Q.1 Given N points in a 2D plane, How many
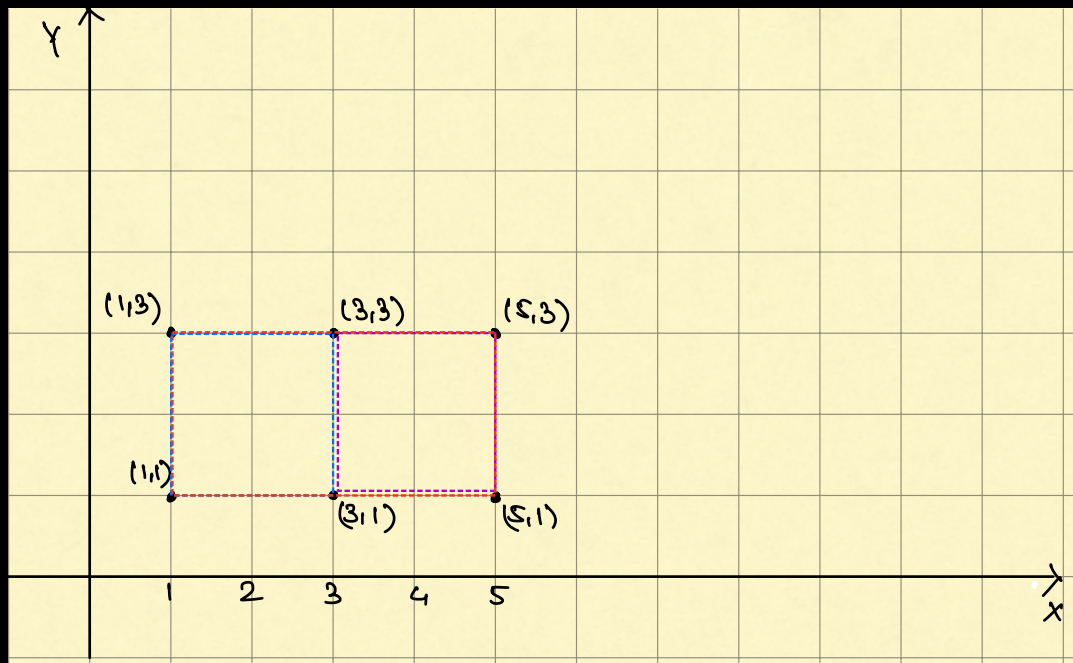
rectangles can be created that has

