Probability :

Random experiment : Random experiment is an experiment where we know all possible outcomes but we dont know which out come will come exactly when it happens.

Ex: (i) Tossing a fair coin possible out comes are H, T. and any thing can come when we toss the coin.
⇒ Tossing a coin is an random experiment.

(ii) Throwing a die is an random experiment

(iii) drawing a card from 52 playing cards is an radom experiment.

(iv) drawing a ball from different coloured balls.

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Sample Space:

Set of all possible outcomes of random experiment is called as sample space of that random experiment and it is denoted by S.

(i) In tossing one coin $S = \{H, T\}$

in two coins $S = \{HH, HT, TH, TT\}$

Tossing three coins
$= $ tossing one coin 3 times
$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

Here $n(S) = 2^3 = 8$

Illy In tossing n coins $n(S) = 2^n$.

$\{\because$ Every coin has 2 chances H & T$\}$
like this n coins have 2^n chances

(ii) In drawing a card from 52 playing cards $n(S) = 52$

$S = \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K$ with ♤, ♡, ♢, ♧$\}$

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(iii) In Throwing a die $S = \{1, 2, 3, 4, 5, 6\}$

in 2 dies $S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

Here $n(S) = 36$

1st dies have 6 chances & 2nd die has 6 chances

$6 \times 6 = 36$

In Throwing 3 dies $n(S) = 6^3 = 216$

n dies $n(S) = 6^n$.

Simple event & Elementary Event:

Every possible out come of random experiment & Every element in sample space is called as simple event & elementary event.

Ex: getting Heads in tossing one coin is a simple event

getting 3 in Throwing dies is a simple event

Event: Any subset of sample space is called as an event. and it is denoted by capital letters like $E, E_1, E_2, A, B, \ldots$

(or) Collection of some simple events is an event.

Ex: In throwing a die getting even no, getting odd no, prime no, ... are events.

Ex: In throwing a die

$E_1 = \{1, 3, 5\}$ $E_2 = \{2, 6\}$, $E_3 = \{2\}$

all are events.

Equally likely events: 2 or more events are said to be equally likely if they have same chance of occuring

Ex: In throwing a die, getting even no, odd no, prime no are equally likely events.

$\because E = \{2, 4, 6\}$ $O = \{1, 3, 5\}$

$P = \{2, 3, 5\}$

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Mutually exclusive events (Disjoint events):

Occurence of one event stops the occurence of other event then that events are mutually exclusive.

i.e E_i, E_j events are exclusive if

$$E_i \cap E_j = \phi.$$

Ex: Getting odd no, even no in throwing a die on exclusive events

In tossing a coin getting H, T are exclusive

In drawing a card getting diamond, spade, heart, club events exclusive.

exhaustive events:

$E_1, E_2, \ldots E_n$ events are said to be exhaustive if they cover every element of sample space.

i.e $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S.$

Ex: In throwing a die getting even no, odd no are exhaustive

In drawing cards drawing black card, Red card are exhaustive

Compound events: Combination of 2 or more events is said to be compound events.

Ex: (i) Getting Red Card and Ace card from pack cards

(ii) Getting even no and prime no in throwing die.

Sure event (Certain event): If $E = S$ i.e event covers all elements of sample space & then it is called as sure event & certain event.

Ex: In throwing a die number less than & equal to 6 is sure event.

Impossible event: If $E = \phi$. chance of occurence of event is zero then that event is called as impossible event.

Ex: getting more than 7 in throwing die

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Probability of an event :

If E is an event, then chace of occurrence (Happening) of the event E is defined as probability of event E and it is denoted by P(E).

☆ P(E) is a real number between 0 & 1

$P(E) \in [0,1]$

In tossing one coin $S = \{H, T\}$

getting one head probability = 50%
$= \frac{1}{2}$.

getting one tail probability = 50% = $\frac{1}{2}$.

In tossing two coins $S = \{HH, HT, TH, TT\}$

. Probability for getting exactly 2 heads = 25% = $\frac{1}{4}$,

" " " " 2 tails = 25% = $\frac{1}{4}$

" " exactly 1 Head = 50% = $\frac{1}{2}$.

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If sample space S have total 'n' out comes and all are equally likely and E is an event with m favourable comes then $P(E) = \frac{m}{n}$.

$$P(E) = \frac{\text{no. of favourable out comes to } E}{\text{Total out comes in } S}$$

if all out comes are equally likely

Ex:

① If a bag have 99 distinct red balls, 1 white ball. Then

$S = \{R_1, R_2, R_3 \ldots R_{99}, w\}$.

Then Probability for red ball drawn $= \frac{99}{100}$

Probability for white ball drawn $= \frac{1}{100}$.

② If a bag have 99 identical red balls & 1 white ball, then $S = \{R, w\}$

Probability for white ball drawn $= \frac{1}{100}$ $\left(\neq \frac{1}{2}\right)$

" " Red ball drawn $= \frac{99}{100}$ $\left(\neq \frac{1}{2}\right)$

③ In throwing 2 dies setting $\{x, y\}$ probability is same for identical dies & and also distinct dies

Complementary event:

If E is an event non occurrence of E is called as Complementary event of E and it is denoted by E^c (or) E' & $\bar{E}$

and $E' = S - E$.

$$E \cup E' = S, \quad E \cap E' = \phi.$$

E, E' are always exclusive & exhaustive events of sample space.

$$\boxed{P(E) + P(E^c) = 1}$$

Note: (i) $P(E) \in [0, 1]$

If $P(E) = 0$ then E is called as impossible event

and if $P(E) = 1$ then event E is called as sure event.

(2) If E_i, E_j are exclusive events then

$$P(E_i \cap E_j) = 0.$$

$$P(E_i \cup E_j) = P(E_i) + P(E_j)$$

(3) $E_1, E_2, E_3 \ldots E_n$ are exclusive and elementary events of S. Then

$$P(E_1) + P(E_2) + \ldots + P(E_n) = 1$$

(4) If E is an event and m are favourable outcomes and n are total outcomes and $m, n \to \infty$ then

$$P(E) = \lim \left(\frac{m}{n}\right)$$

Ex: Find probability for a chosen no. is even from natural no.

$$\lim_{n \to \infty} \frac{n}{2n}$$ (If total $2n$ natural no.s)

or $$\lim_{n \to \infty} \frac{n}{2n+1}$$ (If total odd no. of natural no.s are available,

And both results are equal to $= \frac{1}{2}$.

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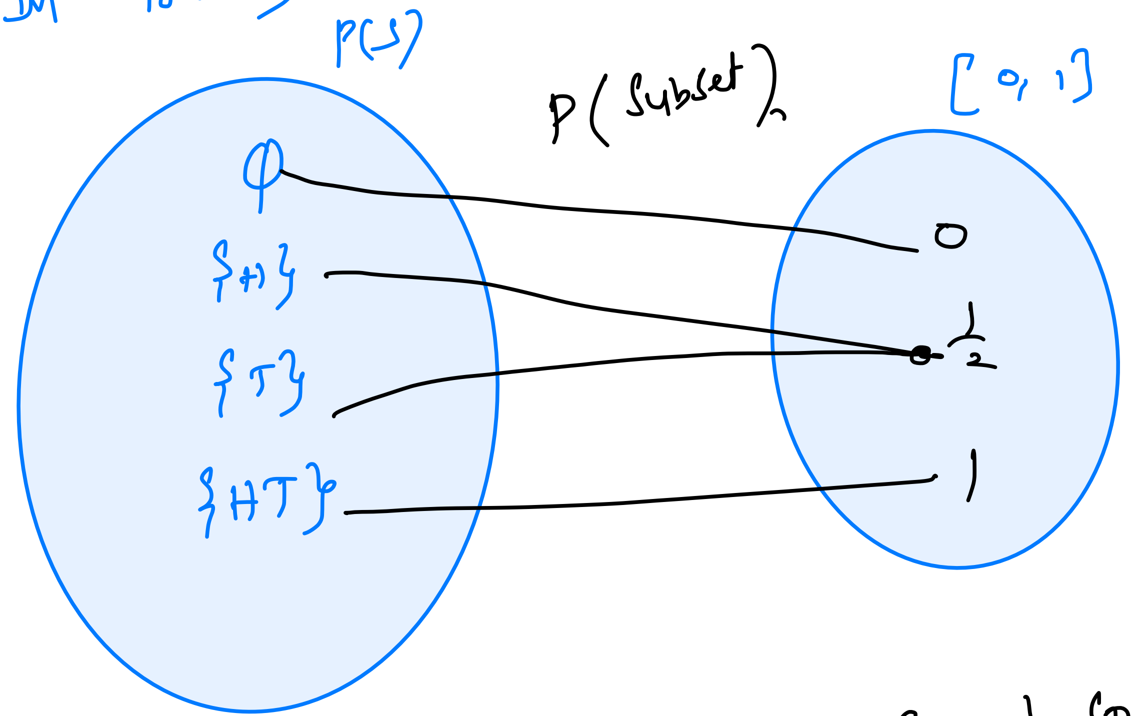
(5) Probability of a random experiment is always a function from set of all subsets of sample space (Power set of S) to [0, 1].

Ex: In tossing one coin $S = \{H, T\}$



as event is subset of sample space every event is an element in domain of function, and every Probability ∈ [0, 1] so Range ⊆ Codomain

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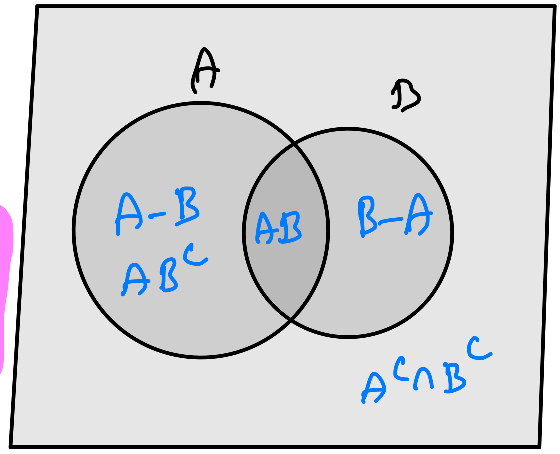
Addition Theorem of Probability:

A, B are 2 events,

Then probability of

atleast one of A, B happens $= P(A\cup B)$



and $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $AB = A\cap B$

and This result is called as addition theorem of probability.

Pf:

$$n(A\cup B) = n(A) + n(B) - n(A\cap B)$$

$$\Rightarrow \frac{n(A\cup B)}{n(S)} = \frac{n(A) + n(B) - n(A\cap B)}{n(S)}$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A\cap B)}{n(S)}$$

$$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

If $A\cap B = \phi$. Then $P(A\cup B) = P(A) + P(B)$.

If exactly only A happens $= P(A-B)$

$$= P(A) - P(A\cap B)$$

$$= P(AB^c)$$

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If exactly one of A, B happens $= P(A \Delta B)$

$= P(A \cup B) - P(A \cap B)$

$= P(A-B) + P(B-A)$

$= P(A) + P(B) - 2P(A \cap B)$

neither A, nor B happens $= P(A^c \cap B^c)$

$= P((A \cup B)^c)$

$= 1 - P(A \cup B)$

$= 1 -$ atleast one of A, B happens

at most one of A, B happens $= 1 -$ both A, B happens

$= 1 - P(A \cap B)$

If A, B, C are three events

atleast one of A, B, C happens $= P(A \cup B \cup C)$

neither of A, B, C happens $= P((A \cup B \cup C)^c)$

$= 1 -$ atleast one of A, B, C happens

$= 1 - P(A \cup B \cup C)$

$= P(A^c \cap B^c \cap C^c)$

atmost 2 of A, B, C happens $= 1 - P(A \cap B \cap C)$

exactly 2 of A, B, C happens $= P(A \cap B) + P(B \cap C) + P(A \cap C) - 3P(A \cap B \cap C)$

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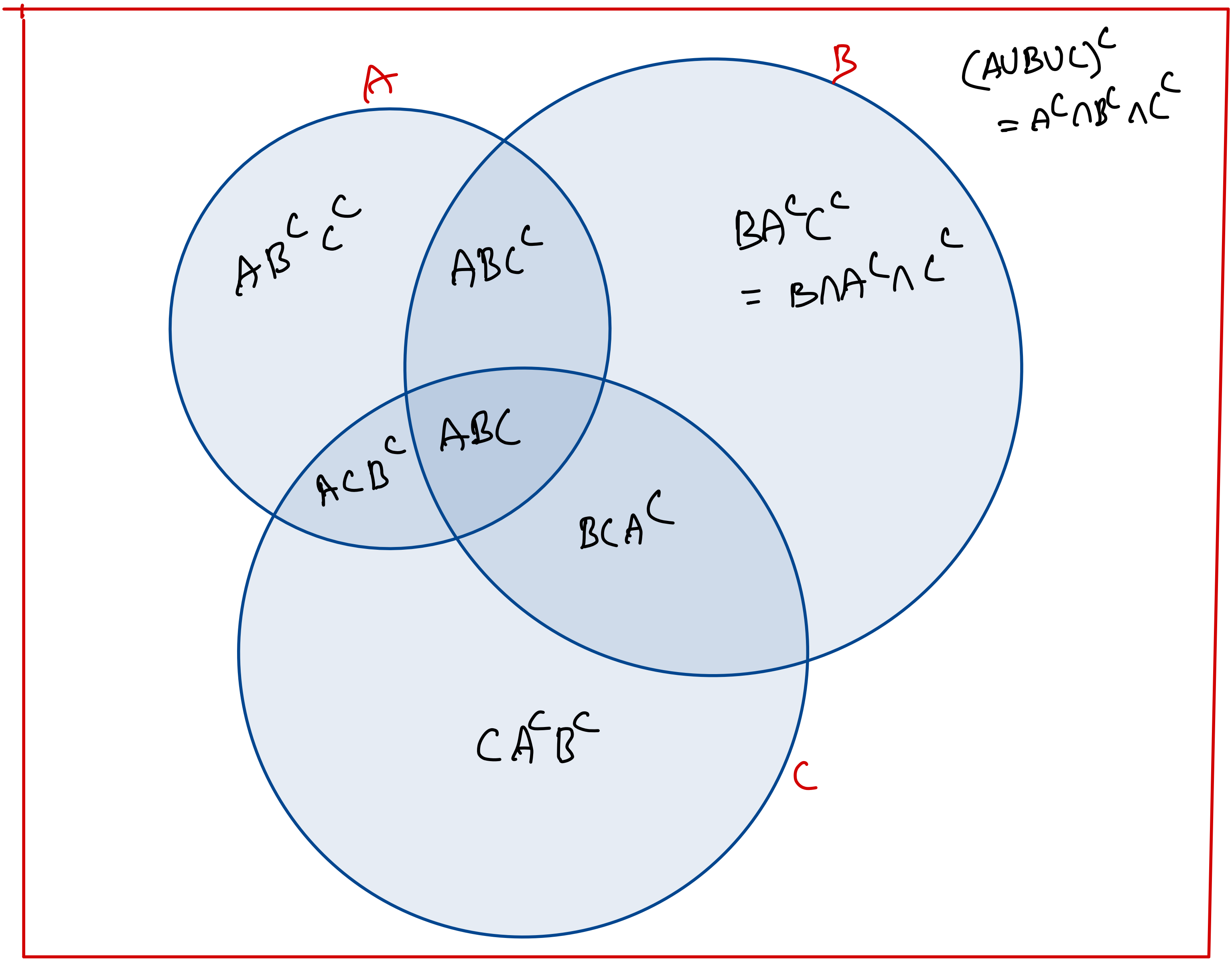

Coins:

If we toss 5 coins what is the probability for

1. Exactly 0 heads — $\frac{5C_0}{2^5}$ = TTTTT
2. " 1 head — $\frac{5C_1}{2^5} = \frac{5}{32}$ (HTTTT, THTTT, TTHTT, TTTHT, TTTTH)
3. " 2 heads — $\frac{5C_2}{2^5}$
4. " 3 heads — $\frac{5C_3}{2^5}$
5. " 4 heads — $\frac{5C_4}{2^5}$
6. " 5 heads — $\frac{5C_5}{2^5}$ (HHHHH)

Total cases = 2^5 (as every coin can show H & T)

For exactly 1 Head out of 5 places choose 1 place and fix it with Head and remaining all are tails.

