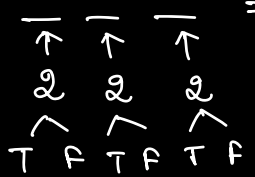
Quiz-1 3 T/F questions
Count the total no ways to answer the questions.

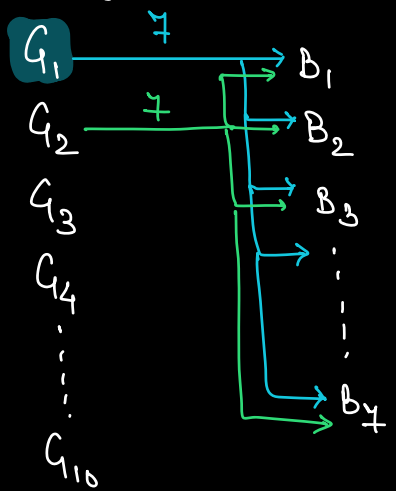
T	F	F
T	F	T
T	T	F
T	T	T

F	F	F
F	F	T
F	T	F
F	T	T



$\Rightarrow 2 \times 2 \times 2 = 8.$

Quiz-2 10 Girls & 7 Boys.
No. of ways of forming a B-G couple.



$G_1 B_1$
$G_1 B_2$
⋮

# of couples $= 10 \times 7$
$= 70.$

Quiz-3

Hyd to Delhi ⇒

1) Hyd to Mumbai ⇒ 3

AND

2) Mumbai to Delhi ⇒ 4

} 3 * 4

Quiz-4

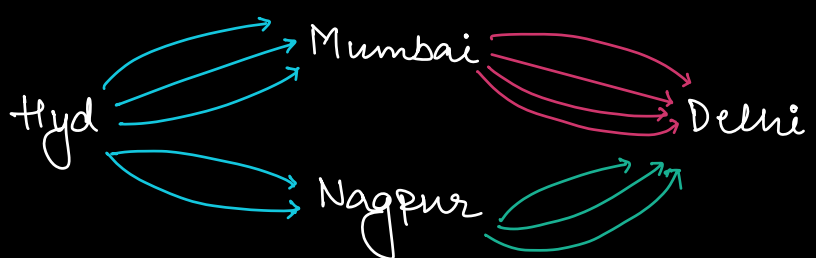
Hyd to Delhi ⇒ 1) Hyd to Goa ⇒ 2

and

2) Goa to Delhi ⇒ 3

} * = 6

Quiz-5



Hyd to Delhi

Hyd to Delhi via Mumbai OR Hyd to Delhi via Nagpur

Hyd to Delhi via Mumbai ⇒ Hyd to Mumbai ⇒ 3 and Mumbai to Delhi ⇒ 4 } 12

OR

Hyd to Delhi via Nagpur ⇒ Hyd to Nagpur ⇒ 2 and Nagpur to Delhi ⇒ 3 } 6

12 + 6 = 18

AND ⇒ *
OR ⇒ +

Quiz-6 You can gift out of following combo's :-

- [1 pen and 1 Book]

OR

- [1 flower and 1 chocolate]

OR

- 1 Ring

Pen : 3
Book: 5
Flowers : 7
Chocolates = 3
Ring : 3.

Combo 1 (or) Combo 2 (or) Combo 3

3×5 + 7×3 + 3

$\Rightarrow$ 39

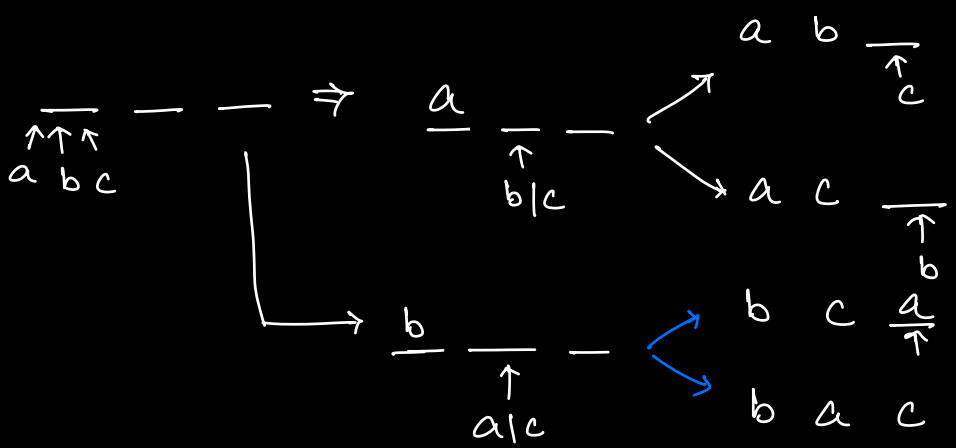
Permutation

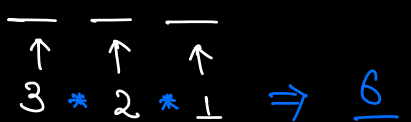
$\Rightarrow$ Arrangement of objects.

