FILTER DESIGN ASSIGNMENT

EE338 : Digital Signal Processing

IIT BOMBAY

Autumn 2020

Group 3

Devank Rajvanshi 18D100009

Contents

Filter Number : 152
Reviewed by : Abhilaksh Maheshwari (18D070035)
Reviewed of : Harshit Shrivastava (18D070011)
Group Number : 3

1 Band-pass Filter Design

1.1 Un-normalized Discrete Time Filter Specifications

Since $152 > 80$;
$m = 152 - 80 = 72$
The passband of this filter is **equiripple**.

$$\begin{aligned} q(72) &= \text{greatest integer lesser than } 0.1 \times 72 = 7 \\ r(72) &= 72 - 10 \times 7 = 72 - 70 = 2 \\ \mathrm{BL}(72) &= 25 + 1.7 \times 7 + 6.1 \times 2 = 49.1kHz \\ \mathrm{BH}(72) &= \mathrm{BL}(72) + 20 = 69.1kHz \end{aligned}$$

The required discrete time filter should have a equiripple passband and all the IIR filters are required to have a monotonic stopband. Therefore, **Chebyschev** approximation would be used to design IIR Filter.

Here we design a **bandpass filter** with the following specifications:

1. **Passband:** Equiripple passband, with $49.1\,\text{kHz} \le f \le 69.1\,\text{kHz}$
2. **Transition band:** 4 kHz on either side of the passband
3. **Stopband:** Monotonic passband, with $0 \le f \le 45.1\,\text{kHz}$ and $f \ge 73.1\,\text{kHz}$
4. **Tolerance:** 0.15 in both passband and stopband (in magnitude)

1.2 Normalized Discrete-Time Filter Specifications

We are given a sampling rate of $f_s = 330\,\text{kHz}$. Specifications in terms of the normalized angular frequencies ω can be found using the following relation with the un-normalized frequencies f:

$$\omega = \frac{f}{f_s} 2\pi \tag{1}$$

1. **Passband:** Equiripple Passband $0.2976\pi \le \omega \le 0.4188\pi$
2. **Transition band:** 0.0242π on either side of the passband
3. **Stopband:** Monotonic Passband $0 \le \omega \le 0.2733\pi$ and $0.4430\pi \le \omega \le \pi$
4. **Tolerance:** 0.15 in both passband and stopband (in magnitude)

Hence, we have the following normalized discrete time specifications:

$$\begin{aligned} \omega_{s1} &= 0.2733\pi \qquad & \omega_{p1} &= 0.2976\pi \\ \omega_{p2} &= 0.4188\pi \qquad & \omega_{s2} &= 0.4430\pi \end{aligned} \tag{2}$$

1.3 Analog Filter Specifications Using the Bilinear Transformation

The bilinear transformation for transforming the given frequencies to the analog frequency domain is given as

$$\Omega = tan(\frac{\omega}{2}) \tag{3}$$

The following table lists the values of the analog frequency transformations for the above normalized filter specifications:

ω	Ω
0	0
0.2733π	0.4578
0.2976π	0.5048
0.4188π	0.7727
0.4430π	0.8352
π	∞

Thus, corresponding analog filter specifications for the same type of analog filter after the bilinear transformations are:

1. **Passband:** Equiripple Passband $0.5048 \leq \Omega \leq 0.7727$
2. **Transition band**: $0.4578 \leq \Omega \leq 0.5048$, $0.7727 \leq \Omega \leq 0.8352$
3. **Stopband:** $0 \leq \Omega \leq 0.4578$ and $0.8352 \leq \Omega < \infty$, Monotonic
4. **Tolerance:** 0.15 in both passband and stopband (in magnitude)

Hence, we have the following transformed analog filter specifications:

$$\begin{aligned} \Omega_{s1} &= 0.4578 \qquad & \Omega_{p1} &= 0.5048 \\ \Omega_{p2} &= 0.7727 \qquad & \Omega_{s2} &= 0.8352 \end{aligned} \tag{4}$$

1.4 Frequency Transformation

For a bandpass filter, the following analog frequency transformation is used for converting the desired bandstop filter (with independent variable Ω) to an analog lowpass filter (with independent variable Ω_L):

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega} \tag{5}$$

here Ω are the analog filter specifications obtained in the previous section and the two parameters B and Ω_0 are as follows:

$$\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.6245$$
$$B = \Omega_{p2} - \Omega_{p1} = 0.2679$$

Hence, we have the following low pass transformed frequency values:

$$\begin{aligned} &\Omega_{L_{s1}} = -1.4716 \qquad && \Omega_{L_{p1}} = -1 \\ &\Omega_{L_{p2}} = 1 && \Omega_{L_{s2}} = 1.3743 \end{aligned} \tag{6}$$

1.5 Frequency Transformed Lowpass Analog Filter Specifications

Following the transformation of frequency as in Eqn. 5, we get the following specifications for a lowpass filter:

1. **Passband:** Equiripple, $0 \leq \Omega \leq \Omega_{L_{p2}} \; \equiv \; 0 \leq \Omega \leq 1$
2. **Transition band:** $\Omega_{L_{p2}} \leq \Omega \leq \min\left(|\Omega_{L_{s1}}|, \Omega_{L_{s2}}\right) \; \equiv \; 1 \leq \Omega \leq 1.3743$
3. **Stopband:** Monotonic, $\min\left(|\Omega_{L_{s1}}|, \Omega_{L_{s2}}\right) \leq \Omega \leq \infty \; \equiv \; 1.3743 \leq \Omega \leq \infty$
4. **Tolerance:** 0.15 in both passband and stopband (in magnitude)

1.6 Analog Lowpass Transfer Function

We need an Analog Filter which has an **equiripple passband** and a **monotonic stopband**. Therefore we need to design using the **Chebyshev approximation**. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = 43.4444$$

Now choosing the parameter ϵ of the Chebyshev filter to be $\sqrt{D_1}$, we get the min value of N

$$N_{min} = \left\lceil \frac{cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{cosh^{-1}(\frac{\Omega_{L_s}}{\Omega L_p})} \right\rceil \tag{7}$$

$$N_{min} = \lceil 3.636 \rceil = 4$$

Now, the poles of the transfer function can be obtained by

$$1 + D_1 \cdot cosh^2(N_{min} \cdot cosh^{-1}(\frac{s}{j})) = 1 + 0.3841 \cdot cosh^2(4cosh^{-1}(\frac{s}{j})) = 0 \tag{8}$$

Solving for this equation, we get the following roots on the complex plane.

