

# CHAPTER-II

## MECHANICAL PROPERTIES OF FLUIDS

### 2 MARK QUESTIONS

**1.Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.**

**ANSWER:**

**Answer: No**

Bernoulli's equation cannot be used to describe the flow of water through a rapid in a river because of the turbulent flow of water. This principle can only be applied to a streamline flow.

**2.Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.**

**ANSWER:**

**Answer: No**

It does not matter if one uses gauge pressure instead of absolute pressure while applying Bernoulli's equation. The two points where Bernoulli's equation is applied should have significantly different atmospheric pressures.

**3.Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?**

**ANSWER:**

**Answer: Yes**

Two vessels having the same base area have identical force and equal pressure acting on their common base area. Since the shapes of the two vessels are different, the force exerted on the sides of the vessels has nonzero vertical components. When these vertical components are added, the total force on one vessel comes out to be greater than that on the other vessel. Hence, when these vessels are filled with water to the same height, they give different readings on a weighing scale.

**4. During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? [Use the density of whole blood from Table 10.1].**

**ANSWER:**

Gauge pressure,  $P = 2000 \text{ Pa}$

Density of whole blood,  $\rho = 1.06 \times 10^3 \text{ kg m}^{-3}$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

Height of the blood container =  $h$

Pressure of the blood container,  $P = h\rho g$

$$\begin{aligned}\therefore h &= \frac{P}{\rho g} \\ &= \frac{2000}{1.06 \times 10^3 \times 9.8} \\ &= 0.1925 \text{ m}\end{aligned}$$

The blood may enter the vein if the blood container is kept at a height greater than 0.1925 m, i.e., about 0.2 m.

## 4 MARK QUESTIONS

**1. Explain why**

- (a) The blood pressure in humans is greater at the feet than at the brain
- (b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km
- (c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

**ANSWER:**

- (a) The pressure of a liquid is given by the relation:

$$P = h\rho g$$

Where,  $P$  = Pressure  $h$  = Height of the

liquid column  $\rho$  = Density of the liquid

$g$  = Acceleration due to the gravity

It can be inferred that pressure is directly proportional to height. Hence, the blood pressure in human vessels depends on the height of the blood column in the body. The height of the blood column is more at the feet than it is at the brain. Hence, the blood pressure at the feet is more than it is at the brain.

(b) Density of air is the maximum near the sea level. Density of air decreases with increase in height from the surface. At a height of about 6 km, density decreases to nearly half of its value at the sea level. Atmospheric pressure is proportional to density. Hence, at a height of 6 km from the surface, it decreases to nearly half of its value at the sea level. (c) When force is applied on a liquid, the pressure in the liquid is transmitted in all directions. Hence, hydrostatic pressure does not have a fixed direction and it is a scalar physical quantity.

**2.A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?**

**ANSWER:**

Mass of the girl,  $m = 50 \text{ kg}$

Diameter of the heel,  $d = 1 \text{ cm} = 0.01 \text{ m}$

Radius of the heel,  $r = \frac{d}{2} = 0.005 \text{ m}$

Area of the heel  $= \pi r^2$

$$= \pi (0.005)^2$$

$$= 7.85 \times 10^{-5} \text{ m}^2$$

Force exerted by the heel on the floor:

$$F = mg$$

$$= 50 \times 9.8$$

$$= 490 \text{ N}$$

Pressure exerted by the heel on the floor:

$$P = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{490}{7.85 \times 10^{-5}}$$

$$= 6.24 \times 10^6 \text{ N m}^{-2}$$

Therefore, the pressure exerted by the heel on the horizontal floor is

$$6.24 \times 10^6 \text{ Nm}^{-2}$$

**3. What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature? Surface tension of mercury at that temperature (20°C) is  $4.65 \times 10^{-1} \text{ N m}^{-1}$ . The atmospheric pressure is  $1.01 \times 10^5 \text{ Pa}$ . Also give the excess pressure inside the drop.**

**ANSWER:**



**Answer:**  $1.01 \times 10^5 \text{ Pa}$ ; 310 Pa

Radius of the mercury drop,  $r = 3.00 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Surface tension of mercury,  $S = 4.65 \times 10^{-1} \text{ N m}^{-1}$

