Chapter 8

Binomial Theorem

The binomial theorem is a significant algebraic concept explored in Chapter 8 of NCERT Class 11 Mathematics. This theorem provides a systematic way to expand expressions of the form (a + b)n, where *a* and *b* are any real numbers or variables, and n is a positive integer.

The binomial theorem simplifies the process of expanding expressions involving binomials, providing a concise and efficient way to represent these expressions without having to multiply out each term individually. By understanding and applying the binomial theorem, students gain insights into the algebraic structure of binomial expressions, and they can easily compute the expansion of expressions raised to any positive integer power.

Exercise 8.1

Expand each of the expressions in Exercises 1 to 5.

1. $(1 - 2x)^5$

Solution:

From binomial theorem expansion, we can write as

$(1 - 2x)^5$

$= {}^5C_0 (1)^5 - {}^5C_1 (1)^4 (2x) + {}^5C_2 (1)^3 (2x)^2 - {}^5C_3 (1)^2 (2x)^3 + {}^5C_4 (1)^1 (2x)^4 - {}^5C_5 (2x)^5$

$= 1 - 5 (2x) + 10 (4x)^2 - 10 (8x^3) + 5 (16 x^4) - (32 x^5)$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

2. $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

Solution:

From the binomial theorem, the given equation can be expanded as

$$\left(\frac{2}{x}-\frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2$$
$$- {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5$$
$$= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32}$$
$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$$

3. $(2x - 3)^6$

Solution:

From the binomial theorem, the given equation can be expanded as

$$(2x-3)^6 = {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_1(2x)^4(3)^2 - {}^4C_3(2x)^3(3)^3$$
$$= 64x^6 - 6\left(32x^5\right)(3) + 15\left(16x^4\right)(9) - 20\left(8x^3\right)(27)$$
$$+ 15\left(4x^2\right)(81) - 6(2x)(243) + 729$$
$$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$$

4. $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Solution:

From the binomial theorem, the given equation can be expanded as

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5 + {}^3C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^3C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2$$
$$= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5}$$
$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^3}$$

5. $\left(x+\frac{1}{x}\right)^6$

Solution:

From the binomial theorem, the given equation can be expanded as

$$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x)'\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2$$
$$+ {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_3(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$
$$= x^4 + 6(x)^3\left(\frac{1}{x}\right) + 15(x)^4\left(\frac{1}{x^2}\right) + 20(x)^3\left(\frac{1}{x^3}\right) + 15(x)^2\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + \frac{1}{x^6}$$
$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

6. Using the binomial theorem, find $(96)^3$.

Solution:

Given $(96)^3$

96 can be expressed as the sum or difference of two numbers, and then the binomial theorem can be applied.

The given question can be written as 96 = 100 – 4

$(96)^3 = (100 - 4)^3$

$= {}^3C_0 (100)^3 - {}^3C_1 (100)^2 (4) - {}^3C_2 (100) (4)^2 - {}^3C_3 (4)^3$

$= (100)^3 - 3 (100)^2 (4) + 3 (100) (4)^2 - (4)^3$

$= 1000000 - 120000 + 4800 - 64$

$= 884736$

7. Using the binomial theorem, find $(102)^5$.

Solution:

Given $(102)^5$

102 can be expressed as the sum or difference of two numbers, and then the binomial theorem can be applied.

The given question can be written as $102 = 100 + 2$

$(102)^5 = (100 + 2)^5$

$= {}^5C_0 (100)^5 + {}^5C_1 (100)^4 (2) + {}^5C_2 (100)^3 (2)^2 + {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100) (2)^4 + {}^5C_5 (2)^5$

$= (100)^5 + 5 (100)^4 (2) + 10 (100)^3 (2)^2 + 5 (100) (2)^3 + 5 (100) (2)^4 + (2)^5$

$= 1000000000 + 1000000000 + 40000000 + 80000 + 8000 + 32$

$= 11040808032$

8. Using the binomial theorem, find $(101)^4$.

Solution:

Given $(101)^4$

101 can be expressed as the sum or difference of two numbers, and then the binomial theorem can be applied.

