4G3: COMPUTATIONAL NEUROSCIENCE

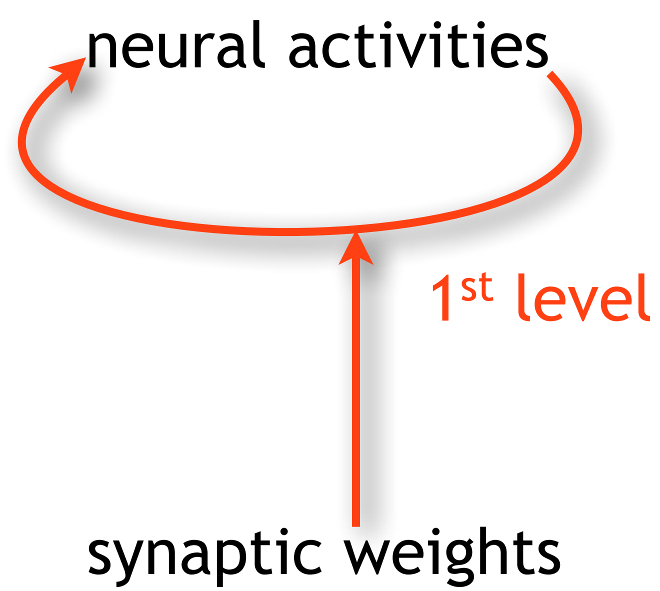
ASSOCIATIVE MEMORY

TWO LEVELS OF NETWORK DYNAMICS

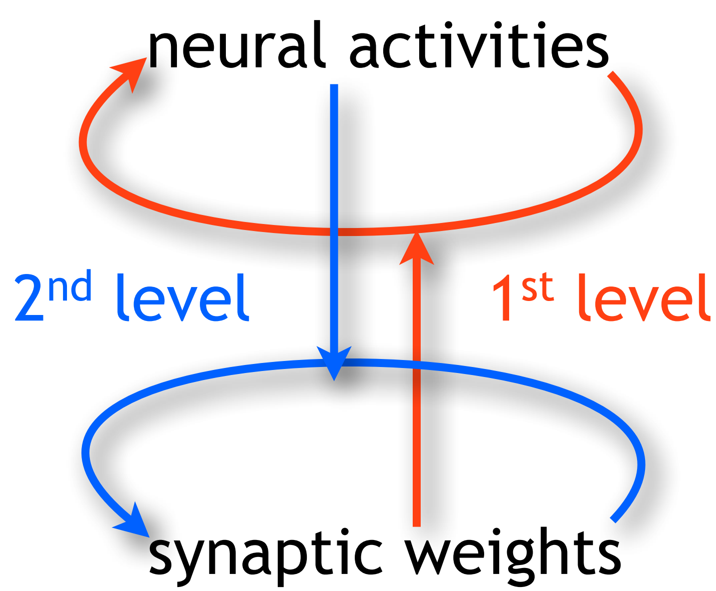
neural activities

synaptic weights

TWO LEVELS OF NETWORK DYNAMICS



TWO LEVELS OF NETWORK DYNAMICS



AUTOASSOCIATIVE MEMORY: AN EXAMPLE

I raised to my lips a spoonful of the tea in which I had soaked a morsel of the cake. ... And suddenly the memory returns. The taste was that of the little crumb of madeleine which on Sunday mornings at Combray, when I went to say good day to her in her bedroom, my aunt Léonie used to give me, dipping it first in her own cup of real or of lime-flower tea.

Marcel Proust: À la recherche du temps perdu

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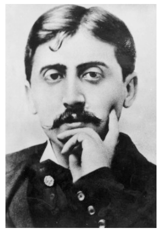
HOW DOES THIS HAPPEN?

MEMORY PROCESSING IN NEURAL NETWORKS

MEMORY PROCESSING IN NEURAL NETWORKS

the Hebbian paradigm



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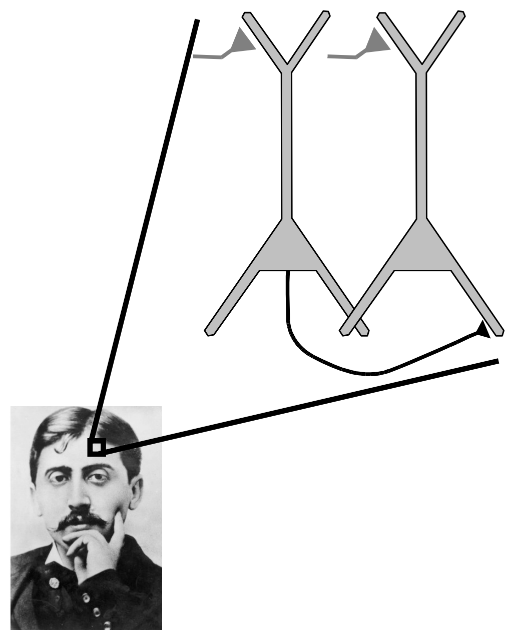

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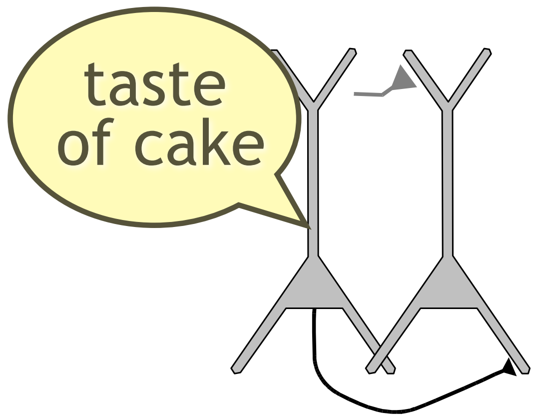

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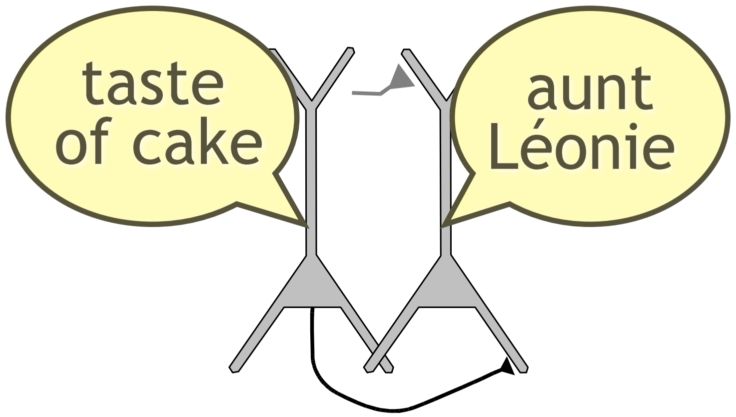

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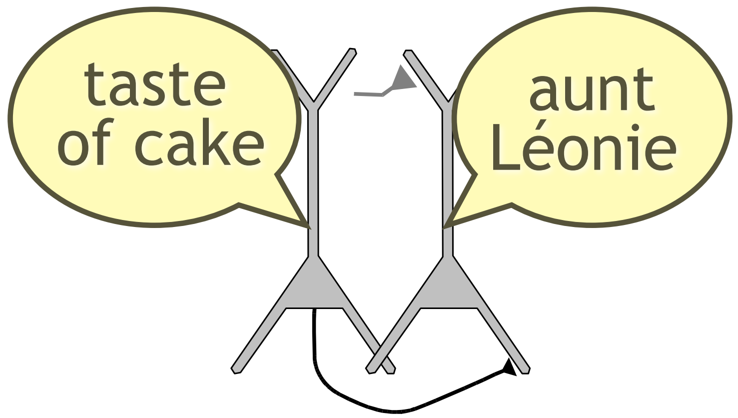
storage



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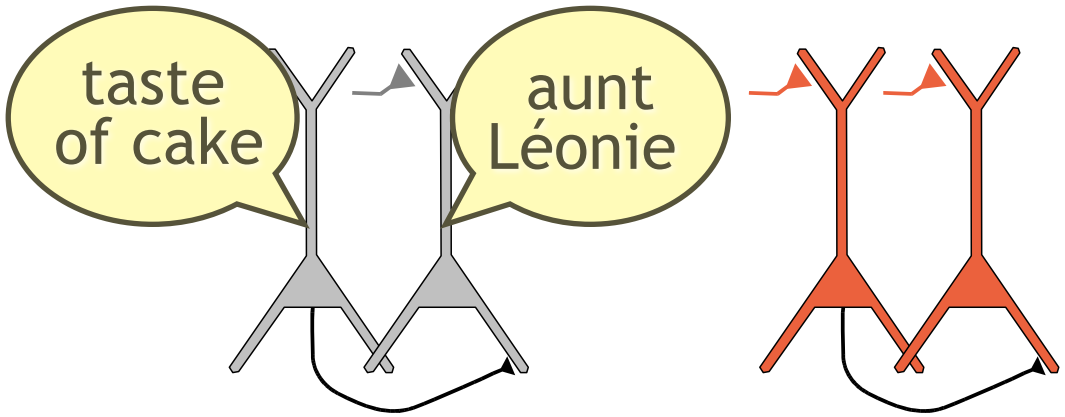

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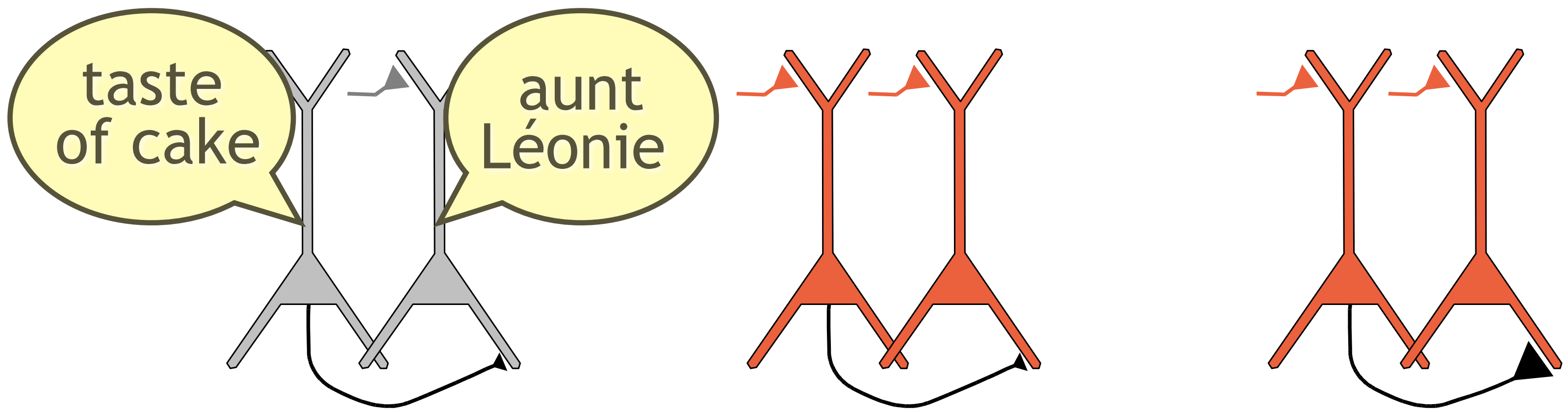

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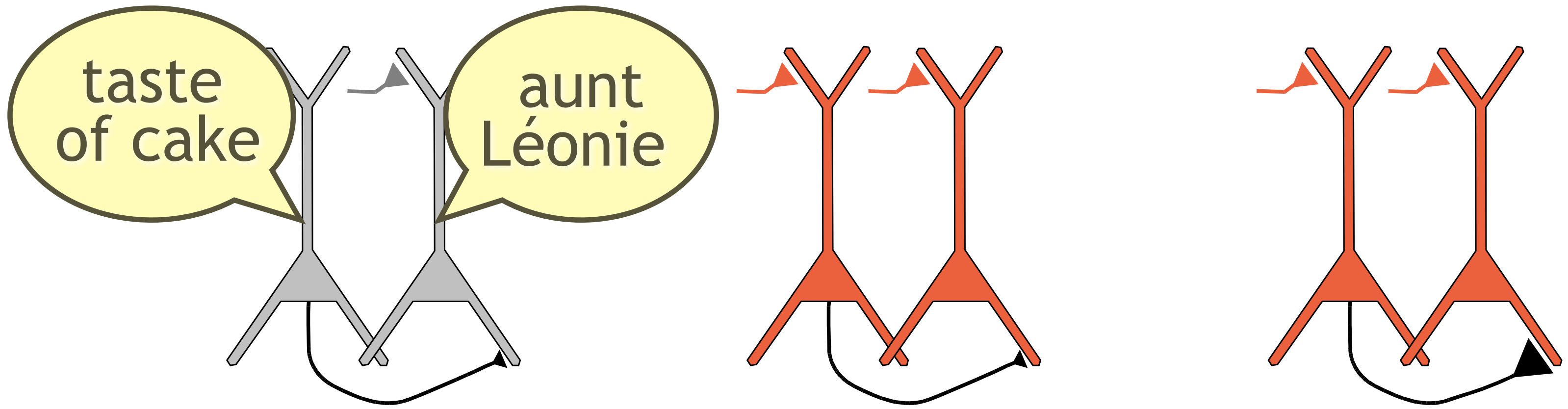

recall

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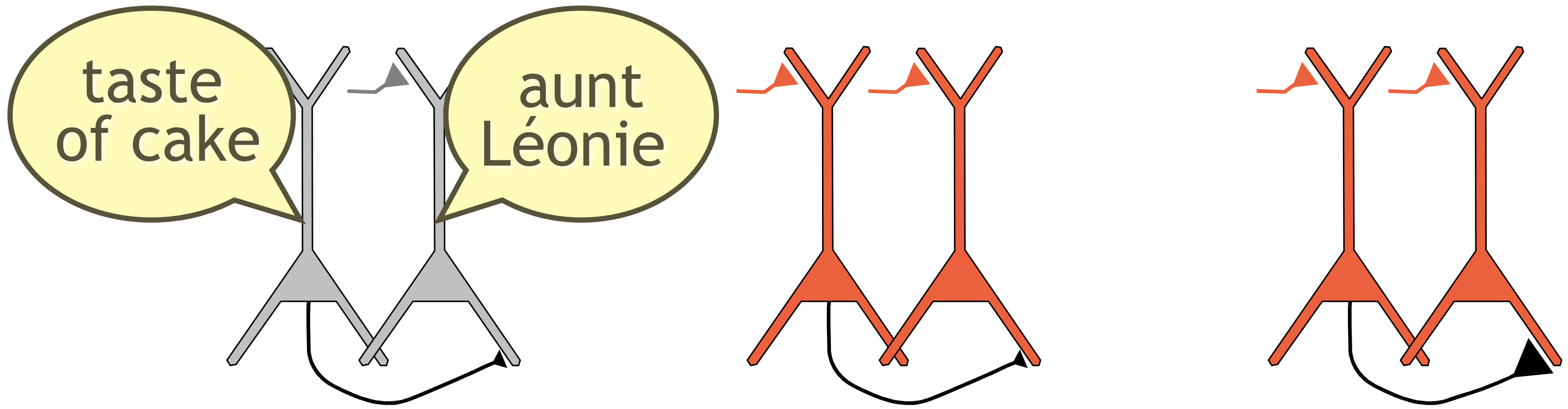

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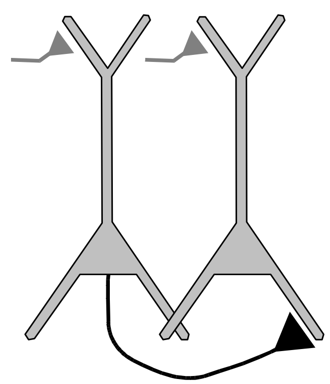

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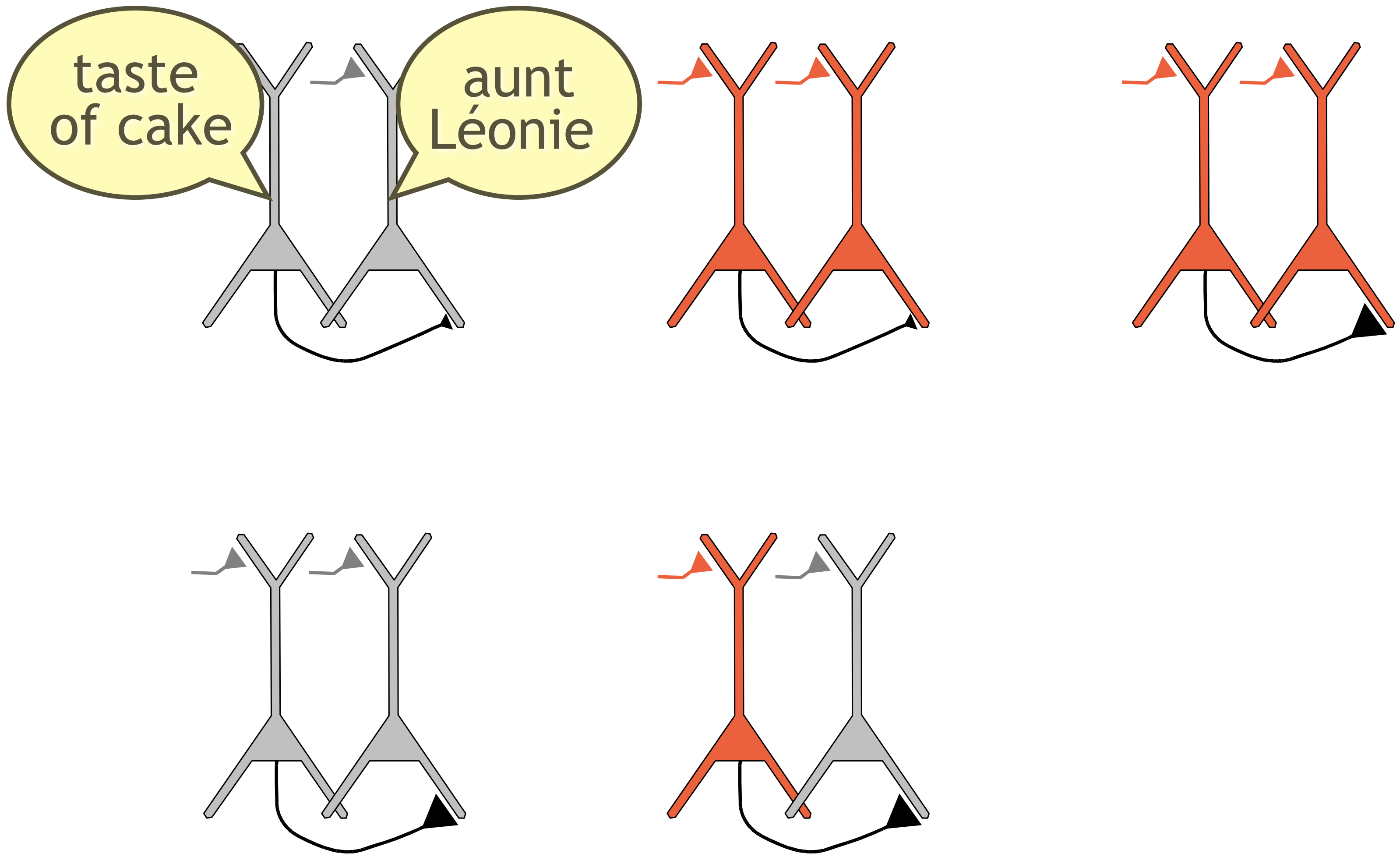

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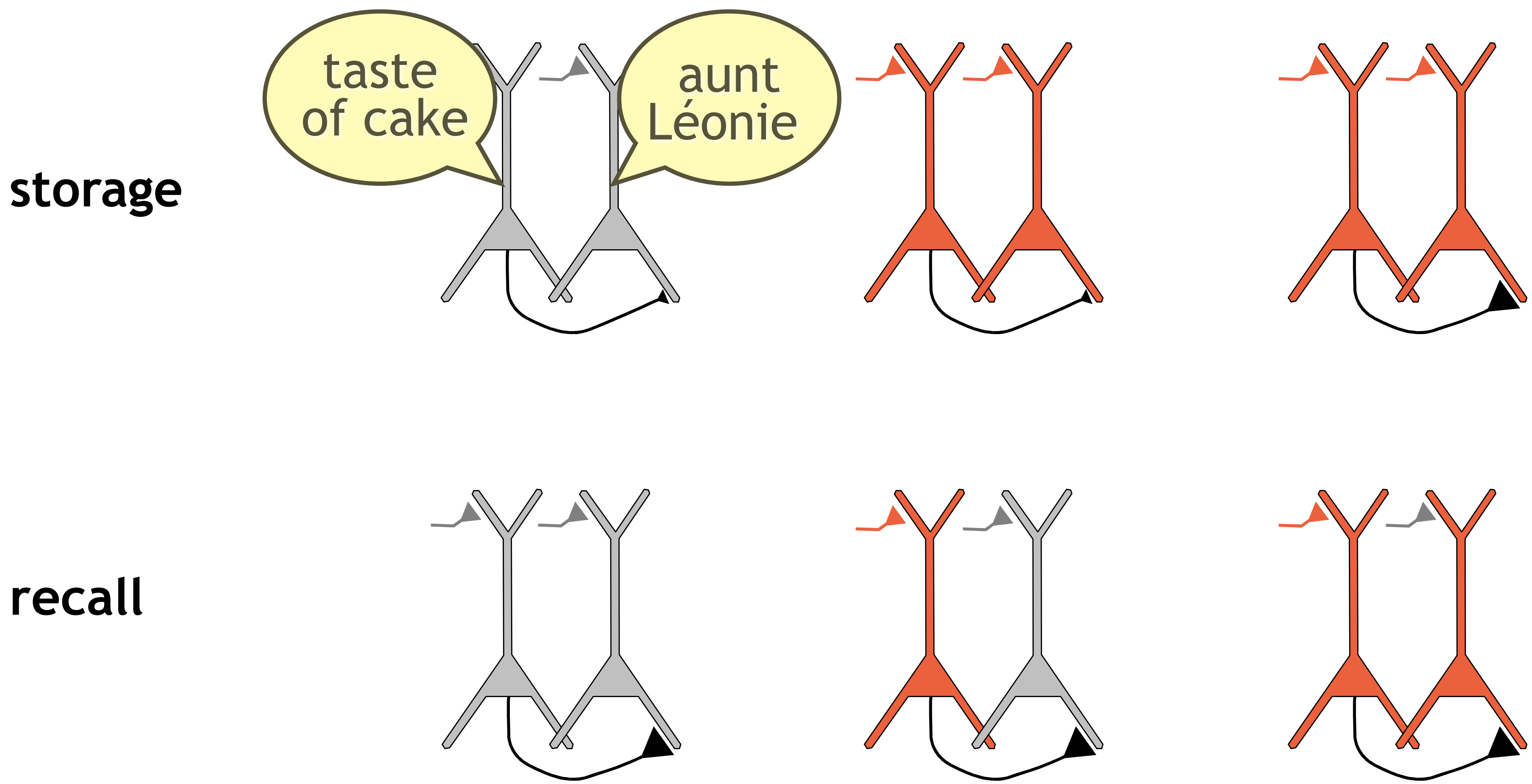

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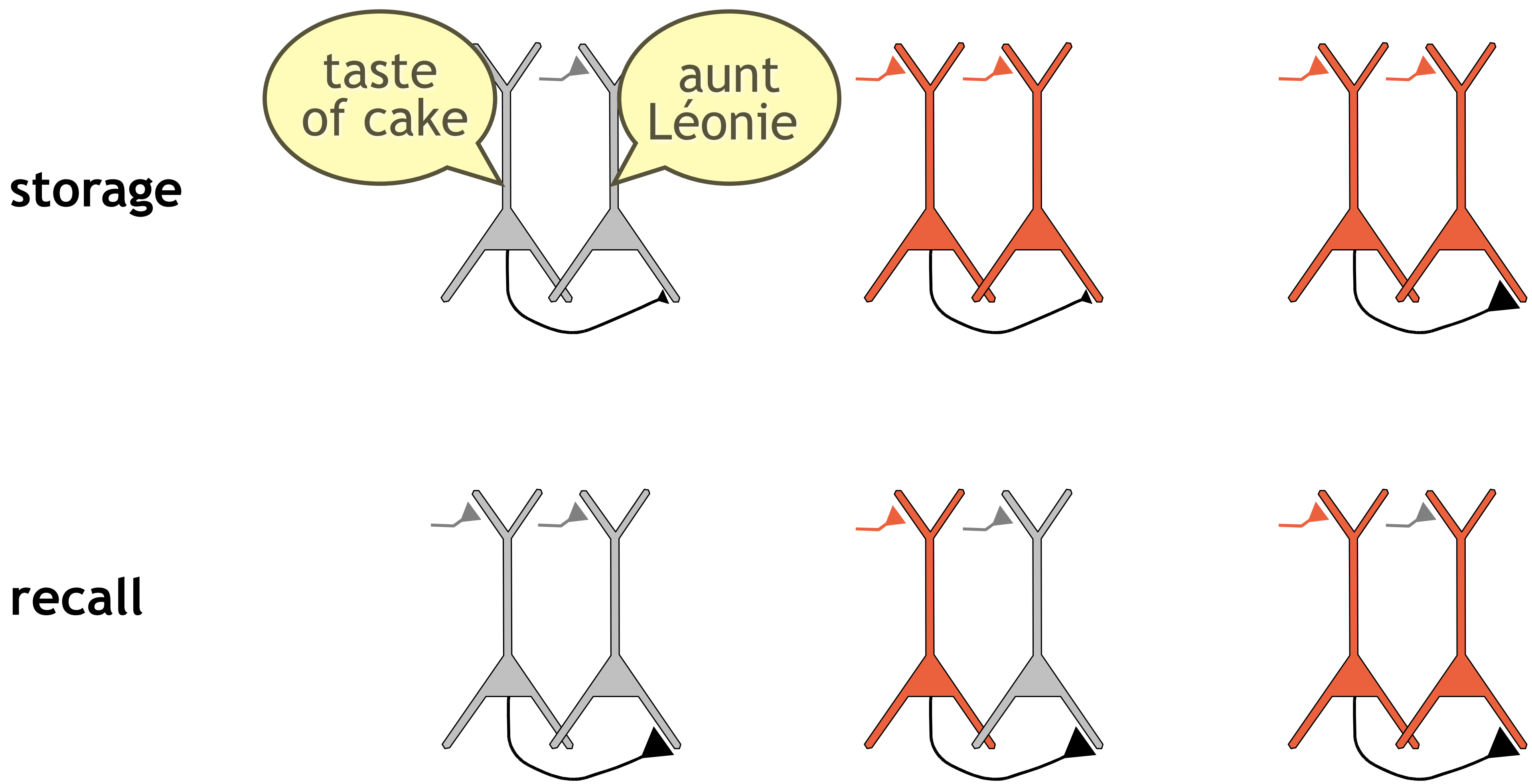

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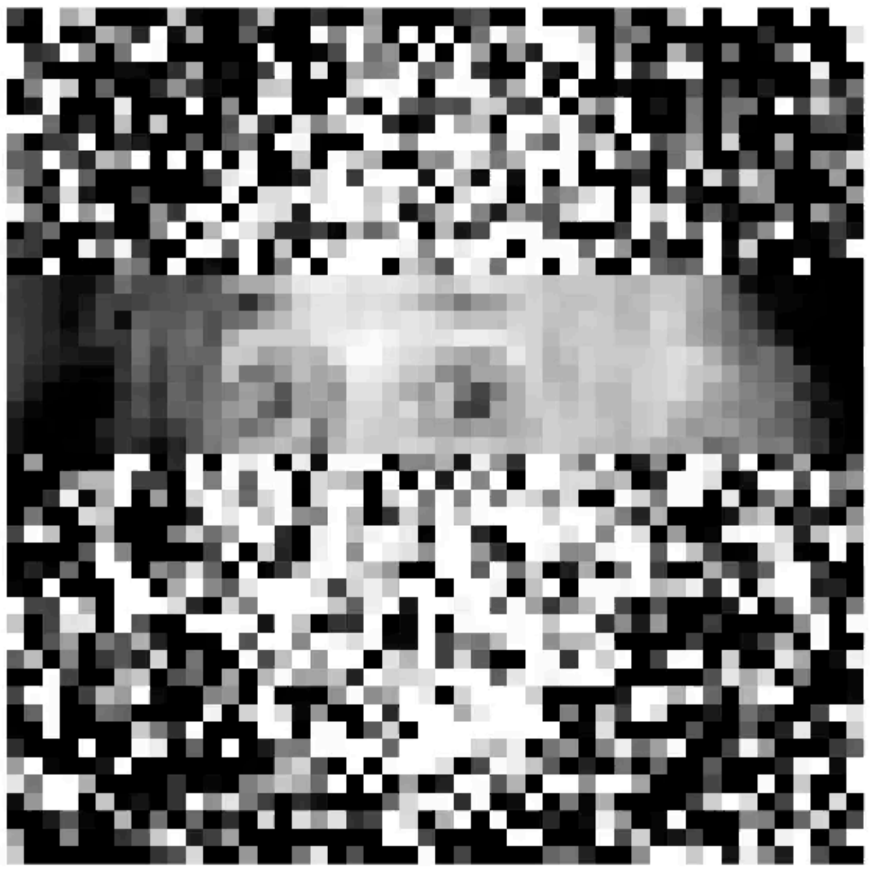

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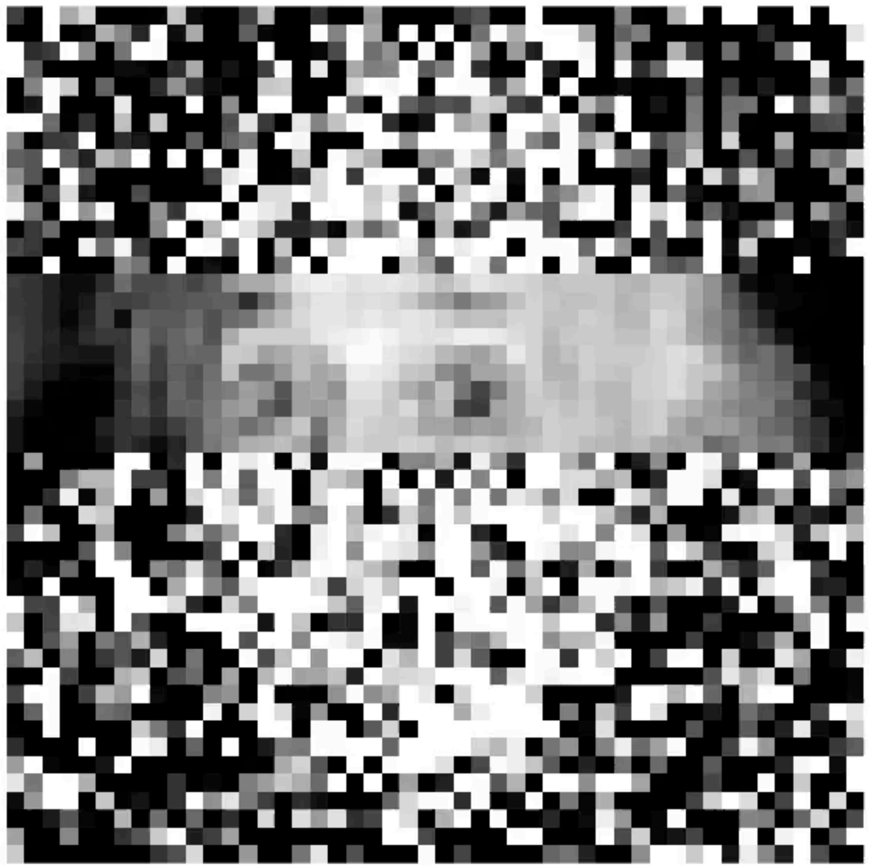

