System of Particles & Rotational Motion Important Questions

Short Questions and Answers [2 Marks Questions]

Q1: What will be the effect on the day-length if the polar ice caps of the planet melt?

Ans: Melting of the polar ice caps of the Earth will produce a huge amount of water that would spread around the planet going farther away from the axis of rotation. This will increase the radius of gyration and hence M.I. To conserve angular momentum, the angular velocity ω(omega) will decrease. So the length of the day($T= 2\pi/\omega$) will increase.

Q2: What is the difference between the center of gravity and C.M?

Ans: The difference between the center of gravity and C.M. is as follows-

Centre of gravity	C.M
The Center of Gravity is the point where the whole body weight is supposed to be concentrated i.e. on this point, the resultant of the gravitational force on all the particles of the body acts.	C.M. refers to the point where the whole of the mass of the body may be supposed to be concentrated to describe its motion as a particle.

Q3: Suppose you have been given two spheres of the same mass and radius, one is solid and the other is hollow. Which of them has a larger moment of inertia about its diameter?

Ans: The hollow sphere will have a greater M.I. value as its entire mass is concentrated at the boundary which is the maximum distance from its axis.

Q4: What is a rigid body?

Ans: A rigid body refers to an object in which the distance between all the constituting particles remains fixed under any influence of external force. Therefore, a rigid body conserves its shape during motion.

Q5: Distinguish between internal and external forces.

Ans: The mutual forces between the particles of a particular system are called internal forces.

The forces exerted by some kind of external source on the particles of the system are to be named external forces.

Q6: Why are spokes fitted in the cycle wheel?

Ans: The cycle wheel is generally constructed in a way that the M.I of the wheel can be increased with minimum possible mass. This can be achieved by using spokes and the M.I is increased to ensure the uniform speed.

Q7: Why cannot a single force balance the torque?

Ans: The effect of torque can be seen to produce angular acceleration and its effect is entirely different from that of the force that causes linear acceleration. Therefore, a single force cannot balance the torque.

Q8.) Prove that the cross product of two parallel vectors is zero.

Ans: AB=AB sinx

If A and B are parallel to each other,

Then, =0°

AB=0

Q9.) A rope of negligible mass is wound around a hollow cylinder of mass 3kg and radius 40cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30N? What is the linear acceleration of the rope? Consider that no slipping takes place.

Ans. The mass of the hollow cylinder is given as, m=3 kg

Radius of the hollow cylinder is given as, r=40cm=0.4m

Applied force on the given rope is given as, F=30N

The moment of inertia of the hollow cylinder about its geometric axis can be given as

I=mr2

I=30.42

I=0.48kgm2

Torque acting on the rope,

τ=Fr

τ=300.4

τ=12Nm

For angular acceleration , torque can also be given by the expression,

τ=I

=τI=120.48

=25 rad s-2

Linear acceleration of the rope can be stated as =ra=0.425=10ms-2.

Q10.) A child sits stationary at one end of a long trolley moving uniformly with a speed on a smooth horizontal floor. What will be the speed of the CM of the (trolley + child) system when the child gets up and runs about on the trolley?

Ans. The child runs about on the trolley with an acceleration that has no effect on the velocity of the centre of mass of the trolley. This occurs due to the force caused by the child's internal motion.

There are no effects on the motion of the bodies on which they are acting when the motion is produced by internal forces. As no external force is involved in the (child + trolley) system, there will be no change in the speed of the centre of mass of the trolley due to the child's motion.

Q11.) Determine which factors affect the moment of inertia of a body.

Ans: The factors that affect the moment of inertia of a body are as follows.

- Position of the axis of rotation
- Mass of the body
- Shape and size of the body

Q12.) A bullet of mass 10g and speed 500m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0m wide and weighs 12kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it. Hint: The moment of inertia of the door about the vertical axis at one end is ML23

Ans.

The mass of the bullet is given as, m=10g=1010-3kg

Velocity of the bullet is given as, =500m/s

Width of the door, L=1.0m

Radius of the door, r=12m

Mass of the door is given, M=12kg

Angular momentum transmitted by the bullet on the door:

=mr

=(10010-3)(500)12=2.5kgm2s-1

Moment of inertia of the door can be given as

I=**ML23**

I=131212=4kgm2

But we have the relation,

=I

I=2.54=0.625rads-1, which is the required angular speed.

