Question 1 – Exoplanet Characterization

In this question, you will estimate the mass and radius of a planet from its radial velocity and transit data.

A mysterious new (and fake!) planet, GJ 8999 b, has been detected orbiting the M dwarf GJ 8999. GJ 8999 is a very small star, with a mass of $0.2M_\odot$ and a radius of $0.2R_\odot$. (If you haven't seen those symbols before, $M_\odot$ and $R_\odot$ are the mass and radius of the Sun, respectively.)

The cunning astronomer you are, you have been measuring transit and radial velocity data of this star to figure out the planet's mass and radius of this planet, so you can publish a paper on the system! Let's characterize this planet now.

a) What is the inclination of GJ 8999 b?

Inclination has to be approximately equal to 90 degree as we are observing a transit and it can only happen if the orbit is nearly edge on.

Therefore $i \approx 90$degrees.

b) New transit data from the Transiting Exoplanet Survey Satellite (TESS) has come in, and it very much looks like we have some exoplanet transits! A plot of the flux from the full 28-day observation period of TESS is shown here, as well as a plot that is zoomed into a single transit.

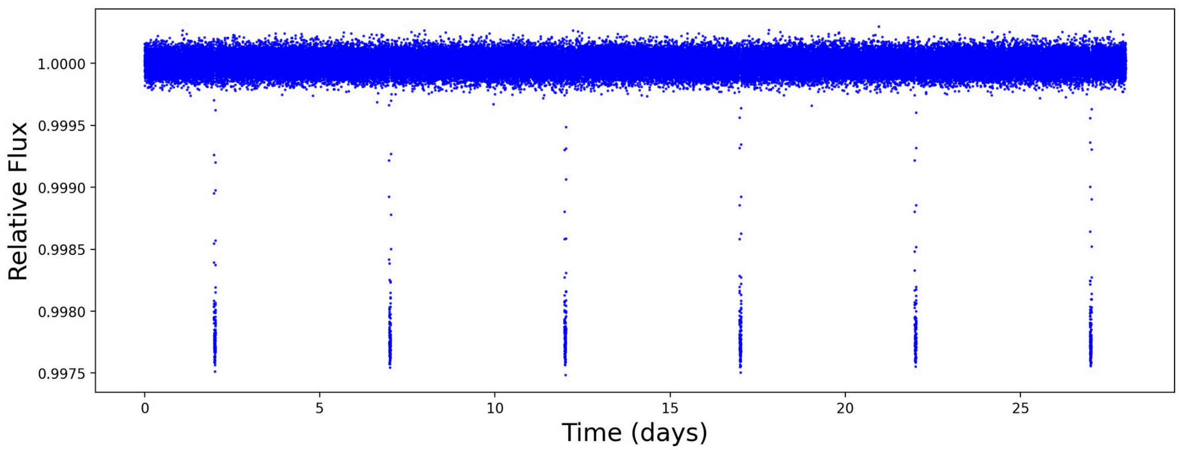

Figure 1: A plot of the flux of GJ 8999 over time over a 28-day period.

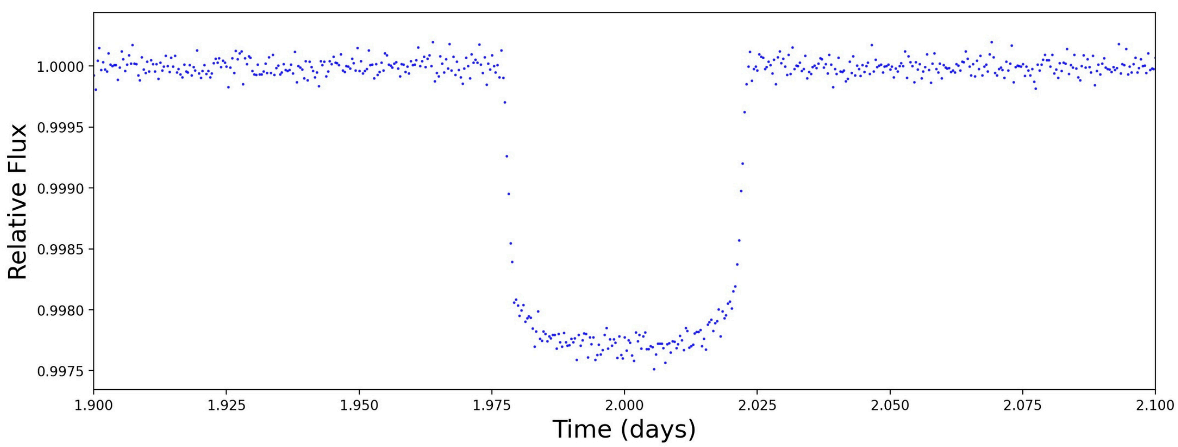

Figure 2: A plot of the flux of GJ 8999 over time, zoomed into a single exoplanet transit.

What is the period of this exoplanet?