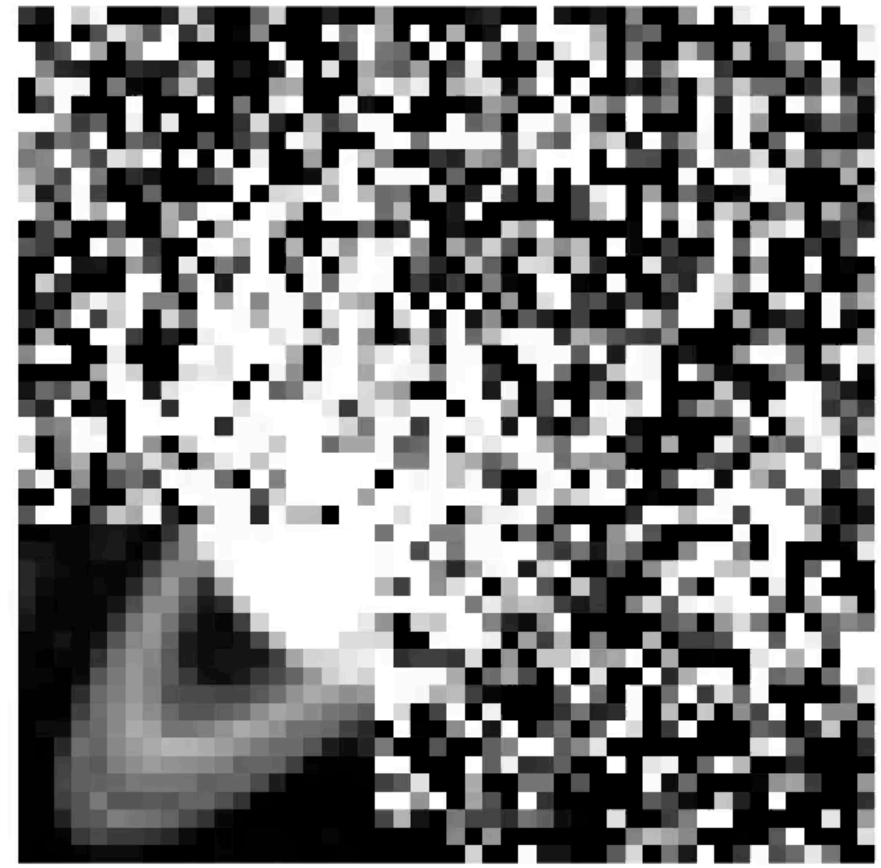

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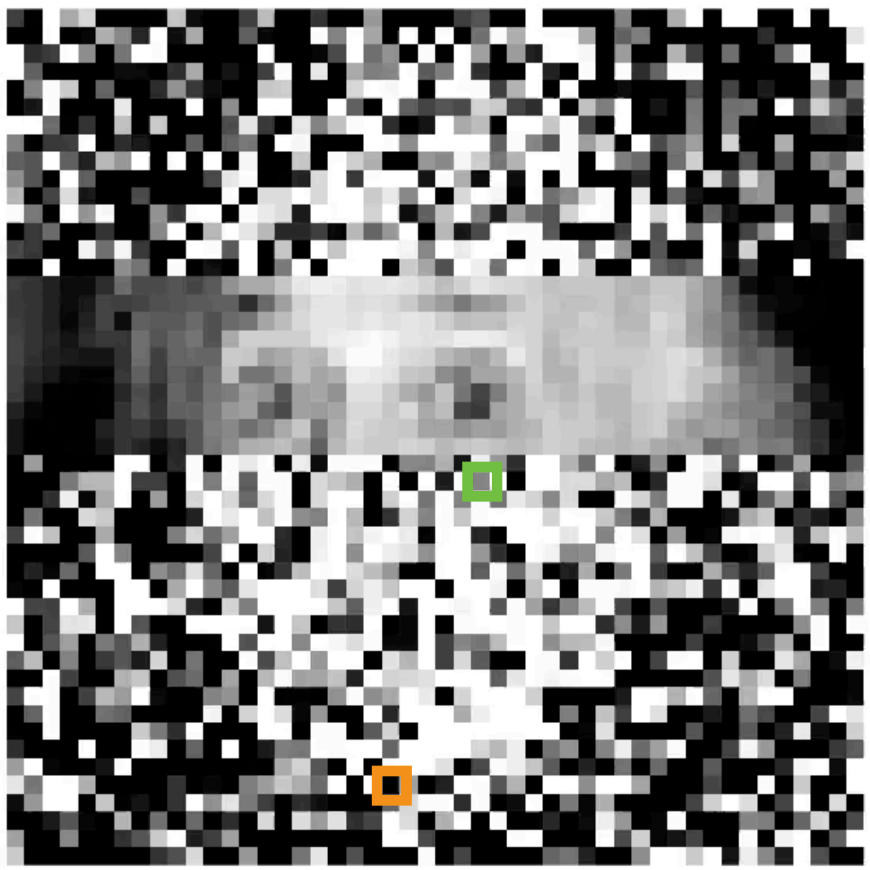

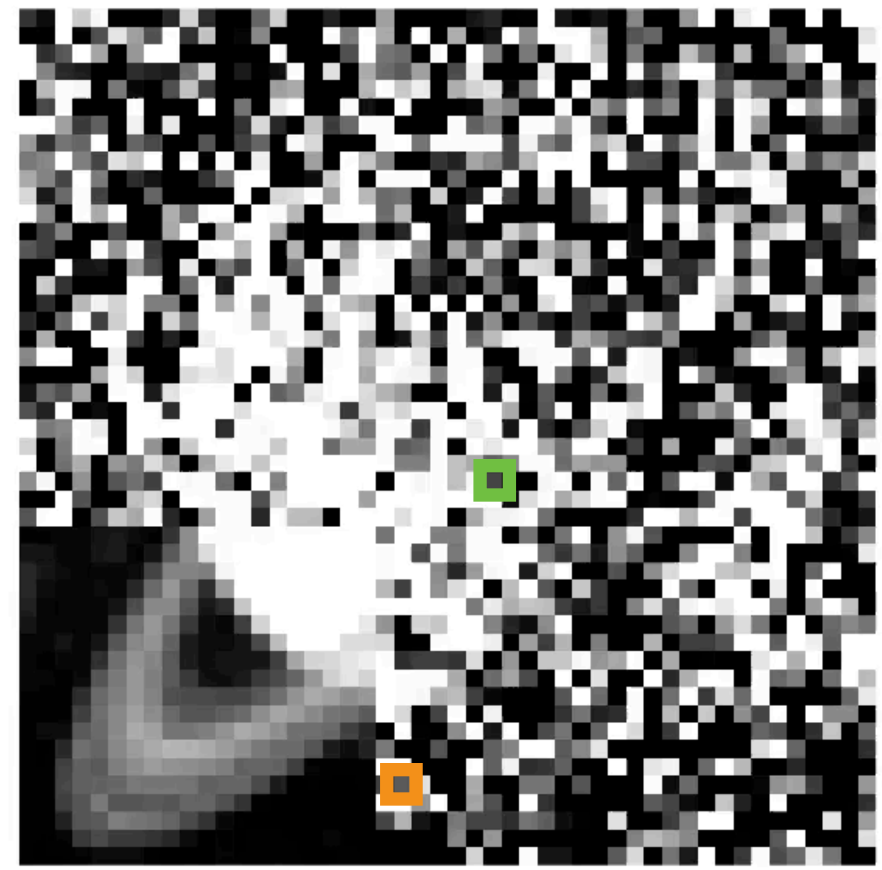

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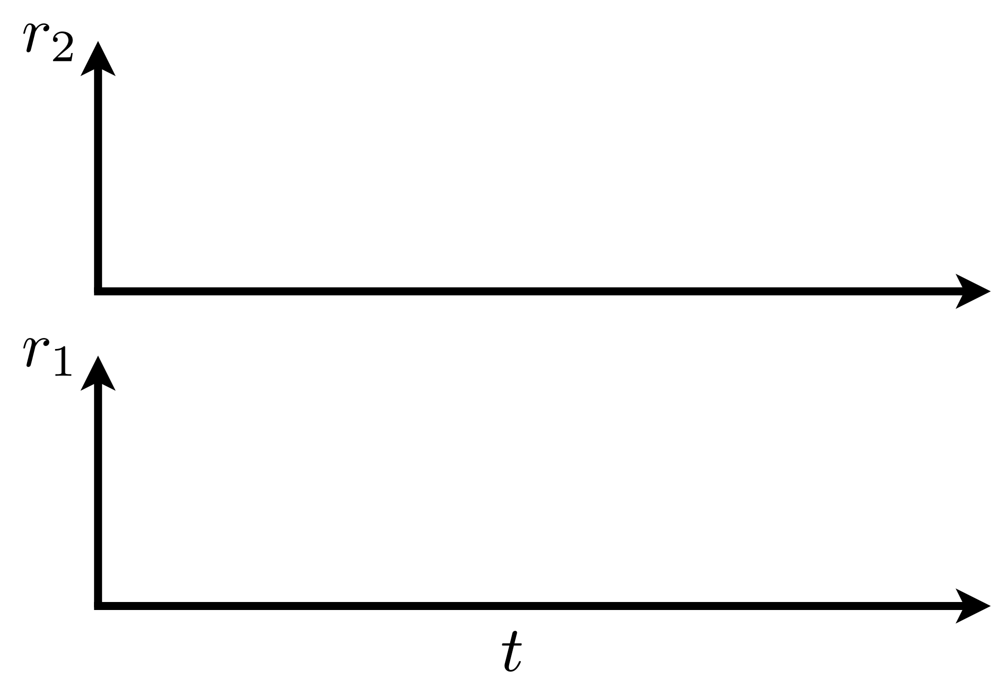

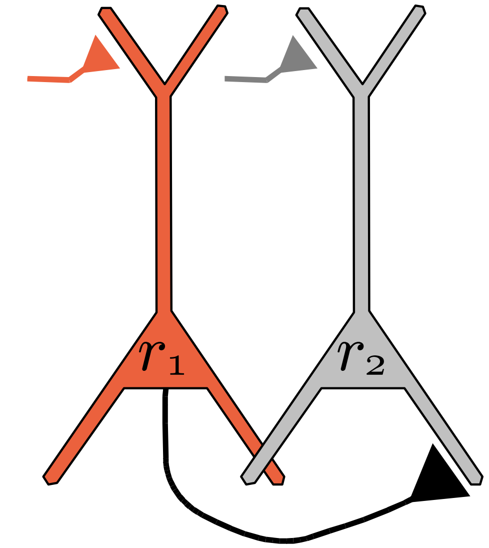

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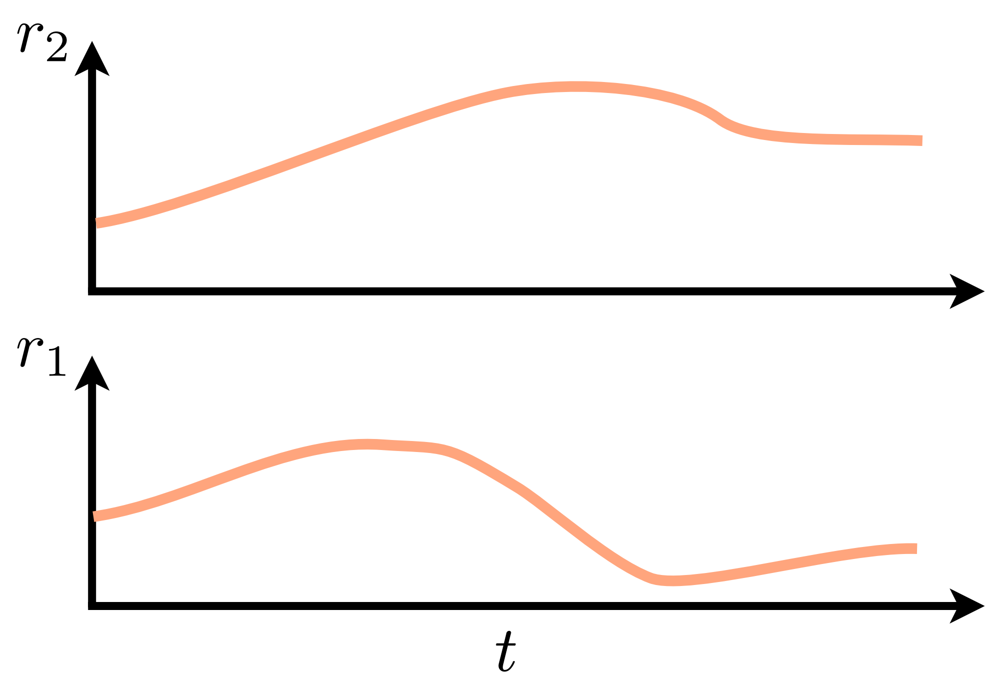

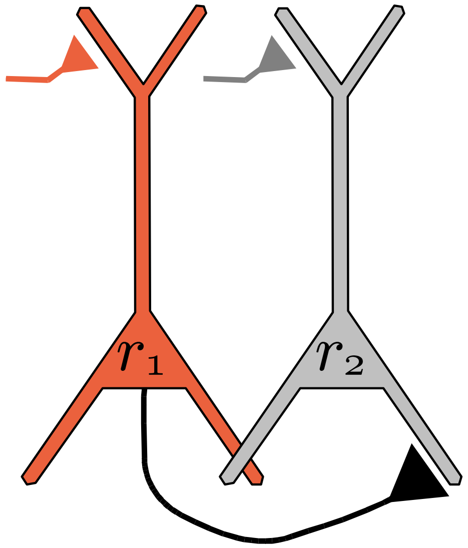

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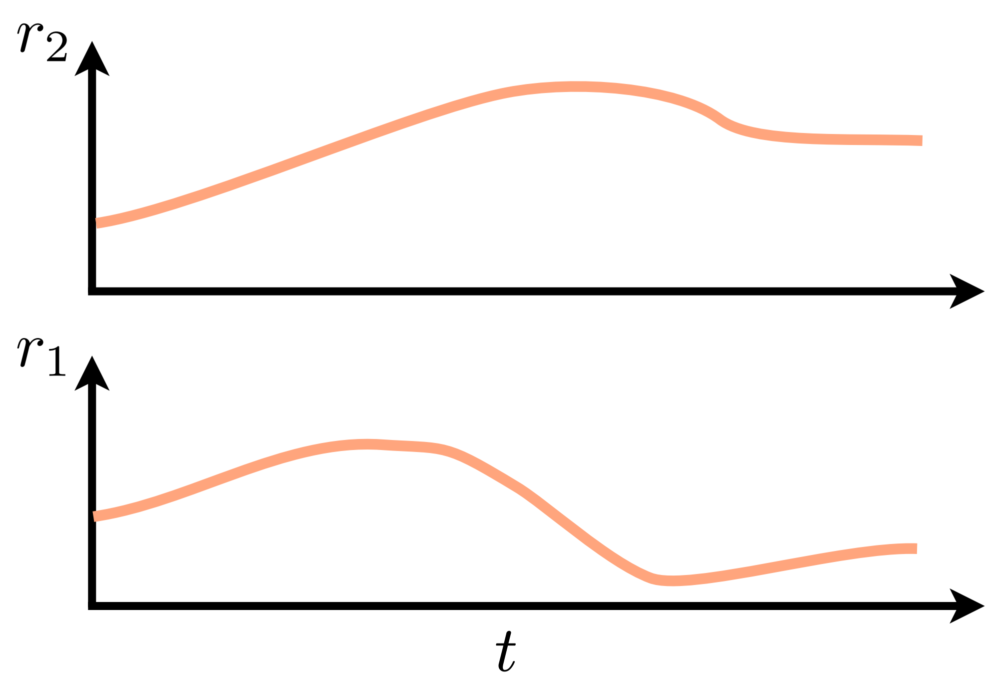

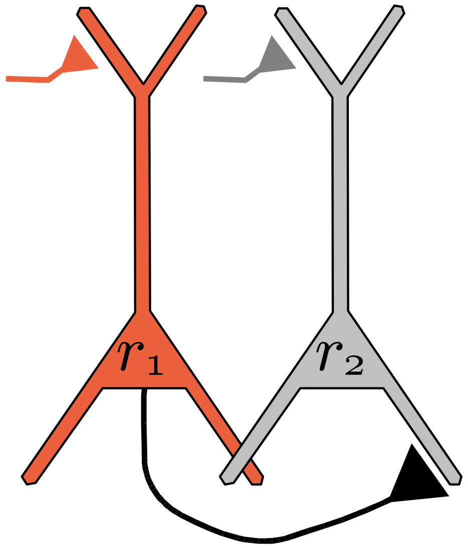

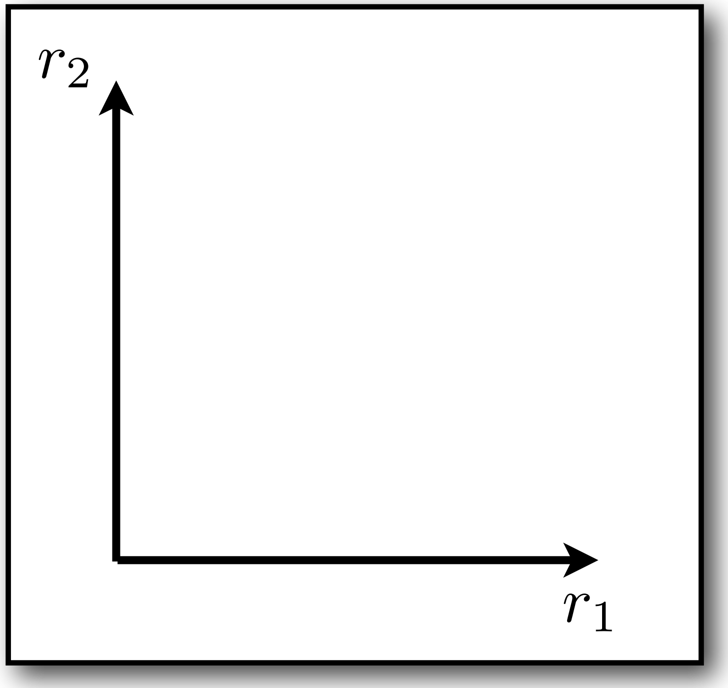

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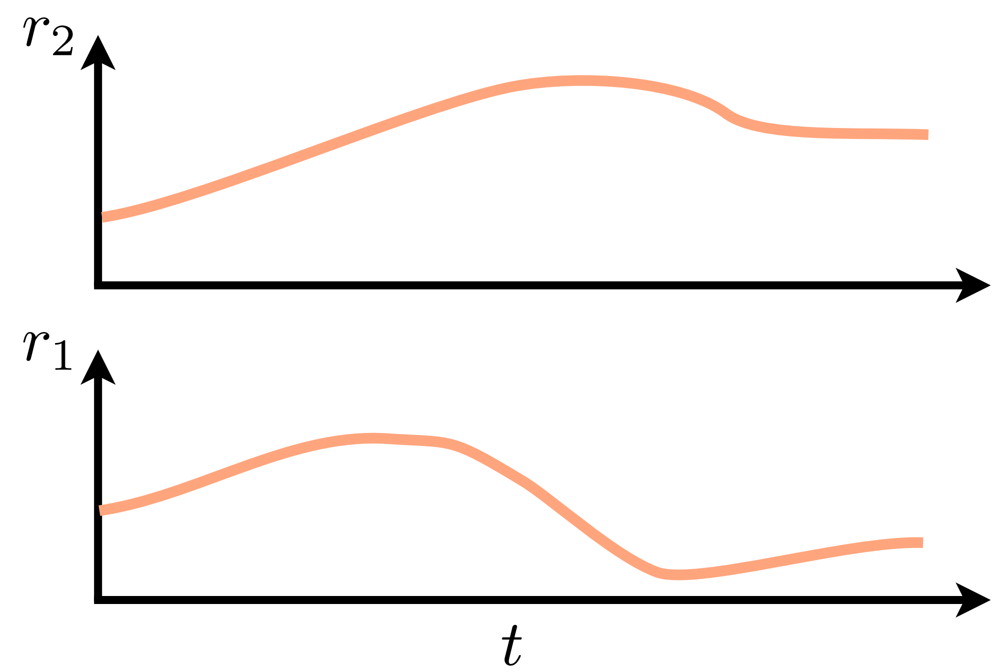

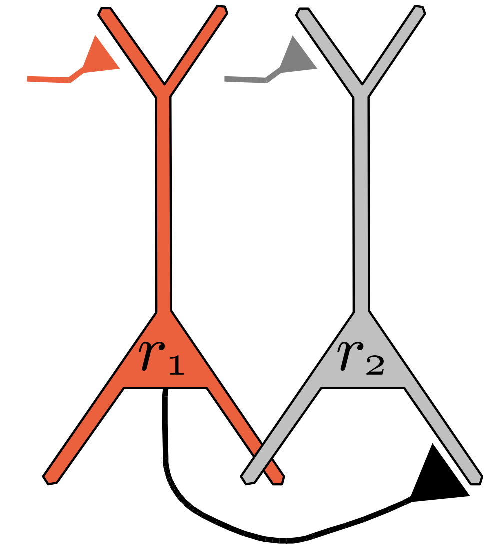

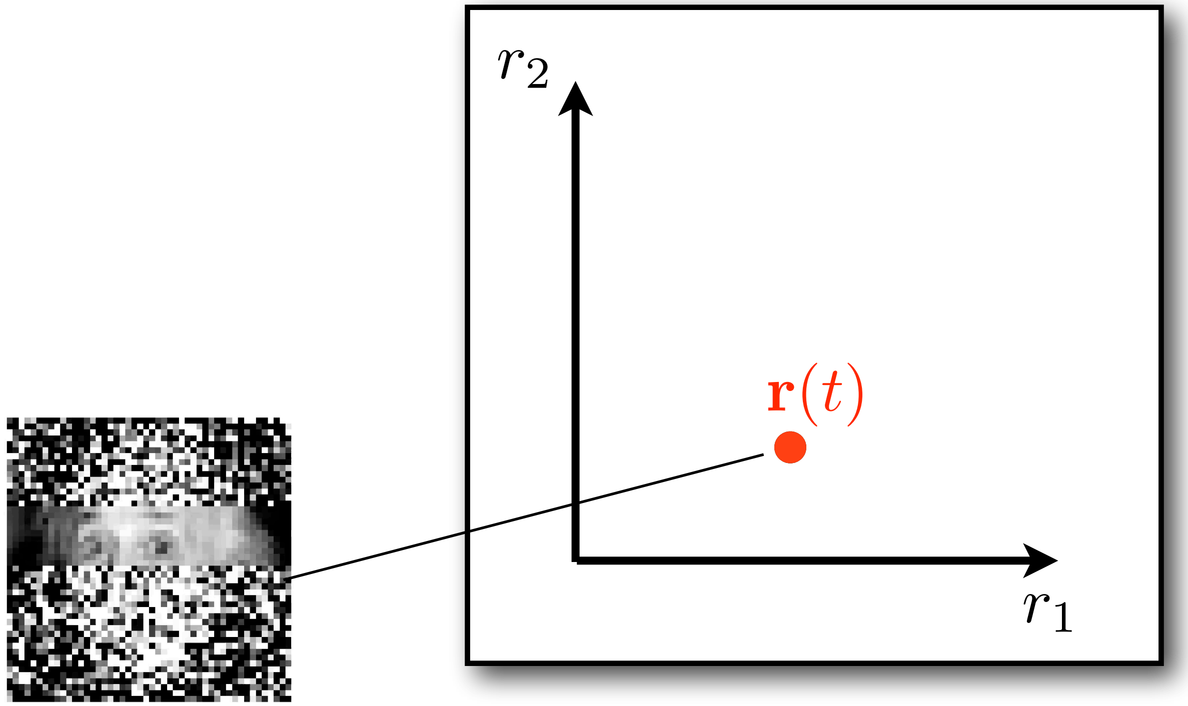

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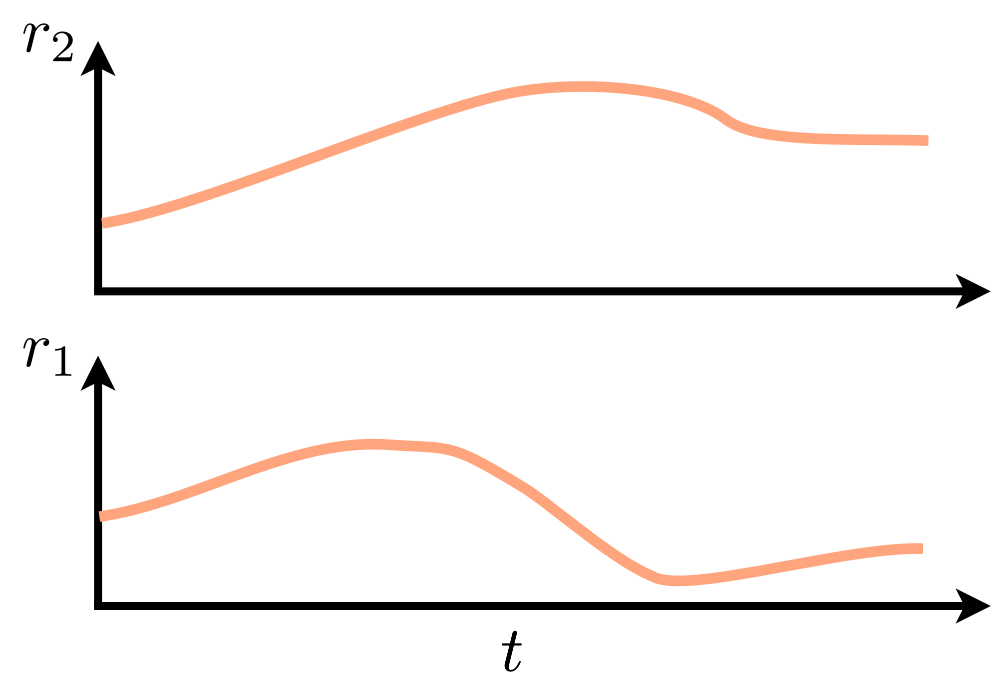

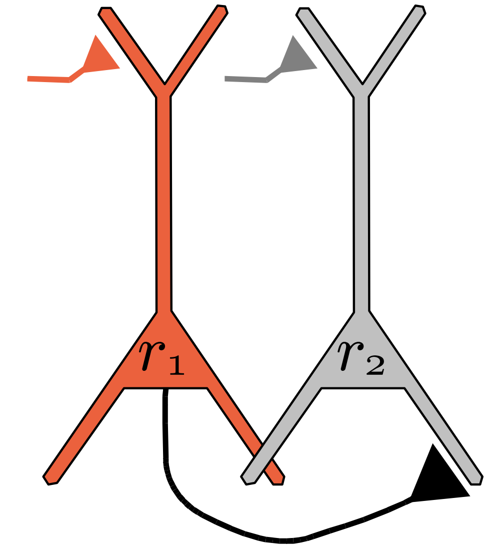

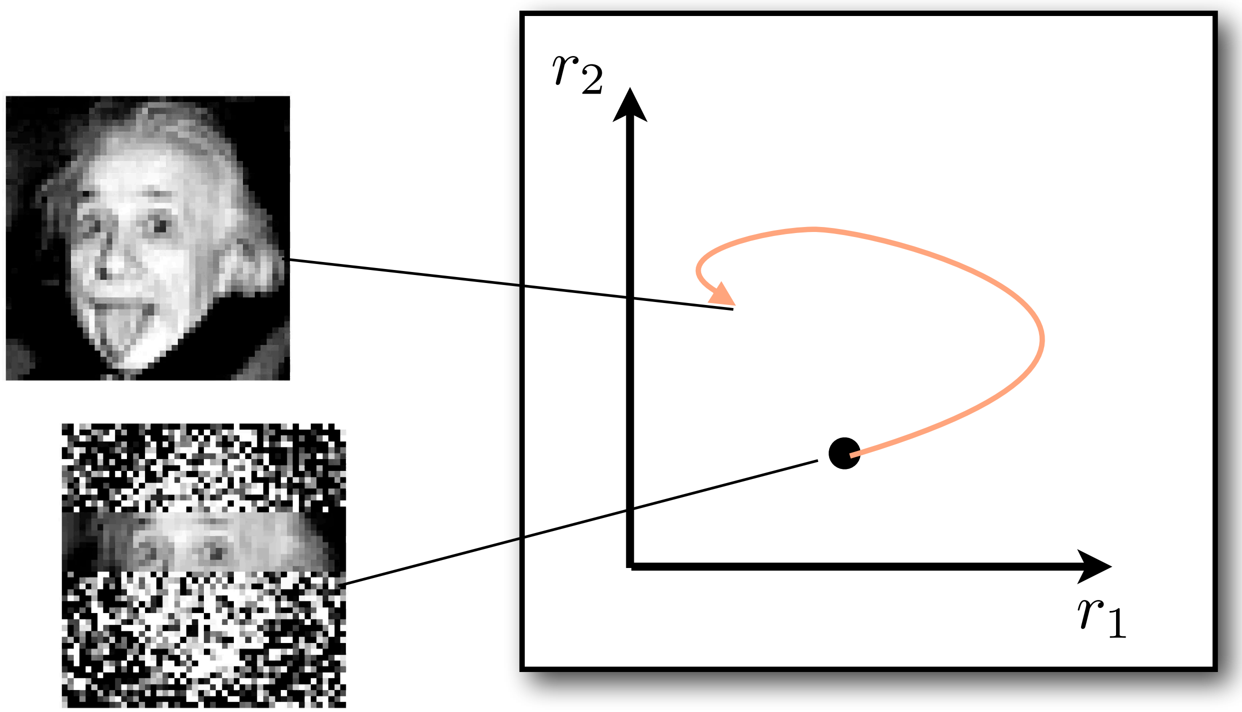

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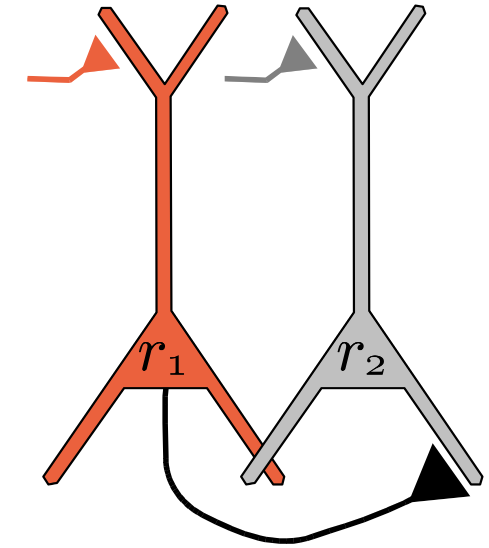

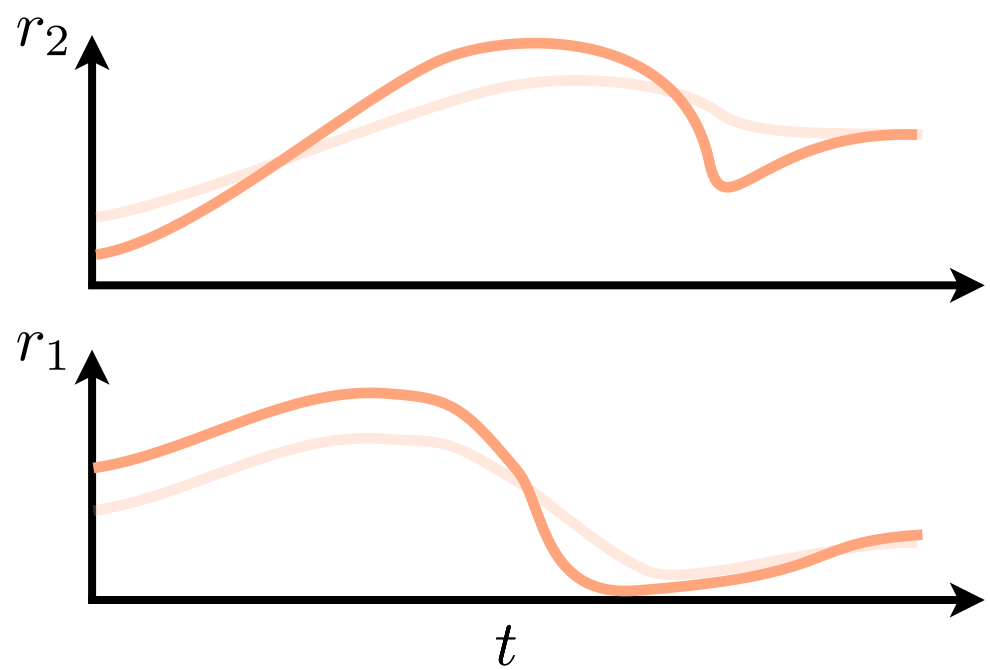

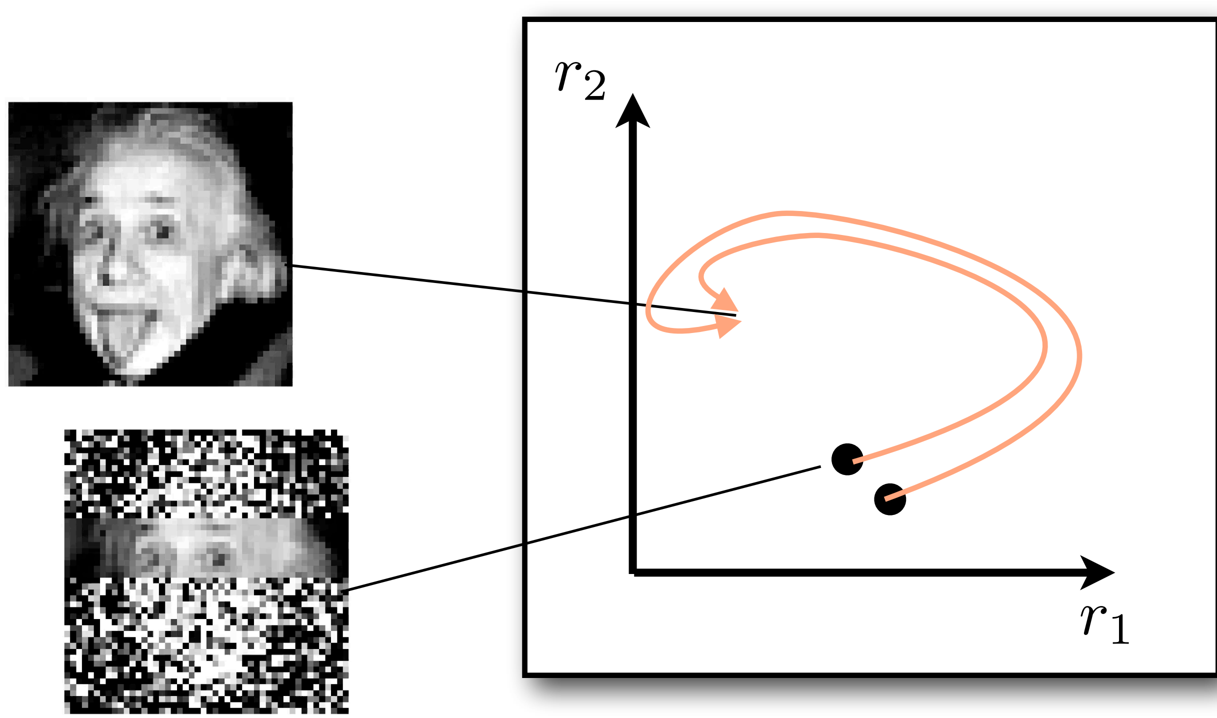

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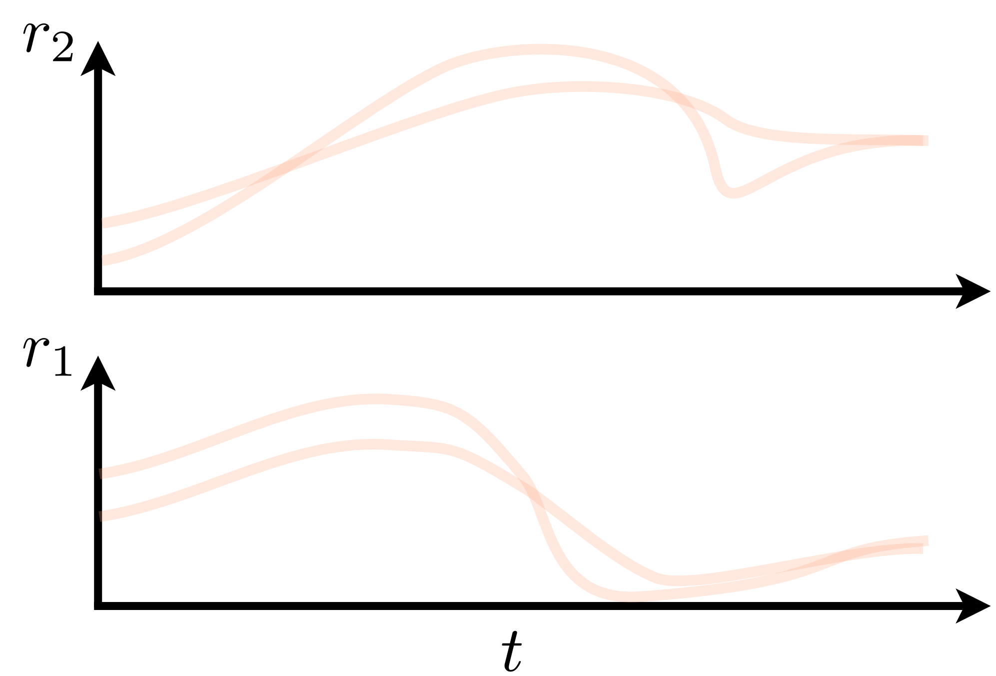

