutually exclusive events.

But,

$P \cup Q$ = (HHH) $\cup$ (TTT)

= {HHH, TTT}

$P \cup Q \neq S$

Since $P \cup Q \neq S$, these are not exhaustive events.

(v) Three events which are mutually exclusive but not exhaustive.

Let us assume 'X' be the event of getting only head,

X = (HHH)

Let us assume 'Y' be the event of getting only tail,

Y = (TTT)

Let us assume 'Z' be the event of getting exactly two heads,

Z= (HHT, THH, HTH)

Now,

$X \cap Y$ = (HHH) $\cap$ (TTT)

= φ

$X \cap Z$ = (HHH) $\cap$ (HHT, THH, HTH)

= φ

$Y \cap Z$ = (TTT) $\cap$ (HHT, THH, HTH)

= φ

Therefore, they are mutually exclusive.

Also,

$X \cup Y \cup Z$ = (HHH TTT, HHT, THH, HTH)

$X \cup Y \cup Z \neq S$

So, X, Y and Z are not exhaustive.

Hence, it is proved that X, Y and X are mutually exclusive but not exhaustive.

6. Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5.

Describe the events

(i) A^I (ii) not B (iii) A or B

(iv) A and B (v) A but not C (vi) B or C

(vii) B and C (viii) $A \cap B^I \cap C^I$

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

In the question, it is given that pair of die is thrown, so the sample space will be

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

As per the condition given the question,

A: getting an even number on the first die.

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

B: getting an odd number on the first die.

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

C: getting the sum of the numbers on the dice ≤ 5

$$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$$

Then,

$$\text{(i) } A' = \left\{ \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix} \right\} = B$$

(ii) $B' = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$ = A

(iii) $A \cup B\ (A\ or\ B) = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} = S$

(iv) A and B (A ∩ B) = ϕ

(v) $A\ but\ not\ C = A - C = \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$

(vi) $B\ or\ C = B \cup C = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$

(vii) $B\ and\ C = B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii)

$$C' = \begin{Bmatrix} (1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2), \\ (4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$$\therefore A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$$

$$= \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5), \\ (4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

7. Refer to question 6 above and state true or false. (Give reasons for your answer)

(i) A and B are mutually exclusive

(ii) A and B are mutually exclusive and exhaustive

(iii) A = B^I

(iv) A and C are mutually exclusive

(v) A and B^I are mutually exclusive.

(vi) A^I, B^I, C are mutually exclusive and exhaustive.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

In the question, it is given that pair of die is thrown, so the sample space will be,

By referring the question 6 above,

$$S = \left\{ \begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5). (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{matrix} \right\}$$

As per the condition given the question,

A: getting an even number on the first die.

$$A = \left\{ \begin{matrix} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5). (4,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{matrix} \right\}$$

B: getting an odd number on the first die.

$$B = \left\{ \begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \end{matrix} \right\}$$

C: getting the sum of the numbers on the dice ≤ 5

$$C = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (3,1), (3,2), (2,3), (4,1)\}$$

(i) A and B are mutually exclusive.

So, $(A \cap B) = \varphi$

So, A & B are mutually exclusive.

Hence, the given statement is true.

(ii) A and B are mutually exclusive and exhaustive.

$$A \cup B = \left\{ \begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5). (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{matrix} \right\} = S$$

$\Rightarrow A \cup B = S$

From statement (I), we have A and B are mutually exclusive.

∴ A and B are mutually exclusive and exhaustive.

Hence, the statement is true.

(iii) A = B

$$B' = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \end{Bmatrix} = A$$

Therefore, the statement is true.

(iv) A and C are mutually exclusive.

We have,

$A \cap C = \{(2, 1), (2, 2), (2, 3), (4, 1)\}$

$A \cap C \neq \varphi$

A and C are not mutually exclusive.

Hence, the given statement is false.

(v) A and B^I are mutually exclusive.

We have,

$A \cap B^I = A \cap A = A$

$\therefore A \cap B^I \neq \varphi$

So, A and B^I are not mutually exclusive.

