

Recursion

Merge sort & Quick sort
Trees (Binary Tree | BST)
Heap
Segment Tree
Dynamic Programming
Backtracking

Observations :

- 1) size keeps decreasing
- 2) All the dolls are same, except size.
- 3) After last doll, we can't break further.

Recursion

⇒ Function calling itself.

⇒ Recursion is a way of solving a problem using smaller sub-problem.

Same problem of
small size.

Ex

$$\text{Sum}(N) = 1 + 2 + 3 + 4 + 5 + \dots + (N-1) + N.$$

Sum(N-1)

$$\text{Sum}(N) = \text{Sum}(N-1) + N$$

Steps to write the recursive code :

- 1) Assume | Trust your recursive code will work
- 2) Main logic : Solve the bigger problem using smaller subproblem.
- 3) Base | Exit Condition
→ Decide when recursion should stop.

```
int sum(int N) {  
    if (N == 1)  
        return 1;  
    // Main logic  
    return sum(N-1) + N;  
}
```

1) assumption :
 $\text{sum}(N) \Rightarrow$ returns sum
of first N natural no's.

2) Main logic :
 $\text{sum}(N) = \text{sum}(N-1) + N$.

3) Exit Condⁿ
 $\text{sum}(1) \rightarrow \textcircled{1}$

Ex Factorial of a number.

$$N! = \underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \dots \cdot (N-1)}_{(N-1)!} \cdot N$$

$$\text{fact}(N) = \text{fact}(N-1) * N$$

```

int fact(int N) {
    if (N == 0)
        return 1;
    return fact(N-1) * N;
}

```

1) Trust :

$$\text{fact}(N) \Rightarrow N!$$

2) Main Logic

$$\text{fact}(N) = \text{fact}(N-1) * N$$

3) Base Cond :-

$$1! = 1$$

$$0! = 1$$

Ex: Fibonacci Series.

<u>Indices</u>	0	1	2	3	4	5	6	7	8	9	10
<u>Series</u>	1	1	2	3	5	8	13	21	34	55	89 ...

$$\text{fib}(N) = \text{fib}(N-1) + \text{fib}(N-2)$$

```

int fib(int N) {
    if (N == 0 || N == 1)
        return 1;
    return fib(N-1)
        +
        fib(N-2);
}

```

1. Assumption

$$\text{fib}(N) \rightarrow N^{\text{th}} \text{ fibonacci no.}$$

2. Main Logic

$$\text{fib}(N) = \text{fib}(N-1) + \text{fib}(N-2)$$

3. Unit Cond

$$\times \text{fib}(1) = \text{fib}(0) + \text{fib}(-1)$$

$$\times \text{fib}(0) = \text{fib}(-1) + \text{fib}(-2)$$

$$\checkmark \text{fib}(2) = \text{fib}(1) + \text{fib}(0)$$

```

int sum(int N) {
    if (N == 1)
        return 1;
    // Main logic
    return sum(N-1) + N;
}

```

3

sum(5) {

// N=5

sum(4) + 5;

3

sum(4) {

// N=4

sum(3) + 4

3

sum(3) {

// N=3

sum(2) + 3;

2

sum(2) {

// N=2

sum(1) + 2;

3

sum(1) {

// N=1

Base case

3

15

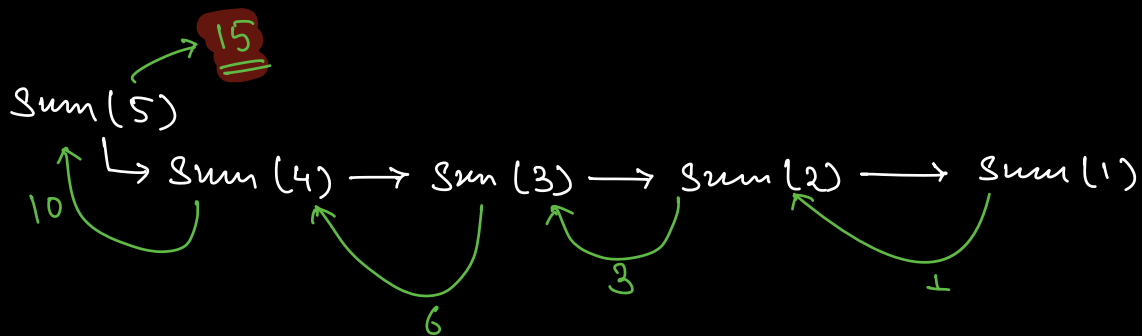
10

6

3

1

1



\Rightarrow LIFO (Last In, First Out)
 \hookrightarrow Stack.

\Rightarrow

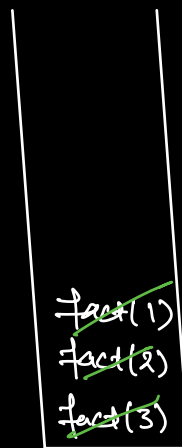
```
int fact(int N) {
    if (N == 0)
        return 1;
    return fact(N-1) * N;
}
```

3
 \rightarrow 6
 $\text{fact}(3)$
 $// N=3$
 $\text{fact}(2) * 3$
3
 \uparrow 2
 \downarrow

$\text{fact}(2)$
 $// N=2$
 $\text{fact}(1) * 2$
3
 \uparrow 1
 \downarrow

$\text{fact}(1)$
 $// N=1$
 $\text{fact}(0) * 1$
3
 \uparrow 1
 \downarrow

$\text{fact}(0)$
 $// N=0$
 $\text{return } 1;$
3

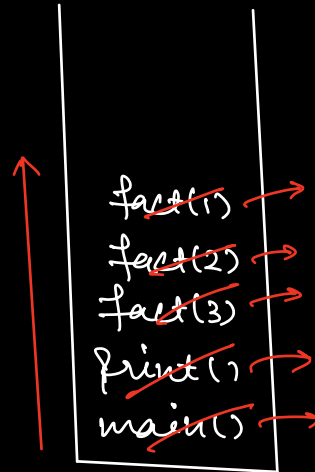


fun[^] Stack |
 Call Stack

⇒

```
main() {  
    print(fact(3))
```

3



Call stack | function stack

Q. Should we count the fun calls getting stored in the call stack in Space complexity?

⇒ YES.

When this call stack overflows ⇒ Stack Overflow exception.

Q: Given a number N, print all the numbers from 1 to N in increasing using recursion.

N=5 \Rightarrow 1, 2, 3, 4, 5

```
void printInc (int N) {  
    if (N == 0) {  
        return;  
    }
```

```
    printInc (N-1);  
    print (N);  
}
```

3

\uparrow
printInc (3) {
 // N=3

printInc (2)
 print (3)

3

printInc (2) {
 // N=2

printInc (1)

\rightarrow print (2)

3

printInc (1) {
 // N=1

printInc (0)

print (1)

\rightarrow printInc (0) {
 // N=0

3

1, 2, 3

1. Assumption

printInc(N) \rightarrow prints all the no's from 1 to N in increasing order.

2. Main logic

printInc(N-1)
print(N)

3. Base Condition

N==0

Q. Given a number N, print all the numbers from 1 to N in decreasing using recursion.

$N \Rightarrow N, N-1, \dots, 3, 2, 1$
recursion.

```
void printDec (int N) {  
    if (N == 0) return;  
    print (N)  
    printDec (N-1)  
}
```

3