

$$\begin{array}{r}
 8734 : 8000 \\
 \quad 700 \\
 \quad \quad 30 \\
 \quad \quad \quad 4 \\
 \hline
 8734
 \end{array}$$

$$8734 : 8 \times (10^3) + 7 \times (10^2) + 3 \times (10^1) + 4 \times (10^0)$$

base = 10

↳ Decimal

base Value

⇒ Every digit is being multiplied by power of 10 (B.V)

⇒ Decimal Number System

↳ Base = 10

→ [0-9] : 10 Unique digits.

Base = 8

↳ Octal Number System.

↳ [0-7]

$$(0132)_8 : 0 \times 8^3 + 1 \times 8^2 + 3 \times 8^1 + 2 \times 8^0$$

$$: 64 + 24 + 2$$

$$: (90)_{10}$$

Decimal equivalent of
(0132)₈

Base = 3

└→ [0-2]

└→ Ternary

$$(1120)_3 : 1 \times 3^3 + 1 \times 3^2 + 2 \times 3^1 + 0 \times 3^0$$

$$: 27 + 9 + 6 + 0$$

$$: (42)_{10}$$

Quiz Base Value of 5th digit in Decimal N.S

$$\begin{array}{ccccccc} \dots & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ \hline & & & & \downarrow & & \downarrow & \downarrow & \downarrow \\ & & & & 10^4 & & 10^2 & 10^1 & 10^0 \end{array}$$

Quiz Base Value of x^{th} digit in Decimal N.S

$$\text{Pow}(10, n-1)$$

Quiz $(125)_8 \rightarrow 1 \times 8^2 + 2 \times 8^1 + 5 \times 8^0$

$$\rightarrow 64 + 16 + 5$$

$$\rightarrow (85)_{10}$$

Quiz Incorrect Octal representation :-

1000001 ✓

6854 ✗

└→ [0-7]

Quiz

$$\begin{aligned} & (02101)_3 \\ & \quad \quad \quad \begin{matrix} 4 & 3 & 2 & 1 & 0 \end{matrix} \quad \text{3} \\ & \rightarrow 0 \times 3^4 + \underbrace{2 \times 3^3}_{54} + \underbrace{1 \times 3^2}_9 + 0 \times 3^1 + \underbrace{1 \times 3^0}_1 \\ & \rightarrow \underline{\underline{(64)_{10}}} \end{aligned}$$

Binary Number System

Base = 2 $\Rightarrow [0, 1]$: 2 unique digits.

$$\begin{aligned} (10110)_2 &= 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^1 \\ &= \underline{\underline{(22)_{10}}} \end{aligned}$$

Decimal to Binary conversion :-

$$\begin{aligned} (28)_{10} &\rightarrow \begin{array}{r|l} 2 & 28 \\ \hline 2 & 14 \\ \hline 2 & 7 \\ \hline 2 & 3 \\ \hline 2 & 1 \\ \hline & 0 \end{array} \begin{array}{l} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{array} \end{array} \quad \begin{array}{l} 2 \sqrt{28} \text{ (14)} \\ \underline{28} \\ 0 \end{array} \quad \begin{array}{l} 28 \\ \swarrow \searrow \\ 14 \\ 0 \end{array}$$

$$\begin{aligned} (11100)_2 &= (28)_{10} \\ &\rightarrow 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 \\ &\rightarrow 16 + 8 + 4 \\ &\rightarrow \underline{\underline{28}} \end{aligned}$$

Quiz

Binary of 37 = ?

$$\begin{array}{r|l} 2 & 37 \\ \hline 2 & 18 \\ \hline 2 & 9 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline 2 & 1 \\ \hline & 0 \end{array} \begin{array}{l} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{array}$$

(100101)₂

Quiz

$$25 \Rightarrow \frac{1}{4} \frac{1}{3} \frac{0}{2} \frac{0}{1} \frac{1}{0}$$

Ex

$$44 \Rightarrow \frac{1}{5} \frac{0}{4} \frac{1}{3} \frac{1}{2} \frac{1}{1} \frac{1}{0}$$

$$32 + 8 + 4 + 2 + 1$$

$$\downarrow \\ 2^5 + 2^3 + 2^2 + 2^1 + 2^0$$

int \Rightarrow 4 Bytes

\Rightarrow 32 bits.

