

$$\begin{array}{cccccccc}
 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 S: & a & a & b & a & c & a & a & b & a \\
 LPS[i] & 0 & 1 & 0 & 1 & 0 & 1 & 2 & 3 & 4
 \end{array}$$

P: aaba $\Rightarrow N$

T: aabacaaaba $\Rightarrow M$

P\$T \Rightarrow aaba\$aabacaaaba

LSP[i] \Rightarrow 0 1 0 1 0 1 2 3 4 0 1 2 3 4

$$TC: O(N+M)^3$$

\Rightarrow KMP (Knuth Morris Prat)

Quiz

$$\begin{array}{cccccc}
 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
 S: & a & b & a & y & a & b & a \\
 LPS[i]: & 0 & 0 & 1 & 0 & 1 & 2 & 3
 \end{array}$$

Quiz

$$\begin{array}{ccccccc}
 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
 S: & \underline{c} & a & c & y & c & a & c & \underline{a} \\
 LPS[i]: & 0 & 0 & 1 & 0 & 1 & 2 & 3 & 2
 \end{array}$$

$LPS[i]$: length of longest prefix that is also a suffix in the string from $[0, i]$.

$s_0 s_1 s_2 s_3 s_4 \dots s_{i-4} s_{i-3} s_{i-2} s_{i-1} s_i s_{i+1} \dots$
↑
5

$$LPS[i] = 5$$

$$s_0 s_1 s_2 s_3 s_4 = s_{i-4} s_{i-3} s_{i-2} s_{i-1} s_i$$

$$(LPS[i-1])_{\min} = 4$$

$$s_0 s_1 s_2 s_3 = s_{i-4} s_{i-3} s_{i-2} s_{i-1}$$

$$LPS[i-1] \geq 4$$

$$LPS[i] = n$$

$$LPS[i-1] \geq n-1$$

$$LPS[i-1] \geq LPS[i] - 1$$

$$LPS[i] \leq LPS[i-1] + 1$$

⇒ Note: The LPS value can atmax increase by 1.

Quiz

$$\text{LPS}[i] = 5$$

$$\text{LPS}[i-1] \gamma = 5 - 1$$

$$\text{LPS}[i-1] \gamma = 4$$

$$\hookrightarrow \underline{\underline{\neq 3}}$$

Ex

(i)

S:

0	1	2	3	4	5	6	
a	b	a	y	a	b	a	?
0	0	1	0	1	2	3	4

LPS[i]:

$$\text{LPS}[i] = ?$$

$$\text{LPS}[i-1] = n = 3$$

$$\text{LPS}[i] \leq n + 1$$

$$\text{LPS}[i] \leq 4$$

\neq

$$\text{LPS}[i] = 4$$

$$s[i] = s[n]$$

||

0	1	2	3	4	5	6	7	8	9
b	c	a	d	c	b	c	a	d	?
0	0	0	0	0	1	2	3	4	

↓
n

$$LPS[i] \leq LPS[i-1] + 1$$

$$LPS[i] \leq 5$$

if $LPS[i] = 5$

→ $s[i]$ should be equal to $s[4]$.

$$s[i] = c$$

$$n = LPS[i-1]$$

$$\text{if } (s[i] == s[n]) \{$$

$$LPS[i] = n + 1;$$

}

2n

						$n-1$	n										(i)
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
S:	c	a	c	y	c	a	c	a	b	c	a	c	y	c	a	c	y
LPS:	0	0	1	0	1	2	3	2	0	1	2	3	4	5	6	7	4

↓
4

$$LPS[16] = ?$$

$$LPS[15] = 7 = n$$

$$LPS[16] \leq 8$$

$$i \neq (S[16] == S[7]) \times$$

$$S_0 S_1 S_2 S_3 S_4 S_5 S_6 = S_9 S_{10} S_{11} S_{12} S_{13} S_{14} S_{15}$$

$$\underline{S_0 S_1 S_2} = S_{13} S_{14} S_{15} \Rightarrow S[16] == S[3] \Rightarrow \checkmark$$

