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91578



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SUPERVISOR'S USE ONLY

## Level 3 Calculus, 2015

### 91578 Apply differentiation methods in solving problems

2.00 p.m. Wednesday 25 November 2015

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

TOTAL

ASSESSOR'S USE ONLY

**QUESTION ONE**ASSESSOR'S  
USE ONLY

- (a) Differentiate  $y = 6 \tan(5x)$ .

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- (b) Find the gradient of the tangent to the function  $y = (4x - 3x^2)^3$  at the point  $(1,1)$ .

*You must use calculus and show any derivatives that you need to find when solving this problem.*

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- (c) Find the values of  $x$  for which the function  $f(x) = 8x - 3 + \frac{2}{x+1}$  is increasing.

*You must use calculus and show any derivatives that you need to find when solving this problem.*

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- (d) For what value(s) of  $x$  is the tangent to the graph of the function  $f(x) = \frac{x+4}{x(x-5)}$  parallel to the  $x$ -axis?

*You must use calculus and show any derivatives that you need to find when solving this problem.*

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**QUESTION TWO**ASSESSOR'S  
USE ONLY

- (a) Differentiate  $f(x) = \sqrt[5]{x - 3x^2}$ .

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- (b) Find the gradient of the normal to the curve  $y = x - \frac{16}{x}$  at the point where  $x = 4$ .

*You must use calculus and show any derivatives that you need to find when solving this problem.*

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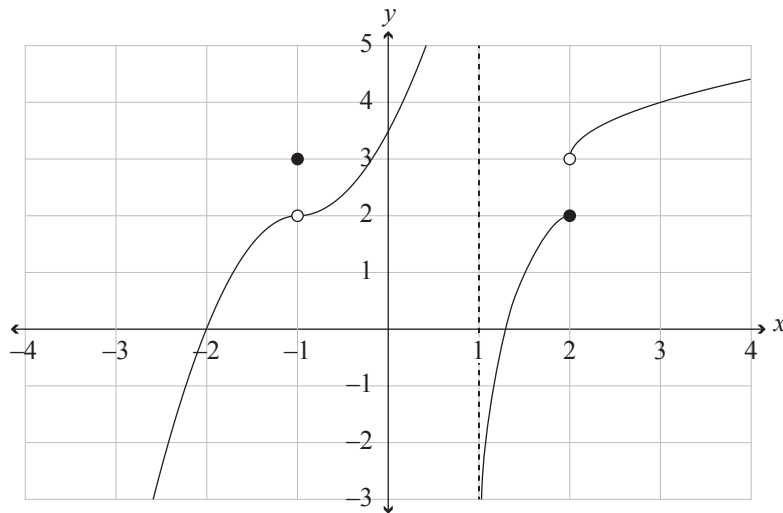
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(c) The graph below shows the function  $y = f(x)$ .



For the function above:

(i) Find the value(s) of  $x$  that meet the following conditions:

1.  $f(x)$  is not defined: \_\_\_\_\_
2.  $f(x)$  is not differentiable: \_\_\_\_\_
3.  $f''(x) > 0$ : \_\_\_\_\_

(ii) What is the value of  $f(-1)$ ? \_\_\_\_\_

State clearly if the value does not exist.

(iii) What is the value of  $\lim_{x \rightarrow 2} f(x)$ ? \_\_\_\_\_

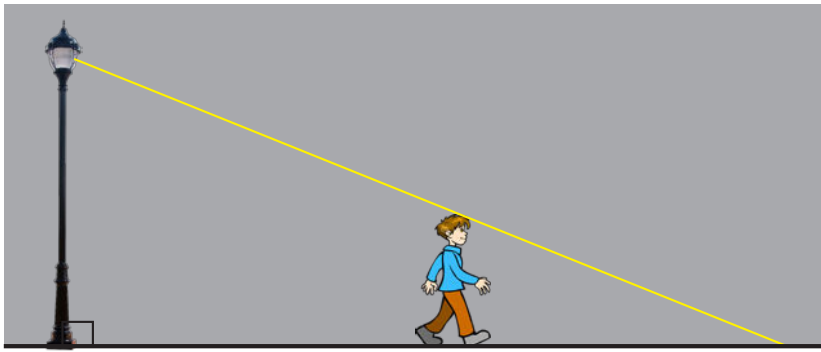
State clearly if the value does not exist.

- (d) A street light is 5 m above the ground, which is flat.

A boy, who is 1.5 m tall, is walking away from the point directly below the streetlight at 2 metres per second.

At what rate is the length of his shadow changing when the boy is 8 m away from the point directly under the light?

*You must use calculus and show any derivatives that you need to find when solving this problem.*



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- A diagram of a square pyramid. The base is a square with side length  $h$ , indicated by a double-headed arrow at the bottom. The height of the pyramid is  $h$ , indicated by a vertical double-headed arrow on the left. The pyramid is shown in a 3D perspective with dashed lines for hidden edges.

A diagram of a rectangular prism. The top horizontal edge is labeled  $h$ . The left vertical edge is labeled  $h$ . The bottom right vertical edge is labeled  $20\text{ cm}$ . Dashed lines indicate the hidden edges of the prism.

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

*You must use calculus and show any derivatives that you need to find when solving this problem.*



**QUESTION THREE**

- (a) For what value(s) of  $x$  does the tangent to the graph of the function  $f(x) = 5 \ln(2x - 3)$  have a gradient of 4?

*You must use calculus and show any derivatives that you need to find when solving this problem.*

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- (b) If  $f(x) = \frac{x}{e^{3x}}$ , find the value(s) of  $x$  such that  $f'(x) = 0$ .

*You must use calculus and show any derivatives that you need to find when solving this problem.*

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- (c) A curve is defined parametrically by the equations  $x = 3 \cos t$  and  $y = \sin 3t$ .

Find the gradient of the normal to the curve at the point where  $t = \frac{\pi}{4}$ .

*You must use calculus and show any derivatives that you need to find when solving this problem.*

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- (d) The equation of motion of a particle is given by the differential equation

$$\frac{d^2x}{dt^2} = -k^2x$$

where  $x$  is the displacement of the particle from the origin at time  $t$ , and  $k$  is a positive constant.

- (i) Show that  $x = A \cos kt + B \sin kt$ , where  $A$  and  $B$  are constants, is a solution of the equation of motion.

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- (ii) The particle was initially at the origin and moving with velocity  $2k$ .

Find the values of  $A$  and  $B$  in the solution  $x = A \cos kt + B \sin kt$ .

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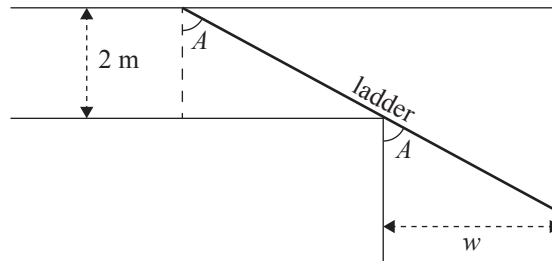
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- At the end it turns  $90^\circ$  into another corridor.



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