Addition

Decimal Number System

$$\begin{array}{r}
 13/10 \quad 10/10 \quad 16/10 \\
 0 \quad 1 \quad 1 \quad 1 \\
 3 \quad 4 \quad 5 \quad 9 \\
 2 \quad 8 \quad 4 \quad 7 \\
 + \\
 \hline
 6 \quad 13 \quad 10 \quad 16 \\
 \hline
 6 \quad 3 \quad 0 \quad 6 \\
 \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\
 6/10 \quad 13/10 \quad 10/10 \quad 16/10
 \end{array}$$

$$\begin{array}{r}
 \text{C} \\
 \hline
 \text{+} \\
 \hline
 \text{S} \\
 \hline
 \text{d}
 \end{array}$$

$$\begin{array}{l}
 d = S \% B \\
 c = S / B
 \end{array}$$

Binary Addition

$$\begin{array}{r}
 1/2 \quad 1/2 \quad 3/2 \quad 2/2 \rightarrow 1/2 \\
 0 \quad 0 \quad 1 \quad 1 \quad 0 \\
 1 \quad 0 \quad 1 \quad 1 \quad 0 \rightarrow 22 \\
 0 \quad 0 \quad 1 \quad 1 \quad 1 \rightarrow 7 \\
 \hline
 1 \quad 1 \quad 3 \quad 2 \quad 1 \\
 \hline
 1 \quad 1 \quad 1 \quad 0 \quad 1 \Rightarrow 29 \\
 \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\
 1/2 \quad 1/2 \quad 3/2 \quad 2/2 \quad 1/2
 \end{array}$$

Ex

$$\begin{array}{cccccc} & 2/2 & 4/2 & & & \\ & \uparrow & \uparrow & \uparrow & \uparrow & \\ 1 & 2 & 1 & 1 & & \\ 0 & 0 & 1 & 0 & 1 & \rightarrow 5 \\ 0 & 0 & 1 & 0 & 0 & \rightarrow 4 \\ 0 & 0 & 0 & 1 & 1 & \rightarrow 3 \\ 0 & 0 & 1 & 1 & 1 & \rightarrow 7 \end{array} \left. \vphantom{\begin{array}{cccccc} 1 & 2 & 1 & 1 & & \\ 0 & 0 & 1 & 0 & 1 & \rightarrow 5 \\ 0 & 0 & 1 & 0 & 0 & \rightarrow 4 \\ 0 & 0 & 0 & 1 & 1 & \rightarrow 3 \\ 0 & 0 & 1 & 1 & 1 & \rightarrow 7 \end{array}} \right\} \underline{\underline{19}}$$

$$\begin{array}{cccccc} 1 & 2 & 4 & 3 & 3 & \\ \hline 1 & 0 & 0 & 1 & 1 & \rightarrow (19) \\ \hline \end{array}$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $1 \times 2 \quad 2 \times 2 \quad 4 \times 2 \quad 3 \times 2 \quad 3 \times 2$

Quiz

$$\begin{array}{r} 1 \quad 1 \\ 10011 \\ 01001 \\ \hline 11100 \end{array}$$

$$\left. \begin{array}{c} \oplus \\ \ominus \\ * \\ / \\ \% \end{array} \right\} \text{TC : 0(1)}$$

Bit Manipulation

→ Operations performed on bits.

→ Execution is faster as they are directly acting on bits.

Bitwise Operators

($\&$, $|$, \sim , \wedge , \ll , \gg)
AND OR NOT XOR Left Shift Right Shift

a	b	$a \& b$	$a b$	$a \wedge b$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

$a \wedge b$: If both operands (a, b) are different the output is 1.

