Indian Institute of Technology Jodhpur
MAL1010, Dec'21-Mar'22
Assginment Sheet 10 (Practice problems)

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1. Sketch the region of integration and evaluate the integral.
   (i) $\int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx dy$
   Ans. $e-2$

   (ii) $\int_1^4 \int_0^{\sqrt{x}} \frac{3}{2} e^{y/\sqrt{x}} dy dx$
   Ans. $7(e-1)$

2. Sketch the region of integration, reverse the order of integration, and evaluate the integral.
   (i) $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dy dx$
   Ans. 2

   (ii) $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$
   Ans. $\frac{e-2}{2}$

3. Find the volume of the solid in the first octant bounded by the coordinate planes, the cylender $x^2+y^2=4$, and the plane $z+y=3$.
   Ans. $\frac{9-\pi}{8}$

4. Evaluate $\int\int_D (x-y)^2 sin^2(x+y) d(x,y)$
   where D is the parallelogram with vertices at $(\pi,0),(2\pi,\pi),(\pi,2\pi),(0,\pi)$.
   Ans. $\frac{\pi^4}{3}$

5. Evaluate $\int_{\ln 6}^{\ln 7} \int_0^{\ln 2} \int_{\ln 4}^{\ln 5} e^{x+y+z} dx dy dz$
   Ans. 1

6. Evaluate $\int_0^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} \int_{x^2+y^2}^{2} xdz\ dy\ dx$
   Sketch the region of integration and evaluate the integral by expressing the order of integration as $dx\ dy\ dz$.
   Ans. $\frac{8\sqrt{2}}{15}$.

7. Let D be the region in xyz-space defined by the inequalities

   $$1 \le x \le 2, \quad 0 \le xy \le 2, \quad 0 \le z \le 1.$$

   Evaluate $\int\int\int_D (x^2y+3xyz) dx\ dy\ dz$ by applying the transofrmation $u=x, v=xy, w=3z$.
   Ans. $2+\ln 8$.

8. Evaluate $\int\int\int |xyz| dxdydz$ over the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$.
Hint: Take $x = au, y = bv, z = cw$, then integrate over appropriate region over uvw-space.
Ans. $\frac{a^2b^2c^2}{6}$

9. Using suitable change of variables, evaluate the following:
$$\int\int\int_D (z^2x^2 + z^2y^2) dx\, dy\, dz$$
where D is the cylindrical region $x^2 + y^2 \leq 1$ bounded by $-1 \leq z \leq 1$.
Ans. $\pi/3$.

10. Evaluate the triple integral
$$\int\int\int_D \exp(x^2 + y^2 + z^2)^{3/2} dx\, dy\, dz$$
where D is the region enclosed by the unit sphere in $\mathbb{R}^3$.
Ans. $4\pi(e - 1)/3$.