CHAPTER-8

Algebraic Expressions and Identities

2MARK Q&A:

Exercise 8.1

Q1. Identify the terms, their coefficients for each of the following expressions. (i) $5xyz^2 - 3zy$ (ii) $1 + x + x^2$ (iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$ (iv) $3 - pq + qr - p$ (v) $(x/2) + (y/2) - xy$ (vi) $0.3a - 0.6ab + 0.5b$

Sl. No.	Expression	Term	Coefficient
i)	$5xyz^2 - 3zy$	Term: $5xyz^2$ Term: $-3zy$	5 -3
ii)	$1 + x + x^2$	Term: *1* Term: *x* *Term:* x^2	1 1 1
iii)	$4x^2y^2 - 4x^2y^2z^2 + z^2$	Term: $4x^2y^2$ Term: $-4\,x^2y^2z^2$ *Term :* z^2	4 -4 1
iv)	$3 - pq + qr - p$	*Term :* 3 - pq qr -p	3 -1 1 -1

v)	(x/2) + (y/2) – xy	*Term :* x/2 Y/2 -xy	½ 1/2 -1
vi)	0.3a – 0.6ab + 0.5b	*Term :* 0.3a -0.6ab 0.5b	0.3 -0.6 0.5

Solution :

237

2. Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories? x + y, 1000, $x + x^2 + x^3 + x^4$, 7 + y + 5x, $2y – 3y^2$, $2y – 3y^2 + 4y^3$, 5x – 4y + 3xy, $4z – 15z^2$, ab + bc + cd + da, pqr, $p^2q + pq^2$, 2p + 2q

Solution:

Let us first define the classifications of these 3 polynomials:

Monomials contain only one term.

Binomials contain only two terms.

Trinomials contain only three terms.

x + y	two terms	Binomial
1000	one term	Monomial

$x + x^2 + x^3 + x^4$	four terms	Polynomial, and it does not fit in the listed three categories
$2y - 3y^2$	two terms	Binomial
$2y - 3y^2 + 4y^3$	three terms	Trinomial
$5x - 4y + 3xy$	three terms	Trinomial
$4z - 15z^2$	two terms	Binomial
$ab + bc + cd + da$	four terms	Polynomial, and it does not fit in the listed three categories
pqr	one term	Monomial
$p^2q + pq^2$	two terms	Binomial
$2p + 2q$	two	Binomial

	terms	
$7 + y + 5x$	three terms	Trinomial

139

3. Add the following.

(i) ab – bc, bc – ca, ca – ab

(ii) a – b + ab, b – c + bc, c – a + ac

(iii) $2p^2q^2 – 3pq + 4$, $5 + 7pq – 3p^2q^2$

(iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, 2lm + 2mn + 2nl

Solution:

i) (ab – bc) + (bc – ca) + (ca-ab)

= ab – bc + bc – ca + ca – ab

= ab – ab – bc + bc – ca + ca

= 0

ii) (a – b + ab) + (b – c + bc) + (c – a + ac)

= a – b + ab + b – c + bc + c – a + ac

= a – a +b – b +c – c + ab + bc + ca

= 0 + 0 + 0 + ab + bc + ca

$= ab + bc + ca$

iii) $2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$

$= (2p^2q^2 - 3pq + 4) + (5 + 7pq - 3p^2q^2)$

$= 2p^2q^2 - 3p^2q^2 - 3pq + 7pq + 4 + 5$

$= - p^2q^2 + 4pq + 9$

iv) $(l^2 + m^2) + (m^2 + n^2) + (n^2 + l^2) + (2lm + 2mn + 2nl)$

$= l^2 + l^2 + m^2 + m^2 + n^2 + n^2 + 2lm + 2mn + 2nl$

$= 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

272

4. (a) Subtract *4a – 7ab + 3b + 12* from *12a – 9ab + 5b – 3*

(b) Subtract *3xy + 5yz – 7zx* from *5xy – 2yz – 2zx + 10xyz*

(c) Subtract *$4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$* from *$18 - 3p - 11q + 5pq - 2pq^2 + 5p^2$*q

Solution:

(a) $(12a - 9ab + 5b - 3) - (4a - 7ab + 3b + 12)$

$= 12a - 9ab + 5b - 3 - 4a + 7ab - 3b - 12$

$= 12a - 4a - 9ab + 7ab + 5b - 3b - 3 - 12$

$= 8a - 2ab + 2b - 15$

b) $(5xy - 2yz - 2zx + 10xyz) - (3xy + 5yz - 7zx)$

$= 5xy - 2yz - 2zx + 10xyz - 3xy - 5yz + 7zx$

$=5xy - 3xy - 2yz - 5yz - 2zx + 7zx + 10xyz$

$= 2xy - 7yz + 5zx + 10xyz$

c) $(18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q) - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$