Therefore, the given statement is false.

(vi) A^{I}, B^{I}, C are mutually exclusive and exhaustive.

Here $A^{I} = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \end{Bmatrix}$

B= $\begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$

And C ={(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)}

$A^{I} \cap B^{I} = \phi$

Hence there is no common element in A' and B'

So they are mutually exclusive.

$B^{I} \cap C = \{(2,1),(2,2),(2,3),(4,1)\}$

$B^{I} \cap C \neq \phi$

B= $\begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$

And C ={(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)}

$A^{I} \cap B^{I} = \phi$

Hence there is no common element in A' and B'

So they are mutually exclusive.

$B^{I} \cap C = \{(2,1),(2,2),(2,3),(4,1)\}$

$B^{I} \cap C \neq \phi$

They are not mutually exclusive.

B^{I} and C are not mutually exclusive.

Therefore, A', B' and C are not mutually exclusive and exhaustive.

So, the given statement is false.

Exercise 16.2

1. Which of the following cannot be a valid assignment of probabilities for outcomes of sample Space S = {ω_1, ω_2, ω_3, ω_4, ω_5, ω_6, ω_7}

Assignment

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	**0.1**	**0.01**	**0.05**	**0.03**	**0.01**	**0.2**	**0.6**
(b)	**1/7**	**1/7**	**1/7**	**1/7**	**1/7**	**1/7**	**1/7**
(c)	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**
(d)	**-0.1**	**0.2**	**0.3**	**0.4**	**-0.2**	**0.1**	**0.3**
(e)	**1/14**	**2/14**	**3/14**	**4/14**	**5/14**	**6/14**	**15/14**

Solution:-

(a) Condition (i): Each of the numbers p(ω_i) is positive and less than zero.
Condition (ii): Sum of probabilities

0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1

Therefore, the given assignment is valid.

b) Condition (i): Each of the numbers p(ω_i) is positive and less than zero.
Condition (ii): Sum of probabilities

= (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7)

= 7/7

= 1

Therefore, the given assignment is valid.

c) Condition (i): Each of the numbers p(ω_i) is positive and less than zero.
Condition (ii): Sum of probabilities

= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7

$= 2.8 > 1$

Therefore, the 2nd condition is not satisfied

Which states that $p(w_i) \leq 1$

So, the given assignment is not valid.

d) The conditions of the axiomatic approach don't hold true in the given assignment, that is

1) Each of the numbers $p(w_i)$ is less than zero but also negative.

To be true, each of the numbers $p(w_i)$ should be less than zero and positive.

So, the assignment is not valid.

e) Condition (i): Each of the numbers $p(\omega_i)$ is positive and less than zero.
Condition (ii): Sum of probabilities

$= (1/14) + (2/14) + (3/14) + (4/14) + (5/14) + (6/14) + (7/14)$

$= (28/14) \geq 1$

The second condition doesn't hold true, so the assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

Here coin is tossed twice, then the sample space is S = (TT, HH, TH, HT)

∴ Number of possible outcomes n (S) = 4

Let A be the event of getting at least one tail.

∴ n (A) = 3

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

P(A) = n(A)/n(S)

= ¾

3. A die is thrown, find the probability of the following events.

(i) A prime number will appear.

(ii) A number greater than or equal to 3 will appear.

(iii) A number less than or equal to one will appear.

(iv) A number more than 6 will appear.

(v) A number less than 6 will appear.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

Here, S = {1, 2, 3, 4, 5, 6}

∴n(S) = 6

(i) A prime number will appear.

Let us assume 'A' be the event of getting a prime number,

A = {2, 3, 5}

Then, n(A) = 3

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(A) = n(A)/n(S)

= 3/6

= ½

(ii) A number greater than or equal to 3 will appear.

Let us assume 'B' be the event of getting a number greater than or equal to 3, B = {3, 4, 5, 6}

Then, n(B) = 4

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(B) = n(B)/n(S)

= 4/6

= 2/3

(iii) A number less than or equal to one will appear.

