

$\% \Rightarrow$ Modulus Operator

\hookrightarrow Remainder

$n \% a \Rightarrow$ Remainder when n is divided by a .

$$10 \% 4 = 2 \quad | \quad 13 \% 5 = 3$$

$$\begin{array}{r} \textcircled{5} \overline{) \textcircled{13}} \textcircled{2} \rightarrow Q \\ \underline{10} \\ \textcircled{3} \rightarrow R \end{array}$$

$d \leftarrow \textcircled{5}$ $D \leftarrow \textcircled{2}$ $R \leftarrow \textcircled{3}$

$$13 = 2 * 5 + 3$$

$$\text{Divident} = \text{Quotient} * \text{divisor} + \text{remainder.}$$

$$\text{Remainder} = \text{Divident} - \text{Quotient} * \text{divisor}$$

\downarrow
largest multiple of
divisor \leq Divident.

$$\begin{aligned} * \quad 13 \% 5 &= 13 - 10 \\ &= 3 \end{aligned}$$

$$\begin{aligned} * \quad 10 \% 4 &= 10 - 2 * 4 \\ &= 10 - 8 = \underline{2} \end{aligned}$$

Quiz

$$150 \% 11 = 7$$

$$\begin{array}{r} 11 \overline{) 150} 13 \\ \underline{11} \\ 40 \\ \underline{33} \\ \textcircled{7} \end{array}$$

Quiz $100 \% 7 \Rightarrow 100 - 98 = 2$

Quiz $-43, -42, -45, -32$
 $-32 > -42 > -43 > -45$

Quiz $-40 \% 7$
 $x = -40 - (\text{largest multiple of } 7 \leq -40)$
 $= -40 - (-42)$
 $= -40 + 42 = 2$

Quiz $-60 \% 9$
 $-60 - (\text{largest multiple of } 9 \leq -60)$
 $-60 - (-63) = 3$

$$A \% M \in [0, M-1]$$

C / C++ / Java / JS

Python

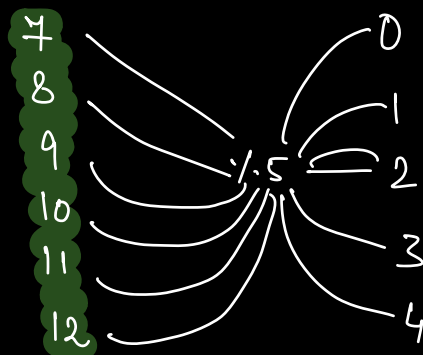
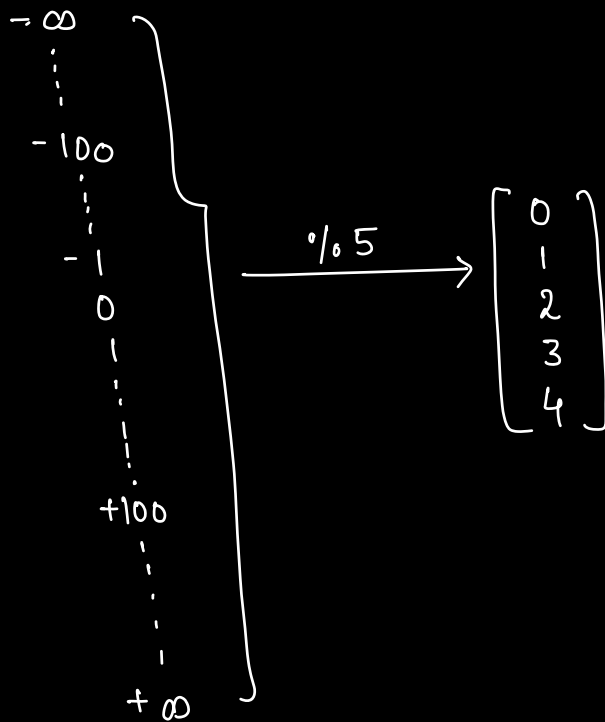
$$-40 \% 7 = -5 \xrightarrow{+7} 2$$

$$-60 \% 9 = -6 \xrightarrow{+9} 3$$

$$-30 \% 4 = -2 \xrightarrow{+4} 2$$

MODULO Operator restricts the range.

$$A \% M \in [0, M-1]$$



→ Hashing / HashTables / Dict / map
→ Consistent Hashing.

