Computational Analysis of Signal Attenuation in First-Order Subwoofer and Tweeter Circuits

Saketh Ayyagari, Victoria Collemi, Nathan Martin, and Krish Shah

Abstract—**Low-pass and high-pass filters, composed of resistor-capacitor circuits, split a signal into amplified and attenuated frequencies determined by a specified cutoff frequency. This divides an audio signal into two different outputs, one managing lower frequencies (e.g. bass) into a "subwoofer" and the other managing higher frequencies (e.g. treble) into a "tweeter". We modeled these filters using their governing differential equations in Python to calculate the effect each filter had on the output voltage amplitude compared to that of the input voltage for a given audio signal. Then, we created Bode Plots to visualize the effect different frequencies can have on the signal output. Our simulations aligned with circuit principles, validating our simulation conditions for other first-order filters.**

Index Terms—**RC circuits, low-pass filter, high-pass filter, Bode plots, transfer function, Python, cutoff frequency, voltage**

I. Introduction

Low-pass and high-pass filters are two resistor-capacitor circuits commonly used in signal processing. The former only allows frequencies below a cutoff frequency, while the latter only allows frequencies higher than the cutoff. The cutoff frequency can be described by:

$$f_c = \frac{1}{2\pi\tau}, \tag{1}$$

where τ is the time constant of the resistor-capacitor circuit. The time constant of the circuit can be described by $\tau = R_{eq}C_{eq}$, where R_{eq} is the equivalent resistance of the circuit and C_{eq} describes the equivalent capacitance; it represents the time needed to charge a capacitor to 63% of its maximum capacity or discharge it to 37% of its maximum capacity. As shown by the cutoff frequency equation, increasing the total resistance or total capacitance lowers the filter's cut-off frequency, whereas decreasing either increases the cutoff frequency. When audio is run through a low-pass filter, the resulting sample contains only those with lower pitches as lower-pitched sounds have lower frequencies. Similarly, when audio is run through a high-pass filter, the result is a sample consisting of higher pitches as they correspond to higher frequencies.

how

We simulated both filters in Python and visualized the effect of each filter on the original signal using a Bode plot. The effect of a specific frequency on the output signal is modeled using a transfer function, which is easier to model than a differential equation and can help determine an output without respect to another variable other than time. These findings have proven useful for soundboards (for which the effects demonstrated in this experiment could be utilized), enhancing clarity in communication devices, and radar equipment [3].

yes but you not doing step response not like freq. response

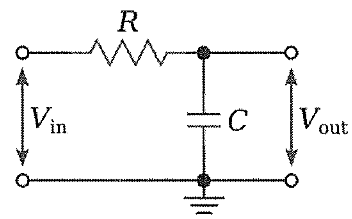

Fig. 1. Low-Pass Filter Schematic [7]

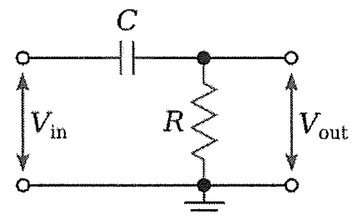

Fig. 2. High-Pass Filter Schematic [8]

II. Methods

We first converted the raw audio file provided by NM [2], from a .MP3 format to a .WAV using an online converter. We then used the `librosa` Python library to process the audio, ensuring that the sample rate remained consistent throughout the analysis [1]. A consistent sample rate is important for accurate signal processing, as it makes sure that each audio sample represents that same time interval. This consistency is necessary for reliable filtering and visualization, avoiding distortions or errors in the output. The audio filtering was done using digital approximations of RC circuit equations to simulate the low-pass and high-pass filters.

The differential equation that describes the low-pass filter is

$$RC\frac{dV_{out}}{dt} + V_{out} = V_{in} \tag{2}$$

, which allows lower frequencies to pass through and attenuates higher frequencies.

The differential equation that describes the high-pass filter is

$$V_{out} + RC\frac{dV_{out}}{dt} = RC\frac{dV_{in}}{dt} \tag{3}$$

, which allows higher frequencies to pass through and attenuates lower frequencies. These equations were numeri-

Can you give transfer function in Laplace domain?

seems too high?