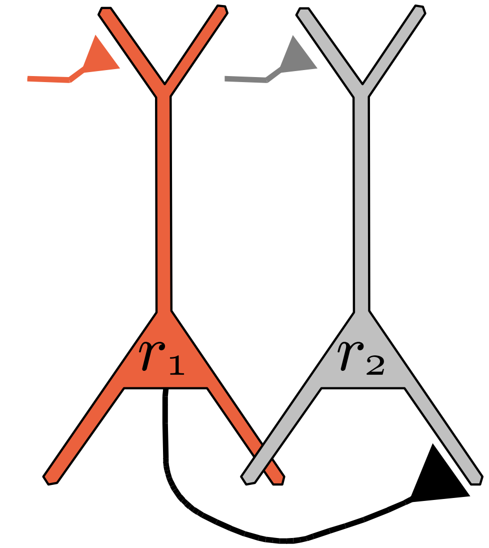

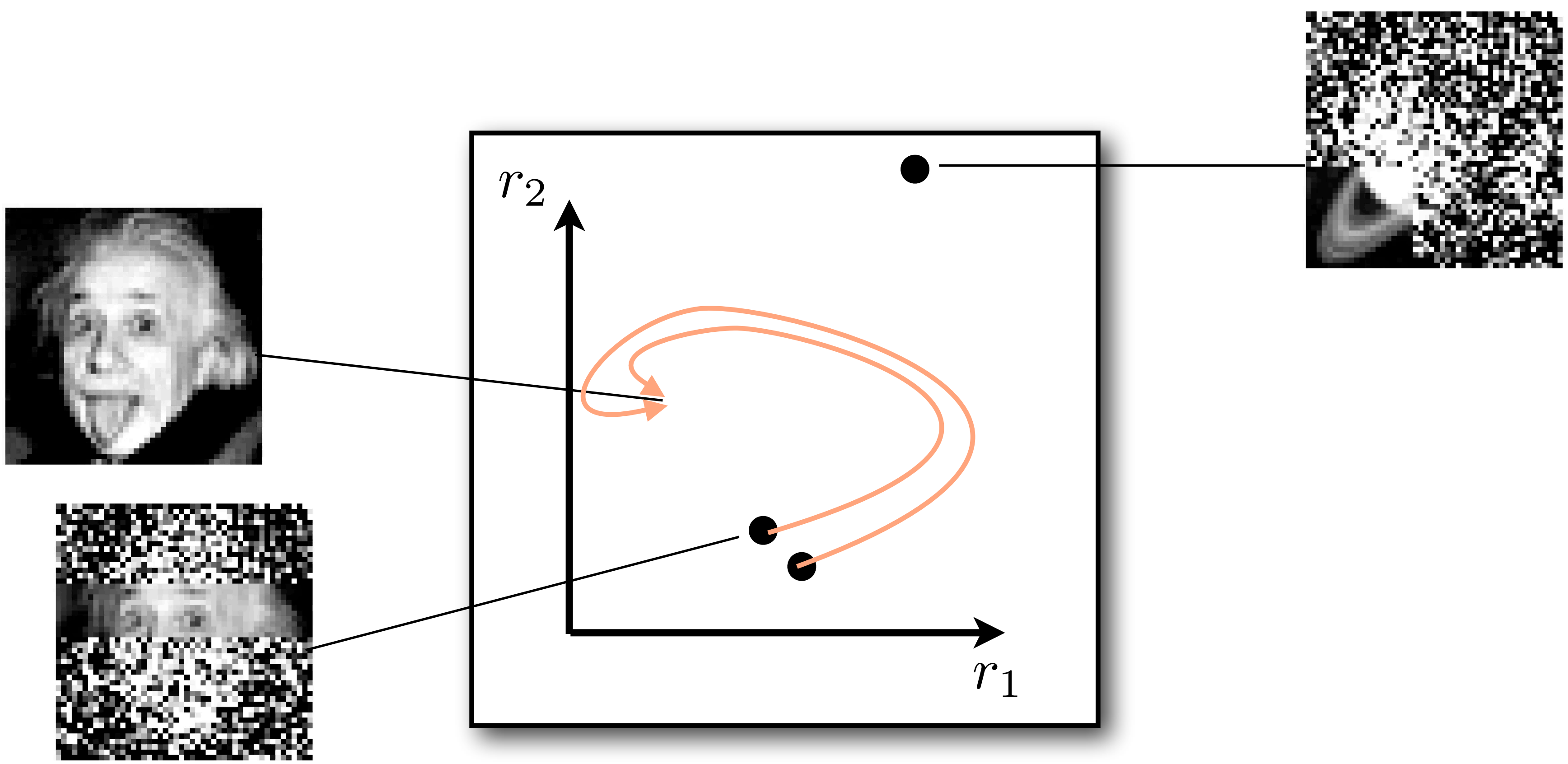

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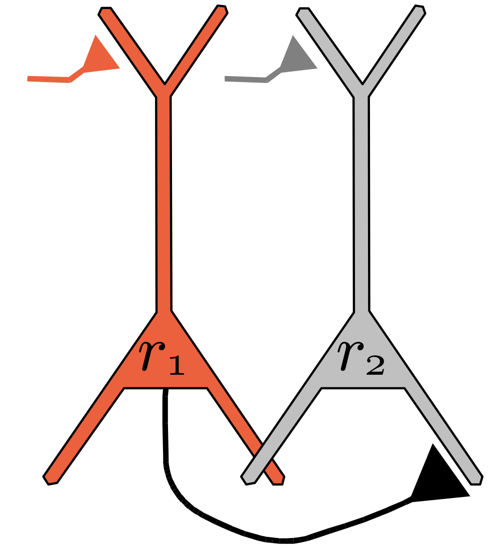

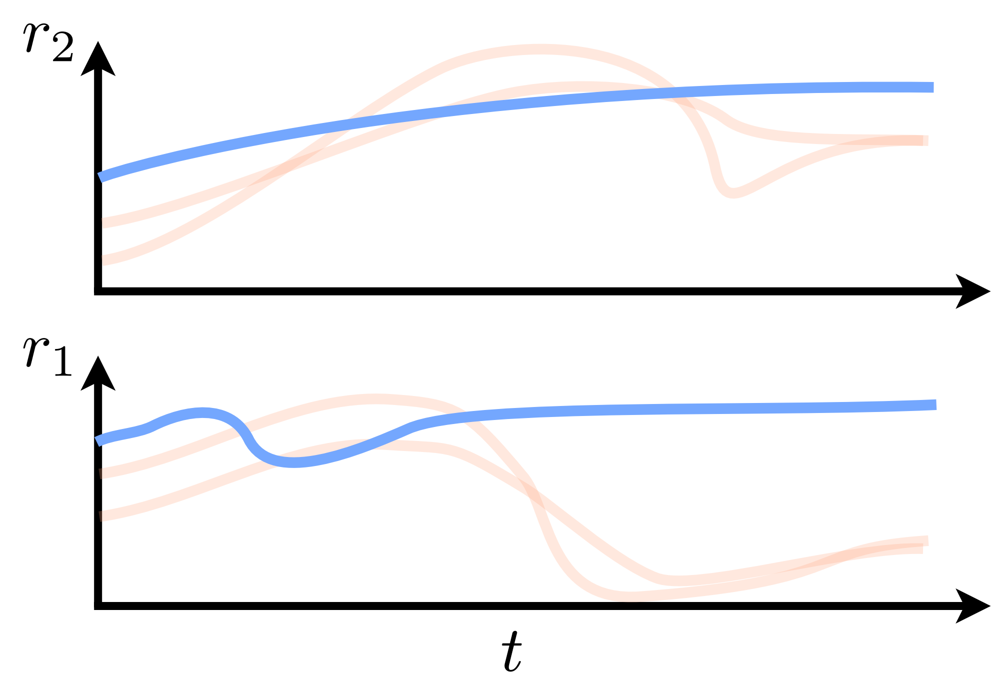

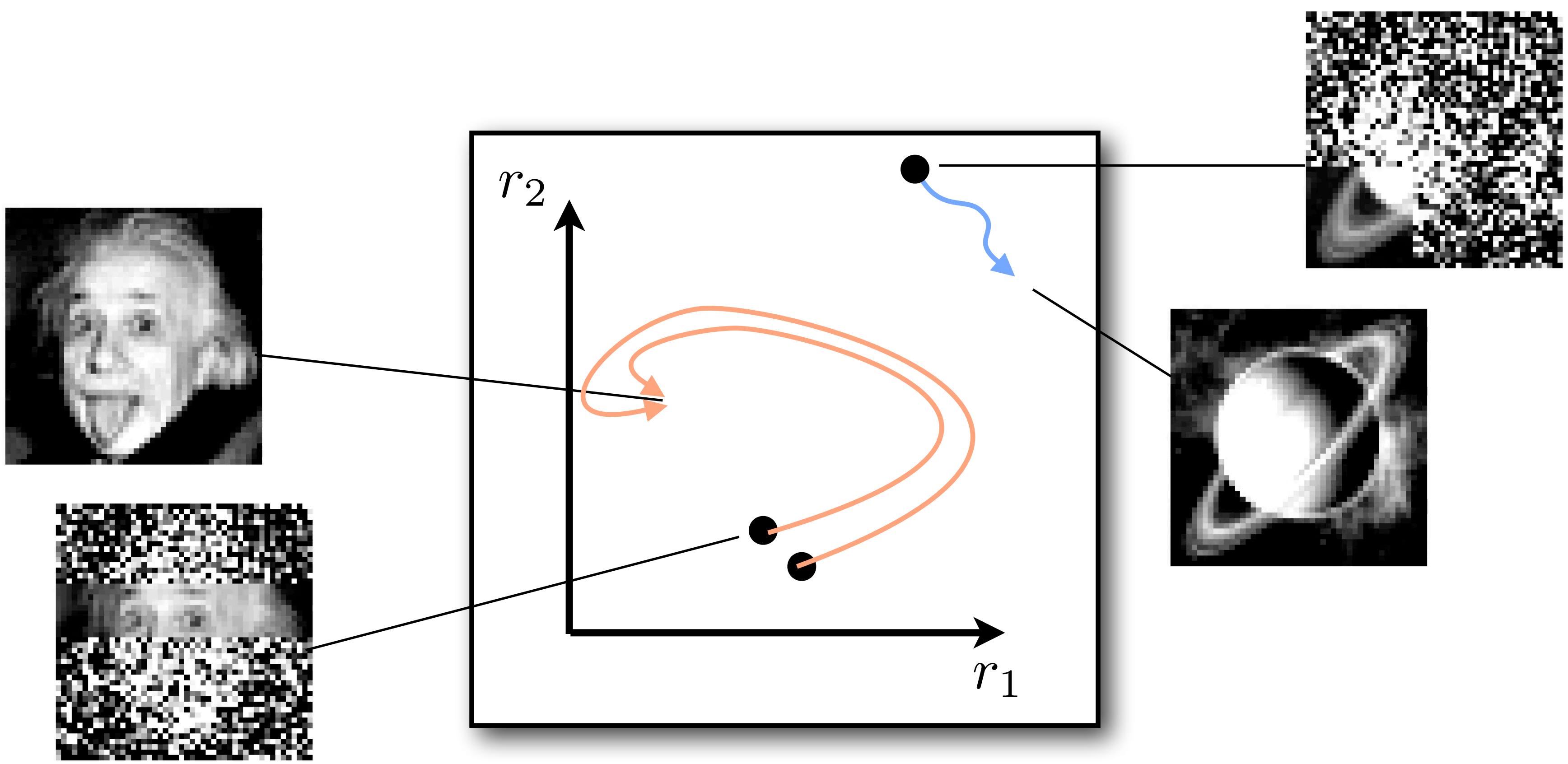

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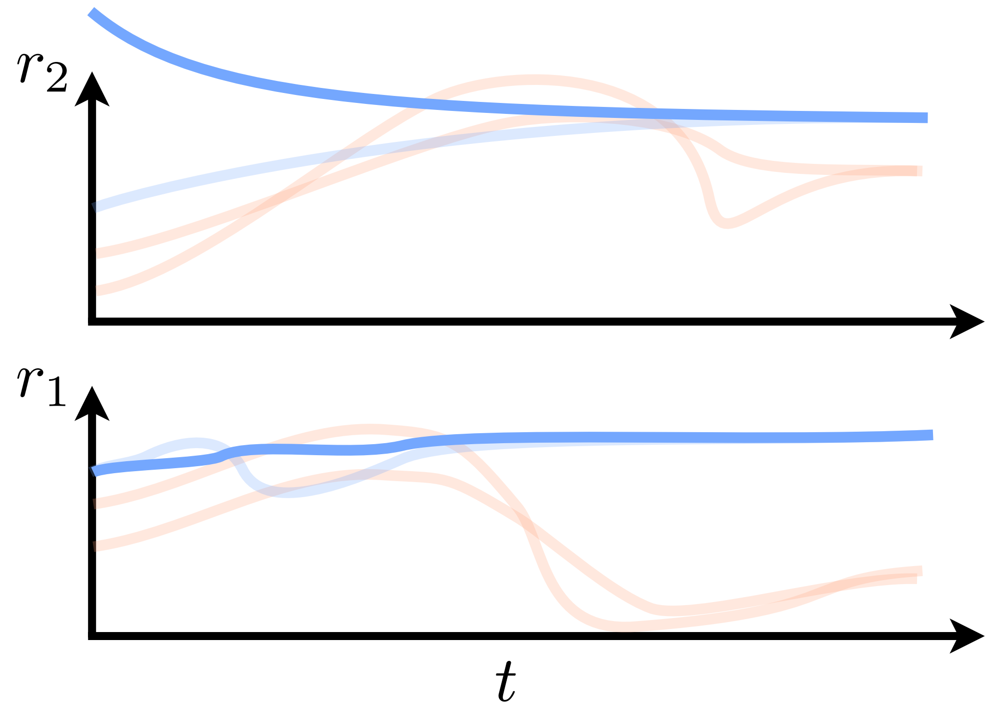

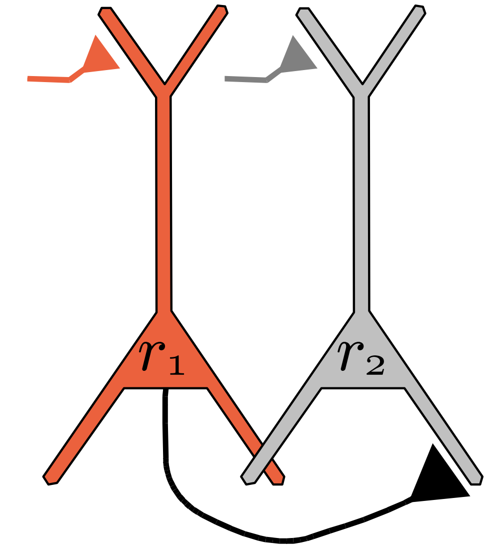

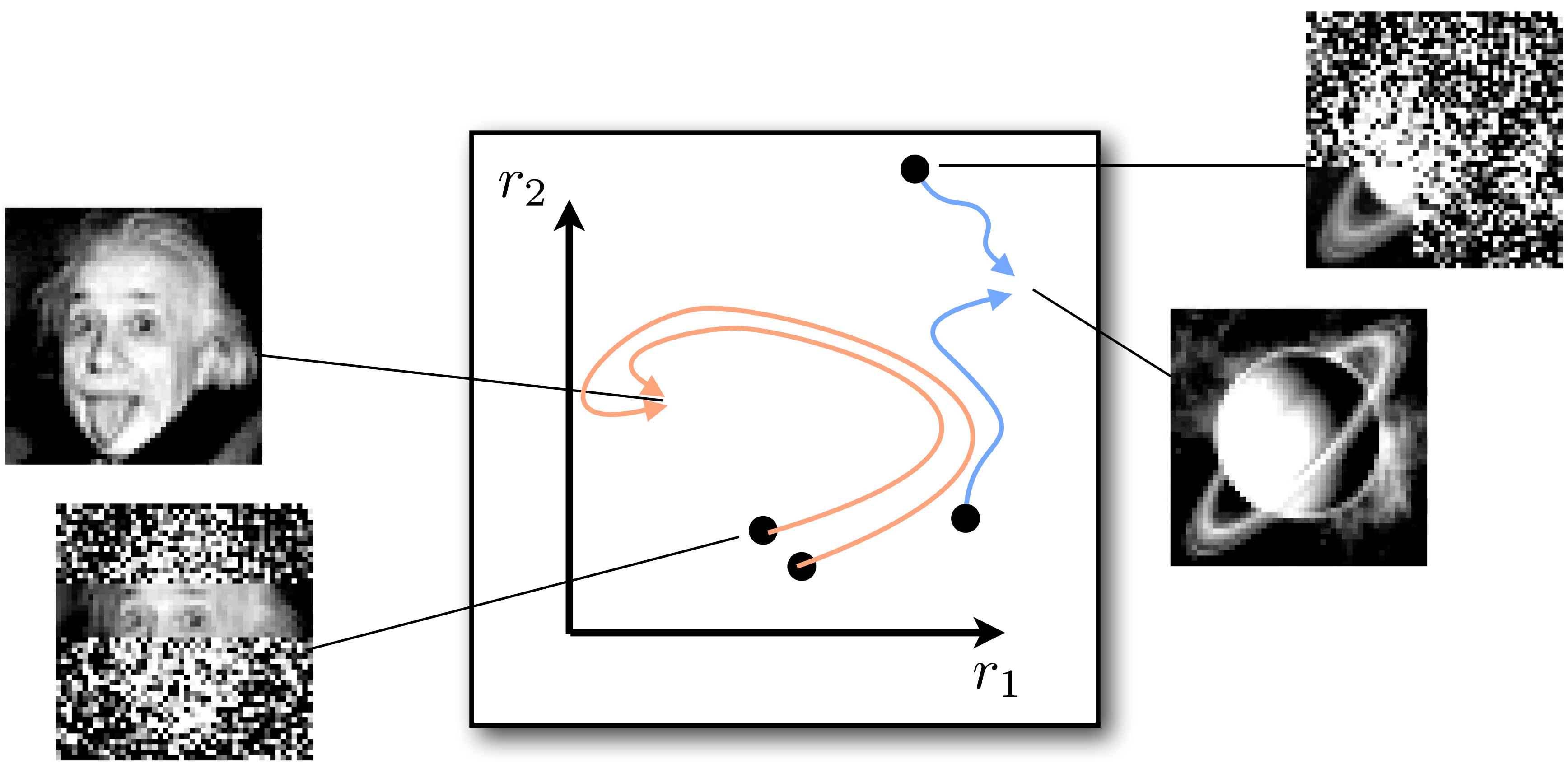

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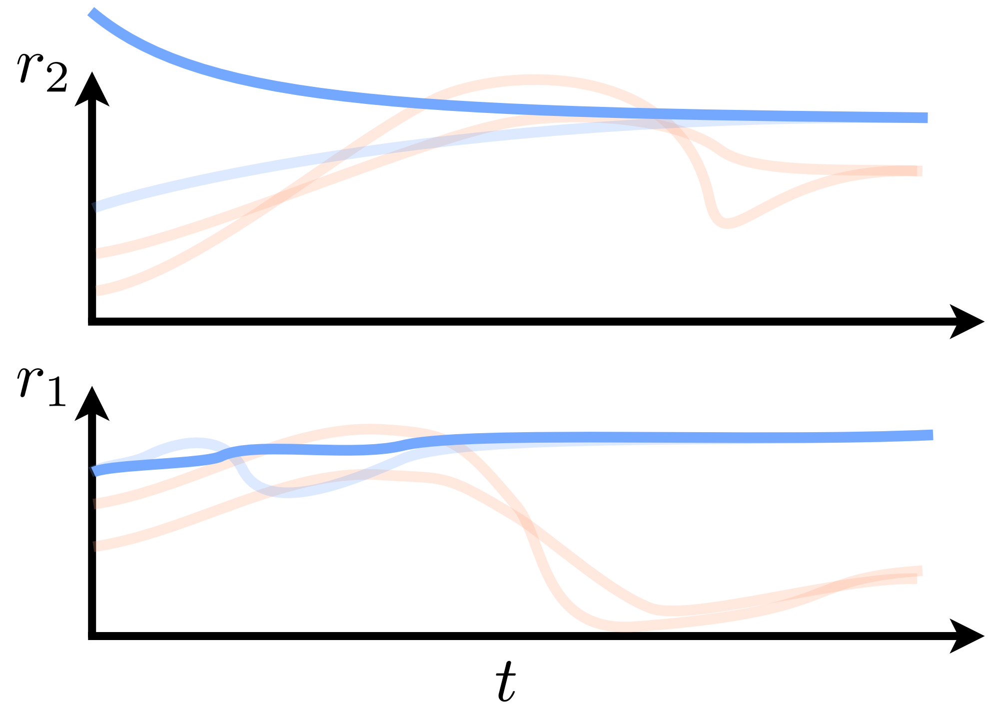

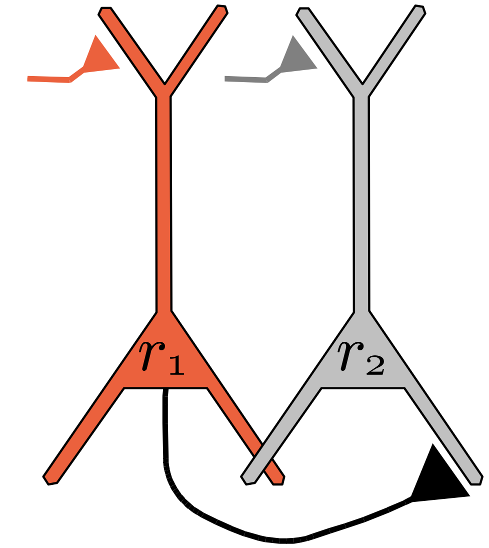

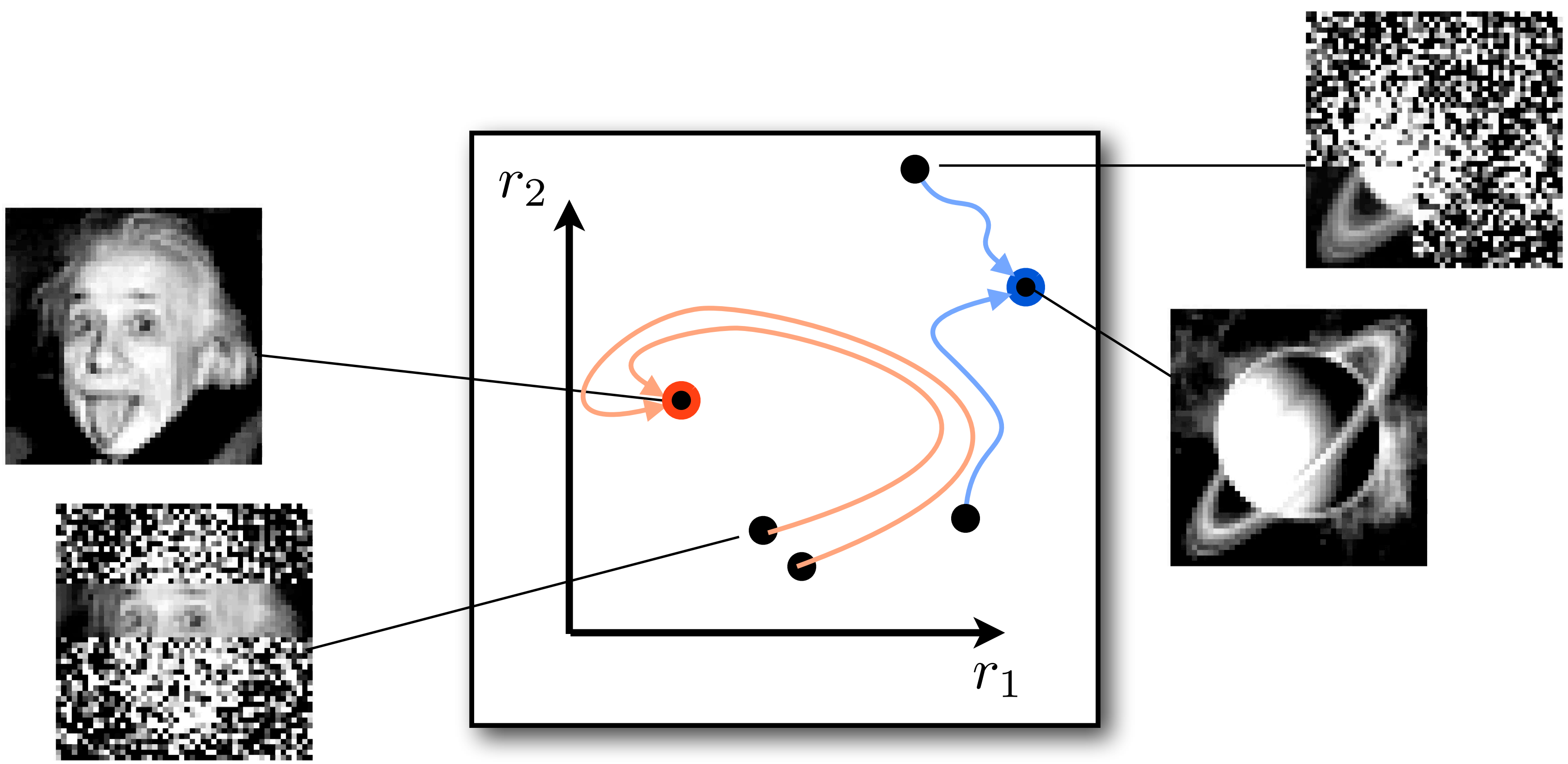

stored patterns should be (point) attractors

THE HOPFIELD (1982) NETWORK

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binary neurons

McCulloch & Pitts, 1943. A logical calculus of the ideas immanent in nervous activity.

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$$r_i(t) \in \{0, 1\} \quad \forall i$$

$$r_i(t + \Delta t) = F\left(\sum_{j=1}^{N} W_{ij}\, r_j(t)\right)$$, asynchronous update

$$F(I) = \begin{cases} 1 & \text{if } I \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

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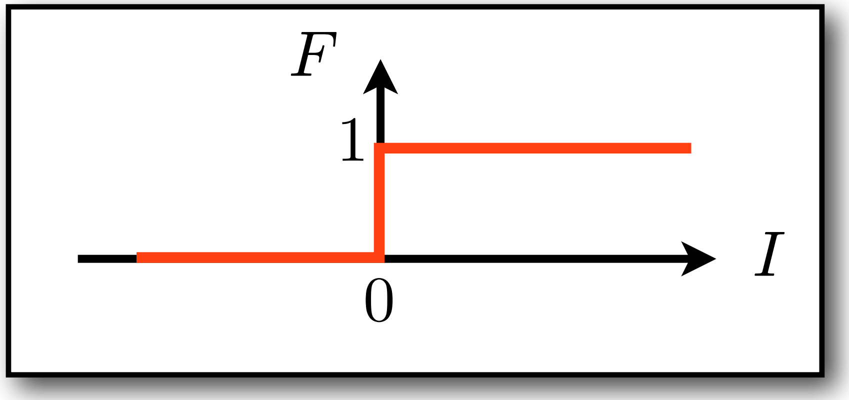

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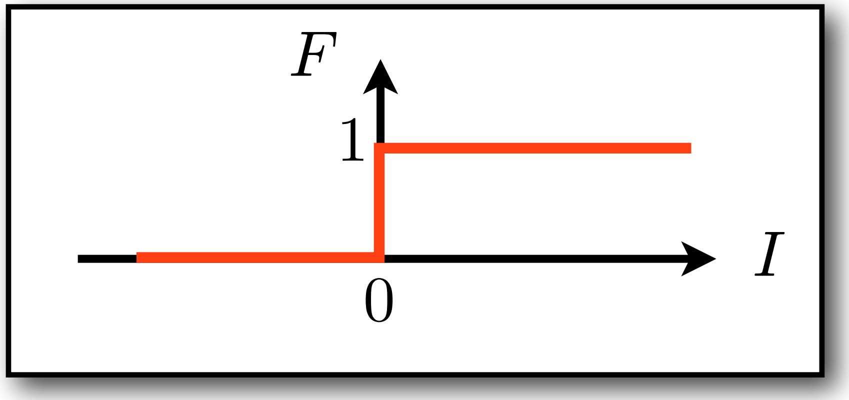

Hebbian synaptic plasticity

Hebb, 1949. The Organization of Behavior.