Modulo Arithmetic

$$1) (a+b) \% M = (a \% M + b \% M) \% M$$

Quiz $(a+b) \% M \in [0, M-1]$

Ex $a=6, b=8, m=10$

$(6+8) \% 10$	$6 \% 10 + 8 \% 10$
$14 \% 10$	$6 + 8$
<u>4</u>	<u>14</u> $\xrightarrow{\% 10}$ $14 \% 10 = \underline{4}$

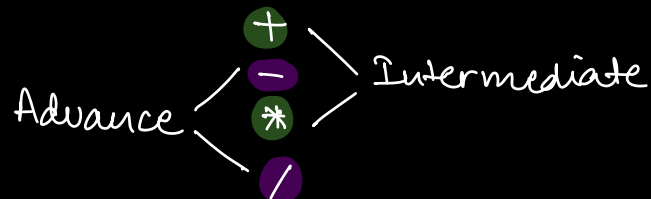
$$\Rightarrow 14 \% 5 = 4$$

$$(14 \% 5) \% 5 = 4 \% 5 = 4$$

$$((14 \% 5) \% 5) \% 5 = \underline{4}$$

$a \% M = (a \% M) \% M$

$$2) (a \times b) \% M = (a \% M \times b \% M) \% M$$



$$\begin{aligned} 3) \quad A \% M &\Rightarrow (A + M) \% M \\ &\Rightarrow (A \% M + M \% M) \% M \\ &\Rightarrow A \% M \end{aligned}$$

$A \% M = (A + n \times M) \% M$

Q. Implement power function

$$\text{pow}(a, n, p) \Rightarrow a^n \% p$$

$$a=2, n=5, p=4 \Rightarrow 2^5 \% 4 = \underline{4}$$

$$a=3, n=4, p=6 \Rightarrow 3^4 \% 6 = 81 \% 6 = \underline{3}$$

```
int pow(a, n, p) {  
    int ans = 1;  
    for (i = 1; i <= n; i++) {  
        ans = ans * a;  
    }  
    return ans % p;  
}
```

$$a^n = \underbrace{a \times a \times a \dots a}_{n \text{ times}}$$

3

2n $a=10, n=40, p$

$$\text{ans} = \underline{10^{40}} \% p \quad \times$$

```
int pow(a, n, p) {  
    int ans = 1;  
    for (i = 1; i <= n; i++) {  
        ans = (ans * a) % p;  
    }  
    return ans % p;  
}
```

3

This multiplication can also overflow.

$$\rightarrow ans = (ans \cdot /P * a \cdot /P) \cdot /P$$

$$a = 2, n = 4, P = 10$$

$$ans = 1$$

$$ans = (1 \cdot /10 \times 2 \cdot /10) \cdot /10 = 2$$

$$ans = (2 \cdot /10 \times 2 \cdot /10) \cdot /10 = 4$$

$$ans = (4 \cdot /10 \times 2 \cdot /10) \cdot /10 = 8$$

$$ans = (8 \cdot /10 \times 2 \cdot /10) \cdot /10 = (8 \times 2) \cdot /10 \\ = \underline{\underline{6}}$$

$$ans = (ans \cdot /P * a \cdot /P) \cdot /P$$

$$\text{eg } P = 10^9 + 7$$

$$ans \cdot /P \in [0, P-1]$$

$$a \cdot /P \in [0, P-1]$$

$$ans = (\underbrace{ans \cdot /P}_{P-1} * \underbrace{a \cdot /P}_{P-1}) \cdot /P$$

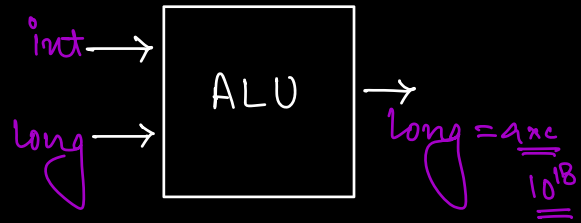
$$ans = (\text{long})(ans \cdot /P) * a \cdot /P \cdot /P$$

$$TC: O(N)$$

$$SC: \underline{\underline{O(1)}}$$

2m
 long ans;
 int a = 10^9 , b = 10^9
 long c = 10^9 , d = 10^9

ans = a * b ✗
 ans = a * c ✓
 ans = c * d ✓



Quiz No. not divisible by 3

231
 4562
 7821
 1026

Quiz

4351
 3521

7326 → (7 + 3 + 2 + 6) = 18 ÷ 3 = 0
 8236

Divisibility rule for 3.

