Chapter-4

Quadratic Equations

2 MARKS QUESTIONS

1. Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

Solutions:

(i) Given,

$(x + 1)^2 = 2(x - 3)$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 7 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$

Therefore, the given equation is a quadratic equation.

(ii) Given, $x^2 - 2x = (-2)(3 - x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 4x + 6 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$

Therefore, the given equation is a quadratic equation.

2. Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Solutions:

(i) Let us consider,

The breadth of the rectangular plot = x m

Thus, the length of the plot = (2x + 1) m

As we know,

Area of rectangle = length × breadth = 528 m^2

Putting the value of the length and breadth of the plot in the formula, we get,

$(2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x – 528 = 0$

Therefore, the length and breadth of the plot satisfy the quadratic equation, $2x^2 + x – 528 = 0$, which is the required representation of the problem mathematically.

3. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 – 3x – 10 = 0$

Solutions:

(i) Given, $x^2 – 3x – 10 =0$

Taking L.H.S.,

=>$x^2 – 5x + 2x – 10$

=>$x(x – 5) + 2(x – 5)$

=>$(x – 5)(x + 2)$

The roots of this equation, $x^2 – 3x – 10 = 0$ are the values of x for which $(x – 5)(x + 2) = 0$

Therefore, $x – 5 = 0$ or $x + 2 = 0$

=> $x = 5$ or $x = -2$

4. Find two numbers whose sum is 27 and product is 182.

Solution:

Let us say the first number is x, and the second number is 27 – x.

Therefore, the product of two numbers

$x(27 – x) = 182$

$\Rightarrow x^2 – 27x – 182 = 0$

$\Rightarrow x^2 – 13x – 14x + 182 = 0$

$\Rightarrow x(x – 13) -14(x – 13) = 0$

$\Rightarrow (x – 13)(x -14) = 0$

Thus, either, x = -13 = 0 or x – 14 = 0

⇒ x = 13 or x = 14

Therefore, if first number = 13, then second number = 27 – 13 = 14

And if first number = 14, then second number = 27 – 14 = 13

Hence, the numbers are 13 and 14.

5. Find two consecutive positive integers, the sum of whose squares is 365.

Solution:

Let us say the two consecutive positive integers are x and $x + 1$.

Therefore, as per the given questions,

$x^2 + (x + 1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) -13(x + 14) = 0$

$\Rightarrow (x + 14)(x - 13) = 0$

Thus, either, $x + 14 = 0$ or $x - 13 = 0$,

$\Rightarrow x = -14$ or $x = 13$

Since the integers are positive, x can be 13 only.

$\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

6. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

Solution:

On comparing the given equation with $ax^2 + bx + c = 0$, we get,

a = 2, b = -7 and c = 3

By using the quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow x = (7 \pm \sqrt{(49 - 24)})/4$

$\Rightarrow x = (7 \pm \sqrt{25})/4$

$\Rightarrow x = (7 \pm 5)/4$

$\Rightarrow x = (7+5)/4$ or $x = (7-5)/4$

$\Rightarrow x = 12/4$ or $2/4$

$\therefore\ x = 3$ or $1/2$

7. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.
(i) $2x^2 - 3x + 5 = 0$

Solutions:

(i) Given,

$2x^2 - 3x + 5 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -3$ and $c = 5$

We know, Discriminant = $b^2 - 4ac$

$= (-3)^2 - 4(2)(5) = 9 - 40$

$= -31$

As you can see, $b^2 - 4ac < 0$

Therefore, no real root is possible for the given equation, $2x^2 - 3x + 5 = 0$

8. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution:

Let us say the marks of Shefali in Maths be x.

Then, the marks in English will be 30 – x.

As per the given question,

$(x + 2)(30 - x - 3) = 210$

$(x + 2)(27 - x) = 210$

$\Rightarrow -x^2 + 25x + 54 = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

$\Rightarrow x^2 - 12x - 13x + 156 = 0$

$\Rightarrow x(x - 12) -13(x - 12) = 0$

$\Rightarrow (x - 12)(x - 13) = 0$

$\Rightarrow x = 12, 13$

Therefore, if the marks in Maths are 12, then marks in English will be 30 – 12 = 18, and if the marks in Maths are 13, then marks in English will be 30 – 13 = 17.

