Indian Institute of Technology Jodhpur
MAL1010, Dec'21-Mar'22
Tutorial Sheet 5

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1. Consider the function $f : [0,1] \to \mathbb{R}$ defined by
$$f(x) := \begin{cases} 0, & x \in [0,1] \setminus \{\frac{1}{2}\} \\ 1, & x = \frac{1}{2} \end{cases}$$
Then show that f is Riemann integrable.
2. Give an example of bounded function which is not integrable.
3. Let $f : [0,1] \to \mathbb{R}$ such that
$$f(x) := \begin{cases} \frac{1}{n}, & x = \frac{1}{n} \\ 0, & x \neq \frac{1}{n} \end{cases}$$
Show that f is integrable and find $\int_0^1 f(x)dx$.
4. Give an example of integrable function with infinitely many discontinuous points.
5. Let
$$g_n(y) = \begin{cases} \frac{ny^{n-1}}{1+y}, & 0 \leq y < 1 \\ 0, & y = 1 \end{cases}$$
Then prove that $\lim_{n\to\infty} \int_0^1 g_n(y)dy = \frac{1}{2}$ whereas $\int_0^1 \lim_{n\to\infty} g_n(y)dy = 0$.
6. Let p be fixed number and let f be a continuous function on $\mathbb{R}$ that satisfies the equation $f(x+p) = f(x)$ for every $x \in \mathbb{R}$. Show that the integral $\int_a^{a+p} f(t)dt$ has the same value for every real number a.
7. Let f be a continuous function on $[0, \pi/2]$ and $\int_0^{\pi/2} f(t)dt = 0$. Show that there exists a real number $c \in (0, \pi/2)$ such that $f(c) = 2\cos c$.
8. Evaluate $\lim_{n\to\infty} S_n$ by showing that S_n is an approximate Riemann sum for a suitable function over a suitable interval, where
(i) $S_n = \sum_{i=1}^{n} \frac{1}{\sqrt{1+(i/n)}} \left(\frac{i}{n} - \frac{i-1}{n}\right)$ (ii) $S_n = \sum_{i=1}^{n} \frac{n}{i^2+n^2}$