Chapter -9

Circles

Exercise: 9.1

1marks Questions

1. Fill in the blanks.

(i) The centre of a circle lies in **interior** of the circle. (exterior/ interior)

(ii) A point whose distance from the centre of a circle is greater than its radius lies in **exterior** of the circle. (exterior/ interior)

(iii) The longest chord of a circle is a **diameter** of the circle.

(iv) An arc is a **semicircle** when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and **chord** of the circle.

(vi) A circle divides the plane, on which it lies, in **3 (three)** parts.

2. Write True or False. Give reasons for your solutions.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only a finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is the diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Solution:

(i) **True.** Any line segment drawn from the centre of the circle to any point on it is the radius of the circle and will be of equal length.

(ii) **False.** There can be infinite numbers of equal chords in a circle.

(iii) **False.** For unequal arcs, there can be major and minor arcs. So, equal arcs on a circle cannot be said to be major arcs or minor arcs.

(iv) **True.** Any chord whose length is twice as long as the radius of the circle always passes through the centre of the circle, and thus, it is known as the diameter of the circle.

(v) **False.** A sector is a region of a circle between the arc and the two radii of the circle.

(vi) **True.** A circle is a 2d figure, and it can be drawn on a plane.

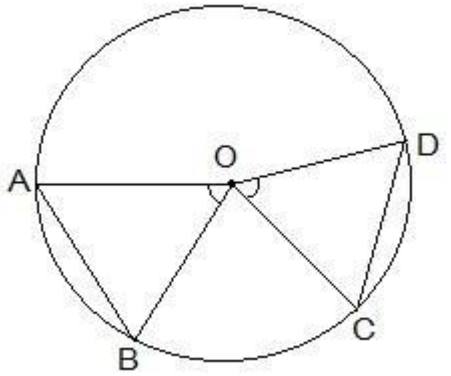
Exercise: 09.2

2marks Questions

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Solution:

To recall, a circle is a collection of points whose every point is equidistant from its centre. So, two circles can be congruent only when the distance of every point of both circles is equal from the centre.



For the second part of the question, it is given that AB = CD, i.e., two equal chords.

Now, it is to be proven that angle AOB is equal to angle COD.

Proof:

Consider the triangles ΔAOB and ΔCOD.

OA = OC and OB = OD (Since they are the radii of the circle.)

AB = CD (As given in the question.)

So, by SSS congruency, ΔAOB ≅ ΔCOD

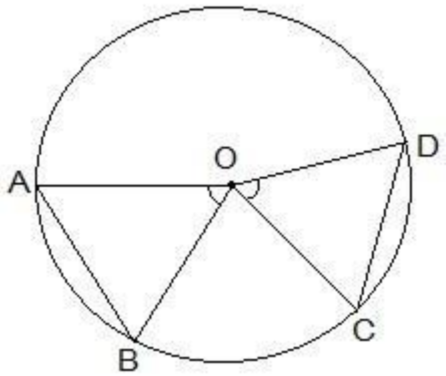
∴ By CPCT, we have,

∠AOB = ∠COD (Hence, proved).

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Solution:

Consider the following diagram.



Here, it is given that ∠AOB = ∠COD, i.e., they are equal angles.

Now, we will have to prove that the line segments AB and CD are equal, i.e., AB = CD.

Proof:

In triangles AOB and COD,

∠AOB = ∠COD (As given in the question.)

OA = OC and OB = OD (These are the radii of the circle.)

So, by SAS congruency, ΔAOB ≅ ΔCOD

∴ By the rule of CPCT, we have,

AB = CD (Hence, proved.)

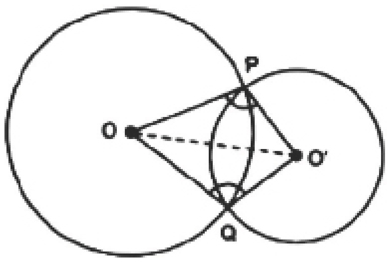
Exercise: 9.3

2marks Questions

1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution:

Consider the following diagram.



In ΔPOO' and ΔQOO'

OP = OQ (Radius of circle 1)

O'P = O'Q (Radius of circle 2)

OO' = OO' (Common arm)

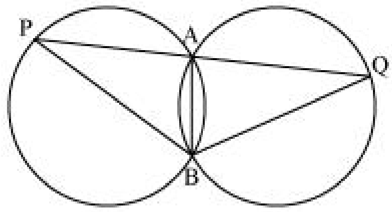
So, by SSS congruency, ΔPOO' ≅ ΔQOO'

Thus, ∠OPO' = ∠OQO' (proved).

3. Two congruent circles intersect each other at points A and B. Through A, any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Solution:

The diagram will be



Here, ∠APB = ∠AQB (as AB is the common chord in both the congruent circles.)

Now, consider ΔBPQ.