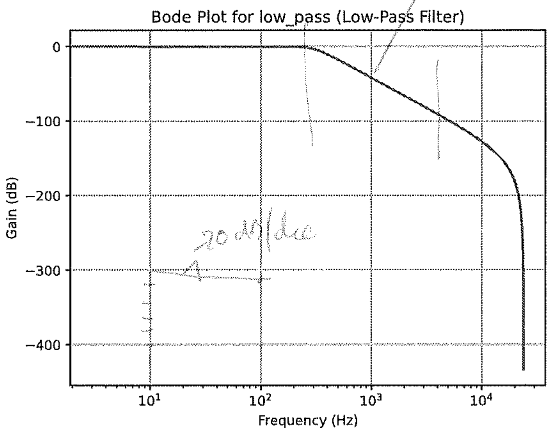

Fig. 3. Low-Pass Filter Bode Plot (all audio effects produced similar results) [9]

Wrong roll off

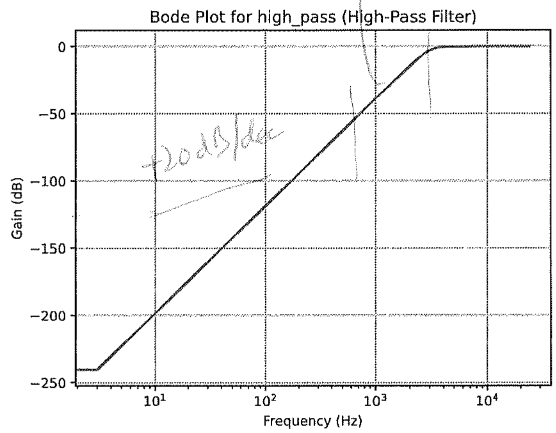

Fig. 4. High-Pass Filter Bode Plot (all audio effects produced similar results) [9]

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cally solved using recursive filtering, a technique where each output depends on both current inputs and previous outputs, to extract low frequencies for subwoofers and high frequencies for tweeters.

In addition to crossover filtering, other audio effects were applied to the signal to modify its frequency content (spectral characteristics). Distortion was introduced by clipping the signal amplitude to create a non-linear response. The pitch change was performed using the `pitch_shift()` function, which changes the spectral frequency content of the signal. The Cartoon Effect was achieved by shifting the signal $+8$ semitones, increasing its pitch. The Robot Effect was done by shifting the signal -3 semitones, lowering its pitch to obtain a mechanical sound.

To see how the filters behave, we computed the input voltage V_{in} and output voltage V_{out} for each processed signal. Then, we plotted them over time to visualize the signal transformation for each filtering stage. The filters were evaluated using Bode plots which show the Gain (in dB) vs Frequency (in Hz) response. The magnitude response in decibels (dB) is:

$$H(f) = 20 \log_{10} |\frac{V_{out}(f)}{V_{in}(f)}| \tag{4}$$

The `freqz()` function was used to compute the digital frequency response to get proper logarithmic scaling.

III. Results

A. Low-Pass Filter

For the low-pass simulations, the capacitor was in series with the resistor and connected to ground. The low-pass circuit was modeled with a 1 kΩ resistor and a $0.53\mu F$ capacitor.

As seen in Fig. 3 , the experimental cutoff frequency (at -3 dB) occurred at approximately 300 Hz across all of the audio samples, demonstrating the simulation's consistency across all frequency ranges for each of the audio effects applied to NM's original recording. This also indicates that frequencies above 300 Hz were attenuated by the filter. We also observed the transition periods across Fig. 3 were within the same frequency range (250 - 400 Hz) and linear with a negative slope. The slope of the transition period across all samples was approximately -20 dB/Decade. In Fig. 3 , we observed the amplitude of the output voltage of the simulated low-pass filter was lower than that of the input voltage across the duration of the audio samples.

B. High-Pass Filter

For high-pass simulations, the resistor was in series with the capacitor and connected to ground. The high-pass circuit was also modeled with a 1 kΩ resistor, but modeled with a 0.053 μF capacitor to lower its cutoff frequency.

