Chapter 7

Permutations and Combinations

The curriculum for Class 11 Maths Chapter 7 presents the ideas of permutation and combination in an engaging way. Students will solve the practice questions by applying new mathematical concepts and ideas that they have learned. Use the NCERT solutions for each of the exercises in this chapter to improve the process. Learn how to approach and resolve these issues in order to receive a higher score in

Mathematical ideas called permutations and combinations are used to count and compute various object groupings. Here's a quick rundown:

Permutations:

Permutations refer to the different arrangements of a set of objects.

The number of permutations of n things different things taken r at a time, where repetition is not allowed is denoted by p where n! denotes the factorial of n.

Combinations:

Combinations refer to the different selections of a set of objects, where the order does not matter.

Here n is the total number of objects r is the number of objects to be selected, and n! denotes the fractional of n (the product of all positive integers up to n).

Exercise 7.1

1. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5, assuming that

(i) Repetition of the digits is allowed?

(ii) Repetition of the digits is not allowed?

Solution:

(i) Let the 3-digit number be ABC, where C is at the units place, B at the tens place and A at the hundreds place.

Now, when repetition is allowed,

The number of digits possible at C is 5. As repetition is allowed, the number of digits possible at B and A is also 5 at each.

Hence, the total number possible 3-digit numbers $=5 \times 5 \times 5 =125$

(ii) Let the 3-digit number be ABC, where C is at the units place, B at the tens place and A at the hundreds place.

Now, when repetition is not allowed,

The number of digits possible at C is 5. Suppose one of 5 digits occupies place C; now, as the repletion is not allowed, the possible digits for place B are 4, and similarly, there are only 3 possible digits for place A.

Therefore, the total number of possible 3-digit numbers$=5 \times 4 \times 3=60$

2. How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, and 6 if the digits can be repeated?

Solution:

Let the 3-digit number be ABC, where C is at the unit's place, B at the tens place and A at the hundreds place.

As the number has to be even, the digits possible at C are 2 or 4 or 6. That is, the number of possible digits at C is 3.

Now, as repetition is allowed, the digit possible at B is 6. Similarly, at A, also, the number of digits possible is 6.

Therefore, The total number of possible 3-digit numbers = $6 \times 6 \times 3 = 108$

3. How many 4-letter codes can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

Solution:

Let the 4-letter code be 1234.

In the first place, the number of letters possible is 10.

Suppose any 1 of the ten occupies place 1.

Now, as repetition is not allowed, the number of letters possible at place 2 is 9. Now, at 1 and 2, any 2 of the 10 alphabets have been taken. The number of alphabets left for place 3 is 8, and similarly, the number of alphabets possible at 4 is 7.

Therefore, the total number of 4-letter codes$=10 \times 9 \times 8 \times 7=5040$

4. How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once?

Solution:

Let the five-digit number be ABCDE. Given that the first 2 digits of each number are 67. Therefore, the number is 67CDE.

As repetition is not allowed and 6 and 7 are already taken, the digits available for place C are 0,1,2,3,4,5,8,9. The number of possible digits at place C is 8. Suppose one of them is taken at C; now the digits possible at place D is 7. And similarly, at E, the possible digits are 6.

$\therefore$ The total five-digit numbers with given conditions $= 8 \times 7 \times 6 = 336$

5. A coin is tossed 3 times, and the outcomes are recorded. How many possible outcomes are there?

Solution:

Given A coin is tossed 3 times, and the outcomes are recorded.

The possible outcomes after a coin toss are head and tail.

The number of possible outcomes at each coin toss is 2.

∴ The total number of possible outcomes after 3 times = $2 \times 2 \times 2 = 8$

6. Given 5 flags of different colors, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Solution:

Given 5 flags of different colors.

We know the signal requires 2 flags.

The number of flags possible for the upper flag is 5.

Now, as one of the flags is taken, the number of flags remaining for the lower flag in the signal is 4.

The number of ways in which signal can be given = $5 \times 4 = 20$

Exercise 7.2

1. Evaluate
(i) 8!

(ii) 4! – 3!

Solution:

(i) Consider 8!

We know that $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 40320$

(ii) Consider 4!-3!

$4!-3! = (4 \times 3!) – 3!$

The above equation can be written as

$= 3! (4-1)$

$= 3 \times 2 \times 1 \times 3$

$= 18$

2. Is 3! + 4! = 7!?

Solution:

Consider LHS 3! + 4!

