CHAPTER-11

Direct and Indirect proportions

2MARK Q&A

Exercise 11.1

1. Which of the following are in inverse proportion?

(i) The number of workers on a job and the time to complete the job.

(ii) The time taken for a journey and the distance travelled at a uniform speed.

(iii) Area of cultivated land and the crop harvested.

(iv) The time taken for a fixed journey and the speed of the vehicle.

(v) The population of a country and the area of land per person.

Solution:

(i) The number of workers and the time to complete the job is in inverse proportion because less workers will take more time to complete a job, and more workers will take less time to complete the same job.

(ii) Time and distance covered in direct proportion.

(iii) It is a direct proportion because more are of cultivated land will yield more crops.

(iv) Time and speed are in inverse proportion because if time is less, speed is more.

(v) It is an inverse proportion. If the population of a country increases, the area of land per person decreases.

2. In a Television game show, the prize money of Rs.1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners:

No. of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs)	1,00,000	50,000	----	----	----	----	----

Solution:

Here, the number of winners and prize money are in inverse proportion because winners are increasing, and prize money is decreasing.

When the number of winners is 4, each winner will get =100000/4 = Rs. 25,000

When the number of winners is 5, each winner will get =100000/5 = Rs. 20,000

When the number of winners is 8, each winner will get =100000/8 = Rs. 12,500

When the number of winners is 10, each winner will get = 100000/10 = Rs. 10,000

When the number of winners is 20, each winner will get = 100000/20 = Rs. 5,000

3. Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table:

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	...	...	...

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40 degree?

Solution:

Here, the number of spokes is increasing, and the angle between a pair of consecutive spokes is decreasing. So, it is an inverse proportion, and the angle at the centre of a circle is 360 degree.

When the number of spokes is 8, then the angle between a pair of consecutive spokes = 360/8 = 45 degree

When the number of spokes is 10, then the angle between a pair of consecutive spokes = 360/10= 36 degree.

When the number of spokes is 12, then the angle between a pair of consecutive spokes = 360/12 = 30 degree.

No. of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

(i) Yes, the number of spokes and the angles formed between a pair of consecutive spokes is in inverse proportion.

(ii) When the number of spokes is 15, then the angle between a pair of consecutive spokes = 360/15= 24 degree.

(iii) The number of spokes would be needed = 360/40 = 9

4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of children is reduced by 4?

Solution:

Each child gets = 5 sweets

24 children will get 24×5 = 120 sweets.

Total number of sweets = 120

If the number of children is reduced by 4, then children left = 24-4 = 20

Now, each child will get sweets = 120/20 = 6 sweets

5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

Solution:

Let the number of days be x.

Total number of animals = 20+10 = 30

Animals	20	30
Days	6	x

Here, the number of animals and the number of days are in inverse proportion.

$$\frac{20}{30}=\frac{x}{6}$$

$$30\times x=20\times 6$$

$$x=\frac{20\times 6}{30}$$

x = 4

Hence, the food will last for four days.

6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If he uses 4 persons instead of three, how long should they take to complete the job?

Solution:

Let the time taken to complete the job be x.

Persons	3	4
Days	4	x

Here, the number of persons and the number of days are in inverse proportion.

¾ = x/4

3×4 = 4x

x = 3×4/4

x = 3

Hence, 4 persons will complete the job in 3 days.

7. A batch of bottles was packed in 25 boxes, with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?

Solution:

Let the number of boxes be x.

No. of bottles in each box	12	20
Boxes	25	x

Here, the number of bottles and the number of boxes are in inverse proportion.

$12/20 = x/25$

$12 \times 25 = 20x$

$x = 12 \times 25/20 = 15$

Hence, 15 boxes would be filled.

8. A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Solution:

Let the number of machines required be x.

Days	63	54
Machines	42	x

Here, the number of machines and the number of days are in inverse proportion.

$63/54 = x/42$

$63 \times 42 = 54x$

$x = 63 \times 42/54$

$x = 49$

Hence, 49 machines would be required.

9. A car takes 2 hours to reach a destination by travelling at the speed of 60 km/hr. How long will it take when the car travels at the speed of 80 km/hr?

Solution :

Let the number of hours be x.

Speed (in km/hr)	60	80
Time (in hours)	2	x

Here, the speed of the car and time are in inverse proportion.

60/80 = x/2

60×2 = 80x

x = 60×2/80

x = 3/2 = $1\frac{1}{2}$ hrs.

Hence the car will take $1\frac{1}{2}$ hours to reach its destination.

10. Two persons could fit new windows in a house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now?

(ii) How many persons would be needed to fit the windows in one day?

Solution:

(i) Let the number of days be x.

Persons	2	1
Days	3	x

Here, the number of persons and the number of days are in inverse proportion.

