Chapter 9

Sequences and Series

NCERT Class 11 Chapter 9 on Sequences and Series. However, I can provide you with a general overview of the topic based on typical curriculum structures.

Sequences:

A sequence is an ordered list of numbers. It can be finite or infinite. In mathematics, sequences are often denoted as 1,2,3,…,$a1,a2,a3,…,an$, where ai represents the $i^{th}$of the sequence. Sequences can be arithmetic, geometric, or more general.

A sequence is an ordered list of numbers, often generated according to a specific rule and It can be finite or infinite. Common types of sequences include arithmetic sequences (where the difference between consecutive terms is constant) and geometric sequences (where the ratio between consecutive terms is constant).

Series:

A series is the sum of the terms of a sequence.

The sum of the first n terms of a sequence is called an n-term series.

Common types of series include arithmetic series and geometric series.

Arithmetic Progression (AP):

In an arithmetic sequence, each term after the first is obtained by adding a constant difference to the previous term.

The general form of an arithmetic sequence is a,a+d,a+2d,….,where a is the first term, d is the common difference.

Geometric Progression (GP):

In a geometric sequence, each term after the first is obtained by multiplying the previous term by a constant ratio.

The general form of a geometric progression is $a,ar,ar2,\ldots$, where a is the first term, r is the common ratio.

Sum of n Terms of an AP:

The sum of the first n terms of an arithmetic sequence (also called an arithmetic progression) is given by the formula $Sn=2n[2a+(n-1)d]$, where Sn is the sum of first n terms, a a is the first term, d is the common difference, and n is the number of terms.

Sum of n Terms of a GP:

The sum of the first n terms of a geometric sequence (also called a geometric progression) is given by the formula $Sn=r-1a(rn-1)$, where Sn is the sum of the first n terms, a is the first term, r is the common ratio, and n is the number of terms.

Exercise 9.1

Write the first five terms of each of the sequences in Exercises 1 to 6 whose nth terms are:

1. a_n = n (n + 2)

Solution:

Given,

n^{th} term of a sequence $a_n = n\ (n + 2)$

On substituting n = 1, 2, 3, 4, and 5, we get the first five terms

$a_1 = 1(1 + 2) = 3$

$a_2 = 2(2 + 2) = 8$

$a_3 = 3(3 + 2) = 15$

$a_4 = 4(4 + 2) = 24$

$a_5 = 5(5 + 2) = 35$

Hence, the required terms are 3, 8, 15, 24, and 35.

2. $a_n = n/n+1$

Solution:

Given the n^{th} term, $a_n = n/n+1$

On substituting $n = 1, 2, 3, 4, 5$, we get

$$a_1 = \frac{1}{1+1} = \frac{1}{2},\ a_2 = \frac{2}{2+1} = \frac{2}{3},\ a_3 = \frac{3}{3+1} = \frac{3}{4},\ a_4 = \frac{4}{4+1} = \frac{4}{5},\ a_5 = \frac{5}{5+1} = \frac{5}{6}$$

Hence, the required terms are 1/2, 2/3, 3/4, 4/5 and 5/6.

3. $a_n = 2^n$

Solution:

Given the n^{th} term, $a_n = 2^n$

On substituting $n = 1, 2, 3, 4, 5$, we get

$a_1 = 2^1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$

$a_5 = 2^5 = 32$

Hence, the required terms are 2, 4, 8, 16, and 32.

4. **$a_n = (2n – 3)/6$**

Solution:

Given the n^{th} term, $a_n = (2n – 3)/6$

On substituting $n = 1, 2, 3, 4, 5$, we get

$$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$$
$$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$$
$$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$$
$$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$$
$$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$$

Hence, the required terms are -1/6, 1/6, 1/2, 5/6 and 7/6..

5. $a_n = (-1)^{n-1} 5^{n+1}$

Solution:

Given the n^{th} term, $a_n = (-1)^{n-1} 5^{n+1}$

On substituting *n* = 1, 2, 3, 4, 5, we get

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Hence, the required terms are 25, –125, 625, –3125, and 15625.

6. $a_n = n\dfrac{n^2+5}{4}$

Solution:

On substituting *n* = 1, 2, 3, 4, 5, we get the first 5 terms.

$$a_1 = 1\cdot\frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$$
$$a_2 = 2\cdot\frac{2^2+5}{4} = 2\cdot\frac{9}{4} = \frac{9}{2}$$
$$a_3 = 3\cdot\frac{3^2+5}{4} = 3\cdot\frac{14}{4} = \frac{21}{2}$$
$$a_4 = 4\cdot\frac{4^2+5}{4} = 21$$
$$a_5 = 5\cdot\frac{5^2+5}{4} = 5\cdot\frac{30}{4} = \frac{75}{2}$$

Hence, the required terms are 3/2, 9/2, 21/2, 21 and 75/2.

Find the indicated terms in each of the sequences in Exercises 7 to 10 whose n^{th} terms are:

7. $a_n = 4n - 3$; a_{17}, a_{24}

Solution:

Given,

The n^{th} term of the sequence is $a_n = 4n - 3$

On substituting *n* = 17, we get

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Next, on substituting *n* = 24, we get

$a_{24} = 4(24) - 3 = 96 - 3 = 93$

8. $a_n = n^2/2^n$; a^7

Solution:

Given,

The n^{th} term of the sequence is $a_n = n^2/2^n$

Now, on substituting $n = 7$, we get

$a_7 = 7^2/2^7 = 49/\ 128$

9. $a_n = (-1)^{n-1}\ n^3$; a_9

Solution:

Given,

The n^{th} term of the sequence is $a_n = (-1)^{n-1}\ n^3$

On substituting $n = 9$, we get

$a_n = \frac{n(n-2)}{n+3}; a_{20}$ $a_9 = (-1)^{9-1}\ (9)^3 = 1 \times 729 = 729.$

10. $a_n = \frac{n(n-2)}{n+3}; a_{20}$

Solution:

On substituting $n = 20$, we get

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20(18)}{23} = \frac{360}{23}$$

Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

11. $a_1 = 3$, $a_n = 3a_{n-1} + 2$ for all $n > 1$

Solution:

Given, $a_n = 3a_{n-1} + 2$ and $a_1 = 3$

Then,

$a_2 = 3a_1 + 2 = 3(3) + 2 = 11$

$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$

$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$

$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$

Thus, the first 5 terms of the sequence are 3, 11, 35, 107 and 323.

