Indian Institute of Technology Jodhpur
MAL1010, End exam, Part-2, 04 Mar'22

Time: 3:00PM to 4:45PM Marks: 25

Instructions: (1). This question paper contains FIVE questions. Answer **ALL** questions.
(2). Write your Name & Roll no. on top of each page.
(3). Answers of subparts of a question should appear together.
(4). Support your calculations/conclusions always with proper explanation. (5). All the best!

Q.1. (i) Evaluate $\int_C (2x^3 - y^3)dx + (x^3 + y^3)dy$, where C is the unit circle $x^2 + y^2 = 1$ with counter clockwise orientation.
(ii) Let D be the region cutout of the solid $S = \{(x, y, z) \in \mathbb{R}^3 : 2x^2 + y^2 + z^2 \leq 4\}$ by the elliptic cylender $E = \{(x, y, z) \in \mathbb{R}^3 : 2x^2 + y^2 = 1\}$, i.e., Let D be the region inside the solid S and the cylender E. Find the volume of D.

[3+3]

Q.2. (i) Show that the vector Field $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ on $\mathbb{R}^3$ is conservative. Also find the work done by F in moving an object from $(1, -2, 1)$ to $(3, 1, 4)$
(ii) Compute the line integral

$$\int_C \frac{dx + dy}{|x| + |y|}$$

where C is the square with vertices $(1, 0), (0, 1), (-1, 0)$, and $(0, -1)$ traversed in once in the clockwise direction.

[3+3]

Q.3. Let $F(x, y, z) = (-y^3, x^3, -z^3)$ for $(x, y, z) \in \mathbb{R}^3$. Let C denote the intersection of the cylender $x^2 + y^2 = 1$ and the plane $x + y + z = 1$, oriented by the anticlockwise motion on the projection $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ of C on the xy plane. By using Stokes theorem, find $\int_c \vec{F}.dS$.

[4]

Q.4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) := \begin{cases} (x^2 + y^2)\cos(\frac{1}{x^2+y^2}), & if \ \ (x, y) \neq (0, 0), \\ 0, & if \ \ (x, y) = (0, 0). \end{cases}$$

(i) Do the partial derivatives f_x and f_y exist at every point of $\mathbb{R}^2$?
(ii) Discuss the continuity of f_x and f_y at $(0, 0)$.
(iii) Is the function f differentiable at $(0, 0)$?
Justify your answers.

[5]

Q.5. Let $p, q \in \mathbb{R}$ such that $p > 1$, $q > 1$. Use Lagrangers multiplier method to show that

$$\frac{1}{p} + \frac{1}{q} \leq \frac{x^p}{p} + \frac{y^q}{q}, \quad for\ all\ \ x, y \in (0, \infty)$$

with $xy = 1$.

[4]