Indian Institute of Technology Jodhpur
MAL1010, Dec'21-Mar'22
Tutorial Sheet 8

Q.1. Find the natural domains and range of the following functions of two variables.
(i) $f(x,y) = \frac{xy}{x^2-y^2}$ (ii) $f(x,y) = \ln(x^2+y^2)$

Q.2. Describe the level curves and the contour lines for the following functions.
(i) $f(x,y) = y - x$ (ii)$f(x,y) = xy$ (iii) $f(x,y) = x^2 - y^2$

Q.3. Examine the continuity of the following functions $f : \mathbb{R}^2 \to \mathbb{R}$ at $(0,0)$.
(i)

$$f(x,y) := \begin{cases} \frac{\sin^2(x-y)}{|x|+|y|} & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(ii)

$$f(x,y) := \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Q.4. Show that the following functions are not continuous at $(0,0)$.
(i)

$$f(x,y) := \begin{cases} \frac{x^2y}{x^4+y^2} & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(ii)

$$f(x,y) := \begin{cases} \frac{x^4-y^2}{x^4+y^2} & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Q.5. Show that the function defined by $f(x,y) = \frac{2xy}{x^2+y^2}$ when $(x,y) \neq (0,0)$ and $f(0,0) = 0$ is not continuous at $(0,0)$, however the partial derivatives are exist at $(0,0)$.

Q.6. Let $f(x,y)$ be defined in $S = \{(x,y) \in \mathbb{R}^2 : a < x < b, c < y < d\}$. Suppose that the partial derivatives of f exist and are bounded in S. Then show that f is continuous in S.

Q.7. Examine the differentiability of f at $(0,0)$.

$$f(x,y) := \begin{cases} xy\,\frac{x^2-y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Q.8. Show that the following function is not differentiable at $(0,0)$ but the directional derivatives in all directions at that point exist.
$f(x,y) = \frac{x^2y}{x^4+y^2}$ when $(x,y) \neq 0$ and $f(0,0) = 0$.

Q.9. Show that for the function $f(x,y) = \frac{x}{y}$ if $y \neq 0$ and zero if $y = 0$, the directional derivative at a point with respect to some vector may exist and with respect to other vector may not exist. Is it differentiable at $(0,0)$.

Q.10. $f(x,y) = \frac{y}{|y|}\sqrt{x^2+y^2}$ if $y \neq 0$ and $f(x,y) = 0$ if $y = 0$. Show that f is continuous at $(0,0)$, it has all directional derivatives at $(0,0)$ but it is not differentiable at $(0,0)$.

Q.11. In general, f_{xy} need not be equal to f_{yx}. Give an example of such function.
Hint: $f(x,y) = \frac{y}{|y|}\sqrt{x^2+y^2}$ if $(x,y) \neq 0$ and $f(0,0) = 0$.