Gravitation

2 Marks Answers and Questions

Q1. Explain gravitation.

Ans: Any two bodies will be attracted to one another by the force of gravity, also known as gravity. There is an attraction between everything in the cosmos, however most of the time the force is too faint to be seen due to the extreme distance between the objects.

Q2. Name two factors that influence whether or not a planet has an atmosphere.

Ans: The existence of an atmosphere on a planet is determined by two factors.

1. The temperature on the planet's surface
2. The presence of gravitational acceleration on the planet's surface

Q3. At the centre of the Earth, a body is weightless. Why?

Ans: G = 0 in the centre of the Earth.

It is determined by:

w = mg

Q3. At the centre of the Earth, a body is weightless. Why?

Ans: G = 0 in the centre of the Earth.

It is determined by:

w = mg

⇒w = 0

The body is therefore weightless at the centre of the Earth.

Q4.What is kepler's law of periods? Show it mathematically?

Ans. It states that the square of the period of revolution of a planet around the sun is proportional of a planet to the cube of the semi-major axis of the elliptical orbit.

i.e. T2 $\propto$ R3

T2 = KR3

where T is time period of evolution

R is the length of semi major axis

K is constant for all planets

Q5.With two characteristics of gravitational force?

Ans.(1) It is a central force

(2) It is a conservation force

(3) It obeys inverse square law.

(4) It is a universal force and is always attractive in nature.

Q6.Assuming earth to be a uniform sphere finds an expression for density of earth in terms of g nd G?

Ans. Since g = $\frac{GM}{R^2}$

If earth is uniform sphere of mean density P

g = $\frac{G}{R^2}\left(\frac{4}{3}\pi R^3 P\right)$

g = $\frac{4}{3}\pi GRP$

$\Rightarrow$ P = $\frac{3g}{4\pi GR}$

Q7.If radius of earth is 6400km, what will be the weight of 1 quintal body if taken to the height of 1600 km above the sea level?

Ans.R = 6400km = 6400 x 103m

h = 1600km

w = mg = 1 quintal = 100 kg = 100x9.8 N

weight (w) = mgh

w = mg $\left(\frac{R}{R+h}\right)^2$

w = 100x9.8 $\left(\frac{6400}{1600+6400}\right)^2$

w = 64x9.8N = 64kg

Q8.A man can jump 1.5 m high on earth. Calculate the height he may be able to jump on a planet whose density is one fourth that of the earth and Whose radius is one third of the earth.

Ans. $g = \frac{4}{3}\pi GR\rho$

$g' = \frac{4}{3}\pi GR'\rho'$

The gain in PE at the highest point will be same in both cases. Hence
mg"h" = mgh

$$h' = \frac{mgh}{mg'} = \frac{m\times\frac{4}{3}\pi GR\rho h}{m\frac{4}{3}\pi GR'\rho'}$$

$$= \frac{R\rho h}{R'\rho'} = \frac{3R'\times 4\rho'\times 1.5}{R'\times\rho'}$$

= 18 m

4 Marks Answers and Questions

Q1. Assume that there are 2.5 1011 stars in our galaxy, each with a solar mass. How long will it take a star 50,000 light years from the galactic centre to make one revolution? Assume that the Milky Way is 105ly in diameter.

Ans: Mass of the Milky Way galaxy, M = 2.5 × 1011 solar mass

Solar Mass = Mass of the Sun = 2.0 × 1036 kg

Mass of our galaxy, M = 2.5 × 1011 × 2 × 1035

M = 5 × 1041 kg

Diameter of the Milky Way, d = 105 ly

Radius of the Milky Way, r = 5 × 104 ly

1 ly = 9.46 × 1015m

r = 5 × 104 × 9.46 × 1015

r = 4.73 × 1020m

Since a star rotates around the galactic centre of the Milky Way, its time period is given by

T=42r3GM12=4×3.1424.73310606.67×10-11×5×1041

T=125.27×103012

T = 1.12 × 1016s

As we know,

1 year = 365 × 24 × 60 × 60s

We get,

1s = 1365246060years

1.12 × 1016s = 1.121016365246060

∴1.121016s=3.55108years

Thus, the star will take 3.55 × 108 years to complete one revolution.

