Option Pricing Analysis for American Airlines (AAL)

Financial Derivatives Project Report

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Financial Derivatives Pricing

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1 Data Collection and Initial Analysis

1.1 Data Download

Data for American Airlines (AAL) stock was downloaded from Yahoo Finance with maximum available price history from September 2005 to May 2025. The data begins on September 27, 2005 with an opening price of \$19.84 and closing price of \$18.19.

1.2 Price Visualization

Figure 1 shows the historical price movement of AAL stock over the entire period. Notable features include significant price fluctuations over the 20-year period, with current price levels around \$10.52 as of April 2025.

Figure 1: AAL Stock Price History (2005-2025)

1.3 Log Returns Analysis

Daily log returns were calculated using the formula:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

Figure 2 presents the time series of log returns, highlighting the volatility clustering phenomenon typical in financial markets.

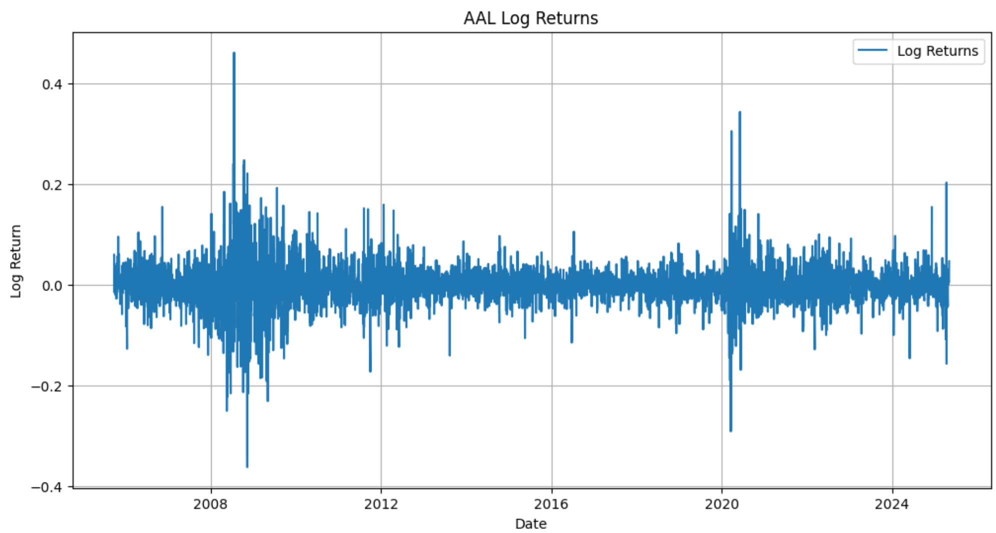


Figure 2: AAL Daily Log Returns (2005-2025)

2 Normality Testing for Log Returns

2.1 Visual Analysis

2.1.1 QQ Plot

Figure 3 shows the QQ plot for AAL log returns compared against theoretical normal distribution quantiles.

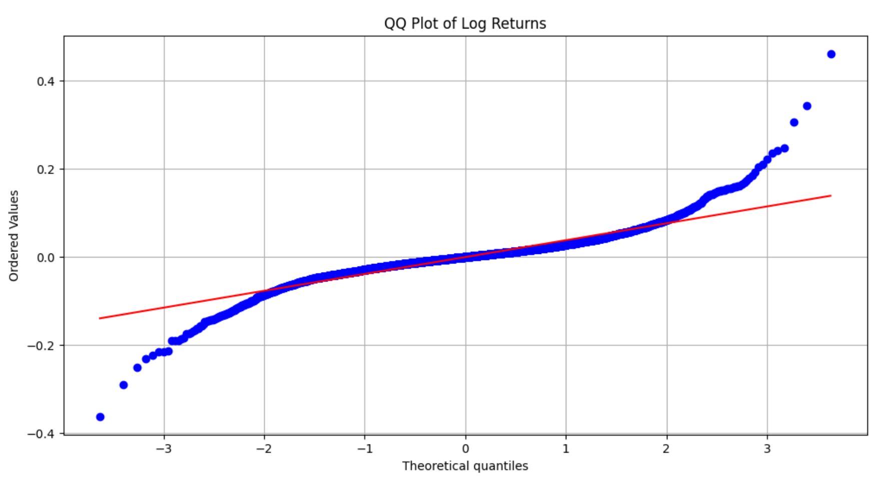


Figure 3: QQ Plot of AAL Log Returns

2.1.2 Histogram with Normal Distribution

Figure 4 displays the histogram of log returns with a normal distribution overlay.

