PHYSICS

Class XII

Chapter 7-Alternating Current

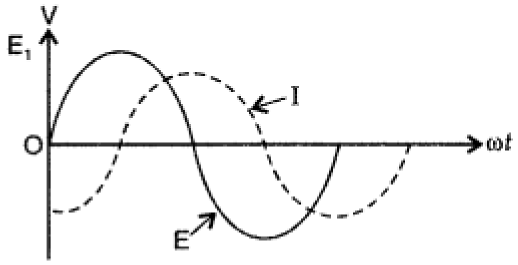
1 Mark Questions

Question 1.
Define the term 'wattless current'.
Answer:
Wattless current is that component of the circuit current due to which the power consumed in the circuit is zero.

Question 2.
Mention the two characteristic properties of the material suitable for making core of a transformer.
Answer:
Characteristic properties of material suitable for core of a transformer :

- It should have high permeability
- It should have low hysteresis loss.
- It should have low coercivity/retentivity.
- It should have high resistivity. (Any two)

Question 3.
When an ac source is connected across an ideal inductor, show on a graph the nature of variation of the voltage and the current over one complete cycle.
Answer:



Question 4.
A heating element is marked 210 V, 630 W. What is the value of the current drawn by the element when connected to a 210 V dc source?
Answer:

$$I = \frac{P}{V} = \frac{630}{210} = \mathbf{3A}$$

Question 5.
A heating element is marked 210 V, 630 W. Find the resistance of the element when connected to a 210 V dc source.
Answer:

$$\text{Resistance} = \frac{V^2}{P} = \frac{(210)^2}{630} = \frac{210 \times 210}{630} = \mathbf{70\ \Omega}$$

Question 6.
Why is the core of a transformer laminated?
Answer:
The core of a transformer is laminated to minimize eddy currents in the iron core.

Question 7.
Why is the use of a.c. voltage preferred over d.c. voltage? Give two reasons.
Answer:
a.c. voltage is preferred over d.c. voltage because of following reasons :

1. it can be stepped-up or stepped-down by a transformer.
2. carrying losses are much less.

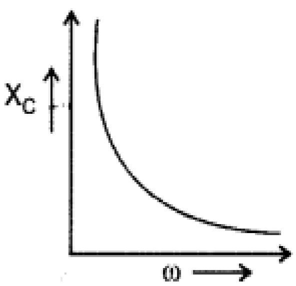
Question 8.
Define capacitor reactance. Write its S.I. units.
Answer:
'Capacitor reactance' is defined as the opposition to the flow of current in ac circuits offered by a capacitor.

$$X_C = \frac{1}{\omega_C}$$

S.I. Unit : Ohm.

Question 9.
A variable frequency AC source is connected to a capacitor. Will the displacement current change if the frequency of the AC source is decreased?
Answer:
On decreasing the frequency of AC source, reactance, xC=1ωC will increase, which will lead to decrease in conduction current. In this case
$I_D = I_C$
Hence, displacement current will decrease.

Question 10.
Plot a graph showing variation of capacitive reactance with the change in the frequency of the AC source.
Answer:
Graph showing a variation of x_c capacitive reactance with the change in frequency of AC source.

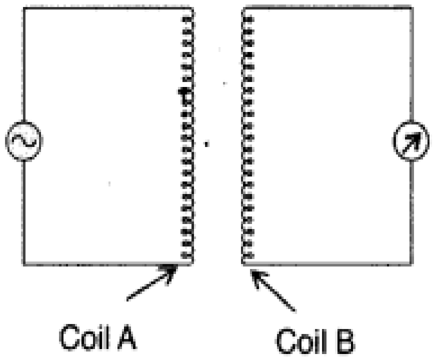


Question 11.
Define 'quality factor' of resonance in series LCR circuit. What is its SI unit?
Answer:
Quality factor (Q) is defined as, Q = ω0LR
It gives the sharpness of the resonance circuit. It has no SI unit.

Question 15.
For an ideal inductor, connected across a sinusoidal ac voltage source, state which one of the following quantity is zero :
(i) Instantaneous power
(ii) Average power over full cycle of the ac voltage source
Answer:
Average power over full cycle of the ac voltage source is zero, when connected with an ideal inductor.

2 Mark Questions

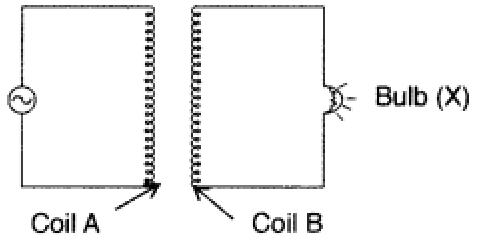
Question 1.
The circuit arrangement as shown in the diagram shows that when an a.c. passes through the coil A, the current starts flowing in the coil B.



(i) State the underlying principle involved.
(ii) Mention two factors on which the current produced in the coil B depends.
Answer:
(i) It is based on the principle of "mutual induction".
(ii) Two factors are:

- distance between the coils.
- orientation of the coils.
- Number of turns in the coil, (any two)

Question 2.
The figure given shows an arrangement by which current flows through the bulb (X) connected with coil B, when a.c. is passed through coil A.



(i) Name the phenomenon involved.
(ii) If a copper sheet is inserted in the gap between the coils, explain, how the brightness of the bulb would change.
Answer:
(i) The phenomenon involved is mutual induction.
(ii) When the copper sheet is inserted, eddy currents are set up in it which opposes the passage of magnetic flux. The induced emf in coil B decreases. This decreases the brightness of the bulb.

Question 21.
A 15.0 μF capacitor is connected to 220 V, 50 Hz source. Find the capacitive reactance and the rms current.
Answer:

Capacitive reactance, X_C

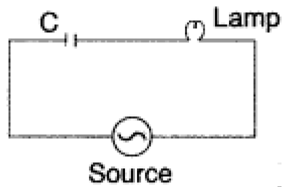
$$= \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times 3.14 \times 50 \times (15 \times 10^{-6})}$$

$$= \frac{10000}{3.14 \times 15} = \frac{1000}{4.71} = \mathbf{212.3\ \Omega}$$

$$\text{and } \mathbf{rms\ current} = \frac{V}{X_C} = \frac{220}{212.3} = \mathbf{1.04\ A}$$

Question 3.
An electric lamp having coil of negligible inductance connected in series with a capacitor and an a.c. source is glowing with certain brightness. How does the brightness of the lamp change on reducing the
(i) capacitance, and
(ii) the frequency? Justify your Answer.