⑥ atleast 2 heads

$$= \frac{5C_2 + 5C_3 + 5C_4 + 5C_5}{2^5}$$

(or) 1 − exactly 0 head − exactly 1 head

$$= 1 - \left(\frac{5C_0 + 5C_1}{2^5}\right)$$

⑦ atmost 2 heads

$$\frac{5C_0 + 5C_1 + 5C_2}{2^5}$$

In tossing 10 coins find the probability for exactly 5 consecutive Heads.

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1 2 3 4 5 6 $\underline{7\ 8\ 9\ 10}$ 2^4

H H H H H T

1 2 3 4 5 6 7 $\underline{8\ 9\ 10}$ 2^3

T H H H H H T

$\underline{1}$ 2 $\underbrace{3\ 4\ 5\ 6\ 7}_{H}$ 8 $\underline{9\ 10}$ 2^3

T T

$\underbrace{1\ 2}$ 3 $\underbrace{4\ 5\ 6\ 7\ 8}_{H}$ 9 $\underbrace{10}$ 2^3

T T

$\underbrace{1\ 2\ 3}$ 4 $\underbrace{5\ 6\ 7\ 8\ 9}_{H}$ $\underbrace{10}_{T}$ 2^3

T

$\underline{1\ 2\ 3\ 4}$ $\underbrace{5}_{T}$ $\underbrace{6\ 7\ 8\ 9\ 10}_{H}$ 2^4

$$\frac{2\cdot 2^4 + 4\cdot 2^3}{2^{10}} = \frac{2^6}{2^{10}} = \frac{1}{2^4} = \frac{1}{16}.$$

In tossing $2n$ coins exactly n consecutive Heads probability =

$\underbrace{1\ 2\ 3\ 4\ 5 \cdots n}_{H}$ $\underbrace{n+1}_{T}$ ______ 2^{n-1}

$\underline{1}$ $\underbrace{2\ 3\ 4 \cdots n+1}_{H}$ $\underline{n+2}$ ______ 2^{n-2}

T T

$\vdots$

______ $\underbrace{n}_{T}$ $\underbrace{n+1\ n+2 \cdots 2n}_{H}$ ______ 2^{n-1}

Total Cases $n+1$

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$$\text{Total possibilities} = \frac{2 \cdot 2^{n-1} + (n-1) \cdot 2^{n-2}}{2^{2n}}$$

$$= \frac{4 + n - 1}{2^{n+2}}$$

$$\boxed{\text{Exactly } n \text{ Consecutive Heads in } 2n \text{ coins} = \frac{n+3}{2^{n+2}}}$$

Find Probability for atleast n Consecutive Heads in $2n$ coins

atleast $5 \to$ Consecutive Heads in tossing 10 coins.

H H H H H _ _ _ _ _ $= 2^5$

T H H H H H _ _ _ _ $= 2^4$

_ T H H H H H _ _ _ $= 2^4$

_ _ T H H H H H _ _ $= 2^4$

_ _ _ T H H H H H _ $= 2^4$

_ _ _ _ T H H H H H $= 2^4$.

$$\text{Total} = \frac{2^5 + 5 \cdot 2^4}{2^{10}} = \frac{7}{2^6} = \frac{7}{64}$$

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atleast n Consecutive Heads in 2n coins

$$= \frac{2^n + n \cdot 2^{n-1}}{2^{2n}} = \frac{n+2}{2^{n+1}}$$

Non leap year = 365 days

= 52 weeks + 1 extra day.

In Non leap year one of the day is coming 53 times and remaining all are coming 52 times.

⇒ In non leap year Probability of

52 Sundays = $6/7$

53 Sundays = $1/7$.

Leap year = 366 days

= 52 weeks + 2 extra days.

these 2 extra days are {M, Tue} {Tue, wed} {wed, Thu} {Thu, Fri} {Fri, Sat} {Sat, Sun} {Sun, Mon} like this 7 chances are there.

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Probability for 53 Sundays = $\frac{2}{7}$.

" " 52 Sundays = $\frac{5}{7}$.

In 53 Sundays that 2 extra days are
{S, M} or {Sa, Su}

Probability for 5 Sundays in February month of a leap year = $\frac{1}{7}$.

Probability for 4 Sundays in February in the leap year = $\frac{6}{7}$.

Problems on dies:

2 dies are thrown. Then find the probability for the sum of the numbers on dies is 2, 3, 4 . . . 12.

$P(2) = \frac{1}{36}$ $\quad P(6) = \frac{5}{36}$ $\quad P(10) = \frac{3}{36}$

$P(3) = \frac{2}{36}$ $\quad P(7) = \frac{6}{36}$ $\quad P(11) = \frac{2}{36}$

$P(4) = \frac{3}{36}$ $\quad P(8) = \frac{5}{36}$ $\quad P(12) = \frac{1}{36}$.

$P(5) = \frac{4}{36}$ $\quad P(9) = \frac{4}{36}$

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$P(\text{Sum atleast } 10) = \frac{6}{36}$

$P(\text{sum} \leq 7) = \frac{21}{36} = \left(\frac{1+2+3+\cdots+6}{36}\right)$

$P(\text{Sum is even}) = \frac{(3\times 3)+(3\times 3)}{36} = \frac{1}{2}$

$P(\text{Sum is odd}) = \frac{1}{2}.$

$P(\text{Sum is prime number}) = \frac{15}{36}$

$P(\text{both numbers are prime}) = \frac{9}{36}$

$P(\text{atleast one number is prime}) = 1 - \text{no prime} = 1 - \frac{1}{4} = \frac{3}{4}.$

$P(\text{Product is odd}) = \frac{1}{4}.$

In throwing 3 dies Probability for sum of the numbers on 3 dies is

$P(3) = P(18) = 1/216$ $({}^2C_2)$

$P(4) = P(17) = 3/216$ $({}^3C_2)$

$P(5) = P(16) = 6/216$ $({}^4C_2)$

$P(6) = P(15) = 10/216$ $({}^5C_2)$

$P(7) = P(14) = 15/216$ $({}^6C_2)$

$P(8) = P(13) = 21/216$ $({}^7C_2)$

$P(9) = P(12) = 25/216$ $({}^8C_2 - 3)$

$P(10) = P(11) = 27/216$ $({}^9C_2 - 9)$

Sum of 3 dies is 7 is equal to number of positive integral solutions for

$x + y + z = 7$ Ans: ${}^6C_2 = 15$

$x + y + z = 9$ solutions ${}^8C_2 - 3$ $\left(\begin{matrix}7 & 1 & 1 \\ 1 & 7 & 1 \\ 1 & 1 & 7\end{matrix}\right)$

$= 28 - 3$

$= 25.$

$x + y + z = 10$ solutions ${}^9C_2 - 9$

$= 36 - 9$

$= 27$

Except $8\,1\,1 - 3$

$7\,2\,1 - 6$

P(13) Favourable outcomes are no. of Solutions of

$x+y+z=13$ $\quad 1 \le x, y, z \le 6$

let $x = 7-a$ where $a, b, c \in \{1, 2, 3 .. 6\}$

$y = 7-b$

$z = 7-c$

$7-a+7-b+7-c = 13$

$\Rightarrow a+b+c = 21-13$

$\Rightarrow a+b+c = 8$ where $a, b, c \in \{1, \dots, 6\}$

$\Rightarrow P(13) = P(8).$

Probability for i) Sum of the numbers is even $= \frac{1}{2}$

sum of numbers is odd $= \frac{1}{8} = \left(\frac{27}{216}\right)$

(ii) product of numbers is odd

(iii) product of numbers is prime $= \frac{9}{216}$

(iv) $P(\text{Sum} \le 10) = P(\text{Sum} \ge 11) = \frac{1}{2}$

(v) P(Product is divisible by 5)

= P(at least one no. is 5)

$= 1 -$ P (No no. is 5)

$= 1 - \frac{5 \times 5 \times 5}{6 \times 6 \times 6} = \frac{216-125}{216} = \frac{91}{216}$

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(vi) P (at least one number is repeating)

$= 1 - P$ (No number is repeated)

$= 1 - \frac{6 \times 5 \times 4}{6^3} = 1 - \frac{20}{36} = \frac{16}{36} = \frac{4}{9}$

(vii) Probability for number of ordered pairs (a,b,c) where $a,b,c \in \{1, \dots 6\}$ and a,b,c are in A.P

Ans: $\frac{18}{216}$.

Problems on Numbers:

$A = \{1, 2, 3 \dots 10\}$

Three numbers are choosen from A.

Probability for all are even $= \frac{{}^5C_3}{{}^{10}C_3}$

all are odd $= \frac{{}^5C_3}{{}^{10}C_3}$

Let $a, b, c \in A$

and $P(a < b < c) = \frac{{}^{10}C_3}{10^3}$

$P(a \le b \le c) = \frac{{}^{12}C_3}{10^3}$

$P(a < b \le c) = \frac{{}^{11}C_3}{10^3}$

(a, b, c) forms an A.P $= \frac{(5 \times 5) \times 2}{10^3}$

· Probability for b, c chosen from A s.t $x^2+bx+c=0$ eq have real roots.
(b can be equal to c also)

Total possibility $= 10 \times 10 = 100$

$b^2 - 4c \geq 0$

$\Rightarrow c \leq \frac{b^2}{4}$.

If $b=1$	0	
$b=2$	$c \leq 1$	1 chance
$b=3$	$c \leq \frac{9}{4}$	2 chances
$b=4$	$c \leq 4$	4
$b=5$	$c \leq \frac{25}{4}$	6
$b=6$	$c \leq 9$	9 chan
$b=7$	$c \leq \frac{49}{4}$	10
$b=8, 9, 10$		10 chances
		62 chances

$\Rightarrow$ Probability $= \frac{62}{100}$.

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Problems on Balls:

A bag contain 5 Red 6 black balls. Every time one ball is drawn without replacement. What is the probability for

① First ball is Red — $\frac{5}{11}$

② First 2 balls are Red — $\frac{5}{11}\cdot\frac{4}{10} = \frac{2}{11}$

③ First 3 balls are black — $\frac{6\cdot5\cdot4}{11\cdot10\cdot9} = \frac{4}{33}$

④ All Red balls are over by 7th draw.
(i.e 5th Red will come in 7th draw)

$$\underbrace{\begin{array}{cccccc} R & B & R & R & R & B \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array}}_{4\,Red} \quad \frac{7}{R}$$

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$$6C_4 \times \frac{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1\cdot\cancel{6}\cdot\cancel{5}}{11\cdot\cancel{10}\cdot\cancel{9}\cdot\cancel{8}\cdot7\cdot\cancel{6}\cdot\cancel{5}}$$

$$\frac{\cancel{15}\,5}{11\times\cancel{4}\,2\times7} = \frac{5}{154}$$

All 5 Red balls are arranged in First 7 positions

$$\frac{\frac{6!}{2!\,4!}}{\frac{11!}{5!\,6!}} = 15\times\frac{1\times2\times3\times4\times5}{11\times10\times9\times8\times7} = \frac{5}{154}$$

All arrangement

5R 6B

(5) Probability for second drawn ball is Red,

$$RR + BR = \frac{5}{11}\cdot\frac{4}{10} + \frac{6}{11}\cdot\frac{5}{10}$$

$$= \frac{20+30}{110} = \frac{5}{11}$$

(6) Probability for Third drawn ball is Red,

$$RRR + RBR + BRR + BBR = \frac{5}{11}$$

1 2 3 4 5 6 7 8 9 10 11

R

In remaining 10 places 4 Reds 6 blacks will come in $\frac{10!}{4!\,6!}$ arrangements

1 2 3 4 5 6 7 8 9 10 11

R

Remaining 10 places 4 Red 6 black will come in $\frac{10!}{4!\,6!}$ places

If a bag have m Red n Black balls

Then probability for red in ith attempt $1 \leq i \leq m+n$ $= \frac{m}{m+n}$.

ith attempt black ball probability

$$= \frac{n}{m+n}$$

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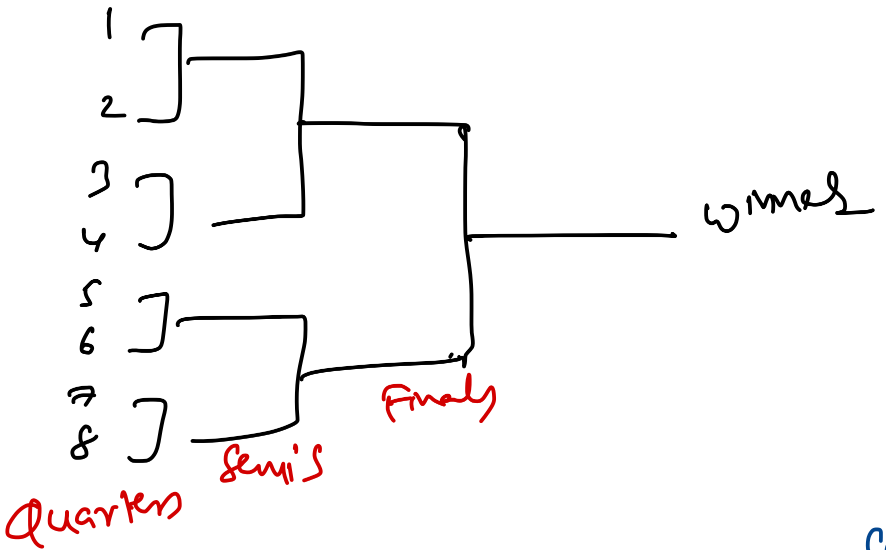
Games:

$P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$ 8 players are playing a knock out tennis tournament. and every player have equal chances of winning. Then probability for P_1 wins the tournament

Ans: $\frac{1}{8}$.