Computing the left-hand side, we get

$3! + 4! = (3 \times 2 \times 1) + (4 \times 3 \times 2 \times 1)$

$= 6 + 24$

$= 30$

Again, considering RHS and computing, we get

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

Therefore, LHS $\neq$ RHS

Therefore, $3! + 4! \neq 7!$

3. Compute

$$\frac{8!}{6! \times 2!}$$

Solution:

Given $\frac{8!}{6! \times 2!}$

Expanding all the factorials and simplifying we get

$$\frac{8!}{6! \times 2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1}$$

$$\Rightarrow \frac{8!}{6! \times 2!} = \frac{8 \times 7}{2} = 28$$

4. If $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$ **find x.**

Solution:

Consider LHS and by computing we get

$$\frac{1}{6!}+\frac{1}{7!} = \frac{1}{6!}+\frac{1}{7 \times 6!}$$

$$\Rightarrow \frac{7+1}{7 \times 6!} = \frac{8}{7!}$$

Equating LHS to RHS to get the value of x

$$\frac{8}{7!} = \frac{x}{8!}$$

$$\Rightarrow \frac{8}{7!} = \frac{x}{8 \times 7!}$$

On rearranging we get

$$\Rightarrow 8 \times 8 = x$$

$$\Rightarrow x = 64.$$

5. Evaluate

$$\frac{n!}{(n-r)!},$$

When
(i) n = 6, r = 2
(ii) n = 9, r = 5

Solution:

(i) Given n = 6 and r = 2

Putting the value of n and r we get

$\frac{6!}{(6-2)!}$

$\Rightarrow \frac{6!}{4!} = \frac{6 \times 5 \times 4!}{4!} = 6 \times 5 = 30.$

(ii) Given n = 9 and r = 5

Putting the value of n and r we get

$\frac{9!}{(9-5)!}$

$\Rightarrow \frac{9!}{4!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!} = 9 \times 8 \times 7 \times 6 \times 5 = 15120.$

Exercise 7.3

1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Solution:

Total number of digits possible for choosing = 9

Number of places for which a digit has to be taken = 3

As there is no repetition allowed,

$\Rightarrow$ No. of permutations = ${}^{9}_{3}P = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9\times8\times7\times6!}{6!} = 504.$

2. How many 4-digit numbers are there with no digit repeated?

Solution:

To find the four-digit number (digits do not repeat),

We will have 4 places where 4-digits are to be put.

So, at the thousand's place = There are 9 ways as 0 cannot be at the thousand's place = 9 ways

At the hundredth's place = There are 9 digits to be filled as 1 digit is already taken = 9 ways

At the ten's place = There are now 8 digits to be filled as 2 digits are already taken = 8 ways

At unit's place = There are 7 digits that can be filled = 7 ways

The total number of ways to fill the four places = 9 × 9 × 8 × 7 = 4536 ways

So, a total of 4536 four-digit numbers can be there with no digits repeated.

3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

Solution:

An even number means that the last digit should be even.

The number of possible digits at one's place = 3 (2, 4 and 6)

⇒ Number of permutations=

$${}^{3}_{1}P = \frac{3!}{(3-1)!} = 3$$

One of the digits is taken at one's place; the number of possible digits available = 5

⇒ Number of permutations=

$${}^{5}_{2}P = \frac{5!}{(5-2)!} = \frac{5\times4\times3!}{3!} = 20.$$

Therefore, the total number of permutations =3 × 20=60

4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, if no digit is repeated. How many of these will be even?

Solution:

Total number of digits possible for choosing = 5

Number of places for which a digit has to be taken = 4

As there is no repetition allowed,

⇒ Number of permutations =

$${}^{5}_{4}P = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120.$$

The number will be even when 2 and 4 are in one's place.

The possibility of (2, 4) at one's place = 2/5 = 0.4

The total number of even numbers = 120 × 0.4 = 48

5. from a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman, assuming one person cannot hold more than one position?

Solution:

Total number of people in committee = 8

Number of positions to be filled = 2

⇒ Number of permutations =
$^8_2P = \frac{8!}{(8-2)!} = \frac{8!}{6!} = 56.$

6. Find n if $^{n-1}P_3 : {}^{n}P_3 = 1: 9$.

Solution:

Given equation can be written as

$$\frac{^{n-1}_{3}P}{^{n}_{4}P} = \frac{1}{9}$$

By substituting the values we get

$$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$$

On simplification

$$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

⇒ n=9.