Answer:
Brightness of lamp ∝ I_0,
Assuming zero resistance and zero inductance of lamp

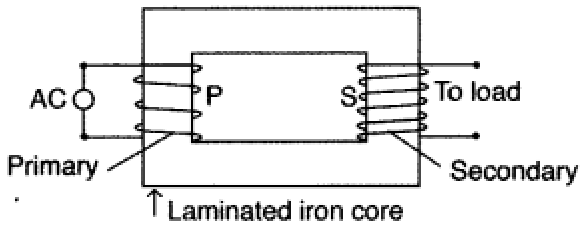
$$I_0 = \frac{E_0}{X_C} = \frac{V_0}{1/\omega C} = E_0\,(2\pi\nu)\,C$$

On reducing C or v; It would decrease
∴ Brightness of the lamp will decrease.

Question 4.
State the principle of working of a transformer. Can a transformer be used to step up or step down a d.c. voltage? Justify your Answer.
Answer:
Transformer works on the principle of mutual induction, i.e., when a changing current is passed through one of the two inductively coupled coils, an induced emf is set up in the other coil.

No, transformer cannot be used to step up or step down a d.c. voltage because d.c. voltage cannot produce a change in magnetic flux.

Question 5.
Mention various energy losses in a transformer.
Answer:
(i) A transformer is an electrical device for converting an alternating current at low voltages into that at high voltage or vice versa.
If it increases the input voltage, it is called step- up-transformer.



Principle : It works on the principle of mutual induction i.e., "when a changing current is passed through one of the two inductively coupled coils, an induced emf is set up in the other coil."

Working : As the alternating current flows through the primary, it generates an alternating magnetic flux in the core which also passes through the secondary. This changing flux sets up an induced emf in the secondary, also a self- induced emf in the primary. If there is no leakage of magnetic flux, then flux linked with each turn
of the primary will be equal to that linked with each turn of the secondary.

$$V_P = -N_P\frac{d\phi}{dt} \qquad \text{and} \qquad V_S = -N_S\frac{d\phi}{dt}$$

…where [N_P and N_S are number of turns in the primary and secondary respectively,
V_P and V_S are their respective voltages]

$$\therefore \quad \frac{V_S}{V_P} = \frac{N_S}{N_P} \qquad \ldots(i)$$

This ratio NSNP is called the turns ratio.
Assuming the transformer to be ideal one, so that there are no energy losses, then
Input power = output power

$V_p I_p = V_S I_S$
…where [I_P and I_S are the current in the primary and secondary respectively

$$\frac{V_S}{V_P} = \frac{I_P}{I_S} \quad \ldots(ii)$$

From (*i*) and (*ii*), we get $\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$

In a step up transformer, $N_s > N_p$ i.e., the turns ratio is greater than 1 and therefore $V_s > V_p$.
The output voltage is greater than the input voltage.

Main assumptions :

1. The primary resistance and current are small.
2. The same flux links both with the primary and secondary windings as the flux leakage from due core is negligible (small).
3. The terminals of the secondary are open or the current taken from it, is small, (any two)

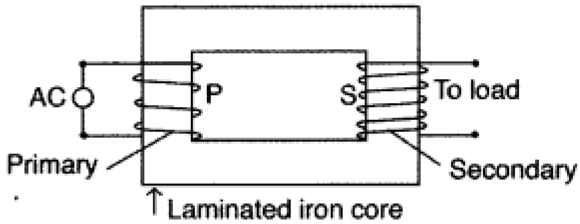
For long distance transmission, the voltage output of the generator is stepped-up (so that current is reduced and consequently, IR loss is reduced). It is transmitted over long distance and is stepped- down at distributing substations at consumers' end.

(ii) Two sources of energy loss in a transformer:
1. Copper loss": Some energy is lost due to heating of copper wires used in the primary and secondary windings. This power loss (= I^2R) can be minimised by using thick copper wires of low resistance.

2. Eddy current loss : The alternating magnetic flux induces eddy currents in the iron core which leads to some energy loss in the form of heat. This loss can be reduced by using laminated iron core.

(iii) No, a step up transformer does not violate law of conservation of energy because whatever is gained in voltage ratio is lost in the current ratio and vice-versa. It steps up the voltage while it steps down the current.

Question 6.
State the underlying principle of a transformer.
How is the large scale transmission of electric energy over long distances done with the use of transformers?
Answer:
A transformer is an electrical device for converting an alternating current at low voltages into that at high voltage or vice versa.
If it increases the input voltage, it is called step- up-transformer.



Principle : It works on the principle of mutual induction i.e., “when a changing current is passed through one of the two inductively coupled coils, an induced emf is set up in the other coil.”

Working : As the alternating current flows through the primary, it generates an alternating magnetic flux in the core which also passes through the secondary. This changing flux sets up an induced emf in the secondary, also a self- induced emf in the primary. If there is no leakage of magnetic flux, then flux linked with each turn
of the primary will be equal to that linked with each turn of the secondary.

$$V_P = -N_P \frac{d\phi}{dt} \quad \text{and} \quad V_S = -N_S \frac{d\phi}{dt}$$

…where [N_p and N_s are number of turns in the primary and secondary respectively,
V_p and V_s are their respective voltages]

$$\therefore \quad \frac{V_S}{V_P} = \frac{N_S}{N_P} \qquad \ldots(i)$$

This ratio NSNP is called the turns ratio.
Assuming the transformer to be ideal one, so that there are no energy losses, then
Input power = output power
$V_pI_p = V_SI_S$
…where [I_P and I_S are the current in the primary and secondary respectively

$$\frac{V_S}{V_P} = \frac{I_P}{I_S} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get $\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$

In a step up transformer, $N_s > N_p$ i.e., the turns ratio is greater than 1 and therefore $V_s > V_p$.
The output voltage is greater than the input voltage.

