Testing Newton's second law with a half-Atwood machine: a meta-analysis

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Several studies have recently tested the validity of Newton's second law using a half-Atwood machine consisting of a cart and a hanging mass connected by a string over a pulley, including studies conducted with constant force and varying cart mass, as well as studies in which the hanging mass provided varying external force on the system. In this meta-analysis of nine such studies, we non-dimensionalized and combined the results, finding that the results in the aggregate support Newton's second law; and that friction was not the dominant source of experimental error in prior work.

I. INTRODUCTION

Newton's second law states that the sum of the forces is equal to the time rate of change of momentum [1]; for systems where mass is constant this is commonly written as [2, 3]:

$$\sum \vec{F} = m\vec{a}. \tag{1}$$

To verify (1), it is typical apply the same constant force to different masses and observe the resulting acceleration to check for inverse proportionality [4–8]; or alternatively to subject a mass to different forces and check the acceleration is proportional [9–11]. A common experimental setup is to use a half-Atwood machine (Fig. 1) to allow systematically varying either the force or the system mass with other factors held constant.

For the system in Fig. 1, the equations of motion can be easily derived by writing Newton's second law for both the total cart mass (m_1) and the hanging mass (m_2) [2, 3]:

$$T - m_1 g\mu = m_1 a, \tag{2}$$
$$m_2 g - T = m_2 a, \tag{3}$$

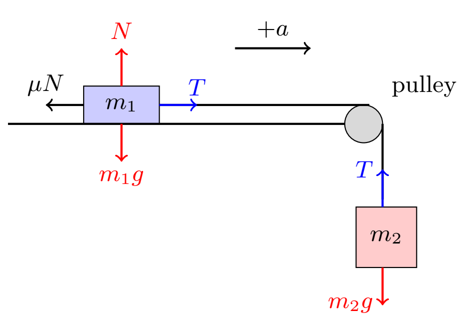

FIG. 1. Half-Atwood machine setup for testing Newton's second law, modified from [6].

where T is tension in the string, $g = 9.81\,\mathrm{m\,s^{-2}}$ is the acceleration of gravity, and μ is a coefficient of friction associated with the cart. Combining (2) and (3) gives the acceleration of the system [2, 3]:

$$a = \frac{m_2}{m_1+m_2}g - \frac{m_1}{m_1+m_2}\mu g. \tag{4}$$

Several studies have recently tested the validity of (1), including studies conducted with constant force and varying cart mass [4–8], as well as studies in which the hanging mass provided varying external force on the system [9–11]. Curiously, all the studies [9–11] conducted with varying force ($m_2 g$) also had varying total system mass ($m_1 + m_2$), potentially confounding the effects of force versus mass. [12] did not systematically vary m_1 or m_2 and was limited in conclusions regarding (1); while [13] had no means to measure accceleration. [11] exceeded the acceleration of gravity g in observed results, while [9] observed extremely low values of system acceleration, possibly due to incorrectly recorded mass m_1. Many studies also suffered from having only minimal replicates of measured data. All [4–12] claimed to be affected by friction; some also noted timing inaccuracies, factors related to the clean release of the system from rest, and various other "human error[s]" as sources of experimental error.

Here, we wish to combine the results of [4–12]. The combined dataset could potentially overcome many of the shortfalls of each individual study, including items like release, timing inaccuracies, and "human error" since these are presumably randomized out among the different workers. With a larger dataset it may also be possible to test the claim the friction affected the results. Since some previous studies [4–8] were done with constant force, while others were done with varying hanging mass [9–11], we nondimensionalize (4), to combine the various datasets into a single coherent picture and collapse four independent variables (m_1, m_2, g, and μ) into two ($\hat{m}$, μ).

Defining nondimensional system acceleration $\hat{a} = \frac{a}{g}$ and nondimensional hanging mass $\hat{m} = \frac{m_2}{m_1+m_2}$ gives

$$\hat{a} = \hat{m} - (1-\hat{m})\mu, \tag{5}$$

$$\hat{a} = (1+\mu)\hat{m} - \mu, \tag{6}$$

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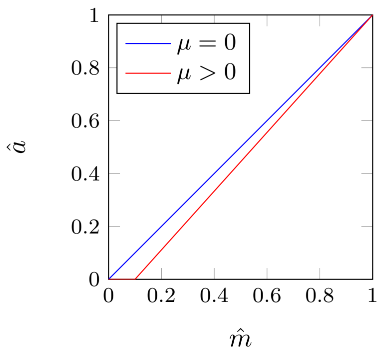

FIG. 2. Predicted relationships between $\hat{m}$ and $\hat{a}$ for case with friction, (7) (red) and frictionless case, (9) (blue).

$$\hat{a} = \begin{cases} (1+\mu)\hat{m} - \mu & \hat{m} \geq \dfrac{\mu}{1+\mu}, \\ 0 & \hat{m} < \dfrac{\mu}{1+\mu}. \end{cases} \tag{7}$$

The condition $\hat{m} = \dfrac{\mu}{1+\mu}$ is equivalent to $m_2 = \mu m_1$; below this critical value static friction will keep the system from moving.

For the case of no friction, (4) reduces to

$$a = \frac{m_2}{m_1 + m_2} g, \tag{8}$$

while (7) becomes

$$\hat{a} = \hat{m}. \tag{9}$$

Fig. 2 shows predictions for $\hat{a}$ based on whether friction is significant (7) or not (9). If the measured data fail to follow either of these relationships, then the models from (4) are wrong, indicating either missing or unmodeled forces, or that (1) is invalid in this system.