$= 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - 4p^2q + 3pq - 5pq^2 + 8p - 7q + 10$

=18+10 -3p+8p -11q – 7q + 5 pq+ 3pq- 2pq^2 – 5pq^2 + 5 p^2 q – 4p^2 q

$= 28 + 5p - 18q + 8pq - 7pq^2 + p^2q$

Exercise 8.2

1. Find the product of the following pairs of monomials.

(i) 4, 7p

(ii) – 4p, 7p

(iii) – 4p, 7pq

(iv) $4p^3$, – 3p

(v) 4p, 0

Solution:

(i) 4 , 7 p = $4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = (-4 \times 7) \times (p \times p) = -28p^2$

(iii) $-4p \times 7pq = (-4 \times 7)(p \times pq) = -28p^2q$

(iv) $4p^3 \times -3p = (4 \times -3)(p^3 \times p) = -12p^4$

(v) $4p \times 0 = 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths, respectively.

(p, q) ; (10m, 5n) ; ($20x^2$, $5y^2$) ; ($4x$, $3x^2$) ; (3mn, 4np)

Solution:

Area of rectangle = Length x breadth. So, it is multiplication of two monomials.

The results can be written in square units.

(i) $p \times q = pq$

(ii) $10m \times 5n = 50mn$

(iii) $20x^2 \times 5y^2 = 100x^2y^2$

(iv) $4x \times 3x^2 = 12x^3$

(v) $3mn \times 4np = 12mn^2p$

3. Complete the following table of products:

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

Solution:

First monomial / Second monomial	2x	-5y	$3x^2$	-4xY	$7x^2Y$	$-9x^2y^2$
2x	$4x^2$	-10xy	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
-5y	-10xy	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
-4xy	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27\ x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. Obtain the volume of rectangular boxes with the following length, breadth and height, respectively.

(i) $5a, 3a^2, 7a^4$

(ii) $2p, 4q, 8r$

(iii) $xy, 2x^2y, 2xy^2$

(iv) $a, 2b, 3c$

Solution:

Volume of rectangle = length x breadth x height. To evaluate volume of rectangular boxes, multiply all the monomials.

(i) $5a \times 3a^2 \times 7a^4 = (5 \times 3 \times 7)(a \times a^2 \times a^4) = 105a^7$

(ii) $2p \times 4q \times 8r = (2 \times 4 \times 8)(p \times q \times r) = 64pqr$

(iii) $y \times 2x^2y \times 2xy^2 = (1 \times 2 \times 2)(x \times x^2 \times x \times y \times y \times y^2) = 4x^4y^4$

(iv) $a \times 2b \times 3c = (1 \times 2 \times 3)(a \times b \times c) = 6abc$

5. Obtain the product of

(i) xy, yz, zx

(ii) a, $-a^2$, a^3

(iii) 2, 4y, $8y^2$, $16y^3$

(iv) a, 2b, 3c, 6abc

(v) m, – mn, mnp

Solution:

(i) $xy \times yz \times zx = x^2 y^2 z^2$

(ii) $a \times -a^2 \times a^3 = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 1024\, y^6$

(iv) $a \times 2b \times 3c \times 6abc = 36a^2 b^2 c^2$

(v) $m \times -mn \times mnp = -m^3 n^2 p$

Exercise 8.3

1. Carry out the multiplication of the expressions in each of the following pairs.

(i) 4p, q + r

(ii) ab, a – b

(iii) a + b, $7a^2b^2$

(iv) $a^2 – 9$, 4a

(v) pq + qr + rp, 0

Solution:

(i)4p(q + r) = 4pq + 4pr

(ii)ab(a – b) = a^2 b – a b^2

(iii)(a + b) ($7a^2b^2$) = $7a^3b^2 + 7a^2b^3$

(iv) ($a^2 – 9$)(4a) = $4a^3 – 36a$

(v) (pq + qr + rp) × 0 = 0 (Anything multiplied by zero is zero)

2. Complete the table.

	First expression	Second expression	Product
(i)	a	$b + c + d$	...
(ii)	$x + y - 5$	$5xy$	...
(iii)	p	$6p^2 - 7p + 5$	...
(iv)	$4p^2q^2$	$p^2 - q^2$	...
(v)	$a + b + c$	abc	...