Hence, the corresponding series is

$3 + 11 + 35 + 107 + 323$ …….

12. $a_1 = -1$, $a_n = a_{n-1}/n$, $n \geq 2$

Solution:

Given,

$a_n = a_{n-1}/n$ and $a_1 = -1$

Then,

$a_2 = a_1/2 = -1/2$

$a_3 = a_2/3 = -1/6$

$a_4 = a_3/4 = -1/24$

$a_5 = a_4/5 = -1/120$

Thus, the first 5 terms of the sequence are -1, -1/2, -1/6, -1/24 and -1/120.

Hence, the corresponding series is

$-1 + (-1/2) + (-1/6) + (-1/24) + (-1/120) + \ldots\ldots.$

13. $a_1 = a_2 = 2$, $a_n = a_{n-1} - 1$, $n > 2$

Solution:

Given,

$a_1 = a_2$, $a_n = a_{n-1} - 1$

Then,

$a_3 = a_2 - 1 = 2 - 1 = 1$

$a_4 = a_3 - 1 = 1 - 1 = 0$

$a_5 = a_4 - 1 = 0 - 1 = -1$

Thus, the first 5 terms of the sequence are 2, 2, 1, 0 and -1.

The corresponding series is

$2 + 2 + 1 + 0 + (-1) + \ldots\ldots$

14. The Fibonacci sequence is defined by

$1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}$, $n > 2$

Find a_{n+1}/a_n, for $n = 1, 2, 3, 4, 5$

Solution:

Given,

$1 = a_1 = a_2$

$a_n = a_{n-1} + a_{n-2}$, $n > 2$

So,

$a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

$a_6 = a_5 + a_4 = 5 + 3 = 8$

Thus,

$$\text{For } n = 1, \frac{a_n + 1}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$$
$$\text{For } n = 2, \frac{a_n + 1}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2$$
$$\text{For } n = 3, \frac{a_n + 1}{a_n} = \frac{a_4}{a_3} = \frac{3}{2}$$
$$\text{For } n = 4, \frac{a_n + 1}{a_n} = \frac{a_5}{a_4} = \frac{5}{3}$$
$$\text{For } n = 5, \frac{a_n + 1}{a_n} = \frac{a_6}{a_5} = \frac{8}{5}$$

Exercise 9.2

1. Find the 20th and *n*th terms of the G.P. 5/2, 5/4, 5/8, ………

Solution:

Given G.P. is 5/2, 5/4, 5/8, ………

Here, a = First term = 5/2

r = Common ratio = (5/4)/(5/2) = ½

Thus, the 20th term and nth term

$$a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$$
$$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$$

2. Find the 12th term of a G.P. whose 8th term is 192, and the common ratio is 2.

Solution:

Given,

The common ratio of the G.P., $r = 2$

And, let a be the first term of the G.P.

Now,

$a_8 = ar^{8-1} = ar^7$

$ar^7 = 192$

$a(2)^7 = 192$

$a(2)^7 = (2)^6\,(3)$

So,

$$a = \frac{(2)^6 \times 3}{(2)^7} = \frac{3}{2}$$

Hence,

$$a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$$

3. The 5th, 8th and 11th terms of a G.P. are *p*, *q* and *s*, respectively. Show that $q^2 = ps$.

Solution:

Let's take a to be the first term and r to be the common ratio of the G.P.

Then, according to the question, we have

$a_5 = a\,r^{5-1} = a\,r^4 = p$ … (i)

$a_8 = a\,r^{8-1} = a\,r^7 = q$ … (ii)

$a_{11} = a\,r^{11-1} = a\,r^{10} = s$ … (iii)

Dividing equation (ii) by (i), we get

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \quad \text{..... (iv)}$$

On dividing equation (iii) by (ii), we get

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$

$$r^3 = \frac{s}{q} \quad \text{..... (v)}$$

Equating the values of r^3 obtained in (iv) and (v), we get

$$\frac{q}{p} = \frac{s}{q}$$

$$q^2 = ps$$

Hence proved.

4. The 4th term of a G.P. is the square of its second term, and the first term is –3. Determine its 7th term.

Solution:

Let's consider a to be the first term and r to be the common ratio of the G.P.

Given, $a = -3$

And we know that,

$a_n = ar^{n-1}$

So, $a_4 = ar^3 = (-3)\, r^3$

$a_2 = a\, r^1 = (-3)\, r$

Then, from the question, we have

$(-3)\, r^3 = [(-3)\, r]^2$

$\Rightarrow -3r^3 = 9\, r^2$

$\Rightarrow r = -3$

$a_7 = a\, r^{7-1} = a\, r^6 = (-3)\, (-3)^6 = -\, (3)^7 = -2187$

Therefore, the seventh term of the G.P. is –2187.

5. Which term of the following sequences:

(a) 2, 2√2, 4,… is 128 ? (b) √3, 3, 3√3,… is 729 ?

(c) 1/3, 1/9, 1/27, … is 1/19683 ?

Solution:

(a) The given sequence, 2, 2√2, 4,…

We have,

a = 2 and r = 2√2/2 = √2

Taking the n^{th} term of this sequence as 128, we have

$$a_n = ar^{n-1}$$
$$(2)\left(\sqrt{2}\right)^{n-1} = 128$$
$$(2)(2)^{\frac{n-1}{2}} = (2)^7$$
$$(2)^{\frac{n-1}{2}+1} = (2)^7$$
$$\frac{n-1}{2}+1 = 7$$
$$\frac{n-1}{2} = 6$$
$$n-1 = 12$$
$$n = 13$$

Therefore, the 13^{th} term of the given sequence is 128.