Q2. A body weighs 63N while it is on the Earth's surface. What is the magnitude of the Earth's gravitational pull on it at a height equal to one-half the radius of the Earth?

Ans: Weight of the body, W = 63N

Acceleration due to gravity at height h from the Earth’s surface

Re = Radius of the Earth

For h=Re2, gravity at h is given by

gh= g1+hRe2

gh= g1+122

gh= 49g

Weight of a body of mass m at height h is given as”

W’ = mgh

W’ = 49mg

W’ = 49W

W’ = 49×63

W’ = 28N

Therefore, at a height equal to half the Earth’s radius, the gravitational force exerted on the body by the Earth is 28N.

Q3. A projectile can escape from the Earth’s surface at a speed of 11.2 km/s. With three times this speed, a body is thrust out. What is the speed of a body travelling far from Earth? Ignore the sun’s and other planets’ presence.

Ans: The escape velocity of the projectile from the Earth, vesc=11.2kms-1

Projectile velocity of the projectile, vp=3vesc

Mass of the projectile = m

Projectile’s velocity far from the Earth = vf

Projectile’s total energy on the Earth = 12mvp2– 12mvesc2

The projectile’s gravitational potential energy far from the Earth is 0.

Total energy of the projectile far from the Earth = 12mvf2

From the law of energy conservation, we have

12mvp2– 12mvesc2= 12mvf2

We get,

vf=vp2–vesc2

vf=3vesc2–vesc2

vf=8vesc

vf=8×11.2

vf=31.68kms-1

Therefore, the speed of the body far from the Earth's surface is 31.68kms-1

Q4. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth"s gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^{6} m; G = $6.67\times10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$.

Ans. Mass of the Earth, M = 6.0×10^{24} kg

Mass of the satellite, m = 200 kg

Radius of the Earth, Re = 6.4×10^{6} m

Universal gravitational constant, G = $6.67\times10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$

Height of the satellite, h = 400 km = $4\times10^{5}\ m = 0.4\times10^{6}\ m$

Total energy of the satellite at height h $= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e + h}\right)$

Orbital velocity of the satellite, v = $\sqrt{\frac{GM_e}{R_e + h}}$

Total energy of height, h $= \frac{1}{2}m + \left(\frac{GM_e}{R_e + h}\right) - \frac{GM_e m}{R_e + h} = -\frac{1}{2}\left(\frac{GM_e m}{R_e + h}\right)$

The negative sign indicates that the satellite is bound to the Earth. This is called bound energy of the satellite.

Energy required to send the satellite out of its orbit = – (Bound energy)

$$= \frac{1}{2}\frac{GM_e m}{(R_e + h)}$$

$$= \frac{1}{2}\times\frac{6.67\times10^{-11}\times6.0\times10^{24}\times200}{\left(6.4\times10^{6} + 0.4\times10^{6}\right)}$$

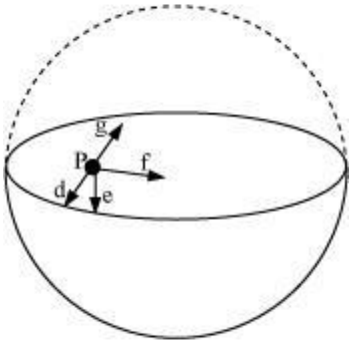
$$= \frac{1}{2}\times\frac{6.67\times6\times2\times10}{6.4\times10^{6}} = 5.9\times10^{9} J$$

Q5. Choose the correct answer from among the given ones:

For the problem 8.10, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Ans.(ii)Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient $\left(\frac{dV}{dr}\right)$ is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.

If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle at an arbitrary point P will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at an arbitrary point P of the hemispherical shell has the direction as indicated by arrow **e**.

Q6.A satellite is revolving is a circular path close to a planet of density P. find an expression for its period of revolution?