(Order matters)

$(i, j) \neq (j, i)$

Quiz-1 a, b & c





$3 * 2 * 1 \Rightarrow 6$

Quiz-8 4 characters → a, b, c & d

_ _ _ _
↑ ↑ ↑ ↑
4 × 3 × 2 × 1 ⇒ 24

⇒ No. of ways to arrange N distinct characters at N distinct places :-

$$N \times (N-1) \times (N-2) \times \dots \times 1 = N!$$

Quiz-9 # of ways to arrange 0 distinct characters :

3, 100 rs notes → 3!

0 notes

$0! = 1$

$1! = 1$

Quiz-10 5 distinct characters
Count the # of ways to arrange them in 2 places.

a, b, c, d & e

5 * 4 → 20

a : {b, c, d, e}
b : {a, c, d, e}
c : {a, b, d, e}
d : {a, b, c, e}
e : {a, b, c, d}

5 distinct characters
Count the # of ways to arrange them in 3 places.

$$\underline{\quad}\ \underline{\quad}\ \underline{\quad} \Rightarrow 5 \times 4 \times 3$$
$$\uparrow\ \uparrow\ \uparrow$$
$$5\ \ 4\ \ 3$$

N distinct characters & 3 positions $\Rightarrow N \times (N-1) \times (N-2)$

N distinct characters & 4 positions $\Rightarrow N(N-1)(N-2)(N-3)$

$\vdots$

N distinct characters & r positions $\Rightarrow$

$$N(N-1)(N-2)\cdots\cdots(N-r+1) \quad (N-(r-1))$$

$$\frac{N(N-1)(N-2)\cdots(N-r+1) * (N-r)(N-r-1) * \cdots\cdots 1}{(N-r)(N-r-1) * \cdots\cdots \times 1}$$

# of ways to arrange N objects at r places $= \frac{N!_0}{(N-r)!_0}$

$= {}^{N}P_{r}$

Combination :

$(i,j) = (j,i)$

Selection of Objects (Order doesn't matter)

Quiz-11 Select 3 cricketers from 4 cricketers.

P_1 P_2 P_3 P_4

P_1 P_2 P_3

P_1 P_2 P_4

P_1 P_3 P_4

P_2 P_3 P_4

} 4 ways. Selection

No. of ways of arranging 3 players out of 4 :-

P_1 P_2 P_3	P_1 P_2 P_4	P_1 P_3 P_4	P_2 P_3 P_4
P_1 P_3 P_2	P_1 P_4 P_2	P_1 P_4 P_3	P_2 P_4 P_3
P_2 P_1 P_3	P_2 P_1 P_4	P_3 P_1 P_4	P_3 P_2 P_4
P_2 P_3 P_1	P_2 P_4 P_1	P_3 P_4 P_1	P_3 P_4 P_2
P_3 P_1 P_2	P_4 P_1 P_2	P_4 P_1 P_3	P_4 P_2 P_3
P_3 P_2 P_1	P_4 P_2 P_1	P_4 P_3 P_1	P_4 P_3 P_2
P_1 P_2 P_3	P_1 P_2 P_4	P_1 P_3 P_4	P_2 P_3 P_4

Total arrangement = $\underline{24} = \frac{4!}{(4-3)!}$

No. of selections = $\frac{24}{3!} = \boxed{4}$

ways to arrange N objects at r places

$= {}^{N}P_{r} = \frac{N!}{(N-r)!}$

ways to arrange r objects at r places = $r!$

Ways to select (r) items from (N) items

$= \frac{N!}{(N-r)!} \times \frac{1}{r!}$

$= \frac{N!}{r!(N-r)!} = \underline{{}^{N}C_{r}}$

$$\boxed{{}^{N}C_{r} = \frac{{}^{N}P_{r}}{r!}}$$

Properties

→ $^{N}C_{1} = N = \frac{N!}{(N-1)!\,1!} = N$

→ $^{N}C_{0} = \frac{N!}{(N-0)!\,0!} = 1$

(Not selecting anything)

Quiz-12

$$^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \text{----} + {}^{n}C_{n} = 2^{N}$$

$^{n}C_{0}$ → Not selecting anything

$^{n}C_{1}$ → ways of selecting 1 out of N.