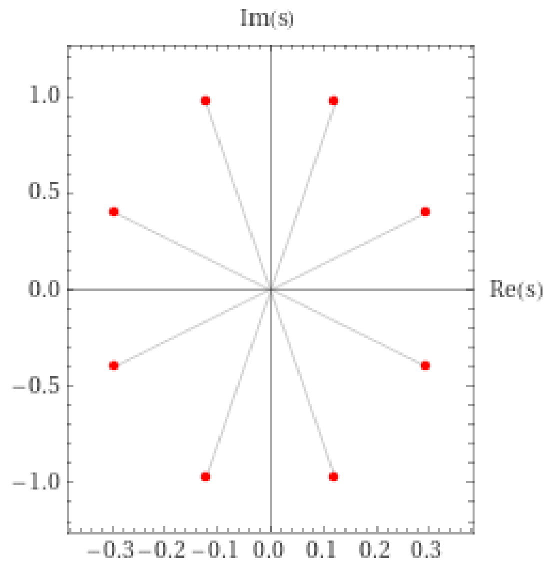

Figure 1: Roots of the magnitude sqaured system function (Plotted using Wolfram Alpha)

The above figure shows the poles of the Magnitude Squared System function. We'll only consider the roots with negative real part as our stable solutions corresponding to $H_{analog}(s_L)$.

$$p_1 = -0.1222 - 0.9698\iota$$

$$p_2 = -0.1222 + 0.9698\iota$$

$$p_3 = -0.2949 + 0.4017\iota$$

$$p_4 = -0.2949 - 0.4017\iota$$

Using the above poles lying in the left half plane and the fact that N is even, we can write the Analog Lowpass Transfer Function as follows:

$$H_{analog,LPF}(s_L) = \frac{(-1)^4 p_1 \cdot p_2 \cdot p_3 \cdot p_4}{\sqrt{1+D_1}(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)} \tag{9}$$

$$H_{analog,LPF}(s_L) = \frac{0.2017}{(s_L^2 + 0.2444 \cdot s_L + 0.9554)(s_L^2 + 0.5898 \cdot s_L + 0.2483)}$$

1.7 Analog Bandpass Transfer Function

To go from the analog lowpass transfer function to the analog bandpass transfer function, following transformation is used:

$$s_L = \frac{s^2 + \Omega_0^2}{Bs} \tag{10}$$

Substituting the values of the parameters $B = 0.2679$ and $\Omega_0 = 0.6245$

$$s_L = \frac{s^2 + 0.39}{0.2679s}$$

Hence we have the analog bandpass transfer function as:

$$H_{\text{analog,BPF}}(s) = H_{\text{analog,LPF}}\left(\frac{s^2 + \Omega_0^2}{Bs}\right) \tag{11}$$

Using the above substitution, we get $H_{analog,BPF}(s)$ from $H_{analog,LPF}(s_L)$ of the form :

$$H_{analog,BPF}(s) = \frac{0.0010s^4}{D(s)} \tag{12}$$

$D(s)$ is a 8 degree polynomial in s and the coefficients of the same are tabulated below. MATLAB codes referenced here were used to obtain these expressions.

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
8	1	7	0.22347	6	1.6566
5	0.2735	4	0.9892	3	0.1066
2	0.2519	1	0.01325	0	0.0231

Table 1: Coefficients of $D(s)$, the denominator of Analog Bandpass Transfer Function

1.8 Discrete-time filter transfer function

Finally, to go from the analog bandpass transfer function to the discrete-time transfer function that is needed, we use the transformation

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \tag{13}$$

Hence we have the discrete-time transfer function

$$H_{\text{discrete,BPF}}(z) = H_{\text{analog,BPF}}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) \tag{14}$$

Using the above substitution, we get $H_{discrete,BPF}(z)$ from $H_{analog,BPF}(s)$ of the form :

$$H_{discrete,BPF}(z) = \frac{N(z)}{D(z)} \tag{15}$$

$N(z)$ and $D(z)$ are 8 degree polynomials in z and the coefficients of the same are tabulated below. MATLAB codes referenced here were used to obtain these expressions.

Degree	Coefficient*(10^{-4})	Degree	Coefficient*(10^{-4})	Degree	Coefficient*(10^{-4})
8	2.2894	7	0	6	-9.1577
5	0	4	13.7365	3	0
2	-9.1577	1	0	0	2.2894

Table 2: Coefficients of $N(z)$, the numerator of Discrete Bandpass Transfer Function

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
8	1	7	-3.3119	6	7.6865
5	-11.2453	4	12.8849	3	-10.3893
2	6.5608	1	-2.6089	0	0.7281

Table 3: Coefficients of $D(z)$, the denominator of Discrete Bandpass Transfer Function

1.9 IIR Bandpass Filter Response

The plot in Fig.2 shows the magnitude response of the filter designed as a function of the frequency. It can be seen that the magnitude does indeed follow the specifications outlined in Sec. 1.1.