The given question can be written as $101 = 100 + 1$

$(101)^4 = (100 + 1)^4$

$= {}^4C_0\,(100)^4 + {}^4C_1\,(100)^3\,(1) + {}^4C_2\,(100)^2\,(1)^2 + {}^4C_3\,(100)\,(1)^3 + {}^4C_4\,(1)^4$

$= (100)^4 + 4\,(100)^3 + 6\,(100)^2 + 4\,(100) + (1)^4$

$= 100000000 + 4000000 + 60000 + 400 + 1$

$= 104060401$

9. Using the binomial theorem, find $(99)^5$m.

Solution:

Given $(99)^5$

99 can be written as the sum or difference of two numbers then the binomial theorem can be applied.

The given question can be written as 99 = 100 -1

$(99)^5 = (100 – 1)^5$

$= {}^5C_0\,(100)^5 – {}^5C_1\,(100)^4\,(1) + {}^5C_2\,(100)^3\,(1)^2 – {}^5C_3\,(100)^2\,(1)^3 + {}^5C_4\,(100)\,(1)^4 – {}^5C_5\,(1)^5$

$= (100)^5 – 5\,(100)^4 + 10\,(100)^3 – 10\,(100)^2 + 5\,(100) – 1$

$= 1000000000 – 5000000000 + 10000000 – 100000 + 500 – 1$

$= 9509900499.$

10. Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Solution:

By splitting the given 1.1 and then applying the binomial theorem, the first few terms of $(1.1)^{10000}$ can be obtained as

$(1.1)^{10000} = (1 + 0.1)^{10000}$

$= (1 + 0.1)^{10000}\,C_1\,(1.1)$ + other positive terms

$= 1 + 10000 \times 1.1$ + other positive terms

$= 1 + 11000$ + other positive terms

> 1000

$(1.1)^{10000} > 1000$

11. Find $(a + b)^4 – (a – b)^4$. Hence, evaluate

$$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 .$$

Solution:

Using the binomial theorem, the expression $(a + b)^4$ and $(a – b)^4$ can be expanded

$(a + b)^4 = {}^4C_0\, a^4 + {}^4C_1\, a^3 b + {}^4C_2\, a^2 b^2 + {}^4C_3\, a b^3 + {}^4C_4\, b^4$

$(a – b)^4 = {}^4C_0\, a^4 – {}^4C_1\, a^3 b + {}^4C_2\, a^2 b^2 – {}^4C_3\, a b^3 + {}^4C_4\, b^4$

Now $(a + b)^4 – (a – b)^4 = {}^4C_0\, a^4 + {}^4C_1\, a^3 b + {}^4C_2\, a^2 b^2 + {}^4C_3\, a b^3 + {}^4C_4\, b^4 – [{}^4C_0\, a^4 – {}^4C_1\, a^3 b + {}^4C_2\, a^2 b^2 – {}^4C_3\, a b^3 + {}^4C_4\, b^4]$

$= 2\, ({}^4C_1\, a^3 b + {}^4C_3\, a b^3)$

$= 2\, (4a^3 b + 4ab^3)$

$= 8ab\, (a^2 + b^2)$

Now by substituting $a = \sqrt{3}$ and $b = \sqrt{2}$, we get

$(\sqrt{3} + \sqrt{2})^4 – (\sqrt{3} – \sqrt{2})^4 = 8\, (\sqrt{3})\, (\sqrt{2})\, \{(\sqrt{3})^2 + (\sqrt{2})^2\}$

$= 8\, (\sqrt{6})\, (3 + 2)$

$= 40\, \sqrt{6}$

12. Find $(x + 1)^6 + (x – 1)^6$. Hence or otherwise evaluate

$$\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$$

Solution:

Using binomial theorem, the expressions $(x + 1)^6$ and $(x – 1)^6$ can be expressed as

$(x + 1)^6 = {}^6C_0\, x^6 + {}^6C_1\, x^5 + {}^6C_2\, x^4 + {}^6C_3\, x^3 + {}^6C_4\, x^2 + {}^6C_5\, x + {}^6C_6$

