Testing Newton's Second Law ✓

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Abstract

awk.

The purpose of this experiment is to investigate the relationship between net force and acceleration of a two-mass system. This system was composed of a cart carrying varying masses and connected by a string to a counterweight over a pulley. For each cart mass, three trials were conducted, recording the time the cart required to move 0.5m from its initial position, for which acceleration was calculated. Results suggest that as the cart's mass increased, its acceleration decreased, in such a way that it agrees with the inverse relationship between mass and acceleration as stated by Newton's Second Law. These findings align with the dependency of acceleration on the abstraction of "net force" predicted by that law: $\sum \vec{F} = m\vec{a}$.

as predicted by?

awk

INTRODUCTION

The equation $\sum \vec{F} = m\vec{a}$, commonly cited as Newton's Second Law, represents a principle first written down by Sir Isaac Newton in his *Principia* (1687), a principle that offers an explanation to how the motion of macroscopic systems can change. The equation directly relates the acceleration of a chosen system (or, in particular, the system's center of mass) to the net "force" on that system, an abstract quantity that is proportional to the system's acceleration by a factor of the system's total mass. Here, "force" and "acceleration" are vector quantities, allowing the above equation to be applied separately to any set of basis directions (i.e. axis) one chooses. Thus, verifying Newton's Second Law as a reasonable model (at least for sufficiently similar systems) would be valuable for deriving the masses or accelerations of bodies in nature.

good.

If Newton's Second Law is not accurate within our experiment's degree of precision:

$$H_0 : \sum \vec{F} \neq m\vec{a} \quad (1)$$

Alternatively, if Newton's Second Law is accurate within our limits of precision:

$$H_A : \sum \vec{F} = m\vec{a} \quad (2)$$

This lab investigated the relationship between force, mass, and acceleration through an experiment that utilizes a two-mass pulley system. By setting up a cart connected to a hanging mass over a pulley and releasing it, the resulting acceleration of the carts and masses can then be observed and measured. We tested the null hypothesis by conducting multiple replicates of the above and using the data that resulted to independently calculate acceleration via kinematic equations (the baseline method) and via net force equations from Newton's Second Law (the calculation method being tested).

hard to follow

Good

Materials & Methods

METHODOLOGIES

Materials Used

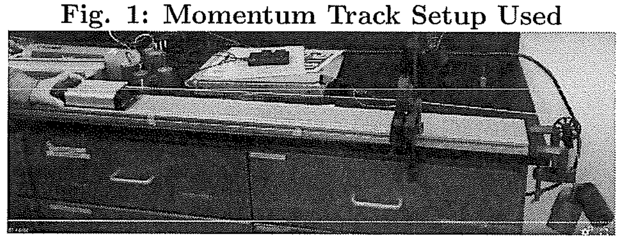
how?

The experiment used a 0.5 kg cart and near frictionless rail, in particular the PASCO Dynamics Systems Basic Smart Cart Metal Track 1.2 m System for the rail and the PASCO Dynamics Systems Scientific ME-9454 Dynamic Collision Cart for the corresponding cart, respectively. Three 1 kg masses were used to show the effects of differing masses on the acceleration of the cart. In addition, a pulley, string, and counterweight were used to move the cart along its rail. Three stopwatches and a metronome were used to time the different trials of the cart. Irwin trigger clamps were used to secure the rail to a level table.

why? how?

PROCEDURE

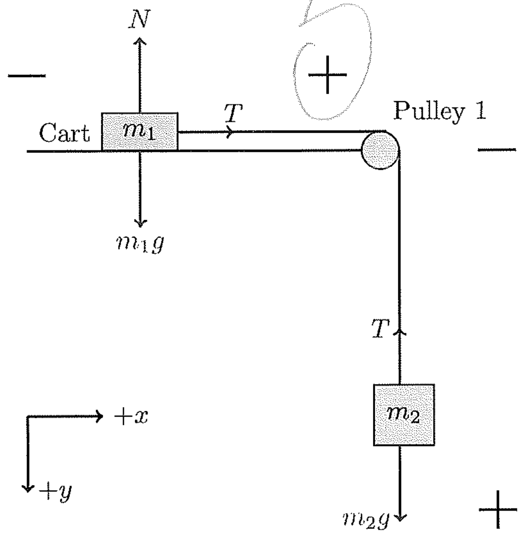
Fig. 1: Momentum Track Setup Used



Refer to Fig() in text.

