

$A \div B \Rightarrow$ Remainder when A is divided B.

14 \div 5 \Rightarrow

Divident
Divisor
Quotient
Remainder

$$\text{Divident} = \text{Divisor} * \text{Quotient} + \text{Remainder}$$

$$\text{Remainder} = \text{Divident} - \text{Divisor} * \text{Quotient}$$

$$\rightarrow 14 \div 5 \Rightarrow 14 - 5 = 9$$
$$9 - 5 = \underline{4}$$

Division is a repeated subtraction.

$$1) \underbrace{30 \div 4}_2 = 30 - 4 = 23 - 4 = 16 - 4 = 9 - 4 = \underline{2}$$
$$= 30 - 4 \times 4$$

$$2) 40 \div 6 = 4 = 40 - 6 \times 6$$

$$\text{Remainder} = \text{Divident} - \left(\text{largest multiple of divisor} \leq \text{divident} \right)$$

#

$$a \% M \in [0, M-1]$$

$$\begin{array}{c}
 -\infty \\
 \vdots \\
 100 \% 3 = 1 \\
 10 \% 3 = 1 \\
 3 \% 3 = 0 \\
 4 \% 3 = 1 \\
 5 \% 3 = 2 \\
 \vdots \\
 +\infty
 \end{array}
 \left. \vphantom{\begin{array}{c} -\infty \\ \vdots \\ 100 \% 3 = 1 \\ 10 \% 3 = 1 \\ 3 \% 3 = 0 \\ 4 \% 3 = 1 \\ 5 \% 3 = 2 \\ \vdots \\ +\infty \end{array}} \right\} \xrightarrow{\% 3} 0, 1, \underline{\underline{2}}$$

\uparrow
 $\underline{\underline{M-1}}$

Modulo Arithmetic :

$$1) (A + B) \% M = (A \% M + B \% M) \% M$$

$$\begin{array}{l|l}
 A=4, B=5, C=6 & 4 \% 6 + 5 \% 6 \\
 (4+5) \% 6 & 4 + 5 \\
 9 \% 6 & \underline{\underline{9}} \\
 = \underline{\underline{3}} &
 \end{array}$$

$$2) (A * B) \% M = (A \% M * B \% M) \% M$$

$$\begin{array}{l}
 \underline{\underline{Ex}} \quad 100 \% 6 = 4 \\
 (100 \% 6) \% 6 = 4 \% 6 = 4 \\
 ((100 \% 6) \% 6) \% 6 = 4
 \end{array}$$

$$\begin{aligned}
 3) \quad A \% M &\Rightarrow (A + M) \% M \\
 &\Rightarrow (A \% M + \cancel{M \% M}) \% M \\
 &\Rightarrow A \% M
 \end{aligned}$$

