Microcontroller Application Series
PID Control 3/3

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PID Controller Tuning

- What is Tuning?
- PID Tuning Objectives and Methods
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What is Tuning?

- **Tuning:** Process in which one or more parameters of a device or model are adjusted upwards or downwards to achieve an improved or specified result. (BusinessDictionary)

PID Tuning Objectives

- **Tuning** is the choosing of the parameters K_p, K_i, and K_d, for a PID controller.
- It is obviously the crucial issue in the overall controller design.
- Controller parameters are tuned such that the closed-loop control system would be stable and would meet specific objectives:
  - (**SR**) stability robustness;
  - (**SLTt**) set-point following and tracking performance at transient, including rise-time, overshoot, and settling time;
  - (**Rss**) regulation performance at steady-state, including load disturbance rejection;
  - (**R**) robustness against plant modeling uncertainty;
  - (**NAR**) noise attenuation and robustness against environmental uncertainty.

- A major advantage of the PID controller is that its parameters have a clear physical meaning (recall the simulation results of analog PID)

TABLE I
EFFECTS OF INDEPENDENT P, I, AND D TUNING

Closed-Loop Response	Rise Time	Overshoot	Settling Time	Steady-State Error	Stability
Increasing K_P	Decrease	Increase	Small Increase	Decrease	Degrade
Increasing K_I	Small Decrease	Increase	Increase	Large Decrease	Degrade
Increasing K_D	Small Decrease	Decrease	Decrease	Minor Change	Improve

- These considerations allow tuning the controller by a **trial-and-error** procedure. However, this can be time consuming, and the final result can be far from the optimum. More importantly, the achieved performance depends on the operator’s skill.

- Empirical ranges of K_p, T_i, T_d for typical processes

Process type	K_p	T_i /min	T_d /min
Flow	1~2.5	0.1~1	
Temperature	1.6~5	3~10	0.5~3
Pressure	1.4~3.5	0.4~3	
Level	1.25~5		

- Empirical data make tuning of the parameters K_p, T_i, T_d much simpler. Nevertheless, changing one parameter can influence effects of the other two.

- **Tuning methods for PID controllers:**
- **Analytical methods**—Parameters are calculated from analytical or algebraic relations between a plant model and an objective. These can lead to an easy-to-use formula and can be suitable for use with online tuning, but the objective needs to be in an analytical form and the model must be accurate.
- **Heuristic methods**—These are evolved from practical experience in manual tuning and from artificial intelligence (including expert systems, fuzzy logic and neural networks). They can serve in the form of a formula or a rule base for online use, often with tradeoff design objectives.
- **Frequency response methods**—Frequency characteristics of the controlled process are used to tune the PID controller. These are often offline and academic methods, where the main concern of design is stability robustness.

- **Optimization methods**—Parameters are obtained ad hoc using an offline numerical optimization method for a single composite objective or using computerized heuristics or an evolutionary algorithm for multiple design objectives. These are often time-domain methods and mostly applied offline.
- **Adaptive tuning methods**—These are for automated online tuning, using one or a combination of the previous methods based on real-time identification.

✓ The above classification does not set an artificial boundary and some methods applied in practice may belong to more than one category.
✓ No methods are generic and can be quickly applied to the design of onboard or onchip controllers for a wide range of devices.
✓ However, no tuning method so far can replace the simple Ziegler-Nichols (Z-N) method in terms of familiarity and ease of use to start with.

Ziegler-Nichols Methods

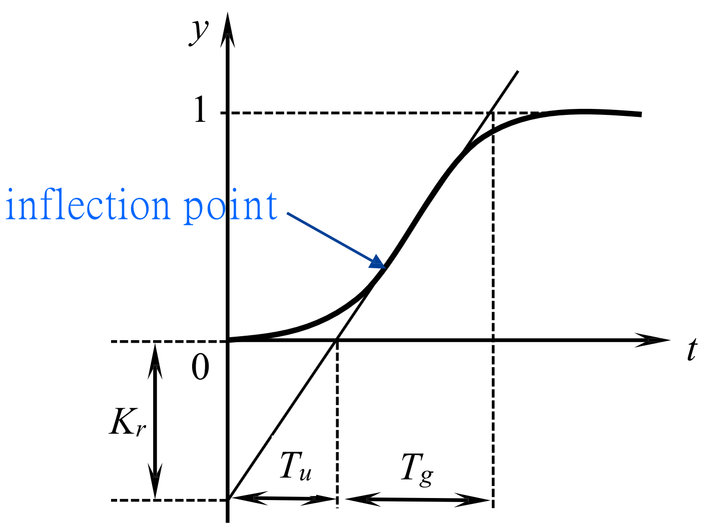
- Ziegler-Nichols (Z-N) method, developed in the 1940's, is the oldest and most used method of tuning.
- It is a heuristic method of tuning a PID controller with the goal of providing satisfactory load disturbance rejection (**Rss performance**).

