CHAPTER-10

Exponents and Powers

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Exercise 10.1

1. Express the following numbers in standard form.

(i) 0.0000000000085

(ii) 0.00000000000942

(iii) 6020000000000000

(iv) 0.00000000837

(v) 31860000000

Solution:

(i) $0.0000000000085 = 0.0000000000085\times(10^{12}/10^{12}) = 8.5\times10^{-12}$

(ii) $0.00000000000942 = 0.00000000000942\times(10^{12}/10^{12}) = 9.42\times10^{-12}$

(iii) $6020000000000000 = 6020000000000000\times(10^{15}/10^{15}) = 6.02\times10^{15}$

(iv) $0.00000000837 = 0.00000000837\times(10^{9}/10^{9}) = 8.37\times10^{-9}$

(v) $31860000000 = 31860000000\times(10^{10}/10^{10}) = 3.186\times10^{10}$

2. Express the following numbers in the usual form.

(i) 3.02×10^{-6}

(ii) 4.5×10^{4}

(iii) 3×10^{-8}

(iv) 1.0001×10^{9}

(v) 5.8×10^{12}

(vi) 3.61492×10^{6}

Solution:

(i) $3.02\times10^{-6} = 3.02/10^{6} = 0\ .00000302$

(ii) $4.5\times10^{4} = 4.5\times10000 = 45000$

(iii) $3\times10^{-8} = 3/10^{8} = 0.00000003$

(iv) $1.0001\times10^{9} = 1000100000$

(v) $5.8\times10^{12} = 5.8\times1000000000000 = 5800000000000$

(vi) $3.61492\times10^{6} = 3.61492\times1000000 = 3614920$

3. Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to 1/1000000 m.
(ii) Charge of an electron is 0.000, 000, 000, 000, 000, 000, 16 coulomb.
(iii) Size of bacteria is 0.0000005 m
(iv) Size of a plant cell is 0.00001275 m
(v) Thickness of a thick paper is 0.07 mm

Solution:

(i) 1 micron = 1/1000000

$= 1/10^{6}$

$= 1\times10^{-6}$

(ii) Charge of an electron is 0.00000000000000000016 coulombs

$= 0.00000000000000000016\times10^{19}/10^{19}$

$= 1.6\times10^{-19}$ coulomb

(iii) Size of bacteria = 0.0000005

$= 5/10000000 = 5/10^{7} = 5\times10^{-7}$ m

(iv) Size of a plant cell is 0.00001275 m

$= 0.00001275\times10^{5}/10^{5}$

$= 1.275\times10^{-5}$m

(v) Thickness of a thick paper = 0.07 mm

$0.07 \text{ mm} = 7/100 \text{ mm} = 7/10^2 = 7\times10^{-2} \text{ mm}$

4. In a stack, there are 5 books, each having a thickness of 20 mm and 5 paper sheets, each having a thickness of 0.016 mm. What is the total thickness of the stack?

Solution:

Thickness of one book = 20 mm

Thickness of 5 books = 20×5 = 100 mm

Thickness of one paper = 0.016 mm

Thickness of 5 papers = 0.016×5 = 0.08 mm

Total thickness of a stack = 100+0.08 = 100.08 mm

$= 100.08\times10^2/10^2 \text{ mm}$

$= 1.0008\times10^2 \text{ mm}$

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Exercise 10.2

1. Evaluate:

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $(1/2)^{-5}$

Solution:

(i) $3^{-2} = (1/3)^2$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= 1/9$

(ii) $\mathbf{(-4)^{-2} = (1/-4)^2}$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= 1/16$

(iii) $\mathbf{(1/2)^{-5} = (2/1)^5}$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= 2^5$

$= 32$

2. Simplify and express the result in power notation with a positive exponent:

(i) $(-4)^4 \div (-4)^8$

(ii) $(1/2^3)^2$

(iii) $-(3)^4 \times (5/3)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Solution:

(i)