Main assumptions :

1. The primary resistance and current are small.
2. The same flux links both with the primary and secondary windings as the flux leakage from due core is negligible (small).
3. The terminals of the secondary are open or the current taken from it, is small, (any two)

For long distance transmission, the voltage output of the generator is stepped-up (so that current is_ reduced and consequently, IR loss is reduced). It is transmitted over long distance and is stepped- down at distributing substations at consumers’ end.

Question 7.
A light bulb is rated 100 W for 220 V ac supply of 50 Hz. Calculate
(i) the resistance of the bulb;
(ii) the rms current through the bulb.
Answer:

$$(i)\ P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{220 \times 220}{100} = \mathbf{484\ \Omega}$$

$$(ii)\ I_{rms} = \frac{V_{rms}}{R} = \frac{220}{484} = \mathbf{0.45\ ampere}$$

Question 8.
A light bulb is rated 200 W for 220 V ac supply of 50 Hz. Calculate
(i) the resistance of the bulb;
(ii) the rms current through the bulb.
Answer:

$$(i)\ P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{220 \times 220}{100} = \mathbf{484\ \Omega}$$

$$(ii)\ I_{rms} = \frac{V_{rms}}{R} = \frac{220}{484} = \mathbf{0.45\ ampere}$$

Hint: (i) 242Ω
(ii) I_{rms} = 0.90 atmosphere

Question 9.
A light bulb is rated 150 W for 220 V ac supply of 60 Hz. Calculate
(i) the resistance of the bulb; ,
(ii) the rms current through the bulb.
Answer:

$$(i)\ P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{220 \times 220}{100} = \mathbf{484\ \Omega}$$

$$(ii)\ I_{rms} = \frac{V_{rms}}{R} = \frac{220}{484} = \mathbf{0.45\ ampere}$$

Hint :
(i) P = 322.67 Ω
(ii) I_{rms} = 0.68 ampere

Question 10.
An alternating voltage given by V = 140 sin 314 t is connected across a pure resistor of 50 Ω. Find
(i) the frequency of the source.
(ii) the rms current through the resistor.

Answer:

(*i*) $V_0 = 140\text{ V}, \quad \omega = 314$

$\Rightarrow 2\pi\nu = 314 \qquad \therefore\ \nu = \dfrac{314}{2\pi} = \mathbf{50\ Hz.}$

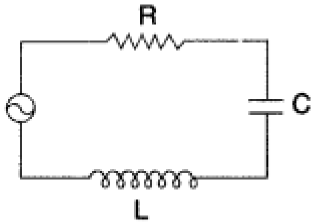
(*ii*) $I_{rms} = \dfrac{V_{rms}}{R}$ **Where** $[V_{rms} = \dfrac{V}{\sqrt{2}}]$

$= \dfrac{V_0/\sqrt{2}}{R} = \dfrac{V_0}{\sqrt{2}R} = \dfrac{140}{\sqrt{2}\times 50} = \dfrac{140}{1.414\times 50}$

$= \mathbf{1.98\ A} \approx \mathbf{2A}$

4 Mark Questions

Question 1.
The figure shows a series LCR circuit with L = 5.0 H, C = 80 μF, R = 40 Ω connected to a variable frequency 240 V source. Calculate
(i) The angular frequency of the source which drives the circuit at resonance.
(ii) The current at the resonating frequency.
(iii) The rms potential drop across the capacitor at resonance.



Answer:

Angular frequency, $\omega = \dfrac{1}{\sqrt{LC}}$

(*i*) $\omega = \dfrac{1}{\sqrt{5\times 80\times 10^{-6}}} = \dfrac{1}{\sqrt{400\times 10^{-6}}}$

$= \dfrac{1}{20\times 10^{-3}} = \dfrac{1000}{20} = \mathbf{50\ radians/sec.}$

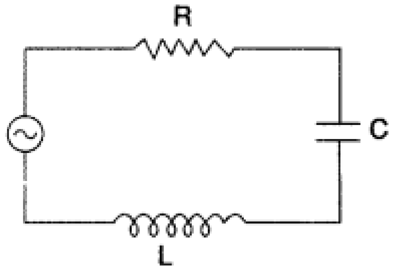
(*ii*) $I_{rms} = \dfrac{V_{rms}}{R} = \dfrac{240}{40}\text{A} = \mathbf{6A}$

(*iii*) V_{rms} across capacitor $= 6\times\dfrac{1}{\omega C}$

$= 6\times\dfrac{1}{50\times 80\times 10^{-6}}\text{ V}$

$= \dfrac{6\times 10^6}{4\times 10^3}\text{V} = \dfrac{6000}{4}\text{V} = \mathbf{1500\ V}$

Question 2.
A series LCR circuit with L = 4.0 H,C = 100 μF and R = 60 Ω. is connected to a variable frequency 240 V source as shown in the figure.
Calculate :
(i) the angular frequency of the source which derives the circuit at resonance;
(ii) the current at the resonating freqency;
(iii) the rms potential drop across the inductor at



Answer:

$$\omega = \frac{1}{\sqrt{LC}}$$

(*i*) Angular frequency, ω

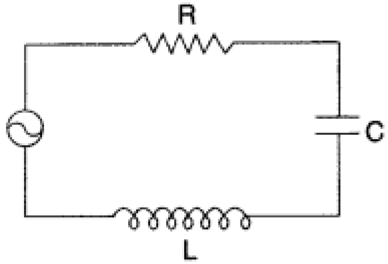
$$= \frac{1}{\sqrt{4 \times 100 \times 10^{-6}}} = \frac{1}{\sqrt{4 \times 10^{-4}}}$$

$$= \frac{1}{2 \times 10^{-2}} = \frac{100}{2} = \mathbf{50\ radians/sec}$$

(*ii*) Current resonance, $I_{rms} = \frac{V_{rms}}{R} = \frac{240}{60} = \mathbf{4A}$

(*iii*) V_{rms}, across capacitor, $R = IX_L = I_\omega L$
$= 4 \times 50 \times 4 = \mathbf{800\ V}$

Question 3.
The figure shows a series LCR circuit with L = 10.0 H, C = 40 μF, R = 60 Ω connected to a variable frequency 240 V source.



