Q.1 Longest Increasing Subsequence (LIS)

Given an array, find the length of longest increasing subsequence (LIS)

Note :- Increasing subsequence means strictly increasing.

↳ Order matters.

A : { 9 2 4 3 10 } ⇒ 2, 4, 10
2, 3, 10

LIS ⇒ 3.

A : { 2 -1 6 3 7 9 }

↳ { 2 3 7 9 }
{ 2 6 7 9 }
{ -1 6 7 9 }
{ -1 3 7 9 }

LIS = 4.

No. of subsequences = 2^N

$a_0 \quad a_1 \quad a_2 \quad - - - - \quad a_{n-1}$

I E I E

⇒ 2^N

⇒ Empty sequence is also a subsequence.

⇒ Archive : Intermediate DSA : Subsequences & subset

Brute force

1) Backtracking $\Rightarrow O(2^N)$
2) Bit Masking $\Rightarrow O(N \cdot 2^N)$

$\Rightarrow$ $\boxed{N <= 10^4}$

NOTE : If we have a recursive / backtracking sol^n and the constraints are high then ~~that~~ problem is a D.P. problem.

$\Rightarrow$

A[12] :

	0	1	2	3	4	5	6	7	8	9	10	11
A[12]	10	3	12	7	2	9	11	20	11	13	6	8

LIS[0-i] $\Rightarrow$ length of LIS from index 0 to i

$$
\begin{tikzcd}
 & \text{LIS}[0-11] \arrow[dl, "\text{inc}"'] \arrow[dr, "\text{exc}"] & \\
\text{LIS}[0-10] + 1 & & \text{LIS}[0-10]
\end{tikzcd}
$$

$\{a_1, a_2 \ldots \underbrace{a_x\} \; 8}_{a_x < 8}$

$\Rightarrow$ here, we don't know the ending of a subseq.

LIS[i] : length of longest Increasing Subsequence from index 0 to i ending at index i.

{A[i]}

	0	1	2	3	4	5	6	7	8	9	10	11
A[12] :	10	3	12	7	2	9	11	20	11	13	6	8
LIS[12] :	1	1	2	2	1	3	4	5	4	5	2	3
	{10}	{3}	{10, 12} {3, 12}	{3, 7}	{2}	{3, 7, 9}	{3, 7, 9, 11}	{3 7 9 11 20}	{3 7 9 11}	{3 7 9 11 13}	{2, 6} {3, 6}	{2, 6, 8}

final ans ⇒ MAX of LIS[].

dp[i] = length of LIS ending at index i.

dp Expression :

$$\boxed{dp[i] = \text{Max}\left\{ \forall_{j=i-1}^{0} dp[j] \right\} + 1 \quad A[i] > A[j]}$$

dp[0] = 1

int dp[N];

int LIS(int arr[ ], N){

```
    int dp[N];
    dp[0] = 1;
    for( i= 1; i< N; i++ ){
        s = 0
        for( j= i-1; j>=0; j-- ){
            if( A[i] > A[j] ){
                s = max(s, dp[j]);
            }
        }
        dp[i] = s+1;
    }
    return max(dp[ ]);
}
```

$$\left.\begin{aligned} TC &: O(N^2) \\ SC &: O(N) \end{aligned}\right\}\ \text{LIS}$$

Optimise

$O(N\log N)$

Q.2 N Houses.

Given N Houses & cost associated to color each house with R|G|B. Find min cost to color all the houses s.t no two adjacent houses have same color

N=3

	1	2	3
R	5	8	4
G	2	1	5
B	6	9	7

G R G ⇒ 15 (2+8+5)

G B R ⇒ 15

B G B ⇒ 14

R G R ⇒ $\boxed{10}$ ✱ ans.

Try out all the possibilities.