$(x - 1)^6 = {}^6C_0\, x^6 - {}^6C_1\, x^5 + {}^6C_2\, x^4 - {}^6C_3\, x^3 + {}^6C_4\, x^2 - {}^6C_5\, x + {}^6C_6$

Now, $(x + 1)^6 - (x - 1)^6 = {}^6C_0\, x^6 + {}^6C_1\, x^5 + {}^6C_2\, x^4 + {}^6C_3\, x^3 + {}^6C_4\, x^2 + {}^6C_5\, x + {}^6C_6 - [{}^6C_0\, x^6 - {}^6C_1\, x^5 + {}^6C_2\, x^4 - {}^6C_3\, x^3 + {}^6C_4\, x^2 - {}^6C_5\, x + {}^6C_6]$

$= 2\, [{}^6C_0\, x^6 + {}^6C_2\, x^4 + {}^6C_4\, x^2 + {}^6C_6]$

$= 2\, [x^6 + 15x^4 + 15x^2 + 1]$

Now by substituting $x = \sqrt{2}$, we get

$(\sqrt{2} + 1)^6 - (\sqrt{2} - 1)^6 = 2\, [(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1]$

$= 2\, (8 + 15 \times 4 + 15 \times 2 + 1)$

$= 2\, (8 + 60 + 30 + 1)$

$= 2\, (99)$

$= 198.$

13. Show that $9^{n+1} - 8n - 9$ is divisible by 64 whenever n is a positive integer.

Solution:

In order to show that $9^{n+1} - 8n - 9$ is divisible by 64, it has to be shown that $9^{n+1} - 8n - 9 = 64\, k$, where k is some natural number.

Using the binomial theorem,

$(1 + a)^m = {}^mC_0 + {}^mC_1\, a + {}^mC_2\, a^2 + \ldots + {}^mC_m\, a^m$

For a = 8 and m = n + 1 we get

$(1 + 8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1\, (8) + {}^{n+1}C_2\, (8)^2 + \ldots + {}^{n+1}C_{n+1}\, (8)^{n+1}$

$9^{n+1} = 1 + (n + 1)\, 8 + 8^2\, [{}^{n+1}C_2 + {}^{n+1}C_3\, (8) + \ldots + {}^{n+1}C_{n+1}\, (8)^{n-1}]$

$9^{n+1} = 9 + 8n + 64\, [{}^{n+1}C_2 + {}^{n+1}C_3\, (8) + \ldots + {}^{n+1}C_{n+1}\, (8)^{n-1}]$

$9^{n+1} - 8n - 9 = 64\, k$

Where k = $[{}^{n+1}C_2 + {}^{n+1}C_3\,(8) + \ldots + {}^{n+1}C_{n+1}\,(8)^{n-1}]$ is a natural number

Thus, $9^{n+1} - 8n - 9$ is divisible by 64 whenever n is a positive integer.

Hence proved.

14. Prove that

$$\sum_{r=0}^{n} 3^r\,{}^nC_r = 4^n$$

Solution:

By Binomial Theorem

$$\sum_{r=0}^{n} \binom{n}{r} a^{n-r}b^r = (a+b)^n$$

On right side we need 4^n so we will put the values as,
Putting b = 3 & a = 1 in the above equation, we get

$$\sum_{r=0}^{n} \binom{n}{r} (1)^{n-r}(3)^r = (1+3)^n$$

$$\sum_{r=0}^{n} \binom{n}{r} (1)(3)^r = (4)^n$$

$$\sum_{r=0}^{n} \binom{n}{r} (3)^r = (4)^n$$

Hence Proved.

2Marks Questions & Answers

1. Expand $\left(x^2 + \frac{3}{x}\right)^4$ x ≠ 0

Ans: By using binomial theorem, we have

$$\left(x^2 + \frac{3}{x}\right)^4 = {}^4C_0\,(x^2)^4 + {}^4C_1\,(x^2)^3\left(\frac{3}{x}\right) + {}^4C_2\,(x^2)^2\left(\frac{3}{x}\right)^2 + {}^4C_3\,(x^2)\left(\frac{3}{x}\right)^3 + {}^4C_4\left(\frac{3}{x}\right)^4$$

$$= x^8 + 4.x^6.\frac{3}{x} + 6.x^4.\frac{9}{x^2} + 4.x^2.\frac{27}{x^3} + \frac{81}{x^4}$$

$= \boldsymbol{x^8 + 12x^5 + 54x^2 + \frac{108}{x} + \frac{81}{x^4}}$.

