Chapter-11

AREAS RELATED TO CIRCLES

2 MARKS QUESTIONS

1. Find the area of a sector of a circle with a radius 6 cm if the angle of the sector is 60°.

Solution:

It is given that the angle of the sector is 60°

We know that the area of sector = $(\theta/360°)\times\pi r^2$

∴ area of the sector with angle 60° = $(60°/360°)\times\pi r^2$ cm^2

= $(36/6)\pi$ cm^2

= $6\times22/7$ cm^2 = $132/7$ cm^2

2. An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming the umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Solution:

The radius (r) of the umbrella when flat = 45 cm

So, the area of the circle (A) = πr^2 = $(22/7)\times(45)^2$ =6364.29 cm^2

Total number of ribs (n) = 8

∴ The area between the two consecutive ribs of the umbrella = A/n

6364.29/8 cm^2

Or, The area between the two consecutive ribs of the umbrella = 795.5 cm^2

3. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution:

Given,

Radius (r) = 25 cm

Sector angle (θ) = 115°

Since there are 2 blades,

The total area of the sector made by wiper = $2\times(\theta/360°)\times\pi\, r^2$

$= 2\times(115/360)\times(22/7)\times25^2$

$= 2\times158125/252\ cm^2$

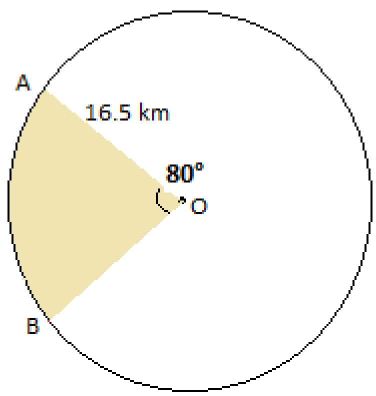
$= 158125/126 = 1254.96\ cm^2$

4. To warn ships of underwater rocks, a lighthouse spreads a red-coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.

(Use π = 3.14)

Solution:

Let O bet the position of the lighthouse.



Here, the radius will be the distance over which light spreads.

Given radius (r) = 16.5 km

Sector angle (θ) = 80°

Now, the total area of the sea over which the ships are warned = Area made by the sector

Or, Area of sector $= (\theta/360°)\times\pi r^2$

$= (80°/360°)\times\pi r^2\ km^2$

$= 189.97\ km^2$

5. Tick the correct solution in the following:

The area of a sector of angle p (in degrees) of a circle with radius R is

(A) $p/180 \times 2\pi R$

(B) $p/180 \times \pi R^2$

(C) $p/360 \times 2\pi R$

(D) $p/720 \times 2\pi R^2$

Solution:

The area of a sector = $(\theta/360°)\times\pi r^2$

Given, $\theta = p$

So, the area of sector = $p/360\times\pi R^2$

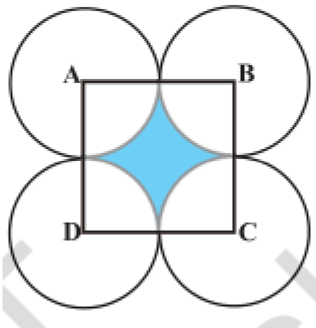
Multiplying and dividing by 2 simultaneously,

$= (p/360)\times2/2\times\pi R^2$

$= (2p/720)\times2\pi R^2$

So, option (D) is correct.

6. In Fig. 12.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



Solution:

Side of square = 14 cm

Four quadrants are included in the four sides of the square.

∴ radius of the circles = 14/2 cm = 7 cm

Area of the square ABCD = 14^2 = 196 cm^2

Area of the quadrant = $(\pi R^2)/4$ cm^2 = $(22/7) \times 7^2/4$ cm^2

= 77/2 cm^2

Total area of the quadrant = 4×77/2 cm^2 = 154cm^2

Area of the shaded region = Area of the square ABCD – Area of the quadrant

= 196 cm^2 – 154 cm^2

= 42 cm^2

4 MARKS QUESTIONS

1. Find the area of a quadrant of a circle whose circumference is 22 cm.

Solution:

Circumference of the circle, C = 22 cm (given)

It should be noted that a quadrant of a circle is a sector which is making an angle of 90°.