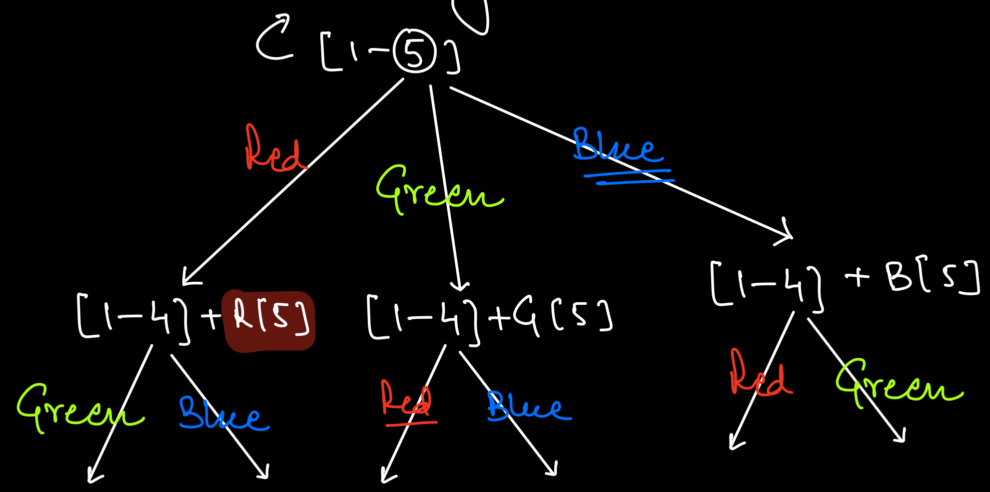
$3 \times 3 \times 3 \cdots 3 \Rightarrow 3^N$

_ _ _ _ _ _ _

R 2 2 2 - - - 2
G
B
⇓
3

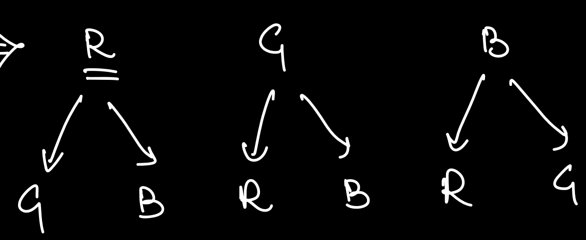
$\Rightarrow 3 \times 2^{N-1}$

Min Cost of painting the houses from [1-5]



[1-5] 5th ⇒

dp[5]



Note: We need to know the color of each house along with min cost.

R → 0 , G → 1 , B → 2

dp[i][R] ⇒ Min cost of painting 1 to i houses if ith house is painted with Red.

dp[i][G] ⇒ Min cost of painting [1-i] houses if ith house is painted with Green

dp[i][B] ⇒ Min cost of painting [1-i] houses if ith house is painted with Blue

$$
\left\{\begin{aligned}
dp[i][0] &= R[i] + \min(dp[i-1][1], dp[i-1][2]) \\
dp[i][1] &= G[i] + \min(dp[i-1][0], dp[i-1][2]) \\
dp[i][2] &= B[i] + \min(dp[i-1][0], dp[i-1][1])
\end{aligned}\right.
$$

Minimum

* Base Case ⇒ $i = 0$

$$dp[0][0] = dp[0][1] = dp[0][2] = 0$$

* dp table size

int dp[N+1][3];

Code

R[ ]
G[ ]
B[ ]

} 1 based indexing

```
int dp[N+1][3];
dp[0][0] = dp[0][1] = dp[0][2] = 0;
for (i = 1; i <= N; i++) {
      dp[i][0] = R[i] + min(dp[i-1][1], dp[i-1][2]);
      dp[i][1] = G[i] + min(dp[i-1][0], dp[i-1][2]);
      dp[i][2] = B[i] + min(dp[i-1][0], dp[i-1][1]);
}
return min(dp[n][0], dp[n][1], dp[n][2]);
```

row

TC : O(N)

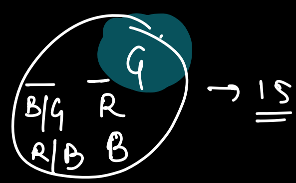
SC : O(N)

Todo

SC can be optimised, as we only need 6 variables at a time.