2. Compute$(98)^5$.

Ans: We express 98 as the sum or difference of two numbers whose powers are easier to calculate, and then use Binomial Theorem. Write 98 = 100 – 2
Therefore, $(98)^5 = (100 - 2)^5$

$= {}^5C_0\,(100)^5 - {}^5C_1\,(100)^4\,.2 + {}^5C_2\,(100)^3 2^2 - {}^5C_3\,(100)^2\,(2)^3 + {}^5C_4\,(100)\,(2)^4 - {}^5C_5\,(2)^5$

$= 10000000000 - 5 \times 100000000 \times 2 + 10 \times 1000000 \times 4 - 10 \times 10000 \times 8 + 5 \times 100 \times 16 - 32$

= 10040008000 – 1000800032

= 9039207968.

3. Which is larger (1.01)1000000 or 10,000?

Ans: Splitting 1.01 and using binomial theorem to write the first few terms we have $(1.01)^{1000000} = (1 + 0.01)^{1000000}$

$= {}^{1000000}C_1 + {}^{1000000}C_1\,(0.01)$ + other positive terms

$= 1 + 1000000 \times 0.01$ + other positive terms

$= 1 + 10000$ + other positive terms

> 10000

Hence $(1.01)^{1000000} > 10000$.

4. using binomial theorem, prove that 6^n – 5n always leaves remainder 1 when divided by 25.

Ans: For two numbers a and b if we can find numbers q and r such that a = bq + r, then we say that b divides a with q as quotient and r as remainder. Thus, in order to show that 6^n – 5n leaves remainder 1 when divided by 25, we prove that 6^n – 5n = 25k + 1, where k is some natural number.

We have

$$(1+a)^n = {}^nC_0 + {}^nC_1\, a + {}^nC_2\, a^2 + \dots + {}^nC_n\, a^n$$

For a = 5, we get

$$(1+5)^n = {}^nC_0 + {}^nC_1\, 5 + {}^nC_2\, 5^2 + \ldots + {}^nC_n\, 5^n$$

i.e. $(6)^n = 1 + 5n + 5^2 \,.\, {}^nC_2 + 5^3 \,.\, {}^nC_3 + \ldots + 5^n$

i.e. $6^n - 5n = 1 + 5^2\,({}^nC_2 + {}^nC_3\, 5 + \ldots + 5^{n-2})$

Or $6^n - 5n = 1 + 25\,({}^nC_2 + 5.\, {}^nC_3 + \ldots + 5^{n-2})$

Or $6^n - 5n = 25k + 1$ where $k = {}^nC_2 + 5.\, {}^nC_3 + \ldots + 5^{n-2}$.

This shows that when divided by 25, $6^n - 5n$ leaves remainder 1.

5. If a and b are distinct integers, prove that a – b is a factor of $a^n - b^n$, whenever n is a positive integer. [Hint write $a^n = (a-b+b)^n$ and expand]

Ans: In order to prove that (a-b) is a factor of $(a^n - b^n)$, it has to be proved that $a^n - b^n = k(a-b)$, where k is some natural number.

It can be written that, a = a – b + b

$$\therefore a^n = (a-b+b)^n = [(a-b)+b]^n$$

$$= {}^nC_0\,(a-b)^n + {}^nC_1\,(a-b)^{n-1}b + \ldots + {}^nC_2\,(a-b)b^{n-1} + {}^nC_n\, b^n$$

$$= (a-b)^n + {}^nC_1\,(a-b)^{n-1}b + \ldots + {}^nC_{n-1}\,(a-b)b^{n-1} + b^n$$

$$a^n - b^n = (a-b)\,[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}\, b^{n-1}]$$

$$a^n - b^n = k\,(a-b)$$

Where, $k = [(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}\, b^{n-1}]$.