$(-4)^5 \div (-4)^8$

$= (-4)^5/(-4)^8$

$\left[\because \ a^m \div a^n = a^{m-n}\right]$

$= (-4)^{5-8}$

$= 1/(-4)^3$

(ii) **$(1/2^3)^2$**

$= 1^2/(2^3)^2$

$\left[\because \ \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$

$= 1/2^{3\times 2} = 1/2^6$

$\left[\because \ \left(a^m\right)^n = a^{m\times n}\right]$

(iii) $\mathbf{-(3)^4\times(5/3)^4}$

$$(-3)^4\times\left(\frac{5}{3}\right)^4=(-3)^4\times\frac{5^4}{3^4}$$

$$\left[\because\ \left(\frac{a}{b}\right)^m=\frac{a^m}{a^n}\right]$$

$= (-1)^4\times3^4\times(5^4/3^4)$

$$\left[\because\ (ab)^m=a^mb^m\right]$$

$= 3^{(4-4)}\times5^4$

$$\left[\because\ a^m\div a^n=a^{m-n}\right]$$

$= 3^0\times5^4 = 5^4$

$$\left[\because\ a^0=1\right]$$

(iv) $\left(3^{-7}\div3^{-10}\right)\times3^{-5}$

$= (3^{-7}/3^{-10})\times 3^{-5}$

$= 3^{-7-(-10)}\times 3^{-5}$

$$\left[\because\ a^m\div a^n=a^{m-n}\right]$$

$= 3^{(-7+10)}\times3^{-5}$

$= 3^3\times3^{-5}$

$= 3^{(3+-5)}$

$$\left[\because\ a^m\times a^n=a^{m+n}\right]$$

$= 3^{-2}$

$=1/3^2$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

(v) $2^{-3}\times(-7)^{-3}$

$= (2\times-7)^{-3}$

(Because $a^m\times b^m = (ab)^m$)

$= 1/(2\times-7)^3$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= 1/(-14)^3$

3. Find the value of:

(i) $(3^0+4^{-1})\times2^2$

(ii) $(2^{-1}\times4^{-1})\div2^{-2}$

(iii) $(1/2)^{-2}+(1/3)^{-2}+(1/4)^{-2}$

(iv) $(3^{-1}+4^{-1}+5^{-1})^0$

(v) $\{(-2/3)^{-2}\}^2$

Solution:

(i)$(3^0+4^{-1})\times2^2 = (1+(1/4))\times2^2$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= ((4+1)/4\ ) \times 2^2$

$= (5/4) \times 2^2$

$= (5/2^2) \times 2^2$

$= 5 \times 2^{(2-2)}$

$\left[\because\ a^m \div a^n = a^{m-n}\right]$

$= 5 \times 2^0$

$= 5 \times 1 = 5$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

(ii)$(2^{-1} \times 4^{-1}) \div 2^{-2}$

$= [(1/2) \times (1/4)] \div (1/4)$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= (1/2 \times 1/2^2\ ) \div 1/4$

$= 1/2^3 \div 1/4$

$= (1/8) \times (4)$

$= 1/2$

(iii) **$(1/2)^{-2} + (1/3)^{-2} + (1/4)^{-2}$**

$= (2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2}$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= 2^{(-1\times-2)}+3^{(-1\times-2)}+4^{(-1\times-2)}$

$\left[\because \ \left(a^m\right)^n = a^{m\times n}\right]$

$= 2^2+3^2+4^2$

$= 4+9+16$

$=29$

(iv) $(3^{-1}+4^{-1}+5^{-1})^0$

$= 1$

$\left[\because \ a^0 = 1\right]$

(v) **$\{(-2/3)^{-2}\}^2 = (-2/3)^{-2\times2}$**

$\left[\because \ \left(a^m\right)^n = a^{m\times n}\right]$

$= (-2/3)^{-4}$

$= (-3/2)^4$

$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$

$= 81/16$

4. Evaluate:

(i) $(8^{-1}\times5^{3})/2^{-4}$

(ii) $(5^{-1}\times2^{-2})\times6^{-1}$

Solution:

(i) $(8^{-1}\times5^{3})/2^{-4}$

$$\frac{8^{-1}\times5^{3}}{2^{-4}}=\frac{\left(2^{3}\right)^{-1}\times5^{3}}{2^{-4}}=\frac{2^{-3}\times5^{3}}{2^{-4}}$$

$$\left[\because\ \left(a^{m}\right)^{n}=a^{m\times n}\right]$$

=

$$2^{-3-(-4)}\times5^{3}=2^{-3+4}\times5^{3}$$

$$\left[\because\ a^{m}\div a^{n}=a^{m-n}\right]$$

$= 2\times125 = 250$

(ii) $(5^{-1}\times2^{-2})\times6^{-1}$

$$\left(5^{-1}\times2^{-1}\right)\times6^{-1}=\left(\frac{1}{5}\times\frac{1}{2}\right)\times\frac{1}{6}$$

$$\left[\because\ a^{-m}=\frac{1}{a^{m}}\right]$$

$= (1/10)\times1/6$

$= 1/60$

5. Find the value of *m* for which $5^{m} \div 5^{-3} = 5^{5}$

Solution:

$5^{m} \div 5^{-3} = 5^{5}$

$5^{(m-(-3))} = 5^{5}$

$\left[\because \ a^{m} \div a^{n} = a^{m-n}\right]$

$5^{m+3} = 5^{5}$

Comparing exponents on both sides, we get

$m+3 = 5$

$m = 5-3$

$m = 2$

6. Evaluate:

(i)

$$\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$$

(ii)

$$\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$$

Solution:

(i)

$$\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}=\left\{\left(\frac{3}{1}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\}$$

$$\left[\because\ a^{-m}=\frac{1}{a^{m}}\right]$$

= 3-4

= -1

(ii)

$$\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}=\frac{5^{-7}}{8^{-7}}\times\frac{8^{-4}}{5^{-4}}$$

$$\left[\because\ \left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}\right]$$

=

$$5^{-7-(-4)}\times 8^{-4-(-7)}$$

$$\left[\because\ a^{m}\div a^{n}=a^{m-n}\right]$$

$=$

$5^{-7+4}\times 8^{-4+7}$

$=5^{-3}\times 8^{3}\ =$

$\frac{8^3}{5^3}$

$\left[\because\ a^{-m}=\frac{1}{a^m}\right]$

$=$ 512/125

7. Simplify the following:

(i)

$\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}\quad (t\neq 0)$

(ii)

$\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$

Solution:

(i)

$\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}$

$=\frac{5^2\times t^{-4}}{5^{-3}\times 5\times 2\times t^{-8}}$

$=\frac{5^{2-(-3)-1}\times t^{-4-(-8)}}{2}$

$\left[\because\ a^m\div a^n=a^{m-n}\right]$

$=$

$$\frac{5^{2+3-1}\times t^{-4+8}}{2}=\frac{5^4\times t^4}{2}=$$

$$\frac{625}{2}t^4$$

(ii)

$$\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$$

$$=\frac{3^{-5}\times(2\times 5)^{-5}\times 5^3}{5^{-7}\times(2\times 3)^{-5}}$$

$$=\frac{3^{-5}\times 2^{-5}\times 5^{-5}\times 5^3}{5^{-7}\times 2^{-5}\times 3^{-5}}$$

$$\left[\because\ \ (ab)^m=a^m b^m\right]$$

$=$

$$\frac{3^{-5}\times 2^{-5}\times 5^{-5+3}}{5^{-7}\times 2^{-5}\times 3^{-5}}=\frac{3^{-5}\times 2^{-5}\times 5^{-2}}{5^{-7}\times 2^{-5}\times 3^{-5}}$$

