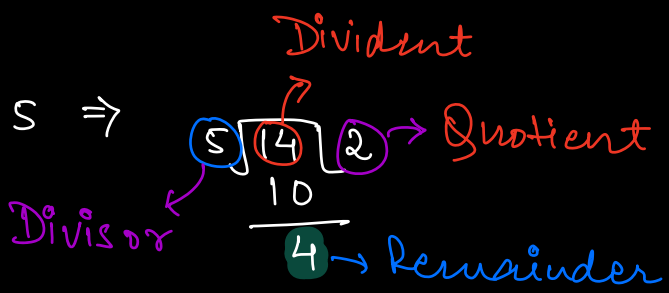
# A %. B ⇒ Remainder when A is divided B.

14 %. 5 ⇒




Divident = Divisor * Quotient + Remainder

**Remainder = Divident - Divisor * Quotient**

→ 14 %. 5 ⇒ 14 - 5 = 9

9 - 5 = 4

# Division is a repeated subtraction.

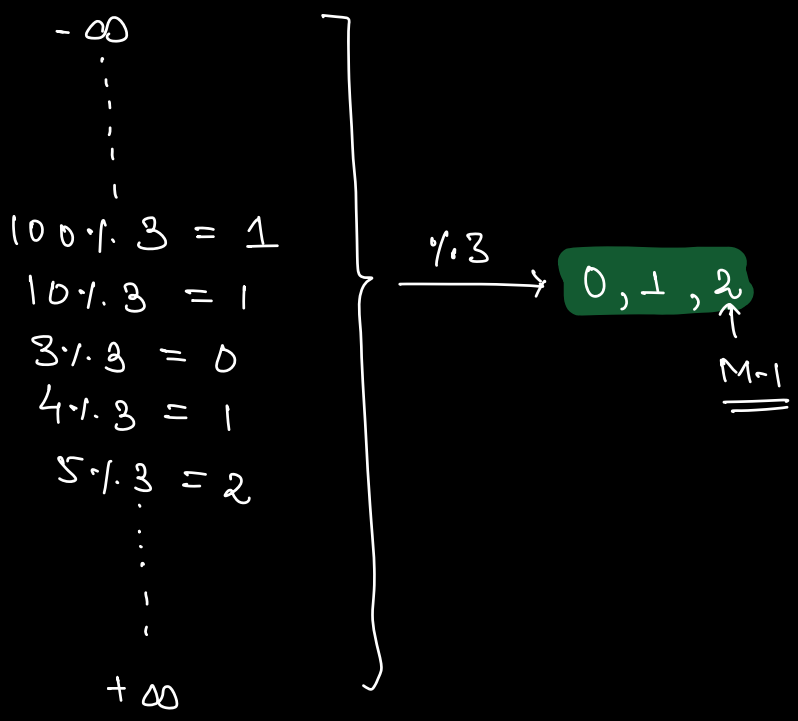
1) 30 %. 7 = 30 - 7 = 23 - 7 = 16 - 7 = 9 - 7 = 2

2 = 30 - 7 × 4

2) 40 %. 6 = 4 = 40 - 6 * 6

**Remainder = Divident - (largest multiple of divisor <= divident)**

# $a \% M \in [0, M-1]$




# Modulo Arithmetic :

1) $(A + B) \% M = (A \% M + B \% M) \% M$

$A = 4, B = 5, C = 6$

$(4 + 5) \% 6$

$9 \% 6$

$= 3$

$4 \% 6 + 5 \% 6$

$4 + 5$

$9$

2) $(a * b) \% M = (A \% M * B \% M) \% M$

Ex $100 \% 6 = 4$

$(100 \% 6) \% 6 = 4 \% 6 = 4$

$((100 \% 6) \% 6) \% 6 = 4$

3) $A \,\%\, M \Rightarrow (A+M) \,\%\, M$

$\Rightarrow (A \,\%\, M + M \,\%\, M) \,\%\, M$

$M \,\%\, M = 0$

$\Rightarrow A \,\%\, M$

$$\boxed{A \,\%\, M = (A + n*M) \,\%\, M}$$

4) $(A-B) \,\%\, M = (A \,\%\, M - B \,\%\, M + M) \,\%\, M$

$A=8,\ B=4,\ M=5$

$(8-4) \,\%\, 5$ | $(8 \,\%\, 5 - 4 \,\%\, 5) \,\%\, 5$

$4 \,\%\, 5$ | $(3-4) \,\%\, 5$

$= 4$ | $(-1) \,\%\, 5$

$\downarrow$

$(-1+5) \,\%\, 5 = 4 \,\%\, 5 = 4$

Ex

$-8 \,\%\, 5$

$a \,\%\, 5 \in [0,4]$

C/C++/Java/JS

$-(8 \,\%\, 5)$

$-3$

Python

$-8 \,\%\, 5 \Rightarrow 2$

$+5$

$+M$

Q. A, B

$A > B$

Find M, s.t $A \% M = B \% M$ ; $M > 1$

Ex $A = 16$, $B = 4$

$16 \% M = 4 \% M$

$M = 2, 3, 4, 6, \ldots\ldots$

$\Rightarrow$ $A \% M = B \% M$

$A \% M - B \% M = 0$

$(A - B) \% M = 0$

$x \% y = 0$

$\Rightarrow x$ is a multiple of $y$.