Atmospheric pressure,  $P_0 = 1.01 \times 10^5 \text{ Pa}$

Total pressure inside the mercury drop

= Excess pressure inside mercury + Atmospheric pressure

$$= \frac{2S}{r} + P_0$$

$$= \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} + 1.01 \times 10^5$$

$$= 1.0131 \times 10^5$$

$$= 1.01 \times 10^5 \text{ Pa}$$

Excess pressure  $= \frac{2S}{r}$

$$= \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310 \text{ Pa}$$

**4.Toricelli's barometer used mercury. Pascal duplicated it using French wine of density  $984 \text{ kg m}^{-3}$ . Determine the height of the wine column for normal atmospheric pressure.**

**ANSWER:**

10.5 m

Density of mercury,  $\rho_1 = 13.6 \times 10^3 \text{ kg/m}^3$

Height of the mercury column,  $h_1 = 0.76 \text{ m}$

Density of French wine,  $\rho_2 = 984 \text{ kg/m}^3$

Height of the French wine column =  $h_2$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

The pressure in both the columns is equal, i.e.,

Pressure in the mercury column = Pressure in the French wine column

$$\rho_1 h_1 g = \rho_2 h_2 g$$

$$h_2 = \frac{\rho_1 h_1}{\rho_2}$$

$$= \frac{13.6 \times 10^3 \times 0.76}{984}$$

$$= 10.5 \text{ m}$$

Hence, the height of the French wine column for normal atmospheric pressure is 10.5 m.

**5.A vertical off-shore structure is built to withstand a maximum stress of  $10^9 \text{ Pa}$ . Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.**

**ANSWER:**

**Answer:** Yes

The maximum allowable stress for the structure,  $P = 10^9 \text{ Pa}$

Depth of the ocean,  $d = 3 \text{ km} = 3 \times 10^3 \text{ m}$

Density of water,  $\rho = 10^3 \text{ kg/m}^3$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

The pressure exerted because of the sea water at depth,  $d = \rho dg$

$$= 3 \times 10^3 \times 10^3 \times 9.8$$

$$= 2.94 \times 10^7 \text{ Pa}$$

The maximum allowable stress for the structure ( $10^9 \text{ Pa}$ ) is greater than the pressure of the sea water ( $2.94 \times 10^7 \text{ Pa}$ ). The pressure exerted by the ocean is less than the pressure that the structure can withstand. Hence, the structure is suitable for putting up on top of an oil well in the ocean.

**6.A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is  $425 \text{ cm}^2$ . What maximum pressure would the smaller piston have to bear?**

**ANSWER:**

The maximum mass of a car that can be lifted,  $m = 3000 \text{ kg}$

Area of cross-section of the load-carrying piston,  $A = 425 \text{ cm}^2 = 425 \times 10^{-4} \text{ m}^2$

The maximum force exerted by the load,  $F = mg$

$$= 3000 \times 9.8$$

$$= 29400 \text{ N}$$

The maximum pressure exerted on the load-carrying piston,  $P = \frac{F}{A}$

$$= \frac{29400}{425 \times 10^{-4}}$$

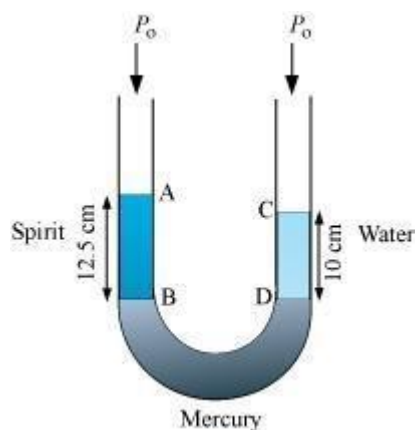
$$= 6.917 \times 10^5 \text{ Pa}$$

Pressure is transmitted equally in all directions in a liquid. Therefore, the maximum pressure that the smaller piston would have to bear is  $6.917 \times 10^5 \text{ Pa}$ .

**7.A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?**

**ANSWER:**

The given system of water, mercury, and methylated spirit is shown as follows:



Height of the spirit column,  $h_1 = 12.5 \text{ cm} = 0.125 \text{ m}$

Height of the water column,  $h_2 = 10 \text{ cm} = 0.1 \text{ m}$   $P_0 =$

Atmospheric pressure  $\rho_1 =$  Density of spirit  $\rho_2 =$

Density of water

Pressure at point B =  $P_0 + h_1 \rho_1 g$

Pressure at point D =  $P_0 + h_2 \rho_2 g$

Pressure at points B and D is the same.

$$P_0 + h_1 \rho_1 g = h_2 \rho_2 g$$



$$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$$

$$= \frac{10}{12.5} = 0.8$$

Therefore, the specific gravity of spirit is 0.8.