(ii) Given the sequence, √3, 3, 3√3,…

We have,

a = √3 and r = 3/√3 = √3

Taking the n^{th} term of this sequence to be 729, we have

$$a_n = ar^{n-1}$$
$$\therefore ar^{n-1} = 729$$
$$\left(\sqrt{3}\right)\left(\sqrt{3}\right)^{n-1} = 729$$
$$(3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)^6$$
$$(3)^{\frac{1}{2}+\frac{n-1}{2}} = (3)^6$$

Equating the exponents, we have

$$\frac{1}{2}+\frac{n-1}{2} = 6$$
$$\frac{1+n-1}{2} = 6$$
$$\therefore n = 12$$

Therefore, the 12th term of the given sequence is 729.

(iii) Given sequence, 1/3, 1/9, 1/27, …

a = 1/3 and r = (1/9)/(1/3) = 1/3

Taking the nth term of this sequence to be 1/19683, we have

$$a_n = ar^{n-1}$$
$$\therefore ar^{n-1} = \frac{1}{19683}$$
$$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$$
$$\left(\frac{1}{3}\right)^{n} = \left(\frac{1}{3}\right)^{9}$$
$$n = 9$$

Therefore, the 9th term of the given sequence is 1/19683.

6. For what values of *x*, the numbers -2/7, x, -7/2 are in G.P?

Solution:

The given numbers are -2/7, x, -7/2

Common ratio = x/(-2/7) = -7x/2

Also, common ratio = (-7/2)/x = -7/2x

$$\therefore \frac{-7x}{2} = \frac{-7}{2x}$$

$$x^2 = \frac{-2 \times 7}{-2 \times 7} = 1$$

$$x = \sqrt{1}$$

$$x = \pm 1$$

Therefore, for $x = \pm 1$, the given numbers will be in G.P.

7. Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015 …

Solution:

Given G.P., 0.15, 0.015, 0.00015, …

Here, $a = 0.15$ and $r = 0.015/0.15 = 0.1$

We know that, $S_n = \frac{a(1-r^n)}{1-r}$

$$\therefore S_{20} = \frac{0.15\left[1-(0.1)^{20}\right]}{1-0.1}$$

$$= \frac{0.15}{0.9}\left[1-(0.1)^{20}\right]$$

$$= \frac{15}{90}\left[1-(0.1)^{20}\right]$$

$$= \frac{1}{6}\left[1-(0.1)^{20}\right]$$

8. Find the sum to *n* terms in the geometric progression √7, √21, 3√7, ….

Solution:

The given G.P. is √7, √21, 3√7, ….

Here,

a = √7 and

$$r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\therefore S_n = \frac{\sqrt{7}\left[1-\left(\sqrt{3}\right)^n\right]}{1-\sqrt{3}}$$

$$= \frac{\sqrt{7}\left[1-\left(\sqrt{3}\right)^n\right]}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \qquad \text{(By rationalizing)}$$

$$= \frac{\sqrt{7}\left(1+\sqrt{3}\right)\left[1-\left(\sqrt{3}\right)^n\right]}{1-3}$$

$$= \frac{-\sqrt{7}\left(1+\sqrt{3}\right)}{2}\left[1-(3)^{\frac{n}{2}}\right]$$

$$= \frac{\sqrt{7}\left(1+\sqrt{3}\right)}{2}\left[(3)^{\frac{n}{2}}-1\right]$$

9. Find the sum to *n* terms in the geometric progression 1, -a, a^2, $-a^3$ …. (if a ≠ -1)

Solution:

The given G.P. is 1, -a, a^2, $-a^3$ ….

Here, the first term = $a_1 = 1$

And the common ratio = $r = -a$

We know that,

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$\therefore S_n = \frac{1\left[1-(-a)^n\right]}{1-(-a)} = \frac{\left[1-(-a)^n\right]}{1+a}$$

10. Find the sum to *n* terms in the geometric progression x^3, x^5, x^7, … (if x ≠ ±1)

Solution:

Given G.P. is x^3, x^5, x^7, …

Here, we have $a = x^3$ and $r = x^5/x^3 = x^2$

We know that, $S_n = \frac{a(1-r^n)}{1-r}$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{x^3\left[1-(x^2)^n\right]}{1-x^2} = \frac{x^3(1-x^{2n})}{1-x^2}$$

11. Evaluate: $\sum_{k=1}^{11}(2+3^k)$

Solution:

$$\sum_{k=1}^{11}(2+3^k)=\sum_{k=1}^{11}(2)+\sum_{k=1}^{11}3^k=2(11)+\sum_{k=1}^{11}3^k=22+\sum_{k=1}^{11}3^k \quad ...(1)$$

$$\sum_{k=1}^{11}3^k=3^1+3^2+3^3+...+3^{11}$$

We can see that, the terms of this sequence 3, 3^2, 3^3, ... forms a G.P

And, we know

$$S_n=\frac{a(r^n-1)}{r-1}$$

$$S_{11}=\frac{3\left[(3)^{11}-1\right]}{3-1}$$

$$S_{11}=\frac{3}{2}(3^{11}-1)$$

$$\therefore \sum_{k=1}^{11}3^k=\frac{3}{2}(3^{11}-1)$$

On substituting the above value in equation (1), we get

$$\sum_{k=1}^{11}(2+3^k)=22+\frac{3}{2}(3^{11}-1)$$

12. The sum of the first three terms of a G.P. is 39/10, and their product is 1. Find the common ratio and the terms.

Solution:

Let a/r, a, ar be the first three terms of the G.P.

a/r + a + ar = 39/10 …… (1)

(a/r) (a) (ar) = 1 …….. (2)

From (2), we have

$a^3 = 1$

Hence, a = 1 [Considering real roots only]

Substituting the value of a in (1), we get

1/r + 1 + r = 39/10

$(1 + r + r^2)/r = 39/10$

$10 + 10r + 10r^2 = 39r$

$10r^2 – 29r + 10 = 0$

$10r^2 – 25r – 4r + 10 = 0$

$5r(2r – 5) – 2(2r – 5) = 0$

$(5r – 2) (2r – 5) = 0$

Thus,

$r = 2/5$ or $5/2$

Therefore, the three terms of the G.P. are 5/2, 1 and 2/5.

