Chapter 10

Straight Lines

"Straight Lines," delves into the fundamental concepts related to lines and their equations in a plane. The chapter commences with an introduction to the basics of a line, exploring different forms of its equation, such as the slope-intercept form, point-slope form, two-point form, and intercept form. Students learn to calculate the angle between two lines and understand the conditions for lines to be parallel or perpendicular**.** the distance formula for determining the distance between two points and the section formula for dividing a line segment in a given ratio.

Throughout the discussions, the slope of a line is emphasized, and students learn to find the slope given the line's equation. The chapter concludes with a thorough exploration of various forms of equations of a line, encouraging students to convert equations from one form to another. The general equation of a line, expressed as Ax + By + C = 0, is presented, providing a comprehensive understanding of the chapter's content.

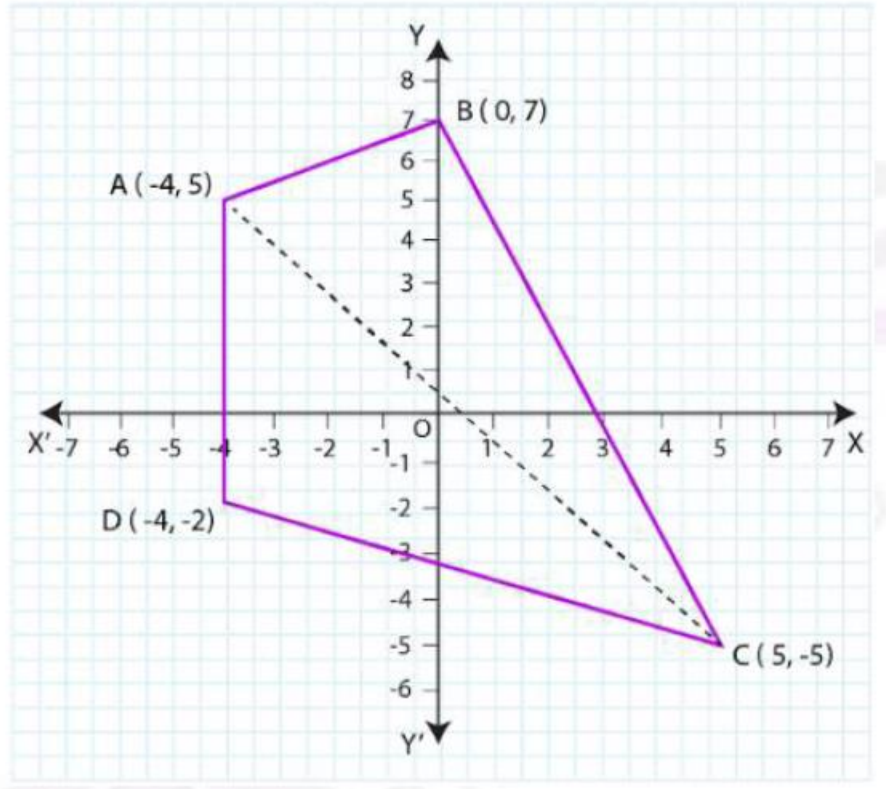
EXERCISE 10.1

1. Draw a quadrilateral in the Cartesian plane whose vertices are (– 4, 5), (0, 7), (5, – 5) and (– 4, –2). Also, find its area.

Solution:

Let ABCD be the given quadrilateral with vertices A (-4,5), B (0,7), C (5.-5) and D (-4,-2).

Now, let us plot the points on the Cartesian plane by joining the points AB, BC, CD, and AD, which give us the required quadrilateral.



To find the area, draw diagonal AC.

So, area (ABCD) = area (ΔABC) + area (ΔADC)

Then, area of triangle with vertices (x_1,y_1) , (x_2, y_2) and (x_3,y_3) is

Are of Δ ABC = ½ $[x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)]$

= ½ [-4 (7 + 5) + 0 (-5 – 5) + 5 (5 – 7)] unit^2

= ½ [-4 (12) + 5 (-2)] unit^2

= ½ (58) unit^2

= 29 unit^2

Are of Δ ACD = ½ $[x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)]$

= ½ [-4 (-5 + 2) + 5 (-2 – 5) + (-4) (5 – (-5))] unit^2

= ½ [-4 (-3) + 5 (-7) – 4 (10)] unit^2

= ½ (-63) unit^2

= -63/2 unit^2

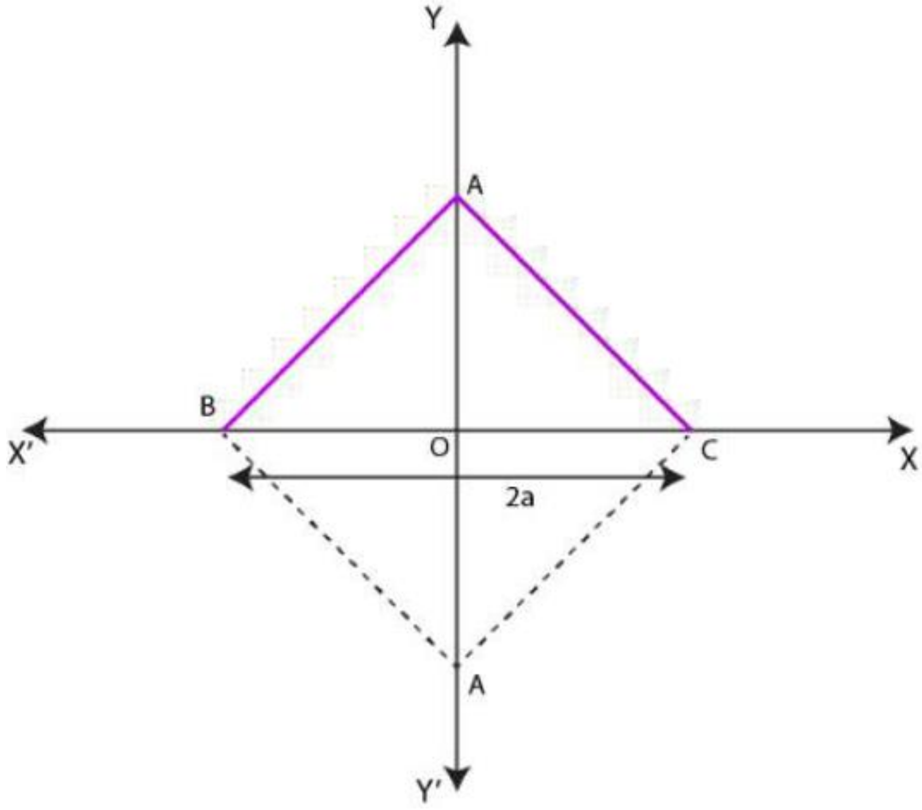
Since area cannot be negative, area Δ ACD = 63/2 unit^2

Area (ABCD) = 29 + 63/2

= 121/2 unit^2

2. The base of an equilateral triangle with side 2a lies along the y-axis such that the mid-point of the base is at the origin. Find the vertices of the triangle.

Solution:



Let us consider ABC, the given equilateral triangle with side 2a.

Where, AB = BC = AC = 2a

In the above figure, assuming that the base BC lies on the x-axis such that the mid-point of BC is at the origin, i.e., BO = OC = a, where O is the origin.

The coordinates of point C are (0, a) and that of B are (0,-a).

The line joining a vertex of an equilateral Δ with the mid-point of its opposite side is perpendicular.

So, vertex A lies on the y –axis.

By applying Pythagoras' theorem,

$(AC)^2 = OA^2 + OC^2$

$(2a)^2 = a^2 + OC^2$

$4a^2 - a^2 = OC^2$

$3a^2 = OC^2$

$OC = \sqrt{3}a$

Co-ordinates of point C = $\pm \sqrt{3}a, 0$

∴ The vertices of the given equilateral triangle are (0, a), (0, -a), ($\sqrt{3}$a, 0)

Or (0, a), (0, -a) and (-$\sqrt{3}$a, 0)

3. Find the distance between P (x_1, y_1) and Q (x_2, y_2) when: (i) PQ is parallel to the y-axis, (ii) PQ is parallel to the x-axis.

Solution:

Given:

Points P (x_1, y_1) and Q(x_2, y_2)

(i) When PQ is parallel to the y-axis, then $x_1 = x_2$

So, the distance between P and Q is given by

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(y_2 - y_1)^2}$

$= |y_2 - y_1|$

(ii) When PQ is parallel to the x-axis, then $y_1 = y_2$

So, the distance between P and Q is given by =

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x_2 - x_1)^2}$

$= |x_2 - x_1|$

4. Find a point on the x-axis which is equidistant from points (7, 6) and (3, 4).

Solution:

Let us consider (a, 0) to be the point on the x-axis that is equidistant from the point (7, 6) and (3, 4).