$\Rightarrow y$ is a factor of $x$.

M is a factor of A-B.

↳ return A-B. $\Rightarrow O(1)$

2 is also a must factor apart from A-B but A- B is biggest factor

Q. Given an Array of size N, calculate the no. of pairs $(i, j)$ s.t $(A[i] + A[j]) \% M = 0$ & $i \neq j$. pair(i,j) is considered to be same as pair(j, i).

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| A: | 4 | 7 | 6 | 5 | 5 | 3 |

$M = 3$

(0,3)
(0,4)
(1,3)
(1,4)
(2,5)

} 5 pairs.

```
Count = 0
for( i = 0; i < N; i++ ) {
    for( j = i+1; j < N; j++ ) {
        if( (A[i] + A[j]) % M == 0)
            Count++;
    }
}
```

TC: $O(N^2)$
SC: $O(1)$

**Quiz** A : {13, 14, 22, 3, 32, 19, 16} $M = 4$.

| i | j |
|---|---|
| 0 | 3 |
| 0 | 5 |
| 1 | 2 |
| 4 | 6 |

$\Rightarrow$ 4 pairs.

$\Rightarrow$ $(A[i] + A[j]) \% M = 0$

$(\underbrace{A[i] \% M}_{x} + \underbrace{A[j] \% M}_{y}) \% M = 0$

$(x + y) \% M = 0$

$x \in [0, M-1]$

$y \in [0, M-1]$

$x + y = M$

$\rightarrow$ $x+y$ should be divisible by $M$.

$A = 13, \; B = ?, \; M = 4$

$(A + B) \% 4 = 0$

$\downarrow$

$(A \% 4 + B \% 4) \% 4 = 0$

$1$

$(1 + x) \% 4 = 0$

$\downarrow$

$x = B \% 4 \in [0, 3]$

0, 1, 2 or 3

Ex $A = 35$, $B = ?$, $M = 10$

$(A + B) \% 10 = 0$

$(A \% 10 + B \% 10) \% 10 = 0$

$A \% 10 = 5$, $B \% 10 = ?$

$(5 + y) \% 10 = 0$

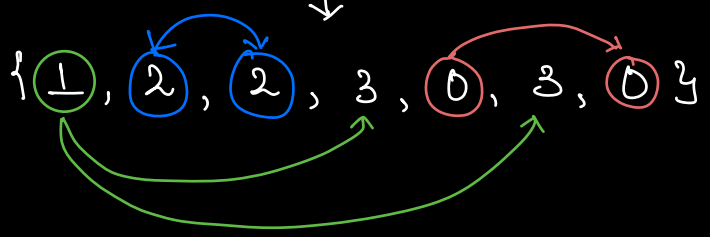
$\Downarrow$

$[0, 9]$

A : { 13, 14, 22, 3, 32, 19, 16 } $M = 4$.

(indices: 0, 1, 2, 3, 4, 5, 6)

$\downarrow \% M$



A : { 29, 11, 21, 14, 2, 5, 4, 6, 23, 13, 26, 14, 18, 15, 30, 35, 50, 20, 40 }

(indices 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18)

M = 10

% M = 10

[0, M-1]

HashMap

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 1 | 2 | 2 | 3 | 2 | 1 | 1 | 1 |
| 30, 50, 20, 40 | 11, 21 | 2 | 23, 13 | 4, 14 | 5, 15, 35 | 6, 26 | 14 | 18 | 29 |

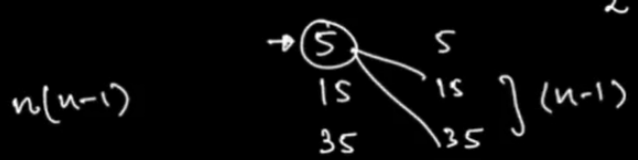
$\frac{C(C-1)}{2}$

(4, 6)
(4, 26)
(14, 6)
(14, 26)

5, 15, 35 → $cnt C_2$

$\frac{cnt(cnt-1)}{2}$

(11, 29)
(21, 29)
(2, 18)
(23, 14)
(13, 14)



```
HashMap<int, int> hm;
for (i = 0; i < N; i++) {
    hm[A[i] % M]++;
}
ans = 0
ans += (hm[0] * (hm[0] - 1)) / 2;
i = 1, j = M - 1;
while (i < j) {
    ans += (hm[i] * hm[j]);
    i++
    j--
}
if (M % 2 == 0) {
    ans += (hm[M/2] * (hm[M/2] - 1)) / 2;
}
return ans;
```

O(N)

O(M)

hm.get(i)

TC: O(N + M)

SC: O(M)

Q: Given an Array of all distinct integers where $\{0 <= A[i] <= N-1\}$, N is the size of the Array.

