$A \% B \Rightarrow$ Remainder when $\underline{\underline{A}}$ is divided $\underline{\underline{B}}$.

$14 \% 5 \Rightarrow$

Divident

$$\begin{array}{r|l} 5 \,\big)\, 14 & 2 \\ 10 & \\ \hline 4 & \end{array}$$

Divisor (5) · Divident (14) · 2 → Quotient · 4 → Remainder

Divident = Divisor * Quotient + Remainder

$$\boxed{\text{Remainder} = \text{Divident} - \text{Divisor} * \text{Quotient}}$$

→ $14 \% 5 \Rightarrow 14 - 5 = 9$

$9 - 5 = \underline{\underline{4}}$

Division is a repeated subtraction.

1) $\underbrace{30 \% 7}_{\underline{\underline{2}}} = 30 - 7 = 23 - 7 = 16 - 7 = 9 - 7 = \underline{\underline{2}}$

$= 30 - 7 \times 4$

2) $40 \% 6 = 4 = 40 - 6 * 6$

$$\boxed{\text{Remainder} = \text{Divident} - \left(\begin{array}{l}\text{largest multiple of} \\ \text{divisor} <= \text{divident}\end{array}\right)}$$

$a \% M \in [0, M-1]$

$$\left.\begin{array}{l} -\infty \\ \vdots \\ 100 \% 3 = 1 \\ 10 \% 3 = 1 \\ 3 \% 3 = 0 \\ 4 \% 3 = 1 \\ 5 \% 3 = 2 \\ \vdots \\ +\infty \end{array}\right\} \xrightarrow{\% 3} \boxed{0, 1, 2}$$

$M-1$

Modulo Arithmetic :

1) $(A+B) \% M = (A \% M + B \% M) \% M$

$A = 4$, $B = 5$, $C = 6$

$(4+5) \% 6$

$9 \% 6$

$= 3$

$4 \% 6 + 5 \% 6$

$4 + 5$

9

2) $(a * b) \% M = (A \% M * B \% M) \% M$

Ex: $100 \% 6 = 4$

$(100 \% 6) \% 6 = 4 \% 6 = 4$

$((100 \% 6) \% 6) \% 6 = 4$

3) $A \% M \Rightarrow (A+M) \% M$

$\Rightarrow (A \% M + \cancel{M \% M}) \% M$

0

$\Rightarrow A \% M$

$$\boxed{A \% M = (A + n*M) \% M}$$

4) $(A-B) \% M = (A \% M - B \% M + M) \% M$

$A = 8, \ B = 4, \ M = 5$

$(8-4) \% 5$	$(8 \% 5 - 4 \% 5) \% 5$
$4 \% 5$	$(3-4) \% 5$
$= 4$	$(-1) \% 5$

$\downarrow$

$(-1+5) \% 5 = 4 \% 5 = 4$

Ex

$-8 \% 5$

$a \% 5 \in [0, 4]$

C/C++/Java/JS

$-(8 \% 5)$

-3

Python

$-8 \% 5 \Rightarrow 2$

+5

+M

Q. A, B

$A > B$

find M, s.t $A \% M = B \% M$; $M > 1$

Ex $A = 16$, $B = 4$

$16 \% M = 4 \% M$

$M = 2, 3, 4, 6, \dots\dots\dots\dots$

$\Rightarrow$ $A \% M = B \% M$

$A \% M - B \% M = 0$

$(A - B) \% M = 0$

$x \% y = 0$

$\Rightarrow$ x is a multiple of y.

$\Rightarrow$ y is a factor of x.

M is a factor of $A - B$.

return $A - B$. $\Rightarrow O(1)$

Q. Given an Array of size N, calculate the no. of pairs (i, j) s.t $(A[i] + A[j]) \% M = 0$ & $i \,!= j$. pair(i, j) is considered to be same as pair(j, i).

$$A: \{\overset{0}{4}, \overset{1}{7}, \overset{2}{6}, \overset{3}{5}, \overset{4}{5}, \overset{5}{3}\} \quad M = 3$$

$$\left.\begin{array}{l}(0,3)\\(0,4)\\(1,3)\\(1,4)\\(2,5)\end{array}\right\} \text{5 pairs.}$$

```
count = 0
for( i = 0; i < N; i++) {
    for( j = i+1 ; j < N; j++ ) {
        if( (A[i] + A[j]) % M == 0)
            count++;
    }
}
```

$$\boxed{\begin{array}{l}TC: O(N^2)\\SC: O(1)\end{array}}$$

Quiz A: $\{\overset{0}{13}, \overset{1}{14}, \overset{2}{22}, \overset{3}{3}, \overset{4}{32}, \overset{5}{19}, \overset{6}{16}\}$ $M = 4$.

i	j
0	3
0	5
1	2
4	6

$\Rightarrow$ 4 pairs.