$\Rightarrow \frac{(n-1)!}{n \times (n-1)!} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9}$

$\Rightarrow$ n=9.

7. Find r if

(i)$^5P_r = 2\,^6P_{r-1}$

(ii) $^5P_r = {}^6P_{r-1}$

Solution:

(i) $^5_rP = 2\;{}^6_{r-1}P$

Substituting the values we get

$\Rightarrow \frac{5!}{(5-r)!} = 2\,\frac{6!}{(7-r)!}$

The above equation can be written as

$\Rightarrow \frac{(7-r)!}{(5-r)!} = 2\,\frac{6!}{5!}$

On simplifying we get

$\Rightarrow (7-r)\,(6-r) = 2\,(6)$

$\Rightarrow 42 - 13r + r^2 = 12$

$\Rightarrow r^2 - 13r + 30 = 0$

$\Rightarrow r^2 - 10r - 3r + 30 = 0$

$\Rightarrow r\,(r - 10) - 3(r - 10) = 0$

$\Rightarrow (r-3)\,(r-10) = 0$

r= 3 or r=10

But r = 10 is rejected, as in ^5_rP, r cannot be greater than 5.

Therefore, r = 3.

(ii) ${}^{5}_{r}P = {}^{6}_{r-1}P$

The above equation can be written as

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$

$\Rightarrow \frac{(7-r)!}{(5-r)!} = \frac{6!}{5!}$

$\Rightarrow (7 - r)\ (6 - r) = 6$

$\Rightarrow 42 - 13r + r^2 = 6$

$\Rightarrow r^2 - 13\ r + 36 = 0$

$\Rightarrow r^2 - 9r - 4r + 36 = 0$

$\Rightarrow r\ (r - 9) - 4(r - 9) = 0$

$\Rightarrow (r-4)\ (r-9) = 0$

r= 4 or r=9

But r=9 is rejected, as in ${}^{5}_{r}P$, r cannot be greater than 5.

Therefore, r=4.

8. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Solution:

Total number of different letters in EQUATION = 8

Number of letters to be used to form a word = 8

$\Rightarrow$ Number of permutations =
${}^{8}_{8}P = \frac{8!}{(8-8)!} = \frac{8!}{0!} = 40320.$

9. How many words, with or without meaning, can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.
(i) 4 letters are used at a time,

(ii) All letters are used at a time,
(iii) All letters are used, but the first letter is a vowel.

Solution:

(i) Number of letters to be used =4

⇒ Number of permutations =
$^{6}_{4}P = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 360.$

(ii) Number of letters to be used = 6

⇒ Number of permutations =
$^{6}_{6}P = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 720.$

(iii) Number of vowels in MONDAY = 2 (O and A)

⇒ Number of permutations in vowel =
$^{2}_{1}P = 2$

Now, the remaining places = 5

Remaining letters to be used =5

⇒ Number of permutations =
$^{5}_{5}P = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 120.$

Therefore, the total number of permutations = 2 × 120 =24

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Solution:

Total number of letters in MISSISSIPPI =11

Letter Number of occurrence

M	1
I	4
S	4
P	2

⇒ Number of permutations = $\frac{11!}{1!4!4!2!} = 34650$

We take that 4 I's come together, and they are treated as 1 letter,

∴ Total number of letters=11 – 4 + 1 = 8

⇒ Number of permutations = $\frac{8!}{1!4!2!} = 840$

Therefore, total number of permutations where four I's don't come together = 34650-840=33810.

11. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) Words start with P and end with S,

(ii) Vowels are all together,
(iii) There are always 4 letters between P and S?

Solution:

(i) Total number of letters in PERMUTATIONS =12

The only repeated letter is T; 2times

The first and last letters of the word are fixed as P and S, respectively.

Number of letters remaining =12 – 2 = 10

⇒ Number of permutations =
$\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$

(ii) Number of vowels in PERMUTATIONS = 5 (E, U, A, I, O)

Now, we consider all the vowels together as one.

Number of permutations of vowels = 120

Now, the total number of letters = 12 – 5 + 1= 8

⇒ Number of permutations =
$\frac{{}^{8}_{8}P}{2!} = \frac{8!}{2(8-8)!} = \frac{8!}{2} = 20160.$

Therefore, the total number of permutations = 120 × 20160 = 2419200

(iii) The number of places is as 1 2 3 4 5 6 7 8 9 10 11 12

There should always be 4 letters between P and S.