Replace $A[i] \Longrightarrow A[A[i]]$

Ex

$A[5] : \{3, 2, 4, 1, 0\}$ (indices 0 1 2 3 4)

$A[0] = A[A[0]] = A[3] = 1$

$A[1] = A[A[1]] = A[2] = 4$

$A[2] = A[A[2]] = A[4] = 0$

$A[3] = A[A[3]] = A[1] = 2$

$A[4] = A[A[4]] = A[0] = 3$

$\rightarrow \{1, 4, 0, 2, 3\}$

Quiz

$A : \{1, 6, 3, 5, 4, 2, 0\}$ (indices 0 1 2 3 4 5 6)

$A[0] = A[A[0]] = A[1] = 6$

$A[1] = A[A[1]] = A[6] = 0$

$A[2] = A[A[2]] = A[3] = 5$

$A[3] = A[A[3]] = A[5] = 2$

$A[4] = A[A[4]] = A[4] = 4$

$A[5] = A[A[5]] = A[2] = 3$

$A[6] = A[A[6]] = A[0] = 1$

$\{6, 0, 5, 2, 4, 3, 1\}$

* B[N]

```
for( i → 0 to N )
    B[i] = A[A[i]]
```

TC : O(N)

SC : O(N)

# Constant Extra Space.

x A[5] : { 3, 2, 4, 1, 0 }

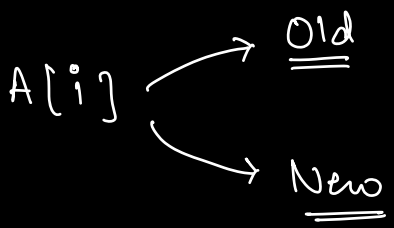
(0) (1) (2) (3) (4)

A[0] = A[A[0]]
= A[3]
= 1

A[4] = A[A[4]]
= A[0]

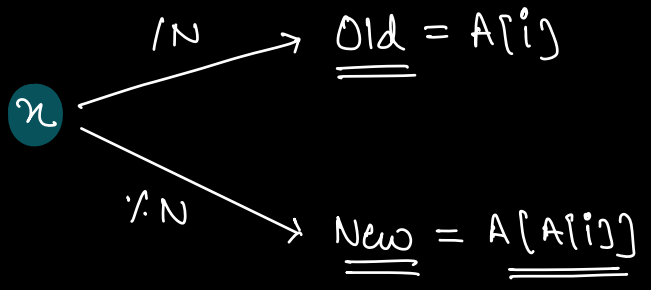
⇒ Day = 0, hr = 0

| | Days | hrs |
|---|---|---|
| 23 hrs | 0 | 23 |
| 46 hrs | 1 | 22 |
| 100 hrs | 4 | 4 |
| 125 hrs | 5 | 5 |
| n hrs | n/24 | n%24 |
| | Quotient | remainder. |




$0 <= A[i] <= N-1$

$$\begin{cases} A[i] \% N = A[i] \\ \frac{A[i]}{N} = 0 \end{cases}$$




$$\frac{x}{N} = \frac{A[i] * N + A[A[i]]}{N} \rightarrow x$$

$$= \frac{A[i] * N}{N} + \frac{A[A[i]]}{N} \rightarrow 0$$

$$x \% N = (A[i] * N + A[A[i]]) \% N$$

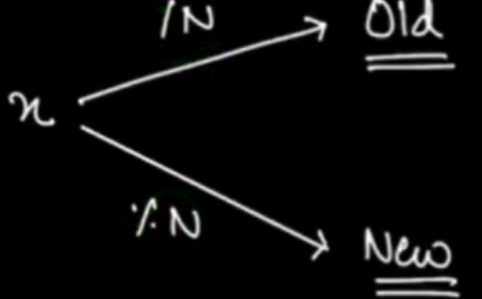
$$\downarrow \% N$$

$$0$$

$$= A[A[i]] \% N \Rightarrow A[A[i]]$$

$$[0, N-1]$$

$N$




$x = Old * N + new.$

$\Downarrow$ $A[i]$

$\Downarrow$ $A[A[i]]$

A[i] ⟶ A[i] * N + A[A[i]]

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| A : | 3 | 1 | 4 | 6 | 5 | 0 | 2 |

N = 7

↓ *N

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| A : | 3×7 | 1×7 | 4×7 | 6×7 | 5×7 | 0×7 | 2×7 |
| | +6 | +1 | +5 | +2 | +0 | +3 | +4 |

↓ + A[A[i]/7] / 7

A : { 3·7+6, 1·7+1, 4·7+5, 6·7+2, 5·7+0, 0·7+3, 2·7+4 }

↓ %N

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A : | 6 | 1 | 5 | 2 | 0 | 3 | 4 |

1)
```
for( i = 0; i < N; i++)
    A[i] *= N;
```

2)
```
for( i = 0; i < N; i++){
    index = A[i] % 7;
    value = A[index] % 7;
    A[i] += value;
}
```

3)
```
for( i = 0; i < N; i++)
    A[i] %= N;
```

45mins - 1hr

| TC : O(N) |
| --- |
| SC : O(1) |

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