Q.1 Given an unsorted array of integers of size N, find the first missing natural number.

FB, Google
MS, LI
Apple,
Oracle,
.....

Natural no's : 1, 2, 3, 4, - - - - - -

A[5] : { 3, -2, 1, 2, 7 } ⇒ 4

Quiz A : { 2, 4, -1, -6, 3, 7, 8, 4, -3, 0 }

↳ 1

Quiz A : { 1, 0, -5, -6, 4, 2 }

↳ 3

Quiz A : { 1, 2, 5, 6, 4, 3 }

N = 6 ⇒ 7

Ex A : { -1, 0, 1, 4, 8, 2, 1, 3 }

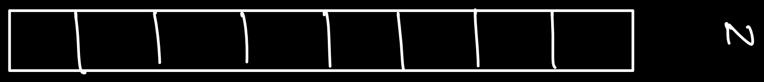
⇒ 5

A : { -2, -3, -4, -5, 0 }

↳ 1

Brute force

For all natural no's, check if it is present in the Array or not.



$1 \longrightarrow O(N)$

$2 \longrightarrow O(N)$

$3 \longrightarrow O(N)$

$\vdots$

$ans \longrightarrow O(N)$

# of iterations $\Rightarrow ans * N$

$ans \in [1, N+1]$

TC: $O(N^2)$

SC: $O(1)$

```
for( i = 1; i <= N; i++){
        search i in the     } → O(N)
        Array linearly.     }
}
return N+1;
```

2. Use SET | MAP to search.

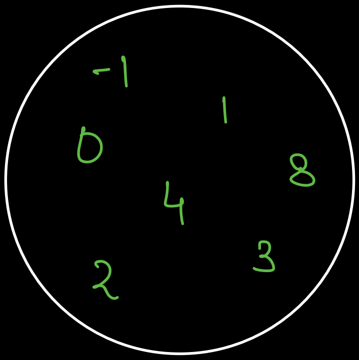
```
HashSet<int> set;
// Insert Array elements into set.
for( i= 1; i <= N; i++) {
        if ( !set.contains(i) )   } → O(1)
            return i;
}
return N+1;
```

TC : O(N)

SC : O(N)

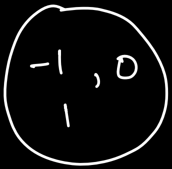
A : {-1, 0, 1, 4, 8, 2, 1, 3}

Set



1, 2, 3, 4, 5 ×

A : {-1, 0, 1}   N = 3



1 ✓, 2 ×

Sort

A[5] : {3, -2, 1, 2, 3, 7}

↓ Sort

{-2, 1, 2, 3, 3, 7}

4 is Missing.

TC : O(NlogN)

SC : O(logN) | O(N)

Quick Sort | Merge Sort

Constraint : Constant Extra space

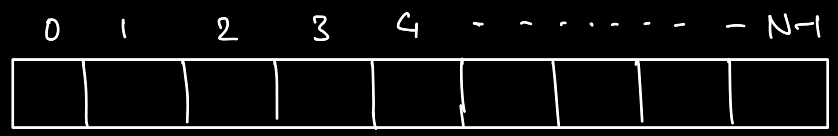
SC : O(1)

→ Input array modification is allowed.

⇒ Use array indices to mark the presence of elements.

HOW ?

Size → N



ele   index

1 → 0

2 → 1

3 → 2

⋮

N → N-1

I. Only distinct +ve no's are present.

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \{-7 & -6 & -2 & 1 & 3\} \end{array}$$

N = 5

1 → 0

2 → 1

3 → 2

4 → 3

5 → 4

↓

ele > N
→ skip

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \{-7 & -6 & -2 & 1 & 3\} \\ \uparrow & \uparrow & \uparrow & \uparrow & \end{array}$$

A[i] < 0
→ i+1 is present in the Array.