Ex $a = 4, b = 3$

$a \& b$: $\begin{array}{r} 100 \\ 011 \\ \hline 000 \end{array} \Rightarrow \underline{0}$	$a b$: $\begin{array}{r} 100 \\ 011 \\ \hline 111 \end{array}$	$a \wedge b$: $\begin{array}{r} 100 \\ 011 \\ \hline 111 \end{array}$
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$$a = 13 \Rightarrow 1101$$

$$b = 10 \Rightarrow 1010$$

Quiz

$$a \& b: \begin{array}{r} 1101 \\ 1010 \\ \hline 1000 \end{array} \Rightarrow 8$$

Quiz

$$a | b: \begin{array}{r} 1101 \\ 1010 \\ \hline 1111 \end{array} \Rightarrow 15$$

Quiz

$$a' b: \begin{array}{r} 1101 \\ 1010 \\ \hline 0111 \end{array} \Rightarrow 7$$

NOT (\neg) Operator

→ Single Bit Operator
→ Toggle the bit.

a	$\neg a$
0	1
1	0

Ex

$$a = 100$$

$$\neg a = \begin{array}{ccc} \downarrow \downarrow \downarrow \\ 011 \end{array}$$

Quiz

$$a = 11$$

$$a \mid 1 \Rightarrow$$

$$\begin{array}{r} \boxed{\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}} \\ \text{OR} \\ \hline \boxed{\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}} \Rightarrow 11 \end{array}$$

$$\begin{array}{l} 1 \mid 0 = 1 \\ 0 \mid 0 = 0 \\ a \mid 0 = a \end{array}$$

$$a \mid 1 = 1$$

Quiz

$$a = 10$$

$$a \mid 1 \Rightarrow$$

$$\begin{array}{r} \boxed{\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}} \\ \text{OR} \\ \hline \begin{array}{cccc} 1 & 0 & 1 & 1 \end{array} \Rightarrow 11 \end{array}$$

$$n \mid 1 \begin{cases} \rightarrow n & \text{if } n \text{ is Odd} \\ \rightarrow n+1 & \text{if } n \text{ is Even} \end{cases}$$

Quiz

$$a \& 1$$

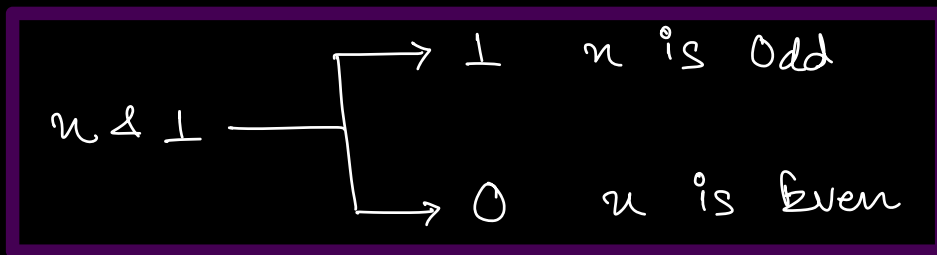
$$a = 11$$

$$\begin{array}{r} \boxed{\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}} \\ \& \\ \hline 0001 \end{array}$$

$$a = 10$$

$$\begin{array}{r} \boxed{\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}} \\ \& \\ \hline 0000 \end{array}$$

$$\begin{array}{l} n \& 0 = 0 \\ n \& 1 = n \end{array}$$



1) if $(n \cdot 1 \cdot 2 == 1) \Rightarrow n$ is Odd

2) if $(n \cdot 1 == 1) \Rightarrow n$ is Odd

Quiz

$a \wedge 1$

$$\begin{array}{r}
 a = 11 \Rightarrow \begin{array}{c} 1011 \\ 0001 \end{array} \\
 \wedge \\
 \hline
 1010 \Rightarrow 10
 \end{array}$$