Long Questions and Answers [4 Marks]

Q1.) A circular ring of diameter 40cm and mass 1kg is rotating about an axis normal to its plane and passing through the centre with a frequency of 10 rotations per second. Calculate the angular momentum about its axis of rotation.

Ans. From the question we can write,

R=402cm=20cm=0.2m

M=1kg

=10 rotations/sec

Now, Moment of inertia can be calculated as,

M.I.=MR2=1(0.2)2=0.04kgm2

=2=210=20rad/s

Angular momentum can be measured as

L=0.0420

L=2.51kgm2/s

Q2.) Four particles of mass 1kg, 2kg, 3kg, and 4kg are placed at the four vertices A,B,C, and D of the square of side 1m. Find the position of the centre of mass of the particle.

Ans. From the given data we can infer,

m1=1kg at x1,y1=(0,0)

m2=2kg at x2,y2=(1,0)

m3=3kg at x3,y3=(1,1)

m4=4kg at x4,y4=(0,1)

Now,

Xcm=m1x1+ m2x2+ m3x3+ m4x4, m1+ m2+ m3+ m4=0.5m

Ycm=m1y1+ m2y2+ m3y3+ m4y4m1+ m2+ m3+ m4=0.7m

Therefore, the centre of mass is located at 0.5m,0.7m

Q3.) A hoop of radius 2m weighs 100kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20m/s. How much work has to be done to stop it?

Ans. Given that,

Radius of the hoop is given as, r=2m

Mass of the hoop is, m=100 kg

Velocity of the hoop is,

=20cm/s=0.2m/s

Total energy of the hoop = Translational KE + Rotational KE

Er=12m2+12I2

Moment of inertia of the hoop about its centre can be given as I=mr2

Er=12m2+12(mr2)2

But we have the formula,

=r

E1=12m2+12(mr2)2

E1=12m2+12mr22

The work needed to be done for halting the hoop is same to the total energy of the hoop.

Hence, the required work to be done can be given as,

W=m2=1000.22=4J

Q4.) State whether the statements mentioned below are true or false, along with suitable reasons;

(i). The instantaneous acceleration of the point of contact during rolling is zero.

Ans: False

During the rolling of a body, its instantaneous acceleration is not equivalent to zero and has a certain value.

(ii). A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.

Ans: True.

A frictional force will work between the body and the surface, resulting in the rolling of the body. The torque provided by this frictional force is needed for rolling. In the absence of frictional force, the body slides from the inclined plane due to its own weight.

(iii) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.

Ans: False

Frictional force works in the opposite direction of motion of the centre of mass of a body and the motion of the centre of mass is in the backward direction. Thus, the frictional force works in the forward direction.

(iv). The instantaneous speed of the point of contact during rolling is zero.

Ans: True

The rotation of a body around an axis that goes across the point of contact of the body with the ground is rolling. Thus, the instantaneous speed of the point of contact while rolling is zero.

(v). For perfect rolling motion, the work done against friction is zero.

Ans: True

The frictional force that works at the lowermost point turns to zero when the rolling motion starts. Hence, the work done against friction is zero as well.

Q5: What are the important features of angular momentum?

Ans: The following are the important features of angular momentum-

- The angular momentum of a particle with respect to a point provides an idea of the strength of its rotational tendency about the point.
- The magnitude of the angular momentum can be defined in terms of mass and velocity of the particle and its distance from the reference point i.e. L = mvr.
- The vector concept of angular momentum is important and useful. Its direction is the axial direction portrayed by the right-handed rule.

Q6: What is the physical significance of M.I?

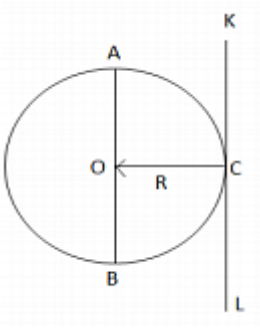
Ans: The inability of a body to change its state of uniform rotation about an axis is called rotational inertia or M.I of that concerned body.

It plays a similar part in a rotatory motion as is being played by the mass in translational motion i.e. it is a type of rotational analogue of mass.

Very Long Questions and Answers [7 Marks]

Q1: The moment of inertia of a solid sphere about a tangent is 7/5 MR2. What will be the moment of inertia about its diameter?

