

\* Currency Exchange

Indian Currency.

1    2    5    10    20    50    100    200    500    2000

Total: 5548

Min. no. of coins/notes to get the required cash.

|       | <u>Notes/Coins.</u> | Amount left |
|-------|---------------------|-------------|
| 2000  | 2                   | 1548        |
| 500   | 3                   | 48          |
| 20    | 2                   | 8           |
| 5     | 1                   | 3           |
| 2     | 1                   | 1           |
| 1     | 1                   | 0           |
| <hr/> |                     |             |
|       |                     | (10)        |



Min # of notes/coins required  
to make a total 5548 rs.

\* Why greedy works in Indian Currency?

⇒ Every denomination is at least greater than or equal to at least twice of its previous currency.

$$n \text{ Rs} \Rightarrow n > 500$$

i)  $n - 500 = y \Rightarrow 1 \text{ notes .}$

ii)  $n - 2 \times 200 \Rightarrow 2 \text{ notes of } 200 \text{ Rs .}$

⇒ Greedy works in Indian Currency

\* Currency : 1 10 18

Target : 20 Rs

Greedy

$$1 \times 18 + 2 \times 1$$

3 coins.

Correct

$$2 \times 10 \Rightarrow 2 \text{ coins}$$

$$\Rightarrow 20$$

Q.

| Food items     | Proteint content | Protein/kg |
|----------------|------------------|------------|
| Tomato: 20 kg  | 200              | 10         |
| Apple: 15 kg   | 180              | 12         |
| Onion: 50 kg   | 250              | 5          |
| Chicken: 10 kg | 150              | 15         |
| Potato: 25 kg  | 200              | 8          |
| Mango: 12 kg   | 132              | 11         |
| Seafood: 5 kg  | 100              | 20         |

- We can pick max of 70 kg
- We can pick ~~1 kg~~ 1 kg from each item.
- Pick the items s.t we get max protein.

⇒ Knapsack Problem  
(Fractional Knapsack)

Idea 1:

Take the item based on Max Protein.

$$\begin{array}{l} 50 \text{ kg} \\ \text{Onion} \\ (250) \end{array} + \begin{array}{l} 20 \text{ kg} \\ \text{Tomato} \\ (200) \end{array} \Rightarrow \begin{array}{l} 450 \text{ units} \\ \text{Protein} \end{array} \quad \times$$

⇒ Greedy based on Max protein content.

Idea 2 : Take items based on protein/kg.

|                | Seafood | Chicken | Apple | Mango | Tomato | Potato |
|----------------|---------|---------|-------|-------|--------|--------|
| Item wt        | 5       | 10      | 15    | 12    | 20     | 8      |
| <u>Protein</u> | 100     | 150     | 180   | 132   | 200    | 64     |

826

⇒ Greedy based on Max protein/kg.

⇒ Can we get more than 826 units of protein?

NO, because we are picking Max protein possible for each kg.

# Properties of Greedy :

1. for optimization related problems.



Max profit | Min cost | Coins | .....

2. Based on what parameter we want to apply greedy.

3. Check for any counter examples where Greedy won't work.

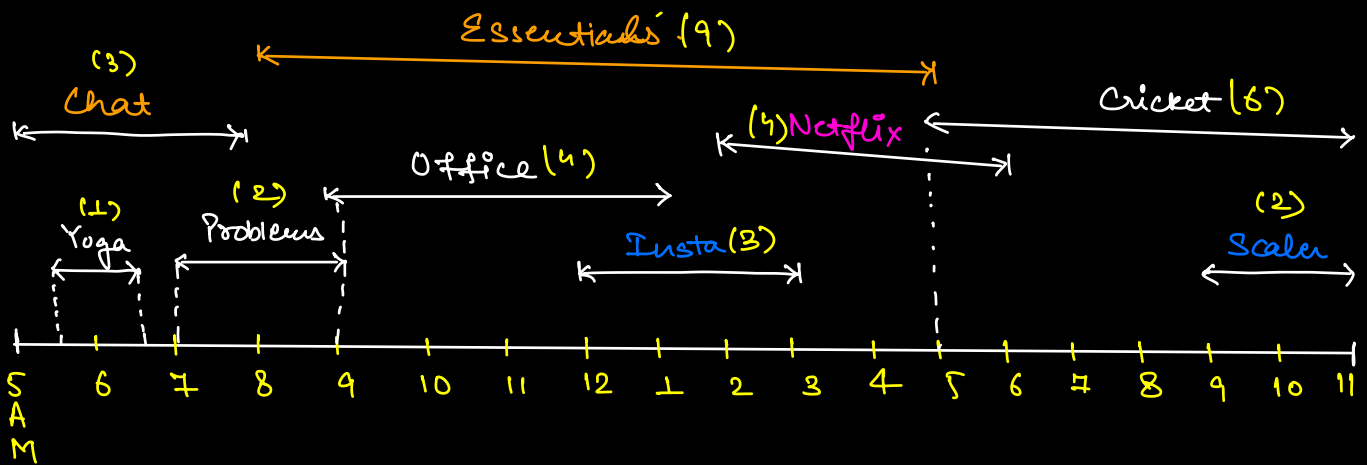
\* Ex. of Greedy Algorithms.

