Chapter 12

Introduction to Three Dimensional Geometry

This chapter introduces students to the realm of three-dimensional space, exploring the coordinates and geometric properties of points, lines, and planes in three dimensions. It serves as a bridge between the familiar two-dimensional Cartesian coordinate system and the more complex three-dimensional space. The chapter begins with a revision of the basic concepts of coordinate geometry, extending them to three axes—X, Y, and Z—enabling the description of points in space using ordered triplets.

The chapter further explores the equations of lines and planes in three-dimensional space, providing students with tools to analyze the relationships and intersections between these geometric elements. Concepts such as direction cosines, direction ratios, and the angle between two lines in space are introduced, contributing to a comprehensive understanding of spatial geometry.

Exercise 12.1

1. A point is on the x-axis. What are its y-coordinate and z-coordinates?

Solution:

If a point is on the x-axis, then the coordinates of y and z are 0.

So the point is (x, 0, 0).

2. A point is in the XZ-plane. What can you say about its *y*-coordinate?

Solution:

If a point is in the XZ plane, then its y-co-ordinate is 0.

3. Name the octants in which the following points lie:
(1, 2, 3), (4, –2, 3), (4, –2, –5), (4, 2, –5), (– 4, 2, –5), (– 4, 2, 5), (–3, –1, 6) (2, –4, –7).

Solution:

Here is the table which represents the octants:

Octants	I	II	III	IV	V	VI	VII	VIII
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

(i) (1, 2, 3)

Here, x is positive, y is positive, and z is positive.

So, it lies in the I octant.

(ii) (4, -2, 3)

Here, x is positive, y is negative, and z is positive.

So, it lies in the IV octant.

(iii) (4, -2, -5)

Here, x is positive, y is negative, and z is negative.

So, it lies in the VIII octant.

(iv) (4, 2, -5)

Here, x is positive, y is positive, and z is negative.

So, it lies in the V octant.

(v) (-4, 2, -5)

Here, x is negative, y is positive, and z is negative.

So, it lies in VI octant.

(vi) (-4, 2, 5)

Here, x is negative, y is positive, and z is positive.

So, it lies in the II octant.

(vii) (-3, -1, 6)

Here, x is negative, y is negative, and z is positive.

So, it lies in the III octant.

(viii) (2, -4, -7)

Here, x is positive, y is negative, and z is negative.

So, it lies in the VIII octant.

4. Fill in the blanks:
(i) The x-axis and y-axis, taken together, determine a plane known as _______.
(ii) The coordinates of points in the XY-plane are of the form _______.
(iii) Coordinate planes divide the space into ______ octants.

Solution:

(i) The x-axis and y-axis, taken together, determine a plane known as **XY Plane**.

(ii) The coordinates of points in the XY-plane are of the form **(x, y, 0)**.

(iii) Coordinate planes divide the space into **eight** octants.

Exercise 12.2

1. Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1)

(ii) (–3, 7, 2) and (2, 4, –1)

(iii) (–1, 3, –4) and (1, –3, 4)

(iv) (2, –1, 3) and (–2, 1, 3)

Solution:

(i) (2, 3, 5) and (4, 3, 1)

Let P be (2, 3, 5) and Q be (4, 3, 1)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 2, y_1 = 3, z_1 = 5$

$x_2 = 4, y_2 = 3, z_2 = 1$

Distance PQ = $\sqrt{[(4 - 2)^2 + (3 - 3)^2 + (1 - 5)^2]}$

$= \sqrt{[(2)^2 + 0^2 + (-4)^2]}$

$= \sqrt{[4 + 0 + 16]}$

$= \sqrt{20}$

$= 2\sqrt{5}$

∴ The required distance is $2\sqrt{5}$ units.

(ii) (–3, 7, 2) and (2, 4, –1)

Let P be (– 3, 7, 2) and Q be (2, 4, –1)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = -3$, $y_1 = 7$, $z_1 = 2$

$x_2 = 2$, $y_2 = 4$, $z_2 = -1$

Distance PQ = $\sqrt{[(2 - (-3))^2 + (4 - 7)^2 + (-1 - 2)^2]}$

$= \sqrt{[(5)^2 + (-3)^2 + (-3)^2]}$

$= \sqrt{[25 + 9 + 9]}$

$= \sqrt{43}$

∴ The required distance is $\sqrt{43}$ units.