⇒ If sum of all digits is divisible by 3 then the number will be divisible by 3.

$$(4372) \cdot 1.3$$

$$\rightarrow (4 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 2 \times 10^0) \cdot 1.3$$

$$\left[(4 \times 10^3) \cdot 1.3 + (3 \times 10^2) \cdot 1.3 + (7 \times 10) \cdot 1.3 + (2 \times 10^0) \cdot 1.3 \right] \cdot 1.3$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$(4 \cdot 1.3 \times 10^3 \cdot 1.3) \cdot 1.3 + (3 \cdot 1.3 \times 10^2 \cdot 1.3) \cdot 1.3 + (7 \cdot 1.3 \times 10 \cdot 1.3) \cdot 1.3 + (2 \cdot 1.3) \cdot 1.3$$

$$\left. \begin{array}{l} 10 \cdot 1.3 \\ 10^2 \cdot 1.3 \\ 10^3 \cdot 1.3 \\ \vdots \\ 10^n \cdot 1.3 \end{array} \right\} \textcircled{1} \Rightarrow 4 \cdot 1.3$$

$$(4 \cdot 1.3 \times 1) \cdot 1.3$$

$$\Rightarrow (4 \cdot 1.3 + 3 \cdot 1.3 + 7 \cdot 1.3 + 2 \cdot 1.3) \cdot 1.3$$

$$\boxed{(a \cdot 1.3 + b \cdot 1.3) \cdot 1.3 \rightarrow (a+b) \cdot 1.3}$$

$$\Rightarrow (4 + 3 + 7 + 2) \cdot 1.3$$

Sum of digits.

HW \Rightarrow Divisibility rule of 4, 5 & 8.

Q. Given a number A in form of an Array of size N, & a no. p return A.p.P

A:

1	2	3	4	4
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p = 4

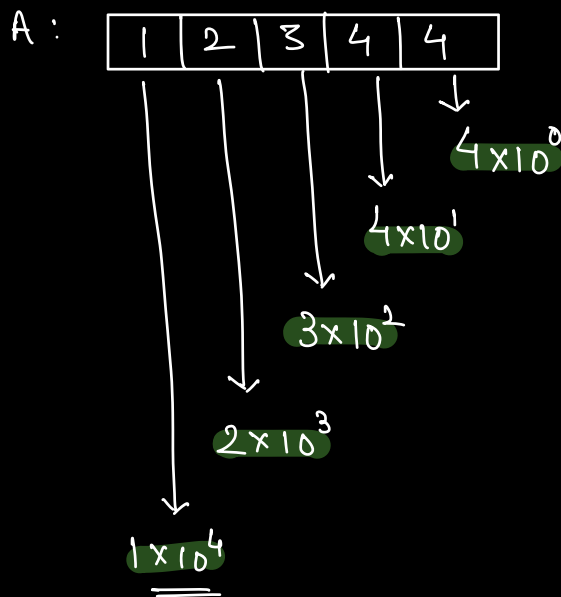
} A.p.P = 12344.p.4
= 0

Constraints

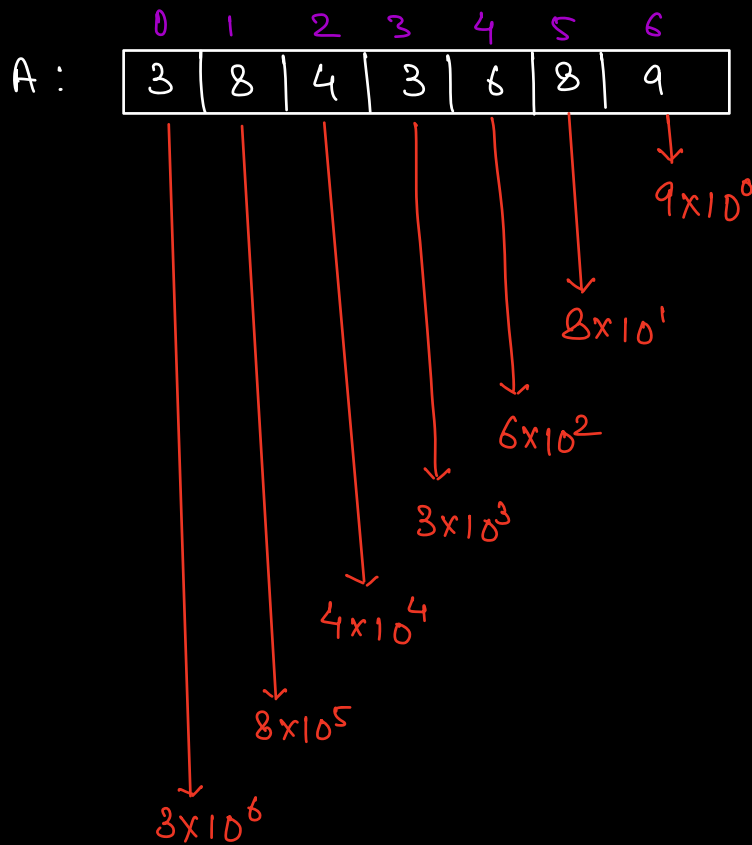
$1 \leq P \leq 10^9$ $1 \leq N \leq 10^5$
--

int $\Rightarrow 10^9$
 $\Rightarrow 10$ digits

long $\Rightarrow 10^{18}$
 $\Rightarrow 19$ digits



Ex



$$N-1 \Rightarrow 10^0$$

$$N-2 \Rightarrow 10^1$$

$$N-3 \Rightarrow 10^2$$

...

