# Indian Institute of Technology Jodhpur
# MAL1010, Dec'21-Mar'22
# Tutorial Sheet 6

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1. (a) Find the area bounded by the curves:
   (i) $f_1(x) = x^4 - 2x^2$ and $f_2(x) = 2x^2$.
   (ii) $x = 3y - y^2$ and $x + y = 3$.

   (b) Find the area of the region $R$ between the graphs of $f$ and $g$ over the intervals.
   (iii) $f(x) = x(x-2), \; g(x) = x/2$, over the interval $[0, 2]$.
   (iv) $f(x) = 2, \; g(x) = x^3/4$, over the interval $[-1, 2]$.

2. Find the area of the region bounded by the given curves in each of the following cases.

   (i) $\sqrt{x} + \sqrt{y} = 1, \; x = 0$ and $y = 0$.

   (ii) $y = x^2 - 2$ and $y = 2$.

   (iii) $y = x^2$ and $y = -x^2 + 4x$

   (iv) $y = x^4 - 4x^2 + 4$ and $y = x^2$.

3. The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$.

4. The base of certain solid is the disk $x^2 + y^2 \leq a^2$. Each section of the solid cut out by a plane perpendicular to the y-axis is an isosceles right triangle with one leg in the base of the solid. Find the volume of the solid.

5. Find the volume generated by revolving the region bounded by $y = \sqrt{x}$, $y = 2$ and $x = 0$ about the x-axis, by washer method.

6. Consider the solid obtained by revolving the region bounded by the functions

   $$y = x^2 + x + 1, \quad y = 1, \quad \& \quad x = 1$$

   about the line $x = 2$. Find the volume of the solid by shell method.

7. Find the length of the curve given by $\{(a\cos^3 t, a\sin^3 t) : t \in [0, 2\pi]\}$ for some $a > 0$.

8. Find the length of the cardioid $r = a(1 + \cos\theta)$, where $a > 0$, $0 \leq \theta \leq 2\pi$.

9. Find the length of the curve $y = (\frac{x}{2})^{2/3}$ from $x = 0$ to $x = 2$.

10. For the curve $y = \frac{x^3}{3} + \frac{1}{4x}, \;\; 1 \leq x \leq 3$, find the area of the surface obtained by revolving it about the line $y = -1$.