13. How many terms of G.P. 3, 3^2, 3^3, … are needed to give the sum 120?

Solution:

Given G.P. is 3, 3^2, 3^3, …

Let's consider that *n* terms of this G.P. be required to obtain the sum 120.

We know that,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Here, $a = 3$ and $r = 3$

$$S_n = 120 = \frac{3(3^n - 1)}{3 - 1}$$

$$120 = \frac{3(3^n - 1)}{2}$$

$$\frac{120 \times 2}{3} = 3^n - 1$$

$$3^n - 1 = 80$$

$$3^n = 81$$

$$3^n = 3^4$$

Equating the exponents, we get $n = 4$

Therefore, four terms of the given G.P. are required to obtain the sum 120.

14. The sum of the first three terms of a G.P. is 16, and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to *n* terms of the G.P.

Solution:

Let's assume the G.P. to be $a, ar, ar^2, ar^3, \ldots$

Then, according to the question, we have

$a + ar + ar^2 = 16$ and $ar^3 + ar^4 + ar^5 = 128$

$a(1 + r + r^2) = 16 \ldots (1)$ and,

$ar^3(1 + r + r^2) = 128 \ldots (2)$

Dividing equation (2) by (1), we get

$$\frac{ar^3(1+r+r^2)}{a(1+r+r^2)} = \frac{128}{16}$$

$r^3 = 8$

$r = 2$

Now, using $r = 2$ in (1), we get

$a(1 + 2 + 4) = 16$

$a(7) = 16$

$a = 16/7$

Now, the sum of terms is given as

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_n = \frac{16}{7}\frac{(2^n - 1)}{2 - 1} = \frac{16}{7}(2^n - 1)$$

15. Given a G.P. with *a* = 729 and 7th term 64, determine S7.

Solution:

Given,

$a = 729$ and $a_7 = 64$

Let r be the common ratio of the G.P.

Then, we know that, $a_n = a\, r^{n-1}$

$a_7 = ar^{7-1} = (729)r^6$

$\Rightarrow 64 = 729\, r^6$

$r^6 = 64/729$

$r^6 = (2/3)^6$

r = 2/3

And we know that

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

So,

$$S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}}$$

$$= 3\times 729\left[1-\left(\frac{2}{3}\right)^7\right]$$

$$= (3)^7\left[\frac{(3)^7-(2)^7}{(3)^7}\right]$$

$$= (3)^7-(2)^7$$

$$= 2187-128$$

$$= 2059$$

16. Find a G.P. for which the sum of the first two terms is –4 and the fifth term is 4 times the third term.

Solution:

Consider *a* to be the first term and *r* to be the common ratio of the G.P.

Given, $S_2 = -4$

Then, from the question, we have

$$S_2 = -4 = \frac{a(1-r^2)}{1-r} \qquad ...(1)$$

And,

$a_5 = 4 \times a_3$

$ar^4 = 4ar^2$

$r^2 = 4$

$r = \pm 2$

Using the value of r in (1), we have

$$-4 = \frac{a\left[1-(2)^2\right]}{1-2} \text{ for } r = 2$$

$$-4 = \frac{a(1-4)}{-1}$$

$$-4 = a(3)$$

$$a = \frac{-4}{3}$$

$$\text{Also, } -4 = \frac{a\left[1-(-2)^2\right]}{1-(-2)} \text{ for } r = -2$$

$$-4 = \frac{a(1-4)}{1+2}$$

$$-4 = \frac{a(-3)}{3}$$

$$a = 4$$

Therefore, the required G.P is

-4/3, -8/3, -16/3, …. Or 4, -8, 16, -32, ……

17. If the 4th, 10th and 16th terms of a G.P. are *x, y* and *z*, respectively. Prove that *x, y, and z* are in G.P.

Solution:

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_4 = a\,r^3 = x$ … (1)

$a_{10} = a\,r^9 = y$ … (2)

$a_{16} = a\,r^{15} = z$ … (3)

On dividing (2) by (1), we get

$$\frac{y}{x} = \frac{ar^9}{ar^3} \Rightarrow \frac{y}{x} = r^6$$

And, on dividing (3) by (2), we get

$$\frac{z}{y} = \frac{ar^{15}}{ar^9} \Rightarrow \frac{z}{y} = r^6$$

$$\frac{y}{x} = \frac{z}{y}$$

Therefore, x, y, z are in G. P.

18. Find the sum to *n* terms of the sequence, 8, 88, 888, 8888…

Solution:

Given sequence: 8, 88, 888, 8888…

This sequence is not a G.P.

But, it can be changed to G.P. by writing the terms as

$S_n = 8 + 88 + 888 + 8888 +$ …………….. to n terms

$$= \frac{8}{9}[9 + 99 + 999 + 9999 + \text{..........to } n \text{ terms}]$$

$$= \frac{8}{9}\left[(10-1)+(10^2-1)+(10^3-1)+(10^4-1)+\text{........to } n \text{ terms}\right]$$

$$= \frac{8}{9}\left[(10+10^2+\text{......} n \text{ terms})-(1+1+1+\text{....} n \text{ terms})\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{9}-n\right]$$

$$= \frac{80}{81}(10^n-1)-\frac{8}{9}n$$

19. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, 1/2.

Solution:

The required sum = 2 x 128 + 4 x 32 + 8 x 8 + 16 x 2 + 32 x ½

= 64[4 + 2 + 1 + ½ + $1/2^2$]

Now, it's seen that

4, 2, 1, ½, $1/2^2$ is a G.P.

With the first term, a = 4

Common ratio, r =1/2

We know,

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\therefore S_5 = \frac{4\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}} = \frac{4\left[1-\frac{1}{32}\right]}{\frac{1}{2}} = 8\left(\frac{32-1}{32}\right) = \frac{31}{4}$$

Therefore, the required sum = 64(31/4) = (16)(31) = 496

20. Show that the products of the corresponding terms of the sequences a, ar, ar^2, …ar^{n-1} and A, AR, AR^2, … AR^{n-1} form a G.P, and find the common ratio.