Possible places of P and S are 1 and 6, 2 and 7, 3 and 8, 4 and 9, 5 and 10, 6 and 11, 7 and 12

Possible ways =7,

Also, P and S can be interchanged,

No. of permutations =2 × 7 =14

The remaining 10 places can be filled with 10 remaining letters,

∴ No. of permutations =
$\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$

Therefore, the total number of permutations = 14 × 1814400 =25401600

Exercise 7.4

1. If $^nC_8 = {^nC_2}$, find $^nC_2$.

Solution:

Given $^nC_8 = {^nC_2}$

We know that if $^nC_r = {^nC_p}$ then either r = p or r = n – p

Here $^nC_8 = {^nC_2}$

$\Rightarrow 8 = n - 2$

On rearranging we get

$\Rightarrow n = 10$

Now,

$\therefore {^nC_2} = {^{10}C_2} = \frac{10!}{2!(10-2)!}$ $\left(\because {^nC_r} = \frac{n!}{r!(n-r)!}\right)$

$\Rightarrow {^{10}C_2} = \frac{10\times9\times8!}{2\times1\times8!} = \frac{90}{2} = 45$

2. Determine n if
(i) $^{2n}C_3 : {^nC_3} = 12 : 1$
(ii) $^{2n}C_3 : {^nC_3} = 11 : 1$

Solution:

(i) Given: $^{2n}C_3 : {^nC_3} = 12 : 1$

The above equation can be written as

$$\Rightarrow \frac{^{2n}C_3}{^nC_3} = \frac{12}{1}$$

Substituting the formula we get

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n!}{3!\,(2n-3)!}}{\frac{n!}{3!\,(n-3)!}} = \frac{12}{1}$$

Expanding the factorial we get

$$\Rightarrow \frac{\frac{2n \times (2n-1) \times (2n-2) \times (2n-3)!}{3!\,(2n-3)!}}{\frac{n \times (n-1) \times (n-2) \times (n-3)!}{3!\,(n-3)!}} = \frac{12}{1}$$

On simplifying

$$\Rightarrow \frac{\frac{2n \times (2n-1) \times (2n-2)}{3!}}{\frac{n \times (n-1) \times (n-2)}{3!}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times2\times(n-1)}{n\times(n-1)\times(n-2)} = \frac{12}{1}$$

On multiplying we get

$$\Rightarrow \frac{4\times n\times(2n-1)}{n\times(n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{4\times(2n-1)}{(n-2)} = \frac{12}{1}$$

Simplifying and computing

$\Rightarrow 4 \times (2n - 1) = 12 \times (n - 2)$

$\Rightarrow 8n - 4 = 12n - 24$

$\Rightarrow 12n - 8n = 24 - 4$

$\Rightarrow 4n = 20$

$\therefore n = 5$

(ii) Given: ${}^{2n}C_3 : {}^{n}C_3$ =11: 1

$$\Rightarrow \frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)\times(2n-3)!}{3!(2n-3)!}}{\frac{n\times(n-1)\times(n-2)\times(n-3)!}{3!(n-3)!}} = \frac{11}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)}{3!}}{\frac{n\times(n-1)\times(n-2)}{3!}} = \frac{11}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times2\times(n-1)}{n\times(n-1)\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{4\times n\times(2n-1)}{n\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{4\times(2n-1)}{(n-2)} = \frac{11}{1}$$

$\Rightarrow 4\times (2n - 1) = 11 \times (n - 2)$

$\Rightarrow 8n - 4 = 11n - 22$

$\Rightarrow 11n - 8n = 22 - 4$

$\Rightarrow 3n = 18$

$\therefore n = 6$

3. How many chords can be drawn through 21 points on a circle?

Solution:

Given 21 points on a circle.

We know that we require two points on the circle to draw a chord.

∴ The number of chords is are

$\Rightarrow {}^{21}C_2 =$
$$\frac{21!}{2!(21-2)!} = \frac{21\times20\times19!}{2!\times19!} = \frac{21\times20}{2\times1} = \frac{420}{2} = 210$$

∴ The total number of chords that can be drawn is 210

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Solution:

Given 5 boys and 4 girls in total.

We can select 3 boys from 5 boys in ${}^5C_3$ ways.

Similarly, we can select 3 boys from 54 girls in ${}^4C_3$ ways.