2 sides parallel to x-anis

and

2 sides parallel to y-anis

$\Rightarrow \underline{3}$
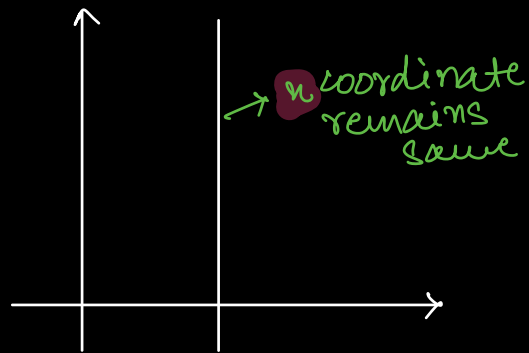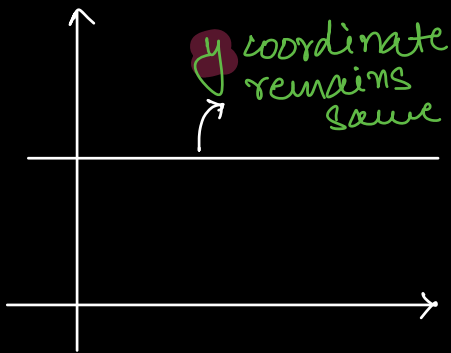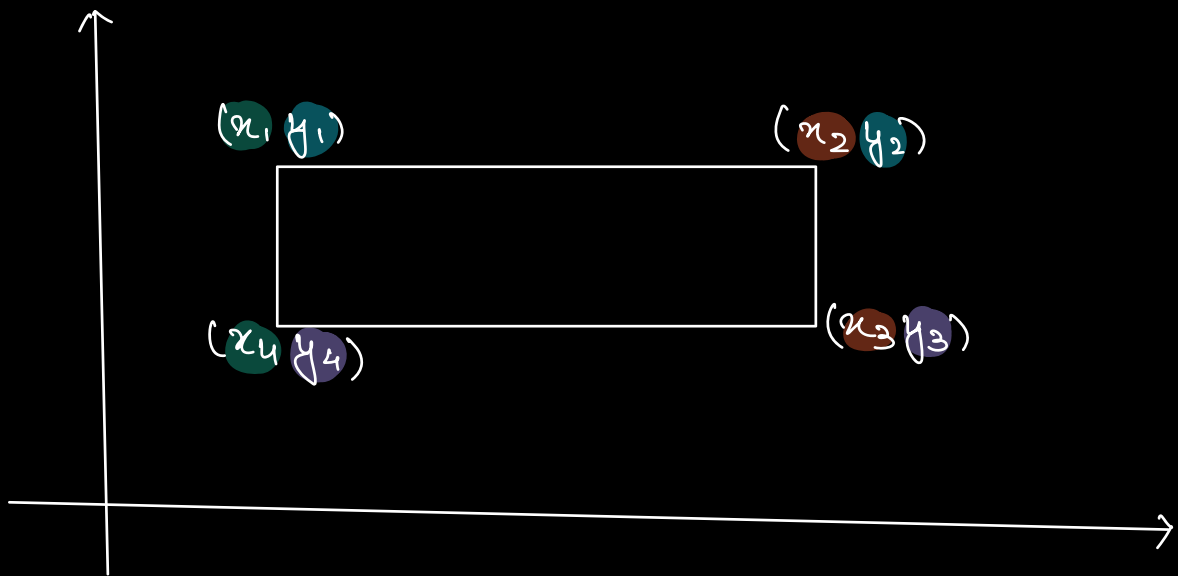
<u>Input</u>

X: [1 , 3 , 5 , 1 , 3 , 5]

Y: [1 , 1 , 1 , 3 , 3 , 3]

A: [(1,1) , (3,1) , (5,1) , (1,3) , (3,3) , (5,3)]

(x_1, y_1) (x_2, y_2)

(x_4, y_4) (x_3, y_3)

y coordinate
remains
same

x coordinate
remains
same

If a line is parallel to x-axis :

↳ Every coordinate on this line will have same y-coordinate

$$\Rightarrow y_1 = y_2 \;\; \&\& \;\; y_3 = y_4$$

If a line is parallel to y-axis :