**8. In problem 10.9, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury = 13.6)**

**ANSWER:**

Height of the water column,  $h_1 = 10 + 15 = 25$  cm

Height of the spirit column,  $h_2 = 12.5 + 15 = 27.5$  cm

Density of water,  $\rho_1 = 1 \text{ g cm}^{-3}$

Density of spirit,  $\rho_2 = 0.8 \text{ g cm}^{-3}$

Density of mercury =  $13.6 \text{ g cm}^{-3}$

Let  $h$  be the difference between the levels of mercury in the two arms.

Pressure exerted by height  $h$ , of the mercury column:

$$= h\rho g$$

$$= h \times 13.6g \dots (i)$$

Difference between the pressures exerted by water and spirit:

$$= h_1\rho_1g - h_2\rho_2g$$

$$= g(25 \times 1 - 27.5 \times 0.8)$$

$$= 3g \dots (ii)$$

Equating equations (i) and (ii), we get:

$$13.6 h_g = 3g h = 0.220588 \approx$$

$$0.221 \text{ cm}$$

Hence, the difference between the levels of mercury in the two arms is 0.221 cm.

## 8 MARK QUESTIONS

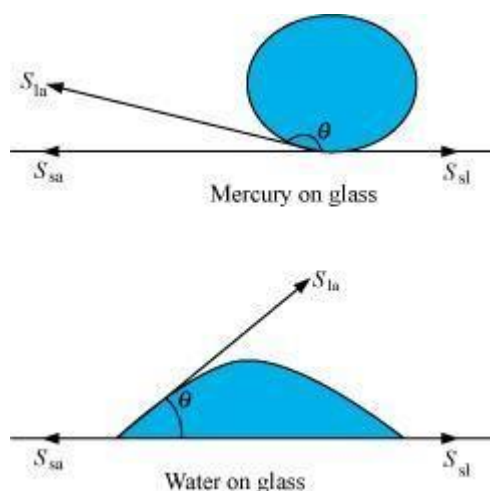
### 1.Explain why

- (a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.
- (b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)
- (c) Surface tension of a liquid is independent of the area of the surface
- (d) Water with detergent dissolved in it should have small angles of contact.
- (e) A drop of liquid under no external forces is always spherical in shape

### ANSWER:

- (a) The angle between the tangent to the liquid surface at the point of contact

and the surface inside the liquid is called the angle of contact ( $\theta$ ), as shown in the given figure.



$S_{la}$ ,  $S_{sa}$ , and  $S_{sl}$  are the respective interfacial tensions between the liquid-air, solid-air, and solid-liquid interfaces. At the line of contact, the surface forces between the three media must be in equilibrium, i.e.,

$$\cos \theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

The angle of contact  $\theta$ , is obtuse if  $S_{sa} < S_{la}$  (as in the case of mercury on glass). This angle is acute if  $S_{sl} < S_{la}$  (as in the case of water on glass).

**(b)** Mercury molecules (which make an obtuse angle with glass) have a strong force of attraction between themselves and a weak force of attraction toward solids. Hence, they tend to form drops.

On the other hand, water molecules make acute angles with glass. They have a weak force of attraction between themselves and a strong force of attraction toward solids. Hence, they tend to spread out.

**(c)** Surface tension is the force acting per unit length at the interface between the plane of a liquid and any other surface. This force is independent of the area of the liquid surface. Hence, surface tension is also independent of the area of the liquid surface.

**(d)** Water with detergent dissolved in it has small angles of contact ( $\theta$ ). This is because for a small  $\theta$ , there is a fast capillary rise of the detergent in the cloth. The

capillary rise of a liquid is directly proportional to the cosine of the angle of contact ( $\theta$ ). If  $\theta$  is small, then  $\cos\theta$  will be large and the rise of the detergent water in the cloth will be fast.

**(e)** A liquid tends to acquire the minimum surface area because of the presence of surface tension. The surface area of a sphere is the minimum for a given volume. Hence, under no external forces, liquid drops always take spherical shape.