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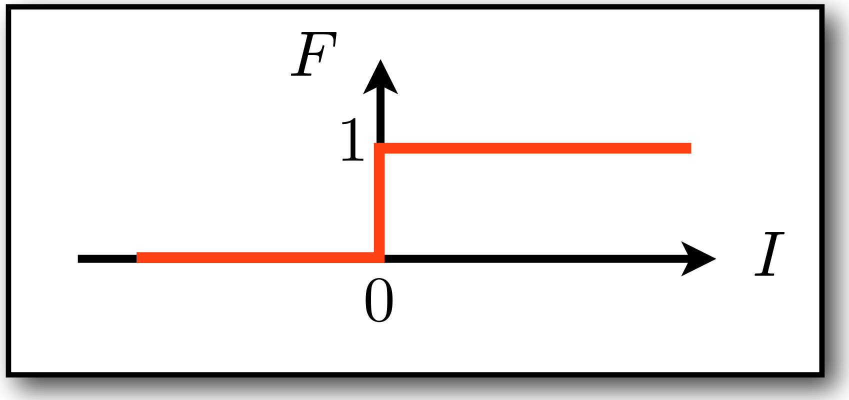

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uncorrelated balanced patterns:
$$r_i^{(m)} \perp r_{j\neq i}^{(m)} \perp r_j^{(m' \neq m)}$$
$$\mathrm{P}\left(r_i^{(m)} = 0\right) = \mathrm{P}\left(r_i^{(m)} = 1\right) = \frac{1}{2}$$

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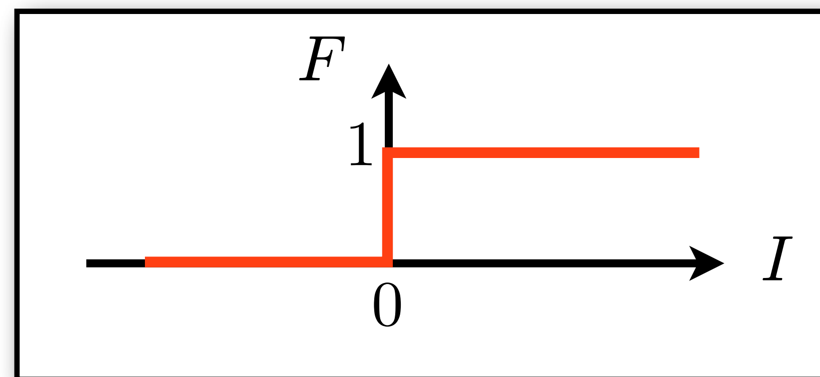

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$$W_{ij} = \sum_{m=1}^{M} \left(r_i^{(m)} - \frac{1}{2}\right)\left(r_j^{(m)} - \frac{1}{2}\right)$$

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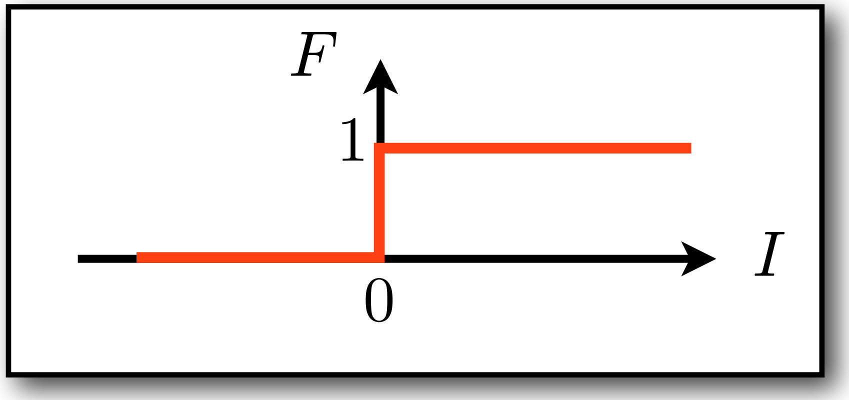

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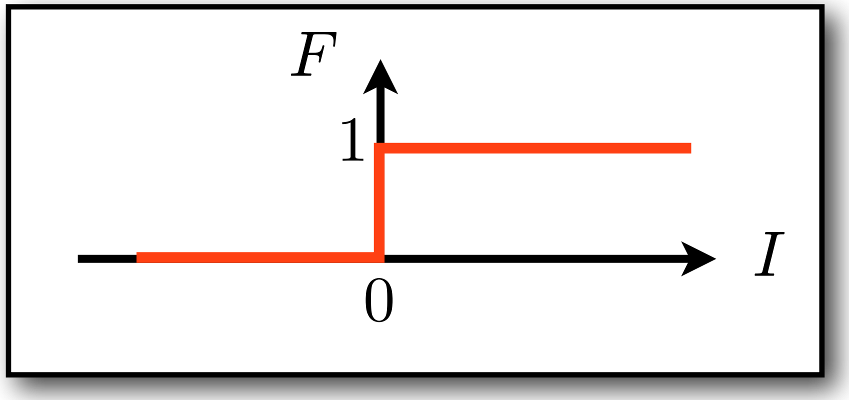

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activities are forced onto the network:
dynamics do not play a role

ENERGY FUNCTION

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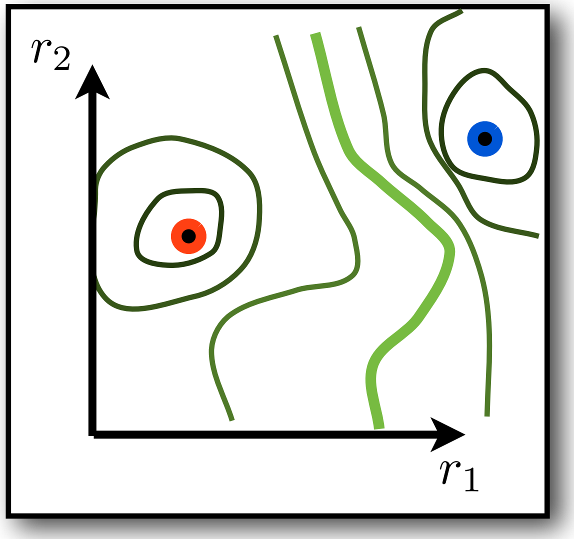

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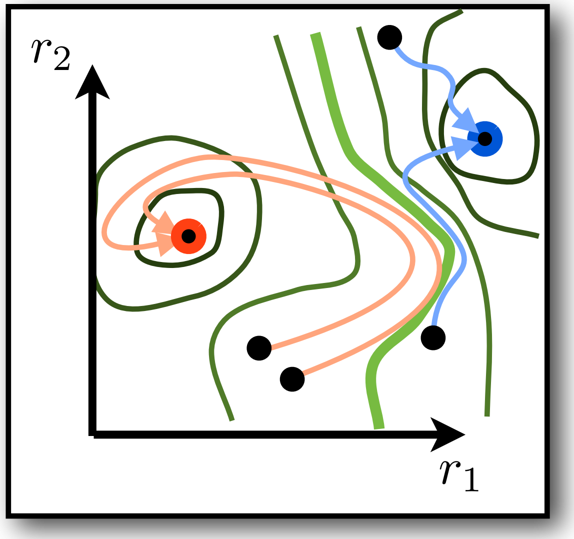

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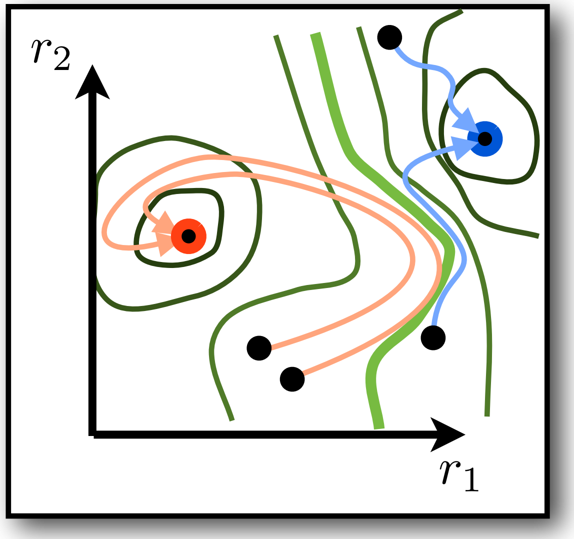

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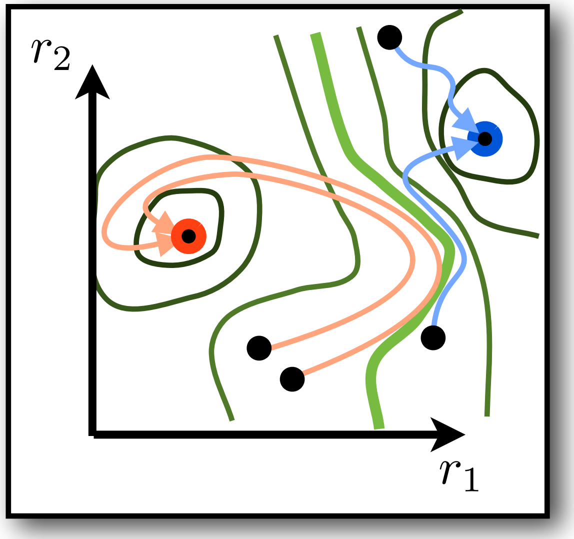

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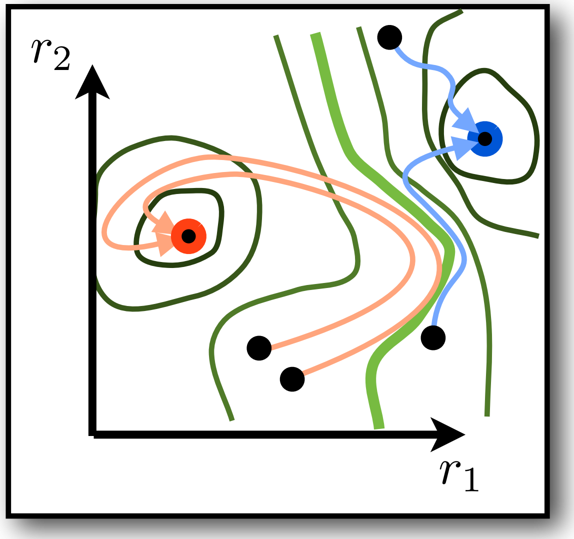

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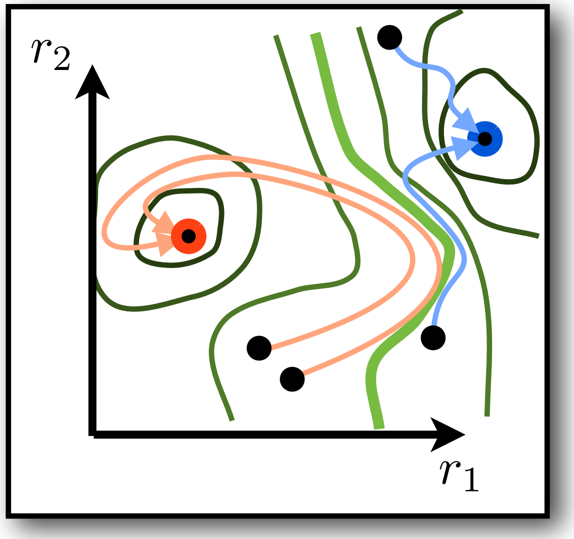
in reality: only corners



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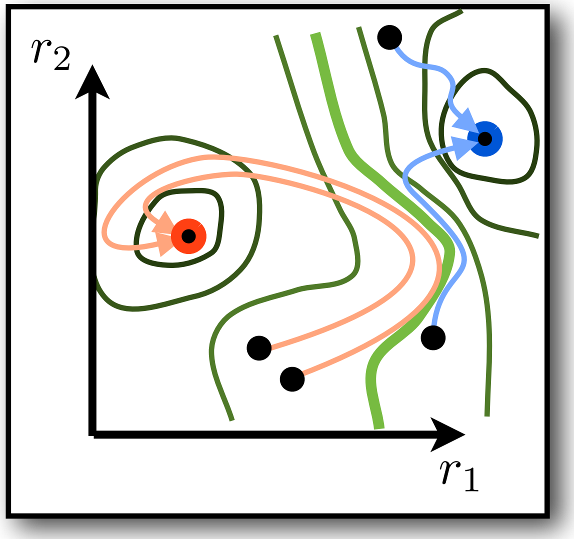

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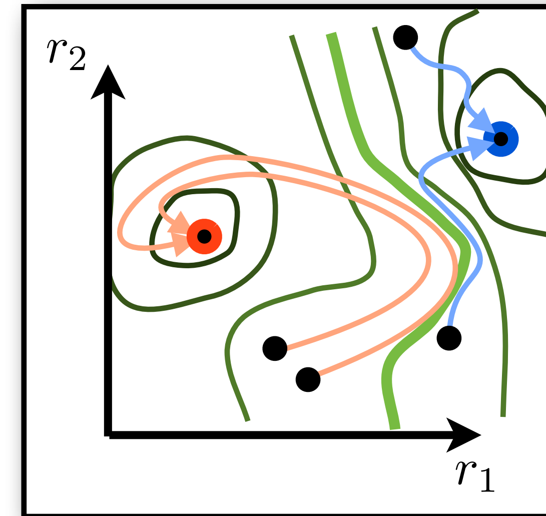

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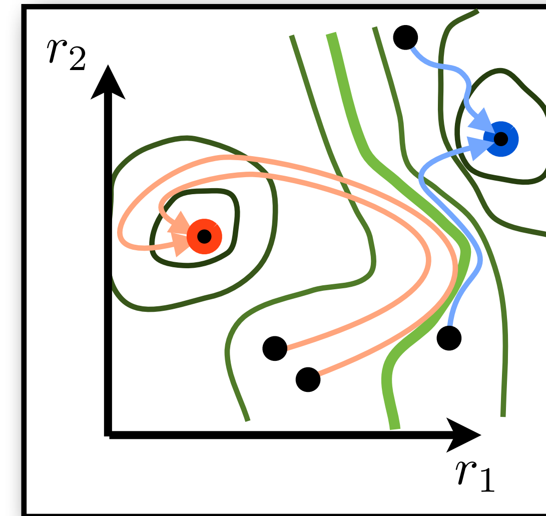

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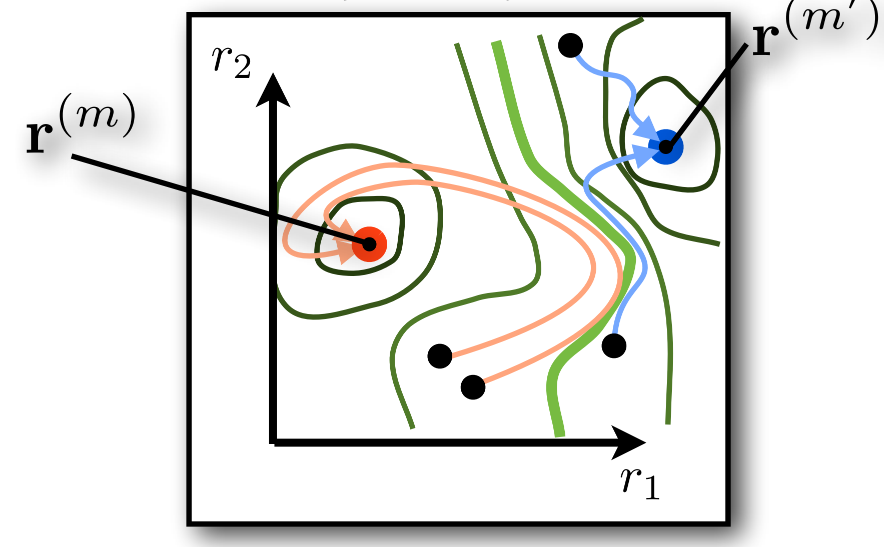
$$\boxed{E(\mathbf{r}(t+\Delta t)) - E(\mathbf{r}(t)) \quad \leq \quad 0}$$

ENERGY FUNCTION

2. stored patterns are fixed points

in reality: only corners



ENERGY FUNCTION

2. stored patterns are fixed points

$$
\begin{aligned}
H_k(t) &= \sum_{j \neq k} W_{kj}\, r_j(t) = \sum_{j \neq k} r_j(t) \sum_m \left(r_k^{(m)} - \frac{1}{2}\right)\left(r_j^{(m)} - \frac{1}{2}\right) \\
&= \sum_m \left(r_k^{(m)} - \frac{1}{2}\right) \sum_{j \neq k} r_j(t) \left(r_j^{(m)} - \frac{1}{2}\right)
\end{aligned}
$$

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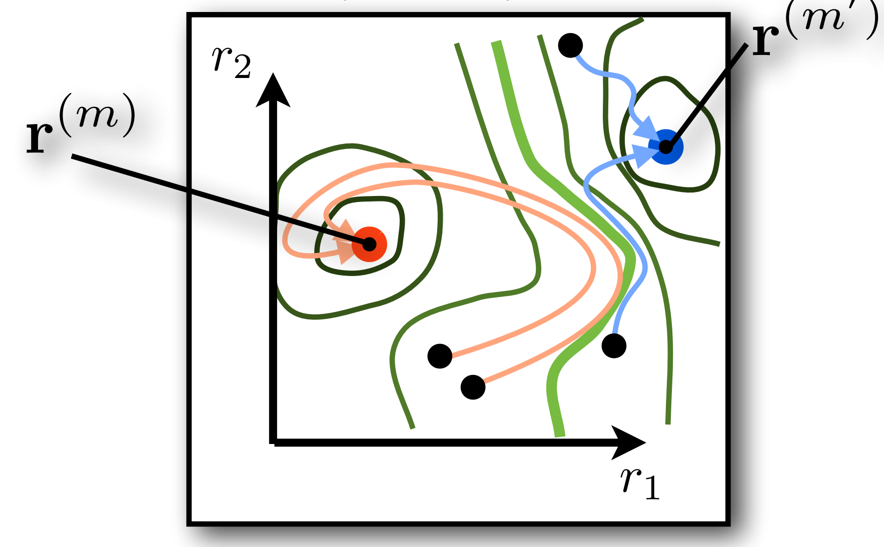

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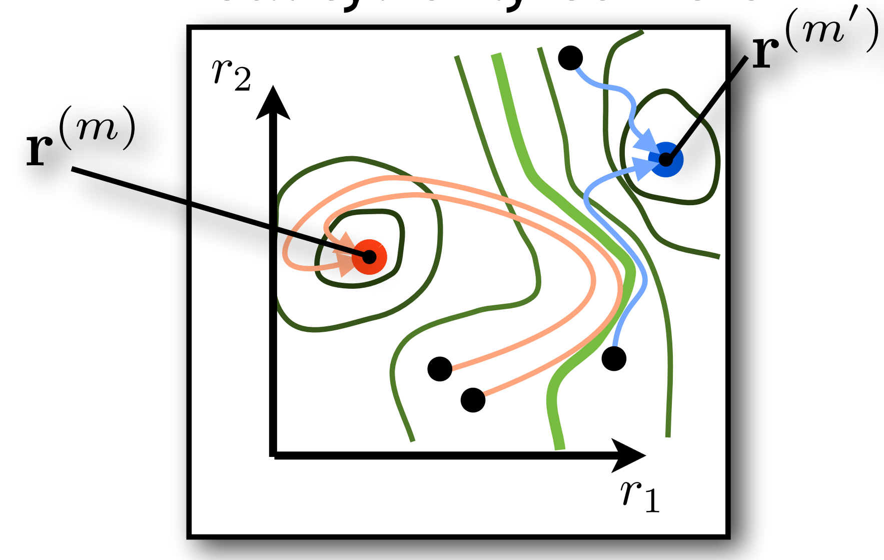

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when in memory state μ: $r_j(t) = r_j^{(\mu)}$

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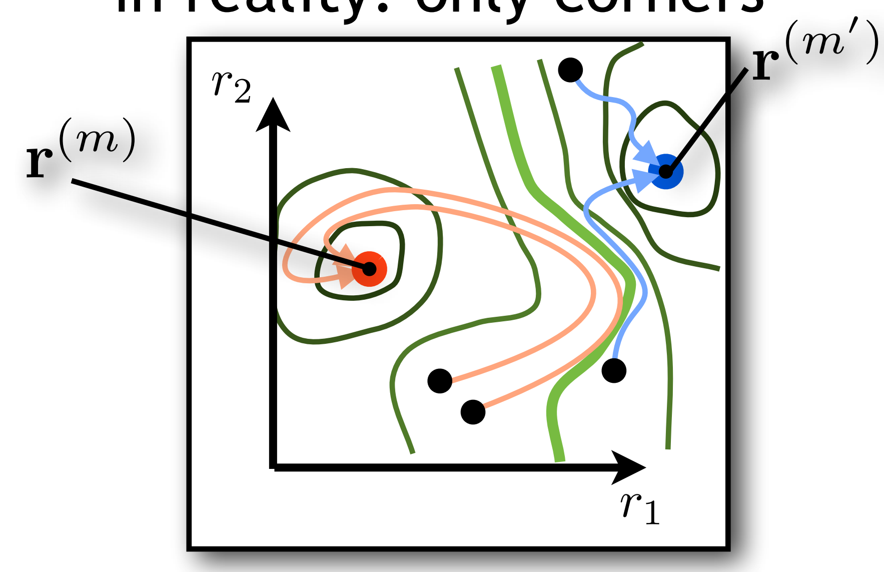

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averaging

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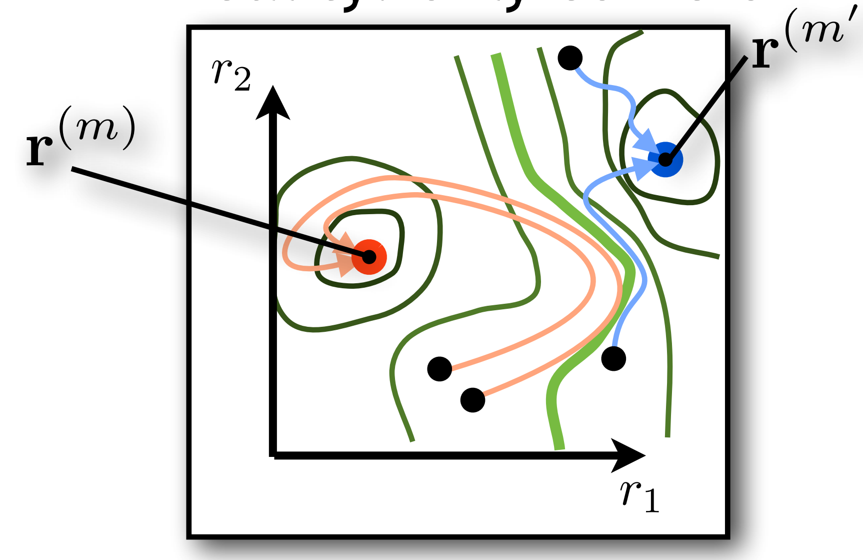

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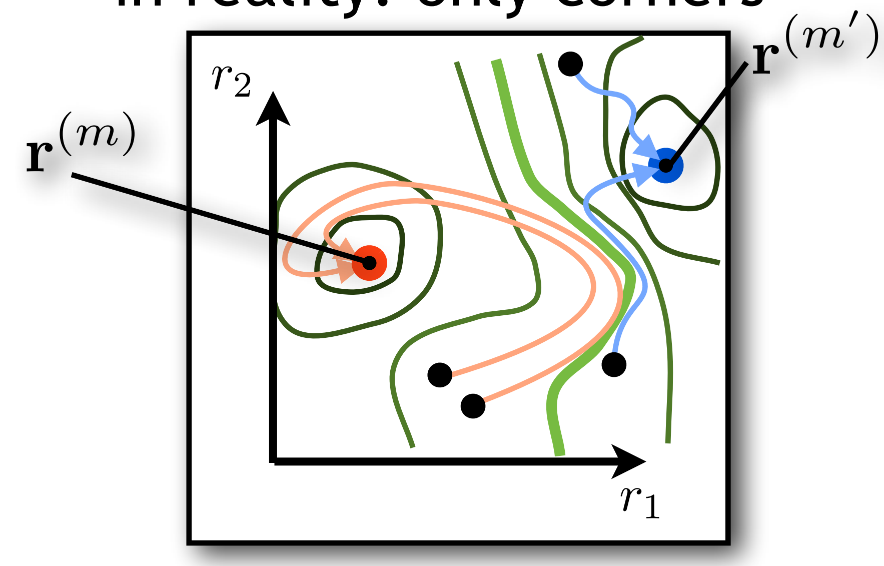
ENERGY FUNCTION

2. stored patterns are fixed points

in reality: only corners



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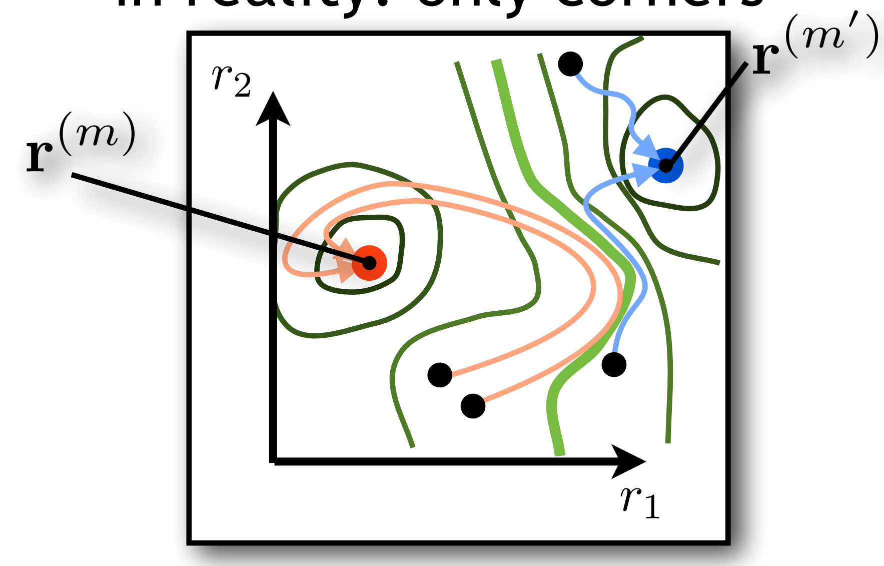
ENERGY FUNCTION

stable?

2. stored patterns are fixed points

in reality: only corners



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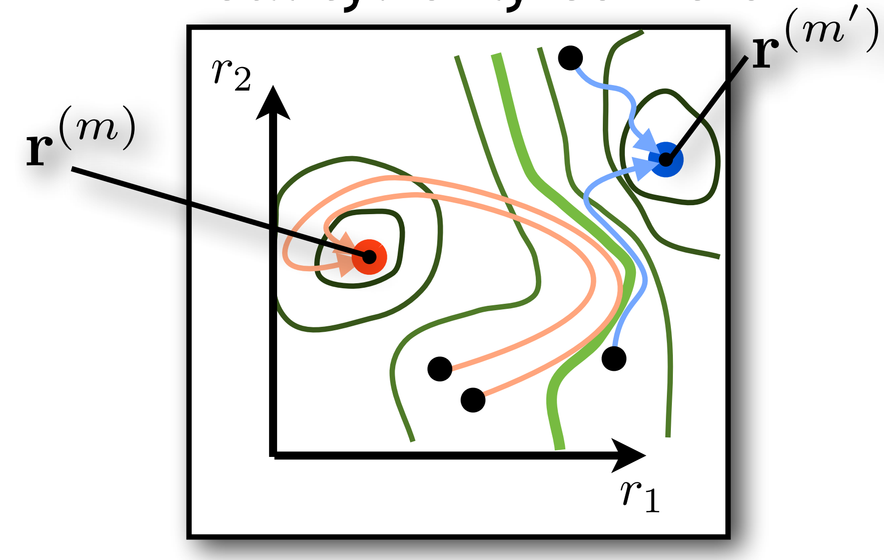
ENERGY FUNCTION

stable?

2. stored patterns are fixed points on average

in reality: only corners



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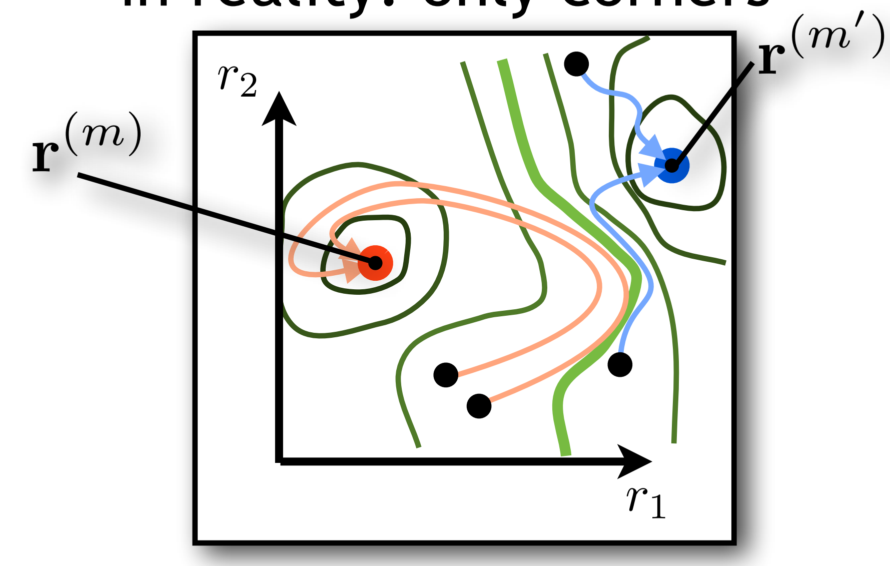
ENERGY FUNCTION

stable?

2. stored patterns are fixed points on average
   (other stable fixed points?)

$$
\begin{aligned}
H_k(t) &= \sum_{j\neq k} W_{kj}\, r_j(t) = \sum_{j\neq k} r_j(t) \sum_m \left(r_k^{(m)} - \frac{1}{2}\right)\left(r_j^{(m)} - \frac{1}{2}\right) \\
&= \sum_m \left(r_k^{(m)} - \frac{1}{2}\right) \sum_{j\neq k} r_j(t) \left(r_j^{(m)} - \frac{1}{2}\right)
\end{aligned}
$$

in reality: only corners



when in memory state μ: $r_j(t) = r_j^{(\mu)}$

$$
H_k(t) = \underbrace{\left(r_k^{(\mu)} - \frac{1}{2}\right) \sum_{j\neq k} r_j^{(\mu)} \left(r_j^{(\mu)} - \frac{1}{2}\right)}_{\text{signal}} + \underbrace{\sum_{j\neq k} r_j^{(\mu)} \sum_{m\neq\mu} \left(r_j^{(m)} - \frac{1}{2}\right)\left(r_k^{(m)} - \frac{1}{2}\right)}_{\text{noise}}
$$

averaging

$$
\langle H_k(t)\rangle_{\mathbf{r}^{(m)}} = \left(r_k^{(\mu)} - \frac{1}{2}\right) \underbrace{\sum_{j\neq k} r_j^{(\mu)} \left(r_j^{(\mu)} - \frac{1}{2}\right)}_{K_+ \geq 0} + \sum_{j\neq k} r_j^{(\mu)} (M-1) \underbrace{\left\langle \left(r_j^{(m)} - \frac{1}{2}\right)\right\rangle}_{=0} \underbrace{\left\langle \left(r_k^{(m)} - \frac{1}{2}\right)\right\rangle}_{=0}
$$

$$
\langle H_k(t)\rangle_{\mathbf{r}^{(m)}} = \left(r_k^{(\mu)} - \frac{1}{2}\right) K_+
$$

$$
\boxed{
\begin{aligned}
r_k(t) = r_k^{(\mu)} = 1 \quad &\rightarrow \quad \langle H_k(t)\rangle > 0 \rightarrow r_k(t+\Delta t) \approx 1 = r_k^{(\mu)} \\
r_k(t) = r_k^{(\mu)} = 0 \quad &\rightarrow \quad \langle H_k(t)\rangle < 0 \rightarrow r_k(t+\Delta t) \approx 0 = r_k^{(\mu)}
\end{aligned}
}
$$

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- spurious attractors
- stored patterns are unstable fixed points
- stored patterns are not fixed points

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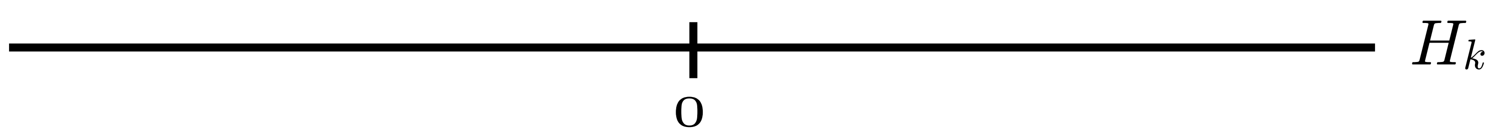

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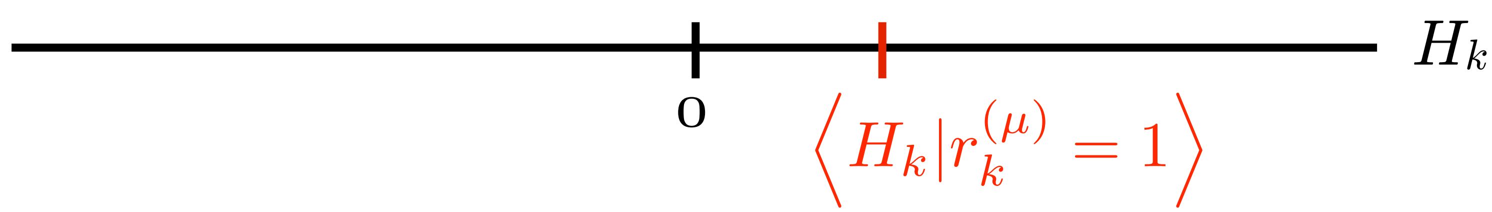