↳ Every coordinate on this line will have same x-coordinate

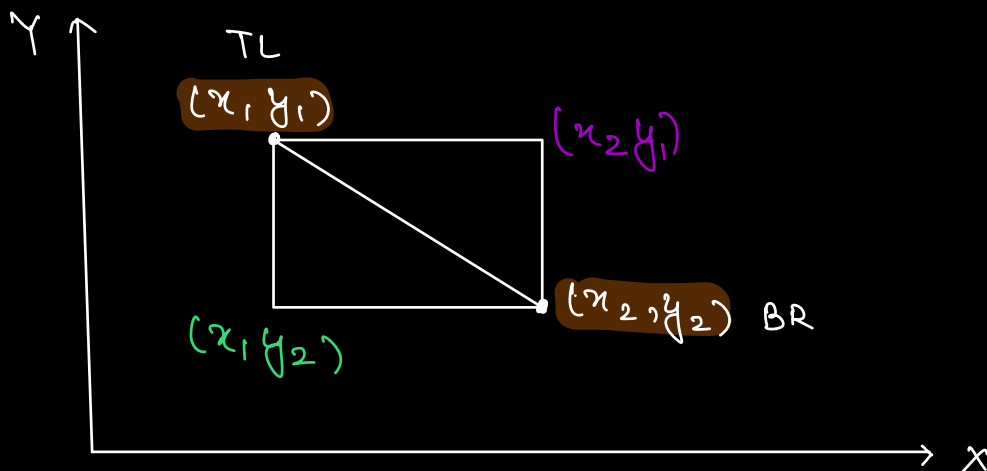$$\Rightarrow x_1 = x_4 \;\; \&\& \;\; x_2 = x_3.$$
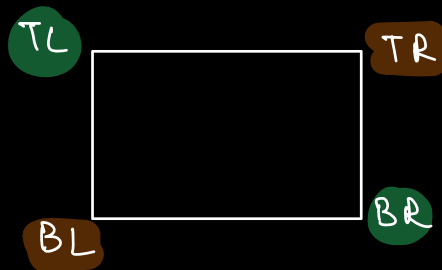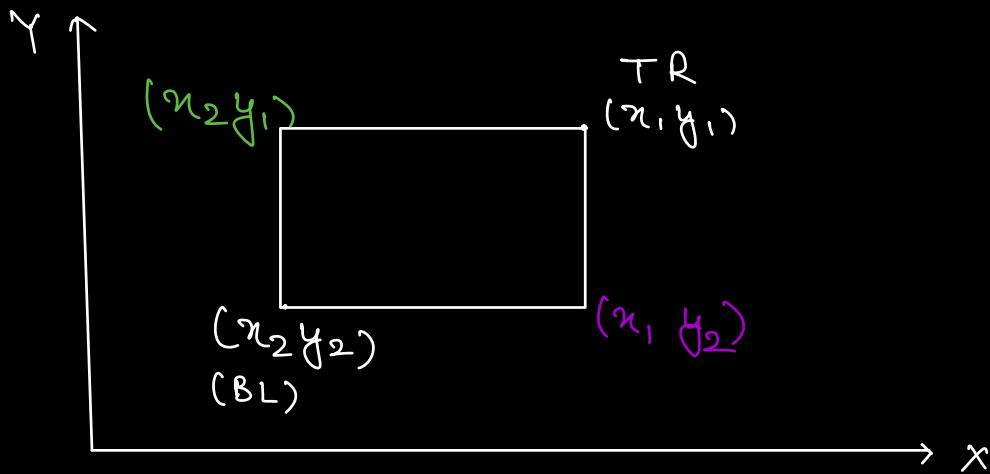
Brute force :

Iterate over all sets of 4 Coordinates &

check if $\left(\begin{array}{l} x_1 = x_4 \ \&\& \ x_2 = x_3 \ \&\& \\ y_1 = y_2 \ \&\& \ y_3 = y_4 \end{array}\right)$

$\Rightarrow$ Count ++

$$TC : O(N^4)$$

$$SC : O(1)$$

TL
TR
BL
BR

Y ↑

TL

(x_1, y_1)

(x_2, y_1)

(x_2, y_2) BR

(x_1, y_2)

→ X

* If (x_1, y_1) & (x_2, y_2) are diagonally opposite coordinates of a rectangle.

$$\text{OR}$$

$$(TL \& BR) \text{ or } (TR \& BL)$$

then there must exists two points (x_1, y_2) & (x_2, y_1) in the set of given points.

\# Iterate over all the pair of points : $\rightarrow O(N^2)$

$$(x_1, y_1) \quad \& \quad (x_2, y_2)$$

if $(x_1 \; != x_2 \;\&\& \; y_1 \; != y_2)\{$

    if other 2 points are present $\Rightarrow$
                  count ++

$\}$

$$(x_1, y_2)$$
$$(x_2, y_1)$$

⇒ To search the points in the input, we can store all points in the Set/Map.

```
if ( Set·contains (x₁, y₂) &&
        Set·contains (x₂, y₁) ) {
        count ++
    }
```
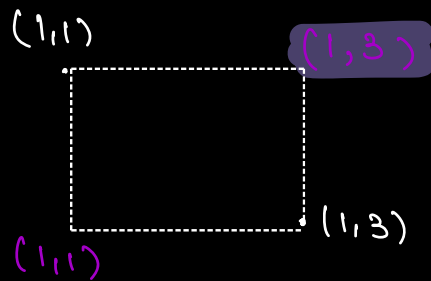
3