Ans.If satellite revolvers around the earth of radius r

T = $\frac{2\pi r}{\upsilon}$

where υ is orbital velocity

where υ = $\sqrt{\frac{Gm}{r}}$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{Gm}{r}}} = 2\pi\sqrt{\frac{r^3}{Gm}}$$

If a satellite is revolving near the plant's surface then r = R radius of planet and

M $= \frac{4}{3}\pi R^3 P$

T = $2\pi\sqrt{\frac{R^3}{G.\frac{4}{3}\pi R^3 P}}$

$$T = 2\pi\sqrt{\frac{3}{4\pi GP}}$$

$$T = \sqrt{\frac{3\pi}{GP}}$$

7 Marks Answers and Questions

Q1. One of Jupiter's satellites, IO, has an orbital period of 1.769 days and an orbital radius of 4.22 108 metres. Prove that Jupiter's mass is around one thousandth that of the sun.

Ans: Given,

Orbital period of IO=TIO = 1.769 days = 1.769×24×60×60s

Orbital radius IO = RIO=4.22×108m

Satellite IO is revolving around Jupiter.

Mass of Jupiter is given by

MJ = 42RIO3GTIO2 …. 1

Where,

MJ is the mass of Jupiter.

G is the Universal gravitational constant.

The orbital period of the Earth, Te = 365.25 days = 365.25 × 25 × 60 × 60s

The orbital radius of the Earth, Re = 1AU = 1.496 × 1011m

Mass of the Sun is given as

Ms=42Re3GTe2 …. 2

MSMJ=42Re3GTe2GTIO242RIO3=Re3RIO3TIO2Te2

MSMJ=1.769×24×60×60365.25×24×60×6021.496×10114.22×1083

MSMJ=1045.04

∴MSMJ~1000

We get,

MS ~ 1000 × MJ

The mass of Jupiter is therefore estimated to be approximately one thousandth that of the Sun.

Q2. A rocket is launched vertically at a speed of 5 km/s-1 from the surface of the Earth. How far does the rocket travel before coming back to Earth? Earth's mass is 6.0 × 1024 kg; its mean radius is 6.4 × 106 m; and its gravitational constant is 6.67 × 10-11 Nm2kg-2.

Ans: Distance from the Earth's centre = 8 × 106m

Velocity of the rocket, v = 5kms-1 = 5 × 103ms-1

Mass of the Earth, Me = 6.0 × 1024kg

Radius of the Earth, Re = 6.4 × 106m

Height reached by rocket mass m is h

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

= 12mv2+-GMemRe

At highest point h, v = 0

Then, Potential energy = –GMemRe+h

Total energy of the rocket = 0+GMemRe+h= –GMemRe+h

From the law of conservation,

Total energy of the rocket at the Earth's surface = Total energy of rocket at height h

Then,

12mv2+-GMemRe=-GMemRe+h

12v2=GMe1Re–1Re+h

12v2=GMeRe+h-ReRe(Re+h)

12v2=GMehReRe+hReRe

12v2=gRehRe+h

Where,

g = GMRe2=9.8ms-2 is the acceleration due to gravity on the Earth's surface.

Clearly,

v2(Re+h) = 2gReh

v2Re = h (2gRe – v2)

h = Rev22gRe–v2

h = 6.4×25×1012100.44×106

h = 1.6 106m

Height achieved by the rocket with respect to the Earth's centre, H is given by: H = Re + h

H = 6.4 106 + 1.6 106

We obtain,

H = 8.0 106m

The distance the rocket travels before landing on Earth is 8.0 106 metres.