Calculate:
(i) The angular frequency of the source which drives the circuit at resonance.
(ii) The current at the resonating frequency.
(iii) The rms potential drop across the inductor at resonance.
Answer:

$$\omega = \frac{1}{\sqrt{LC}}$$

(*i*) Angular frequency, ω

$$= \frac{1}{\sqrt{4 \times 100 \times 10^{-6}}} = \frac{1}{\sqrt{4 \times 10^{-4}}}$$

$$= \frac{1}{2 \times 10^{-2}} = \frac{100}{2} = \mathbf{50\ radians/sec}$$

(*ii*) Current resonance, $I_{rms} = \frac{V_{rms}}{R} = \frac{240}{60} = \mathbf{4A}$

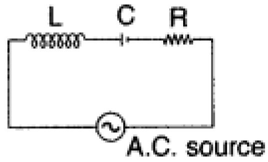
(*iii*) V_{rms}, across capacitor, $R = IX_L = I_\omega L$

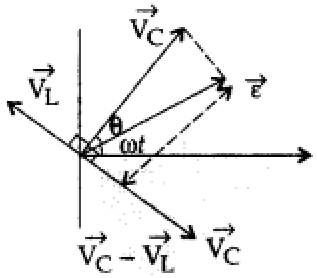
$$= 4 \times 50 \times 4 = \mathbf{800\ V}$$

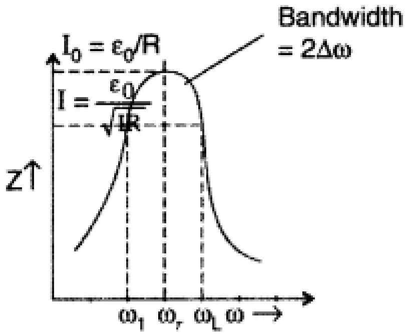
(i) Angular frequency, to = 50 ‘radiance/sec.
(ii) I_{rms} = 4A
(iii) $V_{rms} = I_{rms}X_c$ = 2000 V

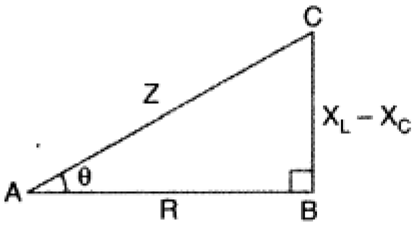
Question 4.
A series LCR circuit is connected to an ac source. Using the phasor diagram, derive the expression for the impedance of the circuit. Plot a graph to show the variation of current with frequency of the source, explaining the nature of its variation.

Answer:





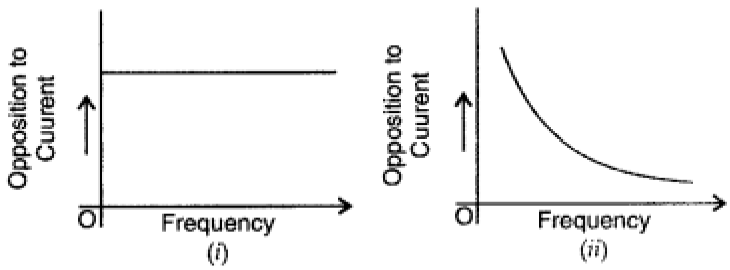




In ΔABC

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

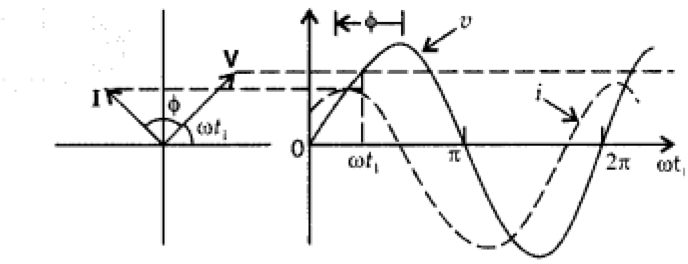
Question 5.
(a) The graphs
(i) and
(ii) shown in the figure represent variation of opposition offered by the circuit elements, X and Y, respectively to the flow of alternating current vs. the frequency of the applied emf. Identify the elements X and Y.



(b) Write the expression for the impedance offered by the series combination of these two elements connected to an ac source of voltage $V = V_0 \sin \omega t$.
Show on a graph the variation of the voltage and the current with 'out' in the circuit.
Answer:
(*a*) 'X' is *Resistor*.
'Y' is *Capacitor*.

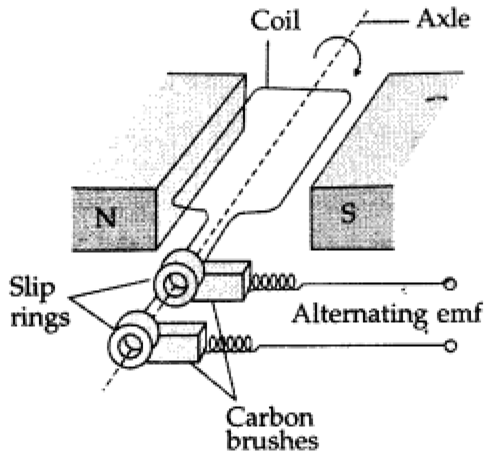
(*b*) Impedance, $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

...where $\left[\phi = \tan^{-1} \frac{X_c}{R} = \frac{1}{\omega CR}\right.$



Question 6.
Draw a sketch showing the basic elements of an a.c. generator. State its principle and explain briefly its working.
Answer:
(a) Principle of A.C. generator : The working of an a.c. generator is based on the principle of electromagnetic induction. When a closed coil is rotated in a uniform magnetic field with its axis perpendicular to the magnetic field, the magnetic flux linked with the coil changes and an induced emf and hence a current is set up in it.

