Chapter – 2

Relations and Functions

Units and measures are fundamental concepts in the field of science, engineering, and everyday life. They provide a standardized way to quantify and express physical quantities, ensuring clear communication and understanding. The use of consistent units facilitates accurate measurements and promotes universal comprehension across different contexts.

It begins by introducing the notion of physical quantities, classifying them into base and derived quantities. The chapter emphasizes the importance of standard units and introduces students to the International System of Units (SI), elucidating the significance of base and derived units.

Measurement techniques and the inherent uncertainties in measurements are explored, emphasizing the concepts of precision and accuracy. The chapter likely covers topics such as significant figures, rounding off, and understanding errors in measurements. The idea of dimensional analysis is introduced, showcasing its utility in verifying the correctness of equations and deriving relationships between physical quantities.

EXERCISE 2.1

1. If $\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$, find the values of *x* and *y*.

Solution:

Given,

$$\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$$

As the ordered pairs are equal, the corresponding elements should also be equal.

Thus,

x/3 + 1 = 5/3 and y – 2/3 = 1/3

Solving, we get

x + 3 = 5 and 3y – 2 = 1 [Taking L.C.M. and adding] x = 2 and 3y = 3

Therefore,

x = 2 and y = 1

2. If set A has 3 elements and set B = {3, 4, 5}, then find the number of elements in (A × B).

Solution:

Given, set A has 3 elements, and the elements of set B are {3, 4, and 5}.

So, the number of elements in set B = 3

Then, the number of elements in (A × B) = (Number of elements in A) × (Number of elements in B)

= 3 × 3 = 9

Therefore, the number of elements in (A × B) will be 9.

3. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Solution:

Given, G = {7, 8} and H = {5, 4, 2}

We know that,

The Cartesian product of two non-empty sets P and Q is given as

P × Q = {(*p*, *q*): *p* ∈ P, *q* ∈ Q}

So,

G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

4. State whether each of the following statements is true or false. If the statement is false, rewrite the given statement correctly.

(i) If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}

(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that $x \in$ A and $y \in$ B.

(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ

Solution:

(i) The statement is false. The correct statement is

If P = {m, n} and Q = {n, m}, then

P × Q = {(m, m), (m, n), (n, m), (n, n)}

(ii) True

(iii) True

5. If A = {–1, 1}, find A × A × A.

Solution:

The A × A × A for a non-empty set A is given by

A × A × A = {(a, b, c): a, b, $c \in$ A}

Here, it is given A = {–1, 1}

So,

A × A × A = {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1), (1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

6. If A × B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.

Solution:

Given,

A × B = {(a, x), (a, y), (b, x), (b, y)}

We know that the Cartesian product of two non-empty sets, P and Q is given by:

$P \times Q = \{(p, q): p \in P, q \in Q\}$

Hence, A is the set of all first elements, and B is the set of all second elements.

Therefore, $A = \{a, b\}$ and $B = \{x, y\}$

7. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times C$ is a subset of $B \times D$

Solution:

Given,

$A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$

(i) To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Now, $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

Thus,

L.H.S. $= A \times (B \cap C) = A \times \Phi = \Phi$

Next,

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

Thus,

R.H.S. $= (A \times B) \cap (A \times C) = \Phi$

Therefore, L.H.S. = R.H.S.

Hence verified

(ii) To verify: $A \times C$ is a subset of $B \times D$

First,

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

And,

B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

Now, it's clearly seen that all the elements of set A × C are the elements of set B × D.

Thus, A × C is a subset of B × D.

Hence verified

8. Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

Solution:

Given,

A = {1, 2} and B = {3, 4}

So,

A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}

Number of elements in A × B is *n*(A × B) = 4

We know that,

If C is a set with $n(C) = m$, then $n[P(C)] = 2^m$.

Thus, the set A × B has $2^4 = 16$ subsets.

And these subsets are as given below:

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

9. Let A and B be two sets such that n(A) = 3 and n (B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y and z are distinct elements.

Solution:

Given,

n(A) = 3 and n(B) = 2; and (x, 1), (y, 2), (z, 1) are in A × B.

We know that,

A = Set of first elements of the ordered pair elements of A × B

B = Set of second elements of the ordered pair elements of A × B

So, clearly, x, y, and z are the elements of A; and

1 and 2 are the elements of B.