→ ans ⇒ count/2

$X: [1, 3, 5, 1, 3, 5]$

$Y: [1, 1, 1, 3, 3, 3]$

$$x_1 \, y_1 = (1,1)$$
$$x_2 \, y_2 = (3,1)$$

$$\left(x_1 \; != \; x_2 \; \&\& \right.$$
$$\left. y_1 \; != \; y_2 \right)$$

$x_1 \, y_1$
$(1,1)$

$x_2 \, y_2$
$(3,3)$

$(1,1)$ $(1,3)$

$(1,1)$ $(1,3)$

$(1,3)$ $(3,3)$

$(1,1)$ $(3,1)$

TC: O(N²)
SC: O(N)

(1,3) → "1_3"

"1_3"

Set ⟨String⟩ s;
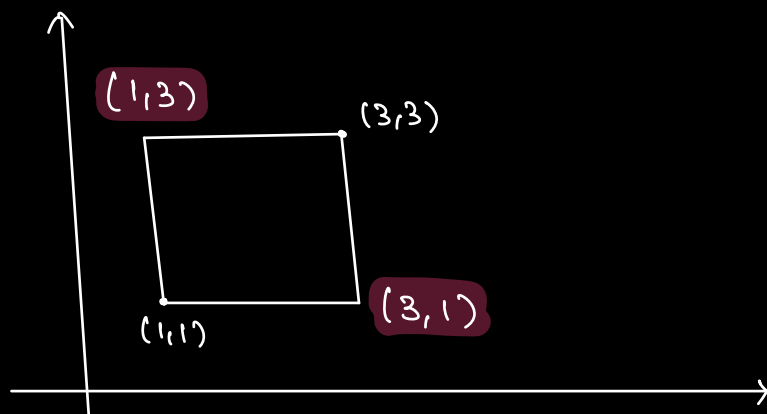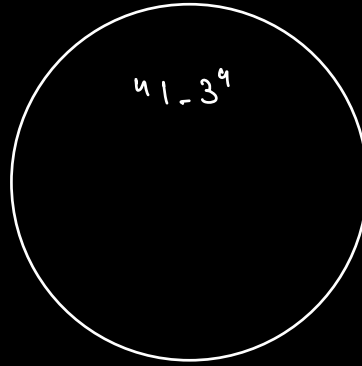
← search(x_y)

* Class Point {
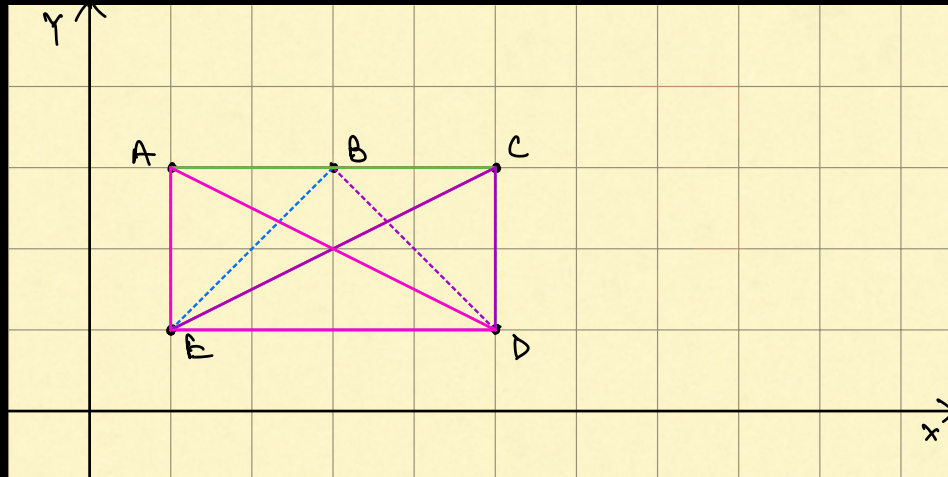   int x;
    int y;

  3

Set ⟨Point⟩ S;

⇒ Overide equals() &
   hashCode() methods.