Q3. A head-on collision is imminent between two stars, each of which has a solar mass of one solar mass (=2 1030 kg). Their speeds are barely perceptible at a distance of 109 kilometres. What is the rate of their collision? Each star has a 104 km radius. Assume that up until their collision, the stars won't change. (Use the known value of G)

Ans: Mass of each star, M = 2 × 1030 kg

Radius of each star, R = 104 km = 107m

Distance between the stars, r = 109 km = 1012m

For negligible speeds, v = 0

Total energy of two stars separated at distance r is given by

TE=-GMMr+12mv2

TE= GMMr+0 ……(i)

Now, consider the case when the stars are about to collide:

Velocity of the stars = v

Distance between the centres of the stars = 2R

Total kinetic energy of both stars = 12Mv2+12Mv2=Mv2

Total potential energy of both stars = -GMM2R

Total energy of the two stars = Mv2–GMM2R …..(ii)

From the law of conservation of energy,

Mv2–GMM2R=-GMMR

v2=6.67×10-10×2×1030–11012+12×107

v2=13.34×1019–10-12+5×10-8

v2=13.34×1019×5×10-8

v2=6.67×1012

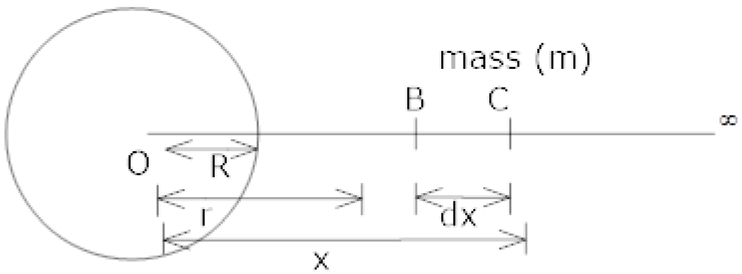
v=6.67×1012

v=2.58×106m/s

The two stars collide at a speed of 2.58 × 106 m/s.

Q4.Define Gravitational potential energy Hence deduces an expression for gravitational potential energy of a body placed at a point sear the surface of earth?

Ans. It is defined as the work done in bringing a body from infinity to that point.



A body of mass (m) lying at a distance x from earth of mass (m)

$F = \frac{GMm}{x^2}$

If the body is displaced through a distance dx then

$dw = Fdx = \frac{GMm}{x^2}dx$

Total work done

$W = \int_{\infty}^{r} \frac{GMm}{x^2}dx$

$w = GMm \int_{\infty}^{r} \frac{1}{x^2}\, dx$

$w = GMm \left|\frac{-1}{x}\right|_{\infty}^{r}$

$w = -GMm \left|\frac{1}{r} - \frac{1}{\infty}\right|$

w = $\frac{-GMm}{r}$

This work done is equal to the gravitational potential energy

i.e.

$$w = U_g = \frac{-GMm}{r}$$

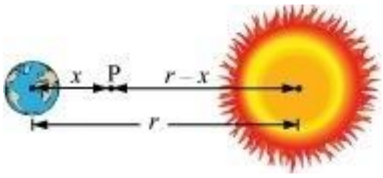
Q5. A rocket is fired from the earth towards the sun. At what distance from the earth"s centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

Ans. Mass of the Sun, M_s = 2×10^{30} kg

Mass of the Earth, M_e = 6×10^{24} kg

Orbital radius, r = 1.5×10^{11} m

Mass of the rocket = m



Let x be the distance from the centre of the Earth where the gravitational force acting on satellite P becomes zero.

From Newton"s law of gravitation, we can equate gravitational forces acting on satellite P under the influence of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} Gm\frac{M_e}{x^2}$$

$$\left(\frac{r-x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\frac{r-x}{x} = \left(\frac{2\times10^{30}}{60\times10^{24}}\right)^{\frac{1}{2}} = 577.35$$

$$1.5\times10^{11} - x = 577.35x$$

$$577.35x = 1.5\times10^{11}$$

$$x = \frac{1.5\times10^{11}}{578.35} = 2.59\times10^{8}m$$

Q6. A rocket is fired vertically with a speed of 5 km s^{-1} from the earth"s surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0×10^{24} kg; mean radius of the earth = 6.4×10^{6} m; G= 6.67×10^{-11} N m^{2} kg^{-2}.