∠APB = ∠AQB

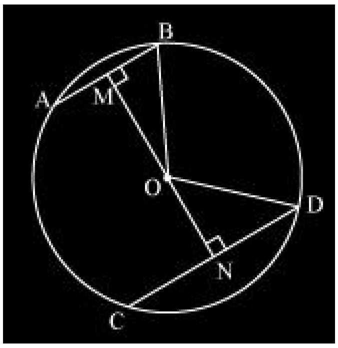
So, the angles are opposite to equal sides of a triangle.

∴ BQ = BP

5marks Questions

1. Two chords AB and CD of lengths 5 cm and 11 cm, respectively, of a circle, are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6, find the radius of the circle.

Solution:



Here, OM ⊥ AB and ON ⊥ CD are drawn, and OB and OD are joined.

We know that AB bisects BM as the perpendicular from the centre bisects the chord.

Since AB = 5 so,

BM = AB/2 = 5/2

Similarly, ND = CD/2 = 11/2

Now, let ON be x.

So, OM = 6−x.

Consider ΔMOB,

$OB^2 = OM^2+MB^2$

Or,

$$OB^2 = 36+x^2-12x+\frac{25}{4} \qquad \ldots (1)$$

Consider ΔNOD,

$OD^2 = ON^2 + ND^2$

Or

$$OD^2 = x^2+\frac{121}{4} \qquad \ldots (2)$$

We know, OB = OD (radii)

From equation 1 and equation 2, we get

$$36+x^2-12x+\frac{25}{4} = x^2+\frac{121}{4}$$
$$12x = 36+\frac{25}{4}-\frac{121}{4}$$
$$= \frac{144+25-121}{4}$$
$$12x = \frac{48}{4} = 12$$
$$x = 1$$

Now, from equation (2), we have,

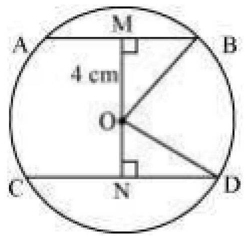
$OD^2 = 1^2 + (121/4)$

Or OD = $(5/2)\times\sqrt{5}$ cm

2. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance 4 cm from the centre, what is the distance of the other chord from the centre?

Solution:

Consider the following diagram.



Here, AB and CD are 2 parallel chords. Now, join OB and OD.

Distance of smaller chord AB from the centre of the circle = 4 cm

So, OM = 4 cm

MB = AB/2 = 3 cm

Consider ΔOMB.

$OB^2 = OM^2+MB^2$

Or, OB = 5cm

Now, consider ΔOND.

OB = OD = 5 (Since they are the radii.)

ND = CD/2 = 4 cm

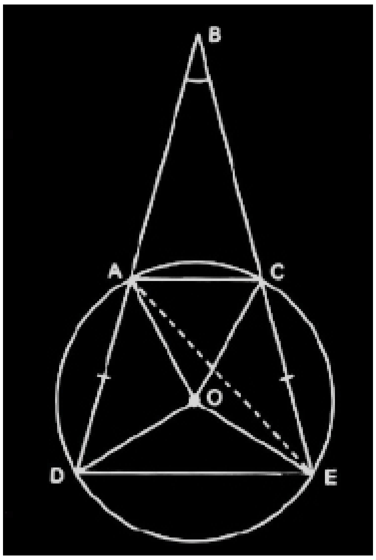
Now, $OD^2= ON^2+ND^2$

Or, ON = 3 cm

3. Let the vertex of an angle ABC be located outside a circle, and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Solution:

Consider the diagram.



Here AD = CE

We know any exterior angle of a triangle is equal to the sum of interior opposite angles.

So,

$\angle DAE = \angle ABC + \angle AEC$ (in ΔBAE) ————————-(i)

DE subtends $\angle DOE$ at the centre and $\angle DAE$ in the remaining part of the circle.

So,

$\angle DAE = (½)\angle DOE$ ————————-(ii)

Similarly, $\angle AEC = (½)\angle AOC$ ————————-(iii)

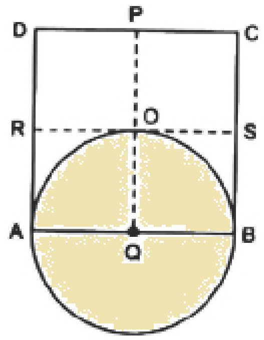
Now, from equations (i), (ii), and (iii), we get

$(½)\angle DOE = \angle ABC + (½)\angle AOC$

Or, $\angle ABC = (½)[\angle DOE - \angle AOC]$ (Hence, proved)

4. Prove that the circle drawn with any side of a rhombus as diameter passes through the point of intersection of its diagonals.

Solution:



To prove: A circle drawn with Q as the centre will pass through A, B and O (i.e., QA = QB = QO).