As n(A) = 3 and n(B) = 2, it is clear that set A = {x, y, z} and set B = {1, 2}

10. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

Solution:

We know that,

If n(A) = p and n(B) = q, then n(A × B) = pq.

Also, n(A × A) = n(A) × n(A)

Given,

n(A × A) = 9

So, n(A) × n(A) = 9

Thus, n(A) = 3

Also, given that the ordered pairs (–1, 0) and (0, 1) are two of the nine elements of A × A.

And, we know in A × A = {(a, a): a ∈ A}

Thus, –1, 0, and 1 have to be the elements of A.

As $n(A) = 3$, clearly A = {–1, 0, 1}

Hence, the remaining elements of set A × A are as follows:

(–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0), and (1, 1)

Exercise 2.2

1. Let A = {1, 2, 3, …, 14}. Define a relation R from A to A by R = {(x, y): $3x - y = 0$, where $x, y \in A$}. Write down its domain, co domain and range.

Solution:

The relation R from A to A is given as:

$R = \{(x, y): 3x - y = 0, \text{ where } x, y \in A\}$

$= \{(x, y): 3x = y, \text{ where } x, y \in A\}$

So,

R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Now,

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of R = {1, 2, 3, 4}

The whole set A is the co domain of the relation R.

Hence, Co domain of R = A = {1, 2, 3, …, 14}

The range of R is the set of all second elements of the ordered pairs in the relation.

Hence, Range of R = {3, 6, 9, 12}

2. Define a relation R on the set N of natural numbers by R = {(x, y): $y = x + 5$, x is a natural number less than 4; $x, y \in$ N}. Depict this relationship using roster form. Write down the domain and the range.

Solution:

The relation R is given by:

R = {(x, y): $y = x + 5$, x is a natural number less than 4, $x, y \in$ **N**}

The natural numbers less than 4 are 1, 2, and 3.

So,

R = {(1, 6), (2, 7), (3, 8)}

Now,

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of R = {1, 2, 3}

The range of R is the set of all second elements of the ordered pairs in the relation.

Hence, Range of R = {6, 7, 8}

3. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd; $x \in$ A, $y \in$ B}. Write R in roster form.

Solution:

Given,

A = {1, 2, 3, 5} and B = {4, 6, 9}

The relation from A to B is given as

R = {(x, y): the difference between x and y is odd; $x \in$ A, $y \in$ B}

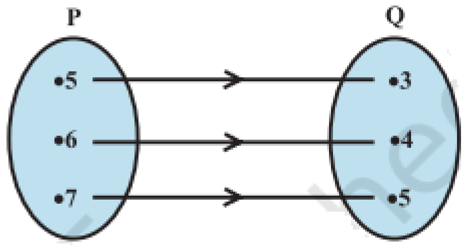
Thus,

R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

4. The figure shows a relationship between the sets P and Q. Write this relation

(i) in set-builder form (ii) in roster form

What is its domain and range?



Solution:

From the given figure, it's seen that

P = {5, 6, 7}, Q = {3, 4, 5}

The relation between P and Q:

Set-builder form

(i) R = $\{(x, y): y = x - 2; x \in P\}$ or R = $\{(x, y): y = x - 2 \text{ for } x = 5, 6, 7\}$

Roster form

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by

$\{(a, b): a, b \in A, b$ is exactly divisible by $a\}$.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R

Solution:

Given,

A = {1, 2, 3, 4, 6} and relation R = {(a, b): a, b ∈ A, b is exactly divisible by a}

Hence,

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

6. Determine the domain and range of the relation R defined by

R = {(x, x + 5): x ∈ {0, 1, 2, 3, 4, 5}}.

Solution:

Given,

Relation R = {(x, x + 5): x ∈ {0, 1, 2, 3, 4, 5}}

Thus,

R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

So,

Domain of R = {0, 1, 2, 3, 4, 5} and,

Range of R = {5, 6, 7, 8, 9, 10}

7. Write the relation R = {(x, x^3): x is a prime number less than 10} in roster form.

Solution:

Given,

Relation R = {(x, x^3): x is a prime number less than 10}

The prime numbers less than 10 are 2, 3, 5, and 7.

Therefore,

R = {(2, 8), (3, 27), (5, 125), (7, 343)}

8. Let A = {*x*, *y*, z} and B = {1, 2}. Find the number of relations from A to B.

Solution:

Given, A = {*x*, *y*, z} and B = {1, 2}

Now,

A × B = {(*x*, 1), (*x*, 2), (*y*, 1), (*y*, 2), (*z*, 1), (*z*, 2)}

As *n*(A × B) = 6, the number of subsets of A × B will be 2^6.