4 MARKS QUESTIONS

1. Represent the following situations in the form of quadratic equations:

(i) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Solution:

(i) Let us consider,

Age of Rohan's = x years

Therefore, as per the given question,

Rohan's mother's age = x + 26

After 3 years,

Age of Rohan's = x + 3

Age of Rohan's mother will be = x + 26 + 3 = x + 29

The product of their ages after 3 years will be equal to 360, such that

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x + 87 – 360 = 0$

$\Rightarrow x^2 + 32x – 273 = 0$

Therefore, the age of Rohan and his mother satisfies the quadratic equation, $x^2 + 32x – 273 = 0$, which is the required representation of the problem mathematically.

2. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

Let us say the base of the right triangle is x cm.

Given, the altitude of right triangle = $(x - 7)$ cm

From Pythagoras' theorem, we know,

$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$

$\therefore x^2 + (x - 7)^2 = 13^2$

$\Rightarrow x^2 + x^2 + 49 - 14x = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x - 12) + 5(x - 12) = 0$

$\Rightarrow (x - 12)(x + 5) = 0$

Thus, either $x - 12 = 0$ or $x + 5 = 0$,

$\Rightarrow x = 12$ or $x = -5$

Since sides cannot be negative, x can only be 12.

Therefore, the base of the given triangle is 12 cm, and the altitude of this triangle will be $(12 - 7)$ cm = 5 cm.

3. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs.90, find the number of articles produced and the cost of each article.

Solution:

Let us say the number of articles produced is x.

Therefore, cost of production of each article = Rs $(2x + 3)$

Given the total cost of production is Rs.90

$\therefore x(2x + 3) = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$

$\Rightarrow (2x + 15)(x - 6) = 0$

Thus, either $2x + 15 = 0$ or $x - 6 = 0$

$\Rightarrow x = -15/2$ or $x = 6$

As the number of articles produced can only be a positive integer, x can only be 6.

Hence, the number of articles produced = 6

Cost of each article = $2 \times 6 + 3$ = Rs 15

4. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

It is given that

Distance = 360 km

Consider x as the speed, then the time taken

t = 360/x

If the speed is increased by 5 km/h, the speed will be (x + 5) km/h.

Distance will be the same.

t = 360/(x + 5)

We know that

Time with original speed – Time with increased speed = 1

360/x – 360/(x + 5) = 1

LCM = x (x + 5)

[360 (x + 5) – 360x]/x(x + 5) = 1
360 x + 1800 – 360x = x (x + 5)

$x^2 + 5x = 1800$

$x^2 + 5x – 1800 = 0$

$x^2 + 45x – 40x – 1800 = 0$

x (x + 45) – 40 (x + 45) = 0

(x – 40) (x + 45) = 0

x = 40 km/hr

As we know, the value of speed cannot be negative.

Therefore, the speed of the train is 40 km/h.

5. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.
(i) $2x^2 - 3x + 5 = 0$
(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
(iii) $2x^2 - 6x + 3 = 0$

Solutions:

(i) Given,

$2x^2 - 3x + 5 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -3$ and $c = 5$

We know, Discriminant $= b^2 - 4ac$

$= (-3)^2 - 4(2)(5) = 9 - 40$

$= -31$

As you can see, $b^2 - 4ac < 0$

Therefore, no real root is possible for the given equation, $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 3$, $b = -4\sqrt{3}$ and $c = 4$

We know, Discriminant $= b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4)$

$= 48 – 48 = 0$

As $b^2 – 4ac = 0$,

Real roots exist for the given equation, and they are equal to each other.

Hence, the roots will be $–b/2a$ and $–b/2a$.

$–b/2a = -(-4\sqrt{3})/2\times3 = 4\sqrt{3}/6 = 2\sqrt{3}/3 = 2/\sqrt{3}$

Therefore, the roots are $2/\sqrt{3}$ and $2/\sqrt{3}$.