6. Find an approximation of $(0.99)^5$ using the first three terms of its expansion

Ans: $0.99 = 1 - 0.01$

$$\therefore (0.99)^5 = (1-0.01)^5$$

$$= {}^5C_0\,(1)^5 - {}^5C_1\,(1)^4(0.01) + {}^5C_2\,(1)^3(0.01)^2$$

$$= 1 - 5(0.01) + 10(0.01)^2$$

$$= 1 - 0.05 + 0.001$$

$=1.001 - 0.05$

$= 0.951$

Thus, the value of $(0.99)^5$ is approximately 0.951.

7. Evaluate $(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$.

Ans: Firstly, the expression $(a + b)^6 - (a - b)^6$ is simplified by using binomial theorem. This can be done as

$$(a + b)^6 = {}^6C_0\, a^6 + {}^6C_1\, a^5 b + {}^6C_2\, a^4 b^2 + {}^6C_3\, a^3 b^3 + {}^6C_4\, a^2 b^4 + {}^6C_5\, a^1 b^5 + {}^6C_6\, b^6$$

$$= a^6 + 6a^5 b + 15a^4 b^2 + 20a^3 b^3 + 15a^2 b^4 + 6ab^5 + b^6$$

$$(a - b)^6 = {}^6C_0\, a^6 - {}^6C_1\, a^5 b + {}^6C_2\, a^4 b^2 - {}^6C_3\, a^3 b^3 + {}^6C_4\, a^2 b^4 - {}^6C_5\, a^1 b^5 + {}^6C_6\, b^6$$

$$= a^6 - 6a^5 b + 15a^4 b^2 - 20a^3 b^3 + 15a^2 b^4 - 6ab^5 + b^6$$

$$\therefore (a + b)^6 - (a - b)^6 = 2[6a^5 b + 20a^3 b^3 + 6ab^5]$$

Putting $a = \sqrt{3}$ $b = \sqrt{2}$, We obtain

$$(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6 = 2[6(\sqrt{3})^5(\sqrt{2}) + 20(\sqrt{3})^3(\sqrt{2})^3 + 6(\sqrt{3})(\sqrt{2})^5]$$

$$= 2[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6}]$$

$$= 2\times 198\sqrt{6}$$

$$= 396\sqrt{6}$$

8. Find the value of $(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4$

Ans: Firstly, the expression $(x + y)^4 - (x - y)^4$ is simplified by using binomial theorem. This can be done as

$$(x + y)^4 = {}^4C_0\, x^4 + {}^4C_1\, x^3 y + {}^4C_2\, x^2 y^2 + {}^4C_3\, x y^3 + {}^4C_4\, y^4$$

$$= x^4 + 4x^3 y + 6x^2 y^2 + 4xy^3 + y^4$$

$$(x - y)^4 = {}^4C_0\, x^4 - {}^4C_1\, x^3 y + {}^4C_2\, x^2 y^2 - {}^4C_3\, x y^3 + {}^4C_4\, y^4$$

$$= x^4 - 4x^3 y + 6x^2 y^2 - 4xy^3 + y^4$$

$\therefore (x+y)^4 - (x-y)^4 = 2(x^4 + 6x^2y^2 + y^4)$

Putting $x = a^2 \quad y = \sqrt{a^2-1}$, we obtain

$(a^2 + \sqrt{a^2-1})^4 + (a^2 - \sqrt{a^2-1})^4 = 2[(a^2)^4 + 6(a^2)^2(\sqrt{a^2-1})^2 + (\sqrt{a^2-1})^4]$

$= 2[a^8 + 6a^4(a^2-1) + (a^2-1)^2]$

$= 2[a^8 + 6a^6 + 6a^4 + a^4 - 2a^2 + 1]$

$= 2[a^8 + 6a^6 - 5a^4 - 2a^2 + 1]$

$=2a^8 + 12a^6 - 10a^4 - 4a^2 + 2$

9.Find the coefficient of x^5 in the product $(1+2x)^6(1-x)^7$ using binomial theorem.

