SF2 Interim Report

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1 Introduction

Image Processing is a very exciting field in the domain of Signal Processing. In this lab various characteristics of filtered images are explored and are then used to design effective compression techniques for the images.

2 Images

Raw images are simple 2d matrices of numbers. Colormaps are used to map these numbers to what colour is displayed on the screen. Colormaps can be of various different types. In the lab linear and histogram equalization based maps were used.

3 Image Filtering

The simple *low-pass filter* used in the lab has a half cosine impulse response in 1 dimension. *Separable* 2-D filtering can be used to get a 2D low-pass filter by separately filtering the rows and columns. In the lab the output from filtering rows first or filtering columns first was observed and it was concluded that the output in the 2 cases was the same.

High-pass filtered image were created by subtracting the low-pass filtered image from the original image.

3.1 Comparison at different filter lengths

As the filter length is increased the cut-off point of low-pass filter increases and that of the high-pass filter decreases. Hence the low-pass filtered image becomes blurry as filter length increases. The high-pass filtered image accounts for more and more of the relevant features as the filter length increases.

3.2 Energy Content

The energy content of the high-pass filtered image was found to be much lesser than that of low-pass filtered images. Following observations in Section 3.1 the energy content of the high pass filtered images increased with increasing filter length, however it is still much less than that of low-pass filtered images.

4 Laplacian Pyramid

The Laplacian pyramid is a energy compaction scheme based on the observation made in Section 3.22.

Low-pass images have much lower bandwidth than the original image and can hence be sub-sampled without loss of much information. This forms the basis of the Laplacian Pyramid scheme. Matlab scripts were written to encode images as Laplacian pyramids and to reconstruct them from these pyramids. It was observed that without quantization there was no error between the original image and the reconstructed images.

5 Quantization and coding efficiency

For effective data compression the bits needed to store the decimated low-pass image and the errors at each stage of the pyramid need to be lower than that required for the original image. The total entropy of each image was found and was equated to the number of bits needed to store the image, this was done assuming that an efficient coding technique is used.

The number of bits required to store the original image was found to be more than that required for storing the error and the sub-sampled image for a single stage Laplacian pyramid. Hence it was shown that using Laplacian pyramids can achieve data compression. From a information theory perspective the compression is achieved since the low-pass image has highly correlated adjacent matrix locations hence losing alternate pixels does not lose much information. This structure of the low-pass filtered image is intrinsically exploited when they are sub-sampled reducing the number of bits required for storing them. A Markov chain based technique for finding image correlations might exploit this structure well, achieving high compression ratios but this was not explored within the scope of the lab.

The images were then quantized to a step size of 17 to further reduce their entropy and the number of bits required to store them. This reduced the storage required for the one stage Laplacian pyramid scheme and the original image. The effect on the Laplacian pyramid scheme was however much stronger.

As the number of layers of the pyramid are increased the number of bits required for storing the image in the Laplacian pyramid scheme further decreases. This is because each low-passed image(Xn) can be treated as a new image which is then compressed by using an extra stage of the pyramid.

To assess the quality of the quantized image the standard deviation between the quantized image and the original image is used as a measure (r.m.s error).

The r.m.s error at the same amount of quantization is much higher for the image reconstructed from the Laplacian pyramid scheme than that of the directly quantized image. This is because small changes due to quantization in the smaller images of the Laplacian pyramid have a much higher effect on the r.m.s error than the same change in larger images.

To get the same r.m.s error as a direct quantizer of step size 17, a step size of 10.45 is needed to quantize the images in the Laplacian pyramid. This gives a Compression ratio of ***1.58.*** The images produced by direct quantization and by quantizing the Laplacian pyramid look different even when they have the same r.m.s error. The fine color variation in the sky due to the clouds is mostly lost. The direct-quantized image has a lot of high frequency color variations at certain points in the sky(where the intensity thresholds are located), this pattern is not observed in the Laplacian pyramid reconstructed image. These variations arise in the individual quantized images of the Laplacian pyramid but are mostly suppressed in the final reconstructed image.

Equal MSE criterion: To reduce the effect of this problem and optimally quantize the Laplacian pyramid an equal MSE(mean squared error) criterion can be used. In this, the images at various stages of the Laplacian pyramid are quantized so that they contribute approximately equally to the total MSE. This is achieved by measuring the *"impulse response"* of different stages in the reconstructed images. As expected the impulse responses of the smaller images in the Laplacian pyramid are higher than the larger ones. Hence smaller step sizes are used to quantize the smaller images in this scheme. This improves the Compression ratio to around **1.98** which is much better than than the original scheme.

6 Effect of filter length

In sections 4 and 5 above $\mathbf{h}$ is $\frac{1}{4}\,[1\;2\;1]$, in this section $\mathbf{h}$ is changed to $\frac{1}{16}\,[1\;4\;6\;4\;1]$, which has a lower cut-off frequency than the original filter. This altered the impulse response of different images, which grew slower than sections 4 and 5 due to the low cutoff nature of the filter. Using optimized step sizes a Compression ratio **1.77** was obtained which is less than that obtained in section 5. This can be explained by the lower cut-off frequency of the filter which causes the low-pass images to have lower energies making the Laplacian Pyramid scheme less effective.

7 Conclusion

- A separable 2d low pass filter is easy to implement and works better than an equivalent non-separable filter.
- As the length of half-cosine filters increases their cut-off frequency decreases and the low-pass filtered images obtained using them get more blurry.
- In general low-pass images have more energy than high-pass images
- A Laplacian pyramid is an excellent method to achieve image compression
- Quantizing images can reduce the storage space they require.
- When quantizing images in the Laplacian pyramid an equal MSE criterion can be used to optimize the compression ratio. This can achieve compression ratios of around 2.
- A longer filter reduces the effectiveness of the Laplacian pyramid method.

Appendix

(a) High-pass for N=5,15

(b) Low-pass for N=5,15

Figure 1: High-pass and Low-pass filtered images for different filter lengths

Filter length	Low-pass	High-Pass
5	$1.28 \text{x} 10^9$	$2.52 \text{x} 10^7$
15	$1.25 \text{x} 10^9$	$4.80 \text{x} 10^7$
25	$1.24 \text{x} 10^9$	$5.79 \text{x} 10^7$

Table 1: Comparing energy of Low-pass and High-pass images for different filter lengths

(a) Direct Quantized

(b) Laplacian Scheme with quantization

Figure 2: Direct and Laplacian Quantized images with same r.m.s error