- **Z-N step response method (First method)**
  - Based on a registration of the open-loop step response of the process
  - The response of the plant to a unit-step input is obtained experimentally.

- **Z-N frequency response method (Second method)**
  - Based on the frequency response of the process

Z-N step response method

- 1. Obtain the step response of the process experimentally;
- 2. Draw a tangent to the step response at the inflection point (the point where the slop has its maximum);
- 3. Compute the delay time T_u and time constant K_r, which are determined by the intersections between the tangent and the coordinate axes;
- 4. Using T_w, K_r to determine the gains K_p, T_i, T_d by referring to the table below;
- 5. Implement the digital PID controller.



Controller	K_p	T_i	T_d
P	$1/K_r$	–	–
PI	$0.8/K_r$	$3T_u$	–
PID	$1.2/K_r$	$2T_u$	$0.42T_u$

Controller parameters for Z-N step response method

Example 1

- Consider the an electrical heating furnace

$$P(s) = \frac{167}{1170s+1} e^{-70s}$$

Design a digital PID controller with sampling time T=0.1 by applying the Ziegler-Nichols step response method.

- **Solution:**
  - **Z-N step response method:**
  - Unit step response:

$$Y(s) = \frac{167}{1170s+1} e^{-70s} \times \frac{1}{s} \longrightarrow y(t) = 167\left(1 - e^{-(t-70)/1170}\right) 1_{t\geq 70}$$

  - Inflection point: $(70, 0)$

$$K_r = \frac{167 \times 70}{1170} \approx 9.99, \quad T_u = 70$$

  - PID parameters:

$$K_{\mathrm{p}} = \frac{1.2}{K_r} \approx 0.12,\ T_{\mathrm{i}} = 2 \times 70 = 140(sec),\ T_{\mathrm{d}} = 0.42 \times 70 = 29.4(sec)$$

- Digital PID controller: $T = 0.1$
- Position algorithm:

$$u_k = 0.12\left(e_k + \frac{1}{1400}\sum_{j=1}^{k} e_j + 294(e_k - e_{k-1})\right)$$

- Velocity algorithm:

$$\Delta u_k = 0.12\left(e_k - e_{k-1} + \frac{1}{1400}e_k + 294(e_k - 2e_{k-1} + e_{k-2})\right)$$

- Modified digital PID controller to account for proportional kick, derivative kick, integral windup, noise, etc.