So,

$$\sqrt{(7-a)^2 + (6-0)^2} = \sqrt{(3-a)^2 + (4-0)^2}$$

$$\sqrt{49 + a^2 - 14a + 36} = \sqrt{9 + a^2 - 6a + 16}$$

$$\sqrt{a^2 - 14a + 85} = \sqrt{a^2 - 6a + 25}$$

Now, let us square on both sides; we get,

$a^2 - 14a + 85 = a^2 - 6a + 25$

$-8a = -60$

$a = 60/8$

$= 15/2$

∴ The required point is (15/2, 0)

5. Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points P (0, – 4) and B (8, 0).

Solution:

The co-ordinates of the mid-point of the line segment joining the points P (0, – 4) and B (8, 0) are (0+8)/2, (-4+0)/2 = (4, -2)

The slope 'm' of the line non-vertical line passing through the point (x_1, y_1) and

(x_2, y_2) is given by $m = (y_2 - y_1)/(x_2 - x_1)$ where, $x \neq x_1$

The slope of the line passing through (0, 0) and (4, -2) is (-2-0)/(4-0) = -1/2

∴ The required slope is -1/2.

6. Without using Pythagoras' theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right-angled triangle.

Solution:

The vertices of the given triangle are (4, 4), (3, 5) and (–1, –1).

The slope (m) of the line non-vertical line passing through the point (x_1, y_1) and

(x_2, y_2) is given by $m = (y_2 - y_1)/(x_2 - x_1)$ where, $x \neq x_1$

So, the slope of the line AB $(m_1) = (5-4)/(3-4) = 1/-1 = -1$

The slope of the line BC $(m_2) = (-1-5)/(-1-3) = -6/-4 = 3/2$

The slope of the line CA $(m_3) = (4+1)/(4+1) = 5/5 = 1$

It is observed that $m_1.m_3 = -1.1 = -1$

Hence, the lines AB and CA are perpendicular to each other.

∴ given triangle is right-angled at A (4, 4)

And the vertices of the right-angled Δ are (4, 4), (3, 5) and (-1, -1)

7. Find the slope of the line, which makes an angle of 30° with the positive direction of the y-axis measured anticlockwise.

Solution:

We know that if a line makes an angle of 30° with the positive direction of the y-axis measured anti-clock-wise, then the angle made by the line with the positive direction of the x-axis measured anti-clock-wise is 90° + 30° = 120°

∴ The slope of the given line is tan 120° = tan (180° – 60°)

= – tan 60°

= $-\sqrt{3}$

8. Find the value of x for which the points (x, – 1), (2, 1) and (4, 5) are collinear.

Solution:

If the points (x, – 1), (2, 1) and (4, 5) are collinear, then the Slope of AB = Slope of BC

Then, (1+1)/(2-x) = (5-1)/(4-2)

2/(2-x) = 4/2

2/(2-x) = 2

2 = 2(2-x)

2 = 4 – 2x

2x = 4 – 2

2x = 2

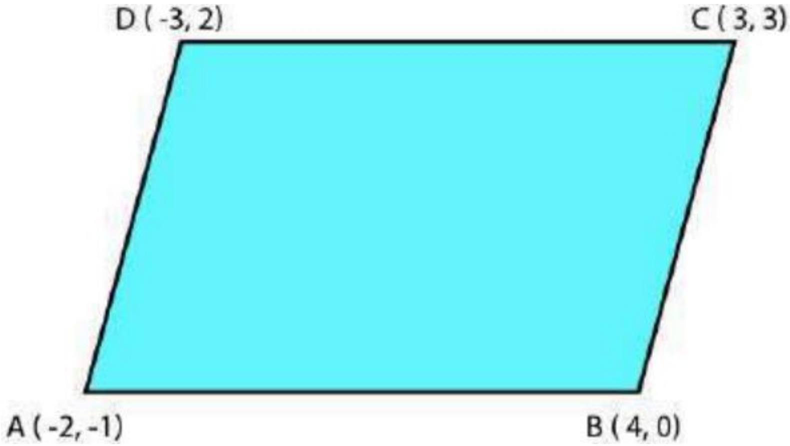
x = 2/2

= 1

∴ The required value of x is 1

9. Without using the distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (–3, 2) are the vertices of a parallelogram.

Solution:



Let the given point be A (-2, -1) , B (4, 0) , C (3, 3) and D (-3, 2)

So now, the slope of AB = (0+1)/(4+2) = 1/6

The slope of CD = (3-2)/(3+3) = 1/6

Hence, the Slope of AB = Slope of CD

∴ AB ∥ CD

Now,

The slope of BC = (3-0)/(3-4) = 3/-1 = -3

The slope of AD = (2+1)/(-3+2) = 3/-1 = -3

Hence, the Slope of BC = Slope of AD

∴ BC ∥ AD

Thus, the pair of opposite sides are quadrilateral are parallel, so we can say that ABCD is a parallelogram.

Hence, the given vertices, A (-2, -1), B (4, 0), C(3, 3) and D(-3, 2) are vertices of a parallelogram.

10. Find the angle between the x-axis and the line joining the points (3, –1) and (4, –2).

Solution:

The Slope of the line joining the points (3, -1) and (4, -2) is given by

$m = (y_2 - y_1)/(x_2 - x_1)$ where, $x \neq x_1$

m = (-2 –(-1))/(4-3)

= (-2+1)/(4-3)

= -1/1

= -1

The angle of inclination of the line joining the points (3, -1) and (4, -2) is given by

tan θ = -1

$\theta = (90° + 45°) = 135°$

∴ The angle between the x-axis and the line joining the points (3, –1) and (4, –2) is 135°.

11. The slope of a line is double the slope of another line. If the tangent of the angle between them is 1/3, find the slopes of the lines.

Solution:

Let us consider 'm_1' and 'm' be the slope of the two given lines such that $m_1 = 2m$

We know that if θ is the angle between the lines l1 and l2 with slope m_1 and m_2, then

$$\tan\theta = \left|\frac{(m_2 - m_1}{(1 + m_1 m_2)}\right|$$

Given here that the tangent of the angle between the two lines is 1/3

So,

$$\frac{1}{3} = \left|\frac{m-2m}{1+2m \times m}\right| = \left|\frac{-m}{1+2m^2}\right|$$

$$\frac{1}{3} = \frac{m}{1+2m^2}$$

Now, case 1:

$$\frac{1}{3} = \frac{-m}{1+2m^2}$$

$1+2m^2 = -3m$

$2m^2 +1 +3m = 0$

$2m (m+1) + 1(m+1) = 0$

$(2m+1) (m+1)= 0$

$m = -1$ or $-1/2$

If $m = -1$, then the slope of the lines are -1 and -2

If $m = -1/2$, then the slope of the lines are $-1/2$ and -1

Case 2:

$\frac{1}{3} = \frac{-m}{1+2m^2}$

$2m^2 – 3m + 1 = 0$

$2m^2 – 2m – m + 1 = 0$

$2m (m – 1) – 1(m – 1) = 0$

m = 1 or 1/2

If m = 1, then the slope of the lines are 1 and 2

If m = 1/2, then the slope of the lines are 1/2 and 1

∴ The slope of the lines are [-1 and -2] or [-1/2 and -1] or [1 and 2] or [1/2 and 1]

12. A line passes through (x_1, y_1) and (h, k). If the slope of the line is m, show that $k – y_1 = m (h – x_1)$.

Solution:

Given: the slope of the line is 'm'.

The slope of the line passing through (x_1, y_1) and (h, k) is $(k – y_1)/(h – x_1)$

So,

$(k – y_1)/(h – x_1) = m$

$(k – y_1) = m (h – x_1)$

Hence, proved.

13. If three points (h, 0), (a, b) and (0, k) lie on a line, show that a/h + b/k = 1

Solution:

Let us consider if the given points A (h, 0), B (a, b) and C (0, k) lie on a line.

Then, the slope of AB = slope of BC

(b – 0)/(a – h) = (k – b)/(0 – a)

By simplifying, we get

-ab = (k-b) (a-h)

-ab = ka- kh –ab +bh

ka +bh = kh

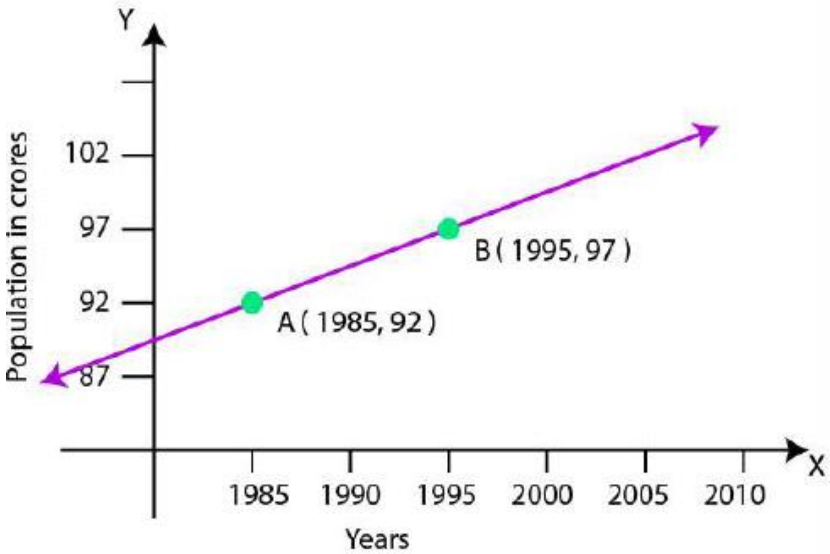
Divide both sides by kh; we get

ka/kh + bh/kh = kh/kh

a/h + b/k = 1

Hence, proved.