Let us assume ‘C’ be the event of getting a number less than or equal to 1,

C = {1}

Then, n (C) = 1

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(C) = n(C)/n(S)

= 1/6

(iv) A number more than 6 will appear.

Let us assume ‘D’ be the event of getting a number more than 6, then

D = {0)}

Then, n (D) = 0

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(D) = n(D)/n(S)

= 0/6

= 0

(v) A number less than 6 will appear.

Let us assume 'E' be the event of getting a number less than 6, then

E= (1, 2, 3, 4, 5)

Then, n (E) = 5

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(E) = n(E)/n(S)

= 5/6

4. A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card

Solution:-

From the question, it is given that there are 52 cards in the deck.

(a) Number of points in the sample space = 52 (given)

∴n(S) = 52

(b) Let us assume 'A' be the event of drawing an ace of spades.

A= 1

Then, n (A) = 1

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(A) = n(A)/n(S)

= 1/52

(c) Let us assume 'B' be the event of drawing an ace. There are four aces.

Then, n (B)= 4

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(B) = n(B)/n(S)

= 4/52

= 1/13

(d) Let us assume 'C' be the event of drawing a black card. There are 26 black cards.

Then, n (C) = 26

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(C) = n(C)/n(S)

= 26/52

= ½

5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

So, the sample space S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),(6, 6)}

Then, n(S) = 12

(i) Let us assume 'P' be the event having the sum of numbers as 3.

P = {(1, 2)},

Then, n (P) = 1

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(P) = n(P)/n(S)

= 1/12

(ii) Let us assume 'Q' be the event having the sum of the number as 12.

Then Q = {(6, 6)}, n (Q) = 1

P(Event) = Number of outcomes favorable to the event/ Total number of possible outcomes

∴P(Q) = n(Q)/n(S)

= 1/12

6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Solution:-

From the question, it is given that there are four men and six women on the city council.

Here total members in the council = 4 + 6 = 10,

Hence, the sample space has 10 points.

∴ n (S) = 10

The number of women is 6 … [given]

Let us assume 'A' be the event of selecting a woman.

Then n (A) = 6

P(Event) = Number of outcomes favorable to the event/Total number of possible outcomes

∴P(A) = n(A)/n(S)

= 6/10 … [divide both numerator and denominators by 2]

= 3/5

7. A fair coin is tossed four times, and a person wins Rs 1 for each head and loses Rs 1.50 for each tail that turns up.

From the sample space, calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

But, now coin is tossed four times, so the possible sample space contains,

S = (HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH,

TTTH, TTHT, THTT, HTTT, TTTT)

As per the condition given the question, a person will win or lose money depending upon the face of the coin, so

(i) For 4 heads = 1 + 1 + 1 + 1 = ₹ 4

So, he wins ₹ 4.

(ii) For 3 heads and 1 tail = 1 + 1 + 1 – 1.50

= 3 – 1.50

= ₹ 1.50

So, he will be winning ₹ 1.50.

(iii) For 2 heads and 2 tails = 1 + 1 – 1.50 – 1.50

= 2 – 3

= – ₹ 1

So, he will be losing ₹ 1.

(iv)For 1 head and 3 tails = 1 – 1.50 – 1.50 – 1.50

= 1 – 4.50

= – ₹ 3.50

So, he will be losing Rs. 3.50.

(v) For 4 tails = – 1.50 – 1.50 – 1.50 – 1.50

= – ₹ 6

So, he will be losing Rs. 6.