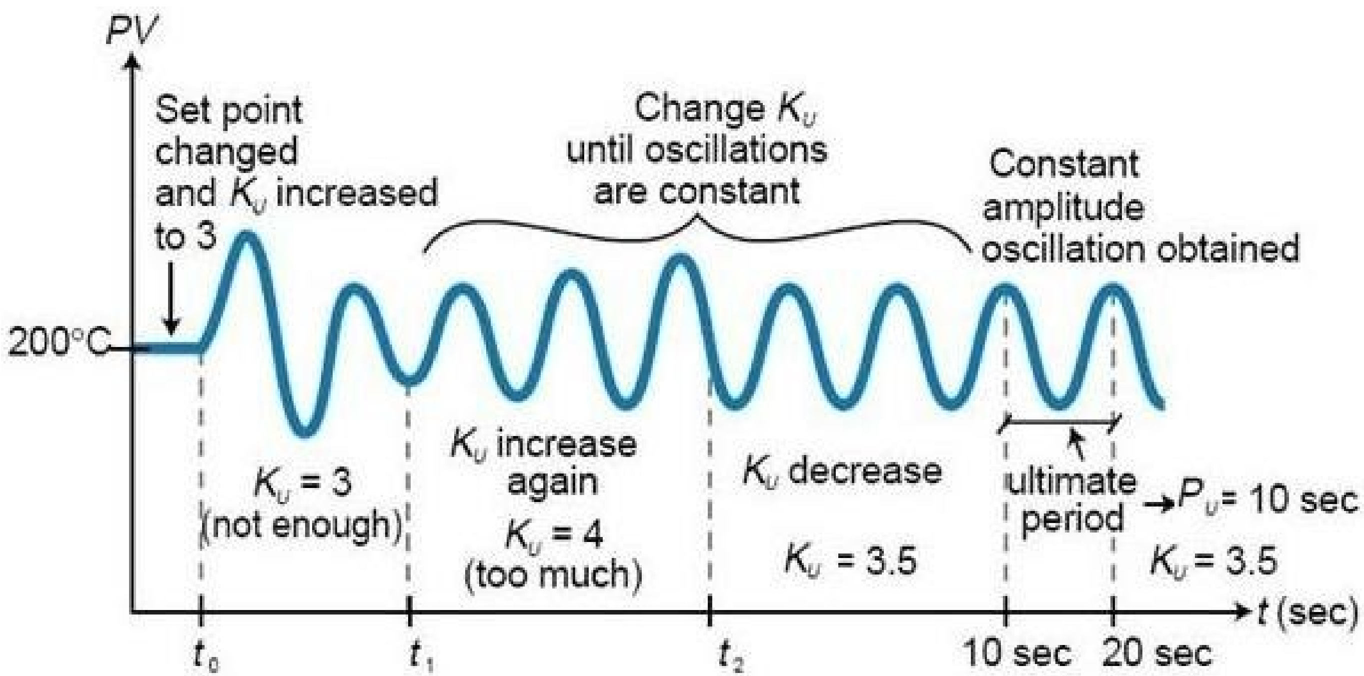
Z-N frequency response method

- 1. Set K_p to some low value (with $T_i = \infty$ and $T_d = 0$ at this stage). So the control action is proportional only;
- 2. Implement the P controller, increase the proportional gain K_p until the output of the control loop starts oscillating with a constant amplitude (**if any**). The value of K_p at this point is referred to as critical gain K_r and the period of the oscillation is T_r;
- 3. Using K_r, Tr to determine the gains K_p, T_i and T_d according to the table below;
- 4. Implement the complete digital PID controller.

Controller	K_p	T_i	T_d
P	$0.5K_r$	–	–
PI	$0.45K_r$	$0.85T_r$	–
PID	$0.6K_r$	$0.5T_r$	$0.12T_r$

Controller parameters for Z-N step response method

- **How to determine the critical gain K_r and the period of the oscillation T_r?**



Source: ControlsWiki

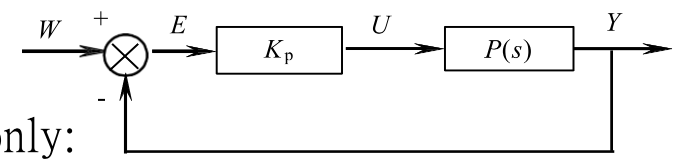
Example 2

- Design a digital PID controller with sampling time $T = 0.1$ for the plant

$$P(s) = \frac{1}{(s+1)(s+3)(s+5)}$$

by applying the Ziegler-Nichols frequency response method.

- **Solution:**
  - **Z-N frequency response method:**
  - Transfer function in the presence of P control only:



$$G(s) = \frac{Y(s)}{W(s)} = \frac{K_p P(s)}{1 + K_p P(s)}$$

  - The characteristic equation with P control is

$$1 + K_p P(s) = 0 \quad \longrightarrow \quad s^3 + 9s^2 + 23s + 15 + K_p = 0$$

  - We can determine the limiting gain for stability (before oscillations) by use of the Routh-Hurwitz condition. (Recall that increasing K_p degrades the stability)

- The Routh array

$$\begin{array}{ccc} s^3 & 1 & 23 \\ s^2 & 9 & 15+K_p \\ s^1 & 192-K_p & 0 \\ s^0 & 15+K_p & \end{array}$$

- Range of K_p for stability: $15 < K_\mathrm{p} < 192$.
- For this system to oscillate, there must be a solution of the characteristic function $1 + K_\mathrm{p}P(s) = 0$ for K_p **real and positive**, and $s = \pm \mathrm{j}\omega$ **pure imaginary**
- When $K_\mathrm{p} = 192$, we have imaginary roots since the s^1 row is identically 0. Indeed, the corresponding auxiliary equation $9s^2 + 15 + 192 = 0$ has roots $s = \pm \mathrm{j}\sqrt{23} \approx \pm\, \mathrm{j}4.8$.
- PID Parameters:

$$K_r = 192,\ T_r = \frac{2\pi}{4.8} \approx 1.31 \quad \Longrightarrow \quad \begin{cases} K_\mathrm{p} = 0.6K_r = 115.2 \\ T_\mathrm{i} = 0.5T_r = 0.66 \\ T_\mathrm{d} = 0.12T_r = 0.16 \end{cases}$$

- Digital PID controller: $T = 0.1$
- Position algorithm:

$$u_k = 115.2\left(e_k + 0.15\sum_{j=1}^{k} e_j + 1.6(e_k - e_{k-1})\right)$$

- Velocity algorithm:

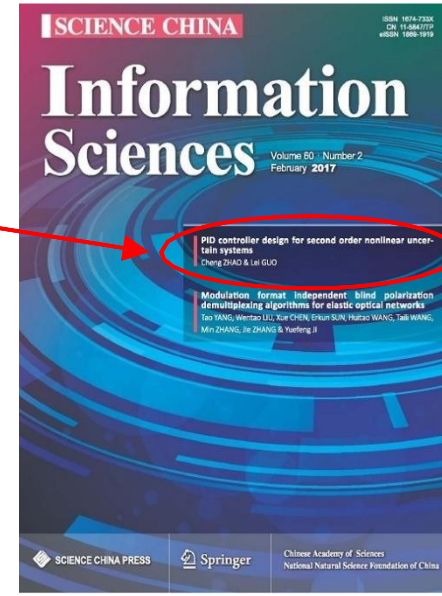
$$\Delta u_k = 115.2\left[e_k - e_{k-1} + 0.15e_k + 1.6(e_k - 2e_{k-1} + e_{k-2})\right]$$

- Modified digital PID controller to account for proportional kick, derivative kick, integral windup, noise, etc.

PID Tuning: Theoretical Guideline

- Recent theoretical guideline for PID tuning:

  Cheng Zhao, Lei Guo, "PID controller design for second order nonlinear uncertain systems", SCIENCE CHINA Information Sciences 60, 022201 (2017)



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- Prof. **Lei Guo**: currently the Director of the National Center for Mathematics and Interdisciplinary Sciences, CAS.

- 1998, Fellow of the IEEE
- 2001, Member of the Chinese Academy of Sciences (中国科学院院士)
- 2002, Fellow of the Academy of Sciences for the Developing World (第三世界科学院院士)
- 2007, Foreign Member of the Royal Swedish Academy of Engineering Sciences (瑞典皇家工程科学院外籍院士)
- 2007, Fellow of the IFAC

- **Model of a moving body**: Newton’ s second law

$$ma(t) = f(p, v, t) + u(t)$$

- where m is mass, $p(t)$, $v(t)$, $a(t)$ are the positon, velocity and acceleration of the moving body, respectively, $u(t)$ is control action.

$f(p, v, t)$ nonlinear function, unknown

- **Objective**: For any constant setpoint w, design the parameters K_p, K_i, K_d of the ideal PID controller

$$u(t) = K_\mathrm{p} e(t) + K_\mathrm{i} \int_0^t e(s)\,\mathrm{d}s + K_\mathrm{d} \frac{\mathrm{d}e(t)}{\mathrm{d}t}$$

such that the steady-state error is zero, i.e,

$$\lim_{t\to\infty} (p(t) - w) = 0, \quad \lim_{t\to\infty} v(t) = 0$$

- **SR & Rss performance**.

- **Theorem:** Consider the system with any unknown $f(p, v)$ satisfying

$$f(p,0,t)=f(p,0,0),\quad \frac{\partial f}{\partial p}\le L_1,\ \left|\frac{\partial f}{\partial v}\right|\le L_2,\quad L_1,L_2>0$$

  Then we can choose $K_{\mathrm{p}}, K_{\mathrm{i}}, K_{\mathrm{d}}$,

$$K_{\mathrm{p}}>L_1,\ K_{\mathrm{i}}>0,\ K_{\mathrm{d}}>L_2,\ (K_{\mathrm{p}}-L_1)(K_{\mathrm{d}}-L_2)-K_{\mathrm{i}}>L_2\sqrt{K_{\mathrm{i}}(K_{\mathrm{d}}+L_2)}$$

  such that the closed-loop system achieves

$$\lim_{t\to\infty}(p(t)-w)=0,\quad \lim_{t\to\infty}v(t)=0$$

  for any initial value $p(0)\,v(0)$ and any constant setpoint w.

- **Remarks:** (1) The integral parameter K_{i} of the PID controller can be taken arbitrarily small; (2) Without such bounds it would not be possible for PID control to achieve global stabilization in general.

Summary

- Tuning methods for PID controllers:
  - Analytical methods
  - Heuristic methods
  - Frequency response methods
  - Optimization methods
  - Adaptive tuning methods
- Ziegler-Nichols method
  - Step response method (First method)
  - Frequency response method (Second method)
- Recent theoretical guidelines

References

- Kiam Heong Ang, Gregory Chong, and Yun Li, **PID control system analysis, design, and technology**, IEEE Transactions on Control Systems Technology, vol. 13, no. 4, pp.559-576, 2005.
- Genke Yang, and Jianying Xie, **Micro-Computer Control Technology**, 4th ed. Changsha: National Defense Industry Press, 2016 (in Chinese).
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- Cheng Zhao, Lei Guo, **PID controller design for second order nonlinear uncertain systems**, SCIENCE CHINA Information Sciences 60, 022201 (2017)

Thanks for your attention!

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