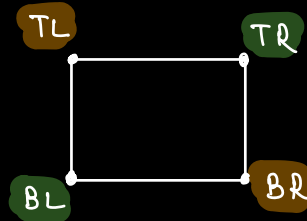


Q.1 Given a matrix of size $N \times M$, for each query Q , find the sum of a given submatrix.

	0	1	2	3	4	5
0	7	1	-6	3	12	-2
1	10	5	-2	0	9	4
2	6	4	-3	8	11	3
3	13	-8	-5	12	4	6
4	3	2	1	9	3	9
5	4	3	-2	6	8	8



TL & BR

TL BR
 (a_1, b_1) (a_2, b_2)

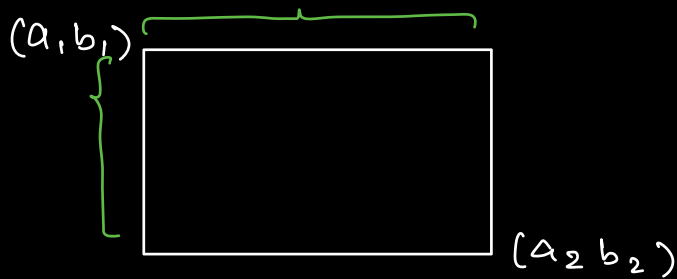
Q1: $(1, 2)$ $(4, 3)$ $\Rightarrow 20$

Q2: $(1, 1)$ $(4, 5)$ $\Rightarrow \underline{\underline{\text{sum}}}$

\vdots

Q (a, b) (c, d)

Brute Force



for each query, iterate from (a_1, b_1) to (a_2, b_2)

sum = 0

```
for (i = a1 ; i ≤ a2 ; i++) {  
    for (j = b1 ; j ≤ b2 ; j++) {  
        sum += mat[i][j];  
    }  
}
```

3
print(sum);

Q times.

TC: $O(Q \cdot N \cdot M)$

SC: $O(1)$

Optimization

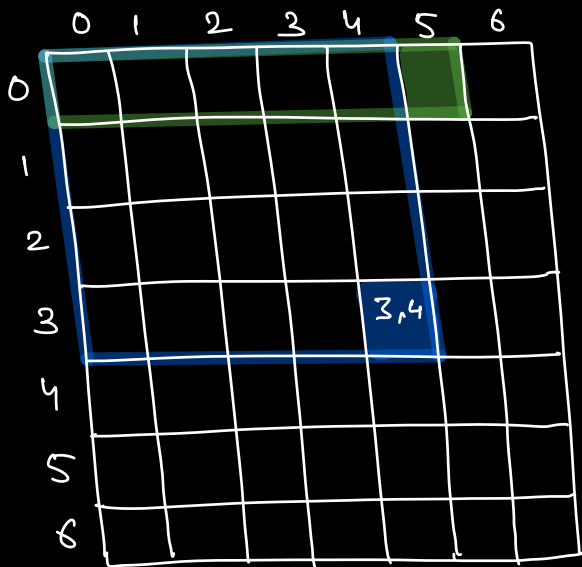
→ Sum of a range in 2D matrix.

→ PS.

⇒ PS[i] : Sum of elements from 0 to i.
(1D Array)

⇒ 2D matrix

$PS[i][j]$: Sum from $(0,0)$ to (i,j)

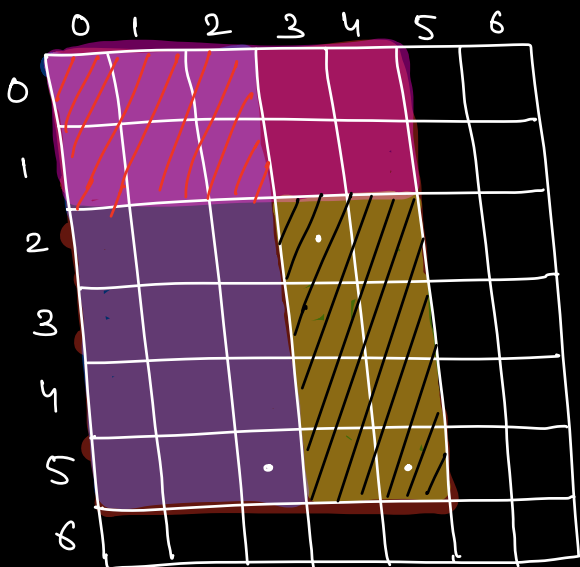


$PS[3,4] \Rightarrow$ Sum from $(0,0)$ to $(3,4)$

$PS[0][5] \Rightarrow$ Sum from $(0,0)$ to $(0,5)$

$PS[i][j] \rightarrow$ Already built.