Let the radius of the circle = r

As $C = 2\pi r = 22$,

$R = 22/2\pi$ cm = 7/2 cm

∴ area of the quadrant = $(\theta/360°) \times \pi r^2$

Here, $\theta = 90°$

So, $A = (90°/360°) \times \pi\, r^2\ cm^2$

$= (49/16)\ \pi\ cm^2$

$= 77/8\ cm^2 = 9.6\ cm^2$

2. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution:

Length of minute hand = radius of the clock (circle)

∴ Radius (r) of the circle = 14 cm (given)

Angle swept by minute hand in 60 minutes = 360°

So, the angle swept by the minute hand in 5 minutes = 360° × 5/60 = 30°

We know,

Area of a sector = (θ/360°) × πr^2

Now, the area of the sector making an angle of 30° = (30°/360°) × πr^2 cm^2

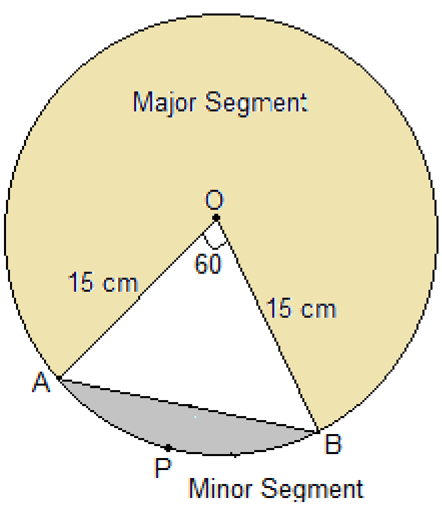
= (1/12) × $\pi 14^2$

= (49/3)×(22/7) cm^2

= 154/3 cm^2

3. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:



Given,

Radius = 15 cm

$\theta = 60°$

So,

Area of sector OAPB = $(60°/360°) \times \pi r^2$ cm^2

= $225/6$ πcm^2

Now, ΔAOB is equilateral as two sides are the radii of the circle and hence equal and one angle is 60°

So, Area of ΔAOB = $(\sqrt{3}/4) \times a^2$

Or, $(\sqrt{3}/4) \times 15^2$

∴ Area of ΔAOB = 97.31 cm^2

Now, the area of minor segment APB = Area of OAPB – Area of ΔAOB

Or, the area of minor segment APB = $((225/6)\pi - 97.31)$ cm^2 = 20.43 cm^2

And,

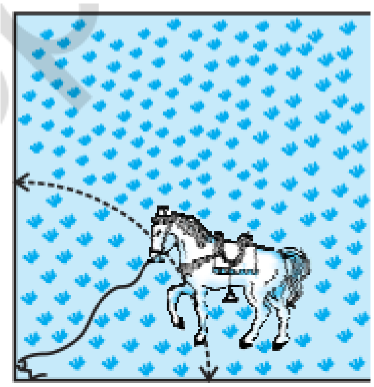
Area of major segment = Area of the circle – Area of the segment APB

Or, area of major segment = $(\pi \times 15^2) - 20.4$ = 686.06 cm^2

4. A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope (see Fig) Find

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)



Solution:

As the horse is tied at one end of a square field, it will graze only a quarter (i.e. sector with $\theta = 90°$) of the field with a radius 5 m.

Here, the length of the rope will be the radius of the circle, i.e. $r = 5$ m

It is also known that the side of the square field = 15 m

(i) Area of circle = $\pi r^2 = 22/7 \times 5^2 = 78.5$ m^2

Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle) = 78.5/4 = 19.625 m^2

(ii) If the rope is increased to 10 m,

Area of circle will be = $\pi r^2 = 22/7 \times 10^2 = 314$ m^2

Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle)

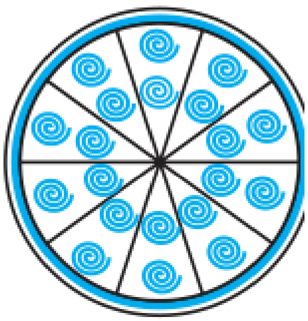
= 314/4 = 78.5 m^2

∴ increase in the grazing area = 78.5 m^2 – 19.625 m^2 = 58.875 m^2

5. A brooch is made with silver wire in the form of a circle with a diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors, as shown in Fig. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.