(iii) (–1, 3, –4) and (1, –3, 4)

Let P be (– 1, 3, – 4) and Q be (1, – 3, 4)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = -1$, $y_1 = 3$, $z_1 = -4$

$x_2 = 1$, $y_2 = -3$, $z_2 = 4$

Distance PQ = $\sqrt{[(1 - (-1))^2 + (-3 - 3)^2 + (4 - (-4))^2]}$

$= \sqrt{[(2)^2 + (-6)^2 + (8)^2]}$

$= \sqrt{[4 + 36 + 64]}$

$= \sqrt{104}$

$= 2\sqrt{26}$

∴ The required distance is $2\sqrt{26}$ units.

(iv) (2, –1, 3) and (–2, 1, 3)

Let P be (2, – 1, 3) and Q be (– 2, 1, 3)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = 2$, $y_1 = -1$, $z_1 = 3$

$x_2 = -2$, $y_2 = 1$, $z_2 = 3$

Distance PQ = $\sqrt{[(-2 - 2)^2 + (1 - (-1))^2 + (3 - 3)^2]}$

$= \sqrt{[(-4)^2 + (2)^2 + (0)^2]}$

$= \sqrt{[16 + 4 + 0]}$

$= \sqrt{20}$

$= 2\sqrt{5}$

∴ The required distance is $2\sqrt{5}$ units.

2. Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Solution:

If three points are collinear, then they lie on the same line.

First, let us calculate the distance between the 3 points

i.e., PQ, QR and PR

Calculating PQ

P ≡ (–2, 3, 5) and Q ≡ (1, 2, 3)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = -2$, $y_1 = 3$, $z_1 = 5$

$x_2 = 1, y_2 = 2, z_2 = 3$

Distance PQ = $\sqrt{[(1 - (-2))^2 + (2 - 3)^2 + (3 - 5)^2]}$

$= \sqrt{[(3)^2 + (-1)^2 + (-2)^2]}$

$= \sqrt{[9 + 1 + 4]}$

$= \sqrt{14}$

Calculating QR

Q ≡ (1, 2, 3) and R ≡ (7, 0, – 1)

By using the formula,

Distance QR = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 1, y_1 = 2, z_1 = 3$

$x_2 = 7, y_2 = 0, z_2 = -1$

Distance QR = $\sqrt{[(7 - 1)^2 + (0 - 2)^2 + (-1 - 3)^2]}$

$= \sqrt{[(6)^2 + (-2)^2 + (-4)^2]}$

$= \sqrt{[36 + 4 + 16]}$

$= \sqrt{56}$

$= 2\sqrt{14}$

Calculating PR

P ≡ (–2, 3, 5) and R ≡ (7, 0, – 1)

By using the formula,

Distance PR = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = -2, y_1 = 3, z_1 = 5$

$x_2 = 7$, $y_2 = 0$, $z_2 = –1$

Distance PR = $\sqrt{[(7 – (-2))^2 + (0 – 3)^2 + (-1 – 5)^2]}$

$= \sqrt{[(9)^2 + (-3)^2 + (-6)^2]}$

$= \sqrt{[81 + 9 + 36]}$

$= \sqrt{126}$

$= 3\sqrt{14}$

Thus, PQ = $\sqrt{14}$, QR = $2\sqrt{14}$ and PR = $3\sqrt{14}$

So, PQ + QR = $\sqrt{14} + 2\sqrt{14}$

$= 3\sqrt{14}$

= PR

∴ The points P, Q and R are collinear.

3. Verify the following:
(i) (0, 7, –10), (1, 6, –6), and (4, 9, –6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) are the vertices of a right-angled triangle.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) are the vertices of a parallelogram.

Solution:

(i) (0, 7, –10), (1, 6, –6), and (4, 9, – 6) are the vertices of an isosceles triangle.

