Verifying Newton's second law: the relationship between force, mass, and acceleration

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This experiment investigates the relationship betwen mass, net force, and acceleration in accordance with Newton's second law of motion. A cart was set up on a near-frictionless plane with a pulley system and accelerated by a constant pulling force generated by a 0.100 kg weight. For each of six mass configurations, three trials were conducted, recording the time taken for the cart to travel a fixed distance of 0.800 m. Acceleration was calculated independently for each trial to capture variability, with averages and standard deviations computed as additional supporting evidence. Results demonstrated a clear inverse relationship: the average acceleration decreased from approximately 1.70 m s^{-2} at 0 g to 0.57 m s^{-2} at 1.000 kg, with low standard deviations indicating consistency across trials. For the 0 g mass confiiguration, calculated acclerations for individual trials ranged from 1.37 m s^{-2} to 1.93 m s^{-2}, while for the 1.000 kg mass, accelerations ranged from 0.44 m s^{-2} to 0.66 m s^{-2}. These findings confirm an inverse relationship between mass and acceleration under a constant force, aligning with Newton's prediction that $\sum \vec{F} = m\vec{a}$ and supporting the law's applicability in controlled experimental settings.

I. INTRODUCTION

Newton's second law of motion describes the relationship between force, mass, and acceleration [1]:

$$\sum \vec{F} = m\vec{a}, \tag{1}$$

where $\vec{F}$ is the force in newtons (N), m is the mass in kilogram (kg), and $\vec{a}$ is the acceleration in m s^{-2}. For a given force, an object's acceleration is inversely proportional to its mass. As mass increases, the acceleration decreases [1]:

$$\vec{a} = \frac{\sum \vec{F}}{m}. \tag{2}$$

The significance of (1) lies in its ability to predict how objects will accelerate when subjected to different forces.

We seek to verify Newton's second law and hypothesize that as the mass of the cart increases, the acceleration will decrease, consistent with (2). To test this hypothesis, we conducted multiple trials in which known masses were placed on the cart, and a constant force was applied via a 0.1 kg weight. By comparing the accelerations for different masses, we examined the relationship between mass and acceleration to verify Newton's second law [1].

II. METHODS AND MATERIALS

A. Cart acceleration tests

Acceleration tests ($n = 18$) were conducted using a one-dimensional cart system along a fixed 0.800 m track. The experimental setup included a wheeled cart with a base mass of $m_c = 0.500$ kg and a near-frictionless track (both PASCO Scientific; Roseville, CA) to ensure consistent performance with minimal resistance for accurate measurements. Additional masses of $m_1 =$ 0.020 kg, 0.050 kg, 0.100 kg, 0.200 kg, 0.500 kg and 1.000 kg were used to vary the cart's total mass. Hanging mass $m_2 = 0.100$ kg was suspended using the pulley to apply a constant gravitational force on the system.

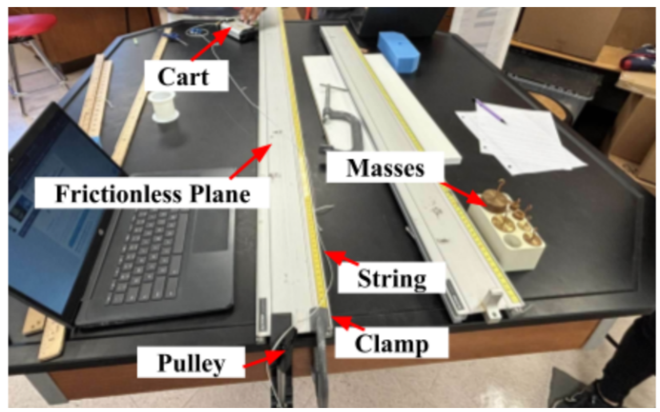

FIG. 1. Cart system consisting of a low friction 0.8 m track with a 0.500 kg wheeled cart; additional masses m_1, and a pulley system with a hanging mass m_2.

The cart was released from a designated starting point 0.800 m from the endpoint, and the time taken to travel the distance was recorded using a stopwatch with 0.01 s precision. Each mass configuration was tested in three separate trials to account for measurement variability. For each setup, the average time and corresponding standard deviation were calculated from the three trials to summarize the timing data, presented as mean ± one standard deviation [2].