Solution:

To be proved: The sequence, aA, $arAR$, ar^2AR^2, …$ar^{n-1}AR^{n-1}$, forms a G.P.

Now, we have

$$\frac{\text{Second term}}{\text{First term}} = \frac{arAR}{aA} = rR$$
$$\frac{\text{Third term}}{\text{Second term}} = \frac{ar^2AR^2}{arAR} = rR$$

Therefore, the above sequence forms a G.P., and the common ratio is rR.

21. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Solution:

Consider a to be the first term and r to be the common ratio of the G.P.

Then,

$a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

From the question, we have

$a_3 = a_1 + 9$

$ar^2 = a + 9$ … (i)

$a_2 = a_4 + 18$

$ar = ar^3 + 18$ … (ii)

So, from (1) and (2), we get

$a(r^2 – 1) = 9$ … (iii)

$ar\ (1– r^2) = 18$ … (iv)

Now, dividing (4) by (3), we get

$$\frac{ar\left(1-r^2\right)}{a\left(r^2-1\right)}=\frac{18}{9}$$

-r = 2

r = -2

On substituting the value of *r* in (i), we get

$4a = a + 9$

$3a = 9$

$\therefore a = 3$

Therefore, the first four numbers of the G.P. are 3, 3(– 2), $3(-2)^2$, and $3(-2)^3$

i.e., 3¸–6, 12, and –24.

22. If the p^{th}, q^{th} and r^{th} terms of a G.P. are *a, b* and *c*, respectively. Prove that $a^{q-r}\, b^{r-p}\, c^{p-q} = 1$

Solution:

Let's take *A* to be the first term and *R* to be the common ratio of the G.P.

Then, according to the question, we have

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

Then,

$a^{q-r}\, b^{r-p}\, c^{p-q}$

$= A^{q-r} \times R^{(p-1)\,(q-r)} \times A^{r-p} \times R^{(q-1)\,(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= Aq^{\,-r+r-p+p-q} \times R^{\,(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

= 1

Hence proved.

23. If the first and the n^{th} term of a G.P. are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Solution:

Given the first term of the G.P is a, and the last term is b.

Thus,

The G.P. is $a, ar, ar^2, ar^3, \ldots ar^{n-1}$, where r is the common ratio.

Then,

$b = ar^{n-1}$ … (1)

P = Product of n terms

$= (a)(ar)(ar^2) \ldots (ar^{n-1})$

$= (a \times a \times \ldots a)(r \times r^2 \times \ldots r^{n-1})$

$= a^n r^{1+2+\ldots(n-1)}$ … (2)

Here, 1, 2, …$(n - 1)$ is an A.P.

So,

$$1 + 2 + \ldots\ldots\ldots + (n-1) = \frac{n-1}{2}[2+(n-1-1)\times 1] = \frac{n-1}{2}[2+n-2] = \frac{n(n-1)}{2}$$

And, the product of n terms P is given by,

$$P = a^n r^{\frac{n(n-1)}{2}}$$

$$\therefore P^2 = a^{2n} r^{n(n-1)}$$

$$= \left[a^2 r^{(n-1)}\right]^n$$

$$= \left[a \times ar^{n-1}\right]^n$$

$$= (ab)^n \qquad [\text{Using } (1)]$$

24. Show that the ratio of the sum of the first *n* terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Solution:

Let *a* be the first term and *r* be the common ratio of the G.P.

$$\text{Sum of first n terms} = \frac{a(1-r^n)}{(1-r)}$$

Since there are *n* terms from $(n+1)^{th}$ to $(2n)^{th}$ term,

Sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term

$$= \frac{a_{n+1}(1-r^n)}{(1-r)}$$

$a_{n+1} = ar^{n+1-1} = ar^n$

Thus, the required ratio =

$$\frac{a(1-r^n)}{(1-r)} \times \frac{(1-r)}{ar^n(1-r^n)}$$

$$= \frac{1}{r^n}$$

Thus, the ratio of the sum of the first *n* terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

25. If a, b, c and d are in G.P., show that $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.

Solution:

Given a, b, c, d are in G.P.

So, we have

$bc = ad$ … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

Taking the R.H.S., we have

R.H.S.

$= (ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [Using (1)]

$= [ab + d(a + c)]^2$

$= a^2b^2 + 2abd(a + c) + d^2(a + c)^2$

$= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [Using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$

[Using (2) and (3) and rearranging terms]

$= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2)$

$= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

= L.H.S.

Thus, L.H.S. = R.H.S.

Therefore,

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$

26. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Solution:

Let's assume G_1 and G_2 to be two numbers between 3 and 81 such that the series 3, G_1, G_2, 81 forms a G.P.

And let a be the first term and r be the common ratio of the G.P.

Now, we have the 1st term as 3 and the 4th term as 81.

$81 = (3)\,(r)^3$

$r^3 = 27$

$\therefore r = 3$ (Taking real roots only)

For $r = 3$,

$G_1 = ar = (3)\,(3) = 9$

$G_2 = ar^2 = (3)\,(3)^2 = 27$

Therefore, the two numbers which can be inserted between 3 and 81 so that the resulting sequence becomes a G.P are 9 and 27.

27. Find the value of *n* so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between *a* and *b*.

Solution:

We know that,

The G. M. of *a* and *b* is given by √ab.