A[3] > 0
→ 3+1 = ④ missing

Ex

$$A : \{\overset{0}{-8}, \overset{1}{-1}, \overset{2}{-4}, \underset{\underset{abs(-2) = 2}{\uparrow}}{\overset{3}{-2}}, \overset{4}{6}, \underset{\underset{abs(-3) = 3}{\uparrow}}{\overset{5}{-3}}\} \qquad N = 6$$

ind

$1 \longrightarrow 0$

$2 \longrightarrow 1$

$3 \longrightarrow 2$

$\vdots$

$N \longrightarrow N-1$

$$\{\overset{0}{-8}, \overset{1}{-1}, \overset{2}{-4}, \overset{3}{-2}, \underset{\uparrow}{\overset{4}{6}}, \overset{5}{-3}\}$$

$A[4] > 0$

$\Rightarrow$ 4+1 is missing

Edge :- If -ve no's are also present.

$$A : \{\overset{0}{-8}, \overset{1}{1}, \overset{2}{5}, \underset{\underset{X}{\uparrow}}{\overset{3}{-2}}, \overset{4}{-6}, \overset{5}{3}\} \qquad N = 6$$

$ans \in [1, N+1]$

-ve $\Rightarrow$ replace with no > N
(N+1)

$$A : \{8, 1, 5, \underset{7}{\cancel{-2}}, 6, 3\} \qquad N = 6$$

$$\{\overset{0}{-8}, \overset{1}{1}, \overset{2}{-5}, \overset{3}{7}, \underset{\underset{abs(-6) = 6}{\uparrow}}{\overset{4}{-6}}, \underset{\underset{abs(-3) = 3}{\uparrow}}{\overset{5}{-3}}\}$$

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ \{-8, & 1, & -5, & 7, & -6, & -3\} \end{array}$$

↑

2 is missing

A[1] > 0

Ex

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \{\cancel{-1}, & \cancel{-2}, & 1, & 3, & 4, & 8, & \cancel{0}\} \\ 8 & 8 & & & & & 8 \end{array} \quad N = 7$$

↓

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \{-8 & 8 & -1 & -3 & 4 & 8 & 8\} \end{array}$$

↑

ele = 1

index = ele - 1

= 0

↓

$$\begin{array}{ccccccc} & i+1 & & & & & \\ 0 & ① & 2 & 3 & 4 & 5 & 6 \\ \{-8 & 8 & -1 & -3 & 4 & 8 & 8\} \end{array}$$

↑

A[1] > 0

1 + 1 = 2

TC : O(N)

SC : O(1)

Duplicates.

A : { 3 6 -~~5~~ 3 -~~4~~ 1 } N=6

	0	1	2	3	4	5
A :	3	6	-~~5~~ 7	3	-~~4~~ 7	1

N=6

↓

0	1	2	3	4	5
-3	6	-7	3	7	-1
		↑	↑		↑

-1*abs(A[index])

0	1	2	3	4	5
-3	6	-7	3	7	-1
	↑				

A[1] > 0

2 is missing.

Ex

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	6	1	2	3	3	1	0	4	2	9	8	8	16	-1

N=15

A[i] <= 0 ⇒ N+1

↓

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-7	-6	-1	-2	3	-3	-1	-16	-4	2	9	8	8	16	16
														↑

↓

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-7	-6	-1	-2	3	-3	-1	-16	-4	2	9	8	8	16	16
				↑										

5 is missing

Q.2 Maximum Absolute Difference.