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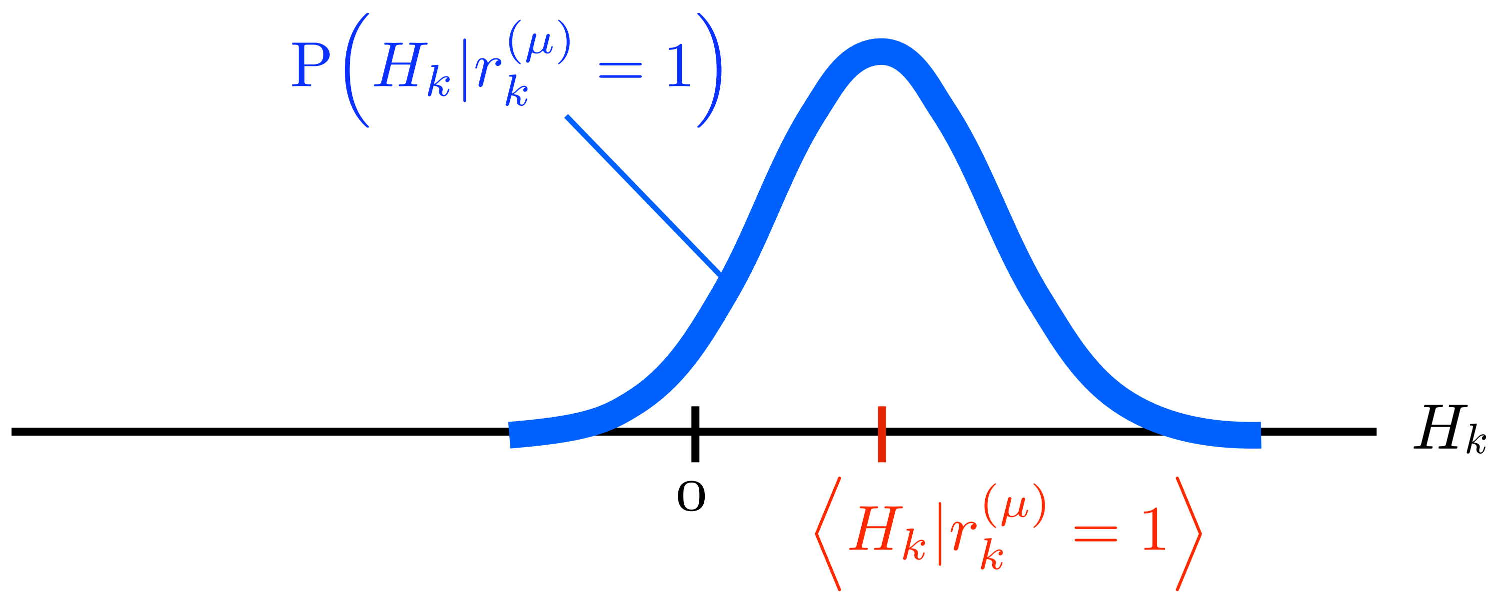

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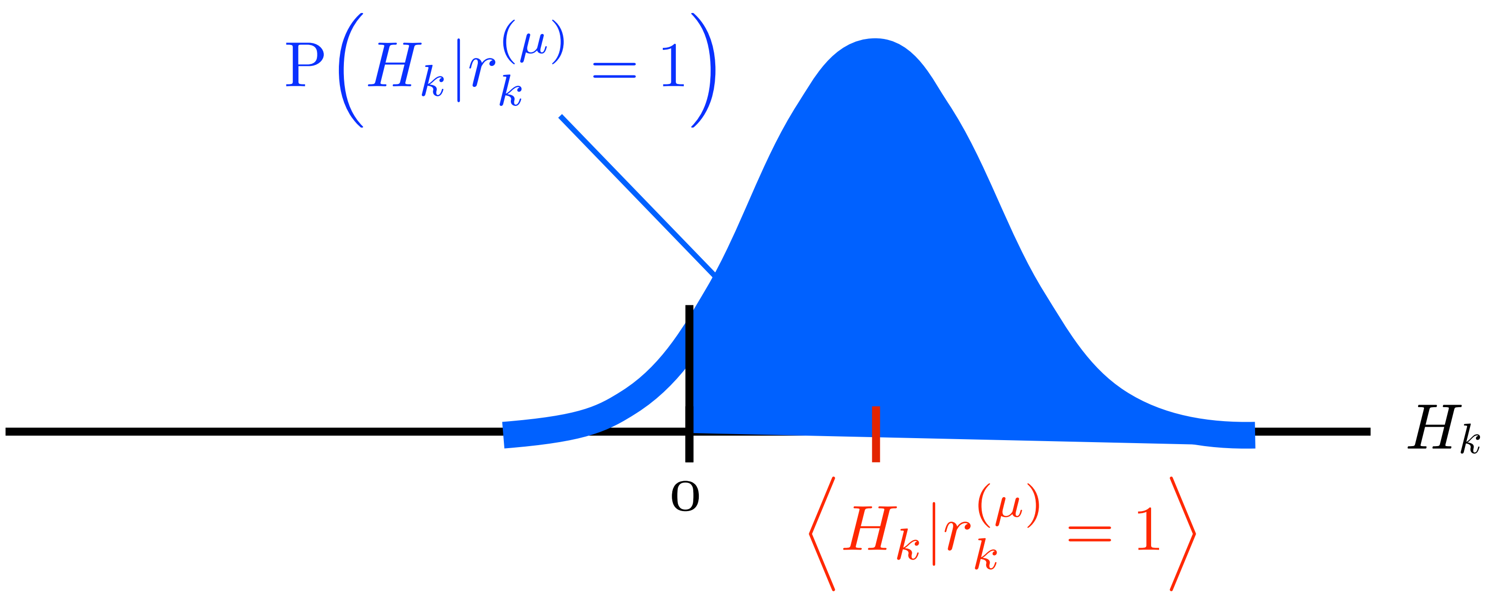

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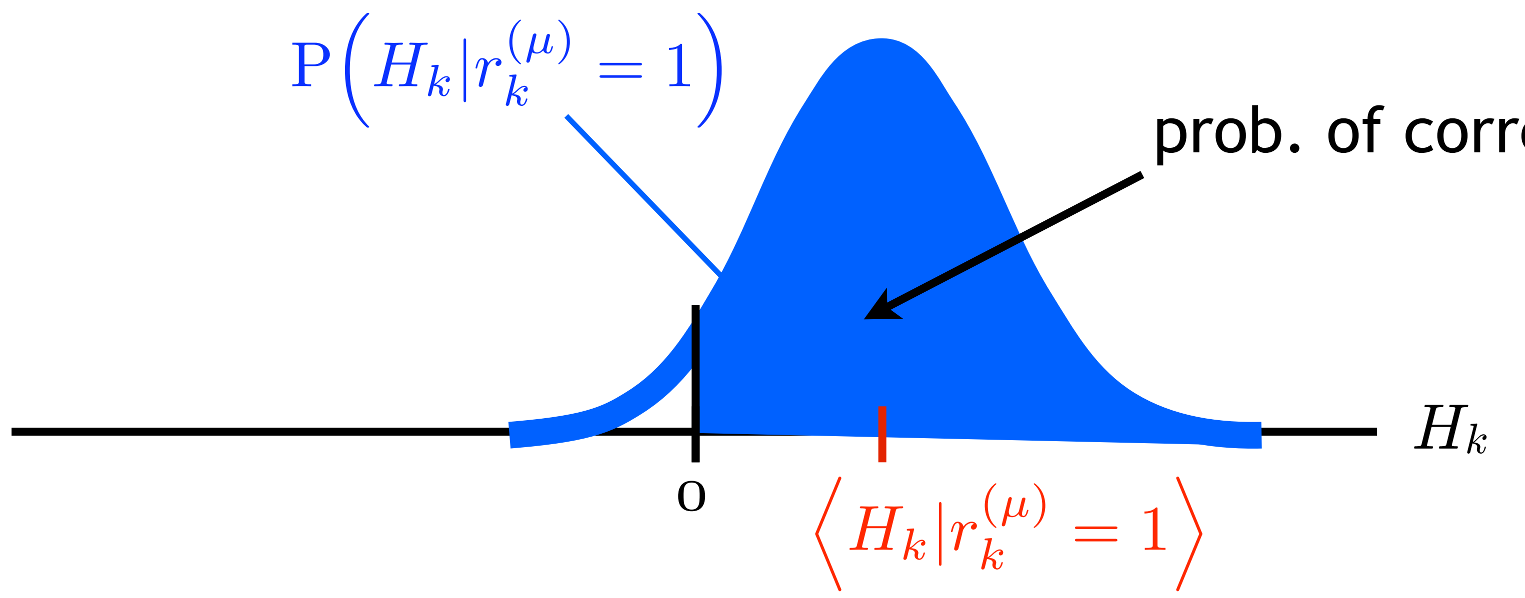

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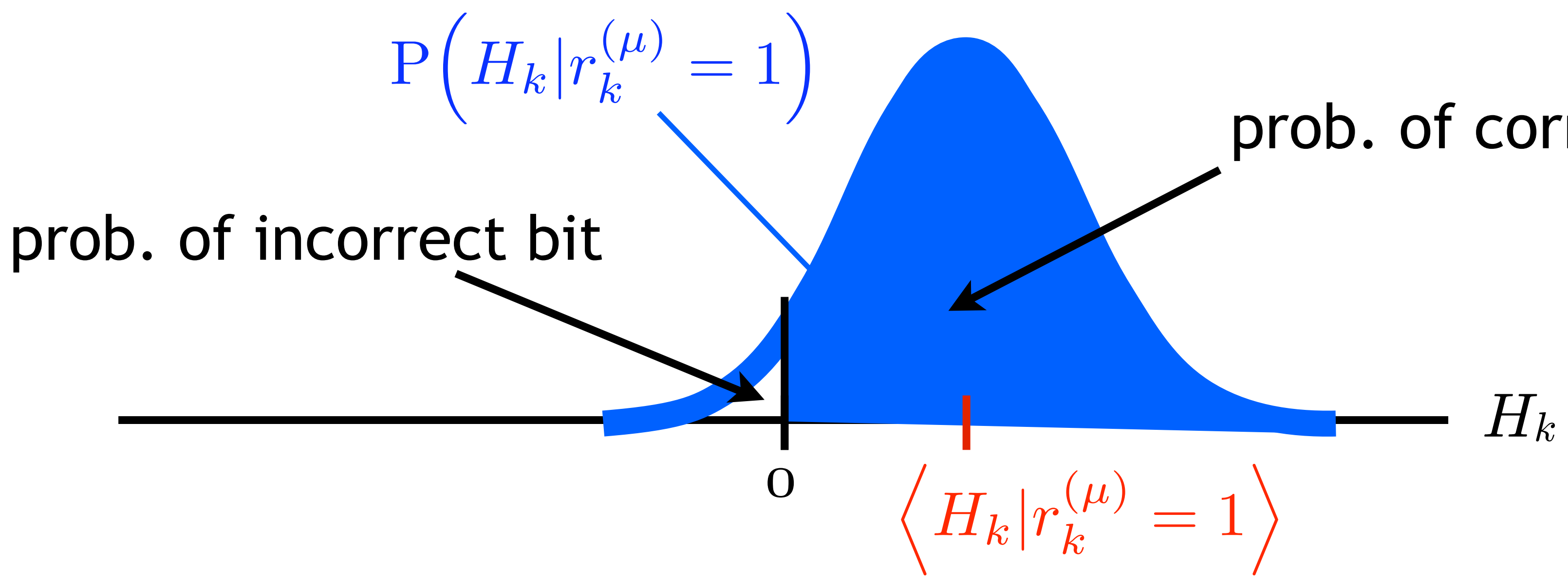

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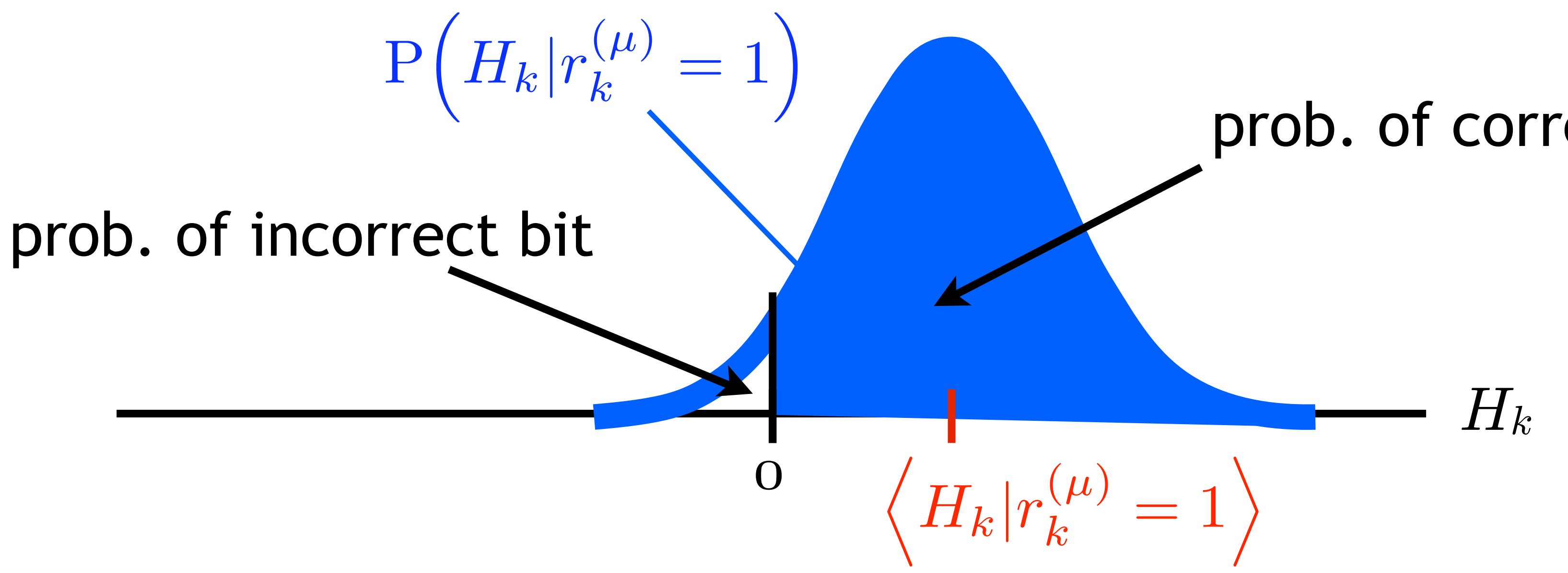

is Gaussianity justified?

$$H_k|r_k^{(\mu)} = \left(r_k^{(\mu)}-\frac{1}{2}\right)\sum_{j\neq k} r_j^{(\mu)}\left(r_j^{(\mu)}-\frac{1}{2}\right)+\sum_{j\neq k} r_j^{(\mu)}\sum_{m\neq\mu}\left(r_j^{(m)}-\frac{1}{2}\right)\left(r_k^{(m)}-\frac{1}{2}\right)$$

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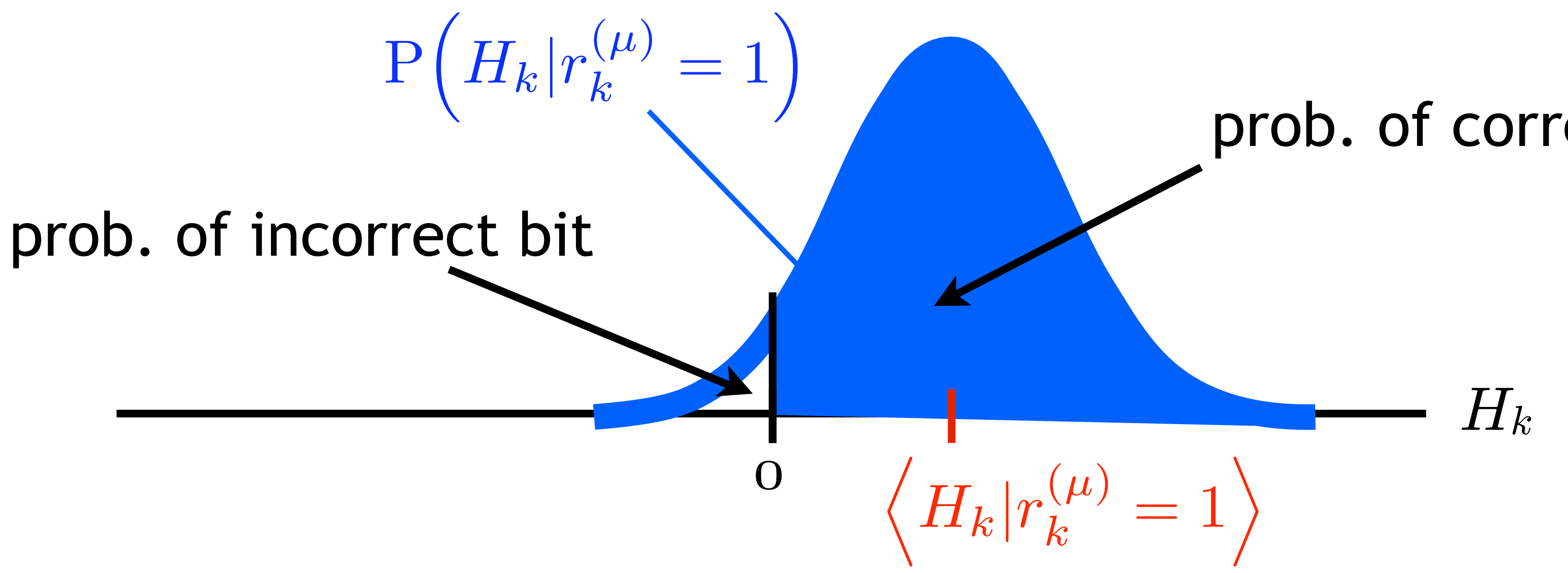

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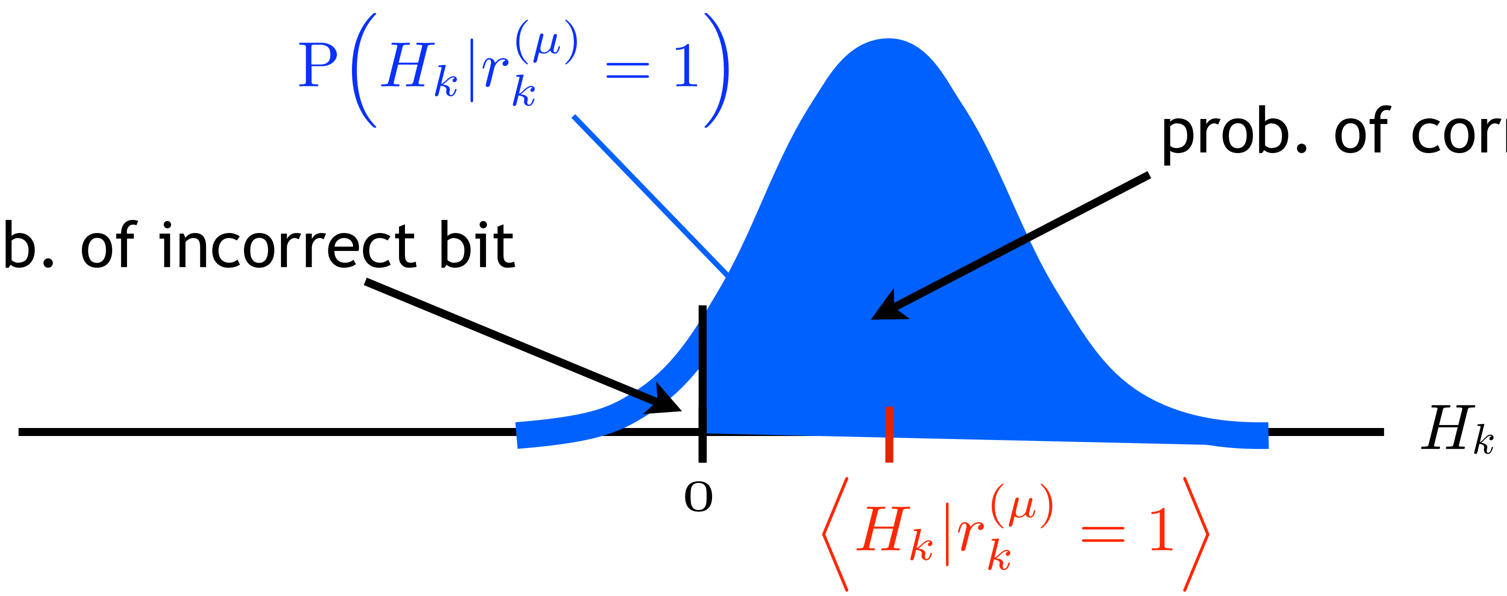

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sum of many independent random terms → central limit theorem

CAPACITY CALCULATIONS

$$H_k = \underbrace{\left(r_k^{(\mu)} - \frac{1}{2}\right) \sum_{j \neq k} r_j^{(\mu)} \left(r_j^{(\mu)} - \frac{1}{2}\right)}_{\text{signal}} + \underbrace{\sum_{j \neq k} r_j^{(\mu)} \sum_{m \neq \mu} \left(r_j^{(m)} - \frac{1}{2}\right) \left(r_k^{(m)} - \frac{1}{2}\right)}_{\text{noise}}$$

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$$\langle \mathrm{Var}[H_k]_{\mathbf{r}^{(m)}} \rangle_{r_j^{(\mu)}} = \underbrace{\langle \mathrm{Var}[\text{signal}] \rangle}_{0} + \langle \mathrm{Var}[\text{noise}] \rangle$$

$$= (N-1) \underbrace{\left\langle r_j^{(\mu)} \right\rangle}_{1/2} (M-1) \underbrace{\mathrm{Var}\left[\left(r_j^{(m)} - \frac{1}{2}\right) \left(r_k^{(m)} - \frac{1}{2}\right)\right]}_{1/16}$$

$$= \frac{(M-1)(N-1)}{32}$$

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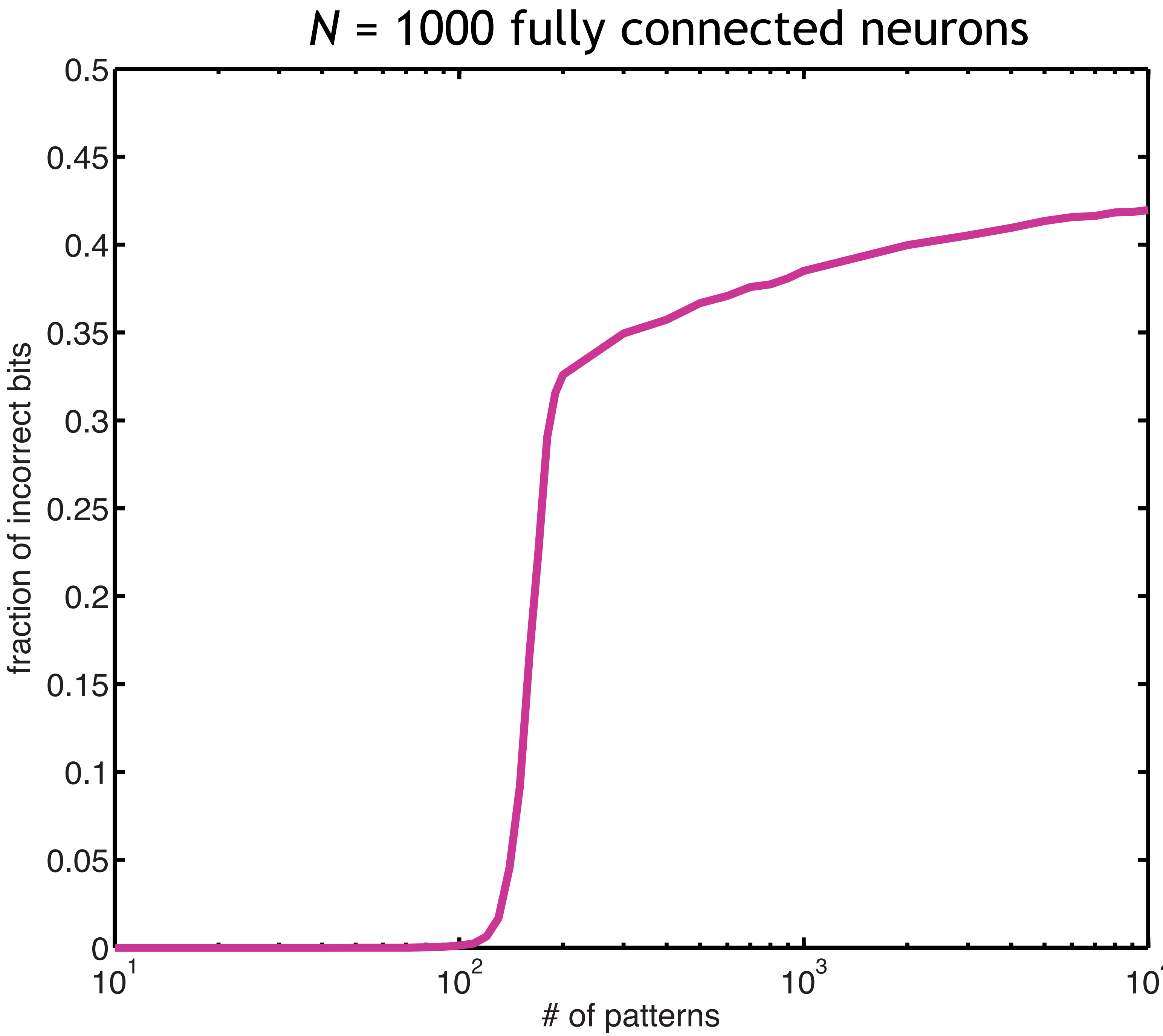
$$\boxed{\mathrm{SNR} = \frac{\langle \langle H_k \rangle \rangle^2}{\langle \mathrm{Var}[H_k] \rangle} = \frac{1}{2} \frac{N-1}{M-1}}$$

SOME MORE SOPHISTICATED STATISTICAL PHYSICS ...



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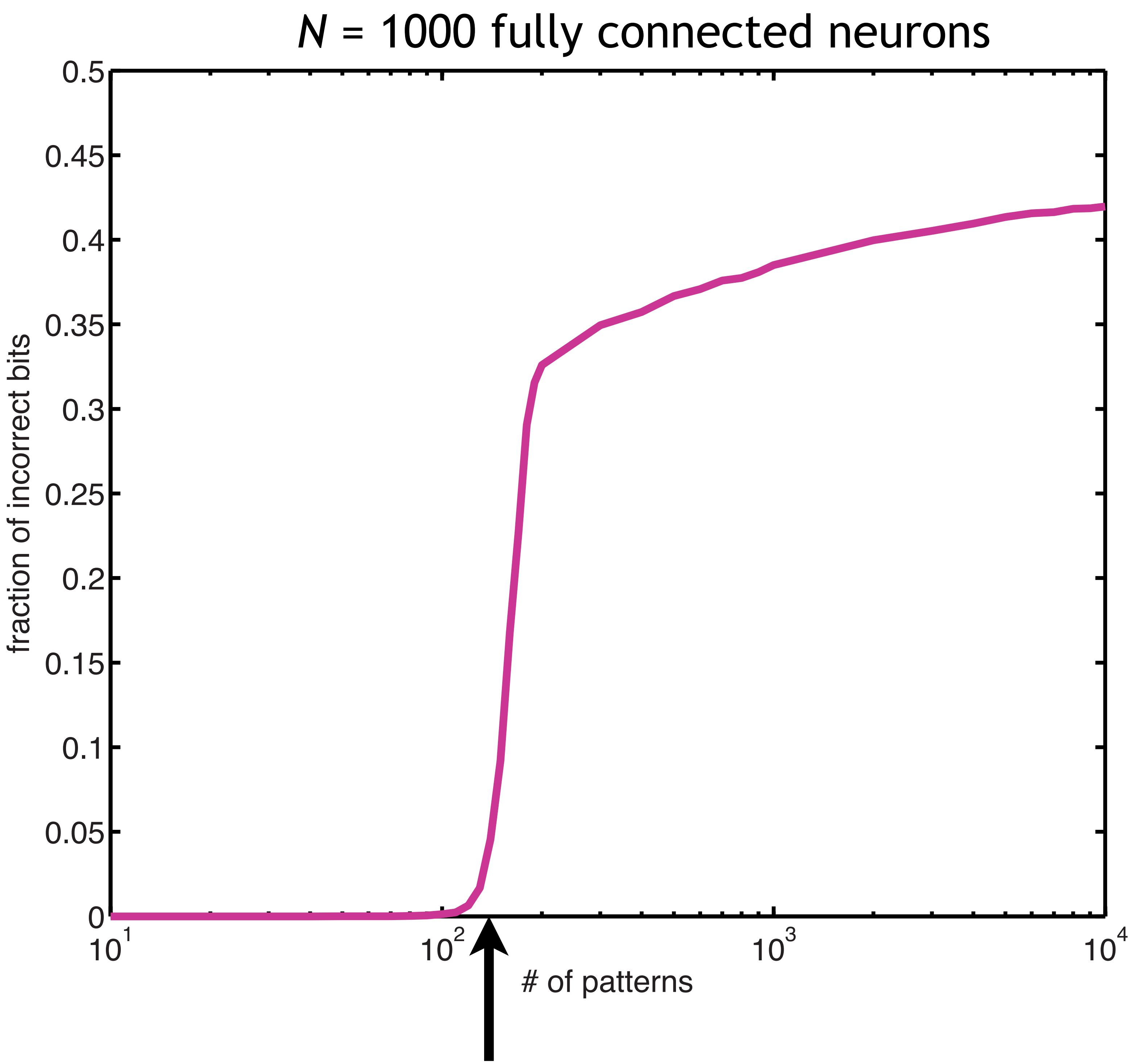


analytically predicted

phase transition at # patterns = 0.138 × # synapses / cell

CONTINUOUS DYNAMICS

Hopfield, 1984

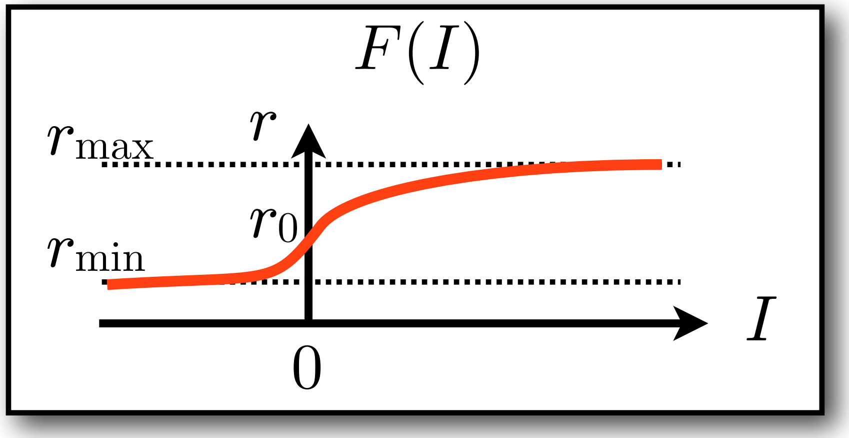
analogue neurons

$$\tau_\mathrm{s} \frac{dI_i}{dt} = -\alpha I_i(t) + \sum_{j \neq i} W_{ij} r_j(t)$$, synchronous update

$$r_i(t) = F(I_i(t))$$

F is ‘sigmoidal’

- monotonically increasing
- bounded by r_{min} and r_{max}



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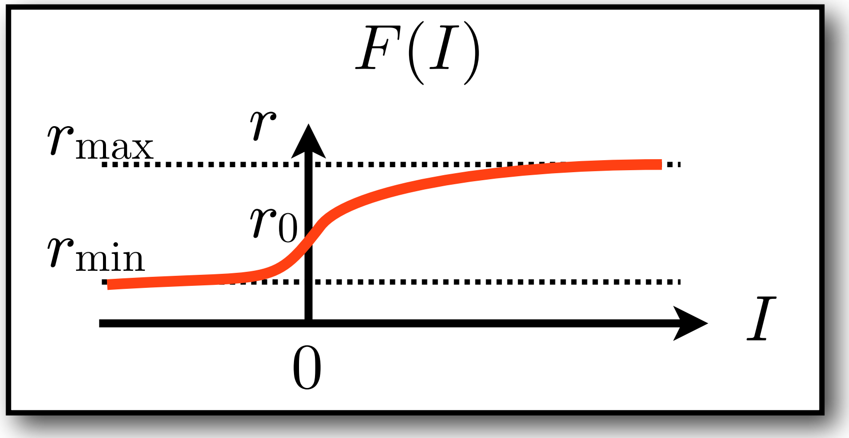

synaptic plasticity as in the binary network → symmetric weight matrix

stored patterns are still binary

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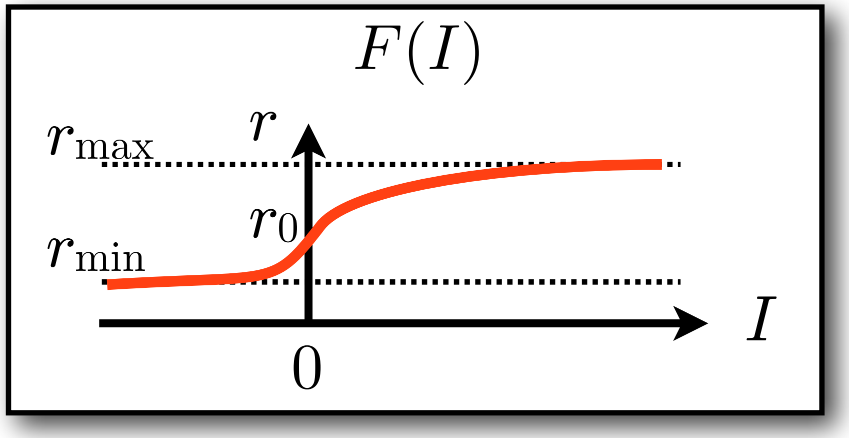