Then from the question, we have

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\sqrt{ab}$$

By squaring both sides, we get

$$\frac{\left(a^{n+1}+b^{n+1}\right)^2}{\left(a^n+b^n\right)^2}=ab$$

$$a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=(ab)\left(a^{2n}+2a^nb^n+b^{2n}\right)$$
$$a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=a^{2n+1}b+2a^{n+1}b^{n+1}+ab^{2n+1}$$
$$a^{2n+2}+b^{2n+2}=a^{2n+1}b+ab^{2n+1}$$
$$a^{2n+2}-a^{2n+1}b=ab^{2n+1}-b^{2n+2}$$
$$a^{2n+1}(a-b)=b^{2n+1}(a-b)$$
$$\left(\frac{a}{b}\right)^{2n+1}=1=\left(\frac{a}{b}\right)^0$$
$$2n+1=0 \qquad \text{(Equting the exponents)}$$
$$n=\frac{-1}{2}$$

28. The sum of two numbers is 6 times their geometric mean; show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Solution:

Consider the two numbers to be *a* and *b*.

Then, G.M. = √ab.

From the question, we have

$$a+b=6\sqrt{ab} \qquad \ldots(1)$$

$$\Rightarrow (a+b)^2=36(ab)$$

Also,

$$(a-b)^2=(a+b)^2-4ab=36ab-4ab=32ab$$

$$\Rightarrow a-b=\sqrt{32}\sqrt{ab}$$

$$=4\sqrt{2}\sqrt{ab} \qquad \ldots(2)$$

On adding (1) and (2), we get

$$2a=\left(6+4\sqrt{2}\right)\sqrt{ab}$$

$$a=\left(3+2\sqrt{2}\right)\sqrt{ab}$$

Substiuting the value of a in (1), we get

$$b=6\sqrt{ab}-\left(3+2\sqrt{2}\right)\sqrt{ab}$$

$$b=\left(3-2\sqrt{2}\right)\sqrt{ab}$$

$$\frac{a}{b}=\frac{\left(3+2\sqrt{2}\right)\sqrt{ab}}{\left(3-2\sqrt{2}\right)\sqrt{ab}}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Therefore, the required ratio is $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$

29. If A and G be A.M. and G.M., respectively, between two positive numbers, prove that the

$$A \pm \sqrt{(A+G)(A-G)}$$

Numbers are.

Solution:

Given that A and G are A.M. and G.M. between two positive numbers.

And, let these two positive numbers be a and b.

So,

$$AM = A = \frac{a+b}{2} \quad ...(1)$$

$$GM = G = \sqrt{ab} \quad ...(2)$$

From (1) and (2), we get

$a + b = 2A$ … (3)

$ab = G^2$ … (4)

Substituting the value of a and b from (3) and (4) in the identity $(a - b)^2 = (a + b)^2 - 4ab$, we have

$(a - b)^2 = 4A^2 - 4G^2 = 4\ (A^2 - G^2)$

$(a - b)^2 = 4\ (A + G)\ (A - G)$

$$(a-b) = 2\sqrt{(A+G)(A-G)} \quad ...(5)$$

From (3) and (5), we get

$$2a = 2A + 2\sqrt{(A+G)(A-G)}$$

$$\Rightarrow a = A + \sqrt{(A+G)(A-G)}$$

Substituting the value of a in (3), we have

$$b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$$

Therefore, the two numbers are $A \pm \sqrt{(A+G)(A-G)}$.

30. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of the 2^{nd} hour, 4^{th} hour and n^{th} hour?

Solution:

Given the number of bacteria doubles every hour. Hence, the number of bacteria after every hour will form a G.P.

Here we have, $a = 30$ and $r = 2$

So, $a_3 = ar^2 = (30)(2)^2 = 120$

Thus, the number of bacteria at the end of 2^{nd} hour will be 120.

And, $a_5 = ar^4 = (30)(2)^4 = 480$

The number of bacteria at the end of 4^{th} hour will be 480.

$a_{n+1} = ar^n = (30)\ 2^n$

Therefore, the number of bacteria at the end of n^{th} hour will be $30(2)^n$.

31. What will Rs 500 amount to in 10 years after its deposit in a bank which pays an annual interest rate of 10% compounded annually?

Solution:

Given,

The amount deposited in the bank is Rs 500.

At the end of first year, amount = Rs $500(1 + 1/10)$ = Rs 500 (1.1)

At the end of 2^{nd} year, amount = Rs 500 (1.1) (1.1)

At the end of 3^{rd} year, amount = Rs 500 (1.1) (1.1) (1.1) and so on….

Therefore,

The amount at the end of 10 years = Rs 500 (1.1) (1.1) … (10 times)

= Rs $500(1.1)^{10}$

32. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Solution:

Let's consider the roots of the quadratic equation to be a and b.

Then, we have

$$\text{A.M.} = \frac{a+b}{2} = 8 \Rightarrow a+b = 16 \qquad \ldots(1)$$
$$\text{G.M.} = \sqrt{ab} = 5 \Rightarrow ab = 25 \qquad \ldots(2)$$

We know that,

A quadratic equation can be formed as,

$x^2 - x$ (Sum of roots) + (Product of roots) = 0

$x^2 - x(a + b) + (ab) = 0$

$x^2 - 16x + 25 = 0$ [Using (1) and (2)]

Therefore, the required quadratic equation is $x^2 - 16x + 25 = 0$

2Marks Questions & Answers

1. 150 workers were engaged to finish a job in a certain no. of days. 4 workers dropped out on a second day, 4 more workers dropped out on the third day and so on. It took 8 more days to finish the work and find the no. of days in which the work was completed?

Ans: A = 150, d = -4

$s_n = \frac{n}{2}[2\times150+(n-1)(-4)]$

If total workers who would have worked for all n days, 150(n - 8)

$\therefore \frac{n}{2}[300 + (n - 1)(-4)] = 150(n - 8)$

$\Rightarrow n = 25.$

2. Prove that the sum of n terms of the series
$11 + 103 + 1005 + \ldots\ldots \text{is} \frac{10}{9}(10^n - 1) + n^2$.

Ans: Sn = 11 + 103 + 1005 + ... + n terms

$Sn = (10 + 1) + (10^2 + 3) + (10^3 + 5) + \ldots. + (10n + (2n - 1))$

$Sn = \frac{10(10^n - 1)}{10-1} + \frac{n}{2}(1 + 2n - 1)$

$= \frac{10}{9}(10^n - 1) + n^2$.