Thus, the number of relations from A to B is 2^6.

9. Let R be the relation on Z defined by R = {(*a*, *b*): *a*, *b* ∈ Z, *a* – *b* is an integer}. Find the domain and range of R.

Solution:

Given,

Relation R = {(*a*, *b*): *a*, *b* ∈ Z, *a* – *b* is an integer}

We know that the difference between any two integers is always an integer.

Therefore,

Domain of R = Z and Range of R = Z

Exercise 2.3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

Solution:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

As 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation can be called a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

As 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation can be called a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

It's seen that the same first element, i.e., 1, corresponds to two different images, i.e., 3 and 5; this relation cannot be called a function.

2. Find the domain and range of the following real function:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{(9 - x^2)}$

Solution:

(i) Given,

$f(x) = -|x|, x \in R$

We know that,

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$$

As $f(x)$ is defined for $x \in R$, the domain of f is R.

It is also seen that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

Therefore, the range of f is given by $(-\infty, 0]$.

(ii) $f(x) = \sqrt{(9 - x^2)}$

As $\sqrt{(9 - x^2)}$ is defined for all real numbers that are greater than or equal to –3 and less than or equal to 3, for $9 - x^2 \geq 0$.

So, the domain of $f(x)$ is $\{x: -3 \leq x \leq 3\}$ or $[-3, 3]$.

Now,

For any value of x in the range $[-3, 3]$, the value of $f(x)$ will lie between 0 and 3.

Therefore, the range of $f(x)$ is $\{x: 0 \leq x \leq 3\}$ or $[0, 3]$.

3. A function *f* is defined by *f*(*x*) = 2*x* – 5. Write down the values of

(i) *f*(0), (ii) *f*(7), (iii) *f*(–3)

Solution:

Given,

Function, $f(x) = 2x - 5$

Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

4. The function 't', which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$**.**

Find (i) *t* (0) (ii) *t* (28) (iii) *t* (–10) (iv) The value of C, when *t*(C) = 212

Solution:

Given function, $t(C) = \frac{9C}{5} + 32$

So,

(i) $t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$

(iii) $t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$

(iv) Given that, $t(C) = 212$

$$\therefore 212 = \frac{9C}{5} + 32$$

$$\Rightarrow \frac{9C}{5} = 212 - 32$$

$$\Rightarrow \frac{9C}{5} = 180$$

$$\Rightarrow 9C = 180 \times 5$$

$$\Rightarrow C = \frac{180 \times 5}{9} = 100$$

Therefore, the value of t when $t(C) = 212$, is 100.

5. Find the range of each of the following functions:

(i) $f(x) = 2 - 3x$, $x \in$ R, $x > 0$

(ii) $f(x) = x^2 + 2$, x is a real number

(iii) $f(x) = x$, x is a real number

Solution:

(i) Given,

f(x) = 2 – 3x, $x \in$ R, $x > 0$

Here the values of f(x) for various values of real numbers x > 0 can be given as

x	0.01	0.1	0.9	1	2	2.5	4	5	...
f(x)	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	...

It can be observed that the range of f is the set of all real numbers less than 2.
Range of f = $(-\infty, 2)$

We have,

$x > 0$

So,

$3x > 0$

$-3x < 0$ [Multiplying by -1 on both sides, the inequality sign changes]

$2 - 3x < 2$

Therefore, the value of $2 - 3x$ is less than 2.

Hence, Range = $(-\infty, 2)$

(ii) Given,

$f(x) = x^2 + 2$, x is a real number

Here the values of f(x) for various values of real numbers x can be given as

x	0	±0.3	±0.8	±1	±2	±3	...
f(x)	2	2.09	2.64	3	6	11	...

It can be oberserved that the range of f is the set of all real numbers greater than 2.
Range of f = $[2, \infty)$

We know that,

$x^2 \geq 0$

So,

$x^2 + 2 \geq 2$ [Adding 2 on both sides]

Therefore, the value of $x^2 + 2$ is always greater or equal to 2, for x is a real number.

Hence, Range = $[2, \infty)$

(iii) Given,

$f(x) = x$, x is a real number

Clearly, the range of f is the set of all real numbers.