(iii) $2x^2 – 6x + 3 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -6$, $c = 3$

As we know, Discriminant $= b^2 – 4ac$

$= (-6)^2 – 4 (2) (3)$

$= 36 – 24 = 12$

As $b^2 – 4ac > 0$,

Therefore, there are distinct real roots that exist for this equation, $2x^2 – 6x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$= (-(-6) \pm \sqrt{(-6^2-4(2)(3))}\ )/ 2(2)$

$= (6\pm2\sqrt{3}\ )/4$

$= (3\pm\sqrt{3})/2$

Therefore, the roots for the given equation are $(3+\sqrt{3})/2$ and $(3-\sqrt{3})/2$

6. Find the values of *k* for each of the following quadratic equations so that they have two equal roots.
(i) $2x^2 + kx + 3 = 0$
(ii) $kx(x – 2) + 6 = 0$

Solutions:

(i) $2x^2 + kx + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get,

$a = 2$, $b = k$ and $c = 3$

As we know, Discriminant $= b^2 – 4ac$

$= (k)^2 – 4(2)(3)$

$= k^2 – 24$

For equal roots, we know,

Discriminant = 0

$k^2 – 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) $kx(x – 2) + 6 = 0$

or $kx^2 – 2kx + 6 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get

$a = k$, $b = – 2k$ and $c = 6$

We know, Discriminant $= b^2 – 4ac$

$= (– 2k)^2 – 4(k)(6)$

$= 4k^2 – 24k$

For equal roots, we know,

$b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k\ (k - 6) = 0$

Either $4k = 0$ or $k = 6 = 0$

$k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

7. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2? If so, find its length and breadth.

Solution:

Let the breadth of the mango grove be l.

The length of the mango grove will be $2l$.

Area of the mango grove = $(2l)\ (l) = 2l^2$

$2l^2 = 800$

$l^2 = 800/2 = 400$

$l^2 - 400 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = 0$, $c = 400$

As we know, Discriminant = $b^2 - 4ac$

=> $(0)^2 - 4 \times (1) \times (-400) = 1600$

Here, $b^2 - 4ac > 0$

Thus, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.

$l = \pm 20$

As we know, the value of length cannot be negative.

Therefore, the breadth of the mango grove = 20 m

Length of mango grove = 2 × 20 = 40 m

8. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their age in years was 48.

Solution:

Let's say the age of one friend is x years.

Then, the age of the other friend will be (20 – x) years.

Four years ago,

Age of First friend = $(x - 4)$ years

Age of Second friend = $(20 - x - 4) = (16 - x)$ years

As per the given question, we can write,

$(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = -20$ and $c = 112$

Discriminant = $b^2 – 4ac$

$= (-20)^2 – 4 \times 112$

$= 400 – 448 = -48$

$b^2 – 4ac < 0$

Therefore, there will be no real solution possible for the equations. Hence, the condition doesn't exist.

9. Is it possible to design a rectangular park of perimeter 80 and an area of 400 m2? If so, find its length and breadth.

Solution:

Let the length and breadth of the park be l and b.

Perimeter of the rectangular park = $2 (l + b) = 80$

So, $l + b = 40$

Or, $b = 40 – l$

Area of the rectangular park = $l \times b = l(40 – l) = 40l – l^2 = 400$

$l^2 – 40l + 400 = 0$, which is a quadratic equation.

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = -40$, $c = 400$

Since, Discriminant = $b^2 – 4ac$

$=(-40)^2 – 4 \times 400$

= 1600 – 1600 = 0

Thus, $b^2 - 4ac = 0$

Therefore, this equation has equal real roots. Hence, the situation is possible.

The root of the equation,

$l = -b/2a$

$l = -(-40)/2(1) = 40/2 = 20$

Therefore, the length of the rectangular park, $l = 20$ m

And the breadth of the park, $b = 40 - l = 40 - 20 = 20$ m.

7 MARKS QUESTIONS

1. Check whether the following are quadratic equations:

(i) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(ii) $(x - 3)(2x + 1) = x(x + 5)$

(iii) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(iv) $x^2 + 3x + 1 = (x - 2)^2$

(v) $(x + 2)^3 = 2x(x^2 - 1)$

(vi) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Solutions:

(i) Given, $(x - 2)(x + 1) = (x - 1)(x + 3)$

By multiplication,

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow 3x - 1 = 0$

The above equation is not in the form of $ax^2 + bx + c = 0$

Therefore, the given equation is not a quadratic equation.

(ii) Given, $(x - 3)(2x + 1) = x(x + 5)$

By multiplication,

$\Rightarrow 2x^2 - 5x - 3 = x^2 + 5x$

$\Rightarrow x^2 - 10x - 3 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$

Therefore, the given equation is a quadratic equation.