14. Consider the following population and year graph (Fig 10.10), find the slope of the line AB and using it, find what will be the population in the year 2010?



Solution:

We know that line AB passes through points A (1985, 92) and B (1995, 97).

Its slope will be (97 – 92)/(1995 – 1985) = 5/10 = 1/2

Let ‘y’ be the population in the year 2010. Then, according to the given graph, AB must pass through point C (2010, y)

So now, slope of AB = slope of BC

$$\frac{1}{2} = \frac{y - 97}{2010 - 1995}$$

15/2 = y – 97

y = 7.5 + 97 = 104.5

∴ The slope of line AB is 1/2, while in the year 2010, the population will be 104.5 crores.

EXERCISE 10.2

In Exercises 1 to 8, find the equation of the line which satisfies the given conditions.

1. Write the equations for the x-and y-axes.

Solution:

The y-coordinate of every point on the x-axis is 0.

∴ The equation of the x-axis is y = 0.

The x-coordinate of every point on the y-axis is 0.

∴ The equation of the y-axis is y = 0.

2. Passing through the point (– 4, 3) with slope 1/2

Solution:

Given:

Point (-4, 3) and slope, m = 1/2

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y – y_0 = m (x – x_0)$

So, y – 3 = 1/2 (x – (-4))

y – 3 = 1/2 (x + 4)

2(y – 3) = x + 4

2y – 6 = x + 4

$x + 4 – (2y – 6) = 0$

$x + 4 – 2y + 6 = 0$

$x – 2y + 10 = 0$

∴ The equation of the line is $x – 2y + 10 = 0$

3. Passing through (0, 0) with slope m.

Solution:

Given:

Point (0, 0) and slope, m = m

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y – y_0 = m (x – x_0)$

So, $y – 0 = m (x – 0)$

$y = mx$

$y – mx = 0$

∴ The equation of the line is $y – mx = 0$

4. Passing through (2, 2√3) and inclined with the x-axis at an angle of 75º.

Solution:

Given: point (2, 2√3) and θ = 75°

Equation of line: $(y – y_1) = m (x – x_1)$

where, m = slope of line = tan θ and (x_1, y_1) are the points through which line passes

∴ m = tan 75°

75° = 45° + 30°

Applying the formula:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

$$\tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ . \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$\tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Let us rationalizing we get,

$$\tan 75^\circ = \frac{3 + 1 + 2\sqrt{3}}{3 - 1} = 2 + \sqrt{3}$$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_1, y1), only if its coordinates satisfy the equation $y – y_1 = m (x – x_1)$

Then, y – 2√3 = (2 + √3) (x – 2)

y – 2√3 = 2 x – 4 + √3 x – 2 √3

y = 2 x – 4 + √3 x

(2 + √3) x – y – 4 = 0

∴ The equation of the line is (2 + √3) x – y – 4 = 0

5. Intersecting the x-axis at a distance of 3 units to the left of origin with slope –2.

Solution:

Given:

Slope, m = -2

We know that if a line L with slope m makes x-intercept d, then the equation of L is

y = m(x − d).

If the distance is 3 units to the left of the origin, then d = -3

So, y = (-2) (x – (-3))

y = (-2) (x + 3)

$y = -2x – 6$

$2x + y + 6 = 0$

∴ The equation of the line is $2x + y + 6 = 0$

6. Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30º with the positive direction of the x-axis.

Solution:

Given: $\theta = 30°$

We know that slope, $m = \tan \theta$

$m = \tan30° = (1/\sqrt{3})$

We know that the point (x, y) on the line with slope m and y-intercept c lies on the line only if $y = mx + c$

If the distance is 2 units above the origin, $c = +2$

So, $y = (1/\sqrt{3})x + 2$

$y = (x + 2\sqrt{3}) / \sqrt{3}$

$\sqrt{3}\ y = x + 2\sqrt{3}$

$x – \sqrt{3}\ y + 2\sqrt{3} = 0$

∴ The equation of the line is $x – \sqrt{3}\ y + 2\sqrt{3} = 0$

7. Passing through the points (–1, 1) and (2, – 4).

Solution:

Given:

Points (-1, 1) and (2, -4)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{-4 - 1}{2 - (-1)}(x - (-1))$$

y – 1 = -5/3 (x + 1)

3 (y – 1) = (-5) (x + 1)

3y – 3 = -5x – 5

3y – 3 + 5x + 5 = 0

5x + 3y + 2 = 0

∴ The equation of the line is 5x + 3y + 2 = 0

8. The vertices of ΔPQR are P (2, 1), Q (–2, 3) and R (4, 5). Find the equation of the median through the vertex R.

Solution:

Given:

Vertices of ΔPQR, i.e., P (2, 1), Q (-2, 3) and R (4, 5)

Let RL be the median of vertex R.

So, L is a midpoint of PQ.

We know that the midpoint formula is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

∴ L = $\left(\frac{2-2}{2}, \frac{1+3}{2}\right)$ = (0, 2)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\therefore y - 5 = \frac{2 - 5}{0 - 4}(x - 4)$$

y – 5 = -3/-4 (x-4)

$(-4)(y - 5) = (-3)(x - 4)$

$-4y + 20 = -3x + 12$

$-4y + 20 + 3x - 12 = 0$

$3x - 4y + 8 = 0$

∴ The equation of median through the vertex R is $3x - 4y + 8 = 0$

9. Find the equation of the line passing through (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).

Solution:

Given:

Points are (2, 5) and (-3, 6).

We know that slope, $m = (y_2 - y_1)/(x_2 - x_1)$

$= (6 - 5)/(-3 - 2)$

$= 1/-5 = -1/5$

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

Then, $m = (-1/m)$

$= -1/(-1/5)$

$= 5$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), only if its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

Then, $y - 5 = 5(x - (-3))$

$y - 5 = 5x + 15$

$5x + 15 - y + 5 = 0$

$5x - y + 20 = 0$

∴ The equation of the line is 5x – y + 20 = 0

10. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1: n. Find the equation of the line.

Solution:

We know that the coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\left(\frac{1(2)+n(1)}{1+n}, \frac{1(3)+n(0)}{1+n}\right) = \left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

We know that slope, $m = (y_2 - y_1)/(x_2 - x_1)$

= (3 – 0)/(2 – 1)

= 3/1

= 3

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

Then, m = (-1/m) = -1/3

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), only if its coordinates satisfy the equation $y - y_0 = m (x - x_0)$

Here, the point is

$$\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

$$\left(y - \frac{3}{1+n}\right) = \frac{-1}{3}\left(x - \frac{2+n}{1+n}\right)$$

3((1 + n) y – 3) = (-(1 + n) x + 2 + n)

3(1 + n) y – 9 = – (1 + n) x + 2 + n

(1 + n) x + 3(1 + n) y – n – 9 – 2 = 0

(1 + n) x + 3(1 + n) y – n – 11 = 0

∴ The equation of the line is $(1 + n) x + 3(1 + n) y - n - 11 = 0$

11. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Solution:

Given: the line cuts off equal intercepts on the coordinate axes, i.e., $a = b$

We know that equation of the line intercepts a and b on the x-and the y-axis, respectively, which is

$x/a + y/b = 1$

So, $x/a + y/a = 1$

$x + y = a$ … (1)

Given: point (2, 3)

$2 + 3 = a$

$a = 5$

Substitute value of 'a' in (1), we get

$x + y = 5$

$x + y - 5 = 0$

∴ the equation of the line is $x + y - 5 = 0$

12. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.

Solution:

We know that equation of the line-making intercepts a and b on the x-and the y-axis, respectively, is $x/a + y/b = 1$. … (1)

Given: sum of intercepts = 9

$a + b = 9$

$b = 9 - a$

Now, substitute the value of b in the above equation, and we get

$x/a + y/(9 - a) = 1$

Given: the line passes through point (2, 2)

So, $2/a + 2/(9 - a) = 1$

$[2(9 - a) + 2a] / a(9 - a) = 1$ $[18 - 2a + 2a] / a(9 - a) = 1$
$18/a(9 - a) = 1$

$18 = a (9 - a)$

$18 = 9a - a^2$

$a^2 - 9a + 18 = 0$

Upon factorizing, we get

$a^2 - 3a - 6a + 18 = 0$

$a (a - 3) - 6 (a - 3) = 0$

$(a - 3) (a - 6) = 0$

$a = 3$ or $a = 6$

Let us substitute in (1)

Case 1 ($a = 3$):

Then $b = 9 - 3 = 6$

$x/3 + y/6 = 1$

$2x + y = 6$

$2x + y - 6 = 0$

Case 2 ($a = 6$):

Then $b = 9 - 6 = 3$

$x/6 + y/3 = 1$

$x + 2y = 6$

$x + 2y - 6 = 0$

∴ The equation of the line is $2x + y - 6 = 0$ or $x + 2y - 6 = 0$

13. Find the equation of the line through the point (0, 2), making an angle 2π/3 with the positive x-axis. Also, find the equation of the line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Solution:

Given:

Point (0, 2) and $\theta = 2\pi/3$

We know that $m = \tan \theta$

$m = \tan (2\pi/3) = -\sqrt{3}$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), only if its coordinates satisfy the equation $y - y_0 = m (x - x_0)$

$y - 2 = -\sqrt{3} (x - 0)$

$y - 2 = -\sqrt{3} x$

$\sqrt{3} x + y - 2 = 0$

Given, the equation of the line parallel to the above-obtained equation crosses the y-axis at a distance of 2 units below the origin.