Now, the sample space of amounts is

S= {4, 1.50, 1.50, 1.50, 1.50, – 1, – 1, – 1, – 1, – 1, – 1, – 3.50, – 3.50, – 3.50, – 3.50, – 6}

Then, n (S) = 16

P (winning ₹ 4) = 1/16

P (winning ₹ 1.50) = 4/16 … [divide both numerator and denominator by 4]

= ¼

P (winning ₹ 1) = 6/16 … [divide both numerator and denominator by 2]

= 3/8

P (winning ₹ 3.50) = 4/16 … [divide both numerator and denominator by 4]

= ¼

P (winning ₹ 6) = 1/16

= 3/8

8. Three coins are tossed once. Find the probability of getting

(i) 3 heads (ii) 2 heads (iii) at least 2 heads

(iv) at most 2 heads (v) no head (vi) 3 tails

(vii) Exactly two tails (viii) no tail (ix) at most two tails

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

But, now three coins are tossed, so the possible sample space contains

S = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

Where s is sample space and here n(S) = 8

(i) 3 heads

Let us assume ‘A’ be the event of getting 3 heads.

n(A)= 1

∴P(A) = n(A)/n(S)

= 1/8

(ii) 2 heads

Let us assume ‘B’ be the event of getting 2 heads.

n (A) = 3

∴P(B) = n(B)/n(S)

= 3/8

(iii) at least 2 heads

Let us assume ‘C’ be the event of getting at least 2 heads.

n(C) = 4

∴P(C) = n(C)/n(S)

= 4/8

= ½

(iv) at most 2 heads

Let us assume 'D' be the event of getting at most 2 heads.

n(D) = 7

∴P(D) = n(D)/n(S)

= 7/8

(v) no head

Let us assume 'E' be the event of getting no heads.

n(E) = 1

∴P(E) = n(E)/n(S)

= 1/8

(vi) 3 tails

Let us assume 'F' be the event of getting 3 tails.

n(F) = 1

∴P(F) = n(F)/n(S)

= 1/8

(vii) Exactly two tails

Let us assume 'G' be the event of getting exactly 2 tails.

n(G) = 3

∴P(G) = n(G)/n(S)

= 3/8

(viii) no tail

Let us assume 'H' be the event of getting no tails.

n(H) = 1

∴P(H) = n(H)/n(S)

= 1/8

(ix) at most two tails

Let us assume 'I' be the event of getting at most 2 tails.

n(I) = 7

∴P(I) = n(I)/n(S)

= 7/8

9. If 2/11 is the probability of an event, what is the probability of the event 'not A'.

Solution:-

From the question, it is given that 2/11 is the probability of an event A.

i.e. P (A) = 2/11

Then,

P (not A) = 1 – P (A)

= 1 – (2/11)

= (11 – 2)/11

= 9/11

10. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that the letter is (i) a vowel (ii) a consonant

Solution:-

The word given in the question is 'ASSASSINATION'.

Total letters in the given word = 13

Number of vowels in the given word = 6

Number of consonants in the given word = 7

Then, the sample space n(S) = 13

(i) a vowel

Let us assume 'A' be the event of selecting a vowel.

n(A) = 6

∴P(A) = n(A)/n(S)

= 6/13

(ii) Let us assume 'B' be the event of selecting the consonant.

n(B)= 7

∴P(B) = n(B)/n(S)

= 7/13

11. In a lottery, a person chooses six different natural numbers at random, from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [Hint order of the numbers is not important.]

Solution:-

From the question, it is given that

Total numbers of numbers in the draw = 20

Numbers to be selected = 6

$\therefore$ n (S) = 20_{c_6}

Let us assume 'A' be the event that six numbers match with the six numbers already fixed by the lottery committee.

$\therefore\ n(A) = 6_{c_6} = 1$

Probability of winning the prize

$$P(A) = \frac{n(A)}{n(S)} = \frac{6_{c_6}}{20_{c_6}} = \frac{6!14!}{20!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 14!}{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14!}$$

$$= \frac{1}{38760}$$

12. Check whether the following probabilities P(A) and P(B) are consistently defined

(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6

(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

Solution:-

(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6

P(A ∩ B) > P(A)

Therefore, the given probabilities are not consistently defined.

(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

Then,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

0.8 = 0.5 + 0.4 – P(A ∩ B)

Transposing – P(A ∩ B) to LHS and it becomes P(A ∩ B) and 0.8 to RHS, and it becomes – 0.8.