Solution:

Diameter (D) = 35 mm

Total number of diameters to be considered= 5

Now, the total length of 5 diameters that would be required = 35×5 = 175

Circumference of the circle = 2πr

Or, C = πD = 22/7×35 = 110

Area of the circle = πr^2

Or, A = $(22/7)\times(35/2)^2$ = 1925/2 mm^2

(i) Total length of silver wire required = Circumference of the circle + Length of 5 diameter

= 110+175 = 285 mm

(ii) Total Number of sectors in the brooch = 10

So, the area of each sector = total area of the circle/number of sectors

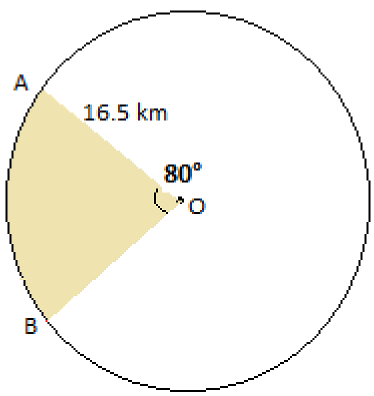
∴ Area of each sector = (1925/2)×1/10 = 385/4 mm^2

6. To warn ships of underwater rocks, a lighthouse spreads a red-coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.

(Use π = 3.14)

Solution:

Let O bet the position of the lighthouse.



Here, the radius will be the distance over which light spreads.

Given radius (r) = 16.5 km

Sector angle (θ) = 80°

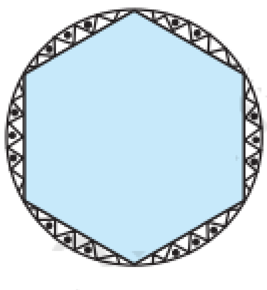
Now, the total area of the sea over which the ships are warned = Area made by the sector

Or, Area of sector = $(\theta/360°)\times\pi r^2$

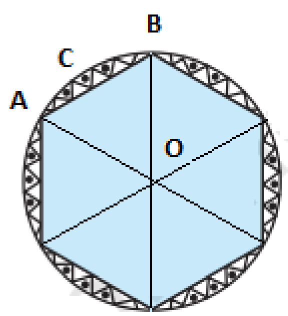
= $(80°/360°)\times\pi r^2$ km^2

= 189.97 km^2

7. A round table cover has six equal designs, as shown in Fig. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2. (Use √3 = 1.7)



Solution:



Total number of equal designs = 6

AOB= 360°/6 = 60°

The radius of the cover = 28 cm

Cost of making design = ₹ 0.35 per cm^2

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔAOB is an equilateral triangle. So, its area will be $(\sqrt{3}/4)\times a^2$ sq. units

Here, a = OA

∴ Area of equilateral $\Delta AOB = (\sqrt{3}/4)\times 28^2 = 333.2\ cm^2$

Area of sector ACB = $(60°/360°)\times\pi r^2\ cm^2$

$= 410.66\ cm^2$

So, the area of a single design = the area of sector ACB – the area of ΔAOB

$= 410.66\ cm^2 - 333.2\ cm^2 = 77.46\ cm^2$

∴ area of 6 designs = $6\times 77.46\ cm^2 = 464.76\ cm^2$

So, total cost of making design = $464.76\ cm^2 \times$ Rs.0.35 per cm^2

= Rs. 162.66

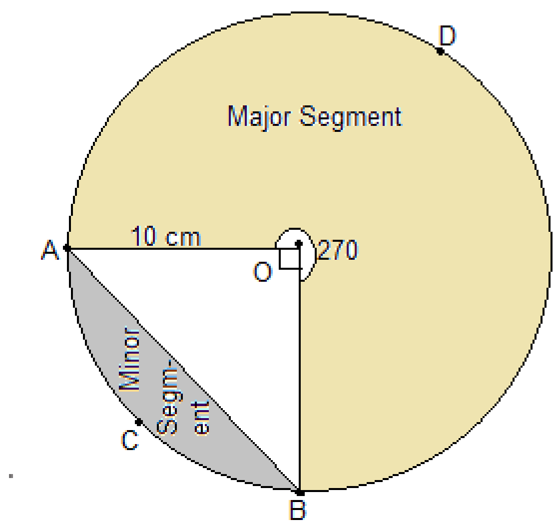
7 MARKS QUESTIONS

1. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) minor segment

(ii) major sector. (Use π = 3.14)

Solution:



Here, AB is the chord which is subtending an angle 90° at the centre O.

