# UNIVERSITY INSTITUTE OF COMPUTING

## MASTER OF COMPUTER APPLICATIONS

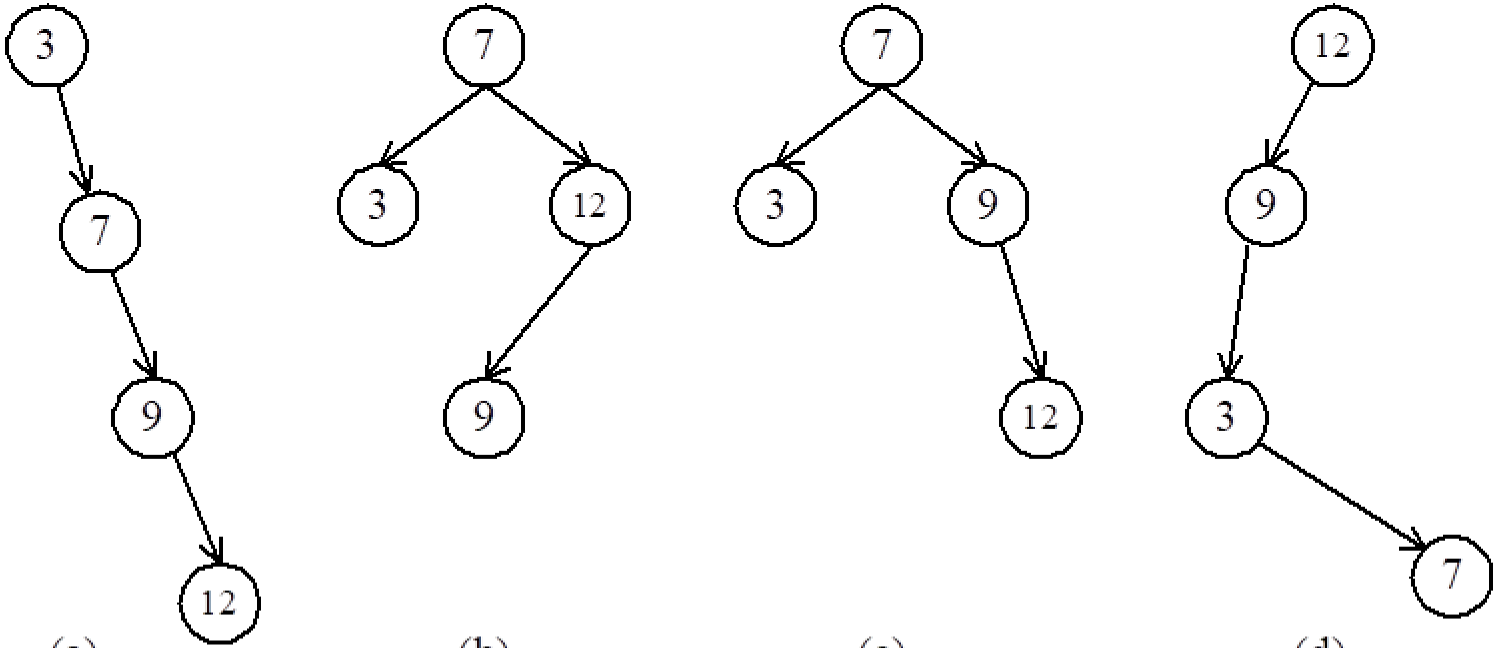
## Design and Analysis of Algorithms

24CAT-611

DISCOVER . LEARN . EMPOWER

# Optimal binary search trees

e.g. binary search trees for 3, 7, 9, 12;




(a) (b) (c) (d)