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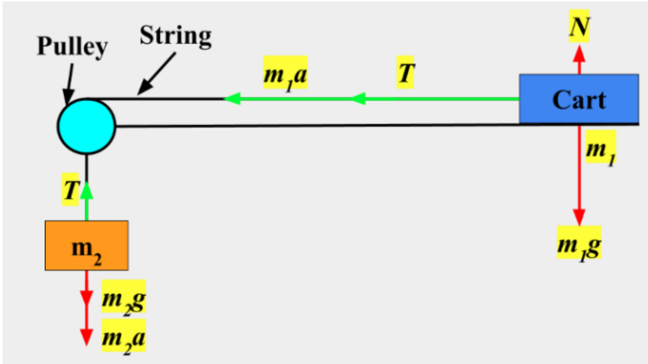

FIG. 2. Free body diagram of the cart system in Fig. 1. The cart (m_1) is connected to a hanging mass (m_2) through an ideal string, which is hung over a pulley, with forces labeled to represent the tension (T), gravitational force (mg), and normal force (N) acting on the system. Friction is assumed to be negligible.

B. Analyses of acceleration

To calculate the acceleration from our measurements, we used kinematics assuming uniform acceleration [1]:

$$a_{meas} = \frac{2d}{t^2}, \tag{3}$$

where a is the acceleration, $d = 0.800\,\mathrm{m}$ is the distance traveled, and t is the time taken for the cart to travel that distance.

We compared our measurements to the acceleration predicted by analysis of the free body diagram in Fig. 2 [1]:

$$a_{pred}(m_1) = \frac{m_2}{m_1 + m_2 + m_c} g, \tag{4}$$

where m_2 is the hanging $0.100\,\mathrm{kg}$ mass providing a constant gravitational force on the system, $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational acceleration, and $m_c = 0.5\,\mathrm{kg}$ is the empty mass of the cart. Independent variable m_1 is the additional mass in the cart in kg, which we varied from $0\,\mathrm{kg}$ to $1.000\,\mathrm{kg}$ in order to probe the relationship between F, m, and a.

III. RESULTS

Table I summarizes the measured time t for the cart to travel $0.8\,\mathrm{m}$ from rest, along with the resulting acceleration a from (3). $n = 3$ for each value of m_1; the hanging mass $m_2 = 0.1\,\mathrm{kg}$, and the empty cart mass $m_c = 0.5\,\mathrm{kg}$ so that the total accelerating system mass is $m_1 + m_2 + m_c$. Results are shown as mean ± one standard deviation.

Fig. 3 presents the relationship between acceleration a and mass m_1 for the cart system under the constant applied force.

TABLE I. Measured time (s) for cart to travel $0.8\,\mathrm{m}$ from rest, and corresponding acceleration ($\mathrm{m\,s^{-2}}$) for varying values of m_1. Hanging mass $m_2 = 0.1\,\mathrm{kg}$, empty cart mass $m_c = 0.5\,\mathrm{kg}$; total accelerating system mass is $m_1 + m_2 + m_c$. $n = 3$ replicates for each value of m_1. Results are shown as mean ± 1 s.d.

m_1 (kg)	t (s)	a_{meas} ($\mathrm{m\,s^{-2}}$)
0.000	0.98 ± 0.09	1.70 ± 0.29
0.020	1.03 ± 0.08	1.53 ± 0.23
0.050	1.15 ± 0.03	1.21 ± 0.07
0.100	1.18 ± 0.03	1.16 ± 0.06
0.200	1.26 ± 0.06	1.01 ± 0.09
0.500	1.33 ± 0.04	0.91 ± 0.06
1.000	1.68 ± 0.19	0.58 ± 0.12