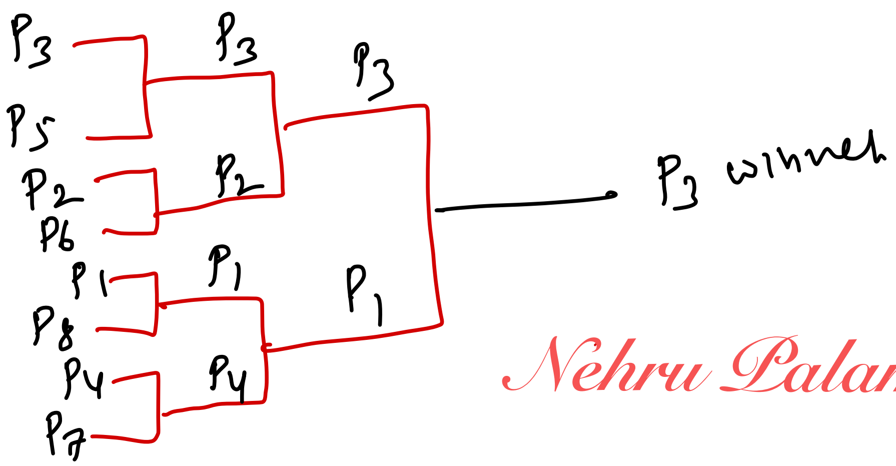


Method 1: P_1 wins Quarters, Semi's & Finals and it is possible in $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Method 2: (or)

$$\frac{\text{Favorable chance (only } P_1 \text{ win)}}{\text{Total chances } (P_1, P_2 \ldots P_8) \text{ in this one of the person will win}} = \frac{1}{8}$$

Method 3:



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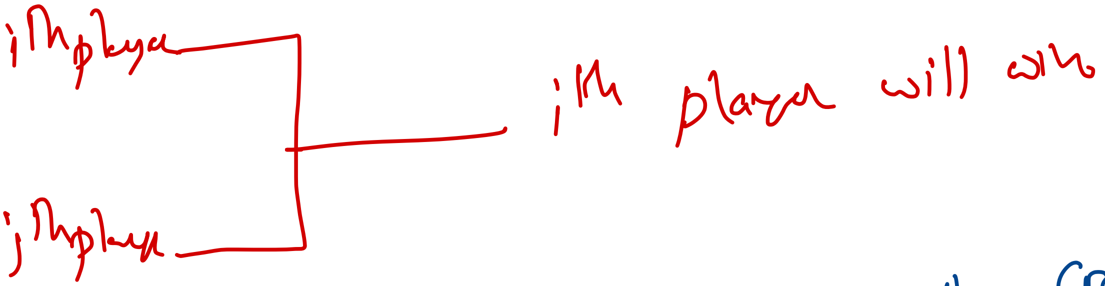
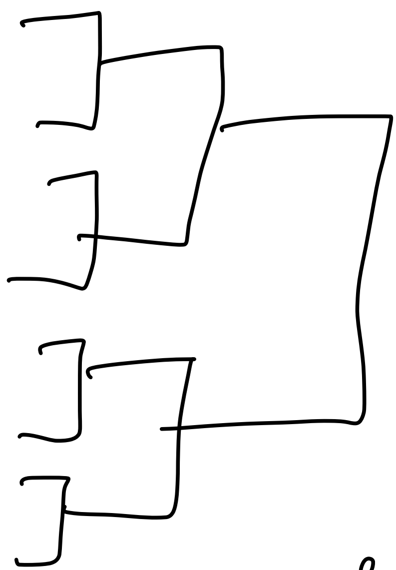


As P_1 is winner P_1 must come in 1st position, with this Favourable arrangements are $= 7!$

total arrangements $= 8!$

$$\Rightarrow P(P_1 \text{ wins}) = \frac{7!}{8!} = \frac{1}{8}.$$

<u>method 4:</u>



P_1 can choose any of 8 positions in this. total 8 chances for winning of tournament only 1st place is possible $\Rightarrow P(P_1 \text{ wins}) = \frac{1}{8}$

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P_1 reaches Final:

① P_1 wins First 2 matchs $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

8₂

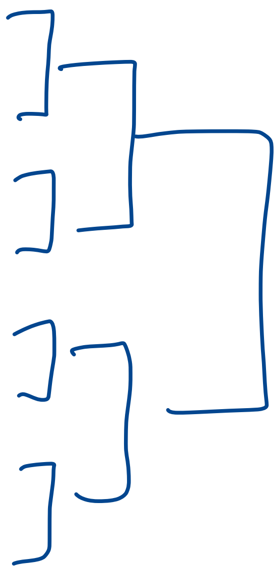
② Any 2 can come to final and it have $8C_2$ chances

As P_1 must reach final other opposite person can have 7 chancs

$$= \frac{7}{8C_2} = \frac{7}{28} = \frac{1}{4}.$$

(or) ③ $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.

(or)



total = $8!$ arrangements

1st (or) 5th position

or favourable to reach final $2 \times 7!$ arrangements

$\Rightarrow P(P_1 \text{ reaches Final})$

$$= \frac{2 \times 7!}{8!} = \frac{1}{4}$$

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Method 4

P_1 can choose any of 8 places and in that 2 places (1st & 5th) are favourable $= \dfrac{2C_1}{8C_1} = \dfrac{1}{4}$.

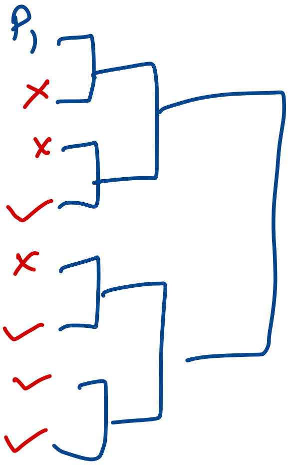
P_1, P_2 both reaches final:

Ans: $\dfrac{1}{8C_2}\left[\dfrac{(P_1,P_2)}{(\text{out of } 8 \text{ choose } 2)}\right]$

P_1, P_2 have 2 chances (1st & 5th position)

$$= \frac{2\times 6!}{8!} = \frac{2}{8\times 7} = \frac{1}{28}$$

P_1 wins the tournament & P_1 is not playing with P_4.



$$= \frac{4\times 6!}{8!} = \frac{4}{8\times 7} = \frac{1}{14}$$

$P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$ are 8 players playing a knock out tournament. If P_i plays with P_j always P_i will win if $i<j$. Then find probabilities for $P_2, P_3, P_4, P_5, P_6, P_7, P_8$ for reaches 2nd round of Final

P_2 reach second round $= 6/7$

P_3 " " $= 5/7$

P_4 " " $= 4/7$

P_5 " " $= 3/7$

P_6 " " $= 2/7$

P_7 " " $= 1/7$

P_8 " " $= 0$

P_7 reaches final $= 0$

P_6 reaches final $= 0$

P_5 reaches final:

Conduction of first round $\dfrac{{}^8C_2 \cdot {}^6C_2 \cdot {}^4C_2 \cdot {}^2C_2}{4!}$ ways

$$= \frac{8!}{\cancel{6!}\,2!} \times \frac{\cancel{6!}}{\cancel{4!}\,2!} \times \frac{\cancel{4!}}{\cancel{2!}\,2!} \times \frac{1}{4!}$$

$$= \frac{8 \times 7 \times 6 \times 5}{16} = 105$$

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Favourable to P_5 s.t P_5 can reach to final,

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6\ P_7\ P_8$

let P_5 choose one of the person from P_6, P_7, P_8 and the remaining 2 person will play each other and it can be done in $3C_1$ ways. and between remaining 4 persons we can conducted matches in 3 different ways.

so total possibilities for first round

$3 \times 3 = 9.$

and in second round P_1, P_3, P_5, P_7 persons are available, and probability for P_5 win second round is $\frac{1}{3}$.

$\Rightarrow$ Total probability $= \frac{9}{105} \times \frac{1}{3} = \frac{1}{35}.$

P_4 reaches Final:

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6\ P_7\ P_8$

$$\frac{4C_1 \times 3C_2 \times 3}{105} \times \frac{1}{3} = \frac{4}{35}$$

$P_4, P_6, P_1, (P_2/P_3)$ — second round

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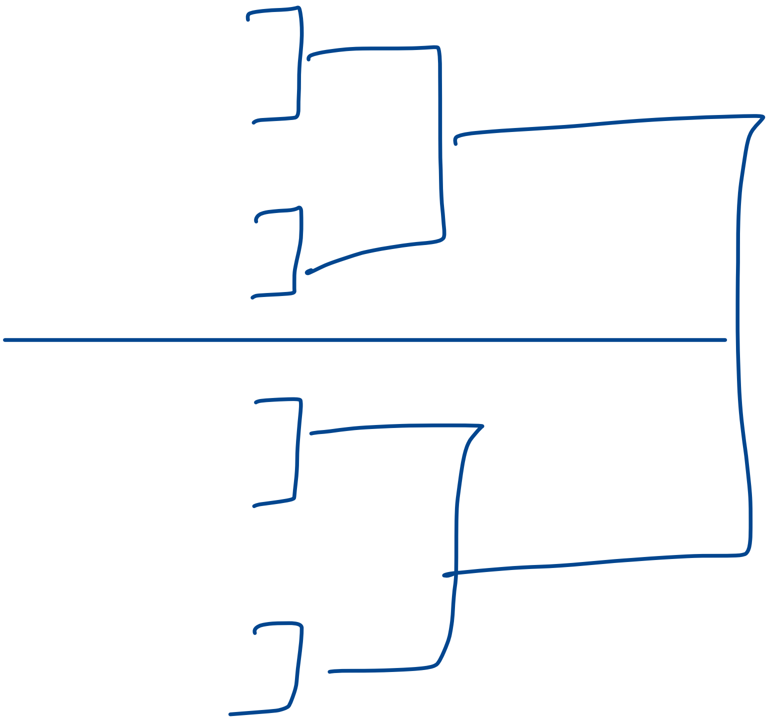
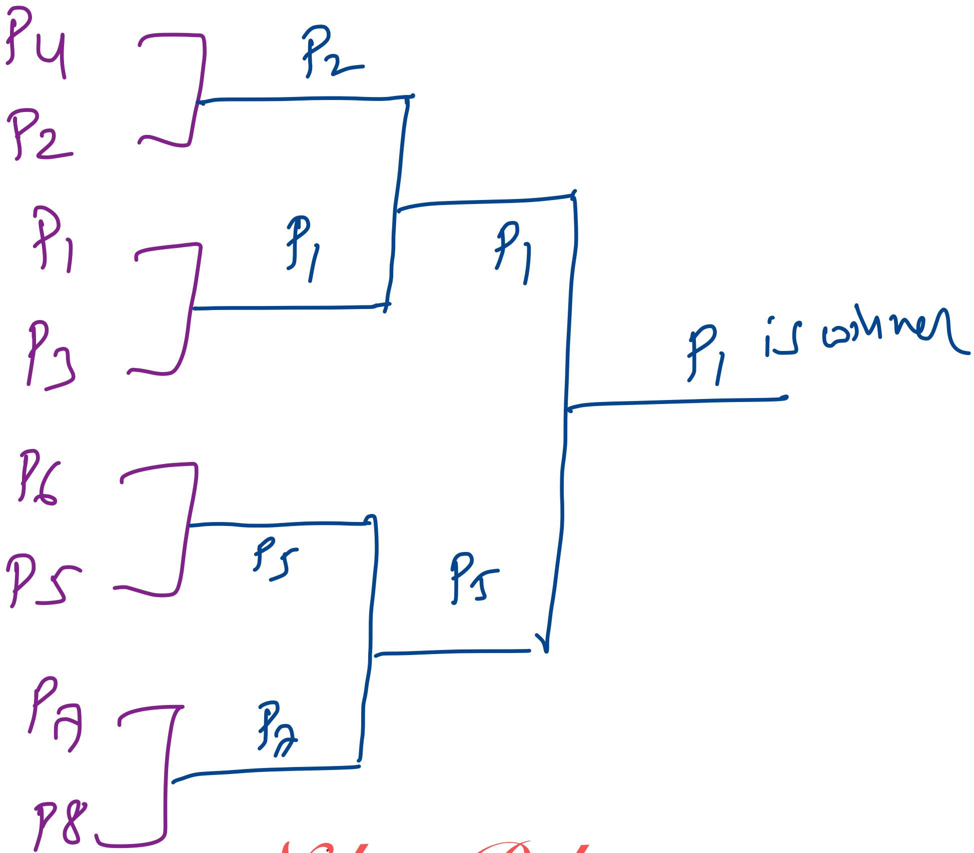
Method 2: (Arrangment method):



P_5 reaches Final:



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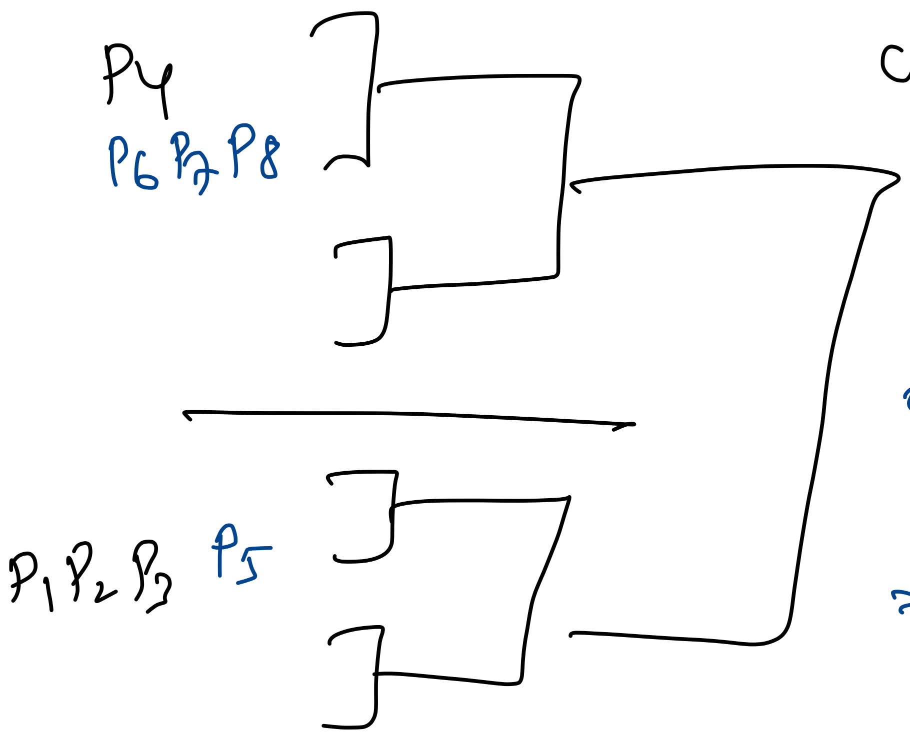
Here P_5 reach final if P_1, P_2, P_3, P_4 will play in First 4 positions and P_5 will play in any of down (or) P_5 in First 4 positions of last 4 positions. and P_1, P_2, P_3, P_4 in last 4 positions. and it can be done in

$$\frac{\overset{P_1P_2P_3P_4}{4!} \times \overset{P_5P_6P_7P_8}{4!} \times 2}{8!}$$

$$= \frac{24 \times \cancel{2}}{\cancel{8} \times 7 \times \cancel{6} \times 5} = \frac{1}{35}.$$

P4 reach Final:



$$\underbrace{2}_{\text{choosing above/below half}} \times \underbrace{{}^4C_1}_{\text{choosing one person for } P_1, P_2 P_3 \text{ group}} \times \overset{\text{arranging upper half}}{4!} \times \underbrace{4!}_{\text{below half}}$$

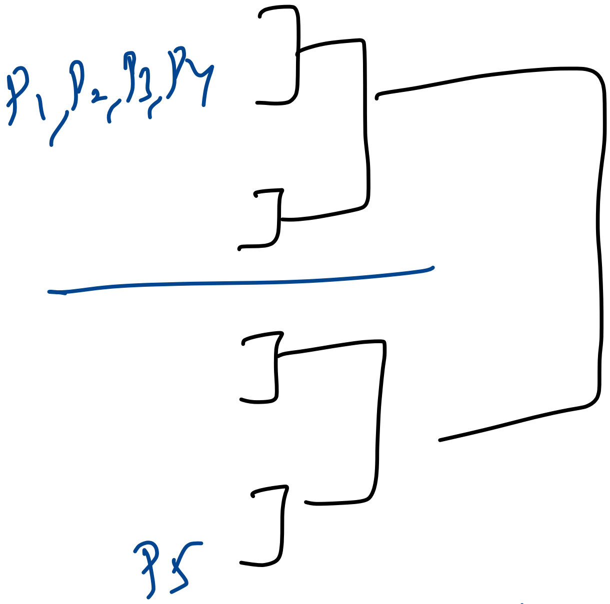
Ans: $\frac{8 \times 4! \times 4!}{8!}$

$$= \frac{\cancel{8} \times \cancel{24}\ 4}{\cancel{8} \times 7 \times \cancel{6} \times 5}$$

$$= \frac{4}{35}.$$

Combination method:

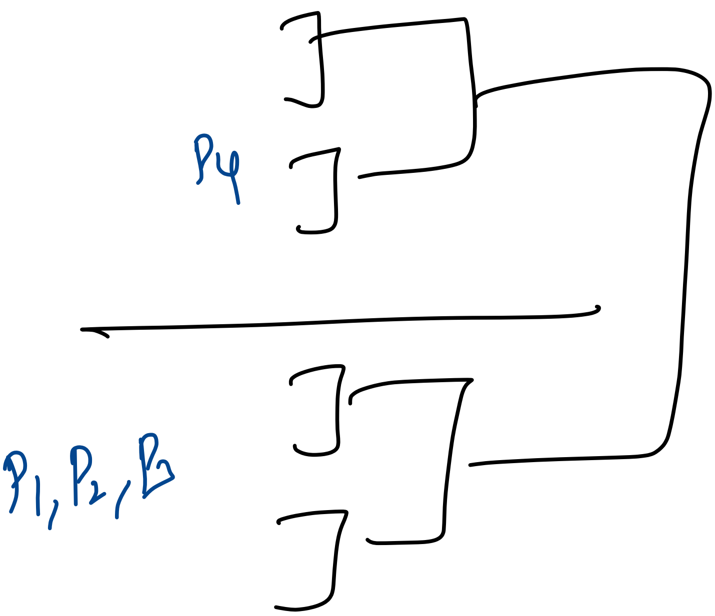
P_5 reaches Final:



P_1, P_2, P_3, P_4 can choose any of 7 places in $^7C_4$ ways and P_5 reach final if P_1, P_2, P_3, P_4 choose 4 places in different half plane of P_5 and it can be done in $^4C_4$ ways.

$$\text{Probability} = \frac{^4C_4}{^7C_4} = \frac{1}{35}$$

P_4 reaches Final:



P_1, P_2, P_3 can chose any of 3 positions from 7 positions in $^7C_3$ ways, and P_4 reaches final if P_1, P_2, P_3 choose 3 places from 4 in the other half plane.

it can be done with probability $\dfrac{^4C_3}{^7C_3} = \dfrac{4}{35}$.