2/1 = x/3

6 = x

Or

x = 6 days

(ii) Let the number of persons be x.

Persons	2	x
Days	3	1

Here, the number of persons and the number of days are in inverse proportion.

2/x = 1/3

6 = x

Or

x = 6 persons

11. A school has 8 periods a day, each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Solution:

Let the duration of each period be x.

Period	8	9
Duration of period (in minutes)	45	x

Here, the number of periods and the duration of periods are in inverse proportion.

8/9 = x/45

8×45 = 9x

x = 40

Hence, the duration of each period would be 40 minutes.

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Exercise 11.2

1. The following are the car parking charges near a railway station up to,

4 hours – Rs.60

8 hours – Rs.100

12 hours – Rs.140

24 hours – Rs.180

Check if the parking charges are in direct proportion to the parking time.

Solution:

Charges per hour:

C1 = 60/4 = Rs. 15

C2 = 100/8 = Rs. 12.50

C3 = 140/12 = Rs. 11.67

C4 = 180/24 = Rs.7.50

Here, the charges per hour are not the same, i.e. C1 ≠ C2 ≠ C3 ≠ C4

Therefore, the parking charges are not in direct proportion to the parking time.

2. A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of the base. In the following table, find the parts of the base that need to be added.

Parts of red pigment	1	4	7	12	20
Parts of base	8	----	----	----	----

Solution:

Let the ratio of parts of red pigment and parts of the base be a/b.

Case 1: Here, $a_1 = 1$, $b_1 = 8$

$a_1/b_1 = 1/8 = k$ (say)

Case 2: When $a_2 = 4$, $b_2 = ?$

$$k = \frac{a_2}{b_2}$$

$b_2 = a_2/k = 4/(1/8) = 4 \times 8 = 32$

Case 3: When $a_3 = 7$, $b_3 = ?$

$$k = \frac{a_3}{b_3}$$

$b_3 = a_3/k = 7/(1/8) = 7 \times 8 = 56$

Case 4: When $a_4 = 12$, $b_4 = ?$

$$k = \frac{a_4}{b_4}$$

$b_4 = a_4/k = 12/(1/8) = 12 \times 8 = 96$

Case 5: When $a_5 = 20$, $b_5 = ?$

$$k = \frac{a_5}{b_5}$$

$b_5 = a_5/k = 20/(1/8) = 20 \times 8 = 160$

When combining results for all the cases, we get

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

3. In Question 2 above, if 1 part of a red pigment requires 75 mL of the base, how much red pigment should we mix with 1800 mL of the base?

Solution:

Let the parts of red pigment mix with 1800 mL base be x.

Parts of red pigment	1	x
Parts of base	75	1800

Since it is in direct proportion,

$$\frac{1}{75} = \frac{x}{1800}$$

$$\Rightarrow$$

$$75 \times x = 1 \times 1800$$

$$x = \frac{1 \times 1800}{75} = 24$$

Hence, with the base 1800 mL, 24 parts of the red pigment should be mixed.

4. A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Solution:

Let the number of bottles filled in five hours be x.

Here, the ratio of hours and bottles is in direct proportion.

$$\frac{6}{840} = \frac{5}{x}$$

$6x = 5 \times 840$

$x = 5 \times 840/6 = 700$

Hence, the machine will fill 700 bottles in five hours.

5. A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm, as shown in the diagram. What is the actual length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?

Solution:

Let the enlarged length of bacteria be x.

Actual length of bacteria = $5/50000 = 1/10000$ cm = 10^{-4} cm

Length	5	x
Enlarged length	50,000	20,000

Here, the length and enlarged length of bacteria are in direct proportion.

$$\frac{5}{50000} = \frac{x}{20000}$$

$$\Rightarrow$$

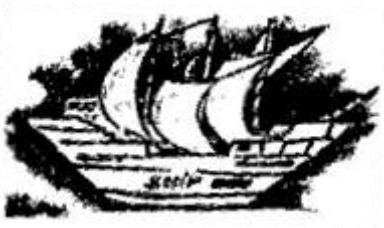
$$x \times 50000 = 5 \times 20000$$

$$x = \frac{5 \times 20000}{50000}$$

x= 2cm

Hence, the enlarged length of bacteria is 2 cm.

6. In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?



Solution:

Let the length of the model ship be x.

Length of actual ship (in m)	12	28
Length of model ship (in cm)	9	x

Here, the length of the mast and the actual length of the ship are in direct proportion.

$$\frac{12}{9} = \frac{28}{x}$$

$\Rightarrow$

$x \times 12 = 28 \times 9$

$x = \frac{28 \times 9}{12}$

x = 21 cm

Hence, the length of the model ship is 21 cm.

7. Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in

(i) 5 kg of sugar? (ii) 1.2 kg of sugar?