So, the point = (0, -2) and $m = -\sqrt{3}$

From point slope form equation,

$y - (-2) = -\sqrt{3} (x - 0)$

$y + 2 = -\sqrt{3} x$

$\sqrt{3} x + y + 2 = 0$

∴ The equation of the line is $\sqrt{3} x + y - 2 = 0$, and the line parallel to it is $\sqrt{3} x + y + 2 = 0$

14. The perpendicular from the origin to a line meets it at the point (–2, 9). Find the equation of the line.

Solution:

Given:

Points are origin (0, 0) and (-2, 9).

We know that slope, $m = (y_2 – y_1)/(x_2 – x_1)$

$= (9 – 0)/(-2-0)$

$= -9/2$

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

$m = (-1/m) = -1/(-9/2) = 2/9$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y – y_0 = m (x – x_0)$

$y – 9 = (2/9) (x – (-2))$

$9(y – 9) = 2(x + 2)$

$9y – 81 = 2x + 4$

$2x + 4 – 9y + 81 = 0$

$2x – 9y + 85 = 0$

∴ The equation of the line is $2x – 9y + 85 = 0$

15. The length L (in centimeters) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L= 125.134 when C = 110, express L in terms of C.

Solution:

Let us assume 'L' along X-axis and 'C' along Y-axis; we have two points (124.942, 20) and (125.134, 110) in XY-plane.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$C - 20 = \frac{110 - 20}{125.134 - 124.942}(L - 124.942)$$

$$C - 20 = \frac{90}{0.192}(L - 124.942)$$

$$0.192\,(C - 20) = 90\,(L - 124.942)$$

$$L = \frac{0.192}{90}(C - 20) + 124.942$$

$\therefore$ The required relation is $L = \frac{0.192}{90}(C - 20) + 124.942$

16. The owner of a milk store finds that he can sell 980 liters of milk each week at Rs. 14/liter and 1220 liters of milk each week at Rs. 16/litre. Assuming a linear relationship between the selling price and demand, how many liters could he sell weekly at Rs. 17/liter?

Solution:

Assuming the relationship between the selling price and demand is linear.

Let us assume the selling price per litre along X-axis and demand along Y-axis, we have two points (14, 980) and (16, 1220) in XY-plane.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$$y - 980 = \frac{240}{2}(x - 14)$$

$y - 980 = 120\,(x - 14)$

$y = 120\,(x - 14) + 980$

When x = Rs 17/litre,

$y = 120\,(17 - 14) + 980$

y = 120(3) + 980

y = 360 + 980 = 1340

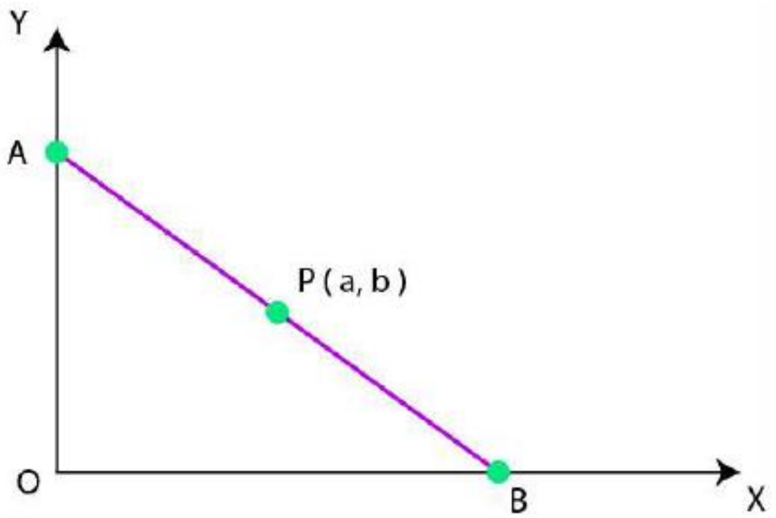
∴ The owner can sell 1340 litres weekly at Rs. 17/litre.

17. P (a, b) is the mid-point of a line segment between axes. Show that the equation of the line is x/a + y/b = 2

Solution:

Let AB be a line segment whose midpoint is P (a, b).

Let the coordinates of A and B be (0, y) and (x, 0), respectively.



We know that the midpoint is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Since P is the midpoint of (a, b),

$\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = (a, b)$

$\left(\frac{x}{2}, \frac{y}{2}\right) = (a, b)$

a = x/2 and b = y/2

x = 2a and y = 2b

A = (0, 2b) and B = (2a, 0)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$

$y - 2b = \frac{0-2b}{2a-0}(x - 0)$

$y - 2b = \frac{-2b}{2a}(x)$

$y - 2b = \frac{-b}{a}(x)$

a (y – 2b) = -bx

ay – 2ab = -bx

bx + ay = 2ab

Divide both sides with ab, then

$$\frac{bx}{ab} + \frac{ay}{ab} = \frac{2ab}{ab}$$
$$\frac{x}{a} + \frac{y}{b} = 2$$

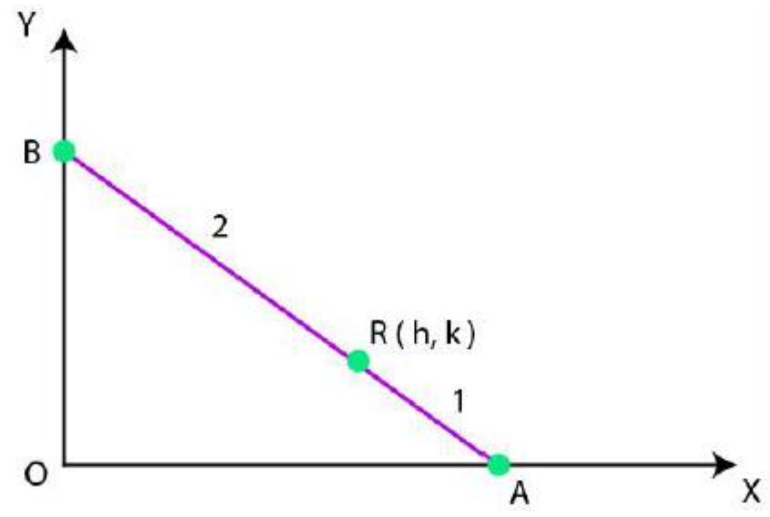
Hence, proved.

18. Point R (h, k) divides a line segment between the axes in the ratio 1: 2. Find the equation of the line.

Solution:

Let us consider AB to be the line segment, such that r (h, k) divides it in the ratio 1: 2.

So, the coordinates of A and B be (0, y) and (x, 0), respectively.



We know that the coordinates of a point dividing the line segment join the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n is

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$
$$\left(\frac{1(0)+2(x)}{1+2}, \frac{1(y)+2(0)}{1+2}\right) = (h, k)$$
$$\left(\frac{2x}{3}, \frac{y}{3}\right) = (h, k)$$

h = 2x/3 and k = y/3

x = 3h/2 and y = 3k

∴ A = (0, 3k) and B = (3h/2, 0)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 3k = \frac{0 - 3k}{\frac{3h}{2} - 0}(x - 0)$$

3h(y – 3k) = -6kx

3hy – 9hk = -6kx

6kx + 3hy = 9hk

Let us divide both sides by 9hk, and we get,

2x/3h + y/3k = 1

∴ The equation of the line is given by 2x/3h + y/3k = 1

19. By using the concept of the equation of a line, prove that the three points (3, 0), (– 2, – 2) and (8, 2) are collinear.

Solution:

According to the question,

If we have to prove that the given three points (3, 0), (– 2, – 2) and (8, 2) are collinear, then we have to also prove that the line passing through the points (3, 0) and (– 2, – 2) also passes through the point (8, 2).

By using the formula,

The equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 0 = \frac{-2 - 0}{-2 - 3}(x - 3)$$
$$y = \frac{-2}{-5}(x - 3)$$

-5y = -2 (x – 3)

-5y = -2x + 6

2x – 5y = 6

If 2x – 5y = 6 passes through (8, 2),

2x – 5y = 2(8) – 5(2)

= 16 – 10

= 6

= RHS

The line passing through points (3, 0) and (– 2, – 2) also passes through the point (8, 2).