$\Rightarrow$ $(A[i] + A[j]) \% M = 0$

$(\underbrace{A[i] \% M}_{x} + \underbrace{A[j] \% M}_{y}) \% M = 0$

$(x + y) \% M = 0$ $\begin{cases} x \in [0, M-1] \\ y \in [0, M-1] \end{cases}$ $\boxed{x + y = M}$

$\rightarrow$ $x+y$ should be divisible by M.

$A = 13$, $B = ?$, $M = 4$

$(A + B) \% 4 = 0$

$\downarrow$

$(\underbrace{A \% 4}_{1} + \underbrace{B \% 4}) \% 4 = 0$

$(1 + x) \% 4 = 0$

$\downarrow$

$x = B \% 4 \in [0, 3]$

0, 1, 2 or 3

Ex

$A = 35, \ B = ?, \ M = 10$

$(A + B) \% 10 = 0$

$$(\underbrace{A \% 10}_{5} + \underbrace{B \% 10}_{?}) \% 10 = 0$$

$$(5 + y) \% 10 = 0$$

$$\Downarrow$$

$$[0, 9]$$

$$A : \{\overset{0}{13}, \overset{1}{14}, \overset{2}{22}, \overset{3}{3}, \overset{4}{32}, \overset{5}{19}, \overset{6}{16}\} \qquad M = 4.$$

$$\downarrow \% M$$

$$\{1, 2, 2, 3, 0, 3, 0\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A :	29	11	21	17	2	5	4	6	23	13	26	14	18	15	30	35	50	20	40

A : {29, 11, 21, 17, 2, 5, 4, 6, 23, 13, 26, 14, 18, 15, 30, 35, 50, 20, 40}

M = 10

% M = 10

[0, M-1]

HashMap

i

0	1	2	3	4	5	6	7	8	9
4	2	1	2	2	3	2	1	1	1
30, 50, 20, 40	11, 21	2	23, 13	4, 14	5, 15, 35	6, 26	17	18	29

$\frac{C(C-1)}{2}$

(4, 6)
(4, 26)
(14, 6)
(14, 26)

(11, 29)
(21, 29)
(2, 18)
(23, 17)
(13, 17)

5
15
35

$\text{cnt}C_2 = \boxed{\frac{\text{cnt}(\text{cnt}-1)}{2}}$

$\frac{\quad}{2}$

$n(n-1)$

5   5
15   15
35   35

$\left.\begin{matrix}15\\35\end{matrix}\right\}(n-1)$

```
HashMap<int, int> hm;
for( i=0; i< N; i++ ){                O(N)
        hm[A[i] % M]++;
}
ans = 0
ans += ( hm[0] * (hm[0]-1) / 2 );

i=1 , j= M-1;
while ( i < j ) {                     O(M)
        ans += (hm[i] * hm[j]);
        i++                hm.get(i)
        j--
}
if (M % 2 == 0){
        ans += ( hm[M/2] * (hm[M/2]-1) / 2 );
}
return ans;
```

TC: O(N + M)

SC: O(M)

Q: Given an Array of all distinct integers where $\{0 <= A[i] <= N-1\}$, N is the size of the Array.

Replace $A[i] \Longrightarrow A[A[i]]$

Ex

$$A[5] : \{\overset{0}{3}, \overset{1}{2}, \overset{2}{4}, \overset{3}{1}, \overset{4}{0}\}$$

$A[0] = A[A[0]]$
$= A[3]$
$= 1$

$A[1] = A[A[1]]$
$= A[2]$
$= 4$

$A[2] = A[A[2]]$
$= A[4]$
$= 0$

$A[3] = A[A[3]]$
$A[3] = A[1] = 2$

$A[4] = A[A[4]]$
$= A[0]$
$= 3$

$\rightarrow \{1, 4, 0, 2, 3\}$

Quiz

$$A : \{\overset{0}{1}, \overset{1}{6}, \overset{2}{3}, \overset{3}{5}, \overset{4}{4}, \overset{5}{2}, \overset{6}{0}\}$$