II. METHODS AND MATERIALS

Data from [4–12] for m_1, m_2, and a_{meas} were combined for analysis and plotting in R [14] using the `ggplot2` library [15]. The original data from [4–12] were collected using half-Atwood machine setups (PASCO Scientific; Roseville, CA); accelerations a_{meas} were estimated by timing the time t_{meas} for the system to move a distance d from rest. For data from [9], m_1 was corrected to be 2.5 kg. Data from [11] at highest two values of m_2 show timing issues with very short times, resulting in non-physical estimates that exceed the acceleration of gravity; these were removed from analysis.

The mass was nondimensionalized using

$$\hat{m} = \frac{m_2}{m_1 + m_2}, \tag{10}$$

while the measured system acceleration was nondimensionalized using

$$\hat{a} = \frac{a}{g}. \tag{11}$$

These were plotted in R using the `ggplot2` library; further statistical analyses were carried out using a linear model (`lm`) [14, 15]. Data and code are available at https://github.com/devangel77b/426ahacker-lab2.1.

III. RESULTS

Fig. 3 shows the datasets from [4–12] in nondimensional form (10) and (11). For these data, the best fit line is $\hat{a} = 0.98\hat{m}$ (linear regression, $R^2 = 0.9777$, $df = 95$, $p < 2 \times 10^{-16}$). The slope is not significantly different from 1.0 (t-test, $p = 0.227$).

IV. DISCUSSION

As shown in Fig. 3, the combined results of [4–12] are most consistent with (8) and (9). The slope is not significantly different from one, and the intercept is not significantly different from zero; these do not support the models with friction from (4) and (7). Friction does not appear to be the dominant source of experimental error, contrary to what was claimed in [4–12].

The results are consistent with Newton's second law. The models of (8) and (9) were derived from analysis of the free body diagrams in Fig. 1 using (1); good agreement of Fig. 3 with the predictions of (9) supports the findings of [4–12] and the validity of Newton's second law in this system.

The largest and most notable experimental error appears to be at high values of m_2 in data from [11]. It is possible that the highest values of m_2 and a becomes prone to timing errors during data gathering. Most of the measured data from [4–12] are for $\hat{m} < 0.3$; while data above this range comes mainly from one study [11]. Future measurements with this system should focus on filling in gaps in the region $0.3 < \hat{m} < 1$.

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APH and LR devised the analyses and wrote the manuscript.

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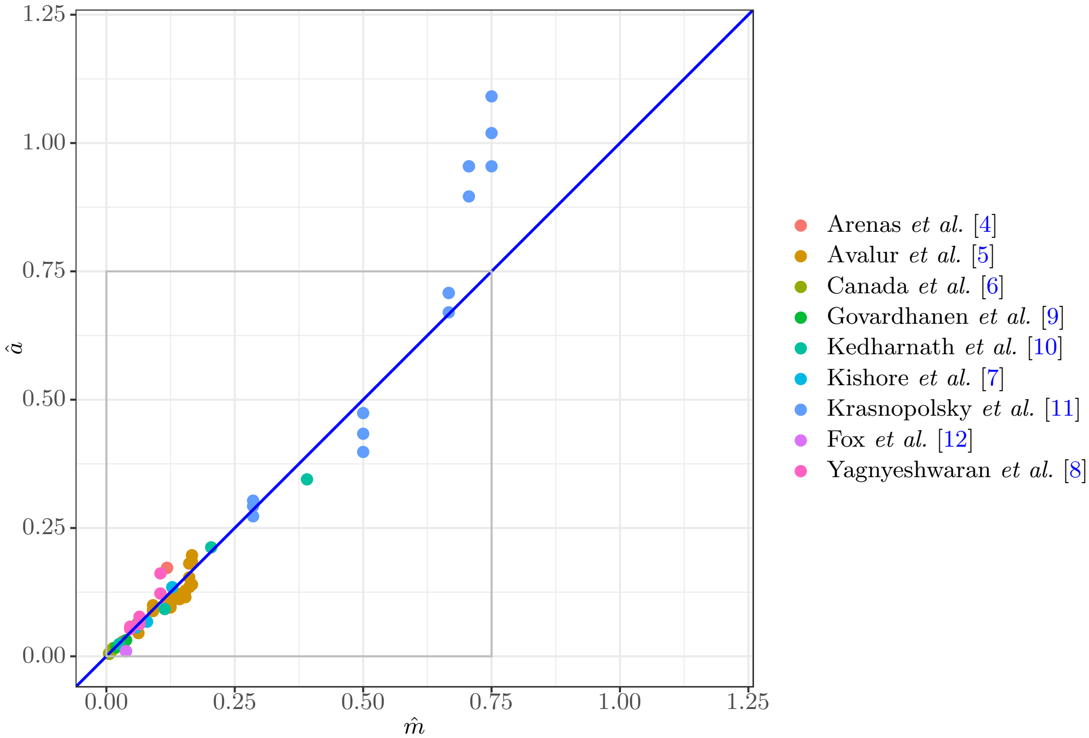

FIG. 3. Datasets from [4–12] in nondimensional form, $\hat{a} = \dfrac{a}{g}$ and $\hat{m} = \dfrac{m_2}{m_1 + m_2}$. Best fit line is $\hat{a} = 0.98\hat{m}$ (linear regression, $R^2 = 0.9777$, $df = 95$, $p < 2 \times 10^{-16}$), slope is not significantly different from 1.0 (t-test, $p = 0.227$), providing support for (1), (8) and (9). Data from [9] uses corrected value of $m_1 = 2.5\,\text{kg}$. Data from [11] at highest two values of m_2 (top, right) show timing issues with very short times and results that exceed the acceleration of gravity and so are removed from analysis. Gray box shows area where analysis is valid.

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