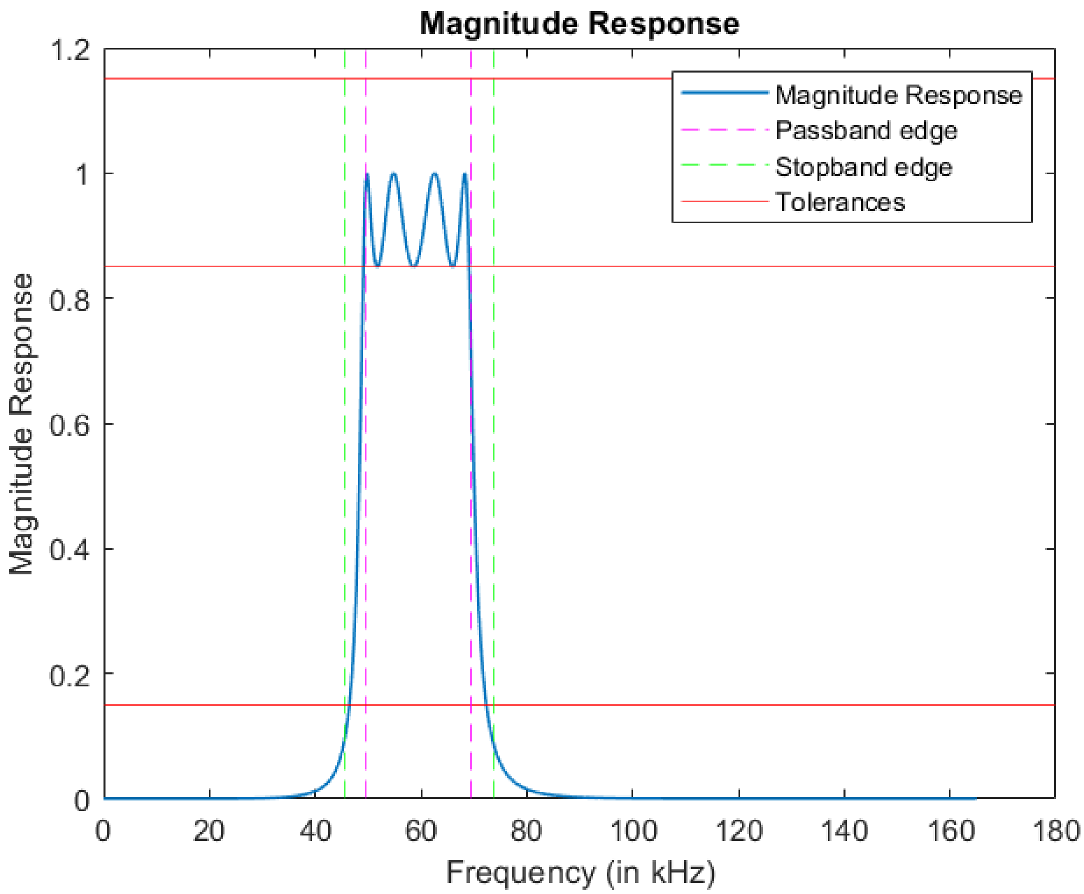

Figure 2: Magnitude response of the desired discrete-time bandpass filter

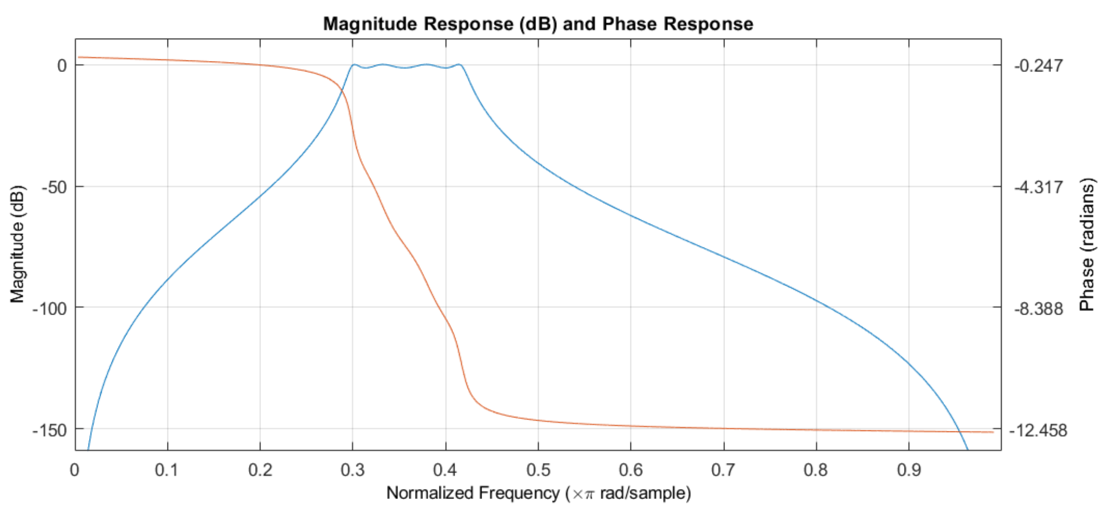

Figure 3: Magnitude response ($Log - scale$) of the designed filter on normalised frequency axis

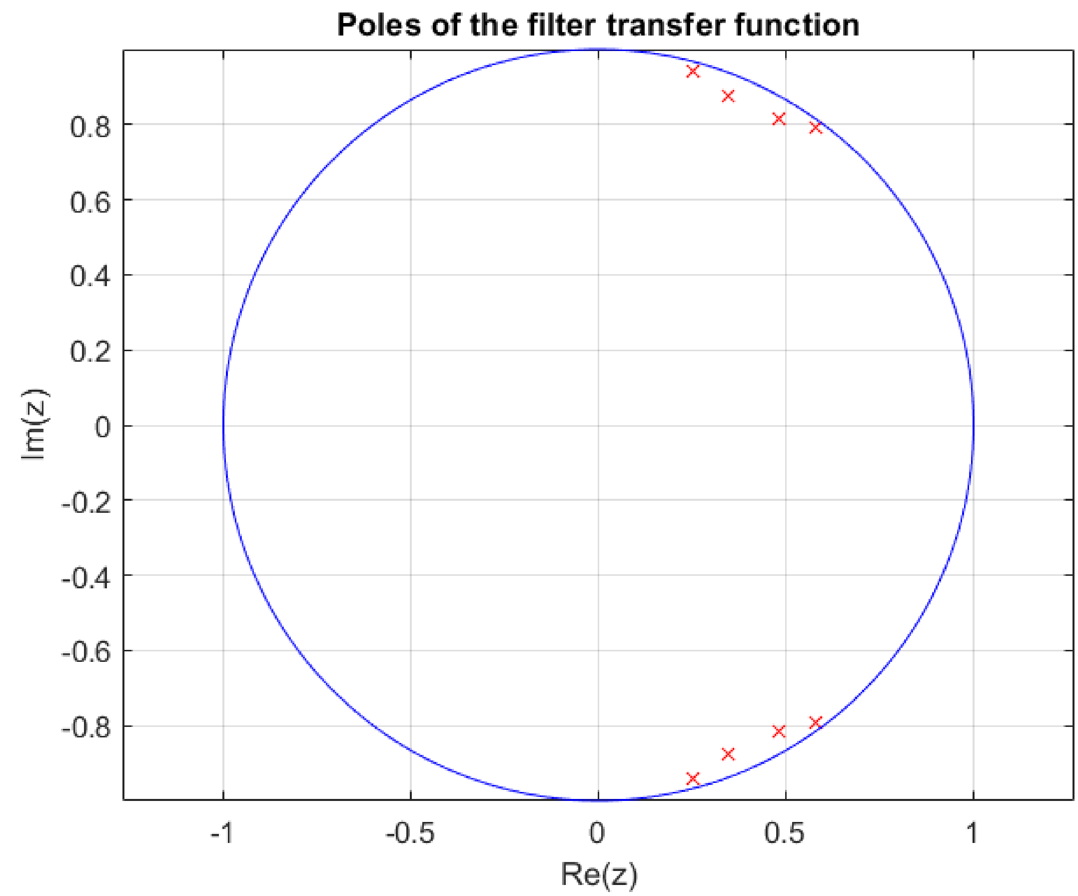

Figure 4: Pole-Zero Plot

1.10 FIR filter transfer function

We now realize the same specifications as a Finite Impulse Response transfer function, using a Kaiser window. We first design this window,

$$W(k) = \frac{I_0\left(\beta N\sqrt{1-(\frac{k}{N})^2}\right)}{I_0(\beta N)} \tag{16}$$

where $I_0(\cdot)$ is the modified Bessel function of order 0, $\beta > 0$ is the shape parameter, and $2N+1$ is the length of the window, calculated as follows:

$$(2N+1) \geq 1 + \frac{A-8}{2.2385\Delta\omega_T}, \qquad \alpha = \beta N = \begin{cases} 0 & A < 21 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) & 21 \leq A < 50 \\ 0.1102(A-8.7) & 51 \leq A \end{cases} \tag{17}$$

where $A = -20\log\delta$, with δ the stricter tolerance, and $\Delta\omega_T$ is the width of the transition band. Here we have $\delta = 0.15$ and $\Delta\omega_T = 0.0242\pi$ (Sec. 1.2), which gives us

$$2N+1 \geq 50.8173 \qquad \alpha = 0 \implies \beta = 0 \tag{18}$$

For designing the bandpass filter according to specifications, we first require an ideal bandpass filter. This is obtained by subtracting two lowpass filters of appropriate frequencies. We take the midpoint of the transition band in each case as the cutoff frequencies for the ideal lowpass filter. Then we apply the Kaiser window of length $2N+1 \geq 50.8173$ and $\beta = 0$, and adjust $2N+1$ such that the passband and stopband specifications are met. A minimum value of $N = 25$ satisfies the first condition, but to meet the filter stopband and passband specifications, we further increase the value of N iteratively. Finally,

$$N = 25 + 8 = 33$$

Following is the time domain sequence $h[n]$ corresponding to the filter.

```
FIR_BandPass =

  Columns 1 through 10

    0.0155   -0.0020   -0.0140   -0.0080    0.0026    0.0026   -0.0014    0.0046    0.0136    0.0057

  Columns 11 through 20

   -0.0174   -0.0256   -0.0021    0.0274    0.0256   -0.0048   -0.0243   -0.0130    0.0047    0.0026

  Columns 21 through 30

   -0.0039    0.0122    0.0334    0.0122   -0.0477   -0.0702   -0.0021    0.0928    0.0917   -0.0264

  Columns 31 through 40

   -0.1307   -0.0883    0.0621    0.1455    0.0621   -0.0883   -0.1307   -0.0264    0.0917    0.0928

  Columns 41 through 50

   -0.0021   -0.0702   -0.0477    0.0122    0.0334    0.0122   -0.0039    0.0026    0.0047   -0.0130

  Columns 51 through 60

   -0.0243   -0.0048    0.0256    0.0274   -0.0021   -0.0256   -0.0174    0.0057    0.0136    0.0046

  Columns 61 through 67

   -0.0014    0.0026    0.0026   -0.0080   -0.0140   -0.0020    0.0155
```

Figure 5: Time Domain Sequence Values

The transfer function, from which the pseudo-linear phase is clear to see, i.e. the phase is linear throughout except for the discontinuities due to the extra negative sign since the magnitude response is always positive.