It is given that the radius (r) of the circle = 10 cm

(i) Area of minor sector = $(90/360°)\times\pi r^2$

$= (¼)\times(22/7)\times 10^2$

Or, the Area of the minor sector = $78.5\ cm^2$

Also, the area of $\Delta AOB = ½\times OB\times OA$

Here, OB and OA are the radii of the circle, i.e., = 10 cm

So, the area of ΔAOB = ½×10×10

= 50 cm^2

Now, area of minor segment = area of the minor sector – the area of ΔAOB

= 78.5 – 50

= 28.5 cm^2

(ii) Area of major sector = Area of the circle – Area of he minor sector

= (3.14×10^2)-78.5

= 235.5 cm^2

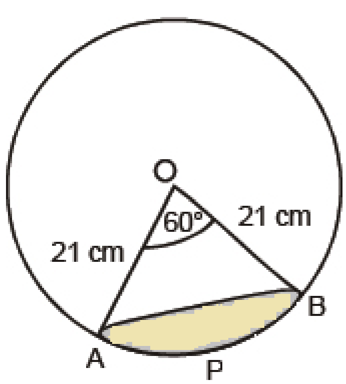
2. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc

(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord

Solution:



Given,

Radius = 21 cm

$\theta = 60°$

(i) Length of an arc = $\theta/360° \times$ Circumference($2\pi r$)

∴ Length of an arc AB = $(60°/360°)\times 2\times(22/7)\times 21$

$= (1/6)\times 2\times(22/7)\times 21$

Or Arc AB Length = 22cm

(ii) It is given that the angle subtended by the arc = 60°

So, the area of the sector making an angle of 60° = $(60°/360°)\times\pi\, r^2$ cm^2

$= 441/6\times 22/7$ cm^2

Or, the area of the sector formed by the arc APB is 231 cm^2

(iii) Area of segment APB = Area of sector OAPB – Area of ΔOAB

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔOAB is an equilateral triangle. So, its area will be $\sqrt{3}/4\times a^2$ sq. Units.

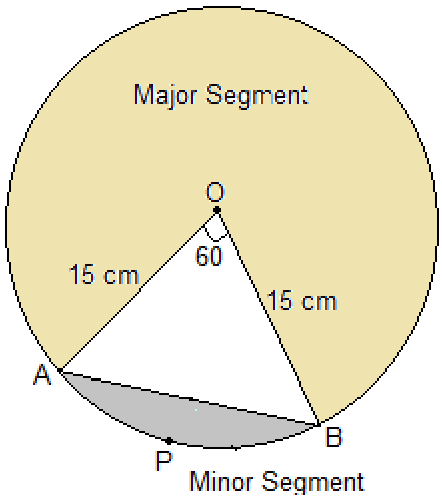
The area of segment APB = $231-(\sqrt{3}/4)\times(OA)^2$

$= 231-(\sqrt{3}/4)\times 21^2$

Or, the area of segment APB = $[231-(441\times\sqrt{3})/4]$ cm^2

3. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:



Given,

Radius = 15 cm

θ = 60°

So,

Area of sector OAPB = $(60°/360°)\times\pi r^2$ cm^2

= $225/6\ \pi cm^2$

Now, ΔAOB is equilateral as two sides are the radii of the circle and hence equal and one angle is 60°

So, Area of ΔAOB = $(\sqrt{3}/4)\times a^2$

Or, $(\sqrt{3}/4)\times 15^2$

∴ Area of ΔAOB = 97.31 cm^2

Now, the area of minor segment APB = Area of OAPB – Area of ΔAOB

Or, the area of minor segment APB = ((225/6)π – 97.31) cm^2 = 20.43 cm^2

And,

Area of major segment = Area of the circle – Area of the segment APB

Or, area of major segment = (π×15^2) – 20.4 = 686.06 cm^2

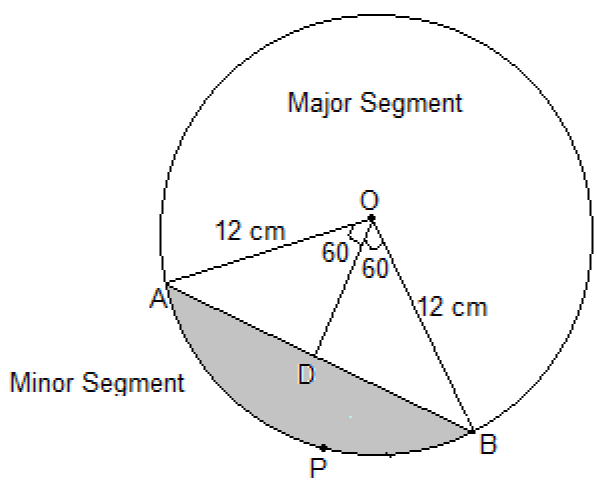
4. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:

Radius, r = 12 cm

Now, draw a perpendicular OD on chord AB, and it will bisect chord AB.