P(A ∩ B) = 0.9 – 0.8

= 0.1

Therefore, P(A ∩ B) < P(A) and P(A ∩ B) < P(B)

So, the given probabilities are consistently defined.

13. Fill in the blanks in the following table.

	P(A)	**P(B)**	**P(A ∩ B)**	**P(A ∪ B)**
(i)	**1/3**	**1/5**	**1/15**	**….**
(ii)	**0.35**	**…..**	**0.25**	**0.6**
(iii)	**0.5**	**0.35**	**….**	**0.7**

Solution:-

From the given table,

(i) P(A) = 1/3, P(B) = 1/5, P(A ∩ B) = 1/15, P(A ∪ B) = ?

We know that,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

= (1/3) + (1/5) – (1/15)

= ((5 + 3)/15) – (1/15)

= (8/15) – (1/15)

= (8 – 1)/15

= 7/15

(ii) P(A) = 0.35, P(B) = ?, P(A ∩ B) = 0.25, P(A ∪ B) = 0.6

Then,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

0.6 = 0.35 + P(B) – 0.25

Transposing – 0.25, 0.35 to LHS, and it becomes 0.25 and – 0.35.

P(B) = 0.6 + 0.25 – 0.35

= 0.5

(iii) P(A) = 0.5, P(B) = 0.35, P(A ∪ B) = 0.7, P(A ∩ B) = ?

Then,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

0.7 = 0.5 + 0.35 – P(A ∩ B)

Transposing – P(A ∩ B) to LHS and it becomes P(A ∩ B) and 0.7 to RHS, and it becomes – 0.7.

P(A ∩ B) = 0.85 – 0.7

= 0.15

14. Given P(A) = 5/3 and P(B) = 1/5 . Find P(A or B), if A and B are mutually exclusive events.

Solution:-

From the question, it is given that

P(A) = 5/3 and P(B) = 1/5

Then, P(A or B), if A and B are mutually exclusive

P(A∪B) or P(A or B) = P(A) + P(B)

= (3/5) + (1/5)

= 4/5

15. If E and F are events such that P(E) = ¼ , P(F) = ½ and P(E and F) = 1/8 , find

(i) P(E or F), (ii) P(not E and not F)

Solution:-

From the question, we have P(E) = ¼ , P(F) = ½ and P(E ∩ F) = 1/8

(i) P(E or F) i.e., P(E∪F) = P(E) + P(F) – P(E ∩ F)

= ¼ + ½ – (1/8)

= 5/8

(ii) $P(\text{not E and not F}) = P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$

= 1 – (5/8)

= (8 – 5)/8

= 3/8

16. Events E and F are such that P(not E or not F) = 0.25, State whether E and F are mutually exclusive.

Solution:-

From the question it is given that, P(not E and not F) = 0.25

So, $P(\bar{E} \cup \bar{F}) = 0.25$

Then we have,

$\Rightarrow P(\overline{E \cap F}) = 0.25$

$\Rightarrow 1 - P(E \cap F) = 0.25$

$\Rightarrow P(E \cap F) = 1 - 0.25$

$= 0.75$

$P(E \cap F) \neq 0$

Hence, E and F are not mutually exclusive events.

17. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine (i) P(not A), (ii) P(not B) and (iii) P(A or B)

Solution:-

From the question, it is given that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16.

(i) $P(\text{not } A) = 1 - P(A)$

$= 1 - 0.42$

$= 0.58$

(ii) $P(\text{not } B) = 1 - P(B)$

$= 1 - 0.48$

$= 0.52$

(iii) $P(A \text{ not } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16$

$= 0.74$

18. In Class XI of a school, 40% of the students study Mathematics, and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Solution:-

Let us assume 'A' be the event that the student is studying mathematics and 'B' be the event that the student is studying biology.

So, $P(A) = 40/100$

$= 2/5$

And, $P(B) = 30/100$

$= 3/10$

Then, $P(A \cap B) = (10/100)$

$= 1/10$, $P(A \cap B)$ is probability of studying both mathematics and biology.