Hence, proved. The given three points are collinear.

EXERCISE 10.3

1. Reduce the following equations into slope-intercept form and find their slopes and the y-intercepts.
(i) x + 7y = 0

(ii) 6x + 3y – 5 = 0
(iii) y = 0

Solution:

(i) x + 7y = 0

Given:

The equation is x + 7y = 0

The slope-intercept form is represented in the form ‘$y = mx + c$’, where m is the slope and c is the y-intercept.

So, the above equation can be expressed as

$y = -1/7x + 0$

∴ The above equation is of the form $y = mx + c$, where $m = -1/7$ and $c = 0$

(ii) $6x + 3y – 5 = 0$

Given:

The equation is $6x + 3y – 5 = 0$

The slope-intercept form is represented in the form ‘$y = mx + c$’, where m is the slope and c is the y-intercept.

So, the above equation can be expressed as

$3y = -6x + 5$

$y = -6/3x + 5/3$

$= -2x + 5/3$

∴ The above equation is of the form $y = mx + c$, where $m = -2$ and $c = 5/3$

(iii) $y = 0$

Given:

The equation is $y = 0$

The slope-intercept form is given by ‘$y = mx + c$’, where m is the slope and c is the y-intercept.

$y = 0 \times x + 0$

∴ The above equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$

2. Reduce the following equations into intercept form and find their intercepts on the axes.

(i) $3x + 2y - 12 = 0$

(ii) $4x - 3y = 6$

(iii) $3y + 2 = 0$

Solution:

(i) $3x + 2y - 12 = 0$

Given:

The equation is $3x + 2y - 12 = 0$

The equation of the line in intercept form is given by $x/a + y/b = 1$, where 'a' and 'b' are intercepted on the x-axis and the y-axis, respectively.

So, $3x + 2y = 12$

Now, let us divide both sides by 12; we get

$3x/12 + 2y/12 = 12/12$

$x/4 + y/6 = 1$

∴ The above equation is of the form $x/a + y/b = 1$, where $a = 4$, $b = 6$

The intercept on the x-axis is 4.

The intercept on the y-axis is 6.

(ii) $4x - 3y = 6$

Given:

The equation is $4x - 3y = 6$

The equation of the line in intercept form is given by $x/a + y/b = 1$, where 'a' and 'b' are intercepted on the x-axis and the y-axis, respectively.

So, $4x - 3y = 6$

Now, let us divide both sides by 6; we get

4x/6 – 3y/6 = 6/6

2x/3 – y/2 = 1

x/(3/2) + y/(-2) = 1

∴ The above equation is of the form x/a + y/b = 1, where a = 3/2, b = -2

The intercept on the x-axis is 3/2.

The intercept on the y-axis is -2.

(iii) 3y + 2 = 0

Given:

The equation is 3y + 2 = 0

The equation of the line in intercept form is given by x/a + y/b = 1, where 'a' and 'b' are intercepted on the x-axis and the y-axis, respectively.

So, 3y = -2

Now, let us divide both sides by -2; we get

3y/-2 = -2/-2

3y/-2 = 1

y/(-2/3) = 1

∴ The above equation is of the form x/a + y/b = 1, where a = 0, b = -2/3

The intercept on the x-axis is 0.

The intercept on the y-axis is -2/3.

3. Find the distance of the point (–1, 1) from the line 12(x + 6) = 5(y – 2).

Solution:

Given:

The equation of the line is 12(x + 6) = 5(y – 2).

12x + 72 = 5y – 10

12x – 5y + 82 = 0 … (1)

Now, compare equation (1) with the general equation of line Ax + By + C = 0, where A = 12, B = –5, and C = 82

Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Given point (x_1, y_1) = (-1, 1)

∴ Distance of point (–1, 1) from the given line is

$$d = \frac{|12 \times (-1) + (-5) \times 1 + 82|}{\sqrt{12^2 + (-5)^2}} = \frac{|-12 - 5 + 82|}{\sqrt{144 + 25}} = \frac{|65|}{\sqrt{169}} = \frac{65}{13} \text{units}$$

$= 5$ units

∴ The distance is 5 units.

4. Find the points on the x-axis whose distances from the line x/3 + y/4 = 1 are 4 units.

Solution:

Given:

The equation of the line is x/3 + y/4 = 1

4x + 3y = 12

4x + 3y – 12 = 0 …. (1)

Now, compare equation (1) with the general equation of line Ax + By + C = 0, where A = 4, B = 3, and C = -12

Let (a, 0) be the point on the x-axis whose distance from the given line is 4 units.

So, the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$4 = \frac{|4a - 12|}{\sqrt{16 + 9}} = \frac{|4a - 12|}{5}$$

$|4a – 12| = 4 \times 5$

$\pm (4a – 12) = 20$

$4a – 12 = 20$ or $– (4a – 12) = 20$

$4a = 20 + 12$ or $4a = -20 + 12$

$a = 32/4$ or $a = -8/4$

$a = 8$ or $a = -2$

∴ The required points on the x-axis are (-2, 0) and (8, 0)

5. Find the distance between parallel lines.
(i) 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0

(ii) l(x + y) + p = 0 and l (x + y) – r = 0

Solution:

(i) 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0

Given:

The parallel lines are 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0.

By using the formula,

The distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Where, $A = 15$, $B = 8$, $C_1 = -34$, $C_2 = 31$

Distance between parallel lines is

$$d = \frac{|-34 - 31|}{\sqrt{15^2 + 8^2}}$$

$$= \frac{|-65|}{\sqrt{225 + 64}}$$

$$= \frac{65}{\sqrt{289}}$$

$= 65/17$

∴ The distance between parallel lines is 65/17

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$

Given:

The parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$

$lx + ly + p = 0$ and $lx + ly - r = 0$

By using the formula,

The distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Where, $A = l$, $B = l$, $C_1 = p$, $C_2 = -r$

Distance between parallel lines is

$$d = \frac{|p - (-r)|}{\sqrt{l^2 + l^2}}$$

$$= \frac{|p + r|}{\sqrt{2}l}$$

$$= \frac{|p+r|}{l\sqrt{2}}$$

∴ The distance between parallel lines is $|p+r|/l\sqrt{2}$

6. Find the equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point (–2, 3).

Solution:

Given:

The line is $3x - 4y + 2 = 0$

So, $y = 3x/4 + 2/4$

$= 3x/4 + ½$

Which is of the form $y = mx + c$, where m is the slope of the given line.

The slope of the given line is 3/4

We know that parallel lines have the same slope.

∴ Slope of other line = m = 3/4

The equation of line having slope m and passing through (x_1, y_1) is given by

$y - y_1 = m (x - x_1)$

∴ The equation of the line having slope 3/4 and passing through (-2, 3) is

$y - 3 = ¾ (x - (-2))$

$4y - 3 \times 4 = 3x + 3 \times 2$

$3x - 4y = 18$

∴ The equation is $3x - 4y = 18$

7. Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

Solution:

Given:

The equation of line is $x - 7y + 5 = 0$

So, y = 1/7x + 5/7 [which is of the form y = mx + c, where m is the slope of the given line.]

The slope of the given line is 1/7

The slope of the line perpendicular to the line having slope m is -1/m

The slope of the line perpendicular to the line having a slope of 1/7 is -1/(1/7) = -7

So, the equation of the line with slope -7 and the x-intercept 3 is given by y = m(x – d)

y = -7 (x – 3)

y = -7x + 21

7x + y = 21

∴ The equation is 7x + y = 21

8. Find angles between the lines √3x + y = 1 and x + √3y = 1.

Solution:

Given:

The lines are √3x + y = 1 and x + √3y = 1

So, y = -√3x + 1 … (1) and

y = -1/√3x + 1/√3 …. (2)

The slope of the line (1) is m_1 = -√3, while the slope of the line (2) is m_2 = -1/√3

Let θ be the angle between two lines.

So,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\sqrt{3} - \left(-\frac{1}{\sqrt{3}}\right)}{1 + (-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)}\right| = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2 \times \sqrt{3}}\right|$$

$$= 1/\sqrt{3}$$

$\theta = 30°$

∴ The angle between the given lines is either 30° or 180°- 30° = 150°

9. The line through the points (h, 3) and (4, 1) intersects the line 7x − 9y −19 = 0. At the right angle. Find the value of h.

Solution:

Let the slope of the line passing through (h, 3) and (4, 1) be m_1

Then, $m_1 = (1-3)/(4-h) = -2/(4-h)$

Let the slope of line $7x – 9y – 19 = 0$ be m_2

$7x – 9y – 19 = 0$

So, $y = 7/9x – 19/9$

$m_2 = 7/9$

Since the given lines are perpendicular,

$m_1 \times m_2 = -1$

$-2/(4-h) \times 7/9 = -1$

$-14/(36-9h) = -1$

$-14 = -1 \times (36 – 9h)$

$36 – 9h = 14$

$9h = 36 – 14$

$h = 22/9$

∴ The value of h is 22/9

10. Prove that the line through the point (x_1, y_1) and parallel to the line Ax + By + C = 0 is A (x – x_1) + B (y – y_1) = 0.