P_3 reaches final:

P_1, P_2: $\dfrac{4C_2}{7C_2} = \dfrac{6 \times 2}{7 \times 6} = \dfrac{2}{7} = \dfrac{10}{35}$

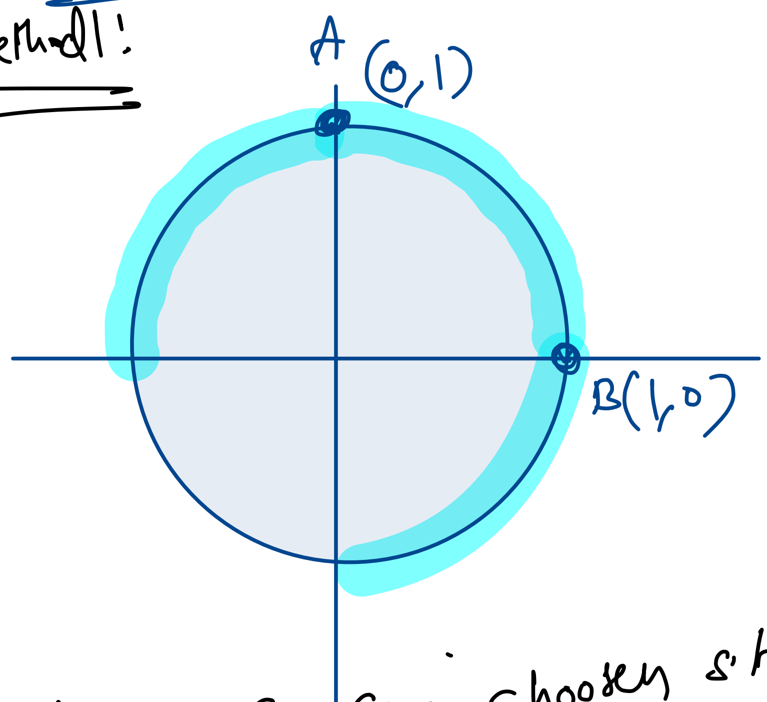
P_2 reaches final:

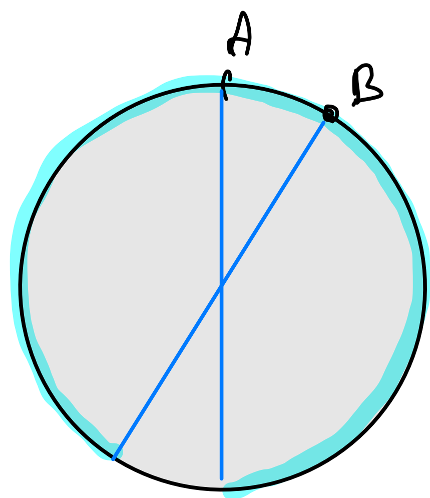
P_1: $\dfrac{4C_1}{7C_1} = \dfrac{4}{7} = \dfrac{20}{35}$

Three points are chosen from points on circle. Then find the probability so that 3 points lies on a semicircle.

Ans: $\frac{3}{4}$.

Method 1:





Here C can chosen s.t A, B, C lies on semicircle, Probability $= \frac{3}{4}$.

When B tends to A
Probability $\to 1$

$\frac{3}{4} + x$

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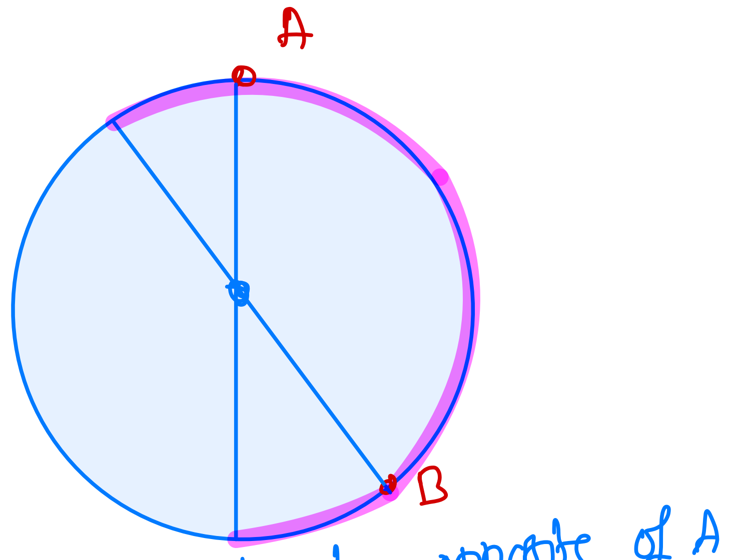
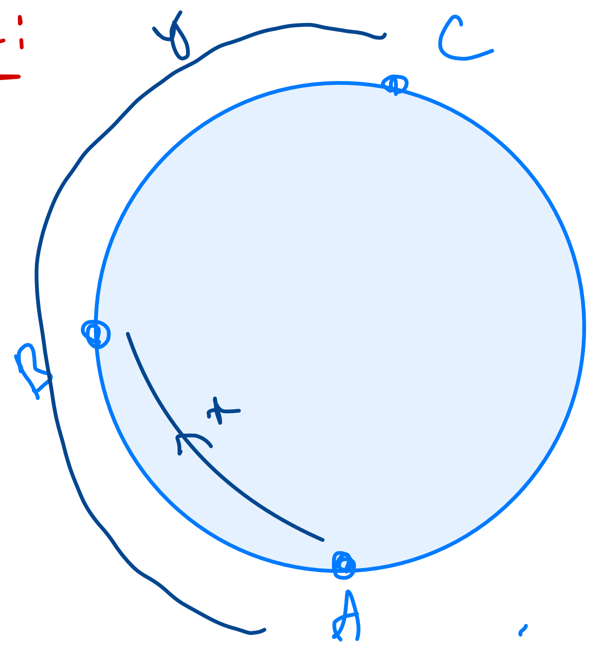


when B tends to opposite of A

Probability tends to $\frac{1}{2}$. $\frac{3}{4} \sim x$

So average of all probability $= \frac{3}{4}$.

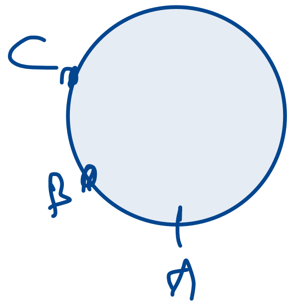
Method2:



let $r=1$

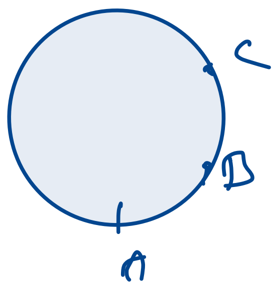
let x, y are distances from A to B, C points in clockwise direction,

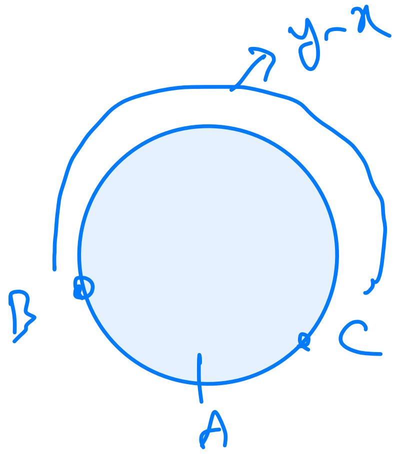
Then $x, y \in (0, 2\pi)$



If $x, y \in (0, \pi)$ then A, B, C lies on same semicircle.

If $x, y > \pi$ then also A, B, C lies on same semicircle.

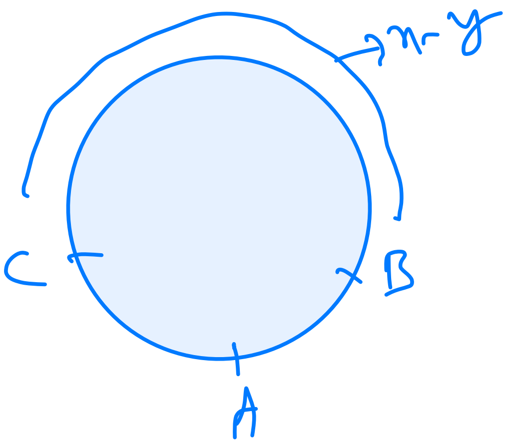




$x < \pi$, $y > \pi$, $y - x > \pi$

$\Downarrow$

$\overline{BC}$ distance



$x > \pi$, $y < \pi$, $x - y > \pi$

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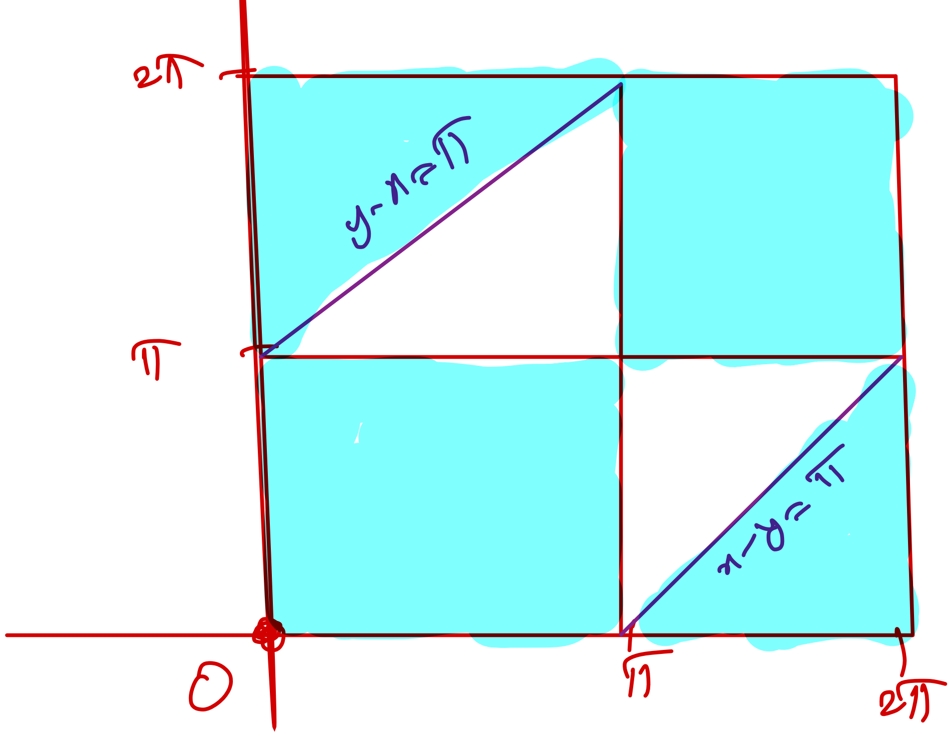
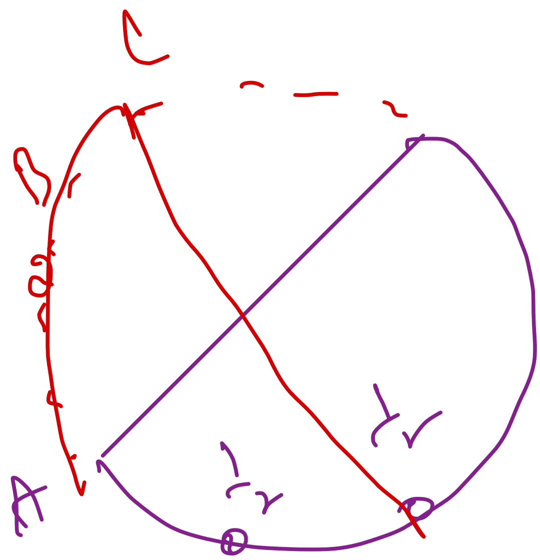
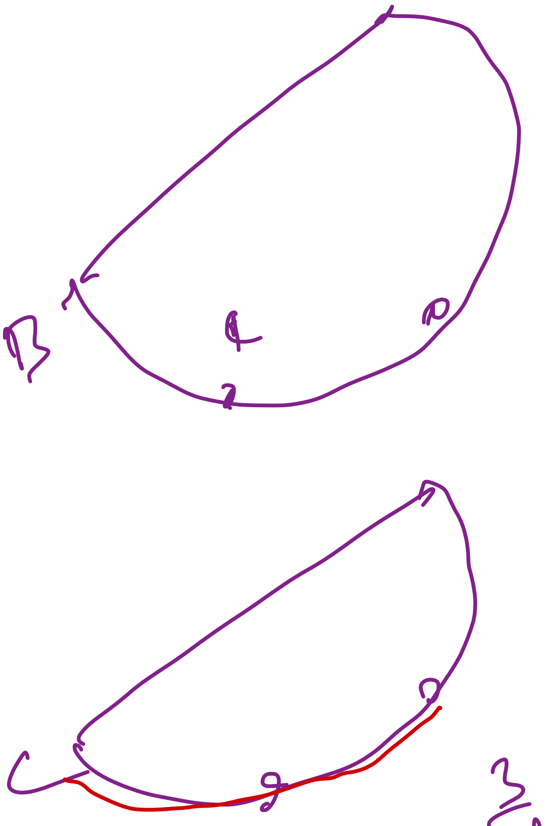
Probability $= \dfrac{\text{Shaded Area}}{\text{Total}}$