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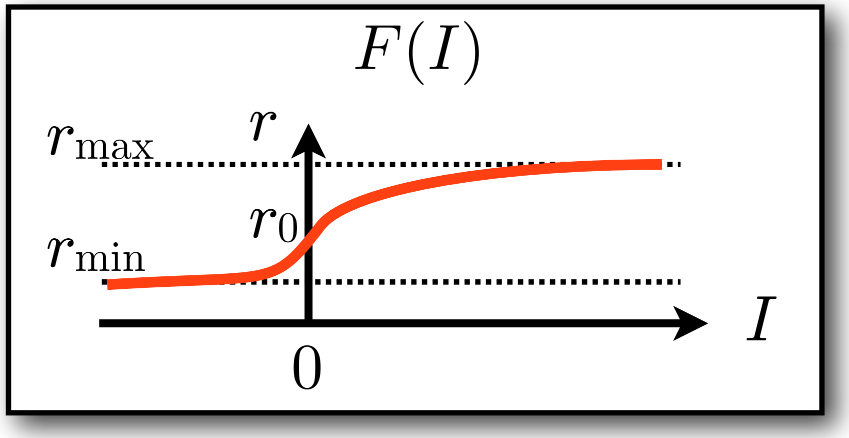

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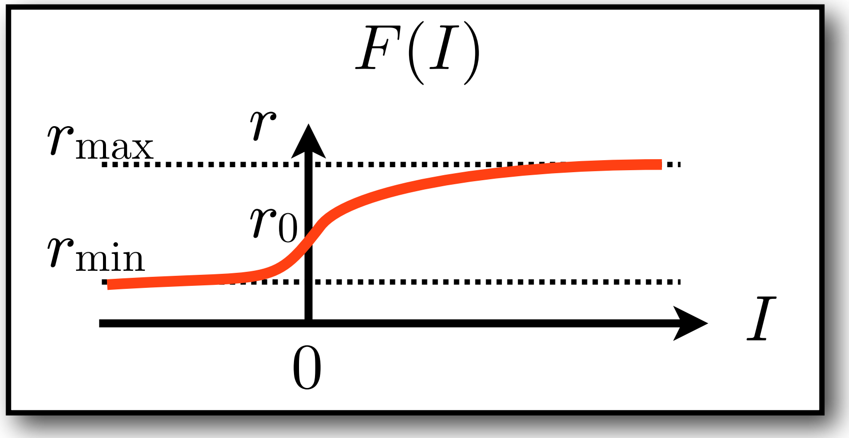

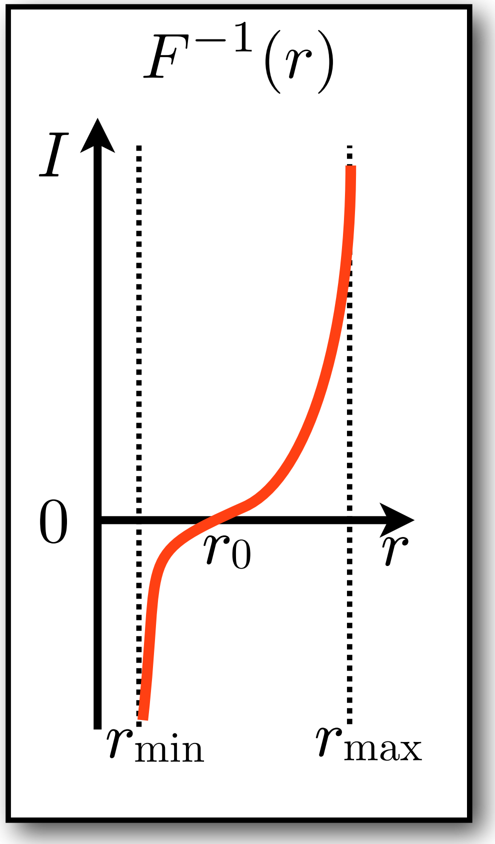

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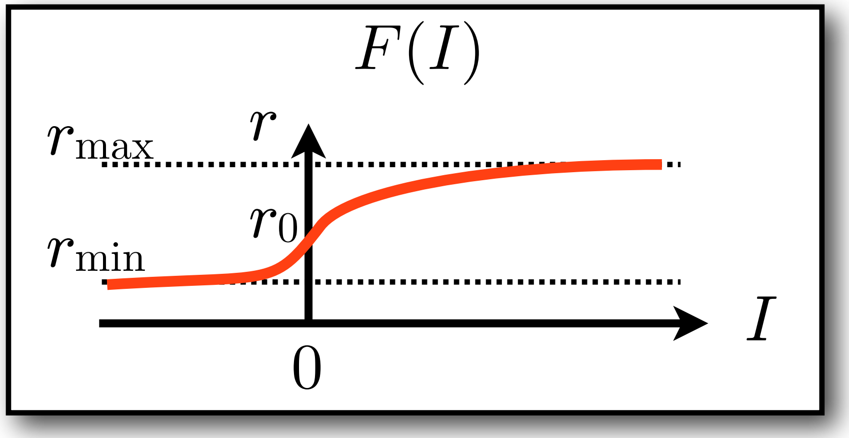

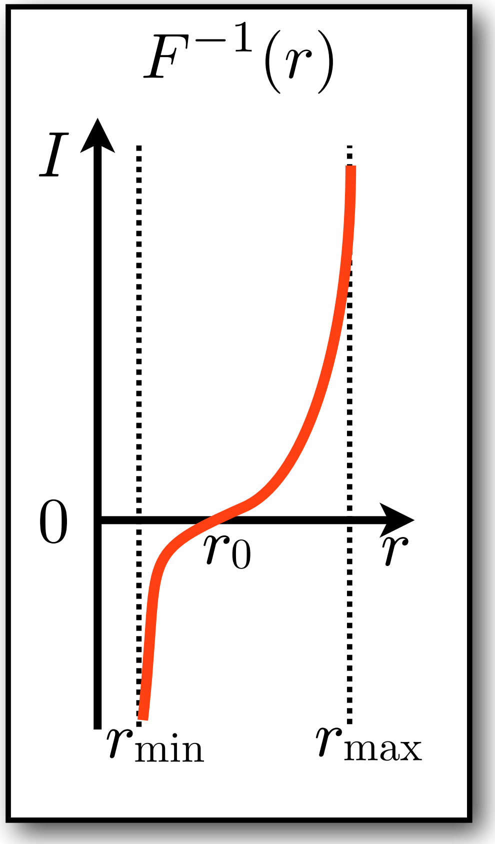

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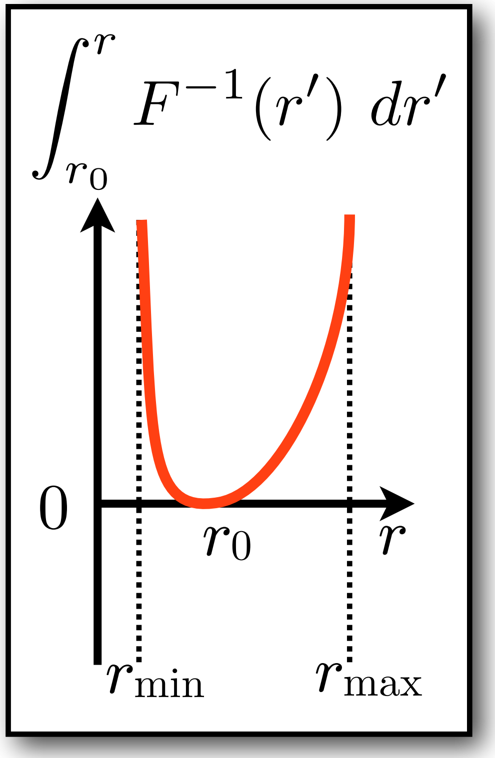

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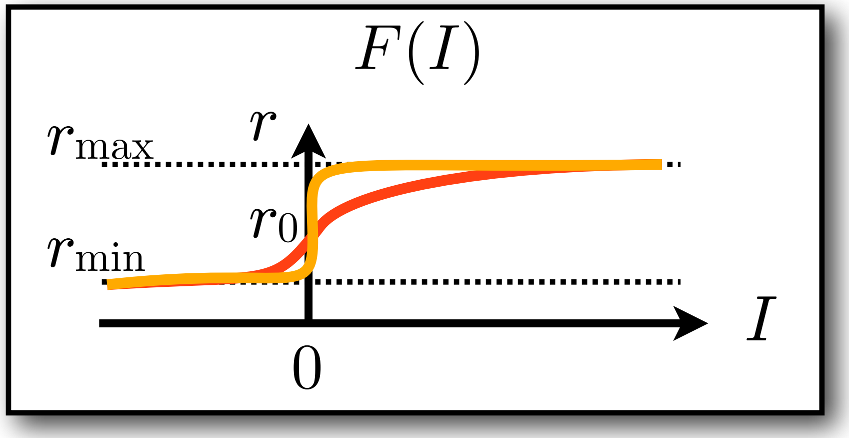

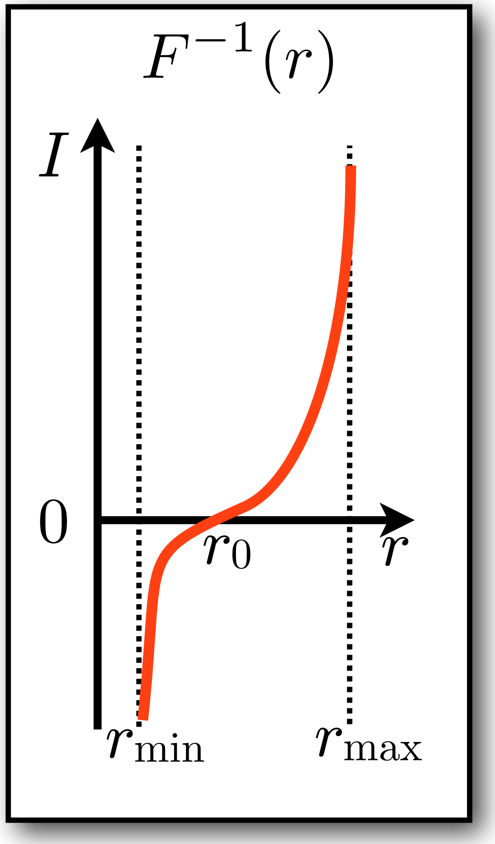

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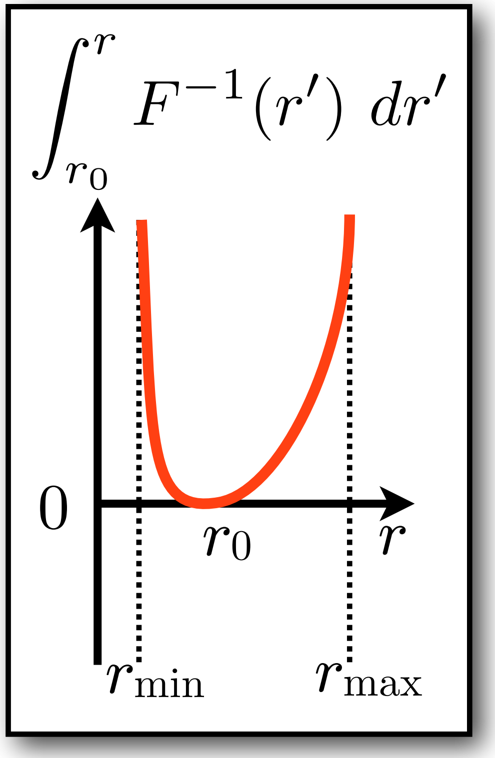

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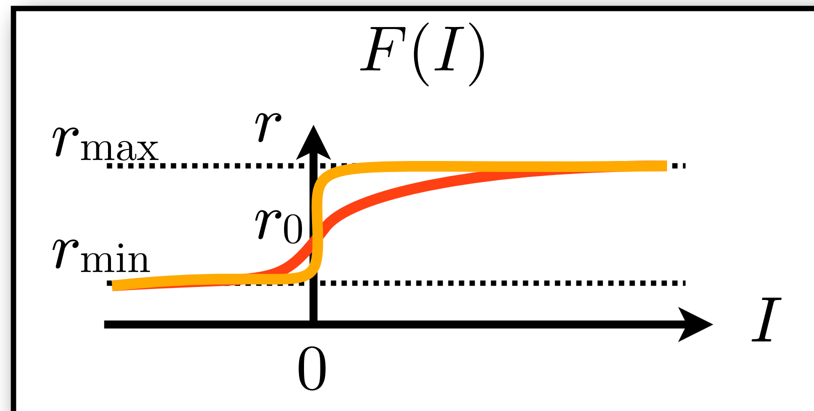

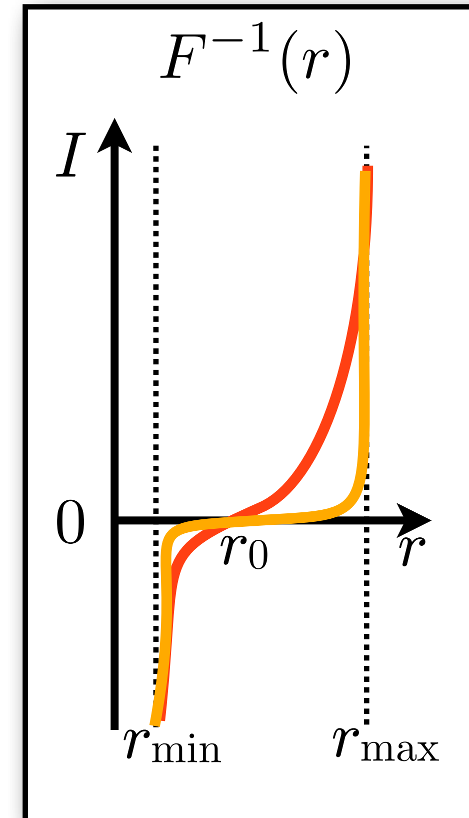

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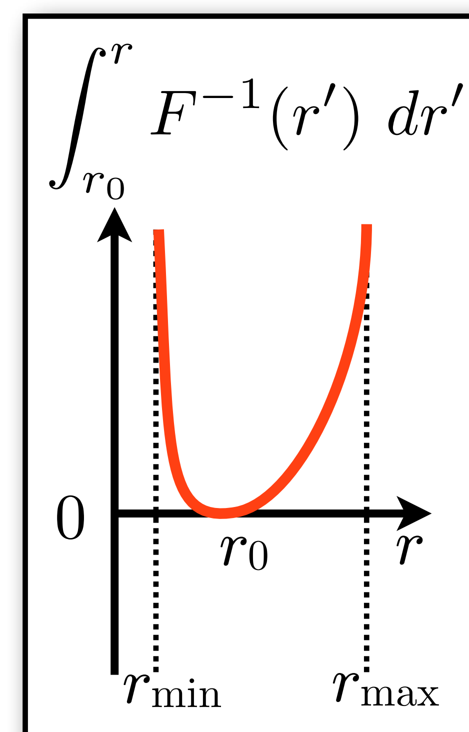

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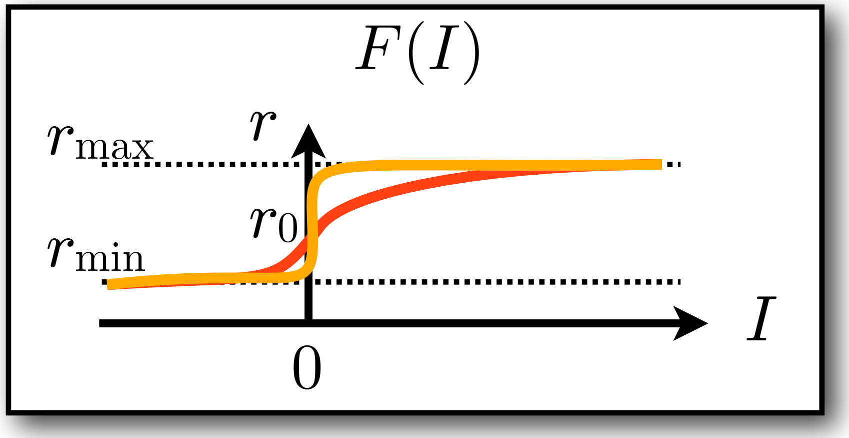

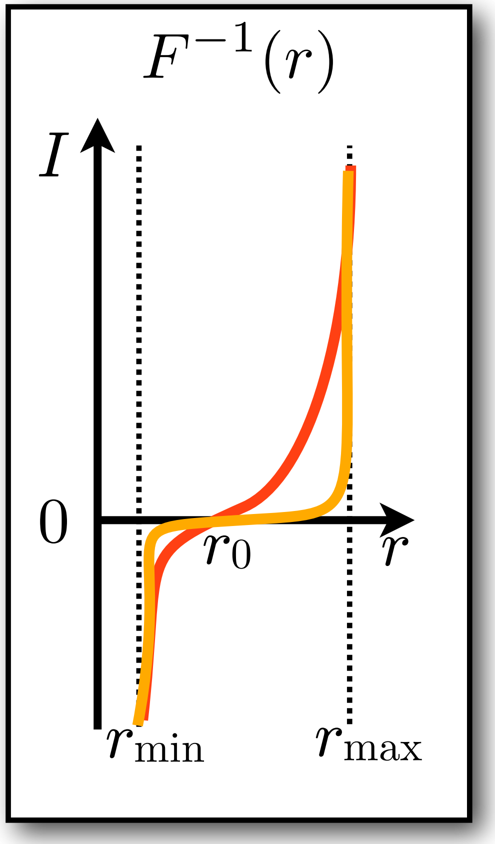

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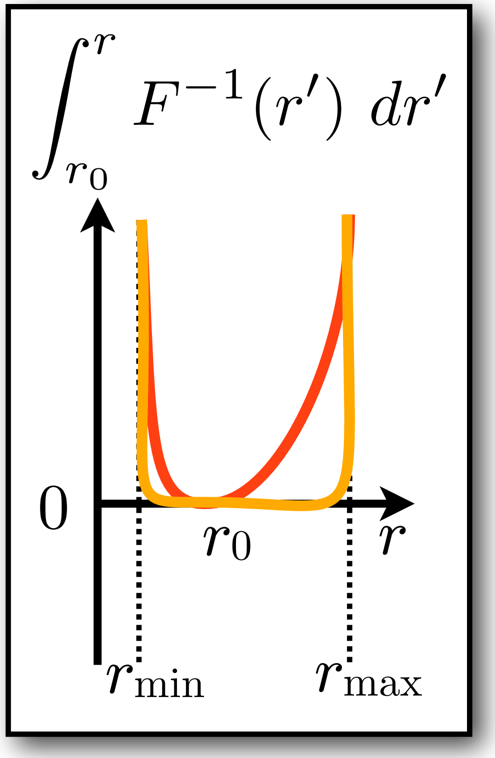


CONTINUOUS DYNAMICS

Hopfield, 1984

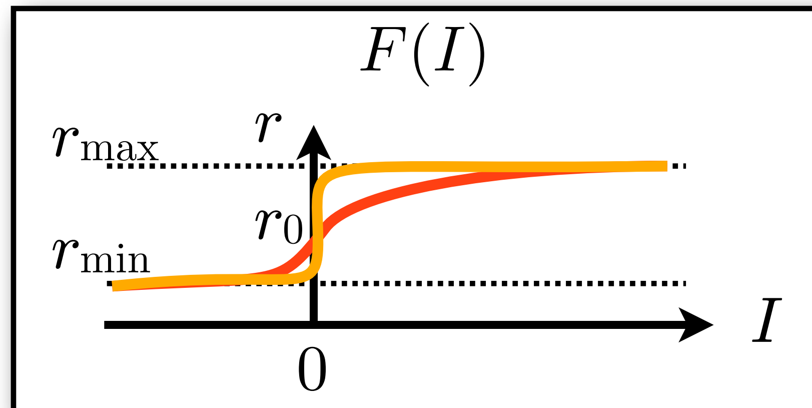
analogue neurons

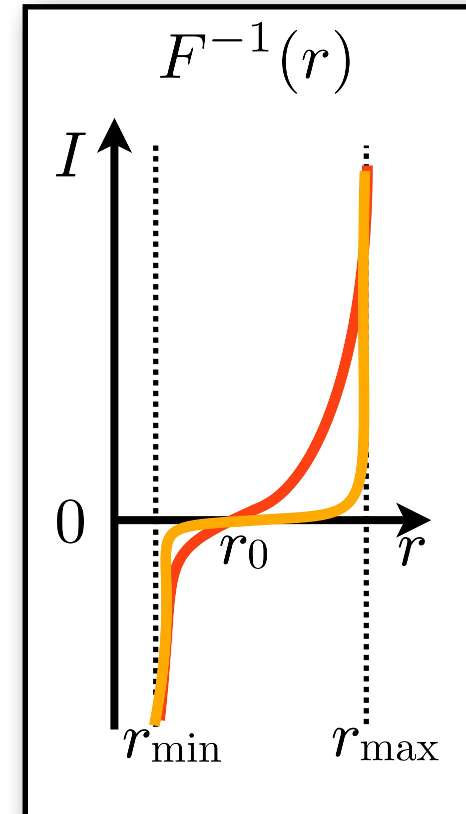
$$\tau_s \frac{dI_i}{dt} = -\alpha I_i(t) + \sum_{j \neq i} W_{ij} r_j(t) \text{ , synchronous update}$$

$$r_i(t) = F(I_i(t))$$

F is ‘sigmoidal’

- monotonically increasing
- bounded by r_{min} and r_{max}





synaptic plasticity as in the binary network → symmetric weight matrix

stored patterns are still binary

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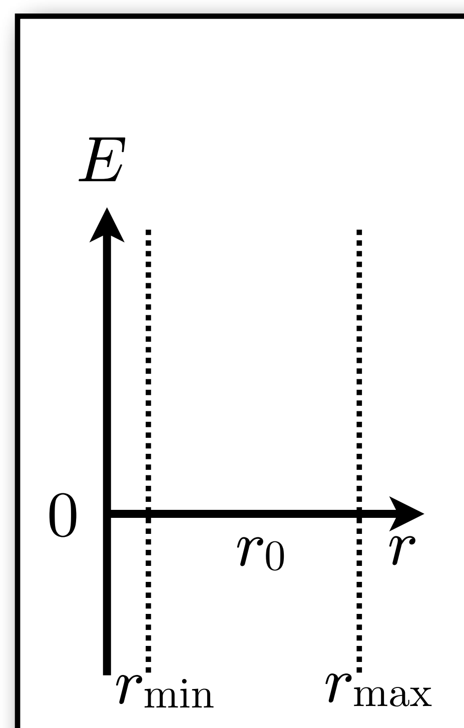

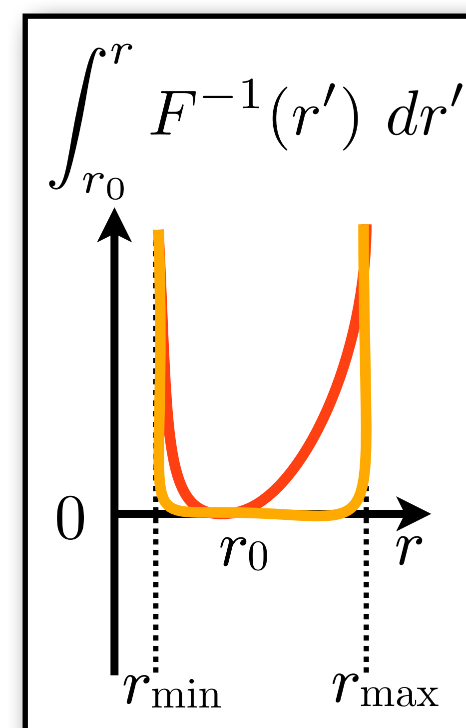

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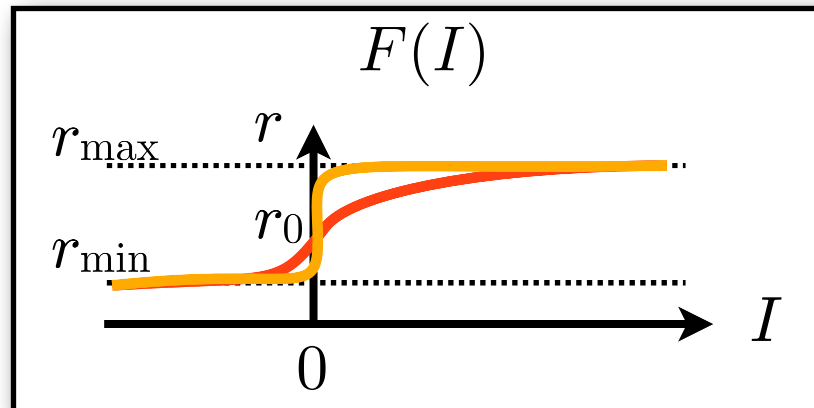

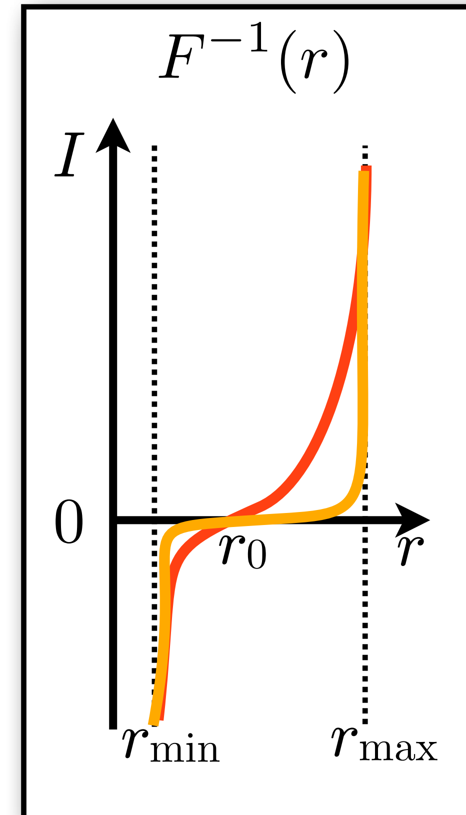

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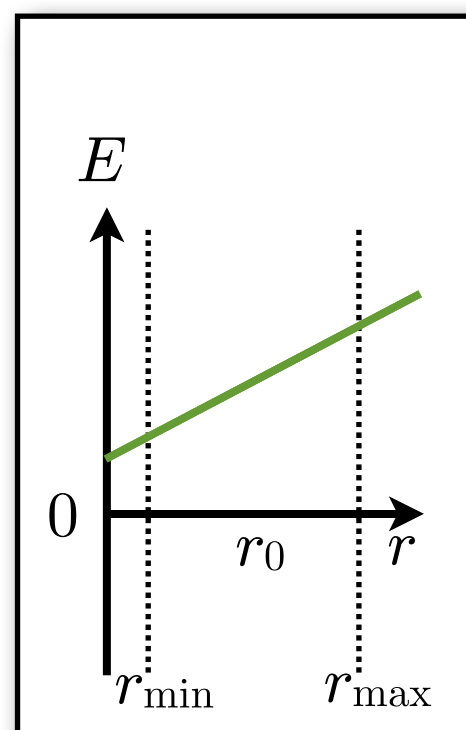

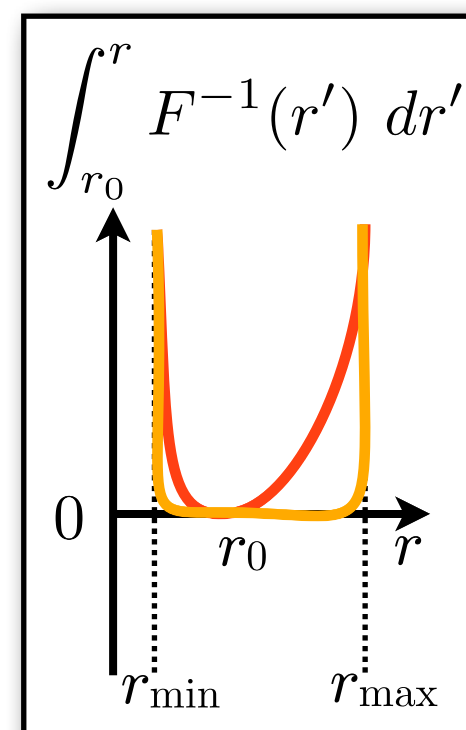

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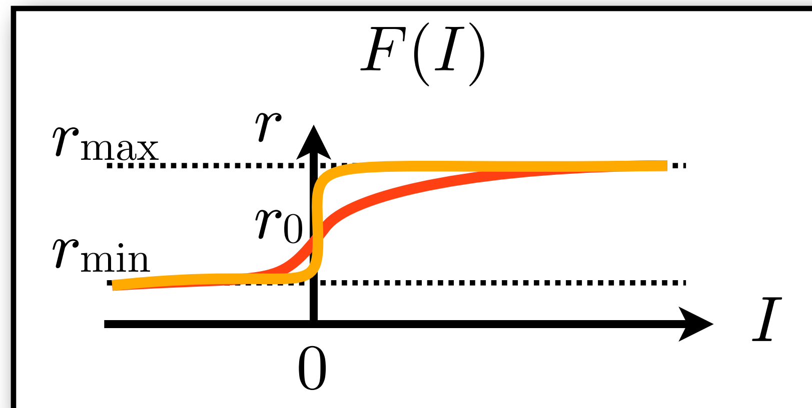

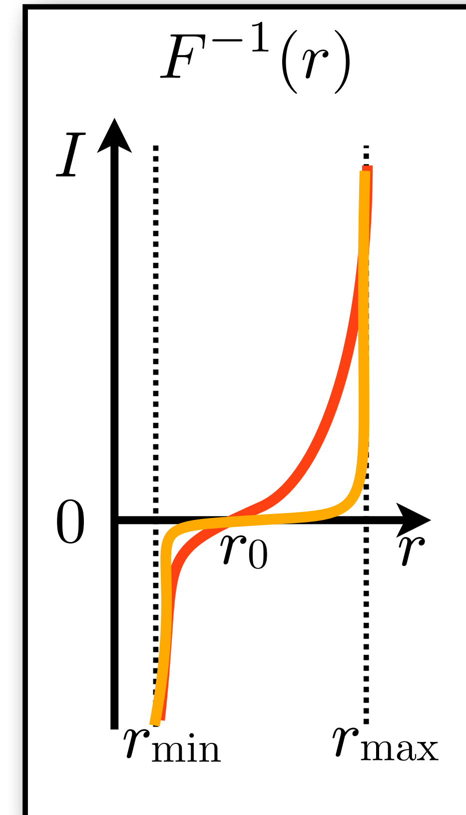

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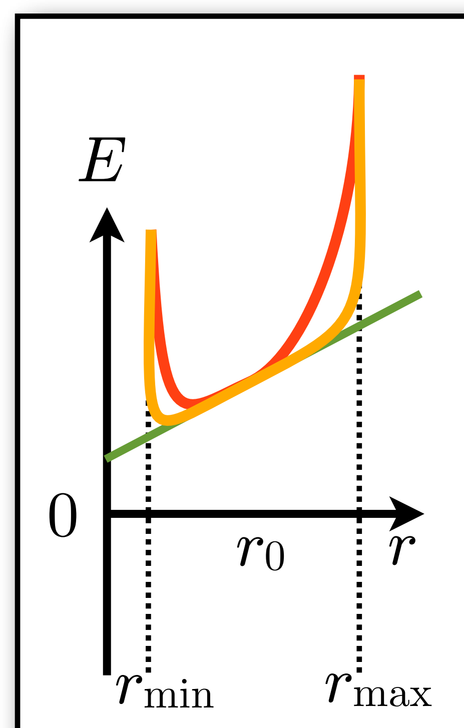

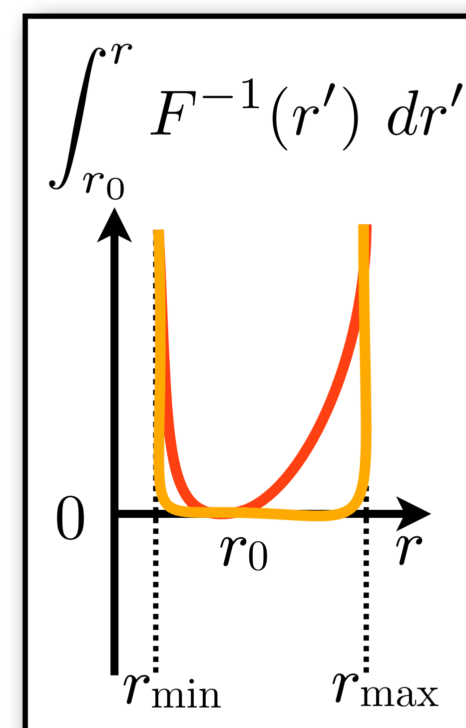

HOPFIELD NETWORK IN OPERATION

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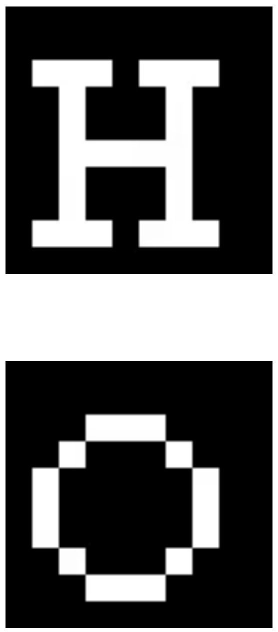
stored patterns