Solution:

Let the slope of line $Ax + By + C = 0$ be m

$Ax + By + C = 0$

So, $y = -A/Bx – C/B$

$m = -A/B$

By using the formula,

Equation of the line passing through point (x_1, y_1) and having slope $m = -A/B$ is

$y – y_1 = m (x – x_1)$

$y – y_1= -A/B (x – x_1)$

$B (y – y_1) = -A (x – x_1)$

$\therefore A(x – x_1) + B(y – y_1) = 0$

So, the line through point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A (x – x_1) + B (y – y_1) = 0$

Hence, proved.

11. Two lines passing through point (2, 3) intersects each other at an angle of 60^o. If the slope of one line is 2, find the equation of the other line.

Solution:

Given: $m_1 = 2$

Let the slope of the first line be m_1

And let the slope of the other line be m_2.

The angle between the two lines is 60°.

So,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan 60° = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$

$$\sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

i.e.,

$$\sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\sqrt{3}\,(1 + 2m_2) = 2 - m_2 \text{ or } \sqrt{3}\,(1 + 2m_2) = -(2 - m_2)$$

$$\sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$

$$m_2(2\sqrt{3} + 1) = 2 - \sqrt{3} \text{ or } m_2(2\sqrt{3} - 1) = -(2 + \sqrt{3})$$

$$m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)} \text{ or } m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$$

So now let us consider

Case 1: When

$$m_2 = \frac{2-\sqrt{3}}{(2\sqrt{3}+1)}$$

The equation of the line passing through point (2, 3) and having a slope m_2 is

$$y - 3 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1}\right)(x - 2)$$

$$(2\sqrt{3} + 1)y - 3(2\sqrt{3} + 1) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$$

$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3$$

$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = 8\sqrt{3} - 1$$

∴ Equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = 8\sqrt{3} - 1$

Case 2: When

$$m_2 = \frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}$$

The equation of the line passing through point (2, 3) and having a slope m_2 is

$$y - 3 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}(x - 2)$$

$$y - 3 = \frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}(x-2)$$

$$(2\sqrt{3}-1)y - 3(2\sqrt{3}-1) = -(2+\sqrt{3})x + 2(2+\sqrt{3})$$

$$(2\sqrt{3}-1)y + (2+\sqrt{3})x = 4 + 2\sqrt{3} + 6\sqrt{3} - 3$$

$$(2\sqrt{3}-1)y + (2+\sqrt{3})x = 8\sqrt{3} + 1$$

∴ Equation of the other line is $(2\sqrt{3}-1)y + (2+\sqrt{3})x = 8\sqrt{3} + 1$

12. Find the equation of the right bisector of the line segment joining the points (3, 4) and (–1, 2).

Solution:

Given:

The right bisector of a line segment bisects the line segment at 90°.

End-points of the line segment AB are given as A (3, 4) and B (–1, 2).

Let the mid-point of AB be (x, y).

x = (3-1)/2= 2/2 = 1

y = (4+2)/2 = 6/2 = 3

(x, y) = (1, 3)

Let the slope of line AB be m_1

m_1 = (2 – 4)/(-1 – 3)

= -2/(-4)

= 1/2

And let the slope of the line perpendicular to AB be m_2

m_2 = -1/(1/2)

= -2

The equation of the line passing through (1, 3) and having a slope of –2 is

$(y – 3) = -2 (x – 1)$

$y – 3 = – 2x + 2$

$2x + y = 5$

∴ The required equation of the line is $2x + y = 5$

13. Find the coordinates of the foot of the perpendicular from the point (–1, 3) to the line 3x – 4y – 16 = 0.

Solution:

Let us consider the coordinates of the foot of the perpendicular from (-1, 3) to the line $3x – 4y – 16 = 0$ be (a, b)

So, let the slope of the line joining (-1, 3) and (a, b) be m_1

$m_1 = (b-3)/(a+1)$

And let the slope of the line $3x – 4y – 16 = 0$ be m_2

$y = 3/4x – 4$

$m_2 = 3/4$

Since these two lines are perpendicular, $m_1 \times m_2 = -1$

$(b-3)/(a+1) \times (3/4) = -1$

$(3b-9)/(4a+4) = -1$

$3b – 9 = -4a – 4$

$4a + 3b = 5$ …….(1)

Point (a, b) lies on the line $3x – 4y = 16$

$3a – 4b = 16$ ……..(2)

Solving equations (1) and (2), we get

$a = 68/25$ and $b = -49/25$

∴ The coordinates of the foot of perpendicular are (68/25, -49/25)

14. The perpendicular from the origin to the line y = mx + c meets it at the point (–1, 2). Find the values of m and c.

Solution:

Given:

The perpendicular from the origin meets the given line at (–1, 2).

The equation of the line is y = mx + c

The line joining the points (0, 0) and (–1, 2) is perpendicular to the given line.

So, the slope of the line joining (0, 0) and (–1, 2) = 2/(-1) = -2

The slope of the given line is m.

$m \times (-2) = -1$

$m = 1/2$

Since point (-1, 2) lies on the given line,

$y = mx + c$

$2 = 1/2 \times (-1) + c$

$c = 2 + 1/2 = 5/2$

∴ The values of m and c are 1/2 and 5/2, respectively.

15. If p and q are the lengths of perpendiculars from the origin to the lines x cos θ − y sin θ = k cos 2θ and x sec θ + y cosec θ = k, respectively, prove that $p^2 + 4q^2 = k^2$

Solution:

Given:

The equations of the given lines are

$x \cos\theta - y \sin\theta = k \cos 2\theta$ …………………… (1)

$x \sec\theta + y \operatorname{cosec}\theta = k$ ………………….. (2)

Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

So now, compare equation (1) to the general equation of line i.e., Ax + By + C = 0, we get

A = cos θ, B = -sin θ, and C = -k cos 2θ

It is given that p is the length of the perpendicular from (0, 0) to line (1).

$$p = \frac{|A\times 0+B\times 0+C|}{\sqrt{A^2+B^2}} = \frac{|C|}{\sqrt{A^2+B^2}} = \frac{|-k\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}} = k\cos 2\theta$$

p = k cos 2θ

Let us square on both sides we get,

$P^2 = k^2 \cos^2 2\theta$ ………………… (3)

Now, compare equation (2) to the general equation of line i.e., Ax + By + C = 0, we get

A = sec θ, B = cosec θ, and C = -k

It is given that q is the length of the perpendicular from (0, 0) to line (2)

$$q = \frac{|A \times 0 + B \times 0 + C|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|C|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|-k|}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}}$$

$$= \frac{k}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}}$$

$$= \frac{k}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}} = \frac{k\cos\theta\sin\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} = k\cos\theta\sin\theta$$

q = k cos θ sin θ

Multiply both sides by 2, and we get

2q = 2k cos θ sin θ = k × 2sin θ cos θ

2q = k sin 2θ

Squaring both sides, we get

$4q^2 = k^2 \sin^2 2\theta$ …………………(4)

Now add (3) and (4); we get

$p^2 + 4q^2 = k^2 \cos^2 2\theta + k^2 \sin^2 2\theta$

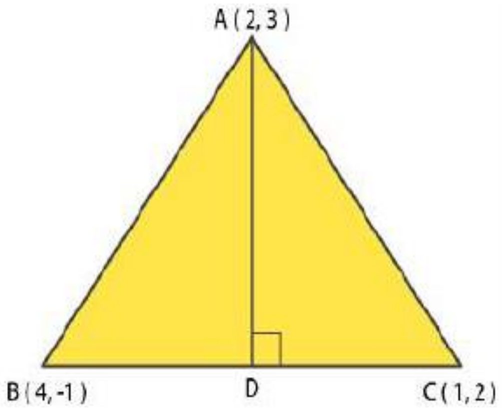
$p^2 + 4q^2 = k^2 (\cos^2 2\theta + \sin^2 2\theta)$ [Since, $\cos^2 2\theta + \sin^2 2\theta = 1$]

$\therefore p^2 + 4q^2 = k^2$

Hence proved.

16. In the triangle ABC with vertices A (2, 3), B (4, –1) and C (1, 2), find the equation and length of altitude from vertex A.

Solution:



Let AD be the altitude of triangle ABC from vertex A.

So, AD is perpendicular to BC.

Given:

Vertices A (2, 3), B (4, –1) and C (1, 2)

Let the slope of the line BC = m_1

$m_1 = (-1 - 2)/(4 - 1)$

$m_1 = -1$

Let the slope of the line AD be m_2

AD is perpendicular to BC.