Thus,

Range of f = R

2 Marks Questions & Answers

1. If (a – 1, b + 5) = (2, 3), find a and b.

Ans: Given (a – 1, b + 5) = (2, 3),

Then,

$a - 1 = 2$

$b + 5 = 3$

Therefore,

$a = 3$

$b = -2$

2. If A = {1, 3, 5} and B = {2, 3},

- **Find A × B**

Ans: The Cartesian product of sets implies that if the two sets are non-empty sets, then they will be ordered pairs. Therefore, the Cartesian product of A and B is the set of ordered pairs (a, b), so that a ∈ A, b ∈ B, is represented by A × B.

Hence, A × B = {(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)}

- **Find B × A**

Ans: Since B × A is the Cartesian product set of B and A, where b ∈ B, a ∈ A, and (b, a) ∈ B × A are all positive integers, we get

B × A = {(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)}

3. A real function f is defined by f(x) = 2x − 5. Then the value of f(−3) is

(a) 0

(b) 1

(c) -11

(d) None of the above

Ans: Given, f(x) = 2x – 5

Substituting x = -3, we get:

= – 6 – 5

= –11

Therefore, the correct answer is (c).

Let f **and** g **be two real valued functions, defined by,** f(x) = x^2, g(x) =3x+2.

4. Find the value of (f+ g) (−2).

Ans: The given functions are f(x) = x^2g(x)= 3x+2

Therefore, (f+ g) (x)=f(x)+g(x)= x^2+3x+2

So, substituting x=−2, we obtain

(f+ g)(−2)= $(-2)^2$+ 3(−2) + 2

=4−6+2

=0.

Hence, (f+ g) (−2) = 0.

5. Find the value of (f−g) (1)

Ans: The given functions are f(x) = $(x)^2$, g(x) =3x+2

Therefore,

(f−g) (x)= f(x) −g(x)

=$(x)^2$ − (3x+2)

=$(x)^2$ −3x−2

Substituting x=1, we get

(f−g)(1)= $(1)^2$ −3 (1)−2

=1−3−2

=−2−2

=−4.

Hence, (f−g) (1) =−4.

6. Find the value of (fg) (−1)

Ans: The given functions are f(x) =x^2, g(x) =3x+2

Therefore, (fg) (x) =f(x) g(x) = x^2 (3x+2)

Substituting X=−1, we have

(fg) (−1)= $(-1)^2$ {3(−1) + 2}

=−3+2

=−1.

Hence, (fg) (−1) =−1.

7. Find the value of ($\frac{f}{g}$)(0)

Ans: The given functions are f(x) = $(x)^2$, g(x)=3x+2

Then, **(** $\frac{f}{g}$) (x)= $\frac{f(x)}{g(x)}$) = $\frac{x2}{3x+2}$ **.**

Substituting, x=0, we get

$\left(\frac{f}{g}\right)(0)=\frac{(0)2}{3(0)+2}=0$

That is, $\frac{f}{g}(0)=0$.

8. If f(x) =x^3**, find the value of** $\frac{f(5)-f(1)}{5-1}$**.**

Ans: It is given that,

f(x)= x^3

Substituting x=1we get,

f(1)= 1^3**,**= 1 and

Substituting x=5, we obtain

f(5)= 5^3**,** =125.

Therefore, $\frac{f(5)-f(1)}{5-1}=\frac{125-1}{4}=\frac{124}{4}=31$

Hence, the value of $\frac{f(5)-f(1)}{5-1}$ is 31.

- **Find the range of the relations in Question 9 and 10.**

9. R= {(a, b): a, b∈N and 2a+b=10}

Ans: The pair of values of (a, b) ∈N for which the equation

2a+b=10 is satisfied are given by

(4,2),(3,4),(2,6),(4,2),(3,4),(2,6), and (1,8)(1,8).

Now, we know that range of a relation is the set of all the images.

Therefore, the range of the given relation R, that is
R={(4,2),(3,4),(2,6),(1,8)} is given by {2,4,6,8}{2,4,6,8}.

10. R= $\{(x,\frac{1}{x}) : x\in z, 0<x<6\}$.

Ans: The pair of values of $(x,\frac{1}{x})$, for all x∈Z, such that 0<x<6 are $(1,1),(2,\frac{1}{2}),(3\ ,,\frac{1}{3}),(4,\frac{1}{4})$ and $(5\ ,\frac{1}{5})$

Since, the range of a relation is the set of all images, therefore, the range of the given relation R is given by {1, 12, 13, 14, 15} {1, 12, 13, 14, 15}.