Solution:

(i) Let sugar crystals be x.

Weight of sugar (in kg)	2	5
No. of crystals	9×10^6	x

Here, the weight of sugar and the number of crystals are in direct proportion.

$\frac{2}{9 \times 10^6} = \frac{5}{x}$

$\Rightarrow$

$x \times 2 = 5 \times 9 \times 10^6$

$x = \frac{5 \times 9 \times 10^6}{2}$

=

$22.5 \times 10^6 = 2.25 \times 10^7$

Hence, the number of sugar crystals is 2.25×10^7.

(ii) Let sugar crystals be x.

Here, the weight of sugar and the number of crystals are in direct proportion.

Weight of sugar (in kg)	2	1.2
No. of crystals	9×10^5	x

$$\frac{2}{9\times10^5}=\frac{1.2}{x}$$

$$\Rightarrow$$

$$x\times2=1.2\times9\times10^5$$

$$x=\frac{1.2\times9\times10^5}{2}$$

$$=$$

$$0.6\times9\times10^5=5.4\times10^5$$

Hence, the number of sugar crystals is 5.4×10^6.

8. Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on the road for 72 km. What would be her distance covered on the map?

Solution:

Let the distance covered in the map be x.

Actual distance (in km)	18	72
Distance covered in map (in cm)	1	x

Here, the actual distance and distance covered in the map are in direct proportion.

$$\frac{18}{1}=\frac{72}{x}$$

$$\Rightarrow$$

$$x\times 18=72\times 1$$

$$x=\frac{72\times 1}{18}$$

x = 4 cm

Hence, the distance covered on the map is 4 cm.

9. A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time (i) the length of the shadow cast by another pole 10 m 50 cm high (ii) the height of a pole which casts a shadow 5 m long.

Solution:

Here, the height of the pole and the length of the shadow are in direct proportion.

And 1 m = 100 cm

5 m 60 cm = 5×100+60 = 560 cm

3 m 20 cm = 3×100+20 = 320 cm

10 m 50 cm = 10×100+50 = 1050 cm

5 m = 5×100 = 500 cm

(i) Let the length of the shadow of another pole be x.

Height of pole (in cm)	560	1050
Length of shadow (in cm)	320	x

$$\frac{560}{320}=\frac{1050}{x}$$

$$\Rightarrow$$

$$x\times 560=1050\times 320$$

$$x=\frac{1050\times 320}{560}$$

x= 600 cm = 6m

Hence, the length of the shadow of another pole is 6 m.

(ii) Let the height of the pole be x.

Height of pole (in cm)	560	x
Length of shadow (in cm)	320	500

$$\frac{560}{320}=\frac{x}{500}$$

$$\Rightarrow$$

$$x\times 320=560\times 500$$

$$x=\frac{560\times 500}{320}$$

= 875 cm = 8 m 75 cm

Hence, the height of the pole is 8 m 75 cm.

10. A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

Solution:

Let the distance covered in 5 hours be x km.

1 hour = 60 minutes

Therefore, 5 hours = 5×60 = 300 minutes

Distance (in km)	14	x
Time (in minutes)	25	300

Here, the distance covered and time are in direct proportion.

$$\frac{14}{25} = \frac{x}{300}$$

$\Rightarrow 25x = 300(14)$

$$x = \frac{14 \times 300}{25}$$

x = 168

Therefore, the truck can travel 168 km in 5 hours.

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Exercise 11.3

Multiple-choice questions and answers:

Question 1: Which of the following is an example of direct proportion?

a) The time it takes to travel a certain distance at a constant speed.

b) The cost of buying more items at a fixed price per item.

c) The amount of flour needed to bake a cake as the number of eggs increases.

d) The temperature of a liquid decreases as it is heated.

Answer 1:

a) The time it takes to travel a certain distance at a constant speed.

Question 2: If the price of 3 books is $30, what is the price of 7 books if the relationship between the number of books and the cost is directly proportional?

a) $70

b) $21

c) $10

d) $90

Answer 2:

a) $70

Question 3:In an indirect proportion, as one quantity increases, the other quantity:

a) Decreases

b) Stays the same

c) Increases

d) Fluctuates

Answer 3:

a) Decreases

Question 4: If the length of a rectangle is doubled, and the area remains the same, what type of proportion is this?

a) Direct

b) Indirect

c) Neither

d) Inverse

Answer 4:

a) Direct

Question 5: The speed of a car is inversely proportional to the time it takes to travel a certain distance. If the car's speed is doubled, what happens to the time it takes to cover the same distance?

a) It is halved.

b) It remains the same.

c) It doubles.

d) It quadruples.

Answer 5:

a) It is halved.