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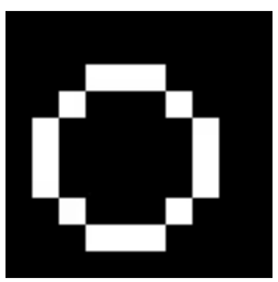

noisy input

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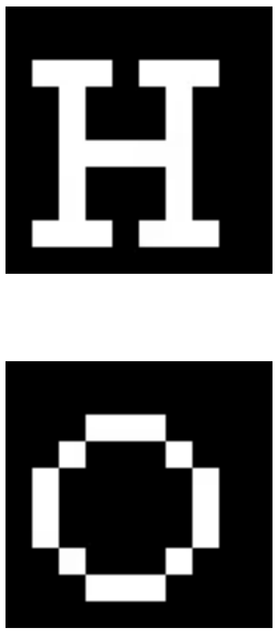

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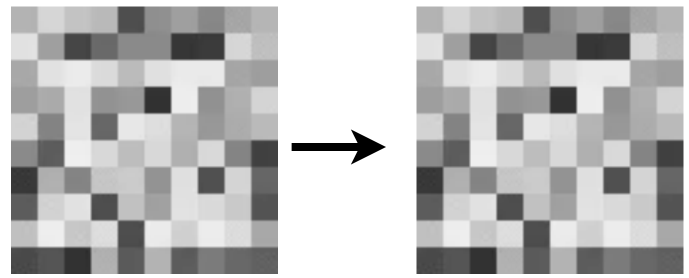

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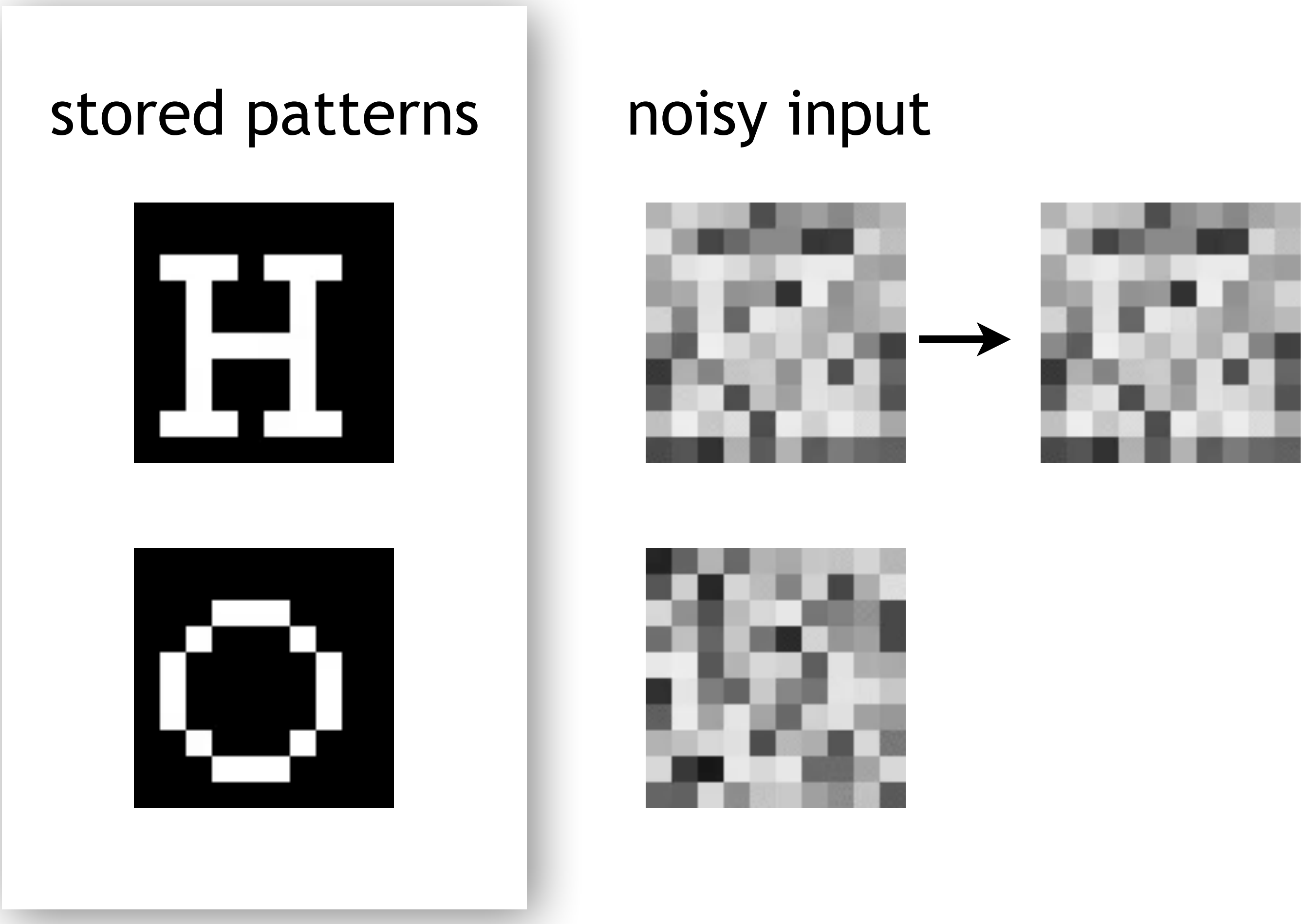

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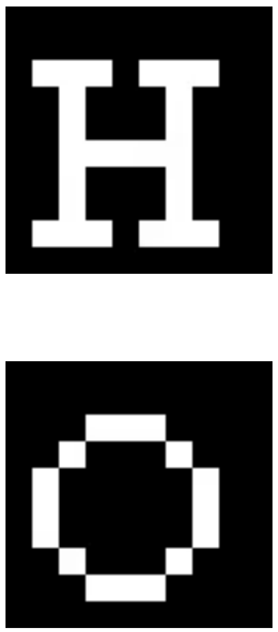

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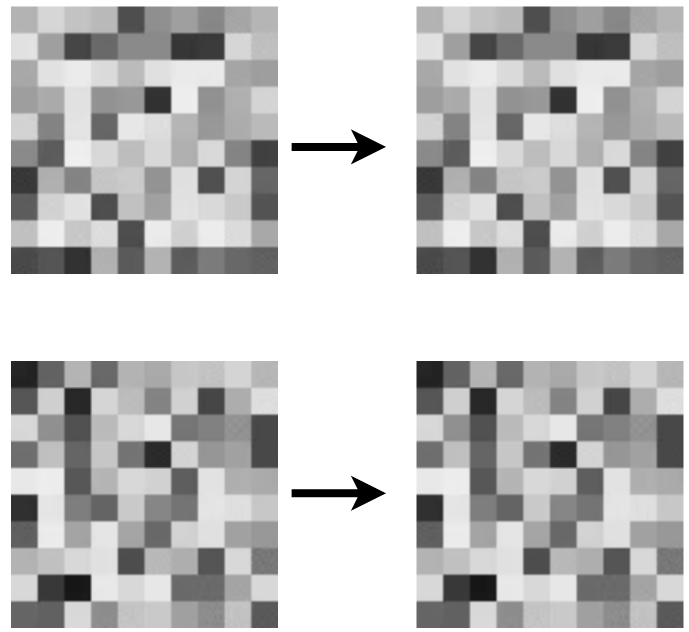

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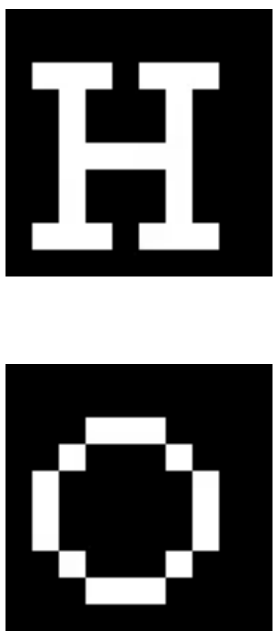
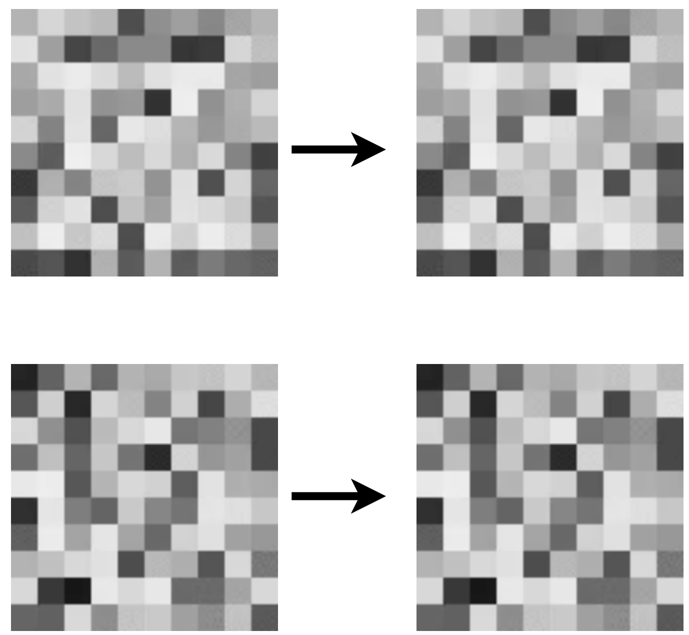
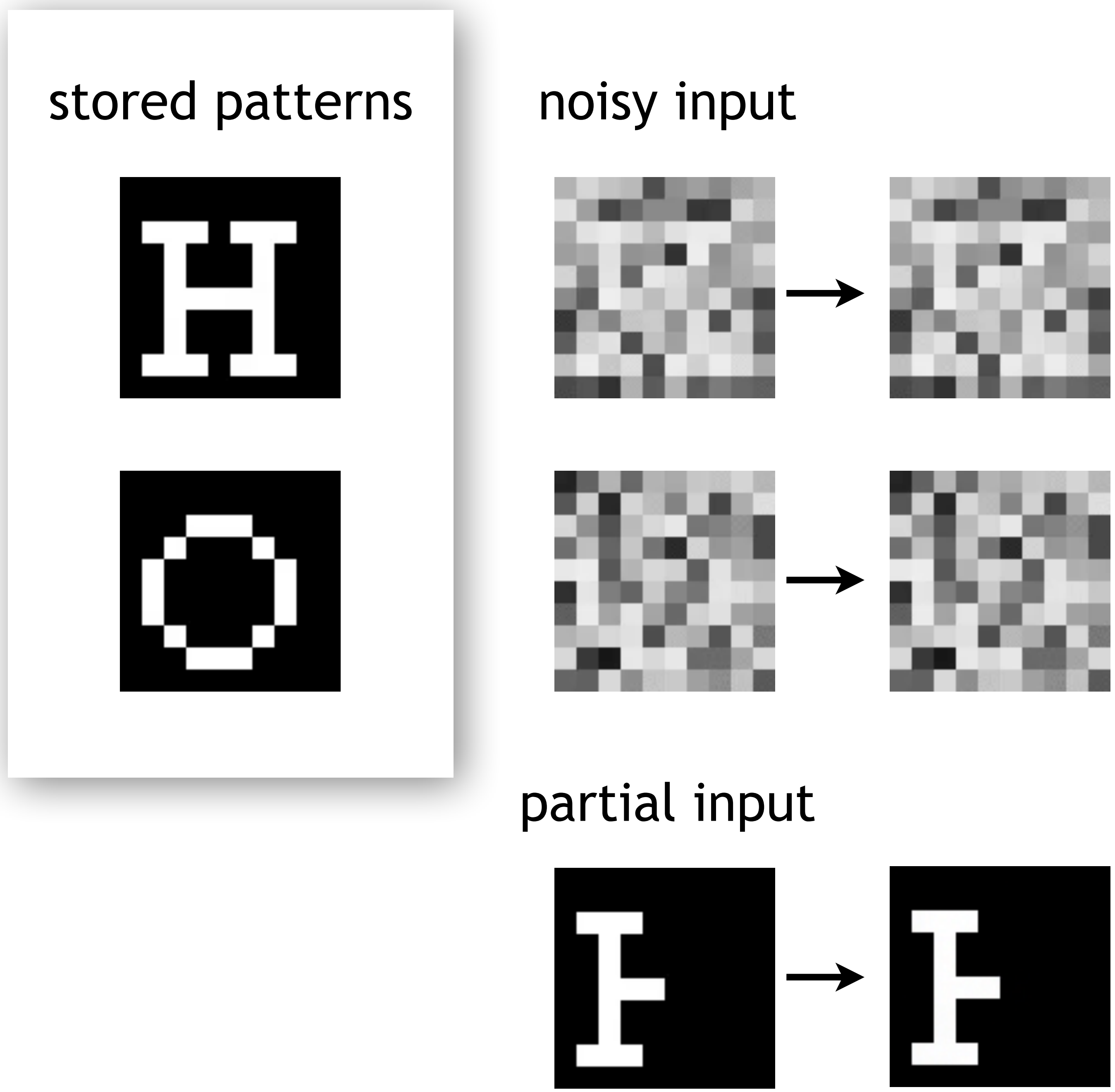


noisy input



partial input

HOPFIELD NETWORK IN OPERATION



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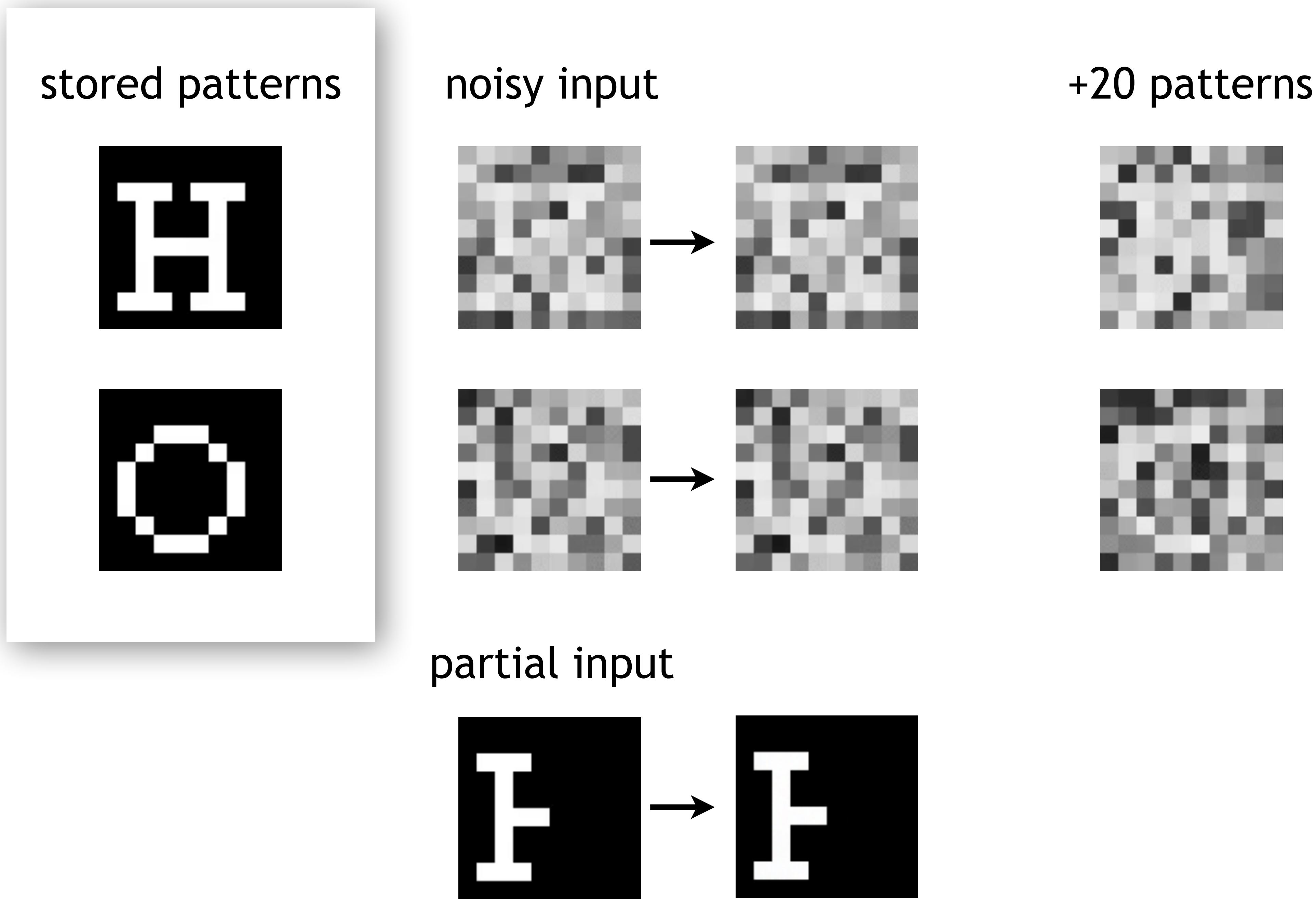

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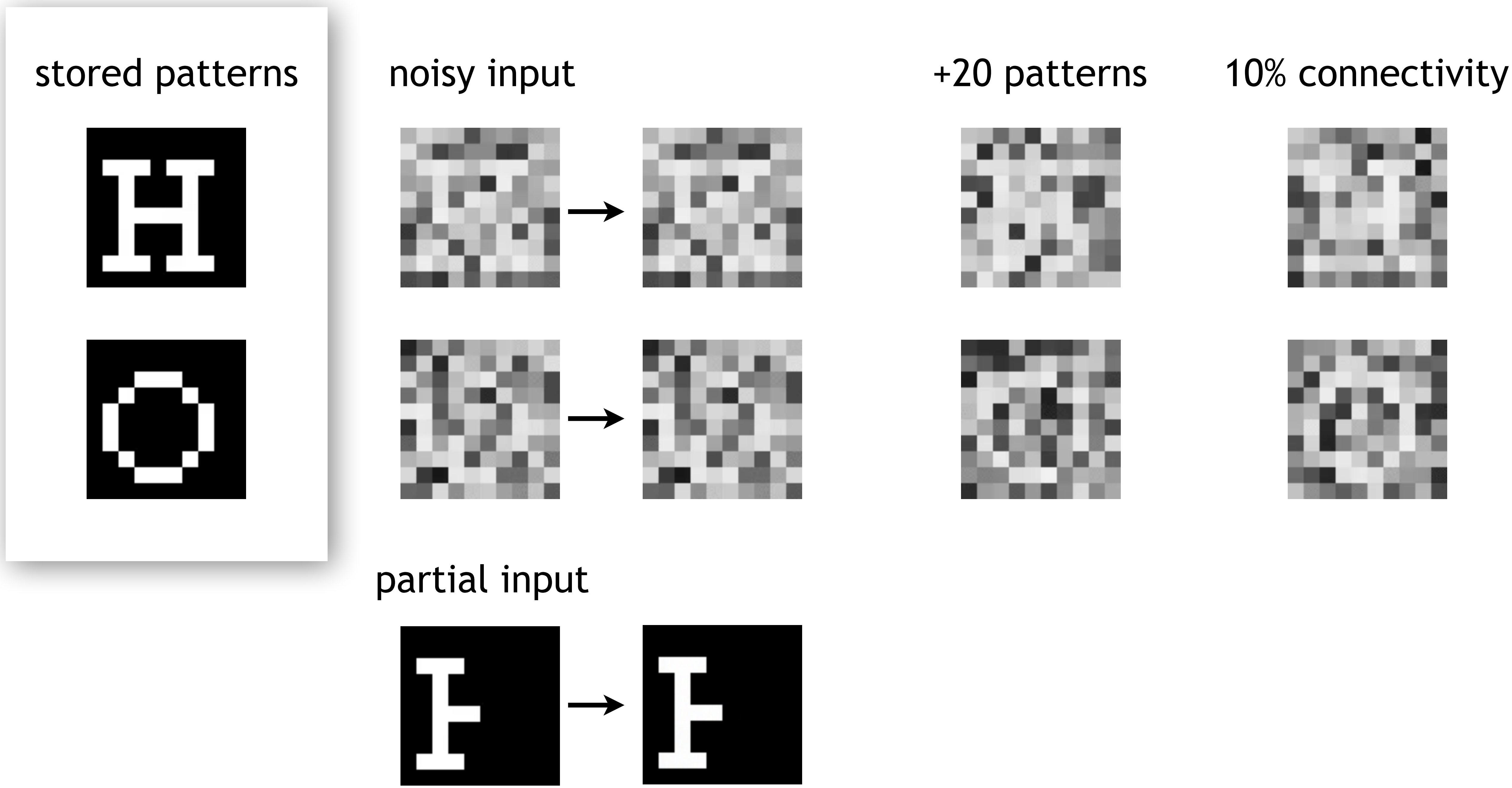


EXPERIMENTAL DATA

NMDA receptors in CA3 and pattern completion in the Morris water maze

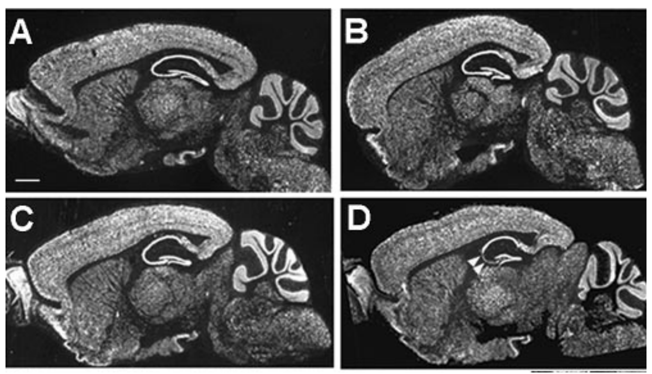
Nakazawa et al, 2002

histology

control mutant

5 weeks

18 weeks



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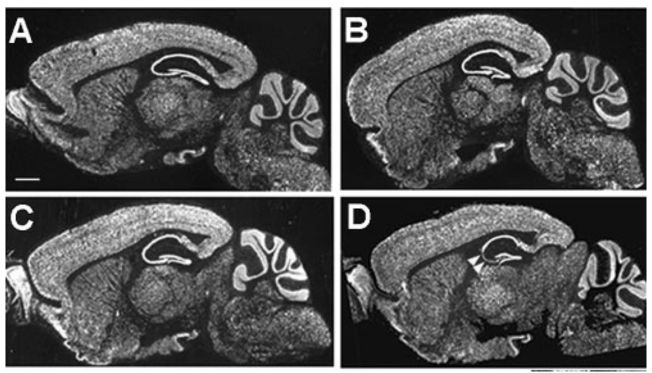

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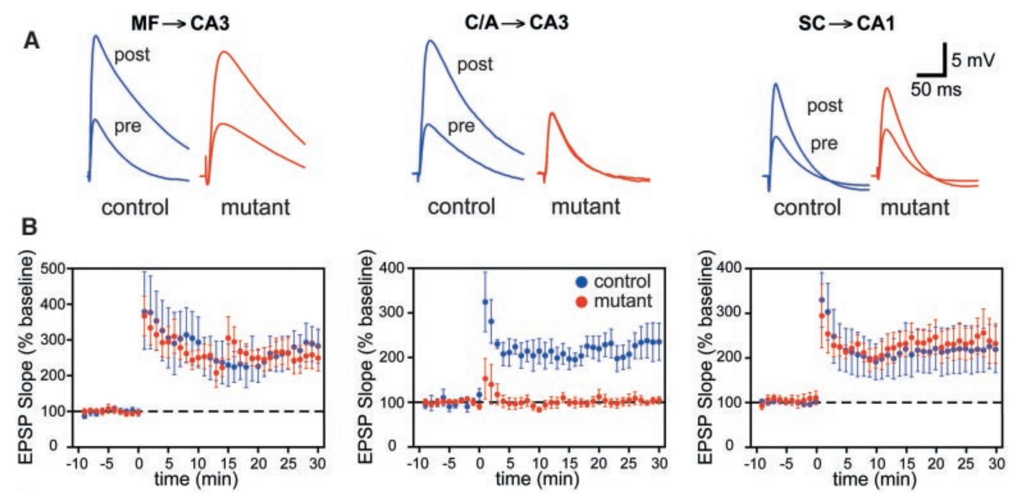

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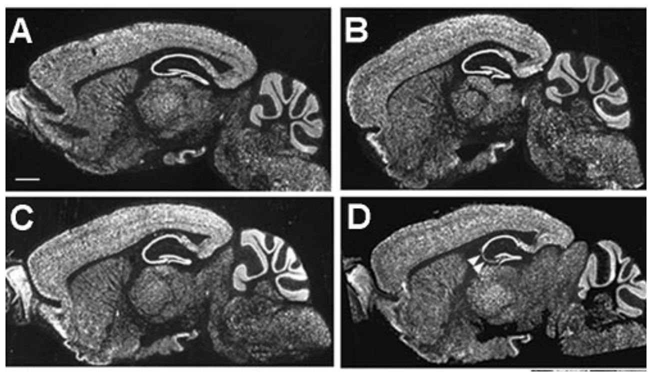

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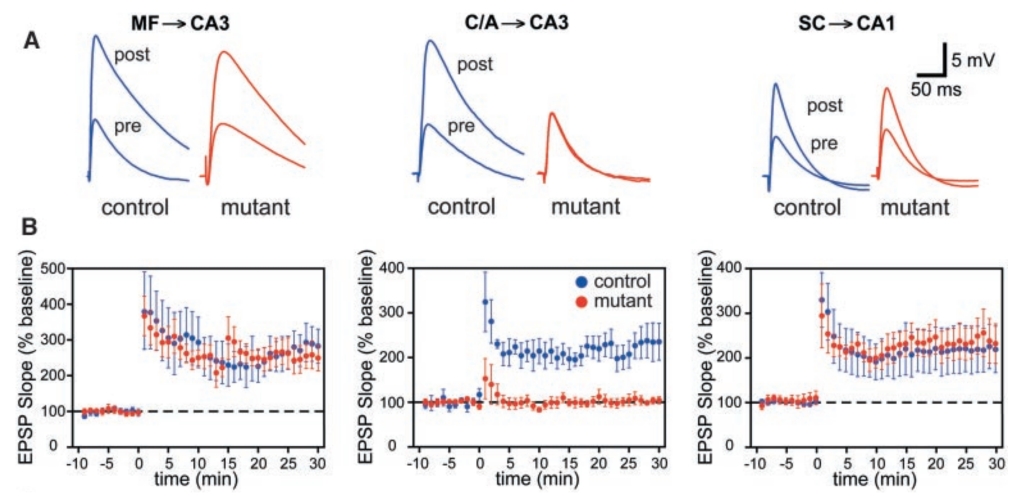

behaviour

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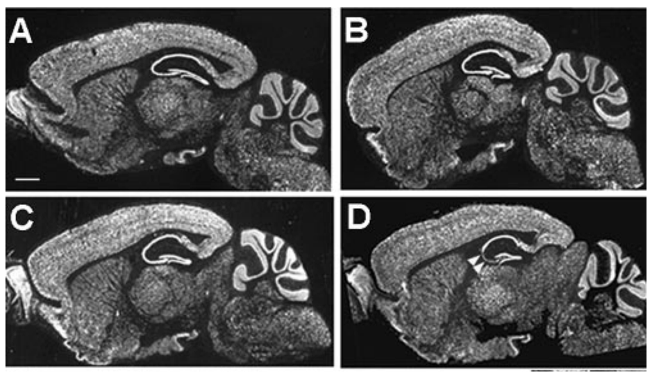

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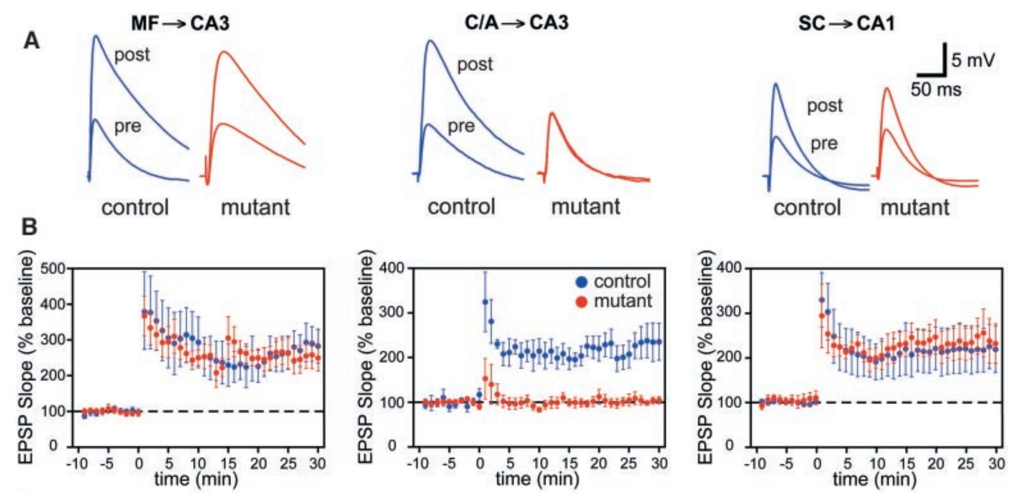

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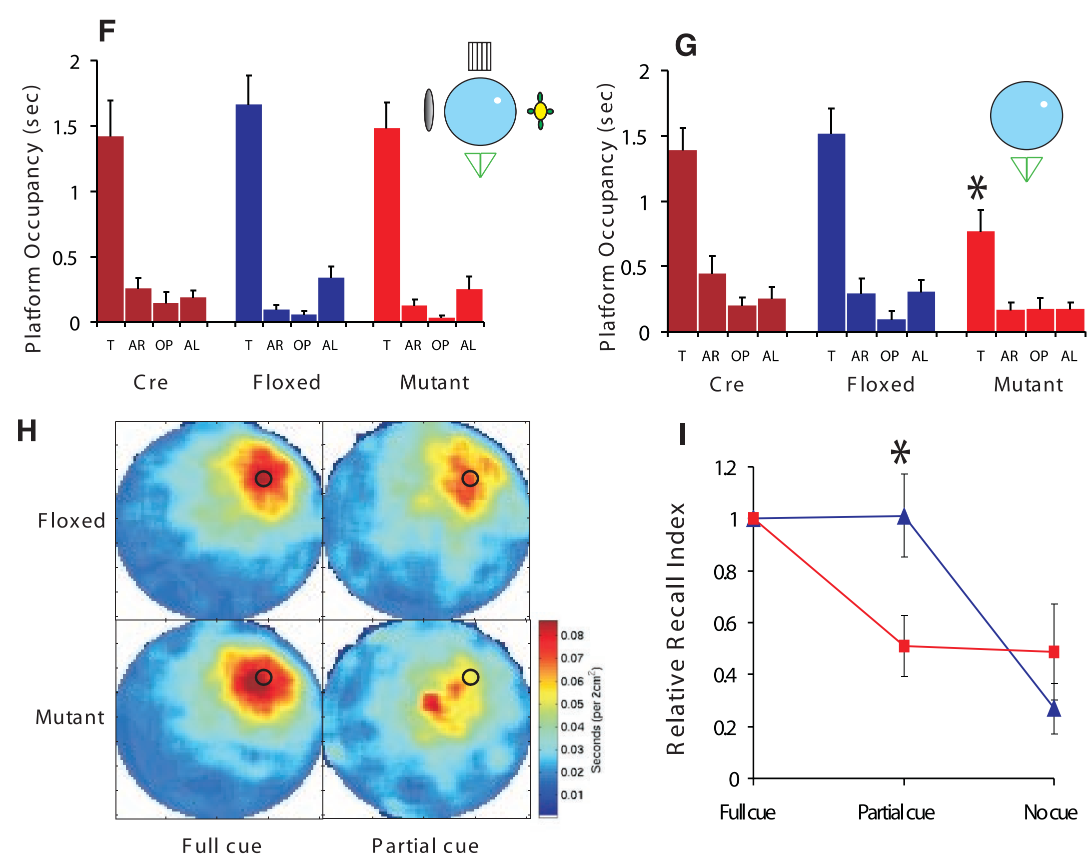

EXPERIMENTAL DATA

attractor dynamics in the population activity hippocampal place cells

Wills et al, 2005

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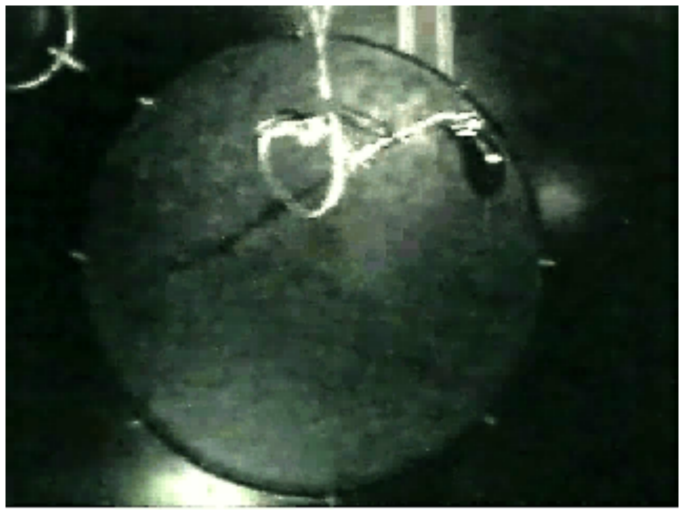