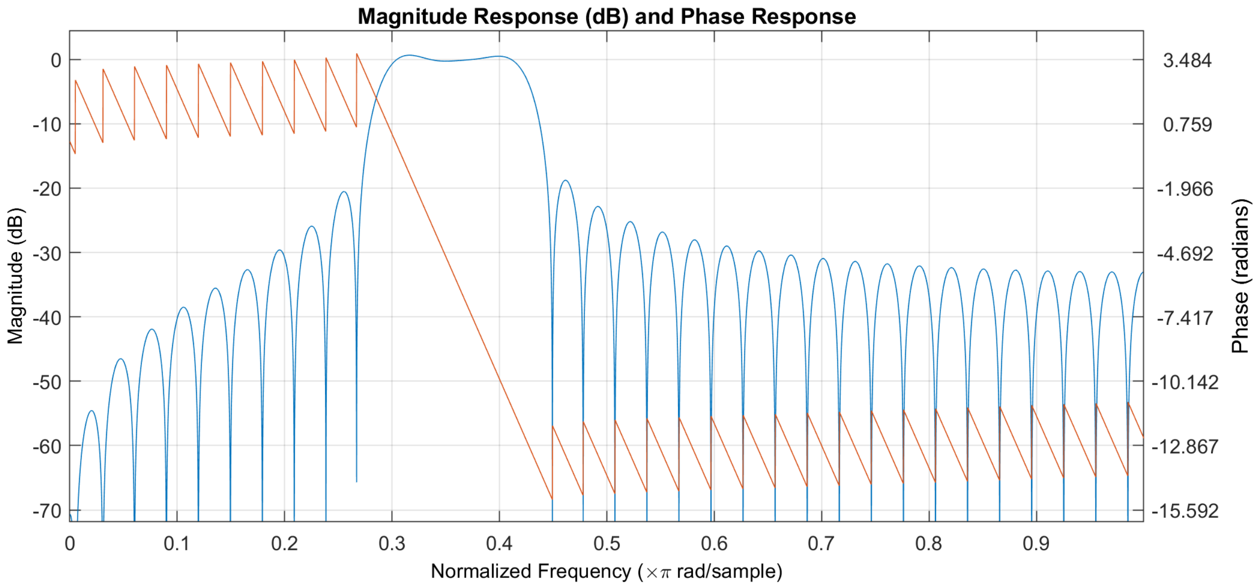

Figure 6: Transfer function of the bandpass filter desired, with Magnitude response shown in blue and Phase response in red

The magnitude response is shown from which it can be seen that the specifications are followed.

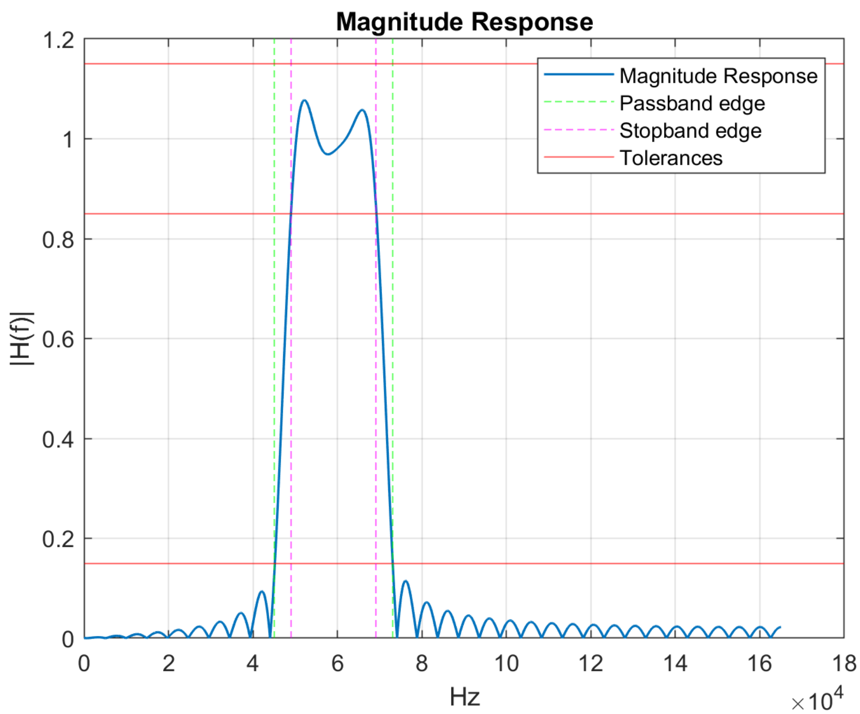

Figure 7: Magnitude response of the bandpass filter desired

Time series of the FIR is

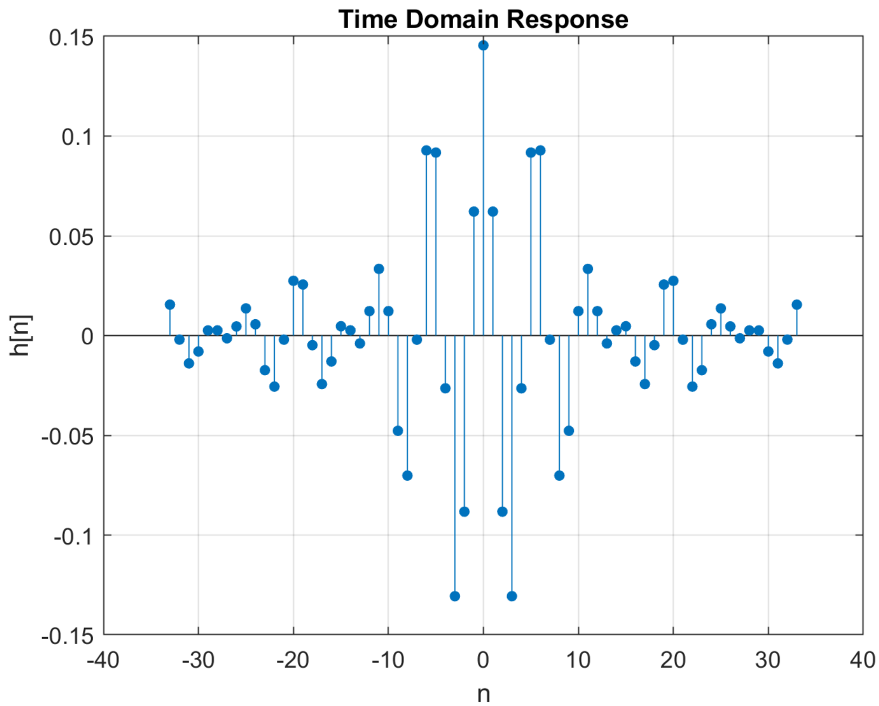

Figure 8: Time series $h[n]$ of bandpass filter

2 Band-Stop Filter Design

2.1 Un-normalized discrete-time filter specifications

Since $152 > 80$;
$m = 152 - 80 = 72$
The passband of this filter is **monotonic**. We are to design a **bandstop filter** with the following specifications:

$$\begin{aligned}
q(72) &= \text{greatest integer lesser than } 0.1 \cdot 72 = 7 \\
r(72) &= 72 - 10 \cdot 7 = 72 - 70 = 2 \\
\text{BL}(72) &= 25 + 1.9 \cdot 7 + 4.1 \cdot 2 = 46.5 \\
\text{BH}(72) &= \text{BL}(72) + 20 = 66.5
\end{aligned}$$