Google
MS

Given an Array, find the max value of

$$f(i,j) = |A[i] - A[j]| + |i-j|$$

$$|x| \rightarrow abs(x)$$

$$A : \{\overset{0}{1}, \overset{1}{3}, \overset{2}{-1}\}$$

i	j	
0	0	$\rightarrow |1-1| + |0-0| = 0$
0	1	$\rightarrow |1-3| + |0-1| = 3$
0	2	$\rightarrow |1-(-1)| + |0-2| = 4$
1	0	$\rightarrow |3-1| + |1-0| = 3$
1	1	$\rightarrow |3-3| + |1-1| = 0$
1	2	$\rightarrow |3-(-1)| + |1-2| = \boxed{5}$
2	0	$\rightarrow |-1-1| + |2-0| = 4$
2	1	$\rightarrow |-1-3| + |2-1| = 5$
2	2	$\rightarrow \underline{\underline{0}}$

1) $f(i,j) = |A[i] - A[j]| + |i-j|$

$$f(i,i) = 0$$

$$\boxed{|x| = |-x|}$$

2) $f(i, j) = f(j, i)$

Brute force

```
ans = 0/-∞
for(i=0; i<N; i++){
    for(j=0; j<N; j++){
        ans= max(ans, x);
    }
}
```

→ Upper triangular OR Lower triangular

```
ans = 0/-∞
for(i=0; i<N; i++){
    for(j=i+1; j<N; j++){
        ans= max(ans, x);
    }
}
```

TC: $O(N^2)$

SC: $O(1)$

$$\Rightarrow \quad \underbrace{|x|}_{\geq 0} \rightarrow \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

|4| = 4, |-7| = -(-7)

$|-7| \Rightarrow -(-7) = 7$

$|8| = 8$

$$A : \{ \overset{0}{1}, \overset{1}{3}, \overset{2}{-1} \}$$

3×3 pairs

$$\begin{array}{c|c|c|c} i \backslash j & 0 & 1 & 2 \\ \hline 0 & 0 & \begin{matrix}3\\(0,1)\end{matrix} & \begin{matrix}4\\(0,2)\end{matrix} \\ \hline 1 & \begin{matrix}3\\(1,0)\end{matrix} & 0 & \begin{matrix}5\\(1,2)\end{matrix} \\ \hline 2 & \begin{matrix}4\\(2,0)\end{matrix} & \begin{matrix}5\\(2,1)\end{matrix} & 0 \end{array}$$

$\underline{i < j}$

$i > j$

$$f(i,j) = \underbrace{|A[i] - A[j]|} + |i - j|$$

→ Upper triangular

$$i < j \longrightarrow i - j < 0$$

$$|\underbrace{i - j}_{<0}| \Rightarrow -(i-j) = -i + j$$

i) $A[i] - A[j] \geq 0$

$$|A[i] - A[j]| = A[i] - A[j]$$

$$\begin{aligned} f(i,j) &= (A[i] - A[j]) + (-(i-j)) \\ &= A[i] - A[j] - i + j \\ &= (A[i] - i) - (A[j] - j) \end{aligned}$$

ii) $A[i] - A[j] < 0$

$$|A[i] - A[j]| = -(A[i] - A[j])$$

$$\begin{aligned} f(i,j) &= -(A[i] - A[j]) + (-(i-j)) \\ &= -A[i] + A[j] - i + j \end{aligned}$$

$$f(i,j) = (A[j] + j) - (A[i] + i)$$

Ex

$$|A| - B \begin{cases} A - B \\ -A - B \end{cases} \rightarrow \max(A-B, -A-B)$$

1) $A = 6, \; B = 2$

$A - B = 6 - 2 = 4$

$-A - B = -6 - 2 = -8$

$|6| - 2 = 4$

2) $A = -7, \; B = 4$

$A - B = -7 - 4 = -11$

$-A - B = -(-7) - 4$

$= 3$

$|-7| - 4 = 3$

3) $A = 5, \; B = -1$

$A - B = 5 - (-1) = 6$

$-A - B = -5 - (-1) = -4$

$|5| - (-1) = \underline{\underline{6}}$

$$|x-y| \begin{cases} x-y \\ -(x-y) \\ -x+y \end{cases} \Big\} \max(x-y,\,-x+y)$$

|x| -> Mod of anything will increase. It will convert negative to positive value.