As seen in Fig. 4 , the experimental cutoff frequency (at -3 dB) occurred at approximately 3000 Hz across all audio samples, demonstrating the consistency of the simulation across all frequency ranges in each of the audio effects applied to the original NM recording. This also indicates that frequencies below 3000 Hz were attenuated by the filter. We also observed the transition periods across Fig. 4 were within the same frequency range (2500 - 4000 Hz) and linear with a positive slope, which was approximately +20 dB/Decade. This value is the same magnitude as the transition slope for the Low-Pass Filter simulations, demonstrating a similar mathematical relationship across the two filter circuits. In Fig. 4 , we observed the amplitude of the output voltage of the simulated high-pass filter was at least that of the input voltage across the duration of the audio samples.

IV. Discussion

Based on the values of the resistor and capacitors from the Low-Pass and High-Pass filters, the theoretical time constants for each circuit are $5.3 \times 10^{-4}s$ and $5.3 \times 10^{-5}s$, respectively (Low-Pass: $R = 1k\Omega, C_1 = 0.53\mu F$; High-Pass: $R = 1k\Omega, C_2 = 0.053\mu F$). Referencing (1) and the respective component values given, the theoretical cutoff frequencies for the low-pass and high-pass filters are 300.29 Hz and 3002.92

Your Bode plots do not match your circuits

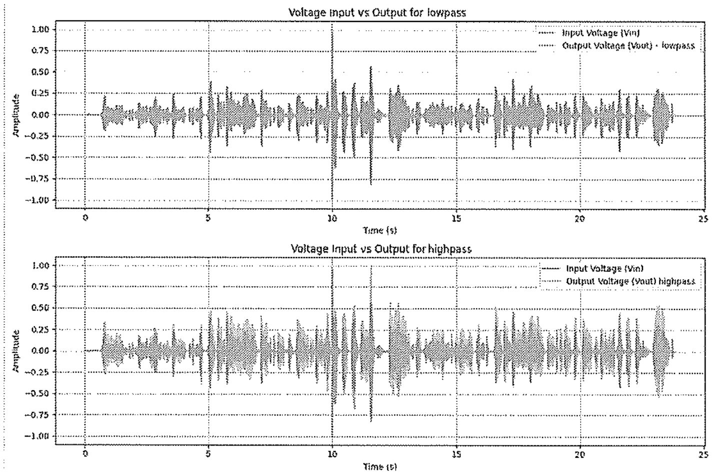

Fig. 5. Top: Input VS Output signal through low-pass filter. Bottom: Input VS Output signal through high-pass filter. Blue lines indicate input voltage while orange lines indicate output voltage.)

Hz, respectively. We then calculated the percent error δ of our observed cutoff frequency values using the equation:

$$\delta = \frac{f_a - f_e}{f_e} \times 100 \tag{5}$$

where f_a is the experimental cutoff frequency and f_e is the expected cutoff frequency. We found that $\delta_{LowP} = -0.096\%$, indicating our experimental frequency for the low-pass circuit was very close to the expected value in the simulations. Similarly, we found that $\delta_{HighP} = -0.097\%$, indicating our experimental frequency for the high-pass circuit was also very close to the expected value.

When (4) is zero or infinity, there exists either zeros or poles in the domain of the transfer function, which can cause changes in slope in the Bode plots for each circuit, with zeros and poles indicating positive and negative slopes, respectively [5]. To be consistent with the filters, the slope in the transition periods of Figs. 3 and 4 should be -20 dB/Decade for low-pass filters and +20 dB/Decade for high-pass filters [4]. Our experimental results are consistent with the expected behavior of both filters since the slope values are the same as the theoretical values predicted by (4).

In Fig 5 , as the amplitude of the input voltage increases, the amplitude of the output voltage through the low-pass filter generally decreases, while that of the high-pass filter generally increases. This behavior is consistent with (4), further implying each first-order filter effectively attenuates the proper frequencies. However, an anomaly occurs at $t = 10s$, where the low-pass and high-pass output voltages are greater than and less than the input voltage, respectively.