$$i = 16,$$

$$LPS[i-1] = 7 = n$$

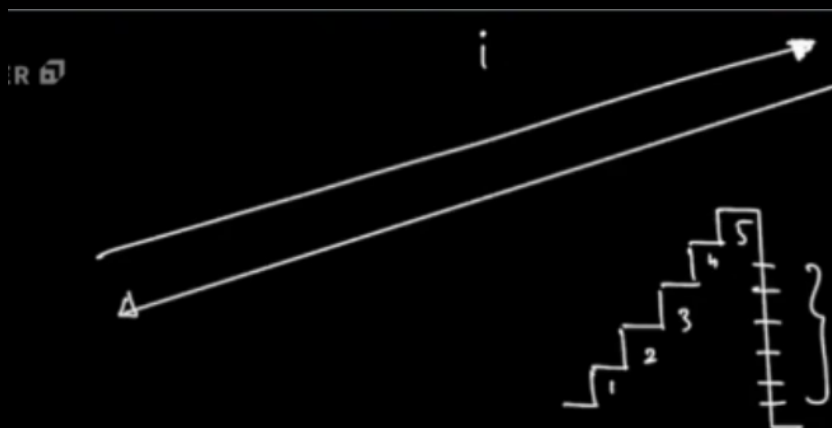
n	$i \neq (S[i] == S[n])$	n_{new}
7	$S[16] == S[7]$ $y == a \times$	$n = LPS[n-1]$ $= LPS[6] = \underline{\underline{3}}$
3	$S[16] == S[3]$ $y == y \checkmark$ $\Rightarrow LPS[i] = n+1$ $= \underline{\underline{4}}$	

(n)											(i)												
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
a	b	c	a	b	d	a	b	c	a	b	e	a	b	c	a	b	d	a	b	c	a	b	c
0	0	0	1	2	0	1	2	3	4	5	0	1	2	3	4	5	6	7	8	9	10	11	3

$$i = 23, \text{LPS}[i] = ?$$

$$n = \text{LPS}[i-1] = 11$$

n	$i \neq (\text{S}[i] == \text{S}[n])$	n_{new}
11	$\text{S}[23] == \text{S}[11]$ $c == e ? \quad \times$	$n = \text{LPS}[n-1]$ $= \underline{5}$
5	$\text{S}[23] == \text{S}[5]$ $c == d \quad \times$	$n = \text{LPS}[n-1]$ $= \text{LPS}[4] = 2$
2	$\text{S}[23] == \text{S}[2]$ $c == c \quad \checkmark$ $\text{LPS}[i] = n+1$ $= \underline{\underline{3}}$	



every index will be touched at max 1 by increasing 1 by one and also by decreasing

```

int LPS[N];
LPS[0] = 0;
for (i = 1; i < N; i++) {
    n = LPS[i-1];
    while (s[i] != s[n]) {
        if (n == 0) { n = -1; break; }
        n = LPS[n-1];
    }
    LPS[i] = n + 1;
}

```

TC: $O(N)$

✱

a	b	c	d	e	f
0	0	0	0	0	0

a	a	b	c	a	d	e
0	1	0	0	1	0	

a	a	a	a	a	a	a
0	1	2	3	4	5	6

a	b	a	b	c	a	b	c	d
0	0	1	2	0				

Pattern Matching

$P \Rightarrow (N)$

$T \Rightarrow M \gg N.$

$S: P \& T \Rightarrow N+M$

\hookrightarrow find LPS[] for (S) .

\Rightarrow Count occurrences of N in LPS[].

TC $\Rightarrow O(N+M)$

SC $\Rightarrow O(N+M)$

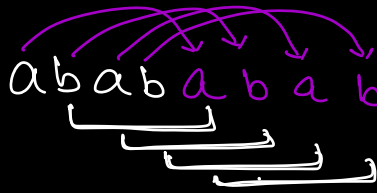
Q. Given a string, Count the no. of different rotations ($\leq N$) that gives us the original string back.

$S: abcd$
 $bcda$
 $cdab$
 $dabc$
 $abcd$ $\Rightarrow 1$

S: abab
 baba
 abab
 baba
 abab $\Rightarrow \textcircled{2}$

S: aaa
 aaa
 aaa
 aaa $\Rightarrow \textcircled{3}$

S: abacd
 bacda
 acdab
 cdaba
 dabac
 abacd $\Rightarrow \textcircled{1}$

\Rightarrow  \Rightarrow Count the no. of occurrences of abab in this string

<p>P: abab</p> <p>T: abababab \rightarrow <u>S+S.</u></p>
--

T: abababab.

P: abab

$P\$T \Rightarrow abab\$ \boxed{abab} abab.$

$\downarrow \downarrow$
 $0 \ 0 \ 1 \ 2 \ 0 \ 1 \ 2 \ 3 \ 4 \ 3 \ 4 \ 3 \ 4$
 \uparrow

Count - 1

7/8: a a a

T: a a a a a a

$\Rightarrow P\$T \Rightarrow a a a \$ \boxed{a a a a a a}$

$0 \ 1 \ 2 \ 0 \ 1 \ 2 \ 3 \ 3 \ 3 \ 3$
 $\Rightarrow 4 - 1 = \underline{\underline{3}}$

———— * ————

$\overbrace{abababab}^{\text{0}}$
 $\underbrace{abababab}_{1st}$
 $\underbrace{abababab}_{2nd}$
 $\underbrace{abababab}_{3rd}$
 $\underbrace{abababab}_{4th}$