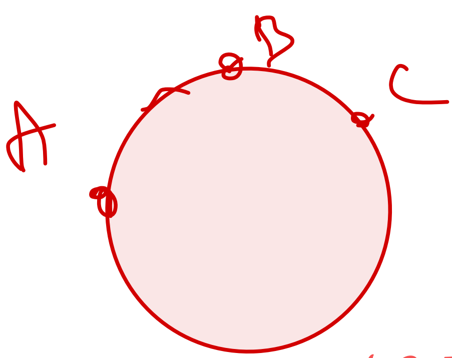
$= \dfrac{3}{4}$



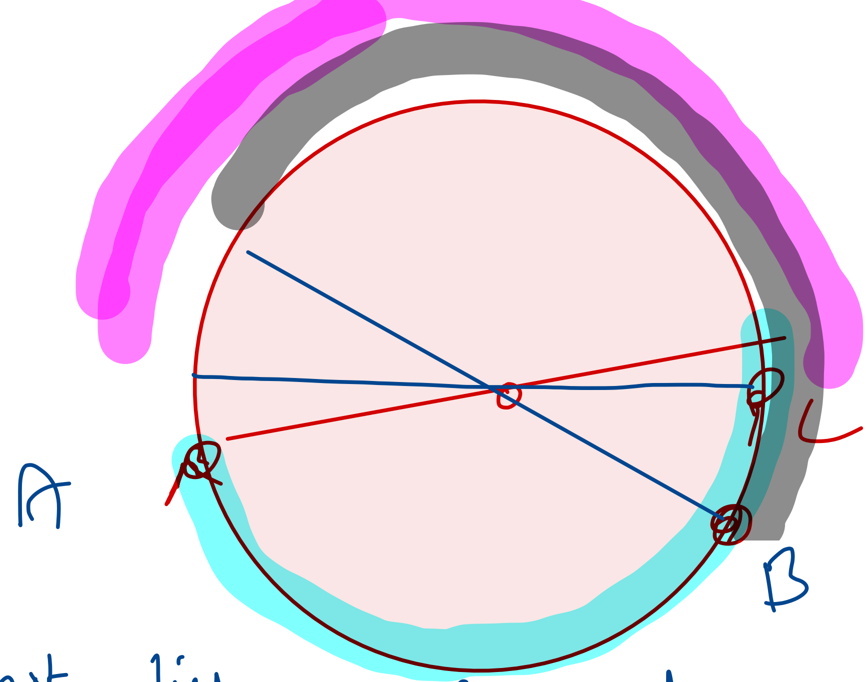
Method 3:





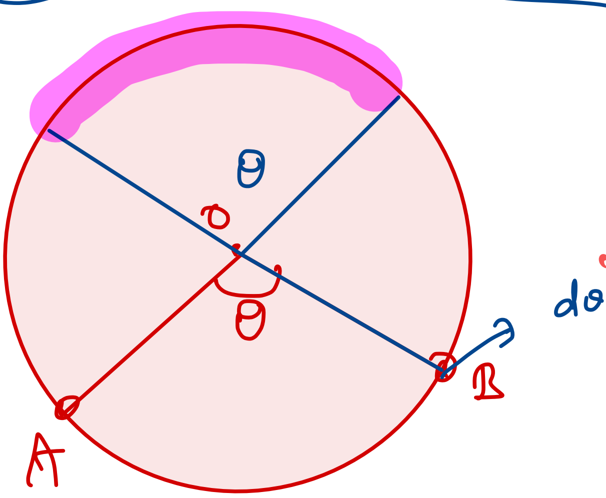


$3 \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$.



Method 4:

A, B, C not lies on semicircle:



$$\frac{\cancel{2}\times d\theta}{\cancel{2}\pi} \times \frac{\theta}{2\pi} \quad \text{where } \theta \in (0,\pi)$$

$$= \frac{1}{2\pi^2} \cdot \int_0^{\pi} \theta \cdot d\theta$$

$$= \frac{\theta^2}{2} \times \frac{1}{2\pi^2} \Big|_0^{\pi}$$

$$= \frac{1}{4}.$$

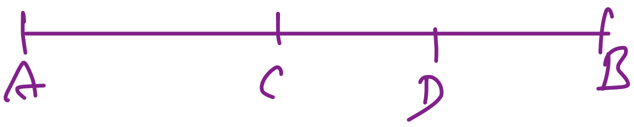
A, B, C lies on semicircle $= 1 - \frac{1}{4} = \frac{3}{4}$.

P(ΔABC obtuse) = 3/4

P(ΔABC acute) = 1/4

If n points on chosen on circle then n points lies on same semicircle then probability $= \dfrac{n}{2^{n-1}}$

AB is a rod having length l units.



If we are cutting AB rod at 2 point C, D (C is leftside of D) Then find the length AC, CD, DB forms probability for a triangle.

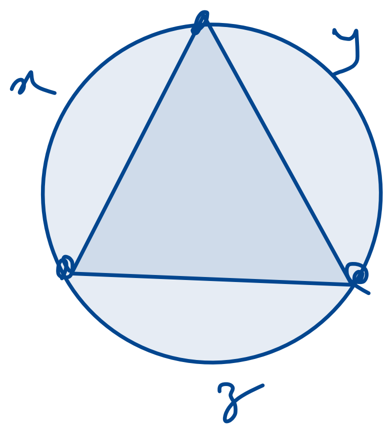
[Ans: $\frac{1}{4}$]

circumference of circle $= l$.

$x+y > z$.

$x+y > l-(x+y)$

$x+y > \frac{l}{2}$



Δ is Acute angle triangle

Probability $= \frac{1}{4}$.

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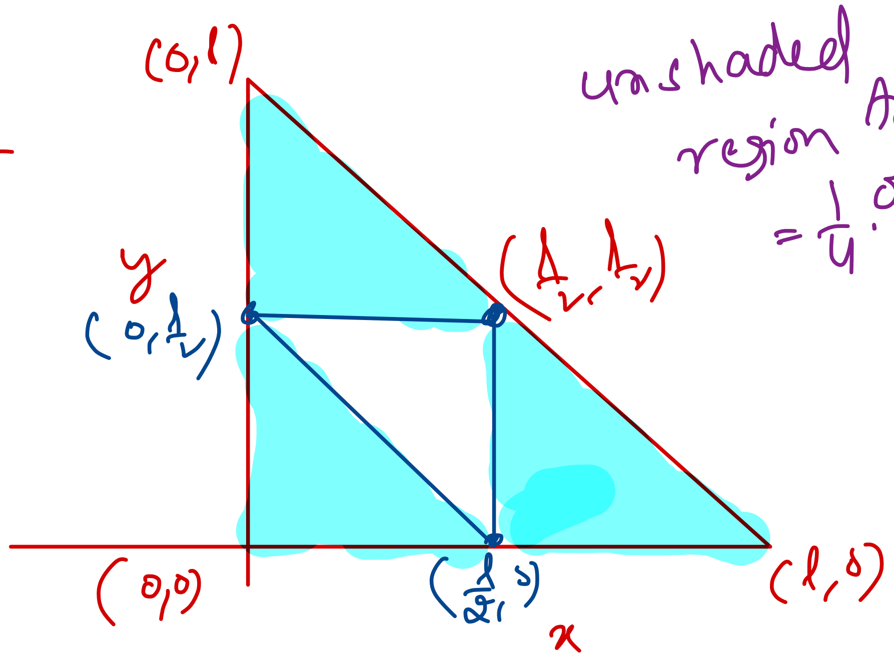
$x \in (0, l)$ $y \in (0, l)$ & $x+y < l$



Here shaded region represent Probability for not forming triangle

unshaded region Area $= \frac{1}{4}$ of total area.

If $x > \frac{l}{2}$. Then $y+z < \frac{l}{2}$

$y+z < x$

Sum of two sides less Than third side is not possible

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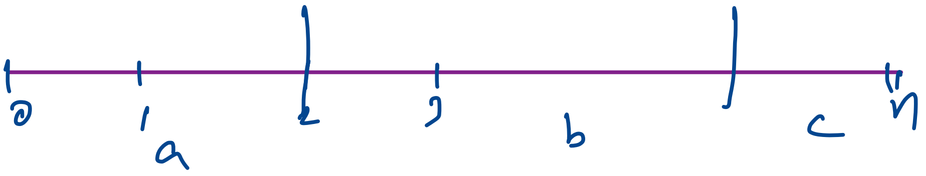
||ly $y > \frac{l}{2}$ Then $x+z < y$ no triangle with sides as x, y, z

If $x < \frac{l}{2}$ & $y < \frac{l}{2}$ & $x+y < \frac{l}{2}$ Then no triangle with sides x, y, z

$\Rightarrow$ Required Probability $= \frac{1}{4}$.



$a + b + c = n$

and $a \geq 1,\ b \geq 1\ \ c \geq 1$

and also $a+b > c$, $b+c > a$ $a+c > b$

$n-1-c > c \Rightarrow c < \frac{n-1}{2}$, $b < \frac{n-1}{2}$, $a < \frac{n-1}{2}$.

Conditional event:

If event A occurs always occurence of event B then it is said to be conditional event. and it is denoted by $\frac{A}{B}$ or A/B (A given B)

Ex: In Throwing 2 dies getting 2 on a die when sum of numbers is 7. is a conditional even,

Here $A \rightarrow$ getting 2

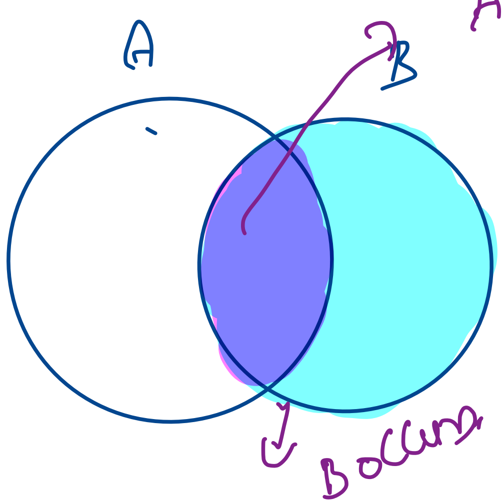
$B \rightarrow$ sum is 7

$A/B \rightarrow$ getting 2 when sum is 7.

B/A → sum is 7 when one of the die shows 2.

and $\boxed{P(A/B) = \dfrac{P(A\cap B)}{P(B)}}$ ∵ $P(B) \neq 0$

A occurs after B occurs



$$\boxed{P\left(\frac{A}{B}\right) = \frac{n(A\cap B)}{n(B)} = \frac{P(A\cap B)}{P(B)}}$$

Here as alredy B occurs then after that A is occuring so sample space is reduced to B and favourable space is which is common to both A, B.

Note:

In A/B event A occurs after event B (or) possible results of B are given then chance of occurrence of A in that given results.

Ex: Find the probability to get 2 when sum of the numbers on two dies is given as 7.

$$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{n(A\cap B)}{n(B)}$$

$B =$ sum is 7 $= \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}$

$A =$ getting 2 $= \{(2,1)(2,2) \ldots (2,6)(1,2)(3,2) \ldots (6,2)\}$

$A\cap B = \{(2,5)(5,2)\}$.

$$P(A/B) = \frac{2}{6}.$$

Find the probability to get sum 7 when atleast one of two dies are thrown and the die shows 2.

$$P(B/A) = \frac{P(B\cap A)}{P(A)} = \frac{2/36}{11/36} = \frac{2}{11}.$$

$\Rightarrow P(A/B), P(B/A)$ need not be equal.

A Bag have 6 Red balls 4 white Balls every time one ball is drawn without replacement.

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If event A represent First ball is Red
event B represent Second ball is Red,

$P(A) = \frac{6}{10}$, $P(B) = \frac{6}{10}$.

B/A = Second ball is Red when First ball red is given.

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{6}{10} \cdot \frac{5}{9}}{\frac{6}{10}} = \frac{5}{9}.$$

Multiplication theorem of Probability:

If A, B are two events then

$$\boxed{P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)}$$

Pf: From conditional probability definition

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(B) \cdot P(A/B)$$

& $$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B/A)$$

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$$P(A \cap B \cap C) = P(A \cap B) \cdot P\left(\frac{C}{A \cap B}\right)$$

$$= P(A) \cdot P(B/A) \cdot P\left(\frac{C}{B \cap A}\right)$$

$$P(A_1 \cap A_2 \cap A_3 \ldots \cap A_n) = P(A_1) \cdot P(A_2/A_1) \cdot P\left(\frac{A_3}{A_1 \cap A_2}\right) \ldots P\left(\frac{A_n}{A_1 \cap A_2 \ldots A_{n-1}}\right)$$

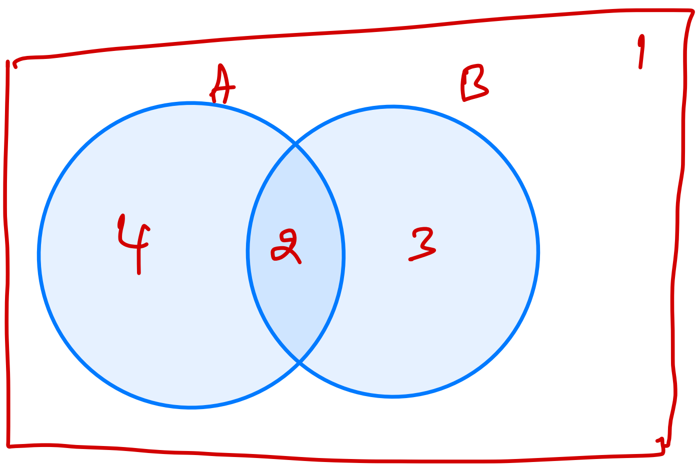
Independent events:

A, B are said to be independent events if chance of occuring one event is not effecting chance of occuring of other event.

$P\left(\frac{A}{B}\right) = P(A)$ & $P\left(\frac{B}{A}\right) = P(B)$

Then A, B are said to be independent events.

$S = \{1, 2, 3, 4\}$



A represents multiple of 2 $= \{2, 4\}$

B represents prime numbers $= \{2, 3\}$

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$P(A) = \frac{2}{4} = \frac{1}{2}$, $P(B) = \frac{2}{4} = \frac{1}{2}$.

$P(A/B) = 1/2$, $P(B/A) = 1/2$

$\Rightarrow$ A, B are independent events

$$P\left(\frac{A}{B}\right) = P(A)$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\Rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B)}$$

If A, B are independent events Then

$$P(A \cap B) = P(A) \cdot P(B)$$

Note: 2 or more students are solving a problem & shooting a target, Throwing a die . . are independent events.

** If A, B are independent events Then $P(A \cap B) = P(A) \cdot P(B)$

(i) A, B^C are independent

(ii) A^C, B are independent

(iii) A^C, B^C are independent.

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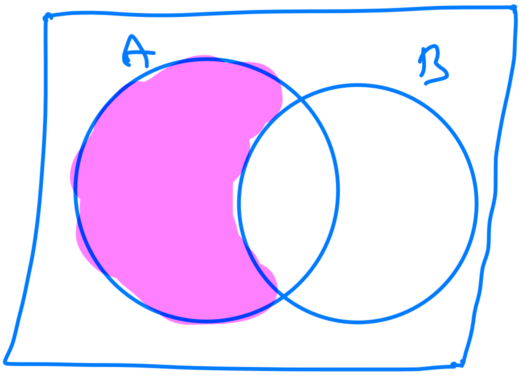

$$P(A \cap B^c) = P(A-B)$$
$$= P(A) - P(A \cap B)$$
$$= P(A) - P(A).P(B)$$
$$= P(A)(1-P(B))$$
$$= P(A).P(B^c)$$

$\Rightarrow$ A, B^c are independent events.

$\|^y$ A^c, B are also independent

(iii) $P(A^c \cap B^c) = P((A \cup B)^c)$
$$= 1 - P(A \cup B)$$
$$= 1 - P(A) - P(B) + P(A \cap B)$$
$$= 1 - P(A) - P(B) + P(A).P(B)$$
$$= (1-P(A))(1-P(B))$$
$$= P(A^c).P(B^c)$$

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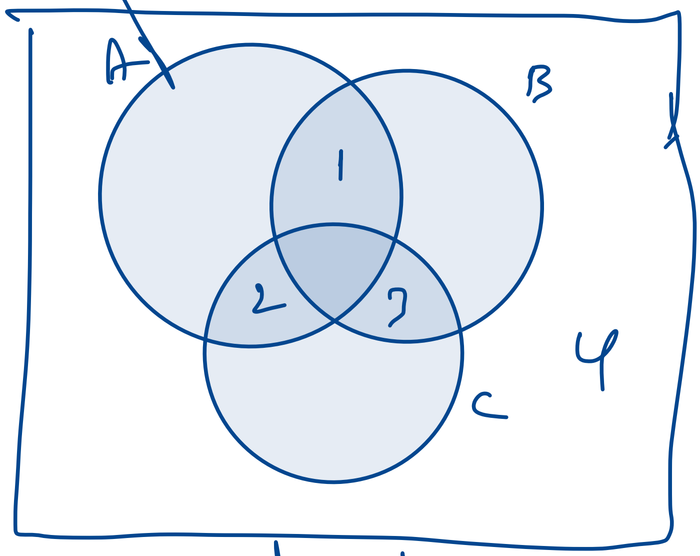
Pair wise independent:

$E_1, E_2, E_3, \ldots E_n$ events are said to be pairwise independent if $P(E_i \cap E_j) = P(E_i).P(E_j)$ $\forall\, i, j$

mutually independent (Independent)

$E_1, E_2, \ldots E_n$ are said to be independent if $P(E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_n) = P(E_1).P(E_2) \ldots P(E_n)$

and they are pair wise independent.



