Regional geoid modelling

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December 6, 2020

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Abstract The report investigates the use of remove-restore approach to compute regional geoid models in US region(state of Colorado), with usage of GGM model. Detailed analysis was performed, accounting outcomes received in each step of the Remove-Restore method as well as their influence on the results of the geoid calculation. Calculations and analysis were realized in own program developed in octave software. Resources were gathered from GRAV-D data project, EGM2008 model and SRTM Data. For geoid undulation Stokes approach was followed, with spherical variant of stokes kernel.

Keywords geoid calculation · remove restore approach · Stokes integral

1 Introduction

The true shape of the Earth varies slightly from the mathematically smooth ellipsoidal surface. Variations in the density of the Earth from place to place causes variation in the value of gravity in different locations, which alters the gravitational pull at those regions, causing regions to bulge above or below a

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reference ellipsoid. This undulating shape is called a geoid as a representation of the earth's gravity field.

2 Remove-restore method

The remove-restore method is commonly used in geodesy in the local estimation of the gravity field. From the observed gravity values, first the effect of a global gravity field model is removed. Also other anomalies such as terrain reduction(δ_{gt}), correction to account for the gravitational attraction ($\delta g_a tm$) are removed. Due to these removals, long wave undulations are now absent in out gravity field, which is more accurate. Then the process is to start adding the undulations, (N_{GGM}) corresponding to the removed long-wavelength gravity anomaly of the GGM model, indirect effect($\delta N_{indirect}$) of the terrain reduction Text with citations [0] and [0].
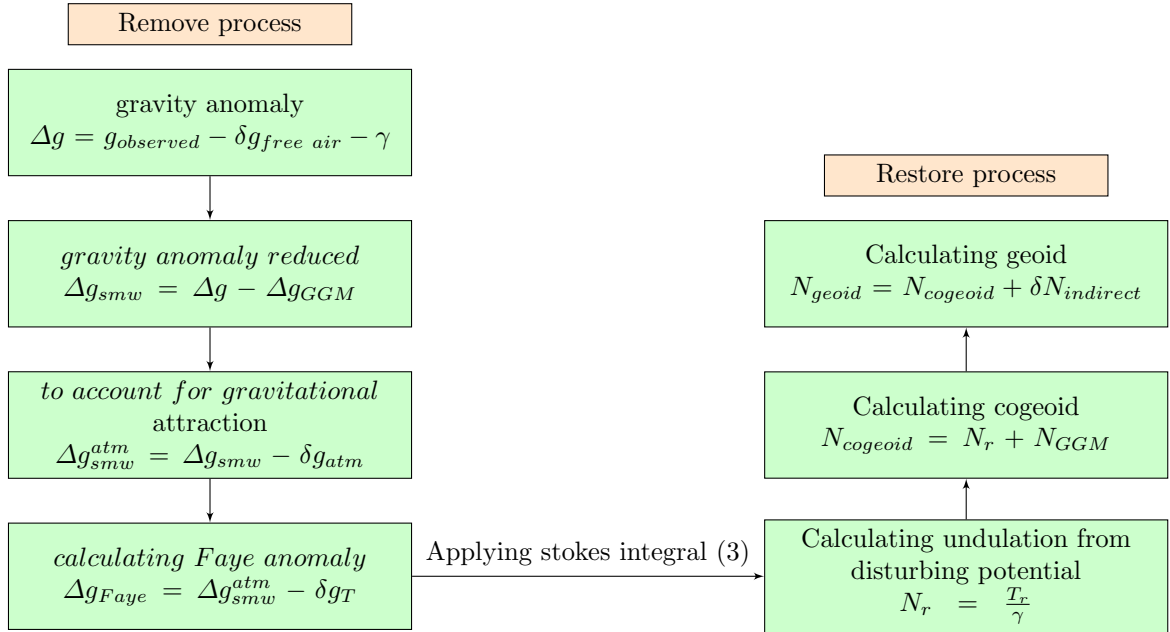
2.1 Remove

The observed airborne value of gravity at a particular point (P) is determined ($g_{observed}$)

Various anomalies needed to be removed :

- $\delta g_{freeair}$
- Δg_{GGM}

$- \delta g_{atm}$
$- \delta g_t$

Final expression for reduced gravity anomaly:

$$\Delta g_{Faye} = g_{observed} - \delta g_{free\ air} - \Delta g_{GGM} - \delta g_{atm} - \delta g_t - \gamma \qquad (1)$$

where,

$$\gamma = \frac{a\gamma_a cos^2\phi + b\gamma_b sin^2\phi}{\sqrt{a\cos^2\phi + b\ sin^2\phi}}\ (Somigliana - Pizzetti\ formula)$$

a = semi-major axis, b = semi-minor axis, γ_a = gravity at equator, γ_b = gravity at poles, ϕ = latitude in degrees

