

1 2 ^ n

<< >>
left right

1 0 0 2²
1 0 0 0 2³

$a = 10$
 $a \ll 1$
 $a \ll 2$
 $a \ll 3$
 $a \ll 4$
 $a \ll 5$

$\Rightarrow 10 \quad 10 \times 2^0$
 $\Rightarrow 20 \quad 10 \times 2^1$
 $\Rightarrow 40 \quad 10 \times 2^2$
 $\Rightarrow 80 \quad 10 \times 2^3$
 $\Rightarrow 160 \quad 10 \times 2^4$
 $\Rightarrow 320 \quad 10 \times 2^5$

$2^3 + 2^4 = 2(2^2 + 2^3)$
 $2^5 + 2^6 = 2(2^4 + 2^5)$

every low

$$a \ll n = a \times 2^n$$

(Assuming no overflow)

$$a = 1$$

$$1 \ll 1 = 1 \times 2^1 = 2$$

$$1 \ll 2 = 1 \times 2^2 = 2^2$$

$$1 \ll 3 = 1 \times 2^3 = 2^3$$

$$1 \ll n = 2^n$$

$a = 10$	<u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>1</u> <u>0</u> <u>1</u> <u>0</u>	$\Rightarrow 10$	Integer division
$a >> 1$	<u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>1</u> <u>0</u> <u>1</u>	$\Rightarrow 5$	$10/2^1$
$a >> 2$	<u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>1</u> <u>0</u>	$\Rightarrow 2$	$10/2^2$
$a >> 3$	<u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>1</u>	$\Rightarrow 1$	$10/2^3$
$a >> 4$	<u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u>	$\Rightarrow 0$	$10/2^4$

$$a \gg n = \frac{a}{2^n}$$

Quiz 1

$$15 \ll 2$$

$$\begin{array}{cccccccc}
 \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} \\
 \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{0}
 \end{array} \Rightarrow 60$$

$$15 \ll 2 = 15 \times 2^2 = 60$$

Quiz 2

$$29 \gg 2$$

$$= \frac{29}{2^2} = 7$$

Quiz 3

$$2^n = ?$$

$$a \ll n = a \times 2^n$$

$$2 \ll n = 2 \times 2^n = 2^{n+1}$$

$$1 \ll n = 1 \times 2^n = 2^n$$

Quiz 4

$$5^n = ?$$

$$5 * (1 \ll n) = 5 * 2^n \quad \times$$

$$5 \ll n = 5 \times 2^n \quad \times$$

$$5 \ll (n-1) = 5 \times 2^{n-1} \quad \times$$

Q. Given 2 positive no. N & j . Check if j^{th} bit in N is set or unset.

1 \rightarrow set
0 \rightarrow unset

$$N = 4$$

$$j = 1$$

$$\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{array}$$

return false

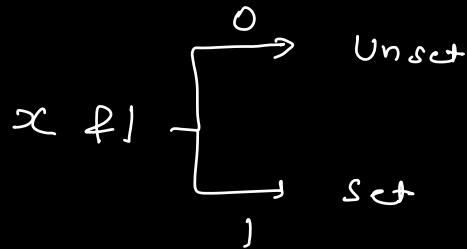
$$N = 10$$

$$\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ \hline 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{array}$$

$j = 2 \rightarrow$ false

$j = 3 \rightarrow$ True

Check if the rightmost bit is set / unset



N: 7 6 5 4 3 2 1 0
 0 0 0 0 1 0 1 0

i=3

N >> 1 0 0 0 0 0 1 0 1

N >> 2 0 0 0 0 0 0 1 0

N >> 3 0 0 0 0 0 0 0 1

$(N \gg i) \& 1$

boolean checkbit(N, i)

if $((N \gg i) \& 1 == 0)$

return False

else

return True

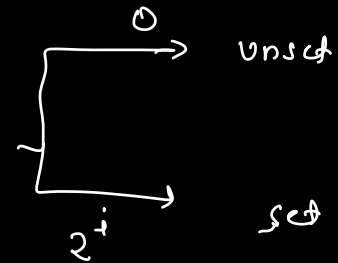
N: 7 6 5 4 3 2 1 0
 0 0 0 0 1 0 1 0

0 0 0 0 0 0 0 1
 +
 0 0 0 0 0 0 0 1

 0 0 0 0 0 0 0 1

$$\begin{array}{r}
 00001010 \\
 \& \\
 00001000 \\
 \hline
 00001010
 \end{array}$$

$N \& (1 \ll i)$



boolean checkbit(N, i)

IF $(N \& (1 \ll i) == 0)$
 return False
 else
 return True

$$\begin{array}{r}
 \begin{array}{ccccccc}
 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\
 N = & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0
 \end{array} \\
 \& \\
 \begin{array}{ccccccc}
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
 \end{array} \\
 \hline
 \begin{array}{ccccccc}
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
 \end{array}
 \end{array}$$

Q. Given two number N, i. Set the ith bit.

$N = 4$

$i = 3$

$\begin{array}{ccccccc}
 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
 \end{array}$

00001100 = 12 ans.