Let us consider the points,

P(0, 7, –10), Q(1, 6, –6) and R(4, 9, –6)

If any 2 sides are equal, it will be an isosceles triangle

So, first, let us calculate the distance of PQ, QR

Calculating PQ

$P \equiv (0, 7, -10)$ and $Q \equiv (1, 6, -6)$

By using the formula,

Distance $PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here,

$x_1 = 0, y_1 = 7, z_1 = -10$

$x_2 = 1, y_2 = 6, z_2 = -6$

Distance $PQ = \sqrt{[(1 - 0)^2 + (6 - 7)^2 + (-6 - (-10))^2]}$

$= \sqrt{[(1)^2 + (-1)^2 + (4)^2]}$

$= \sqrt{[1 + 1 + 16]}$

$= \sqrt{18}$

Calculating QR

$Q \equiv (1, 6, -6)$ and $R \equiv (4, 9, -6)$

By using the formula,

Distance $QR = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 1, y_1 = 6, z_1 = -6$

$x_2 = 4, y_2 = 9, z_2 = -6$

Distance $QR = \sqrt{[(4 - 1)^2 + (9 - 6)^2 + (-6 - (-6))^2]}$

$= \sqrt{[(3)^2 + (3)^2 + (-6+6)^2]}$

$= \sqrt{[9 + 9 + 0]}$

$= \sqrt{18}$

Hence, PQ = QR

18 = 18

2 sides are equal

∴ PQR is an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) are the vertices of a right-angled triangle.

Let the points be

P(0, 7, 10), Q(– 1, 6, 6) & R(– 4, 9, 6)

First, let us calculate the distance of PQ, OR and PR

Calculating PQ

P ≡ (0, 7, 10) and Q ≡ (– 1, 6, 6)

By using the formula,

Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = 0, y_1 = 7, z_1 = 10$

$x_2 = -1, y_2 = 6, z_2 = 6$

Distance PQ = $\sqrt{[(-1 - 0)^2 + (6 - 7)^2 + (6 - 10)^2]}$

$= \sqrt{[(-1)^2 + (-1)^2 + (-4)^2]}$

$= \sqrt{[1 + 1 + 16]}$

$= \sqrt{18}$

Calculating QR

Q ≡ (1, 6, –6) and R ≡ (4, 9, –6)

By using the formula,

Distance QR = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 1, y_1 = 6, z_1 = -6$

$x_2 = 4$, $y_2 = 9$, $z_2 = –6$

Distance QR = $\sqrt{[(4 – 1)^2 + (9 – 6)^2 + (-6 – (-6))^2]}$

= $\sqrt{[(3)^2 + (3)^2 + (-6+6)^2]}$

= $\sqrt{[9 + 9 + 0]}$

= $\sqrt{18}$

Calculating PR

P ≡ (0, 7, 10) and R ≡ (– 4, 9, 6)

By using the formula,

Distance PR = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So, here

$x_1 = 0$, $y_1 = 7$, $z_1 = 10$

$x_2 = – 4$, $y_2 = 9$, $z_2 = 6$

Distance PR = $\sqrt{[(-4 – 0)^2 + (9 – 7)^2 + (6 – 10)^2]}$

= $\sqrt{[(-4)^2 + (2)^2 + (-4)^2]}$

= $\sqrt{[16 + 4 + 16]}$

= $\sqrt{36}$

Now,

$PQ^2 + QR^2 = 18 + 18$

= 36

= PR^2

By using the converse of Pythagoras theorem,

∴ The given vertices P, Q & R are the vertices of a right–angled triangle at Q.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) are the vertices of a parallelogram.

Let the points: A(–1, 2, 1), B(1, –2, 5), C(4, –7, 8) & D(2, –3, 4)

ABCD can be vertices of parallelogram only if opposite sides are equal.

i.e., AB = CD and BC = AD

First, let us calculate the distance

Calculating AB

A ≡ (–1, 2, 1) and B ≡ (1, –2, 5)

By using the formula,

Distance AB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = -1$, $y_1 = 2$, $z_1 = 1$

