Assignment 2
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2)
a). Avg. absolute visual magnitude of globular clusters ~ 3.5

$$m - M = 5\log_{10}\left(\frac{d}{10}\right)$$

~~R~~ relation b/w apparent magnitude, absolute magnitude

Average apparent magnitude of globular cluster ~ 18.5

$$-3.5 + 18.5 = 5\log_{10}\left(\frac{d}{10}\right)$$

$$3 = \log\left(\frac{d}{10}\right)$$

$$d \sim 10^3 \times 10 \text{ pc}$$

$$\simeq 10^4 \text{ pc}$$

From internet; 8.5×10^3 pc

b). Clusters are bound by gravitational forces which makes its distribution confined to distances close to the earth and sun distance. It is hard to measure the distances by parallax so we approximate it as being at the same distance from Earth.

3).
a). $L \propto M^3$ L = luminosity, M = Mass

$$\Rightarrow L \propto T_{eff}^6 \quad [\, M \propto T_{eff}^2 \,]$$

Wein's displacement law: $\lambda_{max} T = 2.89 \times 10^{-3}$ mK

$$\frac{M_{star}}{M_\odot} = \frac{(T_{eff}(star))^2}{(T_{eff}\odot)^2}$$

$$9 = \left(\frac{T_{eff}^{star}}{T_{eff}^{\odot}}\right)^2$$

$T_{eff}^{\odot} = 6000$ K

$\Rightarrow T_{eff}^{(star)} = 6000\,K \times \sqrt{9}$

$= 18000\,K$

$\lambda_{max} = \dfrac{2.89 \times 10^{-3}}{18 \times 10^{3}} = 1.605 \times 10^{-7}\,m$

$= 160.5\,nm$

UV $\rightarrow$ light

For $M = 0.25\,M_\odot$

$T_{eff} = 0.5 \times 6000K = 3000\,k$

$\lambda_{max} = \dfrac{2.89 \times 10^{-3}}{3} = 0.963 \times 10^{-6}\,m$

Infrared

4).

a). Virial theorem : $2E_T + E_G = 0$

E_T = Thermal energy

E_G = Gravitational energy

$E_{Total} = E_G + E_T = \frac{1}{2} E_G$

$\Rightarrow$ half of Gravitational potential energy is released.

self energy for star : $-\int_{0}^{R} \frac{GM}{r} 4\pi r^2 \rho\, dr$

$= \dfrac{3GM^2}{5R}$

For accreting, $\emptyset$

$\Delta E_{Total} = \frac{1}{2} \times \frac{GM}{R} \Delta M$

$\cancel{\Delta E_{Total}} = \dfrac{dE}{dt} = L = \dfrac{GM}{2R}\dot{m}$

$\dot{m}$ = mass accretion rate

$$\langle \sigma v \rangle \propto \frac{S(E)}{T^{2/3}} e^{-3\left(\frac{e^4}{32\epsilon_0^2 \pi \hbar^2} \frac{m Z_1^2 Z_2^2}{T}\right)^{1/3}}$$

so $r \Delta E = n_1 n_2 \langle \sigma v \rangle \Delta E$

ΔE = energy generation rate / unit volume

$$\epsilon = C \rho X_1 X_2 \frac{1}{T^{2/3}} e^{-3\left(\frac{e^4}{32\epsilon_0^2 k \hbar^2} \frac{m Z_1^2 Z_2^2}{T}\right)^{1/3}}$$

$X_1, X_2 \rightarrow$ mass fraction of (projectile, target)

$\rho X_1, \rho X_2 \rightarrow$ density fraction of particles

n_1, n_2 are respectively $\frac{\rho X_1}{m_p}$ and $\frac{\rho X_2}{m_Z}$

$$\epsilon = \frac{C \rho X_1 X_2}{T^{2/3}} e^{-3\left(\frac{e^4}{32\epsilon_0^2 k \hbar^2} \frac{m Z_1^2 Z_2^2}{T}\right)^{1/3}}$$

For p-p chain

$$\epsilon_{pp} = \frac{2.4}{10} \rho X^2 \left(\frac{10^6}{T}\right)^{2/3} e^{-33.8\left(\frac{10^6}{T}\right)^{1/3}} \; \mathrm{W/kg}$$

X = mass fraction of H_2

For CNO cycle,

$$\epsilon_{CNO} = 8.7 \times 10^{20} \rho X_{CNO} X \left(\frac{10^6}{T}\right)^{2/3} e^{-152.3\left(\frac{10^6}{T}\right)^{1/3}}$$

At center, sun has temperature

$T_c = 1.56 \times 10^7 \,\mathrm{K}$

$\rho_c = 1.48 \times 10^5 \,\mathrm{kg/m^3}$

$$\langle \sigma v \rangle \propto \frac{S(E)}{T^{2/3}} e^{-3\left(\frac{e^4}{32\epsilon_0^2 k \hbar^2}\frac{m z_1^2 z_2^2}{T}\right)}$$

$$S\epsilon = r\Delta E = n_1 n_2 \langle \sigma v \rangle \Delta E$$

ΔE = energy generator rate / unit volume

$$\epsilon = C\rho X_1 X_2 \frac{1}{T^{2/3}} e^{-3\left(\frac{e^4}{32\epsilon_0^2 k \hbar^2}\frac{m z_1^2 z_2^2}{T}\right)^{1/3}}$$