$P(A) = \frac{1}{2}$

$P(B) = \frac{1}{2}$

$P(C) = \frac{1}{2}$

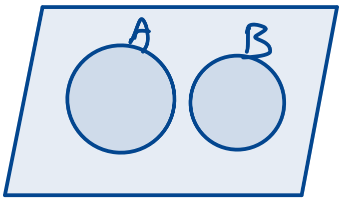
$P(A \cap B) = \frac{1}{4} = P(A) \cdot P(B)$

Here A, B, C are not independent but They are pairwise independent

Note: nonempty Disjoint sets are always dependent

$[\because P(A \cap B) = 0$ but $P(A) \cdot P(B) \neq 0.]$

occurence of A stops occurence of B.



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A — Probabilities — Late

40 Bike — $1/3$ — $1/10$ — 4 $\rightarrow P\left(\frac{L}{B}\right)$

60 Scooty — $1/2$ — $1/4$ — 15 $\rightarrow P\left(\frac{L}{S}\right)$

20 Car — $\frac{1}{6}$ — $3/5$ — 12 $\rightarrow P\left(\frac{L}{C}\right)$

$$P(L) = \frac{1}{3} \times \frac{1}{10} + \frac{1}{2} \times \frac{1}{4} + \frac{1}{\cancel{6}\,2} \times \frac{\cancel{3}}{5}$$

Total probability theorem

$$= \frac{1}{30} + \frac{1}{8} + \frac{1}{10}$$

$$= \frac{4 + 15 + 12}{120} = \frac{31}{120}$$

Bayes theorem

Reverse probability theorem

$$\frac{\frac{3}{30}}{\frac{31}{\cancel{120}\,4}} = \frac{12}{31} \rightarrow P\left(\frac{C}{L}\right)$$

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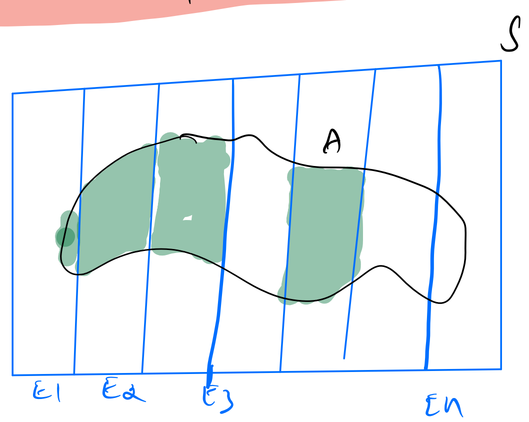
Total Probability Theorem:

Let S be the sample space and $E_1, E_2, E_3 \ldots E_n$ are 'n' mutually exclusive and exhaustive events of this sample space. [i.e $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$ & $E_i \cap E_j = \phi \ \forall i \neq j$] Then If A is any event of sample space then

$$P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + \ldots + P(E_n) \cdot P\left(\frac{A}{E_n}\right)$$

$$P(A) = \sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{A}{E_i}\right)$$

Pf:



$$A = A \cap S$$

$$= A \cap (E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n)$$

$$= (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup \ldots \cup (A \cap E_n)$$

Here $A \cap E_1$, $A \cap E_2$, $A \cap E_3$, .. $A \cap E_n$ are exclusive events.

$P(A \cup B) = P(A) + P(B)$ if A, B are exclusive.

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$$\Rightarrow \quad P(A) = P\left((A\cap E_1)\cup(A\cap E_2)\cup(A\cap E_3)\cup\cdots\cup(A\cap E_n)\right)$$

$$= P(A\cap E_1) + P(A\cap E_2) + \cdots + P(A\cap E_n)$$

$$= P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right) + \cdots + P(E_n)\cdot P\left(\frac{A}{E_n}\right)$$

$$\Rightarrow \quad \boxed{P(A) = \sum_{i=1}^{n} P(E_i)\cdot P\left(\frac{A}{E_i}\right)}$$

Baye's Theorem: (Reverse Probability theorem)

Let S be the sample space and $E_1, E_2, E_3, \ldots E_n$ are n mutually exclusive & exhaustive events of sample space and A is an event in sample space then

$$P\left(\frac{E_k}{A}\right) = \frac{P(E_k)\cdot P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i)\cdot P\left(\frac{A}{E_i}\right)}$$

Pf: $$P\left(\frac{E_k}{A}\right) = \frac{P(E_k\cap A)}{P(A)}$$ [$\because$ Conditional probability def]

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$$= \frac{P(E_k) \cdot P\left(\frac{A}{E_k}\right)}{P(A)}$$ [multiplication Theorem]

$$= \frac{P(E_k) \cdot P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{A}{E_i}\right)}$$ [Total probability Theorem]

$$\therefore \boxed{P\left(\frac{E_k}{A}\right) = \frac{P(E_k) \cdot P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{A}{E_i}\right)}}$$

A Person use bike, Scooty, Car to goto his office with probabilities $\frac{1}{2}, \frac{1}{6}, \frac{1}{3}$ respectively and he is late to office when he use bike, Scooty, Car with probability $\frac{1}{5}, \frac{1}{4}, \frac{5}{6}$

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respectively Then find probability for

(i) He is late to office

(ii) If he is already late to office find probability for he uses car.

$$P(\text{Bike}) = \frac{1}{2} \qquad P\left(\frac{\text{Late}}{\text{Bike}}\right) = \frac{1}{5}$$

$$P(\text{Scooty}) = \frac{1}{6} \qquad P\left(\frac{\text{Late}}{\text{Scooty}}\right) = \frac{1}{4}$$

$$P(\text{Car}) = \frac{1}{3} \qquad P\left(\frac{\text{Late}}{\text{Car}}\right) = \frac{5}{6}.$$

$\frac{1}{2} + \frac{1}{6} + \frac{1}{3} = 1$

From total probability Theorem

$$P(\text{Late}) = P(L) = P(B) \cdot P\left(\frac{L}{B}\right) + P(S) \cdot P\left(\frac{L}{S}\right) + P(C) \cdot P\left(\frac{L}{C}\right)$$

$$= \frac{1}{2} \cdot \frac{1}{5} + \frac{1}{6} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{5}{6}$$

$$= \frac{1}{10} + \frac{1}{24} + \frac{5}{18}$$

$$= \frac{36 + 15 + 100}{360} = \frac{151}{360}$$

$$P\left(\frac{\text{Car}}{\text{Late}}\right) = \frac{P(C) \cdot P\left(\frac{L}{C}\right)}{P(L)} = \frac{\frac{1}{3} \times \frac{5}{6}}{\frac{151}{360}}$$

$$= \frac{100}{151}$$

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In FIITJEE madhapur 25% of the boys and 10% of the girls completed maths work book. The girls strength is 60% of the total strength in college. If a student selected at random is completed his maths work book then find probability for that student is girl student.

$$P\left(\frac{G}{W}\right) = \frac{P(G \cap W)}{P(W)}$$

$$= \frac{P(G) \cdot P\left(\frac{W}{G}\right)}{P(B) \cdot P\left(\frac{W}{B}\right) + P(G) \cdot P\left(\frac{W}{G}\right)}$$

$$= \frac{\frac{60}{100} \times \frac{10}{100}}{\frac{40}{100} \times \frac{25}{100} + \frac{60}{100} \times \frac{10}{100}}$$

$$= \frac{600}{1000 + 600} = \frac{3}{8}.$$

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In a test an examinee either guess or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the ans is $\frac{1}{6}$. The probability that his answer is correct given that he copied it is $\frac{1}{8}$. Then find the probability that he knew the answer to a question given that he correctly answered it.

$$P(G) = \frac{1}{3} \qquad P\left(\frac{M}{G}\right) = \frac{1}{4}$$

$$P(C) = \frac{1}{6} \qquad P\left(\frac{M}{C}\right) = \frac{1}{8}$$

$$P(K) = \frac{1}{2} \qquad P\left(\frac{M}{K}\right) = 1$$

$$P\left(\frac{K}{M}\right) = ?$$

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$$P\left(\frac{K}{M}\right) = \frac{P(K \cap M)}{P(M)}$$

$$= \frac{P(K) \cdot P\left(\frac{M}{K}\right)}{P(K) \cdot P\left(\frac{M}{K}\right) + P(G) \cdot P\left(\frac{M}{G}\right) + P(C) \cdot P\left(\frac{M}{C}\right)}$$

$$= \frac{\frac{1}{2} \cdot 1}{\frac{1}{2} \cdot 1 + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{6} \cdot \frac{1}{8}}$$

$$= \frac{\frac{1}{2}}{\frac{24+4+1}{48}} = \underline{\underline{\frac{24}{29}}}$$

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$$P\left(\frac{K}{M}\right) = \frac{P(K \cap M)}{P(M)}$$

$$= \frac{P(K) \cdot P\left(\frac{M}{K}\right)}{P(K) \cdot P\left(\frac{M}{K}\right) + P(G) \cdot P\left(\frac{M}{G}\right) + P(C) \cdot P\left(\frac{M}{C}\right)}$$

$$= \frac{\frac{1}{2} \cdot 1}{\frac{1}{2} \cdot 1 + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{6} \cdot \frac{1}{8}}$$

$$= \frac{\frac{1}{2}}{\frac{24+4+1}{48}} = \frac{24}{29}.$$

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$$P\left(\frac{K}{M}\right) = \frac{P(K\cap M)}{P(M)}$$

$$= \frac{P(K)\cdot P\left(\frac{M}{K}\right)}{P(K)\cdot P\left(\frac{M}{K}\right) + P(G)\cdot P\left(\frac{M}{G}\right) + P(C)\cdot P\left(\frac{M}{C}\right)}$$

$$= \frac{\frac{1}{2}\cdot 1}{\frac{1}{2}\cdot 1 + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{6}\cdot\frac{1}{8}}$$

$$= \frac{\frac{1}{2}}{\frac{24+4+1}{48}} = \underline{\underline{\frac{24}{29}}}.$$

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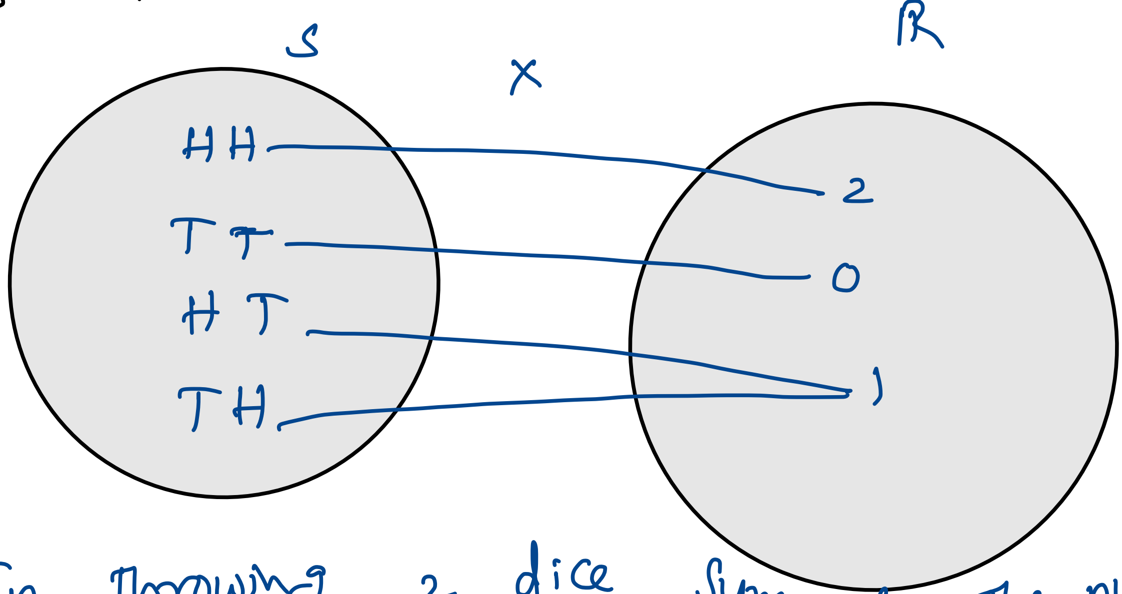
Random Variable :

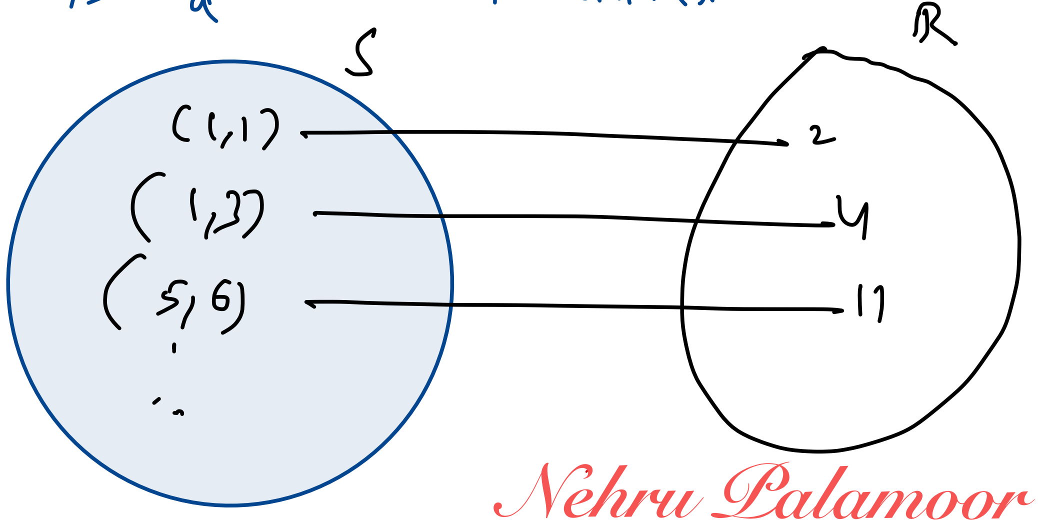
Random Variable is a function from sample space of an experiment to Real number set. and it is denoted by X.

$X: S \to \mathbb{R}$ is always a function.

Ex: (i) In tossing 2 coins no. of heads is a random variable



(ii) In throwing 2 dice sum of the numbers on dies is a random variable.