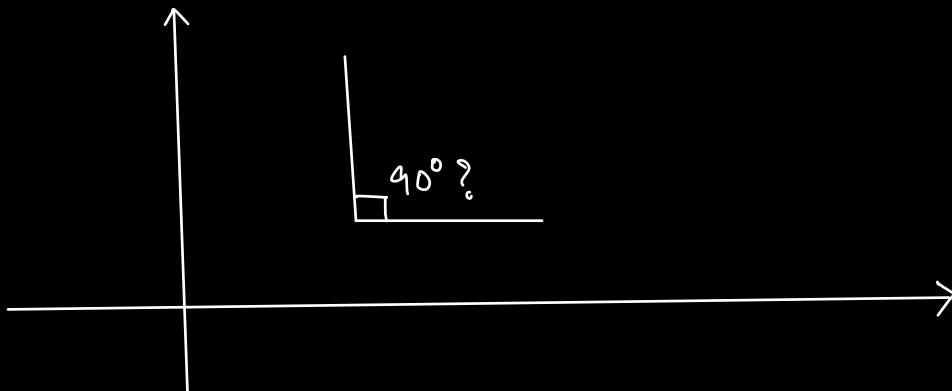**Q.** Given N points in a 2D plane, count the no. of right angled triangles for which

* One non hypotenuse side is parallel to x-axis AND.

* Other non hypotenuse side is parallel to y-axis

right angled triangles :-

1) BAE
2) BCD
3) CAE
4) ACD
5) CDE
6) AED
} 6 △'s.

$90°$?

Obs:

If one side is parallel to x-axis & other side is parallel to y-axis then $\triangle$ will be a right angled $\triangle$.

(x_3, y_3)

(x_1, y_1)

(x_2, y_2)

$(x_1 = x_3)$ && $(y_1 = y_2)$

$\dot{v}, \dot{j}$

$\begin{bmatrix} i, j \\ j, i \end{bmatrix}$

$\dot{i}, \dot{j}$

$\begin{bmatrix} i \to 0 \text{ to } N \\ j \to 0 \text{ to } N \end{bmatrix} \to C \sim |2$

$\begin{pmatrix} i \to 0 \\ j \to i+1 \end{pmatrix}$

$1 + 1 + 1 + 1$

B A

A B
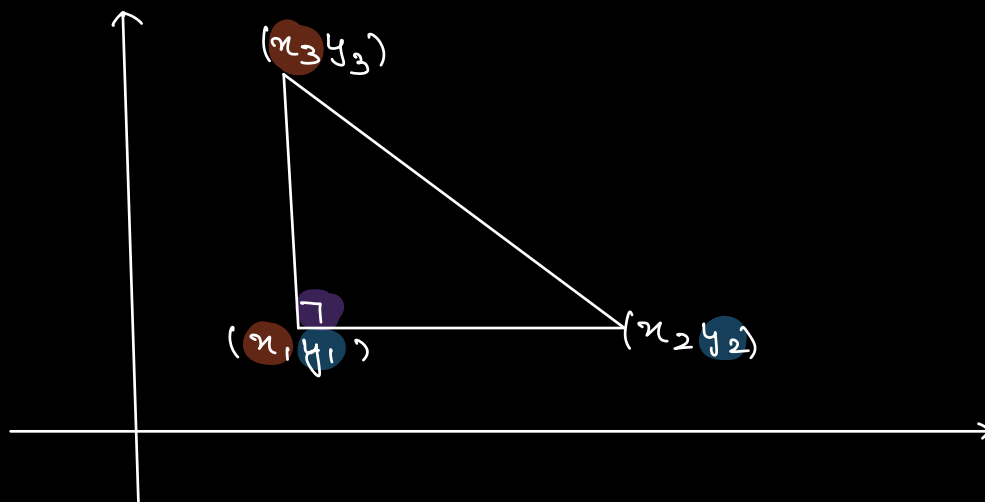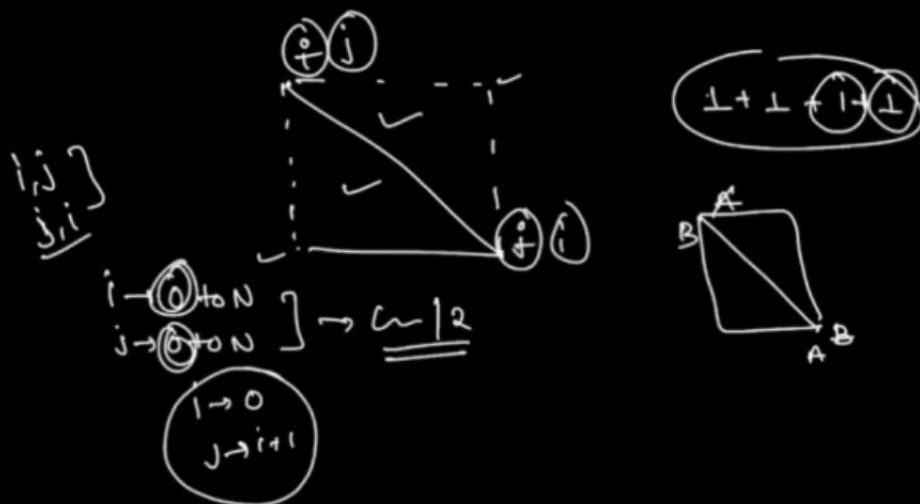
Brute force

```
for ( i = 0; i < N; i++ ) {        ⇒ (A)
    for ( j = 0; j < N; j++ ) {    ⇒ (B)
        if ( i == j ) continue;
        for ( K = 0; K < N; K++ ) {   ⇒ (C)
            if ( i == K || j == K ) continue;
            if ( X[i] == X[K] && Y[i] = Y[j] )
                     n₁          n₃      y₁      y₂
                Count ++
```