Here, Probability of studying mathematics or biology will be given by $P(A \cup B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= (2/5) + (3/10) - (1/10)$

$= 6/10$

$= 3/5$

Hence, (3/5) is the probability that the student will studying mathematics or biology

19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8, and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Solution:-

Let us assume the probability of a randomly chosen student passing the first examination is 0.8 be P(A).

And also, assume the probability of passing the second examination is 0.7 be P(B).

Then,

$P(A\cup B)$ is the probability of passing at least one of the examinations.

Now,

$P(A\cup B) = 0.95$, $P(A)=0.8$, $P(B)=0.7$

$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.95 = 0.8 + 0.7 - P(A\cap B)$

Transposing $- P(A \cap B)$ to LHS and it becomes $P(A \cap B)$ and 0.95 to RHS, and it becomes

$- 0.95$

$P(A\cap B) = 1.5 - 0.95$

$= 0.55$

Hence, 0.55 is the probability that the student will pass both examinations.

20. The probability that a student will pass the final examination in both English and Hindi is 0.5, and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Solution:-

Let us assume the probability of passing the English examination is 0.75 be P(A).

And also, assume the probability of passing the Hindi examination is P(B).

Here given, $P(A) = 0.75$, $P(A \cap B) – 0.5$, $P(A^I \cap B^I) = 0.1$

We know that, $P(A^I \cap B^I) = 1 – P(A \cup B)$

Then, $P(A \cup B) = 1 – P(A^I \cap B^I)$

$= 1 – 0.1$

$= 0.9$

$\therefore P(A \cup B) = P(A) + P(B) – P(A \cap B)$

$0.9 = 0.75 + P(B) – 0.5$

Transposing 0.75, – 0.5 to LHS, and it becomes – 0.75, 0.5.

$P(B) = 0.9 + 0.5 – 0.75$

$= 0.65$

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) The student opted for NCC or NSS.

(ii) The student has opted for neither NCC nor NSS.

(iii) The student has opted for NSS but not NCC.

Solution:-

From the question, it is given that

The total number of students in the class = 60

Thus, the sample space consists of $n(S) = 60$

Let us assume that the students opted for NCC to be 'A'.

And also, assume that the students opted for NSS to be 'B'.

So, $n(A) = 30$, $n(B) = 32$, $n(A\cap B) = 24$

We know that, $P(A) = n(A)/n(S)$

$= 30/60$

$= 1/2$

$P(B) = n(B)/n(S)$

$= 32/60$

$= 8/15$

$P(A\cap B) = n(A\cap B)/n(S)$

$= 24/60$

$= 2/5$

Therefore, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

(i) The student opted for NCC or NSS.

P (A or B) = P(A) + P(B) –P(A and B)

P(A∪B) = P(A) + P(B) – P(A∩B)

= ½ + (8/15) – (2/5)

= 19/30

(ii) P(student opted neither NCC nor NSS)

P(not A and not B) = $P(A^I \cap B^I)$

We know that, $P(A^I \cap B^I)$ = 1 – P(A∪B)

= 1 – (19/30)

= 11/30

(iii) P(student opted NSS but not NCC)

n(B – A) = n(B) – n (A∩B)

⇒ 32 – 24 = 8

The probability that the selected student has opted for NSS and not NCC is

= (8/60) = 2/15

2Marks Questions & Answers

1. Three coins are tossed simultaneously to list the sample space for the event.
Ans: S = HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

2. 20 cards are numbered from 1 to 20. One card is then drawn at random. What is the prob. of a prime number?

Ans: Let 'E' be the event of getting prime number and S be the sample space.

Therefore, n(S) n(S) = {1, 2, 3…..20}

n(E)n(E)= {2,3,5,7,11,13,17,19}

Probability P (E) = $\frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$.

3. If A and B are two mutually exclusive events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$. Find P (A or B).

Ans: We know that,

$$P(AorB) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \emptyset$$

$$= \frac{5}{6}$$

4. What is the chance that a leap year, selected at random, will contain 53 Sundays.

Ans: The total number of days in a leap year is 366 and there are 52 complete weeks and two days over.

