

Q.1 Given an unsorted array of integers of size N, find the first missing natural number.

FB, Google  
MS, LI  
Apple,  
Oracle,  
.....

Natural no's : 1, 2, 3, 4, - - - - -

A[S] : { 3, -2, 1, 2, 4 }  $\Rightarrow$  4

Quiz

A : { 2, 4, -1, -6, 3, 7, 8, 4, -3, 0 }

$\hookrightarrow$  1

Quiz

A : { 1, 0, -5, -6, 4, 2 }

$\hookrightarrow$  3

Quiz

A : { 1, 2, 5, 6, 4, 3 }

N=6

$\Rightarrow$  7

Ex

A : { -1, 0, 1, 4, 8, 2, 1, 3 }

$\Rightarrow$  5

A : { -2, -3, -4, -5, 0 }

$\hookrightarrow$  1

## Brute Force

For all natural no's, check if it is present in the Array or not.



1  $\longrightarrow$   $O(N)$

2  $\longrightarrow$   $O(N)$

3  $\longrightarrow$   $O(N)$

$\vdots$

ans  $\longrightarrow$   $O(N)$

# of iterations  $\Rightarrow$  ans \* N

ans  $\in [1, N+1]$

TC:  $O(N^2)$

SC:  $O(1)$

for ( $i = 1$ ;  $i \leq N$ ;  $i++$ ) {

search  $i$  in the  
Array linearly.  $\} \rightarrow O(N)$

3  
return  $N+1$ ;

2. Use SET / MAP to search.

HashSet<int> set;

// Insert Array elements into set.

```
for (i = 1; i <= N; i++) {  
    if (!set.contains(i)) } → O(1)  
    return i;  
}
```

3

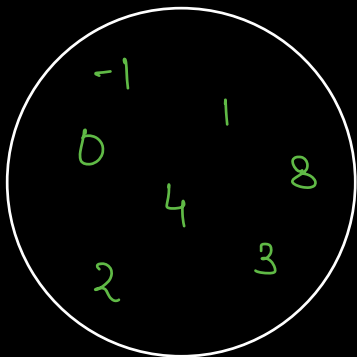
return N+1;

TC:  $O(N)$

SC:  $O(N)$

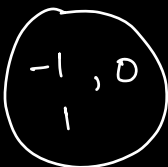
A: { -1, 0, 1, 4, 8, 2, 1, 3 }

Set



1, 2, 3, 4, 5  
x

A: { -1, 0, 1 }    N = 3



1, 2  
✓ x

Sort

A[5]: { 3, -2, 1, 2, 3, 4 }

↓ Sort

{ -2, 1, 2, 3, 3, 4 }

↑ 4 is Missing.

TC:  $O(N \log N)$

SC:  $O(\log N)$  /  $O(N)$

Quick  
Sort

Merge Sort

Constraint: Constant Extra space

SC:  $O(1)$

→ Input array modification is allowed.

⇒ Use array indices to mark the presence of elements.

HOW?

Size  $\rightarrow 2^{11}$

0 1 2 3 4 - - - - - N-1

--	--	--	--	--	--	--	--	--

ele      inden

$$1 \rightarrow 0$$
$$2 \rightarrow 1$$
$$3 \rightarrow 2$$
$$N \rightarrow N-1$$

I. Only distinct +ve no's are present.

$$f = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{matrix} -4 \\ -6 \\ -2 \\ 1 \\ 3 \end{matrix} & \begin{matrix} -6 \\ -2 \\ 1 \\ 3 \\ 3 \end{matrix} & \begin{matrix} -2 \\ 1 \\ 3 \\ 3 \\ 3 \end{matrix} & \begin{matrix} 1 \\ 3 \\ 3 \\ 3 \\ 3 \end{matrix} & \begin{matrix} 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{matrix} \end{matrix}$$
$$N = 5$$
$$1 \rightarrow 0$$
$$2 \rightarrow 1$$
 $3 \rightarrow 2$  $4 \rightarrow 3$ 
$$S \rightarrow$$

ele > N  
→ skip

$$\begin{array}{cccccc} & 0 & 1 & 2 & 3 & 4 \\ f & -4 & -6 & -2 & 1 & 3 \\ & \uparrow & \uparrow & \uparrow & \uparrow & \end{array}$$
$$A[i] < 0$$

→  $i+1$  is present in the Array.

$$A[3] > 0$$

→  $3 + 1 = 4$   
missing  
=

Ex 3

A : {<sup>0</sup>-8, <sup>1</sup>-1, <sup>2</sup>-4, <sup>3</sup>-2, <sup>4</sup>6, <sup>5</sup>-3}

N = 6

1 → ind  
0

2 → 1

3 → 2

⋮

N → N-1

abs(-2)  
= 2

abs(-3)  
= 3

{<sup>0</sup>-8, <sup>1</sup>-1, <sup>2</sup>-4, <sup>3</sup>-2, <sup>4</sup>6, <sup>5</sup>-3}

A[4] > 0

⇒ 4+1 is

missing

Edge :- If -ve no's are also present.

