## **Understanding DC Motors**

## Introduction

The DC, permanent magnet, brushed electric motor is the workhorse of small, powered mechanical systems. Examples are the small motor and the gear motor in the ME2011 Robot kit are shown below.





The motor on the left is a gearmotor because the silver area is a gearhead that is coupled to the actual, gold-colored motor. Small motors like to spin fast with low torque. The gearing reduces the shaft speed and increases the torque.

When the motor leads are connected to a source of DC power, the shaft spins. Small motors run best at a preferred voltage, which is listed on the data sheet. Common preferred voltages are 3, 6, 12 and 24 Volts. If a voltage much lower than the preferred is applied, the motor may not be able to overcome its internal friction. If a voltage much larger is applied, the motor may heat up.

Some web resources on dc motors: (1) Wikipedia, (2) from a class at MIT, (3) from Micromo, a motor company.

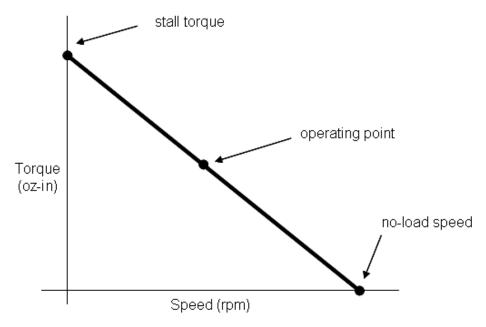
The four essential facts that dictate the basic properties of a DC motor:

- 1. For a fixed load, the shaft speed is proportional to the applied voltage.
- 2. For a fixed voltage, the shaft speed is proportional to the torque load applied to the shaft.
- 3. The shaft torque is proportional to the applied current, no matter what the voltage
- 4. There is internal electrical resistance and internal mechanical friction.

To "feel" some of these properties, find a motor and some batteries of different voltages (e.g. 12, 9, and 1.5). Connect each battery to the motor and observe how the shaft speed at no load changes with applied voltage. Then, for one battery, squeeze down on the shaft with your fingers (check first that the shaft is smooth) while the motor is spinning. As you increase the load applied to the shaft by squeezing, you will see (or hear!) the motor speed drop. The motor speed when nothing is touching the shaft is called the no-load speed. Depending on the motor and the battery voltage, you may be able to squeeze hard enough to get the shaft to stop turning, stalling the motor. At stall, the amount of torque you are applying with your fingers is called the stall torque.

## **Torque-Speed Curve**

Motors have torque-speed curves. For a fixed input voltage from a battery, the motor speed slows down as it is loaded. With no load on the shaft (free-running), the motor runs at the no-load speed (NLS), the fastest possible speed for that voltage. When the shaft is fully loaded and not allowed to move, the speed is zero and the motor is producing its stall torque (ST), the maximum possible torque. At stall torque, the current drawn out of battery is at its maximum, as is motor heating. Motors should be operated at stall only for brief periods of time (seconds) to save on batteries and to keep the motor from melting. The chart below shows an ideal motor torque-speed curve. The point labeled as the operating point indicates the speed the motor will run when driving a specific operating load. As that load changes, the operating point slides up and down the torque-speed curve.

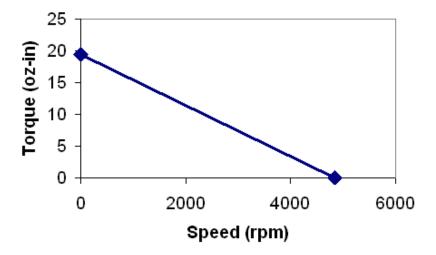


Notice how the torque speed curve is a straight line that connects the no-load speed with the stall torque. Sometimes you can find the torque-speed curve, or the no-load speed and stall torque on the motor data sheet, other times you have to find them through experiments.

For example, here is a portion of the data sheet for a series of 10 Watt, brushed DC motors from Maxon, one of the leading suppliers of high-performance small DC motors used for precision applications such as robotics and space probes.