1. **Stopband:** $46.5\,\text{kHz} \leq f \leq 66.5\,\text{kHz}$, Monotonic
2. **Transition band:** 4 kHz on either side of the stopband
3. **Passband:** $0 \leq f \leq 42.5\,\text{kHz}$ and $f \geq 70.5\,\text{kHz}$, Monotonic
4. **Tolerance:** 0.15 in both passband and stopband (in magnitude)

2.2 Normalized Discrete-Time Filter Specifications

We are given a sampling rate of $f_s = 260\,\text{kHz}$. Specifications in terms of the normalized angular frequencies ω can be found using the following relation with the un-normalized frequencies f:

$$\omega = \frac{f}{f_s} 2\pi \tag{19}$$

1. **Stopband:** Monotonic $0.3577\pi \leq \omega \leq 0.5115\pi$
2. **Transition band:** 0.0242π on either side of the passband
3. **Passband:** Monotonic $0 \leq \omega \leq 0.327\pi$ and $0.5423\pi \leq \omega \leq \pi$
4. **Tolerance:** 0.15 in both passband and stopband (in magnitude)

Hence, we have the following normalized discrete time specifications:

$$\begin{aligned}
\omega_{p1} &= 0.327\pi \qquad \omega_{s1} = 0.3577\pi \\
\omega_{s2} &= 0.5115\pi \qquad \omega_{p2} = 0.5423\pi
\end{aligned} \tag{20}$$

2.3 Analog filter specifications using bilinear transformation

We now convert the normalized angular frequency ω to the analog frequency Ω, related by

$$\Omega = \tan\left(\frac{\omega}{2}\right) \tag{21}$$

The following table lists the values of the analog frequency transformations for the above normalized filter specifications:

ω	Ω
0	0
0.327π	0.564
0.3577π	0.6295
0.5115π	1.0369
0.5423π	1.1426
π	∞

Thus, corresponding analog filter specifications for the same type of analog filter after the bilinear transformations are:

1. **Stopband:** $0.6295 \leq \Omega \leq 1.0369$, Monotonic
2. **Transition band:** $0.564 \leq \Omega \leq 0.6295$, $1.0639 \leq \Omega \leq 1.1426$
3. **Passband:** $0 \leq \Omega \leq 0.564$ and $1.1426 \leq \Omega < \infty$, Monotonic
4. **Tolerance:** 0.15 in both passband and stopband (in magnitude)

Hence, we have the following transformed analog filter specifications:

$$\begin{aligned} \Omega_{p1} &= 0.564 \qquad & \Omega_{s1} &= 0.6295 \\ \Omega_{s2} &= 1.0369 \qquad & \Omega_{p2} &= 1.1426 \end{aligned} \tag{22}$$

2.4 Frequency transformation

For a band-stop filter, the following analog frequency transformation is used for converting the desired bandstop filter (with independent variable Ω) to an analog lowpass filter (with independent variable Ω_L):

$$\Omega_L = \frac{B\,\Omega}{\Omega_0^2 - \Omega^2} \tag{23}$$

here Ω are the analog filter specifications obtained in the previous section and the two parameters B and Ω_0 are as follows:

$$B = \Omega_{p2} - \Omega_{p1} = 0.5786$$

$$\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.8028$$

It is clear that $\Omega = 0^+$ corresponds to $\Omega_L = 0^+$, $\Omega = \Omega_0^-$ corresponds to $\Omega_L = \infty$, $\Omega = \Omega_0^+$ corresponds to $\Omega_L = -\infty$, and $\Omega = \infty$ corresponds to $\Omega_L = 0^-$. Hence we have a one-to-one transformation of the bandstop filter to an equivalent lowpass filter (symmetry of the lowpass filter about $\Omega_L = 0$ is guaranteed by the choice of Ω_0). B is chosen such that the lowpass passband edge occurs at $\Omega_L = \pm 1$, to make calculations easier.
The values of important frequencies after transformation are

$$\begin{aligned} \Omega_{L_{p1}} &= 1 \qquad & \Omega_{L_{s1}} &= 1.4674 \\ \Omega_{L_{s2}} &= -1.3929 \qquad & \Omega_{L_{p2}} &= -1 \end{aligned} \tag{24}$$

2.5 Frequency transformed lowpass analog filter specifications

Following the transformation of frequency as in Eqn. 23, we get specifications for a lowpass filter:

1. **Passband:** $0 \leq \Omega \leq \Omega_{L_{p1}} \;\equiv\; 0 \leq \Omega \leq 1$, Monotonic
2. **Transition band:** $\Omega_{L_{p1}} \leq \Omega \leq \min\left(\Omega_{L_{s1}},\, |\Omega_{L_{s2}}|\right) \;\equiv\; 1 \leq \Omega \leq 1.3929$
3. **Stopband:** $\min\left(\Omega_{L_{s1}},\, |\Omega_{L_{s2}}|\right) \leq \Omega \leq \infty \;\equiv\; 1.3929 \leq \Omega \leq \infty$, Monotonic
4. **Tolerance:** 0.15 in both passband and stopband (in magnitude)

2.6 Analog lowpass transfer function

We need an Analog Filter which has a **monotonic passband** and a **monotonic stopband**. Therefore we need to design using the **Butterworth approximation**. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = 43.4444$$

Now using the inequality on the order N of the filter for the Butterworth Approximation

$$N_{min} = \left\lceil \frac{log(\sqrt{\frac{D_2}{D_1}})}{log(\frac{\Omega_{L_s}}{\Omega L_p})} \right\rceil \tag{25}$$

$$N_{min} = \lceil 7.134 \rceil = 8$$

The cut-off frequency(Ω_c) of the Analog LPF should satisfy

$$\frac{\Omega_{L_p}^{2N}}{D_1} \leq \Omega_c^{2N} \leq \frac{\Omega_{L_s}^{2N}}{D_2} \tag{26}$$

$$1.061 \leq \Omega_c \leq 1.100$$

We take $\Omega_c = 1.08$, which lies within the range.
Now we find the poles of the magnitude squared system function, given by solutions to the equation

$$1 + \left(\frac{s}{j\Omega_c}\right)^{2N} = 0 \tag{27}$$

Solving for this equation, we get the following roots on the complex plane.

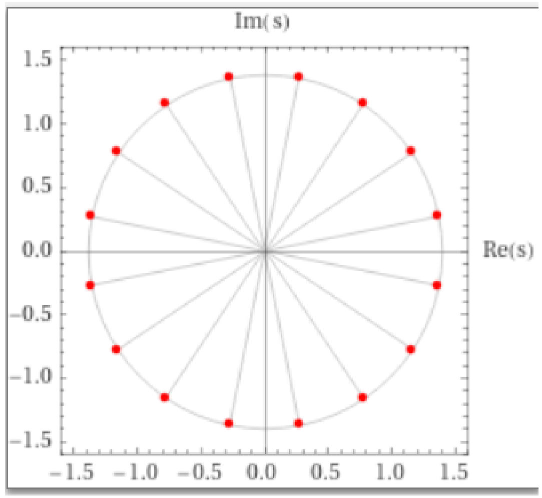

Figure 9: Roots of the magnitude sqaured system function (Plotted using Wolfram Alpha)

The above figure shows the poles of the Magnitude Squared System function. We'll only consider the roots with negative real part as our stable solutions corresponding to $H_{analog}(s_L)$.