Ans:



From the diagram we can say that a tangent is drawn at the point of C of the given solid sphere of mass M and radius of R.

Now, a diameter AOB∥KCl

Then according to the Theorem of Parallel Axis, it can be inferred

$I_1 = I - M(OC)^2$

M.I about the tangent can be written as $I_1 = 7/5\ MR^2$

$I = I_1 - M(OC)^2$

$\Rightarrow I = 7/5\ MR^2 - MR^2$

$\Rightarrow I = 2/5\ MR^2$

Therefore, $2/5\ MR^2$ is the moment of inertia about the diameter.

Q2: A circular ring having a diameter of 40cm and mass of 1 kg is rotating about an axis normal to its plane and passing through the center with a frequency of 10 rotations pers second. Now, calculate the angular momentum about the axis of rotation.

Ans: From the question we can obtain,

R = 40/2 cm = 20 cm = 0.2 m

M = 1 kg

v = 10 rotations / sec

Now, we can calculate the moment of inertia as follows:

M.I = MR^2 = 1 × $(0.2)^2$ = 0.04 kg sq.m

ω = 2π × 10 = 20π rad / sec

Thus, angular momentum can be calculated as,

⇒ L= 0.04 × 20π

⇒ L = 2.51 kg sq. m / sec

Q3: Using the expression of power and K.E. of rotational motion, derive the relation τ = Iα.

Ans: As we know that power is given by

$$P = \tau\omega \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Also, we know that,

$$K.E. = 1/2\ I\omega^2 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Now, the power of rotational motion is equal to time rate of work done during the rotational motion. Since the work done is stored in the form of kinetic energy,

$$p = d/dt\ (K.E.\ of\ rotational\ motion)$$

Using the equations (i) and (ii), we have,

$$\tau(\omega) = d/dt\left(1/2 I(\omega)^2\right) = 1/2 I\, d/dt(\omega^2)$$

$$= 1/2 \times I \times 2\omega d\omega/dt$$

$$= I\omega\alpha \quad (\because d\omega/dt = \alpha) \qquad \ldots (iii)$$

From (i) and (iii), we get:

$$\tau\omega = I\omega\alpha$$

$$\tau = I\alpha.$$

Q4: Three homogeneous rigid bodies namely a solid sphere, a solid cylinder and a hollow cylinder are placed at the top of an inclined plane. If they're

released from the rest at the same elevation and roll without slipping, which one will be reaching the bottom first and which one will be reaching the last?

Ans: Let us assume that a_1, a_2, a_3 are respective linear accelerations respectively,

We know that $$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$

Here, $$I_1 = \frac{2}{5}m_1R^2$$

$$I_2 = \frac{1}{2}m_2R^2$$

$$I_3 = m_3R^2$$

$$\therefore \quad a_1 = \frac{g\sin\theta}{1+\frac{\frac{2}{5}m_1R^2}{m_1R^2}} = \frac{5}{7}\, g\sin\theta = 0.71\, g\sin\theta$$

Therefore the solid sphere will reach first and the hollow cylinder will reach last.

Q5: Three separate particles of masses of 1g, 2g, and 3g have their center of mass at the precise point (2,2,2). What should be the position of the fourth particle of mass 4g so that the combined center of mass may be at the point (0,0,0).

Ans: Let's assume that r_1, r_2, r_3 are the position vectors of the three mass particles in consideration w.r.t origin.

Then their center of mass is given by

$$\mathbf{r} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_1+m_2+m_3}$$

Here, $m_1 = 1g$, $m_2 = 2g$, $m_3 = 3g$

$$\mathbf{r} = 2\hat{i} + 2\hat{j} + 2\hat{k}$$

$$= \frac{\mathbf{r}_1 + 2\mathbf{r}_2 + 3\mathbf{r}_3}{1+2+3}$$

or $$\mathbf{r}_1 + 2\mathbf{r}_2 + 3\mathbf{r}_3 = 12\left(\hat{i}+\hat{j}+\hat{k}\right) \qquad \ldots. (i)$$

If r' be the position vector of the new C.M. then

r'= 0

And r_4 be the position vector of the fourth particle from the origin, then

$$0 = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{1\mathbf{r}_1 + 2\mathbf{r}_2 + 3\mathbf{r}_3 + 4\mathbf{r}_4}{1+2+3+4}$$

$$\text{or} \quad \mathbf{r}_1 + 2\mathbf{r}_2 + 3\mathbf{r}_3 + 4\mathbf{r}_4 = 0$$

$$\text{or} \quad 12\left(\hat{i}+\hat{j}+\hat{k}\right) + 4\mathbf{r}_4 = 0$$

$$\text{or} \quad \mathbf{r}_4 = -3\left(\hat{i}+\hat{j}+\hat{k}\right)$$

Thus the new particle should be at (-3, -3, ,-3).