signs here are a little confusing. you intend to show direction of positive a... I think

Fig. 2. Free Body Diagram of the System



past tense in methods

To measure the acceleration of the cart on a near-frictionless track while examining the effect of mass on acceleration, a cart is attached to a small mass (0.02 kg) providing a pull. The track is securely clamped to a level surface, and the cart, initially unloaded, without any masses, is tied to a rope on the pulley with the small mass attached to the end. Starting from the 0.3 m mark the cart travels to the 0.8 m mark, traveling a total distance of 0.5 m, where a 0.5 kg mass is setup to indicate the stopping point and to stop the cart. Three timers start in sync with a metronome set up to tick at 120 bpm and stop when the cart reaches the mark. The trials were conducted three times per mass setting (1 kg, 2 kg, and 3 kg), incrementally adding 1 kg to the cart and keeping the small, hanging mass constant.

What does this accomplish?

RESULTS

A plot of the average travel times for the cart and its load versus the mass of the cart and load is shown in Fig. 3. Note that each trial utilized three timers for each mass, and so each point is the mean of three times. Various regressions were fitted to the data (via the least square method), with the closest best-fit line shown in Fig. 3 (a square root regression).

Newton's Second Law (Experimental Method):

$$\sum \vec{F} = m\vec{a} \tag{3}$$

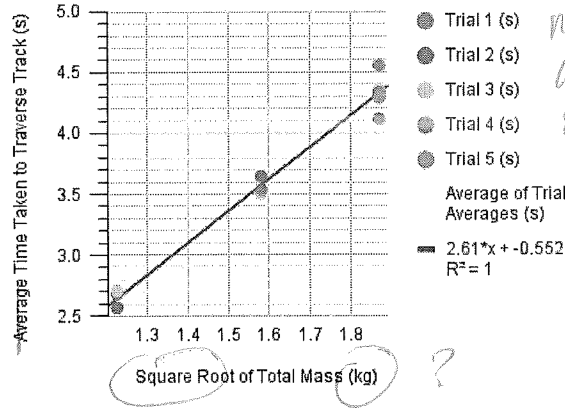
not explained

Kinematics (Baseline Method):

Why? Which is best model?

$$\vec{x}(t) = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \tag{4}$$

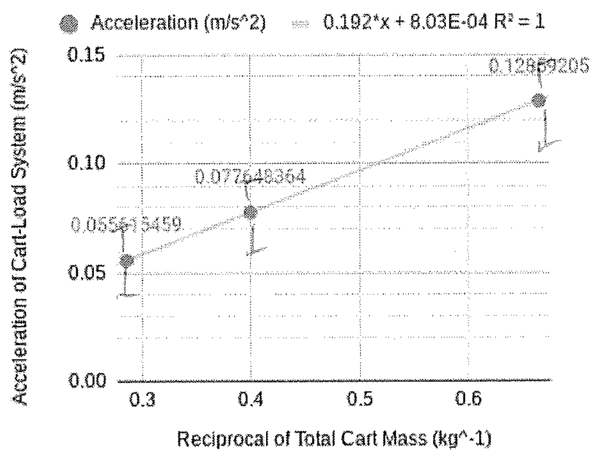
Theoretical Relation: $t = (\sqrt{\frac{2\Delta x}{||\sum \vec{F}||}})(\sqrt{m})$



not showing avgs you're showing all 5 reps (better)

?

Fig. 3: Time to Accelerate 0.5m vs. Mass of Cart/Weight System - Square Root Fit



is R^2 best stat here? p value?

