__/__/__

Devansh Agarwal (102103620)

Q1.

mean = $\theta 1$, var = $\theta 2$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e \wedge \left\{\frac{-(x-\mu)^2}{2\sigma^2}\right.$$

Joint Density of $(x_1, x_2, x_3 \ldots)$,

$$L(\theta_1, \theta_2 ; x_1, x_2, x_3 \ldots) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\left(\frac{x_i-\theta_1}{2\theta_2^2}\right)^2}$$

Taking log on both sides,

$$\cancel{\log} \ln[L(\theta_1, \theta_2)] = \ln\left((2\pi\theta_2)^{-n/2} \cdot e^{\frac{-\Sigma(x_i-\theta_1)^2}{2\theta_2^2}}\right)$$

$$\ln[L(\theta_1, \theta_2)] = \sum_{i=1}^{n}\left(-\frac{(x_i-\theta_1)^2}{2\theta_2^2} - \frac{1}{2}\ln(2\pi\theta_2^2)\right)$$

Partial Derivative,

For θ_1

$$\frac{\partial \ln L}{\partial \theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_1) = 0$$

$$\Rightarrow \boxed{\theta_1 = \frac{\Sigma x_i}{n}}$$

For θ_2

$$\frac{\partial \ln L}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\Sigma(x_i - \theta_1)^2 = 0$$

$$\Rightarrow \boxed{\theta_2 = \frac{1}{n}\Sigma(x_i - \theta_1)^2}$$

Q2. $B(m, \theta) \rightarrow$ Binomial Distribution

$m \rightarrow$ +ve integer

$$f(x) = {}^{m}C_{x}\, p^{x} (1-p)^{m-x}$$

$$L(\theta; x_i, x_2, x_3 \ldots) = \prod_{i=1}^{m} P(x_i \mid m, P)$$

$$L(\theta) = \prod_{i=1}^{m} \left({}^{m}C_{x} \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i}\right)$$

Taking ln both sides,

$$\ln(L(\theta)) = \sum_{i=1}^{m} \cancel{\log} \ln({}^{n}C_{x_i}) + \sum_{i=1}^{m} x_i \ln\theta + \sum_{i=1}^{m} (m - x_i) \ln(1-\theta)$$

Differentiate w.r.t θ,

$$\frac{d(\ln(L))}{d\theta} = \frac{1}{\theta}\sum_{i=1}^{n} x_i + \frac{1}{1-\theta}\sum_{i=1}^{m} (m-x_i)(-1) = 0$$

$$\Rightarrow \frac{1}{\theta}\sum_{i=1}^{m} x_i = \frac{1}{1-\theta}\sum_{i=1}^{m}(m - x_i)$$

$$(1-\theta)\sum x_i = \theta \sum (m - x_i)$$

$$\boxed{\theta = \frac{\sum x_i}{m}}$$

$$\Rightarrow \boxed{\theta = \overline{x}}$$