**2.Fill in the blanks using the word(s) from the list appended with each statement:**

- (a)** Surface tension of liquids generally . . . with temperatures (increases / decreases)
- (b)** Viscosity of gases. .. with temperature, whereas viscosity of liquids . . . with temperature (increases / decreases)
- (c)** For solids with elastic modulus of rigidity, the shearing force is proportional to . . . , while for fluids it is proportional to . .. (shear strain / rate of shear strain)
- (d)** For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoulli's principle)
- (e)** For the model of a plane in a wind tunnel, turbulence occurs at a ... speed for turbulence for an actual plane (greater / smaller)

**ANSWER:**

**(a)** decreases

The surface tension of a liquid is inversely proportional to temperature.

**(b)** increases; decreases

Most fluids offer resistance to their motion. This is like internal mechanical friction, known as viscosity. Viscosity of gases increases with temperature, while viscosity of liquids decreases with temperature.

**(c)** Shear strain; Rate of shear strain

With reference to the elastic modulus of rigidity for solids, the shearing force is proportional to the shear strain. With reference to the elastic modulus of rigidity for fluids, the shearing force is proportional to the rate of shear strain.

**(d)** Conservation of mass/Bernoulli's principle

For a steady-flowing fluid, an increase in its flow speed at a constriction follows the conservation of mass/Bernoulli's principle.

**(e)** Greater

For the model of a plane in a wind tunnel, turbulence occurs at a greater speed than it does for an actual plane. This follows from Bernoulli's principle and different Reynolds' numbers are associated with the motions of the two planes.

### **3.Explain why**

**(a)** To keep a piece of paper horizontal, you should blow over, not under, it

**(b)** When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers

**(c)** The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection

**(d)** A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel

**(e)** A spinning cricket ball in air does not follow a parabolic trajectory **ANSWER:**

**(a)** When air is blown under a paper, the velocity of air is greater under the paper than it is above it. As per Bernoulli's principle, atmospheric pressure reduces under the paper. This makes the paper fall. To keep a piece of paper horizontal, one should blow over it. This increases the velocity of air above the paper. As per Bernoulli's principle, atmospheric pressure reduces above the paper and the paper remains horizontal.

**(b)** According to the equation of continuity:

$$\text{Area} \times \text{Velocity} = \text{Constant}$$

For a smaller opening, the velocity of flow of a fluid is greater than it is when the opening is bigger. When we try to close a tap of water with our fingers, fast jets of water gush through the openings between our fingers.

This is because very small openings are left for the water to flow out of the pipe. Hence, area and velocity are inversely proportional to each other.

**(c)** The small opening of a syringe needle controls the velocity of the blood flowing out. This is because of the equation of continuity. At the constriction point of the syringe system, the flow rate suddenly increases to a high value for a constant thumb pressure applied.

**(d)** When a fluid flows out from a small hole in a vessel, the vessel receives a backward thrust. A fluid flowing out from a small hole has a large velocity according to the equation of continuity:

$$\text{Area} \times \text{Velocity} = \text{Constant}$$

According to the law of conservation of momentum, the vessel attains a backward velocity because there are no external forces acting on the system.

**(e)** A spinning cricket ball has two simultaneous motions – rotatory and linear. These two types of motion oppose the effect of each other. This decreases the velocity of air flowing below the ball. Hence, the pressure on the upper side of the ball becomes lesser than that on the lower side. An upward force acts upon the ball. Therefore, the ball takes a curved path. It does not follow a parabolic path.

**4. Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is  $4.0 \times 10^{-3} \text{ kg s}^{-1}$ , what is the pressure difference between the two ends of the tube? (Density of glycerine =  $1.3 \times 10^3 \text{ kg m}^{-3}$  and viscosity of glycerine =  $0.83 \text{ Pa s}$ ). [You may also like to check if the assumption of laminar flow in the tube is correct].**

**ANSWER:**

**Answer:**  $9.8 \times 10^2 \text{ Pa}$

Length of the horizontal tube,  $l = 1.5 \text{ m}$

Radius of the tube,  $r = 1 \text{ cm} = 0.01 \text{ m}$

Diameter of the tube,  $d = 2r = 0.02 \text{ m}$

Glycerine is flowing at a rate of  $4.0 \times 10^{-3} \text{ kg s}^{-1}$ .