i) Prim's & Kruskal's Algo  $\Rightarrow$  MST

ii) Dijkstra's Algo.

iii) Huffman Coding

# Activity Selection.



1. Once you start a task, we need to complete it.

2. At any given time, we can only perform one task.

3. Max. no. of tasks we can do.

ans  
Yoga  
Problem Solving  
Office  
Netflix  
Scaler.

\* Greedy

1. Pick the tasks with MIN duration.

→ Yoga  
→ Problem  
→ Scaler  
→ Duster } 4

2. Pick the tasks with MIN start time.

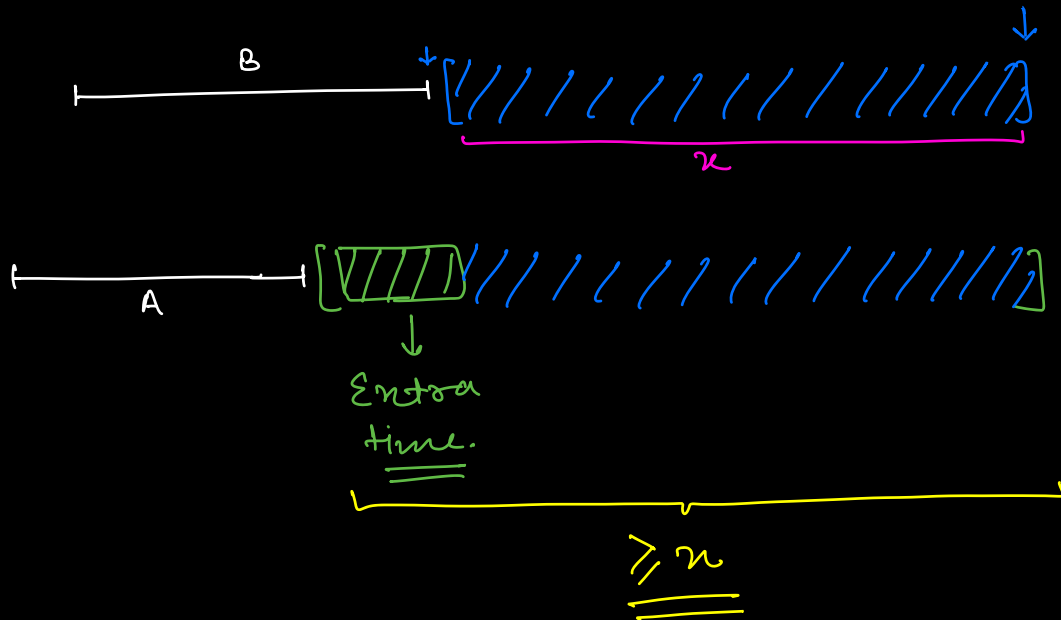
Chat  
Essentials  
Cricket } X

↓  
Starts earlier.

3. Pick the task that ends early.

Yoga  
PS  
Office  
Netflix  
Scaler } ✓

## # Correctness of logic



⇒ By picking the tasks that ends first we are leaving more slots/time to do more tasks.

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## Job Scheduling

- 1) Given  $N$  tasks to complete.
- 2) Deadline assigned for each task, day on or before we can do the task.
- 3) Payment is assigned to each task.
- 4) On any given day we can perform only ① task & each task takes 1 day.
- 5) Find max payment we can get.