∴ The number of ways a team of 3 boys and 3 girls can be selected is ${}^5C_3 \times {}^4C_3$

$\Rightarrow {}^5C_3 \times {}^4C_3 =$
$$\frac{5!}{3!(5-3)!} \times \frac{4!}{3!(4-3)!} = \frac{5!}{3!\times2!} \times \frac{4!}{3!\times1!}$$

$\Rightarrow {}^5C_3 \times {}^4C_3 = 10 \times 4 = 40$

∴ The number of ways a team of 3 boys and 3 girls can be selected is ${}^5C_3 \times {}^4C_3 = 40$ ways

5. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Solution:

Given 6 red balls, 5 white balls and 5 blue balls.

We can select 3 red balls from 6 red balls in ${}^6C_3$ ways.

Similarly, we can select 3 white balls from 5 white balls in ${}^5C_3$ ways.

Similarly, we can select 3 blue balls from 5 blue balls in ${}^5C_3$ ways.

∴ The number of ways of selecting 9 balls is ${}^6C_3 \times {}^5C_3 \times {}^5C_3$

$$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} \times \frac{5!}{3!(5-3)!} = \frac{6!}{3!\times3!} \times \frac{5!}{3!\times2!} \times \frac{5!}{3!\times2!}$$

$$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6\times5\times4\times3!}{3!\times3!} \times \frac{5\times4\times3!}{3!\times2!} \times \frac{5\times4\times3!}{3!\times2!} = \frac{120}{3\times2\times1} \times \frac{20}{2\times1} \times \frac{20}{2\times1} = 20 \times 10 \times 10 = 2000$$

∴ The number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour is ${}^6C_3 \times {}^5C_3 \times {}^5C_3 = 2000$

6. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Solution:

Given a deck of 52 cards.

There are 4 Ace cards in a deck of 52 cards.

According to the question, we need to select 1 Ace card out of the 4 Ace cards.

∴ The number of ways to select 1 Ace from 4 Ace cards is ${}^4C_1$

⇒ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards)

∴ The number of ways to select 4 cards from 48 cards is ${}^{48}C_4$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4!}{1!(4-1)!} \times \frac{48!}{4!(48-4)!} = \frac{4!}{1!\times3!} \times \frac{48!}{4!\times44!}$$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4\times3!}{1!\times3!} \times \frac{48\times47\times46\times45\times44!}{4!\times44!} = \frac{4}{1} \times \frac{4669920}{24} = 4 \times 194580 = 778320$$

∴ The number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination is 778320.

7. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution:

Given 17 players, in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers.

There are 5 players that can bowl, and we can require 4 bowlers in a team of 11.

∴ The number of ways in which bowlers can be selected is: ${}^5C_4$

Now, other players left are = 17 – 5(bowlers) = 12

Since we need 11 players in a team and already 4 bowlers have been selected, we need to select 7 more players from 12.

∴ The number of ways we can select these players is: ${}^{12}C_7$

∴ The total number of combinations possible is: ${}^5C_4 \times {}^{12}C_7$

$$\Rightarrow {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!(5-4)!} \times \frac{12!}{7!(12-7)!} = \frac{5!}{4!\times 1!} \times \frac{12!}{7!\times 5!}$$

$$\Rightarrow {}^5C_4 \times {}^{12}C_7 = \frac{5\times 4!}{1!\times 4!} \times \frac{12\times 11\times 10\times 9\times 8\times 7!}{5!\times 7!} = \frac{5}{1} \times \frac{95040}{120} = 5 \times 792 = 3960$$

∴ The number of ways we can select a team of 11 players where 4 players are bowlers from 17 players is 3960.

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Solution:

Given a bag contains 5 black and 6 red balls

The number of ways we can select 2 black balls from 5 black balls is ${}^5C_2$

The number of ways we can select 3 red balls from 6 red balls is ${}^6C_3$

The number of ways 2 black and 3 red balls can be selected is ${}^5C_2 \times {}^6C_3$

$$\therefore {}^5C_2 \times {}^6C_3 = \frac{5!}{2!(5-2)!} \times \frac{6!}{3!(6-3)!} = \frac{5!}{2!\times 3!} \times \frac{6!}{3!\times 3!}$$

$$\Rightarrow {}^5C_2 \times {}^6C_3 = \frac{5\times 4\times 3!}{2!\times 3!} \times \frac{6\times 5\times 4\times 3!}{3!\times 3!} = \frac{20}{2} \times \frac{120}{6} = 10 \times 20 = 200$$

∴ The number of ways in which 2 black and 3 red balls can be selected from 5 black and 6 red balls is 200.

9. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Solution:

Given 9 courses are available and 2 specific courses are compulsory for every student.

Here, 2 courses are compulsory out of 9 courses, so a student needs to select 5 – 2 = 3 courses

∴ The number of ways in which 3 ways can be selected from 9 – 2(compulsory courses) = 7 are ${}^7C_3$

$$\therefore {}^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!\times 4!}$$

$$\Rightarrow {}^7C_3 = \frac{7\times 6\times 5\times 4!}{3!\times 4!} = \frac{210}{6} = 35$$

∴ The number of ways a student selects 5 courses from 9 courses where 2 specific courses are compulsory is 35.

2Marks Questions & Answers

1. Find the value of n such that

(i) ${}^nP_5 = 42\ {}^nP_3 = n>4$ (ii) $\frac{nP4}{n-1P4} = \frac{5}{3}$ n>4

Ans: (i) Given that

${}^nP_5 = 42\ {}^nP_3$

Or n (n – 1) (n – 2) (n – 3) (n – 4) = 42 n (n – 1) (n – 2)

Since n > 4 so n (n – 1) (n – 2) ≠ 0

Therefore, by dividing both sides by n (n – 1) (n – 2), we get

(n – 3 (n – 4) = 42

Or n^2 – 7n – 30 = 0

Or n^2 – 10n + 3n – 30

Or (n – 10) (n + 3) = 0

Or n – 10 = 0 or n + 3 = 0

Or n = 10 or n = – 3

As *n* cannot be negative, so *n* = 10.

(ii) Given that $\frac{\mathbf{nP4}}{\mathbf{n-1P4}} = \frac{\mathbf{5}}{\mathbf{3}}$

Therefore 3n (n – 1) (n – 2) (n – 3) = 5(n – 1) (n – 2) (n – 3) (n – 4)

Or 3n = 5 (n – 4) [as (n – 1) (n – 2) (n – 3) ≠ 0, n > 4]

Or n = 10.

2. Find r, if 5 ^4P_r = 6 $^5P_{r-1}$.

Ans: We have 5 ^4P_r = 6 $^5P_{r-1}$

Or 5× $\frac{4!}{(4-r)!}$ = 6× $\frac{5!}{(5-r+1)!}$

Or $\frac{5!}{(4-r)!} = \frac{6 \times 5!}{(5-r+1)(5-r)(5-r-1)!}$

Or (6 – r) (5 – r) = 6

Or r^2 – 11r + 24 = 0

Or r^2 – 8r – 3r + 24 = 0

Or (r – 8) (r – 3) = 0

Or r = 8 or r = 3.

Hence r = 8, 3.

3. If $^nC_9 = {}^nC_8$ Find $^nC_{17}$.

Ans: We have

$$^nC_9 = {}^nC_8$$

i.e., $$\frac{n!}{9!(n-9)!} = \frac{n!}{(4-8)!8!}$$

Or $\frac{1}{9} = \frac{1}{n-8}$ or n – 8 = 9 or n = 17

Therefore $^nC_{17} = {}^nC_{17} = 1$.

4. How many numbers greater than 1000000 can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

Ans: Since, 1000000 is a 7-digit number and the number of digits to be used is also 7. Therefore, the numbers to be counted will be 7-digit only. Also, the numbers have to be greater than 1000000, so they can begin either with 1, 2 or 4.

The number of numbers beginning with $1 = \frac{6!}{3!2!} = \frac{4\times5\times6}{2} = 60$, as when 1 is fixed at the extreme left position, the remaining digits to be rearranged will be 0, 2, 2, 2, 4, 4, in which there are 3, 2s and 2, 4s.

Total numbers beginning with

$$2 = \frac{6!}{2!2!} = \frac{3\times4\times5\times6}{2} = 180.$$

And total numbers beginning with

$$4 = \frac{6!}{3!} = 4\times5\times6 = 120.$$

5. In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

Ans: Let us first seat the 5 girls. This can be done in 5! Ways. For each such arrangement, the three boys can be seated only at the cross marked places.

$$\times\ G \times G \times G \times G \times G \times.$$

There are 6 cross marked places and the three boys can be seated in $^6P_3$ ways.