# Optimal binary search trees

n identifiers : $a_1 < a_2 < a_3 < \ldots < a_n$

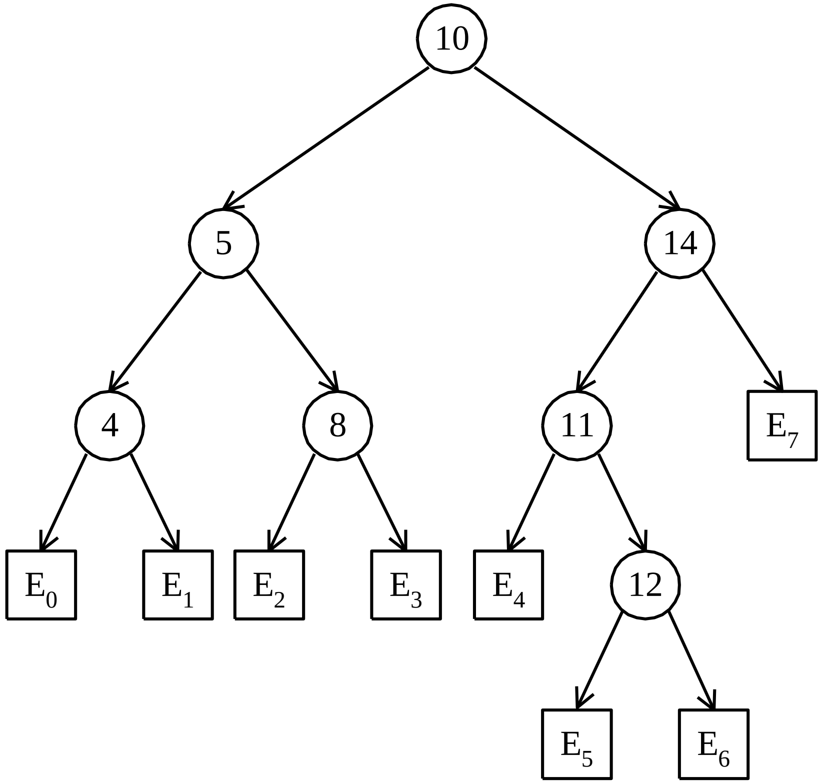
$P_i$, $1 \le i \le n$ : the probability that $a_i$ is searched.

$Q_i$, $0 \le i \le n$ : the probability that x is searched

where $a_i < x < a_{i+1}$ ($a_0 = -\infty$, $a_{n+1} = \infty$).

$$\sum_{i=1}^{n} P_i + \sum_{i=1}^{n} Q_i = 1$$

# Optimally Binary Search Tree




Identifiers : 4, 5, 8, 10, 11, 12, 14
Internal node : successful search, $P_i$
External node : unsuccessful search, $Q_i$

- The expected cost of a binary tree:

- The level of the root : 1

# The dynamic programming approach

Let C(i, j) denote the cost of an optimal binary search tree containing $a_i,\ldots,a_j$ .

The cost of the optimal binary search tree with $a_k$ as its root :

# Computation relationships of subtrees

e.g. n=4

Time complexity : $O(n^3)$

when j-i=m, there are (n-m) C(i, j)'s to compute.

Each C(i, j) with j-i=m can be computed in O(m) time.

# Matrix-chain multiplication

n matrices $A_1, A_2, \ldots, A_n$ with size

$p_0 \times p_1, p_1 \times p_2, p_2 \times p_3, \ldots, p_{n-1} \times p_n$

To determine the multiplication order such that # of scalar multiplications is minimized.

To compute $A_i \times A_{i+1}$, we need $p_{i-1}p_ip_{i+1}$ scalar multiplications.

e.g. n=4, $A_1$: $3 \times 5$, $A_2$: $5 \times 4$, $A_3$: $4 \times 2$, $A_4$: $2 \times 5$

$((A_1 \times A_2) \times A_3) \times A_4$, # of scalar multiplications:

$3 * 5 * 4 + 3 * 4 * 2 + 3 * 2 * 5 = 114$

$(A_1 \times (A_2 \times A_3)) \times A_4$, # of scalar multiplications:

$3 * 5 * 2 + 5 * 4 * 2 + 3 * 2 * 5 = 100$

$(A_1 \times A_2) \times (A_3 \times A_4)$, # of scalar multiplications:

$3 * 5 * 4 + 3 * 4 * 5 + 4 * 2 * 5 = 160$

# PRACTICE PROBLEM BASED ON 0/1 KNAPSACK PROBLEM-

Let m(i, j) denote the minimum cost for computing

$A_i \times A_{i+1} \times \ldots \times A_j$

Computation sequence :

Time complexity : $O(n^3)$

# References

1. https://webpages.uncc.edu › ras › courses › OB

Books:

1. Introduction to Algorithms by Coreman, Leiserson, Rivest, Stein.
2. Fundamentals of Algorithms by Ellis Horwitz
3. Computer Algorithms/C++ by Sartaj Sahni, Sanguthevar Rajasekaran

# THANK YOU