$$A \% M = (A + n * M) \% M$$

$$4) (A - B) \% M = (A \% M - B \% M + M) \% M$$

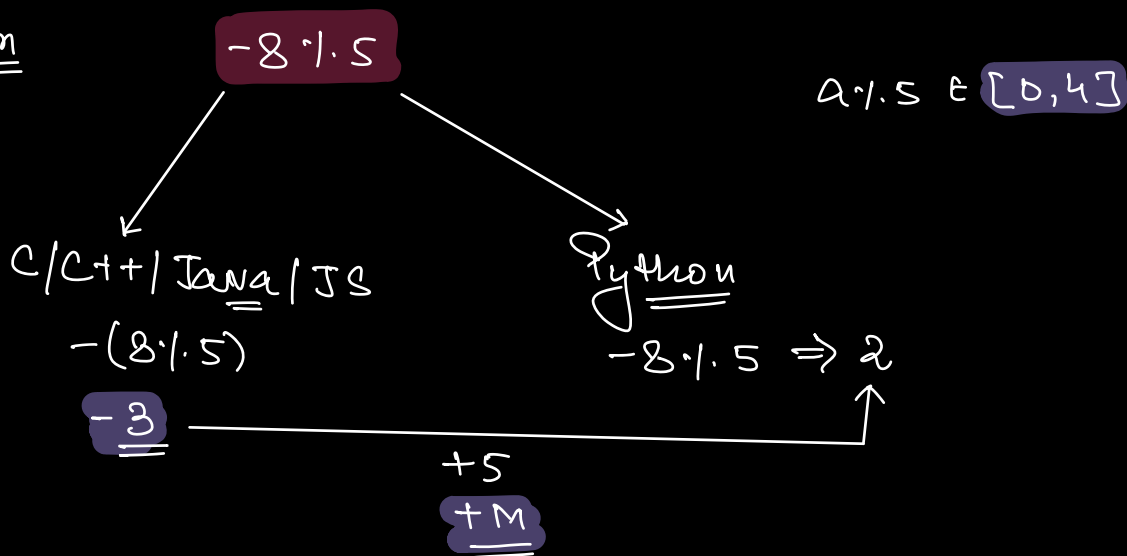
$$A=8, B=4, M=5$$

$$\begin{array}{l|l}
 (8-4) \% 5 & (8 \% 5 - 4 \% 5) \% 5 \\
 4 \% 5 & (3 - 4) \% 5 \\
 = \underline{4} & (-1) \% 5
 \end{array}$$

$$\underline{(-1) \% 5}$$

$$(-1 + 5) \% 5 = 4 \% 5 = \underline{4}$$

Ex



Q.

A, B

$A > B$

Find M , s.t $A \% M = B \% M$; $M > 1$

Ex

$A = 16, B = 4$

$$16 \% M = 4 \% M$$

$M = 2, 3, 4, 6, \dots$

\Rightarrow

$$A \% M = B \% M$$

$$A \% M - B \% M = 0$$

$$(A - B) \% M = 0$$



M is a factor of $A - B$.

\rightarrow return $A - B$ \Rightarrow $O(1)$

$$n \% y = 0$$

$\Rightarrow n$ is a multiple
of y .

$\Rightarrow y$ is a factor of n .

Q. Given an Array of size N , calculate the no. of pairs (i, j) s.t. $(A[i] + A[j]) \% M = 0$ & $i \neq j$. pair (i, j) is considered to be same as pair (j, i) .

$A: \{4, 4, 6, 5, 5, 3\}$ $M = 3$

$(0, 3)$
 $(0, 4)$
 $(1, 3)$
 $(1, 4)$
 $(2, 5)$

5 pairs.

Count = 0

for ($i = 0$; $i < N$; $i++$) {

for ($j = i+1$; $j < N$; $j++$) {

if $((A[i] + A[j]) \% M == 0)$

Count++;

}

3

TC: $O(N^2)$

SC: $O(1)$

Quiz A: {⁰13, ¹14, ²22, ³3, ⁴32, ⁵19, ⁶16} M = 4.

i	j
0	3
0	5
1	2
4	6

\Rightarrow 4 pairs.

$$\Rightarrow (A[i] + A[j]) \% M = 0$$

$$(\underbrace{A[i] \% M}_x + \underbrace{A[j] \% M}_y) \% M = 0$$

$$(x + y) \% M = 0 \quad \begin{matrix} x \in [0, M-1] \\ y \in [0, M-1] \end{matrix} \quad \leftarrow \boxed{x + y = M}$$

\rightarrow x + y should be divisible by M.

$$A = 13, B = ?, M = 4$$

$$(A + B) \% 4 = 0$$

$$\downarrow$$

$$(\underbrace{A \% 4}_1 + \underbrace{B \% 4}_x) \% 4 = 0$$

$$(1 + x) \% 4 = 0$$

$$\downarrow$$

$$x = B \% 4 \in [0, 3]$$

$$0, 1, 2 \text{ or } 3$$

3

$$A = 35, B = ?, M = 10$$

$$(A + B) \cdot 10 = 0$$

$$(\underbrace{A \cdot 10}_5 + \underbrace{B \cdot 10}_?) \cdot 10 = 0$$

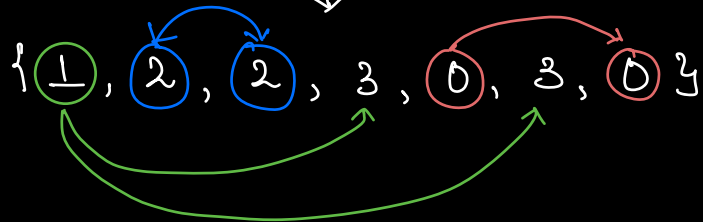
$$(5 + ?) \cdot 10 = 0$$

\Downarrow

$$[0, 9]$$

$$A: \{ \overset{0}{1}3, \overset{1}{1}4, \overset{2}{2}2, \overset{3}{3}, \overset{4}{3}2, \overset{5}{1}9, \overset{6}{1}6 \} \quad \underline{\underline{M = 4.}}$$

$\downarrow \cdot M$



A: {⁰29, ¹11, ²21, ³14, ⁴2, ⁵5, ⁶4, ⁷6, ⁸23, ⁹13, ¹⁰26, ¹¹14, ¹²18, ¹³15, ¹⁴30, ¹⁵35, ¹⁶50, ¹⁷20, ¹⁸40}

M=10

% M=10

Hash Map

[0, M-1]