Multiple Choice Questions

1. If f(x) = x^3 – (1/x^3), then f(x) + f(1/x) is equal to

(a) $2x^3$

(b) $2/x^3$

(c) 0

(d) 1

Correct option: (c) 0

Solution:

Given,

$f(x) = x^3 – (1/x^3)$

Now,

$f(1/x) = (1/x)^3 – 1/(1/x)^3$

$= (1/x^3) – x^3$

Thus, $f(x) + f(1/x) = x^3 – (1/x^3) + (1/x^3) – x^3 = 0$

2. Let n (A) = m, and n (B) = n. Then the total number of non-empty relations that can be defined from A to B is

(a) m^n

(b) $n^m – 1$

(c) $mn – 1$

(d) $2^{mn} – 1$

Correct option: (d) $2^{mn} – 1$

Solution:

Given,

n(A) = m and n(B) = n

We know that,

n(A x B) = n(A). n(B) = mn

Total number of relations from A to B = Number of subsets of A x B = 2^{mn}

So, the total number of non-empty relations from A to B = $2^{mn} - 1$.

3. If $f(x) = x^2 + 2$, $x \in R$, then the range of f(x) is

(a) $[2, \infty)$

(b) $(-\infty, 2]$

(c) $(2, \infty)$

(d) $(-\infty, 2) \cup (2, \infty)$

Correct option: (a) $[2, \infty)$

Solution:

Given,

$f(x) = x^2 + 2$

We know that the square of any number is positive, i.e. greater than or equal to 0.

So, $x^2 \geq 0$

Adding 2 on both sides,

$x^2 + 2 \geq 0 + 2$

$f(x) \geq 2$

Therefore, f(x) range is $[2, \infty)$.

4. What will be the domain for which the functions $f(x) = 2x^2 – 1$ and $g(x) = 1 – 3x$ are equal?

(a) {-2, 1}

(b) {1/2, -2}

(c) [2, 12]

(d) (-1, 2)

Correct option: (b) {1/2, -2}

Solution:

Given,

$f(x) = 2x^2 – 1$

$g(x) = 1 – 3x$

Now,

$f(x) = g(x)$

$\Rightarrow 2x^2 – 1 = 1 – 3x$

$\Rightarrow 2x^2 + 3x – 2 = 0$

$\Rightarrow 2x^2 + 4x – x – 2 = 0$

$\Rightarrow 2x(x + 2) – 1(x + 2) = 0$

$\Rightarrow (2x – 1) (x + 2) = 0$

Thus the domain for which the function f (x) = g (x) is {1/2, -2}.

5. If $[x]^2 – 5 [x] + 6 = 0$, where [.] denotes the greatest integer function, then

(a) $x \in [3, 4]$

(b) $x \in (2, 3]$

(c) $x \in [2, 3]$

(d) $x \in [2, 4)$

Correct option: (d) $x \in [2, 4)$

Solution:

Given,

$[x]^2 – 5 [x] + 6 = 0$, where [.] denotes the greatest integer function. $[x]^2 – 5[x] + 6 = 0$ $[x]^2 – 2[x] – 3[x] + 6 = 0$ $[x]([x – 2) – 3([x] – 2) = 0$
$([x] – 2)([x] – 3) = 0$

When $[x] = 2, 2 \leq x < 3$

When $[x] = 3, 3 \leq x < 4$

From the above, $x \in [2, 4)$.

6. If f(x) = ax + b, where a and b are integers, f(–1) = – 5 and f(3) = 3, then a and b are equal to

(a) $a = – 3, b = –1$

(b) $a = 2, b = – 3$

(c) $a = 0, b = 2$

(d) $a = 2, b = 3$

Correct option: (b) $a = 2, b = – 3$

Solution:

Given,

$f(x) = ax + b$

And

$f(-1) = -5$

$a(-1) + b = -5$

$-a + b = -5$….(i)

Also, $f(3) = 3$

$a(3) + b = 3$

$3a + b = 3$….(ii)

From (i) and (ii),

$a = 2, b = -3$

7. The domain of the function $f(x) = x/(x^2 + 3x + 2)$ is

(a) [-2, -1]

(b) R – {1, 2}

(c) R – {-1, -2}

(d) R – {2}

Correct option: (c) R – {-1, -2}

Solution:

Given f(x) is a rational function of the form g(x)/h(x), where g(x) = x and h(x) = $x^2 + 3x + 2$.

