

Q Given a matrix of size $N \times M$, calculate the sum of all submatrix sum.

Google
MS
Apple.

$$\text{Sum of all subarray sum} = \underbrace{\left(\text{No. of times } A[i] \text{ is present in all subarrays} \right)}_{\text{Contribution of } A[i]} * A[i]$$

$$= \sum_{i=0}^{N-1} a[i] * n$$

Sum of all submatrix sum =

$$\underbrace{\left(\text{No. of times } \text{mat}[i][j] \text{ is present in all submatrices} \right)}_{\text{Contribution of } \text{mat}[i][j]} * \text{mat}[i][j]$$

Contribution of $\text{mat}[i][j]$

$$\text{mat} = \begin{bmatrix} 4 & 9 & 6 \\ 5 & -1 & 2 \end{bmatrix} \quad 2 \times 3 \text{ matrix}$$

$$\begin{array}{llll}
 [4] \rightarrow 4 & [4, 9] \rightarrow 13 & [4] \rightarrow 4 & [4, 9] \rightarrow 17 \\
 [5] \rightarrow 5 & [9, 6] \rightarrow 15 & [5] \rightarrow 5 & [5, -1] \rightarrow 17 \\
 [9] \rightarrow 9 & [5, -1] \rightarrow 4 & [9] \rightarrow 9 & [9, 6] \rightarrow 16 \\
 [-1] \rightarrow -1 & [-1, 2] \rightarrow 1 & [-1] \rightarrow -1 & [-1, 2] \rightarrow 1 \\
 [6] \rightarrow 6 & [4, 9, 6] \rightarrow 19 & [6] \rightarrow 6 & [4, 9, 6] \rightarrow 25 \\
 [2] \rightarrow 2 & [5, -1, 2] \rightarrow 6 & [2] \rightarrow 2 & [5, -1, 2] \rightarrow 6
 \end{array}$$

Sum of all submatrix sums = 166

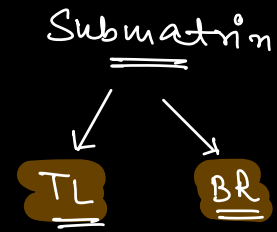
Contribution Technique

$$\begin{aligned}
 \text{Sum} &= 4 \times 6 + 9 \times 8 + 6 \times 6 + 5 \times 6 + (-1) \times 8 + 2 \times 6 \\
 &= 24 + 72 + 36 + 30 - 8 + 12 \\
 &= \underline{\underline{166}}
 \end{aligned}$$

No. of times $\text{mat}[i][j]$ is appearing is how many submatrices.

	0	1	2	3	4	5
0	✓	✓	✓	✓		
1	✓	✓	✓	✓		
2	✓	✓	✓	✓	✓	✓
3				✓	✓	✓
4				✓	✓	✓

5x6



mat[2][3]

No. of choices for TL = 12

No. of choices for BR = 9

No. of submatrices in which mat[2][3] will be present = 12×9
= 108

Quiz

	0	1	2	3	4
0	✓	✓	✓		
1	✓	✓	✓	✓	✓
2			✓	✓	✓
3			✓	✓	✓

No. of choices for TL = 6

No. of choices for BR = 9

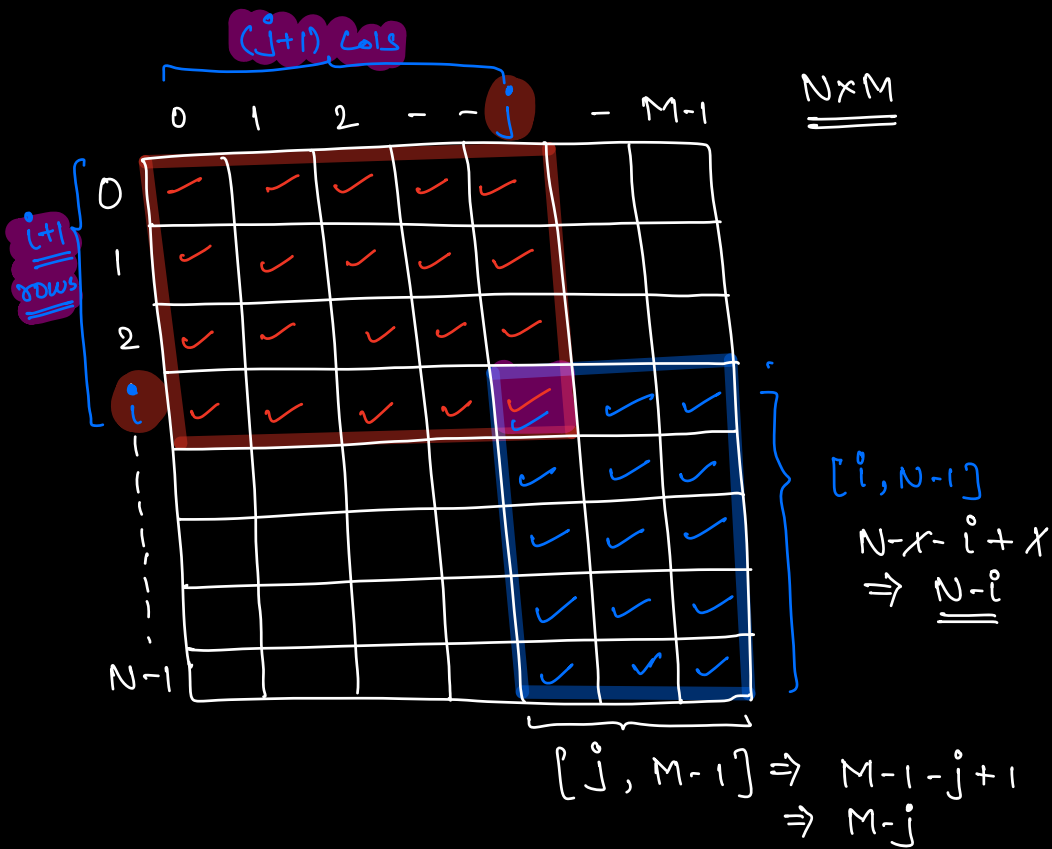
No. of submatrices in which mat[1][2] will be present = 54