0 | 1 = 1

1 | 1 = 1

```
  00000101
+ 00001000
-----
  00001101
```

```
int setbit (N, j)
{
    return N | (1 << j)
}
```

```
int setbit (N, j)
{
    if (checkbit(N, j) == true)
        return N
    else
        return N + (1 << j)
}
```

Q. Toggle j^{th} bit of N

0 ^ 1 \rightarrow 1

1 ^ 1 \rightarrow 0

return $N \wedge (1 \ll i)$

$N = 10$

$i = 3$

```
      00001010
    ^ 00001000
    -----
      00000010
```

Tower Research

Q. Given a +ve no N . Toggle all the bits starting from the right until rightmost set bit.

$N = 20$

```
10100
10011
```

$N = 24$

```
11000
10111
```

32 bits \rightarrow int

for ($i = 0$; $i < 32$; $i++$)

if (checkbit(N, i) == false)

setbit(N, i)

else

{ toggle(N, i)

break

1000
0999

1000
↓ -1
0111

N

(N-1)

3: 011

010 = 2

15: 1111

1110 = 14

6: 110

101 = 5

8: 1000

0111 = 7

20: 10100

10011 = 19

24: 11000

10111 = 23

int toggleUntilRightmost (N)

{
return N-1

Google, Amazon

Q. Given +ve N. Count of no. of set bits of N

N=10

1010

⇒

2

N=8

1000

⇒

1

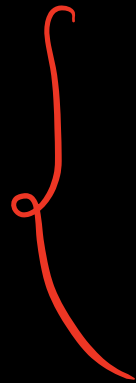
N=15

1111

⇒

4


```
int countSetBits ( int N)
```



```
int cnt = 0
```

TC: O(1)

```
for( j=0 ; j<32 ; j++)
```

```
{ if ( checkbit( N, j) == true)
    cnt++ }
```

```
return cnt
```

```
int countSetBits ( int N)
```

16



```
int cnt = 0
```

```
while( N > 0 )
```

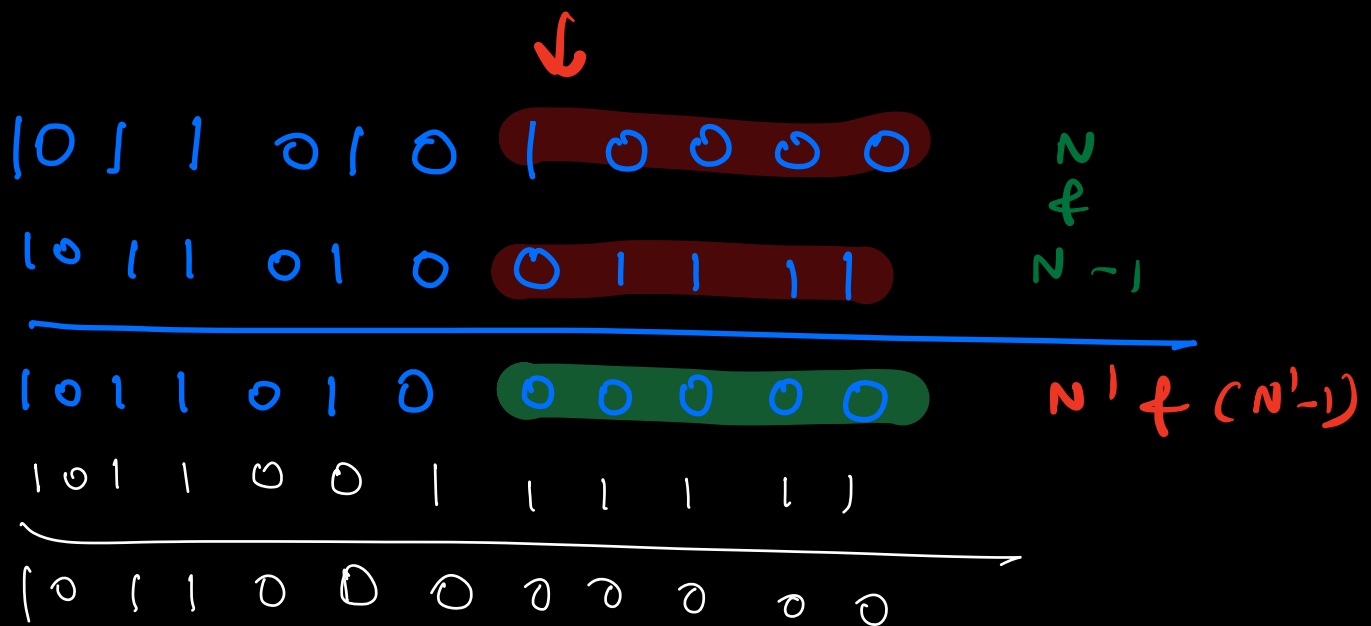
```
{ if ( N & 1 == 1 ) cnt++
  N = N >> 1 }
```

```
return cnt
```

TC: $\log N \leq 32$

000000010000
00-----001000

$N = 10^9$



$N \neq (N-1) \Rightarrow$ rightmost set bit
 will be unset

```

int countSetBits (int N)
    16
{
    int cnt = 0;
    while (N > 0)
    {
        N = N & (N-1);
        cnt++;
    }
    return cnt;
  
```

TC: $O(\log N)$

iteration

1 0 0 0 0

1

1 1 1 1 1

5

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

$N \leq 10^9$

1 1

32

$N = 16$

No of bits $\log N$

1 0 0 0 0

$N = 1024$

11

1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

X

X

Doubts.

$$Z = 2^{32}$$

0 1 0 0 0 , — — —
0 0 1 0 , — — —

↓ 32

၁၂၁၂

9 0 1

↓

୦ ୧ ୨ ୩ ୪ ୫ ୬ ୭ ୮ ୯ ୧୦

$$\begin{array}{r} \wedge \quad 1 \quad 1 \quad 0 \\ \quad 1 \quad 1 \quad 0 \\ \hline \quad 0 \quad 0 \quad 0 \end{array}$$

```

int countSetBits ( int N)
{
    int cnt = 0
    while ( N > 0 )
    {
        N = N & (N-1)
        cnt++
    }
    return cnt
}

```

N = 21

```

    1 0 1 0 1
      ↓
N = 1 0 1 0 0
      ↓
    1 0 0 0 0
      ↓
    0 0 0 0 0

```

cnt = 0
~~1~~
~~2~~
3