$m_1 \times m_2 = -1$

$-1 \times m_2 = -1$

$m_2 = 1$

The equation of the line passing through the point (2, 3) and having a slope of 1 is

$y - 3 = 1 \times (x - 2)$

$y - 3 = x - 2$

$y - x = 1$

Equation of the altitude from vertex A = $y - x = 1$

Length of AD = Length of the perpendicular from A (2, 3) to BC

The equation of BC is

$y + 1 = -1 \times (x - 4)$

$y + 1 = -x + 4$

$x + y - 3 = 0$ …………………(1)

Perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Now, compare equation (1) to the general equation of the line, i.e., $Ax + By + C = 0$; we get

Length of AD = $\frac{|1\times2+1\times3-3|}{\sqrt{1^2+1^2}} = \frac{|2|}{\sqrt{2}} = \sqrt{2}$ units

[where, A = 1, B = 1 and C = -3]
∴ The equation and the length of the altitude from vertex A are $y - x = 1$ and

$\sqrt{2}$ units, respectively.

17. If p is the length of the perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $1/p^2 = 1/a^2 + 1/b^2$

Solution:

The equation of a line whose intercepts on the axes are a and b is x/a + y/b = 1

bx + ay = ab

bx + ay – ab = 0 ………………..(1)

Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Now compare equation (1) to the general equation of line i.e., Ax + By + C = 0, we get

A = b, B = a and C = -ab

If p is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (1), we get

$$p = \frac{|A \times 0 + B \times 0 - ab|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|-ab|}{\sqrt{a^2+b^2}}$$

Now, square on both sides; we get

$$p^2 = \frac{(-ab)^2}{a^2 + b^2}$$

$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2}$$

$$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$$

$$\therefore \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

$\therefore 1/p^2 = 1/a^2 + 1/b^2$

Hence, proved.

2Marks Questions & Answers

1. Find the slope of the lines passing through the point $(3,-2)$ and $(-1,4)$
Ans: Slope of the line = m

The given points are $(3,-2)$ and $(-1,4)$

Let (x_1, y_1) be $(3,-2)$ and (x_2,y_2) be $(-1,4)$

$m=\frac{y_2-y_1}{x_2-x_1}$

$\Rightarrow \frac{4-(-2)}{-1-3}$

$\Rightarrow \frac{6}{-4} = -\frac{3}{2}$

Therefore, the slope of the line passing through the point $(3,-2)$ and $(-1,4)$ is $-\frac{3}{2}$

2. Three points $P(h,k), Q(x_1,y_1)$ and $R(x_2,y_2)$ lie on a line. Show that

$(h-x_1)(y_2-y_1) = (k-y_1)(x_2-x_1)$
Ans: The given points are $P(h,k), Q(x_1,y_1)$ and $R(x_2,y_2)$.

Since, the points P,Q and R lie on a line. Therefore, they are collinear. Hence,

Slope of PQ= Slope of QR

$\Rightarrow \frac{y_1-k}{x_1-h} = \frac{y_2-y_1}{x_2-x_1}$ [∵Slope =y2−y1x2−x1]

$\Rightarrow \frac{k-y_1}{h-x_1} = \frac{y_2-y_1}{x_2-x_1}$ (On cross multiplication)

$(h-x_1)(y_2-y_1) = (k-y_1)(x_2-x_1)$

3. Write the equation of the line through the points $(1, -1)$ and $(3, 5)$
Ans: We know that equation of line through two points

$(x_1,y_1)\&(x_2,y_2)$is

y−y1=y2−y1x2−x1(x−x1)

Since the equation of the line through the points $(1,-1)$ and $(3,5)$.
Here,

$x_1=1, y_1=-1$

$x_2=3$ and $y_2=5$

Putting values

$(y-(-1))=\frac{5-(-1)}{3-1}(x-1)$

$y+1=\frac{5+1}{2}(\boldsymbol{x}-\mathbf{1})$

$y+1=\frac{6}{2}(x-1)$

$y+1=3(x-1)$

$y+1=3x-3$

$y-3x+1+3=0$

$y-3x+4=0$

Hence the required equation is $y-3x+4=0$

4. Find the measure of the angle between the lines $x+y+7=0$ and $x-y+1=0$.

Ans: The given equation of lines are $x+y+7=0$ and $x-y+1=0$.
Express the given equation as slope-intercept form $y=mx+c$
Where,

(Slope)m = coefficient of x

$x+y+7=0$

$m_1=\frac{-1}{1}$

$x-y+1=0$

$m_2=\frac{-1}{-1}=1$

Product of these two slopes is -1 therefore; the lines are at right angles.

5. Find the equation of the line that has $y-$intercept 4 and is perpendicular to the line $y=3x-2$.

Ans: The given equation of the is $y=3x-2$.

Express the given equation as slope-intercept form $y=mx+c$

where,

(Slope)m= coefficient of x

$m_1=3$

When the lines are perpendicular. Then the product of slope is -1−1
.

$\therefore m_1.m_2=-1$

$3.\ m2=-1$

$m_2=-\frac{1}{3}$

Given,

$Y-$intercept of other line is 4.

Therefore,

The required equation of the line using the slope-intercept form $y=mx+c$

$y=-13x+4$.

6. Find the distance between the parallel lines $3x-4y+7=0$ and $3x-4y+5=0$.

Ans: Given,

Equations of the parallel lines are $3x-4y+7=0$ and $3x-4y+5=0$.

The general equation of the parallel lines is given by

$Ax+By+C_1=0$ and $Ax+By+C_2=0$
.

On comparing the given equation of two parallel lines with general equation, We get

$A=3$, $B=-4, C_1=7$ and $C_2=5$.

Distance between two parallel lines is

$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$

$\Rightarrow \frac{|7-5|}{\sqrt{(3)^2+(-4)^2}}$

$\Rightarrow \frac{2}{\sqrt{9+16}}$

$\Rightarrow 25.$

7. Find the slope of the line, which makes an angle of $30°$ with the positive direction of $y-$axis measured anticlockwise.

Ans: Given,

Line which makes an angle of $30°$ with the positive direction of $y-$axis measured anticlockwise.

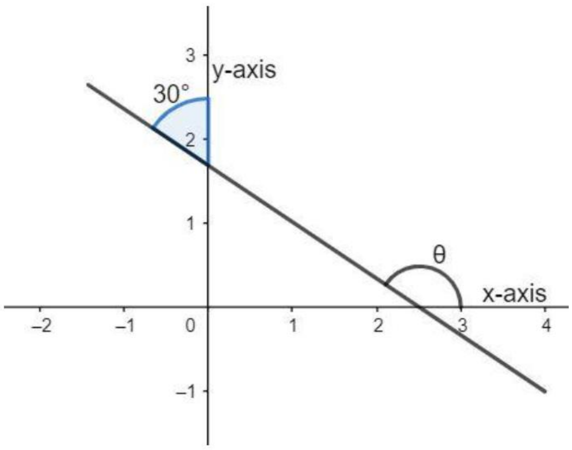
Let θ be the inclination of the line,

$\theta=120°$

(slope) $m=\tan 120°$

$=\tan(90+30)$

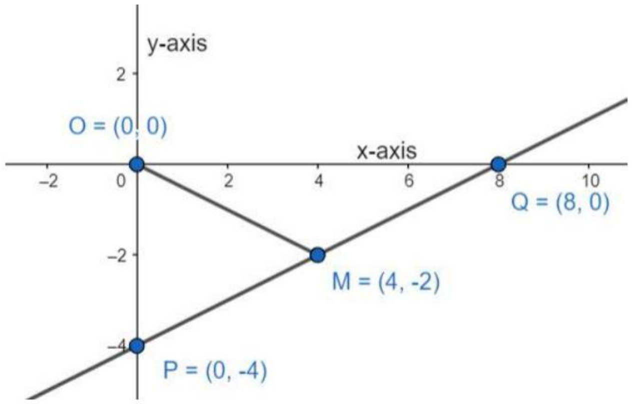
$=-\sqrt{3}$



8. Find the slope of a line, which passes through the origin, and the midpoint of the line segment joining the point $P\ (0,-4)$ and $Q(8,0)$

Ans: Given,

Line segment joining the point $P\ (0,-4)$ and $Q(8,0)$.



Let M be the midpoint of segment PQ then

$$M = \left(\frac{x1+x2}{2} + \frac{y1+y2}{2}\right)$$

$$= \left(\frac{0+8}{2}, \frac{-4+0}{2}\right)$$

$$= (4, -2)$$

Slope of line joining origin $O\ (0, 0)$ and the mid-point $M\ (4,-2)$,

$$OM = \frac{y2-y1}{x2-x1}$$

$$= \frac{-2-0}{4-0}$$

$$= \frac{-1}{2}.$$

9. Reduce the equation $\sqrt{3}$–x+y−8=0 into normal form. Find the values p and ω.

Ans: Given,

Equation of a line $\sqrt{3}$–x+y−8=0

Let $\sqrt{3}$–x+y=8…….(i)

$\sqrt{\left(\sqrt{3}\right)^2 + (1)^2} = 2$

Dividing (i) by 2

$\frac{\sqrt{3}}{2} + \frac{y}{2} = 4$

$x\cos 30° + y \cdot \sin 30° = 4 \ldots\ldots.(ii)$

On comparing the above equation with the standard from,
$x\cos\omega + y\sin\omega = p$

Where,

P is the perpendicular distance from the origin

ω is the angle between perpendicular and the positive x−axis

$p = 4$

$\omega = \frac{\pi}{6}$.