	0	1	2	3	4	5
0	✓	✓	✓	✓	✓	
1	✓	✓	✓	✓	✓	✓
2					✓	✓
3					✓	✓
4					✓	✓

Contribution of $\text{mat}[1][4]$

$$= 10 \times 8$$

$$= \underline{\underline{80}}$$



No. of choices for TL = $(i+1) \times (j+1)$

No. of choices for BR = $(N-i) * (M-j)$

No. of times $\text{mat}[i][j]$ is present in submatrices = $(i+1) \times (j+1) * (N-i) * (M-j)$

Code

```
for (i = 0; i < N; i++) {  
    for (j = 0; j < M; j++) {  
        TL = (i+1) * (j+1)  
        BR = (N-i) * (M-j)  
        ans += (TL * BR * mat[i][j]);  
    }  
}  
return ans;
```

TC: $O(N \cdot M)$

SC: $O(1)$

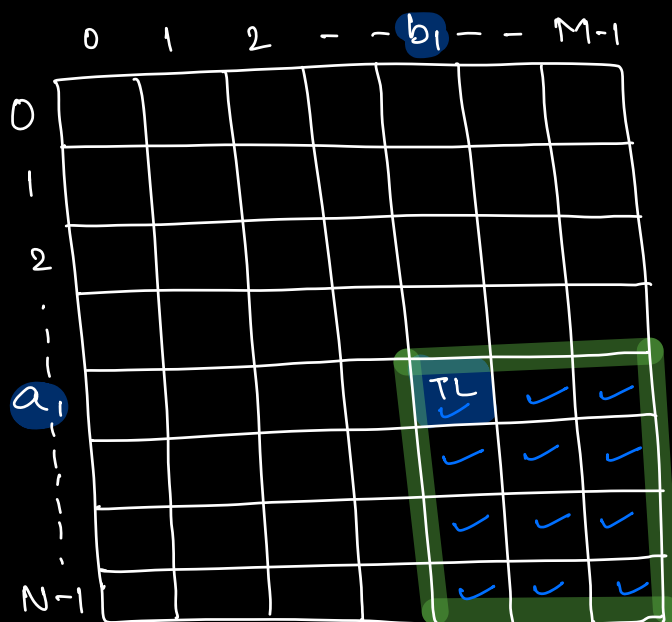
Q. Given a matrix of size $N \times M$, find the max submatrix sum.
Google

-2	3	1
3	6	-3

2×3

Brute force

Create all the submatrices & find the max submatrix sum.



$N \times M$

Submatrices.

TL BR

TL $\Rightarrow (a_1, b_1)$

BR $\Rightarrow (a_2, b_2)$

\downarrow \rightarrow
 $[a_1, N-1]$ $[b_1, M-1]$

```
for (a1 = 0; a1 < N; a1++) {
```

```
    for (b1 = 0; b1 < M; b1++) {
```

```
        // TL: (a1, b1)
```

```
        for (a2 = a1; a2 < N; a2++) {
```

```
            for (b2 = b1; b2 < M; b2++) {
```

```
                // BR: (a2, b2)
```

Using PS, find the sum
of submatrix.

3 3 3 3

TC: $O(N^2 \times M^2)$

SC: $O(N \cdot M)$

↳ PS[][] matrix.

	0	1	2
0			
1			
2			

TL
(a₁, b₁) ≡ (0, 0) →

BR
(0, 0)
(0, 1)
(0, 2)
(1, 0)
(1, 1)
(1, 2)
(2, 0)
(2, 1)
(2, 2)

Optimization:-

- ① Find max submatrix sum where submatrix starts at row = 0 & ends at row = N-1.

	0	1	2	3	4(M-1)
0					
1					
(N-1)2					

3x5

N x M

$$5 + 4 + 3 + \dots + 1$$

$$\Rightarrow M + (M-1) + \dots + 2 + 1$$

$$\Rightarrow \frac{M(M+1)}{2}$$

	0	1	2	3	4
0	-3	4	2	2	9
1	-9	-3	3	3	-3
2	-1	6	-4	4	-10



↓ Column Sum

-13	7	1	9	-4
-----	---	---	---	----



max sum.

① find max submatrix sum where submatrix starts at row = 0 & ends at any row

	0	1	2	3	4
0	-3	4	2	2	9
1	-9	-3	3	3	-3
2	-1	6	-4	4	-10

arr

0	0	0	0	0
---	---	---	---	---

start_row = 0, end_row = 0

arr

-3	4	2	2	9
----	---	---	---	---

 sum = 14

→ find max subarray sum in 0th row.

start_row = 0, end_row = 1

-12	1	5	5	6
-----	---	---	---	---

 sum = 14
→ find max subarray sum

start_row = 0, end_row = 2

-13	7	1	9	-4
-----	---	---	---	----

 sum = 14
→ find max subarray sum

```
for (start_row = 0; start_row < N; start_row++) {
    arr[M] = {0}
```

```
for (end_row = start_row; end_row < N; end_row++) {
```

```
    for (j = 0; j < M; j++) {
```

```
        arr[j] += mat[end_row][j];
```

3

```
    ans = max(ans, kadanes(arr, M));
```

3

↪ O(M)

3

TC: O(N²M)

O(N*(N*(M+M)))

SC: O(M)



Thank You

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Doubts