Now $h(x) \neq 0$

$\Rightarrow x^2 + 3x + 2 \neq 0$

$\Rightarrow x^2 + x + 2x + 2 \neq 0$

$\Rightarrow x(x + 2) + 2(x + 1)$

$\Rightarrow (x + 1)(x + 2) \neq 0$

$\Rightarrow x \neq -1, x \neq -2$

Therefore, the domain of the given function is R – {– 1, – 2}.

8. The range of $f(x) = \sqrt{(25 - x^2)}$ is

(a) (0, 5)

(b) [0, 5]

(c) (-5, 5)

(d) [1, 5]

Correct option: (b) [0, 5]

Solution:

Given,

$f(x) = \sqrt{(25 - x^2)}$

The domain of f(x) is [-5, 5] since the given function is defined only when $(25 - x^2) \geq 0$.

Let $y = \sqrt{(25 - x^2)}$

$y^2 = 25 - x^2$

or $x^2 = 25 - y^2$

Since $x \in [-5, 5]$, the range of f(x) is [0, 5].

9. The domain and range of the real function f defined by f(x) = (4 – x)/(x – 4) is given by

(a) Domain = R, Range = {–1, 1}

(b) Domain = R – {1}, Range = R

(c) Domain = R – {4}, Range = {– 1}

(d) Domain = R – {– 4}, Range = {–1, 1}

Correct option: (c) Domain = R – {4}, Range = {– 1}

Solution:

To find the domain, consider the denominator $\neq 0$

$(x - 4) \neq 0$

$x \neq 4$

So, domain = R – {4}

Now,

$f(x) = (4 – x)/(x – 4)$

$= -1(x – 4)/(x – 4)$

$= -1$

Therefore, the range of $f(x) = -1$.

10. The domain and range of the function f given by $f(x) = 2 – |x −5|$ is

(a) Domain = R+ , Range = $(– \infty, 1]$

(b) Domain = R, Range = $(– \infty, 2]$

(c) Domain = R, Range = $(– \infty, 2)$

(d) Domain = R+ , Range = $(– \infty, 2]$

Correct option: (b) Domain = R, Range = $(– \infty, 2]$

Solution:

Given,

$f(x) = 2 – |x – 5|$

Now x is defined for all real numbers.

Hence the domain of f is R.

To find the range, consider $|x – 5| \geq 0$

or

$-|x – 5| \leq 0$

Adding 2 on both sides,

$2 – |x – 5| \leq 2$

$\Rightarrow f(x) \leq 2$

Hence, the range of f(x) is $(-\infty, 2]$.

Summary

In this Chapter, we studied about relations and functions. The main features of this Chapter are as follows:

- Ordered pair A pair of elements grouped together in a particular order.
- Cartesian product A × B of two sets A and B is given by A × B = {(a, b): a ∈ A, b ∈ B} In particular R × R = {(x, y): x, y ∈ R} and R × R × R = {(x, y, z): x, y, z ∈ R}
- If (a, b) = (x, y), then a = x and b = y.
- If n(A) = p and n(B) = q, then n(A × B) = pq.
- A × φ = φ Æ In general, A × B ≠ B × A.
- Relation A relation R from a set A to a set B is a subset of the Cartesian product A × B obtained by describing a relationship between the first element x and the second element y of the ordered pairs in A × B.
- The image of an element x under a relation R is given by y, where (x, y) ∈ R,
- The domain of R is the set of all first elements of the ordered pairs in a relation R.
- The range of the relation R is the set of all second elements of the ordered pairs in a relation R.
- Function A function f from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B. We write f: A→B, where f(x) = y.
- A is the domain and B is the co domain of f. The range of the function is the set of images.
- A real function has the set of real numbers or one of its subsets both as its domain and as its range.
- Algebra of functions For functions f : X → R and g : X → R, we have

$$(f + g)(x) = f(x) + g(x), x \in X$$

$$(f - g)(x) = f(x) - g(x), x \in X$$

$$(f.g)(x) = f(x) .g(x), x \in X$$

$$(kf)(x) = k(f(x)), x \in X, \text{ where k is a real number.}$$

$$(f/g)x = f(x)/g(x), x \in X, g(x) \neq 0$$