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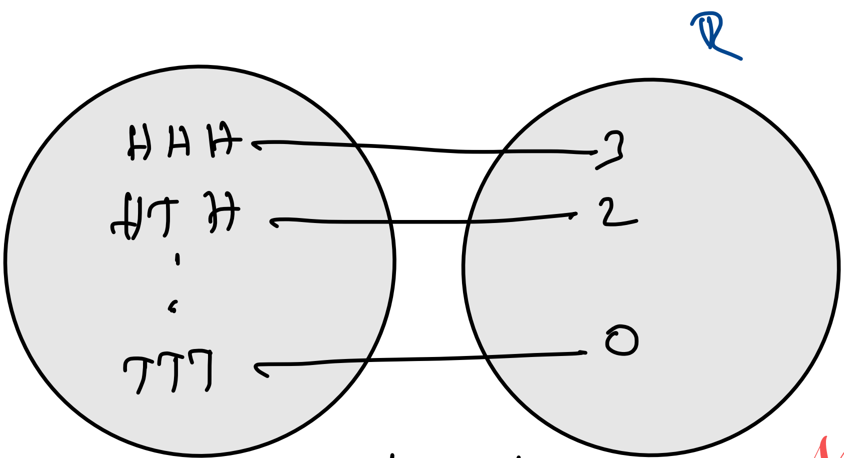
discrete, continuous random variable:

If $X(S)$ is finite & countable infinite then that variable is called as discrete random variable, If $X(S)$ is uncountable infinite & $X(S) = (a,b)$ or $[a,b]$ then $X(S)$ is continuous random variable

Ex: If sample is finite then always it is discrete random variable.

no. of heads on tossing 3 coins is a discrete random variable.

choosing a number from $[0,1]$ and mapping itself is a continuous random variable



discrete

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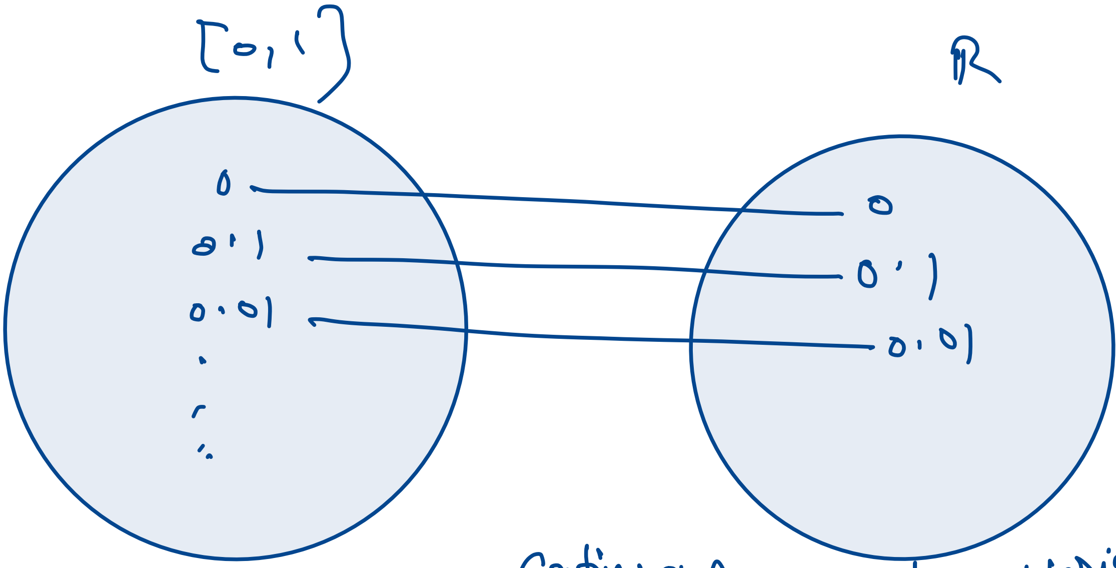
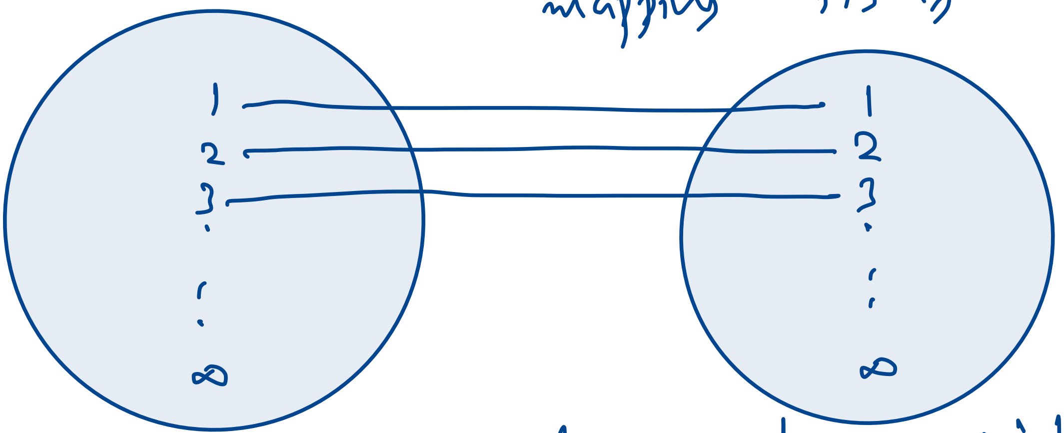


Continuous random variable

choosing a natural no from natural nos and mapping itself



discrete random variable

Probability Distribution Function:

Probability distribution Function (P.D.F) is a function from $X(S)$ (Range of random variable) to $[0,1]$. and it is denoted by $\mathcal{P}$.

If $X(S) = \{x_1, x_2, \dots x_n\}$ then $P(X = x_i)$ represent chance of getting number x_i.

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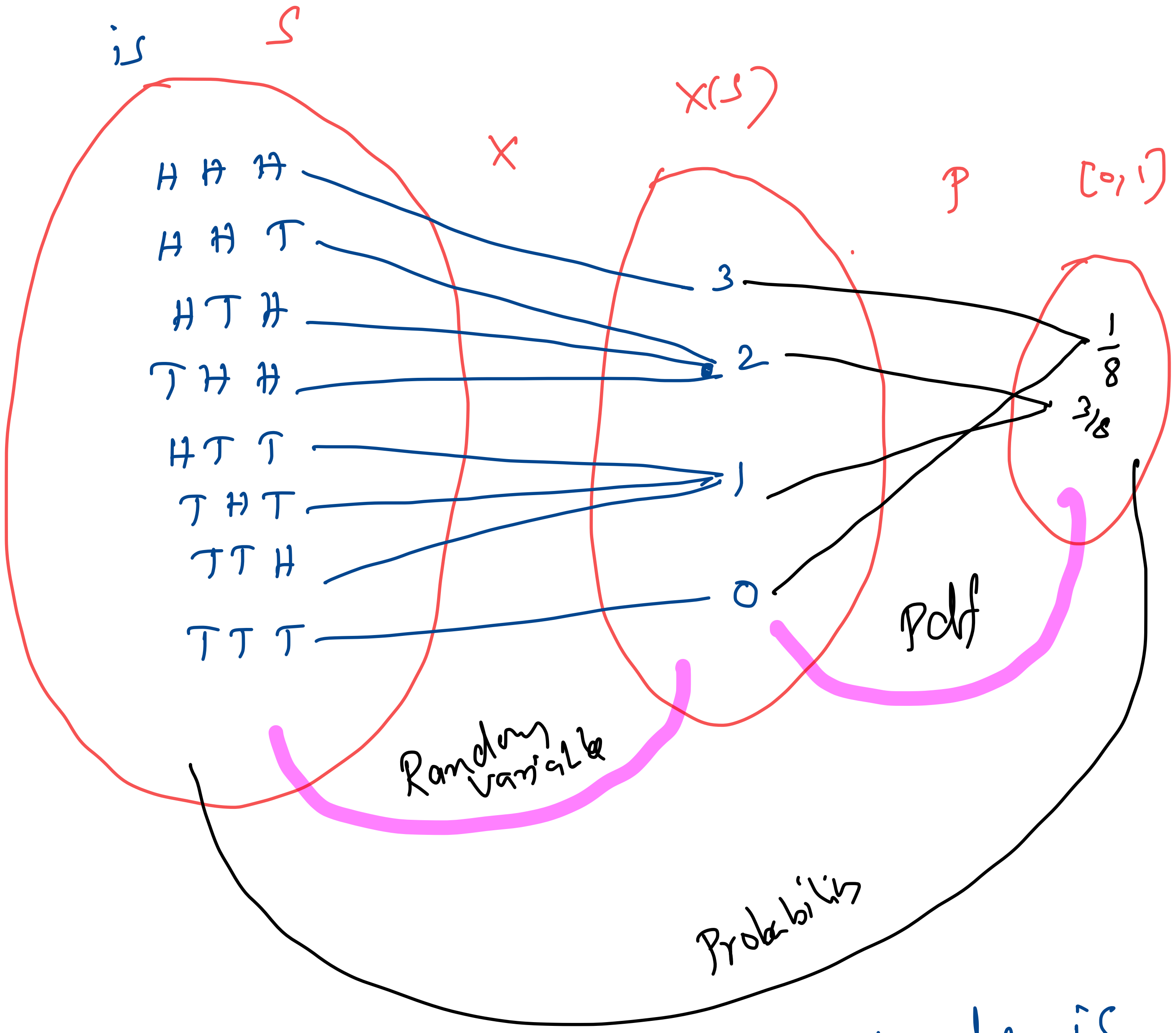
Ex: In tossing 3 coins no. of heads is a random variable then its pdf is



In tossing 3 coins no. of heads is a random variable then its pdf is

x_i	0	1	2	3
$P(X=x_i)$	$1/8$	$3/8$	$3/8$	$1/8$

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In throwing 2 dies sum of the numbers is a random variable then pdf of this random variable is

$X = x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(X = x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Note: If $X(S) = \{x_1, x_2, x_3, \ldots x_n\}$

then $P(x_1) + P(x_2) + \cdots + P(x_n) = 1$

i.e $\boxed{\sum_{i=1}^{n} P(X = x_i) = 1}$

① If pdf of a random variable is

$X = x_i$	0	1	2
$P(X = x_i)$	$3c^3$	$4c - 10c^2$	$5c - 1$

then find c.

$$3c^3 + 4c - 10c^2 + 5c - 1 = 1$$

$$\Rightarrow 3c^3 - 10c^2 + 9c - 2 = 0$$

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$$(c-1)(3c^2 - 7c + 2) = 0$$

$$(c-1)(c-2)(3c-1) = 0$$

but $c = 1, 2$ are not possible $[\because 3c^3 \notin (0,1)]$

$$\Rightarrow c = 1/3.$$

In above problem find $P(X \leq 1)$

$$= P(X=0) + P(X=1)$$

$$= 1 - P(X=2)$$

$$= 1 - (5c - 1)$$

$$= 2 - \frac{5}{3} = \underline{\underline{\frac{1}{3}}}.$$

Mean of the random variable:

Range of $X = \{x_1, x_2, \ldots x_n\}$.

If $\sum_{i=1}^{n} x_i P(X = x_i)$ is finite then this value is called as mean or expectation of random variable, and it is denoted by μ.

$$\boxed{\mu = \sum_{i=1}^{n} x_i P(X = x_i)}$$

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Variance of the random Variable:

If $\sum_{i=1}^{n} (x_i - \mu)^2 \, p(X = x_i)$ is finite then This value is called as variance of random variable and it is denoted by σ^2.

$$\boxed{\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot p(X = x_i)}$$

And σ is called as standard deviation.

Theorem: $\sigma^2 + \mu^2 = \sum_{i=1}^{n} x_i^2 \, p(X = x_i)$

Pf:
$$\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 \, p(X = x_i)$$

$$= \sum_{i=1}^{n} (x_i^2 + \mu^2 - 2x_i\mu) \, p(X = x_i)$$

$$= \sum_{i=1}^{n} x_i^2 \cdot p(X = x_i) + \mu^2 \cdot \sum_{i=1}^{n} p(X = x_i) - 2\mu \cdot \sum_{i=1}^{n} p(X = x_i) \cdot x_i$$

$$= \sum_{i=1}^{n} x_i^2 \, p(X = x_i) + \mu^2 \cdot 1 - 2\mu \cdot \mu$$

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$$\Rightarrow \boxed{\sigma^2 + \mu^2 = \sum_{i=1}^{n} x_i^2 \cdot p(X = x_i)}$$

A Cubical dies is thrown. random Variable is number on the die. Then find mean and Variance of this random variable

$X = x_i$	1	2	3	4	5	6
$p(X = x_i)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

$$\text{Mean} = \mu = \sum_{i=1}^{n} x_i \, p(X = x_i)$$

$$= 1\cdot\frac{1}{6} + 2\cdot\frac{1}{6} + 3\cdot\frac{1}{6} + 4\cdot\frac{1}{6} + 5\cdot\frac{1}{6} + 6\cdot\frac{1}{6}$$

$$= \frac{6\times 7}{2\times 6} = 3.5$$

<u>Variance:</u>

$$\sigma^2 + \mu^2 = \sum_{i=1}^{n} x_i^2 \, p(X = x_i)$$

$$\sigma^2 + \left(\frac{7}{2}\right)^2 = 1^2\cdot\frac{1}{6} + 2^2\cdot\frac{1}{6} + 3^2\cdot\frac{1}{6} + 4^2\cdot\frac{1}{6} + 5^2\cdot\frac{1}{6} + 6^2\cdot\frac{1}{6}$$

$$\sigma^2 + \frac{49}{4} = \frac{1}{6}\cdot\frac{\cancel{6}\times 7\times 13}{\cancel{6}}$$

$$\sigma^2 = \frac{91}{6} - \frac{49}{4} = \frac{182 - 147}{12} = \frac{35}{12}$$

$$\sigma^2 = \frac{35}{12}, \qquad \sigma = \sqrt{\frac{35}{12}}$$

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$X = x_i$	1	2	3	4	5
$P(X = x_i)$	K	2K	3K	4K	5K

Then find value of k, Mean, Variance

(i) $k = \frac{1}{15}$

(ii) Mean $= \frac{11}{3}$

(iii) Variance $= \frac{14}{9}$.

Bernouli's trial:

An experiment is said to be bernoulis trial if it gives only 2 results like success, failure $s \cap f = \varphi$. with probabilities p, q. and $p+q=1$ and success probability is same if we do that experiment any no. of times. and every trial is independent to compare with other trials.

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Ex: (i) In throwing a die getting 5 is success then $p = \frac{1}{6}$, $q = \frac{5}{6}$ and it is same in every experiment, so this trial is a Bernouli's trial.

(ii) In tossing a coin of getting heads is success then $p = \frac{1}{2}$, $q = \frac{1}{2}$. This trial is also a bernouli's trial.

Binomial Distribution

Let a Bernouli trial be conducted 'n' number of times. Then no. of successes are like $0, 1, 2, \ldots n$. and This no. of success is a random variable.

if success probability is p and failure probability is q then

probability of exactly r successes is

$$nC_r \cdot p^r \cdot q^{n-r} \quad \text{where } r = 0, 1, 2 \ldots n$$

And pdf is like

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$X=r$	0	1	2	i	n
$P(X=r)$	$nC_0 \cdot q^n \cdot p^0$	$nC_1 \cdot p \cdot q^{n-1}$	$nC_2 \cdot p \cdot q^{n-2}$	$nC_i\, p^i\, q^{n-i}$	$nC_n \cdot p^n \cdot q^0$

Here every term is term in $(q+p)^n$ binomial expansion, This distribution is called as binomial distribution. with parameters n, p.