2 No load speed rpm 5350 5310 5230 4850 4980 4780 5190 5510 5610 3 No load current mA 79.6 44.3 38.6 26.2 21.7 17.2 14.3 11.6 64 4 Nominal speed rpm 4910 4510 4230 3820 3940 3740 4150 4470 4450 5 Nominal torque (max. continuous torque) mNm 11.4 20.9 24.0 29.1 28.8 28.9 28.8 28.6 26 Nominal current (max. continuous current) A 1.50 1.50 1.50 1.26 1.03 0.823 0.668 0.529 0.7 Stall torque mNm 138 139 126 137 138 133 144 152 18 Starting current A 17.2 9.73 7.72 5.82 4.83 3.72 3.28 2.76 19 Max. efficiency % 87 87 86 87 87 87 87 88 68 87 87 87 87 88 87 87 87 88 87 87 87 88 88												
Values at nominal voltage           1 Nominal voltage         V         4.5         8.0         9.0         12.0         15.0         18.0         24.0         32.0         4           2 No load speed         rpm         5350         5310         5230         4850         4980         4780         5190         5510         56           3 No load current         mA         79.6         44.3         38.6         26.2         21.7         17.2         14.3         11.6         6           4 Nominal speed         rpm         4910         4510         4230         3820         3940         3740         4150         4470         44           5 Nominal torque (max. continuous torque)         mNm         11.4         20.9         24.0         29.1         28.8         28.9         28.8         28.6         2           6 Nominal current (max. continuous current)         A         1.50         1.50         1.50         1.26         1.03         0.823         0.668         0.529         0.           7 Stall torque         mNm         138         139         126         137         138         133         144         152         1           8 Starting current				118740	118741	118742	118743	118744	118745	118746	118747	118748
1 Nominal voltage       V       4.5       8.0       9.0       12.0       15.0       18.0       24.0       32.0       4         2 No load speed       rpm       5350       5310       5230       4850       4980       4780       5190       5510       56         3 No load current       mA       79.6       44.3       38.6       26.2       21.7       17.2       14.3       11.6       6         4 Nominal speed       rpm       4910       4510       4230       3820       3940       3740       4150       4470       44         5 Nominal torque (max. continuous torque)       mNm       11.4       20.9       24.0       29.1       28.8       28.9       28.8       28.6       2         6 Nominal current (max. continuous current)       A       1.50       1.50       1.50       1.26       1.03       0.823       0.668       0.529       0.         7 Stall torque       mNm       138       139       126       137       138       133       144       152       1         8 Starting current       A       17.2       9.73       7.72       5.82       4.83       3.72       3.28       2.76       1 <t< td=""><td>Mo</td><td>tor Data</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	Mo	tor Data										
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3 No load current	1	Nominal voltage	V	4.5	8.0	9.0	12.0	15.0	18.0	24.0	32.0	48.0
4 Nominal speed rpm 4910 4510 4230 3820 3940 3740 4150 4470 44 5 Nominal torque (max. continuous torque) mNm 11.4 20.9 24.0 29.1 28.8 28.9 28.8 28.6 2 6 Nominal current (max. continuous current) A 1.50 1.50 1.50 1.26 1.03 0.823 0.668 0.529 0. 7 Stall torque mNm 138 139 126 137 138 133 144 152 1 8 Starting current A 17.2 9.73 7.72 5.82 4.83 3.72 3.28 2.76 1 9 Max. efficiency % 87 87 86 87 87 87 87 88 6 87 87 87 88 6 87 87 87 88 1 81 1 1 Terminal resistance Ω 0.261 0.822 1.17 2.06 3.10 4.84 7.31 11.6 3 1 1 Terminal inductance mH 0.0275 0.0882 0.115 0.238 0.353 0.551 0.832 1.31 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	2	No load speed	rpm	5350	5310	5230	4850	4980	4780	5190	5510	5070