3. Between 1 and 31, m number have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and (m – 1)th numbers is 5:9. Find the value of m.

Ans: Let 1, A_1, A_2,…,A_m, 31 are in A.P.

a = 1, a_n = 31

$a_{m+2} = 314$

an = a + (n - 1)d

31 = a + (m + 2 - 1)d

$d = \frac{30}{m+1}$

$\frac{A7}{Am-1} = \frac{5}{9}(given)$

$\Rightarrow \frac{1+7(\frac{30}{m+1})}{1+(m-1)(\frac{30}{m+1})} = \frac{5}{9}$

$\Rightarrow$ m = 1

4. Write the first three terms in each of the following sequences defined by the following: (i) a_n = 2n + 5, (ii) $a_n = \frac{(n-3)}{4}$.

Ans: (i) Here a_n = 2n + 5

Substituting n = 1, 2, 3, we get

$a_1 = 2(1) + 5 = 7$, $a_2 = 9$, $a_3 = 11$

Therefore, the required terms are 7, 9 and 11.

(ii) Here $\boldsymbol{a_n} = \frac{(n-3)}{4}$. Thus, $\boldsymbol{a_1} = \frac{(1-3)}{4} = -\frac{1}{2}$, $\boldsymbol{a_2} = -\frac{1}{4}$ $\boldsymbol{a_3}$ **=0.**

Hence, the first three terms are and $-\frac{1}{2}, -\frac{1}{4}$, 0.

5. What is the 20th term of the sequence defined by

an = (n – 1) (2 – n) (3 +n) ?

Ans: Putting n = 20, we obtain

a_{20} = (20 – 1) (2 – 20) (3 + 20)

= 19 × (– 18) × (23) = – 7866.

6. Let the sequence $\boldsymbol{a_n}$ be defined as follows: $\boldsymbol{a_1}$ = 1, $\boldsymbol{a_n} = \boldsymbol{a_{n-1}} + 2$ for n ≥ 2. Find first five terms and write corresponding series.

Ans: We have

$\boldsymbol{a_1}$ = 1, $\boldsymbol{a_2} = a_1 + 2 = 1 + 2 = 3$, $\boldsymbol{a_3}$ = a2 + 2 = 3 + 2 = 5,

$\boldsymbol{a_4} = a_3 + 2 = 5 + 2 = 7$, $\boldsymbol{a_5} = a_4 + 2 = 7 + 2 = 9$.

Hence, the first five terms of the sequence are 1,3,5,7 and 9.

The corresponding series is 1 + 3 + 5 + 7 + 9 +...

7. A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

Ans: Here a = 2, r = 2 and n = 10

Using the sum formula

$$\boldsymbol{s_n} = \frac{a(r^n-1)}{r-1}$$

We have

$$\boldsymbol{s_{10}} = 2(2^{10} - 1) = 2046$$

Hence, the number of ancestors preceding the person is 2046.

8. Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

Ans: Let $\boldsymbol{G_1}$, $\boldsymbol{G_2}$, $\boldsymbol{G_3}$ be three numbers between 1 and 256 such that

1,$\boldsymbol{G_1}$, $\boldsymbol{G_2}$, $\boldsymbol{G_3}$, 256 is a G.P.

Therefore 256 = r^4 giving r = ± 4 (Taking real roots only)

For r = 4, we have $\boldsymbol{G_1}$= ar = 4, $\boldsymbol{G_2}$= ar^2 = 16, $\boldsymbol{G_3}$= ar^3= 64

Similarly, for r = – 4, numbers are – 4, 16 and – 64.

Hence, we can insert 4, 16, 64 between 1 and 256 so that the resulting sequences are in G.P

9.Find the sum of first n terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+\ldots$

Ans: Here a = 1 and r = $\frac{2}{3}$. Therefore

$$\boldsymbol{s_n} = \frac{a(1-r^n)}{1-r} = \frac{\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}}$$

$$= 3\left[1-\left(\frac{2}{3}\right)^n\right]$$

In particular, $\boldsymbol{s_5} = 3\left[1-\left(\frac{2}{3}\right)^n\right] = 3 \times \frac{211}{243} = \frac{211}{81}$.

10. Find the sum of the sequence 7, 77, 777, 7777,… to n terms.

Ans: This is not a G.P., however, we can relate it to a G.P. by writing the terms as

$\boldsymbol{s_n}$ = 7 + 77 + 777 + 7777 + ... to n terms

= $\frac{7}{9}$ [9+99+999+9999+…to n term]

= $\frac{7}{9}$ [(10–1) + (10^2–1) + (10^3–1) + (10^4–1) +…n terms]

$= \frac{7}{9}$ [(10+10^2+10^3+…n terms) − (1+1+1+... terms)]

$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1} - n\right] = \frac{7}{9}\left[\frac{10(10^n-1)}{9} - n\right]$

Multiple Choice Questions

1) If a, 4, b are in Arithmetic Progression; a, 2, b are in Geometric Progression; then a, 1, b are in

a. A.P
b. G.P
c. H.P
d. None of these

Answer: (c) H.P

Explanation:

Given that a, 4, b are in A.P

Hence, 4-a = b-4

a+b = 8 …(1)

Also, given that a, 2, b are in G.P.

Hence, 2/a = b/2

So, ab = 4…(2)

If a, 1, b are in H.P, then 1 = 2(ab)/(a+b) ..(3)

Now substitute (1) and (2) in (3)

1 = 2(4) /(8)

1 = 8/8

1=1.

Therefore, a, 1, b are in H.P.

2) If “a” is the first term and “r” is the common ratio, then the nth term of a G.P is:

a. ar^n
b. ar^{n-1}
c. $(ar)^{n-1}$
d. None of these

Answer: (b) ar^{n-1}

Explanation:

If “a” is the first term and “r” is the common ratio, the terms of infinite G.P are written as a, ar, ar^2, ar^3, ar^4, …ar^{n-1}.

Hence, the nth term of a G.P is ar^{n-1}.

Therefore, option (b) is the correct answer.