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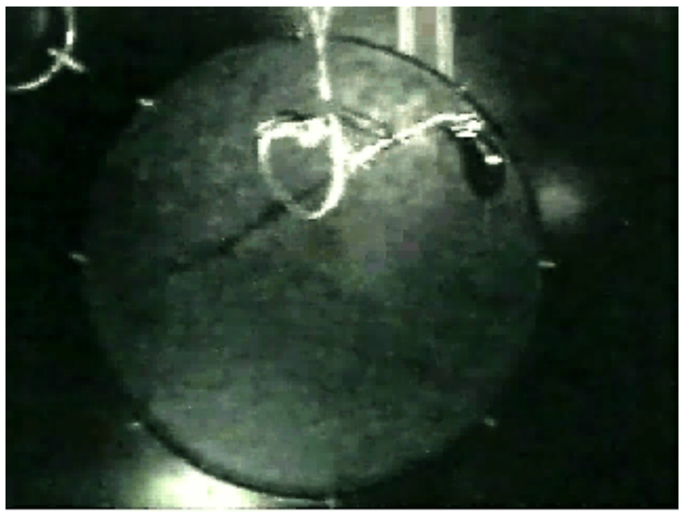

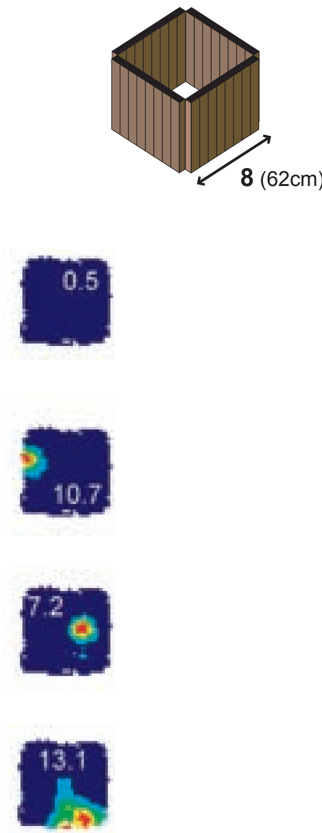

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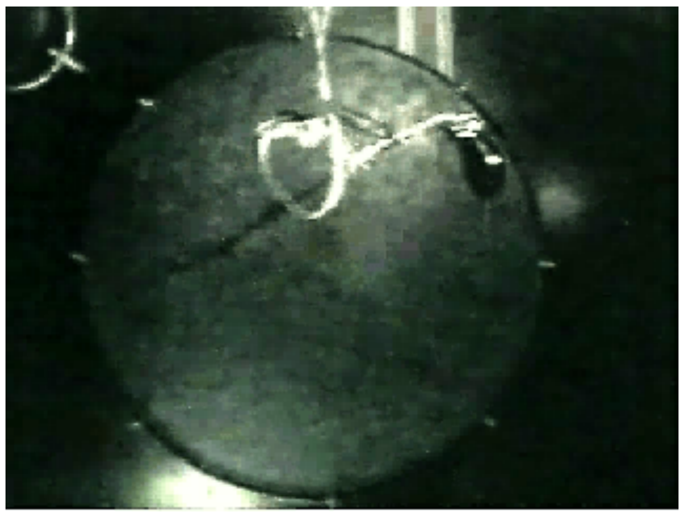

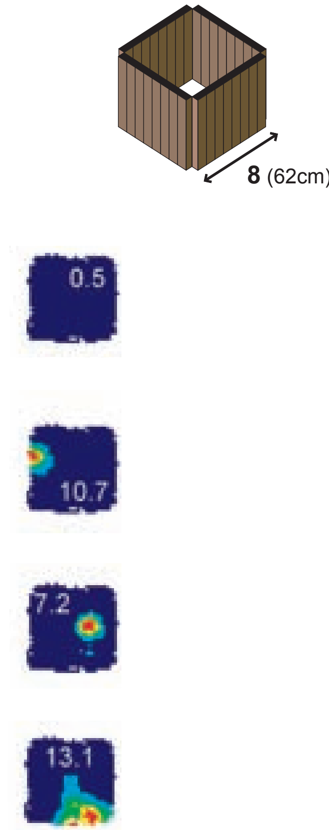

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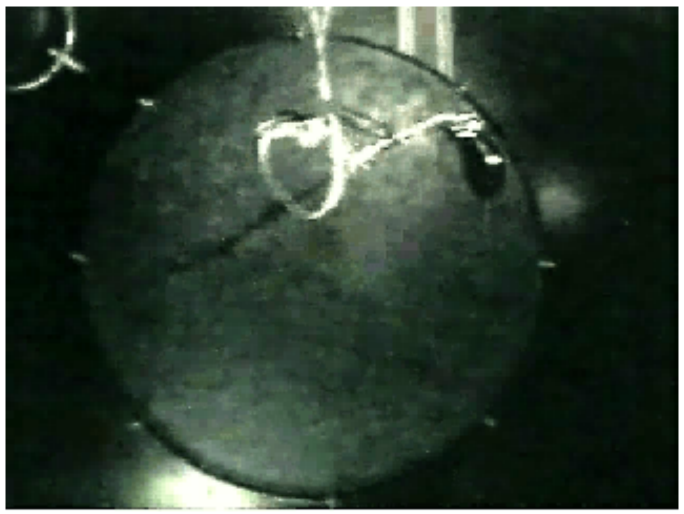

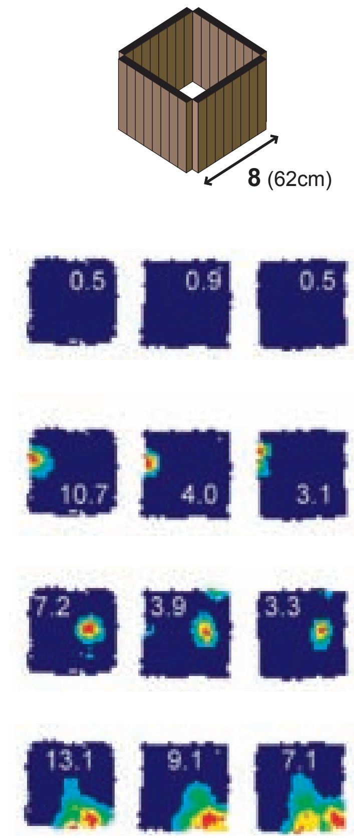

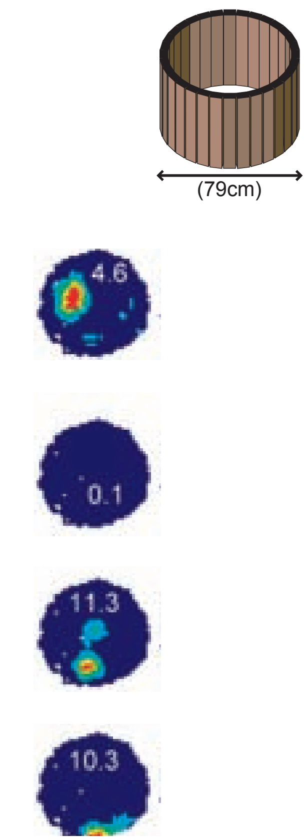

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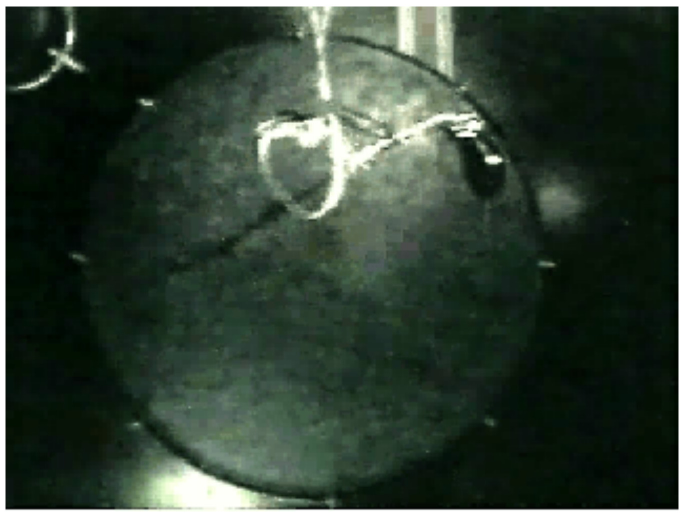

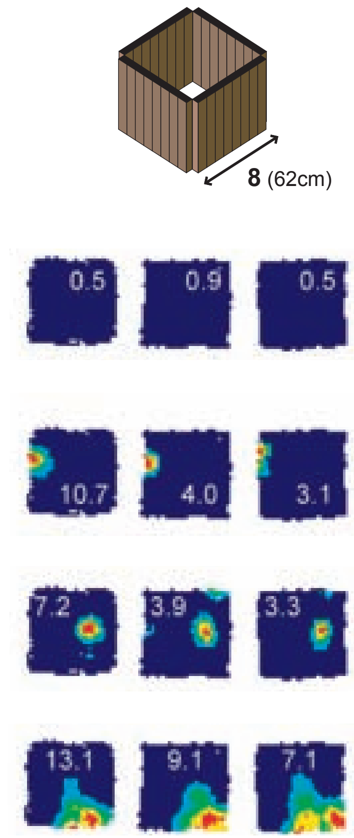

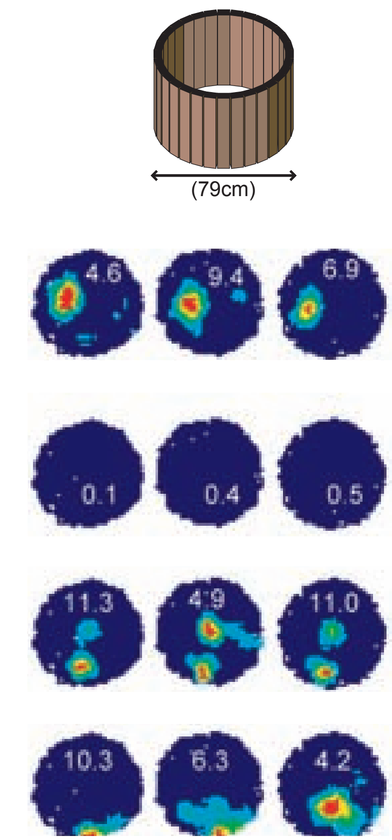

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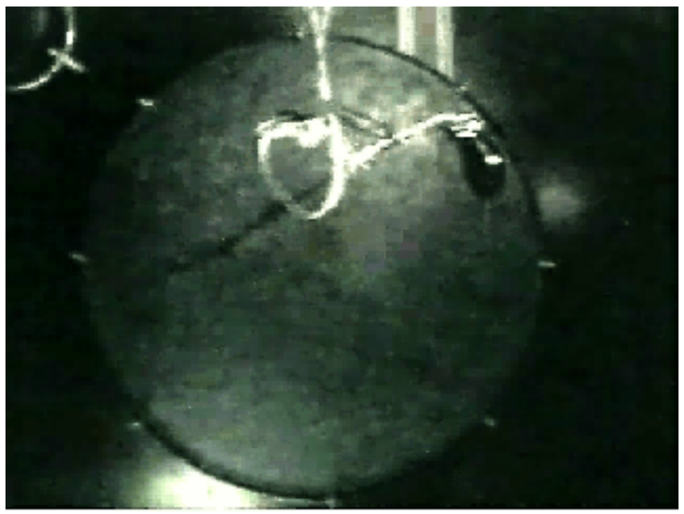

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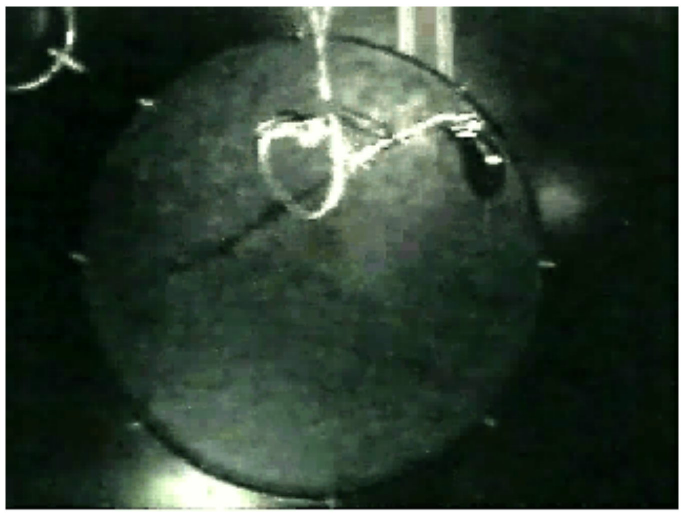

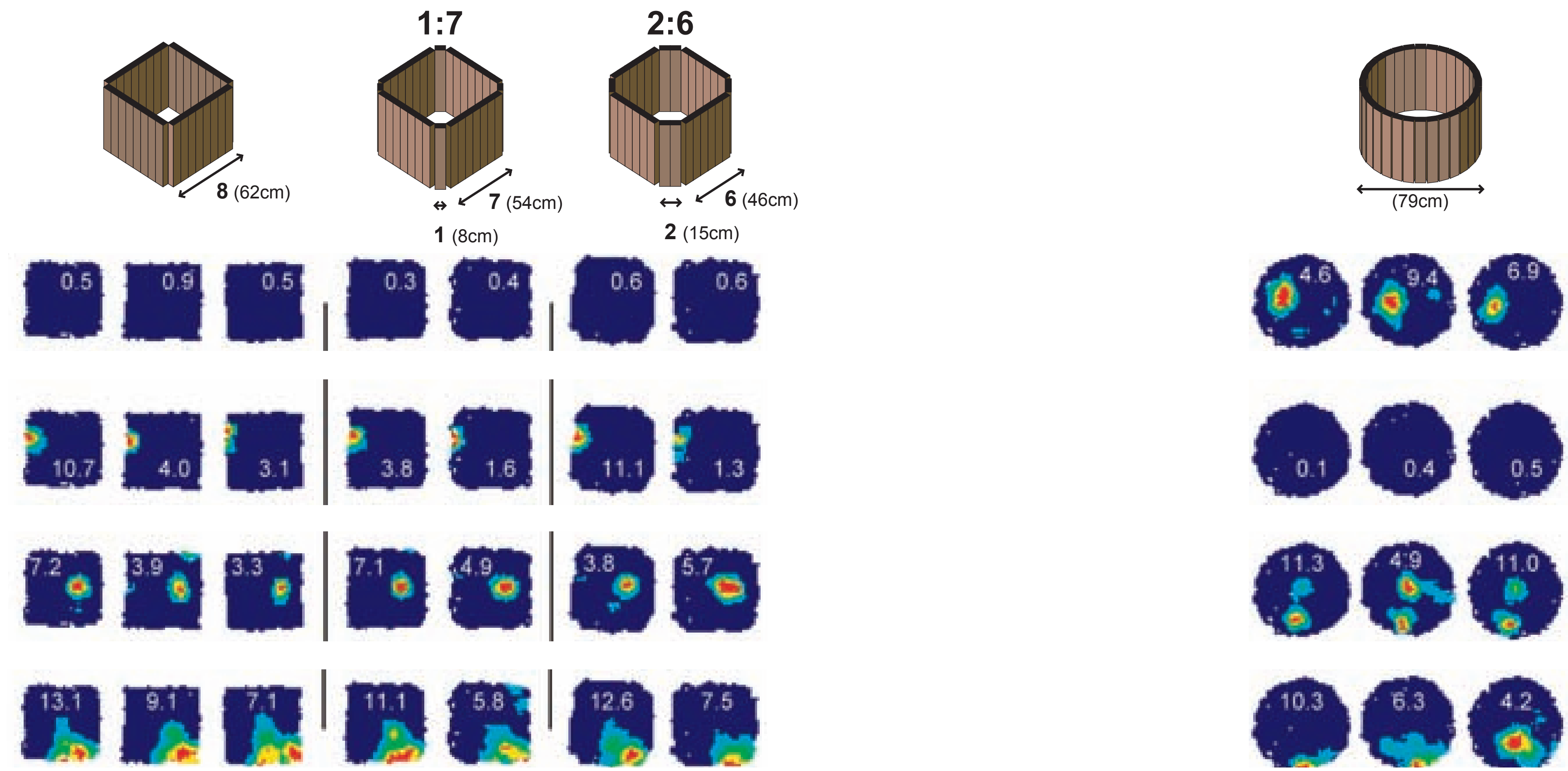

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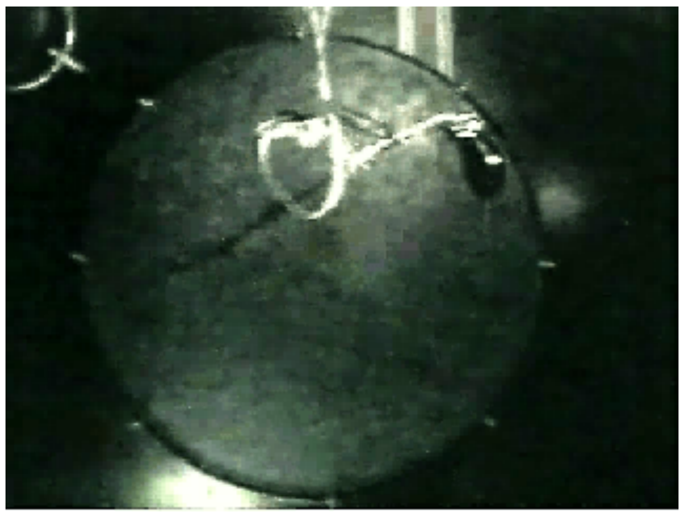

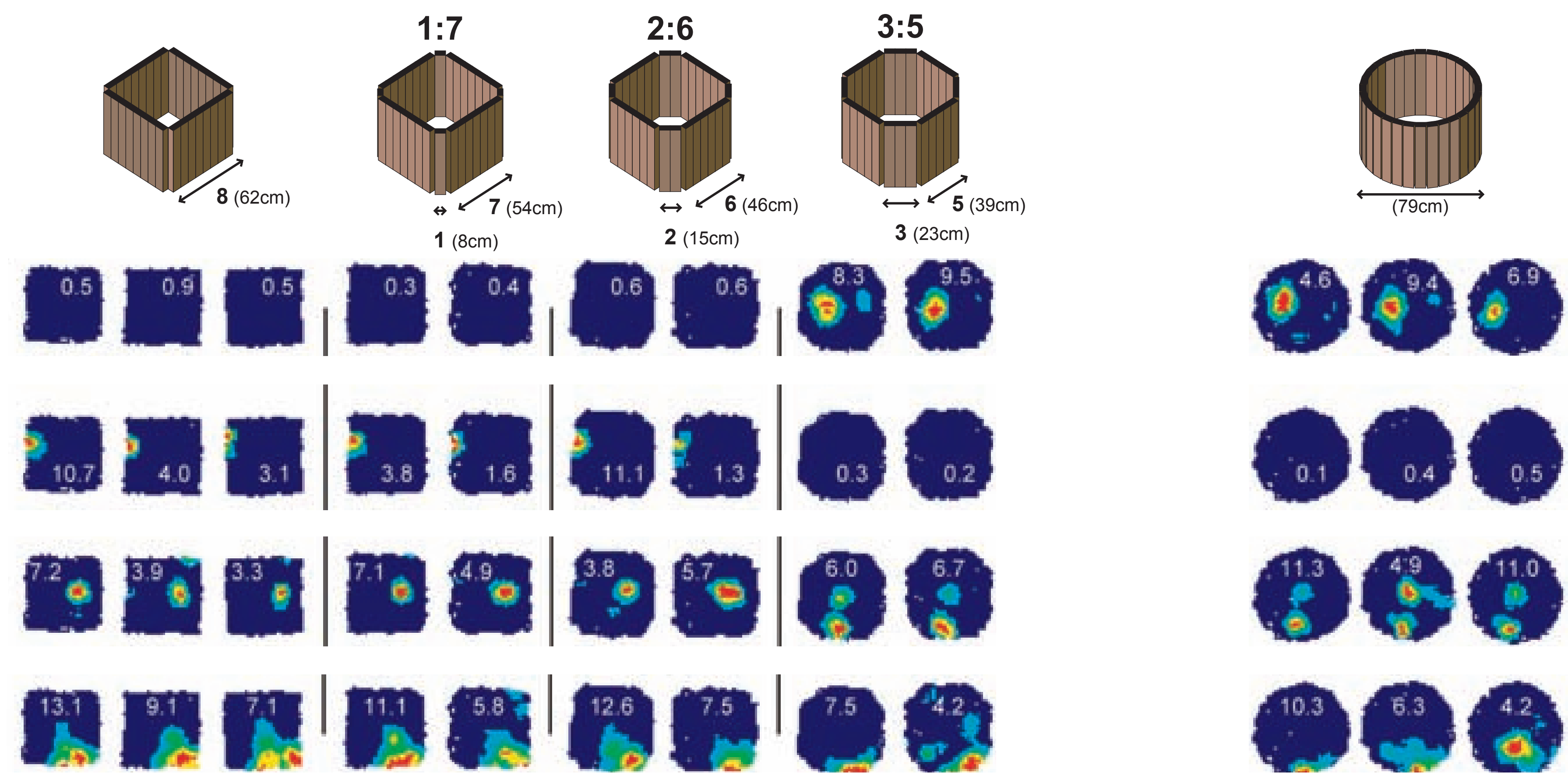

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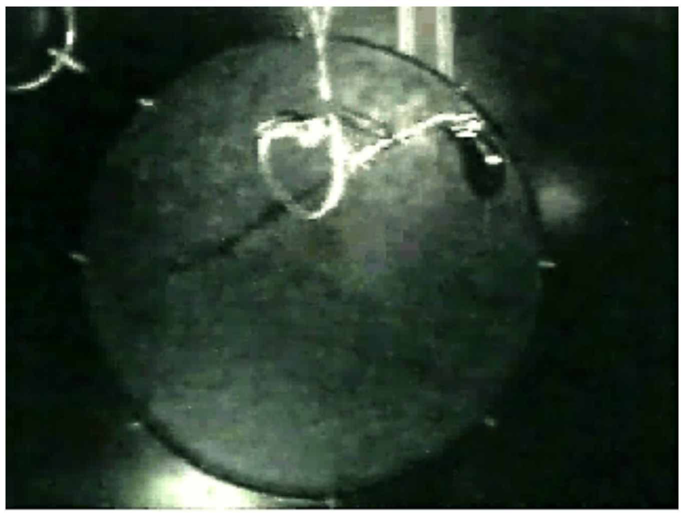

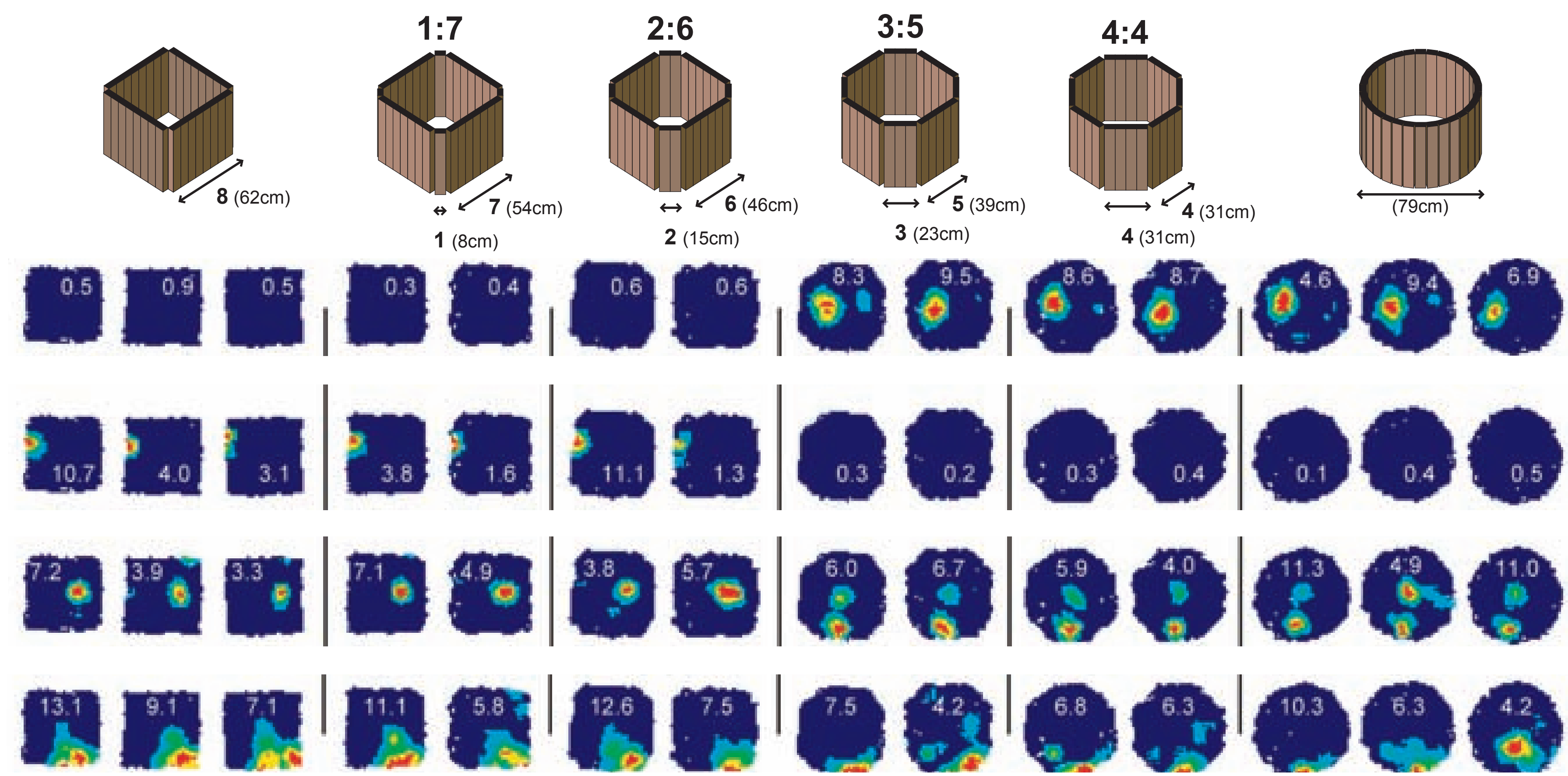

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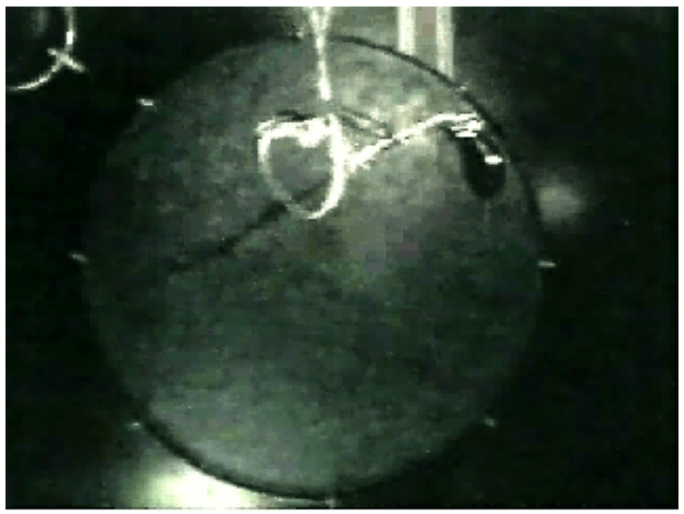

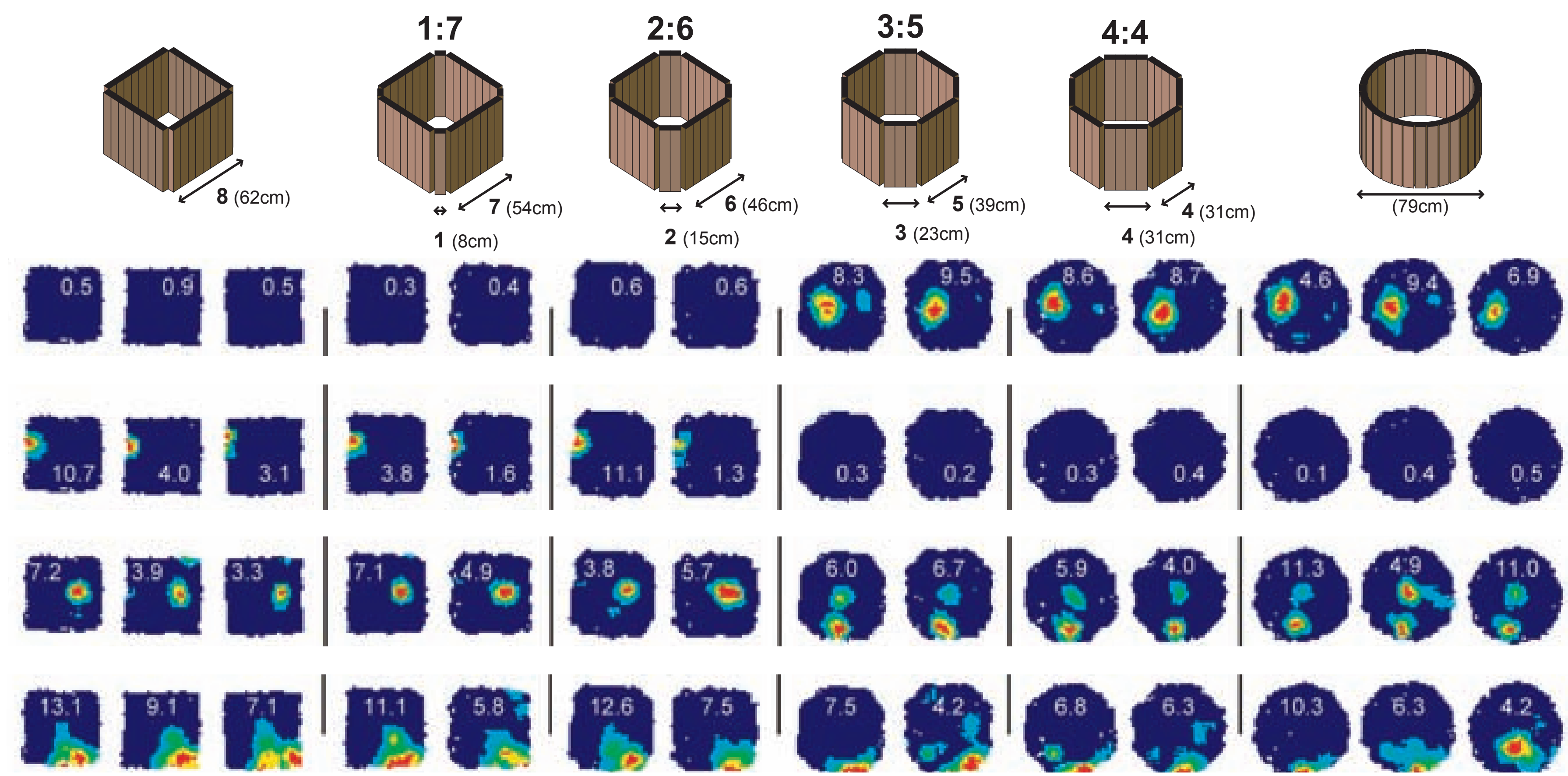

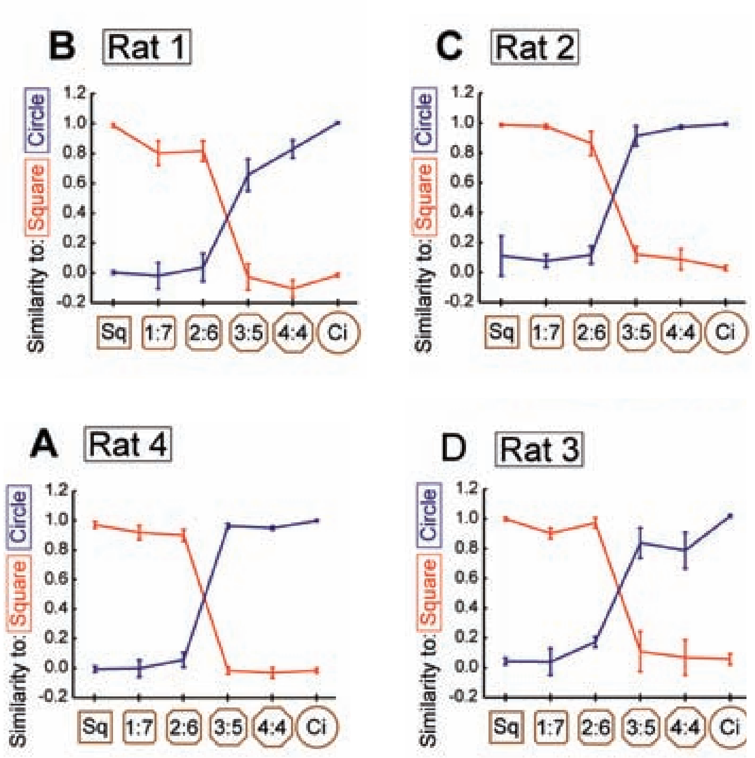

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