$x_2 = 1$, $y_2 = -2$, $z_2 = 5$

Distance AB = $\sqrt{[(1 - (-1))^2 + (-2 - 2)^2 + (5 - 1)^2]}$

$= \sqrt{[(2)^2 + (-4)^2 + (4)^2]}$

$= \sqrt{[4 + 16 + 16]}$

$= \sqrt{36}$

$= 6$

Calculating BC

B ≡ (1, –2, 5) and C ≡ (4, –7, 8)

By using the formula,

Distance BC = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 1$, $y_1 = -2$, $z_1 = 5$

$x_2 = 4$, $y_2 = -7$, $z_2 = 8$

Distance BC = $\sqrt{[(4 - 1)^2 + (-7 - (-2))^2 + (8 - 5)^2]}$

= $\sqrt{[(3)^2 + (-5)^2 + (3)^2]}$

= $\sqrt{[9 + 25 + 9]}$

= $\sqrt{43}$

Calculating CD

C ≡ (4, –7, 8) and D ≡ (2, –3, 4)

By using the formula,

Distance CD = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = 4$, $y_1 = -7$, $z_1 = 8$

$x_2 = 2$, $y_2 = -3$, $z_2 = 4$

Distance CD = $\sqrt{[(2 - 4)^2 + (-3 - (-7))^2 + (4 - 8)^2]}$

= $\sqrt{[(-2)^2 + (4)^2 + (-4)^2]}$

= $\sqrt{[4 + 16 + 16]}$

= $\sqrt{36}$

= 6

Calculating DA

D ≡ (2, –3, 4) and A ≡ (–1, 2, 1)

By using the formula,

Distance DA = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = 2$, $y_1 = -3$, $z_1 = 4$

$x_2 = -1$, $y_2 = 2$, $z_2 = 1$

Distance DA = $\sqrt{[(-1 - 2)^2 + (2 - (-3))^2 + (1 - 4)^2]}$

= $\sqrt{[(-3)^2 + (5)^2 + (-3)^2]}$

= $\sqrt{[9 + 25 + 9]}$

= $\sqrt{43}$

Since AB = CD and BC = DA (given),

In ABCD, both pairs of opposite sides are equal.

∴ ABCD is a parallelogram.

4. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Solution:

Let A (1, 2, 3) & B (3, 2, –1)

Let point P be (x, y, z)

Since it is given that point P(x, y, z) is equal distance from point A(1, 2, 3) & B(3, 2, –1)

i.e. PA = PB

First, let us calculate

Calculating PA

P ≡ (x, y, z) and A ≡ (1, 2, 3)

By using the formula,

Distance PA = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = x$, $y_1 = y$, $z_1 = z$

$x_2 = 1$, $y_2 = 2$, $z_2 = 3$

Distance PA = $\sqrt{[(1 - x)^2 + (2 - y)^2 + (3 - z)^2]}$

Calculating PB

$P \equiv (x, y, z)$ and $B \equiv (3, 2, -1)$

By using the formula,

Distance PB $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So, here

$x_1 = x, y_1 = y, z_1 = z$

$x_2 = 3, y_2 = 2, z_2 = -1$

Distance PB $= \sqrt{[(3 - x)^2 + (2 - y)^2 + (-1 - z)^2]}$

Since PA = PB

Square on both sides, we get

$PA^2 = PB^2$

$(1 - x)^2 + (2 - y)^2 + (3 - z)^2 = (3 - x)^2 + (2 - y)^2 + (-1 - z)^2$

$(1 + x^2 - 2x) + (4 + y^2 - 4y) + (9 + z^2 - 6z)$

$(9 + x^2 - 6x) + (4 + y^2 - 4y) + (1 + z^2 + 2z)$

$-2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$4x - 8z = 0$

$x - 2z = 0$

∴ The required equation is $x - 2z = 0$.

5. Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(–4, 0, 0) is equal to 10.