$$p_1 = -0.211 - 1.059\iota$$

$$p_2 = -0.211 + 1.059\iota$$

$$p_3 = -0.600 - 0.898\iota$$

$$p_4 = -0.600 + 0.898\iota$$

$$p_5 = -0.898 - 0.600\iota$$

$$p_6 = -0.898 + 0.600\iota$$

$$p_7 = -1.059 - 0.211\iota$$

$$p_8 = -1.059 + 0.211\iota$$

Using the above poles lying in the left half plane and the fact that N is even, we can write the Analog Lowpass Transfer Function as follows:

$$H_{analog,LPF}(s_L) = \frac{(\Omega_c)^N}{(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)(s_L - p_5)(s_L - p_6)(s_L - p_7)(s_L - p_8)} \tag{28}$$

2.7 Analog bandstop transfer function

To go from the analog lowpass transfer function to the analog band-stop transfer function, following transformation is used:

$$s_L = \frac{Bs}{s^2 + \Omega_0^2} \tag{29}$$

Substituting the values of the parameters $B = 0.5786$ and $\Omega_0 = 0.8028$

$$s_L = \frac{0.5786s}{s^2 + 0.8028^2}$$

We can convert the lowpass transfer function to the desired bandstop transfer function.

$$H_{\text{analog,BSF}}(s) = H_{\text{analog,LPF}}(s_L) = H_{\text{analog,LPF}}\left(\frac{0.5786s}{0.8028^2 + s^2}\right) \tag{30}$$

$$H_{analog,BSF}(s) = \frac{N(s)}{D(s)} \tag{31}$$

$N(s)$ and $D(s)$ are 16 degree polynomials in s and the coefficients of the same are tabulated below. MATLAB codes referenced here were used to obtain these expressions.

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
16	1.00	15	0	14	5.15
13	0	12	11.62	11	0
10	14.986	9	0	8	12.07
7	0	6	6.22	5	0
4	2.00	3	0	2	0.36
1	0	0	0.029	blank	blank

Table 4: Coefficients of $N(s)$, the numerator of Analog Band-stop Transfer Function

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
16	1	15	2.74	14	8.92
13	15.74	12	28.32	11	35.73
10	44.24	9	41.60	8	37.93
7	26.80	6	18.37	5	9.56
4	4.88	3	1.75	2	0.63
1	0.12	0	0.029	blank	blank

Table 5: Coefficients of $D(s)$, the denominator of Analog Band-stop Transfer Function

2.8 Discrete-time filter transfer function

Finally, to go from the analog bandstop transfer function to the discrete-time transfer function that is needed, we use the transformation

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \tag{32}$$

Hence we have the discrete-time transfer function

$$H_{\text{discrete,BPF}}(z) = H_{\text{analog,BPF}}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = \frac{N(z)}{D(z)} \tag{33}$$

$N(z)$ and $D(z)$ are 16 degree polynomials in z and the coefficients of the same are tabulated below. MATLAB codes referenced here were used to obtain these expressions.

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
16	0.19	15	-0.66	14	2.54
13	-5.52	12	11.88	11	-18.46
10	27.75	9	-32.44	8	36.44
7	-32.44	6	27.75	5	-18.46
4	11.88	3	-5.52	2	2.54
1	-0.66	0	0.19	blank	blank

Table 6: Coefficients of $N(z)$, the numerator of Discrete Band-stop Transfer Function

Degree	Coefficient	Degree	Coefficient	Degree	Coefficient
16	1	15	-2.76	14	8.13
13	-14.08	12	23.85	11	-29.82
10	35.86	9	-34.078	8	31.035
7	-22.694	6	15.9	5	-8.77
4	4.67	3	-1.81	2	0.69
1	-0.15	0	0.037	blank	blank

Table 7: Coefficients of $D(z)$, the denominator of Discrete Band-stop Transfer Function

2.9 IIR Band-stop Filter Response

The plot in Fig.10 shows the magnitude response of the filter designed as a function of the frequency. It can be seen that the magnitude does indeed follow the specifications outlined in Sec. 2.1.

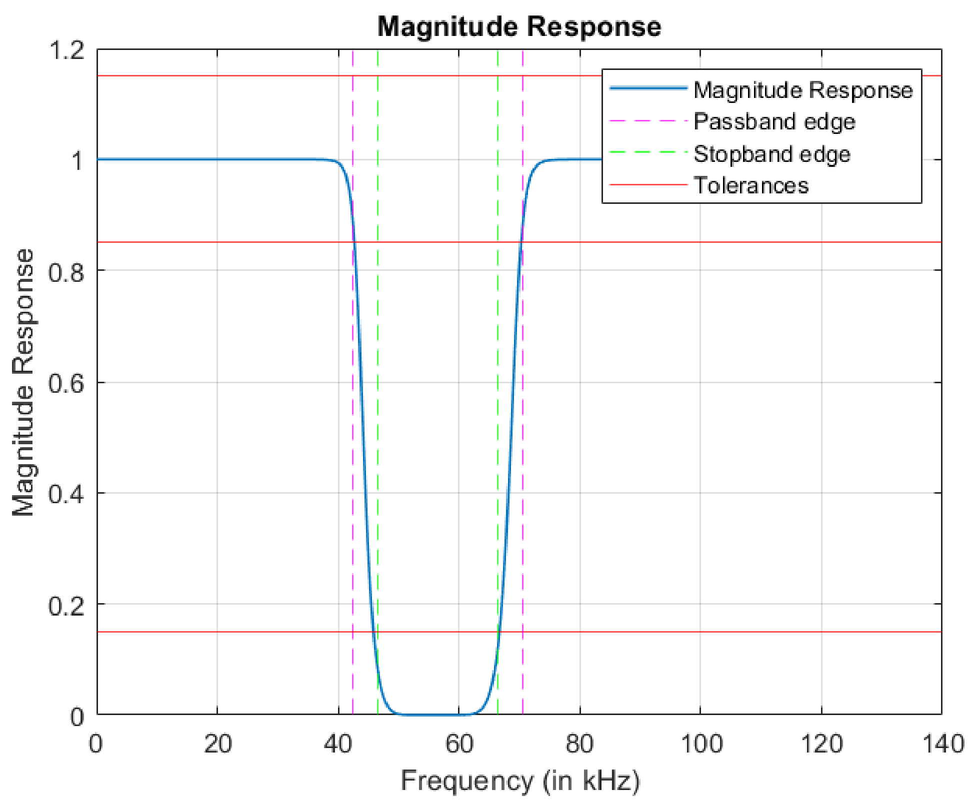

Figure 10: Magnitude response of the desired discrete-time band-stop filter

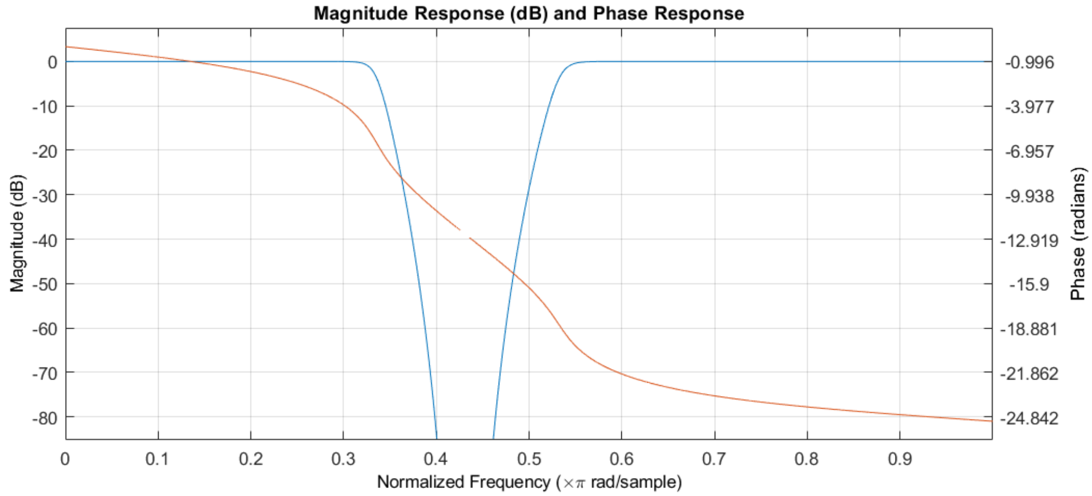

Figure 11: Magnitude response ($Log - scale$) of the designed filter on normalised frequency axis