$$f(i,j) = \max\Big((A[i]-i) - (A[j]-j),\; (A[j]+j) - (A[i]+i) \Big)$$

$\rightarrow \; \uparrow x - y \downarrow \Rightarrow$ maximise

$$\left.\begin{array}{l} A[i]-i \rightarrow \max \\ A[j]-j \rightarrow \min \end{array}\right\} (A[i]-i) - (A[j]-j) \uparrow$$

$$\left.\begin{array}{l} A[i]+i \rightarrow \min \\ A[j]+j \rightarrow \max \end{array}\right\} (A[j]+j) - (A[i]+i) \uparrow$$

A :

0	1	2
1	3	-1

B :

1	4	1
j	i	

C :

0	1	2
1	2	-3

$B[i] \rightarrow A[i]+i$

$C[i] = \underline{\underline{A[i]-i}}$

$\max(2-(-3),\; 4-1)$

$\max(5,3) \Rightarrow \underline{\underline{5}}$

$A[i]+i$

A :

0	1	2
1	3	-1

$max = \cancel{-\infty}\ \cancel{1}\ 4$

$min = \cancel{\infty}\ 1$

$A[i]-i$

A :

0	1	2
1	3	-1

$M = \cancel{-\infty}\ \cancel{1}\ 2$

$m = \cancel{\infty}\ \cancel{1}\ -3$

$$max(\ 2-(-3)\ ,\ 4-1\ )$$

$$max(\ 5,\ 3) \Rightarrow \underline{\underline{5}}$$

TC : O(N)

SC : O(1)

Q. Given a row-wise & col-wise sorted matrix, find if element K is present in the matrix.

Amazon

Bloom-rberg

NxM

	0	1	2	3
0	5	10	15	20
1	6	12	18	24
2	7	14	21	28
3	8	16	24	34

K = 100 → false

K = 14 → true.

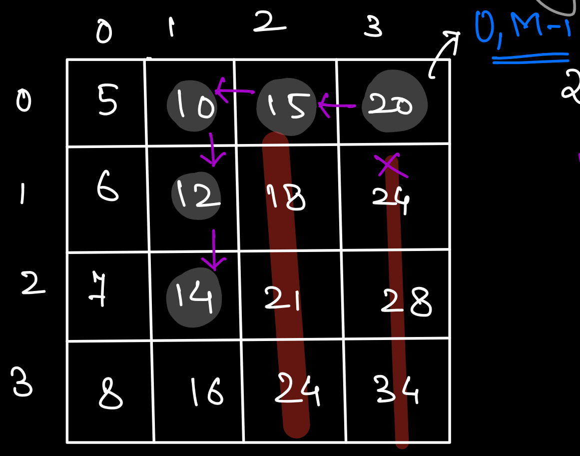
1. TC : $O(N \cdot M)$

SC : $O(1)$

2. TC : $O(N \cdot \log M)$ ⇒ B.S on each row.

OR

$O(M \cdot \log N)$ ⇒ B.S on each col.



	0	1	2	3
0	5	10	15	20
1	6	12	18	24
2	7	14	21	28
3	8	16	24	34

NxM

K = 14

20 > 14

15 > 14

10 < 14

12 < 14

14 == 14 → true.

TC : $O(N+M)$

SC : $O(1)$

```
row = 0, col = M-1
while (row < N && col >= 0) {
        if (mat[row][col] == k)
            return true;
        else if (mat[row][col] > k)
                col--
        else
                row++
}
return false;
```

Doubts.

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ -1 & -4 & 8 & -2 \end{array}$$

N = 4

$\uparrow$ abs(-2) = 2

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ -1 & -4 & 8 & -2 \end{array}$$

$\uparrow$ A[2] > 0

2+1 is missing

N = 4

1	4	2	3

$\underbrace{\qquad\qquad}_{[1-N]}$

[1 — N]

1 → ?

2 → ?

3 → ?

4 → ?

⋮