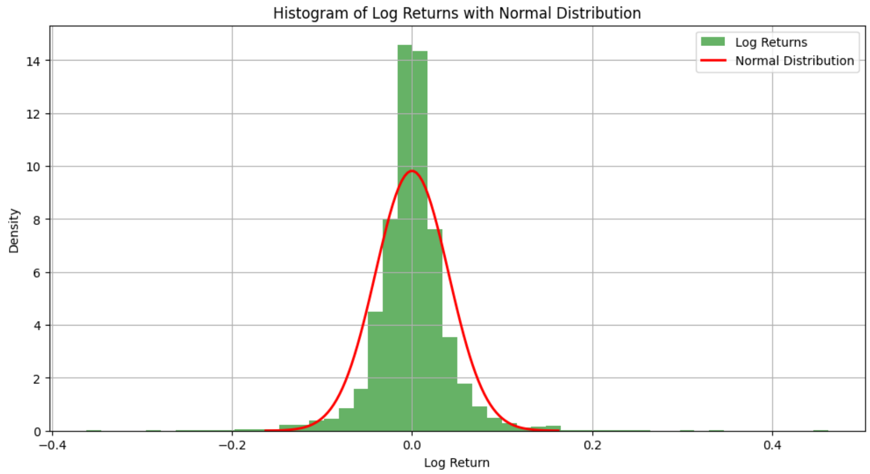


Figure 4: Histogram of AAL Log Returns with Normal Distribution Curve

2.2 Statistical Tests

2.2.1 Jarque-Bera Test

The Jarque-Bera test examines the goodness-of-fit of data to a normal distribution by analyzing skewness and kurtosis.

Statistic	**Value**
Test Statistic	26517.9698
p-value	0.0000

Table 1: Jarque-Bera Test Results

2.2.2 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test compares the empirical distribution function with the cumulative distribution function of a reference distribution.

Statistic	**Value**
Test Statistic	0.091889
p-value	0.0000

Table 2: Kolmogorov-Smirnov Test Results

2.2.3 Anderson-Darling Test

The Anderson-Darling test gives more weight to the tails of the distribution than the Kolmogorov-Smirnov test.

Statistic	Value
Test Statistic	105.1095

Table 3: Anderson-Darling Test Results

2.3 Normality Test Conclusion

Based on the visual evidence and statistical tests, AAL log returns exhibit significant deviations from normality. The QQ plot shows heavy tails, indicating more extreme values than would be expected in a normal distribution. The Jarque-Bera, Kolmogorov-Smirnov, and Anderson-Darling tests all reject the null hypothesis of normality with p-values near zero. This non-normality is typical of financial returns and has important implications for option pricing models.

3 Volatility Estimation

3.1 Historical Volatility

The historical volatility was estimated using the standard deviation of log returns:

$$\sigma_{historical} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(r_i - \bar{r})^2} \times \sqrt{252} \tag{2}$$

where N is the number of observations, r_i is the log return on day i, $\bar{r}$ is the mean log return, and $\sqrt{252}$ annualizes the daily volatility assuming 252 trading days per year.

Metric	Value
Annualized Historical Volatility	64.49%

Table 4: Historical Volatility Estimates

4 Risk-Free Rate Determination

The 3-month U.S. Treasury Bill rate was selected as the risk-free rate for equity option pricing. This rate was programmatically retrieved from the Federal Reserve Economic Data (FRED) database.

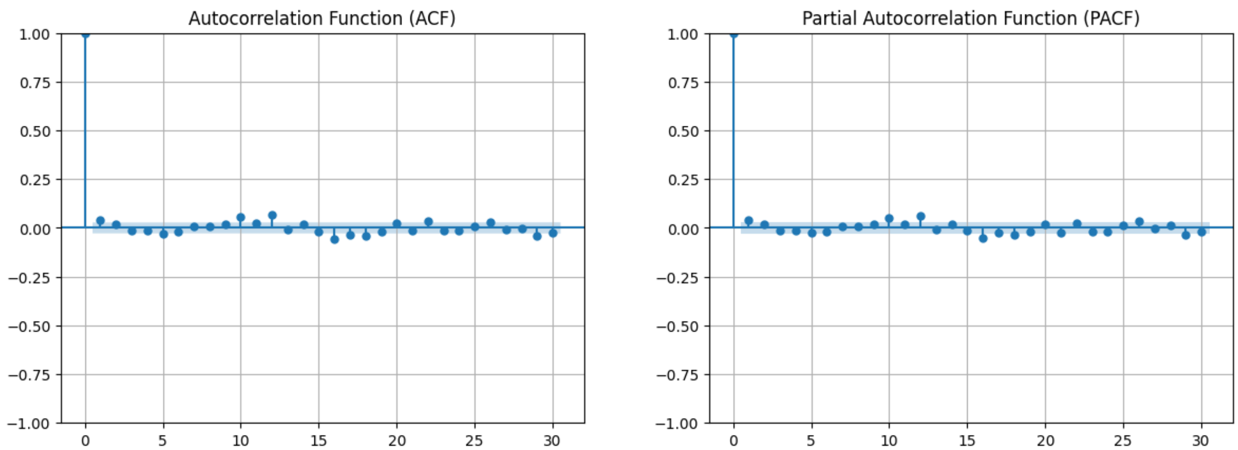
Metric	Value
3-Month Treasury Bill Rate	4.20%
Date Retrieved	May 2, 2025

Table 5: Risk-Free Rate Data

5 Independence Testing of Log Returns

5.1 Autocorrelation Analysis

The autocorrelation function (ACF) and partial autocorrelation function (PACF) were calculated to examine serial dependence in the log returns.