Since all sides of a rhombus are equal,

AB = DC

Now, multiply (½) on both sides.

(½)AB = (½)DC

So, AQ = DP

BQ = DP

Since Q is the midpoint of AB,

AQ= BQ

Similarly,

RA = SB

Again, as PQ is drawn parallel to AD,

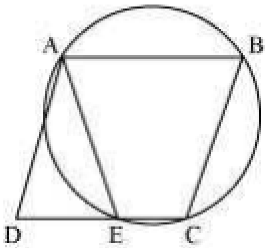
RA = QO

Now, as AQ = BQ and RA = QO, we get

QA = QB = QO (Hence, proved)

5. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

Solution:



Here, ABCE is a cyclic quadrilateral. In a cyclic quadrilateral, the sum of the opposite angles is 180°.

So, ∠AEC+∠CBA = 180°

As ∠AEC and ∠AED are linear pairs,

∠AEC+∠AED = 180°

Or, ∠AED = ∠CBA … (1)

We know in a parallelogram, opposite angles are equal.

So, ∠ADE = ∠CBA … (2)

Now, from equations (1) and (2), we get

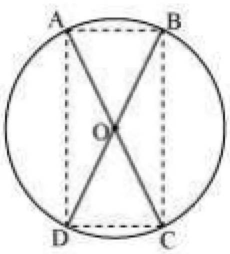
∠AED = ∠ADE

Now, AD and AE are angles opposite to equal sides of a triangle.

∴ AD = AE (proved)

6. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.

Solution:



Here, chords AB and CD intersect each other at O.

Consider ΔAOB and ΔCOD.

$\angle AOB = \angle COD$ (They are vertically opposite angles.)

OB = OD (Given in the question.)

OA = OC (Given in the question.)

So, by SAS congruency, $\Delta AOB \cong \Delta COD$

Also, AB = CD (By CPCT)

Similarly, $\Delta AOD \cong \Delta COB$

Or, AD = CB (By CPCT)

In quadrilateral ACBD, opposite sides are equal.

So, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

So, $\angle A = \angle C$

Also, as ABCD is a cyclic quadrilateral,

$\angle A+\angle C = 180°$

$\Rightarrow \angle A+\angle A = 180°$

Or, $\angle A = 90°$

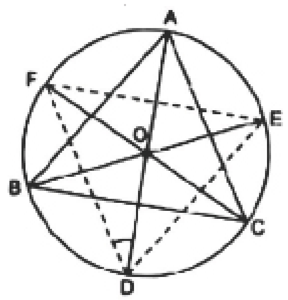
As ACBD is a parallelogram and one of its interior angles is 90°, so, it is a rectangle.

$\angle A$ is the angle subtended by chord BD. And as $\angle A = 90°$, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

7. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F, respectively. Prove that the angles of the triangle DEF are 90°–(½)A, 90°–(½)B and 90°–(½)C.

Solution:

Consider the following diagram.



Here, ABC is inscribed in a circle with centre O, and the bisectors of ∠A, ∠B and ∠C intersect the circumcircle at D, E and F, respectively.

Now, join DE, EF and FD.

As angles in the same segment are equal, so,

∠EDA = ∠FCA —————-(i)

∠FDA = ∠EBA —————-(i)

By adding equations (i) and (ii), we get

∠FDA+∠EDA = ∠FCA+∠EBA

Or, ∠FDE = ∠FCA+∠EBA = (½)∠C+(½)∠B

We know, ∠A +∠B+∠C = 180°

So, ∠FDE = (½)[∠C+∠B] = (½)[180°-∠A]

∠FDE = [90-(∠A/2)]

In a similar way,

∠FED = [90° -(∠B/2)] °

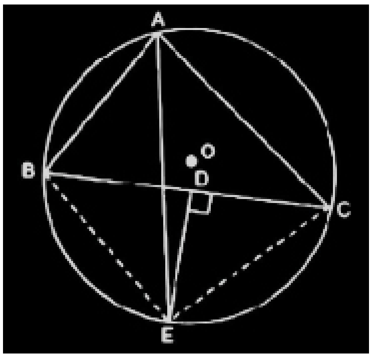
And,

∠EFD = [90° -(∠C/2)] °

8. In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Solution:

Consider this diagram.



Here, join BE and CE.

Now, since AE is the bisector of ∠BAC,

∠BAE = ∠CAE

Also,

∴arc BE = arc EC

This implies chord BE = chord EC

Now, consider triangles ΔBDE and ΔCDE.

DE = DE (It is the common side)

BD = CD (It is given in the question)

BE = CE (Already proved)

So, by SSS congruency, ΔBDE ≅ ΔCDE.

Thus, ∴∠BDE = ∠CDE

We know, ∠BDE = ∠CDE = 180°

Or, ∠BDE = ∠CDE = 90°

∴ DE ⊥ BC (Hence, proved).