$$\left[\because\ \ a^m\times a^n=a^{m+n}\right]$$

$=$

$$3^{-5-(-5)}\times 2^{-5-(-5)}\times 5^{-2-(-7)}$$

$$\left[\because\ \ a^m\div a^n=a^{m-n}\right]$$

$=$

$$3^{-5+5}\times 2^{-5+5}\times 5^{-2+7}=$$

$$3^0\times 2^0\times 5^5$$

$$=1\times 1\times 3125$$

$$\left[\because\ \ a^0=1\right]$$

= 3125

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Exercise 10.3

Multiple-choice question and answers

1. What is the value of 2^4?

a) 8

b) 16

c) 32

d) 64

2. Simplify $3^2 \times 3^3$.

a) 6

b) 9

c) 27

d) 81

3. What is the value of 5^0?

a) 0

b) 1

c) 5

d) 25

4. Which of the following is equivalent to 4^3?

a) 16

b) 64

c) 81

d) 256

5. Evaluate 10^{-2}.

a) 0.01

b) 1

c) 10

d) 100

6. Simplify $(2^4) \div (2^2)$.

a) 4

b) 8

c) 16

d) 64

7. What is the value of 7^1?

a) 7

b) 14

c) 49

d) 1

8. If $2^x = 16$, what is the value of x?

a) 4

b) 3

c) 2

d) 1

9. Calculate $3^{(-3)}$.

a) 1/27

b) 9

c) 27

d) 81

10. Which of the following is equivalent to $6^2 \times 6^{(-3)}$?

a) $6^{(-1)}$

b) $6^{(-5)}$

c) 6^1

d) 6^{-6}

11. What is the value of 9^0?

a) 1

b) 0

c) 3

d) 9

12. Simplify $(5^2)^3$.

a) 125

b) 250

c) 625

d) 2500

13. If $3^x = 27$, what is the value of x?

a) 1

b) 2

c) 3

d) 4

14. Calculate 2^{-4}.

a) 1/16

b) 16

c) 8

d) 0.25

15. What is the value of $10^3 \times 10^{(-2)}$?

a) 1,000

b) 10,000

c) 100

d) 10

Answers:

1. b) 16

2. d) 81

3. b) 1

4. b) 64

5. a) 0.01

6. a) 4

7. a) 7

8. a) 4

9. a) 1/27

10. a) $\mathbf{6^{-1}}$

11. a) 1

12. c) 625

13. c) 3

14. a) 1/16

15. b) 10,000

Exercise 10.4

1. 5^3 can be read as "5 raised to the power of _____."

- Answer: 3

2. $x^0$is equal to _____.

- Answer: 1

3. The value of $6^3 * 6^2$ is $6^{(\ )}$

- Answer:6^5

4. $10^{-3}$is equivalent to _____.

- Answer: $\frac{1}{1000}$or 0.001

5. The exponent in the expression (4^x) is _____.

- Answer: x

6. $2^{(m+n)}$can be expressed as $2^m * 2^n = 2^{(\ )}$

- Answer: $2^{(m+n)}$

7. The number of zeros in the standard form of $7*10^8$ is _____.

- Answer: 8

Summary

1. Exponents:

- Exponents represent repeated multiplication of a number by itself.

- For instance, in \(5^3\), 5 is the base, and 3 is the exponent, indicating that 5 is multiplied by itself 3 times (5 × 5 × 5 = 125).

2. Rules of Exponents:

- Multiplying powers with the same base: $a^m * a^n = a^{m+n}$

-Dividing powers with the same base: $a^m \div a^n = a^{m-n}$

- Power of a power: $(a^m)^n = a^{mn}$

- Power of a product: $(ab)^n = a^n * b^n$

3. Zero and Negative Exponents:

- a^0 (where a ≠ 0) is always equal to 1.

- a^{-n} is the reciprocal of a^n (i.e., $a^{-n} = \frac{1}{a^n}$)

4. Scientific Notation:

- Scientific notation expresses numbers as a product of a number between 1 and 10 and a power of 10.

- For example, 3.5 x 10^4 represents 35,000**.**

5. Application in Real Life:

- Exponents and powers are used in scientific notations, computing large quantities, analyzing growth rates, and understanding logarithms in various fields such as science, finance, and computer science.

6. Solving Equations and Problems:

- Understanding exponents is crucial in solving equations involving unknown powers, simplifying expressions, and solving problems related to growth or decay.

7. Patterns and Sequences:

- Exponents play a key role in understanding patterns and sequences in mathematics, allowing predictions of values in sequences based on powers.

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