Ans. 8×10^{6} m from the centre of the Earth

Velocity of the rocket, v = 5 km/s = 5×10^{3} m/s

Mass of the Earth, $M_e = 6.0\times10^{24} kg$

Radius of the Earth, $R_e = 6.4\times10^{6} m$

Height reached by rocket mass, m = h

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$=\frac{1}{2}mv^2+\left(\frac{-GM_em}{R_e}\right)$$

At highest point h, V=0

And, Potential energy = $-\frac{GM_em}{R_e+h}$

Total energy of the rocket $=0+\left(-\frac{GM_em}{R_e+h}\right)=-\frac{GM_em}{R_e+h}$

From the law of conservation of energy, we have

Total energy of the rocket at the Earth"s surface = Total energy at height h

$$\frac{1}{2}mv^2+\left(-\frac{GM_em}{R_e+h}\right)=-\frac{GM_em}{R_e+h}$$

$$\frac{1}{2}v^2=GM_e\left(\frac{1}{R_e}-\frac{1}{R_e+h}\right)$$

$$GM_e\left(\frac{R_e+h-R_e}{R_e(R_e+h)}\right)$$

$$\frac{1}{2}v^2=\frac{GM_eh}{R_e(R_e+h)}\times\frac{R_e}{R_e}$$

$$\frac{1}{2}v^2=\frac{gR_eh}{R_e+h}$$

Where g = $=\frac{GM}{R_e^2}=9.8\,m/s^2$ Acceleration due to gravity on the Earth’s surface)

$$\therefore v^2(R_e+h)=2gR_eh$$

$$v^2R_e=h(2gR_e-v^2)$$

$$h=\frac{R_ev^2}{2gR_e-v^2}$$

$$=\frac{6.4\times10^6\times(5\times10^3)^2}{2\times9.8\times6.4\times10^6-(5\times10^3)^2}$$

$$h=\frac{6.4\times25\times10^{12}}{100.44\times10^6}=1.6\times10^6m$$

Height achieved by the rocket with respect to the centre of the Earth

$$=R_e+h$$

$$=6.4\times10^6+1.6\times10^6$$

$$=8.0\times10^6m$$

Q7. Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centers of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

Ans. 0;

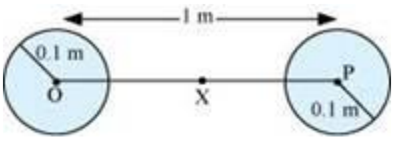
–2.7 × 10–8 J /kg;

Yes;

Unstable

Explanation:

The situation is represented in the given figure:



Mass of each sphere, M = 100 kg

Separation between the spheres, r = 1m

X is the midpoint between the spheres. Gravitational force at point X will be zero. This is because gravitational force exerted by each sphere will act in opposite directions.

Gravitational potential at point X:

$$=\frac{-GM}{\left(\frac{r}{2}\right)}-\frac{GM}{\left(\frac{r}{2}\right)}=-4\frac{GM}{r}$$

$$=\frac{4\times6.67\times10^{-11}\times100}{r}$$

$= -2.67 \times 10^{-8} J / kg$

Any object placed at point X will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effective force in that direction.

MCQs with Answers for Gravity

1.The value of gravitational acceleration on Earth is

(a) 9.8 m/s

(b) 9.8 m/s^2

(c) 8.9 m/s

(d) 8.9 m/s^2

Answer:(b) 9.8 m/s^2

2.What would be the height of an artificial satellite so that it remains stationary with respect to earth's surface?

(a) 36000 km above the earth surface

(b) 40000 km above the earth surface

(c) 26000 km above the earth surface

(d) 63000 km above the earth surface

Answer:(a) 36000 km above the earth surface

3.The value of gravitational acceleration

(a) increases as height increases from the earth.

(b) decreases as height increases from the earth.

(c) remains constant.

(d) None of the above

Answer:(a) increases as height increases form the earth.

4.The centripetal force required by the artificial satellite to revolve around earth is provided by

(a) fuel contained in the satellite

(b) gravitational force due to sun

(c) gravitational force due to earth

(d) Thrust produced by burning fuel

Answer:(c) gravitational force due to earth

5.In which region of earth the weight of a body is slightly greater than the other regions?