3) If a, b, c are in arithmetic progression, then

a. b = a+c
b. 2b = a+c
c. b^2 = a+c
d. $2b^2$ = a+c

Answer: (b) 2b = a+c

Explanation:

Given that a, b, c are in arithmetic progression.

So, the common difference is b-a = c-b

Rearranging the same terms, we get

b+b = c+a

2b = a+c.

Hence, if a, b, c are in A.P, then 2b = a+c.

4) The sum of arithmetic progression 2, 5, 8, …, up to 50 terms is

a. 3775
b. 3557
c. 3757
d. 3575

Answer: (a) 3775

Explanation:

Given A.P. = 2, 5, 8, …

We know that the sum of n terms of an A.P is $S_n = (n/2)[2a+(n-1)d]$

Here, a = 2, d = 3 and n=50.

Now, substitute the values in the formula, we get

$S_{50} = (50/2)[2(2)+(50-1)(3)]$

$S_{50} = 25[4+(49)(3)]$

$S_{50} = 25[4+147]$

$S_{50} = 25(151)$

$S_{50} = 3775$.

Hence, the sum of A.P 2, 5, 8, …up to 50 terms is 3775.

5) The 3rd term of G.P is 4. Then the product of the first 5 terms is:

a. 4^3
b. 4^4
c. 4^5
d. None of these

Answer: (c) 4^5

Explanation:

We know that the terms of infinite G.P are written as a, ar, ar^2, ar^3, ar^4, …ar^{n-1}.

Hence, the 3rd term, (i.e) $ar^2 = 4$

Thus, the product of the first 5 terms = $(a)(ar)(ar^2)(ar^3)(ar^4)$

$= a^5r^{10}$

$= (ar^2)^5$

Now, substitute $ar^2 = 4$ in the above form, we get

Product of the first 5 terms = $(4)^5 = 4^5$.

Hence, option (c) 4^5 is the correct answer.

6) Which of the following is an example of a geometric sequence?

a. 1, 2, 3, 4
b. 1, 2, 4, 8
c. 3, 5, 7, 9
d. 9, 20, 21, 28

Answer: (b) 1, 2, 4, 8

Explanation:

Among the options given, option (b) 1, 2, 4, 8 is an example of a geometric sequence.

We know that in a geometric sequence each term is found by multiplying the previous term by a constant.

In option (b) 1, 2, 4, 8, each term is found by multiplying 2 to the previous term. Here, the common ratio is 2.

7) The next term of the given sequence 1, 5, 14, 30, 55, … is

a. 80
b. 90
c. 91
d. 96

Answer: (c) 91

Explanation: The next term in the sequence 1, 5, 14, 30, 55, … is 91.

Ist term = $1^2 = 1$

2nd term = $1^2+2^2 = 1+4 = 5$

3rd term = $1^2+2^2+3^2 = 1+4+9 = 14$

4th term = $1^2+2^2+3^2+4^2 = 1+4+9+16 = 30$

5th term = $1^2+2^2+3^2+4^2+5^2 = 1+4+9+16+25 = 55$

6th term = $1^2+2^2+3^2+4^2+5^2+6^2 = 1+4+9+16+25+36 = 91$.

Thus, option (c) is the correct answer.

8) If the nth term of an arithmetic progression is 3n-4, then the 10th term of an A.P is

a. 10
b. 12
c. 22
d. 26

Answer: (d) 26

Explanation:

Given that the nth term of A.P = 3n-4.

To find the 10th term of A.P, substitute n = 10

Therefore, 10th term of A.P = 3(10) -4 = 30-4 = 26.

9) 3, 5, 7, 9 is an example of

a. Arithmetic sequence
b. Geometric sequence
c. Harmonic sequence
d. Fibonacci sequence

Answer: (a) Arithmetic sequence

Explanation: 3, 5, 7, 9 is an example of an arithmetic sequence. In this sequence 3, 5, 7, 9, the difference between each term is 2.

(i.e) 5-3 = 2, 7-5 = 2, 9-7 = 2.

Hence 3, 5, 7, 9 is an arithmetic sequence.

10) The first term of a G.P is 1. The sum of the 3rd and 5th terms is 90. Then the common ratio is:

a. 1
b. 2
c. 3
d. 4

Answer: (c) 3

Explanation:

Given that first term of G.P, a = 1.

The sum of the 3rd and 5th term = 90

(i.e) $ar^2+ar^4 = 90$

Substitute a = 1,

$\Rightarrow r^2+r^4 = 90$

$\Rightarrow r^4 + r^2 - 90 = 0$

$\Rightarrow r^4 + 10r^2 - 9r^2 - 90 = 0$

Now, factorize the above equation,

$\Rightarrow r^2 (r^2+10) - 9 (r^2+10) = 0$

$\Rightarrow (r^2-9)(r^2+10) = 0$

$\Rightarrow r^2 = 9$ or $r^2 = -10$

Here, $r^2= -10$ is not possible, as the square of a number cannot be negative.

So, $r^2 = 9$

$r = 3$ or $r= -3$

Therefore, option (c) 3 is the correct answer.

Summary

- By a sequence, we mean an arrangement of number in definite order according to some rule. Also, we define a sequence as a function whose domain is the set of natural numbers or some subsets of the type {1, 2, 3, ...k}. A sequence containing a finite number of terms is called a finite sequence. A sequence is called infinite if it is not a finite sequence.
- Let a_1, a_2, a_3,... be the sequence, then the sum expressed as $a_1 + a_2 + a_3 + \ldots$ is called series. A series is called finite series if it has got finite number of terms.
- A sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout. This constant factor is called the common ratio. Usually, we denote the first term of a G.P. by *a* and its common ratio by *r*. The general or the n^{th} term of G.P. is given by $a_n = ar^{n-1}$. The sum S_n the first n terms of G.P. is given by

  S_n=a $(r^n-1)/r-1$ or a $(1-r^n)/1-r$, if $r \neq 1$
- The geometric mean (G.M.) of any two positive numbers a and b is given by $\sqrt{ab}$ i.e., the sequence a, G, b is G.P.