The 2 days may be (Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday) or (Saturday, Sunday)

P (a leap year has 53 Sunday) = $\frac{2}{7}$.

5. A and B are two mutually exclusive events of an experiment. If P (not A) = 0.65, P (A∪B) = 0.65, P (B) = k, find k.

Ans:

$P(A\cup B) = P(A) + P(B)P(A\cup B) = P(A) + P(B)$

P (A∪B) = 1 - P (not A) + P (B)

0.65 = 1 - 0.65 + k

k = 0.30

6. Three coins are tossed once. Find the probability at most two heads.

Ans: S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

E = {HHT, THH, HTH, HTT, THT, TTH, TTT}

P (E) = $\frac{7}{8}$.

7. Cards are drawn from a well shuffled deck of 52 cards. What is the prob. Of obtaining 3 diamonds and one spade.

Ans:

$\frac{13C3\times\ 13C1}{52C4} = \frac{286}{20825}$ (Since, one ace out of 13 and 3 spades out of 13)

8. One card is drawn from a pack of 52 cards; find the probability that the drawn card is either red or king.

Ans: P= $\frac{26+2}{52}$

$= \frac{28}{52}$

$= \frac{7}{13}$.

9. a coin is tossed three times consider the following event a: No head appears, B: Exactly one head appears and C: At least two heads appears do they form a set of mutually exclusive and exhaustive events.

Ans: S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

From the given data, A={TTT}, B={HTT, THT, TTH}, C={HHT, HTH, THH, HHH}

A∪B∪C = S

Therefore, A, B and C are exhaustive events.

Also, A∩B = ∅, A∩C=∅, C∩C=∅, disjoint i.e. they are mutually exclusive.

10. Find the prob. that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains (i) all king (ii) 3 kings (iii) at least 3 kings

Ans: P (all king) = $\frac{4C4 \times 48C3}{52C7}\ \frac{1}{7735}$

P (3 king) = $\frac{4C4 \times 48C3}{52C7} = \frac{9}{1547}$

P (at least 3 kings) = P (3 king) + P (4 king) =

$$\frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}$$

Multiple Choice Questions

Q.1: What is the total number of sample spaces when a die is thrown 2 times?

A. 6
B. 12

C. 18

D. 36

Answer: D. 36

Explanation: The possible outcomes when a die is thrown are 1, 2, 3, 4, 5, and 6.

Given, a die is thrown two times.

Then, the total number of sample spaces = (6 × 6)

= 36

Q.2: What is the total number of sample spaces when a coin is tossed and a die is thrown?

A. 6

B. 12

C. 8

D. 16

Answer: B. 12

Explanation: The possible outcomes when a coin is tossed are Head (H) or Tail (T).

The possible outcomes when a die is thrown are 1, 2, 3, 4, 5, and 6.

Then, total number of space = (2 × 6) = 12

Q.3: Three identical dice are rolled. What is the probability that the same number will appear on each of them?

A. 1/6

B. 1/36

C. 1/18

D. 3/28

Answer: B. 1/36

Explanation:

Total number of cases = 6^3 = 216

The same number can appear on each of the dice in the following ways:

(1, 1, 1), (2, 2, 2), ………….(3, 3, 3)

So, favorable number of cases = 6

Hence, required probability = 6/216 = 1/36

Q.4: A bag contains 5 brown and 4 white socks. Ram pulls out two socks. What is the probability that both the socks are of the same colour?

A. 9/20

B. 2/9

C. 3/20

D. 4/9

Answer: D. 4/9

Explanation:

Total number of socks = 5 + 4 = 9

Two socks are pulled.

Now, P(Both are same color) = (5C2 + 4C2)/9C2

= {(5×4)/(2×1) + (4×3)/(2×1)}/{(9×8)/(2×1)}

= {(5×4) + (4×3)/}/{(9×8)

= (5 + 3)/(9×2)

= 8/18

= 4/9

Q.5: What is the probability of getting the number 6 at least once in a regular die if it can roll it 6 times?