Def: A discrete random variable X is said to follow a binomial distribution (or simply a binomial variable with parameters n, p where $0<p<1$) if

$$\boxed{P(X=r) = nC_r \cdot p^r \cdot q^{n-r} \text{ where } r \in \{0,1,2..n\}}$$

it is also denoted by $X \sim B(n,p)$

Ex: In throwing a die 4 times getting 3 is success then no. of successes always follow binomial distribution.

r	0	1	2	3	4
$P(X=r)$	$4C_0 \cdot \left(\frac{5}{6}\right)^4$	$4C_1 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^3$	$4C_2 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2$	$4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)$	$4C_4 \cdot \left(\frac{1}{6}\right)^4$

Mean of binomial distribution is np.
and Variance of binomial distribution is npq.

$$\mu = \sum_{i=0}^{n} x_i \, P(X = x_i)$$

$$= \sum_{i=0}^{n} i \cdot nC_i \, p^i \cdot q^{n-i}$$

$$= 0 + \sum_{i=1}^{n} \cancel{i} \cdot \frac{n}{\cancel{i}} (n-1)C_{i-1} \cdot p^i \cdot q^{n-i}$$

$$= \sum_{i=1}^{n} n \cdot (n-1)C_{i-1} \cdot p^{i-1} \cdot p \cdot q^{(n-1)-(i-1)}$$

$$= np \sum_{i=1}^{n} (n-1)C_{i-1} \cdot p^{i-1} \cdot q^{(n-1)-(i-1)}$$

$$= np \cdot (q+p)^{n-1}$$

$$= np \cdot 1 = np$$

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$$\Rightarrow \boxed{\text{Mean } \mu = np}$$

Variance is σ^2 Then $\sigma^2 + \mu^2 = \sum_{i=0}^{n} i^2 \cdot p(x=i)$

$$\Rightarrow \sigma^2 + n^2p^2 = \sum_{i=0}^{n} (i^2 - i + i) \cdot nC_i \cdot p^i \cdot q^{n-i}$$

$$\Rightarrow \sigma^2 + n^2p^2 = \sum_{i=0}^{n} i(i-1) \cdot nC_i \, p^i \cdot q^{n-i} + \sum_{i=0}^{n} i \cdot nC_i \, p^i q^{n-i}$$

$$= \sum_{i=2}^{n} \cancel{i}\cancel{(i-1)} \frac{n}{\cancel{i}} \cdot \frac{n-1}{\cancel{i-1}} \cdot {}^{(n-2)}C_{i-2} \cdot p^2 \cdot p^{i-2} \cdot q^{(n-2)-(i-2)} + np$$

$$= n(n-1) \cdot p^2 \cdot \sum_{i=2}^{n} {}^{(n-2)}C_{i-2} \cdot p^{i-2} \cdot q^{(n-2)-(i-2)} + np$$

$$= n(n-1)p^2 \cdot (q+p)^{n-2} + np$$

$$\Rightarrow \sigma^2 + \cancel{n^2p^2} = \cancel{n^2p^2} - np^2 + np$$

$$\Rightarrow \sigma^2 = np(1-p) = npq$$

$$\boxed{\sigma^2 = npq}$$

In a binomial distribution mean $= 4$ Variance $= 3$ Then find $P(X \leq 2)$, $P(X > 3)$.

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mean $= 4 \Rightarrow np = 4$

Variance $= 3 \Rightarrow npq = 3$

$$\frac{npq}{np} = \frac{3}{4} \Rightarrow q = \frac{3}{4} \Rightarrow p = \frac{1}{4}.$$

$$n \cdot \frac{1}{4} = 4 \Rightarrow n = 16.$$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= 16C_0 \cdot \left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^{16} + 16C_1 \cdot \frac{1}{4} \cdot \left(\frac{3}{4}\right)^{15} + 16C_2 \cdot \left(\frac{1}{4}\right)^2 \cdot \left(\frac{3}{4}\right)^{14}$$

$$P(X>3) = 1 - P(X \le 3)$$
$$= 1 - \left(P(X=0) + P(X=1) + P(X=2) + P(X=3)\right)$$
$$= 1 - \left(16C_0 \cdot \left(\tfrac{1}{4}\right)^0 \left(\tfrac{3}{4}\right)^{16} + 16C_1 \cdot \tfrac{1}{4} \cdot \left(\tfrac{3}{4}\right)^{15} + 16C_2 \cdot \left(\tfrac{1}{4}\right)^2 \left(\tfrac{3}{4}\right)^{14} + 16C_3 \cdot \left(\tfrac{1}{4}\right)^3 \cdot \left(\tfrac{3}{4}\right)^{13}\right)$$

Poisson distribution:

If no. of trials in bernoulli trial is very large ($n \to \infty$) and getting success probability is almost zero. and average no. of success is λ (i.e $p = \frac{\lambda}{n}$) then no of success is always follow poisson distribution and probability for r success is

$$\boxed{P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \quad \text{where } r = 0, 1, 2, \ldots}$$

In Binomial distribution if $p = \frac{\lambda}{n}$ and $n \to \infty$ then it is converted as Poisson distribution.

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$$\lim_{n\to\infty} {}^nC_r \cdot p^r \cdot q^{n-r} = \lim_{n\to\infty} {}^nC_r \cdot \left(\frac{\lambda}{n}\right)^r \cdot \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \lim_{n\to\infty} \underbrace{\frac{n!}{(n-r)!\times n^r}}_{\downarrow\, 1} \cdot \underbrace{\frac{1}{r!}\lambda^r}_{\downarrow\, \frac{\lambda^r}{r!}} \cdot \underbrace{\left(1-\frac{\lambda}{n}\right)^{n-r}}_{\downarrow\, e^{-\lambda}}$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-r} = e^{\lim_{n\to\infty}(n-r)\left(1-\frac{\lambda}{n}-1\right)} = e^{-\lambda}.$$

$$\frac{{}^nC_r}{n^r} = \frac{n\,(n-1)\,(n-2)\cdots(n-(r-1))}{n\cdot n\cdots n}$$

$$= 1\cdot 1\cdot 1\cdots 1 \text{ r times} = 1$$

pdf of Poisson distribution

$r =$	0	1	2	3 ...
$P(X=r)$	$\frac{e^{-\lambda}\cdot\lambda^0}{0!}$	$\frac{e^{-\lambda}\cdot\lambda^1}{1!}$	$\frac{e^{-\lambda}\cdot\lambda^2}{2!}$	$\frac{e^{-\lambda}\cdot\lambda^3}{3!}$ ...

$$\sum_{r=0}^{\infty} P(X=r) = \sum_{r=0}^{\infty}\frac{e^{-\lambda}\cdot\lambda^r}{r!} = e^{-\lambda}\left(1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\cdots\right)$$

$$= e^{-\lambda}\cdot e^{\lambda} = 1$$

Ex: (i) No. of accidents in a road follow poisson distribution

(ii) No. of phone calls received, follow poisson distribution

(iii) No. of mistakes in a book of paper follow poisson distribution.

Mean of poisson distribution $= \lambda$

Variance of poisson distribution $= \lambda$.

Pf: Mean $= np = \cancel{n} \cdot \frac{\lambda}{\cancel{n}} = \lambda$.

Variance $= npq = \cancel{n} \cdot \frac{\lambda}{\cancel{n}} \cdot \left(1 - \frac{\lambda}{n}\right) = \lambda$.

If pdf of a random variable satisfies $P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$ where $r = 0, 1, 2 \ldots$

Then that random variable follows poisson distribution with parameter λ.

Pro: The number of persons joining a cinema ticket counter in a minute follow poisson distribution with parameter 6

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Then find the probability for

(i) No one joins in queue in a particular minute

(ii) atleast two persons are joining in particular minute

(iii) exactly 10 persons joining in particular minute.

Here $\lambda = 6$ $\quad$ $p(x=r) = \dfrac{e^{-6} \cdot 6^r}{r!}$

(i) $p(x=0) = \dfrac{e^{-6} \cdot 6^0}{0!} = e^{-6}$.

(ii) $p(x=2,3,4 \ldots) = 1 - p(x=0) - p(x=1)$

$= 1 - e^{-6} - e^{-6} \cdot 6$

$= 1 - \dfrac{7}{e^6}$

(iii) $p(x=10) = \dfrac{e^{-6} \cdot 6^{10}}{10!}$

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$P(\text{A speaks truth}) = \frac{3}{4}$

$P(\text{B speaks truth}) = \frac{7}{10}.$

$$= \frac{P(\text{truth})}{P(\text{both are saying true}) + P(\text{both saying False})}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4} \times \frac{7}{10}}{\frac{1}{6} \times \frac{3}{4} \times \frac{7}{10} + \frac{5}{6} \times \frac{1}{4} \times \frac{3}{10}}$$

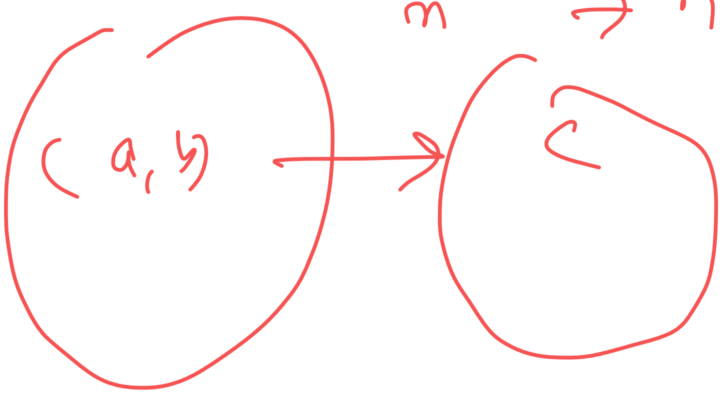
$$= \frac{21}{21+15} = \frac{21}{36}$$

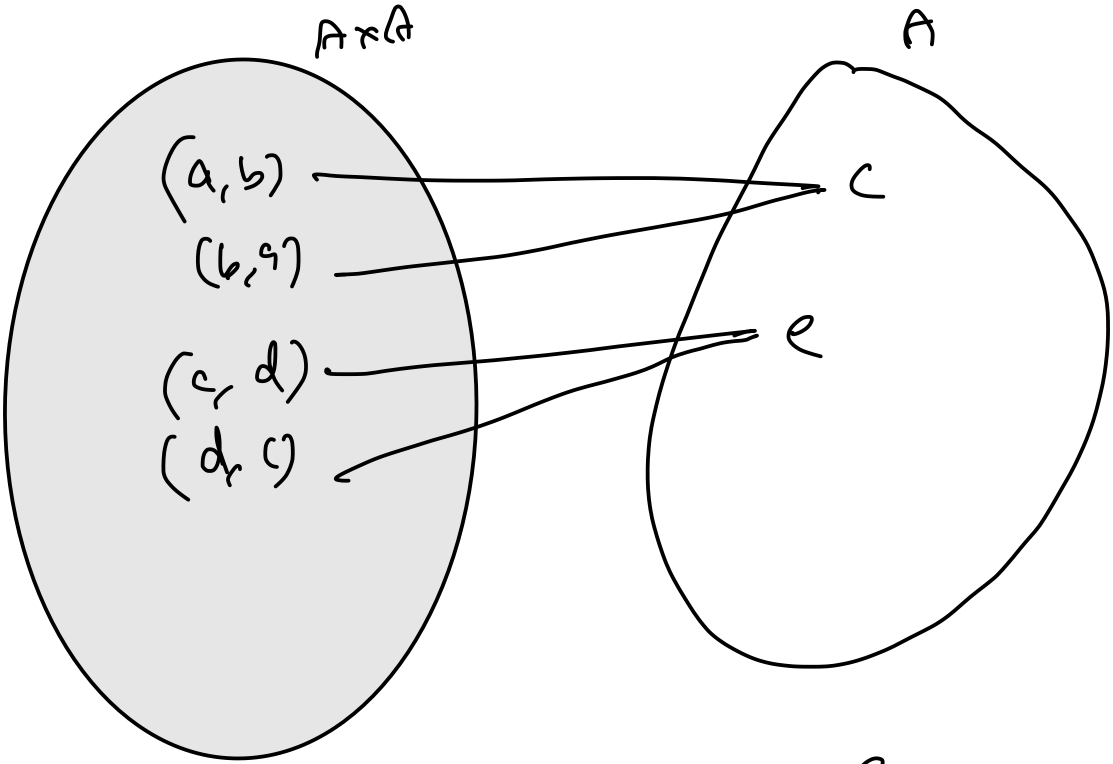
(32) Binary operation is a function from

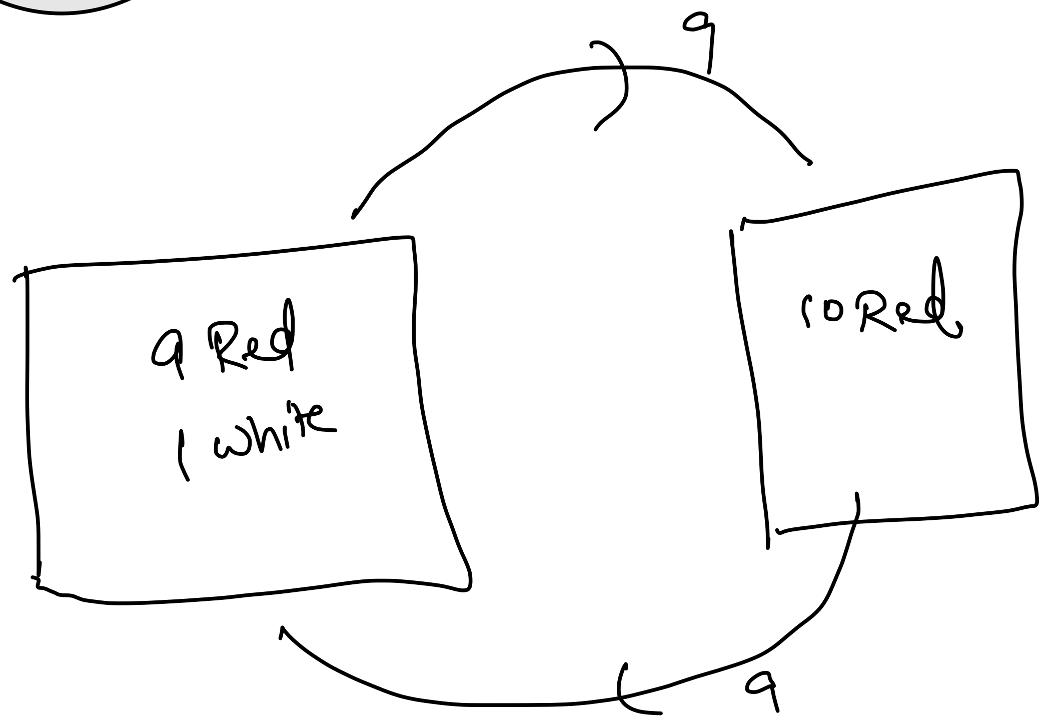
$A \times A \rightarrow A$

$n^2 \rightarrow n$

$$\frac{n^{\frac{n^2+n}{2}}}{n^{n^2}}$$







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WWW $\longrightarrow$ $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

LWWW $\longrightarrow$ $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{2}$

WLWW $\longrightarrow$ $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2}$

WWLW $\longrightarrow$ $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}$

_ _ _ _ W

WWLLW $\longrightarrow$ $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4}$

WLWLW $\longrightarrow$

WLLWW

L W W L W
L W L W W
L L W W W

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$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1/216}{\frac{69}{216}}$$

B → isosceles triangle

$$= \frac{1}{69}$$

1 1 1 — 1

22 {1, 2, 3} = 7

33 {1, 2, 3, 4, 5} = 13

44 {1, . 6} = 16

55 {1. .. 6} = 16

66 {1. . 6} = 16

9 x 9 = 81

S = {0, 1, 2. .. 8}

$$\frac{8 \times 9}{2 \times 9} = 4.$$

0 → 9

1 → 8x2 = 16

2 → 7x2 = 14

3 →

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$A_n = \{1, 2, 3, \ldots, n\}$

P_n denotes no. of subsets of A_n which does not have two consecutive numbers

$P_8 =$ no of subsets of $\{1, 2, 3, \ldots 8\}$ with no two elements are consecutive

Subsets not having 8

Subsets no two elements are consecutive and maximum possible element is 7.

P_7

Subsets having 8

Ex $\{2, 4, 6, 8\}$

After removing 8 here maximum element is 6

P_6

$\Rightarrow \quad P_8 = \underbrace{P_7}_{\text{not having } 8} + \underbrace{P_6}_{\{8 + \text{Max } 6\}}$

$$\boxed{P_n = P_{n-1} + P_{n-2}}$$

$P_6 \rightarrow \{\varphi\}, \{1\}, \{1, 3\}, \{1, 3, 5\}, \{2, 4, 6\}$

$\downarrow$

$\{8\}, \{1, 8\}, \{1, 3, 8\}, \{1, 3, 5, 8\}, \{2, 4, 6, 8\}$

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Thank you

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