Hence, by multiplication principle, the total number of ways

$$= 5! \times {}^6P_3 = 5! \times \frac{6!}{3!}$$

$$= 4 \times 5 \times 2 \times 3 \times 4 \times 5 \times 6 = 14400.$$

6. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Ans: Here, order does not matter. Therefore, we need to count combinations. There will be as many committees as there are combinations of 5 different persons taken 3 at a time.

Hence, the required number of ways = ${}^5C_3 = \frac{5!}{3!2!} = \frac{4 \times 5}{2} = 10$.

Now, 1 man can be selected from 2 men in ${}^2C_1$ ways and 2 women can be selected from 3 women in ${}^3C_2$ ways.

Therefore, the required number of committees.

$$= {}^2C_1 \times {}^3C_2 = \frac{2!}{1!1!} \times \frac{3!}{2!1!} = 6.$$

7. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE?

Ans: In the word INVOLUTE, there are 4 vowels, namely, I, O, E, U and 4 consonants, namely, N, V, L and T.

The number of ways of selecting 3 vowels out of 4 = ${}^4C_3$ = 4.

The number of ways of selecting 2 consonants out of 4 = ${}^4C_2$ = 6.

Therefore, the number of combinations of 3 vowels and 2 consonants is 4 × 6 = 24.

Now, each of these 24 combinations has 5 letters which can be arranged among themselves in 5! Ways.

Therefore, the required number of different words is 24 × 5! = 2880.

8. If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

Ans: In the given word EXAMINATION, there are 11letters out of which, A,I and N 2times and all the other letters appears only once

The words that will be listed before the words starting with E in a dictionary will be the words start with A only

Therefore to get the number of words starting with A, the letter A is fixed at the extreme left position, and then the remaining 10letters taken all at a time are rearranged.

Since there are 2 Is and 2Ns in the remaining 10letters,

Number of words starting with A = $\frac{10!}{2!2!} = 907200.$

Thus the required numbers of words is 907200.

9. Determine the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Ans: From a deck of 52cards, 5-card combinations have to be made in such a way that in each selection of 5cards, there is exactly one king.

In a deck of 52cards there are 4 kings.

1king can be selected out of 4kings in ${}^4C_1$ ways.

4Cards out of the remaining 48cards can be selected in ${}^{48}C_4$ ways

Thus the required number of 5-card combinations is ${}^4C_1 \times {}^{48}C_4$.

10. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Ans: 4Men and 4Women should be seated in a row such that the women occupy the even places.

The 5men can be seated in 5! Ways. For each arrangement, the 4women can be seated only at the cross marked places (so that women occupy the even places).

$$M \times M \times M \times M \times M$$

Therefore, the women can be seated in 4! Ways.

Thus, possible number of arrangements = 4!× 5! = 24 ×120 = 2880.

Multiple Choice Questions

1) The value of P (n, n-1) is

a. n
b. n!
c. 2n
d. 2n!

Answer: (b) n!

Explanation: We know that $P(n, r) = {}^nP_r = n!/(n-r)!$

Hence, $P(n, n-1) = {}^nP_{n-1} = n!/[n-(n-1)]!$

$P(n, n-1) = n!/(n-n+1)! = n!/1! = n!$

Therefore, the value of P(n, n-1) is n!.

Hence, the correct answer is option (b) n!

2) The number of ways in which 8 students can be seated in a line is

a. 5040
b. 50400
c. 40230
d. 40320

Answer: (d) 40320

Explanation: For the 1st position, there are 8 possible choices. For the 2nd position, there are 7 possible choices. For the 3rd position, there are 7 possible choices, etc. And for the eighth position, there is only one possible choice. Hence, this can be written as 8!

(i.e.) $8! = 8\times7\times6\times5\times4\times3\times2\times1=40,320$

Hence, the number of ways in which 8 students can be seated in a line is 40320.

3) If $^{n}P_{5} = 60^{n-1}P_{3}$, the value of n is

a. 6
b. 10
c. 12
d. 16

Answer: (b) 10

Explanation:

Given that $^{n}P_{5} = 60^{n-1}P_{3}$

We know that P (n, r) = $^{n}P_{r}$ = n!/(n-r)!

Now, apply the formula on both sides to get the value of n.

n!/(n-5)! = 60 [(n-1)!/[(n-1)-3]!]

On solving the above equation, we get n= -6 and n=10.

Since the value of n cannot be negative, the value of n is 10.