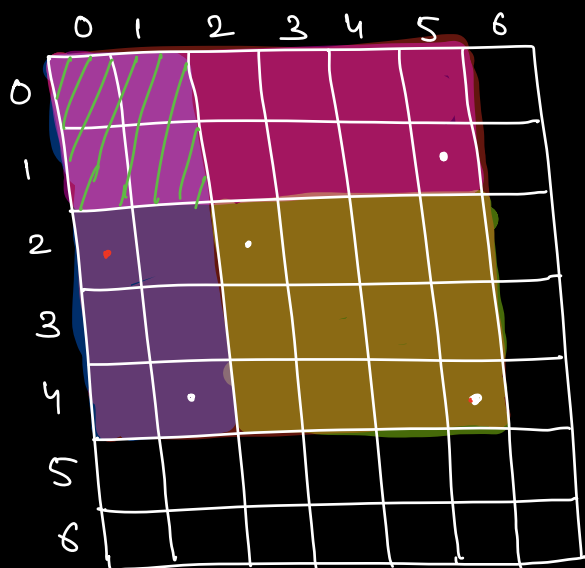
$mat[N][M]$



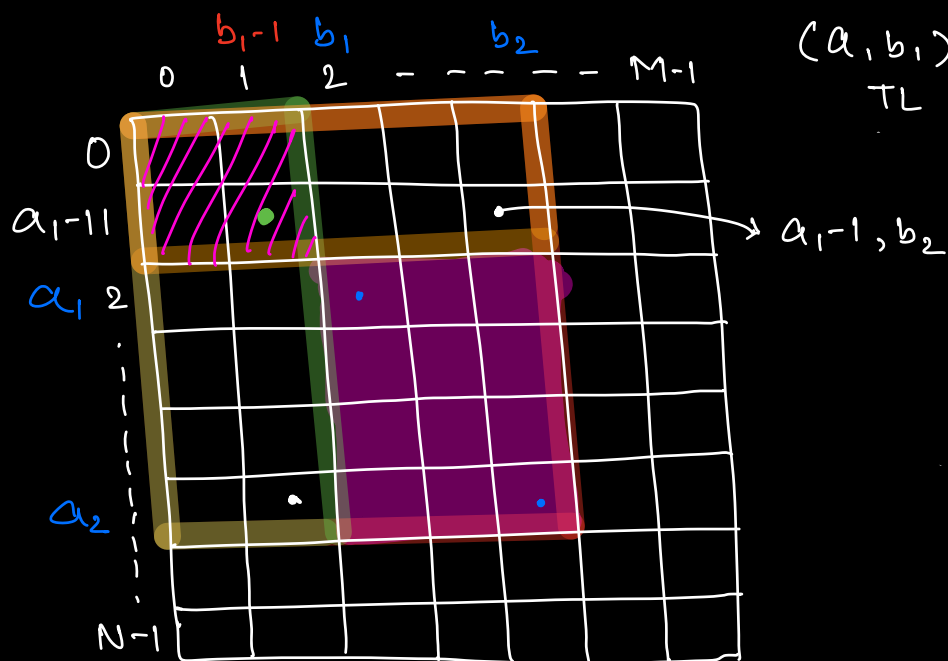
TL : $(2,3)$ }
BR : $(5,4)$ }

$$\text{Sum} = PS[5][4] - PS[5][2] - PS[1][4] + PS[1][2]$$

Q: (2,2) (4,5)
TL BR



$$\text{Sum} = \text{PS}[4][5] - \text{PS}[4][1] \\ - \text{PS}[1][5] + \text{PS}[1][1]$$



$(a_1, b_1) \leftrightarrow (a_2, b_2)$
TL BR

$$\text{Sum} = \text{PS}[a_2][b_2] - \text{PS}[a_2][b_1-1] - \text{PS}[a_1-1][b_2] \\ + \text{PS}[a_1-1][b_1-1]$$

$(a_1, b_1) \quad (a_2, b_2)$

$$\text{sum} = \text{PS}[a_2][b_2]$$

if $(b_1 > 0)$

$$\text{sum} -= \text{PS}[a_2][\underline{b_1 - 1}]$$

≥ 0

if $(a_1 > 0)$

$$\text{sum} -= \text{PS}[\underline{a_1 - 1}][b_2]$$

if $(a_1 > 0 \text{ \& \& } b_1 > 0)$

$$\text{sum} += \text{PS}[\underline{a_1 - 1}][\underline{b_1 - 1}]$$

TC for 1 query $\Rightarrow O(1)$

* How to Build PS matrix.

	0	1	2
0	a_0	a_1	a_2
1	b_0	b_1	b_2
2	c_0	c_1	c_2

PS row wise \rightarrow

	0	1	2
0	a_0	$a_0 + a_1$	$a_0 + a_1 + a_2$
1	b_0	$b_0 + b_1$	$b_0 + b_1 + b_2$
2	c_0	$c_0 + c_1$	$c_0 + c_1 + c_2$

PS col.
wise

a_0	$a_0 + a_1$	$a_0 + a_1 + a_2$
$a_0 + b_0$	$a_0 + a_1 + b_0 + b_1$	$a_0 + a_1 + a_2 + b_0 + b_1 + b_2$
$a_0 + b_0 + c_0$	$a_0 + a_1 + b_0 + b_1 + c_0 + c_1$	$a_0 + a_1 + a_2 + b_0 + b_1 + b_2 + c_0 + c_1 + c_2$

PS Matrix

Overall TC: $\underbrace{O(N \cdot M)}_{\text{PS[[]]} \text{ build}} + \underbrace{O(Q)}_{\text{queries.}}$

SC: $\underbrace{O(N \cdot M)}_{\text{PS[[]]}}$

→ Find PS row wise
 → Find PS col wise any order is possible.

Q.2 Given a matrix of size $N \times M$, calculate the sum of all submatrix sum.
 Google
 MS
Apple.

Sum of all subarray sums = $\left(\begin{matrix} \text{No. of times } A[i] \\ \text{is present in} \\ \text{all subarrays} \end{matrix} \right) * \underline{\underline{A[i]}}$

Tomorrow