(a) At polar region

(b) At equator

(c) Tropic of Cancer

(d) None of this

Answer:(a) At Polar region

6.What is the value of the escape velocity of earth?

(a) 9.8 km/sec

(b) 10 km/sec

(c) 11.2 km/sec

(d) 12 km/sec

Answer:(c) 11.2 km/sec

7.How much time does a polar satellite take to complete one revolution around earth?

(a) 1 hour 30 min

(b) 2 hours

(c) 2 hour 20 min

(d) 3 hour

Answer:(b) 2 hours

8.In case of a planet's motion on its circular orbit

(a) its velocity remains constant in its orbit

(b) its angular velocity remains constant

(c) its total angular momentum remains constant.

(d) radius of orbit remains constant.

Answer:(b) its angular velocity remains constant

9.If the radius of earth is decreased keeping its mass constant, then the length of day will

(a) decrease

(b) Increase

(c) remain same

(d) cannot say

Answer:(a) decrease

10.If the earth would stop rotating then the weight of an object on north pole will

(a) increase

(b) decrease

(c) remain same

(d) be zero

Answer:(c) remain same

11.What is the weight of a body inside an artificial satellite of earth?

(a) It depends on the mass of the body.

(b) It depends on the velocity of satellite

(c) Product of its mass and gravitational acceleration

(d) Zero

Answer:(d) Zero

12.Weight of an object in a free fall is equal to

(a) mass of the object × gravitational acceleration

(b) Zero

(c) greater than when the object was at rest

(d) less than when the object was at rest

Answer:(b) Zero

13.Which of the following is true for an artificial satellite revolving around earth in a circular orbit?

(a) Its linear velocity is constant.

(b) Its acceleration is constant.

(c) Its acceleration keeps on changing.

(d) Its angular velocity is constant.

Answer:(c) Its acceleration keeps on changing.

14.How does the weight of a body change with an increase in the speed of rotation of the earth?

(a) Weight of the body will increase

(b) Weight of the body will decrease

(c) Weight of the body remains constant

(d) Cannot be answered

15.What would be the escape velocity of a planet whose mass and radius are double from earth with the escape velocity of earth taken as Ve.

(a) Ve

(b) 2 Ve

(c) 4 Ve

(d) 16 Ve

Answer:(b) 2 Ve

FILL IN THE BLANKS

1. **The gravitational force is a _________ force.**

Answer: The gravitational force is a central force.

2. **The acceleration due to earth's gravity decreases as __________ increases.**

Answer: The acceleration due to earth's gravity decreases as altitude increases.

3. **The energy of the satellite is __________.**

Answer: The energy of the satellite is negative.

4.The gravitational potential energy is a __________ quantity.

Answer: The gravitational potential energy is a scalar quantity.

5.The escape speed is _________ of mass of the object.

Answer: The escape speed is independant of mass of the object.

6.___________ proposed heliocentric theory.

Answer: Nicolaus copernics proposed heliocentric theory.

7.Due to rotation of the earth, the acceleration due to gravity is maxium at ___________.

Answer: Due to rotation of the earth, the acceleration due to gravity is maxium at poles.

8. ________experimentally determined the value of gravitational constant 'G' using a torsion balance.

Answer: Henry canevdish experimentally determined the value of gravitational constant 'G' using a torsion balance.

9. According to Kepler's second law, the radius vector to a planet from the Sun sweeps out equal areas in equal intervals of time. This law is a consequence of the conservation of ______________.

Answer: According to Kepler's second law, the radius vector to a planet from the Sun sweeps out equal areas in equal intervals of time. This law is a consequence of the conservation of angular momentum.

10. A geostationary satellite is orbiting the earth at a height of 6R above the surface of the earth, where R is the radius of the earth. The time period of another satellite at a height of 2.5 R from the surface of the earth is …… hours.

Answer: A geostationary satellite is orbiting the earth at a height of 6R above the surface of the earth, where R is the radius of the earth. The time period of another satellite at a height of 2.5 R from the surface of the earth is $6\sqrt{2}$h hours.