Solution:

Let A(4, 0, 0) & B(– 4, 0, 0)

Let the coordinates of point P be (x, y, z)

Calculating PA

$P \equiv (x, y, z)$ and $A \equiv (4, 0, 0)$

By using the formula,

Distance $PA = \sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So, here

$x_1 = x, y_1 = y, z_1 = z$

$x_2 = 4, y_2 = 0, z_2 = 0$

Distance $PA = \sqrt{[(4– x)^2 + (0 – y)^2 + (0 – z)^2]}$

Calculating PB,

$P \equiv (x, y, z)$ and $B \equiv (– 4, 0, 0)$

By using the formula,

Distance $PB = \sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So here,

$x_1 = x, y_1 = y, z_1 = z$

$x_2 = – 4, y_2 = 0, z_2 = 0$

Distance $PB = \sqrt{[(-4– x)^2 + (0 – y)^2 + (0 – z)^2]}$

It is given that,

$PA + PB = 10$

$PA = 10 – PB$

Square on both sides, we get

$PA^2 = (10 – PB)^2$

$PA^2 = 100 + PB^2 – 20 PB$

$(4 – x)^2 + (0 – y)^2 + (0 – z)^2$

$100 + (– 4 – x)^2 + (0 – y)^2 + (0 – z)^2 – 20 PB$

$(16 + x^2 – 8x) + (y^2) + (z^2)$

$100 + (16 + x^2 + 8x) + (y^2) + (z^2) – 20\ PB$

$20\ PB = 16x + 100$

$5\ PB = (4x + 25)$

Square on both sides again, we get

$25\ PB^2 = 16x^2 + 200x + 625$

$25\ [(– 4 – x)^2 + (0 – y)^2 + (0 – z)^2] = 16x^2 + 200x + 625$

$25\ [x^2 + y^2 + z^2 + 8x + 16] = 16x^2 + 200x + 625$

$25x^2 + 25y^2 + 25z^2 + 200x + 400 = 16x^2 + 200x + 625$

$9x^2 + 25y^2 + 25z^2 – 225 = 0$

∴ The required equation is $9x^2 + 25y^2 + 25z^2 – 225 = 0$.

2Marks Questions & Answers

1. Find the distance between the points P (1, –3, 4) and Q (– 4, 1, 2).

Ans:

The distance PQ between the points P (1,–3, 4) and Q (– 4, 1, 2) is

$$PQ = \sqrt{(-4-1)^2 + (1+3)^2 + (2-4)^2}$$

$$= \sqrt{25 + 16 + 4}$$

$$= \sqrt{45} = 3\sqrt{5} \text{ units.}$$

2. Show that the points P (–2, 3, 5), Q (1, 2, 3) and R (7, 0, –1) are collinear.

Ans: We know that points are said to be collinear if they lie on a line.

Now, $PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4} = \sqrt{14}$

$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$ and

$PR = \sqrt{(-4-1)^2 + (1+3)^2 + (2-4)^2} = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$

Thus, PQ + QR = PR. Hence, P, Q and R are collinear.

3. Using the section formula, prove that the three Points A(−2,3,5),B(1,2,3) **and** C(7,0,−1) **are collinear.**
Ans: Assume that the given points are collinear and C divides AB in the ratio λ=1.
Then coordinates of C are

$(\frac{\lambda-2}{2+1}, \frac{2\lambda+3}{\lambda+1}, \frac{3\hat{2}+5}{5\lambda+1})$

But the coordinates of C are (3, 0,−1) from the above equations
We get $\lambda = \frac{3}{2}$

Since these equation give the same value of V.

∴ the given points are collinear and C divides AB exactly in the ratio 3: 2.

4. Prove by distance formula that the points X(1,2,3),Y(−1,−1,−1),Z(3,5,7) **are collinear.**

Ans: The Distance

|XY = $\sqrt{(-1-1)^2+(-1-2)^2+(-1-3)^2} = \sqrt{4+9+16} = \sqrt{29}$

Distance

|YZ|= $\sqrt{(3+1)^2+(5+1)^2+(7+1)^2} = \sqrt{16+36+64} = 2\sqrt{29}$

Distance

|XZ|= $\sqrt{(3-1)^2+(5-2)^2+(7-3)^2} = \sqrt{4+9+16} = \sqrt{29}$

∴|YZ|=|XY|+|XZ|

The points X, Y, Z are collinear.

5. Find the equation of set of points P such that PA2 + PB2 = 2k 2 , where A and B are the points (3, 4, 5) and (–1, 3, –7), respectively.