A. $1 – (5/6)^6$

B. $1 – (1/6)^6$

C. $(5/6)^6$

D. $(1/6)^6$

Answer: A. $1 – (5/6)^6$

Explanation:

Let A be the event that 6 does not occur at all.

Now, the probability of at least one 6 occurs = 1 – PA.

= $1 – (5/6)^6$

Q.6: Events A and B are said to be mutually exclusive if:

A. P (A U B) = P A. + P B.

B. P (A ∩ B) = P A. × P B.

C. P(A U B) = 0

D. None of these

Answer: A. P (A U B) = P A. + P B.

Explanation:

If A and B are mutually exclusive events,

Then P(A ∩ B) = 0

Now, by the addition theorem,

P(A U B) = PA. + PB. – P(A ∩ B)

⇒ P(A U B) = PA. + PB.

Q.7: A die is rolled. What is the probability that an even number is obtained?

A. 1/2

B. 2/3

C. 1/4

D. ¾

Answer: A. 1/2

Explanation:

When a die is rolled, total number of outcomes = 6 (1, 2, 3, 4, 5, 6)

Total even number = 3 (2, 4, 6)

So, the probability that an even number is obtained = 3/6 = ½

Q.8: What is the probability of selecting a vowel in the word "PROBABILITY"?

A. 2/11

B. 3/11

C. 4/11

D. 5/11

Answer: B. 3/11

Q.9: An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. What is the probability that they are of different colors?

A.⅖

B.1/15

C.8/15

D.4/15

Answer: C. 8/15

Explanation:

Given that, the total number of balls = 6 balls

Let A and B be the red and black balls, respectively,

The probability that two balls are drawn are different = P (the first ball drawn is red)(the second ball drawn is black)+ P (the first ball drawn is black)P(the second ball drawn is red)

= (2/6)(4/5) + (4/6)(2/5)

=(8/30)+ (8/30)

= 16/30

= 8/15

Q.10: 20 cards are numbered from 1 to 20. If one card is drawn at random, what is the probability that the number on the card is a prime number?

A. ⅕

B. ⅖

C. ⅗

D. 5

Answer: B. ⅖

Explanation:

Let E be the event of getting a prime number.

E = {2, 3, 5, 7, 11, 13, 17, 19}

Hence, P(E) = 8/20 = 2/5.

Summary

In this Chapter, we studied about the axiomatic approach of probability. The main features of this Chapter are as follows:

- **Event:** A subset of the sample space
- **Impossible event :** The empty set
- **Sure event:** The whole sample space
- **Complementary event or 'not event' :** The set A′ or S – A
- **Event A or B:** The set A ∪ B
- **Event A and B:** The set A ∩ B
- **Event A and not B:** The set A – B
- **Mutually exclusive event:** A and B are mutually exclusive if A ∩ B = φ
- Exhaustive and mutually exclusive events: Events E1 , E2 ,..., En are mutually exclusive and exhaustive if $E_1 \cup E_2 \cup ... \cup E_n = S$
  And
  $E_i \cap E_j = \varphi \ \forall\ i \neq j$
- **Probability:** Number P (ωi) associated with sample point ω i such that
  (i) 0 ≤ P (ωi) ≤ 1 (ii) ∑P (ωi) for all ωi ∈ S = 1
  (iii) P (A) = ∑P (ωi) for all ωi ∈A.
  The number P (ωi) is called probability of the outcome ωi.
- **Equally likely outcomes:** All outcomes with equal probability
- **Probability of an event:** For a finite sample space with equally likely outcomes Probability of an event P (A)=$\frac{n(A)}{n(S)}$, where n(A) = number of elements in the set A, n(S) = number of elements in the set S.
- If A and B are any two events, then
  P (A or B) = P (A) + P (B) – P (A and B)

Equivalently, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- If A and B are mutually exclusive, then P(A or B) = P(A) + P(B)
- If A is any event, then

  P (not A) = 1 – P (A).