

# Mapping Electric Potential and Electric Field Lab

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**Abstract**—Equipotential curves (also called isolines) are lines of constant electric potential where a charged object does no work moving along them. These lines can help determine the direction of an electric field (represented by a vector field) as the direction of the field at a point will be perpendicular to an equipotential line. We used two oppositely charged hex nuts to observe the relationship between potential and electric fields and measured the potential at different points on a 6x6 plane. After measuring the potential at every point, we used Matplotlib to generate graphs of both equipotentials and predicted electric field lines. As a result, we were able to experimentally verify the relationship between the potential of a charge and the electric field it causes.

## I. INTRODUCTION

The electric potential  $V$  of a particle can be described as the amount of electrical potential energy per unit coulomb of charge. This can be expressed by the following equation:

$$V = \frac{U_E}{q} = k \frac{Q}{r}$$

where  $U_E$  is the electric potential energy between a charge  $Q$  and a small test charge  $q$ , and  $r$  is the distance from the point of interest to the location of the charge  $Q$ . Multiple points along a plane where the potential  $V$  is equal can be represented by equipotential lines. Using these equipotential lines, the electric field generated by the charge  $Q$  can be described as:

$$E_x = -\frac{dV}{dX}, E_y = -\frac{dV}{dY} \quad (1)$$

where  $X$  and  $Y$  describe directions perpendicular to each other. These equations indicate that a larger magnitude of an electric potential difference with respect to a particular direction results in a larger magnitude of the electric field antiparallel to that direction. This also implies that the direction of the electric field will always point in the direction of decreasing electric potential and be perpendicular to the isolines.

## II. METHODS AND MATERIALS

To measure the electric potential difference, we used a small container, graphing paper, a power supply, 2 metal hex nuts, a digital voltmeter, 4 leads, and a camera (via smartphone). We filled our container with water to approximately 0.01m of water in depth. Then, we measured the voltage of the power supply when set to around 10 volts (V), the real voltage was approximately 8.89 V. After measuring the voltage output and turning off the power supply, we attached 2 leads to connect

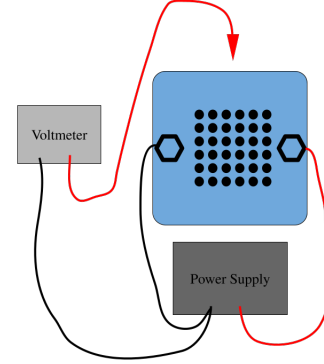


Fig. 1. Diagram of our setup

the power supply to the hex nuts. These were then placed on the left and right sides of the container initially. On the graph paper, we marked out 36 equally spaced points in a 6x6 square to measure the voltage, which was then placed under the container so the center of the square was under the center of the container. The power supply was set to the aforementioned position where we then measured and took photos of the voltage at each of the points. Following the first trial, we rotated the orientation of the hex nuts 90 degrees clockwise, with the positive lead at the top of the container and the negative lead at the bottom. We repeated the same measurement process from the First Trial in the Second.

After we collected our data, we utilized the Numpy and Matplotlib libraries to generate visuals of both the equipotential and electric field lines. To generate coordinate pairs, we used Numpy's meshgrid() method [2], which takes in lists of  $x$  and  $y$  coordinates to generate appropriate pairs for our system. For equipotential lines, we used Matplotlib's contour() method [1], [3], which takes in two matrices generated from the meshgrid() function along with the level at each point and generates level curves showing lines of equal potential. We also used the .colorbar() method to show what colored line represents which quantity of potential. To generate the electric field, we use Matplotlib's quiver() method [4], which takes in the outputs of the meshgrid() function along with matrices containing the components of a vector at each coordinate, therefore generating a vector field.

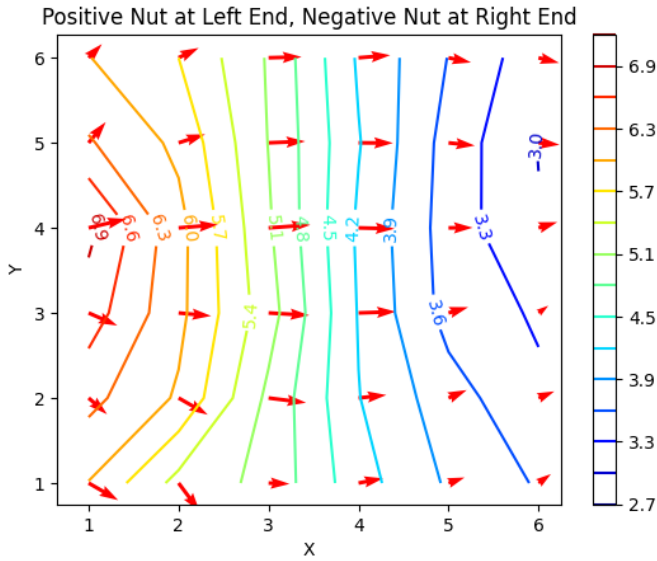


Fig. 2. Equipotential lines and electric field in the first configuration.