2.2 Restore

After the estimation of Faye anomaly Δg_{Faye}, **disturbing potential** is calculated :

$$T_r = \frac{R}{4\pi} \iint_\Omega \Delta g_{Faye} S(\psi)\, d\Omega$$

Now, by burns formula undulation is calculated:

$$N_r = \frac{T_r}{\gamma}$$

by restoring the undulation (N_{GGM}) corresponding to the removed long-wavelength gravity anomaly, cogeoid will be calculated :

$$N_{cogeoid} = N_r + N_{GGM}$$

By adding the indirect effect of the gravimetric terrain reduction value of geoid is obtained :

$$N_{geoid} = N_{cogeoid} + \delta N_{indirect}$$

Paragraph headings Use paragraph headings as needed.

$$a^2 + b^2 = c^2 \qquad (2)$$

3 Stokes formula

Stokes formula for geoid undulation relative to the reference ellipsoid is:

$$N_{\Delta g} = \frac{R}{4\pi\gamma} \iint_\sigma \Delta g(\varphi, \lambda) S(\psi) \, dv \tag{3}$$

Where S(φ) is the Stokes function,R is the mean radius of the Earth and σ denotes the Earth's surface. By using the geoid undulation equation, i.e. the figure of the earth,undulation can be determined from the gravity observation. As we can easily understand from the equation ,we need to know the gravity values on the entire surface of the earth of the geoid determination .It is not practical to get gravity data densely throughout the globe. When a Stokes-Kernel function is given by:

$$S(\psi) = \frac{1}{\sin\frac{\psi}{2}} - 4 - 6\sin\frac{\psi}{2} + 10\sin^2\left(\frac{\psi}{2}\right) - [3 - 6\sin^2\left(\frac{\psi}{2}\right)] - ln[\sin\frac{\psi}{2} + \sin^2\left(\frac{\psi}{2}\right)] \tag{4}$$

where,

$$\sin^2\left(\frac{\psi}{2}\right) = \sin^2\left(\frac{\varphi_p - \varphi}{2}\right) + \sin^2\left(\frac{\lambda_p - \lambda}{2}\right)\cos\phi_p\cos\phi \tag{5}$$

where,
$\varphi = latitude$ $\lambda = longitude$,
$\varphi_p = latitude\ at\ center\ point, \lambda_p = longitude\ at\ center\ point$

Once the disturbing potential is calculated, we can calculate the geoid by following steps mentioned in (sec 2.2)

4 Results and conclusion

Spherical stokes kernel, yields errors, due to the fact that for our assumption, the shape of geoid is assumed to be spherical, for our simplicity. Modified stokes kernels yield better results. For an observed gravity value, the total correction accounted was approx. 0.0022%

5 Important links

References

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2. Askari. A, Precision geoid determination by Fast Fourier Transform solutions of the Kernel functions of the gridded gravity anomalies and distances in the Oman Gulf, Vol-2, Issue-7, July- 2016
3. Earth Positioning System , 12, Ram Prasad mukherjee university, Ranchi
4. noaa-airborne gravity data. `gravihttps://geodesy.noaa.gov/GRAV-D/data_products.shtml`

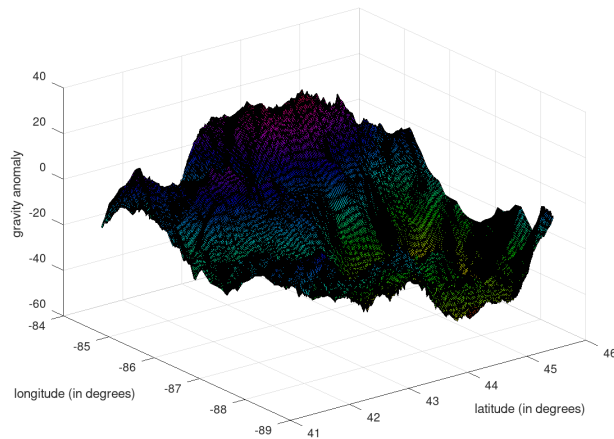Fig. 1: Mesh grid for gravity anomaly(terrain reduction(δg_T) not removed) at EN03 tile(GRAV-D, data project)
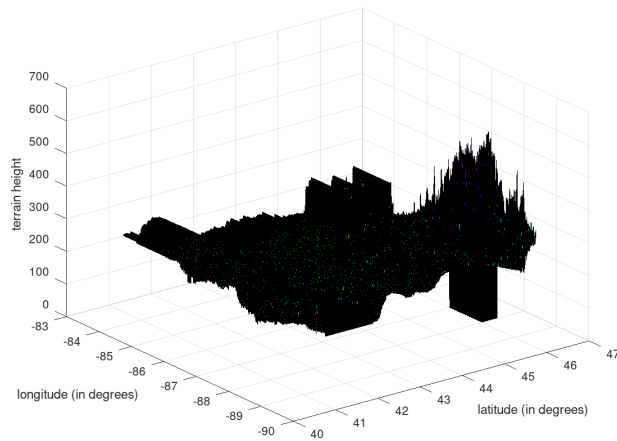

Fig. 2: terrain height(takeen out from DEM model `http://srtm.csi.cgiar.org/srtmdata/`) at EN03 tile(GRAV-D, data project)

long wave undulation data. `https://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/anomalies_dov.html`
for calculating latitude and longitude of a particular region. `http://icgem.gfz-potsdam.de/calcgrid`
for downloading the DEM data. `http://srtm.csi.cgiar.org/srtmdata/`