$$11 \wedge 1 = 10$$

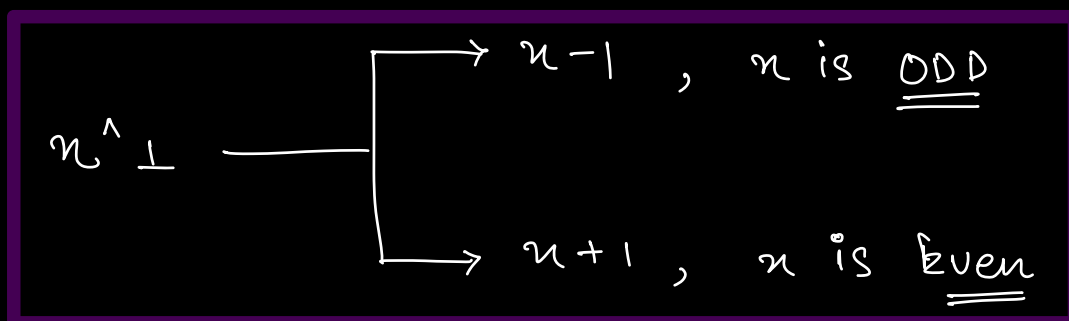
$$\begin{array}{l}
 0 \wedge 1 = 1 \\
 1 \wedge 1 = 0
 \end{array}$$

$$0 \wedge 0 = 0$$

$$1 \wedge 0 = 1$$

$$n \wedge 0 = n$$

$$\begin{array}{r}
 a = 10 \Rightarrow \begin{array}{c} 1010 \\ 0001 \end{array} \\
 \wedge \\
 \hline
 1011 \Rightarrow 11
 \end{array}$$



Properties

$$1) a | a = a$$

$$2) a \& a = a$$

$$3) a^{\wedge} a = 0$$

$$4) a^{\wedge} 0 = a$$

$$5) a^{\wedge} b = b^{\wedge} a$$

$$6) a \& b = b \& a$$

$$7) a | b = b | a$$

Commutative
Property

$$\begin{aligned} 8) \quad a^{\wedge} b^{\wedge} c &= (a^{\wedge} b)^{\wedge} c \\ &= a^{\wedge} (b^{\wedge} c) \\ &= (a^{\wedge} c)^{\wedge} b \end{aligned}$$

Associative

$$9) \text{ If } a^{\wedge} b = k \Rightarrow \begin{aligned} a^{\wedge} k &= b \\ b^{\wedge} k &= a \end{aligned}$$

$$a^{\wedge} b = k$$

$$\downarrow^{\wedge} b$$

$$a^{\wedge} b^{\wedge} b = k^{\wedge} b$$

$$a^{\wedge} (\underbrace{b^{\wedge} b}_0) = b^{\wedge} k$$

$$a^{\wedge} 0 = b^{\wedge} k$$

$$\boxed{a = b^{\wedge} k}$$

$$a^{\wedge} b = k$$

$$\downarrow^{\wedge} a$$

Quiz

$$120^{\wedge} 5^{\wedge} 6^{\wedge} 6^{\wedge} 120^{\wedge} 5$$

$a^{\wedge} a = 0$

$$120^{\wedge} 120^{\wedge} 6^{\wedge} 6^{\wedge} 5^{\wedge} 5$$

$$\downarrow$$
$$(120^{\wedge} / 120)^{\wedge} (6^{\wedge} / 6)^{\wedge} (5^{\wedge} / 5)$$

\swarrow
 0

\swarrow
 0

\swarrow
 0

$$\Rightarrow \underline{\underline{0}}$$

$$\|_{\mathbb{F}_3} : a^{\wedge} a^{\wedge} a^{\wedge} a^{\wedge} b^{\wedge} a^{\wedge} c^{\wedge} a^{\wedge} b = \underline{\underline{c}}$$

$$\|_{\mathbb{F}_3} : a^{\wedge} a^{\wedge} a^{\wedge} a^{\wedge} b^{\wedge} a^{\wedge} c^{\wedge} a^{\wedge} b^{\wedge} c^{\wedge} c = \underline{\underline{c}}$$

————— * —————

$$a^{\wedge} b^{\wedge} c^{\wedge} b^{\wedge} c^{\wedge} a^{\wedge} b^{\wedge} c = \underline{\underline{b^{\wedge} c}}$$