Ex

| Task | Deadline<br>(day) | Payment |
|------|-------------------|---------|
| a    | 3                 | 100     |
| b    | 1                 | 19      |
| c    | 2                 | 27      |
| d    | 1                 | 25      |
| e    | 3                 | 30      |

$$\frac{d}{25} \frac{e}{30} \frac{a}{100} \Rightarrow 155$$

$$\frac{a}{100} \frac{c}{27} \frac{e}{30} \Rightarrow 157$$

$$\frac{a}{\checkmark} \frac{e}{\checkmark} \frac{c}{\times} \quad ]$$

$$\begin{array}{c} \downarrow \\ b \quad d \quad a \\ \hline \checkmark \quad \times \quad \hline \end{array}$$

$$\frac{d}{\checkmark} \frac{c}{\checkmark} \frac{a}{\checkmark} \Rightarrow 152$$



$$\begin{array}{ccc} \underline{c} & \underline{e} & \underline{a} \Rightarrow \checkmark \\ \underline{e} & \underline{c} & \underline{a} \Rightarrow \checkmark \end{array}$$

\* deadline =  $n^{\text{th}}$  day

$\Rightarrow$  We need to complete the task on  $\leq n^{\text{th}}$  day  
i.e. on day 1 | 2 | 3 | ... | n

$\Rightarrow$  Greedy based on Deadline (Sort based on deadline)

|            |    |    |    |     |                   |
|------------|----|----|----|-----|-------------------|
| Task :     | b  | d  | c  | a   | $\downarrow$<br>e |
| deadline : | 1  | 1  | 2  | 3   | 3                 |
| Payment :  | 19 | 25 | 27 | 100 | 30                |

Diary

|               |               |    |
|---------------|---------------|----|
| <del>19</del> | <del>25</del> | 27 |
| 100           | 30            |    |

$\rightarrow$  157

$\rightarrow$  insert  
getMin()  
deleteMin() } Min heap

Ex

|           |     |     |     |     |     |     |     |     |     |     |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Task:     | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
| Deadline: | 2   | 1   | 1   | 1   | 4   | 5   | 4   | 5   | 5   | 2   |
| Payment:  | 200 | 250 | 200 | 350 | 300 | 100 | 250 | 600 | 400 | 150 |

⇒ Sort based on deadline

|           |     |     |     |     |     |     |     |     |     |     |   |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|
| Task:     | 2   | 3   | 4   | 1   | 10  | 5   | 7   | 6   | 8   | 9   | ↓ |
| Deadline: | 1   | 1   | 1   | 2   | 2   | 4   | 4   | 5   | 5   | 5   |   |
| Payment:  | 250 | 200 | 350 | 200 | 150 | 300 | 250 | 100 | 600 | 400 |   |

|                |     |                |
|----------------|-----|----------------|
| <del>250</del> | 350 | <del>200</del> |
| 300            | 250 |                |
| <del>100</del> | 600 |                |
|                | 400 |                |

⇒ 1900

|           |     |     |     |     |     |        |
|-----------|-----|-----|-----|-----|-----|--------|
| Task:     | 2   | 3   | 4   | 1   | 10  | ↓<br>6 |
| Deadline: | 1   | 1   | 1   | 2   | 2   | 5      |
| Payment:  | 250 | 200 | 350 | 200 | 150 | 300    |

$$\begin{array}{|c|} \hline \begin{array}{r} \cancel{250} \quad 350 \quad 200 \\ 300 \end{array} \\ \hline \end{array} \Rightarrow \begin{array}{r} \cancel{550} \quad 850 \\ \hline \hline \end{array}$$

————— \* —————

```

int maxPayment (list<pair<int, int> & data) {
    deadline pay
    sort(list) // Based on the deadline
    MinHeap<int> minH;
    for (i = 0; i < N; i++) {
        pair<int, int> n = data[i];
        dead = n.first;
        pay = n.second;
        if (deadline > minH.size()) {
            // Empty slot is there
            minH.insert(pay);
        }
        else if (pay > minH.getMin()) {
            minH.deleteMin();
            minH.insert(pay);
        }
    }
    ans = 0
    while (minH.size() > 0) {
        ans += minH.getMin();
        minH.deleteMin();
    }
    return ans;
}

```

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TC:  $O(N \log N)$     SC:  $O(N)$