N=3

	1	2	3
R:	5	8	4
G:	2	1	5
B:	6	9	7



dp[4][3]

	R 0	G 1	B 2
0	0	0	0
1	5	2	6
2	10	6	11
3	10	15	13

$dp[1][0] = \underbrace{R[1]}_{5} + \min(dp[0][1], dp[0,2])$

$dp[2][0] = \underbrace{R[2]}_{8} + \min(dp[1][1], dp[1][2])$

$dp[2][1] = \underbrace{G[2]}_{1} + \min(dp[1][0], dp[1][2])$

$$dp[2][2] = B[2] + \min(\underline{dp[1][0]}, dp[1][1])$$

$$dp[3][0] = \underbrace{R[3]}_{4} + \min(dp[2][1], dp[2][2]$$

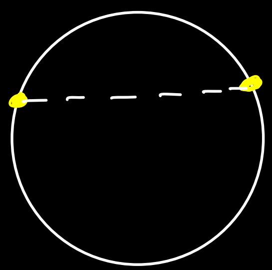
$$dp[3][1] = \underbrace{G[3]}_{5} + \min(dp[2][0], dp[2][2]$$

$$dp[3][2] = \underbrace{B[3]}_{7} + \min(dp[2][0], dp[2][1])$$

Q.3 Interesting Chords.

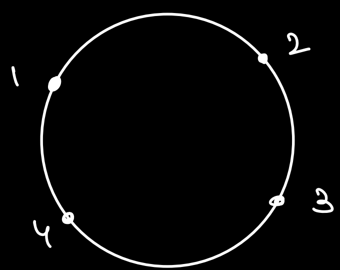
Given 2A no. of points on circle, find no. of ways we can draw A Chords in the circle from 2A points.

Note: No 2 chords should intersect.

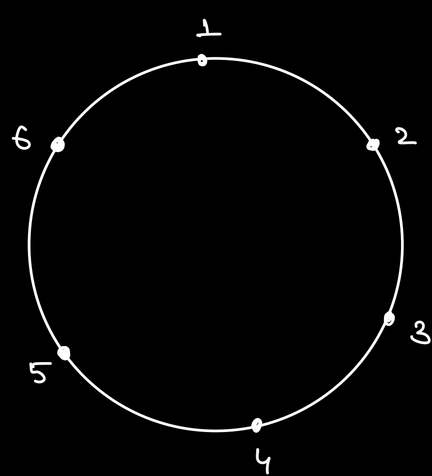


$f(1) = 1$

1 pair of points.