$$1 \Rightarrow 10^{N-2}$$

$$0 \Rightarrow 10^{N-1}$$

$$i \Rightarrow N-1-i$$

$$\underline{A \cdot 10^P} = (3 \times 10^6 + 8 \times 10^5 + 4 \times 10^4 + 3 \times 10^3 + 6 \times 10^2 + 8 \times 10 + 9) \cdot 10^P$$

$$= ((3 \times 10^6) \cdot 10^P + (8 \times 10^5) \cdot 10^P + (4 \times 10^4) \cdot 10^P + (3 \times 10^3) \cdot 10^P$$

$$+ (6 \times 10^2) \cdot 10^P + (8 \times 10 + 9) \cdot 10^P)$$

$$(4 \cdot 10^P \times 10^4 \cdot 10^P) \cdot 10^P$$

$$\text{pow}(10, 4, P)$$

$$(3 \cdot 10^P \times 10^6 \cdot 10^P) \cdot 10^P$$

$$\text{pow}(10, 6, P)$$

$$(8 \cdot 10^P \times 10^5 \cdot 10^P) \cdot 10^P$$

$$\text{pow}(10, 5, P)$$

$$A: \overset{0}{a_0} \overset{1}{a_1} \overset{2}{a_2} \overset{3}{a_3} \dots \overset{n-2}{a_{n-2}} \overset{n-1}{a_{n-1}}$$

$$(a_0 \times 10^{n-1} + a_1 \times 10^{n-2} + a_2 \times 10^{n-3} + \dots + a_{n-1} \times 10^0) \% P$$

$$((a_0 \times 10^{n-1}) \% P + (a_1 \times 10^{n-2}) \% P + \dots + (a_{n-1} \times 10^0) \% P) \% P$$

$$((a_0 \% P * \text{pow}(10, n-1, P)) \% P + (a_1 \% P * \text{pow}(10, n-2, P)) \% P \dots \\ \dots + (a_{n-1} \% P * \text{pow}(10, 0, P)) \% P) \% P$$

$$\Rightarrow \sum_{i=0}^{N-1} (a_i \% P * \text{pow}(10, n-1-i, P)) \% P$$

ans = 0

for (i = 0; i < N; i++) {

ans = ans + (A[i] * pow(10, N-1-i, P)) % P

}

return ans;

$O(N)$

$$\text{pow}(a, n, P) \Rightarrow a^n \% P$$

TC: $O(N^2)$

<u>i</u>	
0	$10^{N-1} \cdot P = (10 \times 10^{N-2}) \cdot P$
1	$10^{N-2} \cdot P = (10 \times 10^{N-3}) \cdot P$
2	$10^{N-3} \cdot P$
3	$10^{N-4} \cdot P$
...	...
N-2	$10^1 \cdot P$
N-1	<u><u>$10^0 \cdot P$</u></u>

i	
N-1	$\Rightarrow 10^0 \cdot P = 1$
N-2	$\Rightarrow 10^1 \cdot P = (10 \times 10^0) \cdot P = (10 \cdot P \times 10^0 \cdot P) \cdot P$
N-3	$\Rightarrow 10^2 \cdot P \Rightarrow (10 \times 10^1) \cdot P = (10 \cdot P \times 10^1 \cdot P) \cdot P$
N-4	$\Rightarrow 10^3 \cdot P \Rightarrow (10 \times 10^2) \cdot P = (10 \cdot P \times 10^2 \cdot P) \cdot P$
N-5	$\Rightarrow 10^4 \cdot P \Rightarrow (10 \times 10^3) \cdot P = (10 \cdot P \times 10^3 \cdot P) \cdot P$

$x = 1, ans = 0$

for ($i = N-1$; $i \geq 0$; $i--$) {

$ans = ans + (A[i] \cdot 10^x * x) \cdot 10^x$

$x = (x + 10) \cdot 10^x$

}

return ans;

TC: $O(N)$

SC: $O(1)$

————— ✱ —————