(b) Let N = number of turns in the coil
A = Area of face of each turn
B = magnitude of the magnetic field
θ = angle which normal to the coil makes with field B at any instant
ω = the angular velocity with which coil rotates



The magnetic flux linked with the coil at any instant f will be,
$\phi = NAB \cos \theta = NAB \cos \omega t$
By Faraday's flux rule, the induced emf is given by,

$$E = -\frac{d\phi}{dt} = \frac{-d}{dt} NAB (\cos \omega t)$$

$$E = NAB (\sin \omega t) . \omega$$

$$\Rightarrow E = E_0 \sin \omega t \qquad \text{...where } [E_0 = NAB\omega$$

When a load of resistance R is connected across the terminals, a current I flows in the external circuit.

$$I = \frac{E}{R} = \frac{E_0 \sin \omega t}{R} = I_0 \sin \omega t$$

$$\text{...where} \left[I_0 = \frac{E_0}{R} \right.$$

(c) $v = 0.5$ Hz; $N = 100$; $A = 0.1\ m^2$; $B = 0.01$ T

$$e_{max} = NAB (2\pi v)$$

$$e_{max} = 100 \times 0.01 \times 0.1 \times (2\pi \times 0.5)$$

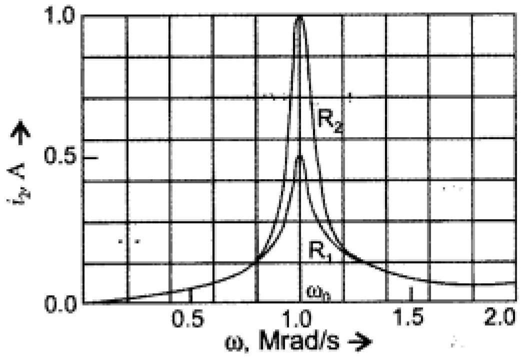
$$\therefore e_{max} = \mathbf{0.314\ volt}$$

7 Marks Questions

Question 1.
In a series LCR circuit connected to an ac source of variable frequency and voltage $v = V_m \sin \omega t$, draw a plot showing the variation of current (I) with angular frequency (ω) for two different values of resistance R_1 and R_2 ($R_1 > R_2$). Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define Q-factor of the circuit and give its significance.

Answer:
(a) Condition for resonance to occur is $X_L = X_C$, and Z = R.
(b) Sharper resonance is produced for R_2
(c) The Q-factor (quality factor) of series resonant circuit is defined as the ratio of the voltage developed across the inductance of capacitance at resonance to the impressed voltage, which is the voltage applied across the R.

$$Q = \frac{\omega_0 L}{R}$$



Significance : Higher the value of Q, the narrower and sharper is the resonance and therefore circuit will be more selective

Question 2.
(i) For a given a.c., i = i_m sin ωt, show that the average power dissipated in a resistor R over a complete cycle is 12i2mR.
(ii) A light bulb is rated at 100 W for a 220 V a.c. supply. Calculate the resistance of the bulb.
Answer:

(*i*) $i = i_m \sin \omega t$, where i_m is peak current, ω is angular frequency

$P = V_{rms} I_{rms} \cos \phi$, R = resistance

In a.c., across resistance V_{rms} and I_{rms} are in phase $\therefore \cos \phi = 1$ $(\phi = 1)$

$$P = V_{rms} I_{rms} \cos 0, \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\Rightarrow P = \frac{V_m i_m}{\sqrt{2} \times \sqrt{2}}, \quad I_{rms} = \frac{i_m}{\sqrt{2}}$$

$$\therefore P = \frac{V_m i_m}{2} \quad \ldots(i)$$

We know $V_m = i_m R$

$\therefore$ Equation (*i*) becomes $\boxed{P = \frac{i_m^2}{2} R}$

(*ii*) P = 100 W, V = 220 V

$$P = \frac{V^2}{R} \quad \therefore R = \frac{V^2}{P} = \frac{220 \times 220}{100} = \mathbf{484\ \Omega}$$

Question 3.
(a) For a given a.c., i = i_m sin ωt, show that the average power dissipated in a resistor R over complete cycle is 12i2mR.
(b) A light bulb is rated at 125 W for 250 V a.c. supply. Calculate the resistance of the bulb.
Answer:

(*i*) $i = i_m \sin \omega t$, where i_m is peak current, ω is angular frequency
$P = V_{rms} I_{rms} \cos \phi$, R = resistance
In a.c., across resistance V_{rms} and I_{rms} are in phase $\quad \therefore \cos \phi = 1 \quad (\phi = 1)$

$$P = V_{rms} I_{rms} \cos 0, \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\Rightarrow \quad P = \frac{V_m i_m}{\sqrt{2} \times \sqrt{2}}, \quad I_{rms} = \frac{i_m}{\sqrt{2}}$$

$$\therefore \quad P = \frac{V_m i_m}{2} \quad \ldots(i)$$

We know $V_m = i_m R$

$\therefore$ Equation (*i*) becomes $\boxed{P = \frac{i_m^2}{2} R}$

(*ii*) P = 100 W, V = 220 V

$$P = \frac{V^2}{R} \quad \therefore \quad R = \frac{V^2}{P} = \frac{220 \times 220}{100} = \mathbf{484\ \Omega}$$

(ii) P = 125 W, V = 250 V
Where P = Power and V = Voltage

$$P = \frac{V^2}{R} \text{ or } R = \frac{V^2}{P}, \ R = \frac{250 \times 250}{125} = 500\ \Omega$$

Question 4.
(a) When an a.c. source is connected to an ideal capacitor show that the average power supplied by the source over a complete cycle is zero.
(b) A lamp is connected in series with a capacitor. Predict your observations when the system is connected first across a d.c. and then an a.c. source. What happens in each case if the capacitance of the capacitor is reduced?
Answer:
(a) Average power associated with a capacitor :
When an a.c. is applied to a capacitor, the current leads the voltage in phase by π2radian. So we write the expressions for instantaneous voltage and current as follows :

$$V = V_0 \sin \omega t, \ I = I_0 \sin \left(\omega t + \frac{\pi}{2}\right) = I_0 \cos \omega t$$

Work done in the circuit in small time dt will be

$$d\omega = pdt = VI\, dt = V_0 I_0 \sin \omega t \cos \omega t\, dt$$
$$= \frac{V_0 I_0}{2} \sin 2\omega t\, dt$$

The average power dissipated per cycle in the capacitor is,

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T dW$$

$$= \frac{V_0 I_0}{2T}\int_0^T \sin 2\omega t\, dt = \frac{V_0 I_0}{2T}\left[\frac{-\cos 2\omega t}{2\omega}\right]_0^T$$

$$\left[\because \omega = \frac{2\pi}{T}\right]$$

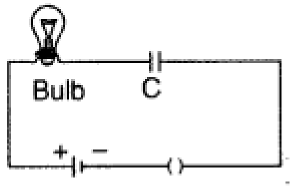
$$= \frac{-V_0 I_0}{4T\omega}\left[\cos\left(\frac{4\pi T}{T}\right)\right]_0^T$$

$$= \frac{-V_0 I_0}{4T\omega}[\cos 4\pi - \cos 0]$$

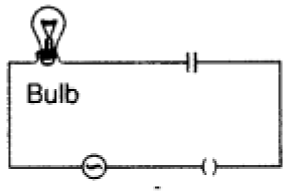
$$= \frac{-V_0 I_0}{4T\omega}[1 - 1] = 0$$

Thus the average power dissipated per cycle in a capacitor is zero.