Ans: Let the coordinates of point P be (x, y, z).

Here $PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$

$PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$

By the given condition $PA^2 + PB^2 = 2k^2$, we have

$(x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = 2k^2$

i.e., $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2k^2 - 109$.

6. The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, – 6), respectively, find the coordinates of the point C.

Ans: Let the coordinates of C be (x, y, z) and the coordinates of the centroid G be (1, 1, 1).

Then

$\frac{x+3-1}{3}$, i.e., x = 1; $\frac{y-5+7}{3}$, i.e., y = 1; $\frac{z+7-6}{3}$, i.e., z = 2.

Hence, coordinates of C are (1, 1, 2).

7.Given that P(3,2,−4),Q(5,4,−6) and R(9,8,−10) are collinear. Find the ratio in which Q divides PR

Ans: Suppose Q divides PR in the ratio λ:1.. Then coordinators of Q are

$\left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right)$

But, coordinates of QQ are (5, 4,−6).(5,4,−6).Therefore

$\frac{9\lambda+3}{2+1}$ =5, $\frac{8\lambda+2}{\lambda+1}$ =4, $\frac{-10\lambda-4}{\lambda+1}$ =6

These three equations give

$\hat{\alpha} = \frac{1}{2}$

So Q divides PR in the ratio $\frac{1}{2}$: 1 or 1:2

8.Determine the points in x y plane which is equidistant from these point A (2,0,3)**T**B(0,3,2) **and** C(0,0,1)
Ans: Since the z coordinate in the xy plane is zero. So, let P(x, y, 0) be a point in xy- plane, such that PA=PB=PC. Now PA=PB

$PA^2 = PB^2$

$\Rightarrow (x-2)^2 + (y-0)^2 + (0-3)^2 = (x-0)^2 + (y-3)^2 + (0-2)^2$

2x−3y=0…..(i)

PB=PC

$\Rightarrow PB^2 = PC^2$

$\Rightarrow (x-0)^2 + (y-3)^2 + (0-2)^2 = (x-0)^2 + (y-0)^2 + (0-1)^2$T

⇒−6y+12=0⇒y=2……..(ii)

Put y=2 in (i) we get x=3

Hence the points required are (3, 2, 0).

Multiple Choice Questions

1: Which octant do the point (−5, 4, 3) lie?

A. Octant I
B. Octant II

C. Octant III

D. Octant IV

Answer: B. Octant II

Explanation: Given (−5, 4, 3) is the point.

Here, the x-coordinate is negative but y and z coordinates are positive. Therefore, (−5, 4, 3) lie in octant II.

Q.2: A point is on the x-axis. Which of the following represent the point?

A. (0, x, 0)

B. (0, 0, x)

C. (x, 0, 0)

D. None of the above

Answer: C. (x, 0, 0)

Explanation: At x-axis, y and z coordinates are zero.

Q.3: Coordinate planes divide the space into ______ octants.

A. 4

B. 6

C. 8

D. 10

Answer: C. 8

Explanation: The coordinate planes divide the three dimensional space into eight octants.

Q.4: What is the distance between the points (2, –1, 3) and (–2, 1, 3)?

A. $2\sqrt{5}$ units

B. 25 units

C. $4\sqrt{5}$ units

D. $\sqrt{5}$ units

Answer: A. $2\sqrt{5}$ units

Explanation: Let the points be P (2, – 1, 3) and Q (– 2, 1, 3)

By using the distance formula,

$PQ = \sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$

So here,

$x_1 = 2, y_1 = – 1, z_1 = 3$

$x_2 = – 2, y_2 = 1, z_2 = 3$

$PQ = \sqrt{[(-2 – 2)^2 + (1 – (-1))^2 + (3 – 3)^2]}$

$= \sqrt{[(-4)^2 + (2)^2 + (0)^2]}$

$= \sqrt{[16 + 4 + 0]}$

$= \sqrt{20}$

$= 2\sqrt{5}$

Therefore, the required distance is $2\sqrt{5}$ units.