Question 6: If the number of workers on a construction site is inversely proportional to the time it takes to complete a project, and it takes 10 workers 12 days to finish, how many days would it take if 8 workers were used?

a) 9 days

b) 15 days

c) 6 days

d) 16 days

Answer 6:

b) 15 days

Question 7: A car rental company charges $40 per day for renting a car. Which type of proportion is this?

a) Direct

b) Indirect

c) Both

d) Neither

Answer 7:

a) Direct

Question 8: If the number of students per classroom is directly proportional to the number of classrooms in a school, and there are 30 students in each classroom, how many students are there in a school with 6 classrooms?

a) 60 students

b) 120 students

c) 180 students

d) 210 students

Answer 8:

b) 120 students

Question 9: If the price of a shirt is directly proportional to the number of shirts purchased, and 4 shirts cost $60, how much will 6 shirts cost?

a) $80

b) $90

c) $120

d) $45

Answer 9:

b) $90

Question 10: A car travels 240 miles in 4 hours. If the car's speed is directly proportional to the time it takes to travel, what is the car's speed in miles per hour (mph)?

a) 60 mph

b) 80 mph

c) 40 mph

d) 120 mph

Answer 10:

a) 60 mph

Question 11: If the price of a book is inversely proportional to the number of books purchased, and 6 books cost $48, how much will 8 books cost?

a) $64

b) $36

c) $60

d) $72

Answer 11:

b) $36

Question 12: The amount of work done is directly proportional to the number of workers employed. If 5 workers can complete a project in 10 days, how many workers are needed to complete the same project in 4 days?

a) 20 workers

b) 8 workers

c) 12 workers

d) 16 workers

Answer 12:

a) 20 workers

Question 13: If the cost of a meal at a restaurant is inversely proportional to the number of people sharing it, and a meal for 2 people costs $40, how much will a meal for 4 people cost?

a) $80

b) $20

c) $60

d) $10

Answer 13:

b) $20

Question 14: If the length of a rectangular garden is directly proportional to its width, and a garden with a length of 6 meters has a width of 2 meters, what is the width of a garden with a length of 9 meters?

a) 3 meters

b) 4 meters

c) 6 meters

d) 12 meters

Answer 14:

a) 3 meters

Question 15: The time it takes to complete a 500-mile journey is inversely proportional to the average speed of a vehicle. If the journey takes 5 hours at a certain speed, how many hours will it take at twice that speed?

a) 2.5 hours

b) 10 hours

c) 20 hours

d) 7.5 hours

Answer 15:

a) 2.5 hours

Exercise 11.4

1. In a direct proportion, as one quantity increases, the other _____.

- Answer: Increases

2. If (x) is directly proportional to (y) and (x = 4) when (y = 6), then when (y = 9), (x) will be _____.

- Answer: (x = 6)

3. In an indirect proportion, as one quantity increases, the other _____.

- Answer: Decreases

4. If (x) is inversely proportional to (y) and (x = 12) when (y = 3), then when (y = 4), (x) will be _____.

- Answer: (x = 9)

5. In a direct proportion, the ratio of the two quantities remains _____.

- Answer: Constant

6. If 10 workers can complete a task in 8 days, then 5 workers can complete the same task in _____ days (assuming direct proportionality).

- Answer: 16 days

7. In an indirect proportion, the product of the two quantities remains _____.

- Answer: Constant

8. If 15 men can build a wall in 10 days, then 30 men can build the same wall in _____ days (assuming indirect proportionality).

- Answer: 5 days

Summary

Direct Proportion:

1. Definition: In a direct proportion, two quantities change in the same direction. As one quantity increases (or decreases), the other also increases (or decreases) in a consistent ratio.

2. Rule: If (x) is directly proportional to (y), it can be represented as (x = ky) where (k) is the constant of proportionality.

3. Example: The more hours you work, the more money you earn. Here, the hours worked and the money earned are directly proportional.

4. Characteristics: In direct proportion, when one quantity doubles, the other also doubles, and if one quantity becomes half, the other becomes half, and so on.

Indirect Proportion:

1. Definition: In an indirect (or inverse) proportion, as one quantity increases (or decreases), the other quantity decreases (or increases) in a consistent ratio.

2. Rule: If (x) is inversely proportional to (y), it can be represented as (x = ky) where (k) is the constant of proportionality.

3. **Example:** The time taken to complete a job is inversely proportional to the number of people working on it. More workers mean less time to finish the job.

4. Characteristics: In an indirect proportion, when one quantity doubles, the other is halved, and if one quantity triples, the other becomes one-third, and so on.

Key Points:

1. Direct and indirect proportions are fundamental concepts in mathematics used to solve problems involving varying quantities.

2. Direct proportions are observed when two quantities change in the same direction, while indirect proportions occur when two quantities change in opposite directions.

3. Proportionality is often expressed as ratios or equations involving constants that relate the varying quantities.

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