(b) (i) In this case, the bulb will glow initially for a very short Bulb duration depending upon its time constant during the charging of capacitor. Once the capacitor is fully charged, it will not allow current to pass, hence the bulb ceases to glow.



(ii) In this case, when connected to a.c. source, the bulb will glow with the same brightness.
When the capacity of capacitor is reduced, it will have no appreciable effect when connected to d.c. source. However, in case when connected to a.c. source, capacitance is reduced, hence



$$\chi_C = \frac{1}{\omega C}$$

χC capactive reactance will increase and thus the brightness of bulb will reduce.

Question 5.
A voltage V = V_0 sin est is applied to a series LCR circuit. Derive the expression for the average power dissipated over a cycle.
Under what condition is
(i) no power dissipated even though the current flows through the circuit,
(ii) maximum power dissipated in the circuit?
Answer:
Average power in LCR circuit :
Let the alternating emf applied to an LCR circuit,
$V = V_0 \sin \omega t$ …(i)
If alternating current developed lags behind the applied emf by a phase angle ϕ
then, $I = I_0 \sin(\omega t - \phi)$ …(ii)
Total work done over a complete cycle is,

$$W = \int_0^T VI\,dt = \int_0^T V_0 \sin \omega t . I_0 \sin(\omega t - \phi)\,dt$$

$$= V_0 I_0 \int_0^T \sin \omega t \sin(\omega t - \phi)\,dt$$

∴ Average power in LCR circuit over complete cycle is,

$$= \frac{V_0 I_0}{2} \int_0^T 2 \sin \omega t \sin(\omega t - \phi)\,dt$$

$$= \frac{V_0 I_0}{2} \int_0^T [\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)]\,dt$$

$[\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B)]$

$$= \frac{V_0 I_0}{2} \int_0^T [\cos \phi - \cos(2\omega t - \phi)]\,dt$$

$$= \frac{V_0 I_0}{2} \left[t \cos \phi - \frac{\sin(2\omega t - \phi)}{2\omega} \right]_0^T$$

$$= \frac{V_0 I_0}{2} [t \cos \phi] = \frac{V_0 I_0}{2} . (\cos \phi) . t$$

Average power in LCR circuit over a complete cycle is,

$$P = \frac{W}{t} = \frac{V_0 I_0}{2} \cos \phi = \frac{V_0}{\sqrt{2}} . \frac{V_0}{\sqrt{2}} \cos \phi$$

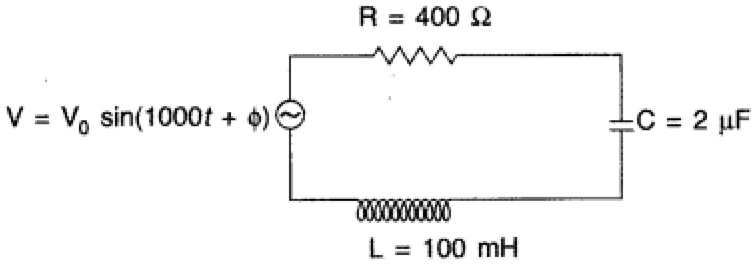
$$\therefore \mathbf{P = E_V I_V \cos \phi}$$

(i) No power is dissipated in purely inductive or purely capacitive circuit, because phase difference between voltage and current is π2 and $\cos \phi = 0$. It is known as wattless current.
(ii) Maximum power is dissipated in a LCR circuit at Resonance, because $X_C - X_L = 0$ and $\phi = 0$, $\cos \phi = 1$
Power = $I^2 Z = I^2 R$

Question 6.
An inductor L of inductance X_L is connected in series with a bulb B and an ac source. How would brightness of the bulb change when
(i) number of turns in the inductor is reduced,
(ii) an iron rod is inserted in the inductor and
(iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit. Justify your Answer in each case.(a) Determine the value of phase difference between the current and the voltage in the given series LCR circuit. (b) Calculate the value of the additional capacitor which may be joined suitably to the capacitor C that would make the power factor of the circuit unity.

Answer:
(i) Increases. $X_L = \omega L$
As number of turns decrease, L decreases, hence current through the bulb increases. Also voltage across bulb increases.
(ii) Decreases : Iron rod increases the inductance which increases X_L, hence current through the bulb decreases./Voltage across the bulb decreases.
(iii) Increases. Under this condition ($X_C = X_L$) the current through the bulb will become maximum.

(a) Determine the value of phase difference between the current and the voltage in the given series LCR circuit.



(*a*) **Given :** $V = V_0 \sin(1000\,t + \phi)$, $R = 400\ \Omega$, $L = 100$ mH, $C = 2\ \mu F$, $\omega = 1000$

$$\theta = \tan^{-1}\left(\frac{\chi_C - \chi_L}{R}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{1000 \times 2 \times 10^{-6}} - 1000 \times 100 \times 10^{-3}}{400}\right)$$

$$= \tan^{-1}\frac{400}{400} = 1 \quad \boldsymbol{\theta = 45°}$$

(b) Let the additional capacitor be C' which is to be connected in parallel with C, to increase the value of combined capacitances; hence resulting into 'capacitive reactance' reduced. In parallel $C_{net} = C + C'$

When the power factor is unity

When $\chi_L = \chi_C$

$$\omega L = \frac{1}{\omega(C + C')}$$

$$C + C' = \frac{1}{\omega^2 L} \qquad C' = \frac{1}{\omega^2 L} - C$$

$$C' = \frac{1}{(1000)^2 \times (100 \times 10^{-3})} - 2 \times 10^{-6}$$

$$C' = (10 \times 10^{-6}) - (2 \times 10^{-6})$$

$$= (10 - 2) \times 10^{-6} = \mathbf{8\mu F}$$

Fill in the Blanks

1. What is the frequency of the AC Mains in India-------------(50 Hz)

2. An alternating current can be produced by----------(Dynamo)

3. -----------------Which of the following can measure an alternating current? (Ammeter)

4. ------------------Which of the following circuits exhibits maximum power dissipation?