N = 6  
A : {<sup>0</sup>-8, <sup>1</sup>1, <sup>2</sup>5, <sup>3</sup>-2, <sup>4</sup>-6, <sup>5</sup>3}

↑  
X

ans ∈ [1, N+1]

-ve ⇒ replace with no > N  
(N+1)

A : {8, 1, 5, ~~-2~~, 6, 3} N = 6  
4

{<sup>0</sup>-8, <sup>1</sup>1, <sup>2</sup>-5, <sup>3</sup>7, <sup>4</sup>-6, <sup>5</sup>-3}

abs(-6)  
= 6

abs(-3)  
= 3

$\{-8, 1, -5, 7, -6, -3\}$



2 is missing  
 $A[1] > 0$

3

$\{-8, -2, 1, 3, 4, 8, 8\}$   $N = 7$



$\{-8, 8, -1, -3, 4, 8, 8\}$



$ele = 1$   
 $index = ele - 1 = 0$



$\{-8, 8, -1, -3, 4, 8, 8\}$   
 $i+1$   
 $A[1] > 0$   
 $1+1 = 2$

TC:  $O(N)$

SC:  $O(1)$

# Duplicates.

A: { 3 6 ~~-5~~ 3 ~~-4~~ 1 } N=6  
~~7~~ ~~7~~

↓  
 { -3 6 ~~-7~~ 3 7 ~~-1~~ }  
 ↑ ↑ ↑

$-1 * \text{abs}(A[\text{index}])$

{ -3 6 -7 3 7 -1 }

↑  
 $A[i] > 0$   
 2 is missing.

15

{ 7 6 1 2 3 3 1 0 4 2 4 8 8 16 -1 }

N=15

$A[i] \leq 0 \Rightarrow N+1$

{ -7 -6 -1 -2 3 -3 -1 -16 -4 2 4 8 8 16 16 } ↑

↓  
 { -7 -6 -1 -2 3 -3 -1 -16 -4 2 4 8 8 16 16 }

↑  
 5 is missing



Q.2 Maximum Absolute Difference.

Google  
MS Given an Array, find the max value of

$$f(i, j) = |A[i] - A[j]| + |i - j|$$

$$|x| \rightarrow \text{abs}(x)$$

$$A: \{ \overset{0}{1}, \overset{1}{3}, \overset{2}{-1} \}$$

i	j	
0	0	$\rightarrow  1-1  +  0-0  = 0$
0	1	$\rightarrow  1-3  +  0-1  = 3$
0	2	$\rightarrow  1-(-1)  +  0-2  = 4$
1	0	$\rightarrow  3-1  +  1-0  = 3$
1	1	$\rightarrow  3-3  +  1-1  = 0$
1	2	$\rightarrow  3-(-1)  +  1-2  = 5$
2	0	$\rightarrow  -1-1  +  2-0  = 4$
2	1	$\rightarrow  -1-3  +  2-1  = 5$
2	2	$\rightarrow 0$

1)  $f(i, j) = |A[i] - A[j]| + |i - j|$

$$f(i, i) = 0$$

$$|x| = |-x|$$

$$2) f(i, j) = f(j, i)$$

Brute force

ans = 0 / -∞

for (i = 0; i < N; i++) {

for (j = 0; j < N; j++) {

ans = max(ans, u);

}

}

→ Upper triangular OR lower triangular

ans = 0 / -∞

for (i = 0; i < N; i++) {

for (j = i+1; j < N; j++) {

ans = max(ans, u);

}

}

TC:  $O(N^2)$

SC:  $O(1)$

⇒

$$\underbrace{|x|}_{>0} \rightarrow \begin{cases} x & x > 0 \quad |4| = 4, |-7| = -(-7) \\ -x & x < 0 \end{cases}$$

$$|-7| \Rightarrow -(-7) = \underline{\underline{7}}$$

$$|8| = \underline{\underline{8}}$$

$$A: \{ \overset{0}{1}, \overset{1}{3}, \overset{2}{-1} \} \quad 3 \times 3 \text{ pairs}$$

	$\overset{j \rightarrow}{0}$	$\overset{j \rightarrow}{1}$	$\overset{j \rightarrow}{2}$	
$\overset{i \rightarrow}{0}$	0	3 (0,1)	4 (0,2)	$\boxed{i < j}$
$\overset{i \rightarrow}{1}$	3 (1,0)	0	5 (1,2)	
$\overset{i \rightarrow}{2}$	4 (2,0)	5 (2,1)	0	$\boxed{i > j}$

$$f(i, j) = \underbrace{|A[i] - A[j]|}_{\text{value difference}} + |i - j|$$

→ Upper triangular

$$i < j \rightarrow i - j < 0$$

$$| \underbrace{i - j}_{<0} | \Rightarrow -(i - j) = -i + j$$

$$i) A[i] - A[j] \geq 0$$

$$|A[i] - A[j]| = A[i] - A[j]$$

$$\begin{aligned} \#(i, j) &= (A[i] - A[j]) + (-(i - j)) \\ &= A[i] - A[j] - i + j \\ &= (A[i] - i) - (A[j] - j) \end{aligned}$$