5 Nominal torque (max. continuous torque)         mNm         11.4         20.9         24.0         29.1         28.8         28.9         28.8         28.6         2           6 Nominal current (max. continuous current)         A         1.50         1.50         1.50         1.26         1.03         0.823         0.668         0.529         0.           7 Stall torque         mNm         138         139         126         137         138         133         144         152         1           8 Starting current         A         17.2         9.73         7.72         5.82         4.83         3.72         3.28         2.76         1           9 Max. efficiency         %         87         87         86         87         87         87         88         87           10 Terminal resistance         Ω         0.261         0.822         1.17         2.06         3.10         4.84         7.31         11.6         3           11 Terminal inductance         mH         0.0275         0.0882         0.115         0.238         0.353         0.551         0.832         1.31         3           12 Torque constant         mNm / A         8.00         14.3         16.4 </td <td>3</td> <td>No load current</td> <td>mA</td> <td>79.6</td> <td>44.3</td> <td>38.6</td> <td>26.2</td> <td>21.7</td> <td>17.2</td> <td>14.3</td> <td>11.6</td> <td>6.95</td>	3	No load current	mA	79.6	44.3	38.6	26.2	21.7	17.2	14.3	11.6	6.95
6 Nominal current (max. continuous current) A 1.50 1.50 1.50 1.26 1.03 0.823 0.668 0.529 0.7 Stall torque mNm 138 139 126 137 138 133 144 152 18 Starting current A 17.2 9.73 7.72 5.82 4.83 3.72 3.28 2.76 19 Max. efficiency % 87 87 86 87 87 87 87 88 87 87 87 87 88 87 87 87	4	Nominal speed	rpm	4910	4510	4230	3820	3940	3740	4150	4470	4030
7 Stall torque mNm 138 139 126 137 138 133 144 152 18 Starting current A 17.2 9.73 7.72 5.82 4.83 3.72 3.28 2.76 19 Max. efficiency % 87 87 86 87 87 87 87 88 87 87 87 88 87 87 87 88 87 87	5	Nominal torque (max. continuous torque)	mNm	11.4	20.9	24.0	29.1	28.8	28.9	28.8	28.6	28.7
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9 Max. efficiency	7	Stall torque	mNm	138	139	126	137	138	133	144	152	140
Characteristics           10 Terminal resistance         Ω         0.261         0.822         1.17         2.06         3.10         4.84         7.31         11.6         3           11 Terminal inductance         mH         0.0275         0.0882         0.115         0.238         0.353         0.551         0.832         1.31         3           12 Torque constant         mNm / A         8.00         14.3         16.4         23.5         28.6         35.8         44.0         55.2         9           13 Speed constant         rpm / V         1190         667         584         406         333         267         217         173         1           14 Speed / torque gradient         rpm / mNm         39.0         38.3         41.6         35.6         36.1         36.0         36.1         36.3         3	8	Starting current	Α	17.2	9.73	7.72	5.82	4.83	3.72	3.28	2.76	1.56
10 Terminal resistance Ω 0.261 0.822 1.17 2.06 3.10 4.84 7.31 11.6 3 11 Terminal inductance mH 0.0275 0.0882 0.115 0.238 0.353 0.551 0.832 1.31 3 12 Torque constant mNm / A 8.00 14.3 16.4 23.5 28.6 35.8 44.0 55.2 9 13 Speed constant rpm / V 1190 667 584 406 333 267 217 173 1 14 Speed / torque gradient rpm / mNm 39.0 38.3 41.6 35.6 36.1 36.0 36.1 36.3 3	9	Max. efficiency	%	87	87	86	87	87	87	87	88	87
11 Terminal inductance     mH     0.0275 0.0882 0.115 0.238 0.353 0.551 0.832 1.31 3       12 Torque constant     mNm / A 8.00 14.3 16.4 23.5 28.6 35.8 44.0 55.2 9       13 Speed constant     rpm / V 1190 667 584 406 333 267 217 173 1       14 Speed / torque gradient     rpm / mNm 39.0 38.3 41.6 35.6 36.1 36.0 36.1 36.3 3		Characteristics										
12 Torque constant     mNm / A     8.00     14.3     16.4     23.5     28.6     35.8     44.0     55.2     9       13 Speed constant     rpm / V     1190     667     584     406     333     267     217     173     1       14 Speed / torque gradient     rpm / mNm     39.0     38.3     41.6     35.6     36.1     36.0     36.1     36.3     3	10	Terminal resistance	Ω	0.261	0.822	1.17	2.06	3.10	4.84	7.31	11.6	30.9
13 Speed constant rpm / V 1190 667 584 406 333 267 217 173 1 14 Speed / torque gradient rpm / mNm 39.0 38.3 41.6 35.6 36.1 36.0 36.1 36.3 3	11	Terminal inductance	mH	0.0275	0.0882	0.115	0.238	0.353	0.551	0.832	1.31	3.48
14 Speed / torque gradient rpm / mNm 39.0 38.3 41.6 35.6 36.1 36.0 36.1 36.3 3	12	Torque constant	mNm / A	8.00	14.3	16.4	23.5	28.6	35.8	44.0	55.2	90.0
	13	Speed constant	rpm / V	1190	667	584	406	333	267	217	173	106
45 Markania Milana annatani	14	Speed / torque gradient rp	m/mNm	39.0	38.3	41.6	35.6	36.1	36.0	36.1	36.3	36.4
15 Mechanical time constant ms 4.74 4.15 4.12 4.00 3.98 3.97 3.97 3.97 3	15	Mechanical time constant	ms	4.74	4.15	4.12	4.00	3.98	3.97	3.97	3.97	3.97
16 Rotor inertia gcm <sup>2</sup> 11.6 10.3 9.45 10.7 10.5 10.5 10.4 1	16	Rotor inertia	gcm <sup>2</sup>	11.6	10.3	9.45	10.7	10.5	10.5	10.5	10.4	10.4