Ans: using Binomial Theorem, the expressions, $(1+2x)^6$and$(1-x)^7$, can be expanded as

$(1+2x)^6 = {}^6C_0 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6$

$= 1 + 6(2x) + 15(2x)^2 + 20(2x)^3 + 15(2x)^4 + 6(2x)^5 + (2x)^6$

$= 1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^4 + 64x^6$

$(1-x)^7 = {}^7C_0 - {}^7C_1(x) + {}^7C_2(x)^2 - {}^7C_3(x)^3 + {}^7C_4(x)^4 - {}^7C_5(x)^5 + {}^7C_6(x)^6 - {}^7C_7(x)^7$

$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$

$\therefore (1+2x)^6 - (1-x)^7$

$= (1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^4 + 64x^6)(1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7)$

The complete Multiplication of the two brackets is not required to be carried out. Only those terms, which involves x^5, are required

The terms containing $\boldsymbol{x^5}$ are

$1(-21x^5) + (12x)(35x^4) + (60x^2)(-35x^3) + (160x^3)(21x^2) + (240x^4)(-7x) + (192x^5)(1) = 171x^5$

Thus the coefficient of x^5 in the given product is 171.

Multiple Choice Questions

1) The coefficient of the middle term in the expansion of $(2+3x)^4$ is:

- **a. 5!**
- **b. 6**
- **c. 216**
- **d. 8!**

Answer: (c) 216

Explanation:

If the exponent of the expression is n, then the total number of terms is n+1.

Hence, the total number of terms is 4+1 = 5.

Hence, the middle term is the 3rd term.

We know that general term of $(x+a)^n$ is $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

Here, n=4, r=2

Therefore, $T_3 = {}^4C_2.(2)^2.(3x)^2$

$T_3 = (6).(4).(9x^2)$

$T_3 = 216x^2$.

Therefore, the coefficient of the middle term is 216.

2) The value of $(126)^{1/3}$ up to three decimal places is

- a. 5.011
- b. 5.012
- c. 5.013
- d. 5.014

Answer: (c) 5.013

Explanation:

$(126)^{⅓}$ can also be written as the cube root of 126.

Hence, $(126)^{1/3}$ is approximately equal to 5.013.

Hence, option (c) 5.013 is the correct answer.

3) If n is even in the expansion of $(a+b)^n$, the middle term is:

a. n^{th} term
b. $(n/2)^{th}$ term
c. $[(n/2)-1]^{th}$ term
d. $[(n/2)+1]^{th}$ term

Answer: (d) $[(n/2)+1]^{th}$ term

Explanation:

In general, if "n" is the even in the expansion of $(a+b)^n$, then the number of terms will be odd. (i.e) n+1.

Hence, the middle term of the expansion $(a+b)^n$ is $[(n/2)+1]^{th}$ term.

4) The largest coefficient in the expansion of $(1+x)^{10}$ is:

a. $10! / (5!)^2$
b. $10! / 5!$
c. $10! / (5!\times4!)^2$
d. $10! / (5!\times4!)$

Answer: (a) $10! / (5!)^2$

Explanation:

Given: $(1+x)^{10}$

The greatest coefficient will always occur in the middle term.

Hence, the total number of terms in an expansion is 11. (i.e. 10+1 = 11)

Therefore, middle term = [(10/2) + 1] = 5+1 = 6th term.

We know that general term of $(x+a)^n$ is $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

Here, n=10, r=5

So, $T6 = {}^{10}C_5.x^5$

Therefore, the coefficient of the greatest term = $^{10}C_5 = 10!/(5!)^2$.

So, option (a) $10!/(5!)^2$ is the correct answer.

5) The coefficient of x^3y^4 in $(2x+3y^2)^5$ is

a. 360
b. 720
c. 240
d. 1080

Answer: (b) 720

Explanation:

Given: $(2x+3y^2)^5$

Therefore, the general form for the expression $(2x+3y^2)^5$ is $T_{r+1} = {}^5C_r.(2x)^r.(3y^2)^{5-r}$

Hence, $T_{3+1} = {}^5C_3\,(2x)^3.(3y^2)^{5-3}$

$T_4 = {}^5C_3\,(2x)^3.(3y^2)^2$

$T_4 = {}^5C_3.8x^3.9y^4$

On simplification, we get

$T_4 = 720x^3y^4$

Therefore, the coefficient of x^3y^4 in $(2x+3y^2)^5$ is 720.