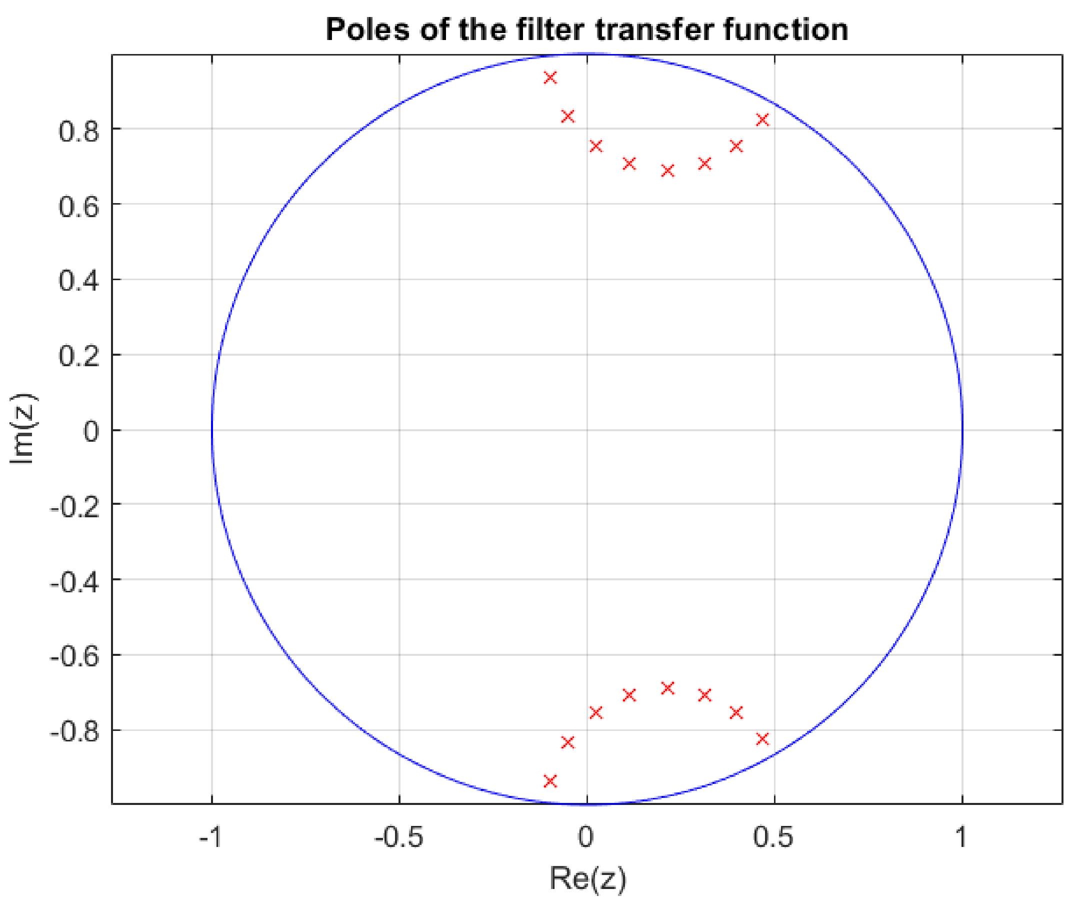

Figure 12: Pole-Zero Plot

2.10 FIR filter transfer function

We now realize the same specifications as a Finite Impulse Response transfer function, using a Kaiser window. We first design this window,

$$W(k) = \frac{I_0\left(\beta N\sqrt{1-(\frac{k}{N})^2}\right)}{I_0(\beta N)}$$

where $I_0(\cdot)$ is the modified Bessel function of order 0, $\beta > 0$ is the shape parameter, and $2N+1$ is the length of the window, calculated as follows:

$$(2N+1) \geq 1 + \frac{A-8}{2.2385\Delta\omega_T}, \qquad \alpha = \beta N = \begin{cases} 0 & A < 21 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) & 21 \leq A < 50 \\ 0.1102(A-8.7) & 51 \leq A \end{cases}$$

where $A = -20\log\delta$, with δ the stricter tolerance, and $\Delta\omega_T$ is the width of the transition band. Here we have $\delta = 0.15$ and $\Delta\omega_T = 0.0308\pi$, which gives us

$$2N+1 \geq 40.142 \qquad \alpha = 0 \implies \beta = 0$$

The band stop impulse response samples were generated by taking differences and sum of all pass and respective low pass response samples.

We now design the bandstop filter as specified, for which we first require an ideal bandstop filter. For this we first make a bandpass filter with the same boundaries by subtracting two lowpass filters of appropriate frequencies, and then subtract this from an allpass filter. Here we have taken the midpoint of the transition band in each case as the cutoff frequencies for the ideal lowpass filters. Then we apply the Kaiser window of length $2N+1 \geq 40.1421$ and $\beta = 0$, and adjust $2N+1$ such that the passband and stopband specifications are met. The first condition is met for a minimum $N = 20$, but to meet the filter specs, we further increase N interatively. Finally,

$$N = 20 + 6 = 26.$$

Following is the time domain sequence $h[n]$ corresponding to the filter.

```
FIR_BandStop =

  Columns 1 through 10

    0.0137    0.0191   -0.0036   -0.0104   -0.0005   -0.0054   -0.0084    0.0161    0.0263   -0.0126

  Columns 11 through 20

   -0.0393   -0.0024    0.0349    0.0131   -0.0137   -0.0022   -0.0071   -0.0345    0.0042    0.0808

  Columns 21 through 30

    0.0347   -0.1082   -0.0994    0.0937    0.1599   -0.0371    0.8154   -0.0371    0.1599    0.0937

  Columns 31 through 40

   -0.0994   -0.1082    0.0347    0.0808    0.0042   -0.0345   -0.0071   -0.0022   -0.0137    0.0131

  Columns 41 through 50

    0.0349   -0.0024   -0.0393   -0.0126    0.0263    0.0161   -0.0084   -0.0054   -0.0005   -0.0104

  Columns 51 through 53

   -0.0036    0.0191    0.0137
```

Figure 13: Time Domain Sequence Values

The transfer function, from which the pseudo-linear phase is clear to see, i.e. the phase is linear throughout except for the discontinuities due to the extra negative sign since the magnitude response is always positive.