(Pure Resistive Circuit)

5. What happens to the inductive reactance when the frequency of the AC supply is--------- increased?(increases)

6. What happens to the quality factor of an LCR circuit -------------if the resistance is increased?(Decreases)

7. Which of the following statements is true about the LCR circuit connected to an AC source at resonance---------------(R equals the applied voltage.)

Multiple choice questions

Q.1. In general in an alternating current circuit
(a) the average value of current is zero
(b) the average value of square of the current is zero
(c) average power dissipation is zero
(d) the phase difference between voltage and current is zero

Answer:(a)

Q.2. The frequency of A.C. mains in India is
(a) 30 c/s
(b) 50 c/s
(c) 60 c/s
(d) 120 c/s

Answer:(b)

Q.3. A.C. power is transmitted from a power house at a high voltage as
(a) the rate of transmission is faster at high voltages
(b) it is more economical due to less power loss
(c) power cannot be transmitted at low voltages
(d) a precaution against theft of transmission lines

Answer:(b)

Q.4. The electric mains supply in our homes and offices is a voltage that varies like a sine function with time such a voltage is called … A… and the current driven by it in a circuit is called the … B… Here, A and B refer to
(a) DC voltage, AC current
(b) AC voltage, DC current
(c) AC voltage, DC voltage
(d) AC voltage, AC current

Answer:(d).

Q.5. Alternating currents can be produced by a
(a) dynamo
(b) choke coil
(c) transformer
(d) electric motor

Answer:(a)

Q.6. The peak value of the a.c. current flowing throw a resistor is given by
(a) $I_0 = e_0/R$
(b) $I = e/R$
(c) $I_0 = e_0$
(d) $I_0 = R/e_0$

Answer:(a)

Q.7. The alternating current can be measured with the help of
(a) hot wire ammeter
(b) hot wire voltmeter
(c) moving magnet galvanometer
(d) suspended coil type galvanometer

Answer:(a)

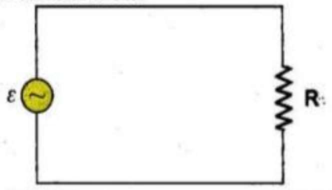
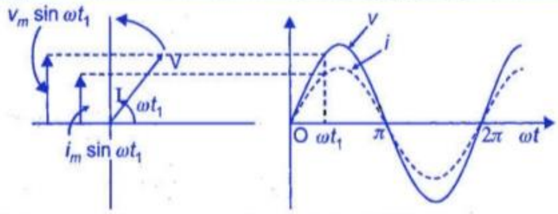
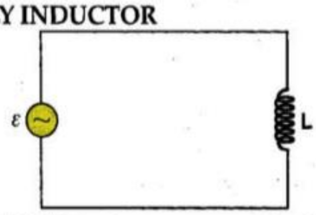
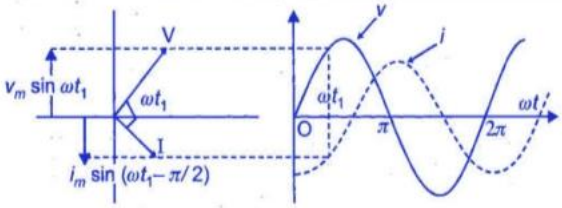
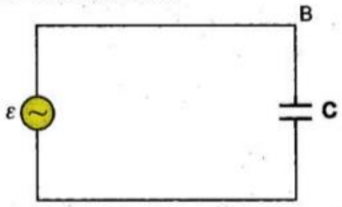
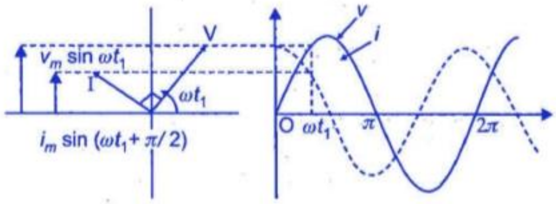
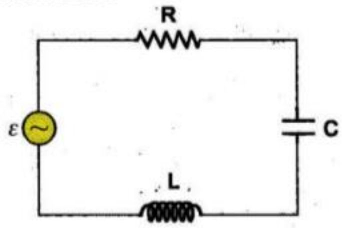
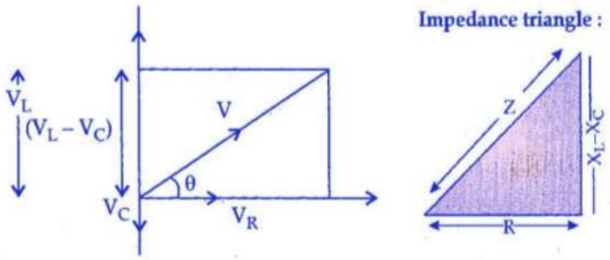
Q.8. Alternating current can not be measured by D.C. ammeter, because
(a) A. C. is virtual
(b) A. C. changes its direction
(c) A. C. can not pass through D.C. ammeter
(d) average value of A. C for complete cycle is zero

Answer:(d)

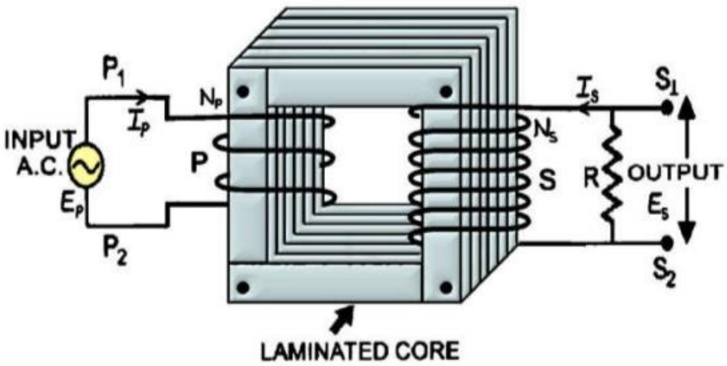
Q.9. The heat produced in a given resistance in a given time by the sinusoidal current $I_0 \sin\omega t$ will be the same as that of a steady current of magnitude nearly
(a) 0.71 I_0
(b) 1.412 I_0
(c) I_0
(d) $\sqrt{I_0}$