10. Without using the Pythagoras theorem show that the points $(4,4),(3,5)$ and $(-1,-1)$ are the vertices of a right angled Δ.

Ans: Let given points be $A(4,4), B(3,5)$ and $C(-1,-1)$

Slope of AB,

$m_1 = \frac{5-4}{3-4} = -1$

Slope of BC,

$m_2 = \frac{-1-5}{-1-3}$

$\Rightarrow \frac{-6}{-4} = \frac{3}{2}$

Slope of AC,

$m_3 = \frac{-1-4}{1-4} = +1$

Since,

Slope of $AB \times$ slope of $AC = -1$

$$m_1 \times m_3 = -1$$

$\Rightarrow AB \perp AC$ (if the product of slope is -1, then the lines are perpendicular)

Hence ΔABC is right angled at A.

Multiple Choice Questions

1) Two lines are said to be perpendicular if the product of their slope is equal to:

a. -1
b. 0
c. 1
d. ½

Answer: (a) -1
Explanation:

When two lines are perpendicular, then the product of their slope is equal to -1. If two lines are perpendicular with slope m_1 and m_2, then $m_1.m_2 = -1$.

2) What is the distance of (5, 12) from the origin?

a. 5 units
b. 8 units
c. 12 units
d. 13 units

Answer: (d) 13 units

Explanation: Let the points be A(0, 0) and B(5, 12).

A (0, 0) = (x_1, y_1)

B(5, 12) = (x_2, y_2)

The distance between two points, AB = $\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$

$AB = \sqrt{[(5-0)^2 + (12-0)^2]}$

$AB = \sqrt{(25+144)}$

$AB = \sqrt{(169)}$

$AB = 13$

Hence, the distance of (5, 12) from the origin is 13 units.

3) Two lines are said to be parallel if the difference of their slope is

a. -1
b. 0
c. 1
d. None of these

Answer: (b) 0

Explanation:

We know that two lines are said to be parallel if their slope is equal. If m_1 and m_2 are the slopes of two parallel lines, then it is represented as $m_1 = m_2$.

Hence, the difference of their slope should be $m_1 - m_2 = 0$.

So, option (b) 0 is the correct answer.

4) The equation of a straight line that passes through the point (3, 4) and perpendicular to the line 3x+2y+5=0 is

a. $2x-3y+6 = 0$
b. $2x+3y+6 = 0$
c. $2x-3y-6 = 0$
d. $2x+3y-6 = 0$

Answer: (a) $2x-3y+6 = 0$

Explanation:

The equation of a straight line perpendicular to $3x+2y+5 = 0$ is $2x-3y+\lambda = 0$ …(1)

This passes through the point (3, 4).

Now, substitute in equation (1), we get

$2(2) – 3(4) +\lambda = 0$

$4-12+\lambda =0$

$-6+\lambda =0$

$\lambda=6$

Substituting $\lambda=6$ in (1), we get $2x-3y+6 = 0$, which is the required equation.

Hence, option (a) $2x-3y+6 =0$ is the correct answer.

5) The slope of a line ax+by+c =0 is

a. a/b
b. -a/b
c. c/b
d. -c/b

Answer: (b) -a/b

Explanation:

We know that the general equation of a line is $ax+by+c = 0$.

Rearranging the equation, we get

$\Rightarrow by = -ax -c$

$\Rightarrow y = (-a/b)x -(c/b)$ …(1)

This is of the form, $y= mx+c$ …(2)

By comparing (1) and (2), we get

Slope, $m = -a/b$.

6) The equation of a line that passes through the points (1, 5) and (2, 3) is:

a. $2x + y – 7 = 0$
b. $2x – y – 7 = 0$
c. $x + 2y – 7 = 0$

d. $2x + y + 7 = 0$

Answer: (a) $2x + y – 7 = 0$

Explanation:

We know that the equation of a line passes through two points (x_1, y_1) and $(x_2\ y_2)$ is

$(y-y_1)/(x-x_1) = (y_2-y_1)/(x_2-x_1)$

$(x_1, y_1) = (1, 5)$

$(x_2, y_2) = (2, 3)$

Now, substitute the values in the formula, we get

$(y-5)/(x-1) = (3-5)/(2-1)$

$(y-5)/(x-1) = (-2)/(1)$

$y-5 = -2(x-1)$

$y-5 = -2x+2$

$2x+y-5-2 =0$

$2x+y-7 =0$

Therefore, the equation of a line that passes through the points (1, 5) and (2, 3) is $2x+y-7=0$.

7) The distance between the lines 3x+4y=9 and 6x+8y=15 is

a. 2/3
b. 3/2
c. 3/10
d. 7/10

Answer: (c) 3/10

Explanation:

Given equations:

$3x + 4y = 9$ …(i)

$6x + 8y = 15$ …(ii)

$\Longrightarrow 3x + 4y = 15/2$

The slope of line (i) is -3/4.

The slope of line (ii) is -3/4.

Since slopes are equal, the lines are parallel.

Hence, the distance between two parallel lines = $|(c_1 – c_2)|/\sqrt{(a^2 + b^2)}$

$= |(9 – (15/2))|/\sqrt{(3^2 + 4^2)}$

$= 3/10$

Therefore, the distance between the lines 3x+4y=9 and 6x+8y=15 is 3/10.

8) The locus of a point, whose abscissa and ordinate are always equal is

a. x-y = 0
b. x+y =1
c. x+y+1 = 0
d. None of the above

Answer: (a) x-y =0

Explanation:

Let the abscissa and ordinate of a point "P" be(x, y)

Given condition: Abscissa = Ordinate

(i.e) x =y

Hence, the locus of a point is x-y = 0.

Therefore, option (a) x-y =0 is the correct answer.

9) If A(6, 4) and B(2, 12) are the two points, then the slope of a line perpendicular to line AB is

a. -2
b. 2

c. ½
d. -½

Answer: (c) ½

Explanation:

Given points: A(6, 4) = (x_1, y_1)

B(2, 12) = (x_2, y_2)

We know that the slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is $(y_2-y_1)/(x_2-x_1)$.

m = (12-4)/(2-6) = 8/-4 = -2.

We know that the slope of two perpendicular lines $m_1.m_2 = -1$.

So, $m_2 = -1/m_1$

Hence, the slope of a line perpendicular to line AB is -1/m = -1/-2 = ½.

10) What can be said regarding a line if its slope is negative?

a. θ is an acute angle
b. θ is an obtuse angle
c. Either the line is x-axis or it is parallel to the x-axis.
d. None of these

Answer: (b) θ is an obtuse angle

Explanation:

The line with a negative slope makes an obtuse angle with a positive x-axis when measured in the anti-clockwise direction.

Summary

- Slope (m) of a non-vertical line passing through the points (x_1 , y_1) and (x_2 , y_2) is given by

$$m=\frac{y2-y1}{x2-x1}=\frac{y1-y2}{x1-x2},\ x_1\neq x_2$$

- If a line makes an angle á with the positive direction of x-axis, then the slope of the line is given by m = tan α, α ≠ 90°.
- Slope of horizontal line is zero and slope of vertical line is undefined.
- An acute angle (say θ) between lines L1 and L2 with slopes m1 and m2 is given by

$$\tan\theta = \left|\frac{m2-m1}{1+m1m2}\right|, 1+m_1m_2\neq 0$$

- Two lines are parallel if and only if their slopes are equal.
- Two lines are perpendicular if and only if product of their slopes is –1.
- Three points A, B and C are collinear, if and only if slope of AB = slope of BC.
- Equation of the horizontal line having distance a from the x-axis is either y = a or y = – a.
- Equation of the vertical line having distance b from the y-axis is either x = b or x = – b.
- The point (x, y) lies on the line with slope m and through the fixed point (x_0 , y_0), if and only if its coordinates satisfy the equation $y - y_0 = m (x - x_0)$.
- Equation of the line passing through the points (x_1 , y_1) and (x_2 , y_2) is given by

$$y-y_1=\frac{y2-y1}{x2-x1}(x-x_1)$$

- The point (x, y) on the line with slope m and y-intercept c lies on the line if and only if y=mx+c.
- If a line with slope m makes x-intercept d. Then equation of the line is y = m (x – d).
- Equation of a line making intercepts a and b on the x-and y-axis, respectively, is $\frac{x}{a}+\frac{y}{b}=1$

- Any equation of the form Ax + By + C = 0, with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line.
- The perpendicular distance (d) of a line Ax + By+ C = 0 from a point (x_1 , y_1) is given by $d=\frac{\mathrm{Ax1+By1+C}}{\sqrt{a^2+b^2}}$
- Distance between the parallel lines Ax + By + C1 = 0 and Ax + By + C2 = 0, is Given by

$$d=\frac{|c1-c2|}{\sqrt{a^2+b^2}}$$