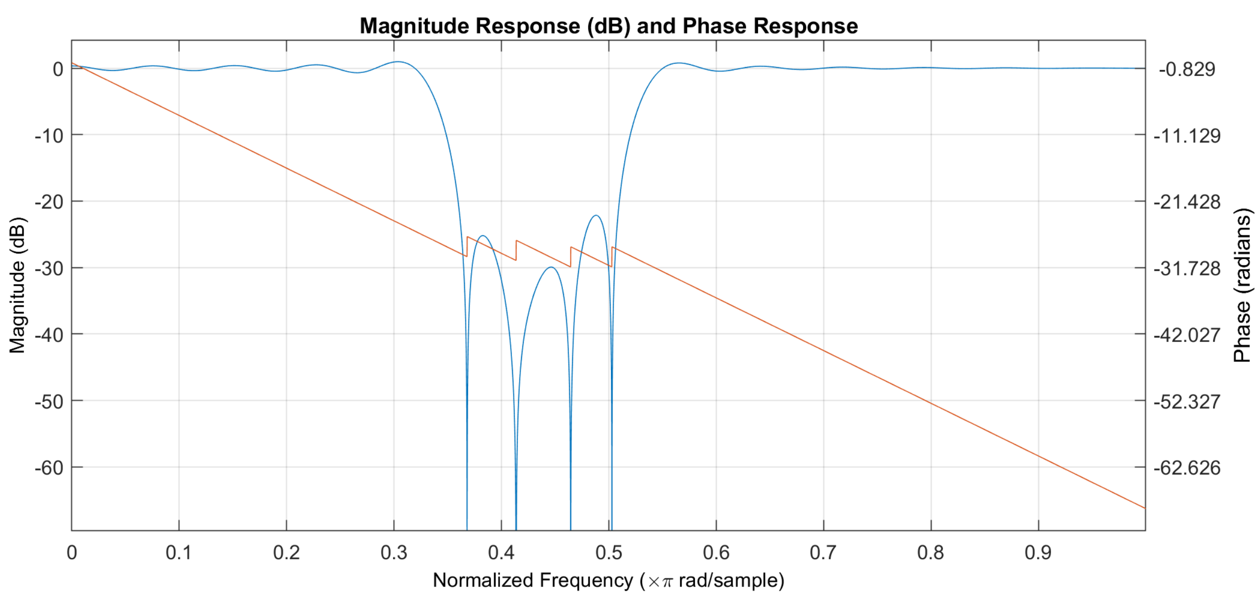

Figure 14: Transfer function of the bandpass filter desired, with Magnitude response shown in blue and Phase response in red

The magnitude response is shown from which it can be seen that the specifications are followed.

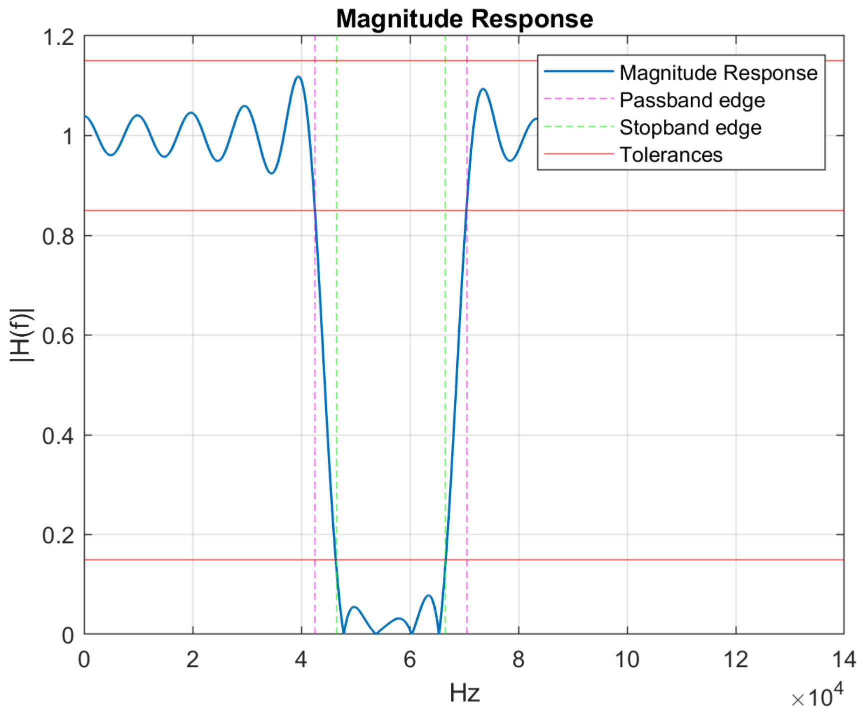

Figure 15: Magnitude response of the bandpass filter desired

Time series of the FIR is

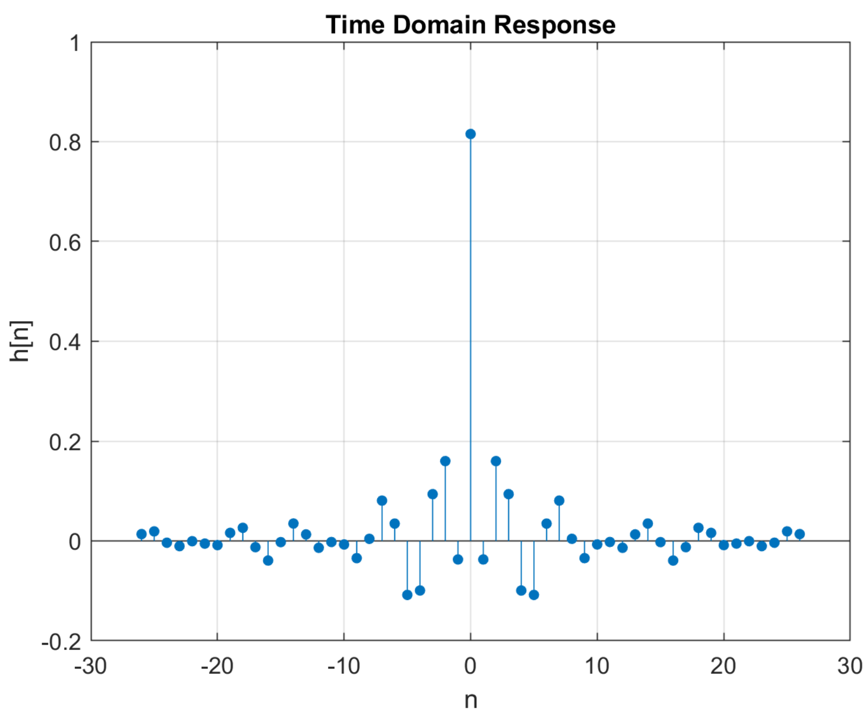

Figure 16: Time series $h[n]$ of bandpass filter

3 Overall comparison between FIR and IIR realizations

Observe that it would be easier to realise IIR filter compared to FIR filter because of less order which basically represents resources requred to be invested.

Furthermore, looking at the frequency responses of IIR and FIR filters, we make these comparisons:

1. The IIR realization has a distorted phase response whereas the FIR realization gives us a perfect pseudo-linear phase response.

2. The IIR filter has a better attenutation in the stopband (we took a monotonic stopband), but at the cost of a less sharp transition band.

3. The FIR realization allows for a sharper transition band because of a higher order.

Finally, regarding the stability, from the pole-zero plot of the IIR realization, we see that some poles are dangerously close to the unit circle, which could lead to unstable systems if numerical precision is reduced. On the other hand there is no such problem for the FIR realization since it involves a finite number of terms only which makes it always stable.

4 Review of my filter design by Abhilaksh Maheshwari

This filter design has been checked by Abhilaksh Maheshwari, 18D070035. In particular,

1. He has checked that I have used the correct specifications according to my assigned filter number, 152;

2. He has checked that the frequency response specifications for the designed FIR and IIR filters have been met, by looking at the plots;

3. He has ascertained that I have completed all parts of the mandatory assignment.

5 Appendix: MATLAB Codes used for the various graphs throughout the report

To allow further use and ease of access, MATLAB codes used in this assignment have been uploaded in a GitHub repository that can be found at the following link:
`https://github.com/devankrajvanshi/DSP-Filter-Design`