$$x_1 \qquad x_3$$
$$y_1 \qquad y_2$$

A B C
A C B
B A C
B C A
C A B
C B A

3

3

3

+-----------------------+
| TC : O(N³) |
| SC : O(1) |
+-----------------------+

$(x_1 y_1)$ .... $(x_2 y_1)$

$(x_1 y_2)$ .... $(x_2 y_2)$
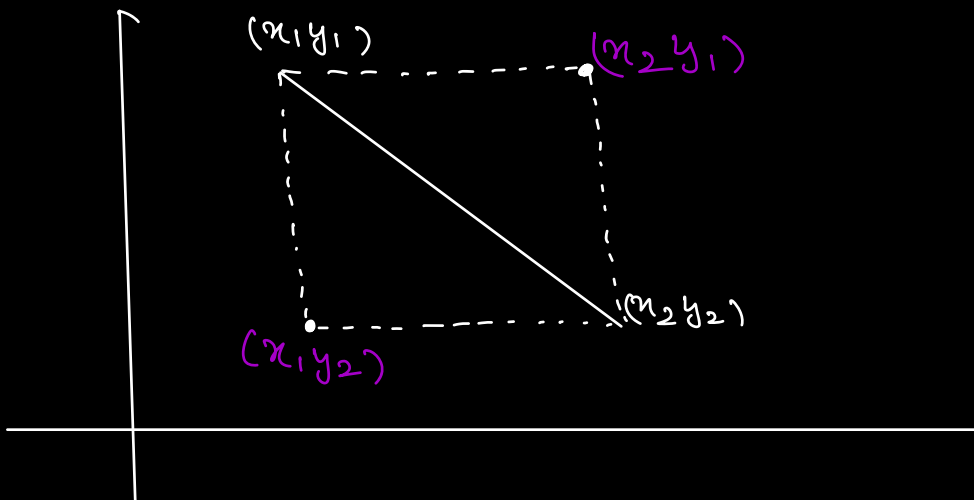
Approach

Iterate over all pair of coordinates (x_1, y_1) $\Rightarrow O(N^2)$

& (x_2, y_2) {Considering these as Hypotaneuse}

$O(1)$
Using
Set.
$$\begin{cases} \text{if } (x_1, y_2) \text{ is present} \Rightarrow \text{Count++} \\ \text{if } (x_2, y_1) \text{ is present} \Rightarrow \text{Count++} \end{cases}$$