So, AD = DB



Now, the area of the minor sector = (θ/360°)×πr^2

= (120/360)×(22/7)×12^2

= 150.72 cm^2

Consider the ΔAOB,

$\angle OAB = 180° - (90° + 60°) = 30°$

Now, $\cos 30° = AD/OA$

$\sqrt{3}/2 = AD/12$

Or, $AD = 6\sqrt{3}$ cm

We know OD bisects AB. So,

$AB = 2 \times AD = 12\sqrt{3}$ cm

Now, $\sin 30° = OD/OA$

Or, $½ = OD/12$

$\therefore$ OD = 6 cm

So, the area of $\Delta AOB = ½ \times$ base $\times$ height

Here, base = AB = $12\sqrt{3}$ and

Height = OD = 6

So, area of $\Delta AOB = ½ \times 12\sqrt{3} \times 6 = 36\sqrt{3}$ cm = 62.28 cm^2

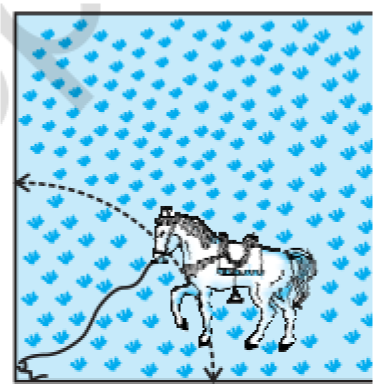
$\therefore$ area of the corresponding Minor segment = Area of the Minor sector – Area of ΔAOB

= 150.72 cm^2 – 62.28 cm^2 = 88.44 cm^2

5. A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope (see Fig). Find

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)



Solution:

As the horse is tied at one end of a square field, it will graze only a quarter (i.e. sector with θ = 90°) of the field with a radius 5 m.

Here, the length of the rope will be the radius of the circle, i.e. r = 5 m

It is also known that the side of the square field = 15 m

(i) Area of circle = $\pi r^2 = 22/7 \times 5^2 = 78.5\ m^2$

Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle) = 78.5/4 = 19.625 m^2

(ii) If the rope is increased to 10 m,

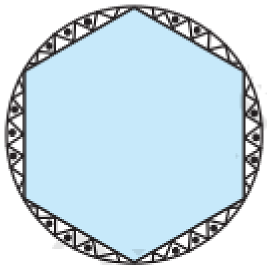
Area of circle will be = $\pi r^2 = 22/7 \times 10^2 = 314\ m^2$

Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle)

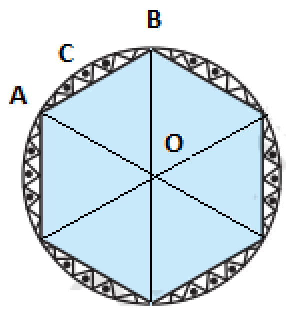
= 314/4 = 78.5 m^2

∴ increase in the grazing area = $78.5\ m^2 - 19.625\ m^2 = 58.875\ m^2$

6. A round table cover has six equal designs, as shown in Fig. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm². (Use √3 = 1.7)



Solution:



Total number of equal designs = 6

AOB= 360°/6 = 60°

The radius of the cover = 28 cm

Cost of making design = ₹ 0.35 per cm^2

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔAOB is an equilateral triangle. So, its area will be $(\sqrt{3}/4)\times a^2$ sq. units

Here, a = OA

∴ Area of equilateral ΔAOB = $(\sqrt{3}/4)\times 28^2 = 333.2$ cm^2

Area of sector ACB = $(60°/360°)\times\pi r^2$ cm^2

= 410.66 cm^2

So, the area of a single design = the area of sector ACB – the area of ΔAOB

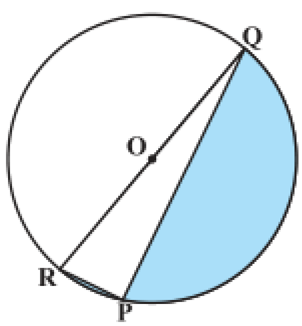
= 410.66 cm^2 – 333.2 cm^2 = 77.46 cm^2

∴ area of 6 designs = 6×77.46 cm^2 = 464.76 cm^2

So, total cost of making design = 464.76 cm^2 ×Rs.0.35 per cm^2

= Rs. 162.66

7. Find the area of the shaded region in Fig, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.



Solution:

Here, P is in the semi-circle, and so,

P = 90°

So, it can be concluded that QR is the hypotenuse of the circle and is equal to the diameter of the circle.