Fig. 4: Cart System Acceleration vs. Reciprocal Mass

Doing the calculations using a 1 kg weight will result in the following calculations (which correspond to the first cluster of points in Fig. 3):

Using our data in Equation (4):

$$x(t) = 0.5m$$
$$x_0 = 0m$$
$$v_0 = 0m/s$$
$$t = 2.65s$$

$$0.5\text{m} = (0\text{m}) + (0\text{m})(2.65\text{s}) + \frac{1}{2}(a)(2.65\text{s})^2$$
$$a = (1/2.65)^2\text{m/s}^2$$
$$a = 0.1424\text{m/s}^2$$

A free body diagram (see Fig 2.) can be drawn of the system with the momentum cart (m_1) and the falling mass (m_2). A system of equations can

Would be stronger w/ more combinations of m_2, m_1

be created by applying Equation (3) to the vectors along the perpendicular axis indicated in Fig. 2. If the string and pulley have both negligible mass and friction, than both tension forces are approximately equal, simplifying calculations. Using known physical constants allows the calculation of a (for more on this system, see Tipler and Mosca).

$$m_1 = 1.5kg, \quad m_2 = 0.02kg, \quad g = 9.8m/s^2$$

$$\sum \vec{F} = m\vec{a} \Rightarrow$$

$$T = m_1 a, \tag{5}$$

$$m_2 a = m_2 g - T \tag{6}$$

$$\Rightarrow T = m_2 g - m_2 a \Rightarrow m_1 a = m_2 g - m_2 a$$

$$\Rightarrow m_1 a + m_2 a = m_2 g \Rightarrow a(m_1 + m_2) = m_2 g$$

$$a = \frac{m_2 g}{m_1 + m_2} \tag{7}$$

$$\Rightarrow a = \frac{0.02\text{kg} * 9.8\text{m/s}^2}{0.02\text{kg} + 1.5\text{kg}} \approx 0.129m/s^2$$

An a of $0.1424m/s^2$ calculated through kinematics and dynamics is relatively close to our a of $0.129m/s^2$ calculated through $\sum \vec{F} = m\vec{a}$. The percent error is:

Good!

$$\left| \frac{a_{calculated} - a_{experimental}}{a_{calculated}} \right| = \tag{8}$$

$$\frac{0.1424m/s^2 - .129m/s^2}{0.1424m/s^2} \approx 0.094 = 9.4\%$$

just under 10%. Repeating the above calculations for 2 kg and 3 kg mass loads ($m_1 = 2.5$ and 3.5, respectively) using both kinematics and force equations yields similar percent errors of about 2.1% and 4.2%, well within the acceptable 10% range for experimental uncertainty. Fig. 4 is a plot of these accelerations versus the cart system's mass.

DISCUSSION

From the coefficient of determination (found by minimizing the squares of residues) in Fig. 4 ($R^2 = 1$), the data strongly suggests that the cart system's acceleration was directly proportional to its reciprocal mass as predicted by Newton's Second Law (where the constant of proportionality is $||\sum \vec{F}||$). This is further supported by an equal R^2 value for Fig. 3, which agrees with the direct proportionality between the time traveled and the square root of the cart system's mass (see Fig. 3 overhead for exact relation) implied by Newton's Second Law and kinematics.

The acceleration derived through (Newtonian) mechanics differed measurably from that calculated through kinematics due to several potential sources of experimental error. For one, more trials may be required to more definitively characterize the seemingly optimal regressions in Fig. 3 as square root and Fig. 4 as linear rather than exponential, etc. Also, the experimental set-up may have had significant friction in various places, such as axial friction in the wheels and pulley or static friction between the rope and the pulley, forces that were not accounted for when acceleration was calculated using Newton's Second Law (which heavily depends upon the accuracies of force measurements). Axial friction, for example, would have rendered the tension forces acting on the cart and counterweight to be unequal, likely reducing the overall acceleration of the system. Another potential source of significant error was the human error associated with our timing methods. Instead of relying on the reaction speeds of the timers and the metronome, a more accurate method might have employed electronics to ensure the timers begin at the same exact instant that the cart is released. Furthermore, we could have stopped the timers using cameras to get more precise timing measurements.

What is broader significance of your findings?

Resources/Bibliography

[1] I. Newton, Philosophiæ Naturalis Principia Mathematica (1687).

[2] P.A. Tipler, and G. Mosca, Physics for Scientists and Engineers (Macmillan Higher Education, 2007).

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