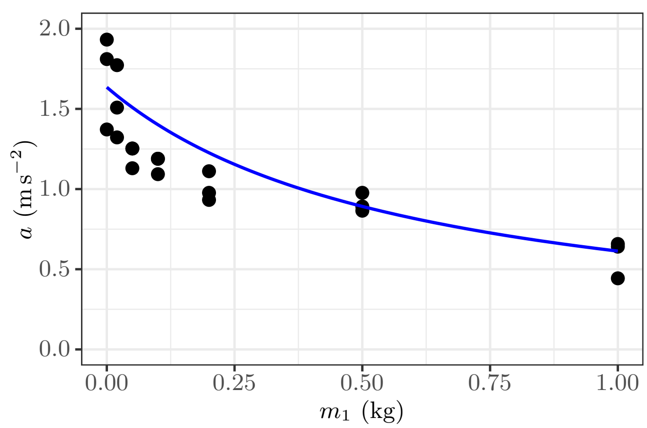

FIG. 3. Measured system acceleration a_{meas} as a function of m_1 using (3) shown by dots; blue line indicates the resulting system acceleration predicted by (4). Hanging mass $m_2 = 0.100\,\mathrm{kg}$; empty cart mass $m_c = 0.500\,\mathrm{kg}$. Total accelerating system mass is $m_1 + m_2 + m_c$.

IV. DISCUSSION

A. Is Newton's second law verified?

As observed in Table I, increasing the mass on top of the cart generally resulted in an increase in the time taken to travel the set distance of $0.80\,\mathrm{m}$. For example, with $0.020\,\mathrm{kg}$, the time recorded across three trials ranged from $0.95\,\mathrm{s}$ to $1.10\,\mathrm{s}$. When the largest mass ($1.000\,\mathrm{kg}$) was added, the time increased, ranging from $1.56\,\mathrm{s}$ to $1.90\,\mathrm{s}$. These individual trial results provide a reliable primary basis for analyzing the effect of mass on time and, subsequently, on acceleration. The trial data clearly show a trend of increasing time with added mass, consistent with Newton's second law [1].

The calculated accelerations, shown in Table I, further reinforce this relationship. By analyzing the individual acceleration values across the three trials for each mass, a clear inverse relationship between mass and acceleration emerges. For instance, with 0.020 kg, the acceleration values across trials ranged from approximately $1.32\,\mathrm{m\,s^{-2}}$ to $1.77\,\mathrm{m\,s^{-2}}$. As the mass increased to 1.000 kg, the acceleration values dropped significantly, ranging from approximately $0.44\,\mathrm{m\,s^{-2}}$ to $0.66\,\mathrm{m\,s^{-2}}$ across trials. This inverse trend across individual measurements strongly supports Newton's second law, where a constant force applied to an increasing mass yields lower acceleration [1].

Fig. 3 further corroborates this trend by plotting individual acceleration values for each trial against the theoretical predicted curve. The individual data points closely follow the expected inverse relationship–for all three trials, as mass increases, acceleration decreases–although some slight deviations from the predicted curve are observed. These minor discrepancies likely result from experimental errors such as slight variations in the release of the cart or timing precision, which will be discussed later. Despite these small deviations, the consistent downward trend in acceleration as mass increases validates the predicted inverse relationship and strongly aligns with Newton's second law [1].

Our findings (Fig. 3; (3) and (4)) demonstrate a consistent inverse relationship between mass and acceleration under constant force. This strong, inverse trend, even in the presence of minor experimental deviations, provides compelling support for Newton's second law, illustrating that as mass increases, acceleration decreases proportionally [1].

B. Sources of experimental error

While the track used in this experiment was near-frictionless, it is essential to acknowledge that some friction is unavoidable. The near-frictionless plane was chosen to minimize the effects of friction on the acceleration measurements, as a lot of friction can introduce significant experimental error by opposing the motion of the cart. Despite this, tiny variations in friction could still have influenced the results.

Timing inaccuracies likely introduced error due to the manual use of a stopwatch, especially at higher masses where precise measurement was required over longer intervals [3]. To improve accuracy, we could use an automated timing system, such as photogates, which would eliminate human reaction time errors and provide precise start and stop measurements [4]. This change would ensure that timing measurements are consistent and highly accurate across trials.

Additionally, slight inconsistencies in the cart's release, such as variations in initial positioning or angle, may have affected the measurements. To fix this, we could use a mechanical release mechanism to standardize the release process [4]. Such a mechanism would ensure that the cart starts from the exact same position and orientation in each trial, minimizing variability due to manual handling. This adjustment would help control for any small discrepancies caused by differences in the release method, leading to more reliable acceleration data.

V. ACKNOWLEDGEMENTS

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DA timed each trial during the experiment and assisted in writing the lab report. JL recorded the data for the experiment, performed the data calculations, created the figures, and also assisted in writing the report. SD prepared the experimental setup and managed the string during each trial. AT tested each weight individually and released the cart in each trial.

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