$f(2) = \{1-2, 3-4\}, \{1-4, 2-3\}$ 2

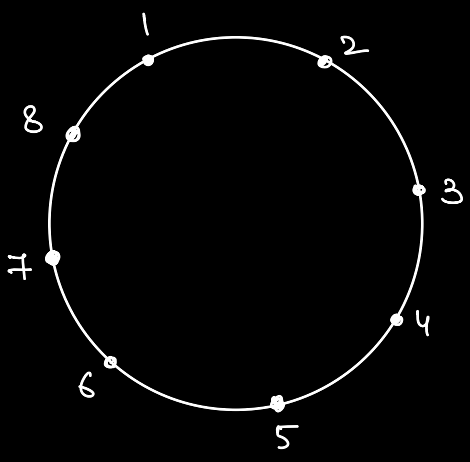


$f(3) =$ 1–2 {3–6, 4–5; 5–6, 3–4}

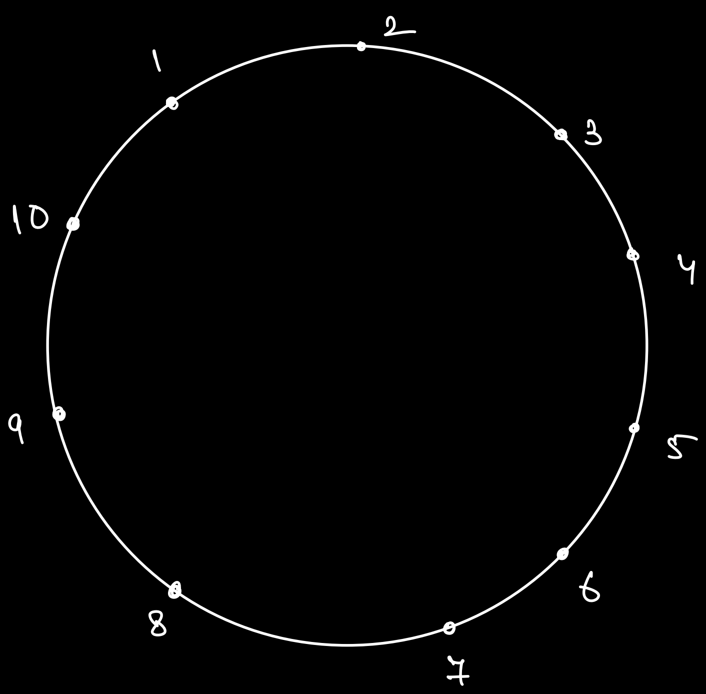
1–4 {2–3, 5–6}

1–6 {2–3, 4–5; 3–4, 2–5}

$f(3) = 5$

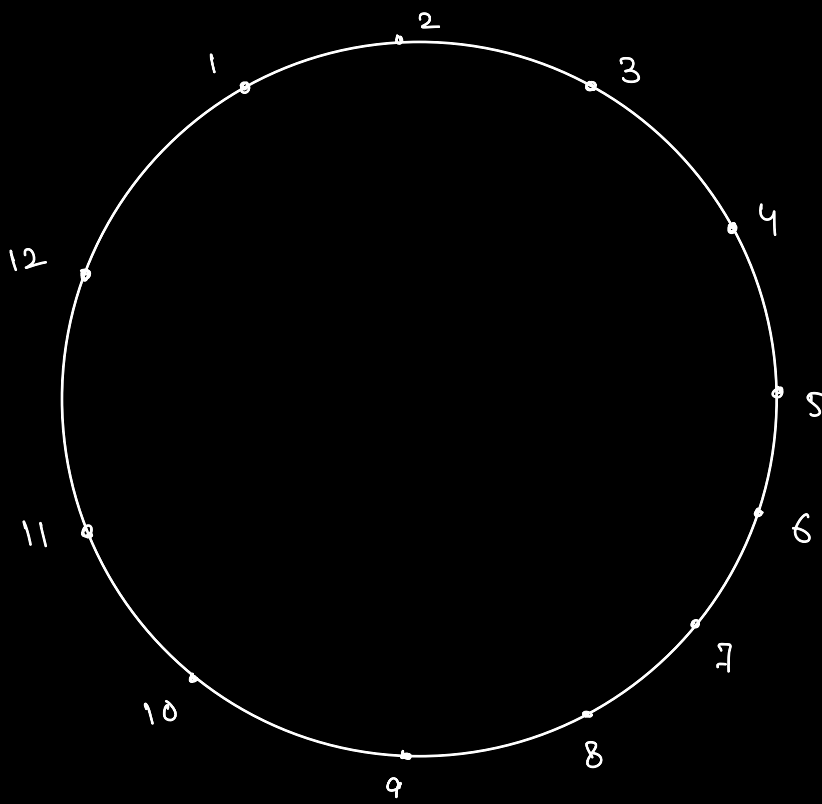


$$f(4) = \underbrace{f(3)}_{1-2} + \underbrace{f(2) \times f(1)}_{1-4} + \underbrace{f(1) \times f(2)}_{1-6} + \underbrace{f(3)}_{1-8}$$



$$f(\underset{\overset{\downarrow}{N}}{5}) = f(4) + f(3) \times f(1) + f(2) \times f(2) + f(1) \times f(3) + f(4)$$

$$\boxed{f(N) = f(N-1) + f(N-2) \cdot f(1) + f(N-2) \cdot f(2) + \cdots \quad f(N-1)}$$



$$f(6) = f(5) + f(4)\cdot f(1) + f(3)\cdot f(2) + f(2)\cdot f(3) + f(1)\cdot f(4) + f(5) \times$$

$$f(6) = f(6-1) + f(6-2)\cdot f(1) + f(6-3)\cdot f(2) + f(6-4)\cdot f(3) + f(6-5)\cdot f(4) + f(5)$$

$$f(6) = f(5)f(0) + f(4)\cdot f(1) + f(3)\cdot f(2) + f(2)\cdot f(3) + f(1)\cdot f(4) + f(5)f(0)$$

$$\boxed{f(0) = 1}$$

int dp[A+1] ;
$\hookrightarrow$ no. of pairs

dp[i] = # of ways to draw i chords using i pair of points.

$$dp[i] = dp[i-1]\times dp[0] + dp[i-2]\times dp[1] + dp[i-3]\times dp[2] + \ldots + dp[0]\times dp[i-1]$$

```
int dp[A+1];
dp[0] = 1
for( i= 1; i <= A; i++ ){
        k=0, s=0
        for( j= i-1 ; j>=0; j-- ) {
                s += dp[j]×dp[k]
             k++
        }
        dp[i] = s;
}
return dp[A]
```

TC: $O(A^2)$

SC: $O(A)$

——— * ———