Q.5: The maximum distance between points (3sin θ, 0, 0) and (4cos θ, 0, 0) is:

(a) 3 units

(b) 4 units

(c) 5 units

(d) Cannot be determined

Answer: (c) 5

Explanation: Let the two points be P (3sin θ, 0, 0) and Q (4cos θ, 0, 0)

Now by distance formula,

$PQ = \sqrt{\{(4\cos θ – 3\sin θ)^2 + (0 – 0)^2 + (0 – 0)^2\}}$

$PQ = \sqrt{\{(4\cos θ – 3\sin θ)^2\}}$

$PQ = 4\cos θ – 3\sin θ$

Now, maximum value of $4\cos θ – 3\sin θ$;

$= \sqrt{\{(4^2 + (-3)^2\}}$

$= \sqrt{(16 + 9)}$

$= \sqrt{25}$

$= 5$

Thus, PQ = 5 units

So, the maximum distance between points ($3\sin\theta$, 0, 0) and ($4\cos\theta$, 0, 0) is 5.

Q.6: The locus represented by $xy + yz = 0$ is:

(a) A pair of perpendicular lines

(b) A pair of parallel lines

(c) A pair of parallel planes

(d) A pair of perpendicular planes

Answer: (d) A pair of perpendicular planes

Q.7: Find the image of (−2, 3, 4) in the y z plane.

A. (-2, 3, 4)

B. (2, 3, 4)

C. (-2, -3, 4)

D. (-2, -3, -4)

Answer: B. (2, 3, 4)

Q.8: The perpendicular distance of the point P (6, 7, 8) from the XY – Plane is:

(a)8

(b)7

(c)6

(d) None of the above

Answer: A. 8

Explanation: Let Q be the foot of perpendicular drawn from the point P (6, 7, 8) to the XY plane.

Thus, the distance of this foot Q from P is z-coordinate of P, i.e., 8 units

Q.9: The image of the point P(1,3,4) in the plane 2x – y + z = 0 is:

(a) (-3, 5, 2)

(b) (3, 5, 2)

(c) (3, -5, 2)

(d) (3, 5, -2)

Answer: (a) (-3, 5, 2)

Explanation:

Let the image of the point P(1, 3, 4) is Q.

The equation of the line through P and normal to the given plane is:

$(x – 1)/2 = (y – 3)/-1 = (z – 4)/1$

Since the line passes through Q, so let the coordinate of Q are (2k + 1, -k + 3, k + 4)

Now, the coordinate of the mid-point of PQ is:

(k + 1, -k/2 + 3, k/2 + 4)

Now, this midpoint lies in the given plane.

$2(k + 1) – (-k/2 + 3) + (k/2 + 4) + 3 = 0$

$\Rightarrow 2k + 2 + k/2 – 3 + k/2 + 4 + 3 = 0$

$\Rightarrow 3k + 6 = 0$

$\Rightarrow k = -2$

Hence, the coordinate of Q is (2k + 1, -k + 3, k + 4) = (-4 + 1, 2 + 3, -2 + 4)

= (-3, 5, 2)

Q.10: The distance of the point P(a, b, c) from the x-axis is:

(a) $\sqrt{(a^2 + c^2)}$

(b) $\sqrt{(a^2 + b^2)}$

(c) $\sqrt{(b^2 + c^2)}$

(d) None of these

Answer: (c) $\sqrt{(b^2 + c^2)}$

Explanation:

The coordinate of the foot of the perpendicular from P on x-axis are (a, 0, 0).

So, the required distance = $\sqrt{\{(a - a)^2 + (b - 0)^2 + (c - 0)^2\}}$

$= \sqrt{(b^2 + c^2)}$

Summary

- In three dimensions, the coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called the x, y and z-axes.
- The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-planes.
- The three coordinate planes divide the space into eight parts known as octants.
- The coordinates of a point P in three dimensional geometry is always written in the form of triplet like (x, y, z). Here x, y and z are the distances from the YZ, ZX and XY-planes.
- (i) Any point on x-axis is of the form (x, 0, 0) (ii) Any point on y-axis is of the form (0, y, 0) (iii) Any point on z-axis is of the form (0, 0, z).
- Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by $PQ=\sqrt{(x2-x1)}^2+(y_2-y_1)^2+(z_2-z_1)^2$