The motor series has parts designed for a range of nominal voltages. Focus your attention on PN 118743, which is a 12 V motor. Moving down the column, the two key entries are the no-load speed, 4850 rpm, and the stall torque, 137 mNm (milli-Newtonmeters). Because we are in America, 0.137 Nm converts to 19.4 oz-in (ounce inches). (<a href="www.onlineconversion.com">www.onlineconversion.com</a> is good for unit conversions.) Using these two numbers and Excel, a torque-speed curve for the 118743 motor running at 12 V can be generated.



If there is no motor data sheet, then the torque-speed curve must be determined through experiments. The no-load speed can be measured for a slow moving shaft by attaching a tape flag to the shaft and using a stop watch to measure the time for 10 to 50 revolutions (the more counts, the more accurate the speed estimate will be). The stall torque can be measured by clamping a lever to the motor and loading the lever with weights until the motor does not move, but this method is difficult as running the motor at stall for more than a few seconds can cause it to overheat. Plus, the motor draws a lot of current at stall, which will quickly drain the battery. A better method is to load the motor down so that it is running at about one-half the no load speed and measure the load torque and the speed. Along with the no-load speed measure, this provides a second point on the torque-speed curve. Given these two points, the equation for the line can be derived. The stall torque is computed from the line equation as the point where the line crosses the vertical axis.

Once the torque-speed curve is known, then the speed can be computed if the load is known, or vice-versa. For example, with the 118743 motor, what will be the motor speed if the shaft is loaded down with a torque of 4.0 oz-in? The equation for the torque-speed line for this motor is

Torque = 
$$-(19.4/4850)$$
 \* Speed + 19.4

or

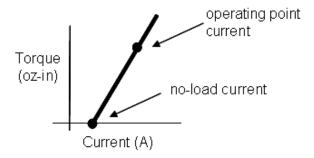
Speed = 
$$(19.4 - Torque) * (4850/19.4)$$

so the speed at 4.0 oz-in of torque is (19.4 - 4) \* (4850/19.4) = 3850 rpm.

Caution: When doing motor calcuations, be careful with and keep track of the units.

## **Torque-Current Curve**

For DC motors, the output torque is proportional to the current going into the motor no matter what the motor speed. The straight-line relation between torque and current is the torque-current curve as shown below.



The slope of the torque-current curve is the motor torque constant  $K_t$  typically expressed in oz-in/amp or oz-in/mA. Note the the no-load current is greater than zero because it takes a certain amount of current to overcome the internal friction of the motor.

The data sheet for the 118743 motor lists the torque constant as 23.5 mNm/A or 3.33 oz-in/A. The stall torque is 137 mNM. Therefore, the amount of current the motor draws at stall is  $I_{stall} = 137/23.5 = 5.8$  A. This would drain your 12 V battery in seconds. Note that, from the data sheet, the no-load current for the 118743 motor is 26 mA, which can be ignored in calculating the stall current. This would not be the case for high friction motors, for example a motor with a substantial gear train, which require significant current to overcome the friction.

If the torque constant is not provided on the data sheet, it can be measured experimentally. Place the motor under a fixed load, for example by having the motor winch up a weight. Measure the motor current with the current function of a digital multi-meter placed in series with the motor. Or, place a 1-ohm resistor in series with the motor and measure the voltage drop across the resistor to find current using Ohm's Law, V = R\*I.

Last modified 22-Sep-2014