$M = 4.0 \times 10^{-3} \text{ kg s}^{-1}$

Density of glycerine,  $\rho = 1.3 \times 10^3 \text{ kg m}^{-3}$

Viscosity of glycerine,  $\eta = 0.83 \text{ Pa s}$

Volume of glycerine flowing per sec:

$$V = \frac{M}{\rho}$$

$$= \frac{4.0 \times 10^{-3}}{1.3 \times 10^3}$$

$$= 3.08 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$$

According to Poiseville's formula, we have the relation for the rate of flow:

$$V = \frac{\pi p r^4}{8 \eta l}$$

Where,  $p$  is the pressure difference between the two ends of the tube

$$\therefore p = \frac{V8\eta l}{\pi r^4}$$

$$= \frac{3.08 \times 10^{-6} \times 8 \times 0.83 \times 1.5}{\pi \times (0.01)^4}$$

$$= 9.8 \times 10^2 \text{ Pa}$$

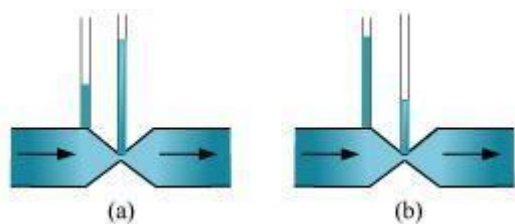
Reynolds' number is given by the relation:

$$R = \frac{4\rho V}{\pi d\eta}$$

$$= \frac{4 \times 1.3 \times 10^3 \times 3.08 \times 10^{-6}}{\pi \times (0.02) \times 0.83} = 0.3$$

Reynolds' number is about 0.3. Hence, the flow is laminar.

**5. Figures 10.23 (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect? Why?**

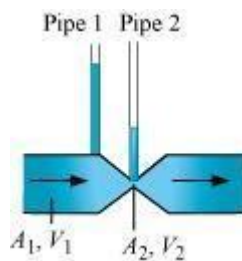


**ANSWER:**

**Answer: (a)**

Take the case given in figure (b).





Where,

$A_1$  = Area of pipe1

$A_2$  = Area of pipe 2

$V_1$  = Speed of the fluid in pipe1

$V_2$  = Speed of the fluid in pipe 2

From the law of continuity, we have:

$$A_1 V_1 = A_2 V_2$$

When the area of cross-section in the middle of the venturimeter is small, the speed of the flow of liquid through this part is more. According to Bernoulli's principle, if speed is more, then pressure is less.

Pressure is directly proportional to height. Hence, the level of water in pipe 2 is less.

Therefore, figure (a) is not possible.

**6. Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is  $7.3 \times 10^{-2} \text{ N m}^{-1}$ . Take the angle of contact to be zero and density of water to be  $1.0 \times 10^3 \text{ kg m}^{-3}$  ( $g = 9.8 \text{ m s}^{-2}$ ).**

**ANSWER:**

Diameter of the first bore,  $d_1 = 3.0 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Hence, the radius of the first bore,  $r_1 = \frac{d_1}{2} = 1.5 \times 10^{-3} \text{ m}$

Diameter of the second bore,  $d_2 = 6.0 \text{ mm}$

Hence, the radius of the second bore,  $r_2 = \frac{d_2}{2} = 3 \times 10^{-3} \text{ m}$

Surface tension of water,  $s = 7.3 \times 10^{-2} \text{ N m}^{-1}$

Angle of contact between the bore surface and water,  $\theta = 0$

Density of water,  $\rho = 1.0 \times 10^3 \text{ kg/m}^{-3}$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

Let  $h_1$  and  $h_2$  be the heights to which water rises in the first and second tubes respectively. These heights are given by the relations:

$$h_1 = \frac{2s \cos \theta}{r_1 \rho g} \quad \dots (i)$$

$$h_2 = \frac{2s \cos \theta}{r_2 \rho g} \quad \dots (ii)$$

The difference between the levels of water in the two limbs of the tube can be calculated as:

$$\begin{aligned} &= \frac{2s \cos \theta}{r_1 \rho g} - \frac{2s \cos \theta}{r_2 \rho g} \\ &= \frac{2s \cos \theta}{\rho g} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] \\ &= \frac{2 \times 7.3 \times 10^{-2} \times 1}{1 \times 10^3 \times 9.8} \left[ \frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}} \right] \\ &= 4.966 \times 10^{-3} \text{ m} \\ &= 4.97 \text{ mm} \end{aligned}$$

Hence, the difference between levels of water in the two bores is 4.97 mm.

## SUMMARY

The study of mechanical properties of fluids is called Hydrostatics. The volume of liquid or gas depends on the pressure acting on it. Since liquids have a fixed volume, the change in volume due to the change in external pressure is less. It is not the same in the case of gases, as they do not have a fixed volume.