

Indian Institute of Technology Jodhpur
MAL1010, Quiz-4, 31 Jan'22

Time: 60Min Marks: 15

Name: Roll No:

Q.1. In the following, identify which are the Riemann integrable functions on $[0, 1]$. Justify your answer.

(a)

$$f(x) := \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

(b)

$$f(x) := \begin{cases} 1, & \text{if } x \in \{\alpha_1, \alpha_2, \dots, \alpha_n\} \\ 0, & \text{otherwise} \end{cases}$$

where $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ are fixed, but arbitrarily chosen numbers from $[0, 1]$.

(c)

$$f(x) := \begin{cases} 0, & \text{if } x \text{ is irrational or } x = 0 \\ \sin qx, & \text{if } x = \frac{p}{q} \end{cases}$$

where p, q are positive and coprime integers.

[3]

Q.2. Let $f : [a, b] \rightarrow \mathbb{R}$ be a bounded and Riemann integrable function. Define for $x \in [a, b]$, $F(x) = \int_a^x f(t)dt$. Which of the following statement(s) is true? Justify your answers.

(a) The function F is uniformly continuous.

(b) The function F is differentiable on (a, b) .

[3]

Q.3. Evaluate $\lim_{n \rightarrow \infty} S_n$ by showing that S_n is an approximate Riemann sum for a suitable function over a suitable interval, where

$$S_n = \frac{1}{n^{16}} (1^{15} + 2^{15} + \dots + n^{15}).$$

[4]

Q.4. Consider the planar region D bounded by the curves $y = e^x$ and $y = \ln x$, and the vertical lines $x = 1$ and $x = e^2$.

(i) Find the area of D .

(ii) Find the volume of the solid generated by revolving the planar region D about the x -axis.

[5]