(iii) Given, $(2x – 1)(x – 3) = (x + 5)(x – 1)$

By multiplication,

$\Rightarrow 2x^2 – 7x + 3 = x^2 + 4x – 5$

$\Rightarrow x^2 – 11x + 8 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

(iv) Given, $x^2 + 3x + 1 = (x – 2)^2$

By using the formula for $(a-b)^2=a^2-2ab+b^2$

$\Rightarrow x^2 + 3x + 1 = x^2 + 4 – 4x$

$\Rightarrow 7x – 3 = 0$

The above equation is not in the form of $ax^2 + bx + c = 0$

Therefore, the given equation is not a quadratic equation.

(v) Given, $(x + 2)^3 = 2x(x^2 – 1)$

By using the formula for $(a+b)^3 = a^3+b^3+3ab(a+b)$

$\Rightarrow x^3 + 8 + x^2 + 12x = 2x^3 – 2x$

$\Rightarrow x^3 + 14x – 6x^2 – 8 = 0$

The above equation is not in the form of $ax^2 + bx + c = 0$

Therefore, the given equation is not a quadratic equation.

(vi) Given, $x^3 – 4x^2 – x + 1 = (x – 2)^3$

By using the formula for $(a-b)^3 = a^3-b^3-3ab(a-b)$

$\Rightarrow$ $x^3 – 4x^2 – x + 1 = x^3 – 8 – 6x^2 + 12x$

$\Rightarrow$ $2x^2 – 13x + 9 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$

Therefore, the given equation is a quadratic equation.

2. Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken

Solutions:

(i) Let us consider,

The breadth of the rectangular plot = x m

Thus, the length of the plot = (2x + 1) m

As we know,

Area of rectangle = length × breadth = 528 m^2

Putting the value of the length and breadth of the plot in the formula, we get,

$(2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

Therefore, the length and breadth of the plot satisfy the quadratic equation, $2x^2 + x - 528 = 0$, which is the required representation of the problem mathematically.

(ii) Let us consider,

The first integer number = x

Thus, the next consecutive positive integer will be = x + 1

Product of two consecutive integers = $x \times (x + 1) = 306$

$\Rightarrow x^2 + x = 306$

$\Rightarrow x^2 + x - 306 = 0$

Therefore, the two integers x and x+1 satisfy the quadratic equation, $x^2 + x - 306 = 0$, which is the required representation of the problem mathematically.

(iii) Let us consider,

The speed of the train = x km/h

And

Time taken to travel 480 km = 480/x km/hr

As per second condition, the speed of train = (x – 8) km/h

Also given, the train will take 3 hours to cover the same distance.

Therefore, time taken to travel 480 km = (480/x)+3 km/h

As we know,

Speed × Time = Distance

Therefore,

$(x - 8)(480/x) + 3 = 480$

$\Rightarrow 480 + 3x - 3840/x - 24 = 480$

$\Rightarrow 3x - 3840/x = 24$

$\Rightarrow x^2 - 8x - 1280 = 0$

Therefore, the speed of the train satisfies the quadratic equation, $x^2 - 8x - 1280 = 0$, which is the required representation of the problem mathematically.

3. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$
(ii) $2x^2 + x - 6 = 0$
(iii) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2} = 0$
(iv) $2x^2 - x + 1/8 = 0$
(v) $100x^2 - 20x + 1 = 0$

Solutions:

(i) Given, $x^2 - 3x - 10 = 0$

Taking L.H.S.,

$\Rightarrow x^2 - 5x + 2x - 10$

$\Rightarrow x(x - 5) + 2(x - 5)$

$\Rightarrow (x - 5)(x + 2)$

The roots of this equation, $x^2 - 3x - 10 = 0$ are the values of x for which $(x - 5)(x + 2) = 0$

Therefore, $x - 5 = 0$ or $x + 2 = 0$

$\Rightarrow x = 5$ or $x = -2$

(ii) Given, $2x^2 + x - 6 = 0$

Taking L.H.S.,

$\Rightarrow 2x^2 + 4x - 3x - 6$

$\Rightarrow 2x(x + 2) - 3(x + 2)$

$\Rightarrow (x + 2)(2x - 3)$

The roots of this equation, $2x^2 + x - 6=0$ are the values of x for which $(x\ x + 2)(2x - 3) = 0$

Therefore, $x + 2 = 0$ or $2x – 3 = 0$

=> $x = -2$ or $x = 3/2$

(iii) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2}=0$

Taking L.H.S.,

=> $\sqrt{2}\, x^2 + 5x + 2x + 5\sqrt{2}$

=> $x (\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)= (\sqrt{2}x + 5)(x + \sqrt{2})$