### III. RESULTS

#### A. Positive Nut (Left), Negative Nut (Right)

For the first trial, the positively charged nut was placed on the left and the negatively charged nut on the right. Our power supply applied a measured voltage of 8.89 V to our experimental system. Therefore, our expected value for the equipotential line equidistant to the two oppositely charged nuts was 4.45 V, exactly half of the total voltage supplied to our system.

Upon analysis of Fig. 2, we observed that the equipotential lines of higher electric potential were closer to the positively charged nut and those of lower electric potential were closer to the negatively charged nut, as expected. Between the y-coordinate values of 3 and 4, we calculated the experimental value of the equipotential line approximately equidistant to the two oppositely charged nuts to be  $4.65V = \frac{4.8+4.5}{2}$ . Thus, we can determine the percent error of our observed voltage value using the following equation:

$$\delta = \left| \frac{v_a - v_e}{v_e} \right| * 100 \quad (2)$$

where  $\delta$  is the percent error,  $v_a$  is the experimental voltage, and  $v_e$  is the expected voltage. Using this equation, we found that  $\delta_1 = 4.49\%$ , indicating that our experimental voltage was close to the expected value. In addition, as the position moved from the location of the positively charged nut to that of the negatively charged nut, the line density of the equipotentials decreased. This indicates that the rate of the change in the electric potential was in the direction of decreasing potential, implying the presence of an electric field.

To obtain an average rate of change of the electric potential in the direction of decreasing potential for this trial, we used the following equation:

$$-\frac{\Delta V}{\Delta s} = -\frac{V_{maxneg} - V_{maxpos}}{d} \quad (3)$$

where  $\Delta V$  is the potential difference between the maximum electric potential near the negatively charged nut and the maximum electric potential near the positively charged nut, and  $d$  is the straight line distance between the two nuts. Referencing Fig. 2 for the electric potential values,  $V_{maxneg} = 3.3V$  and  $V_{maxpos} = 6.9V$ . To determine the value of  $d$ , we can treat the charged nuts as point charges, so  $d$  is the straight-line distance between them since there were six points of measurement separated by approximately 0.01 m (or  $\frac{1}{2}$  inch):  $d = 5 * 0.01 = 0.05$  m.

Therefore, we can substitute the experimental values for  $V_{maxneg}$ ,  $V_{maxpos}$ , and  $d$  to find that the average rate of change of the electric potential in the direction of decreasing potential is 72 V/m.

According to theory, the electric field is perpendicular to the equipotential lines since no work is done when moving along an isoline. To be consistent with the equipotentials in Fig. 2, the electric field must point toward decreasing electric potential. In addition, the magnitude of the electric field at a point in space should be higher in regions where the equipotential lines are more densely packed together (near the positively charged nut) and lower in regions where they are more spaced out (near the negatively charged nut). The direction of the electric field vectors at the points of measurement is also consistent with Fig. 2 since all of the vectors point in the direction of decreasing electric potential (from the positively charged nut to the negatively charged nut). In addition, the regions in which the magnitude of the electric field vectors (represented by the vector length) is at a maximum (between Y-coordinates 3 and 4) are consistent with the isoline density distribution observed in Fig. 2, with areas of higher equipotential line density corresponding to regions of high electric field strength as shown by the lengths of the arrows.

#### B. Positive Nut (Top), Negative Nut (Bottom)

For the second trial, the positively charged nut was placed on the top (North) and the negatively charged nut was placed on the bottom (South). Our power supply applied a measured voltage of 8.46 V to our experimental system. Therefore, our expected value for the equipotential line equidistant to the two oppositely charged nuts was 4.23 V, exactly half of the total voltage supplied to our system.