Mcq-questions-of-system-of-particles-and-rotational-motion

1. A body of M.I. 3 kg m2 rotating with an angular velocity 2 rad/s has the same K.E. as a mass of 12 kg moving with a velocity of

(a) 1 m/s
(b) 2 m/s
(c) 4 m/s
(d) 8 m/s

Answer: (a) 1 m/s

2. A thin uniform circular ring is rolling down an inclined plane of induction 30° without slipping, its linear acceleration along the inclined plane will be

(a) 2/3g
(b) g/3
(c) g/4
(d) g/2

Answer: (c) g/4

3. If a body is rotating about an axis, passing through its centre of mass then its angular momentum is directed along its

(a) Radius
(b) Tangent
(c) Circumference
(d) Axis of rotation

Answer: (d) Axis of rotation

4. Moment of inertia depends on

(a) Distribution of particles
(b) Mass
(c) Position of axis of rotation
(d) All of these

Answer: (d) All of these

5. SI unit of center of mass

(a) Kg
(b) M/s
(c) N
(d) J

Answer: (c) Kg

6. **A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected?**

(a) Moment of inertia
(b) Angular momentum
(c) Angular velocity
(d) Rotation of kinetic energy

Answer: (b) Angular momentum remains constant

7. A particle moves on a circular path with decreasing speed. Choose the correct statement.

(a) Angular momentum remains constant.
(b) Particle moves in a spiral path with decreasing radius.
(c) The direction of angular momentum remains constant.
(d) Both b and c

Answer: (d) Both b and c

8. The centre of mass of a body

(a) lies always on the surface of the body
(b) lies always inside the body
(c) may lie within, outside on the surface of the body
(d) lies always outside the body

Answer: (c) may lie within, outside on the surface of the body

9. The position of centre of mass of a system of particles does not depend upon the

(a) relative distance between the particles
(b) position of the particles

(c) symmetry of the body
(d) mass of particles

Answer: (a)relative distance between particles

10. **The motion of the centre of mass depends on**

(a) total external forces
(b) total internal forces
(c) None of these
(d) Both

Answer: (a) total external forces

11. The centre of mass of two particles lies on the line

(a) joining the particles
(b) perpendicular to the line joining the particles
(c) at any angle to this line
(d) None of these

Answer:(a) joining the particles

Fill in the blanks

1. Centre of mass of a body is …… at which…….is…… .

Answer: Centre of mass of a body is a point at which entire mass is concentrated.

2. In certain cases,there may……. at the …….

Answer:In certain cases,there may no mass at the centre of mass

3. Total linear momentum of a system of particles is equal to …… of the system and velocity of ……..

Answer: Total linear momentum of a system of particles is equal to product of mass of the system and velocity of centre of mass

4. By convention,anticlockwise moments are……..and…….are taken as……

Answer: By convention,anticlockwise moments are positive and clockwise moments are taken as negative

5. Torque due to a force is the product of……and……..of line of action……

Answer: Torque due to a force is the product of force and perpendicular distance of line of action and axis of rotation.

5. Angular momentum of a particle……is……..of the particle……

Answer: Angular momentum of a particle about a given axis is momentum of momentum of the particle about that axis.

6. Angular momentum of a particle about a given axis is……of……and……..of position vector of the particle.

Answer: Angular momentum of a particle about a given axis is twice the product of mass and arial velocity of position vector of the particle.

7. The center of gravity of a body is a point where ……..and……..on the body is……..

Answer: The center of gravity of a body is a point where weight of the body is act and total gravitational torque on the body is zero

8. Mass of a body is ……of……of the body……..

Answer:Mass of a body is measure of inertia of the body in linear motion

9. Moment of inertia of a circular ring about a given axis is……moment of inertia of……..of…….and……..about the same axis.

Answer: Moment of inertia of a circular ring about a given axis is greater than moment of inertia of disc of same mass and same size about the same axis.