Answer:(a)

Diagrams

Type of circuit	Phasor diagram & wave diagram
ONLY RESISTOR 	
ONLY INDUCTOR 	
ONLY CAPACITOR 	
LCR CIRCUIT 	

Transformer



$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$\eta = \frac{E_s I_s}{E_p I_p} \times 100\ \%$$

SUMMARY

- **Alternating Current:**
  The current whose magnitude changes with time and direction reverses periodically is called alternating current. a) Alternating emf E and current I at any time am given by:
  $$E = E_0 \sin \omega t$$
  Where $E_0 = NBA\omega$
  $$I = I_0 \sin(\omega t - \phi)$$
  Where $I_0 = \frac{NBA\omega}{R}$
  $$\omega = 2\pi n = \frac{2\pi}{T}$$
  Where T is the time period.
- **Values of Alternating Current and Voltage**
  a) Instantaneous value:
  It is the value of alternating current and voltage at an instant t.
  b) Peak value:
  Maximum values of voltage E_0 and current I_0 in a cycle are called peak values.
  c) Mean value:
  For complete cycle,
  $$< E >= \frac{1}{T}\int_0^T Edt = 0$$
  $$< I >= \frac{1}{T}\int_0^T Idt = 0$$
  Mean value for half cycle: $E_{mean} = \frac{2E_0}{\pi}$
  d) Root – mean- square (rms) value:
  $$E_{rms} = (< E^2 >)^{1/2} = \frac{E_0}{\sqrt{2}} = 0.707E_0 = 70.7\%E_0$$
  $$I_{rms} = (< I^2 >)^{1/2} = \frac{I_0}{\sqrt{2}} = 0.707I_0 = 70.7\%I_0$$
  RMS values are also called apparent or effective values.
- **Phase difference Between the EMF (Voltage) and the Current in an AC Circuit**
  a) For pure resistance:
  The voltage and the current are in same phase i.e. phase difference $\phi = 0$
  b) For pure inductance:

The voltage is ahead of current by $\frac{\pi}{2}$ i.e. phase difference $\phi = +\frac{\pi}{2}$.

c) For pure capacitance:

The voltage lags behind the current by $\frac{\pi}{2}$ i.e. phase difference $\phi = -\frac{\pi}{2}$

- **Reactance:**

a) Reactance
$$X = \frac{E}{I} = \frac{E_0}{I_0} = \frac{E_{rms}}{I_{rms}} \pm \pi / 2$$

b) Inductive reactance
$$X_L = \omega L = 2\pi nL$$

c) Capacitive reactance
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi nC}$$

- **Impedance:**

Impedance is defined as,
$$Z = \frac{E}{I} = \frac{E_0}{I_0} = \frac{E_{rms}}{I_{rms}} \phi$$

Where ϕ is the phase difference of the voltage E relative to the current I.

a) For L – R series circuit:
$$Z_{RL} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega L^2}$$
$$\tan\phi = \left(\frac{\omega L}{R}\right) or \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

b) For R – C series circuit:
$$Z_{RC} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$
$$\tan\phi = \frac{1}{\omega CR} \quad \text{Or } \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

c) For L – C series circuit:
$$Z_{LCR} = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
$$\tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} \quad \text{Or} \quad \phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

- **Conductance: Reciprocal of resistance is called conductance.**

$$G = \frac{1}{R} mho$$

- **Power in and AC Circuit:**

a) Electric power = (current in circuit) x (voltage in circuit)

P = IE

b) Instantaneous power:

$P_{inst} = E_{inst} \times I_{inst}$

c) Average power:

$$P_{av} = \frac{1}{2} E_0 I_0 \cos\phi = E_{rms} I_{rms} \cos\phi$$

d) Virtual power (apparent power):

$$= \frac{1}{2} E_0 I_0 = E_{rms} I_{rms}$$

- **Power Factor:**

a) Power factor

$$\cos\phi = \frac{P_{av}}{P_v} = \frac{R}{Z}$$

b) For pure inductance

Power factor, $\cos\phi = 1$

c) For pure capacitance

Power factor, $\cos\phi = 0$

d) For LCR circuit

$$\text{Power factor, } \cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$X = \left(\omega L - \frac{1}{\omega C}\right)$$

- **Wattless Current:**

The component of current differing in phase by $\frac{\pi}{2}$ relative to the voltage, is called wattles current.

- **The rms value of wattless current:**

$$= \frac{I_0}{\sqrt{2}} \sin\phi$$

$$= I_{rms} \sin\phi = \frac{I_0}{\sqrt{2}}\left(\frac{X}{Z}\right)$$

- **Choke Coil:**
  a) An inductive coil used for controlling alternating current whose self- inductance is high and resistance in negligible, is called choke coil.
  b) The power factor of this coil is approximately zero.
- **Series Resonant Circuit**
  a) When the inductive reactance (XL) becomes equal to the capacitive reactance (XC) in the circuit, the total impedance becomes purely resistive (Z=R).
  b) In this state, the voltage and current are in same phase ($\phi = 0$), the current and power are maximum and impedance is minimum. This state is called resonance.
  c) At resonance,

  $$\omega_r L = \frac{1}{\omega_r C}$$

  Hence, resonance frequency is,

  $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

  d) In resonance, the power factor of the circuit is one.
- **Half – Power Frequencies:**
  Those frequencies f_1 and f_2 at which the power is half of the maximum power (power at resonance), i.e., f_1 and f_2 are called half – power frequencies.

  $$P = \frac{1}{2}P_{\max}$$

  $$I = \frac{I_{\max}}{\sqrt{2}}$$

  $$\therefore P = \frac{P_{\max}}{2}$$

- **Band – Width:**
  a) The frequency interval between half – power frequencies is called band – width.

  $$\therefore \text{ Bandwidth } \Delta f = f_2 - f_1$$

  b) For a series LCR resonant circuit,

  $$\Delta f = \frac{1}{2\pi}\frac{R}{L}$$

- **Quality Factor (Q):**

  $$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

  $$= \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}}$$

  Or

  $$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{f_r}{(f_2 - f_1)} = \frac{f_r}{\Delta f}$$