$$\underline{\underline{ii)}} A[i] - A[j] < 0$$

$$|A[i] - A[j]| = -(A[i] - A[j])$$

$$\begin{aligned} \#(i, j) &= -(A[i] - A[j]) + (-(i - j)) \\ &= -A[i] + A[j] - i + j \end{aligned}$$

$$\#(i, j) = (A[j] + j) - (A[i] + i)$$

$$\underline{\underline{Ex}} \quad |A| - B \begin{cases} A - B \\ -A - B \end{cases} \rightarrow \max(A - B, -A - B)$$

$$1) A = 6, B = 2$$

$$A - B = 6 - 2 = 4$$

$$-A - B = -6 - 2 = -8$$

$$|6| - 2 = 4$$

$$2) A = -7, B = 4$$

$$A - B = -7 - 4 = -11$$

$$\begin{aligned} -A - B &= -(-7) - 4 \\ &= 3 \end{aligned}$$

$$|-7| - 4 = 3$$

$$3) A = 5, B = -1$$

$$A - B = 5 - (-1) = 6$$

$$-A - B = -5 - (-1) = -4$$

$$|5| - (-1) = \underline{\underline{6}}$$

$$|x-y| \leq \begin{cases} x-y \\ -(x-y) \\ y-x \end{cases} \Rightarrow \max(x-y, -x+y)$$

$$f(i, j) = \max \left( (A[i] - i) - (A[j] - j), (A[j] + j) - (A[i] + i) \right)$$

→  $\uparrow x - y \downarrow \Rightarrow$  maximise

$$\begin{array}{l} A[i] - i \rightarrow \max \\ A[j] - j \rightarrow \min \end{array} \left\{ (A[i] - i) - (A[j] - j) \right\} \uparrow$$

$$\begin{array}{l} A[i] + i \rightarrow \min \\ A[j] + j \rightarrow \max \end{array} \left\{ (A[j] + j) - (A[i] + i) \right\} \uparrow$$

$$A: \begin{array}{c|c|c} 0 & 1 & 2 \\ \hline 1 & 3 & -1 \end{array}$$

↓

$$B: \begin{array}{c|c|c} & j & i \\ \hline 1 & 4 & 1 \end{array}$$

$$B[i] \rightarrow A[i] + i$$

$$C: \begin{array}{c|c|c} 0 & 1 & 2 \\ \hline 1 & 2 & -3 \end{array}$$

$$C[i] = \underline{\underline{A[i] - i}}$$

$$\max(2 - (-3), 4 - 1)$$

$$\max(5, 3) \Rightarrow \underline{\underline{5}}$$

Align + i

	0	1	2
A:	1	3	-1

$$\text{max} = -\infty + 4$$

$$\text{min} = \infty + 1$$

Align - i

	0	1	2
A:	1	3	-1

$$M = -\infty + 2$$

$$m = \infty + -3$$

$$\max(2 - (-3), 4 - 1)$$

$$\max(5, 3) \Rightarrow \underline{\underline{5}}$$

$$TC: O(N)$$

$$SC: O(1)$$

Q. Given a row-wise & col-wise sorted matrix, find if element  $k$  is present in the matrix.

Amazon  
Bloomberg

N x M

	0	1	2	3
0	5	10	15	20
1	6	12	18	24
2	7	14	21	28
3	8	16	24	34

$K = 100 \rightarrow \text{false}$

$K = 14 \rightarrow \underline{\underline{\text{true}}}$

1.

TC:  $O(N \cdot M)$

SC:  $O(1)$

2.

TC:  $O(N \cdot \log M) \Rightarrow$  B.S on each row.

OR

$O(M \cdot \log N) \Rightarrow$  B.S on each col.

	0	1	2	3
0	5	10	15	20
1	6	12	18	24
2	7	14	21	28
3	8	16	24	34

N x M

0, M-1

$20 > \underline{14}$

$K = 14$

$15 > 14$

$10 < 14$

$12 < 14$

$14 == 14 \rightarrow \underline{\underline{\text{true}}}$

TC:  $O(N + M)$

SC:  $O(1)$

```

row = 0, col = M-1
while (row < N & & col >= 0) {
    if (mat[row][col] == k)
        return true;
    else if (mat[row][col] > k)
        col--;
    else
        row++;
}
return false;

```

Doubts.

$$\begin{matrix} 0 & 1 & 2 & 3 \\ [-1 & -4 & 8 & -2] \end{matrix} \quad N=4$$
  
 $\uparrow$   
 $\text{abs}(-2)$   
 $= 2$

$$\begin{matrix} 0 & 1 & 2 & 3 \\ [-1 & -4 & 8 & -2] \end{matrix}$$
  
 $\uparrow$   
 $A[2] > 0$   
 $2+1$  is  
missing

N = 4

1	4	2	3
---	---	---	---

[1-N]

[1-N]

$1 \rightarrow ?$   
 $2 \rightarrow ?$   
 $3 \rightarrow ?$   
 $4 \rightarrow ?$   
 $\vdots$