Since the graph of flux of GJ 8999 over time is given for 28 days and there are 6 dips within that time period, therefore the orbital period of the exoplanet has to be equal to 28/6 which is approximately equal to or slightly less than 5 earth days.
Therefore, Orbital Period approximately equal to or slightly less than 5 earth days.

c) What is the radius of this planet?

$$\Delta F = \left(\frac{R_p}{R_*}\right)^2$$

from the figure 2,

$$\Delta F = 1 - 0.9975$$
$$\Delta F = 0.0025$$

$$0.0025 = \left(\frac{R_p}{R_*}\right)^2$$

$$R_p = R_* \times \sqrt{0.0025}$$

$$= 0.2 R_\odot \times 0.05 \quad \left(\text{given that } R_* = 0.2 R_\odot\right)$$

$$R_p = 0.01 R_\odot$$

$\because$ Sun's diameter is approx. 109 times longer than Earth's diameter.

$$\therefore R_p = 0.01 \times 109 R_E$$

$$= 1.09 R_E$$

$\therefore$ Radius of planet is approximately equal to Earth's radius.

d) Luckily for us, we have gotten some radial velocity data to figure out this planet's mass, too. This data, taken over a period of 30 days, measures the star's Doppler shift as it moves back and forth due to the planet's gravity.

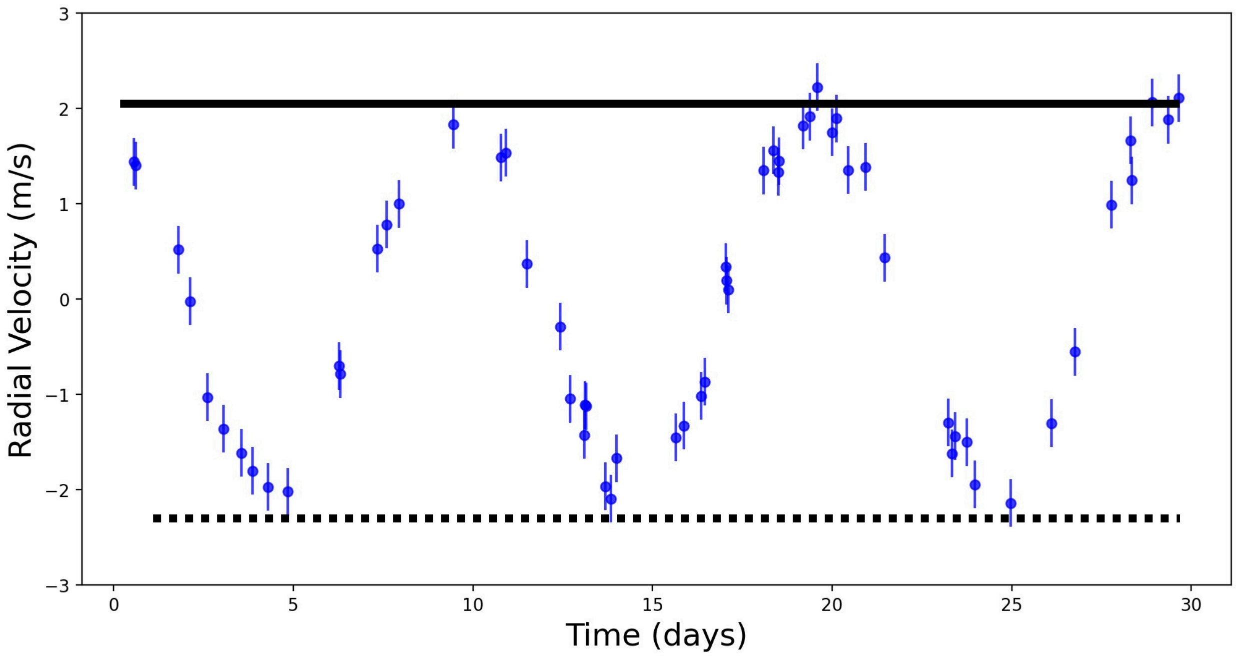

Figure 3: A plot of the radial velocity of GJ 8999 over time.

What is the semi-amplitude K of this planetary signal?

Semi Amplitude (K) can be determined by looking at figure 3.
As visible the Maximum RV is around 2 m/s and Minimum RV is around -2 m/s

Therefore the Semi-Amplitude will be = change in RV/2
= [2 - (-2)]/2
= 2 m/s

e) What is the mass of this planet?