$^{n}C_{2}$ → ways of selecting 2 out of N.

$^{n}C_{n}$ → ways of selecting N out of N.

$\{1, 2, 3\} \rightarrow$

$^3C_0 \rightarrow \{\} \Rightarrow 1$

$^3C_1 \rightarrow \{1\}\ \{2\}\ \{3\} \rightarrow 3$

$^3C_2 \rightarrow \{1,2\}\ \{1,3\}\ \{2,3\} \rightarrow 3$

$^3C_3 \rightarrow \{1, 2, 3\} \rightarrow 1$

All possible Subsets.

$8 = 2^3$

Q. Given 5 players, count the no. of ways of selecting 2 players.

$P_1\ P_2\ P_3\ P_4\ P_5$

$P_1P_2P_3P_4P_5$ $P_2P_3P_1P_4P_5$ $P_3P_4P_1P_2P_5$ $P_4P_5P_1P_2P_3$

$P_1P_3P_2P_4P_5$ $P_2P_4P_1P_3P_5$ $P_3P_5P_1P_2P_4$

$P_1P_4P_2P_3P_5$ $P_2P_5P_1P_3P_4$

$P_1P_5P_2P_3P_4$

# of ways of selecting 2 objects out of 5 = # of ways of rejecting 3 objects out of 5

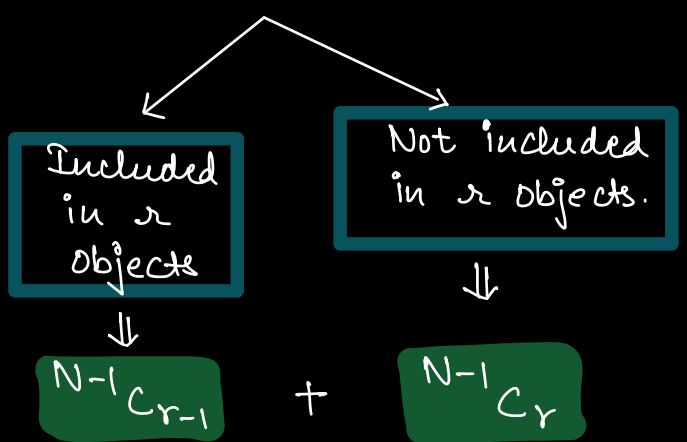
$$^5C_2 = {}^5C_3$$

$$\frac{5!}{3!2!} = \frac{5!}{2!3!} = 10$$

$$^{N}C_{r} = {}^{N}C_{N-r}$$

⇒ Given N objects → Select r out of N objects.

$a_1\ a_2\ a_3\ a_4 \cdots a_N$



$$^{N}C_{r} = {}^{N-1}C_{r-1} + {}^{N-1}C_{r}$$

$$= \frac{(N-1)!}{(N-1-(r-1))! * (r-1)!} + \frac{(N-1)!}{(N-1-r)!\, r!}$$

$$= \frac{(N-1)!}{(N-r)!\,(r-1)!} + \frac{(N-1)!}{(N-r-1)!\, r!}$$

$(N-r)(N-r-1)!$ $\qquad$ $r(r-1)!$

$$= \frac{(N-1)!}{(N-r-1)!\,(r-1)!}\left[\frac{1}{(N-r)} + \frac{1}{r}\right]$$

$$= \frac{(N-1)!}{(N-r-1)!(r-1)!} \left[\frac{r + N-r}{(N-r)\,r} \right]$$

$$= \frac{(N-1)!}{(N-r-1)!(r-1)!} \left(\frac{N}{r(N-r)} \right)$$

$$= \frac{N!}{(N-r)!\,r!} = {}^{N}C_{r}$$

Q. Given N, r, P (prime no.)
Calculate ${}^{N}C_{r} \,\%\, P$

$$\left(\frac{N!}{r!(N-r)!} \right) \% P$$

$$\left(\frac{a}{b}\right) \% m \neq \left(\frac{a \% m}{b \% m}\right) \% m$$

↓

Inverse Modulo.

↳ fermat's little theorem.

pow(a, n, p)
→ $a^n \,\%\, p$

——— * ———