(a) Autocorrelation Function (ACF) (b) Partial Autocorrelation Function (PACF)

Figure 5: Autocorrelation Analysis of AAL Log Returns

5.2 Ljung-Box Test

The Ljung-Box test examines whether a time series exhibits autocorrelation.

Lag	Test Statistic	p-value
10	37.019979	5.614483e-05

Table 6: Ljung-Box Test Results

5.3 Independence Test Conclusion

The ACF and PACF plots show significant spikes at certain lags, and the Ljung-Box test rejects the null hypothesis of independence with a p-value of 5.614483e-05. This suggests that AAL's log returns exhibit some serial correlation, contradicting the random walk hypothesis. This finding has implications for option pricing models that assume independence of returns.

6 Option Pricing Models

6.1 Parameters

European call and put options for AAL were priced with the following parameters:

Parameter	**Value**
Stock Price (S)	$10.52
Strike Price (K)	$10.52 (ATM)
Time to Maturity (T)	0.0767 years
Risk-Free Rate (r)	4.20%
Historical Volatility (σ_H)	64.49%
GARCH Volatility (σ_G)	76.73%

Table 7: Option Pricing Parameters as of May 2, 2025

6.2 Black-Scholes Model

The Black-Scholes model for a European call option is given by:

$$C(S,t) = S \cdot N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{3}$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{4}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{5}$$

Where $N(\cdot)$ is the standard normal cumulative distribution function.

6.3 Cox-Ross-Rubinstein (CRR) Binomial Model

The CRR binomial model approximates the continuous price process with a discrete binomial tree where:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{6}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \tag{7}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{8}$$

Where u is the up factor, d is the down factor, and p is the risk-neutral probability.

6.4 Monte Carlo Simulation Model

The Monte Carlo approach simulates multiple price paths according to:

$$S_T = S_0 e^{(r - \frac{\sigma^2}{2})T + \sigma\sqrt{T}Z} \tag{9}$$

Where $Z \sim N(0,1)$ is a standard normal random variable.

7 Option Pricing Results with Historical Volatility

Using historical volatility (64.49%), the option prices were calculated as follows:

Method	**Call Price**	**Put Price**
Black-Scholes	\$0.764126	\$0.730302
CRR Binomial	\$0.763939	\$0.730115
Monte Carlo	\$0.766021	\$0.730100

Table 8: Option Prices Using Historical Volatility

8 Advanced Volatility Modeling - GARCH(1,1)

8.1 GARCH Model Specification

The GARCH(1,1) model was implemented to capture volatility clustering:

$$r_t = \mu + \epsilon_t \tag{10}$$

$$\epsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1) \tag{11}$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{12}$$

Where ω is the long-term variance, α measures the reaction of conditional variance to market shocks, and β measures the persistence of volatility.

8.2 GARCH Parameter Estimates

Parameter	**Value**
ω	0.0808
α	0.0431
β	0.9508
Persistence ($\alpha + \beta$)	0.9939
Long-run Annualized Volatility	64.49%
GARCH Forecast Volatility	76.73%

Table 9: GARCH(1,1) Model Parameters

8.3 GARCH Volatility Characteristics

The estimated GARCH parameters indicate high volatility persistence ($\alpha + \beta = 0.9939$), suggesting that shocks to volatility have long-lasting effects. The forecast annualized volatility from the GARCH model (76.73%) is significantly higher than the historical volatility (64.49%), suggesting increased market uncertainty ahead for AAL stock.

9 Option Pricing Results with GARCH Volatility

Using GARCH volatility (76.73%), the option prices were recalculated:

Method	**Call Price**	**Put Price**
Black-Scholes	\$0.905443	\$0.871619
CRR Binomial	\$0.905221	\$0.871397
Monte Carlo	\$ 0.907730	\$0.871355

Table 10: Option Prices Using GARCH Volatility

10 Comparative Analysis and Conclusion

10.1 Volatility Comparison

The GARCH volatility estimate (76.73%) is 18.98% higher than the historical volatility (64.49%). This difference significantly impacts option prices, with GARCH-based prices approximately 18.49% higher for calls and 19.35% higher for puts.

10.2 Model Consistency

All three pricing methods (Black-Scholes, CRR Binomial, and Monte Carlo) provide consistent results within each volatility scenario. The minor differences between methods (less than 1%) can be attributed to discretization effects in the binomial model and simulation error in the Monte Carlo method.

10.3 Market Implications

The significantly higher GARCH volatility suggests that:

- The market may experience higher volatility in the future than it has in the past
- Options may be underpriced if only historical volatility is considered
- Hedging strategies should account for potentially higher volatility

10.4 Final Conclusion

The GARCH model provides a more sophisticated and forward-looking estimate of volatility compared to the simple historical approach. Its ability to capture volatility clustering and time-varying characteristics makes it particularly valuable for pricing options in markets with changing volatility regimes. For AAL stock, the evidence suggests that using GARCH volatility leads to more realistic option prices than historical volatility alone, especially given the significant autocorrelation detected in the returns series.