Upon analysis of Fig. 3, we observed that the equipotential lines of higher electric potential were closer to the positively charged nut and those of lower electric potential were closer to the negatively charged nut, as expected. Between the x-coordinate values of 3 and 4, we calculated the experimental value of the equipotential line approximately equidistant to the two oppositely charged nuts to be  $5V = \frac{5.2+4.8}{2}$ . We can determine the percent error of our observed voltage value using Eq(2) from Trial 1. Substituting the necessary values, we found that  $\delta_2 = 18.20\%$ , indicating that our experimental voltage was not close to the expected value. In addition, as

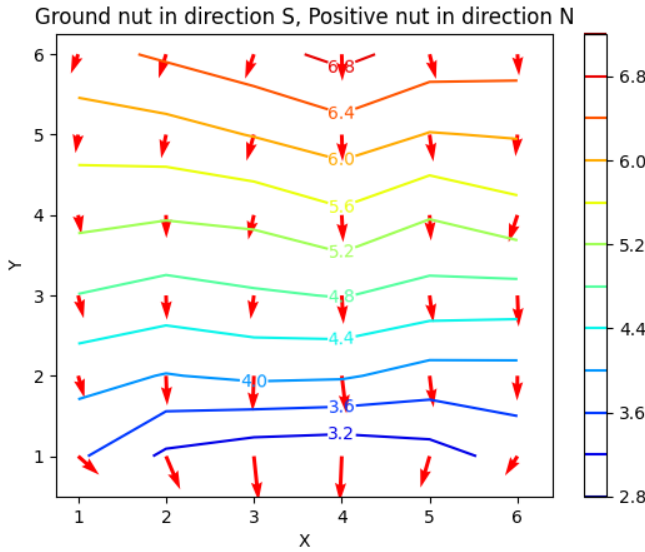


Fig. 3. Equipotential lines and electric field in the second configuration

the position moved from the location of the positively charged nut to that of the negatively charged nut, the line density of the equipotentials decreased. This indicates that the rate of the change in the electric potential was in the direction of decreasing potential, implying the presence of an electric field.

To obtain an average rate of change of the electric potential in the direction of decreasing potential for this trial, we can use Eq (3). Referencing Fig. 3 for the electric potential values,  $V_{maxneg} = 3.2V$  and  $V_{maxpos} = 6.8V$ . Since the value of  $d$  is the same for both trials, we can substitute the experimental values for  $V_{maxneg}$ ,  $V_{maxpos}$ , and  $d$  to find that the average rate of change of the electric potential in the direction of decreasing potential is 72 V/m. This value is the same rate of change of the electric potential as the first trial, demonstrating consistency across both a change in charge orientation (rotating the nuts' locations by 90 degrees) and the potential difference.

According to theory, the electric field is perpendicular to the equipotential lines since no work is done when moving along an isoline. To be consistent with the equipotentials in Fig. 3, the electric field must point in the direction of decreasing electric potential. In addition, the magnitude of the electric field at a point in space should be higher in regions where the equipotential lines are more densely packed together (near the positively charged nut) and lower in regions where they are more spaced out (near the negatively charged nut). As shown in Fig. 3, the direction of the electric field vectors at the points of measurement is consistent with the equipotentials since all of the vectors point in the direction of decreasing electric potential (from the positively charged nut to the negatively charged nut). In addition, the regions in which the magnitude of the electric field vectors (represented by the vector length) is at a maximum is consistent with the isoline density distribution observed in Fig. 3, with areas of higher equipotential line

density corresponding to regions of high electric field strength in Fig. 3.

#### IV. DISCUSSION

Any outlying values of all experimentally determined quantities and the discrete nature of our equipotential lines across experiments are most likely due to the limited number of points measured and measurement errors (e.g. not measuring at the exact point, oscillating voltage values on the voltmeter). Across both experiments, data was collected at only 36 points for each trial, a very small dataset. This may have contributed to the straight, discrete nature of the equipotential lines since there is not enough data to generate curved, smooth lines. In addition, more configurations of, and more measuring points in, the two-nut system should be tested to observe how the isolines and voltage values change with respect to the orientation of the system.

#### V. CONCLUSION

After taking the necessary measurements, we were able to graph both the equipotential lines as well as their respective electric field lines. The resulting graphs were consistent with the relationship between the electric potential of a charged item and its electric field. One thing that we would change within the experiment is that for another trial we would move the nuts to a different position relative to each other to see if we get different equipotential lines and electric fields as a result.

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#### CONTRIBUTIONS

- Saketh: Abstract, Methods and Materials, Data Collection, Graph Generation
- Victoria: Introduction, Results, Discussion, Data Collection, Formatting
- Krish: Data Collection
- Kevin: Methods and Materials, Setup Diagram, Data Collection

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