HOMEWORK 0

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Vectors + Matrices

1. $X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ $\quad y = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\quad z = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Dot product: $y^T z = \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2 + 9 = 11$

2. $Xy = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+12 \\ 1+9 \end{bmatrix} = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$

3. $\det X = 6 - 4 = 2$

$\Rightarrow X^{-1}$ exists

$X^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3/2 & -2 \\ -1/2 & 1 \end{bmatrix}$

4. Rank of $X = 2$

since $\begin{bmatrix} 4 \\ 3 \end{bmatrix} \neq k \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Calculus

1. $y = x^3 + x - 5$

$\frac{dy}{dx} = 3x^2 + 1$

2. $f(x_1, x_2) = x_1 \sin(x_2) e^{-x_1}$

$\nabla f(x) = \begin{pmatrix} \partial_{x_1} f \\ \partial_{x_2} f \end{pmatrix} = \begin{pmatrix} \sin(x_2) e^{-x_1} - x_1 \sin(x_2) e^{-x_1} \\ x_1 \cos(x_2) e^{-x_1} \end{pmatrix}$

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Probability:

1. Mean $= \dfrac{1+1+0+1+0}{5} = 3/5$

2. Variance $= \dfrac{1}{5}\left[\left(\dfrac{3}{5}-1\right)^2 + \left(\dfrac{3}{5}-1\right)^2 + \left(\dfrac{3}{5}-0\right)^2 + \left(\dfrac{3}{5}-1\right)^2 + \left(\dfrac{3}{5}-0\right)^2\right]$

$= \dfrac{1}{5}\left(\dfrac{4}{25} + \dfrac{4}{25} + \dfrac{9}{25} + \dfrac{4}{25} + \dfrac{9}{25}\right)$

$= 6/25$

3. Probability $= \left(\dfrac{1}{2}\right)^5 = \dfrac{1}{32}$

4. Let $P(X=1) = p$

$P(X=0) = 1-p = q$

Probability $= \prod_{i=1}^{5} p^{x_i} (1-p)^{1-x_i}$

Let This be denoted as $f(p)$

$f(p) = \prod_{i=1}^{5} p^{x_i} (1-p)^{(1-x_i)}$

$\log f(p) = \left(\sum_{i=1}^{5} x_i\right) \log(p) + \left(n - \sum_{i=1}^{5} x_i\right) \log(1-p)$

to maximize $\log f(p) \equiv F(p)$,

$\dfrac{dF(p)}{dp} = \dfrac{1}{p} \sum_{i=1}^{5} x_i - \dfrac{1}{1-p}\left(n - \sum_{i=1}^{5} x_i\right) = 0$

$\Rightarrow \dfrac{\sum_{i=1}^{5} x_i - pn}{p(1-p)} = 0$

$pn = \sum_{i=1}^{5} x_i$

$n\log_3$ $\log_e n$

$$p = \frac{1}{n} \sum_{i=1}^{5} x_i$$

$$\sum_{i=1}^{5} x_i = 3$$

$$\Rightarrow p = 3/5$$

5/.

z \ y	a	b	c
T	0.2	0.1	0.2
F	0.05	0.15	0.3

$$p(z=T \text{ and } y=b) = 0.1$$

$$p(z=T \mid y=b) = \frac{p(z=T \cap y=b)}{p(y=b)} = \frac{0.1}{0.1+0.15} = 0.4$$

Big O

1. $f(n) = \ln(n)$
$g(n) = \lg(n)$

Since $f(n) = \log_e n$
$g(n) = \log_2 n$
$\Rightarrow f(n) = c\, g(n)$

c = constant
$c = \log_e 2$

$\frac{\log t}{\log 2}$ $\log e$

$\Rightarrow f(n) = O(g(n))$
and $g(n) = O(f(n))$

2. $f(n) = 3^n$
$g(n) = n^{100}$

Since $3^n >> n^{100}$ as $n \to \infty$

$\Rightarrow g(n) = O(f(n))$

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3. $f(n) = 3^n$
$g(n) = 2^n$

since $3^n > 2^n$ as $n \to \infty$

$\rightarrow g(n) = O(f(n))$

4) $f(n) = 1000n^2 + 2000n + 4000$
$g(n) = 3n^3 + 1$

since $3n^2 + 1 >> 1000n^2 + 2000n + 4000$ as $n \to \infty$

$\Rightarrow f(n) = O(g(n))$