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X = mass fraction of H_0

For CNO cycle,

$$\epsilon_{CNO} = 8.7\times 10^{20} \rho X_{CNO} X \left(\frac{10^6}{T}\right)^{2/3} e^{-152.3\left(\frac{10^6}{T}\right)^{1/3}}$$

At center, sun has temperature

$T_c = 1.56\times 10^7 K$

$\rho_c = 1.48\times 10^5 kg/m^3$

Mass fractions:

$X_H = 0.64$

$X_{He} = 0.34$

$X_{CNO} = 0.015$

$$\epsilon_{pp} = 0.24 \times (0.64)^2 \times 1.48 \times 10^5 \times 0.16\, e^{-33.8 \times 0.?}\ \text{W/kg}$$

$$= 2327.83 \times e^{-18.52}\ \text{W/kg}$$

$$= 3.12 \times 10^{-3}\ \text{W/kg}$$

$$\epsilon_{CNO} = 8.7 \times 10^{20} \times 1.48 \times 10^3 \times 0.015 \times 0.64 \times e^{-152.3 \times 0.4}\ \text{W/kg}$$

$$= 1.977 \times 10^{23} \times e^{-60.92}$$

$$= 6.899 \times 10^{-4}\ \text{W/kg}$$

In sum, 99% pp chain,

1% CNO

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a) $\gamma = 5/3$

$\rho(r_0) = \rho_0$

$T(r_0) = T_0$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Temp. gradient can be assumed according to Schwarzschild stability condn:

$$P = \frac{k_B}{\mu m_H}\rho T$$

For hydrodynamic condⁿ

$$\frac{dP}{dr} = -\frac{GM_\odot \rho}{r^2}$$

$$\frac{dT}{dr} = -\left(1 - \frac{1}{5/3}\right)\frac{T}{\rho}\frac{GM_\odot}{r^2}\frac{\rho \mu m_H}{k_B T}$$

$$= -\frac{2}{5}\frac{GM_\odot}{r^2}\frac{\mu m_H}{k_B}$$

$$\int_{T_0}^{T} dT = -\frac{2}{5}\frac{\mu m_H}{k_B} GM_\odot \int_{r_0}^{r} \frac{1}{r^2}\, dr$$

$$T(r) - T_0 = \frac{2}{5}\frac{GM_\odot \mu m_H}{k_B}\left(\frac{1}{r} - \frac{1}{0.7R_\odot}\right)$$

$$T(r) = T_0 + \frac{2}{5}\frac{GM_\odot \mu m_H}{k}\left(\frac{1}{r} - \frac{10}{7R_\odot}\right)$$

Since $r > 0.7R_\odot$

Temperature decrease from base of convection zone.

$$\frac{dT}{dr} = \frac{2}{5}\frac{T}{P}\frac{dP}{dr}$$

$$\frac{dP}{dr} = \frac{k}{\mu m_H}\left(T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right)$$

$$\frac{dT}{dr} = \frac{2}{5} \times \frac{T}{k\rho T}\mu m_H \times \frac{k}{\mu m_H}\left(T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right)$$

$$\frac{dT}{dr} = \frac{2}{5\rho}\left(T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right)$$

$$\left(\frac{5}{2}\frac{dT}{dr} - \frac{dT}{dr}\right)\rho = T\left(\frac{d\rho}{dr}\right)$$

$$\frac{3}{2}\frac{dT}{T} = \frac{d\rho}{\rho}$$

$$3/2 \int^{T(r)} \frac{dT}{dT} = \int_{\rho_0}^{\rho(r)} \frac{d\rho}{\rho}$$

$$\ln \frac{T(r)}{T_0} = \ln\left(\frac{\rho(r)}{\rho_0}\right)^{2/3}$$

$$\rho(r) = \rho_0\left(\frac{T(r)}{T_0}\right)^{3/2}$$

$$= \rho_0\left[1 + \frac{2}{5}\frac{GM_\odot \mu m_H}{kT_0}\left(\frac{1}{r} - \frac{10}{7R_\odot}\right)\right]^{3/2}$$

Pressure: $\frac{dP}{dr} = \frac{-GM_\odot \rho(r)}{r^2}$

$$P(r) - P_0 = -GM_\odot \rho_0 \int_{r_0}^{r} \frac{\rho(r)}{r'^2}\, dr'$$

b). $X = 0.7$

$$n = \left(2X + \frac{Z}{2}\right)\frac{\rho}{m_n}$$

$$= (1.4 + 0.01)\frac{\rho}{m_n} = \left(1.41\frac{\rho}{m_N}\right)$$

$$\mu = (1.41)^{-1} = 0.709$$

$$\mu m_H = 0.9 \times 10^{-27}\ \text{kg}$$

c). $\frac{dM_r}{dr} = 4\pi r^2 \rho(r)$

$$M = \int_{r_0}^{R_\odot} 4\pi r^2 \rho(r)\, dr$$

$$= \int_{0.7R_\odot}^{R_\odot} 4\pi r^2\left[1 + 2.777\times10^{7}\left(\frac{1}{r} - \frac{10}{7R_\odot}\right)\right]^{3/2}$$

d) solution is not consistent since we have varied $\rho(r)$, m was changing