0	1	2	3	4	5	6	7	8	9
4	2	1	2	2	3	2	1	1	1
30 50 20 40	11 21	2	23 13	4 14	5 15 35	6 26	14	18	29

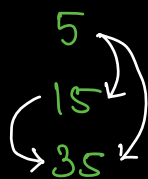
$\frac{C(C-1)}{2}$

(4, 6)

(4, 26)

(14, 6)

(14, 26)



cnt C₂

$\frac{C_{nt}(C_{nt}-1)}{2}$

(11, 29)

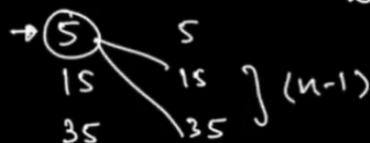
(21, 29)

(2, 18)

(23, 14)

(13, 14)

n(n-1)




```
HashMap<int, int> hm;
```

```
for (i = 0; i < N; i++) {
```

```
    hm[A[i] % M]++;
```

```
}
```

```
ans = 0
```

```
ans += ( (hm[0] * (hm[0] - 1)) / 2 );
```

```
i = 1, j = M - 1;
```

```
while (i < j) {
```

```
    ans += (hm[i] * hm[j]);
```

```
    i++;
```

```
    j--;
```

```
}
```

```
if (M % 2 == 0) {
```

```
    ans += ( (hm[M/2] * (hm[M/2] - 1)) / 2 );
```

```
}
```

```
return ans;
```

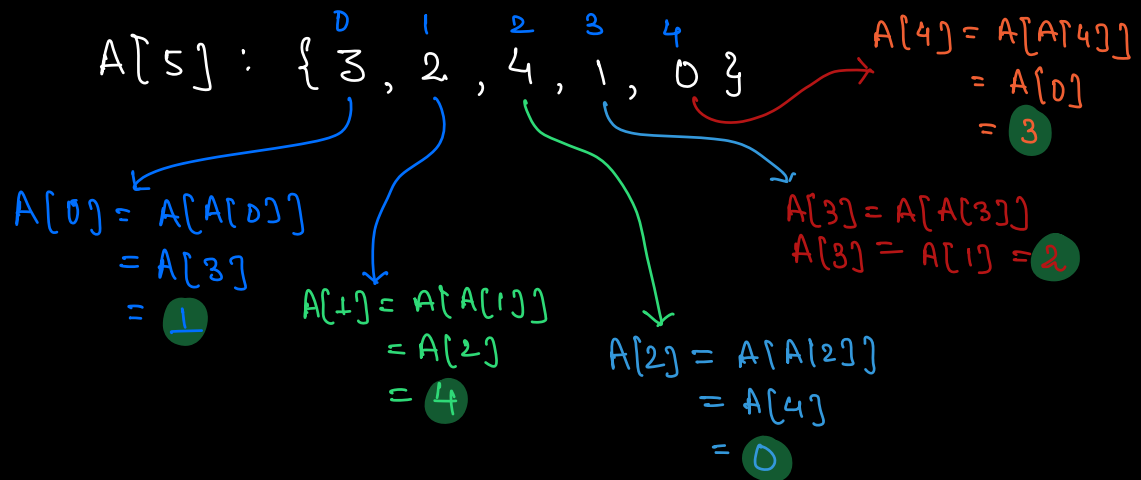
TC: $O(N + M)$

SC: $O(M)$

Q: Given an Array of all distinct integers where $\{0 \leq A[i] \leq N-1\}$, N is the size of the Array.

Replace $A[i] \Rightarrow A[A[i]]$

Ex



$\rightarrow \{1, 4, 0, 2, 3\}$

Quiz

$A : \{ \overset{0}{1}, \overset{1}{6}, \overset{2}{3}, \overset{3}{5}, \overset{4}{4}, \overset{5}{2}, \overset{6}{0} \}$

$A[0] = A[A[0]]$ $= A[1]$ $= 6$	$A[1] = A[A[1]]$ $= A[6]$ $= 0$	$A[2] = A[A[2]]$ $= A[3]$ $= 5$
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$A[3] = A[A[3]]$ $= A[5]$ $= 2$	$A[4] = A[A[4]]$ $= A[4]$ $= 4$	$A[5] = A[A[5]]$ $= A[2]$ $= 3$
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$A[6] = A[A[6]]$
 $= A[0]$
 $= 1$

$\{6, 0, 5, 2, 4, 3, 1\}$

* $B[N]$

for ($i \rightarrow 0$ to N)

$$B[i] = A[A[i]]$$

$$TC: O(N)$$

$$SC: O(N)$$

Constant Extra Space.

