

Commutators & Angular Momentum

$$\begin{aligned}
 [x_i, p_j] &= i\hbar \delta_{ij}, & [x_i, x_j] &= [p_i, p_j] = 0, \\
 [L_i, L_j] &= i\hbar \varepsilon_{ijk} L_k, & \mathbf{J} &= \mathbf{L} + \mathbf{S}, \\
 \mathbf{J}^2 |j, m\rangle &= j(j+1)\hbar^2 |j, m\rangle, & J_z |j, m\rangle &= m\hbar |j, m\rangle, \\
 J_{\pm} |j, m\rangle &= \hbar \sqrt{j(j+1) - m(m \pm 1)} |j, m \pm 1\rangle.
 \end{aligned}$$

Canonical Hamiltonians

$$\begin{aligned}
 \text{Free: } H &= \frac{\mathbf{p}^2}{2m}, & E &= \frac{\hbar^2 k^2}{2m}, \\
 \text{1D Box: } \psi_n &= \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, & E_n &= \frac{\hbar^2 \pi^2}{2mL^2} n^2, \\
 \text{SHO (1D): } H &= \frac{\mathbf{p}^2}{2m} + \frac{1}{2} m \omega^2 x^2, & E_n &= \hbar \omega \left(n + \frac{1}{2} \right), \\
 a &= \sqrt{\frac{m\omega}{2\hbar}} x + \frac{i}{\sqrt{2m\hbar\omega}} p, \quad [a, a^\dagger] = 1, \\
 \psi_0(x) &= \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar} \right), \\
 \text{3D Isotropic SHO: } E_{n,\ell} &= \hbar \omega \left(2n + \ell + \frac{3}{2} \right), \\
 \text{Coulomb: } H &= \frac{\mathbf{p}^2}{2\mu} - \frac{Ze^2}{r}, \quad E_n = -\frac{\mu Z^2 e^4}{2\hbar^2 n^2}, \\
 u''_{E\ell}(r) + \frac{2\mu}{\hbar^2} \left[E - V(r) - \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} \right] u_{E\ell}(r) &= 0, \\
 \text{Rigid rotor: } H &= \frac{\mathbf{L}^2}{2I}, \quad E_\ell = \frac{\hbar^2}{2I} \ell(\ell+1).
 \end{aligned}$$

Minimal Coupling & Gauge

$$\begin{aligned}
 H &= \frac{1}{2m} \left(\mathbf{p} - \frac{q}{c} \mathbf{A} \right) \cdot \left(\mathbf{p} - \frac{q}{c} \mathbf{A} \right) + q\phi \\
 &= \frac{\mathbf{p}^2}{2m} - \frac{q}{2mc} (\mathbf{A} \cdot \mathbf{p} + \mathbf{p} \cdot \mathbf{A}) + \frac{q^2}{2mc^2} \mathbf{A}^2 + q\phi, \\
 \mathbf{B} &= \nabla \times \mathbf{A}, \quad \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{A} - \nabla \phi, \\
 \mathbf{A}' &= \mathbf{A} + \nabla \Lambda, \quad \phi' = \phi - \frac{1}{c} \partial_t \Lambda, \\
 \psi' &= \exp\left(\frac{iq}{\hbar c} \Lambda \right) \psi, \\
 D_0 &= \partial_t + \frac{iq}{\hbar} \phi, \quad \mathbf{D} = \nabla - \frac{iq}{\hbar c} \mathbf{A},
 \end{aligned}$$

Probability Current & Aharonov–Bohm

$$\begin{aligned}
 \partial_t |\psi|^2 + \nabla \cdot \mathbf{j} &= 0, \\
 \mathbf{j} &= \frac{\hbar}{m} \text{Im}(\psi^* \nabla \psi) - \frac{q}{mc} \mathbf{A} |\psi|^2, \\
 \Delta \varphi &= \frac{q}{\hbar c} \oint \mathbf{A} \cdot d\ell = \frac{q\Phi}{\hbar c}.
 \end{aligned}$$

Born–Oppenheimer Ansatz

$$\begin{aligned}
 H &= T_n(\mathbf{R}) + T_e(\mathbf{r}) + V_{ee} + V_{nn}(\mathbf{R}) + V_{en}(\mathbf{r}, \mathbf{R}), \\
 \Psi(\mathbf{r}, \mathbf{R}) &= \psi_e(\mathbf{r}; \mathbf{R}) \chi(\mathbf{R}), \\
 [T_e + V_{ee} + V_{en}(\mathbf{r}, \mathbf{R})] \psi_e(\mathbf{r}; \mathbf{R}) &= E_e(\mathbf{R}) \psi_e(\mathbf{r}; \mathbf{R}), \\
 [T_n + V_{nn}(\mathbf{R}) + E_e(\mathbf{R})] \chi(\mathbf{R}) &= E_{\text{tot}} \chi(\mathbf{R}).
 \end{aligned}$$