$A[0] = A[A[0]]$
$= A[1]$
$= 6$

$A[1] = A[A[1]]$
$= A[6]$
$= 0$

$A[2] = A[A[2]]$
$= A[3]$
$= 5$

$A[3] = A[A[3]]$
$= A[5]$
$= 2$

$A[4] = A[A[4]]$
$= A[4]$
$= 4$

$A[5] = A[A[5]]$
$= A[2]$
$= 3$

$A[6] = A[A[6]]$
$= A[0]$
$= 1$

$\{6, 0, 5, 2, 4, 3, 1\}$

* $B[N]$

for ($i \rightarrow 0$ to N)
  $B[i] = A[A[i]]$

TC : $O(N)$
SC : $O(N)$

Constant Extra Space.

$$\begin{array}{cccccc} & 0 & 1 & 2 & 3 & 4 \\ \times\ A[5] : & \{\cancel{3}, & 2, & 4, & 1, & 0\} \\ & 1 & & & & \end{array}$$

$A[0] = A[A[0]]$
$= A[3]$
$= 1$

$A[4] = A[A[4]]$
$= A[0]$

$\Rightarrow$ Day $= 0$, hr $= 0$

	Days	hrs
23 hrs	0	23
46 hrs	1	22
100 hrs	4	4
125 hrs	5	5
x hrs	$x / 24$ Quotient	$x \% 24$ remainder.

$$A[i] \xrightarrow{\quad} \underline{\underline{\text{Old}}} \qquad A[i] \xrightarrow{\quad} \underline{\underline{\text{New}}}$$

$$\boxed{0 =< A[i] <= N-1}$$

$$\begin{cases} \boxed{A[i] \,\%\, N} = \underline{\underline{A[i]}} \\ \boxed{\dfrac{A[i]}{N}} = 0 \end{cases}$$

$$x \xrightarrow{/N} \underline{\underline{\text{Old}}} = A[i]$$

$$x \xrightarrow{\%N} \underline{\underline{\text{New}}} = \underline{\underline{A[A[i]]}}$$

$$\boxed{\frac{x}{N}} = \frac{\overbrace{A[i] * N + A[A[i]]}^{x}}{N}$$

$$= \frac{A[i] * \cancel{N}}{\cancel{N}} + \underbrace{\cancel{\frac{A[A[i]]}{N}}}_{0}$$

$$\boxed{x \,\%\, N} = (\underbrace{A[i] * N}_{\downarrow \%N \; 0} + A[A[i]]) \,\%\, N$$

$$= \boxed{\underbrace{A[A[i]]}_{[0, N-1]} \,\%\, N} \implies A[A[i]]$$

N

$$x \xrightarrow{/N} \underline{\underline{\text{Old}}} \qquad x \xrightarrow{\%N} \underline{\underline{\text{New}}}$$

$$x = \underset{\Downarrow \atop A[i]}{\text{Old}} * N + \underset{\Downarrow \atop A[A[i]]}{\text{new}}.$$

$$A[i] \longrightarrow A[i] * N + A[A[i]]$$

$$A : \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \{3 & 1 & 4 & 6 & 5 & 0 & 2\} \end{array}$$

$N=7$

$$\Big\downarrow * N$$

$$A : \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \{3 \times 7 & 1 \times 7 & 4 \times 7 & 6 \times 7 & 5 \times 7 & 0 \times 7 & 2 \times 7\} \\ + & + & + & + & + & + & + \\ 6 & 1 & 5 & 2 & 0 & 3 & 4 \end{array}$$

$$\Big\downarrow + \frac{A[A[i]/7]}{7}$$

$$A : \{3 \cdot 7 + 6,\ 1 \cdot 7 + 1,\ 4 \cdot 7 + 5,\ 6 \cdot 7 + 2,\ 5 \cdot 7 + 0,\ 0 \cdot 7 + 3,\ 2 \cdot 7 + 4\}$$

$$\Big\downarrow \% N$$

$$A : \{6 \quad 1 \quad 5 \quad 2 \quad 0 \quad 3 \quad 4\}$$

1)
```
for( i= 0; i< N; i++)
    A[i] *= N;
```

2)
```
for( i= 0; i< N; i++){
    index = A[i] / 7;
    value = A[index] / 7;
    A[i] += value;
}
```

3)
```
for( i= 0; i< N; i++)
    A[i] %= N;
```

45mins-1hr

TC: O(N)

SC: O(1)

*