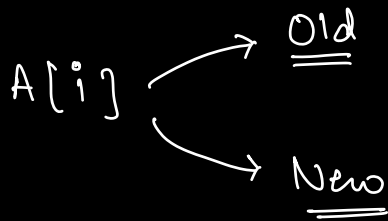
x $A[5]: \{ \overset{0}{\cancel{3}}, \overset{1}{2}, \overset{2}{4}, \overset{3}{1}, \overset{4}{0} \}$

$$\begin{aligned} A[0] &= A[A[0]] \\ &= A[3] \\ &= 1 \end{aligned}$$

$$\begin{aligned} A[4] &= A[A[4]] \\ &= A[0] \end{aligned}$$

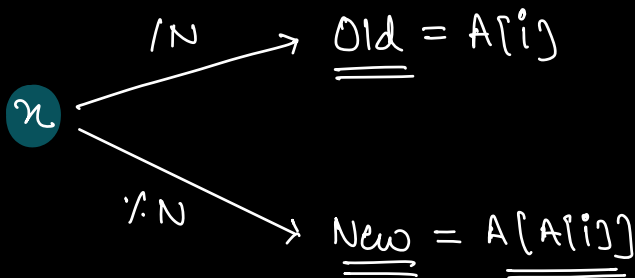
\Rightarrow Day = 0, hrs = 0

	Days	hrs
23 hrs	0	23
46 hrs	1	22
100 hrs	4	4
125 hrs	5	5
n hrs	$n/24$ Quotient	$n \% 24$ <u>remainder.</u>



$$0 \leq A[i] \leq N-1$$

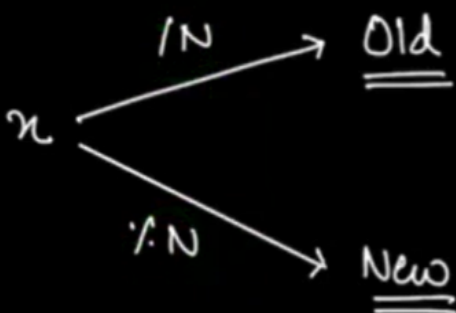
$$\begin{cases} A[i] \% N = \underline{\underline{A[i]}} \\ \frac{A[i]}{N} = 0 \end{cases}$$



$$\begin{aligned} \frac{x}{N} &= \frac{A[i] * N + A[A[i]]}{N} \rightarrow x \\ &= \frac{A[i] * \cancel{N}}{\cancel{N}} + \frac{A[A[i]]}{N} \rightarrow 0 \end{aligned}$$

$$\begin{aligned} x \% N &= (A[i] * N + A[A[i]]) \% N \\ &\quad \downarrow \% N \\ &\quad 0 \\ &= \underbrace{A[A[i]]}_{[0, N-1]} \% N \Rightarrow A[A[i]] \end{aligned}$$

N



$$x = \underset{\downarrow}{\text{Old}} * N + \underset{\downarrow}{\text{New}} = A[i] * N + A[A[i]]$$

$$A[i] \rightarrow A[i] * N + A[A[i]]$$

A: { ⁰3 ¹1 ²4 ³6 ⁴5 ⁵0 ⁶2 }

N=7

↓ *N

A: { ⁰ 3×7
+
6 ¹ 1×7
+
1 ² 4×7
+
5 ³ 6×7
+
2 ⁴ 5×7
+
0 ⁵ 0×7
+
3 ⁶ 2×7
+
4 }

↓ + $\frac{A[A[i]/7]}{7}$

A: { $3 \cdot 7 + 6$, $1 \cdot 7 + 1$, $4 \cdot 7 + 5$, $6 \cdot 7 + 2$, $5 \cdot 7 + 0$, $0 \cdot 7 + 3$, $2 \cdot 7 + 4$ }

↓ %N

A: { 6 1 5 2 0 3 4 }

1) $\text{for}(i=0; i < N; i++)$
 $A[i] * = N;$

2) $\text{for}(i=0; i < N; i++)$
 $\text{index} = A[i] \% 4;$
 $\text{value} = A[\text{index}] \% 4;$
 $A[i] += \text{value};$
}

3) $\text{for}(i=0; i < N; i++)$
 $A[i] /= N;$

45mins-1hr

$TC: O(N)$ $SC: O(1)$

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