∴ QR = D

Using the Pythagorean theorem,

$QR^2 = PR^2+PQ^2$

Or, $QR^2 = 7^2+24^2$

QR= 25 cm = Diameter

Hence, the radius of the circle = 25/2 cm

Now, the area of the semicircle = $(\pi R^2)/2$

= (22/7)×(25/2)×(25/2)/2 cm^2

= 13750/56 cm^2 = 245.54 cm^2

Also, the area of the ΔPQR = ½×PR×PQ

=(½)×7×24 cm^2

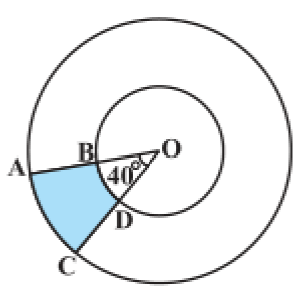
= 84 cm^2

Hence, the area of the shaded region = 245.54 cm^2-84 cm^2

= 161.54 cm^2

8. Find the area of the shaded region in Fig, if the radii of the two concentric circles with centre O are 7 cm and 14 cm, respectively and AOC = 40°.

Solution:



Given,

Angle made by sector = 40°,

Radius the inner circle = r = 7 cm, and

Radius of the outer circle = R = 14 cm

We know,

Area of the sector = $(\theta/360°)\times\pi r^2$

So, Area of OAC = $(40°/360°)\times\pi r^2$ cm^2

= 68.44 cm^2

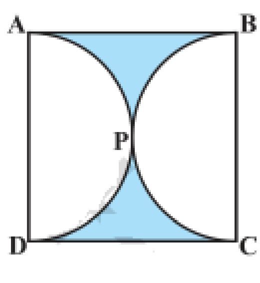
Area of the sector OBD = $(40°/360°)\times\pi r^2$ cm^2

= $(1/9)\times(22/7)\times7^2$ = 17.11 cm^2

Now, the area of the shaded region ABDC = Area of OAC – Area of the OBD

= 68.44 cm^2 – 17.11 cm^2 = 51.33 cm^2

9. Find the area of the shaded region in Fig, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Solution:

Side of the square ABCD (as given) = 14 cm

So, the Area of ABCD = a^2

= 14×14 cm^2 = 196 cm^2

We know that the side of the square = diameter of the circle = 14 cm

So, the side of the square = diameter of the semicircle = 14 cm

∴ the radius of the semicircle = 7 cm

Now, the area of the semicircle = $(\pi R^2)/2$

= $(22/7\times7\times7)/2$ cm^2

= 77 cm^2

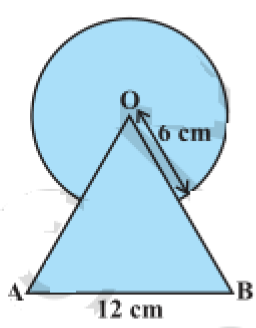
∴ he area of two semicircles = 2×77 cm^2 = 154 cm^2

Hence, the area of the shaded region = Area of the Square – Area of two semicircles

= 196 cm^2 -154 cm^2

= 42 cm^2

10. Find the area of the shaded region in Fig, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as the centre.



Solution:

It is given that OAB is an equilateral triangle having each angle as 60°

The area of the sector is common in both.

The radius of the circle = 6 cm

Side of the triangle = 12 cm

Area of the equilateral triangle = $(\sqrt{3}/4)\,(OA)^2 = (\sqrt{3}/4) \times 12^2 = 36\sqrt{3}\ cm^2$

Area of the circle = $\pi R^2 = (22/7) \times 6^2 = 792/7\ cm^2$

Area of the sector making angle 60° = $(60°/360°) \times \pi r^2\ cm^2$

$= (1/6) \times (22/7) \times 6^2\ cm^2 = 132/7\ cm^2$

Area of the shaded region = Area of the equilateral triangle + Area of the circle – Area of the sector

$= 36\sqrt{3}\ cm^2 + 792/7\ cm^2 - 132/7\ cm^2$

$= (36\sqrt{3} + 660/7)\ cm^2$