TC : $O(N^2)$
SC : $O(N)$

```
for (i = 0; i < N; i++) {  ⇒ x₁ Y₁
    for (j = i+1; j < N; j++) {  ⇒ x₂ Y₂
        if (X[i] == X[j] || Y[i] == Y[j]
                    Continue;
        if ( Set.contains (X[i], Y[j] ))
                    Count++;
        if ( Set.contains (X[j], Y[i]) )
                    Count++;
    }
}
```
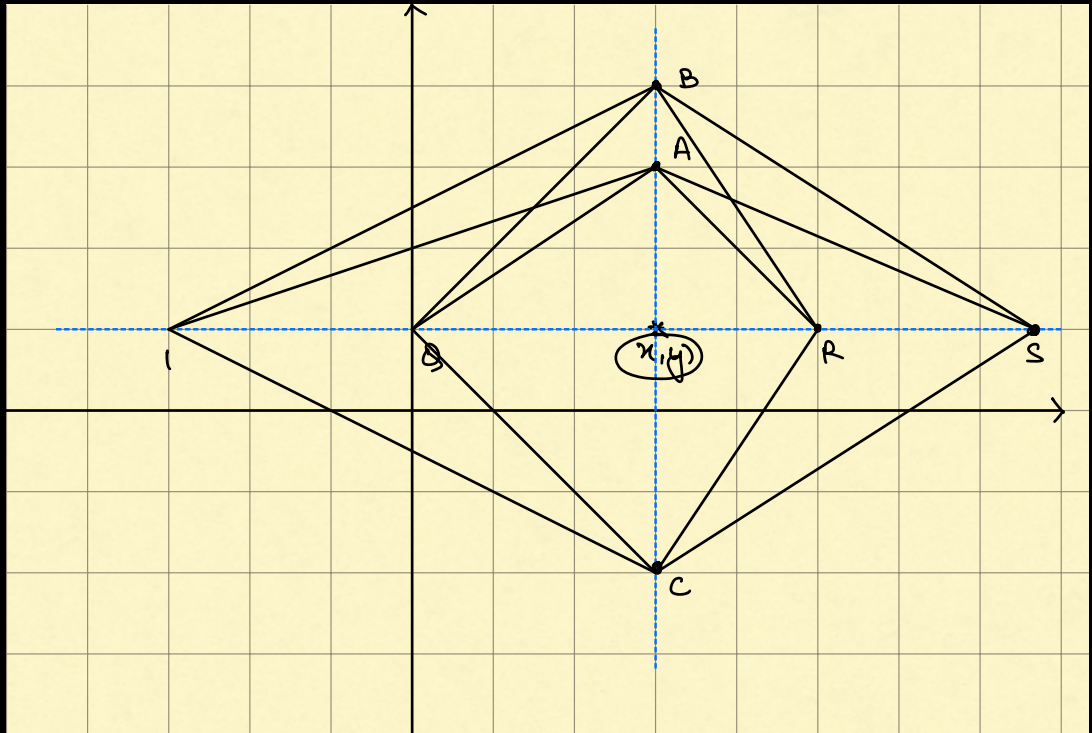
TC : $O(N^2)$
SC : $O(N)$

* for (x, y) :

    If there are ⓝ points having same
    x-coordinate (points on line parallel to y-axis)

               and

    If there are ⓜ points having same
    y-coordinate (points on line parallel to y-axis)

⇒ # of right angled $\triangle$'s with right angle
    at x,y ⇒ $n * m$

(freq of points with
x-coordinates)

x : freq(x)

1 : 2
2 : 1
3 : 2

y : freq(y)

3 : 3
1 : 2

A (1,3)

B (2,3)

C (3,3)

D (3,1)

E (1,1)