6) The largest term in the expansion of $(3+2x)^{50}$, when $x = 1/5$ is

a. 6th term
b. 7th term
c. 8th term
d. None of the above

Answer: (a) 6th term

Explanation:

The greatest term in the expansion of $(x+y)^n$ is the kth term. Where k= [(n+1)y]/[x+y]..(1)

On comparing the given expression with the general form, x = 3, y=2x, n=50

Now, substitute the values in the given expression, we get

Hence, kth term = [(50+1)(2x)]/[3+2x]

When x = ⅕,

Kth term = [(51)(2(⅕))]/[3+2(⅕)] = 6

Hence, the 6th term is the largest term in the expansion of $(3+2x)^{50}$, when x = ⅕.

7) The coefficient of y in the expansion of $(y^2+(c/y))^5$ is:

a. 10c
b. 29c
c. $10c^3$
d. $20c^3$

Answer: (c) $10c^3$

Explanation: Given: $(y^2+(c/y))^5$

We know that general term of $(x+a)^n$ is $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

Here, n=5, r=?

$(y^2+(c/y))^5 = {}^5C_r.(y^2)^r.(c/y)^{5-r}$

$(y^2+(c/y))^5 = 5Cr.\ y^{2r}.\ (c^{5-r}/y^{5-r})$

On solving this, we get r = 2.

Hence, the coefficient of y = ${}^5C_3.c^3 = 10c^3$.

Therefore, option (c) $10c^3$ is the correct answer.

8) The fourth term in the expansion of $(x-2y)^{12}$ is:

a. $-1760\ x^9 \times y^3$
b. $-1670\ x^9 \times y^3$

c. $-7160\ x^9 \times y^3$
d. $-1607\ x^9 \times y^3$

Answer: (a) $-1760\ x^9 \times y^3$

Explanation: We know that the general term of an expansion $(a+b)^n$ is $T_{r+1} = {}^nC_r\ a^{n-r}\ b^r$.

Now, we have to find the fourth term in the expansion $(x-2y)^{12}$

Hence, r = 3, a = x, b = -2y, n= 12.

Now, substitute the values in the formula, we get

$T_{3+1} = {}^{12}C_3\ x^{12-3}\ (-2y)^3$.

On solving this, we get

$T_4 = -1760x^9y^3$.

9) If the fourth term of the binomial expansion of $(px+(1/x))^n$ is 5/2, then

a. n=6, p=6
b. n=8, p=6
c. n=8, p= ½
d. n=6, p=½

Answer: (d) n=6, p=½

Explanation:

Given: $(px+(1/x))^n$

Hence, the fourth term, $T_{3+1} = {}^nC_3(px)^{n-3}(1/x)^3$

Given that the fourth term of the binomial expansion of $(px+(1/x))^n$ is 5/2, which is independent of x.

Hence, $(5/2)= {}^nC_3(px)^{n-3}(1/x)^3$ …(1)

On solving this, we get n=6.

Now, substitute n=6 in $(5/2)= {}^nC_3(p)^3$

$20p^3 = 5/2$

$p^3=⅛$

$p=½$.

Therefore, n=6 and p=½.

10) If n is the positive integer, then $2^{3n} – 7n -1$ is divisible by

a. 7
b. 10
c. 49
d. 81

Answer: (c) 49

Explanation:

Given: $2^{3n} – 7n -1$. It can also be written as $8^n – 7n – 1$

Let $8^n – 7n – 1 =0$

So, $8^n = 7n+1$

$8^n = (1+7)^n$

By applying binomial theorem, we get

$8n – 1 – 7n = 49$ (or) $2^{3n} – 7n -1 = 49$

Hence, $2^{3n} – 7n -1$ is divisible by 49.

Summary

- The expansion of a binomial for any positive integral n is given by Binomial Theorem, which is

$(a + b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + ...+ {}^nC_{n-1}a.b^{n-1} + {}^nC_nb^n$.

- The coefficients of the expansions are arranged in an array. This array is called *Pascal's* triangle.