Hence, option (b) 10 is the correct answer.

4) The number of squares that can be formed on a chessboard is

a. 64
b. 160
c. 204
d. 224

Answer: (c) 204

Explanation:

1×1 grid squares = 8×8 = 64, 2×2 grid squares = 7×7 = 49, 3×3 grid squares = 6×6 = 36 up to 8×8 grid squares = 1×1 = 1.

Hence, the total number of squares that can be formed on a chess board = $8^2+7^2+6^2+\ldots+1^2$

= $1^2+2^2+3^2+\ldots+8^2$

= [n(n+1)(2n+1)]/6

Here, n=8

Hence,

= [8(8+1)(16+1)]/6

= (8×9×17)/6

= 12×17 = 204

Hence, option (c) 204 is the correct answer.

5) The number of ways 4 boys and 3 girls can be seated in a row so that they are alternate is

a. 12
b. 104
c. 144
d. 256

Answer: (c) 144

Explanation: Given that, there are 4 boys and 3 girls.

The only pattern 4 boys and 3 girls are arranged in an alternate way is BGBGBGB.

Therefore, the total number of ways is 4!×3! = 144.

6) The number of ways 10 digit numbers can be written using the digits 1 and 2 is

a. 2^{10}
b. $^{10}C_2$
c. 10!
d. $^{10}C_1+^{9}C_2$

Answer: (a) 2^{10}

Explanation: Given digits are 1 and 2.

Here, each place can be filled in two ways either with 1 or 2 and every place has two chances.

Therefore, the number of ways 10 digit numbers can be written using the digits 1 and 2 is 2^{10}.

Hence, option (a) 2^{10} is the correct answer.

7) A coin is tossed n times, the number of all the possible outcomes is

a. 2n
b. 2^n
c. C(n, 2)
d. P(n, 2)

Answer: (b) 2^n

Explanation:

We know that, when a coin is tossed, we will get either head or tail.

Therefore, the number of all possible outcomes when a coin is tossed n times is 2^n.

8) In how many ways 8 distinct toys can be distributed among 5 children?

a. ${}^8P_5$
b. ${}^5P_8$
c. 5^8
d. 8^5

Answer: (c) 5^8

Explanation:

Given that, the number of toys = 8

The number of children = 5.

Hence, the number of ways 8 distinct toys can be distributed among 5 children is $5\times5\times5\times5\times5\times5\times5\times5=5^8$.

Hence, option (c) 5^8 is the correct answer.

9) There are 10 true-false questions in an examination. These questions can be answered in:

a. 20 ways
b. 100 ways
c. 512 ways
d. 1024 ways

Answer: (d) 1024 ways

Explanation:

Given that there are 10 questions.

Each question can be answered in two ways. (i.e. either true or false).

Hence, the number of ways these questions can be answered is 2^{10}, which is equal to 1024.

10) In how many ways can we paint the six faces of a cube with six different colours?

a. 30
b. 6
c. 6!
d. None of the above

Answer: (a) 30

Explanation:

Now, let us consider the 6 different colors: c1, c2, c3, c4, c5, c6.

Assume that the face of the cube facing up is c1. So, the face of the cube at the bottom can be painted in 5 different ways.

So, 4 faces on the horizontal side of the cube are in circular permutation and they can be painted in (4-1)! ways.

Hence, the total number of ways we can paint the faces of a cube with six different colours is $5\times(4-1)!$ ways.

Hence, $5\times(4-1)!$ ways $= 5\times3! = 5\times3\times2\times1 = 30$ ways.

Summary

- Fundamental principle of counting If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is m × n.
- The number of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by $^{n}p_{r}$ and is given by

  $$^{n}p_{r} = \frac{n!}{(n-r)!}, \text{ where } 0 \leq r \leq n.$$

- $n! = 1 \times 2 \times 3 \times \ldots \times n$
- $n! = n \times (n - 1)\,!$
- The number of permutations of n different things, taken r at a time, where repetition is allowed, is n^{r} .
- The number of permutations of n objects taken all at a time, where p_1 are of first kind, p_2 objects are of the second kind, ..., p_k objects are of the k^{th} kind and rest, if any, are all different is $\frac{n!}{p1!p2!\ldots pk!}$.
- The number of combinations of n different things taken r at a time, denoted by $^{n}c_{r}$ is given by $^{n}c_{r} = \frac{n!}{r!(n-r)!}$, $0 \leq r \leq n$.