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \cdot \frac{m_p \sin i}{(M_* + M_p)^{2/3}} \cdot \frac{1}{\sqrt{1-e^2}}$$

$$K = \frac{m_p \sin i}{(M_*)^{2/3}} \cdot \left(\frac{2\pi G}{P}\right)^{1/3}$$

$$m_p = \frac{K \cdot (M_*)^{2/3} \left(\frac{P}{2\pi G}\right)^{1/3}}{\sin i} \quad (1)$$

let's change everything to SI units

$K = 2\ m/s$

$M_* = 0.2\ M_{\odot} = 0.2 \times 1.989 \times 10^{30} = 3.98 \times 10^{29}\ kg$

$P = 5\ days = 5 \times 86400 = 432000\ seconds$

$G = 6.674 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$

putting all these values in the formula (1)

$m_p \approx 1.07 \times 10^{25}\ kg$

Earth $(M_{\oplus}) = 5.972 \times 10^{24}\ kg$

$\therefore$ mass of planet in terms of $M_{\oplus}$

$m_p \approx 1.8\ M_{\oplus}$

f) So, now that we've found the mass and radius of our planet, let's try to figure out what it's made of!
The following plot shows (very rough) 'mass-radius curves' of rocky exoplanets of different compositions. A planet lying on a given curve has a mass and radius consistent with being made of the corresponding composition.

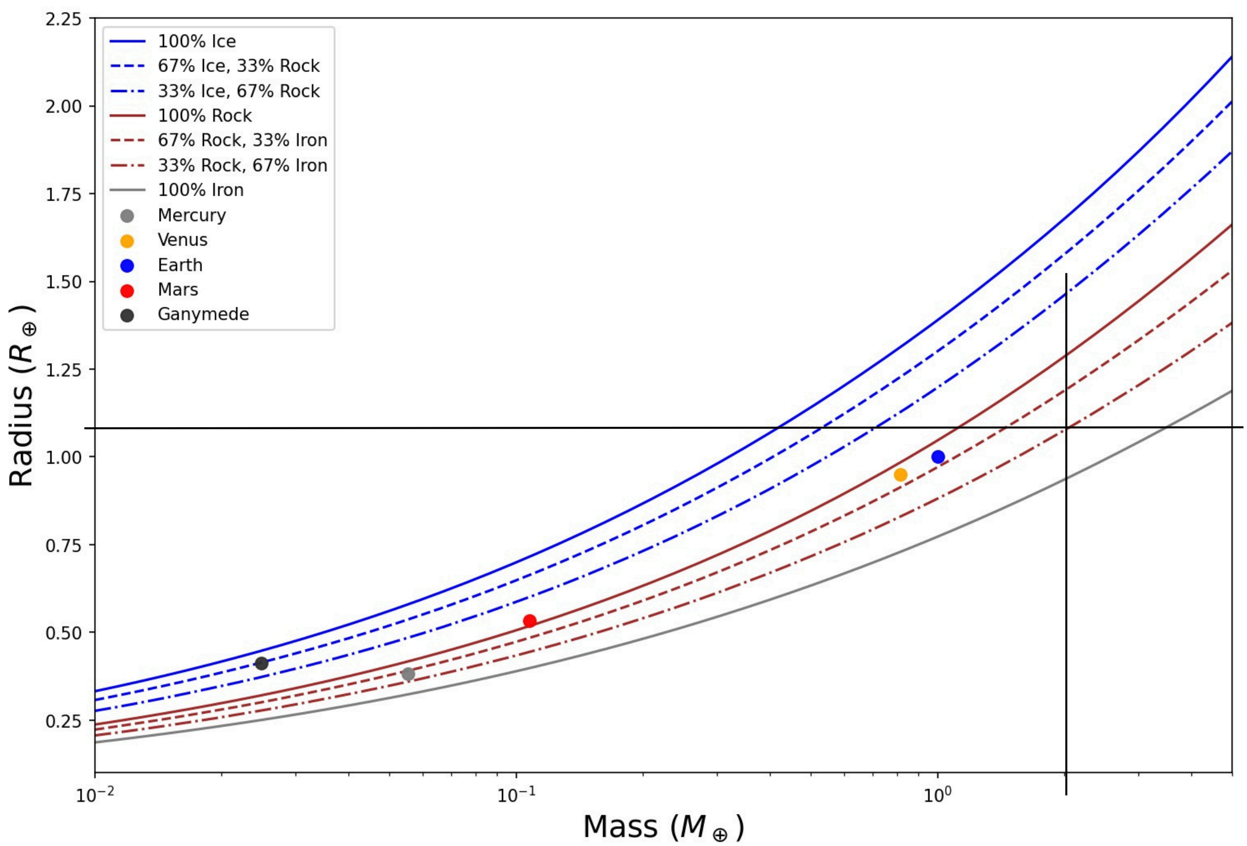

Figure 4: A plot showing the mass-radius curves for different exoplanet compositions.

The five rocky planets (plus Ganymede) are all shown on the plot as well. For example, Earth lies very near the '67% rock, 33% iron' curve, and Earth's composition IS indeed about 67% rock and 33% iron.
With this in mind, what is the composition of GJ 8999 b?

If we plot the mass of GJ 8999 b with it's radius, the point is close to as marked on the graph. Therefore, the composition of GJ 8999 b is close to 67% iron and 33% rock. It makes sense as the plane GJ 8999 b is denser than earth suggesting presence of more iron.