Indian Institute of Technology Jodhpur
MAL1010, Dec'21-Mar'22
Tutorial Sheet 7 (Practice Problems)

Find the volume of the solid in the following problems.

1. The solid lies between the planes perpendicular to the $x-$axis at $x = 0$ and $x = 4$. The cross-sections perpendicular to the axis on the interval $0 \leq x \leq 4$ are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$.   Ans: 16

2. The solid lies between planes perpendicular to the $x-$axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the $x-$axis between these planes are squares whose bases run from the semicircle $y = -\sqrt{1-x^2}$ to the semicircle $y = \sqrt{1-x^2}$.   Ans: $\frac{16}{3}$

3. The base of a solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the $x-$axis. The cross-sections perpendicular to the $x-$axis are equilateral triangles with bases running from $x-$axis to the given curve.   Ans: 8

4. The solid lies between planes perpendicular to the $y-$axis at $y = 0$ and $y = 2$. The cross-sections perpendicular to the $y-$axis are circular disks with diameters running from the $y-$axis to the parabola $x = \sqrt{5}y^2$.   Ans: 8π

5. Find the volume of the solid generated by revolving regions bounded by the lines and given curves about the $x-$axis by Disk Method.
   (i) $y = \sqrt{9-x^2}$, $y = 0$ (ii) $y = \sqrt{\cos x}$, $0 \leq x \leq \pi/2$, $y = 0$, $x = 0$
   (iii) $y = \sec x$, $y = 0$, $x = -\pi/4$, $x = \pi/4$.   Ans: (i) 36π, (ii) π, (iii) 2π.

6. Find the volume of the solid generated by revolving region about the given line.
   (i) The region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the curve $y = \sec x \tan x$, and on the left by the $y-$axis, about the line $y = \sqrt{2}$.
   Ans: $\pi \left(\frac{\pi}{2} + 2\sqrt{2} - \frac{11}{3} \right)$
   (ii) The region in the first quadrant bounded above by the line $y = 2$, below by the curve $y = 2 \sin x$, $0 \leq x \leq \pi/2$, and on the left by the $y-$axis, about the line $y = 2$. Ans: $\pi(3\pi - 8)$

7. Find the volume of the solid generated by revolving the regions bounded by the lines and the given curves about the $y-$axis.
   (i) The region enclosed by $x = \sqrt{\cos(\pi y/4)}$, $-2 \leq y \leq 0$, $x = 0$ Ans: 4
   (ii) $x = \sqrt{2y}/(y^2 + 1)$, $x = 0$, $y = 1$ Ans: $\pi/2$

8. Find the volume of the solid generated by revolving the regions bounded by the lines and curves about the $x - axis$ by Washer Method.
   (i) $y = x^2 + 1$, $y = x + 3$ Ans: $117\pi/5$
   (ii) $y = \sec x$, $y = \tan x$, $x = 0$, $x = 1$ Ans: π

9. Find the volume of the solid generated by revolving each region about the $y-$ axis.
   (i) The region in the first quadrant bounded above by the parabola $y = x^2$, below by the $x-$axis, and on the right by the line $x = 2$.   Ans: 8π
   (ii) The region in the first quadrant bounded on the left by the circle $x^2 + y^2 = 3$, on the right by the line $x = \sqrt{3}$, and above by the line $y = \sqrt{3}$.   Ans: $\pi\sqrt{3}$

10. Find the volume of the solid generated by revolving each region about the given axis.
    (i) The region in the first quadrant bounded above by the curve $y = x^2$, below by the $x-$axis, and on the right by the line $x = 1$, about the line $x = -1$.   Ans: $7\pi/6$
    (ii) The region in the second quadrant bounded above by the curve $y = -x^3$, below by the $x-$axis, and on the left by the line $x = -1$, about the line $x = -2$.   Ans: $3\pi/5$

11. Use the shell method to find the volume of the solids generated by revolving the regions bounded by the curves and lines given in below about the $y-$axis.
    (i) $y = x$, $y = -x/2$, $x = 2$.   Ans: 8π
    (ii) $y = 2x - 1$, $y = \sqrt{x}$, $x = 0$.   Ans: $7\pi/15$
    (iii) $y = 3/(2\sqrt{x})$, $y = 0$, $x = 1$, $x = 4$ Ans: 14π

12. Use the shell method to find the volume of the solid generated by revolving the regions bounded by the curves and lines given in below about the $x-$ axis.
    (i) $x = \sqrt{y}$, $x = -y$, $y = 2$ Ans: $\frac{16\pi}{15}(3\sqrt{2} + 5)$
    (ii) $x = y^2$, $x = -y$, $y = 2$, $y \geq 0$ Ans: $40\pi/3$
    (iii) $y = x$, $y = 2x$, $y = 2$ Ans: $8\pi/3$

13. Compute the volume of the solid generated by revolving the triangular region bounded by the lines $2y = x + 4$, $y = x$, $x = 0$ about
    (i) the $x-$axis using the washer method.   Ans: 16π
    (ii) the $y-$axis using the shell method.   Ans: $32\pi/3$
    (iii) the line $x = 4$ using the shell method.   Ans: $64\pi/3$
    (iv) the line $y = 8$ using the washer method.   Ans: 48π