To evaluate the effectiveness of our simulated filters in isolating their target frequencies, we can determine the theoretical quality factor Q of our circuit using the following equation [6].

$$Q = \frac{f_c}{BW} \tag{6}$$

Where f_c is the center frequency, and BW is the bandwidth. Since we simulated first-order filters, the values of the center frequency and bandwidth are approximately the cutoff frequency of both filters. We found that $Q_{LowP} = Q_{HighP} = 1$, indicating both filters successfully attenuate the desired signals without going beyond the respective cutoff frequencies. The observed consistency of our results for passive first-order filters indicates the validity of using our simulations for devices governed by conventional circuit principles.

Any outlying values of all experimentally determined quantities across simulations are most likely due to the limited number of people and audio filters sampled for testing (i.e. not sampling people with different average pitches and ranges; more audio filters that could have been applied). Across both circuit simulations, we collected data from a very small dataset. Although our results correspond with DC circuit principles, more trials and testing should be conducted to observe if they alter the input-output voltage graphs and the Bode plots. Additionally, our simulations should be implemented and tested on a physical circuit (Figs. 1, 2) using an oscilloscope and a function generator to experimentally verify our results.

V. Conclusion

We simulated and analyzed a 300 Hz low-pass filter and 3000 Hz high-pass filter. Through the anticipated frequency responses of the filters, we confirmed the cutoff frequency and transition slope theory; additionally, our filters had slight deviations from theoretical values. These close results indicate the demonstrated techniques are sufficient for first-order filters. Our quality factor of approximately $Q = 1$ suggests good frequency isolation and indicates the filters operate as expected. Although the overall results were reasonable, few anomalies in the input/output voltage relations will require further research and real-world experiments. Additionally, using more audio samples could allow for a more thorough analysis, along with making the filter design more sophisticated (i.e. second-order filters compared to first-order). Converting our simulation into a physical circuit could also give a deeper understanding of the real-world difficulties and benefits of these filter systems.

Acknowledgments

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References

[1] B. McFee et al., "Librosa: Audio and Music Signal Processing in Python," 2023. [Online]. Available: https://librosa.org/doc/latest/index.html.

[2] ~~Ween. "Mutilated Lips." The Mollusk. Elektra Records, 1997. CD.~~

[3] D. Higginson, "High Pass vs. Lowpass Filters: Which is Best for Your Application?," 2024. https://www.qmicrowave.com/blog/high-pass-vs-lowpass-filters-which-is-best-for-your-application: :text=RF%20and%20electrical%20engineers%20utilize,clear%20and%. Accessed: Feb. 2, 2025.

explain in methods

[4] "Bode Plot Rules Review," Swarthmore College LPSA, https://lpsa.swarthmore.edu/Bode/BodeReviewRules.html. Accessed: Feb. 2, 2025.
[5] "Bode Plots (Real Zeros and Poles)," Bit Driven Circuits, https://bitdrivencircuits.com/Circuit_Analysis/Phasors_AC/FR_bode_4.html. Accessed: Feb. 2, 2025.
[6] "Q Factor and its Relevance in Electrical Circuits," Electronics For You, https://www.electronicsforu.com/technology-trends/learn-electronics/q-factor-relevance-electrical-circuits. Accessed: Feb. 2, 2025.
[7] EC Studio Systems, "Passive Low Pass Filter," EC Studio Systems, Available at: https://ecstudiosystems.com/discover/textbooks/basic-electronics/filters/passive-low-pass-filter/ [Accessed: 27-Jan-2025].
[8] EC Studio Systems, "Passive High Pass Filter," EC Studio Systems, Available at: https://ecstudiosystems.com/discover/textbooks/basic-electronics/filters/passive-high-pass-filter/ [Accessed: 27-Jan-2025].
[9] K. Shah, "Circuits," GitHub, Available at: https://github.com/krishshah-fr/circuits [Accessed: 27-Jan-2025].

code

need more details on
the simulation/link
to code