Solution:

	First expression	Second expression	Product
(i)	a	b + c + d	a(b+c+d) = a×b + a×c + a×d = ab + ac + ad
(ii)	x + y – 5	5xy	5 xy (x + y – 5) = 5 xy x x + 5 xy x y – 5 xy x 5 = 5 x^2y + 5 xy^2 – 25xy
(iii)	p	$6p^2 – 7p + 5$	p (6 p^2-7 p +5) = p× 6 p^2 – p× 7 p + p×5 = 6 p^3 – 7 p^2 + 5 p
(iv)	4 p^2 q^2	$P^2 – q^2$	4p^2 q^2 * (p^2 – q^2) =4 p^4 q^2– 4p^2 q^4
(v)	a + b + c	abc	abc(a + b + c) = abc × a + abc × b + abc × c = a^2bc + ab^2c + abc^2

3. Find the product.

i) $a^2 \times (2a^{22}) \times (4a^{26})$

ii) $(2/3\ xy) \times (-9/10\ x^2y^2)$

(iii) $(-10/3\ pq^3/) \times (6/5\ p^3q)$

(iv) $(x) \times (x^2) \times (x^3) \times (x^4)$

Solution:

i) $a^2 \times (2a^{22}) \times (4a^{26})$

$= (2 \times 4)\ (\ a^2 \times a^{22} \times a^{26}\ )$

$= 8 \times a^{2 + 22 + 26}$

$= 8a^{50}$

ii) $(2xy/3) \times (-9x^2y^2/10)$

$=(2/3 \times -9/10\ )\ (\ x \times x^2 \times y \times y^2\ )$

$= (-3/5\ x^3y^3)$

iii) $(-10pq^3/3) \times (6p^3q/5)$

$= (\ -10/3 \times 6/5\ )\ (p \times p^3 \times q^3 \times q)$

$= (-4p^4q^4)$

iv) $(\ x) \times (x^2) \times (x^3) \times (x^4)$

$= x^{1 + 2 + 3 + 4}$

$= x^{10}$

4. (a) Simplify 3x (4x – 5) + 3 and find its values for (i) x = 3 (ii) x =1/2

(b) Simplify *a* *(a^2+ a + 1)* + *5* and find its value for (i) *a* = *0*, (ii) *a* = *1* (iii) *a* = – *1*.

Solution:

a) 3x (4x – 5) + 3

=3x (4x) – 3x(5) +3

=12x^2 – 15x + 3

(i) Putting x=3 in the equation we gets 12x^2 – 15x + 3 =12(3^2) – 15 (3) +3

= 108 – 45 + 3

= 66

(ii) Putting x=1/2 in the equation we get

12x^2 – 15x + 3 = 12 (1/2)^2 – 15 (1/2) + 3

= 12 (1/4) – 15/2 +3

= 3 – 15/2 + 3

= 6- 15/2

= (12- 15) /2

= -3/2

b) a(a^2 +a +1)+5

= a x a^2 + a x a + a x 1 + 5 =a^3+a^2+a+ 5

(i) putting a=0 in the equation we get $0^3+0^2+0+5=5$

(ii) putting a=1 in the equation we get $1^3 + 1^2 + 1+5 = 1 + 1 + 1+5 = 8$

(iii) Putting a = -1 in the equation we get $(-1)^3+(-1)^2 + (-1)+5 = -1 + 1 – 1+5 = 4$

5. (a) Add: *p (p – q), q (q – r) and r (r – p)*

(b) Add: *2x (z – x – y)* and *2y (z – y – x)*

(c) Subtract: *3l (l – 4 m + 5 n)* from *4l (10 n – 3 m + 2 l)*

(d) Subtract: *3a (a + b + c) – 2 b (a – b + c)* from *4c (– a + b + c)*

Solution:

a) *p (p – q) + q (q – r) + r (r – p)*

$= (p^2 – pq) + (q^2 – qr) + (r^2 – pr)$

$= p^2 + q^2 + r^2 – pq – qr – pr$

b) *2x (z – x – y) + 2y (z – y – x)*

$= (2xz – 2x^2 – 2xy) + (2yz – 2y^2 – 2xy)$

$= 2xz – 4xy + 2yz – 2x^2 – 2y^2$

c) *4l (10 n – 3 m + 2 l) – 3l (l – 4 m + 5 n)*

$= (40ln – 12lm + 8l^2) – (3l^2 – 12lm + 15ln)$

$= 40ln – 12lm + 8l^2 – 3l^2 +12lm -15 ln$

$= 25 ln + 5l^2$

d) *4c (– a + b + c) – (3a (a + b + c) – 2 b (a – b + c))*

$= (-4ac + 4bc + 4c^2) – (3a^2 + 3ab + 3ac – (2ab – 2b^2 + 2bc))$

$=-4ac + 4bc + 4c^2 – (3a^2 + 3ab + 3ac – 2ab + 2b^2 – 2bc)$

$= -4ac + 4bc + 4c^2 – 3a^2 – 3ab – 3ac +2ab – 2b^2 + 2bc$

$= -7ac + 6bc + 4c^2 – 3a^2 – ab – 2b^2$

5MARK Q&A:

Exercise 8.4

1. Multiply the binomials.

(i) (2x + 5) and (4x – 3)

(ii) (y – 8) and (3y – 4)

(iii) (2.5l – 0.5m) and (2.5l + 0.5m)

(iv) (a + 3b) and (x + 5)

(v) $(2pq + 3q^2)$ and $(3pq – 2q^2)$

(vi) $(3/4\ a^2 + 3b^2)$ and $4(a^2 – 2/3\ b^2)$

Solution :

(i) $(2x + 5)(4x – 3)$

$= 2x \times 4x – 2x \times 3 + 5 \times 4x – 5 \times 3$

$= 8x^2 – 6x + 20x - 15$

$= 8x^2 + 14x - 15$

174

ii) (y – 8)(3y – 4)

= y x 3y – 4y – 8 x 3y + 32

= $3y^2$ – 4y – 24y + 32

= $3y^2$ – 28y + 32

88

*(iii) (*2.5l – *0.5m)(2.5l + 0.5m)*

= 2.5l x 2.5 l + 2.5l x 0.5m – 0.5m x 2.5l – 0.5m x 0.5m

= *6.25l^2 + 1.25 lm* – 1.25 lm – 0.25 m^2

= *6.25l^2* – 0.25 m^2

iv) (a + 3b) (x + 5)

= *ax + 5a + 3bx + 15b*

v) *(2pq + 3q^2) (3pq – 2q^2)*

= 2pq x 3pq – 2pq x $2q^2$ + $3q^2$ x 3pq – $3q^2$ x $2q^2$

= $6p^2q^2$ – $4pq^3$ + $9pq^3$ – $6q^4$

= $6p^2q^2$ + $5pq^3$ – $6q^4$

108

(vi) (3/4 a² + 3b²) and 4(a² – 2/3 b²)

=(3/4 a² + 3b²) x 4(a² – 2/3 b²)

$=(3/4\ a^2 + 3b^2) \times (4a^2 - 8/3\ b^2)$

$=3/4\ a^2 \times (4a^2 - 8/3\ b^2) + 3b^2 \times (4a^2 - 8/3\ b^2)$

$=3/4\ a^2 \times 4a^2 - 3/4\ a^2 \times 8/3\ b^2 + 3b^2 \times 4a^2 - 3b^2 \times 8/3\ b^2$

$=3a^4 - 2a^2 b^2 + 12\ a^2\ b^2 - 8b^4$

$= 3a^4 + 10a^2\ b^2 - 8b^4$

2. Find the product.

(i) (5 – 2x) (3 + x)

(ii) (x + 7y) (7x – y)

(iii) $(a^2 + b)$ $(a + b^2)$

(iv) $(p^2 - q^2)$ (2p + q)

Solution:

(i) (5 – 2x) (3 + x)

= 5 (3 + x) – 2x (3 + x)

$=15 + 5x - 6x - 2x^2$

$= 15 - x - 2\ x^2$

(ii) (x + 7y) (7x – y)

= x(7x-y) + 7y (7x-y)