The roots of this equation, $\sqrt{2}\, x^2 + 7x + 5\sqrt{2}=0$ are the values of x for which $(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

Therefore, $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

=> $x = -5/\sqrt{2}$ or $x = -\sqrt{2}$

(iv) $2x^2 – x +1/8 = 0$

Taking L.H.S.,

$=1/8 (16x^2 – 8x + 1)$

$= 1/8 (16x^2 – 4x -4x + 1)$

$= 1/8 (4x(4x – 1) -1(4x – 1))$

$= 1/8 (4x – 1)^2$

The roots of this equation, $2x^2 – x + 1/8 = 0$, are the values of x for which $(4x – 1)^2= 0$

Therefore, $(4x – 1) = 0$ or $(4x – 1) = 0$

$\Rightarrow x = 1/4$ or $x = 1/4$

(v) Given, $100x^2 - 20x + 1=0$

Taking L.H.S.,

$= 100x^2 - 10x - 10x + 1$

$= 10x(10x - 1) -1(10x - 1)$

$= (10x - 1)^2$

The roots of this equation, $100x^2 - 20x + 1=0$, are the values of x for which $(10x - 1)^2= 0$

$\therefore (10x - 1) = 0$ or $(10x - 1) = 0$

$\Rightarrow x = 1/10$ or $x = 1/10$

4. Solve the problems given in Example 1.

Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

Solutions:

(i) Let us say the number of marbles John has = x

Therefore, the number of marble Jivanti has = $45 - x$

After losing 5 marbles each,

Number of marbles John has = $x - 5$

Number of marble Jivanti has = $45 - x - 5 = 40 - x$

Given that the product of their marbles is 124.

$\therefore (x - 5)(40 - x) = 124$

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) -9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

Thus, we can say,

$x - 36 = 0$ or $x - 9 = 0$

$\Rightarrow x = 36$ or $x = 9$

Therefore,

If John's marbles = 36

Then, Jivanti's marbles = 45 – 36 = 9

And if John's marbles = 9

Then, Jivanti's marbles = 45 – 9 = 36

(ii) Let us say the number of toys produced in a day is x.

Therefore, cost of production of each toy = Rs$(55 - x)$

Given the total cost of production of the toys = Rs 750

$\therefore x(55 - x) = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

Thus, either $x - 25 = 0$ or $x - 30 = 0$

$\Rightarrow x = 25$ or $x = 30$

Hence, the number of toys produced in a day will be either 25 or 30.

5. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.
(i) $2x^2 - 3x + 5 = 0$
(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
(iii) $2x^2 - 6x + 3 = 0$

Solutions:

(i) Given,

$2x^2 - 3x + 5 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -3$ and $c = 5$

We know, Discriminant $= b^2 - 4ac$

$= (-3)^2 - 4(2)(5) = 9 - 40$

$= -31$

As you can see, $b^2 - 4ac < 0$

Therefore, no real root is possible for the given equation, $2x^2 - 3x + 5 = 0$

(ii) $3x^2 – 4\sqrt{3}x + 4 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 3$, $b = -4\sqrt{3}$ and $c = 4$

We know, Discriminant = $b^2 – 4ac$

$= (-4\sqrt{3})^2 – 4(3)(4)$

$= 48 – 48 = 0$

As $b^2 – 4ac = 0$,

Real roots exist for the given equation, and they are equal to each other.

Hence, the roots will be $–b/2a$ and $–b/2a$.

$–b/2a = -(-4\sqrt{3})/2×3 = 4\sqrt{3}/6 = 2\sqrt{3}/3 = 2/\sqrt{3}$

Therefore, the roots are $2/\sqrt{3}$ and $2/\sqrt{3}$.

(iii) $2x^2 – 6x + 3 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -6$, $c = 3$

As we know, Discriminant = $b^2 – 4ac$

$= (-6)^2 – 4 (2) (3)$

$= 36 – 24 = 12$

As $b^2 – 4ac > 0$,

Therefore, there are distinct real roots that exist for this equation, $2x^2 – 6x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

= (-(-6) ± √(-6^2-4(2)(3)))/ 2(2)

= (6±2√3)/4

= (3±√3)/2

Therefore, the roots for the given equation are (3+√3)/2 and (3-√3)/2