(x, y)	n	m	Δ's
(1,3)	2	3	6
(2,3)	1	3	3
(3,3)	2	3	6
(3,1)	2	2	4
(1,1)	2	2	4

23 ?
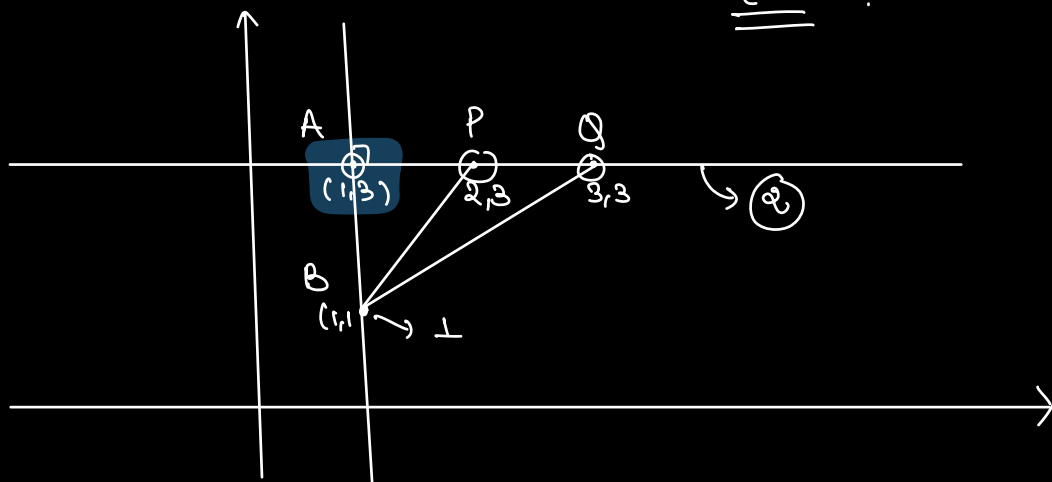
(x, y)	n	m	$\Delta's = (n-1)(m-1)$
(1, 3)	2	3	2
(2, 3)	1	3	0
(3, 3)	2	3	2
(3, 1)	2	2	1
(1, 1)	2	2	1
			6

```
HashMap <int, int> Xmap, Ymap;
for ( i = 0; i < N; i++ ) {
      Xmap [ X[i] ] ++;
      Ymap [ Y[i] ] ++;
3

for ( i = 0; i < N; i++ ) {
      n = Xmap [ X[i] ]
      m = Ymap [ Y[i] ]
      Count += (n-1) * (m-1);
3
```

TC: O(N)
SC: O(N)

TC: O(N³) → O(N²) → O(N)
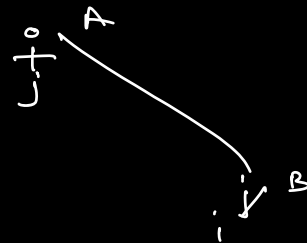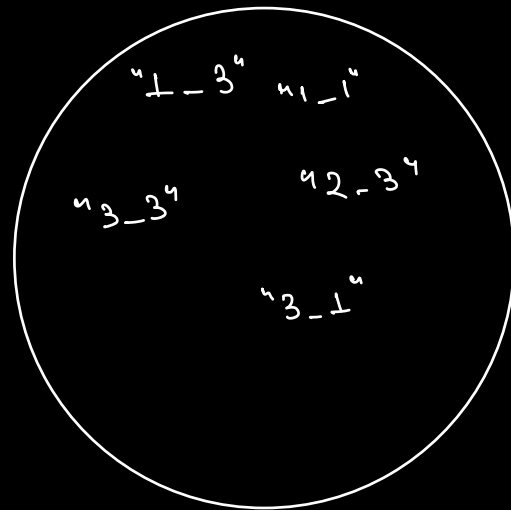SC: O(1) → O(N) → O(N)

— * —

Doubts:

i

A   0 (1, 3) (A, B)

B   1 (2, 3)

C   2 (3, 3)

D   3 (3, 1)

E   4 (1, 1)

$i \to 0,\ j = \emptyset$ $\Rightarrow$ $i\ !=j$
$= (i+1)$

Set < String >

"1_3" "1_1"

"3_3"   "2_3"

"3_1"

$\overset{0}{\underset{j}{i}}$ A

        $\overset{i}{\underset{}{}}$ B

2 Sr

0,0   0,1

B (A)

B ✓ ✓

(B) A