$=7x^2 - xy + 49xy - 7y^2$

$= 7x^2 - 7y^2 + 48xy$

iii) $(a^2 + b)(a + b^2)$

$= a^2 (a + b^2) + b(a + b^2)$

$= a^3 + a^2b^2 + ab + b^3$

$= a^3 + b^3 + a^2b^2 + ab$

iv) $(p^2 – q^2)(2p + q)$

$= p^2 (2p + q) – q^2 (2p + q)$

$=2p^3 + p^2q – 2pq^2 – q^3$

$= 2p^3 – q^3 + p^2q – 2pq^2$

3. Simplify.

(i) $(x^2 – 5)(x + 5) + 25$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

(iii)$(t + s^2)(t^2 – s)$

(iv) $(a + b)(c – d) + (a – b)(c + d) + 2(ac + bd)$

(v) $(x + y)(2x + y) + (x + 2y)(x – y)$

(vi) $(x + y)(x^2 – xy + y^2)$

(vii) $(1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y$

(viii) $(a + b + c)(a + b – c)$

Solution:

i) $(x^2 – 5)(x + 5) + 25$

$= x^3 + 5x^2 – 5x – 25 + 25$

$= x^3 + 5x^2 – 5x$

ii) $(a^2 + 5)(b^3 + 3) + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 5b^3 + 3a^2 + 20$

iii) $(t + s^2)(t^2 – s)$

$= t(t^2 – s) + s^2(t^2 – s)$

$= t^3 – st + s^2t^2 – s^3$

$= t^3 – s^3 – st + s^2t^2$

iv) $(a + b)(c – d) + (a – b)(c + d) + 2(ac + bd)$

$= (a + b)(c – d) + (a – b)(c + d) + 2(ac + bd)$

$=(ac – ad + bc – bd) + (ac + ad – bc – bd) + (2ac + 2bd)$

$= ac – ad + bc – bd + ac + ad – bc – bd + 2ac + 2bd$

$= 4ac$

v) $(x + y)(2x + y) + (x + 2y)(x – y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 – xy + 2xy – 2y^2$

$= 3x^2 + 4xy – y^2$

vi) $(x + y)(x^2 – xy + y^2)$

$= x^3 – x^2y + xy^2 + x^2y – xy^2 + y^3$

$= x^3 + y^3$

vii) $(1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x – 6xy – 16y^2 – 12y – 4.5x + 12y = 2.25x^2 – 16y^2$

viii) $(a + b + c)(a + b – c)$

$= a^2 + ab – ac + ab + b^2 – bc + ac + bc – c^2$

$= a^2 + b^2 – c^2 + 2ab$

Exercise 8.5

1. Use a suitable identity to get each of the following products.

(i) (x + 3) (x + 3)

(ii) (2y + 5) (2y + 5)

(iii) (2a – 7) (2a – 7)

(iv) (3a – 1/2)(3a – 1/2)

(v) (1.1m – 0.4) (1.1m + 0.4)

(vi) $(a^2 + b^2)$ $(- a^2 + b^2)$

(vii) (6x – 7) (6x + 7)

(viii) (- a + c) (- a + c)

(ix) (1/2x + 3/4y) (1/2x + 3/4y)

(x) (7a – 9b) (7a – 9b)

Solution:

(i) $(x + 3)(x + 3) = (x + 3)^2$

$= x^2 + 6x + 9$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

ii) $(2y + 5)(2y + 5) = (2y + 5)^2$

$= 4y^2 + 20y + 25$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

iii) $(2a – 7)(2a – 7) = (2a – 7)^2$

$= 4a^2 – 28a + 49$

Using $(a-b)^2 = a^2 + b^2 – 2ab$

iv) $(3a – 1/2)(3a – 1/2) = (3a – 1/2)^2$

$= 9a^2 - 3a + (1/4)$

Using $(a-b)^2 = a^2 + b^2 – 2ab$

v) $(1.1m – 0.4)(1.1m + 0.4)$

$= 1.21m^2 – 0.16$

Using $(a – b)(a + b) = a^2 – b^2$

vi) $(a^2 + b^2)(– a^2 + b^2)$

$= (b^2 + a^2)(b^2 – a^2)$

$= -a^4 + b^4$

Using $(a - b)(a + b) = a^2 - b^2$

vii) $(6x - 7)(6x + 7)$

$=36x^2 - 49$

Using $(a - b)(a + b) = a^2 - b^2$

viii) $(- a + c)(- a + c) = (- a + c)^2$

$= c^2 + a^2 - 2ac$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

ix) $\left(\frac{1}{2}x + \frac{3}{4}y\right)\left(\frac{1}{2}x + \frac{3}{4}y\right) = \left(\frac{1}{2}x + \frac{3}{4}y\right)^2$

$= (x^2/4) + (9y^2/16) + (3xy/4)$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

x) $(7a - 9b)(7a - 9b) = (7a - 9b)^2$

$= 49a^2 - 126ab + 81b^2$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.

(i) (x + 3) (x + 7)

(ii) (4x + 5) (4x + 1)

(iii) (4x – 5) (4x – 1)

(iv) (4x + 5) (4x – 1)

(v) (2x + 5y) (2x + 3y)

(vi) $(2a^2 + 9)(2a^2 + 5)$

(vii) (xyz – 4) (xyz – 2)

Solution:

(i)$(x + 3)(x + 7)$

$= x^2 + (3+7)x + 21$

$= x^2 + 10x + 21$

ii) $(4x + 5)(4x + 1)$

$= 16x^2 + 4x + 20x + 5$

$= 16x^2 + 24x + 5$

iii) $(4x – 5)(4x – 1)$

$= 16x^2 – 4x – 20x + 5$

$= 16x^2 – 24x + 5$

iv) $(4x + 5)(4x - 1)$

$= 16x^2 + (5-1)4x - 5$

$= 16x^2 + 16x - 5$

v) $(2x + 5y)(2x + 3y)$

$= 4x^2 + (5y + 3y)2x + 15y^2$

$= 4x^2 + 16xy + 15y^2$

vi) $(2a^2 + 9)(2a^2 + 5)$

$= 4a^4 + (9+5)2a^2 + 45$

$= 4a^4 + 28a^2 + 45$

vii) $(xyz - 4)(xyz - 2)$

$= x^2y^2z^2 + (-4 -2)xyz + 8$

$= x^2y^2z^2 - 6xyz + 8$

3. Find the following squares by using the identities.

(i) $(b - 7)^2$

(ii) $(xy + 3z)^2$

(iii) $(6x^2 - 5y)^2$

(iv) $[(2m/3) + (3n/2)]^2$

(v) $(0.4p - 0.5q)^2$

(vi) $(2xy + 5y)^2$

Solution:

Using identities:

$(a - b)^2 = a^2 + b^2 - 2ab$ $(a + b)^2 = a^2 + b^2 + 2ab$

(i) $(b - 7)^2 = b^2 - 14b + 49$

(ii) $(xy + 3z)^2 = x^2y^2 + 6xyz + 9z^2$

(iii) $(6x^2 - 5y)^2 = 36x^4 - 60x^2y + 25y^2$

(iv) $[(2m/3\}) + (3n/2)]^2 = (4m^2/9) + (9n^2/4) + 2mn$

(v) $(0.4p - 0.5q)^2 = 0.16p^2 - 0.4pq + 0.25q^2$

(vi) $(2xy + 5y)^2 = 4x^2y^2 + 20xy^2 + 25y^2$

4. Simplify.

(i) $(a^2 – b^2)^2$

(ii) $(2x + 5)^2 – (2x – 5)^2$

(iii) $(7m – 8n)^2 + (7m + 8n)^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

(v) $(2.5p – 1.5q)^2 – (1.5p – 2.5q)^2$

(vi) $(ab + bc)^2 – 2ab^2c$

(vii) $(m^2 – n^2m)^2 + 2m^3n^2$

Solution:

i) $(a^2 – b^2)^2 = a^4 + b^4 – 2a^2b^2$

ii) $(2x + 5)^2 – (2x – 5)^2$
$= 4x^2 + 20x + 25 – (4x^2 – 20x + 25) = 4x^2 + 20x + 25 – 4x^2 + 20x – 25 = 40x$

iii) $(7m – 8n)^2 + (7m + 8n)^2$
$= 49m^2 – 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$
$= 98m^2 + 128n^2$

iv) $(4m + 5n)^2 + (5m + 4n)^2$
$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
$= 41m^2 + 80mn + 41n^2$

166
v) $(2.5p – 1.5q)^2 – (1.5p – 2.5q)^2$
$= 6.25p^2 – 7.5pq + 2.25q^2 – 2.25p^2 + 7.5pq – 6.25q^2$
$= 4p^2 – 4q^2$

vi) $(ab + bc)^2 – 2ab^2c = a^2b^2 + 2ab^2c + b^2c^2 – 2ab^2c = a^2b^2 + b^2c^2$

117

vii) $(m^2 – n^2m)^2 + 2m^3n^2$

$= m^4 – 2m^3n^2 + m^2n^4 + 2m^3n^2$

$= m^4 + m^2n^4$

5. Show that.

(i) $(3x + 7)^2 – 84x = (3x – 7)^2$

(ii) $(9p – 5q)^2 + 180pq = (9p + 5q)^2$

(iii) $(4/3m – 3/4n)^2 + 2mn = 16/9\ m^2 + 9/16\ n^2$

(iv) $(4pq + 3q)^2 – (4pq – 3q)^2 = 48pq^2$

(v) (a – b) (a + b) + (b – c) (b + c) + (c – a) (c + a) = 0

Solution:

i) LHS $= (3x + 7)^2 – 84x$

$= 9x^2 + 42x + 49 – 84x$

$= 9x^2 – 42x + 49$

= RHS

LHS = RHS

ii) LHS $= (9p – 5q)^2 + 180pq$

$= 81p^2 – 90pq + 25q^2 + 180pq$

$= 81p^2 + 90pq + 25q^2$

RHS $= (9p + 5q)^2$

$= 81p^2 + 90pq + 25q^2$

LHS = RHS

220

(iii) $LHS = \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2 - 2mn + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2$

= RHS

LHS = RHS

iv) LHS = $(4pq + 3q)^2 - (4pq - 3q)^2$

$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$

$= 48pq^2$

= RHS

LHS = RHS

v) LHS = (a – b) (a + b) + (b – c) (b + c) + (c – a) (c + a)

= $a^2 - b^2 + b^2 - c^2 + c^2 - a^2$

= 0

= RHS

6. Using identities, evaluate.

(i) 71^2

(ii) 99^2

(iii) 102^2

(iv) 998^2

(v) 5.2^2

(vi) 297 x 303

(vii) 78 x 82

(viii) 8.9^2

(ix) 10.5 x 9.5

Solution:

i) 71^2

$= (70+1)^2$

$= 70^2 + 140 + 1^2$

$= 4900 + 140 + 1$

= 5041

ii) 99^2

$= (100 - 1)^2$

$= 100^2 – 200 + 1^2$

$= 10000 – 200 + 1$

$= 9801$

iii) 102^2

$= (100 + 2)^2$

$= 100^2 + 400 + 2^2$

$= 10000 + 400 + 4 = 10404$

iv) 998^2

$= (1000 – 2)^2$

$= 1000^2 – 4000 + 2^2$

$= 1000000 – 4000 + 4$

$= 996004$

v) 5.2^2

$= (5 + 0.2)^2$

$= 5^2 + 2 + 0.2^2$

$= 25 + 2 + 0.04 = 27.04$

vi) 297×303

$= (300 – 3)(300 + 3)$

$= 300^2 – 3^2$

$= 90000 – 9$

= 89991

vii) 78 x 82

= (80 – 2)(80 + 2)

= $80^2 – 2^2$

= 6400 – 4

= 6396

viii) 8.9^2

= $(9 – 0.1)^2$

= $9^2 – 1.8 + 0.1^2$

= 81 – 1.8 + 0.01

= 79.21

ix) 10.5 x 9.5

= (10 + 0.5)(10 – 0.5)

= $10^2 – 0.5^2$

= 100 – 0.25

= 99.75

7. Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) $51^2 - 49^2$

(ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

Solution:

i) $51^2 - 49^2$

$= (51 + 49)(51 - 49) = 100 \times 2 = 200$

ii) $(1.02)^2 - (0.98)^2$

$= (1.02 + 0.98)(1.02 - 0.98) = 2 \times 0.04 = 0.08$

iii) $153^2 - 147^2$

$= (153 + 147)(153 - 147) = 300 \times 6 = 1800$

iv) $12.1^2 - 7.9^2$

$= (12.1 + 7.9)(12.1 - 7.9) = 20 \times 4.2 = 84$

8. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find

(i) 103 x 104

(ii) 5.1 x 5.2

(iii) 103 x 98

(iv) 9.7 x 9.8

Solution:

i) 103 x 104

$= (100 + 3)(100 + 4)$

$= 100^2 + (3 + 4)100 + 12$

$= 10000 + 700 + 12$

$= 10712$

ii) 5.1 x 5.2

$= (5 + 0.1)(5 + 0.2)$

$= 5^2 + (0.1 + 0.2)5 + 0.1 \times 0.2$

$= 25 + 1.5 + 0.02$

$= 26.52$

iii) 103 x 98

$= (100 + 3)(100 – 2)$

$= 100^2 + (3-2)100 – 6$

= 10000 + 100 – 6

= 10094

iv) 9.7 x 9.8

= (9 + 0.7)(9 + 0.8)

= 9^2 + (0.7 + 0.8)9 + 0.56

= 81 + 13.5 + 0.56

= 95.06

1MARK Q&A:

Exercise 8.6

Multiple-choice questions and answers:

1. What is the simplified form of the expression 3xs + 2y - x + y?

a) 2x + 3y

b) 4xy

c) 2x - y

d) 4x + 4y

Answer: a) 2x + 3y

2. Which of the following is not an algebraic expression?

a) 3x^2 - 2x + 1

b) 5/4

c) 2x + y

d) 4x - 3y

Answer: b) 5/4

3. If a = 5 and b = 3, what is the value of 2a^2 + 3ab - b^2?

a) 55

b) 36

c) 25

d) 16

Answer: b) 36

4. Which of the following is an identity for all real numbers?

a) (x + y) ^2 = x^2 + y^2

b) (x - y) ^2 = x^2 - y^2

c) (x + y) (x - y) = x^2 - y^2

d) (x + y)(x - y) = x^2 + y^2

Answer: c) (x + y)(x - y) = x^2 - y^2

5. What is the simplified form of the expression 4x^2 + 3x - 2x^2 - 5x + 7?

a) 2x^2 - 2x + 7

b) 2x^2 - 8x + 7

c) 2x^2 - 8x - 7

d) 2x^2 - 2x - 7

Answer: b) 2x^2 - 8x + 7

6. If 2(a + b) = 6, what is the value of (a + b)?

a) 1

b) 2

c) 3

d) 6

Answer: b) 2

7. What is the expansion of (x + 2)(x - 2)?

a) x^2 - 4

b) x^2 + 4

c) x^2 - 2

d) x^2 + 2

Answer: a) x^2 - 4

8. If x = 3, what is the value of x^3 - 2x^2 + 3x - 4?

a) 0

b) 5

c) 10

d) 11

Answer: a) 0

Exercise 8.7

Fill in the blank:

1. An algebraic expression containing only one term is called a _____.

- Answer: Monomial

2. The expression (3x + 5y - 2x + 7y) can be simplified to _____.

- Answer: (x + 12y)

3. The identity (a^2 - b^2)= (a + b)(a - b) is known as _____.

- Answer: Difference of squares

4. The expression (x^2 + 2xy + y^2) can be factored as _____.

- Answer: $(x + y)^2$

5. The constant term in the expression $4x^2$ + 3x - 7 is _____.

- Answer: -7

6. The expression $4(a + b)^2$ expands to _____.

- Answer: ($4a^2$ + 8ab + $4b^2$)

7. The algebraic expression ($2x^3$ - $5x^2$ + 3x - 7) has _____ terms.

- Answer: 4

8. The formula $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ is an example of an _____.

- Answer: Identity

9. The expression (3x + 7) can be classified as a _____.

- Answer: Binomial

10. The coefficient of x in the expression $5x^2$ + 3xy - 2x + 4 is _____.

- Answer: -2

11. $x^3 + y^3$ can be factored as _____.

- Answer: (x + y)(x^2 - xy + y^2)

12. The expression 6xy - 9yz + 4xz has _____ terms.

- Answer: 3

13. The identity $(a + b)^2 = a^2 + 2ab + b^2$ is known as the _____.

- Answer: Square of a binomial

14. The expression $2x^2$ - 5xy + 3x - 4y + 7 contains variables _____.

- Answer: x and y

Summary

1. Monomials, Binomials, and Polynomials:

- A monomial is an algebraic expression with only one term.

- A binomial consists of two unlike terms connected by addition or subtraction.

- A polynomial involves multiple terms joined by addition or subtraction.

2. Variables and Constants:

- Variables are symbols (like x, y, etc.) representing unknown or changing quantities

- Constants are fixed values that do not change in an expression (like numbers).

3. Coefficients and Terms:

- Coefficients are the numerical factors of variables in an expression.

- Terms are the individual parts of an expression separated by addition or subtraction.

4. Simplification and Operations:

- Simplification involves combining like terms and performing arithmetic operations in algebraic expressions.

- Operations include addition, subtraction, multiplication, division, and exponentiation.

Algebraic Identities:

1. Basic Identities:

- Certain algebraic expressions hold true for all values of variables. These are called identities.

- Examples include $(a + b)^2$ = a^2 + 2ab + b^2 and (a^3 + b^3) = (a + b)(a^2 - ab + b^2) .

2. Factorization and Expansion:

- Factorization involves expressing an algebraic expression as a product of its factors.

- Expansion involves multiplying factors to obtain the original expression.

3. Special Products:

- Identifying and applying special products such as the square of a binomial or the difference of squares can simplify expressions.

4. Application and Problem Solving:

- Algebraic identities and expressions are used in problem-solving, equations, modeling real-world situations, and in various mathematical applications.

****